Capacity Assessment of a Cellular Radio System Using a Narrowband Multiuser Detector

Gerard J. M. Janssen, *Member, IEEE,* and Jens Zander, *Member, IEEE*

***Abstract*—With a narrowband multiuser detector (MUD), multiple users can simultaneously share the same channel without using a bandwidth-expanding signature code when sufficient power differences at the receiver are maintained. In this paper, we assess the capacity gain that can be achieved in a hypothetical cellular system with centralized power control, using a narrowband MUD based on successive cancellation. The power-control algorithm is adapted to multiple users per channel in a cell. Simulation results are presented for two heuristic carrier-to-interference ratio-based channel-assignment schemes. The MUD allows for a substantial capacity gain due to a tighter packing of users in a channel. The observed gain compared with a conventional single user-per-channel system, ranges from 1.4 to 5 for cluster sizes $C = 1$ and 7, respectively.**

***Index Terms*—Blocking probability, capacity, cellular radio system, multiuser detection (MUD), transmitter power control.**

I. Introduction

THE VAST increasing number of users in mobile communication systems and the increasing demand for more capacity per user requires exploitation of the available frequency spectrum as efficiently as possible. Cochannel interference is one of the main limiting factors in achieving a high system capacity. A user perceives a good link quality when the carrier-to-interference + noise ratio (CIR) is larger than a required *protection ratio* γ_0. In current second-generation cellular networks, frequency reuse in the spatial domain is used as a means to increase capacity. Cells using the same frequencies are separated sufficiently apart to limit cochannel interference. In the third-generation mobile communication systems, code-division multiple access (CDMA) is applied to increase spectral efficiency by allowing a smaller reuse factor and since it has the ability to be more self organizing. In CDMA, each user signal is coded with a user signature which makes it possible to separate and detect users which occupy simultaneously the same frequency band. This allows frequency reuse in the same cell but at the cost of a large increase of the required bandwidth; i.e., the number of users that can simultaneously operate in this frequency band is substantially less than the processing gain used.

Recently, multiuser detection (MUD) techniques have been proposed for narrowband signals based either on maximum likelihood detection [1] , [2] or on successive cancellation [3], which is less complex to implement. By narrowband, we refer to the fact that no spreading gain or bandwidth-expanding signature code is applied to separate the users like in CDMA. In [4], a narrowband MUD for multiple phase-shift keyed (M-PSK) modulated signals is proposed, which achieves good detection performance for each signal when geometrically related signal power differences at the receiver input are maintained. These techniques, in principle, allow for in-cell channel reuse, i.e., the assignment of multiple signals to the same channel within a cell, at the cost of maintaining sufficient power differences between the signals at the receiver. Here, the inherent link gain differences between users can be exploited: by assigning users with sufficiently different link gains to the same channel, the total transmitted power is only marginally larger than the transmitted power of the signal with the smallest link gain.

In this paper, we asses the system capacity that can be achieved when using narrowband MUD in a hypothetical cellular system with centralized constrained power control [6]. The system capacity is defined here as the average number of users that can be supported by a cell normalized to the number of channels in the system. Power control is an effective method to control cochannel interference in order to achieve a high system capacity as well as a high link quality [6]–[8]. Centralized power control, however, requires that all link gains in the system are known, which is not a realistic assumption for implementation in a practical system, but it results in the optimum transmitted power levels which maximize the minimum CIR in the system [7], [9]. The power differences between the received cochannel signals of a cell that are required for a proper operation of the MUD compensate for the mutual interference these signals cause to each other. It is shown that, provided that the relative power difference between the cochannel signals in a cell are maintained, the centralized power control for multiple users per channel can be limited to controlling the powers of the users with the minor received signal level in each cell, and it results in equal CIR values (CIR-balancing) for these signals. The corresponding power-control algorithm is an adapted version of the one presented in [9]. Since the signal powers of the other in-cell users are directly related to the minor signal power, they can be controlled in a distributed way within each cell.

Channel assignment is the method to allocate a channel to a user in order to maximize system capacity [10]–[12]. We apply two heuristic channel-assignment schemes based on CIR, which aim at maximizing the probability of a fast user assignment and maximizing the number of users in a channel, respectively.

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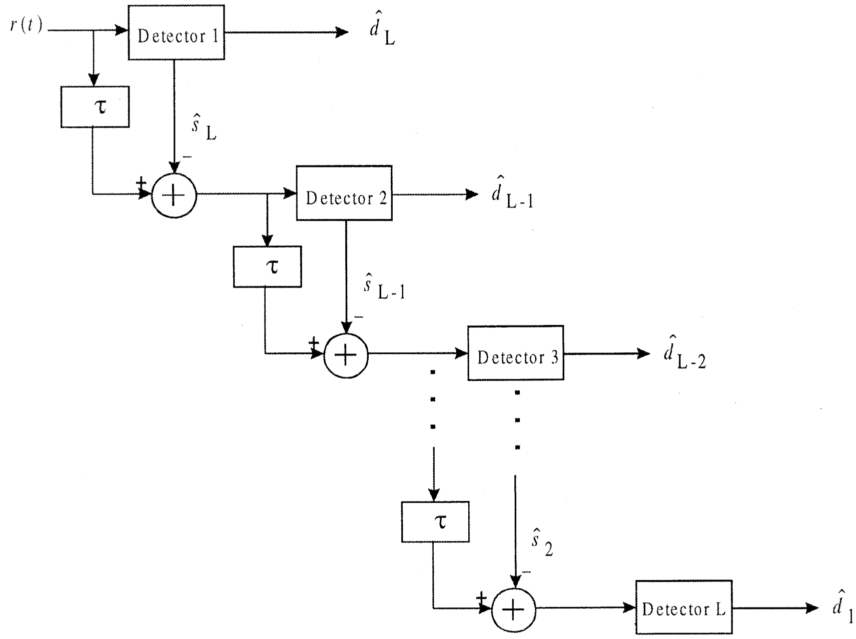

Fig. 1. Principle of the MUD using successive detection and subtraction.

Section II gives a review of the narrowband MUD based on successive cancellation. The system model is described in Section III. The power-control algorithm and channel-allocation schemes are presented in Section IV. In Section V, capacity results obtained from Monte Carlo simulation using Poisson distributed traffic, are compared for a maximum of one, two, and four users per channel. The paper is concluded with a discussion on the implications of the obtained results.

II. Review Narrowband MUD

The narrowband MUD for multiple M-PSK modulated signals, which was originally proposed in [4], is based on successive signal detection, estimation, and cancellation. The structure of the detector is shown in Fig. 1. The major signal is detected and estimated in the first detector of the MUD; this estimate is subtracted from the total input signal. From the remaining signal, subsequently, the next largest signal is detected, estimated, and subtracted, and so on. This principle is known as "onion peeling" [13]. To guarantee a good detection performance for all users, sufficient power differences between the received signals have to be maintained, e.g., by means of a closed-loop power control using a feedback channel from the base station to the mobile users.

Let us consider the uplink of a system with $L = Q_{\max}$ active symbol-synchronous but mutually noncoherent users, all using the same carrier frequency and M-PSK modulation. Perfect parameter estimation is assumed for all signals. Now consider the detection of signal s_q, assuming correct decisions for the first $L - q$ (largest) signals. In that case, after phase compensation the received sample at the input of the detector (see Fig. 1) can be written as

$$y_{n,q} = A_q e^{j(a_{n,q}(2\pi/M))} + \sum_{i=1}^{q-1} A_i e^{j\phi_{n,i}} + N_n \qquad (1)$$

where A_q is the signal amplitude of user q, $a_{n,q} = 0, 1, \ldots, (M-1)$ with equal probability of occurrence and is related to the nth transmitted symbol of user q, $\phi_{n,i} = \theta_i + a_{n,i}(2\pi/M) - \theta_q$ is the total phase of signal $i < q$ during the symbol interval nT_s, and N_n is the sampled additive white Gaussian noise (AWGN). The signal amplitudes are related as $A_1 < A_2 < \cdots < A_L$ and the mutually independent carrier phases θ_i are uniformly distributed over $[0, 2\pi)$. Now ignoring noise, the interference term in the above expression will cause signal-state regions centered around the major signal's states. With q signals left to detect, these regions are annular shaped with an inner circle radius $R_{q,\min} = \max\{A_{q-1} - \sum_{i=1}^{q-2} A_i, 0\}$ and an outer circle radius $R_{q,\max} = \sum_{i=1}^{q-1} A_i$. These annular regions are disjoint if $A_q \sin(\pi/M) - R_{q,\max} > 0$, which is required for successful detection of the symbols of signal s_q. In Fig. 2, the signal-state regions are shown for four independent 4-PSK modulated signals where the receiver is locked to the major one. The task of the first receiver is now to detect the state region belonging to the correct symbol value of the current major signal s_q. After correct cancellation of this signal, the remaining signals will form "clouds" around the next major signal's states, and so on. A symbol error results in partial cancellation or even enhancement of the detected signal, preventing correct detection of all smaller signals. Thus, a symbol error is likely to propagate to all smaller signals. Accurate parameter estimation is an important issue for the MUD based on successive cancellation. In the case of parameter estimation errors, signal cancellation is not perfect, which results in residual interference in the following detection stages. The degrading effect on the symbol error probability (SEP) of inaccurate parameter estimation using training sequences, has been analyzed in [5]. There, it is shown that the degradation caused by estimation errors in the noisy channel is rather small and can be reduced by using longer training sequences.

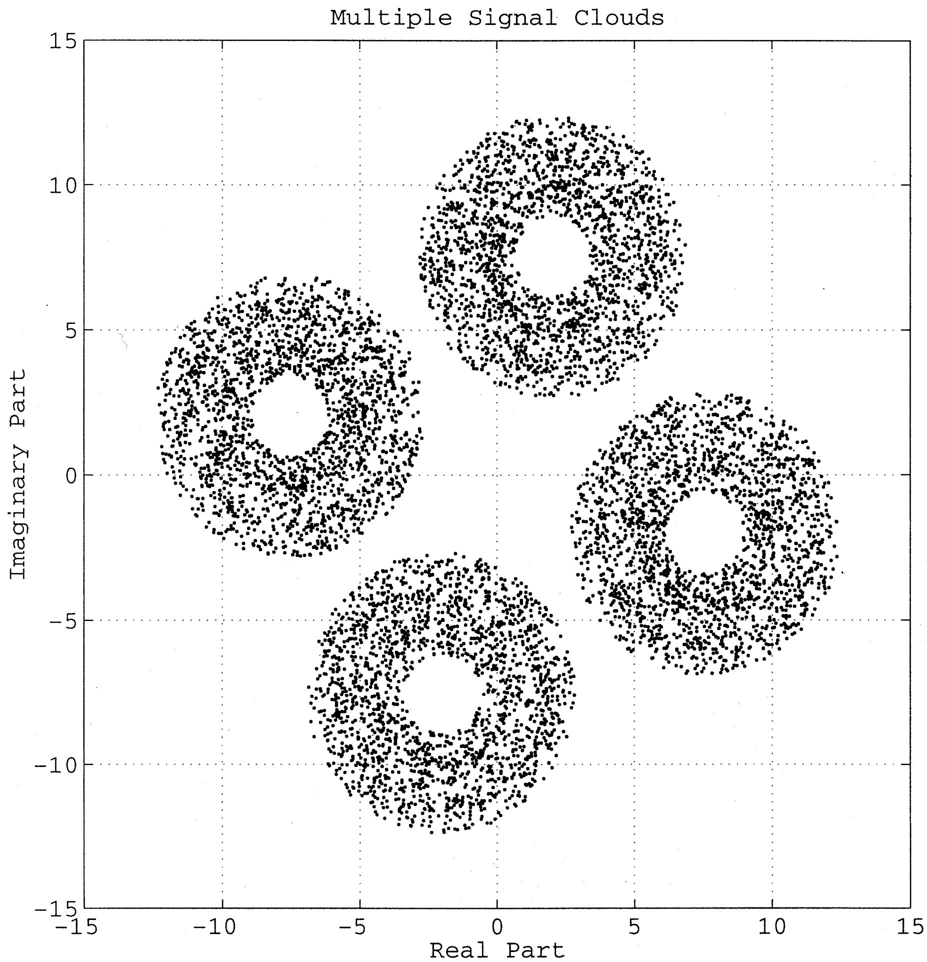

Fig. 2. Annular signal-state regions centered around the dominant signal's states for quadrature-phase-shift keying (QPSK) modulation with $A_k = (2.5)^{k-1}$, $k = 1, \ldots, 4$, and $\theta_4 = -\pi/6$, for many events of the mutually independent signals $s_1 \cdots s_3$.

In [4] and [5], the SEP of the MUD is analyzed. There, it is shown that for signals with geometrically related received amplitudes $A_q = \alpha A_{q-1}$, the minimum distance between the annular regions around two neighboring signal states becomes independent of the signal index q for $\alpha_0 = 1 + 1/\sin(\pi/M)$. This implies for the received power of signal s_q, $P_{rx,\,q} = \beta P_{rx,\,q-1}$ with the power ratio $\beta = \alpha^2$. Note that the minimum distance between two neighboring signal states of an M-PSK signal decreases with the modulation level M, therefore, α_0 increases with increasing M. For $\beta \geq \alpha_0^2$, in principle, an arbitrary number of signals can be stacked, where the minimum distance increases with q for $\beta > \alpha_0^2$. In practice, however, where signal parameter estimation is inaccurate, the number of stacked signals will be limited. In this paper, we assume 4-PSK modulation with $M = 4$ and $\beta = (1 + \sqrt{2})^2 \approx 5.83$ which corresponds to a power ratio of 7.7 dB. If multiple antennas are used, the power ratio β can be substantially reduced [2], [3].

In Fig. 3, the SEPs for each of four 4-PSK signals are shown as a function of E_{s1}/N_0 in the AWGN channel for $\beta = \alpha_0^2$ and taking into account error propagation. The symbol energy of the smallest signal is defined as $E_{s1} \triangleq A_1^2 T_s$. It is observed that the geometrical power ratio does not result in equal performance for all signals, since the larger signals have a substantially lower SEP than the smaller ones. Results on optimum power ratios for equal SEP performance for all signals were presented in [14].

Reception of a compound signal consisting of a number of mutually noncoherent PSK signals is less efficient with respect to receiver input carrier-to-noise ratio (CNR) when compared with that required by a quadrature amplitude modulation scheme with the same bandwidth efficiency. However, in an interference-limited cellular environment, the large variations in link gain can now be further exploited in the following way. A user with a low attenuation can operate at low power in the same channel together with a high-power signal which suffers a high attenuation; however, the low-power signal is received as the major signal at the cost of very low additional interference to the system, thus increasing spectral efficiency.

III. System Model

We study the performance of the narrowband MUD in a large, but finite, cellular radio system which consists of D cells in a hexagonal grid structure. The *total number of channel pairs* available is E, each consisting of an independent up- and down-link channel (implemented as a time or a frequency slot) and we assume cochannel interference and noise to be the only restrictions on the use of the channels. The *reuse factor* or *cluster size* C indicates the number of different groups

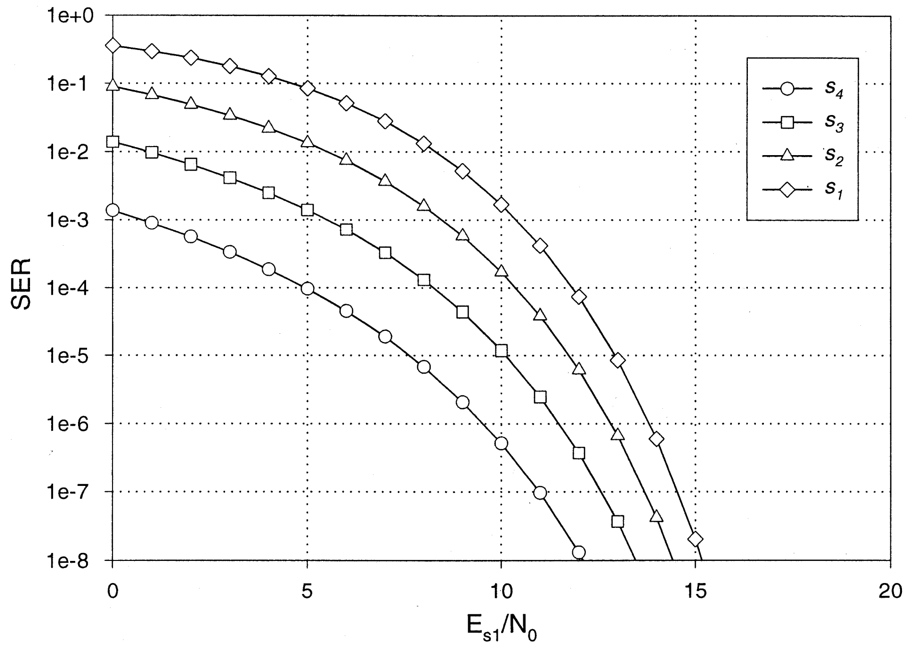

Fig. 3. SEP results for each of the $L = Q_{\max} = 4$ users in the AWGN channel with QPSK modulation and $\beta = 7.7$ dB.

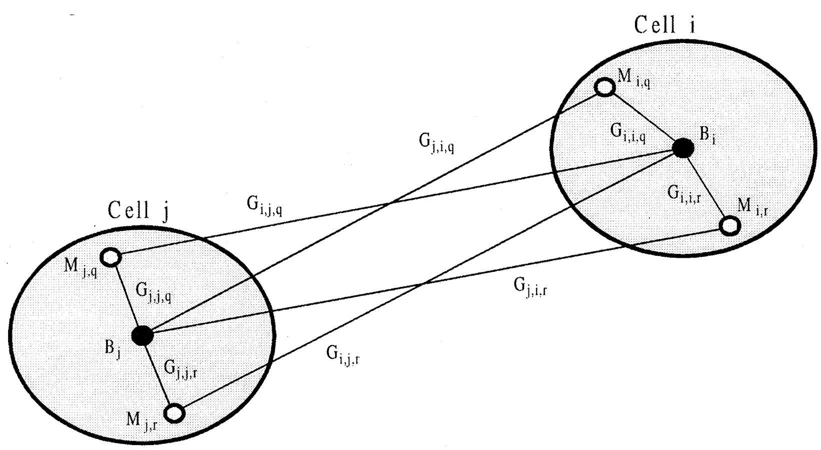

Fig. 4. Link geometry and link gain model for two cochannel cells.

of channel pairs. For a reuse factor C, there are $K = E/C$ channel pairs available per cell and the cochannel set, i.e., the set of cells using the same channels, has size $J = D/C$. Cells that use the same channel are placed symmetrically in the hexagonal grid. The in-cell reuse factor Q_j^k indicates the number of users assigned to channel pair k in cell j. This number is upper-bounded by $Q_{\max}$ the maximum number of signals that can be detected simultaneously by the MUD. In the following, the reference to the channel pair k will be omitted, when we consider an unspecified cochannel set. Here, we study the up-link channels only, where mobile $M_{i,q}(q = 1 \cdots Q_i)$ transmits to base station B_i of cell i, and the received signals are mutually noncoherent. In order to limit model complexity, the users are assumed to be connected to the nearest base station (instead of the base station with the best link gain, as in a practical system), and do not move; i.e., they remain in the same location during a connection.

A. Channel Model

In the radio channel, the signal is attenuated. The *link gain* which is the inverse of the attenuation between user q in cell j and base station B_i of cell i is indicated by $G_{i,j,q}$, as shown in Fig. 4. The link gain is modeled as

$$G_{i,j,q} = \frac{S_{i,j,q}}{d_{i,j,q}^{\delta}} \tag{2}$$

where $S_{i,j,q}$ takes into account the log-normal shadow fading with mean value 0 dB and standard deviation σ dB, $d_{i,j,q}$ is the distance between mobile q in cell j and base station B_i, and δ is the path-loss exponent. For urban environments, a value of σ in the range of 6–10 dB and a value of δ in the range of 3–5 dB provide realistic propagation models. The fast Rayleigh or Rice fading caused by multipath propagation in general cannot be tracked by a power-control scheme and, therefore, is not taken into account here. The degradation caused by fast fading is assumed to be compensated for by application of coding and interleaving or other diversity techniques. The gain matrix $\mathbf{G} = [G_{i,\,j,\,q}]$ is a three-dimensional $J \times J \times Q_{\max}$ matrix. If user q in cell j is not active, the column $\mathbf{G}(:,\, j,\, q) = \mathbf{0}$.

B. Traffic Model and Performance Parameters

In the traffic model, we assume that new users arrive in a cell at arbitrary time instants with exponentially distributed interarrival times and connection durations. The expected values of the interarrival time and connection duration are $1/\lambda$ and $1/\mu$ (unit of time), respectively. The traffic load per cell is taken equal for each cell of the system and is given by $\rho_t = \lambda/\mu$. For increasing traffic load, the probability that a user is blocked, i.e., cannot be supported by the system, will increase. Two types of blocking are identified. *Load-blocking* occurs when we try to assign a new user in cell j, but all K channels are fully loaded at the time of arrival, i.e., all channels in the cell contain $Q_{\max}$ users. In a cochannel set of J cells, the probability of load-blocking as a function of the traffic load $P_{\text{load-block}}(\rho_t)$ is defined as

$$P_{\text{load-block}}(\rho_t) \triangleq \frac{1}{J}\sum_{j=1}^{J} \Pr(Q_j^k = Q_{\max}\ \forall k \in \{1, \ldots, K\} | \rho_t). \quad (3)$$

CIR-blocking occurs when we try to assign a new user to a channel of cell j with $Q_j < Q_{\max}$ active users, but the CIR related to the minor signal, as defined in Section IV, drops below the protection ratio γ_0 in all available channels. This implies that CIR-blocking is a conditional event; i.e., it will only occur under the condition of no load-blocking. In a cochannel set of J cells, this conditional probability of CIR-blocking as a function of the traffic load $P_{\text{CIR-block}}(\rho_t)$, is defined as

$$P_{\text{CIR-block}}(\rho_t) \triangleq \frac{1}{J}\sum_{j=1}^{J} \Pr(\text{CIR}_j^{k'} < \gamma_0\ \forall k' \in \{1, \ldots, K\} | \rho_t) \quad (4)$$

where $\text{CIR}_j^{k'}$ is the CIR in channel k' of cell j, and k' indicates the available channels for which $Q_j^{k'} < Q_{\max}$. Since load-blocking and CIR-blocking are disjoint events, the total blocking probability is given by

$$\begin{aligned} P_{\text{block}} &= P_{\text{load-block}} + [1 - P_{\text{load-block}}]P_{\text{CIR-block}} \\ &\simeq P_{\text{load-block}} + P_{\text{CIR-block}} \end{aligned} \quad (5)$$

where the approximation holds for small values of $P_{\text{load-block}}$.

In practical systems, a small blocking probability is acceptable for the users. As a measure for the system performance, we use the system capacity ρ_c, which is defined as the average traffic load per cell normalized to the total number of channels in the system, at a blocking probability $P_{\text{block}} = 0.05$

$$\rho_c \triangleq \frac{\rho_t(P_{\text{block}} = 0.05)}{E} \quad (6)$$

where $\rho_t(P_{\text{block}} = 0.05)$ indicates the cell traffic load at 5% blocking probability.

IV. Power Control and Channel Assignment

The quality of a radio link is determined by its CIR. Based on intercell interference, the CIR $\Gamma_{i,\,q}$ of user q in cell i with transmission power $P_{i,\,q}$ is given by

$$\Gamma_{i,\,q} = \frac{P_{rx,\,i,\,q}}{\sum\limits_{\substack{j=1\\ j\neq i}}^{J} I_{i,\,j} + \eta_i} \quad (7)$$

where $P_{rx,\,i,\,q} = P_{i,\,q}G_{i,\,i,\,q}$ is the received power, $I_{i,\,j}$ is the total interference received from cochannel cell j and η_i is the received noise power at base station B_i. In order to minimize the interference generated by a cell, the assigned value of q at a particular frequency in a cell increases with increasing link gain; i.e., the minor received user has the smallest link gain. Note that for $q \neq 1$, the effective CIR is less than $\Gamma_{i,\,q}$ due to the presence of smaller not-yet detected intracell cochannel signals.

The received signal powers of users occupying the same channel in a cell are ordered as $P_{rx,\,i,\,Q_{\max}} > P_{rx,\,i,\,Q_{\max}-1} > \cdots > P_{rx,\,i,\,1}$. When these powers are geometrically related, and the proper ratio β between the received signals is maintained, all signals can be detected by the MUD. The power ratio β guarantees a sufficient distance between the cochannel signal states for a good detection performance of all Q_i users if the CIR for the minor received user $(q = 1)$ is higher than the protection ratio γ_0. If not, the minor signal will suffer first, and probably only for $\beta > \alpha_0^2$. Therefore, the interference generated by the cochannel signals of cell i is not accounted for as interference in this cell. In the Appendix, the expression of the CIR of the minor signal $\Gamma_{i,\,1}$ is derived as

$$\Gamma_{i,\,1} = \frac{P_{i,\,1}}{\sum\limits_{j=1}^{J} P_{j,\,1}\overline{A_{i,\,j}} + \eta_i'} \quad (8)$$

where

$$\eta_i' = \frac{\eta_i}{G_{i,i,1}} \quad (9)$$

is the normalized received noise power. The variable $\overline{A_{i,\,j}}$, as given by (25) in the Appendix, is the equivalent link gain normalized to $G_{i,\,i,\,1}$ for the total interference from cell j to cell i, with respect to the minor signal transmit power $P_{j,\,1}$; i.e., $P_{j,\,1}\overline{A_{i,\,j}}$ is related to the total interference at base station B_i caused by all signals in cell j. Unless otherwise stated, CIR refers to the CIR of the minor received signal in the following.

A. Power Control

By applying transmitter power control (TPC), a significant increase of the system capacity is achieved, as shown in [7]–[9]. TPC aims at providing all active cochannel signals in a cellular system with a CIR $\Gamma \geq \gamma_0$ at minimum interference level. Since it is sufficient in the proposed multiuser scenario to make sure that user signal $q = 1$, which is received with the lowest power, has a CIR value $\Gamma_{i,\,1} \geq \gamma_0$, power control can be based on the CIR of the minor signal in each of the cochannel cells. In addition, the power ratio β between the received signals $q =$

$1, \ldots, Q_i$ has to be maintained. In order to satisfy the protection ratio γ_0 on every link, i.e., $\Gamma_{i,1} \geq \gamma_0$ for every i, the system of equations can be written in matrix notation as

$$\overline{\mathbf{A}}\mathbf{P}_1 + \mathbf{N}' \leq \frac{\mathbf{P}_1}{\boldsymbol{\gamma}_0} \tag{10}$$

where matrix $\overline{\mathbf{A}}$ contains the elements $\overline{A_{i,j}}$, the power vector $\mathbf{P}_1 = [P_{1,1}\ P_{2,1}\ \cdots\ P_{J,1}]^{\mathrm{T}}$ contains the transmit powers of the minor received signals of the cochannel set, and the noise vector $\mathbf{N}' = [\eta_1'\ \eta_2'\ \cdots\ \eta_J']^{\mathrm{T}}$. In (10), note that the inequality is interpreted as applying component by component. The $\overline{\mathbf{A}}$ matrix is comparable to the $\mathbf{A}$ matrix as defined in [9] for a single user-per-channel system. By putting $\mathbf{N}' = \mathbf{0}$, we get for the noiseless case $\gamma_0\overline{\mathbf{A}}\mathbf{P}_1 \leq \mathbf{P}_1$ from which we can determine the maximum achievable CIR $\gamma^* = \gamma_0/\mu^*$ where μ^* is the Perron–Frobenius eigenvalue of matrix $\overline{\mathbf{A}}$, as shown in [7] and [9]. The positive eigenvector $\mathbf{P}^*$ belonging to the eigenvalue μ^* is the power vector which results in identical CIR $\Gamma_{j,1} = \gamma^*$ in every cochannel cell, thus CIR-balancing is achieved for all minor received signals irrespective of the number of users Q_j in a cell. Only for an eigenvalue $\mu^* \leq 1$, a valid CIR $\geq \gamma_0$ is obtained for all minor signals in the cochannel set. The probability of obtaining a $\mu^* \leq 1$ decreases when the other-cell interference increases either by increasing the number of signals Q_j assigned to a channel or by decreasing the cluster size C.

In a more realistic scenario, noise is present at the receiver and, in addition the transmit power of a terminal, is constrained to $P_{\max}$, resulting in constrained power control [6]. In order to achieve the required CIR, (10) must hold. From this equation, we can solve for the transmit power vector $\mathbf{P}_1$ of the minor received signals as

$$\mathbf{P}_1 = \left(\frac{\mathbf{I}}{\boldsymbol{\gamma}_0} - \overline{\mathbf{A}}\right)^{-1}\mathbf{N}' \tag{11}$$

where $\mathbf{I}$ is the identity matrix. Only for a positive power vector $\mathbf{P}_1$ with $0 < P_{j,1} \leq P_{\max}$ for every j, the target CIR $\Gamma_{i,1} = \gamma_0$ is obtained for all minor signals in the cochannel cells. When the target CIR cannot be realized, some elements of the calculated power vector $\mathbf{P}_1$ will be negative. The transmit powers of the other in-cell cochannel signals is found from $P_{j,1}$, using (17) of the Appendix.

B. Channel Assignment

Dynamic channel assignment (DCA) is usually applied in a situation where, in principle, all channels are available in each cell (cluster size $C = 1$). Different heuristic DCA schemes based on CIR values have been proposed [10], [11]. Apart from the large potential capacity of the system with a small cluster size, also the self-organizing property of such a system is attractive [12].

In a conventional system, the probability of load-blocking increases with cluster size, whereas CIR-blocking becomes more important for small reuse values. In the system proposed here, the probability of load-blocking is reduced because more users can be assigned to the same channel, but at the cost of more interference. We apply CIR based DCA to achieve efficient combinations of users in the same channel. Two assignment schemes are evaluated.

In Assignment Scheme 1 (AS1), assignment of a new user is attempted in the channel with the highest initial CIR (before assignment of that user) and which has less than $Q_{\max}$ active users. The assignment is successful if the resulting CIR $\geq \gamma_0$, if not then assignment in the channel with the next highest CIR is tried. In this scheme, the probability of a fast assignment (low delay) of a user is maximized. This scheme is slightly different from the scheme proposed by [10], where the CIR after assignment is maximized, which requires trials in all available channels before the assignment can be made.

In Assignment Scheme 2 (AS2), assignment of a new user is attempted in the channel with the lowest initial CIR and which has less than $Q_{\max}$ active users. If the assignment is not successful, then assignment in the channel with the next lowest CIR is tried. A user who will cause little interference is assigned to a channel which already has a low CIR $> \gamma_0$, preserving the channels with high CIR for users who cause more interference. This assignment scheme aims at a tight packing of users in a channel. It is different from the scheme proposed by [11] where all channels are tried and the one with the lowest CIR $\geq \gamma_0$ after assignment is selected, which results in a tighter packing. The AS2 scheme will not always find the lowest CIR $\geq \gamma_0$ and, therefore, not the tightest packing of users, but it results in a faster assignment since the user is assigned to the first channel with CIR $\geq \gamma_0$ after assignment. In a loaded system, AS2 will result in CIR values which are more likely to be close to the protection ratio γ_0 than those obtained with AS1.

Both assignment schemes are centralized, since all link gains are required to estimate the CIR after assignment of a new user and, therefore, are not suitable for direct implementation in a practical system.

V. Simulation Results

The gain in cellular capacity that can be achieved by applying the narrowband MUD has been assessed using Monte Carlo simulations based on the system model, as described in Section III.

A. Simulation Parameters

In the simulated system, the cochannel set has a fixed size $J = 19$, thus the total area covered by the system decreases with decreasing cluster size. The number of available channels $E = 21$, therefore, the number of available channels per cell $K = 3$, 7, and 21, for reuse factor $C = 7$, 3, and 1, respectively. This scenario takes into account the trunking gain that is achieved with more channels available per cell. Depending on the cluster size, between 150 and 600 users are generated in each channel for each value of the traffic load per cell ρ_t. The path-loss exponent and the standard deviation of the shadow fading are taken $\delta = 4$ and $\sigma = 8$ dB, respectively. The protection ratio is set to $\gamma_0 = 12$ dB. Power control is applied with a constraint on the maximum transmit power using (11). The normalized noise power at the base stations are set to $\eta = 10^{-4}$ and $P_{\max} = 1$. This results in an outage probability due to noise $\Pr(\mathrm{CNR} < \gamma_0) = 2 \cdot 10^{-3}$ for a user at the border of a cell with a normalized radius $R = 1$. The power constraint is checked only for the minor signals with $q = 1$. Usually the minor received signals transmit with the largest power, and will

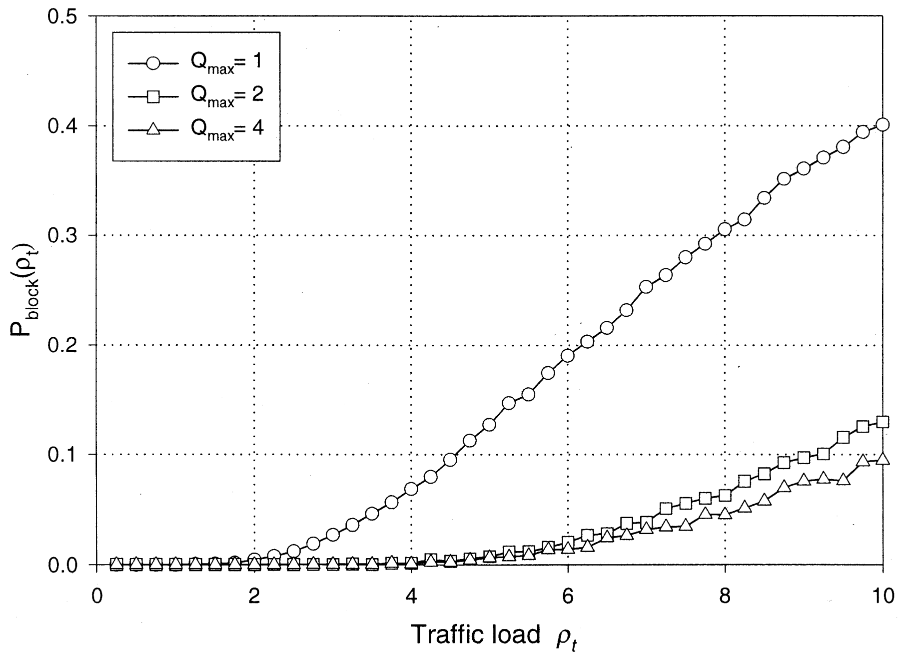

Fig. 5. Total blocking probability for AS1 as a function of ρ_t for $C = 3$, $K = 7$, and $Q_{\max} = 1, 2$, and 4, respectively.

be blocked if $P_{i,1} > P_{\max}$. However, occasionally it will occur that $\beta^{q-1} > G_{j,j,q}/G_{j,j,1}$ for a user $q > 1$, in which case its transmission power $P_{i,q} > P_{i,1}$. In the (rare) cases where $P_{i,q} > P_{\max}$ with $q > 1$, the newly added user is not blocked if CIR $\geq \gamma_0$. The transmission powers are updated at every instant a user enters or leaves the system.

B. Results

Unless indicated otherwise, channel-assignment scheme AS1 is used. It is found that a substantially higher traffic load can be supported when $Q_{\max}$ is larger than 1, as shown in Fig. 5, which gives the total blocking probability $P_{\text{block}} \simeq P_{\text{load\_block}} + P_{\text{CIR\_block}}$ as a function of the cell traffic load ρ_t for cluster size $C = 3$ and different values of $Q_{\max}$. This gain is obtained because the probability of load-blocking, which is dominating the total blocking probability for $Q_{\max} = 1$ for all cluster sizes, significantly decreases when extra users can be added to the same channel. This becomes obvious from Fig. 6, which shows the separate probabilities of load-blocking and CIR-blocking for $Q_{\max} = 1$ and 4. The lower load-blocking probability by increasing $Q_{\max}$, however, is achieved at the cost of increased CIR-blocking probability. The largest gain occurs by increasing $Q_{\max}$ from 1 to 2; a further increase of $Q_{\max}$ results in only marginal improvement, because now CIR-blocking prevents the addition of significantly more users in the same channel.

In Table I, the system capacity ρ_c as defined in Section III for $P_{\text{block}} = 0.05$ is given for different cluster sizes C and different $Q_{\max}$. The *multiuser (detector) gain* G_{MUD}, which reflects the capacity gain due to the MUD, is defined as

$$G_{\text{MUD}} \triangleq \frac{\rho_c(Q_{\max} = 4)}{\rho_c(Q_{\max} = 1)}. \tag{12}$$

In the table, the observed gain with decreasing cluster size (in downward direction) is multiplexing gain obtained by increasing the number of channels K available in a cell. The gain obtained by increasing $Q_{\max}$ for a fixed cluster size C (from left to right) is multiuser gain. For a large cluster size, a high multiuser gain is obtained ($G_{\text{MUD}} = 4.8$ for $C = 7$) because significantly more users can be assigned to the same channel, and so decreasing the load-blocking probability, before CIR-blocking becomes the limiting factor. For small cluster sizes, where CIR-blocking is the main limiting factor, these large gains cannot be achieved. In this case, the trunking gain dominates the multiuser gain. However, the overall multiuser gain is still 30% for reuse $C = 1$ and AS1.

In assignment scheme AS2, we try to assign a new user to the channel with the lowest initial CIR instead of the highest initial CIR, as in AS1; i.e., a user with a good channel is added to the first minimum CIR channel where it fits in, leaving channels with a higher CIR free for future users with a bad channel and which require more power. By applying AS2, we achieve a tighter packing of the signals in the same channel. Fig. 7 shows the load- and CIR-blocking probabilities with AS2 for $C = 1$. In Table II, the system capacity ρ_c is given for different cluster sizes and different $Q_{\max}$ for AS2. Here, a substantial capacity increase is observed in situations with a high CIR-blocking probability. The decrease in CIR-blocking at high traffic load is clearly illustrated when we compare the CIR-blocking for $Q_{\max} = 4$ in Fig. 7 with those of AS1 in Fig. 6 for $C = 1$ at high load values. Using AS2, the multiuser gain for $C = 1$ and $Q_{\max} = 4$ is 44% versus 30% with AS1. For low CIR-blocking situations, either because the load is low or the reuse is high, the gain difference between AS1 and AS2 is not so large.

From these results, we can conclude that application of the narrowband MUD would allow for a substantial increase of capacity in existing cellular systems ($Q_{\max} = 1$) with moderate and large reuse factors. In these systems, which are mainly load-blocking limited, there is substantial opportunity to exploit

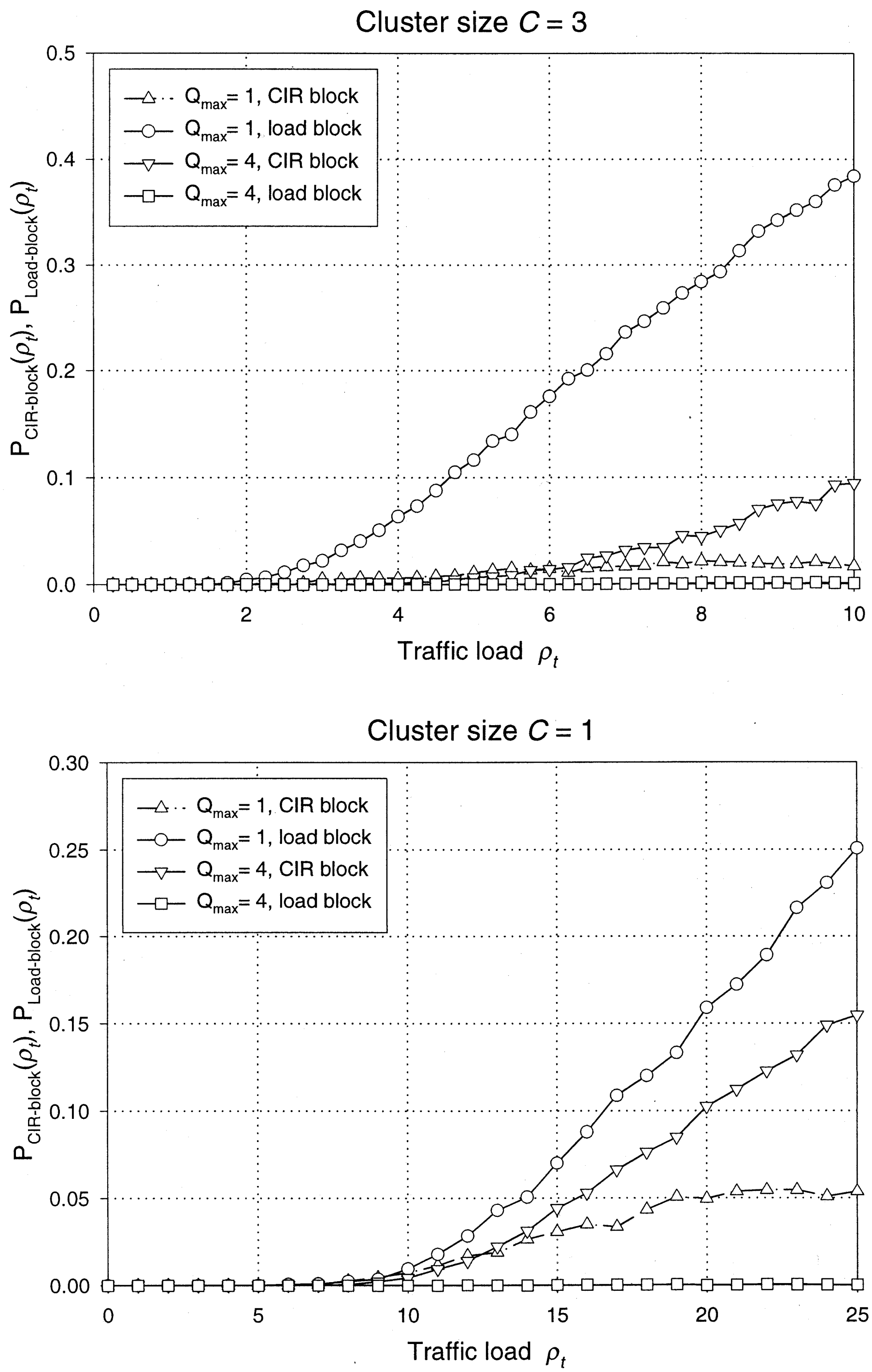

Fig. 6. Load- and CIR-blocking probabilities for AS1 as a function of ρ_t for $C = 3$ and $C = 1$, with $K = 7$ and 21, and $Q_{\max} = 1$ and 4, respectively.

TABLE I
SYSTEM CAPACITY ρ_c AND MULTIUSER GAIN AT $P_{\text{block}} = 0.05$ FOR AS1

C	$Q_{\max}$			G_{MUD}
	1	2	4	
7	0.043	0.138	0.205	4.78
3	0.167	0.345	0.393	2.36
1	0.581	0.738	0.752	1.30

the fact that many users have a rather large CIR. A second important conclusion is that for small reuse factors ($C = 1, 3$), the implementation complexity of the MUD is limited since near maximum system capacity can be obtained with $Q_{\max} = 2$. For larger reuse factors, it may be beneficial to support larger values of $Q_{\max}$.

VI. DISCUSSION

In this paper, we have assessed the capacity gain that can be achieved in the up-link of a hypothetical cellular system by applying a narrowband MUD and centralized constraint power control. The power-control scheme for multiple users

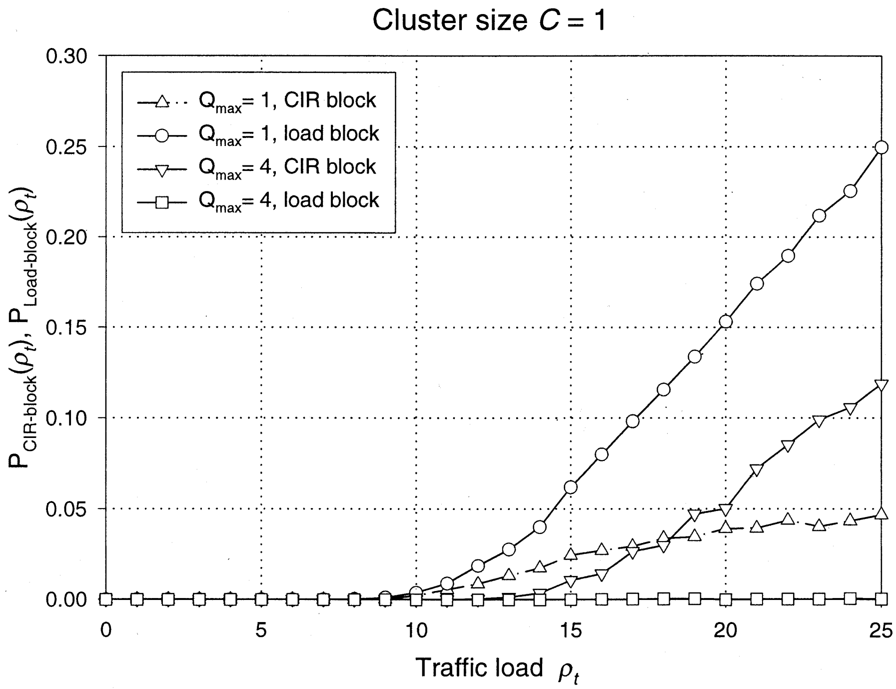

Fig. 7. Load- and CIR-blocking probabilities for AS2 as a function of ρ_t for $C = 1$, with $K = 21$ and $Q_{\max} = 1$ and 4, respectively.

TABLE II
SYSTEM CAPACITY ρ_c AND MULTIUSER GAIN AT $P_{\text{block}} = 0.05$ FOR AS2

C	$Q_{\max}$			G_{MUD}
	1	2	4	
7	0.044	0.137	0.219	5.00
3	0.171	0.381	0.438	2.56
1	0.643	0.879	0.924	1.44

per channel is derived and it is shown to have the same mathematical form as for the single user-per-channel case. Since the signal powers of the users operating in the same channel of a cell are related to each other and maintained within the cell, the intercell power control operates on the minor received signals only.

Simulation results show that a substantial increase of the capacity of existing cellular systems ($Q_{\max} = 1$) is feasible by applying the narrowband MUD. The multiuser gain is especially large at large cluster size ($G_{\text{MUD}} = 5$ at $C = 7$) which is mainly due to the decreasing probability of load-blocking when more users can be assigned to the same channel, at the cost of an increasing probability of CIR-blocking. For small cluster sizes, the MUD still allows a tighter packing of users in the same channel. The largest capacity gain is achieved by increasing $Q_{\max}$ from 1 to 2. For small reuse factor ($C = 1$, 3), choosing $Q_{\max} > 2$ results in only marginal additional gain. This is interesting from an implementation point of view, since near-maximum capacity can be reached with an MUD of limited complexity. For larger reuse factors, a larger capacity gain can be achieved by supporting larger values of $Q_{\max}$. The achievable system capacity depends on the choice of the assignment scheme. The results of two assignment schemes are compared. Assignment to the channel which has initially the lowest CIR (AS2) results in a larger capacity due to a tighter packing of users, compared with assigning the signal to the channel with the highest initial CIR (AS1) which causes a higher blocking probability for future users with a small link gain. The differences between the two schemes becomes apparent when CIR-blocking is high. The achieved multiuser gain is 1.30 with AS1 and 1.44 with AS2 for a cluster size $C = 1$ and $Q_{\max} = 4$.

APPENDIX
DERIVATION OF THE CIR

The CIR is a measure for the quality of a radio link. In the following, we derive the CIR of the signals operating in the same channel of a cochannel set. The CIR of user q in cell i with received power $P_{rx,\,i,\,q}$, based on intercell interference, is given by

$$\Gamma_{i,\,q} = \frac{P_{rx,\,i,\,q}}{\sum_{\substack{j=1 \\ j \neq i}}^{J} I_{i,\,j} + \eta_i} \tag{13}$$

where $I_{i,\,j}$ is the total received interference of cell j and η_i is the received noise power at base station B_i. Let $P_{i,\,q}$ be the power transmitted by user q in cell i, then the received power $P_{rx,\,i,\,q}$ from this signal at base station B_i is

$$P_{rx,\,i,\,q} = P_{i,\,q} G_{i,\,i,\,q}. \tag{14}$$

Note that for $q \neq 1$, the effective CIR will be less than $\Gamma_{i,\,q}$ due to the presence of smaller not-yet detected signals occupying the same channel in a cell. Since the received signal powers of these intracell cochannel users are ordered as $P_{rx,\,i,\,Q_{\max}} > P_{rx,\,i,\,Q_{\max}-1} > \cdots > P_{rx,\,i,\,1}$, and the powers are geometrically related. $P_{rx,\,i,\,q} = \beta P_{rx,\,i,\,q-1} (\beta > 1)$, the received power of user q, can be written as

$$P_{rx,\,i,\,q} = \beta^{q-1} P_{rx,\,i,\,1} \tag{15}$$

and the total received power of the desired users in cell i is equal to

$$P_{rx,i} = \sum_{q=1}^{Q_i} P_{rx,i,q} = \frac{\beta^{Q_i} - 1}{\beta - 1} P_{rx,i,1}. \tag{16}$$

Using (14) and (15), the received power of user q can also be related to the transmitted power of the minor user $q = 1$ as $P_{rx,i,q} = \beta^{q-1} P_{i,1} G_{i,i,1}$, and the transmitted powers of the users in a cell can be expressed as

$$P_{i,q} = \beta^{q-1} \frac{G_{i,i,1}}{G_{i,i,q}} P_{i,1} = \frac{\beta^{q-1} P_{i,1}}{Z_{i,i,q}} \tag{17}$$

where $Z_{i,j,q}$ is the link gain normalized to $G_{i,i,1}$, given by

$$Z_{i,j,q} = \frac{G_{i,j,q}}{G_{i,i,1}}. \tag{18}$$

In a similar way, we find the interference power $I_{i,j,q}$ received by base station B_i from user q of cell j operating at a power level $P_{j,q}$ as $I_{i,j,q} = P_{j,q} G_{i,j,q}$. The total interference power at base station B_i caused by all users operating in cell j is given by

$$I_{i,j} = \sum_{q=1}^{Q_j} P_{j,q} G_{i,j,q}. \tag{19}$$

Now we can express the CIR $\Gamma_{i,q}$ for user q at the base station of cell i as

$$\Gamma_{i,q} = \frac{P_{i,q} G_{i,i,q}}{\sum_{\substack{j=1 \\ j \neq i}}^{J} \sum_{q=1}^{Q_j} P_{j,q} G_{i,j,q} + \eta_i}. \tag{20}$$

Realizing that the degrading effect on the detection performance caused by cochannel users in cell i is compensated for by the power ratio β, this implies that we only need to control the transmit powers $P_{i,1}$ of the minor received signals to achieve a CIR value $\Gamma_{i,1} \geq \gamma_0$. Within a cell the powers of the other users are related to this power according to expression (17). By applying (16) and (17), the CIR value $\Gamma_{i,1}$ for the minor received signal in cell i can be expressed as

$$\begin{aligned} \Gamma_{i,1} &= \frac{P_{i,1} G_{i,i,1}}{\sum_{\substack{j=1 \\ j \neq i}}^{J} \sum_{q=1}^{Q_j} P_{j,q} G_{i,j,q} + \eta_i} \\ &= \frac{P_{i,1} G_{i,i,1}}{\sum_{\substack{j=1 \\ j \neq i}}^{J} \sum_{q=1}^{Q_j} P_{j,1} \beta^{q-1} \frac{G_{j,j,1}}{G_{j,j,q}} G_{i,j,q} + \eta_i} \\ &= \frac{P_{i,1}}{\sum_{\substack{j=1 \\ j \neq i}}^{J} P_{j,1} \sum_{q=1}^{Q_j} \beta^{q-1} \frac{Z_{i,j,q}}{Z_{j,j,q}} + \eta_i'} \\ &= \frac{P_{i,1}}{\sum_{\substack{j=1 \\ j \neq i}}^{J} P_{j,1} \overline{Z_{i,j}} + \eta_i'} \end{aligned} \tag{21}$$

where

$$\eta_i' = \frac{\eta_i}{G_{i,i,1}} \tag{22}$$

is the normalized receiver noise power. The variable $\overline{Z_{i,j}}$ is the equivalent link gain normalized to $G_{i,i,1}$ for the total received interference power from cell j at the base station B_i of cell i with respect to the minor signal power $P_{j,1}$; i.e., $P_{j,1}\overline{Z_{i,j}}$ represents the total received power from the signals in cell j. $\overline{Z_{i,j}}$ is defined as

$$\overline{Z_{i,j}} \triangleq \sum_{q=1}^{Q_j} \beta^{q-1} \frac{Z_{i,j,q}}{Z_{j,j,q}}. \tag{23}$$

Using the fact that the transmitted signals in cell i do not contribute to the interference in this cell, the expression for $\Gamma_{i,1}$ can be further simplified as follows:

$$\Gamma_{i,1} = \frac{P_{i,1}}{\sum_{j=1}^{J} P_{j,1} \overline{A_{i,j}} + \eta_i'} \tag{24}$$

where $\overline{A_{i,j}}$ is defined as

$$\overline{A_{i,j}} = \begin{cases} \overline{Z_{i,j}}, & \text{if } i \neq j \\ 0, & \text{if } i = j \end{cases}. \tag{25}$$

The elements $\overline{A_{i,j}}$ form the $\overline{\mathbf{A}}$ matrix, which becomes equal to the $\mathbf{A}$ matrix, as defined in [9], for a single user per channel system.

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