Reducible complexity in lens design

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ABSTRACT

A major challenge in lens design is the presence of many local minima in the optimization landscape. However, unlike other global optimization problems, the lens design landscape has an additional structure, that can facilitate the design process: many local minima are closely related to minima of simpler problems. For discussing this property, in addition to local minima other critical points in the landscape must also be considered. Usually, in a global optimization problem with M variables one has to perform M-dimensional searches in order to find minima that are different from the known ones. We discuss here simple examples where, due to the special structure that is present, all types of local minima found by other methods can be obtained by a succession of one-dimensional searches. Replacing M-dimensional searches by a set of one-dimensional ones has very significant practical advantages. If the ability to reach solutions by decomposing the search in simple steps will survive generalization to more complex systems, new design tools using this property could have a significant impact on lens design.

Keywords: lens design, saddle points

1. INTRODUCTION

One of the most significant challenges in the lens design process is the complexity of the lens design landscape. We still face today the well-known obstacle that when the complexity of the design task increases the designer gradually loses the ability to solve the problem by an effort of intellect. The most significant source of complexity is the presence of many local minima in the optimization landscape. A non-expert may be surprised that, despite of the fact that the number of local minima increases significantly with the number of components, it is still possible to successfully design systems as complex as, for instance, lithographic objectives. The designer's experience is certainly essential[1] but the fact that complex systems *can* be designed may also be a hint that the lens design landscape has some beneficial, but still insufficiently understood properties that help the designers in their efforts.

In this paper we discuss a feature of the design landscape that is potentially useful in the effort to cope with the complexity challenge. Unlike other global optimization problems, the lens design landscape has a certain structure, where relationships exist between minima in a design space with a certain number of lenses and minima with one lens less.

If we have M optimization variables, a lens design corresponds to a point in a M-dimensional space. Without a-priori knowledge and any special structure, finding such points requires a M-dimensional search and therefore the required computation time of the corresponding search algorithms increases rapidly with M. We will discuss here examples where, due to the additional structure, the existing types or ("shapes") of local minima can be found with a succession of one-dimensional searches. The systems discussed below are simple enough to be studied in detail, but if this reduction of complexity from M-dimensional to one-dimensional searches will survive generalization to more complex systems, new design tools using this property could have a significant impact on lens design.

In Sec. 2 we discuss how such one-dimensional searches are performed, and in Sec 3 we will describe how they are related to the properties of the design landscape and how they can be used to decompose the search for good solutions into simple steps.

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2. SADDLE-POINT CONSTRUCTION

A central concept in our research is that of a saddle point. If a tourist in a mountain landscape wants to go from a mountain valley to another one, he is looking for a mountain pass, which is a saddle point. In a M-dimensional space we also have saddle points. As in the case of local minima, the partial derivatives of the merit function *MF* with respect to all M optimization variables must vanish (for mathematical details see e.g. Ref 2). “Looking” for such M-dimensional “mountain passes” without a-priori knowledge is time-consuming[3]. However, as will be shown in Sec 3. in lens design landscapes many local minima of a problem with a given number of lenses are closely related to minima in the design landscape of simpler problems. This makes it possible to “construct” saddle-points from known minima by adding a meniscus lens (or airspace) to the system[2,4].

For practical purposes, saddle-point construction (SPC) works as follows: We start with an optical system with N surfaces that is already a local minimum in all variables. Although the method works with mirrors and aspheres as well, in this paper we assume for simplicity that all surfaces are spherical lens surfaces. In the original minimum we insert two new surfaces with the same curvature and zero distance between them (Fig. 1a). The pair of surfaces form either a lens meniscus, or an air meniscus (e.g. by splitting an existing lens). With zero distance and equal curvatures, the pair of surfaces disappears optically (i.e. this pair is the “null” element) and does not affect the path of any ray. In a first stage, we add only the two new surface curvatures to the list of variables. For certain values of the meniscus curvatures, the resulting system is a saddle point. We have described earlier two versions of SPC, depending on the way in which the curvatures of the meniscus that correspond to the saddle point are determined.

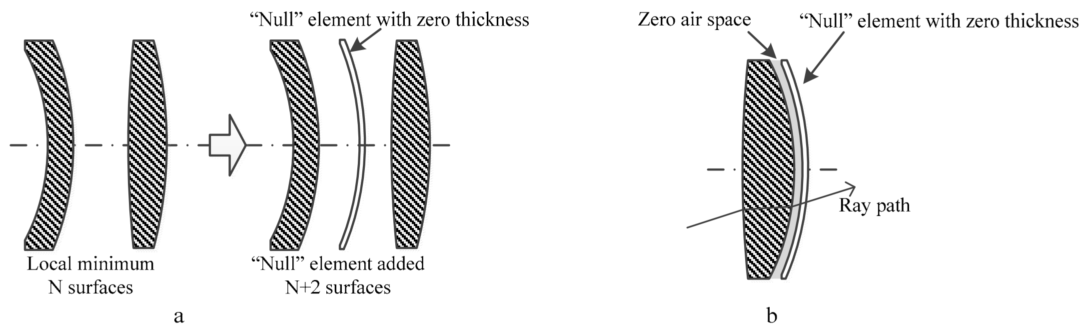

Figure 1. Inserting a “null” element in an existing local minimum. For clarity “null” elements are drawn with small nonzero widths. a) SPC general case (one zero thickness, between the null-element surfaces), b) SPC special case (two consecutive zero thicknesses, for air and glass).

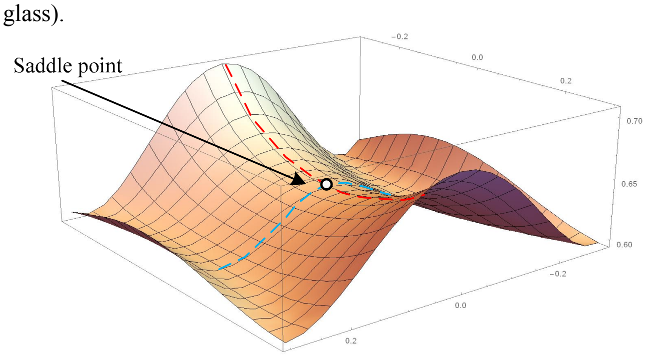

Figure 2. Saddle surface in a 6-lens system. The merit function values are computed with CODEV (using the default error function). Although we typically have many variables, the saddle points obtained with all versions of SPC resemble 2D horse saddles.

If the meniscus is inserted in contact with an existing surface of the starting minimum (the reference surface) and has the same glass as the one at the reference surface, we have proven mathematically that if the three consecutive surfaces in Fig. 1b have the same curvature, then the resulting system is a saddle point (SPC-special case)[4]. In the resulting system it is possible to find analytically two directions in the variable space (see Eqs. 9-11 in Ref. 4) such that, when the merit function is plotted along them, we obtain the well-known shape of a saddle surface: a maximum along one of the two directions and a minimum along the other one (Fig. 2). In all other directions in the variable space the resulting system is a minimum (because we have started from a local minimum).

SPC can be generalized for the case when the null-element position and glass are arbitrary (See Fig. 1a). The details are given in Ref. 2. In a nutshell, because the partial derivatives with respect of the variables of the starting minimum are already zero, only the derivatives with respect to the curvatures of the null-element meniscus need to be considered. In fact, because these two curvatures are not independent, annulling one of them is sufficient. If we denote the first meniscus curvature by *c*, for finding the saddle points it is sufficient to find numerically the values of *c* for which we have

$$\partial MF / \partial c = 0 \tag{1}$$

For each inserted null-element position, the curvature values that lead to saddle points correspond therefore to the intersection points with the horizontal axis of the curve for $\partial MF / \partial c$ that is computed numerically using ray-tracing data (see Fig. 3). Because Eq. (1) is non-linear, it can have more than one solution (or none). The resulting saddle points are similar to the one shown in Fig. 2. Once the saddle points are found, initial systems for subsequent local optimization can be obtained by choosing for each zero crossing in Fig. 3 two points, one to the left, one to the right of the crossing point. Optimization will then lead to two minima, one on each side of the "saddle". The zero-thickness condition for the null element is not a severe limitation, as it may seem, because in the resulting minima the distances between the surfaces (and the glass of the new lens) can be easily changed as desired.

In the example in the next section, all local minima that can be found with other methods in a high- (5-) dimensional merit function landscape can be simply found by using only one-dimensional searches for saddle points (we will call them scans) similar to the one in Fig.3.

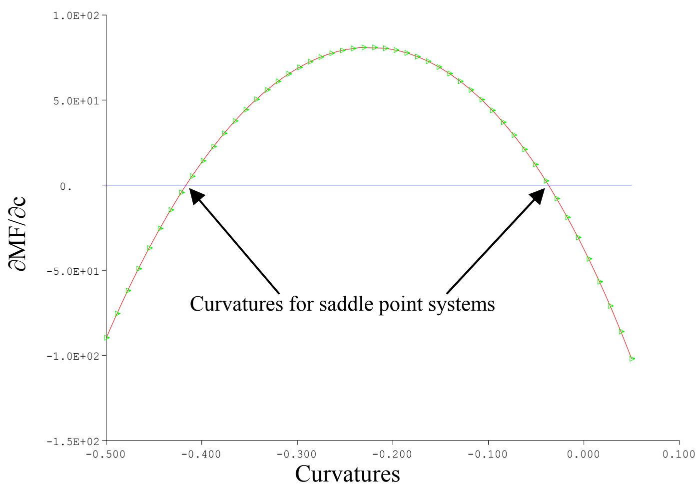

Figure 3. Curvatures of the inserted null element that transforms a local minimum into a saddle point with one lens more. The curve in this figure is obtained with a CODEV macro that can be used for practical purposes.

In principle, the general version of SPC as described in Ref. 2 is sufficient for practical applications. However, below we discuss a technical improvement, called SPC-Intermediate, that in certain situations is simpler than the general version discussed in Ref. 2.

In the computation of $\partial MF / \partial c$ the existing equality constraints (other than those included as weighted operands in the merit function) must be taken into account. If such constraints are present, and they are already satisfied in the original minimum, they are also satisfied when a null element is inserted. However, when for the numerical computation of the derivative c is replaced by $c \pm \Delta c$, (where Δc is the derivative increment) these constraints are in general affected. Restoring the constraints is possible with a "constraints-only" reoptimization[2]. Paraxial equality constraints (e.g. constant total track or effective focal length) are especially important for this discussion. If such constraints are implemented as a "solve" they are satisfied automatically and do not require this extra step, but if a "solve" is not an option then they can be handled in a simpler way.

The meniscus must be inserted as in the case of SPC-Special (a reference surface is chosen in the original minimum, we have two consecutive zero thicknesses, for air and glass, and the meniscus glass is the same as for the reference surface) but we perform the scan as in SPC-General (therefore we call this version SPC-Intermediate).

Consider for instance the situation in Fig. 1b (with SPC-Intermediate, one of the zero crossings in scans like the one in Fig. 3 is always the SPC-Special solution). We have three surfaces in contact, and their total power is given by

$$K_{tot} = K_1 + K_2 + K_3 = (n-1)(-c_1 + c_2 - c_3) \tag{2}$$

where n is the glass refractive index, c are the curvatures and K the powers. The reference surface has index 1, and the two meniscus surfaces have indices 2 and 3. At the saddle point we always have $c_2 = c_3 = c$, but if for computing the derivative

$$\frac{\partial MF}{\partial c} \approx \frac{MF(c+\Delta c) - MF(c-\Delta c)}{2\Delta c} \tag{3}$$

we choose e.g. the 2nd curvature as variable, then during computation we temporarily have $c_2 = c \pm \Delta c$ and $c_3 = c$ which then violates the paraxial constraint. However, the violation of the paraxial constraint during the computation of the derivative can be avoided by performing at the same time a compensating change of the curvature of the reference surface $c_1 = c \pm \Delta c$ that restores K_{tot}. If K_{tot} is kept constant , then all paraxial ray paths, at all surfaces other than these three are left unchanged, and the paraxial constraint is unaffected.

For practical purposes, SPC can be used in two ways. The number of lenses can be increased step-by step by inserting null elements at different positions in a known local minimum (see Fig. 1a). If Eq. (1) is satisfied, we obtain a saddle point that leads to two local minima (blue and black dashed arrows in Fig. 4), and we can choose for further design the better of them. Alternatively, we can switch between local minima that are linked to the same saddle point by first removing a lens (red dashed arrow in Fig. 4, in the mountain landscape analogy at the beginning of this section, the tourist climbs from the first valley to the mountain pass) and then by descending on the opposite side of the saddle to the new local minimum (black dashed arrow).

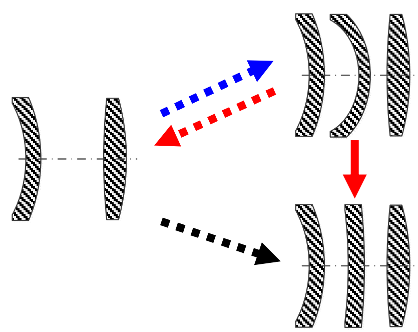

Figure 4. A saddle point obtained with SPC leads after optimization to two minima situated on opposite sides of the saddle. SPC can be done also in reverse (red dashed arrow) by removing a lens, to climb from a local minimum to a saddle point. After descending on the other side of the saddle, the net result is a switch between two minima having the same number of lenses (red continuous arrow).

3. DECOMPOSING THE HIGH-DIMENSIONAL SEARCH FOR NEW LOCAL MINIMA IN ONE-DIMENSIONAL SEARCHES

Rather than limiting the discussion to local minima, for practical purposes it can be useful to consider what we have called earlier "system shapes" [5] A system shape is either an isolated local minimum for which the merit function increases significantly in its immediate neighbourhood, or a set of local minima situated in a flat region of the design landscape. Different local minima that have the same system shape have similar lens drawings, have about the same value of the merit function and are separated by low merit function barriers. Since the performance is similar, any system in the corresponding flat region may be equally satisfactory (or not) for practical applications. If commuting between different minima with the same shape is desired, the low merit function barrier between them can be overcome in different ways, other than SPC.

As shown above, SPC, that is based on one-dimensional searches similar to the one in Fig.3, can be used to obtain in a fully deterministic way different system shapes, either from systems with less lenses, or by switching between system shapes with the same number of lenses. Important questions are then, how many system shapes existing in the landscape can be obtained in this way, by replacing general high-dimensional searches by one-dimensional searches? What percentage of them? Can we at least obtain the shapes that correspond to good solutions? Unfortunately, it is not possible to answer these questions by analysing in detail the design landscape of very complex systems. However, success in the study of design landscapes that are simple enough to be studied in detail gives confidence in the potential of this idea and can lead to clearly defined design strategies. The ultimate confirmation (or invalidation) will then be given by the quality of the practical results obtained with SPC.

We have studied earlier[5] a somewhat idealized monochromatic triplet landscape (which is closely related to the Cooke triplet landscape, but can be studied theoretically more easily) having 22 system shapes. It turned out that all system shapes could be obtained with SPC from 5 doublet minima, which could be all obtained with SPC from only one singlet. Also, any of the 22 triplet system shapes could be obtained from any other one by a finite number of SPC switching steps. Results described in Ref. 5 also strongly suggest that the same properties could also be valid for a Cooke triplet landscape (having curvatures as variables and aperture, field and wavelength specifications typical for a Cooke triplet) for which we have found 16 system shapes (the research is still in progress and the results will be described in detail later). A semi-empirical model derived from third-order thin-lens aberration theory for spherical aberration[6] provides a theoretical explanation in the special case of the idealized monochromatic triplet landscape of the properties mentioned above[7].

In order to see whether this ability of SPC to find many new system shapes can also be observed in problems that are very different from the special case mentioned above, we have first examined the landscape of a wide-angle pinhole lens with a full field of 110 degrees[8]. With other methods (see below) we find three stable solutions (i.e. three system shapes, each consisting of one minimum). We have found that with SPC it is possible to rapidly switch between any of the three solutions, just by inserting and removing lenses with the recipe discussed in the previous section. If optimization started arbitrarily reaches one of the two sub-optimal minima, switching between minima with SPC as shown in Fig. 4 can easily lead to the best of the three minima.

However, since the above landscape had only three minima, we decreased the full field to 60 degrees in order to investigate the performance of SPC when more minima are present in the landscape. The 60-degree minima are triplets with an aperture stop that, as in its 110-degree precursor, is situated in front of the lens and has a very small diameter of only 0.8 mm. While the wide-angle pinhole lens design problem was polychromatic and used more than one glass, for simplicity the 60-degree problem is made monochromatic, uses only one glass, and all three lenses have the same thickness. Local optimization is done with CODEV, and the merit function is the default CODEV error function based on transverse aberrations. We used the lens curvatures as variables, with a solve on the curvature of the last surface to keep the effective focal length equal to 3.5mm. As in the 110-degree case, all lenses are in contact. For the purpose of this study we have disabled the control of edge thickness; the small edge thickness violations observed in some minima can be easily corrected, e.g. by a small increase of axial air separation between lenses. An extensive analysis of the landscape with two different methods (Global Synthesis of CODEV and NETMIN[3], our own software for detecting saddle points without any a-priori knowledge, that for local optimization calls CODEV) has found 10 local minima.

It turns out that all these 10 triplet minima can be obtained rapidly with SPC by starting from only one lens. In the entire procedure, the aperture, field and effective focal length are unchanged, the values being those mentioned above, and in Figures 5 and 6 all lenses have the same thickness. We start with the optimized singlet SG-M in Fig. 5. With SPC we obtain from it two saddle points. After optimization and increasing the thickness of the new lens, both of them lead on one side to the doublet DB-M1. On the other side, the two saddle points lead to DB-M2 and DB-M3, respectively.

The three doublets DB-M1, DB-M2 and DB-M3 have a total of six lenses. We use each of the six lenses to generate a SPC scan that will lead to triplet minima. For each of the six scans, we first double the thickness of the corresponding lens, re-optimize the doublet, and insert an air meniscus at half of the thickness of the chosen doublet lens. We obtain in this way a triplet in which all three lenses have equal thickness and are in contact. In Fig. 6, if we ignore in the saddle-point drawings (the systems SP_X with x=1..11, except SP_{12}) the zero-thickness air meniscus lens of the corresponding scan, the drawings are identical with the six doublet minima resulting from DB-M1, DB-M2 and DB-M3, in which either the first or the second lens has double thickness. For instance, in Scan 1 the two saddle points SP_3 and SP_4 are constructed from a modified version of DB-M1 in which the first lens thickness has been doubled. Both saddle points have an error function value equal to that of the starting doublet, because the air meniscus after the first lens is a null element that has no effect on the ray paths (and can be removed without affecting them).

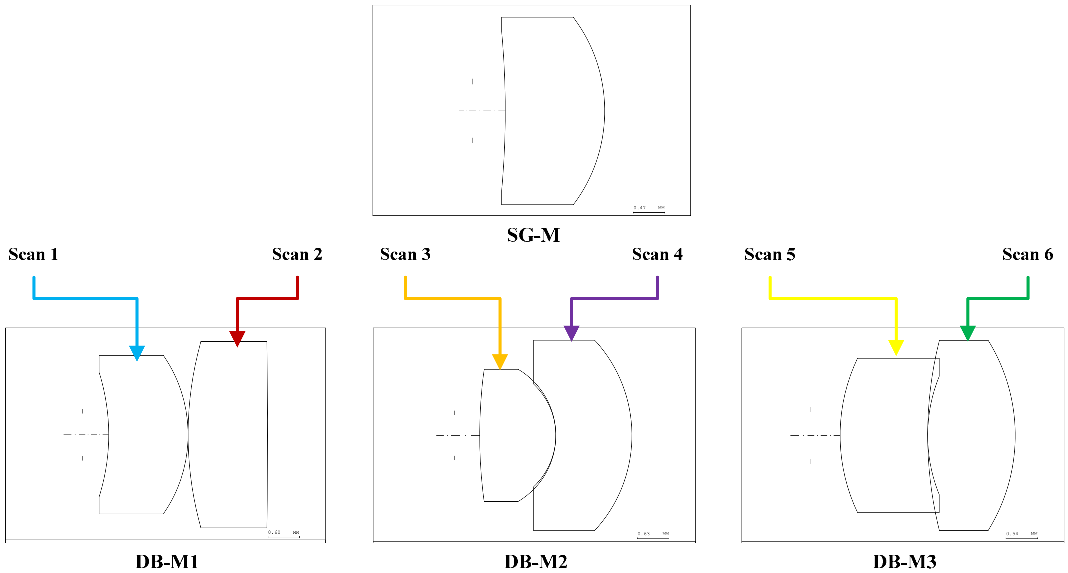

Figure 5. Singlet minimum and three doublet minima obtained from it with SPC. All 10 triplet minima of the 60-degree problem can be obtained with SPC from the three doublets. The coloured arrows indicate the lenses that are used in the scans in Fig. 6.

For each of the six scans we then solve numerically Eq. (1) in order to find the meniscus curvature values that transform the system into a saddle point. The number of solutions differs for different scans. For instance, Scan 2 has four solutions (i.e. the scan curve similar to the one in Fig. 3 has four zero crossings, rather than two, as in Fig. 3), whereas other scans have just one solution. The four saddle points in Scan 2 are SP_1, SP_2, SP_7 and SP_8. It turns out that, with one exception, any two neighboring zero crossings (i.e. saddle points) in each of the six scan curves lead after optimization to a common minimum. Therefore, apart from the one exception, each scan (marked in Figs. 5 and 6 with its own color) leads to a chain of local minima (solid boxes in Fig. 6) alternating with saddle points (dashed boxes).

The exception occurs because of the following reason. When a lens is inserted in an existing minimum (in this case in a doublet in Fig.5) somewhere where the system does not "need" it (i.e. where the new lens is useless and cannot reduce the merit function value) then the decrease of the merit function between a saddle point and the two neighboring local minima is negligibly small. For this reason, the entire chain resulting from Scan 2 (4 saddle points with exactly the same CODEV error function value of 61.8 and 5 minima, the lowest one having an error function of 60.7) leads to a "trench"-like region in the design space where all systems have almost the same value of the merit function. If an optimization starting at a saddle point with a much higher value of the merit function leads to a minimum in the trench, minor changes of the position of the starting point (e.g. the distance to the saddle point) can influence the outcome. Optimizations that are shown in Fig. 6 to lead e.g. to M_1 can therefore easily reach other minima of Scan 2 (M_2, M_3, M_7 or M_8) instead.

The six scans lead to a total number of 11 saddle points (black dashed boxes in Fig. 6). Because of the zero thicknesses between lenses, these 11 saddle points are identical with those found (with a much higher computational effort) with our program NETMIN that detects saddle points without any a-priori knowledge. (NETMIN also detects a 12th saddle-point, SP_{12}, that is not found by SPC, but since there is sufficient redundancy in this network, the lack of ability of SPC to find this saddle point is not critical.) The 11 saddle points found by SPC lead, when optimized on both sides of the saddle, to all 10 local minima detected with other methods (NETMIN and Global Synthesis). The most important conclusion of the 60-degree study is that all 10 minima found by other methods in a 5-dimensional design space can be found systematically and efficiently with a succession of (6) one-dimensional searches.

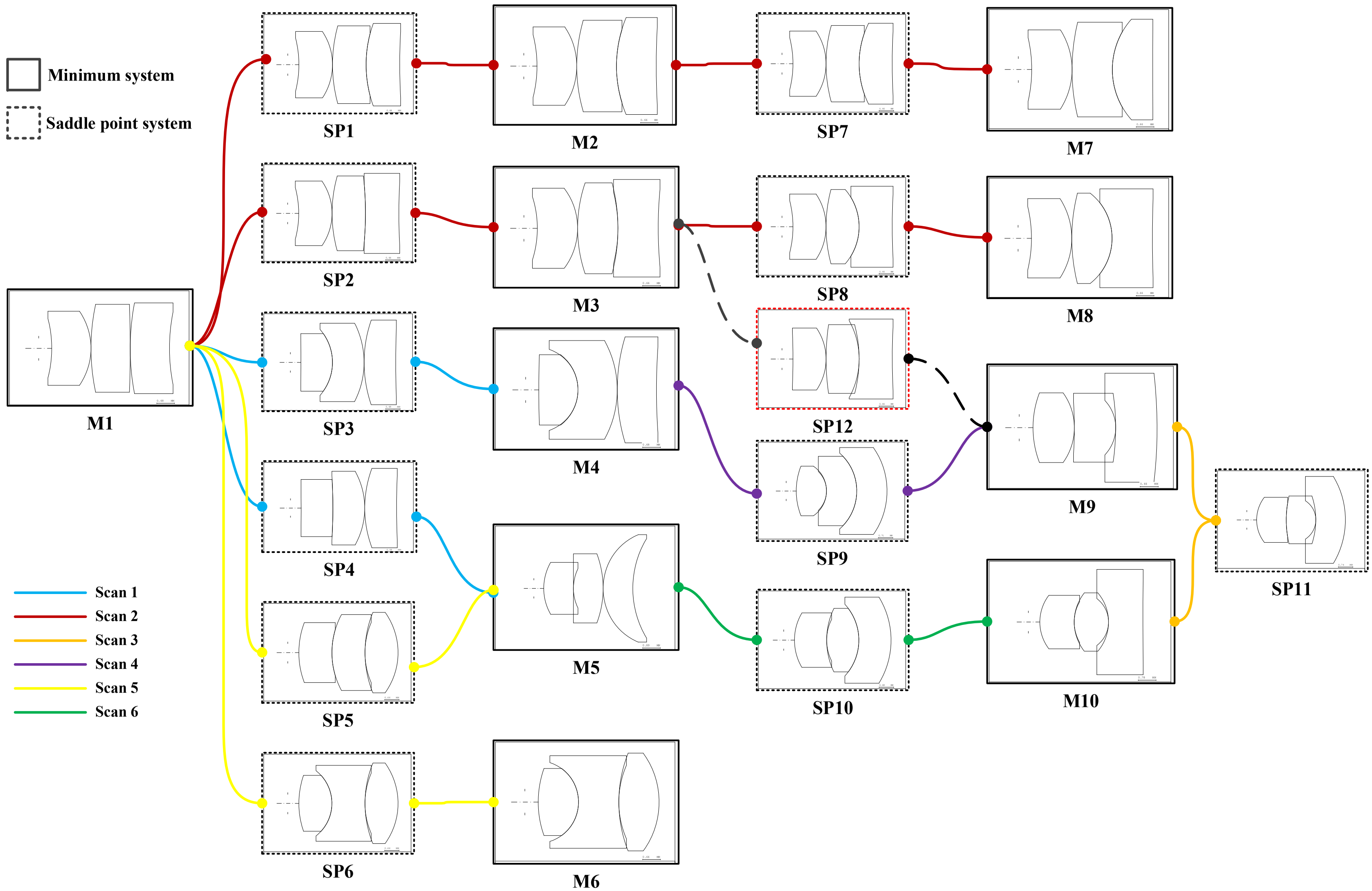

Figure 6. 10 local minima (larger boxes) and 12 saddle points (smaller dashed boxes) in the monochromatic 60 degree triplet landscape. From each saddle point, optimization can roll down on two sides and reach two minima, as shown by the links in the network. 11 saddle points (black dashed boxes) can be obtained with SPC from the three doublets in Fig. 5. The saddle point in the red box cannot be found with SPC, but since there is sufficient redundancy in the network, the lack of ability of SPC to find the red system is not critical. This saddle point and its two dashed links to the neighbouring minima have been obtained with NETMIN.

CONCLUSIONS

Figure 6 illustrates a remarkable structure of the design landscape, the close relationship that can exist between local minima of an optimization problem and local minima with one lens less. The systems SP_1 –SP_{11} are triplet saddle points, but if a pair of surfaces that has no influence at all on the rays or on the merit function (the air null element of the corresponding scan) is removed, they become doublet minima. However, if these doublet minima with an extra null element are optimized on both sides of the saddle (created by the null element), they lead to the triplet minima M_1-M_{10}. For an idealized problem, a first step towards a theoretical model helps understanding why this relationship is valid for all minima that were detected in the landscape of the given problem[7].

Because many local minima in the lens design landscape have such close relationships to local minima of simpler problems, saddle-point construction can be used to find new system shapes by performing a succession of one-dimensional searches. In several simple design landscapes SPC was able to find all, or the vast majority of the existing system shapes. The reduction of complexity resulting from the replacement of high-dimensional searches by one-dimensional searches is essential. In high-dimensional searches the computation time increases rapidly with the number of variables. In contrast, with the present approach we made progress towards an algorithm where the computation time increases much slower.

Although in more complex problems we may encounter design solutions that are not reachable with SPC, the present results increase our confidence that SPC can find important new design solutions and can become a successful practical design tool. High-quality designs have already been obtained with the special[9,10] and with the general version[8] of SPC. SPC will be finally validated if many high-quality systems designs like these will be obtained in this way.

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