

PARTICLE-SPRING METHOD FOR FORM FINDING GRID SHELL STRUCTURES CONSISTING OF FLEXIBLE MEMBERS

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SUMMARY

In this paper a general design method is described that can significantly reduce the effort needed for form finding grid shells consisting of flexible members. This design method is based on particle-spring models for simulating the behaviour of a grid shell during construction. Hereby, the stress limitations that follow from the material properties are taken into account to modify the grid shell geometry with a minimal deviation from a pre-defined target shape. It is demonstrated how a simple design tool can generate the geometry, internal forces and the support reactions with satisfactory accuracy.

Keywords: grid shell, form finding, design tool, timber, particle-springs, dynamic relaxation, large deformations

1. INTRODUCTION

Grid shells that consist of flexible members have, in addition to the favourable properties of shell structures in general, the considerable advantage that they can be erected in very little time. Construction takes place in three steps. First, a flat grid of straight members is laid out, making the creation of the connections very easy. The joints are not fully fixed and allow some sliding of the laths. Second, construction struts push the grid to its shell form by lifting it from within. Third, the edges and connections are fixed and the struts are removed. Prominent examples of grid shells that were constructed this way are the Mannheim grid shell [1] and the Weald and Downland grid shell [2].

The downside of this type of grid shell is that due to the flexibility of the members the final shape is hard to predict with sufficient accuracy. It highly depends on equilibrium between the forces in the laths. The forces in the laths are introduced by the bending process, in addition to self weight and external loading.

Very little is known about the methods that have been used to analyse existing grid shell structures. In design of the earliest grid shells extensive use was made of physical models. Nonetheless, a considerable number of laths broke during

construction. Repairing or replacing broken laths was a time consuming and expensive part of the construction process [1]. In recent grid shell projects far less broken laths were reported which shows that progress has been made [2]. However, most commercially available structural analysis software is ill suited for analysing grid shell structures. For example, very large displacements (nonlinear) are not supported and neither is form finding or the application of initial stresses. Therefore, at Delft University of Technology in the Netherlands, two research projects were undertaken on grid shell design and analysis [3], [4]. The latter of these projects led to the method proposed in this paper.

Barnes [5] has described how dynamic relaxation can be used in form finding tension structures. Killian and Ochsendorf [6] elaborated on this by describing how particle-spring systems can be used in the form finding of shell structures. Hereby they considered axial forces only. For grid shells consisting of flexible members the bending moments are especially important, as shown by Adriaenssens and Barnes [7]. Therefore, their method needed to be extended.

In this paper an intelligent design method is proposed that is capable of form finding and analysing grid shell structures. This method has

been implemented in a computer program and has been applied as a design tool. The method has been demonstrated in several case studies that are included at the end of this paper.

2. GENERAL APPROACH

The process starts by specifying a target shape for a grid shell. This shape can include functional and aesthetic requirements of the geometry. However, this shape does not need to be structurally feasible. The proposed method designs a grid that fits closely to this target shape. Hereby, the method takes the following requirements into account.

- Equilibrium of the grid
- Stress limits of the laths
- Best approximation to the target shape

The grid of laths is modelled by a particle-spring model [6] which consists of particles that are connected by translational and rotational springs (Fig. 1). The particles represent the positions of the connections between the laths. The self weight and the mass of the laths are lumped to these particles. The springs represent the elastic properties of the lath parts between the connections. Several types of springs are possible, where each type corresponds to an action that works on the laths of the grid shell. In this paper the two main actions, namely normal and bending action, are considered. Torsion springs that represent torsion action on the laths have not been included as yet.

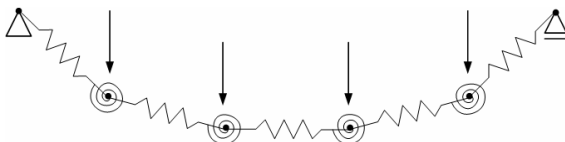


Figure 1. A simply supported lath modelled by particles and springs

First, an initially flat grid is generated. This grid is pulled towards the target shape, which causes the grid to curve and gives some change of the spacing between the grid points. The pulling continues until the limit stress prevents a closer agreement with the target shape. Second, the grid edges are fixed and the pulling forces are removed, after which the final equilibrium geometry is obtained. This approach is similar to the erection method of a grid shell.

The first procedure is called shape approximation. The second procedure is called spring back analysis (Fig. 2). Both procedures are based on dynamic relaxation for finding a shape that is in equilibrium. The result is the geometry of the grid shell as it can stand on its own.

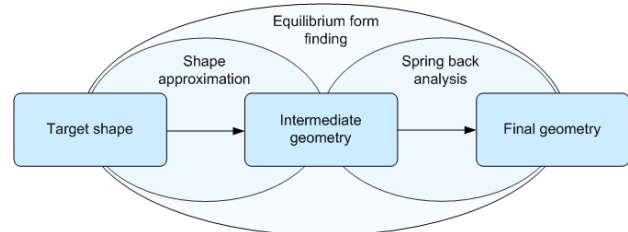


Figure 2. Diagram of the proposed design method

3. GRID SHELL MODEL

Each particle has three coordinates representing its location in space. The particles are connected by translational springs and rotational springs. The translational springs have an initial length equal to the target spacing of the laths. During computation, the actual spring length is the distance between the two particles that the spring connects. The spring exerts forces F onto the particles in the direction of the spring.

$$F = k u \quad (1)$$

In this, u is the difference between the current spring length and the initial length (negative when the distance is smaller than the initial length) and k is the spring stiffness.

Since the deformation of the laths due to bending is much larger than due to extension, the extensional stiffness of the springs does not need to be the exact value of the stiffness of the laths without losing much accuracy. Moreover, it appeared to be useful to implement a non-linear constitutive relation for the translational springs, giving greater speed of calculation without losing numerical stability (Fig. 3).

In addition, a small translational spring stiffness can be specified for simulating sliding of the connections when the grid is pushed into shape. The rotational springs represent bending of the lath parts between the connections. The initial angle of the springs is zero. During a computation these angles change (Fig. 4)

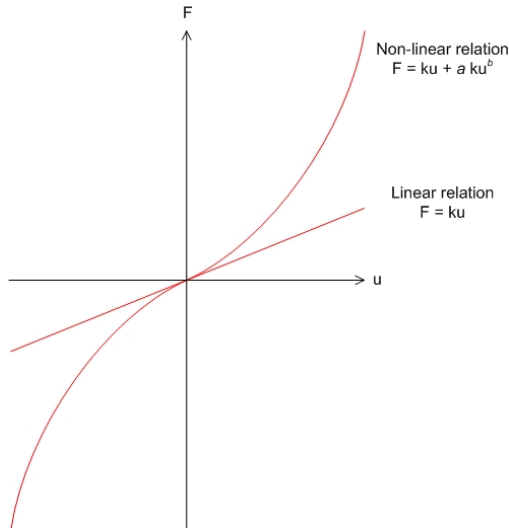


Figure 3. The translational springs can be linear as well as non-linear

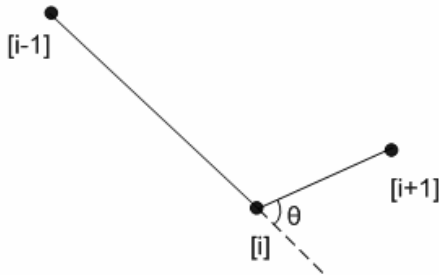


Figure 4. The angle between three particles that are connected by a rotational spring

The moment M due to this angle is

$$M = \frac{EI}{\Delta} \theta \quad (2)$$

where EI is the flexural rigidity of the laths and Δ is the lath spacing (see Appendix). The moment exerts forces on the three particles connected by the rotational spring (Fig. 5).

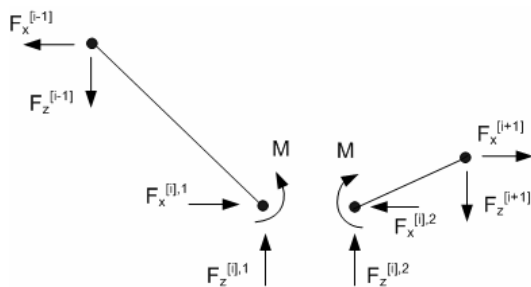


Figure 5. The moment in a rotational spring is converted to equivalent forces acting on the connected particles

4. DYNAMIC RELAXATION

For finding equilibrium states of a particle-spring model the dynamic relaxation method has been applied. Each particle is loaded by internal and external forces. If not in equilibrium, the resultant of all forces on a particle accelerates this particle in the direction of the resultant (Newton’s second law of motion). In some time step this will lead to a new position of the particle. The basic idea of dynamic relaxation is that this dynamic equilibrium will come to rest in a static equilibrium.

In each time step the resultant forces are recalculated for every grid point. The resultant force consists of forces due to bending in the grid point itself, bending in the grid points connected to it, forces due to extension and external forces.

The computation of the particle velocities and displacements is performed by the implicit fourth order Runge Kutta method [8].

The particle mass and the particle damping do not need to have the actual values because the objective is computing the equilibrium situation and not the actual dynamic response. A successful method to prevent oscillations around the equilibrium state is monitoring the total kinetic energy of the system. When a peak is detected all velocities are set to zero [5].

5. APPROXIMATION TO A TARGET SHAPE

As explained in Section 2, the particle-spring system is fitted to a target shape. This is accomplished by adding extra springs – called shaping springs – to the system. Each shaping spring connects a particle to the surface of the target shape. The shaping springs are directed vertically to the target shape. The shaping springs have zero initial length, therefore, they pull the particles towards the target shape (Fig. 6). The bending stresses in the laths are computed from the moments in the particle-spring system. If somewhere the stress limit according to Eurocode 5 [9] is exceeded, the stiffness of the connected shaping spring is reduced. The procedure continues until the internal forces are in equilibrium with the shaping forces. The result is a grid geometry that matches the target shape as accurately as possible and fulfils the stress conditions everywhere (Fig. 7).

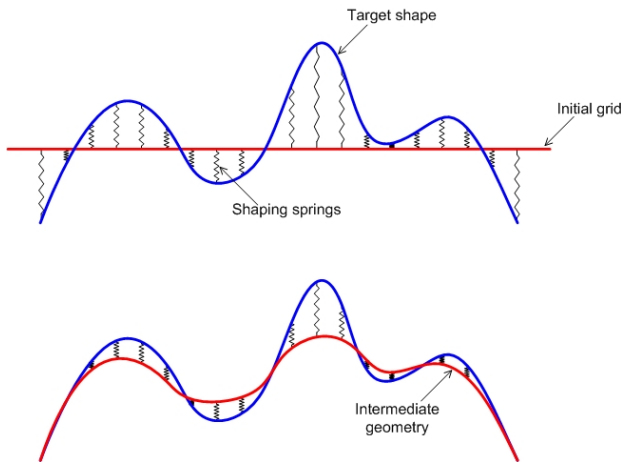


Figure 6. Shape approximation

Normal stresses, shear stresses and torsion stresses are not checked in the proposed method. However, it would not be difficult to implement this too. For reducing torsion stresses the stiffness of two shaping springs need to be reduced. These are located on either side of the grid point on the lath perpendicular to the considered lath part.

6. SPRING BACK ANALYSIS

After the shape approximation is concluded, the edges of the particle-spring system are fixed. The equilibrium length of the translational springs is set to the current local lath spacing Δ . The stiffness of the translational springs can be set to linear elastic with a value of

$$k = \frac{EA}{\Delta} \quad (3)$$

where EA is the axial stiffness of the laths. This represents the fact that the lath connections are fixed now and cannot slide over each other anymore. The shaping springs are removed from the particle-spring system and the final equilibrium geometry is computed with dynamic relaxation (Fig. 8).

7. CASE STUDIES

The proposed method has been implemented in a design tool and then applied to several case studies. Use has been made of the C++ programming language with graphic user interface. The run time strongly depends on the number of laths. The computation times of the examples presented below

are 1 minute for example 1, 10 minutes for example 2 and 30 minutes for example 3, on a modern computer.

Example 1

Figure 9 shows a simple target shape representing a dome like structure.

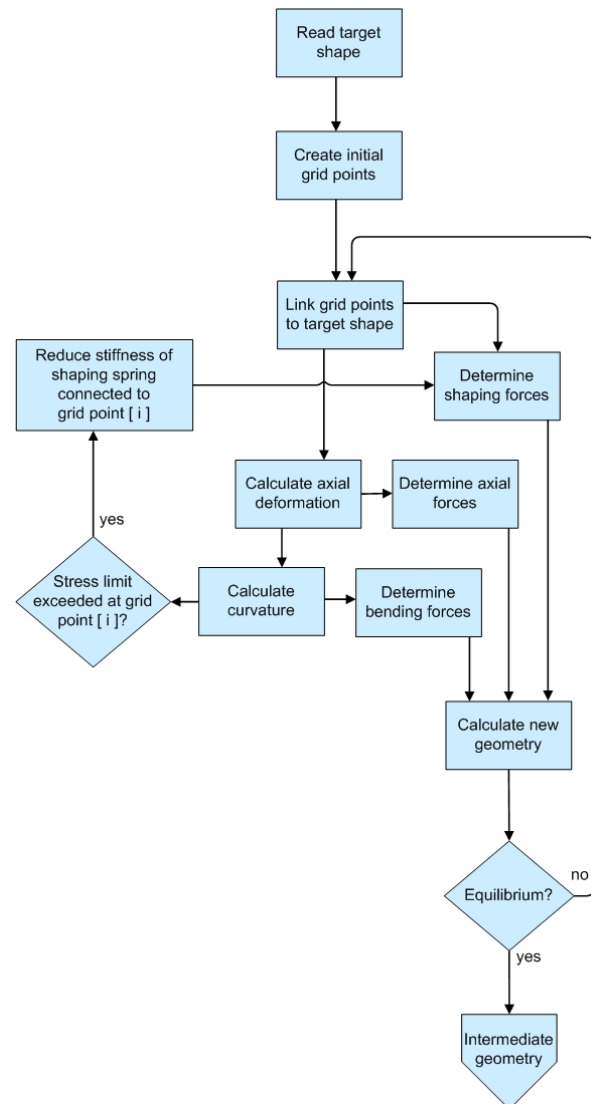


Figure 7. Flow chart of the first calculation procedure (shape approximation)

As input parameters properties are used that are based on timber of strength class C18 (Table 1). Applying the design tool to this example results in the grid geometry shown in Figure 10. The geometry has changed only slightly in the spring back analysis, nevertheless this cannot be neglected (Fig. 11).

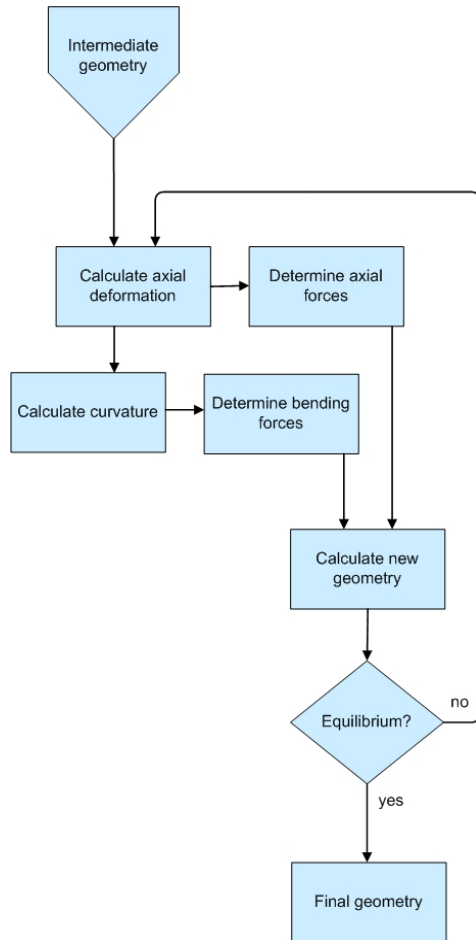


Figure 8. Flow chart of the second calculation procedure (spring back analysis)

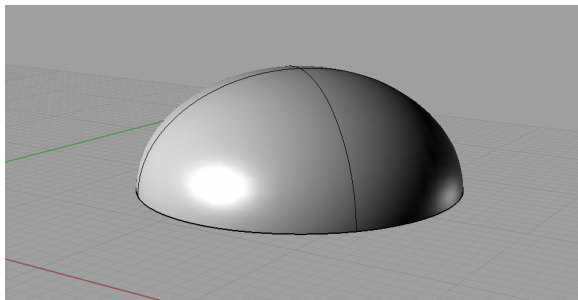


Figure 9. Target shape of example 1

Table 1. Input values of example 1

Grid point spacing	0.8 m
Lath cross section dimensions	34 x 28 mm
Elastic modulus ($E_{0,mean}$)	9000 N/mm ²
Characteristic bending strength ($f_{m,k}$)	18 N/mm ²
Initial spring stiffness of shaping springs	10.0 kN/m
Spring stiffness of translational springs	1100.0 kN/m
Density	320 kg/m ³

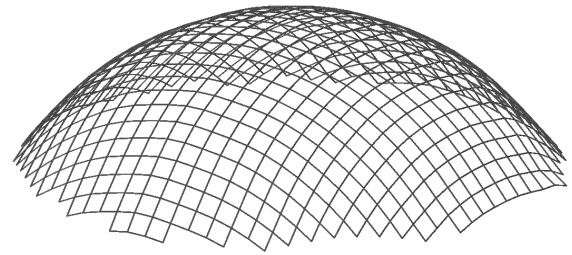


Figure 10. Final grid geometry of example 1

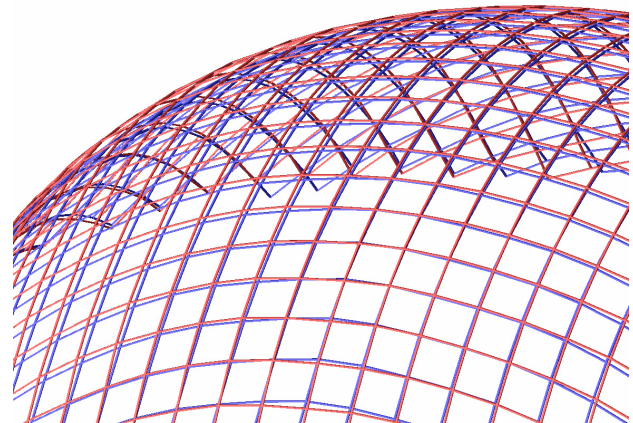


Figure 11. Detail of the grid geometry before (red) and after (blue) the spring back analysis

Example 2

Figure 12 shows a target shape that consists of two intersecting ellipsoids. More detail is not necessary for application of the design tool. The calculation is run twice with values of the mechanical properties corresponding to timber of strength class C22 and D35 (Table 2). For these strength classes Young's moduli are equal, but the bending strength differs. The lath dimensions are kept equal for both calculations. The resulting final geometries are shown in Figure 13 and 14. The calculation based on C22 shows a larger deviation from the target shape than that of D35, because the stiffness of the shaping springs has been reduced more during the shape approximation procedure. This demonstrates that the final geometry depends strongly on the timber strength.

In both instances the design tool has found the grid shell geometry that is as close as possible to the target shape.

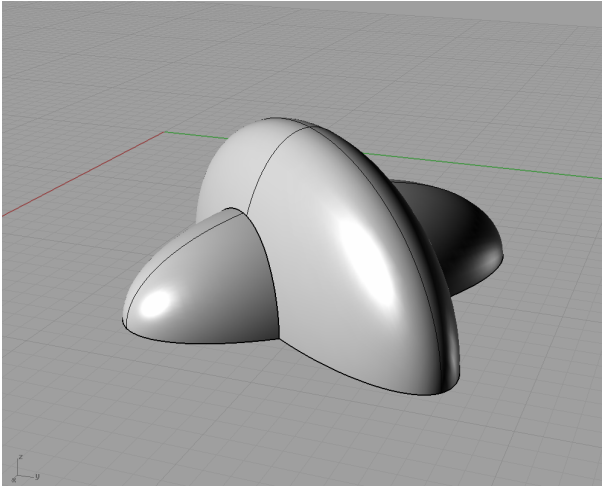


Figure 12 Target shape of example 2

Table 2. Input values of example 2

Grid point spacing	0.6 m
Lath cross section dimensions	34 x 28 mm
Elastic modulus ($E_{0,mean}$)	10000 N/mm ²
Characteristic bending strength ($f_{m,k}$) C22	22 N/mm ²
Characteristic bending strength ($f_{m,k}$) D35	35 N/mm ²
Initial spring stiffness of shaping springs	10.0 kN/m
Spring stiffness of translational springs	1100.0 kN/m
Density (average)	450 kg/m ³

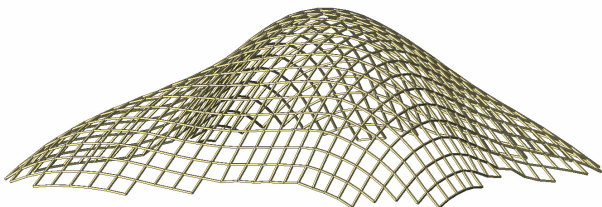


Figure 13. Grid geometry of example 2 for C22

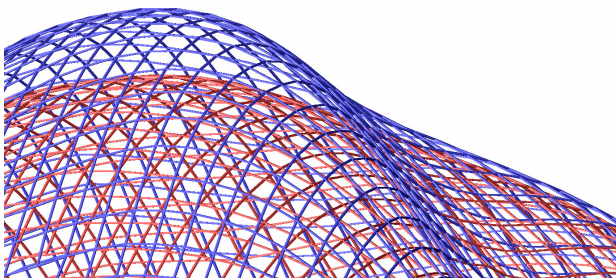


Figure 14. Comparison of generated geometries with timber strength class D35 (blue) and C22 (red)

Example 3

Spectacular shapes can be constructed as a grid shell. The design tool is capable of finding the geometry for very elaborate target shapes. An example is the grid shell in Fig. 15 and 16. For this example the values of the mechanical properties are based on that of a composite material with which very high bending strengths can be obtained (Table 3).

Table 3. Input values of example 3

Grid point spacing	0.3 m
Lath cross section dimensions	50 x 50 mm
Elastic modulus ($E_{0,mean}$)	25000 N/mm ²
Characteristic bending strength ($f_{m,k}$)	450 N/mm ²
Initial spring stiffness of shaping springs	10.0 kN/m
Spring stiffness of translational springs	6250.0 kN/m
Density (average)	1800 kg/m ³

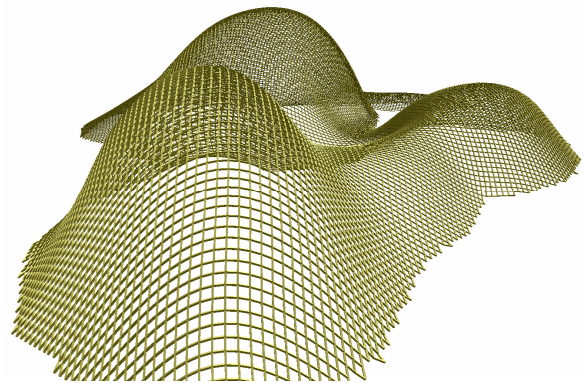


Figure 15. Final grid geometry of example 3

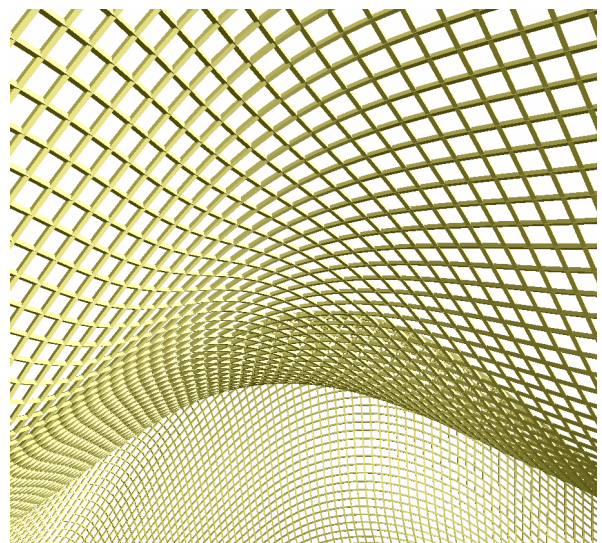


Figure 16. Grid shell geometry viewed from within

8. DISCUSSION

The proposed design method assures that the grid shell geometry can be build with no laths breaking. In general, the final grid geometry needs covering with roofing materials. These can also contribute to the strength of the grid shell structure. The final grid geometry plus roofing materials need checking for load combinations related to the serviceability limit state and ultimate limit state. For this commercially available software for structural analysis can be used. The designed grid shell can be easily imported as a dxf type file.

The proposed method might also be applied to grid shells with triangular spacing or hexagonal spacing. Usually, these shells are made of metal members that are not curved and have no intitial stresses. In this application of the proposed method the initial moments and normal forces have no physical meaning but are used to obtain a regularly shaped grid. However, this idea needs to be tested.

As clearly demonstrated in the examples the target shape does not need to be a nearly feasible grid shell. In fact almost any geometry can be used as a starting point of the form finding process. This is considered a considerable advantage of the proposed method.

9. CONCLUSIONS

This paper presents a new design method for grid shells consisting of flexible members. The method uses particle-spring models that are curved over a target shape. When implemented in user-friendly software the method is fast, reliable and easy to apply. Moreover, the target shape does not need to be specified in great detail, which makes it possible to apply the method in a conceptual design stage.

It is believed that the proposed method removes an important obstacle for realising many more beautiful and efficient grid shell structures.

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APPENDIX: DEFORMATION OF A SINGLE LATH

In this appendix the deformed shape of an initially straight lath is analytically derived and numerically computed. The results have been used to check the

implemented particle-spring method. It may also serve as a bench mark for future implementations of the proposed method.

Figure 17 shows a simply supported lath deformed by two horizontal forces $F = 18093$ N. Before loading the lath was straight with a length $L = 7.6$ m. The flexural rigidity is $EI = 95000$ Nm².

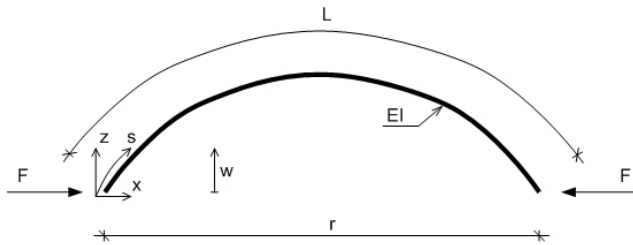


Figure 17. Deformation and loading of the considered lath

In the analytical calculation the principle of minimum potential energy is used to determine the deformed shape. To this end, the deflection w of the lath is assumed as

$$w = a \sin \frac{\pi s}{L} + b \sin \frac{3\pi s}{L} \quad (4)$$

Note that w is a function of s , which runs along the deformed shape of the lath. Note also that if $s = 0$ or $s = L$ then $w = 0$, therefore, w fulfils the kinematic boundary conditions (Fig. 18), which is necessary for application of minimal potential energy.

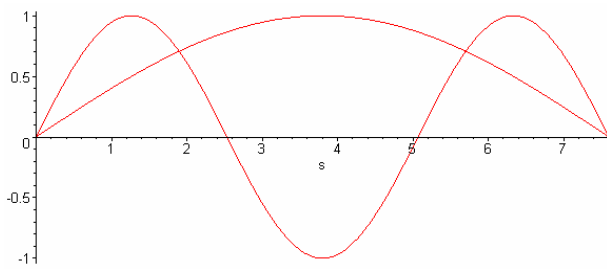


Figure 18 Two components are assumed to make up the deformed shape of the lath ($a = 1, b = 1$)

The values of a and b need to be found for which the system has a minimum potential energy. The potential energy of the system is

$$E_{pot} = \frac{1}{2} EI \int_{s=0}^L \kappa^2 ds - F(L - r) \quad (5)$$

where

$$\kappa = \frac{d\varphi}{ds} = \frac{d}{ds} \arcsin \frac{dw}{ds} \quad (6)$$

$$\kappa = \frac{\frac{d^2 w}{ds^2}}{\sqrt{1 - \left(\frac{dw}{ds}\right)^2}}$$

and

$$r = \int_{s=0}^L \sqrt{1 - \left(\frac{dw}{ds}\right)^2} ds \quad (7)$$

These functions can be evaluated for different values of a and b to find for which the potential energy is at a minimum. This is the case for $a = 2.01$ m and $b = -0.0280$ m. The resulting deformation w is plotted in Fig. 19.

It is noted that the accuracy of the potential energy solution depends on how well the initially assumed functions were chosen.

In the numerical calculation the particle-spring method has been used. The particle distance is 0.2 m. The iterations were continued until the force resultant was less than 0.05 N for each particle. Fig. 19 shows the numerical result as well. The analytical result and the numerical result agree very well.

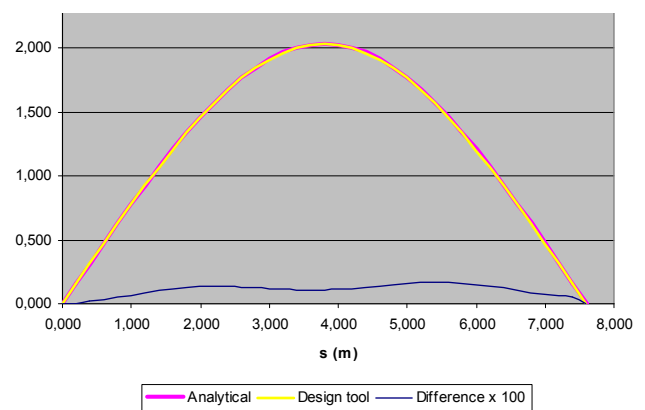


Figure 19. Comparison between the analytical and numerical deformation [m] of an initially straight lath