ADAPTIVE SUPPRESSION OF RFI AND ITS EFFECT ON RADIO-ASTRONOMICAL IMAGE FORMATION

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ABSTRACT

Radio-astronomical observations are increasingly contaminated by interference, and suppression techniques become essential. A powerful candidate for interference mitigation is adaptive spatial filtering. We study the effect of spatial filtering techniques on radio astronomical imaging. Current deconvolution procedures such as CLEAN are shown to be unsuitable to spatially filtered data, and the necessary corrections are derived. To that end, we reformulate the imaging (deconvolution/calibration) process as a sequential estimation of the locations of astronomical sources.

1. INTRODUCTION

Future radio astronomical observations depends on two important factors: Increased resolution and sensitivity, and robustness to the increasingly corrupted electro-magnetic environment. These emission sources generates alot of radio frequency interference (RFI) to the sensitive radio astronomical instruments. Recently many algorithms for on-line suppression of RFI for radio astronomy have been proposed, among these we can find spatial projections [4] generalized sidelobe cancelling, and LMS based adaptive interference cancellation [1]. However no study of the possible effects on the final product (i.e., the image) has been done. In this paper we take initial step in this direction. We reformulate the radio astronomical image formation problem parametrically. This enables us to incorporate spatial filtering techniques into the imaging process, in a natural way. For a more detailed account on this research the reader is referred to [3], which presents a full account on the results, as well as extensive literature overview.

2. ASTRONOMICAL MEASUREMENT EQUATIONS

In this section we describe a simplified mathematical model for the astronomical measurement and imaging process. Our discussion follows the introduction in [5]. We begin with the measurement equation, to reformulate it into a matrix form in the next section. This will allow us to obtain a uniform description of various astronomical imaging operations such as deconvolution and self-calibration.

The signals received from the celestial sphere may be considered as spatially incoherent wideband random noise. Rather than considering the emitted electric field at a location on the celestial sphere, astronomers try to recover the *intensity* $I_f(\mathbf{s})$ in the direction of unit-length vectors $\mathbf{s}$, where f is a specific frequency.

Amir Leshem was supported by the NOEMI project of the STW under contract no. DEL77-4476.e-mail:leshem,allejan@cas.et.tudelft.nl

Let $E_f(\mathbf{r})$ be the received celestial electric field at a location $\mathbf{r}$ on earth. The measured covariance of the electric fields between two identical sensors i and j with locations $\mathbf{r}_i$ and $\mathbf{r}_j$ is called a *visibility* and is (approximately) given by [5]

$$\mathrm{E}\{E_f(\mathbf{r}_i)\overline{E_f(\mathbf{r}_j)}\} = \int_{\text{sky}} I_f(\mathbf{s}) e^{-2\pi j f \mathbf{s}^T(\mathbf{r}_i-\mathbf{r}_j)/c}\, d\Omega\,.$$

($\mathrm{E}\{\cdot\}$ is the mathematical expectation operator, the superscript ^{T} denotes the transpose of a vector, and overbar denotes the complex conjugate). We denote $\mathrm{E}\{E_f(\mathbf{r}_i)\overline{E_f(\mathbf{r}_j)}\}$ by $V_f(\mathbf{r}_i,\mathbf{r}_j)$. Note that it is only dependent on the oriented distance $\mathbf{r}_i-\mathbf{r}_j$ between the two telescopes; this vector is called a baseline.

For simplification, we may sometimes assume that the astronomical sky is a collection of d discrete point sources (maybe unresolved). This gives

$$I_f(\mathbf{s}) = \sum_{l=1}^{d} I_f(\mathbf{s}_l)\delta(\mathbf{s}-\mathbf{s}_l)\,,$$

where $\mathbf{s}_l$ is the coordinate of the l'th source, and thus

$$V_f(\mathbf{r}_i,\mathbf{r}_j) = \sum_{l=1}^{d} I_f(\mathbf{s}_l)\, e^{-2\pi j f \mathbf{s}_l^T(\mathbf{r}_i-\mathbf{r}_j)/c}\,. \tag{1}$$

Upon a proper choice of coordinate systems, for the telescope locations, (u,v,w) and for the image plane (orthogonal to the pointing direction of the telescope (l,m), we obtain after compensating for the delay between the antennas, that the measurement equation in (u,v) coordinates becomes [7]:

$$V_f(u,v) = \iint I_f(\ell,m)\, e^{-2\pi j(u\ell+vm)}\, d\ell dm\,. \tag{2}$$

It has the form of a Fourier transformation. The function $V_f(u,v)$ is sampled at various coordinates (u,v) by first of all taking all possible sensor pairs i,j or baselines $\mathbf{r}_i-\mathbf{r}_j$, and second by realizing that the sensor locations $\mathbf{r}_i$, $\mathbf{r}_j$ are actually time-varying since the earth rotates. Given a sufficient number of samples in the (u,v) domain, the relation can be inverted to obtain an image (the 'map'), which is the topic of section 5.

3. ARRAY SIGNAL PROCESSING FORMULATION

We will now describe the situation from an array signal processing point of view. The signals received by the telescopes are amplified and down-converted to baseband. A time-varying delay for

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every telescope is also introduced, to compensate for the geometrical delay. Following traditional array signal processing practices, the signals at this point are called $x_i(t)$ rather than $E_f(\mathbf{r})$, and are stacked in vectors

$$\mathbf{x}(t) = [x_1(t), \ldots, x_p(t)]^T ,$$

where p is the number of telescopes. These are then processed by a correlation stage.

It will be convenient to assume that $\mathbf{x}(t)$ is first split by a bank of narrow-band sub-band filters into a collection of frequency-components $\mathbf{x}_f(t)$. The main output of the telescope hardware is then a sequence of empirical covariance matrices $\hat{\mathbf{R}}_f(t)$ of cross-correlations of $\mathbf{x}_f(t)$, for a set of frequencies $f \in \{f_k\}$ covering a 10 MHz band or so, and for a set of times $t \in \{t_k\}$ covering up to 12 hours [1]. Each covariance matrix $\hat{\mathbf{R}}_f(t)$ is an estimate of the true covariance matrix $\mathbf{R}_f(t) = \mathrm{E}\{\mathbf{x}_f(t)\mathbf{x}_f(t)^{\mathrm{H}}\}$ and given by:

$$\hat{\mathbf{R}}_f(t) = \frac{1}{N} \sum_{n=0}^{N-1} \mathbf{x}_f(t+nT)\mathbf{x}_f(t+nT)^{\mathrm{H}}, \qquad (3)$$

where the superscript $^{\mathrm{H}}$ denotes a complex conjugate transpose, T is the sample period of $\mathbf{x}_f(t)$ and N is the number of samples over which is averaged. The matrices $\hat{\mathbf{R}}_f(t)$ are stored for off-line spectral analysis and imaging. From now on we consider the sub-bands independently ignoring that they are really connected. Consequently, in future equations we drop the dependence on f in the notation.

The connection of the covariance matrices $\mathbf{R}(t)$ to the visibilities $V(u,v)$ in section 2 is as follows. Each entry $r_{ij}(t)$ of the matrix $\mathbf{R}(t)$ is a sample of this visibility function for a specific coordinate (u,v) corresponding to the baseline vector $\mathbf{r}_i(t) - \mathbf{r}_j(t) = \lambda[u_{ij}(t),\ v_{ij}(t),\ w_{ij}(t)]$ between telescopes i and j at time t:

$$V(u_{ij}(t), v_{ij}(t)) \equiv r_{ij}(t)\,. \qquad (4)$$

3.1. Matrix formulation

For the discrete source model, we can now formulate our measurement equations in terms of matrices. Let $\mathbf{r}_0(t_k)$ be an arbitrary and time-varying reference point, typically at one of the elements of the array, and let us take the (u,v,w) coordinates of the other telescopes with respect to this reference, $\mathbf{r}_i(t) - \mathbf{r}_0(t) = \lambda[u_{i0}(t),\ v_{i0}(t),\ w_{i0}(t)]$, $\quad i = 1, \cdots, p$. Equation (1) can then be written slightly differently in terms of correlation matrices as

$$\mathbf{R}_k \;=\; \mathbf{A}_k \mathbf{B} \mathbf{A}_k^{\mathrm{H}}, \qquad (5)$$

where $\mathbf{A}_k = [\mathbf{a}_k(\ell_1, m_1), \ldots, \mathbf{a}_k(\ell_d, m_d)]$, $\mathbf{R}_k \equiv \mathbf{R}(t_k)$ and

$$\mathbf{a}_k(\ell, m) \;=\; \begin{bmatrix} e^{-2\pi \jmath (u_{10}(t_k)\ell + v_{10}(t_k)m)} \\ \vdots \\ e^{-2\pi \jmath (u_{p0}(t_k)\ell + v_{p0}(t_k)m)} \end{bmatrix} \qquad (6)$$

$$\mathbf{B} \;=\; \begin{bmatrix} I(\ell_1, m_1) & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & I(\ell_d, m_d) \end{bmatrix}$$

The vector function $\mathbf{a}_k(\ell, m)$ is called the *array response vector* in array signal processing. It describes the response of the telescope array to a source in the direction (ℓ, m). As usual, the array response is frequency dependent. In this case, the response is also slowly time-varying due to the earth rotation. Note, very importantly, that the function as shown here is completely known.

More realistically, the array response is less perfect. An important effect is that each telescope may have a different complex receiver gain, $\gamma_i(t)$, dependent on many angle-independent effects such as cable losses, amplifier gains, and (slowly) varying atmospheric conditions. We also have to realize that most of the received signal consists of additive system noise. When this noise is zero mean, independent among the antennas (thus spatially white), and identically distributed, then it has a covariance matrix that is a multiple of the identity matrix, $\sigma^2\mathbf{I}$, where σ^2 is the noise power on a single antenna inside the subband which we consider. Usually the noise is assumed to be Gaussian. The resulting model of the received covariance matrix then becomes

$$\mathbf{R}_k \;=\; \mathbf{\Gamma}_k \mathbf{A}_k \mathbf{B} \mathbf{A}_k^{\mathrm{H}} \mathbf{\Gamma}_k^{\mathrm{H}} \;+\; \sigma^2 \mathbf{I} \qquad (7)$$

where

$$\mathbf{\Gamma}_k = \mathrm{diag}\{\gamma_{1,k}, \ldots, \gamma_{p,k}\} \qquad (8)$$

Assuming that q interferers are present and assuming that we work in sufficiently narrow bands we obtain [4] that the interference contributes to the covariance matrix $\mathbf{R}_k$ a term similar to the astronomical term. The corresponding overall model including astronomical signals, array imperfections, interference and noise is given by:

$$\mathbf{R}_k \;=\; \mathbf{\Gamma}_k \mathbf{A}_k \mathbf{B} \mathbf{A}_k^{\mathrm{H}} \mathbf{\Gamma}_k + (\mathbf{A}_s)_k (\mathbf{R}_s)_k (\mathbf{A}_s)_k^{\mathrm{H}} + \sigma^2 \mathbf{I}\,, k = 0, 1, \cdots . \qquad (9)$$

where we assume that the interference term $\mathbf{A}_s$ is unstructured, and $\mathrm{rk}\mathbf{A}_s = q < p$.

Finally to complete the model we assume that each covariance matrix $\mathbf{R}_k$ has been subject to a linear spatial filter $\mathbf{L}_k$ yielding a filtered covariance matrix $\tilde{\mathbf{R}}_k = \mathbf{L}_k \hat{\mathbf{R}} \mathbf{L}_k^{\mathrm{H}}$. A further discussion of the possible $\mathbf{L}_k$ is given in [4].

4. CLASSICAL INVERSE FOURIER IMAGING

In the previous sections, we discussed spatial filtering techniques. It was shown that an attractive scheme for removing the interference is by projecting it out. However, by doing so we replace the observed visibilities $V(u_i, v_i)$ in the matrix $\mathbf{R}_v$ by some (known) linear combination. In this section, we describe the classical Fourier imaging, as it is implemented in radio astronomy.

The relation between sky brightness $I(\ell, m)$ and visibilities $V(u,v)$ (where u, v are taken at frequency f) is

$$V(u,v) = \iint I(\ell, m)\, e^{-2\pi \jmath (u\ell + vm)}\, d\ell\, dm$$

We have measured V on a discrete set of baselines $\{(u_i, v_i)\}$. The "dirty image" (a lumpy image obtained via direct Fourier inversion possibly modified with some weights c_i) is defined by

$$I_D(\ell, m) \;:=\; \sum_i c_i\, V(u_i, v_i)\, e^{2\pi \jmath (u_i \ell + v_i m)} \qquad (10)$$

It is equal to the 2D convolution of the true image I with a point spread function known as the "dirty beam":

$$I_D(\ell, m) \;=\; \iint I(\ell', m')\, B_0(\ell - \ell', m - m')\, d\ell'\, dm'$$

[1] Many telescope sites including WSRT follow actually a different scheme where the signals are first correlated at several lags and subsequently Fourier transformed. This leads to similar results.

or

$$I_D = I * B_0\,, \qquad B_0(\ell, m) := \sum_i c_i\, e^{2\pi j(u_i \ell + v_i m)}$$

B_0 is the dirty beam, centered at the origin. The weights $\{c_i\}$ are arbitrary coefficients designed to obtain an acceptable beam-shape, with low side lobes, in spite of the irregular sampling.

Specializing to a point source model, $I(\ell, m) = \sum_l I_l\, \delta(\ell - \ell_l, m - m_l)$ where I_l is the intensity of the source at location (ℓ_l, m_l), gives $V(u,v) = \sum_l I_l\, e^{-2\pi j(u\ell_l + v m_l)}$ and

$$I_D(\ell, m) = \sum_l I_l\, B_0(\ell - \ell_l, m - m_l)$$

Thus, every point source excites the dirty beam centered at its location (ℓ_l, m_l).

From the dirty image I_D and the known dirty beam B_0, the desired image I is obtained via a deconvolution process. A popular method for doing this is the CLEAN algorithm [2]. The algorithm assumes that B_0 has its peak at the origin, and consists of a loop in which a candidate location (ℓ_l, m_l) is selected as the largest peak in I_D, and subsequently a small multiple of $B_0(\ell - \ell_l, m - m_l)$ is subtracted from I_D. The objective is to minimize the residual, until it converges to the noise level. The parameter $\gamma \le 1$ is called the loop gain and serves the purpose of interpolation over the grid, λ_l is the estimated power of the source.

Table 1. The CLEAN algorithm with spatial filtering

$$\begin{array}{l}
\text{Compute } \hat{\mathbf{R}}' \text{ using (15)} \\
I'_D(\mathbf{s}) = \sum_{k=1}^{K} \mathbf{a}_k^{\mathrm{H}}(\mathbf{s}) \hat{\mathbf{R}}'_k \mathbf{a}_k(\mathbf{s}) \\
l = 0 \\
\text{while } I'_D \text{ is not noise-like:} \\
\qquad \left[\begin{array}{lcl}
\mathbf{s}_l & = & \arg\max I'_D(\mathbf{s}) \\
\multicolumn{3}{l}{\text{Compute } B(\mathbf{s}, \mathbf{s}_l) \text{ using (16)}} \\
\lambda_l & = & I_D(\mathbf{s}_l)/B(\mathbf{s}_l, \mathbf{s}_l) \\
I'_D(\mathbf{s}) & := & I'_D(\mathbf{s}) - \gamma \lambda_l B(\mathbf{s}, \mathbf{s}_l) \\
l & = & l + 1
\end{array} \right. \\
I = I'_D + \sum_l \gamma \lambda_l B_{synth}(\mathbf{s} - \mathbf{s}_l)
\end{array}$$

5. IMAGING VIA BEAMFORMING TECHNIQUES

In this section, we reformulate the classical inverse-Fourier imaging technique and the CLEAN algorithm for deconvolution in terms of a more general iterative beamforming procedure. This is possible since we have a parametric point-source model, and the prime objective of the deconvolution step is to estimate the location of the point sources. The interpretation of the deconvolution problem as one of direction-of-arrival (DOA) estimation allows access to potentially a large number of algorithms that have been developed for this application.

5.1. CLEAN and sequential beamforming

We set out by showing how CLEAN can be interpreted as an iterative beam-forming procedure.

Let us assume that we have available a collection of measured covariance matrices $\hat{\mathbf{R}}_k$, obtained at times t_k with $k = 1, \cdots, K$, and let us assume the parametric model of (7), i.e.,

$$\mathbf{R}_k = \mathbf{A}_k \mathbf{B} \mathbf{A}_k^{\mathrm{H}} + \sigma^2 \mathbf{I}\,.$$

Here, the unknown parameters are the source locations $\mathbf{s}_l = (\ell_l, m_l)$, $l = 1, \cdots, d$ in each of the $\mathbf{A}_k$, and the source brightness I_l in $\mathbf{B}$. A natural formulation for the estimation of these parameters is to pose it as the solution of a LS cost function, given by

$$[\{\hat{\mathbf{s}}_l\}, \hat{\mathbf{B}}] = \underset{\{\mathbf{s}_l\}, \mathbf{B}}{\arg\min} \sum_{k=1}^{K} \| \hat{\mathbf{R}}_k - \mathbf{A}_k(\{\mathbf{s}_l\})\, \mathbf{B} \mathbf{A}_k^{\mathrm{H}}(\{\mathbf{s}_l\}) - \sigma^2 \mathbf{I} \|_F \tag{11}$$

($\mathbf{B}$ is constrained to be diagonal with positive entries.) This is recognized as the same model as used for DOA estimation in array processing. Note however that the array is moving ($\mathbf{A}_k$ is time-dependent), and that there are many more sources than the dimension of each covariance matrix.

In this notation, the image formation in section 4 can be formulated as follows. Using (4) and (6) and writing $I_D(\mathbf{s}) \equiv I_D(\ell, m)$ and $\mathbf{a}_k(\mathbf{s}) \equiv \mathbf{a}_k(\ell, m)$, we obtain that the dirty image (10) is given by:

$$I_D(\mathbf{s}) = \sum_k \mathbf{a}_k^{\mathrm{H}}(\mathbf{s}) \mathbf{R}_k \mathbf{a}_k(\mathbf{s})\,.$$

(We omitted the optional weighting. Also note that, with noise, we have to replace $\mathbf{R}_k$ by $\mathbf{R}_k - \sigma^2 \mathbf{I}$.) The iterative beam removing in CLEAN can now be posed as an iterative LS fitting between the sky model and the observed visibility [6]. Finding the brightest point $\mathbf{s}_0$ in the image is equivalent to trying to find a point source using classical Fourier beamforming, i.e,

$$\hat{\mathbf{s}}_0 = \arg\max_{\mathbf{s}} \sum_{k=1}^{K} \mathbf{a}_k^{\mathrm{H}}(\mathbf{s}) \left(\mathbf{R}_k - \sigma^2 \mathbf{I} \right) \mathbf{a}_k(\mathbf{s})\,.$$

Thus, the CLEAN algorithm can be regarded as a generalized classical sequential beamformer, where the brightest points are found one by one, and subsequently removed from $\mathbf{R}_k$ until the LS cost function (11) is minimized. An immediate consequence is that the estimated source locations will be biased: a well known fact in array processing. When the sources are well separated the bias is negligible compared to the standard deviation, otherwise it might be significant. This gives an explanation for the poor performance of the CLEAN in imaging extended structures (see e.g., [5]).

5.2. CLEAN with spatial filtering

Let us assume now that we have spatially filtered the covariance matrices $\hat{\mathbf{R}}_k$ by linear operations $\mathbf{L}_k$, for example projections. If we assume that all the interference is removed by the filtering, the measurement equation becomes

$$\tilde{\mathbf{R}}_k := \mathbf{L}_k \mathbf{R}_k \mathbf{L}_k^{\mathrm{H}} = \mathbf{L}_k \left[\mathbf{A}_k(\{\mathbf{s}_l\}) \mathbf{B} \mathbf{A}_k^{\mathrm{H}}(\{\mathbf{s}_l\}) + \sigma^2 \mathbf{I} \right] \mathbf{L}_k^{\mathrm{H}}\,. \tag{12}$$

This modifies the least squares optimization problem to

$$\left[\{\hat{\mathbf{s}}_l\}, \hat{\mathbf{B}} \right] = \underset{\{\mathbf{s}_l\}, \mathbf{B}}{\arg\min} \sum_{k=1}^{K} \| \mathbf{L}_k \left(\hat{\mathbf{R}}_k - \mathbf{R}_k - \sigma^2 \mathbf{I} \right) \mathbf{L}_k^{\mathrm{H}} \|_F\,. \tag{13}$$

and $\mathbf{R}_k(\{\mathbf{s}_l\}, \mathbf{B}) = \mathbf{A}_k(\{\mathbf{s}_l\}) \mathbf{B} \mathbf{A}_k^{\mathrm{H}}(\{\mathbf{s}_l\})$ The cost function is similar to ordinary CLEAN cost function and thus its minimization does not pose stronger computational demands. Indeed, we end up with a deconvolution problem with a space-varying beam, but the

CLEAN algorithm is simply extended to take this into account. Here, we develop the extension more carefully, taking note of the fact that the noise structure after projections is not white anymore.

In the case of spatially filtered signals the classical beamformer follows from the previous by replacing $\mathbf{a}_k(\mathbf{s})$ by the effective array response $\mathbf{L}_k\mathbf{a}_k(\mathbf{s})$, i.e.,

$$I'_D(\mathbf{s}) = \sum_{k=1}^{K} \mathbf{a}_k^{\mathrm{H}}(\mathbf{s})\hat{\mathbf{R}}'_k\mathbf{a}_k(\mathbf{s}), \tag{14}$$

where

$$\hat{\mathbf{R}}'_k = \mathbf{L}_k^{\mathrm{H}}\mathbf{L}_k\,\hat{\mathbf{R}}_k\,\mathbf{L}_k^{\mathrm{H}}\mathbf{L}_k - \sigma^2\mathbf{L}_k^{\mathrm{H}}\mathbf{L}_k\,\mathbf{L}_k^{\mathrm{H}}\mathbf{L}_k \tag{15}$$

Therefore the step of finding the brightest point $\mathbf{s}_0$ in the image can be implemented using FFT in the same way it is implemented in the CLEAN algorithm, but acting on $\hat{\mathbf{R}}'_k$ instead of the original visibilities. Similarly, the contribution of a source at location $\mathbf{s}_0$ in a single covariance matrix $\hat{\mathbf{R}}_k$ is a multiple of $\mathbf{L}_k\mathbf{a}_k(\mathbf{s}_0)\mathbf{a}_k^{\mathrm{H}}(\mathbf{s}_0)\mathbf{L}_k^{\mathrm{H}}$, and hence the response in the dirty image $I'_D(\mathbf{s})$ is given by

$$B(\mathbf{s},\mathbf{s}_0) := \sum_{k=1}^{K} \mathbf{a}_k^{\mathrm{H}}(\mathbf{s})\mathbf{L}_k^{\mathrm{H}}\left(\mathbf{L}_k\mathbf{a}_k(\mathbf{s}_0)\mathbf{a}_k^{\mathrm{H}}(\mathbf{s}_0)\mathbf{L}_k^{\mathrm{H}}\right)\mathbf{L}_k\mathbf{a}_k(\mathbf{s}). \tag{16}$$

This is the space-varying beam. The extended CLEAN algorithm after spatial filtering now follows immediately and is given in table 1.

To test the algorithm, we have taken an array configuration with $p = 14$ telescopes as in WSRT, and generated two equal-powered point sources centered around right ascension 32° and declination 60°, with a signal to noise ratio of -20 dB for each of the sources. To simulate the effect of spatial filtering, we placed an interferer at a fixed terrestrial location (hence varying compared to the look direction of the array), and with INR $= 30$ dB. $K = 100$ sample covariance matrices $\hat{\mathbf{R}}_k$ were generated, uniformly spread along 12 hours, and each based on $N = 1000$ samples. Figure $1(a)$–(c) shows the dirty image without interference present, the effect of the interferer on the dirty image, and the dirty image after estimating and removing the interferer using spatial projections. Clearly, with interference present but not removed, the sources are completely masked out (note the change in scale between the first two figures). After estimating and projecting out the interferer, in the third image, we obtain nominally the same image as in the interference-free case, but the sidelobe patterns are different (as we demonstrated before, they are in fact space-varying).

Acknowledgements

We would like to thank Ed Deprettere and our project partners at NFRA, for the very useful collaboration.

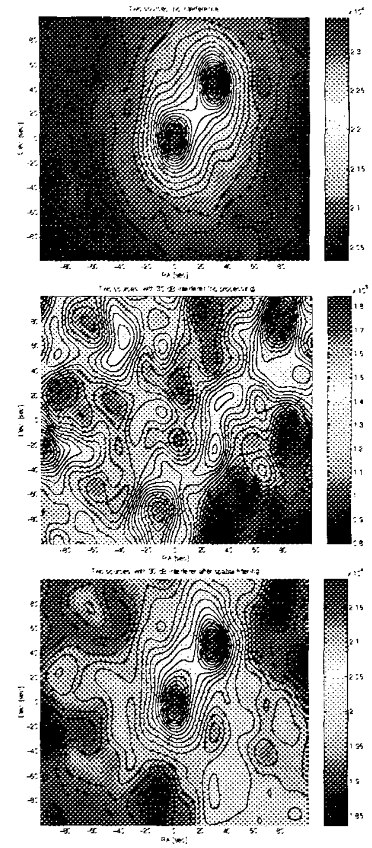

Fig. 1. Dirty images of two closely spaced point sources. (a) no interference; (b) unsuppressed interference (INR $= 30$ dB); (c) after spatial filtering.

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