

A new generic multi-class kinematic wave traffic flow model: Model development and analysis of its properties

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ABSTRACT

We propose and analyze a generic multi-class kinematic wave traffic flow model: Fastlane. The model takes into account heterogeneity among driver-vehicle units with respect to speed and space occupancy: long vehicles with large headways (e.g. trucks) take more space than short vehicles with short headways (e.g. passenger cars). Moreover, and this is what makes the model unique, this effect is larger when the traffic volume is higher. This state dependent space occupancy is reflected in dynamic passenger car equivalent values. The resulting model is shown to satisfy important requirements such as providing a unique solution and being anisotropic. Simulations are applied to compare Fastlane to other multi-class models. Furthermore, we show that the characteristic velocity depends on the truck share, which is one of the main consequences of our modeling approach.

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INTRODUCTION

Traffic flow theory aims to describe human driving behaviour on road networks, including consistent explanations of observed phenomena such as stop and go traffic, capacity drop and traffic hysteresis. Traffic flow models are traditionally classified into microscopic models, which describe the behavior of individual vehicles, mesoscopic models, which describe traffic on the basis of probability distributions of ‘packets’ of vehicles, and macroscopic models, which describe traffic as a continuum flow (1, 2, 3). Our focus is on macroscopic traffic flow models, which are widely used to describe and predict traffic flows in larger networks, both in the context of traffic and transportation planning, as well as in management of traffic operations.

Macroscopic models describe aggregate driving behavior and typically include an average (equilibrium) relation between traffic density ρ (number of vehicles per unit length) and flow q (number of vehicles per unit time). In kinematic wave (KW) models, traffic is assumed to always be in a state described by this fundamental relation. However, observed density-flow plots usually show wide scatter. One reason for this scatter is that not all the data represent steady-state conditions. Higher-order models explain and reproduce (at least partly) this scatter by assuming accordingly that the traffic state tends towards the fundamental relation but is usually not on it, due to for example anticipation and relaxation effects.

A more complete explanation for the scatter in density-flow plots is that it is also related to heterogeneity among drivers and vehicles. Ossen and Hoogendoorn (4) discuss heterogeneity in relation to microscopic models and distinguish between intra- and inter-driver heterogeneity. The first relates to changes in behavior of a single driver over time. Additionally, inter-driver heterogeneity relates to structural differences in behavior and/or capabilities between vehicles and drivers. For example, trucks are usually longer and slower than cars, and have different drive characteristics (e.g. maximum acceleration and deceleration capabilities). As a result, congestion sets in at lower densities if truck shares are higher. These inter-driver differences are used in multi-class (MC) traffic flow models to reproduce scattered density-flow plots. Instead of considering traffic flow as a homogenous flow with homogeneous vehicles and drivers (mixed-class) the heterogeneity of vehicles and drivers is taken into account by dividing them into classes with distinct properties. In (5, 6) MC mesoscopic and MC higher-order macroscopic models are discussed. We focus on multi-class kinematic wave (MC-KW) models, which despite their relative simplicity are also capable of reproducing scattered density-flow plots and some of the spatio-temporal phenomena related to it (1, 7, 8).

Our main contribution is a detailed study of the Fastlane model (First-order fAST muLti-class mAcroscopic traffic flow model for simulation of NEtwork-wide traffic conditions) (9). This model is part of a range of MC-KW models (7, 8, 10, 11, 12, 13, 14, 15). We re-derive the model from a consistent set of principles and analyze its properties. We show that it satisfies certain important requirements and show the consequences of the modeling choices. The consequences include the reproduction of scattered density-flow plots and the dependence of the characteristic velocity in congestion on the traffic composition.

The paper is organized as follows. In the next section we discuss the principles on which the Fastlane model is based and the qualitative requirements that are supposed to be satisfied by it. Then the Fastlane model is specified and it is shown that the model satisfies the requirements. Finally, we show Fastlane’s distinctive properties by simulations, including an analysis of the modeling consequences. The concluding section summarizes the results and indicates future research directions.

PRINCIPLES, REQUIREMENTS AND PROPERTIES

In this section we discuss the principles underlying the Fastlane traffic flow model and the qualitative requirements that should be satisfied by MC-KW models such as Fastlane. We also shortly discuss the expected properties of the model that can be observed in simulations.

Principles

All continuum models, including MC-KW models, are based on the following two principles:

1. Vehicles are conserved.
2. Traffic can be modeled as a continuum flow.

Principle 1 is trivial, in the sense that conservation of vehicles represents the only physical certainty in traffic flow modeling. Based on Principle 2 only aggregated variables of the traffic flow are considered, such as average density ρ (number of vehicles per unit length), average flow q (number of vehicles per unit time) and average vehicle velocity v (m/s or km/h). Principle 2 distinguishes continuum models from microscopic models in which the behavior of individual vehicles is modeled and individual vehicles are traced.

All KW models are derived from the LWR model (16, 17) and are additionally based on the following principles:

3. Traffic flow can be modeled as a single-pipe flow.
4. Traffic is always in equilibrium state.
5. Traffic is always in either of two regimes: free flow or congestion.

Principle 3 states that although in reality there may be multiple lanes and vehicles may overtake each other, this is not modeled explicitly. The consequence of Principle 4 is that drivers adapt their speed *instantaneously* to new traffic conditions (e.g. a change in density). This principle is relaxed in higher-order models. Principle 5 distinguishes two regimes. In free flow, if the density increases the flow also increases. In congestion, if the density increases the flow decreases. The principle is relaxed in the three-regime MC model (14).

An other principle of mixed-class KW models is that *all vehicles behave identically*. This principle is relaxed in MC models, and replaced by the following:

6. Vehicles can be categorized into an arbitrary number of classes, and all vehicles in one class behave identically.

The direct consequence of Principle 6 is that MC models can incorporate different fundamental relations for different vehicle classes. Other differences, such as vehicle length, driving style or destination may also be taken into account.

Model Requirements

We introduce qualitative requirements that should be satisfied by Fastlane or any other MC-KW model. Only after formulating the model based on the *principles*, it can be checked whether the model indeed satisfies the *requirements*. The requirements consist of three groups, related to the model formulation, the fundamental relation and the model dynamics.

The first two requirements are related to the formulation of the model and the uniqueness of the solution:

1. Given ‘permissible’ class-specific densities, (class-specific) velocities and flows are defined uniquely.
2. The model has a unique solution that maximizes flow.

In Requirement 1 permissible class specific densities are nonnegative and the total density is not above a certain threshold (i.e. jam density, see Principle 6). Requirement 2 refers to the entropy condition (18). The entropy condition states that there is only one solution to an initial value problem and that this solution maximizes flow.

We propose the following requirements on the shape of the fundamental relation. They are partly similar to the requirements put forward in (19):

3. Below a certain threshold density (critical density) the velocities of each class are allowed to differ.
4. Below critical density the velocities of relatively fast classes are allowed to decrease with increasing density.
5. At and above critical density the velocity of each class is equal.
6. If the density reaches a certain threshold (jam density), vehicle velocity is zero.
7. If the density of only one class increases, while all other class specific densities remain constant, vehicle velocities do not increase.

Requirements 3–5 are based on observations. For example, (20) describes observation of

decreasing car velocity in free flow. The author also shows that at low densities, velocities of cars and trucks are unequal and that they are equal at high densities. Requirement 5 is also in line with observed low velocity variance at high densities (21). Requirements 6 and 7 may seem trivial. However, not all fundamental relations satisfy Requirement 6. We show that Fastlane only satisfies Requirement 7 if the parameters are chosen appropriately.

Finally, two requirements are related to the dynamics of the model. They put bounds on the velocity of characteristics carrying information or disturbances. A characteristic is a curve in the time-space domain at which a certain variable, such as a class specific density, is constant.

8. Characteristics have finite velocity.

9. Characteristics do not have a larger velocity than vehicles.

Requirement 8 can be interpreted as follows. After a disturbance, such as sudden braking, surrounding vehicles will react. However, it takes some time to react and therefore, not all vehicles react immediately. Requirement 9 implies that the model is anisotropic. An interpretation of this is that vehicles only react to their leader and not to their follower.

Especially the anisotropy requirement has received much attention in the last years. Daganzo (22) initiated an ongoing debate on whether or not certain traffic flow models are anisotropic and whether it is necessary that they are. The main argument to impose this requirement on traffic flow models is that traffic is believed to be anisotropic. Furthermore, if the model is anisotropic, more efficient computational methods can be applied (23, 24).

Properties

We have already discussed that MC models reproduce scattered density-flow plots. A second property of MC models is related to a characteristic velocity in congestion: the congestion wave velocity. This is the velocity with which the downstream front of a congested area propagates. (25) suggests, based on empirical evidence, that the congestion wave velocity depends on the traffic composition. Figure 1 illustrates the theory behind this hypothesis. It shows why the congestion wave velocity depends on the vehicle length and reaction time. We expect that the congestion wave velocity is larger if the share of long vehicles (e.g. trucks) is higher. Thereby neglecting the possible influence of anticipation on multiple vehicles and the influence of headway. We test the hypothesis using simulation studies with several MC-KW models.

MODEL SPECIFICATION

In this section we introduce the generalized MC-KW traffic flow model Fastlane and we show that it satisfies all requirements. Fastlane is based on the principles from the previous section. The model consists of three components which will be discussed in the following order: the conservation of vehicles equation, the fundamental diagram and the link between those two equations using the concepts of effective density and passenger car equivalent (pce) value. A fourth component, namely a node model, can be added to the model to apply it to networks. We only discuss the model for long homogeneous roads, (9, 26) show how Fastlane can be extended to networks.

Conservation of Vehicles

Principle 1 states that vehicles are conserved. Together with the continuum assumption (Principle 2), this leads to the multi-class conservation of vehicles equation:

$$\frac{\partial \rho_u}{\partial t}(x, t) + \frac{\partial q_u}{\partial x}(x, t) = 0, \forall u \in U \quad (1)$$

with ρ_u the class specific density of class u [veh/m] and U the set of all classes. x and t are the the space and time coordinates, respectively. For readability we omit (x, t) from here on. The class specific flow [veh/s] is defined as:

$$q_u = \rho_u v_u \quad (2)$$

with v_u the class specific vehicle velocity [m/s], which is uniquely defined by the fundamental relation (Principle 4). Since it is assumed that traffic can be modeled as a single pipe flow (Principle 3), lanes do not need to be distinguished and flows between lanes are not included explicitly.

Fundamental Relation

Principle 6 leads to different fundamental relations for each class. The Smulders fundamental relation (27) is extended to include multiple vehicle classes, see Figure 2. We have chosen this fundamental relation because it is relatively generic: depending on the parameter choice, it reduces to the Greenshields (parabolic) fundamental relation (28) or the Daganzo (triangular) fundamental relation (29). The Smulders fundamental relation consists of two branches: free flow and congestion (Principle 5). For each class u the velocity is defined by:

$$v_u = v_u(\rho) = \begin{cases} v_{u,\max} - \frac{v_{u,\max} - v_{\text{crit}}}{\rho_{\text{crit}}} \rho & \text{if } 0 \leq \rho < \rho_{\text{crit}} \text{ (free flow)} \\ w \left(\frac{\rho_{\text{jam}}}{\rho} - 1 \right) & \text{if } \rho_{\text{crit}} \leq \rho \leq \rho_{\text{jam}} \text{ (congestion)} \end{cases} \quad (3a)$$

with ρ the ‘effective’ density, which can be thought of as a total density including all classes and will be defined in more detail later. ρ_{jam} is the jam density, ρ_{crit} the critical density, v_{crit} the critical velocity and $v_{u,\max}$ the maximum velocity of class u . $w = \frac{v_{\text{crit}} \rho_{\text{crit}}}{\rho_{\text{jam}} - \rho_{\text{crit}}}$ is the congestion wave parameter. We note that ρ_{jam} , ρ_{crit} , v_{crit} , $v_{u,\max}$ and w are parameters and do not depend on the traffic state.

According to (3), at jam density the velocity is zero and thus the fundamental relation satisfies Requirement 6. The other fundamental relation requirements (Requirement 3–5) are satisfied if its parameters are chosen correctly:

$$v_{\text{crit}} \leq v_{u,\max} \leq v_{1,\max} \leq 2v_{\text{crit}} \quad (4)$$

To understand this, we first discuss the shape of the fundamental relation. The multi-class Smulders density-flow fundamental relation (Figure 2(b)) is a combination of a parabola (free flow) and a straight line (congestion). In free flow, the flow increases if the density increases, in congestion the flow decreases. Both branches intersect at critical density ρ_{crit} and at this density the flow is maximum. However, as Figure 3 shows, this only holds if the fundamental relation parameters are chosen such that $v_{u,\max} \leq 2v_{\text{crit}}$, $\forall u$. Without loss of generality, it is assumed that class 1 is the fastest class.

Theorem 1. *If condition (4) holds, then the following holds:*

1. *Class specific velocities may be unequal if effective density is below critical ($0 \leq \rho < \rho_{\text{crit}}$).*
2. *Velocities are equal if effective density is above or at critical ($\rho_{\text{crit}} \leq \rho \leq \rho_{\text{jam}}$).*
3. *Velocities may be decreasing ($\frac{\partial v_u}{\partial \rho} < 0$) if effective density is below critical ($0 \leq \rho < \rho_{\text{crit}}$).*

Proof. It can readily be concluded from the fundamental relation (3) that in free flow the velocities of two classes are different if their maximum velocity $v_{u,\max}$ is different (Requirement 3 and first part of the theorem). Furthermore, in congestion the velocity of all classes is equal (Requirement 5 and second part of the theorem) and since $v_{\text{crit}} \leq v_{u,\max}$ the class specific velocity in free flow strictly decreases if $v_{\text{crit}} < v_{u,\max}$ (Requirement 4 and third part of the theorem). \square

From the fundamental relation (3) we can furthermore conclude that the velocity does not increase with increasing effective density. Indeed, in free flow

$\frac{\partial v_u}{\partial \rho} = -\frac{v_{u,\max} - v_{\text{crit}}}{\rho_{\text{crit}}} \leq 0$ and in congestion $\frac{\partial v_u}{\partial \rho} = -w \frac{\rho_{\text{jam}}}{(\rho)^2} < 0$. However, from this we may not conclude that the velocity does not increase if a class specific density increases (Requirement 7). In fact, the requirement is only satisfied if furthermore $\frac{d\rho}{d\rho_u} \geq 0$. It may seem obvious that this holds, but below we show that some additional conditions on the model parameters are needed for that. Therefore, first the effective density function is introduced.

Effective Density and PCE-function

The fundamental relation (3), with effective densities ρ , and the conservation equation (1), with class-specific densities ρ_u must be linked to each other to form a complete set of model equations. Therefore, we introduce the effective density, the passenger car equivalent (pce) and the space occupancy. The effective density ρ is a weighted summation of all class specific densities:

$$\rho = \sum_{u \in U} \eta_u(\rho) \rho_u \quad (5)$$

with $\eta_u(\rho)$ the pce-function. A high pce-value η_u indicates that each vehicle in class u (e.g. trucks) contributes a lot to the effective density. Consequently, a small increase in the number of vehicles of class u leads to a large increase in the effective density and a large decrease in the vehicle velocity. We note that this definition of the effective density (5) implies that the effective density is not conserved over time and space and thus there is no equivalent of the continuity equation (1) for the effective density.

The pce-value in (5) depends on the actual traffic state and is based on the relative space occupancy:

$$\eta_u = \frac{\omega_u}{\omega_1} \quad (6)$$

with ω_u the space occupancy of class u and class $u = 1$ the reference class, usually passenger cars. Pce-values and space occupancy are illustrated in Figure 4.

The space occupancy is the road length (in meters) a vehicle needs, this is the vehicle length, plus some minimum headway:

$$\omega_u = L_u + T_u v_u \quad (7)$$

with T_u a parameter that may be interpreted as the minimum time headway. L_u is the gross vehicle length of class u , that is: the length of the vehicle plus the distance between 2 vehicles of this class at standstill. Therefore, and since class 1 is the reference class, $L_1 = l/\rho_{\text{jam}}$. We note that the space occupancy is equal to the safe following distance in Pipes' car following model (30). Figure 4 illustrates that at low densities and high velocities, the minimum headways play the largest role in the space occupancy. They are similar for all types of vehicles and therefore pce-values are relatively low. At high densities and low velocities, the influence of the (physical) vehicle length determines largely how much space is occupied by a vehicle. Since vehicle lengths can differ greatly among classes, pce-values differ as well.

Finally, the condition on the parameters of the space occupancy function is:

$$T_1 \leq \frac{L_1}{w} \leq \frac{L_u}{w} \quad (8)$$

We show later that if the condition holds, the final requirement on the fundamental relation (Requirement 7) holds, as well as the model dynamics requirements (Requirement 8 and 9). However, we first interpret the condition (8), see also Figure 5. The first inequality puts constraints on the minimum time headway of class 1 (T_1) and ensures that the space occupancy takes a realistic value.

Theorem 2. Consider any congested traffic state (i.e. $\rho_{\text{crit}} \leq \rho \leq \rho_{\text{jam}}$) with only vehicles of class 1 (i.e. $\rho_u = 0$ for all $u \neq 1$). If the first inequality of (8) holds, then the space occupancy is not larger than the spacing $\omega_1 \leq s = 1/\rho$.

Proof. There are only vehicles of class 1 and thus the spacing is $s = 1/\rho = 1/\rho_1$.

Because the velocity is decreasing in congestion ($\frac{dv_1}{d\rho} \leq 0$), also the space occupancy (7) is decreasing in congestion:

$$\frac{d\omega_1}{d\rho} = T_1 \frac{dv_1}{d\rho} \leq 0 \quad (9)$$

Therefore, it suffices to show that $\omega_1 \leq s$ at critical density ρ_{crit} . We substitute $\rho_{\text{jam}} = 1/L_1$ and the fundamental relation (3) into the space occupancy function (7). Rewriting the result gives:

$$\omega_1 = L_1 + T_1 w \left(\frac{1}{\rho L_1} - 1 \right) = \frac{1}{\rho} \left[(L_1 - T_1 w) \rho + \frac{T_1 w}{L_1} \right] \quad (10)$$

Furthermore, substituting $\rho = \rho_{\text{crit}} \leq \rho_{\text{jam}} = 1/L_1$ into (10) gives:

$$\omega_1 \leq \frac{1}{\rho} \left[\frac{L_1 - T_1 w}{L_1} + \frac{T_1 w}{L_1} \right] = \frac{1}{\rho} = s \quad (11)$$

□

Theorem 2 implies that, in congestion with only vehicles of class 1, no part of the road can be occupied by two vehicles at the same time. That is: their space occupancies do not overlap.

The second inequality of (8) puts constraints on the gross vehicle lengths L_u and ensures that the pce-values are nondecreasing.

Theorem 3. Consider any congested traffic state (i.e. $\rho_{\text{crit}} \leq \rho \leq \rho_{\text{jam}}$) with only vehicles of class 1 (i.e. $\rho_u = 0$ for all $u \neq 1$). If the second inequality of (8) holds for a certain class u , then the pce-value of class u is nondecreasing: $d\eta_u/d\rho \geq 0$.

Proof. There are only vehicles of class 1 and thus the spacing is $s = 1/\rho = 1/\rho_1$. We apply the pce-function (6) and the space occupancy function (7) and use that $v_u = v_1$ and $dv_u/d\rho = dv_1/d\rho$ to compute the density derivative of the pce-value η_u :

$$\begin{aligned} \frac{d\eta_u}{d\rho} &= \frac{(L_1 + T_1 v_1) T_u \frac{dv_u}{d\rho} - (L_u + T_u v_u) T_1 \frac{dv_1}{d\rho}}{(\omega_1)^2} = \frac{L_1 T_u \frac{dv_1}{d\rho} - L_u T_1 \frac{dv_1}{d\rho}}{(\omega_1)^2} \\ &= \frac{T_1 T_u}{(\omega_1)^2} \left[\frac{L_1}{T_1} - \frac{L_u}{T_u} \right] \frac{dv_1}{d\rho} \end{aligned} \quad (12)$$

We have already shown that the velocity is decreasing ($\frac{dv_1}{d\rho} \leq 0$). Therefore, if and only if, the second inequality of (8) holds, the term between square brackets in (12) is nonpositive and the pce-value is nondecreasing. □

Theorem 3 implies that, if traffic is more congested, trucks (which are longer and have a larger minimum time headway than passenger cars) have a higher pce-value than in free flow. This reflects that at high densities they take relatively more space than in free flow, see Figure 4.

Reformulation of the Effective Density Function

The Fastlane model equations in the previous subsections describing the velocity (3), the effective density (5), the pce-value (6) and the space occupancy (7) form a set of implicit functions:

$$\begin{array}{ccc}
 \rho = \rho(\rho_1, \dots, \rho_U, & \xrightarrow{v_u = v_u(\rho)} & \text{quadratic equation} \\
 \eta_1, \dots, \eta_U) & \searrow & \text{with 2 solutions} \\
 & \omega_u = \omega_u(v_u) & \longrightarrow \rho = \rho^+ \text{ or } \rho = \rho^- \\
 \eta_u = \eta_u(\omega_1, \omega_u) & \nearrow & \longrightarrow \text{1 meaningful} \\
 & & \text{solution} \\
 & & \rho = \rho(\rho_1, \dots, \rho_U)
 \end{array} \quad (13)$$

By combining the equations, a quadratic equation with two solutions is found. One of them leads to unrealistic, not meaningful solutions, for example with negative velocities. The correct solution is selected such that the effective density becomes a uniquely defined function and has a meaningful value:

$$\rho = \begin{cases} \frac{a_1 - \sum_u b_u \rho_u - \sqrt{(a_1 - \sum_u b_u \rho_u)^2 + 4b_1 \sum_u a_u \rho_u}}{-2b_1} & \text{if } b_1 \neq 0 \\ \frac{\sum_u a_u \rho_u}{a_1 - \sum_u b_u \rho_u} & \text{if } b_1 = 0 \end{cases} \quad (14a)$$

with in free flow (if $0 \leq \rho \leq \rho_{\text{crit}}$):

$$a_u = a_u^{\text{ff}} = L_u + T_u v_{u,\text{max}} \quad \text{and} \quad b_u = b_u^{\text{ff}} = -T_u \frac{v_{u,\text{max}} - v_{\text{crit}}}{\rho_{\text{crit}}} \quad (14c)$$

and in congestion (if $\rho_{\text{crit}} \leq \rho \leq \rho_{\text{jam}}$):

$$a_u = a_u^{\text{cn}} = T_u w \rho_{\text{jam}} \quad \text{and} \quad b_u = b_u^{\text{cn}} = L_u - T_u w \quad (14d)$$

With this new effective density function (14), only the class specific densities are needed to calculate the effective density. The velocities, space occupancies and pce-values are incorporated, but their values are not needed anymore. This way, all variables in the model are defined uniquely (Requirement 1). The derivation of the new effective density function (14) from (3), (5), (6) and (7) is not straightforward (31). However, it can be checked relatively easy that they are equivalent by substituting (14) into the fundamental relation (3), substituting the obtained velocity in the space occupancy function (7) and substituting the obtained space occupancy in the pce-function (6). Subsequently, substituting the pce-value into the old effective density function (5), yields the effective density ρ at the right hand side of (5).

With the new formulation of the effective density (14), it is possible to show that the velocity is a nonincreasing function of the class specific densities (Requirement 7).

Theorem 4. *If both condition (4) and (8) hold, then for all classes i and j*

$$\frac{\partial v_i}{\partial \rho_j} \leq 0 \quad (15)$$

We have already shown that $\frac{\partial v_u}{\partial \rho} \leq 0$ for all classes u . It remains to show that $\frac{\partial v_i}{\partial \rho_j} = \frac{\partial v_i}{\partial \rho} \frac{d\rho}{d\rho_j} \leq 0$ for all classes u , because then $\frac{d\rho}{d\rho_u} \geq 0$. The full proof of Theorem 4 can be found in (31).

Lagrangian Formulation of the Model

We reformulate the model in the Lagrangian coordinate system. In this coordinate system the coordinates move with the vehicles of class 1, instead of being fixed in space such as in the traditional Eulerian coordinate system. The reformulation has two main advantages: 1. it becomes easier to assess the model dynamics requirements, and 2. simulations can be done more efficiently, as described in more detail in (24).

Vehicles are numbered in opposite driving direction. The first vehicle gets number $n = 1$, its follower $n = 2$, etc. The main variables of the model are class specific spacing $s_u = 1/\rho_u$ and velocity v_u . The Lagrangian fundamental relation expresses the velocity as a function of the class specific spacings but can be rewritten in Eulerian formulation:

$$v_u^* = v_u^*(s_1, \dots, s_U) = v_u^*(1/\rho_1, \dots, 1/\rho_U) = v_u(\rho_1, \dots, \rho_U) = v_u(\rho(\rho_1, \dots, \rho_U)) \quad (16)$$

The conservation equation (1) is reformulated in Lagrangian coordinates:

$$\frac{\partial s_1}{\partial t} + \frac{\partial v_1}{\partial n} = 0 \quad \text{and} \quad \frac{\partial s_u}{\partial t} + \frac{s_u}{s_1} \frac{\partial v_u}{\partial n} + \frac{v_1 - v_u}{s_1} \frac{\partial s_u}{\partial n} = 0, \quad \forall u \neq 1 \quad (17)$$

To assess the model dynamics requirements, the model is subsequently rewritten in matrix vector notation:

$$\frac{\partial \vec{s}}{\partial t} + \mathbf{J}(\vec{s}) \frac{\partial \vec{s}}{\partial n} = \vec{0} \quad (18)$$

with the vector of class specific spacings $\vec{s} = (s_1, s_2, \dots, s_U)^T$ and Jacobian matrix:

$$\mathbf{J} = \begin{pmatrix} a_{1,1} & \cdots & a_{1,U} \\ \vdots & \ddots & \vdots \\ a_{U,1} & \cdots & a_{U,U} \end{pmatrix} \quad \text{with} \quad a_{i,j} = \begin{cases} \frac{s_i}{s_1} \frac{\partial v_i^*}{\partial s_i} + \frac{v_1 - v_i}{s_1} & \text{for } i = j \text{ (on the diagonal)} \\ \frac{s_i}{s_1} \frac{\partial v_i^*}{\partial s_j} & \text{for } i \neq j \text{ (off the diagonal)} \end{cases} \quad (19)$$

and $v_i^*(s) = v_i^*(1/\rho)$ the Lagrangian fundamental relation. A detailed derivation of this formulation can be found in (42).

Model Dynamics Requirements

We show that the Fastlane model satisfies the model dynamics requirements on finite characteristic velocity (Requirement 8) and anisotropy (Requirement 9). Theory of partial differential equations tells us that the characteristic velocities in the Lagrangian coordinate system are equal to the eigenvalues of the Jacobian (19). The model is anisotropic if the characteristics are not faster than the vehicles of the fastest class. In the Lagrangian coordinate system, the coordinates move with the vehicles of the fastest class. Therefore, the model is anisotropic if the eigenvalues of the Jacobian (19) are nonnegative. The main challenge here is that the eigenvalues can not be computed directly, at least not for the generic case with $U > 4$. However, some theorems from linear algebra can be applied to analyze whether the eigenvalues are finite and the sign of the eigenvalues. The procedure is explained in more detail in (31, 32). It is shown that the Fastlane model satisfies the Requirements 8 and 9 if the class specific velocity is a nonincreasing function of the class specific densities (Requirement 7). We have already shown (Theorem 4) that this is indeed the case if Condition (4) and (8) hold.

Finally, we conclude that if conditions (4) and (8) hold, then the Fastlane model satisfies all requirements, except for Requirement 2 which is not discussed yet. Requirement 2 states that the entropy solution should be selected. It depends on the numerical method whether this requirement is satisfied. For the simulations in the next section, we apply a numerical method that maximizes the flow in pce-units per time unit. This yields a unique entropy solution and thus the requirement is satisfied.

SIMULATIONS AND RESULTS

We provide simulation results to show some of the properties of Fastlane. We study the differences in scatter reproduced in the fundamental diagram between Fastlane and

models with constant pce-values and without pce-values ($\rho = \sum_u \rho_u$). Furthermore, we study the impact of traffic composition on characteristic velocity in congestion in Fastlane and other models.

Before continuing with the simulation setup and its results we shortly discuss the

discretization of the Fastlane model equations.

Discretization

To apply a macroscopic traffic flow model in a simulation tool, its continuous equations are discretized and solved using numerical methods. For discretization, in the original introduction of the Fastlane model (9) the mixed class minimum supply demand method (18, 29) is adapted to include multiple classes (31). The mixed class Godunov method in Lagrangian coordinates (23) has been adapted to include multiple vehicle classes and is shown to be more efficient (24). In simulations time is divided into K time steps of size Δt . Furthermore, vehicles are divided into groups of Δn vehicles. In each time step, the position of each vehicle group is calculated. In the simulations in this section, the parameters of the numerical method are time step size of $\Delta t = 1$ s and vehicle group size of $\Delta n = 0.833$ vehicles.

Simulation Setup

We apply a simple test case consisting of an initial value problem on a long homogeneous road with a queue of length $L = 2000$ m:

$$\rho(x, 0) = \begin{cases} 0 & \text{for } x > 0 \\ \rho_{\text{jam}} & \text{for } -L \leq x \leq 0 \\ \frac{1}{2}\rho_{\text{crit}} & \text{for } x < -L \end{cases} \quad (20)$$

There are 2 vehicle classes: passenger cars and trucks. The initial truck share varies from 0% to 50%. The queue travels backward, its length decreases and after some time (about 1100 s), it is dissolved. This is due to the expansion wave at the front of the queue and a backward traveling shock wave at its tail. In the exact solution, within the queue and in the fan the velocities of both classes are equal. In the rest of the domain the densities are below critical and thus the velocities are unequal. We study the profile of the queue, i.e. the effective density and the class specific densities. Moreover, we study how the profile depends on the pce-value or function and the truck share. Therefore, we run the simulation with the Fastlane

pce-function (6), a large constant pce-value ($\eta_2 = \frac{L_2}{L_1}$), a small constant pce-value ($\eta_2 = \frac{L_2}{2L_1}$) and no class specific pce-value ($\eta_2 = \eta_1 = 1$). This corresponds to models with no pce-value (8, 12) and to models with a constant pce-value (10, 11, 15), though these model mostly use different fundamental relations. All other parameters are kept constant, see Table 1.

Simulation Results

The results of the simulations are shown in Figures 6–8. Figure 6 shows the evolution of the effective densities computed with Fastlane and with initially 20% trucks. Simulation results with constant pce-values (not shown) look very similar to this one. Therefore, we compare the results in a different way. Figure 7 shows cross sections at time $t = 600$ s. The effective density and class specific densities are shown. The results indeed depend on the pce-function or value. The class specific densities in the queue are equal to those with $\eta_2 = L_2/L_1$. This is because in the queue, in Fastlane the same pce-values hold. However, downstream of the queue the pce-values of Fastlane are smaller and therefore its class specific densities are larger. Moreover, Figure 7 shows that the queue travels upstream faster in the Fastlane simulation than with constant pce-values. This implies a higher characteristic velocity, which is studied further in Figure 8. It shows all the density-flow pairs from all simulations. Each subfigure shows the results for one pce-function or value. The fundamental diagrams all show some scatter. With no pce-values ($\eta_2 = \eta_1 = 1$), there is only scatter in the free flow branch of the fundamental diagram. With constant pce-values the scatter in congestion is independent of the actual density. As can be expected, the influence of the truck share is low with low pce-values and it is high with high pce-values. The fundamental diagram reproduced by Fastlane combines both: it shows little scatter (little dependence on truck share) at low densities and the scatter increases gradually for higher densities. This fundamental diagram furthermore

shows that the congestion wave velocity is higher if the truck share is higher. This effect is not observed with constant pce-values.

SUMMARY AND CONCLUSION

We introduced Fastlane as a generic multi-class kinematic wave (MC-KW) traffic flow model. It generalizes previously introduced MC-KW traffic flow models in the sense that it takes into account heterogeneity in vehicle speed and space occupancy. Moreover, and this is what distinguishes Fastlane from other such models, space occupancy of vehicles also depends on the actual traffic state. This is reflected in dynamic pce-values.

We introduced some requirements on continuum traffic flow models, related to for example the existence of a unique solution and anisotropy of traffic flow. It is shown that the Fastlane model satisfies these requirements when appropriate model parameters are chosen. Furthermore, simulation results show distinctive properties of Fastlane related to the model dynamics and the resulting fundamental diagram. They show that scatter in measured density-flow plots can be explained by differences in traffic composition using MC models with dynamic pce-values. Moreover, in Fastlane, high truck shares lead to high congestion wave velocities in congestion.

An important next step in the development of Fastlane is calibration and validation using experimental data and comparison with other models. Therefore, it would also be valuable to study the influence of parameter values (such as maximum velocity $v_{u,\max}$ and critical density ρ_{crit}) on the simulation results. Future research also includes testing our hypothesis on the influence of composition on congestion wave velocity. The model has been applied for traffic state estimation (33) and model predictive control (34). The studies show the added value of a multi-class model because it enables multi-class measurements in state estimation. Furthermore, the model predictive control method is applied to a freeway with many trucks (the A15 close to the port of Rotterdam) and uses control aimed at specific classes (45).

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FIGURE 1 Explanation of influence of truck share on congestion wave velocities by two extreme cases: only trucks (top) and only cars (bottom), all driving from left to right. Downstream of a high density region a vehicle accelerates. The vehicles only react on their leader and accelerate some reaction time τ after the leader accelerates. The velocity with which this ‘acceleration information’ travels upstream is the congestion wave velocity. If there are only trucks, it takes little time (3τ) to reach the upstream end of the graphic. However, when there are only passenger cars it takes more time (7τ), which implies a lower congestion wave velocity.

FIGURE 2 Example of a two-class fundamental diagram. The solid line denotes the velocity or flow of cars, the broken line denotes the velocity or flow of trucks, as a function of the effective density.

FIGURE 3 Density-flow relations. If the critical velocity is relatively large (critical density relatively small) (a), the top of the parabola is not included in the fundamental relation and the maximum flow equals the flow at critical density. However, if the critical velocity is relatively small (i.e. critical density is relatively large) (b), the top of the parabola is included in the fundamental relation and the maximum flow does not equal the flow at critical density. Moreover, part of the free flow branch is decreasing.

FIGURE 4 Illustration of pce-values and space occupancy. Top: free flow, the space occupancy of a truck is similar to the space occupancy of a car, therefore the pce-value of the truck is low. Bottom: congestion, the space occupancy of a truck is much larger than the space occupancy of a car, therefore the pce-value of the truck is high.

FIGURE 5 Schematic view of the condition on the space occupancy parameters, Condition (8). The gray areas illustrate the admissible values of $1/T_1$ and $1/T_u$ respectively.

FIGURE 6 Effective densities computed with Fastlane pce-function and initially 20% trucks. Vertical white lines indicate times for which cross sections are shown in Figure 8.

FIGURE 7 Cross sections of density profiles at $t = 600$ s.

FIGURE 8 Resulting fundamental diagrams. Colors indicate the truck share: • 0%, • 2%, • 5%, • 10%, • 20%, • 50%. Note that the horizontal axis indicates the total density (in vehicles per meter) and not the effective density (in pce-units per meter).

TABLE 1 Parameter Settings for Simulation

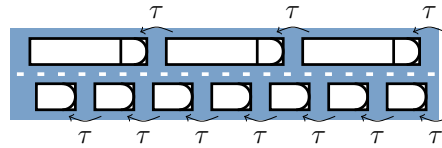


FIGURE 1 Explanation of influence of truck share on congestion wave velocities by two extreme cases: only trucks (top) and only cars (bottom), all driving from left to right. Downstream of a high density region a vehicle accelerates. The vehicles only react on their leader and accelerate some reaction time τ after the leader accelerates. The velocity with which this ‘acceleration information’ travels upstream is the congestion wave velocity. If there are only trucks, it takes little time (3τ) to reach the upstream end of the graphic. However, when there are only passenger cars it takes more time (7τ), which implies a lower congestion wave velocity.

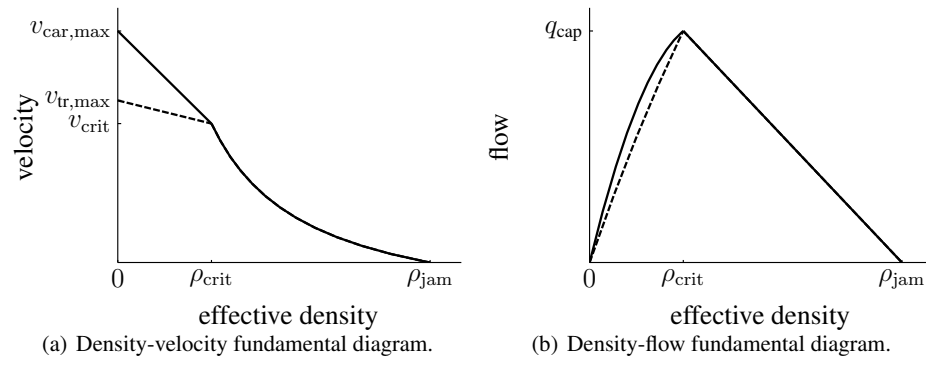


FIGURE 2 Example of a two-class fundamental diagram. The solid line denotes the velocity or flow of cars, the broken line denotes the velocity or flow of trucks, as a function of the effective density.

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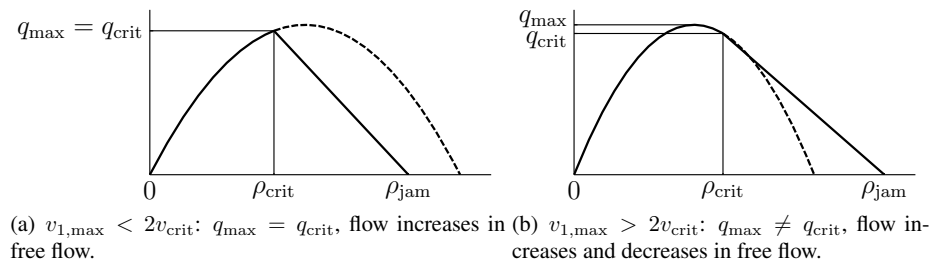


FIGURE 3 Density-flow fundamental relations. If the critical velocity is relatively large (critical density relatively small) **(a)**, the top of the parabola is not included in the fundamental relation and the maximum flow equals the flow at critical density. However, if the critical velocity is relatively small (i.e. critical density is relatively large) **(b)**, the top of the parabola is included in the fundamental relation and the maximum flow does not equal the flow at critical density. Moreover, part of the free flow branch is decreasing.

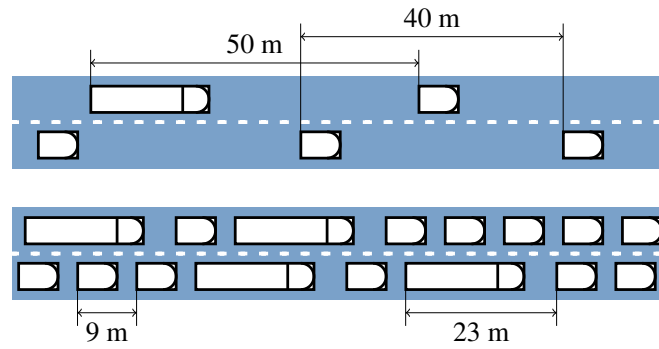


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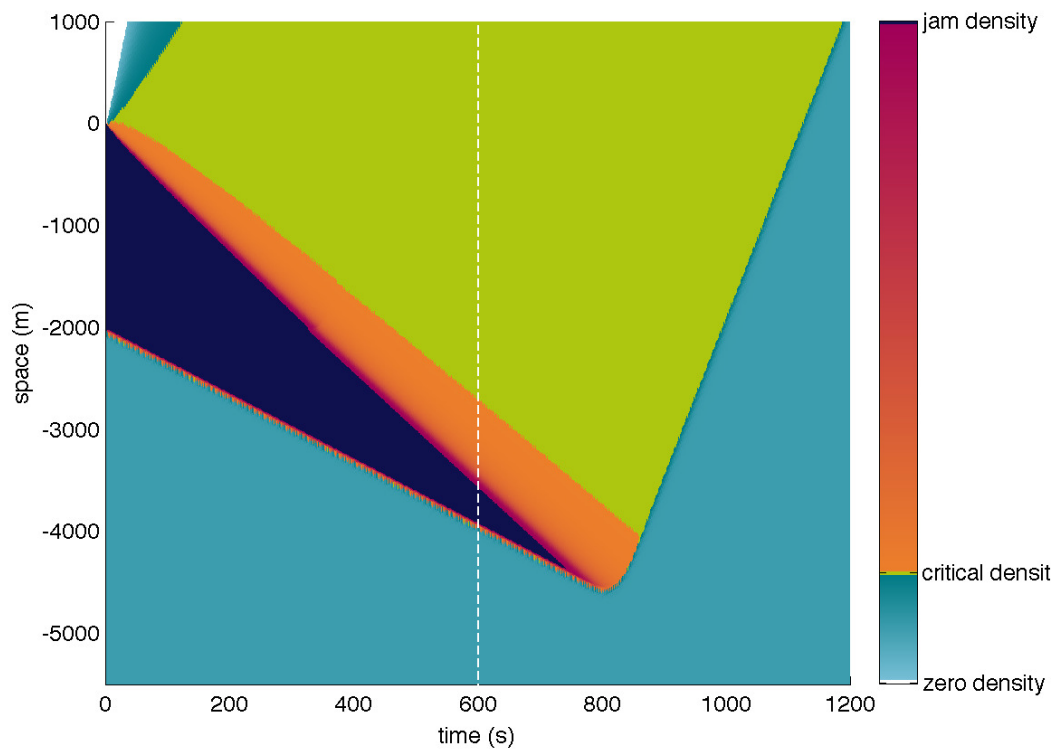
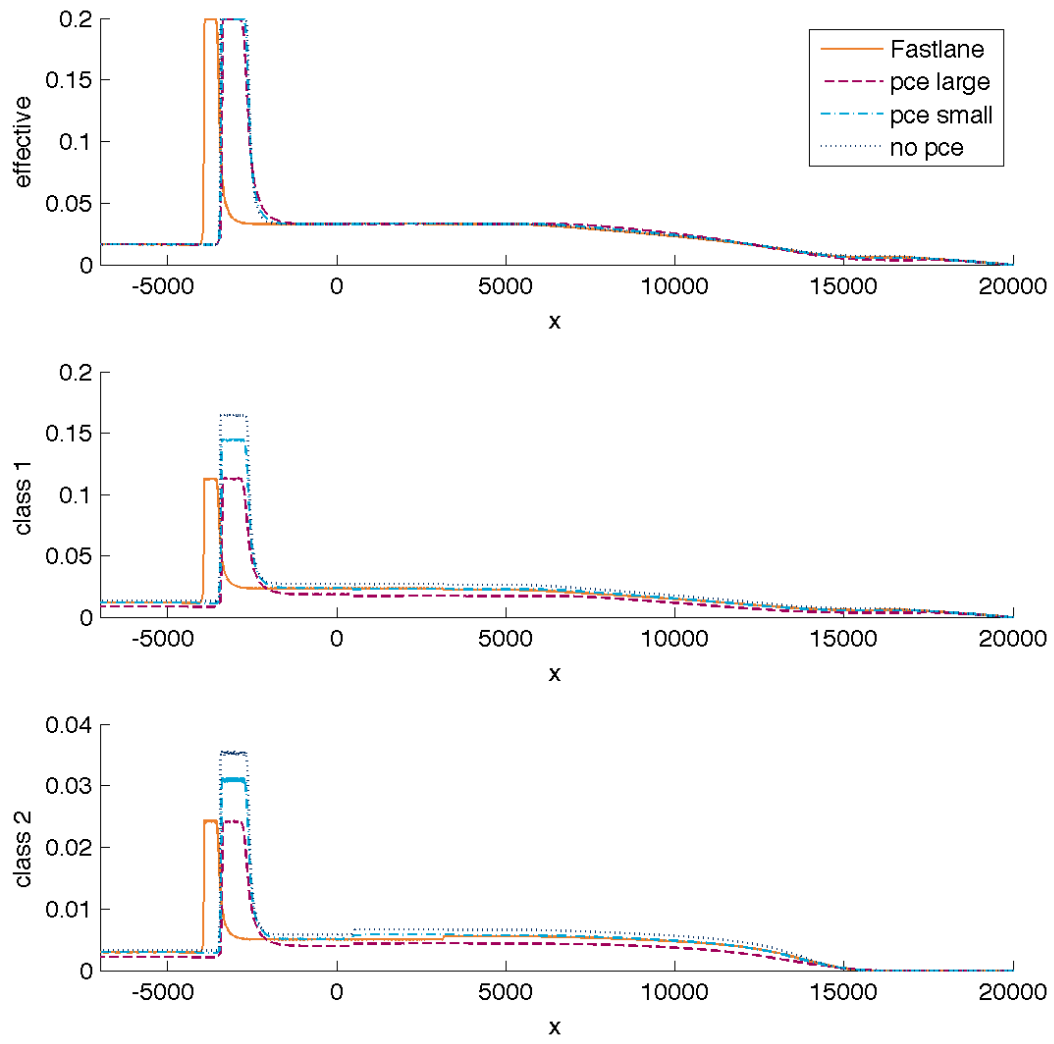


FIGURE 6 Effective densities computed with Fastlane pce-function and initially 20% trucks. Vertical white lines indicate times for which cross sections are shown in Figure 8.

FIGURE 7 Cross sections of density profiles at $t = 600$ s.

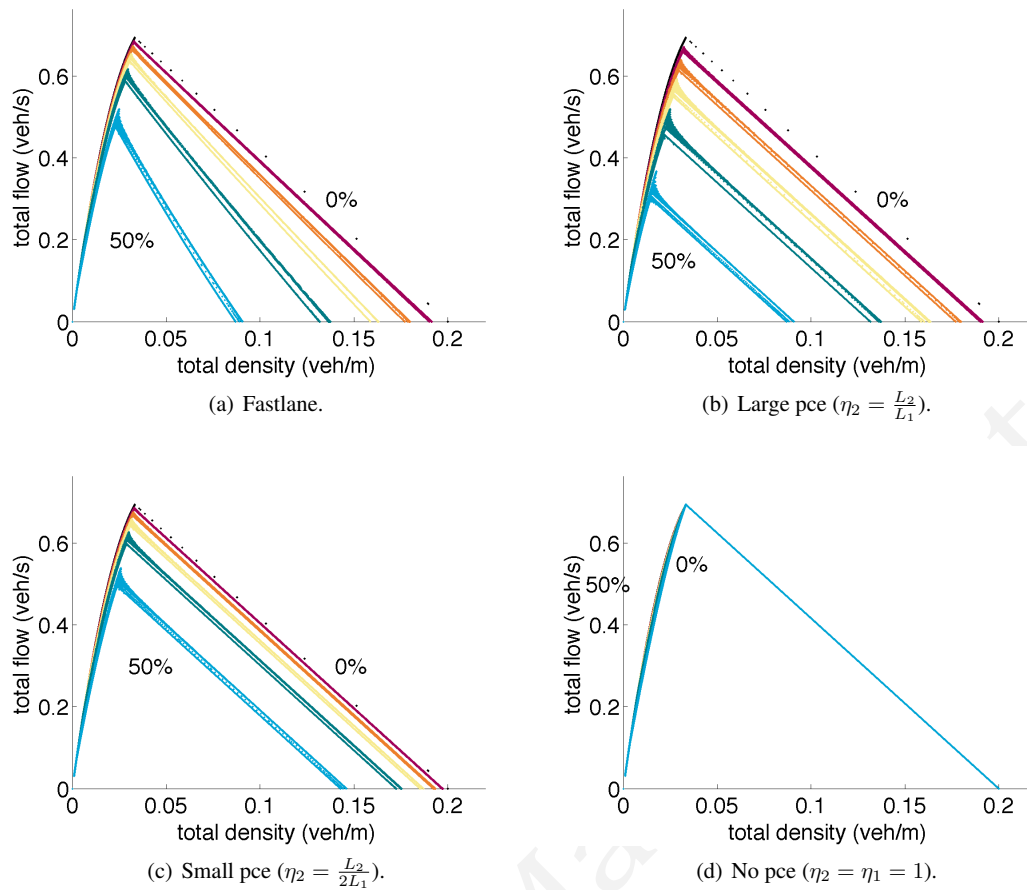


FIGURE 8 Resulting fundamental diagrams. Colors indicate the truck share: • 0%, • 2%, • 5%, • 10%, • 20%, • 50%. Note that the horizontal axis indicates the total density (in vehicles per meter) and not the effective density (in pce-units per meter).

TABLE 1 Parameter Settings for Simulation

parameter	value	unit
$v_{1,\max}$	30	m/s (= 108 km/h)
$v_{2,\max}$	27.5	m/s (= 99 km/h)
v_{crit}	25	m/s (= 90 km/h)
ρ_{crit}	0.0278	pce/(m lane) (= $\rho_{\text{jam}}/6$)
ρ_{jam}	0.1667	pce/(m lane) (= $1/L_1$)
T_1	1	s
T_2	1.5	s
L_1	6	m
L_2	18	m

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