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Adaptive single-stage control for uncertain nonholonomic Euler-Lagrange systems

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Abstract— **This work introduces a new single-stage adaptive controller for Euler-Lagrange systems with nonholonomic constraints. The proposed mechanism provides a simpler design philosophy compared to double-stage mechanisms (that address kinematics and dynamics in two steps), while achieving analogous stability properties, i.e. stability of both original and internal states. Meanwhile, we do not require direct access to the internal states as required in state-of-the-art single-stage mechanisms. The proposed approach is studied via Lyapunov analysis, validated numerically on wheeled mobile robot dynamics and compared to a standard double-stage approach.**

I. Introduction

Nonholonomic Euler-Lagrange (EL) dynamics covers important classes of practical mechanical systems. The most typical example of nonholonomic constraint is the no-slip constraint in mobile robots [1], unicycle [2], [3], and several wheeled vehicles [4]. Control of nonholonomic EL systems is intrinsically more challenging than control of holonomic EL systems. These challenges have generated nonholonomic control methods that are different than the corresponding holonomic versions. For example, in holonomic EL systems it is unnecessary to distinguish between controlling the kinematics (position) and the dynamics (velocity). However, ample literature on nonholonomic EL systems only focuses on the kinematics [1]–[4]. Some literature on nonholonomic EL systems considers both kinematics and dynamics via the so-called double-stage mechanism [5]–[8]: here, the first-stage control is a kinematic tracking problem to track desired trajectories under desired velocities; the second-stage control provides forces and torques as control inputs and generates desired velocities as outputs. Note that holonomic dynamics do not need such a double-stage mechanism [9], [10].

The crucial reason for considering double-stage control in nonholonomic systems are the internal state variables arising from nonholonomic constraints. Such internal state variables allow to transform the original nonholonomic dynamics into lower-dimensional unconstrained dynamics. Let us now discuss a key issue in the few single-stage mechanisms proposed for nonholonomic EL dynamics: [11], [12] stabilize the internal state variables and the constraint forces separately, which means that stability of the original states (generalized coordinates) is not proven. These considerations provide a clear motivation for revisiting and exploring new single-stage mechanisms for nonholonomic EL systems whose design and stability properties can be more consistent with the holonomic scenario. This motivation becomes even more relevant due to the inevitable presence of uncertainties in the systems, requiring a robust or an adaptive control approach. In this work, we focus on the latter (adaptive control), with the objective to reduce the assumptions on the structure of the system uncertainty. In this respect, most literature restricts such uncertainties to have linear-in-the-parameters (LIP) structure: this is the case for both double-stage [7], [13] and single-stage [11], [12]) approaches. A notable exception not requiring LIP is [14], which however is a double-stage mechanism only applicable to the specific application, i.e. it relies on a specific structure of the dynamics. To the best of our knowledge, no single-stage approaches exist for nonholonomic EL dynamics that can tackle state-dependent uncertainties with lack of structural knowledge, in a similar way as it was studied for holonomic EL dynamics. Summarizing, the main contributions of this work are:

- Proposing a new single-stage mechanism for nonholonomic EL systems whose design and stability properties are consistent with the holonomic case. We guarantee stability of both the original states (generalized coordinates) and internal states (transformed coordinates) which state-of-the-art single-stage mechanisms fail to achieve, while avoiding measuring internal states.
- Considering lack of structural knowledge of the system terms. Specifically, our approach can handle state-dependent uncertainties that are either LIP or non-LIP.
- By avoiding structural knowledge, our approach is not restricted to a specific application. Rather, we make use of standard properties of EL systems verified to hold in many systems of practical interest, including nonholonomic mobile robots and unicycles.

The rest of this paper is organized as follows: Section II gives the kinematics and dynamics of a nonholonomic EL system; the control problem is formulated in Section III; the adaptive controller is designed in Section IV with stability analysis in Appendix. A simulation study is in Section V

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with concluding remarks in Section VI.

Basic notations are adopted: I_n is the identity $n \times n$ matrix; $\|\cdot\|$ represents the Euclidean norm of a vector or a matrix; $\underline{\lambda}$ is the minimum eigenvalue of a matrix.

II. Kinematics and Dynamics of a Nonholonomic Euler-Lagrange System

Consider the following nonholonomic underactuated EL dynamics subject to m constraints

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + F(q,\dot{q}) = B(q)\tau + A^T(q)\lambda \tag{1}$$

where $q,\ \dot{q},\ \ddot{q} \in \mathbb{R}^n$ are the generalized coordinates and corresponding velocities and accelerations, $M(q) \in \mathbb{R}^{n\times n}$ is a symmetric and positive-definite mass/inertia matrix, $C(q,\dot{q}) \in \mathbb{R}^{n\times n}$ denotes the Coriolis, centripetal term, $G(q) \in \mathbb{R}^n$ is the gravity force, $F(q,\dot{q}) \in \mathbb{R}^n$ is the friction term, $B(q) \in \mathbb{R}^{n\times r}$ is a full-rank matrix, $A(q) \in \mathbb{R}^{m\times n}$ is the constraint matrix, and $\lambda \in \mathbb{R}^m$ is the vector of constraints multiplier. The literature has shown that several EL systems of interest satisfy the following properties [15]–[17]:

Property 1. *M(q) is a symmetric and positive-definite matrix with* $\underline{m}I_n \le M(q) \le \bar{m}I_n$ *where* $\underline{m},\ \bar{m} \in \mathbb{R}^+$.

Property 2. *There exist* $\bar{c}_1,\ \bar{c}_2,\ \bar{g},\ \bar{f}_1,\ \bar{f}_2 \in \mathbb{R}^+$ *such that* $\|C(q,\dot{q})\| \le \bar{c}_1 + \bar{c}_2\|\dot{q}\|,\ \|G(q)\| \le \bar{g},\ \|F(q,\dot{q})\| \le \bar{f}_1 + \bar{f}_2\|\dot{q}\|$.

Property 3. *The matrix* $\dot{M}(q) - 2C(q,\dot{q})$ *is skew-symmetric, that is,* $x^T(\dot{M} - 2C)x = 0, \forall x \in \mathbb{R}^n$.

In this work, the upper bounds $\underline{m},\ \bar{m},\ \bar{c}_1,\ \bar{c}_2,\ \bar{g},\ \bar{f}_1,\ \bar{f}_2$ in Properties 1-2 are taken to be unknown. Uncertainty in $B(q)$ will be addressed later. We do not assume any LIP structure of the system terms, since terms such as friction/damping may not satisfy LIP assumption in general [18].

As common in the literature, it is assumed that the nonholonomic constraints are independent of time [5]–[7], [12], [19], [20], so that

$$A(q)\dot{q} = 0. \tag{2}$$

Define $r = n - m$, and let $S(q) \in \mathbb{R}^{n\times r}$ be a full-column rank matrix spanning the null space of $A(q)$, i.e.,

$$S^T(q)A^T(q) = 0. \tag{3}$$

The selection of $S(q)$ is not difficult for a given $A(q)$, as shown in [21]. Combining (2) with (3), an internal state variable $u \in \mathbb{R}^r$ can be found such that [7], [22]

$$\dot{q} = S(q)u. \tag{4}$$

Substituting (4) into (1) and premultiplying (1) by S^T, a reduced-order system in terms of u is obtained as:

$$\bar{M}(q)\dot{u} + \bar{C}(q,\dot{q})u + \bar{G}(q,\dot{q}) + \bar{F}(q,\dot{q}) = \bar{B}(q)\tau \tag{5}$$

with $\bar{M}(q) = S^T(q)M(q)S(q) \in \mathbb{R}^{r\times r}, \bar{C}(q,\dot{q}) = S^T(q)(M(q)\dot{S}(q) + C(q,\dot{q})S(q)) \in \mathbb{R}^{r\times r},\ \bar{G}(q) = S^T(q)G(q) \in \mathbb{R}^r,\ \bar{F}(q,\dot{q}) = S^T(q)F(q,\dot{q})$ and $\bar{B}(q) = S^T(q)B(q) \in \mathbb{R}^{r\times r}$.

It can be seen that the degrees of freedom in the system (5) have decreased from n to r. In other words, (5) describes the behavior of the nonholonomic systems in a new set of local coordinates, where $S(q)$ is the Jacobian matrix that transforms the variable u into $\dot{q}$. As a result, the literature has assumed that the system in the new coordinates still satisfies properties analogous to Properties 1-3 ([5], [6], [11]):

Property 1'. $\bar{M}(q)$ *is a symmetric and positive-definite matrix with* $\underline{m}'I_r \le \bar{M}(q) \le \bar{m}'I_r$ *where* $\underline{m}',\ \bar{m}' \in \mathbb{R}^+$.

Property 2'. *There exist* $\bar{c}'_1, \bar{c}'_2,\ \bar{g}',\ \bar{f}'_1,\ \bar{f}'_2, \in \mathbb{R}^+$ *such that* $\|\bar{C}(q,\dot{q})\| \le \bar{c}'_1 + \bar{c}'_2\|\dot{q}\|,\ \|\bar{G}(q)\| \le \bar{g}',\ \|\bar{F}(q,\dot{q})\| \le \bar{f}'_1 + \bar{f}'_2\|\dot{q}\|$.

Property 3'. *The matrix* $\dot{\bar{M}}(q) - 2\bar{C}(q,\dot{q})$ *is skew symmetric, that is,* $x^T(\dot{\bar{M}}(q) - 2\bar{C}(q,\dot{q}))x = 0$.

Proof: Properties 1' and 2' follow directly from Properties 1 and 2, provided that $S(q)$ is bounded (which holds in most cases of practical interest [14], [22]). Regarding Property 3', since $\dot{M} - 2C$ is skew-symmetric from Property 3, it is straightforward to verify that

$$\dot{\bar{M}} - 2\bar{C} = \dot{S}^T MS - (\dot{S}^T MS)^T + S^T(\dot{M} - 2C)S \tag{6}$$

is also skew-symmetric.

The reduced-order system (5) is fully-actuated with r states and r inputs [7], [21]. The following is a sufficient condition proposed in the literature [5]–[7], [12], [21] for the system (5) to be controllable.

Assumption 1. $\bar{B}(q)$ *is full rank, i.e.,* $\mathrm{rank}(\bar{B}) = r$.

It is worth remarking that virtually all nonholonomic dynamics of practical interest [14], [22] verify Assumption 1. To include uncertainty into $\bar{B}$, let us decompose $\bar{B}(q)$ into $\bar{B} = \hat{\bar{B}} + \Delta B$ where $\hat{\bar{B}}$ is the nominal term and ΔB is the unknown part obeying the following assumption [23], [24]:

Assumption 2. *Define* $T = \bar{B}\hat{\bar{B}}^{-1} - I_r$. *There exists a known scalar* $\bar{T} \in \mathbb{R}^+$ *such that*

$$\|T\| \le \bar{T} < 1. \tag{7}$$

Let us discuss which signals are available for feedback: q and $\dot{q}$ can be assumed to be directly available for feedback, which is consistent with the case of holonomic systems [15], [16], [25], [26]. State-of-the-art single-stage mechanisms [11], [12] assume that the internal signal $\dot{u}$ is also directly available, requiring extra sensors. To be consistent with the holonomic case, we aim at a single-stage mechanism where $\dot{u}$ is not used for feedback: meanwhile, the internal state signal u can be calculated as follows, without extra sensors

$$u = [S^T(q)S(q)]^{-1}S^T(q)\dot{q} \tag{8}$$

cf. [11], [12]. Our analysis will prove the boundedness of such internal states.

III. Problem Formulation

The desired trajectory $q^d \in \mathbb{R}^n$ and its derivative $\dot{q}^d \in \mathbb{R}^n$ also satisfy the nonholonomic constraints [7]

$$A(q^d)\dot{q}^d = 0. \tag{9}$$

It is implied from (9) that $\dot{q}^d = S(q^d)u^d$ where $u^d \in \mathbb{R}^r$ is a desired internal state variable. As commonly assumed in the literature, desired trajectories with bounded first and second order derivatives result in $u^d,\ \dot{u}^d$ being also bounded. Define the tracking errors $e_q = q - q^d,\ \dot{e}_q = \dot{q} - \dot{q}^d,\ e_u = u - u^d$.

Problem Formulation. *Design a single-stage adaptive control τ such that, in the presence of state-dependent uncertainty as in Properties 1'-3' and Assumptions 1-2, the tracking errors e_q, e_u are uniformly ultimately bounded (UUB).*

Remark 1. *Even in the holonomic case, the presence of state-dependent uncertainty requires to seek stability in UUB sense [27]–[29]. Therefore, a similar notion is sought also for the nonholonomic case.*

IV. Controller design

Based on e_u and e_q, define the error variable

$$\delta = Ke_u + S^T Pe_q \tag{10}$$

where $K \in \mathbb{R}^{r\times r},\ P \in \mathbb{R}^{n\times n}$ are positive definite matrices chosen by the designer: the control input is designed as

$$\tau = \hat{\bar{B}}^{-1}\big(-\delta - \bar{\tau}\big),\ \bar{\tau} = \rho\,\mathrm{sat}(e_u,\ \varepsilon) \tag{11}$$

with $\mathrm{sat}(e_u, \varepsilon) = \begin{cases} e_u/\|e_u\|, & \|e_u\| \geq \varepsilon, \\ e_u/\varepsilon, & \|e_u\| < \varepsilon. \end{cases}$, and ρ defined later based on the uncertainty analysis.

A. Error Dynamics and Uncertainty Analysis

For compactness, we may omit variable dependency when obvious. Based on (5), the following dynamics are obtained

$$\begin{aligned}\bar{M}\dot{e}_u =& \bar{M}\big(\dot{u} - \dot{u}^d\big) = -\Big(\bar{C}u - \bar{B}\tau + \bar{G} + \bar{F}\Big) - \bar{M}\dot{u}^d \\ =&(\bar{B}\hat{\bar{B}}^{-1} - I_r)(-Ke_u - S^T Pe_q - \bar{\tau}) - Ke_u \\ &- S^T Pe_q - \bar{\tau} - \big(\bar{C}u + \bar{M}\dot{u}^d + \bar{G} + \bar{F}\big) \\ =& -Ke_u - S^T Pe_q - (I_r + T)\bar{\tau} - \bar{C}e_u + \phi \end{aligned} \tag{12}$$

where $\phi = -\bar{C}u^d + \bar{M}\dot{u}^d + \bar{G} + \bar{F} - TKe_u - TS^T Pe_q$, which represents the overall uncertainty.

Define $\xi = [e_u^T\ \ e_q^T\ \ \dot{e}_q^T]^T$. It is implied that $\|e_u\| \leq \|\xi\|,\ \|e_q\| \leq \|\xi\|,\ \|\dot{e}_q\| \leq \|\xi\|$, resulting in an upper bound of the overall uncertainty $\|\phi\|$ as follows:

$$\begin{aligned}\|\phi\| \leq& \|\bar{C}u^d\| + \|\bar{M}\dot{u}^d\| + \|\bar{G}\| + \|\bar{F}\| + \|TKe_u\| + \|TS^T Pe_q\| \\ \leq& \bar{c}_1'\|u^d\| + \bar{c}_2'\|\dot{e}_q\|\|u^d\| + \bar{c}_2'\|\dot{q}^d\|\|u^d\| + \bar{m}'\|\dot{u}^d\| + \bar{g}' \\ &+ \bar{f}_1' + \bar{f}_2'\|\dot{e}_q\| + \bar{f}_2'\|\dot{q}^d + \bar{T}\|K\|\|e_u\| + \bar{T}\|S\|\|P\|\|e_q\| \\ \leq& \theta_0^* + \theta_1^*\|\xi\| \end{aligned} \tag{13}$$

with $\theta_0^* = \bar{g}' + \bar{f}_1' + \bar{m}'\|\dot{u}^d\| + (\bar{c}_1' + \bar{c}_2'\|\dot{q}^d\|)\|u^d\| + \bar{f}_2'\|\dot{q}^d\|,\ \theta_1^* = \bar{f}_2' + \bar{c}_2'\|u^d\| + \bar{T}\|K\|\bar{T})\|S\|\|P\|$.

Remark 2. *Most nonholonomic EL literature requires system uncertainties to have linear-in-the parameters (LIP) structure [7], [13], or to be a priori bounded [8], [22]. In this work, instead of assuming a specific structure for the uncertainty, we made use of Properties 1'-2' to obtain a state-dependent upper bound $\|\phi\|$ in* (13) *regardless of the fact that ϕ is LIP or non-LIP.*

B. Adaptive Laws

According to the form of $\|\phi\|$ in (13), ρ is designed as

$$\rho = \frac{1}{(1-\bar{T})}\big(\sum_{l=0}^{1}\hat{\theta}_l\|\xi\|^l + \gamma\big) \tag{14}$$

with the adaptive laws ($l = 0, 1$)

$$\dot{\hat{\theta}}_l = -\alpha_l\hat{\theta}_l + \|e_u\|\|\xi\|^l \tag{15a}$$

$$\dot{\gamma} = -\big(\epsilon_0 + \epsilon_1\|\xi\|^3\big)\gamma + \epsilon_0\|e_u\| \tag{15b}$$

$$\text{with } \hat{\theta}_0(0), \hat{\theta}_1(0), \gamma(0), \alpha_0, \alpha_1, \epsilon_0, \epsilon_1 \in \mathbb{R}^+$$

The following main stability result holds:

Theorem 1. *Under Properties 1'-3' and Assumptions 1-2, the closed-loop trajectories of* (5) *adopting the single-stage control law* (11), (14) *and adaptive laws* (15)*, are UUB. The tracking errors e_u and e_q are also UUB.*

Proof: See Appendix.

Remark 3. *Note that state-of-the-art single-stage mechanisms [11], [12], only guarantee stability of the internal state errors e_u, without considering the original state errors e_q. However, the stability of e_u is not sufficient to guarantee the stability of e_q. Our stability analysis covers both the original state error and the internal state error.*

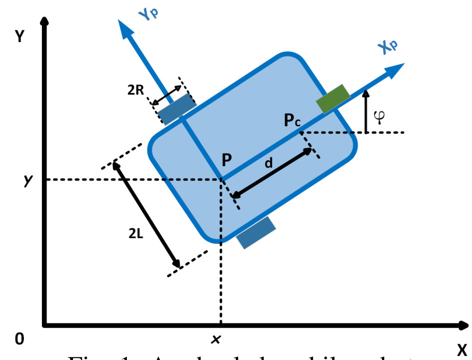

Fig. 1: A wheeled mobile robot

V. Application to Wheeled Mobile Robot

A. The model of wheeled mobile robot

This section considers the tracking problem of a wheeled mobile robot. The robot has two co-axle driving wheels (in dark blue in Fig. 1) and a front passive wheel (in green in Fig. 1), cf [5]. In Fig. 1, P is the geometric center of the left and right driving wheels, and P_c is the center of the mass of the mobile robot. Then, $2L$ is the width of the mobile robot, R is the radius of the driving wheel, d represents the distance from P to P_c.

Consider an inertial Cartesian frame on the plane of motion $\{0, X, Y\}$ with generalized coordinates $q = [x\ y\ \varphi]^T$, where (x, y) is the coordinate of reference point P in the inertial frame, and φ represents the orientation of the robot with respect to the X-axis in the inertial frame. Meanwhile, $\{P, X_P, Y_P\}$ is the coordinate frame in the robot frame.

The driving wheels of the mobile robot satisfy pure roll without slip, i.e. the well-known nonholonomic constraint

$$\dot{y}\cos\varphi - \dot{x}\sin\varphi = 0. \tag{16}$$

Summarizing, the EL dynamics of the wheeled mobile robot in Fig. 1 can be expressed as in (1) with

$$M(q) = \begin{bmatrix} m & 0 & md\sin\varphi \\ 0 & m & -md\cos\varphi \\ md\sin\varphi & -md\cos\varphi & I_c \end{bmatrix}, G(q) = \mathbf{0},$$

$$C(q,\dot{q}) = \begin{bmatrix} 0 & 0 & md\dot{\varphi}\cos\varphi \\ 0 & 0 & md\dot{\varphi}\sin\varphi \\ 0 & 0 & 0 \end{bmatrix}, B(q) = \frac{1}{R}\begin{bmatrix} \cos\varphi & \cos\varphi \\ \sin\varphi & \sin\varphi \\ L & -L \end{bmatrix},$$

$$F = \begin{bmatrix} 0.05(\dot{x} + \sin x) \\ 0.08\dot{y} \\ 0.065(\dot{\varphi} - \sin\varphi) \end{bmatrix}, \ A^T(q) = \begin{bmatrix} \sin\varphi \\ -\cos\varphi \\ 0 \end{bmatrix},$$

$$\lambda = -m(\dot{x}\cos\varphi + \dot{y}\sin\varphi)\dot{\varphi}$$

where m is the mass of the robot, I_c is its inertia moment around the vertical axis at point P_c; $\tau = [\tau_r \ \tau_l]^T$ are the torque acting on the right and left wheels, respectively.

The unconstrained dynamics are obtained as in (5) with:

$$\bar{M} = \begin{bmatrix} m & 0 \\ 0 & I_c \end{bmatrix}, \bar{B} = \frac{1}{R}\begin{bmatrix} 1 & 1 \\ L & -L \end{bmatrix}, \bar{C} = \begin{bmatrix} 0 & md\dot{\varphi} \\ -md\dot{\varphi} & 0 \end{bmatrix},$$

$$\bar{F} = \begin{bmatrix} 0.05(\dot{x}+\sin x)\cos\varphi + 0.08\dot{y}\sin\varphi \\ 0.065(\dot{\varphi} - \sin\varphi) \end{bmatrix}, S(q) = \begin{bmatrix} \cos\varphi & 0 \\ \sin\varphi & 0 \\ 0 & 1 \end{bmatrix}.$$

According to the above $S(q)$, we obtain the kinematics as in (4) where the internal states are $u = [\nu \ \ \omega]^T$ with ν and ω being the linear and angular velocity of the mobile robot at the reference point P in the robot frame, respectively. According to (8), such internal states u can be calculated as

$$u = \begin{bmatrix} \dot{x}\cos\varphi + \dot{y}\sin\varphi \\ \dot{\varphi} \end{bmatrix}.$$

The system parameters are selected as: $d = 0.2m$, $R = 0.13m$, $L = 0.75m$, $m = 3kg$, $I_c = 5.625kg\,m^2$. The knowledge of these system parameters is not required to design the controller, just for simulation. The only nominal parameters used for control design are needed for $\hat{\bar{B}}$, which are $\hat{R} = 0.15m$, $\hat{L} = 0.6m$, giving $\bar{T} = 0.3$ according to Assumption 2. We consider the initial conditions $q(0) = [2.5 \ \ -1.5 \ 0.5]^T$, $\dot{q}(0) = [0 \ 0 \ 0.6]^T$, $\hat{\theta}_0(0) = \hat{\theta}_1(0) = 0.005$, $\gamma(0) = 0.005$. Given the desired trajectory $q^d = [2\sin t \ \ -2\cos t \ \ t]^T$, the desired transformed trajectory can be calculated as $u^d = [2 \ 1]^T$ according to $\dot{q}^d = S(q^d)u^d$.

We select the control parameters as $K = \text{diag}\{14.5, 145\}$, $P = \text{diag}\{14.5, 14.5, 145\}$, $\alpha_0 = 35$, $\alpha_1 = 25$, $\epsilon_0 = 0.0625$, $\epsilon_1 = 25$, $\varepsilon = 1$.

B. Tracking performance

To verify the validity of the proposed single-stage controller, we compare it with the standard double-stage mechanism in [5], and we also consider an external disturbance $\tau_d = [0.1\sin 0.002t \ \ 0.3\cos 0.002t \ \ 0.01\cos 0.002t]^T$ to assess robustness. The performances of the proposed single-stage and the state-of-the-art double-stage controller are in

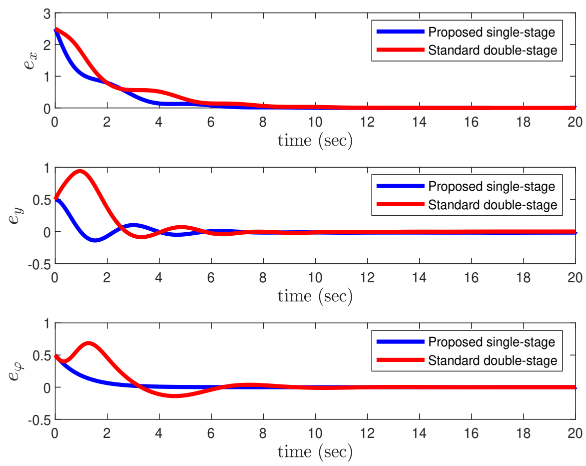

Fig. 2: Comparison between proposed single-stage and state-of-the-art double-stage: Position errors $e_q = [e_x \ e_y \ e_\varphi]^T$

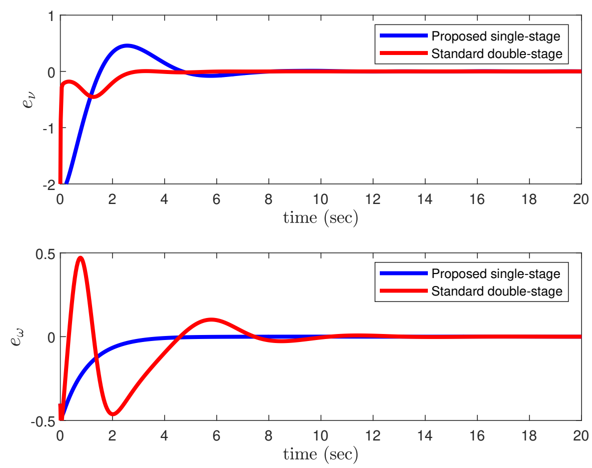

Fig. 3: Comparison between proposed single-stage and state-of-the-art double-stage: Velocity errors $e_u = [e_\nu \ e_\omega]^T$

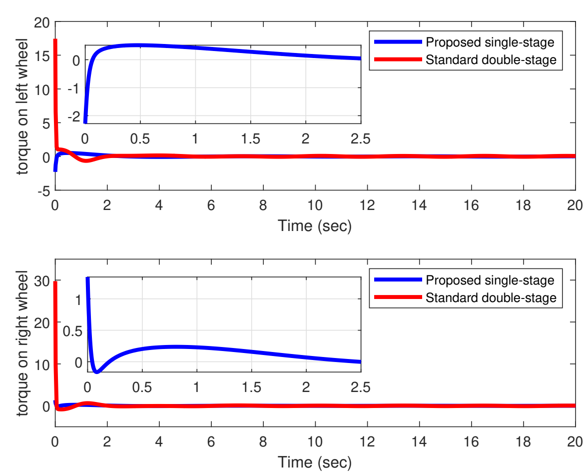

Fig. 4: Comparison between proposed single-stage and state-of-the-art double-stage: Torques on left and right wheels

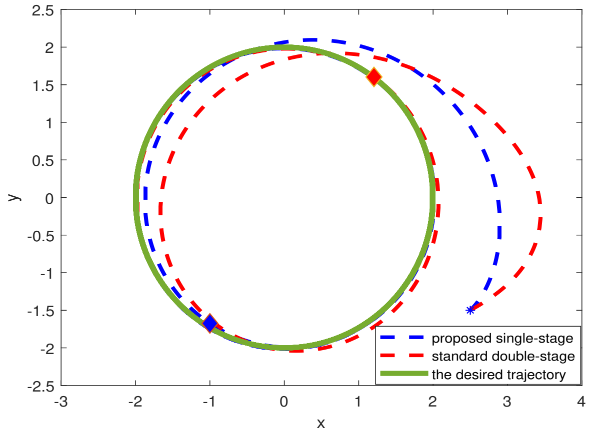

Fig. 5: Comparison between proposed single-stage and state-of-the-art double-stage: Desired path (green solid line) and actual path (blue and red dash lines). The star is the departure and diamonds are the convergence points for both methods

TABLE I: Tracking error and input norms for proposed single-stage and state-of-the-art double-stage mechanism.

	Norm of position error	Norm of velocity error	Norm of input
proposed single-stage	209.17	172.52	63.50
standard double-stage	298.06	116.18	294.17

Figs. 2-5. The norms of tracking errors e_q, e_u and torque input τ are in Table I, showing that the proposed single-stage control is competitive against the state of the art.

VI. Conclusion

A new single-stage mechanism was designed for the tracking problem of nonholonomic EL dynamics. The proposed single-stage mechanism keeps the stability advantage of double-stage mechanisms (i.e. both the original states and the internal states are guaranteed stable), while removing the requirement on accessibility of internal states. Studying more complex unmodelled dynamics and single-stage observer-based design are interesting points for future work.

Appendix

Proof: Consider the Lyapunov function

$$V(t) = \frac{1}{2}\Big[e_u^T\bar{M}e_u + \sum_{l=0}^{1}(\hat{\theta}_l - \theta_l^*)^2 + \frac{\gamma^2}{2\epsilon_0} + e_q^T P e_q\Big]. \quad (17)$$

According to $\dot{q}^d = S(q^d)u^d$, we can infer that

$$\dot{e}_q = \dot{q} - \dot{q}^d = S(q)e_u + \big(S(q) - S(q^d)\big)u^d. \quad (18)$$

Define $\bar{\epsilon}_1 = \frac{\epsilon_1}{\epsilon_0}$. Based on the adaptive law in (15b), we have

$$\begin{aligned}\frac{\gamma\dot{\gamma}}{\epsilon_0} \le& \frac{\gamma}{\epsilon_0}\Big\{-\big(\epsilon_0 + \epsilon_1\|\xi\|^3\big)\gamma + \epsilon_0\|e_u\|\Big\}\\ \le& -\big(1 + \bar{\epsilon}_1\|\xi\|^3\big)\gamma^2 + \|e_u\|\gamma \end{aligned} \quad (19)$$

Using (18) and (19), the time derivative of the Lyapunov function yields

$$\begin{aligned}\dot{V} \le& -e_u^T K e_u - e_u^T(I_r + T)\bar{\tau} + \frac{1}{2}e_u^T(\dot{\bar{M}} - 2\bar{C})e_u\gamma\\ &+ e_u^T\phi + \|e_u\|\gamma + \sum_{l=0}^{1}(\hat{\theta}_l - \theta_l^*)(-\alpha_l\hat{\theta}_l + \|e_u\|\|\xi\|^l)\\ &- \big(1 + \bar{\epsilon}_1\|\xi\|^3\big)\gamma^2 + \|e_q\|\|P\|\big(\|S(q)\| + \|S(q^d)\|\big)\|u^d\|\\ \le& -\underline{\lambda}(K)\|e_u\|^2 - \sum_{l=0}^{1}\alpha_l\hat{\theta}_l(\hat{\theta}_l - \theta_l^*) - \big(1 + \bar{\epsilon}_1\|\xi\|^3\big)\gamma^2\\ &- e_u^T(I_r + T)\rho\,\mathrm{sat}(e_u, \varepsilon) + \Big(\sum_{l=0}^{1}\hat{\theta}_l\|\xi\|^l + \gamma\Big)\|e_u\|\\ &+ \|P\|\big(\|S(q)\| + \|S(q^d)\|\big)\|u^d\|\|\xi\|. \end{aligned} \quad (20)$$

Completing the squares, we have

$$-\alpha_l\hat{\theta}_l(\hat{\theta}_l - \theta_l^*) \le -\frac{1}{2}\alpha_l\big(\hat{\theta}_l - \theta_l^*\big)^2 + \frac{1}{2}\alpha_l{\theta_l^*}^2 \quad (21)$$

where $l = 0,\ 1$. According to adaptive law in (15b), there exists $\underline{\gamma} \in \mathbb{R}^+$ such that with $\gamma \ge \underline{\gamma} > 0$. Substitute (21) into (20), it can be obtained that

$$\begin{aligned}\dot{V} \le& -\underline{\lambda}(K)\|e_u\|^2 - e_u^T(I_r + T)\rho\,\mathrm{sat}(e_u, \varepsilon)\\ &- \frac{1}{2}\alpha_0\big(\hat{\theta}_0 - \theta_0^*\big)^2 - \frac{1}{2}\alpha_1\big(\hat{\theta}_1 - \theta_1^*\big)^2 - \gamma^2 + \frac{1}{2}\alpha_0{\theta_0^*}^2\\ &+ \frac{1}{2}\alpha_1{\theta_1^*}^2 + \|P\|\big(\|S(q)\| + \|S(q^d)\|\big)\|u^d\|\|\xi\|\\ &+ \big(\hat{\theta}_0 + \hat{\theta}_1\|\xi\| + \gamma\big)\|e_u\| - \bar{\epsilon}_1\underline{\gamma}^2\|\xi\|^3. \end{aligned} \quad (22)$$

Scenario 1: When $\|e_u\| \ge \varepsilon$, $\mathrm{sat}(e_u, \varepsilon) = \frac{e_u}{\|e_u\|}$. According to the adaptive law (14), it can be obtained that

$$\begin{aligned}&-e_u^T(I_r + T)\rho\,\mathrm{sat}(e_u, \varepsilon) \le -(1 - \bar{T})\rho\frac{e_u^T e_u}{\|e_u\|}\\ &\le -\big(\hat{\theta}_0 + \hat{\theta}_1\|\xi\| + \gamma\big)\|e_u\|. \end{aligned} \quad (23)$$

The time derivative can be further simplified as

$$\begin{aligned}\dot{V} \le& -\underline{\lambda}(K)\|e_u\|^2 - \frac{1}{2}\alpha_0\big(\hat{\theta}_0 - \theta_0^*\big)^2 - \frac{1}{2}\alpha_1\big(\hat{\theta}_1 - \theta_1^*\big)^2\\ &- \gamma^2 + \frac{1}{2}\big(\alpha_0{\theta_0^*}^2 + \alpha_1{\theta_1^*}^2\big) - \bar{\epsilon}_1\underline{\gamma}^2\|\xi\|^3\\ &+ \|P\|\big(\|S(q)\| + \|S(q^d)\|\big)\|u^d\|\|\xi\|. \end{aligned} \quad (24)$$

Since $\hat{\theta}_0,\ \hat{\theta}_1 > 0$ according to (15a), the definition of the Lyapunov function yields

$$\begin{aligned}V \le& \frac{1}{2}\bar{m}'\|e_u\|^2 + \frac{1}{2}(\hat{\theta}_0 - \theta_0^*)^2 + \frac{1}{2}(\hat{\theta}_1 - \theta_1^*)^2\\ &+ \frac{1}{2\epsilon_0}\gamma^2 + \frac{1}{2}\|e_q\|^2. \end{aligned} \quad (25)$$

Define a scalar $\zeta = \frac{\min\{\underline{\lambda}(K), \alpha_0/2, \alpha_1/2, 1\}}{\max\{\bar{m}', 1/2, 1/2\epsilon_0\}}$, then substitute (25) into (24), so as to obtain

$$\begin{aligned}\dot{V} \le& -\zeta V + \frac{1}{2}\zeta\|\xi\|^2 + \frac{1}{2}\big(\alpha_0{\theta_0^*}^2 + \alpha_1{\theta_1^*}^2\big) - \bar{\epsilon}_1\underline{\gamma}^2\|\xi\|^3\\ &+ \|P\|\big(\|S(q)\| + \|S(q^d)\|\big)\|u^d\|\|\xi\|\\ \le& -\zeta V + Z_1(\|\xi\|) \end{aligned} \quad (26)$$

where $Z_{1(\|\xi\|)} = -\bar{\epsilon}_1\underline{\gamma}^2\|\xi\|^3 + \frac{1}{2}\zeta\|\xi\|^2 + \big(\|S(q)\| + \|S(q^d)\|\big)\|u^d\|\|\xi\| + \frac{1}{2}\big(\alpha_0\theta_0^* + \alpha_1\theta_1^*\big)$. Using Descartes' rules of sign change, the polynomial $Z_1(\|\xi\|)$ has a sole positive root η_1. As the coefficient of highest degree is negative as $-\bar{\epsilon}_1\underline{\gamma}^2$, $Z_1(\|\xi\|) \le 0$ when $\|\xi\| \ge \eta_1$. According to (26),

$$\dot{V} \le -\zeta V \text{ when } \|\xi\| \ge \eta_1. \tag{27}$$

Scenario 2: When $\|e_u\| < \varepsilon$, $\mathrm{sat}(e_u, \varepsilon) = \frac{e_u}{\varepsilon}$. We have

$$-e_u^T(I_r + T)\rho\,\mathrm{sat}(e_u, \varepsilon) \le 0. \tag{28}$$

Similarly to Scenario 1 in (24), according to (28), the time derivative of the Lyapunov function (22) can be rewritten as

$$\begin{aligned}\dot{V} \le &-\underline{\lambda}(K)\|e_u\|^2 - \frac{1}{2}\alpha_0\big(\hat{\theta}_0 - \theta_0^*\big)^2 - \frac{1}{2}\alpha_1\big(\hat{\theta}_1 - \theta_1^*\big)^2 \\ &+ \frac{1}{2}\big(\alpha_0{\theta_0^*}^2 + \alpha_1{\theta_1^*}^2\big) + \|P\|\big(\|S(q)\| + \|S(q^d)\|\big)\|u^d\|\|\xi\| \\ &- \gamma^2 - \bar{\epsilon}_1\underline{\gamma}^2\|\xi\|^3 + \big(\hat{\theta}_0 + \hat{\theta}_1\|\xi\| + \gamma\big)\|e_u\|. \end{aligned} \tag{29}$$

From the input-output properties of the systems in adaptive law (15), there exist scalars $\mathring{\theta}_0, \check{\theta}_0, \mathring{\theta}_1, \check{\theta}_1, \mathring{\gamma}, \check{\gamma} \in \mathbb{R}^+$ so that $\hat{\theta}_0 \le \mathring{\theta}_0 + \check{\theta}_0\|e_u\|$, $\hat{\theta}_1 \le \mathring{\theta}_1 + \check{\theta}_1\|e_u\|\|\xi\|$, $\gamma \le \mathring{\gamma} + \check{\gamma}\|e_u\|$. Since $\|e_u\| \le \varepsilon$, the last term in (29) satisfies

$$\begin{aligned}&\big(\hat{\theta}_0 + \hat{\theta}_1\|\xi\| + \gamma\big)\|e_u\| \\ \le&\check{\theta}_1\varepsilon^2\|\xi\|^2 + \mathring{\theta}_1\varepsilon\|\xi\| + \big(\check{\theta}_0 + \check{\gamma}\big)\varepsilon^2 + \big(\mathring{\theta}_0 + \mathring{\gamma}\big)\varepsilon. \end{aligned} \tag{30}$$

Substituting (30) into (29) gives

$$\dot{V} \le -\zeta V + Z_2(\|\xi\|) \tag{31}$$

where $Z_2(\|\xi\|) = Z_1(\|\xi\|) + \check{\theta}_1\varepsilon^2\|\xi\|^2 + \mathring{\theta}_1\varepsilon\|\xi\| + \big(\check{\theta}_0 + \check{\gamma}\big)\varepsilon^2 + \big(\mathring{\theta}_0 + \mathring{\gamma}\big)\varepsilon$. Analogously to Scenario 1,

$$\dot{V} \le -\zeta V \text{ when } \|\xi\| \ge \eta_2 \tag{32}$$

where η_2 is the sole positive root of $Z_2(\|\xi\|)$ such that $Z_2(\|\xi\|) \le 0$ when $\|\xi\| \ge \eta_2$.

Finally, combining (27) in Scenario 1 and (32) in Scenario 2, we obtain that ξ is UUB with ultimate bound $\max\{\eta_1, \eta_2\}$. Accordingly, $e_u,\ e_q$ are also uniformly ultimately bounded.

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