APPLICATIONS OF LUMINOPHORES IN SANDTRANSPORT-STUDIES

PROEFSCHRIFT
TER VERKRIJGING VAN DE GRAAD VAN DOCTOR IN DE TECHNISCHE WETENSCHAPPEN AAN DE TECHNISCHE HOGESCHOOL TE DELFT OP GEZAG VAN DE RECTOR MAGNIFICUS IR. H. J. DE WIJS, HOOGLERAAR IN DE AFDELING DER MIJNBOUWKUNDE, VOOR EEN COMMISSIE UIT DE SENAAT TE VERDEDIGEN OP WOENSDAG 25 MEI 1966 OM 16 UUR

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DIT PROEFSCHRIFT IS GOEDGEKEURD DOOR DE PROMOTOREN

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EN

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STELLINGEN

1.

Voor de bepaling van het zandtransport in open waterlopen met behulp van merkstoffen, kan thans nog niet algemeen worden aangegeven of radioactieve merkers of luminoforen moeten worden gebruikt.

2.

Bij zandtransport-metingen met merkstoffen voor rivieren zoals de Nederlandse bovenrivieren moeten luminoforen worden gebruikt.

3.

Bij de ontwikkeling van bodemtransportmeters moet uitdrukkelijk rekening worden gehouden met het stochastische karakter van het transportverschijnsel.

4.

De door Colby en Hembree gebruikte benaming „modified Einstein-procedure" werkt verwarrend.

Colby, B. R. and C. H. Hembree, „Computations of total sediment discharge in the Niobrara River". U.S. Dept. of the Interior. Water-Supply-paper No. 1357 U.S. Geol. Survey 1955.

5.

Bij praktische berekeningen met betrekking tot niet-permanent bodemtransport in open waterlopen, is het gebruik van een pseudo-viscositeitsmethode sterk aan te bevelen.

6.

De invloed van regime-wijzigingen op de bodemligging van een rivierensysteem met vrijwel gefixeerde oevers, kan worden bestudeerd met een combinatie van een wiskundig- en een fysisch model. Een opzet waarbij het hoofdaccent op het wiskundig model valt, is aan te bevelen.

7

De afleidingen met betrekking tot de bedvormende afvoer van een rivier, zoals deze voorkomen in het Nedeco-rapport over de Niger en de Benue, kunnen beduidend worden vereenvoudigd door niet a priori uit te gaan van de vergelijking van MEYER-PETER en MUELLER.

Nedeco, „River Studies and recommendations on the improvement of Niger and Benue". North Holland Publishing Company, Amsterdam, 1959.

8.

De door VOLLMERS onderzochte oplossingen voor de aansluiting van een kanaal of een haven aan een bovenrivier, hebben voor Nederlandse omstandigheden vrijwel geen praktische betekenis .

VOLLMERS, H. J. „Systematik der Masznahmen zur Verringerung der Schwebstoffablagerungen in Binnenhafenmündungen". Proefschrift, Karlsruhe, 1963.

9.

Voor het ontwerpen van een optimaliseringstechniek voor waterbeheersingsplannen ten behoeve van waterschappen, kan de door het Waterloopkundig Laboratorium ontwikkelde methode voor de berekening van de waterbeweging in een systeem van open leidingen, als uitgangspunt dienen.

MEYER, TH. J. G. P., C. B. VREUGDENHIL and M. DE VRIES, „A method of computation for non-stationary flow in open-channel networks". I.A.H.R. Leningrad 1965, paper 3.28.

10.

Het is sterk aan te bevelen de student in de weg- en waterbouwkunde tijdens de propaedeuse te onderwijzen in de beginselen van het moderne rekenen.

CONTENTS

ABSTRACT

The quantitative determination of bed-material-transport by means of tracers has been considered. For a stationary-stochastical transport-condition the dispersion of tracers has been described by a diffusion-equation of the gradient-type.

The parameters of the dispersion-model are derived from the measured concentrations by a least-squares procedure. The measuring-technique has been based on the application of fluorescent tracers (luminophores). Some flume-tests show that the method proposed yields fair quantitative answers.

ACKNOWLEDGEMENTS

The author expresses his thanks to Ir. H. J. SCHOEMAKER, director of the Delft Hydraulics Laboratory for the stimulation of this research and the permission for the publication of the results.

The availability of the excellent computer-facilities of the University of Groningen for the numerical treatment of the measurements is also acknowledged.

CHAPTER 1

INTRODUCTION

The transport of sand by running water plays an important role in the design of many hydraulic structures. Problems in coastal-engineering and river-engineering are mostly closely related to the sandtransport phenomena.

Human interference in the character of rivers and coasts is often very expensive. This requires an optimal design i.e. the costs of the works and the maintenance-costs have to be balanced. The optimum design cannot be obtained if the sandtransport phenomena are only described qualitatively. For instance the costs of maintenance-dredging at a harbour-entrance can only be obtained via a quantitative prediction of the siltation at this entrance.

Studies of river-improvements by hydraulic models and/or calculations are important for the prediction of the characteristics of new situations. One of the bases of these studies is formed by the equation of motion for the solid phase. The tremendous amount of literature on these "sandtransport formulas" illustrates the economic importance of this topic. It also demonstrates the little progress that has been made in spite of a great effort. This is due to the complicated nature of the transport-process especially at the interface between the fluid and the solid phase.

The sandtransport formulas have a theoretical and an experimental background. Most experiments have been carried out in laboratory flumes. In that case it is possible to compare the theoretical transport with the actual one.

It can be stated that the measurements of sand-transport in natural channels form the crucial point of the whole set-up. Improvement of the measuring techniques may imply a big step forward. The measurements by mechanical means have only a restricted value, though improvement of these methods is not quite excluded.

In recent years tracer-techniques have been brought forward. Sand-grains marked with radioactive elements or with a fluorescent dye can be added to the transport-material. Detection of the dispersed tracers is possible. Both techniques especially the one with radioactive elements, have made considerable progress. However, only little progress has been obtained with the quantitative interpretation of these measurements. In fact the basic problem remains. For a quantitative interpretation of these tracer-measurements there is a need for a theoretical expression of the dispersion of tracers. However, instead of the true equation of motion for the sand, a more descriptive statistical form has to be found for the tracers. In its turn this expression needs an experimental check.

In this study the applicability of tracer-techniques for riverstudies is considered. Therefore not the evaluation of the equation of motion of the sand is the subject, but the deduction of an adequate measuring technique. This is only possible if the character of the transport-process is taken into consideration. For this reason chapter 2 gives some phenomenological considerations. It deals with the transport-phenomenon present in upper-rivers: a down stream net-movement of sand. A relatively simple measuring-technique is only possible if the average transport is constant during a certain time and in a certain area. For these restricted circumstances chapter 3 gives a theoretical expression for the dispersion of the tracers. The practical aspects of the tracer-techniques with fluorescent tracers (luminophores) are studied in chapter 4. Chapter 5 deals with the general techniques for the interpretation of the measurements. Special attention has been paid to the coupling of a theoretical dispersion-model as developed in the third chapter, and the data obtained by the techniques of chapter 4.

Chapter 6 describes some tracer-experiments. The different interpretation-techniques are applied and a first attempt has been made to study the validity of some theoretical dispersion-models.

CHAPTER 2

PHENOMENOLOGICAL CONSIDERATIONS

2.1 Introduction

The limits of the morphological interest in the transport of sand by running water can be demonstrated in a long straight laboratory-flume. The lower-limit is present if the water-current is hardly able to remove the particles placed on the fixed bottom. The correlation of this "beginning of motion" with the characteristics of the current and the grains is still a topic of intensive study.

The upper-limit is formed by the case in which the current is so strong, that grains added to the current have only a negligible probability to come temporarily at rest at the fixed bottom. With H. A. Einstein [7], this type of transport can be called "wash-load". In alluvial channels wash-load will occur for grains finer than those of the bed-material. Therefore wash-load has a secondary morphological meaning for rivers, as those fine particles may change the properties of the current and influence its transporting capacity.

Between the two given limits, grains may move in different ways. It is essential that the grains are subjected to changing forces and therefore rest-periods of the grains are interchanged with periods in which they have constantly changing velocities. Therefore the sand-movement can only be stationary-stochastical. Only if a relatively long period is concerned, it may be said that the average transported quantity of sand is constant.

Besides the movement of the grains, also the bed-form causes the stochastical character. Dependent on the hydraulic condition and the character of the bed-material, ripples and dunes are present. Pure bed-load transport consists of rolling or sliding grains. Most of them are hidden for a relatively long time at the lee-side of a ripple or dune.

Wash-load on the one hand and pure bed-load on the other hand are types of transport which can be considered to be border-cases. Between these two cases the transport of suspended-load is situated. Most of the time the suspended-load particles are supported by the turbulent fluid. There is a probability that they rest at the river-bed for a certain time. The different types of transport cannot be separated clearly as they are mostly present at the same time and there are gradual transitions between them.

Some aspects of the transport have to be discussed in detail in the next paragraphs. Paragraph 2.2 will deal with the stationary-stochastical movement in every location of the bed. It will be studied whether this movement can be homogeneous for a certain region of the river-bed. If both conditions are fulfilled the elements of the transport-phenomenon can have an ergodic character.

Paragraph 2.3 will deal with the non-steady bed-load transport. Here the movement is not stationary-stochastical.

2.2 Stationary-stochastical movement

An almost ideal stationary-stochastical movement of sand can be obtained in a laboratory. A flume can be supplied with a constant amount of water and sand. If at one point the water-level is kept constant, a stationary-stochastical movement homogeneous in time is obtained, providing the flume has been in operation for a long time.

Moreover the phenomenon is homogeneous along the flume except for the most up-stream and down-stream end. This means that through the unit of width the same average amount of sand is transported, although some wall-effects may be present.

This type of transport: homogeneous in time and place i.e. an ergodical process is an exception. The first deviation of this ideal situation is already present if this stationary-stochastical movement is not obtained in a straight laboratory-flume but in a river-model. Grain-sorting is present in the river-bends: the transport is not homogeneous for the location. This rather complicated phenomenon has been demonstrated by special tests [28].

If a river model with constant width is considered, one may take measurements at locations x_i along a line parallel to the banks. Let those measurements be the local depth h_i, the average velocity v_i in the vertical, and the mean particle size d_i of the bottom material. Each set of h_i, v_i and d_i has to be considered as estimates of the local values $\bar{h}_i$, $\bar{v}_i$, $\bar{d}_i$ of the stationary-stochastical movement. This is due to the presence of ripples and dunes. Therefore a statistical treatment of h_i, v_i and d_i is necessary to come to the conclusions about the behaviour along the river-bed.

Obviously, phase-lags are present between each two of the three elements h, v and d. These phase-lags can be demonstrated in a round-about way via the correlation-coefficient.

Suppose the measurements are taken at locations with constant intervals Δx. The average phase-lag between h and d is supposed to be $k\Delta x$. This means that h_i is related to d_{i+k}.

The elements h and d have a correlation-coefficient r_k with

$$r_k = \frac{\sum_{i=1}^{n-k} (h_i - \bar{h}_k)(d_{i+k} - \bar{d}_k)}{\sqrt{\sum_{i=1}^{n-k} (h_i - \bar{h}_k)^2 \sum_{i=k+1}^{n} (d_i - \bar{d}_k)^2}} \quad \ldots\ldots\ldots\ldots \quad (2.1)$$

in which n denotes the total number of measurements

$$\bar{h}_k = \frac{1}{n-k} \sum_{i=1}^{n-k} h_i \quad \ldots\ldots\ldots\ldots \quad (2\text{-}2)$$

and

$$\bar{d}_k = \frac{1}{n-k} \sum_{i=k+1}^{n} d_i \quad \ldots\ldots\ldots\ldots \quad (2\text{-}3)$$

As a result $r_k = F(k)$ is obtained, in which k has positive values. Also phase-lags in the opposite sense may be postulated by interchanging h and d in eq. (2-1). This treatment can be given to two of the three elements h, v and d, to study the reaction of those quantities on the geometry of river-bends.

Experiments have been carried out for some successive bends of a river-model, to demonstrate the existence of those phase-lags. Two series of measurements, h_i v_i and d_i have been taken. One series at $^1/_4$ of the width from the left bank, the other at $^1/_4$ of the width from the right bank. The measurements were taken at regular intervals $\Delta x \triangleq 250$ m along the river. The results of the calculations are presented in figure 2.1. The correlation-coefficient r_k for the combination h and v shows a maximum at $k = 1^1/_2$. This indicates that the velocity reacts on the presence of the bend more downstream than the water-depth.

The large negative values of r_k are present if k is supposed to be so large, that the depth in an outer-bend is combined with the velocity at the up- or down-stream inner-bend. Larger values of r_k are again present if h and v with a distance of one bend are combined.

The results of those measurements show that for the bends under consideration the velocity reacts as an average at a distance $\triangleq$ $^3/_8$ km more down-stream than the waterdepth. The grain-size reacts another $^1/_8$ km more down-stream.

These phase-lags are also to be expected in natural rivers i.e. in cases for which the stationary-stochastical movement cannot be postulated directly. Though less significant, also in this case the grain-size seems to react more down-stream than the water-depth [28].

These results, though rather giving tendencies than exact figures, may lead to the following conclusions:

1. In a curved alluvial channel phase-lags exist between the time-averages of depth, velocity and grain-size.

2. The transport condition for a stationary-stochastical movement can only be homogeneous for the location if a restricted area is considered. This implies that ergodicity can likewise only be present in a restricted area.

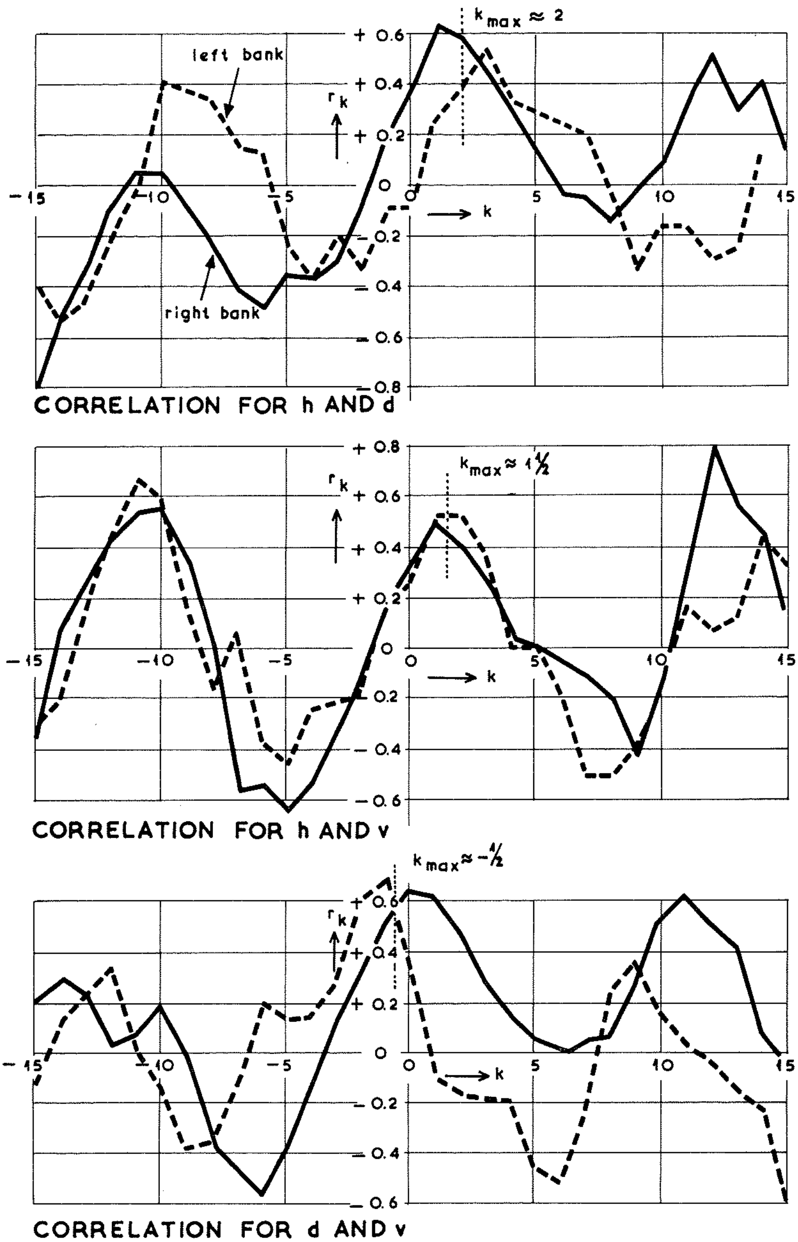

Fig. 2.1 Phase-lags between h, v and d in a river-model.

Besides, the following remark can be made. Due to phase-lags, calculation of the amount of transport from the hydraulic data will contain some error, if those calculations are based on the average values of h, v and d for a whole reach of a river.

After the above given considerations one can try to define the transport-vector $\mathbf{T}$ more specifically for the general case. If x and y are the directions in the plane of the main current and z the direction perpendicular to the x-y-plane, one may define the local temporary grain-velocity $\mathbf{u}(x,y,z,t)$ and the sand-flux.

$$\mathbf{w}(x,y,z,t) = b(x,y,z,t)\, \mathbf{u}(x,y,z,t) \qquad (2\text{-}4)$$

in which b denotes the sand-content by volume.

If ε_0 denotes the porosity of the sand of the bed then $0 < b < 1-\varepsilon_0$. The transport-vector can now be defined. Different definitions are being used. In all cases the transport is defined as a quantity per unit of width and time. Here we choose for the quantity: the total volume of sand and pores after settling. This has the advantage that this quantity is of direct importance for morphological processes.

Therefore

$$\mathbf{T} = (1-\varepsilon_0)^{-1} \int_{-\infty}^{+\infty} \mathbf{w} dz = E \int_{-\infty}^{+\infty} \mathbf{w} dz \qquad (2\text{-}5)$$

The connection with the sometimes used quantity: real mass-rate ($\mathbf{q}_s$) is given by

$$\mathbf{q}_s = \varrho_s (1-\varepsilon_0) \mathbf{T} \qquad (2\text{-}6)$$

Due to the given definition of $\mathbf{T}$ there is a direct relation to the bed-profile η (c.f. eq. 2-18).

For a stationary-stochastical movement the time-average of the transport-vector $\mathbf{T}$ is only dependent on the location (x,y), while moreover the time-average of the component in the z-direction becomes zero. Therefore in this case

$$\left.\begin{aligned}
\bar{T}_1 &= E\theta^{-1} \int_0^{\theta} \int_{-\infty}^{+\infty} w_1(x,y,z,t)\, dz dt = \bar{T}_1(x,y) \\
\bar{T}_2 &= E\theta^{-1} \int_0^{\theta} \int_{-\infty}^{+\infty} w_2(x,y,z,t)\, dz dt = \bar{T}_2(x,y) \\
\bar{T}_3 &= E\theta^{-1} \int_0^{\theta} \int_{-\infty}^{+\infty} w_3(x,y,z,t)\, dz dt = 0
\end{aligned}\right\} \qquad (2\text{-}7)$$

Similar definitions can be given for $\bar{\mathbf{w}}$

This case is present in a river with a stationary-stochastical movement.

The next simplification is obtained for a stationary-stochastical movement in a straight laboratory flume. There the average transport is only present in the x-direction:

$$\left.\begin{array}{l} \bar{T}_1 = \bar{T}_1(y) \\ \bar{T}_2 = 0 \\ \bar{T}_3 = 0 \end{array}\right\} \qquad (2\text{-}8)$$

Moreover if wall-effects can be neglected, $\bar{T}_1$ becomes independent of y.

The following considerations deal with the simplest situation. The movement is supposed to be stationary-stochastical and homogeneous for the x-direction; wall effects are neglected. Therefore the unit of width may be considered with momentary movement of sand only in two directions x and z.

It is usual to define a depth a movement δ as far as tracer-tests are concerned [2, 4, 14]. The depth of movement is then equal to the average depth of the layer in which movement of the grains is sometimes present and with the porosity ε_0.

Thus

$$\delta = E \int_{z_0}^{\infty} \bar{b} dz \quad \text{and} \quad \bar{\bar{b}} = E^{-1} \qquad (2\text{-}9)$$

below z_0 movement of grains does not occur, the porosity below that level is always ε_0.

A logical definition of the average flux is then obtained by

$$\delta^{-1} \int_{-\infty}^{+\infty} \bar{w}_1(z) dz = \bar{\bar{w}}_1 \quad \text{thus} \quad \bar{T} = E\bar{\bar{w}}\delta \qquad (2\text{-}10)$$

The definition of δ given is similar to the one defined by HUBBELL and SAYRE [14] due to the fact that in most cases a bed-level $\eta(x,t)$ can be defined at the location where $b(x,z,t)$ has a steep gradient.

The depth of movement δ is defined by HUBBELL and SAYRE in the following way. Starting at a trough of a ripple they divide the length L in n sections l. Each section l_j ends at a trough which is deeper than the trough at the beginning of l_j. Each section gives a value δ_j and for their weighed mean yields (figure 2.2)

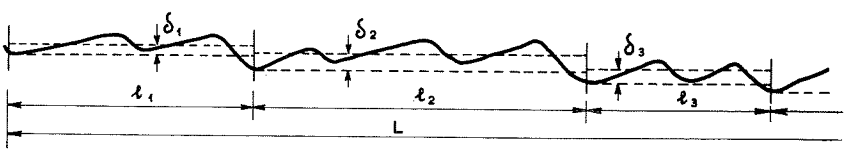

Fig. 2.2 Definition depth of movement δ.

$$\delta = L^{-1} \sum_{j=1}^{n} l_j \delta_j \quad \text{(2-11)}$$

Expressed in the bed-level η and with $L_j = \sum_{j=1}^{n} l_j$ the definition of (2-11) gives

$$\delta_j = l_j^{-1} \int_{L_{j-1}}^{L_{j-1}+l_j} \{\eta(x) - \eta(L_{j-1})\} dx \quad \text{(2-12)}$$

and

$$\delta = L^{-1} \sum_{j=1}^{n} \int_{L_{j-1}}^{L_{j-1}+l_j} \{\eta(x) - \eta(L_{j-1})\} dx \quad \text{(2-13)}$$

or

$$\delta = L^{-1} \int_0^L \eta(x) dx - L^{-1} \sum_{j=1}^{n} \eta(L_{j-1}) \cdot l_j \quad \text{(2-14)}$$

For the special case of a regular ripple-bed it yields

$$\delta = L^{-1} \int_0^L \eta(x) dx - \eta(0) \quad \text{(2-14a)}$$

in which $\eta(0)$ is the level of a trough.

Some remarks have to be made about the transport of heterogeneous bed-material. Instead of the sand-content $b(x,y,z,t)$ we now define the content $a_i(x,y,z,t)$, being the content of sand of the ith fraction. If $p_i(x,y,z,t)$ denotes the local temporary grain-fraction then:

$$a_i(x,y,z,t) = b(x,y,z,t)\ p_i(x,y,z,t)$$

relates the definitions of a_i, b and p_i.

Further

$$\mathbf{w}_i(x,y,z,t) = a_i(x,y,z,t)\ \mathbf{u}_i(x,y,z,t) \quad \text{(2-15)}$$

relates the grain-velocity $\mathbf{u}_i$ of this fraction with the flux $\mathbf{w}_i$. Also

$$\mathbf{T}_i(x,y,z,t) = E \int_{-\infty}^{+\infty} \mathbf{w}_i(x,y,z,t) dz \quad \text{(2-16)}$$

The definition of the average flux $\bar{\bar{\mathbf{w}}}_i$ for the fraction becomes

$$\bar{\bar{\mathbf{w}}}_i = \delta^{-1} \int_{-\infty}^{+\infty} \bar{\mathbf{w}}_i(z) dz \quad \text{(2-17)}$$

The definition of $\bar{T}$ has been given as the average volume of sand per unit of time and width. The volume of sand is measured as a deposit.

This definition has some advantages as $\bar{T}$ is mostly used in relation to the morphology of the bed. With this definition the equation of continuity for the solid phase becomes simpler. For T and therefore for $\bar{T}$ holds

$$\frac{\partial T}{\partial x} = -\frac{\partial \eta}{\partial t} \quad \text{(2-18)}$$

This expression is valid if the variation of suspended-load is negligible.

The eq. (2-18) shows the relation between the transport T and the bed-level η. The variation of T with x can therefore be studied via the variation of η with t.

The simplest hypothetical bed-form is given by triangular dunes of equal

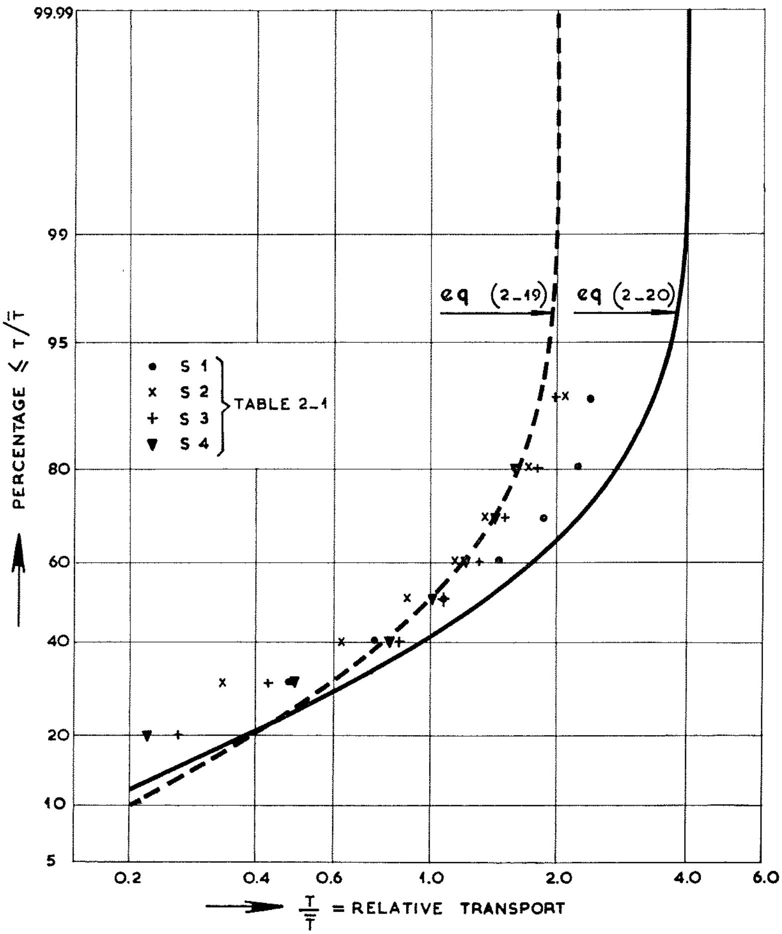

Fig. 2.3 Probability-distribution of transport.

form. In this case the variation of T is periodical and the cumulative distribution $P_1\{T\}$ is given by

$$P_1\{T\} = 0.5T/\bar{T} \quad \text{for} \quad 0 \leqslant T \leqslant 2\bar{T} \qquad (2\text{-}19)$$

This model can be extended for more complicated cases. HAMAMORI [12] constructed a theoretical model for large dunes with small secondary ripples. The secondary ripples which are often present, are much faster than the primary dunes. HAMAMORI arrives at a theoretical cumulative probability-distribution:

$$P_2\{T\} = 0.25\,\frac{T}{\bar{T}}\left[1+\ln\frac{4\bar{T}}{T}\right] \qquad (2\text{-}20)$$

The maximum momentary transport is present if the top of a secondary ripple coincides with the one of a primary dune.

The two schematic probability-distributions of eqs. (2-19) and (2-20) already give a fair description of the physical phenomenon. In a laboratory-flume this distribution has been measured in the following way. At the end of the sand-bed the transport has been measured by a small sand-trap suspended in the main sand-trap. The small sand-trap catches the sand over a width of 0.125 m being $^1/_4$ of the width of the flume. The amount of sand is determined electrically by measuring the weight of the sand and the trap via straingages on the suspension-construction. The small trap is emptied automatically after regular time-intervals Δt. Therefore it is possible to determine the local transport during small time-intervals Δt.

Figure 2.3 represents the distribution-function as it is measured under the conditions of table 2-1.

Table 2-1 Probability of transport; flume experiments.

Symbol	Description	Series 1	Series 2	Series 3	Series 4	Unit
B	width of flume	0.50	0.50	0.50	0.50	m
$\bar{h}$	depth of water	0.39	0.42	0.37	0.39	m
q	discharge	0.200	0.232	0.260	0.300	m^2/s
$\bar{T}$	transport	0.78	1.56	2.22	3.61	$\cdot 10^{-5} m^2/sec$
I	slope	0.35	1.13	1.19	1.58	$\cdot 10^{-3}$
$\bar{d}$	mean grain diameter	0.925	0.925	0.925	0.925	mm
d_{90}	90% grain diameter	1.4	1.4	1.4	1.4	mm
H	mean dune-height	0.08	0.08	0.12	0.23	m
$\bar{\lambda}$	mean dune-length	2.8	2.5	3.5	3.0	m
Δt	time-interval	120	120	120	120	sec

ZWAMBORN [33] describes measurements in the same flume under different transport-conditions. By taking transport-measurements from the large sand-trap he derives:

$\Delta t = 1$ hour	$\bar{T} = 23.0$ L/h	$s_T = 12.5$ L/h
$\Delta t = 2$ hours	$\bar{T} = 23.0$ L/h	$s_T = 7.9$ L/h
$\Delta t = 3$ hours	$\bar{T} = 23.0$ L/h	$s_T = 6.7$ L/h

in which s_T denotes the standard-deviation of T. The standard-deviation is large though the transport has been measured as an average during 3 hours.

In conclusion it can be stated that the presence of ripples and dunes demands a large value of θ in eq. (2-7). It is therefore quite possible that the maximum value, acceptable in practice, is smaller than the one theoretically required. This implies that values of the different magnitudes of the phenomenon, derived with the accepted value of θ are only estimates of the average values.

2.3 Non-steady bed-load transport

The transport-vector $\bar{T}$ is now supposed to vary, even when the average is taken over a long period θ.

Therefore

$$\int_0^\theta T(t')dt' \neq \int_t^{t+\theta} T(t')dt' \text{ for } t \neq 0 \qquad (2\text{-}21)$$

In this chapter this non-steady bed-load transport will be discussed. This implies that the assumption of stationary-stochastical movement has to be dropped. Sedimentation and erosion may be present. As the situation for a straight flume is considered, the homogeneity for the location is not present for the flow direction.

By neglecting the variation perpendicular to the current the phenomenon may be considered to be one-dimensional for the location.

As par. (2.2) discusses the variation of the transport due to ripples and dunes for the stationary-stochastical movement, now the variation of $\bar{T}$ will be studied due to changes of the regime. In this approach the presence of ripples and dunes will be neglected.

The interaction of hydraulic conditions and bed-level can be expressed by a mathematical model [27, 31]. The two independent variables are x and t and the dependent variables are:

$v(x,t)$, the average velocity of the water in a cross-section
$h(x,t)$, the depth of water
$\eta(x,t)$, the bed-level
$T(x,t)$, the transport of sand through a cross-section.

The four dependent variables are related by four equations: the equations of motion and continuity of the two phases, water and sand. The equations of continuity can be expressed as follows.

$$hv_x + vh_x + h_t = 0 \quad \text{(2-22)}$$

$$T_x + \eta_t = 0 \quad \text{(2-23)}$$

It should be noted that eq. (2-23) is based on the assumption that the variation of sand in suspension can be neglected. The subscripts x and t denote differentiation.

The equation of motion for the fluid can have its common one-dimensional form

$$v_t + vv_x + gh_x + g\eta_x = R \quad \text{(2-24)}$$

in which R stands for the hydraulic friction-term.

The equation of motion for the solid phase causes more difficulties. It is, however, assumed that there are only small changes present of a known steady condition. Therefore it may be assumed

$$T = f(v) \quad \text{(2-25)}$$

This implies that grain-sorting and changes of the roughness are supposed to be absent.

The elimination of T from eq. (2-23) by applying the relation (2-25) reduces the system into three partial differential equations of the dependent variables v, h and η. With the expression for the total differentials dv, dh and $d\eta$, a system of six equations is obtained for the six partial derivates.

$$\begin{pmatrix} 1 & v & 0 & g & 0 & g \\ 0 & h & 1 & v & 0 & 0 \\ 0 & f_v & 0 & 0 & 1 & 0 \\ dt & dx & 0 & 0 & 0 & 0 \\ 0 & 0 & dt & dx & 0 & 0 \\ 0 & 0 & 0 & 0 & dt & dx \end{pmatrix} \begin{pmatrix} v_t \\ v_x \\ h_t \\ h_x \\ \eta_t \\ \eta_x \end{pmatrix} = \begin{pmatrix} R \\ 0 \\ 0 \\ dv \\ dh \\ d\eta \end{pmatrix} \quad \text{(2-26)}$$

The three propagation-velocities $c = dx/dt$ are derived from the matrix of the coefficients of eqs. (2-26).

$$-c^3 + 2vc^2 + (gh - v^2 + gf_v)c - vgf_v = 0 \quad \text{(2-27)}$$

The three roots are related by

$$c_1c_2c_3 = -vgf_v \quad \text{(2-28)}$$

The eqs. (2-27) and (2-28) can be put in a dimensionless form by defining three dimensionless parameters:

$$\left.\begin{array}{lll} \varphi = c/v & = & \text{relative celerity} \\ F = v/\sqrt{gh} & = & \text{Froude-number} \\ \psi = f_v/h & = & \text{dimensionless transport-parameter} \end{array}\right\} \quad \text{(2-29)}$$

The third parameter ψ becomes clear if $f(v)$ is expressed in an exponential form

$$f(v) = av^b \quad \text{(2-30)}$$

Hence

$$\psi = \frac{abv^{b-1}}{h} = b\frac{T}{q} \quad \text{(2-31)}$$

This means that ψ is roughly proportional to the ratio of the transport of the solid-, and the fluid-phase. In most cases it yields $3 < b < 7$; furtheron T/q is mostly small enough to state $\psi \ll 1$. For instance for the Dutch Rhine-branches: $\psi = 10^{-5}$ to 10^{-6}.

Introduction of the three dimensionless parameters gives

$$\varphi^3 - 2\varphi^2 + (1 - F^{-2} - \psi F^{-2})\varphi + \psi F^{-2} = 0 \quad \text{(2-32)}$$

and

$$\varphi_1\varphi_2\varphi_3 = -\psi F^{-2} \quad \text{(2-33)}$$

The well-known roots for a fixed-bed can be obtained by inserting $f_v = 0$ in eq. (2-27) or $\psi = 0$ in eq. (2-32), leading to

$$\varphi_1'\varphi_2' = (1 - F^{-2});\ \varphi_3' = 0 \quad \text{(2-34)}$$

It is now assumed that for tranquil flow, with F not too close to unity, the influence of ψ on φ_1 and φ_2 may be neglected.

Thus

$$\varphi_1'\varphi_2' \approx \varphi_1\varphi_2 \quad \text{(2-35)}$$

This means that the relative celerities φ_1 and φ_2 of surface-waves are not effected by the mobility of the bed.

The relative celerity φ_3 of waves at the bed becomes:

$$\varphi_3 = \frac{-\psi F^{-2}}{1 - F^{-2}} = \frac{\psi}{1 - F^2} \quad \text{(2-36)}$$

The relation (2.26) can directly be found from the basic equations if it can be assumed that the water-movement is steady ($v_t = 0$ and $h_t = 0$) i.e. if slow changes of the bottom are assumed to be present with constant discharge.

These theoretical considerations can be applied to two problems. Firstly one may try to compare the theoretical expression for the propagation-velocity c_3 or φ_3 with the celerity of ripples and dunes for steady conditions. Secondly, some aspects of non-steady bed-load transport will be considered.

The considerations of this paragraph are very rough: the presence of ripples and dunes has been neglected. Nevertheless one may take the steady condition with dunes as a border case of the non-steady situation.

By combining the eqs. (2-29, 2-31 and 2-36) the following expression can be found

$$c_3 = bv \frac{T}{q} \frac{1}{1-F^2} \quad \ldots \ldots \quad (2\text{-}37)$$

The coefficient b can now be defined more specifically. Instead of the general form $T = f(v)$ a more refined transport-formula may be used. Usually those formulas can be written as a relation between two dimensionless parameters X and Y.

The transport-parameter

$$X = \frac{T}{d^{3/2}\sqrt{g\Delta}} \quad \ldots \ldots \quad (2\text{-}38)$$

is a combination of the transport-vector and the characteristics of the bed-material viz. the mean-grainsize d and the relative density $\Delta = (\varrho_s - \varrho)/\varrho$ of the material.

The flow-parameter

$$Y = \frac{\Delta d}{\mu' h I} \quad \ldots \ldots \quad (2\text{-}39)$$

relates the characteristics of the bed-material with the factor hI, the product of the water-depth h and the energy-slope I. The coefficient μ', the ripple-factor, gives a more or less empirical correction. FRIJLINK [8] gives the relation.

$$X = 5Y^{-1/2} \exp(-0.27Y) \quad \ldots \ldots \quad (2\text{-}40)$$

By matching $X = \alpha Y^\beta$, similar to eq. (2-30), with eq. (2-40) it can be concluded

$$b = 1 + 0.54Y \quad \ldots \ldots \quad (2\text{-}41)$$

with eq. (2-40) also $b = b(X)$ can be found numerically.

For the interval $5 \cdot 10^{-3} < X < 3 \cdot 10^{-1}$ a good representation is found by

$$b = -4 \log X + 2.5 \quad \ldots \ldots \quad (2\text{-}42)$$

Therefore:

$$c_3 = (-4 \log X + 2.5) T \frac{1}{h(1-F^2)} \quad \ldots \ldots \quad (2\text{-}43)$$

This expression shows the influence of the material via the parameter X on the celerity. Figure 2.4 shows the function $b = b(X)$ based on eq. (2-40) together with the simplified function (2-42). A similar procedure has been applied to the Meyer-Peter equation [16].

The comparison of c_3 via eq. (2-43) and the measured ripple-celerity c has

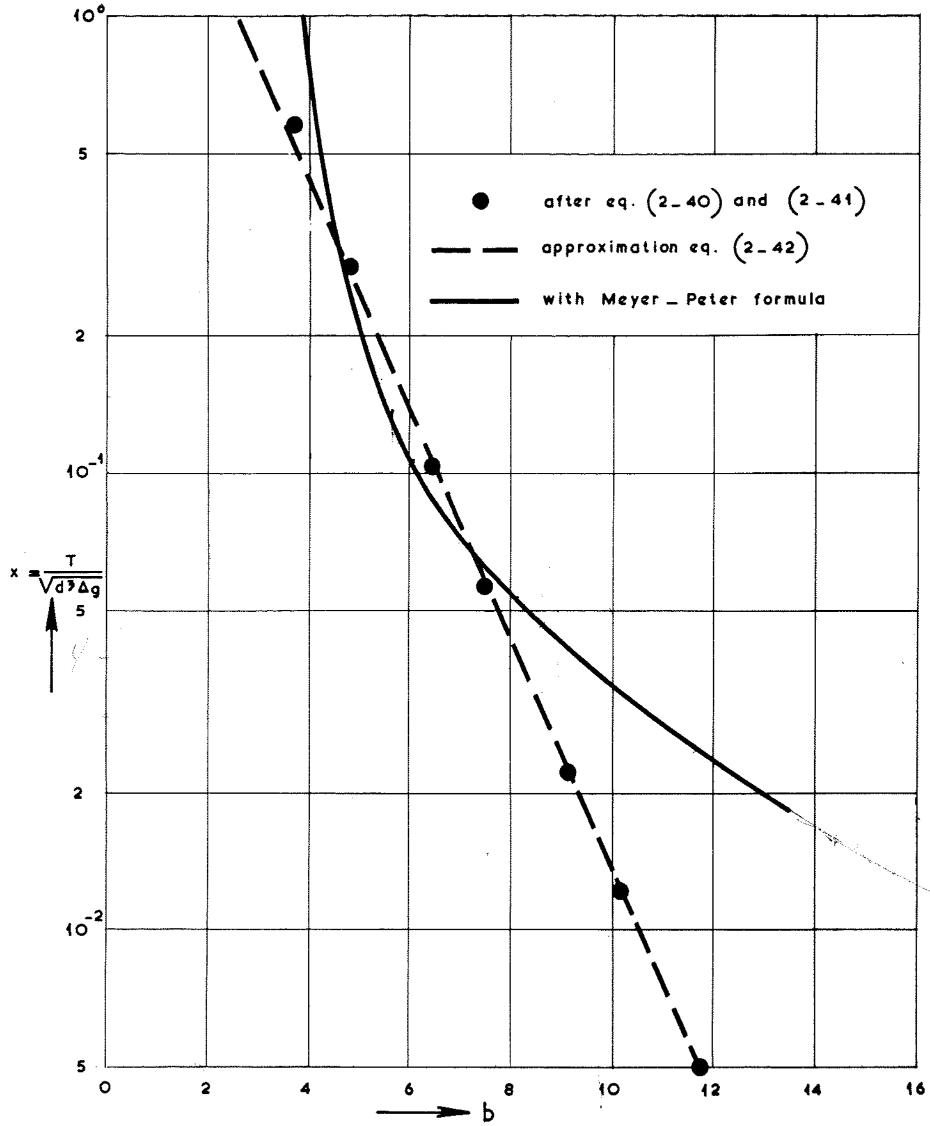

Fig. 2.4 The exponent of the transport-function.

been carried out for the experiments of SHINOHARA and TSUBAKI [23, 25]. From 23 observations the following results are obtained

$$\frac{c_3}{c} = 0.7 \pm 0.2 \quad \text{(2-44)}$$

The significant difference between the calculated and measured value of the celerity may be caused by the following reasons. The deduction of eq. (2-43) implies the assumption of small disturbances. Further the application of c_3, for the celerity of ripples and dunes postulates more or less propagation without deformation. Therefore eq. (2-43) has also been applied, not to the average conditions but to the top of the ripple or dune:

$$c_3' = [2.5 - 4 \log 1.8X] \frac{1.8T}{h - {}^1/_2 H} \left[\frac{1}{1-(1-0.5H/h)^3 F^2} \right] \quad . \; . \; . \quad (2\text{-}45)$$

in which H denotes the ripple-height. The values X, T, h, H and F are the magnitudes for the average situation. The coefficient 1.8 can be found from the form of the ripple [23].

The average value, derived from the same 23 observations, gives now a better agreement

$$\frac{c_3'}{c} = 1.1 \pm 0.4 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (2\text{-}46)$$

A large scattering is present probably mostly caused by scatter of the transport-measurements.

Here, the celerity c_3 of eq. (2-37) has been used as an indicator for the celerity of ripples for a stationary-stochastical movement. Now, the original concept of this paragraph will be discussed with the results of the given derivations.

The magnitude of c_3 determines the speed by which a river-bed is adapted to changes of hydraulic conditions. Calculations and observations show that the given mathematical model gives a reasonable representation of this "adaptation-time" [27, 29, 31, 26]. These studies deal with the border-case in which c_1 and c_2 become infinite. An extension of the procedure, in which three characteristics are indeed used for the calculations, has been described in a Chinese publication [13].

It is known indeed from observations in nature that morphological processes in rivers are mostly very slow. The long adaptation time is also present if one has to reach a stationary-stochastical transport-condition in a laboratory flume.

The celerity c_3 is very small compared to the celerity of a flood wave. The latter is in the order of magnitude of the average water-velocity. Therefore the changes of the discharge Q in the first place govern the non-stationary aspect of the stochastical transport-process. These changes of the discharge are therefore very important as far as the measurements of transport in natural rivers are concerned. At present it gives an important restriction for the quantitative use of tracer-techniques.

At this moment tracer-tests for stationary-stochastical conditions (postulated in the following chapters) already cause many troubles as far as the quantitive interpretation is concerned. For the time being it is therefore necessary to pay only attention to this stationary-stochastical situation.

The practical applicability, however, is enlarged if it is possible to reduce the time of the tracer-tests. The time required is therefore an important criterion for the set-up of the tests.

CHAPTER 3

THE DIFFUSION MODEL APPLIED TO TRACER-EXPERIMENTS

3.1 Introduction

The general set-up of tracer-experiments is to derive conclusions from the dispersion-figures in order to obtain information about the behaviour of the bed-material. This implies that a connection has to be found between the transport-phenomenon and the concentration-distribution as a result of this phenomenon. Therefore a theoretical dispersion-model has to be evaluated. This mathematical model will contain some parameters. Each set of parameters will belong to a specific transport-condition. Derivation of the actual values of these parameters with the concentrations measured will lead to the required answer: the determination of the transport-vector.

This chapter deals with the first step towards this answer: the evaluation of a theoretical dispersion-model. From the considerations of chapter 2 it should be noted that two major aspects have to be taken into account. Firstly the set-up must be made for graded bed-material. Secondly it should be noted that the transport-conditions in actual rivers are only homogeneous for restricted areas and during finite time-intervals. This gives a limitation for the dispersion-model derived in this chapter. Par. 3.2.1 will deal with uniform bed-material; the extension for graded material will be made in par. 3.2.2. The theoretical solutions for some sets of initial-, and boundary-conditions will be derived in par. 3.2.3.

Besides the one-dimensional diffusion-model the two-dimensional case is also considered (par. 3.3).

3.2 The one-dimensional model

3.2.1 *Deduction for homogeneous material*

The one-dimensional model for uniform bed-material will have a restricted value for measurements in practice. Nevertheless this model has to be derived to make extensions possible towards more complicated phenomena. The following major assumptions will be made.

1. The transport-condition is homogeneous in time and place.
2. Variations perpendicular to the main-current will be neglected as far as they are present in the horizontal plane.

3. The bed-material is uniform.
4. The tracer-material has the same transport characteristics as the bed-material.
5. The input of tracer-material is small. It does not influence the transport phenomenon and the tracer-concentrations are small compared to unity.

In this paragraph the second assumption makes it possible to consider the unit of width. The sand-flux can be defined according to par. 2.2. The flux $w(x,z,t)$ in the x-direction will have an average

$$\bar{w}(x,z,t) = \theta^{-1} \int_0^\theta w'(x,z,t+\tau)d\tau \quad \text{(3-1)}$$

According to the first assumption this average value will be independent of x and t if the time-average (or spatial average) is taken over a sufficiently long interval.

Therefore

$$w(x,z,t) = \bar{w}(z) + w'(x,z,t) \quad \text{(3-2a)}$$

and similar for the vertical direction

$$W(x,z,t) = W'(x,z,t) \quad \text{with } \overline{W} = 0 \quad \text{(3-2b)}$$

According to the definitions it yields

$$T = E \int_{-\infty}^{+\infty} w(z)dz = \bar{T} + T'(x,t) \quad \text{(3-3)}$$

The concentration $C(x,z,t)$ will be defined as the ratio of the weight (or volume) of the tracers and the sand.

The transport of tracer-material becomes in the same way

$$E \int_{-\infty}^{+\infty} wCdz \quad \text{(3-4)}$$

If $b(x,z,t)$ is the sand-content, the equation of continuity of tracers for the element $dxdz$ becomes

$$(bC)_t + (wC)_x + (WC)_z = 0 \quad \text{(3-5)}$$

The use of w and W in eq. (3-5) is only possible if the fourth assumption is correct.

The concentration $C(x,z,t)$ can also be seen as a fluctuating magnitude:

$$C(x,z,t) = \bar{C}(x,z,t) + C'(x,z,t) \quad \text{(3-6)}$$

In this case the average has been taken over a period θ, large compared with periods of C but small enough to be sure that $\bar{C}(x,z,t)$ is not changing too much.

So

$$\bar{C}(x,z,t) \approx \bar{C}(x,z,t+\theta) \quad \text{(3-7)}$$

and

$$\overline{C'(x,z,t)} = \theta^{-1} \int_0^\theta C'(x,z,t+\tau)d\tau \approx 0 \quad \text{(3-8)}$$

The equation of continuity now becomes

$$(\bar{b}\bar{C})_t + (b'C')_t + (\bar{b}C')_t + (b'\bar{C})_t + (\bar{w}\bar{C})_x + (w'\bar{C})_x + (\bar{w}C')_x + (w'C')_x + (WC)_z = 0 \quad \text{(3-9)}$$

Averaging over the period θ yields

$$(\bar{b}\bar{C})_t + (\overline{b'C'})_t + (\bar{w}\bar{C})_x + (\overline{w'C'})_x + (\overline{WC})_z = 0 \quad \text{(3-10)}$$

It can now be assumed $\overline{b'C'} = 0$. The following reasoning can be given:
$\overline{b'C'} > 0$ implies $\overline{b'C} > 0$ as $b'\bar{C} = 0$ according to the definition of $\bar{b}$. This would indicate a certain preference for the tracers to be present at places of relatively large sand-contents.
$\overline{b'C'} < 0$ (or $\overline{b'C} < 0$) would indicate a certain preference for the pure sand to be present at places of large sand-content.
The fourth assumption does not admit a behaviour of tracers contradictory to pure sand. Therefore $\overline{b'C'} = 0$ can be concluded.

The equation (3-10) is now integrated in the vertical direction. The last term $(\overline{WC})_z$ then gives no contribution as $\overline{WC}$ disappears at the boundaries of the integral. The integration over the interval $-\infty < z < +\infty$ makes it necessary to define the average concentration $\bar{\bar{C}}$. According to par. 2.2 the depth of movement δ can only be defined roughly.

$$\delta^{-1} \int_{-\infty}^{+\infty} \bar{w}(z)dz = \bar{\bar{w}} \quad \text{and} \quad E\bar{\bar{w}}\delta = \bar{T} \quad \text{(3-11)}$$

The magnitude $\bar{\bar{C}}$ will be defined as the average concentration over the depth of movement δ

$$\int_{-\infty}^{+\infty} \bar{C}dz = \delta\bar{\bar{C}} \quad \text{(3-12)}$$

From eq. (3-10) one gets

$$\int_{-\infty}^{+\infty} (\bar{b}\bar{C})_t dz + \int_{-\infty}^{+\infty} (\bar{w}\bar{C})_x dz + \int_{-\infty}^{+\infty} (\overline{w'C'})_x dz = 0 \quad \text{(3-13)}$$

The first integral can be rewritten by interchanging integration and differentiation

$$\int_{-\infty}^{+\infty} (\bar{b}\bar{C})_t dz = \frac{\partial}{\partial t} \int_{-\infty}^{+\infty} \bar{b}\bar{C}\, dz \quad \text{(3-14)}$$

If the concentration $\bar{C}(z)$ is constant in the vertical direction up to the level z_0, one obtains

$$\frac{\partial}{\partial t} \int_{-\infty}^{+\infty} \bar{b}\bar{C}\, dz = \frac{\partial}{\partial t} \bar{C} \int_{z_0}^{+\infty} \bar{b}\, dz \quad \text{(3-15)}$$

According to the definitions it may be stated

$$\int_{z_0}^{+\infty} \bar{b}\, dz = (1-\varepsilon_0)\delta = \bar{\bar{b}}\, \delta \quad \text{(3-16)}$$

in which ε_0 stands for the porosity of the sand resting in the dunes.

If the concentration $\bar{C}(z)$ is not constant in the vertical direction, but if the distribution-curve is similar for every x, it is necessary to introduce a profile constant K_1 according to:

$$\int_{-\infty}^{+\infty} \bar{b}\bar{C}\, dz = K_1 \bar{\bar{b}} \delta \bar{\bar{C}} \quad \text{(3-17)}$$

The same procedure can be applied to the second integral of eq. (3-13)

$$\int_{-\infty}^{+\infty} (\bar{w}\bar{C})_x dz = K_2 \bar{\bar{w}} \delta \bar{\bar{C}}_x \quad \text{(3-18)}$$

The first assumption gives the reason why $\bar{\bar{w}}$ has been taken independently of the value of x.

The third integral of eq. (3-13) contains $\overline{w'C'}$. The major assumption of the diffusion is now

$$\overline{w'C'} = -D'(z)\bar{C}_x(x,z,t) \quad \text{(3-19)}$$

This implies that the transport by irregular movement of the grains can be described as a diffusion-system of the gradient-type.

Similar to the other integral it yields

$$\int_{-\infty}^{+\infty} [D'(z)\bar{C}_x]_x dz = \frac{\partial^2}{\partial x^2} \int_{-\infty}^{+\infty} D'(z)\bar{C} dz = \frac{\partial^2}{\partial x^2} [K_3 \overline{D'} \delta \bar{\bar{C}}] \quad \text{(3-20)}$$

with $\delta\, \overline{D'} = \int_{-\infty}^{+\infty} D'(z) dz$

This gives as a result

$$\bar{\bar{b}} K_1 \bar{\bar{C}}_t - K_3 \overline{D'}\, \bar{\bar{C}}_{xx} + K_2 \bar{\bar{w}} \bar{\bar{C}}_x = 0 \quad \text{(3-21)}$$

or $$\bar{\bar{C}}_t - D\bar{\bar{C}}_{xx} + K(\bar{w}/\bar{\bar{b}})\bar{\bar{C}}_x = 0 \qquad (3\text{-}22)$$

in which

$$D = (K_3/K_1)\ \overline{D'}\ \bar{\bar{b}}^{-1}$$
$$K = (K_2/K_1) \qquad (3\text{-}23)$$

It may be preferable to write

$$\bar{\bar{C}}_t - D\bar{\bar{C}}_{xx} + \mu\bar{w}\bar{\bar{C}}_x = 0 \qquad (3\text{-}24)$$

in which the coefficient $\mu = K/\bar{\bar{b}}$ becomes $\mu = \bar{\bar{b}}^{-1}$ if the concentration is constant in the vertical direction.

The following remarks can be made.

1. Principally the basic assumption of eq. (3-19) is a substitute for the equation of motion of the sand-particles. Being a hypothesis, the application of this diffusion process can only be justified by experiments.
2. The phenomenon dealt with in this paragraph is essentially two-dimensional. The reduction to a one-dimensional form, eq. (3-24) is only possible after the introduction of a complicated parameter μ. This parameter is difficult to establish if the concentration is not uniformly distributed in the vertical direction.

3.2.2 *Deduction for heterogeneous material*

For heterogeneous bed-material an extension of the above given diffusion-model can be obtained in two ways. The simplest set-up is obtained if the average grain-size distribution $\bar{p}_i$, for grains with diameters d_i, can be established accurately enough to apply tracer-material with the same grading curve. If p_i is not dependent on x, z and t, the same derivation can be used. In that case $\bar{w}$ denotes the average flux of all grain-sizes. However, due to grain-sorting this set-up may give much trouble. In general the basic assumptions will not be justified.

Therefore another approach will be used here. This one implies that the concentrations are determined for each grain-fraction separately. It will be shown that the assumption of the equal size-distribution for sand and tracer-material can be dropped in this case. The only additional assumption is that $\bar{p}_i$ is not dependent on x and t, which is justified for a restricted area in which the homogeneous process is present.

The concentration C_i for the size-fraction i can now be defined in two manners

$$C_{1i} = \frac{\text{weight of tracer-material with } d_i}{\text{weight of all material with } d_i} \qquad (3\text{-}25)$$

and

$$C_{2i} = \frac{\text{weight of tracer-material with } d_i}{\text{weight of all material}} \quad \ldots\ldots\ldots\ldots \quad (3\text{-}26)$$

According to the definitions there is the relation

$$C_{1i}(x,z,t)p_i(x,z,t) = C_{2i}(x,z,t) \quad \ldots\ldots\ldots\ldots \quad (3\text{-}27)$$

At this moment no choice can be made between the two definitions of C_i. The choice will be made later on if the diffusion equation has been derived for the two cases. It can already be remarked, that the use of C_{1i} is only possible if samples are taken according to the common practice for fluorescent tracers. The use of C_{2i} is applicable for all tracer-types.

If $u_i(x,z,t)$ denotes the momentary horizontal grain-velocity, $w_i(x,z,t) = a_i(x,z,t)u_i(x,z,t)$ represents the local horizontal flux, with $a = bp$.

For the sake of simplicity the subscript i can be dropped, as every factor except b is taken for the fraction i.

For the first definition of the concentration C_1 the equation of continuity becomes

$$(aC_1)_t + (auC_1)_x + (aUC_1)_z = 0 \quad \ldots\ldots\ldots\ldots \quad (3\text{-}28)$$

This equation shows much similarity to eq. (3-5); therefore a similar treatment is possible.

The time-averages lead to:

$$(\bar{a}\bar{C}_1)_t + (\overline{a'C'})_t + (\overline{au}\ \bar{C}_1)_x + (\overline{au'C'_1})_x + (\bar{u}\ \overline{a'C'_1})_x + (\overline{aUC_1})_z = 0 \quad (3\text{-}29)$$

Two terms contain the covariance $\overline{a'C_1'}$, this covariance can be neglected for the same reason as $\overline{b'C'}$ is neglected for homogeneous material.

Integration of eq. (3-29) in the vertical direction leads to the elimination of the last term, as $\overline{aUC_1} = 0$ for $z = \pm\infty$.

Thus

$$\frac{\partial}{\partial t}\int_{-\infty}^{+\infty} \bar{a}\bar{C}_1 dz + \frac{\partial}{\partial x}\int_{-\infty}^{+\infty} \overline{au}\bar{C}_1 dz + \frac{\partial}{\partial x}\int_{-\infty}^{+\infty} \overline{au'C_1'} dz = 0 \quad \ldots\ldots \quad (3\text{-}30)$$

Similar to par. 3.2.1 it can be stated

$$\frac{\partial}{\partial t}\int_{-\infty}^{+\infty} \bar{a}\bar{C}_1 dz = K_1'\bar{\bar{p}}\ \bar{\bar{b}}\ \delta\ \bar{\bar{C}}_{1t} \quad \ldots\ldots\ldots\ldots \quad (3\text{-}31)$$

$$\frac{\partial}{\partial x}\int_{-\infty}^{+\infty} \overline{au}\bar{C}_1 dz = K_2'\ \bar{\bar{w}}\ \delta\ \bar{\bar{C}}_{1x} \quad \ldots\ldots\ldots\ldots \quad (3\text{-}32)$$

and

$$\frac{\partial}{\partial x}\int_{-\infty}^{+\infty} \overline{au'C_1'}\ dz = -\frac{\partial^2}{\partial x^2}\int_{-\infty}^{+\infty} D''(z)\bar{C}_1 dz = -\frac{\partial^2}{\partial x^2}[K_3'\ \overline{D''}\ \delta\ \bar{\bar{C}}_1] \quad (3\text{-}33)$$

Thus

$$\bar{\bar{C}}_{1t} - D_1\bar{\bar{C}}_{1xx} + K'(\bar{\bar{w}}\,\bar{\bar{p}}^{-1}\,\bar{\bar{b}}^{-1})\,\bar{\bar{C}}_{1x} = 0 \quad (3\text{-}34)$$

with $D_1 = (K_3'/K_1')\,\overline{D''}\,\bar{\bar{b}}^{-1}\,\bar{\bar{p}}^{-1}$

$$K' = K_2/K_1' \quad (3\text{-}35)$$

Or

$$\bar{\bar{C}}_{1t} - D_1\bar{\bar{C}}_{1xx} + \mu_1\bar{\bar{w}}\,\bar{\bar{C}}_{1x} = 0 \quad (3\text{-}36)$$

with

$$\mu_1 = K'(\bar{\bar{p}}^{-1}\,\bar{\bar{b}}^{-1}) \quad (3\text{-}37)$$

If the second definition of the concentration C_i is used, the deduction of an equation similar to eq. (3-36) is somewhat different. Now the equation of continuity becomes

$$(bC_2)_t + (wp^{-1}C_2)_x + (bUC_2)_z = 0 \quad (3\text{-}38)$$

Here again similarity to eq. (3-5) is present.

And instead of eq. (3-30) it now yields:

$$\frac{\partial}{\partial t}\int_{-\infty}^{+\infty}(\bar{b}\bar{C}_2)\,dz + \frac{\partial}{\partial x}\int_{-\infty}^{+\infty}(\overline{wp^{-1}}\bar{C}_2)\,dz + \frac{\partial}{\partial x}\int_{-\infty}^{+\infty}\overline{bu'C'_2}\,dz = 0 \quad (3\text{-}39)$$

Here

$$\frac{\partial}{\partial t}\int_{-\infty}^{+\infty}\bar{b}\bar{C}_2\,dz = K_1''\,\bar{\bar{b}}\,\delta\,\bar{\bar{C}}_{2t} \quad (3\text{-}40)$$

$$\frac{\partial}{\partial x}\int_{-\infty}^{+\infty}\overline{wp^{-1}}\,\bar{C}_2\,dz = K_2''\,\overline{\overline{wp^{-1}}}\,\delta\,\bar{\bar{C}}_{2x} \quad (3\text{-}41)$$

and

$$\frac{\partial}{\partial x}\int_{-\infty}^{+\infty}\overline{bu'C_2'}\,dz = -\frac{\partial^2}{\partial x^2}\int_{-\infty}^{+\infty}D'''(z)\bar{C}_2\,dz = -\frac{\partial^2}{\partial x^2}[K_3''\overline{D'''}\,\delta\,\bar{\bar{C}}_2] \quad (3\text{-}42)$$

Hence

$$\bar{\bar{C}}_{2t} - D_2\bar{\bar{C}}_{2xx} + K''(\overline{\overline{wp^{-1}}}\cdot\bar{\bar{b}}^{-1})\,\bar{\bar{C}}_{2x} = 0 \quad (3\text{-}43)$$

with

$$K'' = K_2''/K_1'' \quad (3\text{-}44)$$

and $D_2 = (K_3''/K_1'')\overline{D'''}\,\overline{\overline{b^{-1}}}$

Or

$$\bar{\bar{C}}_{2t} - D_2\bar{\bar{C}}_{2xx} + \mu_2\bar{\bar{w}}\bar{\bar{C}}_{2x} = 0 \quad (3\text{-}45)$$

with

$$\mu_2 = K''(\overline{\overline{wp^{-1}}}\,\bar{\bar{b}}^{-1})/\bar{\bar{w}} \quad (3\text{-}46)$$

A comparison between the result for the two definitions of C_i can now be made.

According to eq. (3-27) it yields

$$\bar{C}_2 = \bar{p}\bar{C}_1 + \overline{p'C'_1} \quad (3\text{-}47)$$

According to the assumption that sand and tracer-material behave in a similar way, the covariance $\overline{p'C_1'}$ can be neglected. If one takes the average of eq. (3-47) in the vertical direction, it shows that $\bar{\bar{C}}_1$ and $\bar{\bar{C}}_2$ are directly proportional to each other.

From the structure of eq. (3-36) and eq. (3-45) it may be concluded

$$\mu_1 \equiv \mu_2 \quad \text{and} \quad D_1 \equiv D_2 \quad \ldots\ldots\ldots\ldots\ldots \quad (3\text{-}48)$$

Therefore there is no preference for either one of the definitions of C_i as far as the diffusion-model is concerned. The practical considerations (chapter 4) as well as the determination of the coefficient μ_1 or μ_2 may give some criterion. Later on this subject will therefore be considered again.

3.2.3 *Boundary-, and initial-conditions; solutions*

The use of a theoretical dispersion-model for the interpretation of tracer-measurements needs an analytical expression of the concentration as a function of some parameters. Therefore the derived differential equation will generally not be sufficient. A solution is required, naturally depending on specific boundary-, and initial-conditions.

The simplest case is formed by the solution for an instantaneous source at one location. In practice other solutions may be composed of a number of standard-solutions. Therefore the solution for one instantaneous source has to be considered first of all.

The formulation of the conditions has to be such that applicable analytical solutions are obtained. From the considerations of chapter 2 it can be concluded, that in natural rivers spatial homogeneity is only present in restricted areas. Further a stationary-stochastical transport is only present if a restricted time-period is considered. Both circumstances ask for tracer-measurements rather soon after the injection. The same can be concluded from a third reason. Chapter 4 will deal with the practical aspects of measurements with luminophores. There the accuracy of concentration-measurements will be discussed. It can already be pointed out that only significant concentrations are obtained if measurements are taken rather soon after the injection. Only then the amount of released tracers can be small enough to fulfil the fifth assumption (section 3.2.1) as good as possible.

These reasons make it necessary that the dispersion has not only to be described correctly in the asymptotical case, but more specifically for values of x and t for which the boundary-, and initial-conditions do have a large influence on the solution.

In this connection a remark has to be made regarding the parameters D and $\mu\bar{\bar{w}}$ of the eq. (3-24). These parameters are derived with the assumption of independence from x and t. Obviously this will not hold for small values of x and t i.e. soon after the injection at (0,0). The differential equation is only analytically soluble for varying coefficients if the variation with x and/or t runs according to very specific functions. This does not give very much freedom for hypotheses regarding the behaviour of $D(x,t)$ and $\mu(x,t)\bar{w}(x,t)$. Secondly it has to be noted that in practice the injection-procedure can never be exactly at (0,0). Later on the practical aspects will be discussed in chapter 4.

Both difficulties can be tackled at once by the assumption that the injection is effectuated at (x_0,t), in which x_0 is an unknown distance. And with this additional assumption D and $\mu\bar{w}$ are supposed to be constant for every x and t.

Locally the stationary-stochastical sand-movement shows a temporarily negative transport of sand. The average movement, however, shows only positive transport and so the average tracer-flux should also be positive.

Therefore it has to be stated

$$\mu\bar{w}\bar{\bar{C}}(x_0,t) - D\bar{\bar{C}}_x(x_0,t) = \delta(t)\cdot\tau_*/\delta \quad \text{(3-49)}$$

Here τ_* denotes the volume of released tracers per unit of width. Eq. (3-49) implies a barrier against negative tracer-transport at the fictive location of the injection.

It has been assumed that the material is uniform. Later on the modification will be studied as to heterogeneous material.

The underlying differential equation is

$$\bar{\bar{C}}_t - D\bar{\bar{C}}_{xx} + \mu\bar{w}\bar{\bar{C}}_x = 0 \quad \text{(3-50)}$$

If now the translation $x' = x - x_0$ is assumed, the following system is obtained.

$$\bar{\bar{C}}_t - D\bar{\bar{C}}_{x'x'} + \mu\bar{w}\bar{\bar{C}}_{x'} = 0 \quad \text{(3-51)}$$

$$\mu\bar{w}\bar{\bar{C}}(0,t) - D\bar{\bar{C}}_{x'}(0,t) = \delta(t)\tau_*/\delta \quad \text{(3-52)}$$

$$\bar{\bar{C}}(x'o) = 0 \quad \text{(3-53)}$$

$$\lim_{x'\to\infty} \bar{\bar{C}}(x',t) = 0 \quad \text{(3-54)}$$

One may now, for the sake of simplicity, drop the bars at $\bar{\bar{C}}$ and $\bar{w}$ and the accent of x'. Thus C is supposed to be the average concentration and later on x has to be replaced by $x - x_0$ in the final solution.

The equation (3-53) describes the initial-condition and eq. (3.54) describes the condition that the concentration has to be finite for large values of x.

Thus the solution has to be found for

$$C_t - DC_{xx} + \mu w C_x = 0 \quad \text{(3-55)}$$

with the conditions

$$C(x,0) = 0 \quad \text{(3-56)}$$

$$C(\infty,t) = 0 \quad \text{(3-57)}$$

$$C(0,t) - (D/\mu w)C_x(0,t) = \delta(t)\tau_*/(KT) \quad \text{(3-58)}$$

The solution of eqs. (3-55)–(3-58) can be obtained by means of Laplace-transforms.

If

$$\bar{C}(x,p) = \int_0^\infty e^{-pt} C(x,t)\,dt \quad \ldots \quad (3\text{-}59)$$

then

$$p\bar{C} - D\bar{C}_{xx} + \mu w \bar{C}_x = 0 \quad \ldots \quad (3\text{-}60)$$

for which eq. (3-56) has been used.

The general solution of this ordinary second-order differential equation is formed by

$$\bar{C}(x,p) = A_1(p)e^{\lambda_1 x} + A_2(p)e^{\lambda_2 x} \quad \ldots \quad (3\text{-}61)$$

with

$$\lambda_{1,2} = \frac{\mu w \pm \sqrt{(\mu w)^2 + 4pD}}{2D} \quad \ldots \quad (3\text{-}62)$$

As λ_1 becomes positive, it can be stated $A_1(p) = 0$ in order to fulfil the requirement of finite concentrations.

For $x = 0$ follows from eq. (3-58)

$$\bar{C}(0,p) - (D/\mu w)\bar{C}_x(0,p) = \tau_*/KT \quad \ldots \quad (3\text{-}63)$$

or

$$A_2 = \frac{\tau_*}{KT}\,\frac{\mu w}{\mu w - D\lambda_2} = \frac{\tau_*}{KT}\,\frac{\mu w}{D\lambda_1} \quad \ldots \quad (3\text{-}64)$$

Thus

$$\bar{C}(x,p) = \frac{\tau_*}{KT}\left[\frac{1}{\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{pD}{(\mu w)^2}}}\cdot \exp\left\{\frac{\mu w - \sqrt{(\mu w)^2 + 4pD}}{2D}\,x\right\}\right] \quad \ldots \quad (3\text{-}65)$$

If for the time being $a = x/\sqrt{D}$ and $b = \mu w/(2\sqrt{D})$ is taken it yields

$$\bar{C}(x,p) = \frac{\tau_*}{KT}\,\frac{2b}{b + \sqrt{p + b^2}} \exp\left[ab - a\sqrt{p + b^2}\right] \quad \ldots \quad (3\text{-}66)$$

This expression can be transformed according to CARSLAW and JAEGER [2] into

$$C(x,t) = \frac{\tau_*}{KT}\left[2b \exp(ab - b^2 t)\left\{\frac{1}{\sqrt{\pi t}} \exp\left(-\frac{a^2}{4t}\right) - b \exp(ab + b^2 t)\cdot \right.\right.$$
$$\left.\left.\cdot\, \text{erfc}\left(\frac{a}{2\sqrt{t}} + b\sqrt{t}\right)\right\}\right] \quad \ldots \quad (3\text{-}67)$$

Or

$$C(x,t) = \frac{\tau_*}{KT}\left[\frac{\mu w}{\sqrt{\pi D t}} \exp\left\{-\frac{(x - \mu w t)^2}{(2\sqrt{Dt})^2}\right\} - \frac{(\mu w)^2}{2D} \exp\left(\frac{x\mu w}{D}\right) \text{erfc}\left\{\frac{x + \mu w t}{2\sqrt{Dt}}\right\}\right] \quad (3\text{-}68)$$

It can be studied how the solution in eq. (3-68) is changed if heterogeneous material is present. In this case also the tracer-material will have a grain-size distribution. This grain-size distribution $\bar{q}_i$ is easily determined. It has to be noted that the application of the diffusion model for each size-fraction separately gives the possibility

$$\bar{q}_i \neq \bar{p}_i$$

If the input of tracer-material is again τ_* per unit width, then the input for the ith fraction becomes $\bar{q}_i\tau_*$.

The expression for the instantaneous source becomes for C_1 and C_2 respectively

$$[\mu_1\bar{\bar{w}}\bar{\bar{C}}_1(0,t)]_i - [D_1\bar{\bar{C}}_{1x}(0,t)]_i = \delta(t)q_i\tau_*/(\bar{p}_i\delta) \quad \text{(3-69)}$$

and

$$[\mu_2\bar{\bar{w}}\bar{\bar{C}}_2(0,t)]_i - [D_2\bar{\bar{C}}_{2x}(0,t)]_i = \delta(t)q_i\tau_*/\delta \quad \text{(3-70)}$$

With the two definitions of C it is only possible to arrive at adequate solutions if it is assumed that $\bar{p}_i = \bar{\bar{p}}_i$ at $x = 0$, $t = 0$.

Due to the similarity to homogeneous material, as far as the equations are concerned, one may write directly

$$\bar{\bar{C}}(x,t) = [\tau_*/(K\bar{T})]F_1\{x,t,\mu\bar{\bar{w}},D\} \quad \text{(3-71)}$$

$$\bar{\bar{C}}_{1i}(x,t) = [\bar{q}_i\tau_*/(K_i'\bar{T}_i)]F_{1i}\{x,t,\mu_1\bar{\bar{w}},D_1\} \quad \text{(3-72)}$$

$$\bar{\bar{C}}_{2i}(x,t) = [\bar{p}_i\bar{q}_i\tau_*/(K_i''\bar{T}_i)]F_{1i}\{x,t,\mu_2\bar{\bar{w}},D_2\} \quad \text{(3-73)}$$

The equation (3-71) is the same as eq. (3-68); the other two contain the same function F_1. In addition to the comparison made in par. 3.2.2 regarding $\bar{\bar{C}}_1$ and $\bar{\bar{C}}_2$ it can be stated that $\bar{\bar{C}}_2 = \bar{p}\bar{\bar{C}}_1$ leads to $\mu_1 \equiv \mu_2$ and $D_1 \equiv D_2$ also for the solution. This means that the assumption of $\bar{p}_i$ present at $x = 0$, t $= 0$ over the thickness δ is not introducing deviating concepts in the derivation.

Finally the remark can be made that the source at the unknown location $x = x_0$ leads to the solution for uniform material:

$$C(x,t) = \frac{\tau_*}{KT}\left[\frac{\mu w}{\sqrt{\pi Dt}}\exp\left\{-\left(\frac{x-x_0-\mu wt}{2\sqrt{Dt}}\right)^2\right\} - \frac{(\mu w)^2}{2D}\exp\left(\frac{(x-x_0)\mu w}{D}\right)\cdot\right.$$

$$\left.\cdot\operatorname{erfc}\left(\frac{x-x_0+\mu wt}{2\sqrt{Dt}}\right)\right] = \frac{\tau_*}{KT}F_1\{(x-x_0),t,\mu w,D\} \quad \text{(3-74)}$$

with similar expressions for heterogeneous material.

Each injection-procedure can now be described by summation of a number of standard solutions of eq. (3-68). For a continuous injection of τ, dimension $[L^2T^{-1}]$ one obtains

$$C(x,t) = \frac{\tau}{KT} \int_0^t F_1\{(x-x_0),\, t',\, \mu w,\, D\}\, dt' \quad \text{(3-75)}$$

It has to be noted that eq. (3-75) leads to

$$\lim_{t \to \infty} C(x,t) = \frac{\tau}{KT} \quad \text{(3-75a)}$$

This equation fulfils a physical necessity. At the left hand side C is a ratio between tracers and sand measured in the same way: either true volume, true weight, apparent volume or apparent weight. The right-hand side consists of the ratio τ/T and the parameter K. Both τ and T are supposed to be measured in the same way, viz. by apparent volume. Injection during a long period gives the possibility of complete mixing. Therefore the concentration becomes inside δ independent of z. This asymptotical case leads to $K = 1$.

3.3 The two-dimensional model

3.3.1 *Deduction*

The diffusion-model will now be generalized for the case in which the dispersion of material in the horizontal y-direction is not neglected. So except for the second one, all assumptions made at the beginning of sect. 3.2.1 will also be made here.

The flux of bed-material is now defined with three components

$$\left.\begin{aligned} &\text{in } x\text{-direction} \quad w_1(x,y,z,t) = \bar{w}_1(z) + w_1'(x,y,z,t) \\ &\text{in } y\text{-direction} \quad w_2(x,y,z,t) = w_2'(x,y,z,t) \\ &\text{in } z\text{-direction} \quad W(x,y,z,t) = W'(x,y,z,t) \end{aligned}\right\} \quad \text{(3-76)}$$

The equation of continuity for the tracer-material becomes for the element $dx\, dy\, dz$

$$(bC)_t + (w_1C)_x + (w_2C)_y + (WC)_z = 0 \quad \text{(3-77)}$$

If again

$$C(x,y,z,t) = \bar{C}(x,y,z,t) + C'(x,y,z,t) \quad \text{(3-78)}$$

and taking the average over θ of eq. (3-77), it yields

$$(\bar{b}\bar{C})_t + (\bar{w}_1\bar{C})_x + (\overline{w_1'C'})_x + (\overline{w_2'C'})_y + (\overline{WC})_z = 0. \quad \text{(3-79)}$$

in which $\overline{b'C'}$ has again been neglected.

The last term of eq. (3-79) gives no contribution to the integral over the interval $-\infty < z < +\infty$

Thus

$$\int_{-\infty}^{+\infty} (\bar{b}\bar{C})_t dz + \int_{-\infty}^{+\infty} (\bar{w}_1\bar{C})_x dz + \int_{-\infty}^{+\infty} (\overline{w_1'C'})_x dz + \int_{-\infty}^{+\infty} (\overline{w_2'C'})_y dz = 0 \quad \text{(3-80)}$$

According to par. 3.2 it can be said

$$\int_{-\infty}^{+\infty} (\bar{b}\bar{C})_t dz = K_1\, \bar{\bar{b}}\, \delta\, \bar{\bar{C}}_t \quad \text{(3-81)}$$

and

$$\int_{-\infty}^{+\infty} (\bar{w}_1\bar{C})_x dz = K_2 \bar{\bar{w}}\, \delta\, \bar{\bar{C}}_x \quad \text{(3-82)}$$

Similar to the one-dimensional case, the assumption of a diffusion-model has now to be made as follows.

$$\left.\begin{aligned} \overline{w_1'C'} &= -D'_{11}(z)\bar{C}_x - D'_{12}(z)\bar{C}_y \\ \overline{w_2'C'} &= -D'_{21}(z)\bar{C}_x - D'_{22}(z)\bar{C}_y \end{aligned}\right\} \quad \text{(3-83)}$$

Thus

$$\int_{-\infty}^{+\infty} (\overline{w_1'C'})_x dz + \int_{-\infty}^{+\infty} (\overline{w_2'C'})_y dz = -\frac{\partial^2}{\partial x^2}\int_{-\infty}^{+\infty} D'_{11}(z)\bar{C}dz +$$

$$-\frac{\partial^2}{dxdy}\int_{-\infty}^{+\infty} [D'_{12}(z)+D'_{21}(z)]\bar{C}dz - \frac{\partial^2}{\partial y^2}\int_{-\infty}^{+\infty} D'_{22}(z)\bar{C}dz \quad \text{(3-84)}$$

Or

$$K_1\bar{\bar{C}}_t - K_3\bar{\bar{b}}^{-1}\,\overline{D'_{11}}\,\bar{\bar{C}}_{xx} - [K_4\bar{\bar{b}}^{-1}\,\overline{D'_{12}} + K_5\bar{\bar{b}}^{-1}\,\overline{D'_{21}}]\,\bar{\bar{C}}_{xy} +$$

$$-K_6\bar{\bar{b}}^{-1}\,\overline{D'_{22}}\,\bar{\bar{C}}_{yy} + K_2\bar{\bar{w}}_1\,\bar{\bar{b}}^{-1}\,\bar{\bar{C}}_x = 0 \quad \text{(3-85)}$$

Hence

$$\bar{\bar{C}}_t - D_{11}\bar{\bar{C}}_{xx} - (D_{12}+D_{21})\bar{\bar{C}}_{xy} - D_{22}\bar{\bar{C}}_{yy} + \mu\bar{\bar{w}}_1\bar{\bar{C}}_x = 0 \quad \text{(3-86)}$$

with

$$\left.\begin{aligned} \mu &= K_2/(\bar{\bar{b}}K_1) = K/\bar{\bar{b}} \\ D_{11} &= (K_3/K_1)\,\bar{\bar{b}}^{-1}\,\overline{D'_{11}} \\ D_{12} &= (K_4/K_1)\,\bar{\bar{b}}^{-1}\,\overline{D'_{12}} \\ D_{21} &= (K_5/K_1)\,\bar{\bar{b}}^{-1}\,\overline{D'_{21}} \\ D_{22} &= (K_6/K_1)\,\bar{\bar{b}}^{-1}\,\overline{D'_{22}} \end{aligned}\right\} \quad \text{(3-87)}$$

The definitions of K_i are similar to those for the one-dimensional case.

From the coefficients K_i only K_1 and K_2 need further attention as the others are combined in the diffusion-coefficients D_{ij}. The foregoing assumptions lead to a diffusion system as it is present in anisotropic media. The additional assumptions can lead to simplification of eq. (3-86). This is caused by the fact that the x-axis has been chosen in the direction of the transport-vector. This implies symmetry round the x-axis. Therefore eq. (3-86) must be invariant for the choice of the positive y-axis. This holds for every term of eq. (3-86) except for $(D_{12}+D_{21})\bar{\bar{C}}_{xy}$. For this term invariance is only obtained if either $\bar{\bar{C}}$ is antisymmetrical with respect to the x-axis or $D_{12}+D_{21} = 0$. The first possibility can be dropped and therefore yields

$$\bar{\bar{C}}_t - D_{11}\bar{\bar{C}}_{xx} - D_{22}\bar{\bar{C}}_{yy} + \mu\bar{\bar{w}}_1\bar{\bar{C}}_x = 0 \quad \text{(3-88)}$$

Therefore the choice of the x-axis in the direction of the transport-vector includes that x and y are the principal axis of diffusion [3].

It is obvious that for heterogeneous material expressions can be derived similar to those for the one-dimensional case.

3.3.2 *Solutions for instantaneous sources*

The solution for an instantaneous source at (0,0,0) will be obtained by the application of a boundary-condition similar to the one for the one-dimensional case. By dropping the bars indicating the averages it yields for the simplest case,

$$C_t - D_{11}C_{xx} - D_{22}C_{yy} + \mu w_1 C_x = 0 \quad \text{(3-89)}$$

The initial-condition is

$$C(x,y,0) = 0 \quad \text{(3-90)}$$

The boundary-conditions are

$$C(x,\pm\infty,t) = 0 \quad \text{(3-91)}$$

$$C(\infty,y,t) = 0 \quad \text{(3-92)}$$

and

$$\mu w_1 C(0,y,t) - D_{11}C_x(0,y,t) = \frac{\tau_{**}}{\delta}\,\delta(y)\delta(t) \quad \text{(3-93)}$$

here τ_{**}, the tracer-supply has the dimensions $[L^3]$.

This implies that the resulting flux in $(0,0,t)$ has the direction of the x-axis. The assumption can be based on considerations of symmetry.

By applying Fourier-transforms for the y-direction it yields

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} e^{iky}C(x,y,t)\,dy = \bar{C}(x,k,t) \quad \text{(3-94)}$$

This leads to the system

$$\left.\begin{aligned} &\bar{C}_t - D_{11}\bar{C}_{xx} + k^2 D_{22}\bar{C} + \mu w_1\bar{C}_x = 0 \\ &\bar{C}(x,k,0) = 0 \\ &\bar{C}(\infty,k,t) = 0 \\ &\mu w_1\bar{C}(0,k,t) - D_{11}\bar{C}_x(0,k,t) = \frac{\tau_{**}}{\delta\sqrt{2\pi}}\,\delta(t) \end{aligned}\right\} \quad \text{(3-95)}$$

If Laplace-transforms are applied for t according to

$$\int_0^\infty e^{-pt}\bar{C}(x,k,t)\,dt = \bar{\bar{C}}(x,k,p) \quad \text{(3-96)}$$

it yields

$$\left.\begin{aligned} &p\bar{\bar{C}} - D_{11}\bar{\bar{C}}_{xx} + k^2 D_{22}\bar{\bar{C}} + \mu w_1 \bar{\bar{C}}_x = 0 \\ &\bar{\bar{C}}(\infty,k,p) = 0 \\ &\mu w_1 \bar{\bar{C}}(0,k,p) - D_{11}\bar{\bar{C}}_x(0,k,p) = \frac{\tau_{**}}{\delta\sqrt{2\pi}} \end{aligned}\right\} \quad \text{(3-97)}$$

The solution of the differential equation is

$$\bar{\bar{C}}(x,k,p) = A_1(k,p)e^{\lambda_1 x} + A_2(k,p)e^{\lambda_2 x} \quad \text{(3-98)}$$

with

$$\lambda_{1,2} = \frac{\mu w_1 \pm \sqrt{(\mu w_1)^2 + 4D_{11}(p + k^2 D_{22})}}{2D_{11}} \quad \text{(3-99)}$$

As $\bar{\bar{C}}(\infty,k,p) = 0$ it yields $A_1(k,p) = 0$.

Furtheron

$$\mu w_1 \bar{\bar{C}}(0,k,p) - D_{11}\bar{\bar{C}}_x(0,k,p) = A_2(\mu w_1 - D_{11}\lambda_2) = \frac{\tau_{**}}{\delta\sqrt{2\pi}} \quad \text{(3-100)}$$

Thus

$$A_2 = \frac{2\tau_{**}}{\bar{\bar{b}}\delta\sqrt{2\pi}\{\mu w_1 + \sqrt{(\mu w_1)^2 + 4D_{11}(p + k^2 D_{22})}\}} \quad \text{(3-101)}$$

and

$$\bar{\bar{C}}(x,k,p) = \frac{\tau_{**}}{\delta\sqrt{2\pi}}\frac{1}{\sqrt{D_{11}}}\frac{\exp\left[\frac{x\mu w_1}{2D_{11}} - \frac{x}{\sqrt{D_{11}}}\sqrt{p + \frac{(\mu w_1)^2}{4D_{11}} + k^2 D_{22}}\right]}{\frac{\mu w_1}{2\sqrt{D_{11}}} + \sqrt{p + \frac{(\mu w_1)^2}{4D_{11}} + k^2 D_{22}}} \quad \text{(3-102)}$$

This expression can be transformed according to Carslaw and Jaeger [2] into

$$\bar{C}(x,k,t) = \frac{\tau_{**}}{\delta\sqrt{2\pi}}\cdot\frac{1}{\sqrt{D_{11}}}\exp\left[\frac{x\mu w_1}{2D_{11}} - \left\{\frac{(\mu w_1)^2}{4D_{11}} + k^2 D_{22}\right\}t\right]\cdot\left[\frac{1}{\sqrt{\pi t}}\exp\left\{-\frac{x^2}{4D_{11}t}\right\} + \right.$$

$$\left. - \frac{\mu w_1}{2\sqrt{D_{11}}}\exp\left\{\frac{x\mu w_1}{2D_{11}} + \frac{(\mu w_1)^2 t}{4D_{11}}\right\}\operatorname{erfc}\left\{\frac{x}{2\sqrt{D_{11}t}} + \frac{\mu w_1\sqrt{t}}{2\sqrt{D_{11}}}\right\}\right] =$$

$$= \frac{\tau_{**}}{\delta\sqrt{2\pi}}\exp\left[-k^2 D_{22}t\right]\cdot\left[\frac{1}{\sqrt{\pi D_{11}t}}e^{-\left\{\frac{x-\mu w_1 t}{2\sqrt{D_{11}t}}\right\}^2} - \frac{\mu w_1}{2D_{11}}e^{\left(\frac{x\mu w_1}{D_{11}}\right)}\operatorname{erfc}\left\{\frac{x+\mu w_1 t}{2\sqrt{D_{11}t}}\right\}\right] \quad \text{(3-103)}$$

The Fourier-transform of the factor exp $[-D_{22}k^2t]$ can be obtained by

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} e^{-iky-D_{22}k^2t}dk = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} e^{\left[-\left\{k\sqrt{D_{22}t}-\frac{iy}{2\sqrt{D_{22}t}}\right\}^2-\frac{y^2}{4D_{22}t}\right]} dk =$$

$$= \frac{\exp\left\{-\frac{y^2}{4D_{22}t}\right\}}{\sqrt{D_{22}t}}\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} e^{-u^2}du = \frac{\exp\left[-\frac{y^2}{4D_{22}t}\right]}{\sqrt{2D_{22}t}} \quad . \; . \; . \; . \; . \quad (3\text{-}104)$$

Therefore the solution becomes finally (with $T = w_1\delta$):

$$C(x,y,t) = \frac{\tau_{**}}{2KT\sqrt{\pi D_{22}t}}\, e^{-\frac{y^2}{4D_{22}t}}\left[\frac{\mu w_1}{\sqrt{\pi D_{11}t}}\exp\left\{-\left(\frac{x-\mu w_1 t}{2\sqrt{D_{11}t}}\right)^2\right\} + \right.$$

$$\left. -\frac{(\mu w_1)^2}{2D_{11}}\exp\left(\frac{x\mu w_1}{D_{11}}\right)\operatorname{erfc}\left(\frac{x+\mu w_1 t}{2\sqrt{D_{11}t}}\right)\right] =$$

$$= \frac{\tau_{**}}{KT}F_2\{x, y, t, \mu w_1, D_{11}, D_{22}\} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (3\text{-}105)$$

It is now possible to introduce

$$\Omega(x,t) = \int_{-\infty}^{+\infty} C(x,y,t)dy \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (3\text{-}106)$$

This integration carried out with the help of eq. (3-105) leads to

$$\Omega(x,t) = \frac{\tau_{**}}{KT}\left[\frac{(\mu w_1)}{\sqrt{\pi D_{11}t}}\exp\left\{-\left(\frac{x-\mu w_1 t}{2\sqrt{D_{11}t}}\right)^2\right\} + \frac{(\mu w_1)^2}{4D_{11}}\exp\left(\frac{x\mu w_1}{D_{11}}\right)\operatorname{erfc}\left(\frac{x+\mu w_1 t}{2\sqrt{D_{11}t}}\right)\right]$$

$$= \frac{\tau_{**}}{KT}F_1\{x, t, \mu w_1, D_{11}\} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (3\text{-}107)$$

This is obviously the same expression as the solution for the similar, but one-dimensional case (eq. (3-68)). The result of eq. (3-107) can also be obtained by direct integration of the differential equation and the boundary-condition.

The use of eq. (3-105) or eq. (3-107) depends on the set-up of the tracer-experiments. Integrated measurements for cross-sections, mostly carried out for radioactive tracers, give the possibility to use eq. (3-107). The advantage is that the parameter D_{22} has not to be determined. This important aspect will be given further consideration in chapter 5.

In this case the source can also be placed at the location $x = x_0$ this leads to analogue solutions

$$C(x,y,t) = \frac{\tau_{**}}{KT}F_2\{(x-x_0), y, \mu w_1, D_{11}, D_{22}\} \quad . \; . \; . \; . \; . \; . \; . \quad (3\text{-}108)$$

or

$$\Omega(x,y) = \frac{\tau_{**}}{KT}F_1\{(x-x_0), \mu w_1, D_{11}\} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (3\text{-}109)$$

HOOFDSTUK 4

TRACER-TECHNIQUES

4.1 Introduction

Chapter 3 deals with a theoretical dispersion-model. Under certain circumstances, expressions have been found for the concentrations of tracers.

Now some practical aspects of the tracer-technique will be discussed. The use of fluorescent tracers or luminophores will be postulated. Firstly the preparation of these tracers is considered (par. 4.2). The injection-procedure is very important; it may influence the applicability of any theoretical dispersion-model. Therefore the injection-procedure will have a separate treatment (par. 4.3).

The determination of the concentration demonstrates the typical aspects of fluorescent tracers. Bottom samples have to be taken at different locations (x) and times (t). From these samples the concentration has to be determined. The various aspects of this topic have been discussed in par. 4.4. Special attention has been paid to the relative accuracy of the concentration.

4.2 Fluorescent tracers

The use of fluorescent particles as tracer-material has been introduced by ZENKOVITCH. Coloured particles have already been applied some decades ago, e.g. by EINSTEIN [6]. These particles however are difficult to detect, especially if small grains and small concentrations are present.

Fluorescent tracers or luminophores may be artificial grains or original sand coated with a thin layer containing a fluorescent dye. There is a general tendency towards the use of coated sand. Artificial grains, though similar to sand with regard to density and diameter, do mostly have a rather irregular grain-form. Regularly coated sand does not have this disadvantage.

Many publications describe the fabrication of luminophores. Different techniques are applicable depending on the actual requirements. The main requirements are

a. Tracers have to be identical to the natural sand.
b. The colours have to be easily detectible.
c. The preparation must be easy and inexpensive.

The choice of the tracer-type is governed by the whole set-up of the experiments. The preparation-costs are not dominant if small laboratory-tests have to be carried out. The second requirement is important for small grain-sizes, and if a large number of samples has to be treated. In that case the apparent tendency towards electronic counting may give the need for bright colours of restricted spectra. This question will be considered in greater detail in section 4.4.4.

The comparison of the different tracer-types proposed is difficult for the above mentioned reasons. Therefore only a general rather subjective judgement can be given.

no. 1. The receipt originated from the Hydraulic Research Station at Wallingford U.K. [18] has many advantages. The colours are bright and the grain-form is correct. Though the price of the ingredients is low, the costs of the tracers are relatively high due to the amount of labour.

no. 2. Yasso [32] gives a comparison of seven commercially available coating mixtures. These mixtures consist of a dye, a vehicle and a solvent like the one given at no. 1. No special attention has been paid to the number of suitable colours. Much attention has been paid to the thickness of the coating, the (reasonable) costs and the brightness of the luminophores.

no. 3. The use of casein [9] has also led to a relatively cheap tracer. The brightness of these tracers is not quite large.

no. 4. Several tracer-receipts have been studied by Griesseier and Voigt [10]. Special attention has been paid to the thickness of the coating. No information has been given about the brightness of the grains. It is, however, enigmatical why a double-, or even eight-fold marking is applied for some basic experiments, if a single marking would give bright colours and a resistent coating.

no. 5. Fluorescent paints on a water-basis are applied by the Delft Hydraulics Laboratory. These paints are commercially available. The coating, after drying, is not soluble in water. The resistance of the coating seems to be adequate. The number of colours is restricted.

In summary it can be stated that though tracers of different applicability have been proposed, much progress will be made in future if the other aspects of tracer-experiments have obtained a similar development as the preparation has already undergone.

4.3 Injection-procedures

An injection-procedure must be chosen in order to add the tracers to the sand of the mobile bed. One may state that two important questions form the crucial point of the application of tracers for natural rivers.

1. The location of the injection cannot be known exactly.
2. The intensity of the injection is not known exactly.

Before these questions will be discussed, attention will be paid to the injection-procedures commonly used for laboratory-flumes. Here the sand-bed can be dewatered and part of the sand can be replaced by tracers. The location and the intensity is known in this case, provided that the tracers are placed not too deep. However, also in this case some initial effects may be present during the transport of the tracers. This is caused by the fact that the tracers can never be placed in a natural position. So even in this case of an almost ideal injection-procedure there is a need for a certain "accomodation".

For most flume-studies the flume can be kept in operation for a time long enough to make the accomodation-time negligible. Natural rivers, however, show changes in the hydraulic conditions. It has been stated already that periods of stationary-stochastical movement do have restricted lengths. Therefore the accomodation of the tracers will take place during periods in which measurements have to be taken. This is the more relevant as the injection cannot but be far from ideal and therefore the accomodation-time may be relatively large.

The injection may not disturb the transport-process (basic condition). This implies that the amount of tracers released at one time and in one place has to be restricted. Therefore repeated injections are required to arrive at significant concentrations.

With reference to the two questions raised at the beginning of this paragraph the following can be said. The uncertainty about the *location* of the injection is caused by the fact that the tracers have to be placed on the bed through the running water. Large velocity-gradients near the bottom may cause an unknown initial transport of the tracers which does not belong to the normal transport-mechanism. This effect can be incorporated in the theoretical dispersion-model by introducing the unknown parameter x_0 (section 3.2.3). The model obtains an extra parameter, which is a disadvantage. It makes the model more difficult to handle. In principle, however, this difficulty seems to be soluble.

The second question, that of the *intensity* is more difficult. Part of the injected tracers may be trapped temporarily in a trough behind a crest of a dune or ripple. Some tracers may remain buried for a considerable time as special tests show. Especially if it happens that the injection is carried out in a relatively deep trough. The result of this situation can be demonstrated via the theoretical model. According to eq. 3-74 one may write

$$C = PF_1\{(x-x_0), \mu w, D\} \qquad (4\text{-}1)$$

with $P = \tau_*/KT$.

In this case the parameter P contains the part τ_* of the injection participating in the transport almost immediately. If temporary trapping takes place, τ_* is not known. This implies that even if P could be determined from the measurements of C, no information about KT, the denominator of P, is obtained. It can be concluded that temporary trapping, i.e. uncertainty about the time at which the tracers take part in the process, reduces the applicability of theoretical dispersion-models. In this case one may only try to derive μw and D from the measurements $C(x,t)$; and information about T is only obtained if the depth of movement is measured separately. This requires special measurements (i.e. core-samples or accurate soundings of the bed profile).

The danger of temporary trapping seems to be reduced if a great precaution is taken at the injection-procedure.

Some injection-methods will be discussed in principle:

1. *Container-injections* will only be applicable if only a small amount is released at one time and at some distance from the bed. It implies a certain initial drift of the tracers, expressed in a parameter x_0. Significant concentrations need a number of small injections. For a natural river this number can be obtained either by distribution over different locations perpendicular to the stream, or by injections at different times.

2. *Capsule-injections.* The initial drift of the tracers can be suppressed if the tracers are released very close to the bed and in small portions at the same time and in the same place. In this respect the application of polyvinyl-alcohol foil is very attractive. The dissolution-time is some minutes, dependent on the thickness of the foil and the temperature of the water. The foil is dissolved completely and therefore there is no danger that part of the tracers does not participate in the process. Incomplete dissolution present at other capsule-types was noticed by LEAN and CRICKMORE [15]. Other aspects of the use of P.V.A.-capsules have to be studied. The dropping of these capsules can be divided into two important phases: (1) the landing on the bottom and (2) the movement after the landing. The single capsules must be small for the above given reasons and therefore a free fall through the running water will give a large initial drift. More serious is the behaviour of the capsule after the landing. The capsule naturally filled with dry tracers, has a relatively small density. Special qualitative tests show that small capsules are transported by the current until they come at rest after the crest of a ripple or dune. Therefore dropping without special precaution causes the trouble of the tracers being partially buried.

This precaution may be effectuated by lowering the capsules while suspended in a steel frame as is used for conventional bed-load instruments.

4.4 The determination of the concentration

4.4.1 *Introduction*

The theoretical dispersion-model requires measurements in order to make it possible to deduce the parameters, especially the transport-vector. This paragraph will deal with the practice of the determination of the concentration-figures. The application of fluorescent tracers is postulated, therefore the concentration must be determined from samples taken from the river-bed. The questions arising from the sampling-procedure will be discussed in section 4.4.2. The determination of the concentration from the samples will be discussed in sections 4.4.3 and 4.4.4.

4.4.2 *Sampling-procedure*

The concentrations, necessary to evaluate the required parameters are determined from samples. Therefore the accuracy of the concentration is governed firstly by the sampling-procedure and only in the second place by the handling of the obtained samples.

Chapter 3 considers the theoretical dispersion. Expressions have been found for $\bar{\bar{C}}(x,t)$. This includes averaging in the horizontal direction (or the time) on the one hand and on the other hand an average in the vertical direction. It is obvious that samples must be taken from the depth of movement only. However, for the practical effectuation of the measurements two important questions arise

1. Which concentration is measured?
2. How can an optimal information be obtained as to the concentration-distribution?

The grain-size distribution in natural rivers may influence the answers to these questions. However, for the time being uniform grains will be considered. Bottom-samples must be taken from the depth of movement. These samples may be small compared to the size of a ripple or dune. In that case they are taken from the surface of the sandbed, containing sand being in motion and at rest at that particular moment. Particles at rest are partly those which have not moved for a relatively long time, due to the fact that they are buried temporarily in ripples and dunes. Therefore samples taken from the bed-surface already contain an average over a certain time; grains just arriving at that location are taken as well as grains which have been at rest there for about one ripple-period or even more. On the other hand not every sample is taken from the same horizontal plane. Samples, taken at random are sometimes from sand of the top of ripples and dunes but also samples from the trough are present.

Therefore samples taken from the bed-surface can only be considered as estimates of $\bar{\bar{C}}(x,t)$. The only requirement is that no sand below the depth of movement is kept. From the above consideration it can be concluded, however, that only a rough estimate is obtained and large scatter of the measurements can be expected.

The same reasoning can be applied to non-uniform sand. It is known from measurements in natural rivers that the grain-size distribution of moving sand differs from sand at rest. However, tracers and sand are taken from the same size-fraction and the two components are measured under the same conditions.

The question remains how many samples have to be taken and also the question about the adequate size.

Firstly the size of the samples will be considered. This is one of the factors governing the accuracy of the concentration. (See also section 4.4.4.)

Consider a large quantity of sand with a homogeneous concentration Γ. A sample of N grains with X tracers taken from this quantity follows a binominal probability-distribution [11]:

$$p\{x\} = \binom{N}{X} \Gamma^{X}(1-\Gamma)^{N-X} \quad \text{(4-1)}$$

The measured concentration $C = X/N$ has the following characteristics

$$M\{X/N\} = \Gamma \quad \text{(4-2)}$$

$$\sigma\{X/N\} = \sqrt{\Gamma(1-\Gamma)/N} \quad \text{(4-3)}$$

In which M and σ denote the mean and the standard-deviation respectively of the stochastical variable X/N.

For the relative error $\varrho_\Gamma = \sigma/\Gamma$ holds

$$\varrho_\Gamma \Gamma = \sqrt{\Gamma(1-\Gamma)/N} \quad \text{(4-4)}$$

Or, as $\Gamma \ll 1$

$$\Gamma(\varrho_\Gamma)^2 N = 1 \quad \text{(4-5)}$$

If m_i is the number of grains for the unit mass with grain-diameter d_i and G_i is the mass of the sample then:

$$r_C \approx [CmG]^{-1/2} = X^{-1/2} \quad \text{(4-6)}$$

Here r_C stands for the estimate of ϱ_Γ. It means that the relative error of the concentration is inversely proportional to the square root of the number of tracers of the sample. This implies that the relative error is very greatly dependent on the grain-size, in relation with the sample-size and the concentration. As the factor mentioned here is only one of those determining the final accuracy, no recommendation can yet be given about the sample size. This question will be considered further in section 4.4.4.

The number of samples and especially the most attractive location and time for the samples are dependent on the interpretation-technique. A more detailed study will follow in chapter 5.

4.4.3 *On visual counting*

In principle there are two methods for the visual determination of the tracer-concentration of samples taken from the sand-bed. The first method consists of counting the individual tracers by hand. This simple method becomes tiresome and expensive if:

1. The number of tracers in each sample becomes large.
2. The grain-size becomes small.
3. The number of samples is large.

From the considerations of section 4.4.2 it can be noted that the number of tracers in each sample may not be too small if a satisfactory accuracy of the concentration is necessary. Therefore the conditions 1 and 3 counteract the general application of direct visual counting for quantitative tracer-tests.

The second visual method increases the speed of the determination. This method consist of the application of standard-samples. If standard-samples are prepared for different concentrations, one may compare every sample with the standards and fix the concentration with one of the standards. This method, typical for cases in which the conditions 1 and 3 are present, is, however, relatively inaccurate. Experiments showed that the human eye is only able to discriminate between two concentrations with a ratio of about two. This method is therefore only advisable for qualitative tests.

It may be concluded that other means have to be found for an adequate determination of the concentrations for a large range of circumstances.

4.4.4 *Electronic counting*

In several institutes the disadvantages of visual counting have led to the development of an electronic equipment [10, 17, 24, 32]. Firstly some remarks will be made regarding the set-up described by YASSO [32]. This inexpensive set-up is not very attractive for the problems discussed here. It may be stated that this equipment will not be suitable for small concentrations which have to be applied for various reasons. The range of concentrations $0 < C < 1$ will not be present for the measurements considered here. The interval $0 < C < 10^{-2}$ to 10^{-3} is more reliable in practice. The main reasons for this circumstance are

- large concentrations need a large supply of tracers. This is expensive and it disturbs the transport phenomenon;
- large concentrations require measurements close to the injection-point (in time and/or space). This is very inattractive as the physical considerations require the opposite.

Other equipments e.g. the ones described by TELEKI [24], NACHTIGALL [17], GRIESSEIER and VOIGT [10], as well as the one developed by the Delft Hydraulics Laboratory do have much similarity. A short general description will be given about the method.

The mixture of sand and tracers is transported as a curtain of about one grain thickness. The light of the luminophores, generated by a U-V-source is caught by a photomultiplier.

The signal of the photomultiplier is transformed electronically in order to obtain a distinct pulse for each separate luminophore. These pulses are counted electronically.

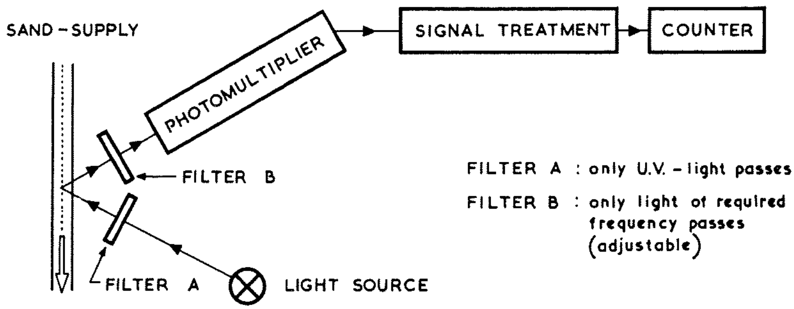

Fig. 4.1 Principle of electronic counter.

The main elements are therefore (see figure 4.1).

a. Mechanical supply system.
b. Radiation with U-V-light
c. The photomultiplier.
d. Signal treatment.
e. Counter.

These elements must have characteristics which enable a harmonic composition. The over-all counting speed is essentially governed by the mechanical system, as a photomultiplier can count upto $3 \cdot 10^4$ regular pulses per second. General improvement of the existing equipment will therefore be obtained by speeding up the regular supply of sand.

An other remark should be made regarding the combination of the elements b., c. and d. Light from the U-V-source must be prevented from entering the photomultiplier by reflection via the sand. Therefore the characteristics of the U-V-source, the tracer and the photomultiplier should be tuned to each other. Corrections can be made by optical filters. In this respect, counting of tracers of various colours in the same sample is important. NACHTIGALL [17] mentions only one colour. The set-up of TELEKI [24] is equipped with four separate channels, each of them is adjusted for a special colour. As this increase of the number of channels determines the cost of the equipment considerably, the D.H.L.-set-up has been basically made for one channel only. By changing the optical filters it is possible to discriminate different tracer-colours.

Some remarks must be made regarding the accuracy of the concentrations measured in this way. This accuracy is determined by three independent factors.

1. The magnitude of the concentration and the size of the sample.
2. The standard-deviation of the counter-efficiency.
3. The conversion of counted particles into weight (or volume).

The first factor has been discussed in section 4.4.2. The relative error determined by this factor is inversely proportional to the square root of the number of tracer-particles.

The second relative error can be determined from the calibration of the counter.

The expression

$$Y = aX \qquad (4\text{-}7)$$

relates the counted number Y with the real number X of tracers in the sample. The counter-efficiency a as well as its standard-deviation s_a will be a function of the grain-size (d), the concentration (C) of the sample and the properties of the counter.

The third error can be determined as follows. If n grains of the size under consideration, represent the mass $g(n)$ with know standard-deviation $s_{g(n)}$, then X grains represent the mass

$$g(X) = \frac{X}{n}\, g(n) \qquad (4\text{-}8)$$

with

$$s_{g(X)} = \sqrt{\frac{X}{n}}\, s_{g(n)} \qquad (4\text{-}9)$$

The relative error in the concentration caused by this factor can therefore be expressed by

$$r_C^2 = \frac{n}{X}\, r^2_{g(n)} \qquad (4\text{-}10)$$

It has to be noted that the factor $nr^2_{g(n)}$ of eq. (4-10) is only determined by the properties of the grains and the sieves but not by the particular choice of n.

The three factors are independent of each other. Therefore from eqs. (4-6, 7 and 10) the total relative error can be expressed by

$$r_C^2 = X^{-1} + r_a^2 + X^{-1} n r^2_{g(n)} \qquad (4\text{-}11)$$

Or

$$r_C^2 = r_a^2 + X^{-1}\{1 + n r^2_{g(n)}\} \qquad (4\text{-}12)$$

This relation is important for the optimal improvement of the accuracy of C. To get more insight in the second term of the righthand member of equation (4-12) the factor $nr^2_{g(n)}$ has been determined for some grain-fractions. The sand, used for the flume-tests described in chapter 6, has been taken for these measurements. Ten times a random selection of 100 grains has been taken. This number $n = 100$ was sufficient to neglect the error in determining the mass. The results are listed in tabel 4-1.

Table 4-1.

Sieve-Fraction 10^{-3} m	$\bar{d}$ 10^{-3} m	n –	$g(n)$ 10^{-6} kg	$s_{g(n)}$ 10^{-6} kg	$nr^2_{g(n)}$ –
0.30–0.42	0.36	100	10.0	0.52	0.28
0.42–0.60	0.51	100	24.4	1.81	0.55
0.60–0.88	0.74	100	69.2	3.85	0.31
0.88–1.7	1.3	100	247	20.7	0.59
1.7 –4.8	3.2	100	1350	18.6	1.9

In general it may be concluded that the influence of the first factor dominates the influence of the third factor. If the ratio of the two sieve-diameters determining the fraction exceeds the normal magnitude of about $\sqrt{2}$ then the third factor becomes more important (see e.g. the fraction 1.7–4.8 mm).

With the help of the data of table 4-1 the maximum relative accuracy of C has been plotted for each fraction versus the product CG of concentration and sample-weight. This plot has been made according to eq. (4-12) for an ideal counter-efficiency ($r_a = 0$), figure 4-2.

The relative error of the counter-efficiency α depends on several factors (1) grain-size, (2) concentration, (3) countingspeed and (4) the number of measurements.

The first two factors are investigated by calibration-tests for samples of 20 grams with a counting-speed of about 0.18 gr/sec. The results are listed in table 4-2. Each value of α and r_a is derived from 10 single observations.

Table 4-2. Counter-efficiency depending on X and d.

	Grain-fraction							
X	0.60–0.88 mm		0.42–0.60 mm		0.30–0.42 mm		0.21–0.30 mm	
	α	r_a	α	r_a	α	r_a	α	r_a
200	0.88	0.02	0.88	0.04	0.87	0.04	0.77	0.04
150	0.91	0.02	0.88	0.04	0.90	0.02	0.84	0.04
100	0.93	0.04	0.91	0.04	0.84	0.05	0.78	0.06
50	0.93	0.03	0.90	0.06	0.86	0.08	0.70	0.06

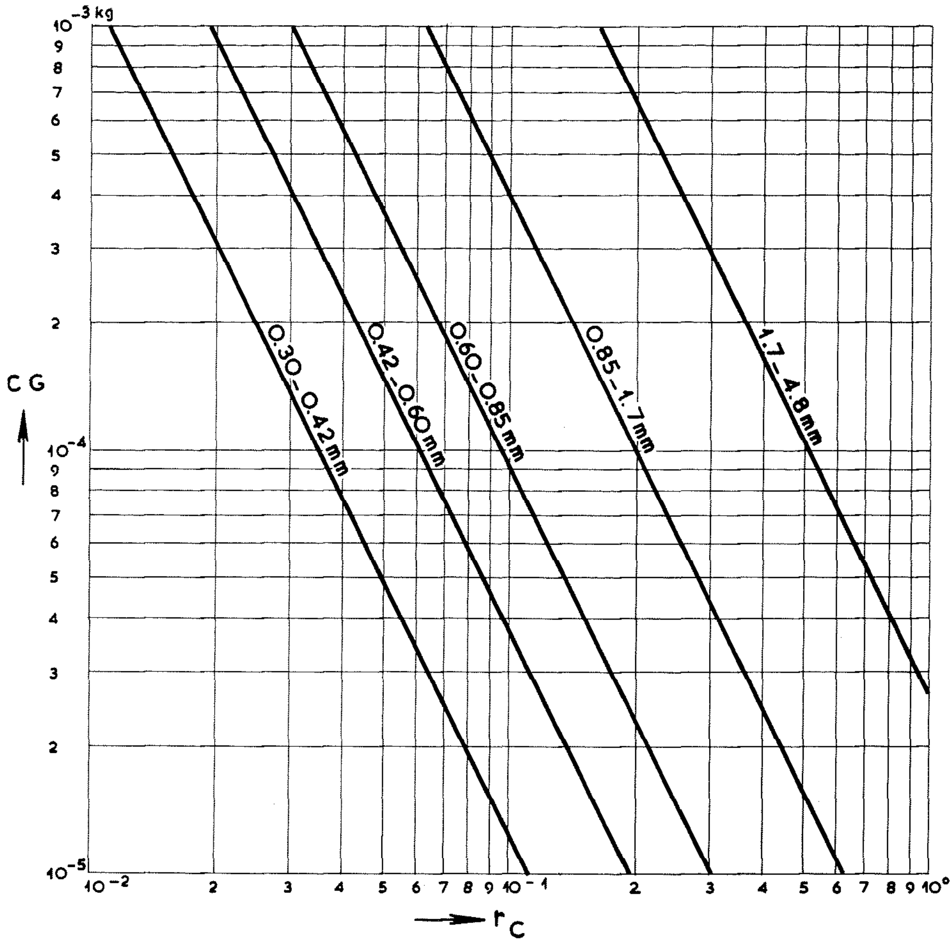

Fig. 4.2 Accuracy of C depending on CG (ideal counter-efficiency).

Further, some tests have been carried out to demonstrate the influence of the counting speed. This has been done for the fraction 0.42–0.60 mm with $X = 100$ and for a sample of 20 grams. The results are listed in tabel 4-3. Also in this case each result has been found from 10 single observations.

Some remarks should be made regarding the results of tables 4-2 and 4-3.

1. Table 4-3 shows that at least for the fraction 0.42–0.60 the counting-speed can be almost ten times as high as the one used for the test of table 4-2, without significant loss of accuracy.
2. A significant discrepancy seems to exist between the figures of the two tables for the circumstances $d = 0.42$–0.60 mm; $X = 100$ and counting-

Table 4-3. The influence of counting-speed

Counting-speed grams/sec	α	r_a
4.04	0.46	0.09
2.41	0.59	0.06
1.13	0.75	0.05
0.526	0.80	0.04
0.177	0.84	0.04
0.0443	0.86	0.04

speed 0.177 g/sec. Probably this has been caused by a factor which has not yet been taken into account. Not each tracer is coated in the same way, therefore irregularities of the coating are present. This is probably also the reason why α varies with X (table 4-2). This variation cannot be significant, it may be caused by the fact that each set of 10 observations has been carried out with different tracers.

3. For the treatment of the samples carried out for the tests described in chapter 6, calibration-curves have been used based on smoothing the values of Y for given values of C. This diminishes the effect of the problem mentioned in the foregoing remark.

Finally a remark has to be made regarding the principle of the electronic equipment. The set-up described above is based on the counting of grains. TELEKI [24] describes briefly that his set-up takes the intensity of the pulses into consideration: i.e. the grain-surface. Deliberately this method has been avoided for the D.H.L.-equipment for the following reasons.

a. Consideration of the intensity of the pulses enlarges the electronic complications.
b. It also enlarges the requirements for the tracer-production. A very regular coating is necessary.
c. The conversion of grain-surface to grain-volume represents a remaining complication. Its influence on the final accuracy of C is only negligible if the sample contains a sufficient large number of tracers.

In conclusion it may be stated from the figures of table 4-2 and 4-3 that the relative accuracy of the counter-efficiency is roughly 5 percent. This can be accepted if the other factors are taken into account. The following reasoning can be given. Suppose $nr^2_{g(n)} = 1/2$ (table 4-1). Then the factors 1 and 3 together give the relative accuracy of 5 percent if $X \approx 6 \cdot 10^2$. This magnitude of X is high. It requires either large samples or large injections. In this case the total value of r_C becomes about 7 percent. As it hardly attractive to enlarge the value of X, a significant decrease of r_a alone will not give a large decrease of r_C. Therefore it does not seem to be economically justified to enlarge the accuracy of the counter-efficiency. A large number of samples will be required in practice to arrive at acceptable errors in the final answers of tracer-tests viz. the transport T.

CHAPTER 5

INTERPRETATION-TECHNIQUES

5.1 Introduction

In recent years different methods have been proposed to arrive at a quantitative interpretation of tracer-measurements. This chapter will deal with a general description of these methods, without describing the measurements in detail. The judgement of the different methods will only be qualitative here. The quantitative properties will be discussed in chapter 6 according to actual measurements.

5.2 Steady dilution

The simplest set-up of tracer-experiments is obtained by applying steady dilution. A constant tracer-supply τ is added to a stationary-stochastical sand-transport $\bar{T}$. An equilibrium concentration $C_0 = \tau/\bar{T}$ is obtained down-stream of the injection-point. Principally the interpretation does not give many difficulties. The disadvantages are formed by physical and technical circumstances.

The character of the transport-process requires a steady condition during a relatively long time. This can be concluded from eq. (3-75a); only for large values of t the tracers become uniformly distributed in the depth of movement, leading to $K = 1$. The diffusion-process must be regarded as very slow especially if it is compared with the diffusion of a soluble matter in the water of a river. Moreover one has to wait for a constant concentration at some distance from the injection-point in order to avoid disturbances from the injection-procedure.

The technical draw-back is caused by the fact that the supply must be given for a long time. This can be overcome by the application of the time-integration method [5, 2]. This method applied for measurements of water-discharges postulates enough mixing to state

$$\bar{T} = \frac{\tau_*}{\int_0^\infty \bar{\bar{C}}(x,t)\,dt} \quad \ldots\ldots\ldots\ldots \quad (5\text{-}1)$$

The physical implications remain the same as eq. (3-75) shows that eq. (5-1) is only correct for $K = 1$ i.e. ideal mixing. This is only true if $\bar{\bar{C}}(x,t)$ is measured at a distance x large enough to ensure sufficient accomodation of the grains; a minimum "accomodation-length" [5] has to be used.

The difficulty of the long time-interval which is required for the steady-dilution is also present here. After a single injection one has to wait until every tracer-particle has passed the location x. In the mean time the character of the transport-process may be changed. It can be stated that the time required is the same for steady-dilution and time-integration [5].

5.3 Spatial integration

A very common procedure for the determination of $\bar{T}$ consists of the separate evaluation of the mean particle speed $\bar{u}$ and the depth of movement δ.

Firstly attention will be paid to the derivation of $\bar{u}$. For a single dropping holds

$$\bar{u} = \frac{1}{t} \frac{\int_0^\infty x\bar{\bar{C}}(x,t)\,dx}{\int_0^\infty \bar{\bar{C}}(x,t)\,dx} \quad \ldots\ldots\ldots\ldots \quad (5\text{-}2)$$

If a number of injections is given, similar expressions can be found. In general the centre of gravity x is calculated and plotted versus the time. This is more cautious as (5-2) already postulates a lineair propagation of the centre of gravity. LEAN and CRICKMORE [15] state that in this case no allowance need be made for accomodation-length, but this statement seems to be questionable (par. 6.3).

The disadvantage of this method is that small concentrations at large distances from the centre of gravity may have great influence on $\bar{u}$. These small concentrations have a restricted accuracy. For fluorescent tracers this is mainly caused by the size of the samples (section 4.4.4). For radioactive tracers these small concentrations are difficult to measure due to the natural background [15]. Also in this case much improvement could be obtained if a theoretical dispersion-model could be used.

Besides the determination of $\bar{u}$ also δ has to be evaluated in order to arrive at the transport $\bar{T}$. In principle there are three methods to derive this depth of movement δ:

1. From analysis of the bed-level $\eta(x)$, (see par. 2.2). This analysis requiring accurate soundings has been used by several investigators [2, 4, 14].
2. By application of core-samples technique [2, 14]. This method seems rather laborious; the irregular character of $\eta(x)$ necessitates a large number of core-samples.

3. The over-all application of the equation of continuity for the tracers [2, 14]. This method is the most attractive. No special measurements are required. However, this method gives uncertain results if part of the injected tracers are temporarily buried.

5.4 Theoretical dispersion-models

In this paragraph not the theoretical dispersion-model itself will be discussed. The aim will be the solution of the following question: How is a theoretical model used in order to obtain the answer about the transport-vector with the measurements?

RANCE [20] has tried to solve this question by putting his model in a dimensionless form and fitting this model with the measurements by trial and error. This "eye-fitting" is somewhat subjective and therefore one may try to obtain a more objective procedure.

It is obvious from the considerations of chapter 4 that the measurements are subject to a large scattering. Each measured concentration is therefore only an estimate of the concentration $\bar{\bar{C}}(x,t)$. In order to compensate the lack of accuracy of the single measurement, it would seem reasonable to take a large number of measurements and apply the method of least-squares.

From par. 4.3 it may be concluded that the injection-procedure has a large influence on the measured concentrations. If the theoretical dispersion-model is constructed via a differential equation (see e.g. chapter 3), one may start trying to evaluate the required information directly from this differential equation.

This procedure has the following characteristics for the diffusion model of chapter 3:

1. The differential equation is linear for two unknowns: μw and D.
2. A separate method is required to evaluate the third unknown δ. One may use one of the methods of par. 5.3.
3. A correct mathematical description of the boundary-condition is not required.

In principle there are two methods to apply the differential equation to the evaluation of two required parameters. The first method consists of the least-squares procedure for the differential equation

$$C_t - AC_{xx} + BC_x = 0 \quad \text{(5-3)}$$

Measurements of $C(x,t)$ for different values of x and t lead to a linear least-squares procedure for A and B. There are, however, two serious draw-backs. The first is formed by the fact that the measured values have to be differentiated numerically, once and even twice. This leads to a considerable loss of accuracy. Secondly the derivative C_t requires the determination of $C(x,t)$ for different values of x and t. The scatter of the single measured values does not permit these direct numerical differentiations. It is obvious that curve-fitting may improve these difficulties. The best curve-fitting, however, is obtained if the values of C are fitted according to the theoretical solution of eq. (5-3). This leads almost automatically to a least-squares procedure for the theoretical solution and not for the basic differential equation.

As differentiation reduces the accuracy of the data, improvement may be obtained by integration of the measured values.

This can be done by multiplication of all the terms of eq. (5-3) by a factor x^i and integration over an interval in which no tracer-sources are present. For this interval can be chosen $a < x < \infty$ with $a > 0$.

One may define

$$\left.\begin{aligned}\int_a^\infty x^i C_t(x,t)\,dx &= \psi_i(t)\\ \int_a^\infty x^i C(x,t)\,dx &= \varphi_i(t)\end{aligned}\right\} \qquad (5\text{-}4)$$

As $\lim\limits_{x\to\infty} C(x,t) = 0$ and $\lim\limits_{x\to\infty} C_x(x,t) = 0$

it yields

$$\int_a^\infty x^i C_x dx = a^i C(a,t) - i\varphi_{i-1}(t) \qquad (5\text{-}6)$$

and

$$\int_a^\infty x^i C_{xx} dx = a^i C_x(a,t) - ia^{i-1}C(a,t) + i(i-1)\varphi(t)_{i-2} \qquad (5\text{-}7)$$

Thus

$$\begin{pmatrix}[i(i-1)\varphi_{i-2} - a^i C_x(a) - ia^{i-1}C(a)] & [i\varphi_{i-1} + a^i C(a)]\\ [i(i+1)\varphi_{i-1} - a^{i+1}C_x(a) - (i+1)a^i C(a)] & [(i+1)\varphi_{i-1} + a^{i+1}C(a)]\end{pmatrix}\begin{pmatrix}A\\ B\end{pmatrix} = \begin{pmatrix}\psi_i\\ \psi_{i+1}\end{pmatrix} \qquad (5\text{-}8)$$

These two linear equations for A and B contain, besides integrated measurements φ_{i-1} and φ_{i-2}, also factors which have roughly the same accuracy as the original material. For instance ψ_i is equal to $\partial\varphi_i/\partial t$ which means that ψ_i can be obtained by integration with respect to x and differentiation with respect to t.

Large values of i increase the contribution of the inaccurate measurements at large values of x. On the other hand small values of i increase the influence of $C(a)$ and $C_x(a)$ on the final result. Therefore some intermediate values of i have to be chosen with a special reference to the particular measurements. This reduces the possibility of introducing more than two values of i in order to apply least-squares procedures for eq. (5-8). The same objection to integrated concentrations with respect to the accuracy (par. 5.3) must be made here.

An important reason for leaving the above given principle is the fact that complete series $C(x,t)$ have to be measured. Further, $C(x)$ must be known at different times t in order to derive the values of $\partial\varphi_i/\partial t$. This system is too rigid for measurements in nature. This can only be accepted if excellent results are expected from this particular method. As this is doubtful other methods have to be found for which less rigid measuring-systems are required, leading to a least-squares procedure with the help of the theoretical solution.

Let $$\Gamma = PF(x,A,B) \qquad (5\text{-}9)$$

be the theoretical dispersion for a fixed value of t. The measured concentrations are $C_i = C(x_i,t)$

In general

$$C_i \neq PF(x_i,A,B) \qquad (5\text{-}10)$$

The least-squares method of evaluating the three parameters P, A and B consists of minimizing the sum S

$$S = \overset{i}{\Sigma}(C_i - \Gamma_i)^2 \qquad (5\text{-}11)$$

This can be tried by solving the set of normal-equations. These normal-equations are obtained by taking the partial derivatives of the sum S with respect to the unknown parameters P, A and B respectively and putting the derivatives equals zero

$$\left.\begin{aligned} \frac{\partial S}{\partial P} &= 2\overset{i}{\Sigma}(C_i - PF_i)(F)_i = 0 \\ \frac{\partial S}{\partial A} &= 2\overset{i}{\Sigma}(C_i - PF_i)(F_A)_i = 0 \\ \frac{\partial S}{\partial B} &= 2\overset{i}{\Sigma}(C_i - PF_i)(F_B)_i = 0 \end{aligned}\right\} \qquad (5\text{-}12)$$

These normal equations are non-linear with respect to the unknowns A and B. The solution has to be obtained by iteration. Therefore a first estimate P_1, A_1, B_1 is needed as a start. The equations are linearized for the unknowns. The solution of these linearized equations (ΔP, ΔA and ΔB) are used for the second estimate. The procedure is repeated until the solution of eq. (5-12) is obtained with sufficient accuracy.

Dependent on the characteristics of the function F, the surface S can become rather complicated even if perfect measurements are used. The surface $S(P,A,B)$ can have local minima which are physically insignificant. One may expect a finite area $G(P,A,B)$ round the significant solution (P_0,A_0,B_0). Inside G a first estimate (P_1,A_1,B_1) can lead to the proper solution. In practice several first estimates may be required to be sure that the correct solution is obtained.

Minimizing the sum S has not yet led to a full proof procedure. Various proposals have been made. In general the calculations are very extensive, especially for complicated functions F. Therefore the help of a large computer is essential. After an extensive study the following procedure happened to be successful.

Let the solution of the linearized eq. (5-12), with the help of the ν-th estimate, be $(\Delta P_\nu, \Delta A_\nu, \Delta B_\nu)$. Now the $(\nu+1)$th estimate is formed by

$$\left.\begin{aligned} P_{\nu+1} &= P_\nu+\lambda\Delta P_\nu \\ A_{\nu+1} &= A_\nu+\lambda\Delta A_\nu \\ B_{\nu+1} &= B_\nu+\lambda\Delta B_\nu \end{aligned}\right\} \qquad (5\text{-}13)$$

The choice $\lambda = 1$ very often does not lead to convergence. Therefore the most adequate value of λ has been determined by minimizing S along the line through (P_ν, A_ν and B_ν) and $(P_{\nu+1}, A_{\nu+1}, B_{\nu+1})_{\lambda=1}$. This local minimizing can be made rather roughly as the next estimate of the parameters has still to be improved. The numerical determination of the derivates is very attractive for these rather awkward functions [19]. Some results of the procedure proposed for measurements will be given in chapter 6.

It may be expected that the number of parameters rules the convergence. Therefore this number should be limited. For a one-dimensional problem, without an initial shift of the source only three parameters are present. This number is increased by one for each of the following cases.

1. The presence of an initial shift of the source (eq. 3-74).
2. The two-dimensional problem (eq. 3-108) if integration of the concentration for cross-sections is not possible.

The use of the least-squares approximation of the dispersion-model seems to be attractive. Only concentrations $C(x,t)$ with sufficient accuracy need to be taken into consideration. No special requirements need to be made with regard to the intervals Δx and Δt. The missing of a sample is not disadvantageous. Therefore the measuring scheme is not fixed, it remains flexible. Unexpected circumstances during the measurements do not lead to the loss of much information.

The accuracy of the final answer is governed by the following factors.

1. The reliability of the theoretical dispersion-model proposed.

2. The accuracy of the concentrations.
3. The number of measurements.

The last two factors can be combined if it is assumed that all the concentrations roughly have the same accuracy. Then the accuracy of the final answer is about $n^{-1/2}$ times that of a single concentration, if n denotes the number of measurements. This implies the general rule that a large number of inaccurate measurements can give the same information as a small number of accurate data.

The method proposed above does use the theoretical dispersion-model intrinsically. Naturally inadequate dispersion-models lead to wrong results. The validity of a theoretical dispersion-model can only be proved by determining the transport-vector from measured concentrations and comparing this value with the one, measured with other means. Obviously flume studies are therefore required.

Some remarks can be made in addition:

1. The application of the least-squares for the solution can be made for concentrations measured at different times and locations.
2. Instead of using the derived parameter P for the determination of T it is also possible to use the derived parameter B together with the depth of movement δ (par. 5.3).

CHAPTER 6

EXPERIMENTS

6.1 General

In this chapter some tracer-experiments are discussed. Naturally the first criterion for the quantitative interpretation is formed by the accuracy of the measured transport. Secondly, the time required to obtain this information is an important feature (par. 2-3). Until now this aspect seems to have got only little attention.

After some experiments with steady dilution (par. 6-2) and spatial integration (par. 6.3) the use of theoretical dispersion-models (par. 6.4) is discussed. Special attention has been paid to the dispersion-model proposed by HUBBELL and SAYRE [14]. Their considerations, published during the preparation of this study, lead to a dispersion-model similar to the one originally described by EINSTEIN [6]. Much attention has been paid to a theoretical comparison between this "Einstein-model" and the diffusion-model described in chapter 3. This comparison has been performed as far as the observations allow.

6.2 Steady dilution

In September 1961 experiments were carried out in the Westervoort-model of the Delft Hydraulics Laboratory. These tests were based on the steady dilution-principle. This river-model of a bifurcation is not the simplest case for this set-up. Although the injection-line, perpendicular to the main current was chosen carefully, variation of $\bar{T}$ over the width of the river-model may be present. At that time luminophores were used according to the Wallingford-receipt [18]. The tracers were supplied almost continuously at different points of the injection-line. The supply of tracers was taken 10^{-3} times the (known) sand-transport. The sand-bed (mean diameter of the grains 0.6 mm) consisted of ripples and dunes.

Samples have been taken at lines perpendicular to the main current with regular intervals Δt, Δx. The samples were taken from the top-level of the bed. The tracer-concentration was determined visually by means of standard-samples (section 4.4.3). Some results are given in figure 6.1. The concentrations presented are obtained by a three-point smoothing of the original measurements. The figure shows that saturation is indeed obtained, but only after a relatively long time.

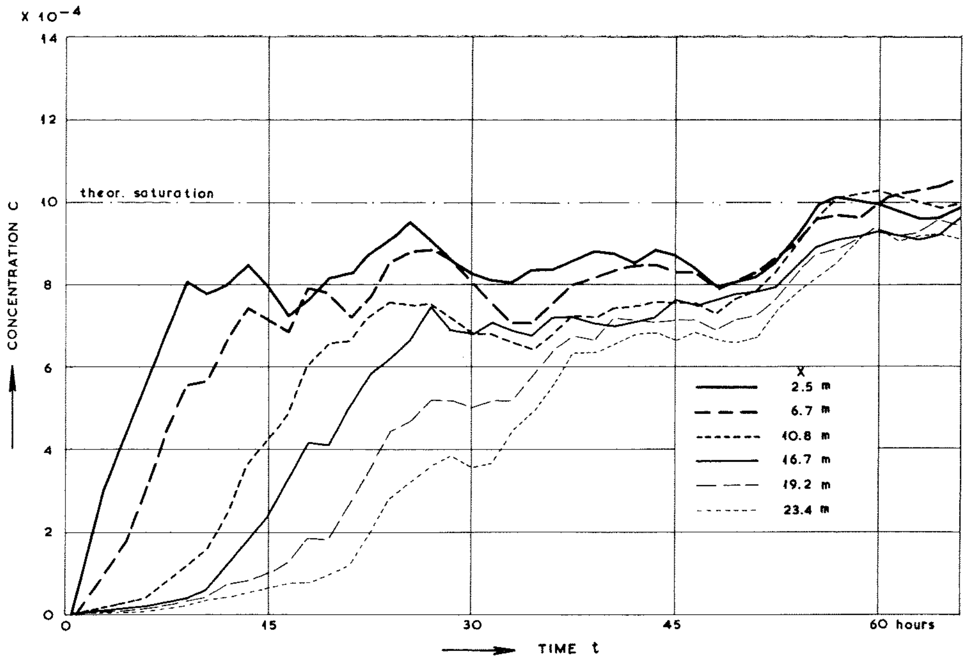

Fig. 6.1 Steady dilution (test S 76-2).

Experiments on steady dilution for the transport of sand under the combined action of currents and waves are performed by RUSSELL et al. [21].

6.3 Spatial integration

Special attention will be paid to the determination of the mean particle-speed by spatial integration (see par. 5.3). For all the laboratory-experiments cited here, the injection of tracers has been arranged by replacing sand of the bed by tracers. In June 1961 a special test of this kind was carried out in the Westervoort-model. At station km 876.8 a strip of sand (width 0.05 m) was removed from the bed and the gap was refilled with a sand-tracer mixture of a concentration 0.25. The measuring procedure was similar to the one for the test described in par. 6.2.

Available data from literature have also been used for the composition of table 6-1. The data consist of the location x_i of the gravity-centre as function of the time t_i. The tests listed as numbers 4 and 5 originated from the same experiments. SAYRE and HUBBELL [22] describe for their North Loup River-experiments how correction of the concentration-curves has been necessary, due to a temporary storage of part of the tracers. Test 4 deals with all their data; test 5 deals with all data except those influenced by corrections.

The derivation of the mean particle speed can be obtained by studying the regression between x_i and t_i.

This has been done by postulating two regression-lines

1. Supposition of a linear regression through the origin:

$$x = b_1 t \quad \text{(6-1)}$$

2. Supposition of a linear regression with stochastically independent parameters [11]:

$$x = a_2 + b_2(t - \bar{t}) \quad \text{(6-2)}$$

In both cases t has been taken as the independent variable because the stochastical variable x (the gravity-centre) has been taken for fixed values of t. In all the cases except 4 and 5 the injection-point at $t = 0$ was also taken as a measured value.

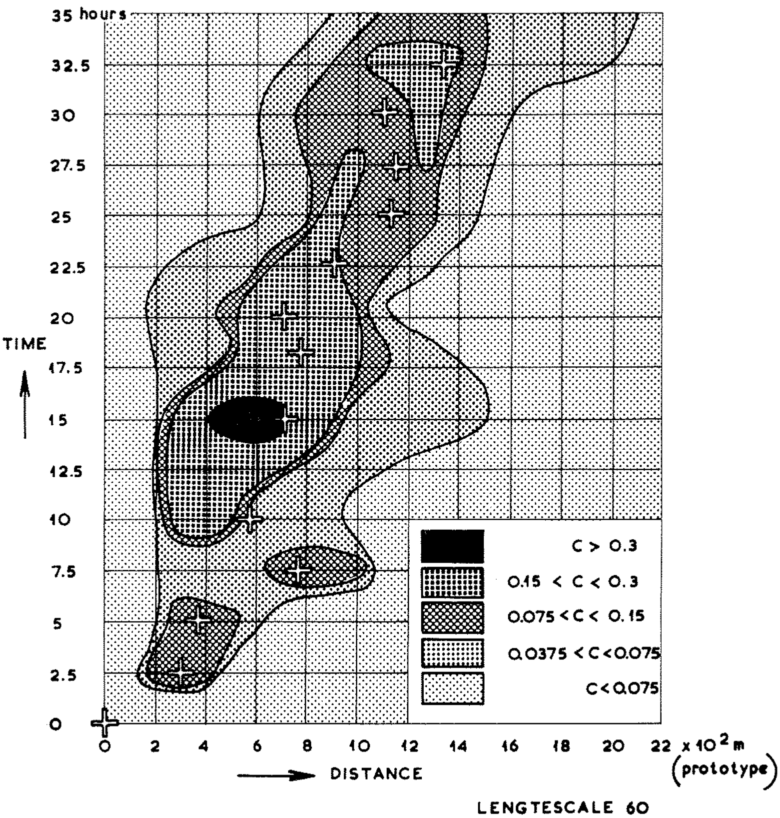

Fig. 6.2 Spatial integration (test S 76-1).

According to HALD [11]

$$b_1 = \frac{\Sigma x_i t_i}{\Sigma (t_i)^2} \quad \ldots \ldots \ldots \ldots \quad (6\text{-}3)$$

$$s_{b_1} = \sqrt{\frac{\Sigma (x_i - b_1 t_i)^2}{(n-2)\Sigma (t_i)^2}} \quad \ldots \ldots \ldots \ldots \quad (6\text{-}4)$$

$$b_2 = \frac{\Sigma x_i (t_i - \bar{t})}{\Sigma (t_i - \bar{t})^2} \quad \ldots \ldots \ldots \ldots \quad (6\text{-}5)$$

$$s_{b_2} = \sqrt{\frac{\Sigma [x_i - \{a_2 + b_2(t_i - \bar{t})\}]^2}{(n-2)\Sigma (t_i - \bar{t})^2}} \quad \ldots \ldots \ldots \ldots \quad (6\text{-}6)$$

with $i = 1, \ldots, n$.

The results are listed in table 6.1. The values derived from the eqs. (6-3) through (6-6) can be used to compare b_1 and b_2.

This comparison has been made in the t-test as it is described by HALD [11]. The results are listed in table 6.1.

Table 6-1. Determination of mean particle-speed.

Case	Regression-line $x = b_1 t$ $b_1 \pm s_{b_1}$ (m/h)	Regression-line $x = a_2 + b_2(t - \bar{t})$ $b_2 \pm s_{b_2}$ (m/h)	Comparison b_1 and b_2	Required time (h)	
1	0.20 ± 0.08	0.047 ± 0.009	$b_1 > b_2$	105	Chatou-test 430, see [2] luminophores, ripples
2	0.70 ± 0.05	0.545 ± 0.06	$b_1 > b_2$	32.5	Test S 76-1, fig. 6.2 luminophores, ripples + dunes 0.6 mm sand
3	0.0211 ± 0.0006	0.0198 ± 0.0010	$b_1 \geqslant b_2$	245.3	HUBBELL, SAYRE [14] +pers. communication radioactive, flume-test
4	1.12 ± 0.04	0.99 ± 0.04	$b_1 > b_2$	287.4	HUBBELL, SAYRE [14, 22], radioactive, North Loup River 0.29 mm sand, dunes
5	1.32 ± 0.06	1.23 ± 0.10	$b_1 = b_2$	117.7	As case 4 but without using data of smaller accuracy [22]

The general tendency is that the mean particle-speed is not constant during the process. The data give the indication that the gravity-centre is moving relatively faster for small values of t. The mean particle-speed is therefore overestimated if relatively short time-periods are chosen for a tracer-test. This does not necessarily include that the derived transport-figures are wrong. At least some tests of CHATOU [2] indicate the understandable behaviour of the depth $\delta(x,t) : \delta$ may be longer if longer time-periods are considered.

It should be remarked that the tests described in table 6.1 dealt with relatively large time-periods. A common duration of these tests was 5 to 10 days. Even for the shortest test (the one of the Westervoort-model) the duration took only half the time of test S 76-2 (steady dilution) mentioned in par. 6.2.

6.4 Properties of dispersion-models

Some authors [2], [15] derive dispersion-models without using them intrinsically for the solution of the problem. Both cases deal with a binomial distribution for a one-dimensional transport in one direction. HUBBELL and SAYRE [14], [22] paid more attention to this problem, though they did not use their model either to derive the transport from their measurements. Nevertheless their theoretical model can be valuable. The basic assumptions for this theoretical model are:

1. One-dimensional transport in one direction.
2. The step-length of the single grains are exponentially distributed with mean step-length k_1^{-1}.
3. The rest-periods of the grains are exponentially distributed with mean duration k_2^{-1}.

These considerations lead to a concentration-distribution similar to the one derived earlier by EINSTEIN [6]:

$$C(x,t) = Pk_1 \exp\left[-(k_1x+k_2t)\right] \sqrt{\frac{k_2t}{k_1x}}\, I_1(2\sqrt{k_1xk_2t}) \quad (6\text{-}7)$$

in which I_1 denotes the modified Bessel-function of the first kind and the first order, and P stands for the intensity of the instantaneous source divided by the product of the depth δ and the width of the bed. HUBBELL and SAYRE derive the parameters k_1 and k_2 by the decay of the relative peak-concentration on the one hand and the propagation of the mode and mean of the concentration $C(x,t)$ on the other . It has to be noted that they arrive at transports by deriving separately the mean particle-speed $\bar{u} = k_2/k_1$ and the depth of movement by spatial integration (par. 5.3 and 6.3).

It has been shown [30] that this "Einstein-model" may be compared with the diffusion-model (chapter 3) by studying the asymptotic behaviour.

From the propagation of the mean-concentration it may be concluded

$$\mu w = k_2/k_1 \quad (6\text{-}8)$$

and the decay of the relative peak-concentration yields

$$D = k_2/k_1^2 \quad (6\text{-}9)$$

Both, the diffusion-model of eq. (3-68) and the Einstein-model of eq. (6-7) behave asymptotically as a Gaussian-distribution of the concentration for fixed (large) values of time.

However, relatively small values of x and t are the most important and therefore the comparison of the two models must be made also for restricted values of x and t.

This comparison can firstly be made for the location of the mean-concentration and that of the peak- concentration, supposing an instantaneous source at (0,0). The location of the centre of gravity is found from

$$x_1(t) = \frac{\int_0^\infty xC(x)\,dx}{\int_0^\infty C(x)\,dx} \qquad (6\text{-}10)$$

The location x_2 of the peak-concentration is found from the condition

$$C_x(x,t) = 0 \qquad (6\text{-}11)$$

The results are respectively:

Diffusion-model:

$$\xi_1 = \eta\left[\frac{1}{\sqrt{\pi\eta}}\exp\left(-{}^1/_4\eta\right) + {}^1/_2 + \left({}^1/_2 + \eta^{-1}\right)\operatorname{erf}\left({}^1/_2\sqrt{\eta}\right)\right] \qquad (6\text{-}12)$$

$$\frac{1}{\eta\sqrt{\pi}}\cdot\frac{\xi_2 - 2\eta}{2\sqrt{\eta}}\exp\left[-\left(\frac{\xi_2+\eta}{2\sqrt{\eta}}\right)^2\right] + {}^1/_2\operatorname{erfc}\left(\frac{\xi_2+\eta}{2\sqrt{\eta}}\right) = 0 \qquad (6\text{-}13)$$

Einstein-model:

$$\xi_1 = \frac{\eta}{1 - \exp(-\eta)} \qquad (6\text{-}14)$$

$$\sqrt{\xi_2}\cdot I_1\{2\sqrt{\xi_2\eta}\} = \sqrt{\eta}\,I_2\{2\sqrt{\xi_2\eta}\} \qquad (6\text{-}15)$$

Here η denotes the dimensionless time

$$\eta = k_2 t = (\mu^2 w^2/D)t \qquad (6\text{-}16)$$

and ξ the dimensionless location

$$\xi = k_1 x = (\mu w/D)x \qquad (6\text{-}17)$$

The dimensionless relative peak-concentration can be expressed as follows, respectively:

For the diffusion-model:

$$C_{\text{peak, rel}} = CD\delta/(\tau_* w) = \frac{1}{\sqrt{\pi\eta}}\exp\left[-\left(\frac{\xi_2-\eta}{2\sqrt{\eta}}\right)^2\right] - {}^1/_2\exp(\xi_2)\operatorname{erfc}\left(\frac{\xi_2+\eta}{2\sqrt{\eta}}\right) \qquad (6\text{-}18)$$

which leads to $C_{\text{peak, rel}} = f(\eta)$ by elimination of ξ_2 from the eqs. (6-13) and (6-18).

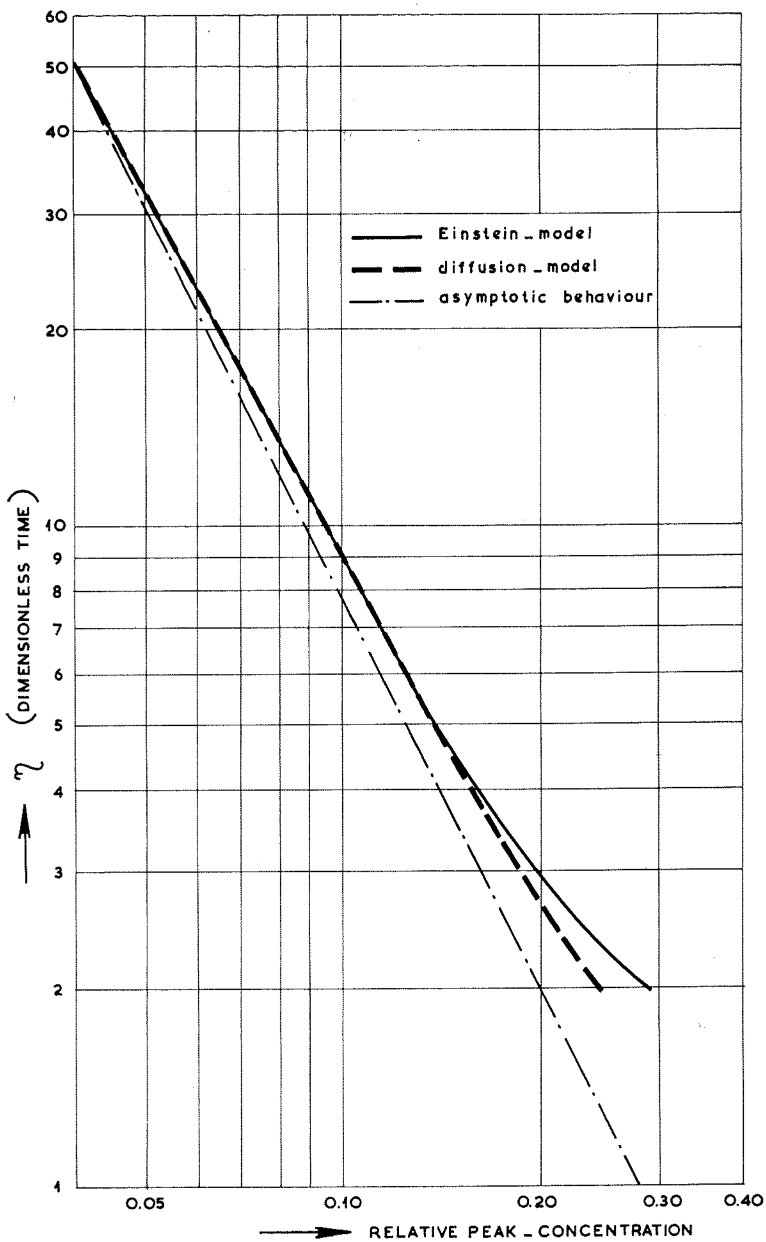

Fig. 6.3 Decay of relative peak-concentration.

For the Einstein-model:

$$C_{\text{peak, rel}} = C/(Pk_1) = [\exp -(\xi_2+\eta)]\sqrt{\eta/\xi_2}\cdot I_1(2\sqrt{\xi_2\eta}) \quad . \quad . \quad . \quad . \quad (6\text{-}19)$$

which leads to $C_{\text{peak, rel}} = f(\eta)$ by elimination of ξ_2 from the eqs. (6-15) and (6-19).

It has been assumed that the instantaneous source is located at (0,0). From the considerations of par. 3.2 and par. 4.3 it follows that the source may be present fictively at $(x_0,0)$. This causes for both models only a simple shift of ξ and also the same shift ξ_0 is present for the location of the mean and the mode of the concentration curve. Of course it does not effect the decay of the relative peak-concentration.

Therefore this decay is only represented by one single curve for each model

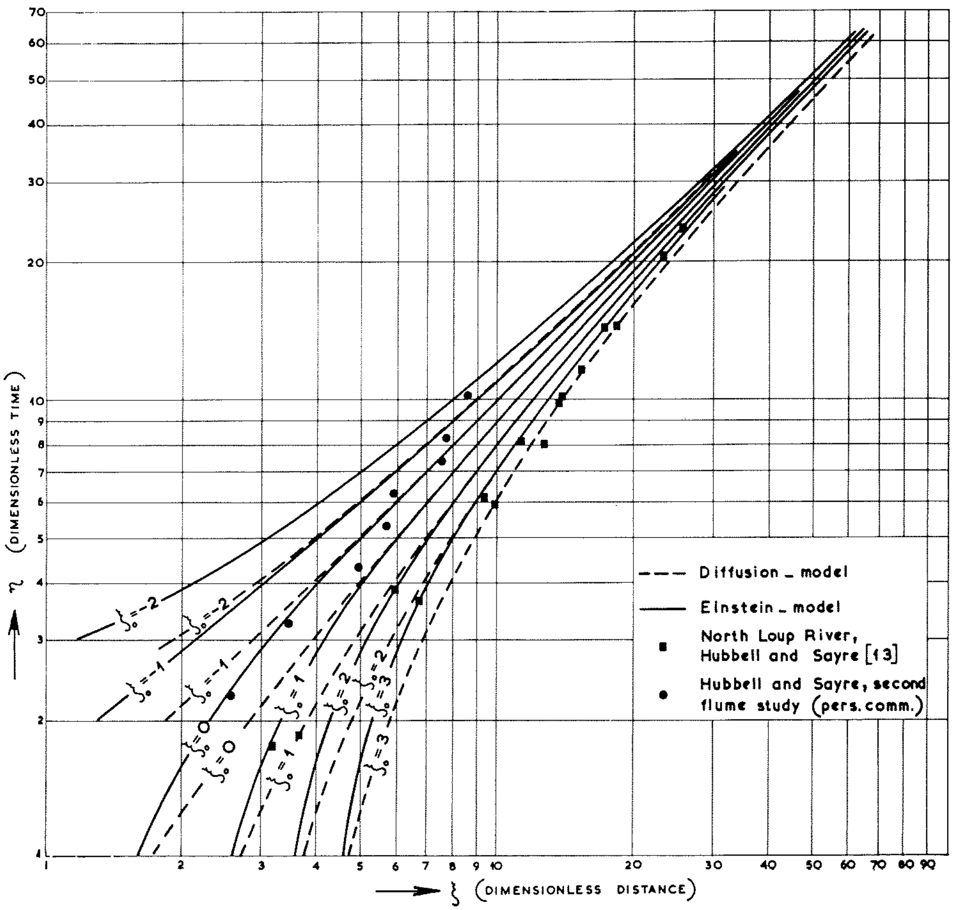

Fig. 6.4 Propagation of the mean.

(figure 6.3), whereas the other relations lead to a family of curves, each for different values of ξ_0. Figure 6.4 represents $\xi_1(\eta)$ for both models dependent on ξ_0. Figure 6.5 gives a similar representation for $\xi_2(\eta)$.

For ξ_1 and ξ_2 this relation is given on a log-log presentation in order to make clear the behaviour with small values of ξ and η.

A direct comparison of the theoretical concentration has been given in figure 6.6. This has been done for the relative concentration: i.e. for the unit of source width and depth of movement. Except for very small values of the time η there is much similarity between the two models. There is, however, a significant shift between each set of curves. This is also demonstrated in figure 6.4 and 6.5 ($\xi_0 = 0$). Figure 6.7 represents the relative tracer-flux.

This shift is significant for the two models. This is further demonstrated in figure 6.8. There the difference between ξ and η for the mode and the mean has been given as functions of η.

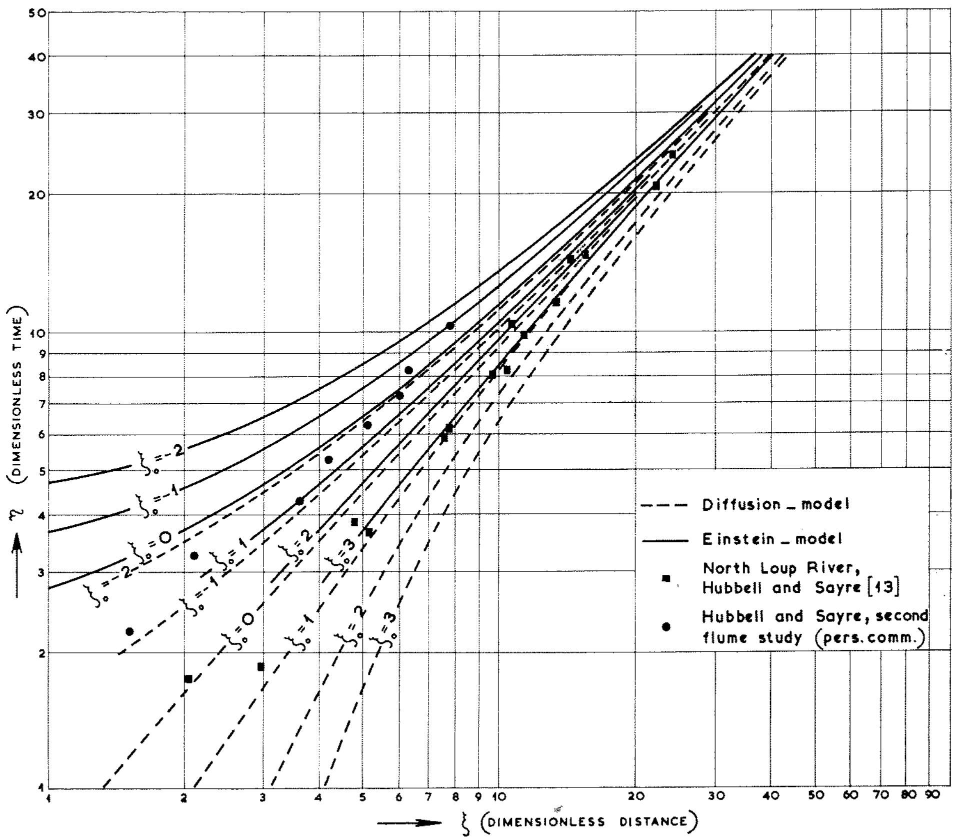

Fig. 6.5 Propagation of the mode.

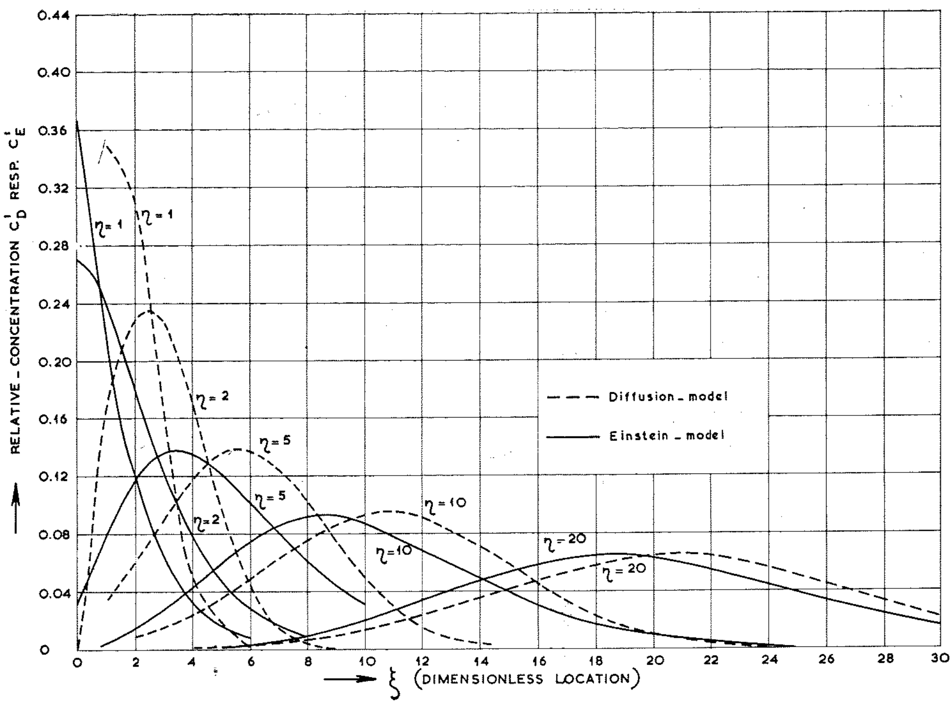

Fig. 6.6 Comparison of concentrations C′(ξ,η).

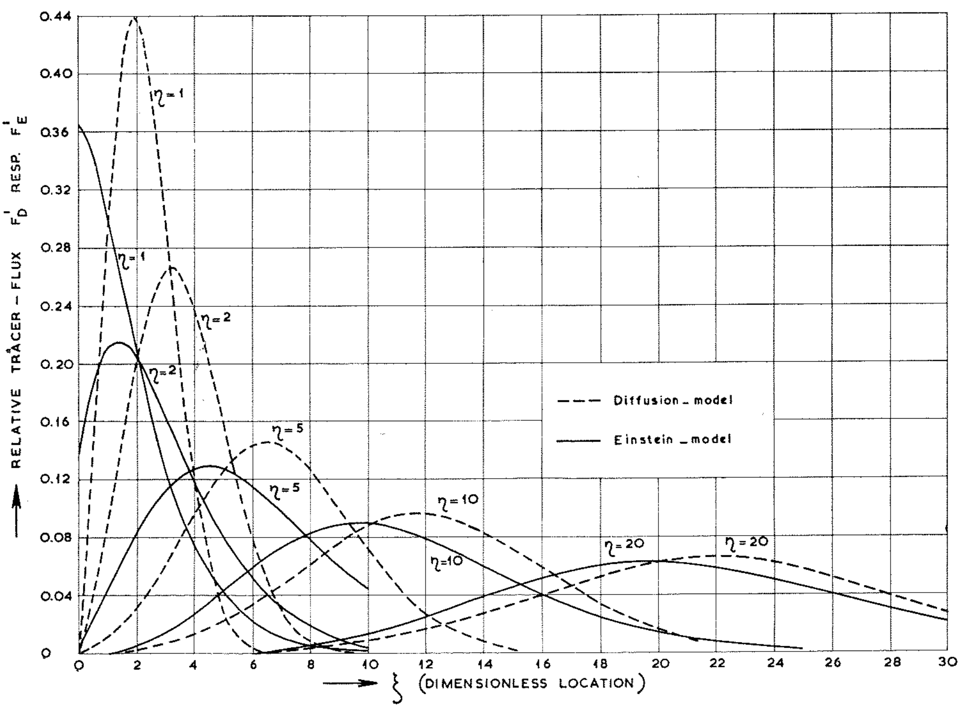

Fig. 6.7 Comparison of flux F′(ξ,η).

The following remarks can be made:

1. Only the mean of the Einstein-model is located on $\xi = \eta$ for large values of η.
2. Only the diffusion model has the characteristic that $\xi_1 - \xi_2 = 0$ for large values of η, i.e. mode and mean coincide.

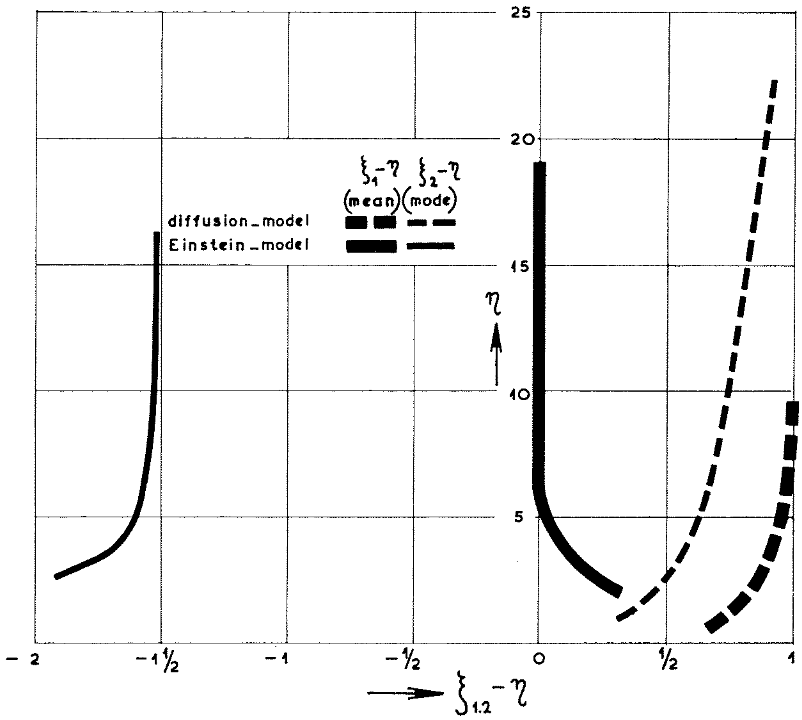

Fig. 6.8 $\xi_{1,2} - \eta = f(\eta)$.

6.5 Application of dispersion-models

6.5.1 *General*

An attempt has been made to test the applicability of some dispersion-models for the quantitative interpretations of the tracer-measurements.

Two tests have been carried out in a laboratory-flune. In each case four estimates for the transport have been obtained from the measurements. The first two estimates are based on the diffusion-model. The last two are based on the Einstein-model. The estimations have the following characteristics:

Method D1 Derivation of $\bar{T}$ from the depth of movement δ and the mean particle-speed derived by a least-squares procedure for the solution of the diffusion-equation for a number of instantaneous sources.

Method D2 This method is similar to D1, but in this case the information about δ has not been used.

Method E1 Similar to D1, but in this case the Einstein-model has been used.

Method E2 The same method as E1, but without the use of δ.

6.5.2 *Experimental set-up*

The experiments have been carried out in a flume wide $B = 0.5$ m. The total length of the sand-bed is about 31 m. The discharge is measured by a circular weir and the sand-transport is measured by apparent volume via a sand-trap at the down-stream-end of the bed. The sand is supplied to the flume by a sand-lift. For each test this lift has been given a constant speed in order to arrive at a stationary-stochastical transport-condition.

Some hydraulic characteristics of the two tests (S 76-V and S76-VI) are listed in table 6.2.

Table 6-2 Transport conditions for tests S 76-V and S 76-VI.

Description	Symbol	Test S 76-V	Test S 76-VI	Unit
discharge	q	0.260	0.180	m^2/s
width	B	0.5	0.5	m
depth	$\bar{h}$	0.45	0.33	m
slope	I	1.56	0.9	10^{-3}
transport	$\bar{T}$	$111 \cdot 10^{-3}$	$33.2 \cdot 10^{-3}$	m^2/h
dune-height	$\bar{H}$	0.13	0.10	m
dune-length	λ	2.3	2.4	m
tracer-injection	τ_*	80	20	$10^{-6}m^2$

Table 6.3 gives information about the grain-size distributions for both tests. Both tests have been carried out with the same sand. The distribution q_i is obtained from samples of the tracers for test S76-V. It may be regarded as a

Table 6-3 Grain-size distributions, tests S 76-V and S 76-VI.

i	Fraction (mm)	q_i	$\bar{\bar{p}}_i$ S 76-V	$\bar{\bar{p}}_i$ S 76-VI
1	0.21–0.30	0.093	0.041	0.096
2	0.30–0.42	0.150	0.196	0.259
3	0.42–0.60	0.230	0.277	0.240
4	0.60–0.88	0.158	0.196	0.141
5	0.88–1.7	0.097	0.148	0.114
6	1.7 –2.4	0.219	0.092	0.100

good estimate. The production of tracers for test S 76-VI has been carried out in the same way as for test S 76-V, by again taking bed-material from the flume.

However, after the tests an apparently slight difference between $\bar{p}_i$ for both tests could be shown. The values of $\bar{p}_i$ can for both tests be regarded as good estimates. The information about $\bar{p}_i$ has been obtained from the large number of samples necessary to determine C_i.

Consequently the use of the given values of q_i for test S 76-VI may be slightly wrong.

To avoid disturbances at the injection-point a number of small injections τ_* has been given. The time-interval between two successive injections amounted to 30 minutes. The injections have been effectuated via a vertical pipe in the axis of the flume ending at a few centimeters from the actual sand-bed. The injection took place at the station (0.0) about 4.5 m down-stream of the beginning of the sand-bed. This distance was required with regard to the length of the dunes in order to have homogeneous conditions down-stream of the injection-point.

At regular time-intervals bottom-samples have been taken at different locations x. These samples were obtained, while the flume was in operation, by sucking some sand from the whole width of the flume. Samples taken in longitudinal directions could not give adequate answers, as the tracers were not injected over the whole width of the flume.

The average mass of the samples was about 60 grams. After sieving the samples, the concentration of each fraction has been determined electronically (see section 4.4.4). The first definition of C according to section 3.2.2 has been used.

The measured concentrations have been treated for each size-fraction separately. These concentrations show much scattering. This could be expected from the considerations of section 4.4.4. For concentrations $C = O(10^{-3})$ and a mass $G = O(10^{-2}\text{ kg})$ for one fraction, figure 4.2 indicates for the mean diameter a relative error of $r_C = O(^1/_2)$.

6.5.3 *Determination of* δ

From the three methods to determine δ (see par 5.3) only the first has been applied. The second method (core-samples) is very laborious due to the irregularity of $\eta(x)$. The third method postulates the absence of burying effects, while the rather inaccurate concentrations do not ensure correct answers after integration.

However, the determination of δ from soundings $\eta(x)$ is not very reliable either. One temporarily deep trough gives a very large contribution to δ. This aspect was present during the test S 76-VI. Therefore the value obtained

$\delta = 7.25$ cm for this test (derived from about 60 single dunes) seems to be questionable.

For test S 76-V the value $\delta = 6.8$ cm was derived from about 60 single dunes. Here the temporary presence of one very deep trough did not occur.

The deviation of δ for test S 76-VI can also be demonstrated with the help of the dune-height $\bar{H}$ (see table 6-2). The values of H were determined from the vertical distance crest-trough. Test S 76-V gives the reliable ratio of $\delta/\bar{H} = 6.8/13 \approx 0.5$; for test S 76-VI a ratio of $\delta/\bar{H} = 7.25/10 \approx 0.7$ was found. This difference cannot be explained by the shape of the dunes as there was no noticeable difference present. The only possibility is that the method applied over-estimates the influence of a relatively deep trough.

A better estimate of δ for test S 76-VI can be obtained by taking also in this case $\delta = {}^1/_2\bar{H}$. This leads to $\delta = 0.05$ m.

6.5.4 *The least-squares-procedure*

The measurements can be divided into series $C(x)$ for a constant value of t. Due to the large scattering of the measurements, a three-point-smoothing has been applied. The sample-size could not be enlarged sufficiently to arrive at a reasonable accuracy of C for the coarsest fractions. The application of a large number of large samples would disturb the transport-process considerably. Therefore only the first four fractions (table 6-3) have been taken into consideration.

After the three-point-smoothing the concentrations have been used for the least-squares approximation of the parameters. The following theoretical functions have been used.

Method D

$$\Gamma_i(X,t) = P_{Di}\Sigma F_i(x,t,\mu w,D) \quad \text{(6-20)}$$

which denotes a sum of solutions according to the diffusion-model of eq. (3-72).

Method E

$$\Gamma_i(x,t) = P_{Ei}\Sigma F_i(x,t,k_1,k_2) \quad \text{(6-21)}$$

denoting a sum of solutions according to the Einstein-model (eq. 6-7). This means that in both cases the absence of an initial drift x_0 has been supposed.

The rather complicated nature of these two functions needs the help of a high-speed computer. The following remarks can be made with regard to the actual computations.

1. There was no need for an accurate first estimate of the parameters in order to get convergence.
2. Test-cases carried out with a high, required accuracy gave insight into the most preferable criteria to stop the iteration-process.

3. The first requirement regards the relative accuracy of the parameters for two successive steps. A relative accuracy of 10^{-1} was sufficient to obtain fair results for the important parameters: P_D and $\mu\bar{w}$ for the diffusion-model, and P_E and k_2/k_1 for the Einstein-model. It appears that the solution is not very sensitive for D or the separate values k_1 and k_2. It could be shown that in succesive steps the separate values of k_1 and k_2 could change considerably with an almost constant ratio k_2/k_1.
4. The circumstances mentioned here led to the introduction of a second criterion. Here the relative accuracy of S between two successive steps has been fixed at 10^{-2}.
   The relatively large scatter of the measurements prevents very small values of S which would make this criterion worthless.
   The practical calculations have been stopped when one of the two criteria was fulfilled.
5. Some calculations stopped when $\lambda = 0$ was reached (see par 5.4). This can be a logical ending if a minimum of S has been reached. However, as this can be only a local minimum without physical meaning, only in some cases the results have been taken into account. A criterion for this decision could be obtained as follows. For normal cases it appeared that the two dispersion-models led for a series $C(x)$ to almost the same minimum value of S. Therefore results obtained via $\lambda = 0$ for one model could be accepted if the other model gave the results without $\lambda = 0$ and with almost the same final value of S.

Table 6-4 Derived parameters for test S 76-V.

		Diffusion-model			Einstein-model				
Series	No. of observ.	P_D 10^{-3}h	$\mu\bar{\bar{w}}$ m/h	D m²/h	P_E 10^{-3}h	k_2/k_1 m/h	k_2/k_1^2 m²/h	k_1 m⁻¹	k_2 h⁻¹
5105.25	24	0.949	2.94	1.9	0.922	3.59	3.7	0.98	3.5
5118.75	26	0.617	2.40	1.7	–	–	–	–	–
mean		0.784	2.67	1.8	0.922	3.59	3.7	0.98	3.5
5205.25	26	0.430	2.38	1.2	0.458	2.39	1.3	1.9	4.5
5209.75	26	0.560	1.97	2.0	0.552	2.06	1.2	1.7	3.6
mean		0.495	2.18	1.6	0.505	2.02	1.2	1.8	4.0
5305.25	23	0.386	1.97	1.3	0.373	2.13	1.3	1.6	3.5
5309.75	26	0.544	1.63	1.6	0.513	1.86	0.6	3.0	5.6
5314.25	25	0.640	1.72	6.4	0.591	1.82	3.1	0.6	1.0
mean		0.523	1.77	3.1	0.492	1.94	1.7	1.7	5.0
5409.75	25	1.083	1.42	1.5	1.075	1.39	0.6	1.3	1.8
5414.25	25	0.582	1.76	2.6	0.569	1.69	1.1	1.6	2.7
5418.75	26	0.823	1.30	7.1	–	–	–	–	–
mean		0.829	1.49	3.7	0.822	1.54	0.8	1.4	2.2

Table 6-5 Derived parameters for test S 76-VI.

		Diffusion-model			Einstein-model				
Series	No. of observ.	P_D 10^{-3}h	$\mu\bar{\bar{w}}$ m/h	D m²/h	P_E 10^{-3}h	k_2/k_1 m/h	k_2/k_1^2 m²/h	k_1 m^{-1}	k_2 h^{-1}
6112.50	26	0.350	1.080	1.1	0.344	1.071	0.4	2.9	3.1
6117.00	26	1.18	0.599	0.1	1.36	0.604	2.0	0.3	0.2
6126.00	25	0.303	0.699	0.1	0.612	0.761	1.0	0.8	0.6
mean		0.611	0.793	0.4	0.772	0.812	1.1	1.3	1.3
6203.75	11	0.220	0.979	0.6	0.212	1.146	0.5	2.4	2.7
6212.50	21	0.282	0.831	0.5	0.281	0.846	2.2	3.8	3.2
6217.00	26	0.749	0.461	1.3	0.916	0.551	1.8	0.3	0.2
6221.50	22	0.394	0.673	0.6	0.383	0.721	4.9	1.4	1.0
mean		0.411	0.736	0.8	0.448	0.816	2.4	2.0	1.8
6308.25	11	0.511	0.521	0.5	0.667	0.613	1.0	0.6	0.4
6312.50	12	0.203	0.951	0.6	0.203	0.975	0.4	2.5	2.4
6321.50	19	0.326	0.580	0.8	0.316	0.658	0.8	0.8	0.5
6335.00	26	0.233	0.642	0.5	0.254	0.682	0.6	1.2	0.8
6339.50	25	0.307	0.569	0.6	0.295	0.607	0.4	1.4	0.8
mean		0.316	0.653	0.6	0.347	0.707	0.6	1.3	1.0
6412.50	10	0.516	0.713	8.7	0.309	0.957	0.8	1.1	1.1
6417.50	14	1.21	0.339	2.3	0.870	0.629	1.7	0.4	0.2
6421.50	17	0.633	0.394	1.9	0.472	0.593	1.0	0.6	0.3
6439.50	25	0.372	0.497	0.3	0.397	0.589	0.2	3.1	1.8
mean		0.683	0.486	3.5	0.512	0.692	0.9	1.7	0.8

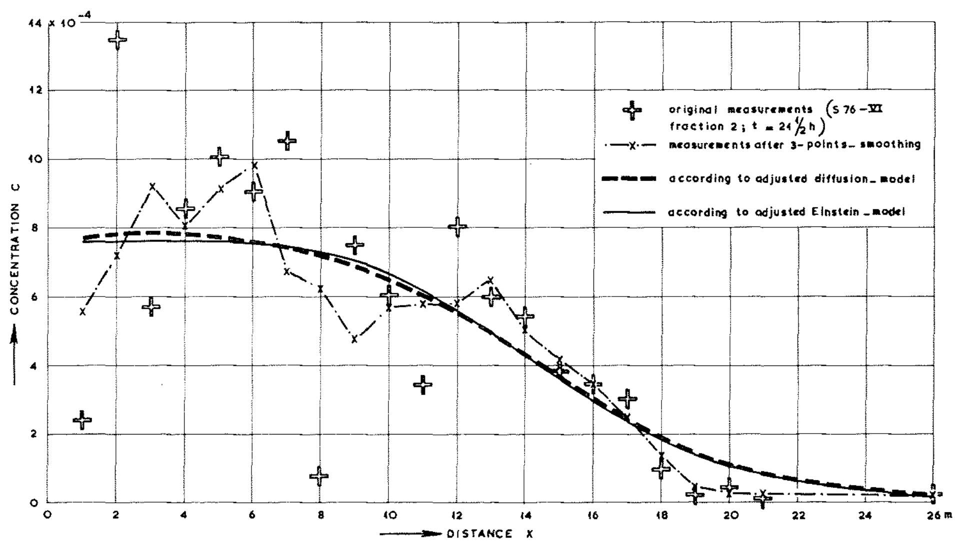

Fig. 6.9 Example of data-handling.

The results of the two tests have been listed in table 6-4 and 6-5 respectively. In these tables the series have been marked by a number composed by the test-number, the fraction-number and the time t.

Fig. 6.9 gives an illustration of the results for series 6221.50, demonstrating the large scatter of the basic-data and even of the data after the three-point smoothing. The coincidence of the curves for the two different models demonstrates that the final value of S is almost the same in the two cases.

6.5.5 *Estimation of $\bar{T}$*

Method D1

The values obtained of $\mu\bar{w}$ demonstrate a significant difference for the various fractions. An estimation of the mean value of $\mu\bar{w}$ can be made for the fractions which have been traced, via weighing with $\bar{p}_i$. Only if the concentration is distributed as an average, uniformly in the vertical direction ($K_i'=1$), this weighed mean indicates the mean particle-speed. About 60% of the sand has been traced. The remaining 40% contains almost exclusively coarse material, that could not be traced accurately enough. The general trend is therefore: application of the derived mean of $\mu\bar{w}$ for all material leads to an over-estimation of $\bar{T}$. The results for the two tests are given in table 6.6.

Method D2

The values obtained of P_i can give an estimation of $\bar{T}_i$ via q_i and τ_*.
According to eq. (3-72)

$$\bar{T}_i = \frac{q_i \tau_*}{K_i' P_i} \quad \text{(6-22)}$$

If it is again supposed $K_i' \approx 1$ then $\bar{T}_i$ can be calculated for all the fractions which have been traced. An estimation of $\bar{T}$ is obtained by assuming that $\Sigma \bar{T}_i / \bar{T}$ is equal to $\Sigma \bar{p}_i$ in which case $i = 1,\ldots,4$.

Table 6-6 Estimations of $\bar{T}$ from tests S 76-V and VI.

Method	Quantity	S 76-V	S 76-VI
sandtrap	transport	$111 \cdot 10^{-3} m^2/h$	$33.2 \cdot 10^{-3} m^2/h$
D 1	δ	0.068 m	0.05 m
	average μw	1.64 m/h	0.67 m/h
	transport	$111 \cdot 10^{-3} m^2/h$	$34 \cdot 10^{-3} m^2/h$
D 2	transport	$112 \cdot 10^{-3} m^2/h$	$41 \cdot 10^{-3} m^2/h$
	δ	0.068 m	0.05 m
E 1	average k_2/k_1	2.00 m/h	0.77 m/h
	transport	$136 \cdot 10^{-3} m^2/h$	$38 \cdot 10^{-3} m^2/h$
E 2	transport	$114 \cdot 10^{-3} m^2/h$	$40 \cdot 10^{-3} m^2/h$

Thus

$$T_i = \frac{\sum_{i=1}^{4} \frac{q_i \tau_*}{P_i}}{\sum_{i=1}^{4} \bar{p}_i} \qquad \text{(6-23)}$$

The results for the two tests are presented in table 6.6.

Method E1

Method E1 is similar to D1. Also in this case the presence of a uniform distribution of the concentration is postulated [14]. The same procedure as for D1 has been used. The results are again given in table 6.6.

Method E2

Although the Einstein-model has been derived for homogeneous material the large similarity with the diffusion model justifies the application of eq. (6-23) also in this case. This leads to a similar procedure arriving at the estimations of $\bar{T}$ given in table 6.6

6.5.6 *Discussion of the results*

1. The least-squares procedure showed that D, k_1 and k_2 may vary considerably while significant values of P, $\mu\bar{w}$ and k_2/k_1 were obtained. It is therefore not expected that a substantial physical value can be attached to D, k_1 and k_2.
2. The values of P for the two models (P_D resp. P_E) show much agreement. By comparing the results for each series $C(x)$ the ratio $P_D/P_E = 1.02 \pm 0.04$ was obtained for both tests. Consequently method D2 and E2 lead to the same estimation of $\bar{T}$.
3. The mean-particle speed derived shows a difference for the separate fractions. The weighed average of the mean particle-speed is therefore only applicable if almost all grain-sizes have been traced. The same remark is true with respect to the estimation of $\bar{T}$ from the parameters P_i.
4. There seems to be a significant difference between k_2/k_1 and $\mu\bar{w}$. This could be expected from the considerations of par. 6.4. Referring to the single dropping presented in fig. 6.6 it may be said that the Einstein-model can only be matched with the diffusion-model if $k_2/k_1 > \mu\bar{w}$. Another possibility is the assumption of an initial drift x_0 (see chapter 3).
5. The results have been obtained during a time-interval of 19 hours for test S 76-V and 40 hours for test S 76-VI. In natural rivers it may be possible to obtain results within one day. This is possible if the transport is not low (small values of $\mu\bar{w}$) and the technical difficulties with regard to the large number of samples can be overcome via an adequate organization.

6. Generally speaking the estimations of $\bar{T}$ agree for test S 76-V remarkably well with the value obtained from the sand-trap. This may be partly accidental. It has already been remarked all methods must give an over-estimation. It is not quite sure if for this reason the conclusion $K_i' < 1$ is justified. The application of method E1 apparently leads to an over-estimation.
7. The estimations of $\bar{T}$ for test S 76-VI are in general larger than the value of $\bar{T}$ measured by the sand-trap. The values derived by methods D1 and E1 are hampered by the small accuracy of δ in this case. The values derived by the methods D2 and E2 are respectively 25 and 20% too high. This may be partly due to inaccurate values of q_i (c.f. section 6.5.2). It is also possible that part of the difference is to be explained by the circumstance that for this test only 60% of the bed-material was traced, whereas for test S 76-V more than 70% was traced. (See also no. 3).

6.6 Remarks

The foregoing experimental survey of course does not permit absolute conclusions. Nevertheless some general remarks can be made.

1. The two different dispersion-models lead to the same results if the transport is estimated via the parameter P. This method includes the hypothesis that no burying-effects are present. This hypothesis becomes more correct if the number of instantaneous sources is increased. The number of sources should therefore be chosen as large as technically possible. This reduces also the possible disturbances of the transport-process.
2. The two dispersion-models lead to different results, if the transport is estimated via the mean particle-speed. According to the tests mentioned in par. 6.5 the diffusion-model seems to be preferable. These two methods are less seriously hampered by burying-effects. However, the determination of δ is not quite satisfactory. Therefore these methods are hardly attractive.
3. For graded bed-material tracing of almost all grain-sizes is required. Tracing of only one fraction i.e. the one containing the mean grean-size is not sufficient, due to the fact that the mean particle-speed is not necessarily the speed of grains with the mean grain-size.
4. In cases with a varying concentration in the vertical direction, the transport can only be estimated with an inaccuracy governed by K_i'. The theoretical considerations of chapter 3 suggest that K_i' is difficult to establish as K_i' is not only determined by $\bar{C}(z)$ but also by $\bar{w}(z)$.

SAMENVATTING

De beweging van vaste stoffen onder de invloed van stromend water speelt een belangrijke rol bij veel waterbouwkundige werken. Ondanks veel onderzoek is het nog steeds niet mogelijk om met behulp van de gegevens over de waterbeweging en het beweeglijke bodemmateriaal een betrouwbare berekening van de aanwezige zandtransporten uit te voeren.

Ten dele vindt deze omstandigheid, als gevolg van de onnauwkeurige bewegingsvergelijking van het zand, zijn oorzaak in het gecompliceerde transportmechanisme zelf. Belangrijk is eveneens dat de meettechniek buiten het laboratorium gebrekkig is. Daardoor kunnen theoretische uitdrukkingen voor de bewegingsvergelijking van het bodemmateriaal slecht worden geverifieerd of aangepast.

In de laatste jaren is er veel onderzoek verricht om transportmetingen uit te voeren die zijn gebaseerd op het gebruik van merkstoffen. In principe wordt daarbij uit het gedrag van gemerkte zandkorrels, toegevoegd aan het bodemmateriaal, gepoogd conclusies te trekken over het gedrag c.q. het transport van niet gemerkte korrels. Deze conclusies moeten dan worden getrokken uit metingen van concentraties aan gemerkte korrels op verschillende plaatsen in de buurt van het injectie-punt en/of op verschillende tijdsduren na de injectie. De kwantitatieve interpretatie van deze gemeten concentraties bevindt zich nog in een beginstadium.

In deze studie is deze kwantitatieve interpretatie als onderwerp gekozen. Er wordt daarbij uitgegaan van het gebruik van fluorescerende merkstoffen (zgn. luminoforen). Na enige fenomenologische beschouwingen (hoofdstuk 2) met betrekking tot het transportmechanisme, wordt in hoofdstuk 3 een theoretisch model ontwikkeld voor de concentratie van merkstoffen afhankelijk van tijd en plaats. Dit model bevat enige onbekende parameters die constant kunnen worden verondersteld voor een beperkt gebied van het zandbed en slechts gedurende beperkte tijdsduur.

In hoofdstuk 4 zijn de grondslagen van de praktische meettechniek aan een beschouwing onderworpen. Het vijfde hoofdstuk beschrijft enkele mogelijke technieken voor de kwantitatieve interpretatie van de metingen. Daarbij wordt speciale aandacht besteed aan het gebruik van theoretische dispersie-modellen om met behulp van de waarnemingen statistisch de onbekende parameters te elimineren.

Hoofdstuk 6 beschrijft de resultaten van enige proeven, die met behulp van de verschillende interpretatie-technieken zijn bewerkt. De gegeven methode gebaseerd op het theoretische model van hoofdstuk 3, blijkt voor deze proeven een redelijke schatting van het opgetreden zandtransport te geven.

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MAIN SYMBOLS

Symbol	Description	Dimension
a	parameter	various
a_i	sand-content of fraction i	[0]
A	general coefficient of diffusion term	$[L^2T^{-1}]$
b	parameter, sand-content	various
B	general coefficient of convective term	$[LT^{-1}]$
c	propagation-velocity or celerity	$[LT^{-1}]$
C	tracer-concentration	[0]
d	grain-diameter	$[L]$
D	coefficient of diffusion	$[L^2T^{-1}]$
E	parameter: $E = \bar{\bar{b}}^{-1} = (1-\varepsilon_0)^{-1}$	[0]
$f(v)$	simplified transport-function	$[L^2T^{-1}]$
F	Froude-number $F = v/\sqrt{gh}$	[0]
$F(\ )$	function of	various
g	acceleration of gravity	$[LT^{-2}]$
G	mass of a sample	$[M]$
h	water-depth	$[L]$
H	ripple-height	$[L]$
i	integer	[0]
I	slope of hydraulic energy-line	[0]
	modified Bessel-function of the first kind	[0]
j	integer	[0]
k	integer	[0]
k_1	constant of step-lengths distribution	$[L^{-1}]$
k_2	constant of rest-periods distribution	$[T^{-1}]$
K	profile-constant	[0]
l	length	$[L]$
L	length	$[L]$
m	number of grains per unit of mass	[0]
$M\{\ \}$	mean of	various
n	integer	[0]
N	number of grains in a sample	[0]
p_i	grain-size distribution of sand	[0]
$P\{\ \}$	accumulative probability	[0]
P	parameter	various
q	water-discharge per unit of width	$[L^2T^{-1}]$
q_s	solid-discharge per unit of width (mass-rate)	$[ML^{-1}T^{-1}]$
q_i	grain-size distribution of tracers	[0]
r	correlation-coefficient; relative error	[0]
R	hydraulic resistance term	$[LT^{-2}]$
s	standard-deviation	various
S	sum of squares of deviations	various
t	time	$[T]$
T	sand-transport per unit of width (measured as settled volume)	$[L^2T^{-1}]$
u	horizontal grain-velocity	$[LT^{-1}]$

U	vertical grain-velocity	$[LT^{-1}]$
v	horizontal water-velocity	$[LT^{-1}]$
w	horizontal sand-flux	$[LT^{-1}]$
W	vertical sand-flux	$[LT^{-1}]$
x	distance in flow-direction	$[L]$
X	dimensionless transport parameter $X = T/(d^{3/2}g^{1/2}\Delta^{1/2})$;	
	number of tracers in a sample	[0]
y	horizontal distance perpendicular to the main current	$[L]$
Y	dimensionless flow parameter $Y = \Delta d/(\mu' hI)$	[0]
z	vertical distance	$[L]$
α	parameter	various
	counter-efficiency	[0]
β	parameter	various
Γ	theoretical concentration	[0]
δ	depth of movement	$[L]$
$\delta(\ )$	delta-function	various
Δ	relative density $\Delta = \varrho_s/(\varrho_s - \varrho)$	[0]
ε	porosity	[0]
ε_0	porosity of sand resting in ripples or dunes	[0]
η	bed-level	$[L]$
	dimensionless time	[0]
θ	time-interval	$[T]$
λ	parameter	various
μ	coefficient	[0]
μ'	ripple-factor	[0]
ξ	dimensionless length	[0]
ϱ	density of water	$[ML^{-3}]$
	theoretical relative error	[0]
ϱ_s	density of sand	$[ML^{-3}]$
σ	theoretical standard-deviation	various
τ	tracer-supply per unit of width and time	$[L^2T^{-1}]$
τ_*	tracer-supply per unit of width	$[L^2]$
τ_{**}	tracer-supply	$[L^3]$
φ	relative celerity $\varphi = c/v$	[0]
	moment of C	various
ψ	relative transport-parameter $\psi = T_v/h$	[0]
	moment of C_t	various
Ω	integrated concentration $\Omega = \int_{-\infty}^{+\infty} C dy$	$[L]$