AN APPROXIMATIVE METHOD FOR THE DETERMINATION OF THE HYDRODYNAMIC COEFFICIENTS OF A SHIP IN CASE OF SWAYING AND YAWING ON SHALLOW WATER

Report no. 75-4

H.L. Fontijn
Delft University of Technology
Department of Civil Engineering
Laboratory of Fluid Mechanics

CONTENTS

SUMMARY

This report gives information about the hydrodynamic coefficients of a box-shaped ship with zero speed of advance in case of pure swaying and yawing on shallow water.
In the horizontal plane a harmonically oscillating motion is imposed on the ship; the hydrodynamic coefficients for the modes of motion of the ship are determined from the exciting forces in a theoretical as well as in an experimental way.
The analytical approach is based on an existing theory for wave generators adapted to the present case; use is made of the strip theory.
The experiments are carried out in the form of forced pure sway and forced pure yaw tests with zero speed of advance.
The theoretical and experimental results for the hydrodynamic coefficients are compared with one another. It is found that the system ship-fluid for swaying can be considered as linear in case of small oscillations, at least for the frequency range examined. In case of very small oscillations for yawing the system ship-fluid may also be considered as linear. Application of the strip theory with neglect of viscous effects is adequate for swaying on shallow water in case of small/moderate to high frequencies. In the frequency range considered the strip theory is not satisfactory for yawing on shallow water.

NOMENCLATURE

1. General conventions

- The subscript s indicates 'ship('s wall)'.
- The subscript kc indicates 'keel clearance'.
- The subscript fl indicates 'fluid'.

2. Coordinate systems

$O\bar{x}\bar{y}\bar{z}$	space fixed right-handed system of Cartesian coordinates; $O\bar{x}\bar{y}$ coincides with the water surface at rest; $O\bar{x}\bar{z}$ coincides with the starting-position (equilibrium position) of the ship's longitudinal plane of symmetry, the origin O is situated at mid-length of the body; the $O\bar{z}$-axis is positive downwards (see fig. 2).
$O\bar{y}\bar{z}$	$O\bar{x}\bar{y}\bar{z}$-coordinate system minus $O\bar{x}$-axis (see fig. 1).
Gxyz	moving right-handed system of Cartesian coordinates fixed with respect to the ship; the origin G is the ship's centre of gravity; Gxz coincides with the longitudinal plane of symmetry of the ship; the Gy-axis is positive to starboard, the Gz-axis is positive downwards (see fig. 2).

3. List of symbols

Symbols not included in the list below are only used at a specific place and are explained where they occur.

$\hat{a}$	amplitude of ship motion; amplitude of sway motion; amplitude of motion of struts of horizontal oscillator.
a_{ij}	hydrodynamic coefficient of mass term in i-equation as a result of motion in j-direction in Gxyz.
a_{yy}	added mass for swaying motion (a'_{yy} = idem per unit length).
$a_{y\psi}$, $a_{\psi y}$	hydrodynamic coupling coefficient of mass (moment of inertia) term.

$a_{\psi\psi}$	added mass moment of inertia for yawing motion.
b_{ij}	hydrodynamic coefficient of damping force in i-equation as a result of motion in j-direction.
b_{yy}	sway damping force coefficient (b'_{yy} = idem per unit length).
$b_{y\psi}$, $b_{\psi y}$	hydrodynamic coupling coefficient of damping moment/force.
$b_{\psi\psi}$	yaw damping moment coefficient.
g	acceleration due to gravity.
h	water depth at rest (mean water level); $h^{+} = h + \varepsilon$ with $\varepsilon \downarrow 0$.
i	$\sqrt{-1}$.
k	keel clearance underneath ship.
l	half distance of struts of horizontal oscillator.
m	mass of ship.
m_0	usual wave number.
m_n	wave number satisfying $(n - \frac{1}{2})\pi < m_n h < n\pi$ $(n = 1,2,3,\ldots)$.
m_y	mass of ship model for horizontal motion.
p	fluid pressure.
r, $\dot{r}$	yaw angular velocity and yaw angular acceleration $(= \dot{\psi}, \ddot{\psi})$.
t	time-coordinate.
v, $\dot{v}$	translational velocitv and translational acceleration of origin of body axes G $(= \dot{y}, \ddot{y})$.
v_{fl}	horizontal velocity component of fluid (in region R).
v_{kc}	horizontal fluid velocity in keel clearance underneath ship.
$\hat{v}_{kc}$	amplitude of v_{kc}.
w_{fl}	vertical velocity component of fluid (in region R).
x	surge motion.
y	sway motion.
$\bar{y}_s$	position of ship's wall.
z	heave motion.
A_0, A_n	coefficients in expression for Φ $(n = 1,2,3,\ldots.)$.
A'_0, A'_n	dimensionless expressions for A_0, A_n $(n = 1,2,3,\ldots.)$.
B	beam of ship (model).
B_0, B_n	coefficients in expression for Φ $(n = 1,2,3,\ldots.)$.
B'_0, B'_n	dimensionless expressions for B_0, B_n $(n = 1,2,3,\ldots.)$.

D	draught of ship (model).
F_s	horizontal hydrodynamic force on ship per unit length.
G	centre of gravity of ship (model).
I_{zz}	mass moment of inertia of ship (model) around Gz-axis.
L	length of ship (model).
N	yaw moment in Gxyz.
N_{osc}	exciting harmonic moment.
R	fluid region in which the Laplacian is (to be) solved.
T	period of harmonic oscillation.
$T_{s,kc}$	simple-harmonic function of time.
U	unity step function.
Y	sway force in Gxyz.
Y_{osc}	exciting harmonic (lateral) force.
$Y_{1,co}, Y_{2,co}$	amplitudes of cosine force components as measured in strut 1 and 2 of the horizontal oscillator.
$Y_{1,const.}, Y_{2,const.}$	constant force components as measured in strut 1 and 2 of the horizontal oscillator.
$Y_{1,si}, Y_{2,si}$	amplitudes of sine force components as measured in strut 1 and 2 of the horizontal oscillator.
δ	phase difference between periodic motion of both the struts and motion of G.
θ	phase difference between fluid motion in keel clearance and ship motion.
ρ	specific mass density of fluid.
$\phi_{s,kc}$	harmonic function.
$\psi, \dot{\psi}, \ddot{\psi}$	yaw angle, yaw angular velocity and yaw angular acceleration.
ψ_0	amplitude of (pure) yaw motion.
ω	circular frequency.
Φ	velocity potential.
Φ_{kc}	velocity potential resulting from motion of mass of water underneath ship.
Φ_s	velocity potential resulting from motion of ship.

GENERAL INTRODUCTION

Ships are becoming larger and larger; as a consequence berthing facilities have to be newly constructed or adapted to the larger units. Up to now reliable design criteria, based on a scientific foundation, are scarcely available: the lacking of good design criteria is the most important reason for making researches into the possibilities of an experimental and / or theoretical determination of the impact forces on berthing facilities.

As berthing manoeuvres and impact phenomena take place mainly in the horizontal plane and on shallow water, only the sway and yaw motions of the ship on shallow water are of importance.

If the impact forces are determined in a theoretical way knowledge of the so-called hydrodynamic coefficients is necessary. If the impact forces are determined in an experimental way insight into the behaviour of the hydrodynamic coefficients will be very useful. Anyhow, the determination of the hydrodynamic coefficients in case of swaying and yawing on shallow water is an important and useful matter.

The report is divided into three parts:

part A: Theoretical determination of hydrodynamic coefficients.

part B: Experimental determination of hydrodynamic coefficients.

part C: Comparison of theory and experiment. Conclusions.

part A: THEORETICAL DETERMINATION OF HYDRODYNAMIC COEFFICIENTS

Section A1: Introduction

In this part of the report it is attempted to provide a (simple) method for estimating hydrodynamic coefficients of a schematized ship in case of swaying and yawing on shallow water.

The ship is considered as a rigid prismatic body with a rectangular cross-section. This schematization is justified by the fact that many sea-going vessels and even more inland ships have a more or less box-like shape, being slightly streamlined at bow and stern.

The ship's forward speed is supposed to be zero or negligible; this assumption is justified in part by the fact that in several problems of interest the forward speed is indeed zero or small (e.g. vessels carrying out berthing operations, etc.). Without doubt a forward speed of the ship will be of great influence on the values of the hydrodynamic quantities. Up to now, however, the knowledge about this phenomenon is still very small. This yields another reason to confine this research project to the case of zero speed of advance.

Special attention is paid to the case when shallowness of the water is of dominant importance, bypassing the intermediate range of 'finite' water depth.

On the schematized ship with zero speed of advance - in the horizontal plane - a harmonically oscillating motion is imposed; then, the hydrodynamic coefficients can be determined from the exciting forces.

In Section A2 a general formulation of the hydrodynamic model is provided. The two very important assumptions made to this point are that the displacement of the ship is small and that the fluid motion is two-dimensional (strip theory). Further it is assumed that the fluid is inviscid and incompressible and moves irrotationally.

Section A3 deals with the harmonic analysis of the ship-fluid system. The mathematical approach is based on an existing theory for wave generators [1] adapted to the present case. The hydrodynamic coefficients for pure swaying and yawing are calculated as a function of circular frequency and keel clearance; a survey of the results obtained theoretically is provided.

Section A2: General formulation of hydrodynamic model

The ship is regarded as a rigid prismatic body with a rectangular cross-section. In rest and during motion the keel clearance of the ship is supposed to remain constant; so heaving, pitching and rolling are neglected. Hereby it is assumed implicitly that the heave, pitch and roll motions - which do occur in reality - do not influence the hydrodynamic quantities for the horizontal plane.

It is supposed that the displacements of the ship are small and that the fluid motion is two-dimensional. Consequently, only motions in planes perpendicular to the longitudinal plane of symmetry of the ship will be considered; the calculations relate to the unit length. Furthermore it will be assumed that the velocities remain low so that the governing equations can be linearized.

The bottom is horizontal. The horizontal dimensions of the water surface are unlimited.

The origin of a fixed rectangular $O\bar{y}\bar{z}$-coordinate system is situated in the still water level. The $O\bar{y}$ - axis is horizontal. The vertical $O\bar{z}$ - axis is positive downwards and coincides with the starting-position of the ship's longitudinal plane of symmetry.
The time-coordinate is represented by t.
The water depth at rest (i.e. the mean water level) is represented by h, the keel clearance by k, the draught, the beam and the length of the ship by D = h - k, B and L, respectively. For a definition sketch see fig. 1.

In virtue of above-stated assumptions the mathematical problem may be formulated in terms of a velocity potential, $\Phi = \Phi(\bar{y},\bar{z},t)$, defined through

$$\nabla\Phi = (\Phi_{\bar{y}}, \Phi_{\bar{z}})$$

where subscripts $\bar{y}$ and $\bar{z}$ indicate partial differentiation with respect

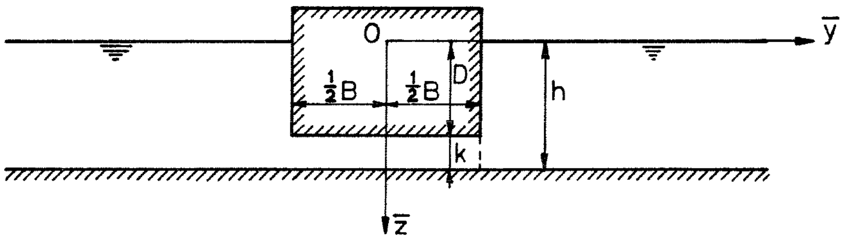

fig.1 – Definition sketch.

to the chosen coordinates. The horizontal and vertical velocity components of a fluid particle with coordinates $\bar{y}$ and $\bar{z}$ at the time t are:

$$v_{fl} = \Phi_{\bar{y}} \quad \text{and} \quad w_{fl} = \Phi_{\bar{z}} \; ;$$

the subscript fl refers to fluid.

The velocity potential Φ must satisfy the Laplace equation

(1) $$\nabla^2\Phi = \frac{\partial^2\Phi}{\partial\bar{y}^2} + \frac{\partial^2\Phi}{\partial\bar{z}^2} = 0$$

in the field of flow, subject to relevant boundary conditions on all boundary surfaces and at infinity.

It is obvious that on each side of the ship a velocity potential exists. As only small motions of the ship are considered it can be stated that these respective velocity potentials are antimetric. Coupling of the fields of flow on both sides of the ship will be done by applying the law of conservation of momentum to the mass of water underneath the ship. As a consequence, it is sufficient to determine the velocity potential only on one side of the ship.

Now define a region R, occupied by fluid, in which the Laplace equation is to be solved:

$$R\begin{cases} \bar{y} > \tfrac{1}{2}B, & 0 \leqq \bar{z} \leqq h \\ \bar{y} = \tfrac{1}{2}B, & 0 \leqq \bar{z} < (h-k), \quad (h-k) \leqq \bar{z} \leqq h . \end{cases}$$

The boundary conditions on all boundary surfaces and at infinity must be known; they are given below.

The free-surface boundary condition states that the pressure along the free-surface, ignoring surface tension, is constant and that fluid particles which are on the free-surface remain there as time passes. This boundary condition is in linearized form:

$$(2) \qquad \frac{\partial^2 \Phi}{\partial t^2} - g\,\frac{\partial \Phi}{\partial \bar{z}} = 0 \quad \text{on} \quad \bar{y} \geqq \tfrac{1}{2}B, \quad \bar{z} = 0,$$

where g = acceleration due to gravity.

At the wall of the ship the horizontal fluid velocity is equal to the velocity of the ship in that direction. As only small displacements of the ship from its starting-position are considered, the boundary condition for the velocities at the ship's wall applies at $\bar{y} = \frac{1}{2}B$ and can be written as:

$$\frac{\partial \Phi}{\partial \bar{y}} = \{U(\bar{z}) - U(\bar{z} - h + k)\}\,\frac{\partial}{\partial t}\,\bar{y}_s \quad \text{on} \quad \bar{y} = \tfrac{1}{2}B,$$

where

$$U(\bar{z}) = \text{unity step function} = \begin{cases} 0 & \text{on} \quad \bar{z} < 0 \\ 1 & \text{on} \quad \bar{z} \geqq 0 , \end{cases}$$

$\bar{y}_s = \bar{y}_s(\bar{z},t)$ = position of ship's wall, the subscript s refers to ship('s wall).

Between the underside of the ship and the bottom (i.e. the keel clearance) the vertical velocities are neglected; the horizontal velocity distribution is supposed to be uniform. On the analogy of the above-mentioned eq. the boundary condition for the velocities across the keel clearance also applies at $\bar{y} = \frac{1}{2}$ B and can be expressed as:

$$\frac{\partial \Phi}{\partial \bar{y}} = \{U(\bar{z} - h + k) - U(\bar{z} - h^{+})\}\, v_{kc} \quad \text{on} \quad \bar{y} = \tfrac{1}{2} B,$$

where

$v_{kc} = v_{kc}(\bar{z},t)$ = horizontal fluid velocity in keel clearance, the subscript kc refers to keel clearance,

$h^{+} = h + \varepsilon$ with ε tending to zero ($\varepsilon \downarrow 0$).

At this stage the functions $\bar{y}_s = \bar{y}_s(\bar{z},t)$ and $v_{kc} = v_{kc}(\bar{z},t)$ are not yet specified. Solving the velocity potential for a particular case relevant expressions for $\bar{y}_s$ and v_{kc} will be prescribed.
The complete boundary condition on the plane $\bar{y} = \frac{1}{2}$ B then can be written as:

(3) $$\frac{\partial \Phi}{\partial \bar{y}} = \{U(\bar{z}) - U(\bar{z} - h + k)\} \frac{\partial}{\partial t} \bar{y}_s + \{U(\bar{z} - h + k) - U(\bar{z} - h^{+})\}\, v_{kc}$$

$$\text{on } \bar{y} = \tfrac{1}{2} B .$$

Supposing the bottom impervious simply implies the boundary condition:

(4) $$\frac{\partial \Phi}{\partial \bar{z}} = 0 \quad \text{on} \quad \bar{y} \geqq \tfrac{1}{2} B, \quad \bar{z} = h .$$

The boundary condition at infinity states that:

(5) $$\Phi(\bar{y},\bar{z},t)\Big|_{\bar{y} \to +\infty} \longrightarrow \text{outgoing dispersive wave},$$

or, at infinity only simple-harmonic waves propagating in positive $\bar{y}$-direction are possible.

As a supplementary condition it is supposed that in region R the function $\Phi(\bar{y},\bar{z},t)$ together with its first derivatives remain finite:

(6) $$\Phi(\bar{y},\bar{z},t), \quad \Phi^{1}(\bar{y},\bar{z},t) \text{ being finite in R;}$$

the superscript 1 means first derivative.

Summarizing: the velocity potential $\Phi(\bar{y},\bar{z},t)$ has to satisfy the homogeneous linear partial differential equation (1) plus a set of non-homogeneous boundary conditions (2), (3), (4) and (5) and the supplementary condition (6). The solution of eqs. (2), (3), (4) and (5) specifies a mixed boundary-value problem for the Laplace equation.

Section A3: Harmonic analysis

A3.1 : Solution of mixed boundary-value problem

The hydrodynamic problem as formulated in Section A2 will be solved for the particular case that a simple-harmonic motion is imposed on the ship. The harmonic oscillations take place about an equilibrium position (viz. the considered starting-position of the ship's longitudinal plane of symmetry) with period T.
Then the position of the ship's wall $\bar{y}_s = \bar{y}_s(\bar{z},t)$ may be represented by

$$\bar{y}_s = \tfrac{1}{2}\,B - i\xi(\bar{z})e^{i\omega t} \quad ,$$

where $\xi(\bar{z})$ = amplitude of ship motion, $\xi(\bar{z}) > 0$, limited and real,

$i = \sqrt{-1}$,

$\omega = \frac{2\pi}{T}$ = circular frequency.

During the oscillation the ship's walls remain vertical. Therefore the amplitude of ship motion is independent of $\bar{z}$ and will be denoted further by $\hat{a}$:

$$\xi(\bar{z}) = \hat{a} \ .$$

As the motion imposed on the ship is harmonic in time, the expression for the horizontal fluid velocity in the keel clearance, v_{kc}, will be of the form:

$$v_{kc} = \hat{v}_{kc}\, e^{i(\omega t - \theta)},$$

where

$\hat{v}_{kc}$ = amplitude of horizontal fluid velocity in keel clearance, $\hat{v}_{kc} > 0$, independent of $\bar{z}$ and real,

θ = phase difference between fluid motion in keel clearance and ship motion, θ = constant and real.

With the above-prescribed expressions for $\bar{y}_s$ and v_{kc} the boundary condition (3) is modified into:

$$\frac{\partial \Phi}{\partial \bar{y}} = \omega[\hat{a}\{U(\bar{z}) - U(\bar{z} - h + k)\} + \frac{\hat{v}_{kc}}{\omega} e^{-i\theta} \{U(\bar{z} - h + k) + - U(\bar{z} - h^{+})\}]\, e^{i\omega t} \quad \text{on} \quad \bar{y} = \tfrac{1}{2} B. \tag{3'}$$

It is essential to note that so far $\hat{v}_{kc}$ and θ are unknown quantities. At determining the velocity potential, however, it is necessary to suppose that $\hat{v}_{kc}$ and θ are known constants. Once having calculated the velocity potential on basis of this supposition, $\hat{v}_{kc}$ and θ can be determined by applying the law of conservation of momentum to the mass of water underneath the ship.

Now it can be stated that the velocity potential $\Phi(\bar{y},\bar{z},t)$ is a simple-harmonic function of time which has to satisfy the Laplace equation (1) plus the set of non-homogeneous boundary conditions (2), (3'), (4) and (5) and the supplementary condition (6).
The problem of determining this velocity potential is that treated by F. Biésel [1]. The solution can be considered as an extension of his work and is outlined in Appendix I. From this Appendix I the general solution for the velocity potential $\Phi(\bar{y},\bar{z},t)$ is seen to be given by:

$$\Phi(\bar{y},\bar{z},t) = i\, \frac{\omega}{m_0} (A_0 + B_0\, e^{-i\theta})\cosh\{m_0(h - \bar{z})\}e^{i(\omega t - m_0\bar{y} + \frac{m_0 B}{2})} + \tag{7}$$

$$- \sum_{n=1}^{\infty} \frac{\omega}{m_n} (A_n + B_n e^{-i\theta}) e^{-m_n \bar{y} + \frac{m_n B}{2}} \cos\{m_n(h - \bar{z})\} e^{i\omega t} ,$$

where

(8^a) m_0 = positive root of $\omega^2 = g m_0 \tanh(m_0 h)$,

(8^b) m_n = positive roots of $\omega^2 = - g m_n \tan(m_n h)$ $(n = 1,2,\ldots; \; m_1 < m_2 < \ldots < m_n < \ldots)$,

(9^a) $$\frac{A_0}{\hat{a}} = A_0' = \frac{2\{\sinh(m_0 h) - \sinh(m_0 k)\}}{m_0 h + \sinh(m_0 h)\cosh(m_0 h)} ,$$

(9^b) $$\frac{A_n}{\hat{a}} = A_n' = \frac{2\{\sin(m_n h) - \sin(m_n k)\}}{m_n h + \sin(m_n h)\cos(m_n h)} ,$$

(10^a) $$\frac{B_0 \omega}{\hat{v}_{kc}} = B_0' = \frac{2 \sinh(m_0 k)}{m_0 h + \sinh(m_0 h)\cosh(m_0 h)} ,$$

(10^b) $$\frac{B_n \omega}{\hat{v}_{kc}} = B_n' = \frac{2 \sin(m_n k)}{m_n h + \sin(m_n h)\cos(m_n h)} .$$

(8^a) and (8^b) are the relationships for the wave numbers: m_0 is the usual wave number and the m_n's satisfy $(n - \frac{1}{2})\pi < m_n h < n\pi$.

A3.2.1 : Determination of hydrodynamic forces

The fluid pressure can be obtained from the linearized equation of Bernoulli for unsteady flow:

$$\frac{\partial \Phi}{\partial t} + \frac{p}{\rho} - g\bar{z} = 0 ,$$

where

$p = p(\bar{y},\bar{z},t)$ = fluid pressure ,

ρ = specific mass density of fluid.

Substituting the velocity potential $\Phi(\bar{y},\bar{z},t)$ as represented by eq. (7) into the equation of Bernoulli the fluid pressure on the vertical plane $\bar{y} = \frac{1}{2}$ B becomes:

$$p(\tfrac{1}{2}B,\bar{z},t) = \rho\frac{\omega^2}{m_0^2}(A_0 + B_0\, e^{-i\theta})\cosh\{m_0(h - \bar{z})\}e^{i\omega t} +$$

$$+\rho\sum_{n=1}^{\infty} i\,\frac{\omega^2}{m_n^2}(A_n + B_n\, e^{-i\theta})\cos\{m_n(h - \bar{z})\}e^{i\omega t} + \rho g\bar{z} .$$

The horizontal force per unit length as exerted by the fluid on the ship's wall in question, $f_s(t)$, then is:

$$f_s(t) = \int_0^{h-k} p(\tfrac{1}{2}\,B,\bar{z},t)\,d\bar{z} =$$

$$= \rho\,\frac{\omega^2}{m_0^2}(A_0 + B_0\, e^{-i\theta})\{\sinh(m_0h) - \sinh(m_0k)\}e^{i\omega t} +$$

$$+ \rho\sum_{n=1}^{\infty} i\,\frac{\omega^2}{m_n^2}(A_n + B_n\, e^{-i\theta})\{\sin(m_nh) - \sin(m_nk)\}e^{i\omega t} +$$

$$+\tfrac{1}{2}\rho g(h - k)^2 ;$$

the first two terms in the right-hand side of this expression represent the hydrodynamic part of the force, the last term is the hydrostatic part.

On each side of the ship a velocity potential exists. These respective velocity potentials are antimetric. The antimetry of the fields of flow on both sides of the ship is the cause of the fact that - at determining the total horizontal force - the hydrostatic contributions to the individual horizontal forces on each wall of the ship cancel. Then the total horizontal (hydrodynamic) force on the ship per unit length, $F_s(t)$, becomes twice the hydrodynamic contribution to the force on a single wall of the ship:

$$(11)\qquad F_s(t) = 2\rho \frac{\omega^2}{m_0^2} (A_0 + B_0\, e^{-i\theta})\{\sinh(m_0 h) - \sinh(m_0 k)\}e^{i\omega t} + + 2\rho \sum_{n=1}^{\infty} i \frac{\omega^2}{m_n^2} (A_n + B_n\, e^{-i\theta})\{\sin(m_n h) - \sin(m_n k)\}e^{i\omega t} ;$$

the real part of $F_s(t)$ is:

$$(11^a)\qquad \mathrm{Re}\{F_s(t)\} = 2\rho \frac{\omega^2}{m_0^2} \{\sinh(m_0 h) - \sinh(m_0 k)\}\{A_0 \cos(\omega t) + + B_0 \cos(\omega t - \theta)\} - 2\rho \sum_{n=1}^{\infty} \frac{\omega^2}{m_n^2} \{\sin(m_n h) - \sin(m_n k)\}\cdot \cdot\{A_n \sin(\omega t) + B_n \sin(\omega t - \theta)\} ,$$

and the imaginary part:

$$(11^b)\qquad \mathrm{Im}\{F_s(t)\} = 2\rho \frac{\omega^2}{m_0^2} \{\sinh(m_0 h) - \sinh(m_0 k)\}\{A_0 \sin(\omega t) + + B_0 \sin(\omega t - \theta)\} + 2\rho \sum_{n=1}^{\infty} \frac{\omega^2}{m_n^2} \{\sin(m_n h) - \sin(m_n k)\}\cdot \cdot\{A_n \cos(\omega t) + B_n \cos(\omega t - \theta)\} .$$

In a similar way it can be derived for the total horizontal (hydrodynamic) force on the mass of water underneath the ship per unit length, i.e. $F_{kc}(t)$:

(12) $$F_{kc}(t) = 2\rho \frac{\omega^2}{m_0^2} (A_0 + B_0\, e^{-i\theta}) \sinh(m_0 k) e^{i\omega t} + 2\rho \sum_{n=1}^{\infty} i \frac{\omega^2}{m_n^2} \cdot (A_n + B_n\, e^{-i\theta}) \sin(m_n k) e^{i\omega t} \,.$$

A3.2.2 : Coupling of fields of flow on both sides of the ship

Coupling of the fields of flow on both sides of the ship will be done by applying the law of conservation of momentum to the mass of water underneath the ship. This procedure renders possible the calculation of the unknown quantities $\hat{v}_{kc}$ and θ, which - up to now - were supposed to be known constants.

Neglecting friction effects the law of conservation of momentum as applied to the mass of water underneath the ship yields:

$$- F_{kc}(t)\, dt = d(M\, v_{kc}) \;,$$

where $M = \rho\, B\, k$ = mass of water underneath the ship per unit ship's length. By substitution of $F_{kc}(t)$ as represented by eq. (12) into this expression one obtains (with $\omega \neq 0$):

$$\frac{2}{m_0^2} (A_0 + B_0\, e^{-i\theta}) \sinh(m_0 k) + \sum_{n=1}^{\infty} i \frac{2}{m_n^2} (A_n + B_n\, e^{-i\theta}) \sin(m_n k) = - i\, B\, k \frac{\hat{v}_{kc}}{\omega} e^{-i\theta};$$

equating the respective real and imaginary parts then gives:

$$\frac{2}{m_0^2} \{A_0 + B_0 \cos(\theta)\} \sinh(m_0 k) + \sum_{n=1}^{\infty} \frac{2}{m_n^2} B_n \sin(m_n k) \sin(\theta) = - B\, k \frac{\hat{v}_{kc}}{\omega} \sin(\theta)$$

and

$$- \frac{2}{m_0^2} B_0 \sinh(m_0 k)\sin(\theta) + \sum_{n=1}^{\infty} \frac{2}{m_n^2} \{A_n + B_n \cos(\theta)\}\sin(m_n k) =$$

$$= - B\, k \frac{\hat{v}_{kc}}{\omega} \cos(\theta) .$$

Introduction of the quantities

(13[a]) $$a_0 = \frac{2 A_0'}{m_0^2} \sinh(m_0 k) \quad , \qquad a_n = \sum_{n=1}^{\infty} \frac{2 A_n'}{m_n^2} \sin(m_n k) ,$$

(13[b]) $$b_0 = \frac{2 B_0'}{m_0^2} \sinh(m_0 k) \quad , \qquad b_n = \sum_{n=1}^{\infty} \frac{2 B_n'}{m_n^2} \sin(m_n k) ,$$

and

(13[c]) $$c = B\, k$$

into these two equations, using the relationships ($9^{a,b}$) and ($10^{a,b}$), finally yields:

$$\frac{\hat{a}\omega}{\hat{v}_{kc}} a_0 + b_0 \cos(\theta) + b_n \sin(\theta) = - c \sin(\theta)$$

and

$$\frac{\hat{a}\,\omega}{\hat{v}_{kc}} a_n - b_0 \sin(\theta) + b_n \cos(\theta) = - c \cos(\theta) \quad .$$

These last two expressions represent a set of two equations with two unknowns, viz. $\hat{v}_{kc}$ and θ. For a_0, a_n, b_0, b_n and c are known in principle, because both A_0' through A_n' and B_0' through B_n' can be calculated for all values of ω from eqs. ($9^{a,b}$) and ($10^{a,b}$) together with eqs. (8^a) and (8^b). The solution for θ and $\hat{v}_{kc}$ can be written in the dimensionless form:

14) $$\tan(\theta) = \frac{a_n b_0 - a_0(b_n + c)}{- a_0 b_0 - a_n(b_n + c)} \quad , \ \theta = \mathrm{atan}\ \{a_n b_0 - a_0\ (b_n + c) \ ,$$

$$- a_0 b_0 - a_n(b_n + c)\} \ ,$$

(15) $$\frac{\hat{v}_{kc}}{\omega\ \hat{a}} = \frac{-\ a_0}{b_0\ \cos(\theta) + (b_n + c)\sin(\theta)} = \frac{a_n}{b_0\ \sin(\theta) - (b_n + c)\cos(\theta)}\ .$$

A3.3: Determination of hydrodynamic coefficients

A3.3.1: Introductory remarks

The space fixed (two-dimensional) rectangular $O\bar{y}\bar{z}$ - coordinate system, as defined in Section A2, is extended now to a (three-dimensional) right-handed Cartesian $O\bar{x}\bar{y}\bar{z}$ - coordinate system, such that the $O\bar{x}$-axis coincides with the equilibrium position of the ship's longitudinal plane of symmetry. The associated moving coordinates fixed with respect to the ship are denoted by (x,y,z), the right-hand convention is applied so that the y-axis is positive to starboard, and y = 0 is taken to be the longitudinal plane of symmetry of the ship; the origin coincides with the ship's centre of gravity G.

Considering merely ship motions in the horizontal plane in addition to which the forward speed is supposed to be zero, a force Y, a moment N, a translational velocity v and an angular velocity r can be indicated; all these quantities are defined in relation to the ship-fixed Gxyz-coordinate system. The angular orientation of the ship is defined by the symbol ψ (yaw): if the Gxyz-coordinate system coincides initially with the fixed $O\bar{x}\bar{y}\bar{z}$ - coordinate system, then the orientation is obtained by a yaw angular displacement with respect to the Gxyz-coordinate system and in the right-hand sense.

For a definition of symbols see fig. 2.

For reasons of simplicity the mass distribution of the ship is supposed to be symmetrical with respect to the plane y = 0.

Neglecting the longitudinal surge motion together with the forces in the surge direction, and supposing that waves and current do not occur, the linearized 'equations of motion' in the Gxyz - coordinate system can be represented by (see refs. [2] and [3]):

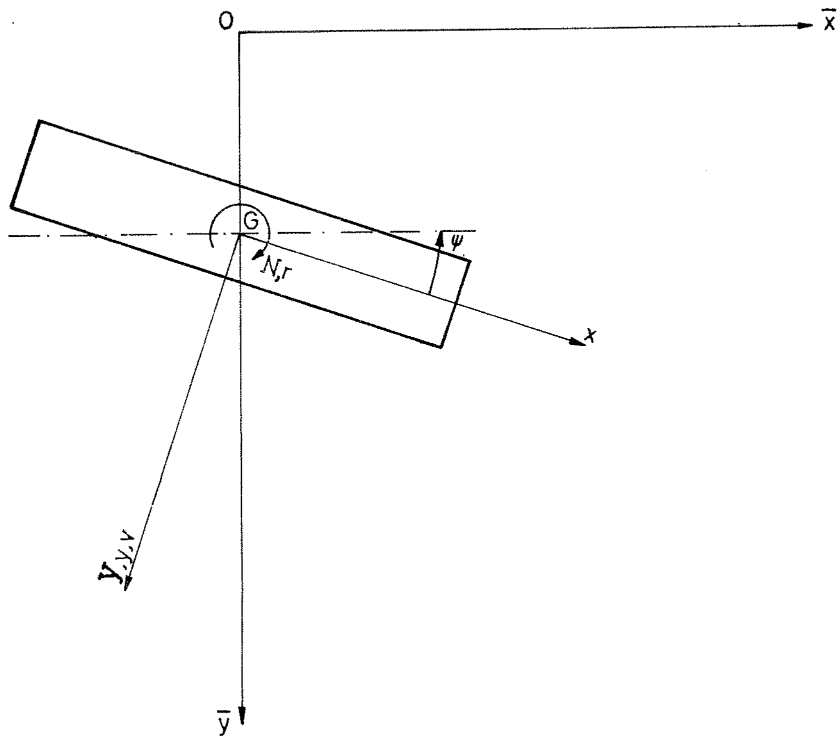

fig.2 – Definition of symbols.

$$(m + a_{yy})\ddot{y} + b_{yy}\,\dot{y} + a_{y\psi}\,\ddot{\psi} + b_{y\psi}\,\dot{\psi} = Y_{osc} \quad \text{(sway motion)} ,$$

(16)

$$(I_{zz} + a_{\psi\psi})\ddot{\psi} + b_{\psi\psi}\,\dot{\psi} + a_{\psi y}\,\ddot{y} + b_{\psi y}\,\dot{y} = N_{osc} \quad \text{(yaw motion)} ,$$

where m = mass of ship,

I_{zz} = mass moment of inertia around the Gz-axis,

a_{ij} = hydrodynamic coefficient of mass term in i-equation as a result of motion in j-direction in Gxyz,

b_{ij} = hydrodynamic coefficient of damping force in i-equation as a result of motion in j-direction in Gxyz,

Y_{osc} = exciting harmonic (lateral) force,

N_{osc} = exciting harmonic moment;

dots over the quantities y and ψ mean derivatives with respect to time. The linearization of these equations is based on the supposition that the unsteady motions are small perturbations of an initial steady state situation (e.g. a situation of rest).

As shown in refs. [2] and [3] the values of the hydrodynamic quantities with respect to the Gxyz - coordinate system, viz. a_{ij} and b_{ij}, in case of purely horizontal motions are not influenced by the position of the centre of gravity G in height. Therefore it may be supposed that the Gxyz - coordinate system in the equilibrium position coincides with the fixed $O\overline{xyz}$ - coordinate system.

The 'equations of motion' in case of pure swaying follow from eq. (16) by equalizing all other motions to zero, i.e. $\dot{\psi} = \ddot{\psi} = 0$:

$$\left.\begin{aligned} (m + a_{yy})\ddot{y} + b_{yy}\,\dot{y} &= Y_{osc} \\ a_{\psi y}\,\ddot{y} + b_{\psi y}\,\dot{y} &= N_{osc} \end{aligned}\right\} \text{pure swaying} \tag{17}$$

Likewise the 'equations of motion' for pure yawing can be determined from eq. (16) by substitution of $\dot{y} = \ddot{y} = 0$:

(13)
$$\left.\begin{aligned} a_{y\psi}\ddot{\psi} + b_{y\psi}\dot{\psi} &= Y_{osc} \\ (I_{zz} + a_{\psi\psi})\ddot{\psi} + b_{\psi\psi}\dot{\psi} &= N_{osc} \end{aligned}\right\} \text{ pure yawing}$$

The two sets of equations (17) and (18) hold only good if all variables depend sinusoidally on time at a same frequency. The hydrodynamic coefficients are considered to pertain to a sinusoidal displacement of the ship with respect to the equilibrium position, and therefore they are functions of the frequency. For a more comprehensive explanation of the character of these so-called 'equations of motion' is referred to refs. [4] and [5].

The hydrodynamic coefficients are independent of the distribution of mass over the ship, they do depend on the geometry of the hull. For this reason in the following the quantities m and I_{zz} may be omitted from the 'equations of motion'.

Supposing that in case of pure swaying a_{yy} and b_{yy} are known for every transverse section of the hull (at x), the hydrodynamic coefficients for the entire ship can be obtained by integration over the length:

(19)
$$\begin{pmatrix} a_{yy} \\ a_{\psi\psi} \end{pmatrix} = \int_L \begin{pmatrix} 1 \\ x^2 \end{pmatrix} a_{yy}(x)\, dx = L \begin{pmatrix} a_{yy}(x) \\ \frac{1}{12} L^2 \ a_{yy}(x) \end{pmatrix} , \begin{pmatrix} b_{yy} \\ b_{\psi\psi} \end{pmatrix} =$$

$$= \int_L \begin{pmatrix} 1 \\ x^2 \end{pmatrix} b_{yy}(x)\, dx = L \begin{pmatrix} b_{yy}(x) \\ \frac{1}{12} L^2 \ b_{yy}(x) \end{pmatrix} ,$$

and

$$\begin{pmatrix} a_{\psi y} \\ \\ b_{\psi y} \end{pmatrix} = \begin{pmatrix} a_{y\psi} \\ \\ b_{y\psi} \end{pmatrix} = \int_L x \begin{pmatrix} a_{yy}(x) \\ \\ b_{yy}(x) \end{pmatrix} dx = 0 \quad .$$

Since the geometry of the hull of the schematized ship is symmetrical with respect to the plane x = 0, this means that $a_{\psi y} = a_{y\psi} = 0$ and $b_{\psi y} = b_{y\psi} = 0$

The 'equations of motion' in the respective cases of pure swaying and pure yawing then can be written as:

$$(17^a) \qquad a_{yy}\,\ddot{y} + b_{yy}\,\dot{y} = Y_{osc} \;,$$

and

$$(18^a) \qquad a_{\psi\psi}\,\ddot{\psi} + b_{\psi\psi}\,\dot{\psi} = N_{osc} \quad .$$

A3.3.2 : Determination of hydrodynamic coefficients for swaying

As indicated in Section A3.1 the horizontal displacement of the ship in case of pure swaying can be represented by $y = -i\,\hat{a}\,e^{i\omega t}$. Substitution of this expression into eq. (17^a) yields:

$$-\,\hat{a}\,\omega^2\,a_{yy}\,\sin(\omega t) + \hat{a}\,\omega\,b_{yy}\,\cos(\omega t) + i\,\{\hat{a}\,\omega^2\,a_{yy}\,\cos(\omega t) + $$
$$+\,\hat{a}\,\omega\,b_{yy}\,\sin(\omega t)\} = Y_{osc} \;.$$

The total horizontal force on the ship per unit length, $F_s(t)$, as resulting from a harmonically oscillating motion (viz. pure swaying) is provided by eq. (11). Consequently it must hold good that

$$Y_{osc} = F_s(t) \;.$$

By combining the two expressions mentioned above and equating the respective real and imaginary parts one obtains:

$$-\,\hat{a}\,\omega^2\,a_{yy}\,\sin(\omega t) + \hat{a}\,\omega\,b_{yy}\,\cos(\omega t) = \mathrm{Re}\,\{F_s(t)\}$$

and

$$\hat{a}\,\omega^2\,a_{yy}\cos(\omega t) + \hat{a}\,\omega\,b_{yy}\sin(\omega t) = \mathrm{Im}\{F_s(t)\} ,$$

where $\mathrm{Re}\{F_s(t)\}$ and $\mathrm{Im}\{F_s(t)\}$ are given by eq. (11^a) and eq. (11^b), respectively. The sum of the terms containing the respective factors $\cos(\omega t)$ and $\sin(\omega t)$ must equal zero; this yields:

$$2\rho\frac{\omega^2}{m_0^2}\{A_0 + B_0\cos(\theta)\}\{\sinh(m_0h) - \sinh(m_0k)\} +$$

$$+ 2\rho\sum_{n=1}^{\infty}\frac{\omega^2}{m_n^2}B_n\sin(\theta)\{\sin(m_nh) - \sin(m_nk)\} = \hat{a}\,\omega\,b_{yy},$$

$$2\rho\frac{\omega^2}{m_0^2}B_0\sin(\theta)\{\sinh(m_0h) - \sinh(m_0k)\} +$$

$$- 2\rho\sum_{n=1}^{\infty}\frac{\omega^2}{m_n^2}\{A_n + B_n\cos(\theta)\}\{\sin(m_nh) - \sin(m_nk)\} =$$

$$= -\hat{a}\,\omega^2\,a_{yy} .$$

Using eqs. ($9^{a,b}$) and ($10^{a,b}$) and the two above expressions the hydrodynamic coefficients of the mass term and the damping force can be derived as functions of the two-dimensional transverse section at x:

(20) $$a_{yy}(x) = -2\rho\frac{1}{m_0^2}\frac{\hat{v}_{kc}}{\omega\hat{a}}\sin(\theta)B_0'\{\sinh(m_0h) - \sinh(m_0k)\} +$$

$$+ 2\rho\sum_{n=1}^{\infty}\frac{1}{m_n^2}\{A_n' + \frac{\hat{v}_{kc}}{\omega\hat{a}}\cos(\theta)B_n'\}\{\sin(m_nh) - \sin(m_nk)\} ,$$

(21) $$b_{yy}(x) = 2\rho\frac{\omega}{m_0^2}\{A_0' + \frac{\hat{v}_{kc}}{\omega\hat{a}}\cos(\theta)B_0'\}\{\sinh(m_0h) - \sinh(m_0k)\} +$$

$$+ 2\rho \sum_{n=1}^{\infty} \frac{\omega}{m_n^2} \frac{\hat{v}_{kc}}{\omega\hat{a}} \sin(\theta)\, B_n' \{\sin(m_n h) - \sin(m_n k)\} .$$

Because of eq. (19) the hydrodynamic coefficients for the entire ship in case of pure swaying then become:

(20[a])
$$a_{yy} = 2\ \rho\ L \left[- \frac{1}{m_0^2} \frac{\hat{v}_{kc}}{\omega\hat{a}} \sin(\theta)\, B_0' \{\sinh(m_0 h) - \sinh(m_0 k)\} + \sum_{n=1}^{\infty} \frac{1}{m_n^2} \{A_n' + \frac{\hat{v}_{kc}}{\omega\hat{a}} \cos(\theta)\, B_n'\}\{\sin(m_n h) - \sin(m_n k)\}\right]$$

and

(21[a])
$$b_{yy} = 2\ \rho\ L \left[\frac{\omega}{m_0^2} \{A_0' + \frac{\hat{v}_{kc}}{\omega\hat{a}} \cos(\theta)\, B_0'\}\{\sinh(m_0 h) - \sinh(m_0 k)\} + \sum_{n=1}^{\infty} \frac{\omega}{m_n^2} \frac{\hat{v}_{kc}}{\omega\hat{a}} \sin(\theta)\, B_n' \{\sin(m_n h) - \sin(m_n k)\}\right] .$$

A3.3.3: Determination of hydrodynamic coefficients for yawing

On account of eq. (19) the hydrodynamic coefficients for the entire ship in case of pure yawing can be written as:

(20[b])
$$a_{\psi\psi} = \frac{1}{12} L^2\ a_{yy}$$

and

(21[b])
$$b_{\psi\psi} = \frac{1}{12} L^2\ b_{yy} ,$$

where a_{yy} and b_{yy} are represented by eq. (20^a) and eq. (21^a), respectively.

A3.3.4: Hydrodynamic coefficients in case $\omega \to 0$ and $\omega \to \infty$

Eqs. (20^a, 21^a) and eqs. (20^b, 21^b) represent the hydrodynamic coefficients for the entire ship in case of pure swaying and pure yawing, respectively. In the case that $\omega \to 0$ or $\omega \to \infty$ the relevant transitions to the limit for the hydrodynamic coefficients have to be determined. Considering the case $\omega \to 0$ one obtains from eqs. (20^a, 20^b) and eqs. (21^a, 21^b):

$$(22^a) \qquad \lim_{\omega\to 0} (a_{yy}) = 2\,\rho\, L\, \{\frac{B(h-k)^2}{2k} + 2\,\frac{h^4}{k^2} \sum_{n=1}^{\infty} \frac{1}{(n\pi)^3}\, \sin^2(n\pi\,\frac{k}{h})\}\ ,$$

$$(22^b) \qquad \lim_{\omega\to 0} (a_{\psi\psi}) = \frac{1}{12}\, L^2 \lim_{\omega\to 0} (a_{yy})$$

and

$$(23^a) \qquad \lim_{\omega\to 0} (b_{yy}) = 0\ ,$$

$$(23^b) \qquad \lim_{\omega\to 0} (b_{\psi\psi}) = \frac{1}{12}\, L^2 \lim_{\omega\to 0} (b_{yy})\ .$$

Likewise one obtains for $\omega \to \infty$:

$$(24^a) \qquad \lim_{\omega\to\infty} (a_{yy}) = 4\,\rho\, Lh^2 \left\{ \sum_{n=1}^{\infty} \frac{[(-1)^n + \sin\{\frac{k}{h}(2n-1)\frac{\pi}{2}\}]^2}{\{(2n-1)\frac{\pi}{2}\}^3} + \right. \\ \left. - \frac{\left[\sum_{n=1}^{\infty} \frac{\sin\{\frac{k}{h}(2n-1)\frac{\pi}{2}\}\,[(-1)^n + \sin\{\frac{k}{h}(2n-1)\frac{\pi}{2}\}]}{\{(2n-1)\frac{\pi}{2}\}^3}\right]^2}{\frac{Bk}{4h^2} + \sum_{n=1}^{\infty} \frac{\sin^2\{\frac{k}{h}(2n-1)\frac{\pi}{2}\}}{\{(2n-1)\frac{\pi}{2}\}^3}} \right\}\ ,$$

$$(24^b) \qquad \lim_{\omega\to\infty} (a_{\psi\psi}) = \frac{1}{12}\, L^2 \lim_{\omega\to\infty} (a_{yy})$$

and

$$(25^a) \qquad \lim_{\omega\to\infty} (b_{yy}) = 0\ ,$$

$$(25^b) \qquad \lim_{\omega\to\infty} (b_{\psi\psi}) = \frac{1}{12}\, L^2 \lim_{\omega\to\infty} (b_{yy})\ .$$

A3.3.5: Hydrodynamic coefficients in case of zero keel clearance

The relevant expressions for the hydrodynamic coefficients in the case under consideration can be derived by taking the transition for $k \to 0$ to the limit from eqs. (20^a, 20^b) and (21^a, 21^b). The results are:

$$(26^a) \qquad a_{yy}\big|_{k=0} = 2\,\rho\,L \sum_{n=1}^{\infty} \frac{1}{m_n^2} A_n'' \sin(m_n h) \ ,$$

$$(27^a) \qquad b_{yy}\big|_{k=0} = 2\,\rho\,L \frac{\omega}{m_0^2} A_0'' \sinh(m_0 h)$$

and

$$(26^b) \qquad a_{\psi\psi}\big|_{k=0} = \frac{1}{12} L^2\, a_{yy}\big|_{k=0} \ ,$$

$$(27^b) \qquad b_{\psi\psi}\big|_{k=0} = \frac{1}{12} L^2\, b_{yy}\big|_{k=0} \ ,$$

where m_0 and m_n are represented by the respective eqs. (8^a) and (8^b),

$$(28^{a,b}) \qquad A_0'' = \frac{2\sinh(m_0 h)}{m_0 h + \sinh(m_0 h)\cosh(m_0 h)} \ , \quad A_n'' = \frac{2\sin(m_n h)}{m_n h + \sin(m_n h)\cos(m_n h)} \ .$$

Likewise it holds good that

$$(29^a) \qquad \lim_{\omega \to 0} \left(a_{yy}\big|_{k=0}\right) = 0 \ ,$$

$$(29^b) \qquad \lim_{\omega \to 0} \left(a_{\psi\psi}\big|_{k=0}\right) = \frac{1}{12} L^2 \lim_{\omega \to 0} \left(a_{yy}\big|_{k=0}\right)$$

and

$$(30^a) \qquad \lim_{\omega \to 0} \left(b_{yy}\big|_{k=0}\right) = 2\,\rho\,Lh\sqrt{gh} \ ,$$

$$(30^b) \qquad \lim_{\omega \to 0} \left(b_{\psi\psi}\big|_{k=0}\right) = \frac{1}{12} L^2 \lim_{\omega \to 0} \left(b_{yy}\big|_{k=0}\right) \ ,$$

whereas

(31[a]) $$\lim_{\omega\to\infty} (a_{yy}|_{k=0}) = 4\ \rho\ Lh^2 \sum_{n=1}^{\infty} \frac{1}{\{(2n-1)\ \frac{\pi}{2}\}^3} \quad ,$$

(31[b]) $$\lim_{\omega\to\infty} (a_{\psi\psi}|_{k=0}) = \frac{1}{12}\ L^2 \lim_{\omega\to 0} (a_{yy}|_{k=0})$$

and

(32[a]) $$\lim_{\omega\to\infty} (b_{yy}|_{k=0}) = 0 \quad ,$$

(32[b]) $$\lim_{\omega\to\infty} (b_{\psi\psi}|_{k=0}) = \frac{1}{12}\ L^2 \lim_{\omega\to\infty} (b_{yy}|_{k=0}) \quad .$$

A3.4: Survey of theoretical results

A3.4.1: Summary of most important formulae

The hydrodynamic coefficients for the entire ship in case of swaying and yawing with non-zero keel clearance are:

- hydrodynamic coefficient of mass term in swaying and yawing:

(20[a]) $$a_{yy} = 2\ \rho\ L\ [-\frac{1}{m_0^{\ 2}}\ \frac{\hat{v}_{kc}}{\omega\hat{a}}\ \sin(\theta)\ B_0{}'\ \{\sinh(m_0h) - \sinh(m_0k)\} +$$

$$+ \sum_{n=1}^{\infty} \frac{1}{m_n^{\ 2}}\ \{A_n{}' + \frac{\hat{v}_{kc}}{\omega\hat{a}}\ \cos(\theta)\ B_n{}'\}\{\sin(m_nh) - \sin(m_nk)\}]\ ,$$

(20[b]) $$a_{\psi\psi} = \frac{1}{12}\ L^2\ a_{yy} \quad ;$$

- hydrodynamic coefficient of damping force/moment in swaying and yawing:

(21[a]) $$b_{yy} = 2\ \rho\ L\ [\frac{\omega}{m_0^{\ 2}}\ \{A_0{}' + \frac{\hat{v}_{kc}}{\omega\hat{a}}\ \cos(\theta)\ B_0{}'\}\{\sinh(m_0h) - \sinh(m_0k)\} +$$

$$+ \sum_{n=1}^{\infty} \frac{\omega}{m_n^2} \frac{\hat{v}_{kc}}{\omega\hat{a}} \sin(\theta)\, B_n' \{\sin(m_n h) - \sin(m_n k)\}] ,$$

(21[b]) $$b_{\psi\psi} = \frac{1}{12} L^2 b_{yy} ;$$

- the relevant quantities in these formulae are:

(8[a]) m_0 = positive root of $\omega^2 = gm_0 \tanh(m_0 h)$,

(8[b]) m_n = positive roots of $\omega^2 = -gm_n \tan(m_n h)$ $(n = 1,2,\ldots;$

$m_1 < m_2 < \ldots < m_n < \ldots)$;

(9[a,b]) $$A_0' = \frac{2\{\sinh(m_0 h) - \sinh(m_0 k)\}}{m_0 h + \sinh(m_0 h)\cosh(m_0 h)} , \quad A_n' = \frac{2\{\sin(m_n h) - \sin(m_n k)\}}{m_n h + \sin(m_n h)\cos(m_n h)} ,$$

(10[a,b]) $$B_0' = \frac{2 \sinh(m_0 k)}{m_0 h + \sinh(m_0 h)\cosh(m_0 h)} , \quad B_n' = \frac{2 \sin(m_n k)}{m_n h + \sin(m_n h)\cos(m_n h)} ;$$

(14) $$\tan(\theta) = \frac{a_n b_0 - a_0(b_n + c)}{- a_0 b_0 - a_n(b_n + c)} , \quad \theta = \mathrm{atan}\,\{a_n b_0 - a_0(b_n + c), - a_0 b_0 - a_n(b_n + c)\} ,$$

(15) $$\frac{\hat{v}_{kc}}{\omega\hat{a}} = \frac{- a_0}{b_0 \cos(\theta) + (b_n + c)\sin(\theta)} = \frac{a_n}{b_0 \sin(\theta) - (b_n + c)\cos(\theta)} ,$$

(13[a]) $$a_0 = \frac{2 A_0'}{m_0^2} \sinh(m_0 k) , \qquad a_n = \sum_{n=1}^{\infty} \frac{2 A_n'}{m_n^2} \sin(m_n k) ,$$

(13[b]) $$b_0 = \frac{2 B_0'}{m_0^2} \sinh(m_0 k) , \qquad b_n = \sum_{n=1}^{\infty} \frac{2 B_n'}{m_n^2} \sin(m_n k) ,$$

(13[c]) $c = Bk;$

- the special limit cases $\omega \to 0$ and $\omega \to \infty$ are:

(22[a]) $$\lim_{\omega\to 0} (a_{yy}) = 2\,\rho\, L \left\{\frac{B(h-k)^2}{2k} + 2\,\frac{h^4}{k^2}\sum_{n=1}^{\infty}\frac{1}{(n\pi)^3}\sin^2\left(n\pi\,\frac{k}{h}\right)\right\} ,$$

(22[b]) $$\lim_{\omega\to 0} (a_{\psi\psi}) = \frac{1}{12} L^2 \lim_{\omega\to 0} (a_{yy}) ,$$

(23[a]) $$\lim_{\omega\to 0} (b_{yy}) = 0 ,$$

(23[b]) $$\lim_{\omega\to 0} (b_{\psi\psi}) = \frac{1}{12} L^2 \lim_{\omega\to 0} (b_{yy}) ;$$

(24[a]) $$\lim_{\omega\to\infty} (a_{yy}) = 4\,\rho\, Lh^2 \left\{ \sum_{n=1}^{\infty} \frac{\left[(-1)^n + \sin\{\frac{k}{h}(2n-1)\frac{\pi}{2}\}\right]^2}{\{(2n-1)\frac{\pi}{2}\}^3} + \right.$$

$$\left. - \frac{\left[\sum_{n=1}^{\infty}\frac{\sin\{\frac{k}{h}(2n-1)\frac{\pi}{2}\}\left[(-1)^n + \sin\{\frac{k}{h}(2n-1)\frac{\pi}{2}\}\right]}{\{(2n-1)\frac{\pi}{2}\}^3}\right]^2}{\frac{Bk}{4h^2} + \sum_{n=1}^{\infty}\frac{\sin^2\{\frac{k}{h}(2n-1)\frac{\pi}{2}\}}{\{(2n-1)\frac{\pi}{2}\}^3}} \right\} ,$$

(24[b]) $$\lim_{\omega\to\infty} (a_{\psi\psi}) = \frac{1}{12} L^2 \lim_{\omega\to\infty} (a_{yy}) ,$$

(25[a]) $$\lim_{\omega\to\infty} (b_{yy}) = 0 ,$$

(25[b]) $$\lim_{\omega\to\infty} (b_{\psi\psi}) = \frac{1}{12} L^2 \lim_{\omega\to\infty} (b_{yy}) .$$

The hydrodynamic coefficients for the entire ship in case of swaying and yawing with zero keel clearance are:

- hydrodynamic coefficient of mass term in swaying and yawing:

(26[a]) $$a_{yy}\big|_{k=0} = 2\,\rho\,L \sum_{n=1}^{\infty} \frac{1}{m_n^2} A_n'' \sin(m_n h) \quad ,$$

(26[b]) $$a_{\psi\psi}\big|_{k=0} = \frac{1}{12} L^2 \, a_{yy}\big|_{k=0} \quad ;$$

- hydrodynamic coefficient of damping force/moment in swaying and yawing:

(27[a]) $$b_{yy}\big|_{k=0} = 2\,\rho\,L \frac{\omega}{m_0^2} A_0'' \sinh(m_0 h) \;,$$

(27[b]) $$b_{\psi\psi}\big|_{k=0} = \frac{1}{12} L^2 \, b_{yy}\big|_{k=0} \;;$$

- the relevant quantities in these formulae are:

(8[a]) $$m_0 = \text{positive root of} \quad \omega^2 = g m_0 \tanh(m_0 h) \quad ,$$

(8[b]) $$m_n = \text{positive roots of} \quad \omega^2 = - g m_n \tan(m_n h) \; (n = 1,2,\ldots; \; m_1 < m_2 < \ldots < m_n < \ldots) \;,$$

(28[a,b]) $$A_0'' = \frac{2\sinh(m_0 h)}{m_0 h + \sinh(m_0 h)\cosh(m_0 h)} \;, \quad A_n'' = \frac{2\sin(m_n h)}{m_n h + \sin(m_n h)\cos(m_n h)} \;;$$

- the special limit cases $\omega \to 0$ and $\omega \to \infty$ are:

(29[a]) $$\lim_{\omega\to 0} \left(a_{yy}\big|_{k=0}\right) = 0 \;,$$

(29[b]) $$\lim_{\omega\to 0} \left(a_{\psi\psi}\big|_{k=0}\right) = \frac{1}{12} L^2 \lim_{\omega\to 0} \left(a_{yy}\big|_{k=0}\right) \quad ,$$

(30[a]) $$\lim_{\omega\to 0} \left(b_{\psi\psi}\big|_{k=0}\right) = 2\,\rho\,Lh\sqrt{gh} \;,$$

(30^b) $$\lim_{\omega\to 0}(b_{\psi\psi}|_{k=0}) = \frac{1}{12} L^2 \lim_{\omega\to 0}(b_{yy}|_{k=0}) \;;$$

(31^a) $$\lim_{\omega\to\infty}(a_{yy}|_{k=0}) = 4\,\rho\, Lh^2 \sum_{n=1}^{\infty} \frac{1}{\{(2n-1)\frac{\pi}{2}\}^3} \,,$$

(31^b) $$\lim_{\omega\to\infty}(a_{\psi\psi}|_{k=0}) = \frac{1}{12} L^2 \lim_{\omega\to\infty}(a_{yy}|_{k=0}) \,,$$

(32^a) $$\lim_{\omega\to\infty}(b_{yy}|_{k=0}) = 0 \,,$$

(32^b) $$\lim_{\omega\to\infty}(b_{\psi\psi}|_{k=0}) = \frac{1}{12} L^2 \lim_{\omega\to\infty}(b_{yy}|_{k=0}) \,.$$

The series occurring in eqs. (20^a), (21^a), (13^a), (13^b), (22^a), (24^a), (26^a) and (31^a) can be proved to be convergent: consequently it is possible to develop a break-off criterion for these series.

A3.4.2: Presentation and discussion of calculational results

Using the above-mentioned expressions calculations are carried out particularly for the hydrodynamic coefficients.
The hydrodynamic coefficients for the swaying motion will be represented in dimensionless form and per unit length by:

$\frac{a'_{yy}}{\rho BD}$ = dimensionless added mass per unit length for swaying motion,

and

$\frac{b'_{yy}}{\rho BD}\sqrt{\frac{B}{g}}$ = dimensionless sway damping force coefficient per unit length,
where the prime used as superscript means 'per unit length'. As for the yawing motion the hydrodynamic coefficients per unit length can be represented by:

$$\frac{a_{\psi\psi}}{\frac{1}{12} L^2 \rho LBD} = \frac{a'_{yy}}{\rho BD} \quad \text{and} \quad \frac{b_{\psi\psi}}{\frac{1}{12} L^2 \rho LBD}\sqrt{\frac{B}{g}} = \frac{b'_{yy}}{\rho BD}\sqrt{\frac{B}{g}} \,,$$

in this case additional calculations are not required.
The results of the calculations are represented in figs. 3,4,5,6^a,6^b and 7:
in figs. 3,4 and 5 $a_{yy}'(\rho BD)^{-1}$ and $b_{yy}'(\rho BD)^{-1}(B/g)^{\frac{1}{2}}$ are plotted versus the dimensionless circular frequency $\omega(B/g)^{\frac{1}{2}}$ with the dimensionless water depth $\frac{h}{D}$ as a parameter;
in figs. 6^a, 6^b and 7 $a_{yy}'(\rho BD)^{-1}$ and $b_{yy}'(\rho BD)^{-1}(B/g)^{\frac{1}{2}}$ are plotted versus the dimensionless keel clearance $\frac{h - D}{D}$ with $\omega(B/g)^{\frac{1}{2}}$ as a parameter.
In both cases the dimensionless parameter $\frac{B}{D}$ has a constant value (viz. $\frac{B}{D} = 2.50$).
The selection of the three values for $\frac{h}{D}$ (viz. 1.333, 1.167 and 1.000) and of the value for $\frac{B}{D}$ (viz. 2.50) is based on experiments as described in part B of this report.
For very large values of $\omega(B/g)^{\frac{1}{2}}$ $a_{yy}'(\rho Bd)^{-1}$ approaches a horizontal asymptote (see fig. 3) and $b_{yy}'(\rho BD)^{-1}(B/g)^{\frac{1}{2}}$ approaches asymptotically the horizontal $\omega(B/g)^{\frac{1}{2}}$ - axis (see fig. 4). The curves of $b_{yy}'(\rho BD)^{-1}(B/g)^{\frac{1}{2}}$ versus $\omega(B/g)^{\frac{1}{2}}$ for the respective values of $\frac{h}{D}$ are going to coincide for large values of $\omega(B/g)^{\frac{1}{2}}$: generally it may be stated that for large values of $\omega(B/g)^{\frac{1}{2}}$ (say $\omega(B/g)^{\frac{1}{2}} > 3.9$) $b_{yy}'(\rho BD)^{-1}(B/g)^{\frac{1}{2}}$ becomes independent of $\frac{h}{D}$.

In fig. 5 $a_{yy}'(\rho BD)^{-1}$ approaches a horizontal asymptote and $b_{yy}'(\rho BD)^{-1}(B/g)^{\frac{1}{2}}$ approaches asymptotically the horizontal $\omega(B/g)^{\frac{1}{2}}$- axis for (very) large values of $\omega(B/g)^{\frac{1}{2}}$. Considering fig. 5 with reference to fig. 4 the statement that for large values of $\omega(B/g)^{\frac{1}{2}}$ $b_{yy}'(\rho BD)^{-1}(B/g)^{\frac{1}{2}}$ becomes independent of $\frac{h}{D}$ may be affirmed.

The relation between the curves of $a_{yy}'(\rho BD)^{-1}$ versus $\omega(B/g)^{\frac{1}{2}}$ for non-zero and zero keel clearance can be understood as follows. For decreasing magnitude of $\frac{h}{D}$ the 'image' of the curve of $a_{yy}'(\rho BD)^{-1}$ versus $\omega(B/g)^{\frac{1}{2}}$ shifts to the left; the branch of the curve on the left of the minimum value of $a_{yy}'(\rho BD)^{-1}$ becomes steeper, the value of $a_{yy}'(\rho BD)^{-1}$ for $\omega(B/g)^{\frac{1}{2}} = 0$ moves along the $a_{yy}'(\rho BD)^{-1}$ - axis in upward direction, the minimum value of $a_{yy}'(\rho BD)^{-1}$ sags down and shifts to smaller values of $\omega(B/g)^{\frac{1}{2}}$, the value which $a_{yy}'(\rho BD)^{-1}$ approaches asymptotically for large $\omega(B/g)^{\frac{1}{2}}$ increases slightly. A limit case is attained for $\frac{h}{D} = 1.000$: the (now degenerated)

branch of the curve on the left of the minimum value of $a_{yy}'(\rho BD)^{-1}$ coincides with the $a_{yy}'(\rho BD)^{-1}$ - axis (the original point of intersection of the curve with the $a_{yy}'(\rho BD)^{-1}$ - axis is at positive infinity), and the branch on the right touches the $\omega(B/g)^{\frac{1}{2}}$ - axis in the origin.
A similar explanation applies to the relation between the curves of $b_{yy}'(\rho BD)^{-1}(B/g)^{\frac{1}{2}}$ versus $\omega(B/g)^{\frac{1}{2}}$ for non-zero and zero keel clearance.
For decreasing magnitude of $\frac{h}{D}$ the 'image' of the curve of $b_{yy}'(\rho BD)^{-1}(B/g)^{\frac{1}{2}}$ versus $\omega(B/g)^{\frac{1}{2}}$ shifts to the left; the branch of the curve on the left of the maximum value of $b_{yy}'(\rho BD)^{-1}(B/g)^{\frac{1}{2}}$ becomes steeper, the maximum value of $b_{yy}'(\rho BD)^{-1}(B/g)^{\frac{1}{2}}$ increases and shifts to smaller values of $\omega(B/g)^{\frac{1}{2}}$.
A limit case is attained for $\frac{h}{D} = 1.000$: the (now degenerated) branch of the curve on the left of the maximum value of $b_{yy}'(\rho BD)^{-1}(B/g)^{\frac{1}{2}}$ coincides with the $b_{yy}'(\rho BD)^{-1}(B/g)^{\frac{1}{2}}$-axis between $b_{yy}'(\rho BD)^{-1}(B/g)^{\frac{1}{2}} = 0$ and $b_{yy}'(\rho BD)^{-1}(B/g)^{\frac{1}{2}}$ = maximum (the original maximum value of the curve is now on the $b_{yy}'(\rho BD)^{-1}(B/g)^{\frac{1}{2}}$-axis).
In figs. 6^a, 6^b and 7 the values already calculated in behalf of the respective figs. 3 and 4 have been marked by crosslets.

In addition to the hydrodynamic coefficients, the dimensionless amplitude of the fluid velocity in the keel clearance (i.e. $\hat{v}_{kc}(\omega\hat{a})^{-1}$) and the phase difference between fluid motion in keel clearance and ship motion (i.e. θ) are calculated as functions of $\frac{h - D}{D}$ with $\omega(B/g)^{\frac{1}{2}}$ as a parameter. The results of these calculations are plotted in figs. 8 and 9, respectively.
From fig. 8 it can be seen that in case of zero keel clearance (i.e. $\frac{h - D}{D} = 0$) a 'fluid velocity underneath the ship' is possible without being necessarily equal to zero. This can be explained from the fact that, caused by the oscillating ship motion, always a pressure drop exists across the ship, whether there is a keel clearance or not. This pressure drop causes a fluid velocity underneath the ship, even if the keel clearance is equal to zero. The discharge through the keel clearance, however, is then zero.

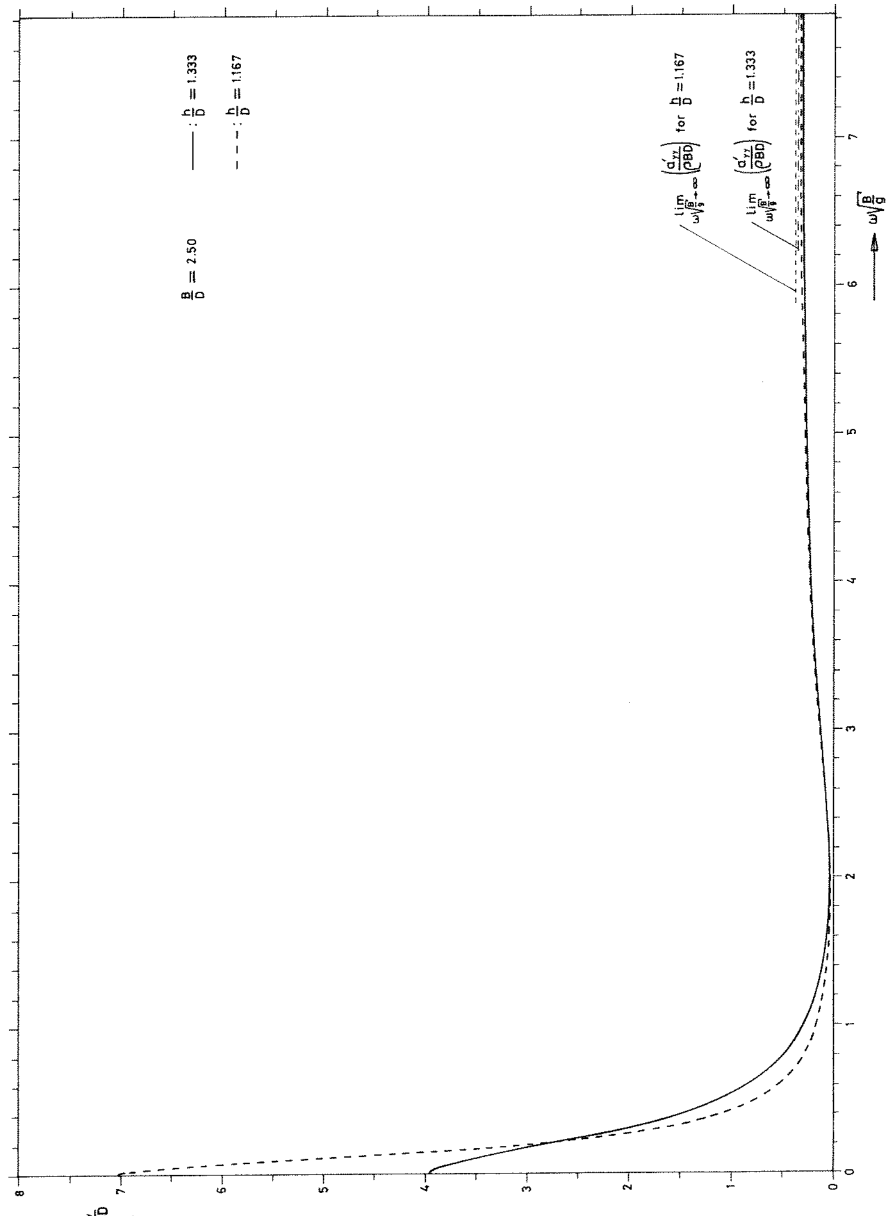

fig.3 – Added mass for swaying motion as function of circular frequency (non-zero keel clearance).

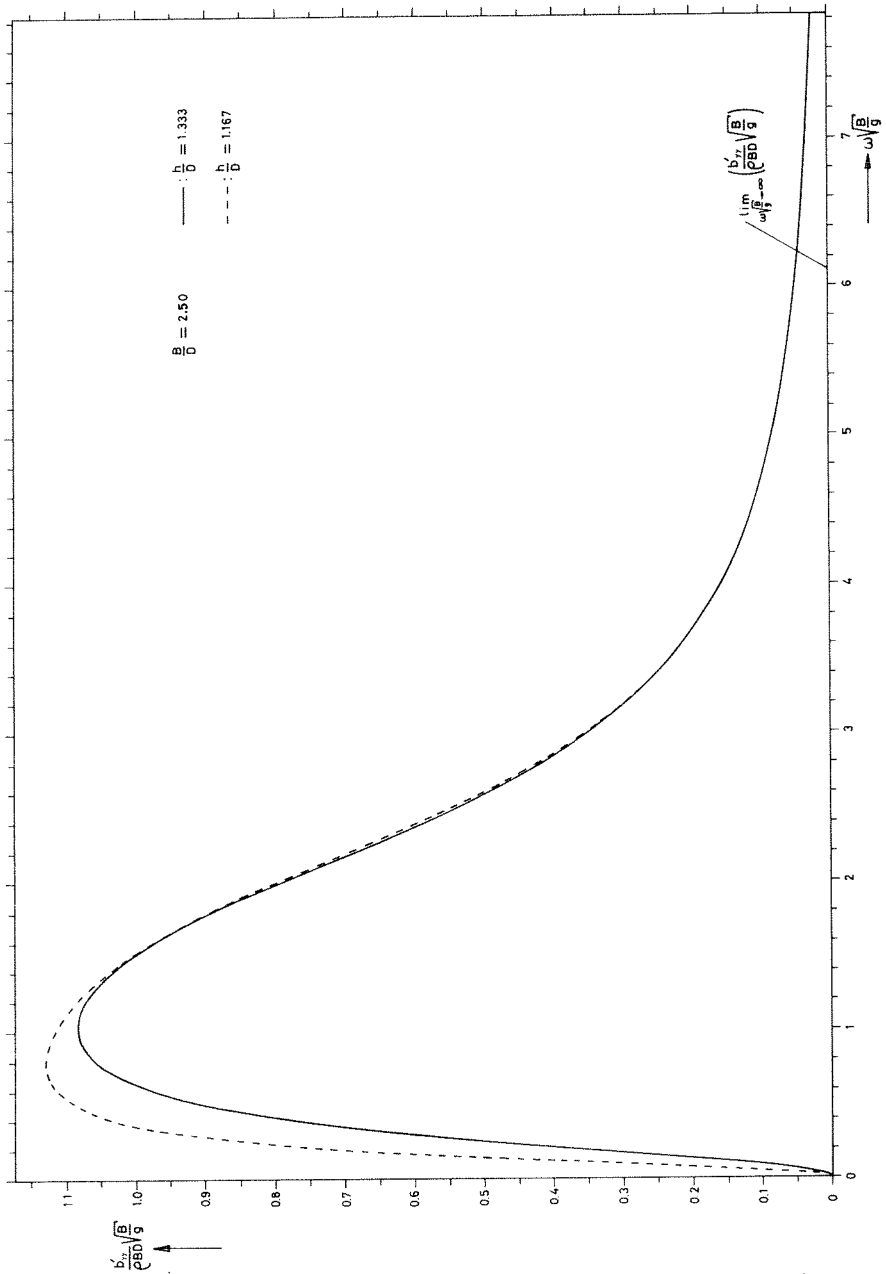

fig. 4 - Sway damping force coefficient as function of circular frequency (non-zero keel clearance).

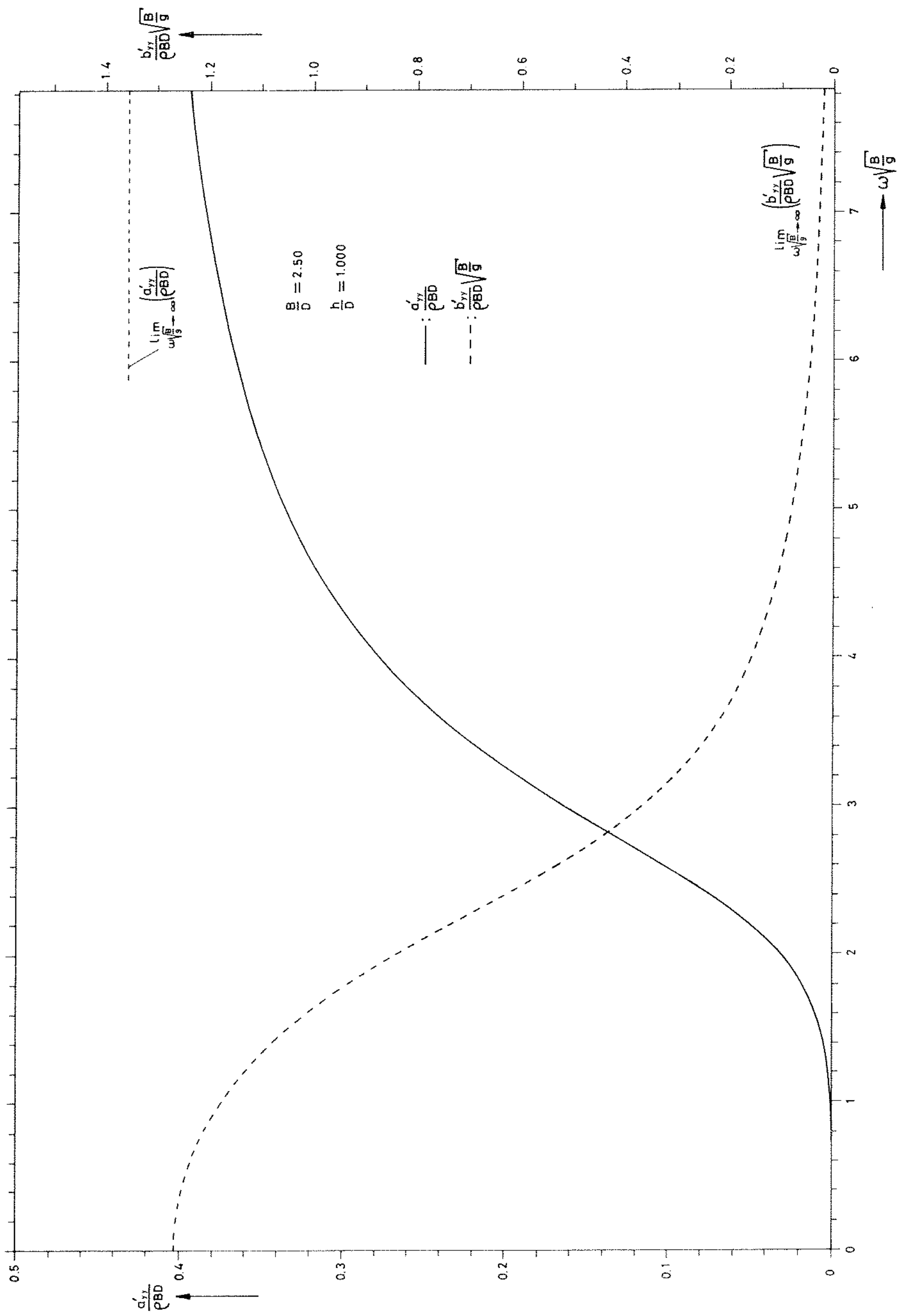

fig. 5 – Added mass for swaying motion and sway damping force coefficient as functions of circular frequency (zero keel clearance).

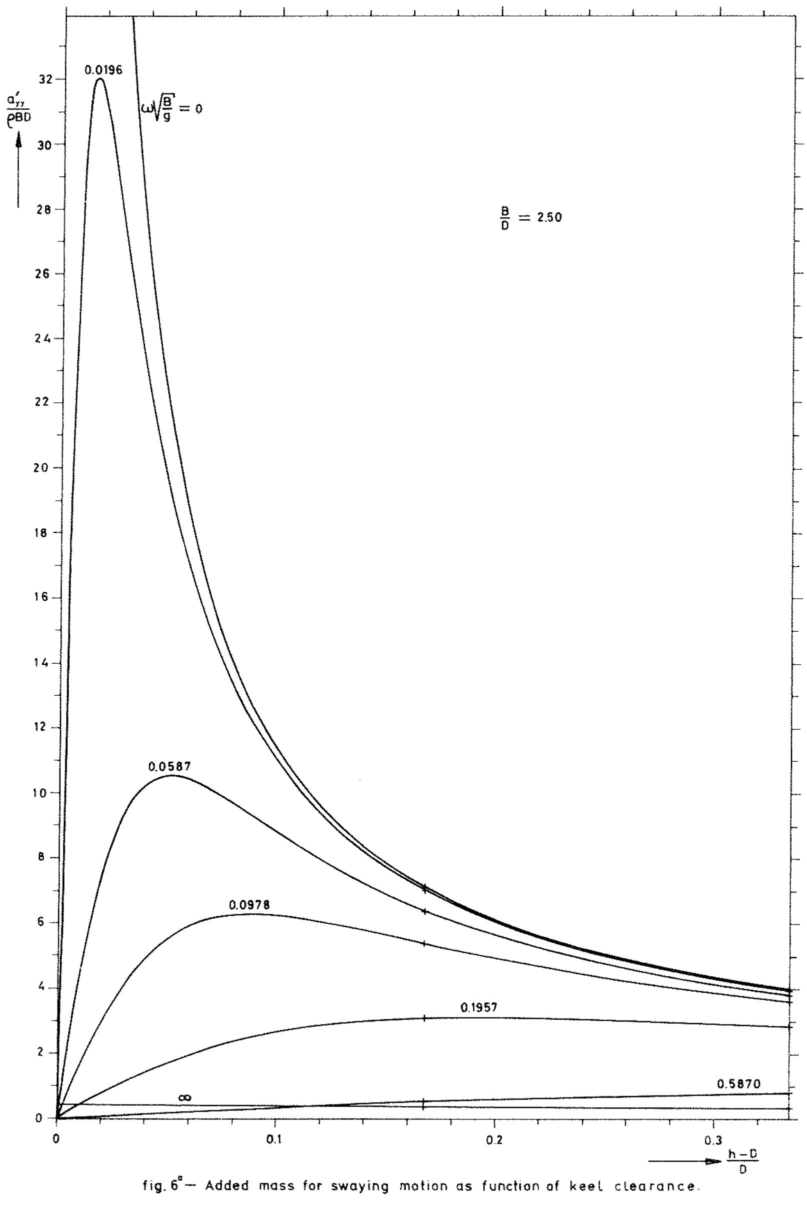

fig. 6[a] — Added mass for swaying motion as function of keel clearance.

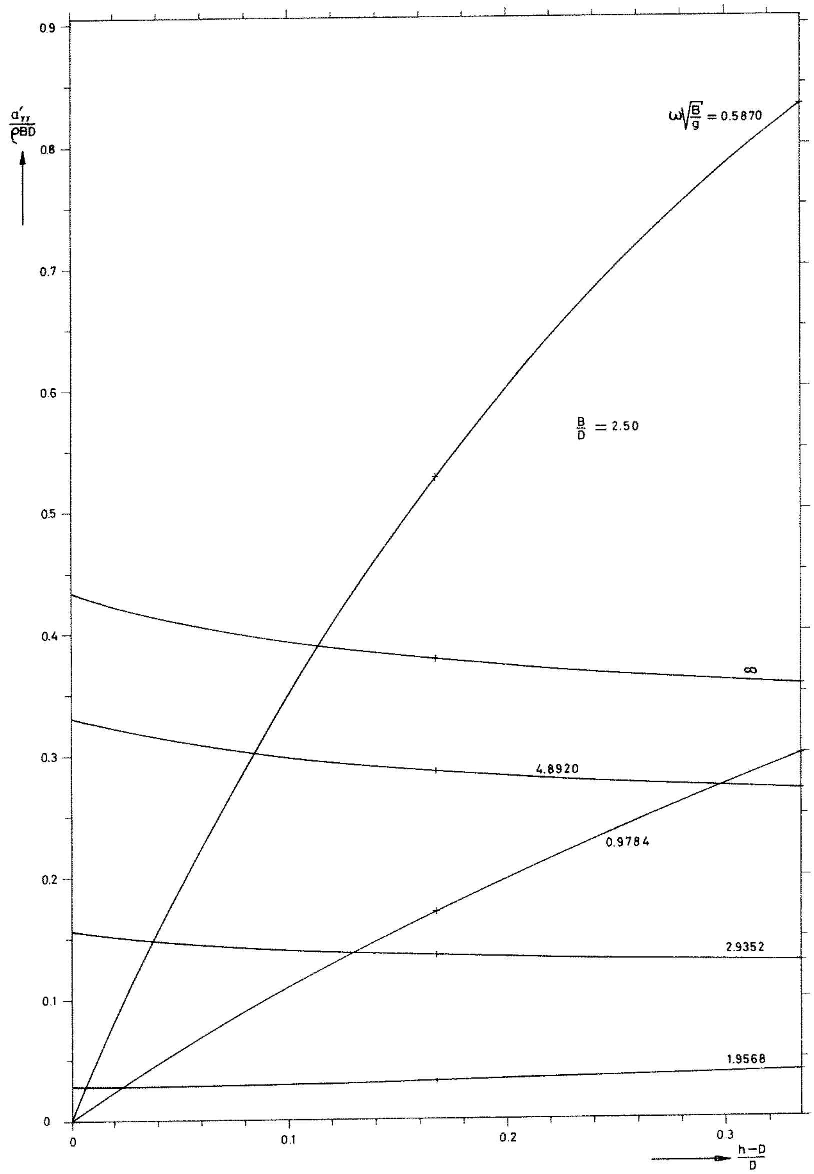

fig.6[b] – Added mass for swaying motion as function of keel clearance.

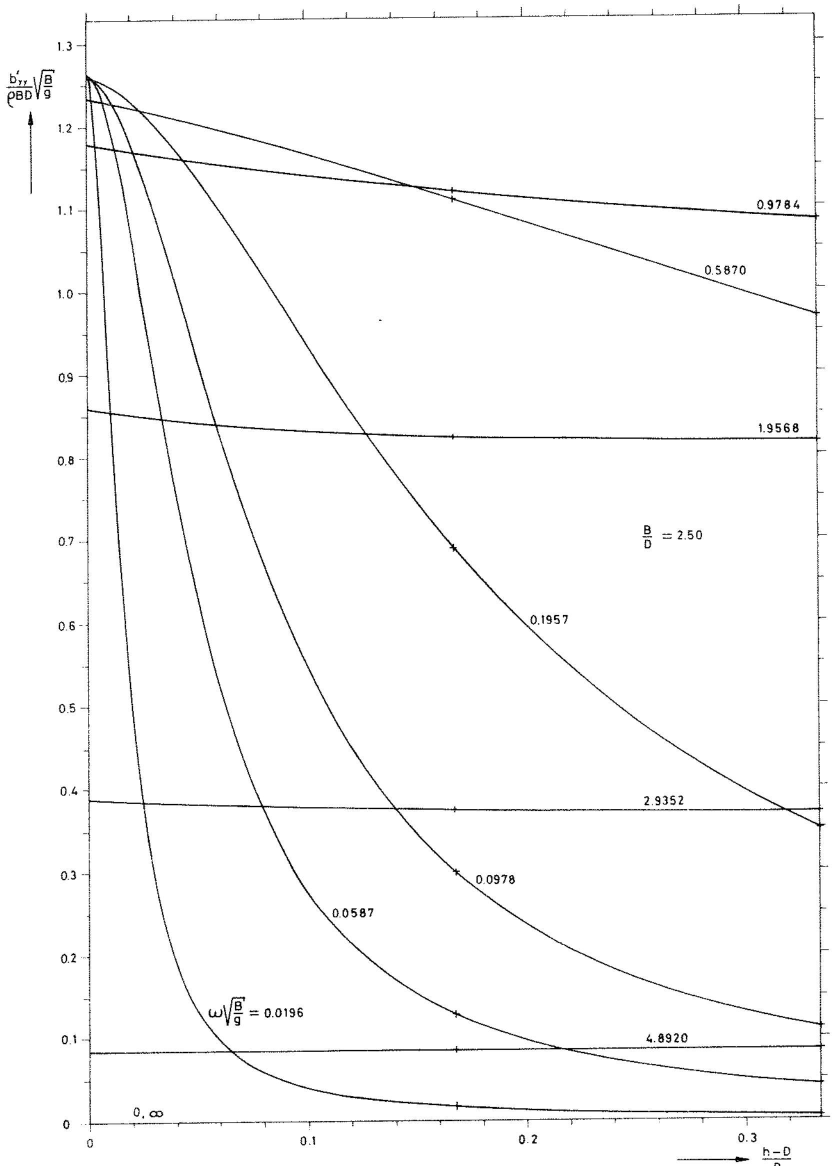

fig. 7 – Sway damping force coefficient as function of keel clearance.

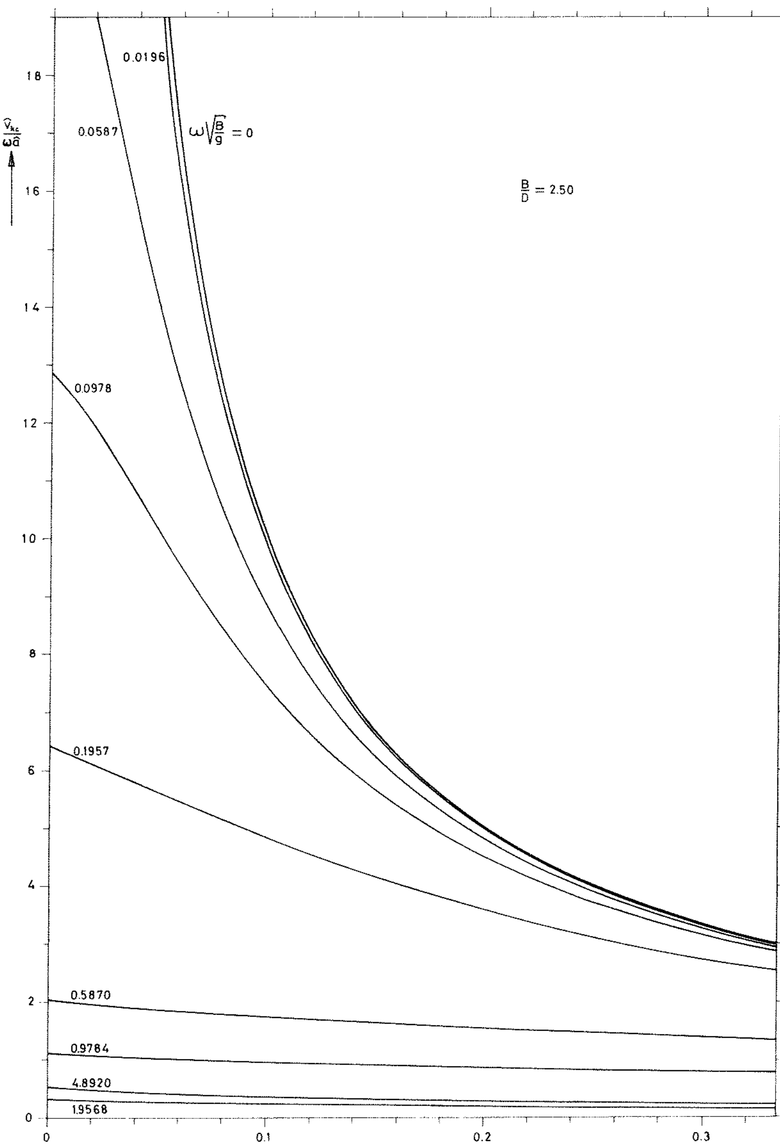

fig. 8 – Amplitude of fluid velocity in keel clearance as function of keel clearance.

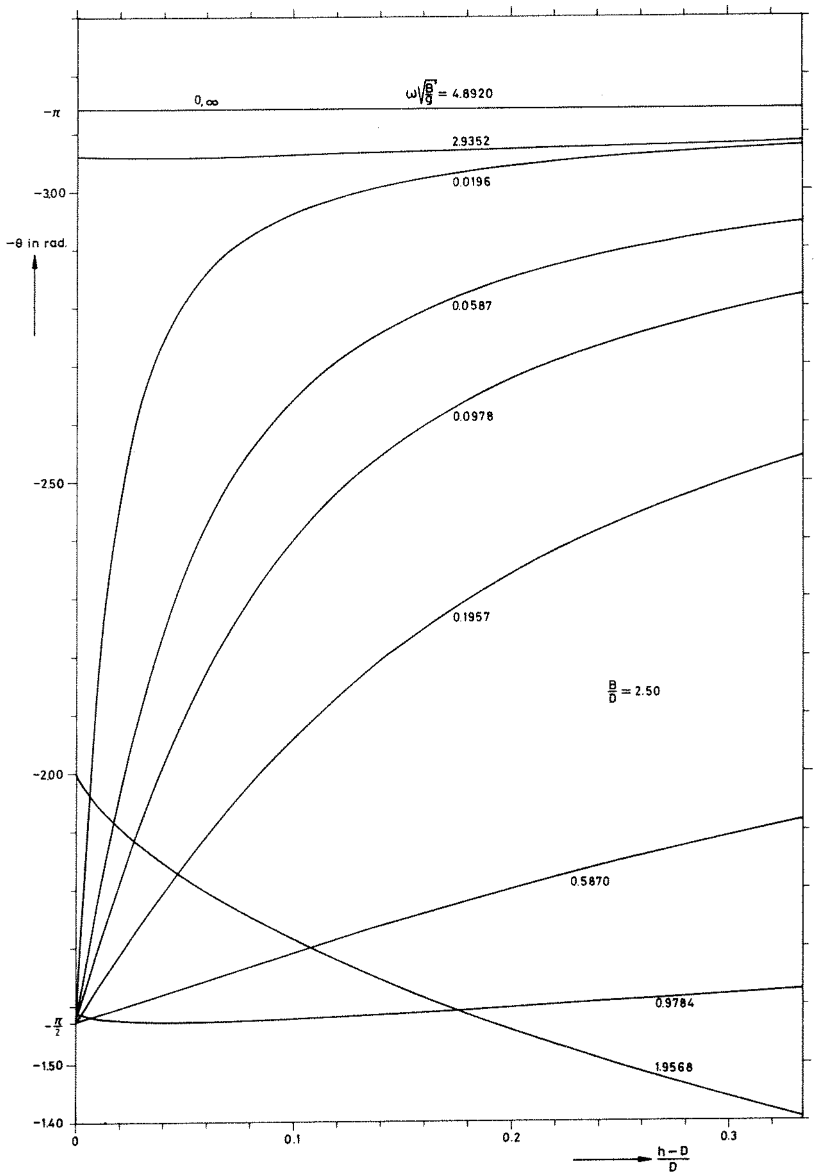

fig. 9 – Phase difference fluid motion in keel clearance – ship motion as function of keel clearance.

part B: EXPERIMENTAL DETERMINATION OF HYDRODYNAMIC COEFFICIENTS

Section B1: Introduction

This part of the report provides information about experiments to determine hydrodynamic coefficients in case of swaying and yawing on shallow water. These experiments were carried out in the form of forced pure sway and forced pure yaw tests with zero speed of advance.

The main purpose of this experimental research is to check the theoretical results derived in part A. For this reason the schematization of the ship (model) is the same as in part A and the forward speed is equal to zero.

In Section B2 an outline of the main particulars of the ship model is provided, together with a survey of the experimental equipment and other test facilities.

In Section B3 the execution of the experiments is outlined; likewise the results obtained experimentally are presented and discussed.

Section B2: Experimental equipment

B2.1: Main particulars of ship model

The ship model is a prismatic body with a rectangular cross-section. The main dimensions (length, beam and draught) are based on a model of the Todd Sixty Series with block coefficient 0.80 and model scale 50. The ship model is made of wood and on the outside sheathed with polyester. The distribution of mass along the length is homogeneous.
The main particulars of the ship model are given in Table 1.

Table 1. Main particulars of ship model.

length (on the water-line)	L	2.438	m
beam	B	0.375	m
draught	D	0.150	m
volume of displacement	L.B.D	0.1371	m^3
area of cross-section	B.D	0.056	m^2
water-line area	L.B	0.924	m^2

lateral plane area	L.D	0.366	m^2
block coefficient		1.000	
centre of gravity (with respect to frame 10)		0	m
centre of gravity in height (with respect to keel point)		0.140	m
mass for horizontal motion	m_y	134.27	kg
mass-moment of inertia	I_{zz}	50.90	kg m^2
radius of gyration		0.610	m

With this ship model harmonic oscillation tests (pure swaying and pure yawing with zero speed of advance) were carried out at calm shallow water with relatively large horizontal dimensions.

The draught of the ship model together with the fact that special attention is paid to the case that shallowness of the water is of dominant importance, explain the selection of the two water depths in these experiments, viz. h = 0.200 m and h = 0.175 m.
The values for the dimensionless water depths then become $\frac{h}{D}$ = 1.333 and $\frac{h}{D}$ = 1.167, respectively; the dimensionless parameter $\frac{B}{D}$ = 2.50. The values of above-mentioned dimensionless parameters are identical to those used in Section A3.4.2.

Further, in the experiments ρ = 1000 kg m^{-3} and g = 9.81 m s^{-2}; the same values for ρ and g were used in the calculations of the theoretical results for the hydrodynamic quantities (Section A3.4.2).

B2.2: Planar-motion mechanism: horizontal oscillator and measuring system

The hydrodynamic coefficients in case of swaying and yawing with zero speed of advance could be determined experimentally by means of the so-called planar-motion mechanism (P.M.M.) from the Shipbuilding Laboratory of the Department of Naval Architecture, Delft University of Technology. This P.M.M. consists of a horizontal ship's oscillator with a coupled measuring system. With this experimental equipment - in the horizontal plane - an arbitrary harmonically oscillating motion can be imposed on a ship model, while at the same time the exciting forces are measured. For (general) details see ref. [6].

Unlike the P.M.M. as described in ref. [6] the version used in these experiments had a degree of freedom in vertical direction, achieved by means of a ball bushing construction in the struts of the horizontal oscillator. This vertical degree of freedom implied that the ship model was allowed to heave and pitch without restraint during the forced horizontal harmonically oscillating motion.

The exciting forces were measured by means of two strain gauge dynamometers. These dynamometers - mounted in the ship model's longitudinal plane of symmetry, at equal distances from the centre of gravity - connected (the struts of) the oscillator to the ship model. Only forces in the plane of the water-line with a direction perpendicular to the longitudinal plane of symmetry of the ship model were measured.

The distance between the centre-lines of the dynamometers was 1.0000 m. One of the dynamometers was fixed, the other admitted some longitudinal sliding. With the P.M.M. used the pure yawing motion - as distinct from the version as described in ref. [6] - needed not to be corrected for a slight swaying motion.

The measuring system forming part of the P.M.M. was able to measure first, second and third harmonic components of the sway and yaw forces as well in amplitude as in phase-relation to the motion of the ship model. This was performed by a mechanical-electronical Fourier-analyzer. For further details see ref. [6].

The maximum amplitudes for the sway and yaw motion were 0.2500 m and 0.4636 rad. (i.e. atan 0.5000). The circular frequency of the oscillatory motions could vary continuously between 0.196 $rad.s^{-1}$ and 3.927 rad. s^{-1}; this corresponds with a period range from 32.0 s to 1.6 s.

The maximum capacity of the dynamometers was about hundred newtons each. The accuracy of the P.M.M. as a measuring device depended mainly on the occurrence of adequately large forces, which had to exceed values from 0.2 N to 0.4 N. Therefore the scale (i.e. actually the dimensions) of the ship model had to be chosen such that particularly for combinations of low frequency and small amplitude measurable forces occurred.

According to Section B2.1 the mass of the ship model, as based on the volume of displacement, amounted to 137.10 kg, whereas the mass for horizontal motion as used in the dynamic tests, m_y, was 134.27 kg. This difference can be explained by the presence of the ball bushing constructions in the struts of the oscillator: the weight of the shafts of the ball bushing constructions plus two times half the weight of the dynamometers did contribute to the weight (or rather the volume of displacement) of the ship model, but they did not contribute to the mass forces on the dynamometers.

B2.3: Other experimental facilities

The pure sway and yaw tests with zero speed of advance were excecuted in the middle of a rectangular basin with relatively large horizontal dimensions, viz. length = 32.34 m and breadth = 13.98 m. The basin had a horizontal bottom and was bounded by vertical walls. The P.M.M. was mounted on a rectangular steel frame of very great rigidity ; this frame had four legs, stood in a fixed position in the basin and was adjustable in height. The dimensions of the horizontal cross-section of the legs were relatively small with respect to the main dimensions of the ship model. The longitudinal plane of symmetry of the ship model in its state of rest (i.e. the equilibrium position) coincided with the respective breadthwise axes of symmetry of the basin and the frame. The distances of the legs of the frame to the ship model were relatively large, even during the oscillations.
For a schematical representation of the model installation see fig. 10.

As a result of the oscillatory motions of the ship model during the dynamic tests waves were generated with a direction of propagation which was mainly parallel to the lengthwise axis of the basin; these waves were reflected against the (short) basin walls.
In order to attain faster wave damping the basin was provided with two simple wave damping constructions in the form of walls of perforated bricks with a relatively great percentage of holes (for the location see fig. 10). The wave damping properties of these perforated walls were rather good for short period waves, but fairly bad for long period waves.

The natural frequencies of the combination frame - P.M.M. in both horizontal and vertical direction turned out to be at least several times greater than the frequencies considered in the experiments; the natural frequencies of the strain gauge dynamometers for different directions were many

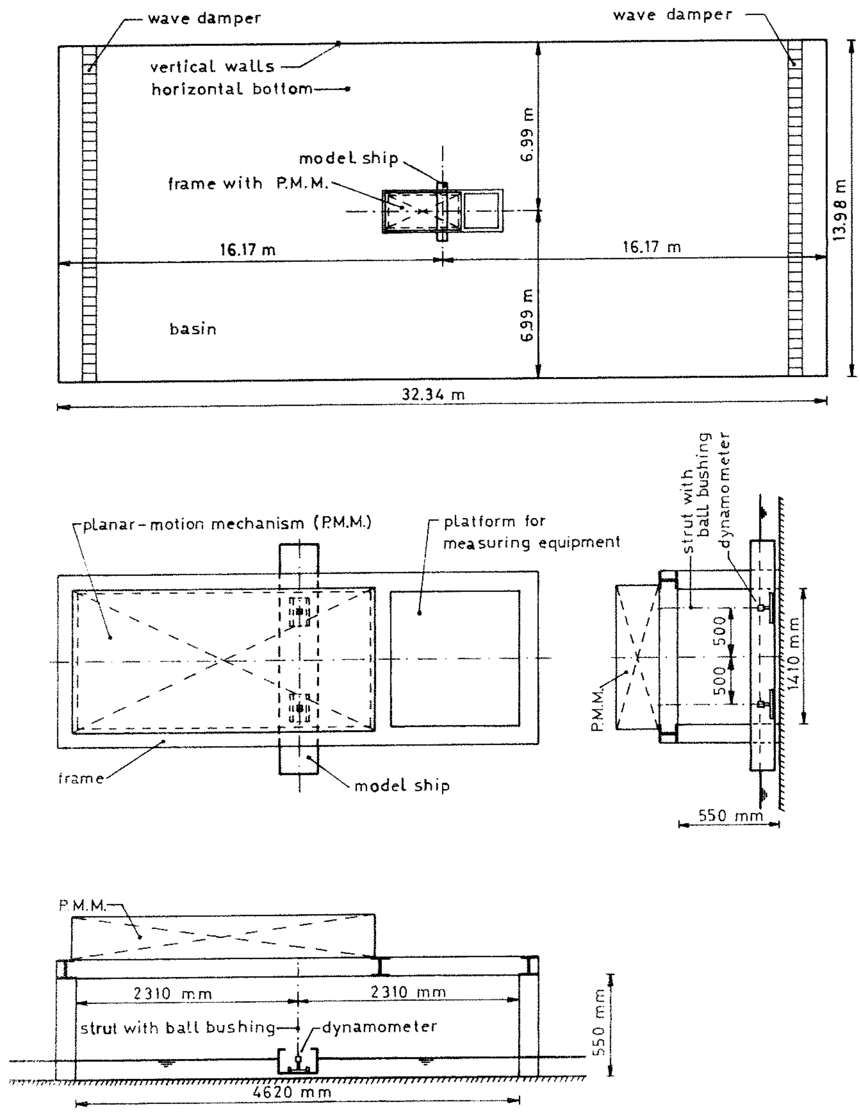

fig.10 - Schematical representation of model installation.

times greater.

Section B3: Execution of model experiments and survey of results

B3.1: General remarks

For the dynamic tests the circular frequency and the amplitude of motion had to be considered as independent variables; for a certain dynamic test they were fixed quantities.
The hydrodynamic coefficients could be determined - as functions of circular frequency and amplitude of motion - from the measurement of the first harmonic components of the lateral exciting forces in a way as is pointed out briefly in Appendix II.

Because of the limitations of the experimental equipment no data could be obtained for values of the circular frequency ω lower than 0.196 $rad.s^{-1}$.

Some combinations of the independent variables (viz. high frequency together with large amplitude) set upper limits to the dynamic tests. One absolute upper limit was formed by the maximum capacity of the dynamometers. Another more relative upper limit was formed by the vertical degree of freedom of the ship model. By this the oscillating ship model could run the risk of touching the bottom for certain combinations of amplitude and frequency. This phenomenon was caused by the velocity effect: the ship model could sink deeper into the water during the oscillating motions than the draught as indicated for the state of rest. For obvious reasons this could not be accepted so that the amplitude of motion had to be bounded as a function of frequency.

Each experiment was started with the water level at rest; it had to be terminated at the moment when the first wave reflections against the (short) basin walls were expected at the walls of the ship model; for, reflected waves arriving at the walls of the ship model during a measurement should influence the test results.

The minimum length of time required for the execution of one dynamic test was equal to the duration of time of the transient phenomena of the experimental equipment plus the period time.
The greater part of the dynamic tests could be carried out before the reflected waves arrived at the walls of the ship model. In a few cases,

however, the model experiments were disturbed by the reflected waves, viz. for $\omega < 0.4$ rad.s^{-1}: this has to be understood in this sense that only the test results for $\omega \geq 0.4$ rad.s^{-1} could be reproduced in a satisfactory way. Although the disturbance of the model experiments by the reflection phenomena was small, the test results for $\omega < 0.4$ rad.s^{-1} have to be considered with some reserve, because they are not completely reliable.

Despite the fact that a perfectly horizontal bottom was tried for, the part of the bottom of the basin covered by the oscillatory motions of the ship model showed differences in height. The water depth as well as the position in height of the bottom were determined with respect to the centre of the bottom of the basin: the bottom under the fore-part of the ship model was tolerably horizontal, whereas the bottom under the hind-part sloped downwards, starting from and mainly in a direction perpendicular to the lengthwise axis of the basin, with a maximum difference in height of $0.5 * 10^{-3}$m. The (possible) inaccuracies in the test results in consequence of this uneveness of the bottom of the basin were accepted. In case of a perfectly horizontal bottom the respective hydrodynamic forces on the fore-part and the hind-part of the ship model have to be equal to one another for reasons of symmetry. Therefore the hydrodynamic coupling coefficients $a_{\psi y}$, $b_{\psi y}$, $a_{y\psi}$ and $b_{y\psi}$ then have to be equal to zero. As a result of the uneven bottom, however, the (absolute) values of the hydrodynamic forces on the fore-part of the ship model (measured by dynamometer 1) turned out to be systematically greater than those on the hind-part (measured by dynamometer 2). This held good for the in-phase components of the forces as well as for the ninety degrees out-of-phase components (for the concepts of in-phase and out-of-phase see Appendix II). For h = 0.175 m these differences were stronger than for h = 0.200 m. The hydrodynamic coupling coefficients were (slightly) different from zero. From the test results it can be verified that in case of a perfectly horizontal bottom, for the frequency range considered in the experiments and for both water depths, the (real) values of a_{yy}, b_{yy}, $a_{\psi\psi}$ and $b_{\psi\psi}$ generally will be (slightly) greater than the measured values: this holds good for b_{yy} and $b_{\psi\psi}$, whereas for a_{yy} and $a_{\psi\psi}$ this roughly holds good up to $\omega = 2.3$ rad.s^{-1} and $\omega = 2.9$ rad.s^{-1}, respectively. The differences from the measured values in case of h = 0.175 m will be somewhat stronger than in case of h = 0.200 m; for both water depths the differences in case of b_{yy} and $b_{\psi\psi}$ will be more significant than in case of a_{yy} and $a_{\psi\psi}$, especially for small amplitudes of motion.

During the oscillatory motions the vertical degree of freedom of the ship model, as a result of the velocity effect, led to a reduction of the original keel clearance. For a certain ω a larger amplitude of motion yielded a larger amplitude of (angular) velocity and as a consequence a deeper sinking of the ship model with respect to the undisturbed water level. The influence of this temporary (i.e. only during the oscillatory motions) reduction of the keel clearance on the test results, however, could not be determined distinctly and unambiguously, the more so as the phenomena of vortex shedding and separation of flow at the 'bow', the 'stern' and the 'bilges' of the ship model came through more explicitly as the amplitude of motion increased.

B3.2: Presentation and discussion of experimental results

The hydrodynamic coefficients, determined experimentally for the entire ship model in the respective cases of pure swaying and pure yawing with zero forward speed, will be represented in dimensionless form by

$\frac{a_{yy}}{\rho LBD}$ = dimensionless added mass for swaying motion,

$\frac{b_{yy}}{\rho LBD}\sqrt{\frac{B}{g}}$ = dimensionless sway damping force coefficient,

and

$\frac{a_{\psi\psi}}{\frac{1}{12} L^2 \rho LBD}$ = dimensionless added mass moment of inertia for yawing motion,

$\frac{b_{\psi\psi}}{\frac{1}{12} L^2 \rho LBD}\sqrt{\frac{B}{g}}$ = dimensionless yaw damping moment coefficient.

The dynamic test results are plotted for several amplitudes of motion as functions of the dimensionless circular frequency $\omega(B/g)^{\frac{1}{2}}$ with the dimensionless water depth $\frac{h}{D}$ as a parameter.

B3.2.1: Test results in case of pure swaying

From the series of dynamic swaying experiments only the test results are given for the following amplitudes of motion: $\hat{a}$ = 0.01, 0.03, 0.05 and 0.10 m. These results are plotted in fig. 11 through fig. 14 as centred symbols.

For both values of $\frac{h}{D}$, $a_{yy}(\rho LBD)^{-1}$ and $b_{yy}(\rho LBD)^{-1}(B/g)^{\frac{1}{2}}$ are subject to large changes when $\omega(B/g)^{\frac{1}{2}}$ increases. For the frequency range considered in the experiments, $a_{yy}(\rho LBD)^{-1}$ in case of the smaller water depth ($\frac{h}{D}$ = 1.167) up to $\omega(B/g)^{\frac{1}{2}}$ = 0.450 (i.e. ω = 2.3 rad.s $^{-1}$) is greater than $a_{yy}(\rho LBD)^{-1}$ in case of the greater water depth ($\frac{h}{D}$ = 1.333); for $\omega(B/g)^{\frac{1}{2}}$ > 0.450 $a_{yy}(\rho LBD)^{-1}$ in case of $\frac{h}{D}$ = 1.167 seems to be somewhat smaller than in case of $\frac{h}{D}$ = 1.333. For $\frac{h}{D}$ = 1.167 $b_{yy}(\rho LBD)^{-1}(B/g)^{\frac{1}{2}}$ is greater than for $\frac{h}{D}$ = 1.333.

For that part of the frequency range where the test results are not influenced by (reflected) waves (i.e. if $\omega(B/g)^{\frac{1}{2}} \geqq 0.08$, or ω > 0.4 rad.s^{-1}) $a_{yy}(\rho LBD)^{-1}$ slightly decreases and $b_{yy}(\rho LBD)^{-1}(B/g)^{\frac{1}{2}}$ slightly increases in case of increasing $\hat{a}$ for certain $\omega(B/g)^{\frac{1}{2}}$. Generally this holds good up to $\hat{a}$ = 0.05 m for both water depths; in case of $\frac{h}{D}$ = 1.167 this is more significant than in case of $\frac{h}{D}$ = 1.333, particularly for lower frequencies. Probably this phenomenon is caused by friction effects of the fluid in the keel clearance. Supposing a friction force for the fluid in the keel clearance which is proportional to certain (positive) power of the underkeel velocity, its influence on the hydrodynamic coefficients becomes greater as the amplitude of motion increases; the influence on the sway damping force coefficient will be more significant than on the added mass for swaying motion. Friction effects in the keel clearance have a similar influence as a reduction of the keel clearance, which reduction is greater as the amplitude of motion (and consequently the friction force) is larger. The respective figs. 6[a] and 7 (theoretical results) show that for $0.08 \leqq \omega(B/g)^{\frac{1}{2}} < 0.57$ a slight decrease of $a_{yy}(\rho LBD)^{-1}$ and a small increase of $b_{yy}(\rho LBD)^{-1}(B/g)^{\frac{1}{2}}$ in case of a reduction of the dimensionless keel clearance can be expected.

During the sway experiments it was observed that for $\hat{a} \geqq$ 0.05 m the phenomena of vortex shedding and separation of flow at the 'bow', the 'stern' and the 'bilges' of the ship model were going to play an increasingly im-

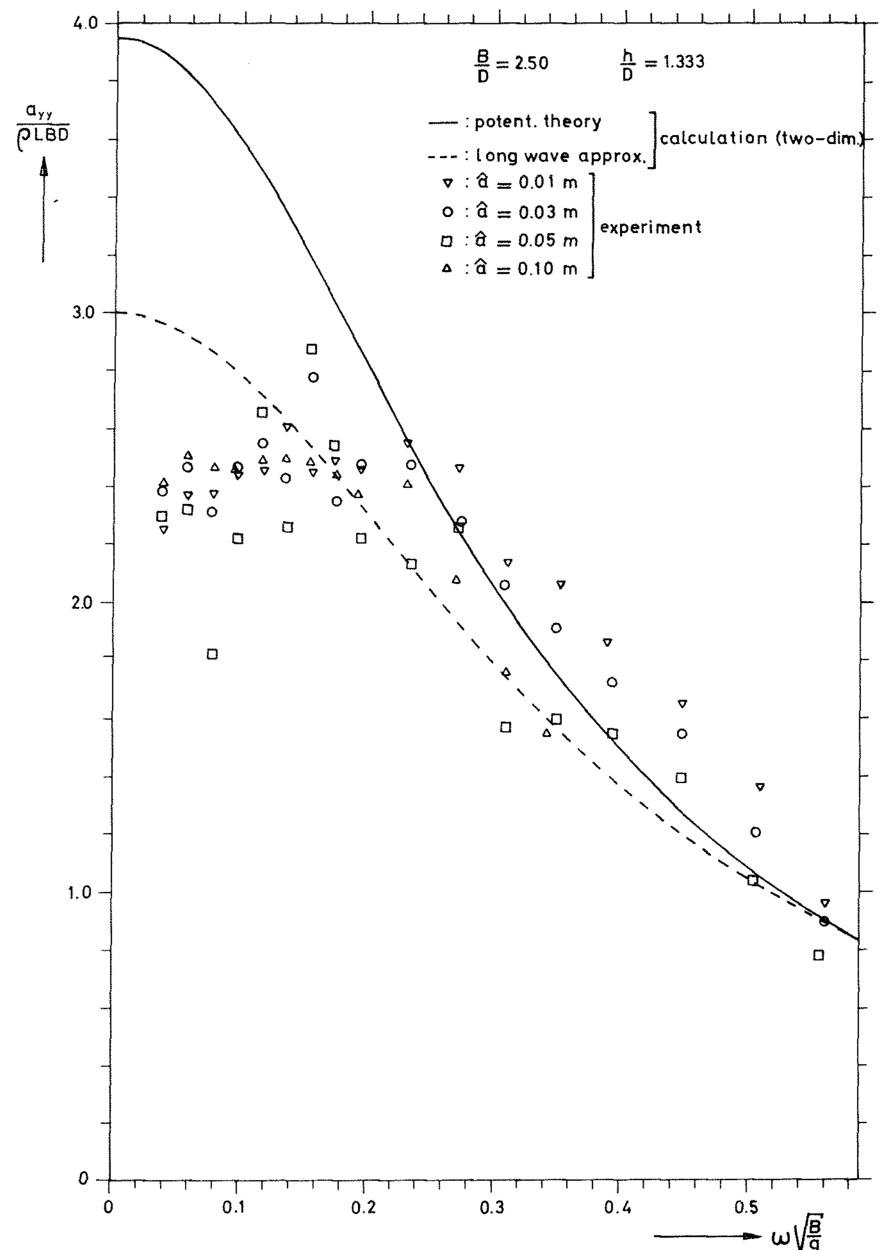

fig. 11 – Added mass for swaying motion $\left(\frac{h}{D} = 1.333\right)$.

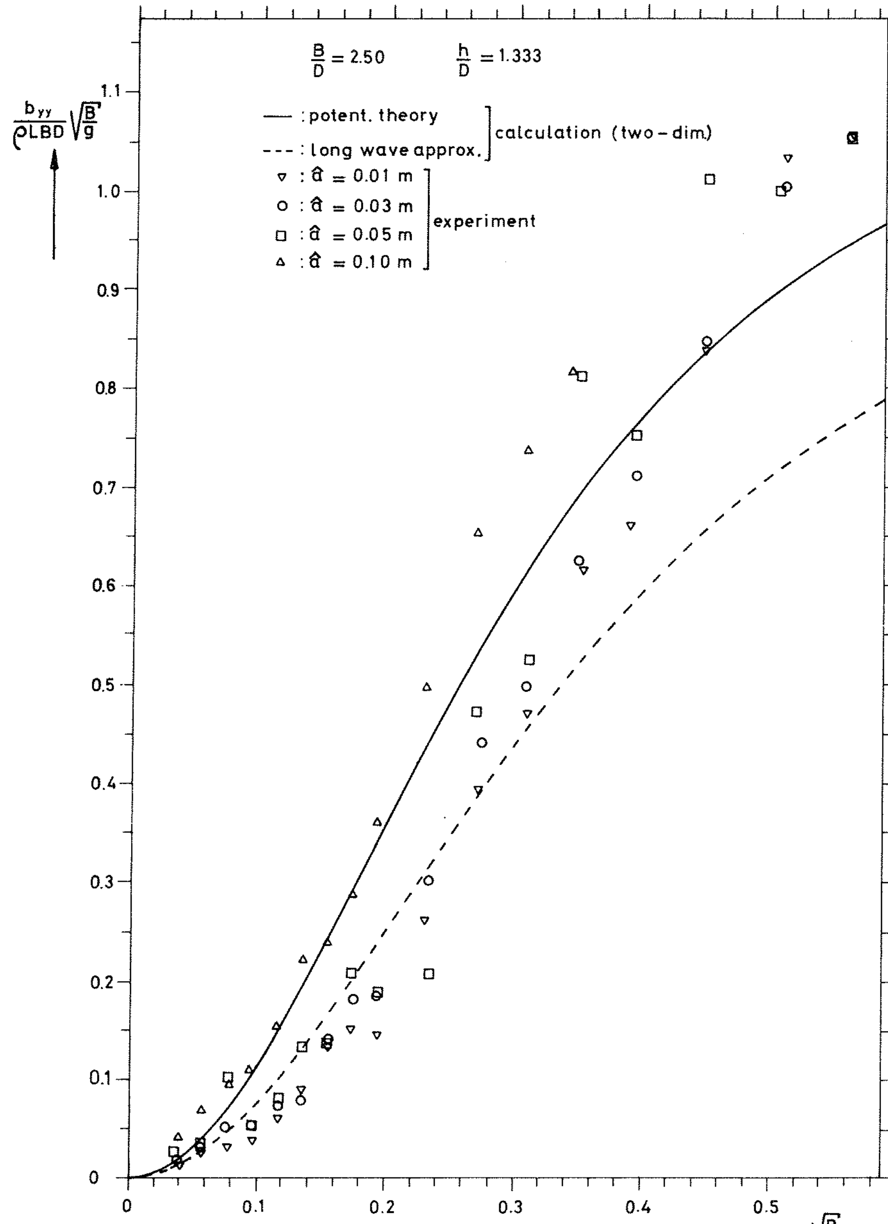

fig.12 – Sway damping force coefficient $\left(\frac{h}{D} = 1.333\right)$.

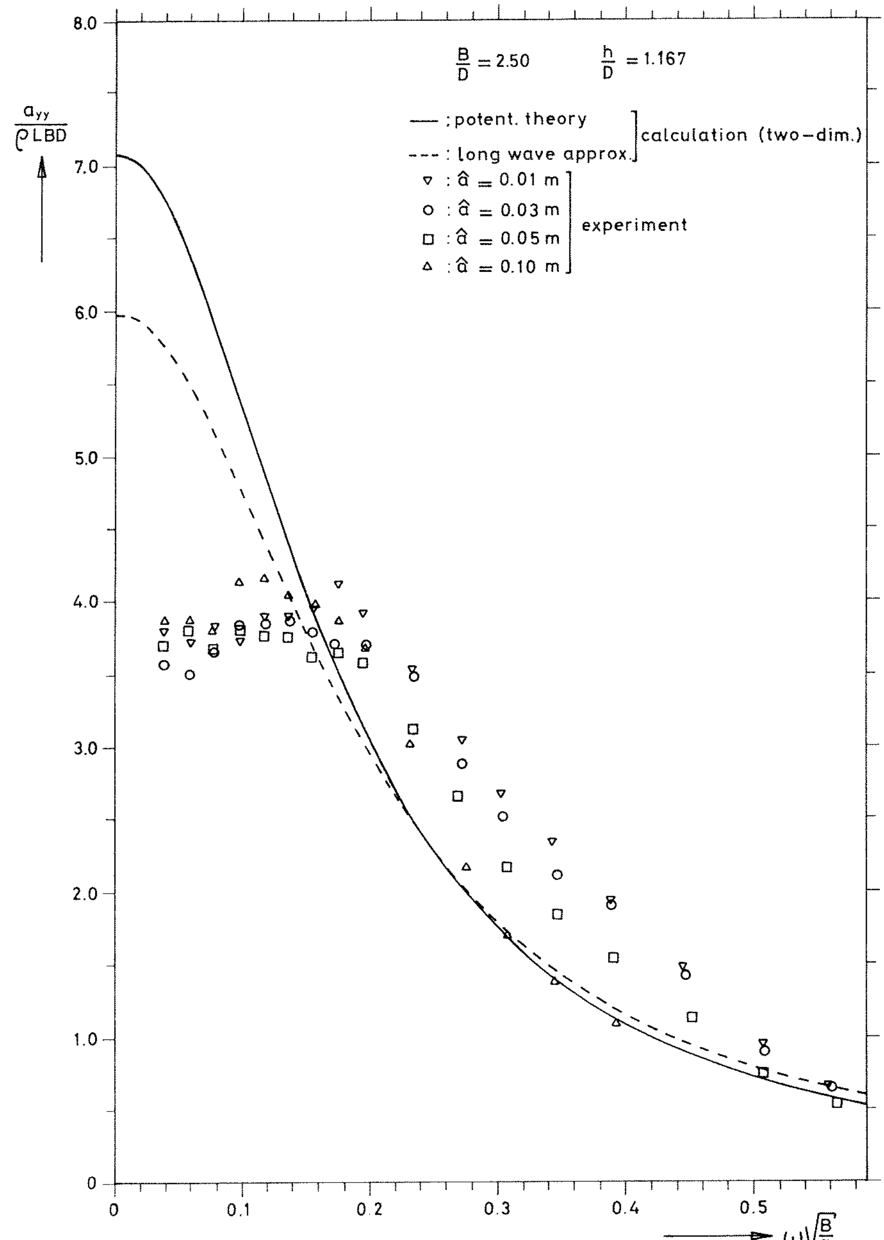

fig. 13 – Added mass for swaying motion $\left(\frac{h}{D} = 1.167\right)$.

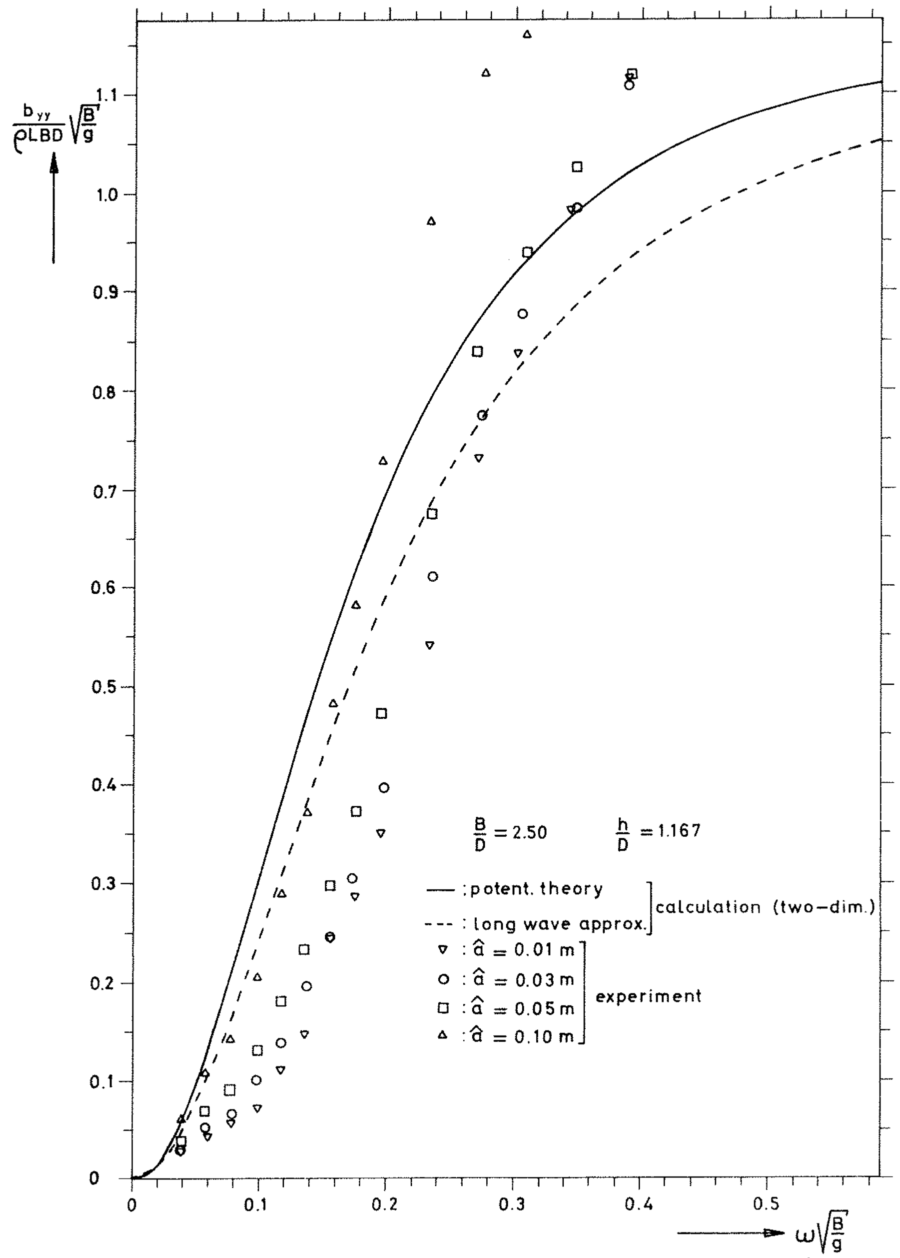

fig.14 – Sway damping force coefficient $\left(\frac{h}{D} = 1.167\right)$.

portant part, such that an extension of the preceding explanation to larger amplitudes of motion does not seem to be justified.

The results of the dynamic sway tests indicate that the system ship-fluid may be considered as being linear (i.e. independent of the amplitude of motion), at least within the frequency range considered in the experiments. Apparently the assumption of linearity of the system ship-fluid in case of swaying is a good working approximation notably for small to moderate amplitudes (say up to $\hat{a}$ = 0.05 m) of model ship forms as used.

B3.2.2: Test results in case of pure yawing

From the series of dynamic yawing experiments only the test results are given for the following amplitudes of motion: $\psi_0 = \left|-\frac{\hat{a}}{l}\right|$ = 0.02, 0.06, 0.10 and 0.20 rad. where l = 0.5000 m. These results are plotted in fig. 15 through fig. 18 as centred symbols.

For both values of $\frac{h}{D}$, $a_{\psi\psi}(\frac{1}{12}L^2\rho LBD)^{-1}$ remains almost constant and $b_{\psi\psi}(\frac{1}{12}L^2\rho LBD)^{-1}(B/g)^{\frac{1}{2}}$ is subject to large changes when $\omega(B/g)^{\frac{1}{2}}$ increases. For the frequency range considered in the experiments, $a_{\psi\psi}(\frac{1}{12}L^2\rho LBD)^{-1}$ in case of the smaller water depth ($\frac{h}{D}$ = 1.167) up to $\omega(B/g)^{\frac{1}{2}}$ = 0.567 (i.e. ω = 2.9 rad.s^{-1}) is greater than $a_{\psi\psi}(\frac{1}{12}L^2\rho LBD)^{-1}$ in case of the greater water depth ($\frac{h}{D}$ = 1.333); for $\omega(B/g)^{\frac{1}{2}}$ > 0.567 $a_{\psi\psi}(\frac{1}{12}L^2\rho LBD)^{-1}$ in case of $\frac{h}{D}$ = 1.167 seems to be somewhat smaller than in case of $\frac{h}{D}$ = 1.167. For $\frac{h}{D}$ = 1.167 $b_{\psi\psi}(\frac{1}{12}L^2\rho LBD)^{-1}(B/g)^{\frac{1}{2}}$ is greater than for $\frac{h}{D}$ = 1.333.

For that part of the frequency range where the test results are not influenced by (reflected) waves (i.e. if $\omega(B/g)^{\frac{1}{2}} \geqq 0.08$, or ω > 0.4 rad.s^{-1}) the values of $a_{\psi\psi}(\frac{1}{12}L^2\rho LBD)^{-1}$ for the various amplitudes of motion coincide - for certain $\omega(B/g)^{\frac{1}{2}}$ - reasonably well; generally this holds good up to ψ_0 = 0.10 rad. for both water depths. During the yaw experiments it was observed that for $\psi_0 \geqq 0.10$ rad. the phenomena of vortex shedding and separation of flow at the 'bow', the 'stern' and the 'bilges' of the ship model were going to play an increasingly important part. This last fact seems to affirm that the influence of the smaller amplitudes of motion on $a_{\psi\psi}(\frac{1}{12}L^2\rho LBD)^{-1}$ is of secondary importance.

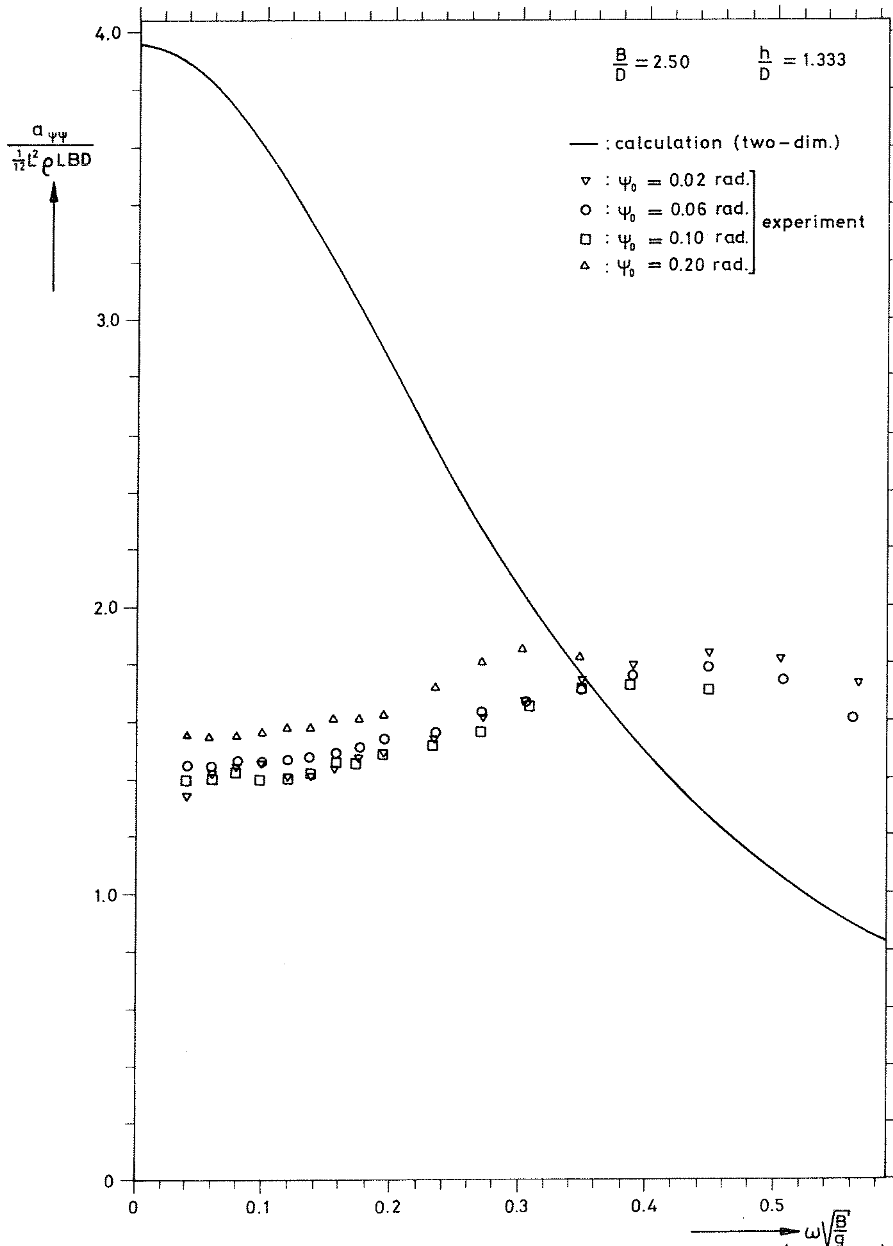

fig.15 – Added mass moment of inertia for yawing motion $\left(\frac{h}{D} = 1.333\right)$.

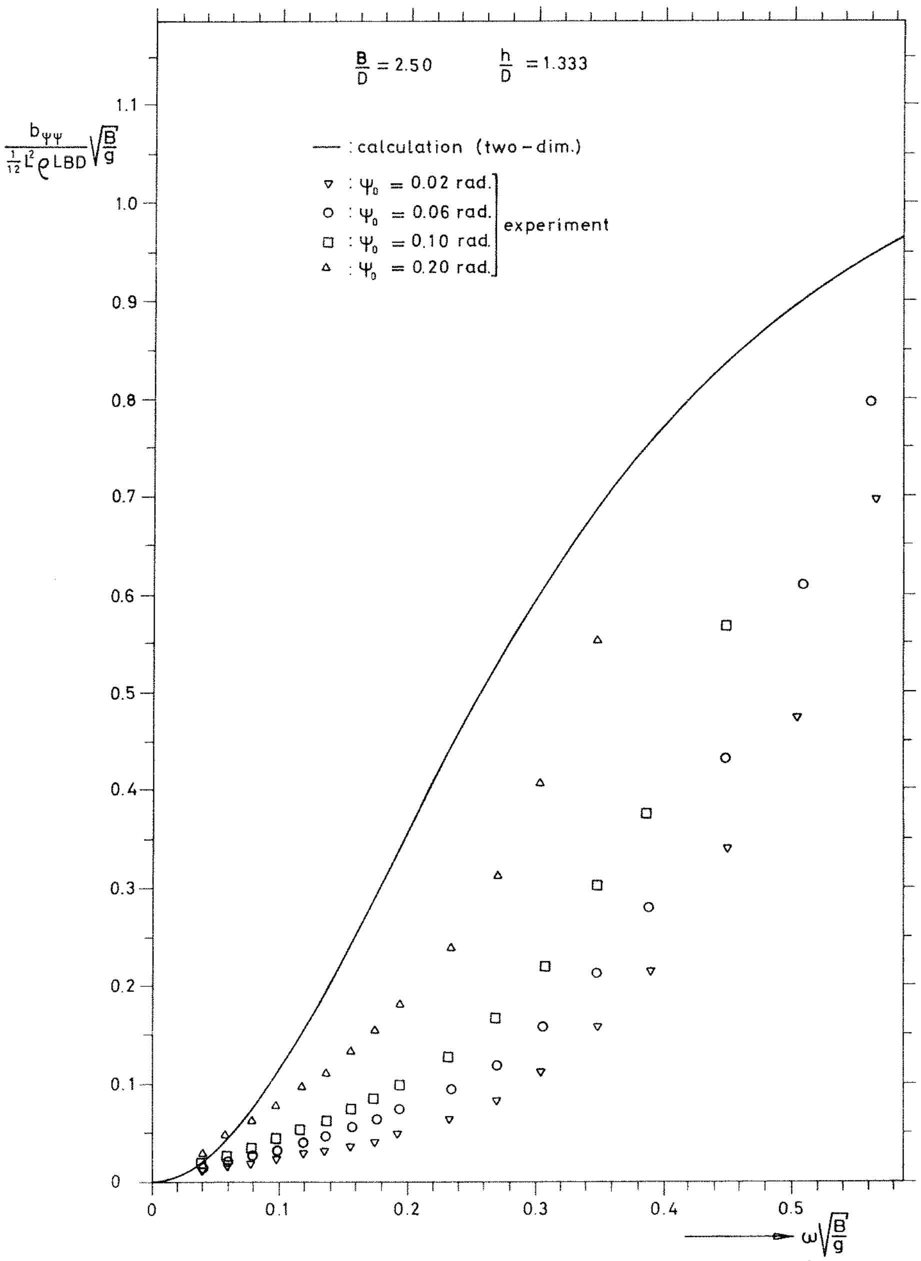

fig.16 — Yaw damping moment coefficient $\left(\frac{h}{D} = 1.333\right)$.

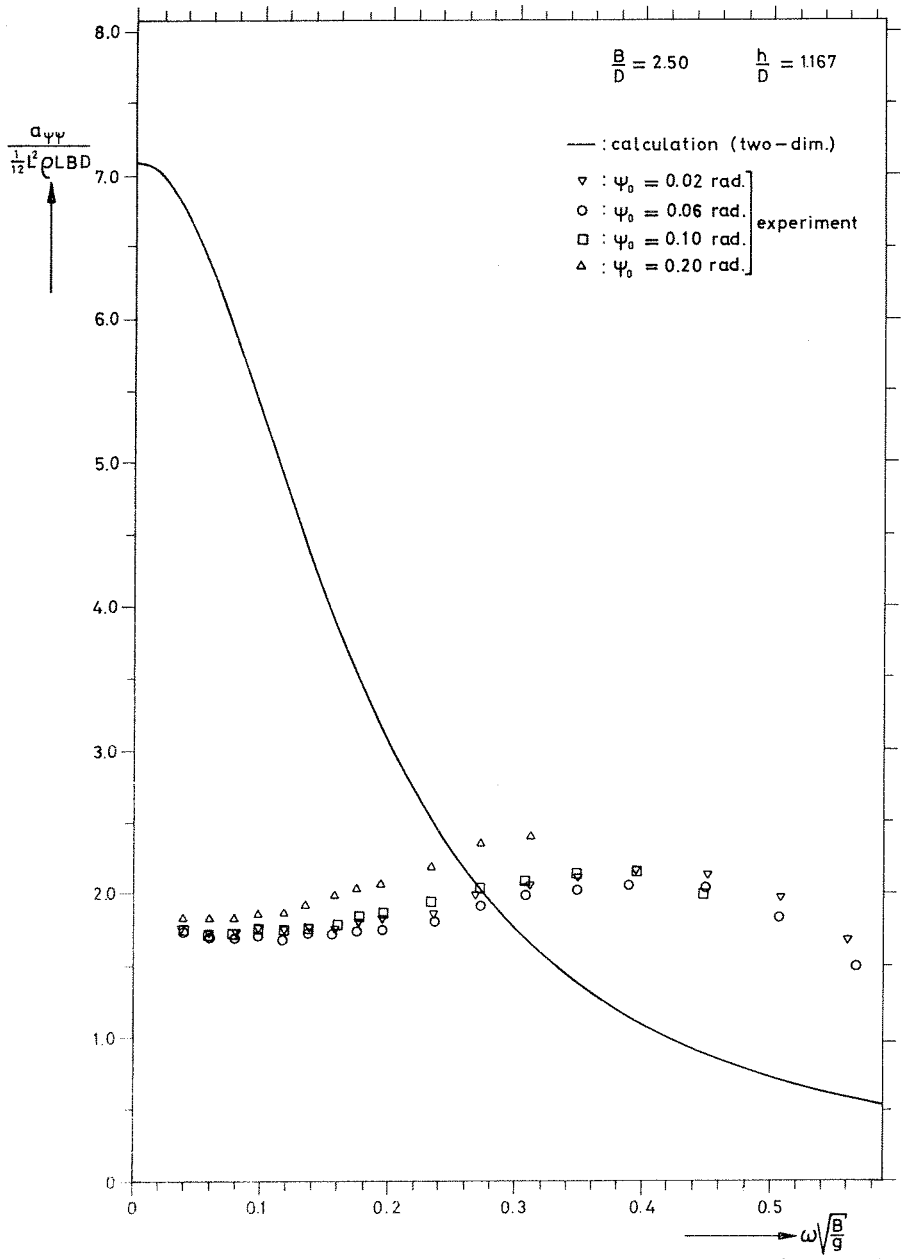

fig.17 — Added mass moment of inertia for yawing motion $\left(\frac{h}{D} = 1.167\right)$.

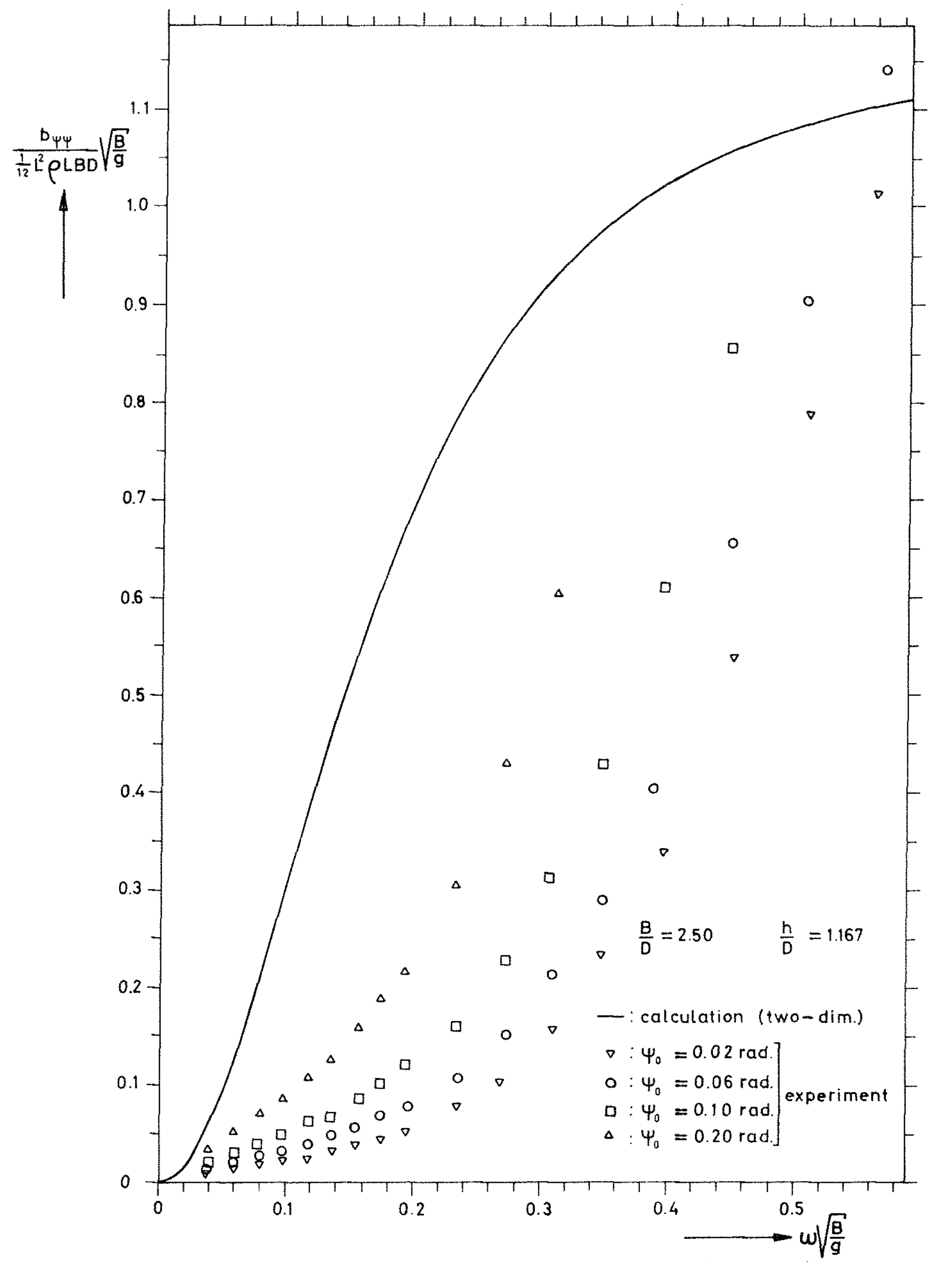

fig.18 — Yaw damping moment coefficient $\left(\frac{h}{D} = 1.167\right)$.

For $\omega(B/g)^{\frac{1}{2}} \geqq 0.08$ $b_{\psi\psi}(\frac{1}{12}L^2\rho LBD)^{-1}(B/g)^{\frac{1}{2}}$ for certain $\omega(B/g)^{\frac{1}{2}}$ (slightly) increases as ψ_0 increases; generally this holds good for both water depths. This phenomenon may be explained in a similar way as in the case of pure swaying, viz. by friction effects in the keel clearance. For $b_{\psi\psi}(\frac{1}{12}L^2\rho LBD)^{-1}(B/g)^{\frac{1}{2}}$ an extension of this explanation to amplitudes of motion $\psi_0 \geqq 0.10$ rad. does not seem justified for reasons of the increasingly important part played by vortex shedding and separation of flow.

The results of the dynamic yaw tests indicate that the added mass moment of inertia for yawing motion - unlike the yaw damping moment coefficient - may be considered as independent of the amplitude of motion, at least within the frequency range considered in the experiments. Nevertheless it will be supposed that the system ship-fluid also in case of yawing is linear, and that this supposition of linearity of the system ship-fluid is a good working approximation notably for (very) small amplitudes of motion (say up to $\psi_0 = 0.06$ rad.) of model ship forms as used.

To conclude with Section B3 a number of three comments have to be made.

One is tempted to assume that the values of the various hydrodynamic coefficients for zero frequency will be the same as those for very low frequencies, but particularly with respect to the added mass for swaying motion and the added mass moment of inertia for yawing motion one has to be very careful with this extrapolation. Concerning the hydrodynamic damping coefficients the problem seems to be less complicated.

It looks like that the symmetry relations $a_{y\psi} = a_{\psi y}$, $b_{y\psi} = b_{\psi y}$ are confirmed by experiments only in the low-frequency domain. At higher frequencies the inequality of the respective hydrodynamic cross-coupling coefficients is probably due to the shedding of vorticity and separation of flow.

Apart from the conducted model experiments the question remains whether at higher frequencies the linear 'equations of motion' (see Appendix II) would satisfy to describe the lateral ship motions. There seem to be objections to the use of the independent variables $\hat{a}$ (ψ_0) and ω. It might be better to carry out the dynamic tests for constant values of the velocity - and acceleration amplitudes, respectively, for these are the variables which are considered in the 'equations of motion' (see Appendix II). In that case suitable combinations of $\hat{a}$ (ψ_0) and ω have to be chosen. A plot of the various hydrodynamic coefficients on a basis of velocity - and acceleration amplitudes, respectively, could be useful to judge the separate effects of both these variables.

part C: COMPARISON OF THEORY AND EXPERIMENT. CONCLUSIONS

Section C1: Introduction

In this part of the report the theoretical and experimental results of the hydrodynamic coefficients - as derived in the respective parts A and B - are compared with one another.

In part A the hydrodynamic coefficients for the three-dimensional ship form are obtained by combining the contributions from the separate cross-sections in a simple stripwise manner. If the hydrodynamic quantities, calculated in this way, are in agreement with test results, then this is an obvious support to two propositions: firstly that the separate contributions for all cross-sections of the ship have been predicted correctly, and secondly that the measure in which the cross-sections influence one another is negligible. For it is considered to be very improbable that zero, first and second order moments of the various quantities are correct, if their distribution along the length of the ship is not.

In presenting the results derived theoretically (see Section A3.4.2) the effects of the strip theory applied and the neglect of viscosity as well as the end effects (i.e. the circulation around 'bow' and 'stern') have not been discussed. Hence, in Section C2, first some remarks will be made with regard to the effects of strip theory and neglect of viscosity on the theoretical results for the hydrodynamic coefficients, before passing on to a comparison of theory and experiment; likewise, the end effects will be discussed.

In Section C3 the theoretical and experimental results for the hydrodynamic coefficients in case of pure swaying and in case of pure yawing are compared with one another.

Some general conclusions are presented in Section C4.

Section C2: General discussion and remarks

The dependence of force and moment coefficients on the frequency of oscillation is caused originally by the wave effects associated with the

unsteady motion of the ship (model) at the free surface and by the vorticity which is shed from the oscillating hull.
There is some evidence to suggest that the presence of the free surface plays a more important role in the damping components of force and moment than in the corresponding added mass (moment of inertia) components. This is to be expected since in an ideal unbounded fluid the hydrodynamic force (moment) is entirely in phase with the (angular) acceleration.

For the low frequency range the viscous effects come into play rather than the free surface effects.
For sufficiently small frequencies the pseudo-steady-state analysis is valid (i.e. pertaining to steady state hydrodynamic forces and moments acting on a ship)(e.g., see Appendix III).

C2.1: Effect of strip theory

By strip theory is simply understood the stringing of a series of two-dimensional elements to construct an approximative solution for a three-dimensional problem: each cross-section of the ship (model) is considered to be part of an infinitely long prismatic body and each two-dimensional problem so constructed is solved separately, after which the solutions are combined in some way to yield a solution for the entire ship. Consequently two stages can be distinguished in the strip theory approach. Firstly, the solution of the two-dimensional problem of oscillating prisms: in this stage the elementary local values of the hydrodynamic coefficients must be determined. Secondly, the combination of these values to approximate the three-dimensional coefficients (at zero forward speed): here physically three-dimensional effects come into play, but only as far as the strip theory neglects them.

Using strip theory it is obvious that the longitudinal translation (surge) cannot be dealt with: therefore this motion has been left out of consideration.

In the two-dimensional theoretical problem as dealt with in part A the cross-section can only perform swaying. A solution for the three-dimensional yawing has been obtained by making the hypothesis that locally this rotational motion is equivalent to a transverse translatory motion of angle times the distance from the axis of rotation.

The strip theory has the great drawback that it neglects the mutual interactions between the various cross-sections. For slender bodies strip theory results logically from the truely three-dimensional theory for high frequencies of motion. Therefore it may be expected that the correctness of this neglect depends primarily on the range of frequencies involved in relation to the size of the body or, in physical terms, on the relative length of the waves generated by the oscillations and the dimensions of the body: short waves will not be affected distinctly by parts of the body being many wave lengths away (and vice versa), but for long waves the same parts are close to the source of the disturbance and will directly attribute to the hydrodynamic phenomena.

Looking at the matter in this physical way another aspect is formed by the phase relation of the motions of the various sections. A phase identity for all sections as with sway motions resembles two-dimensional conditions, while a phase transition of π radians at mid-length as with yaw promotes interference effects.

Naturally the basic principle of strip theory breaks down at the ends of the body.

The above is only a qualitative evaluation; it is very difficult to say where the limits of relatively high frequencies or of long waves are, or to what extent the end effects influence the ultimate results.

C2.2: Effect of neglect of viscosity

Viscous contributions appear in two forms: skin friction and separation of flow. Usually these viscous components are of a non-linear nature.

Skin friction is proportional to some positive power of a velocity (gradient) and will contribute mainly to the damping coefficients, while separation of flow changes the flow pattern about the body to a certain extent so that it may influence both the damping and the added mass (moment of inertia).

Skin friction may be left out of consideration since, probably, it will be small with respect to flow separation, although in unsteady flow motions large velocity gradients and consequently large shear forces may occur.

Separation of flow occurs at relatively sharp edges of the ship (model). This is a source of energy loss due to eddy formation which contributes mainly to the damping coefficients: probably, the shedding of eddies does not seriously affect that part of the pressure distribution which is in phase with the body acceleration. In cases where the damping due to wave radiation is small, the influence of separation of flow cannot be neglected, however; such cases are e.g. the hydrodynamic forces at the ends of the ship in transverse motion, may be the (local) forces on the bilges.

The viscous contributions at the ends of the ship (model) in swaying and yawing may be locally significant, but probably they are negligible with respect to the magnitude of the total damping.

C2.3: End effects: circulation around bow and stern

Strip theory cannot account for the side force and the yaw moment associated with a small keel clearance, because in this case the circulation around 'bow' and 'stern' may become important.

It may be expected that the influence(s) of the ends of the ship does (do) not dominate the integrated pressures due to two-dimensional potential flow, if the ratios of ship's length to wave length are great enough and as long as there is no forward speed: at least the circulation effects will relatively diminish. For short(er) ships deviations can be expected.

In physical terms one thing and another implies the following. Using a two-dimensional theory it is not possible to obtain results always reliable, because three-dimensional effects can be of crucial importance.

In swaying distinction must be made between situations in which fluid easily can pass under the keel of the ship (model), and situations in which most of the fluid must move lengthwise, passing around the ends of the ship. Only in the former situation added mass and damping coefficients may be calculated in a simple stripwise manner, neglecting three-dimensional effects. In the latter situation, which has as its extreme case that of a grounded ship touching the bottom along its whole length, the given theory holds good only for an infinitely long ship (model) and, as a consequence, the results must be regarded with utmost care. Therefore it is possible to state in advance that for a ship with finite keel clearance

and finite length a two-dimensional theory produces values for the hydrodynamic coefficients in case of swaying which are too high.
The hydrodynamic coefficients for yawing are determined from those for swaying in a simple stripwise manner. In yawing the lengthwise motion of fluid passing around the ends of the ship (i.e. circulation) is rather important - certainly in case of a finite keel clearance - so that (very) inaccurate or even wrong results for the hydrodynamic coefficients can be expected from a two-dimensional theory.

Section C3: Comparison of theoretical and experimental results

C3.1: Comparison of theoretical and experimental results in case of pure swaying

The theoretical results for the hydrodynamic coefficients in case of pure swaying are plotted as solid lines in the figs. 11 through 14. This implies that the theoretical results as presented in Section A3.4.2 (figs. 3 and 4) are restricted to the frequency range considered in the experiments.

Making allowance for the fact that - as a result of the uneven bottom in the experiments - the (real) values of a_{yy} and b_{yy} generally will be (slightly) greater than the measured values (see Section B3.1), the agreement between the theoretically derived and the experimentally determined results for the hydrodynamic coefficients in case of swaying is quite satisfactory; notably this holds good for moderate to high frequencies (i.e. if $0.15 < \omega(B/g)^{\frac{1}{2}} < 0.58$) for both values of $\frac{h}{D}$. For the low frequency range (say $\omega(B/g)^{\frac{1}{2}} < 0.15$) the theoretical and experimental results do not agree. This is caused by the fact that in the theoretical determination of the hydrodynamic coefficients the respective effects of strip theory and neglect of viscosity as well as the so-called end effects are not taken into account. In addition to this the experimental results for $\omega(B/g)^{\frac{1}{2}} < 0.08$ were influenced by (reflected) waves. It is very difficult to evaluate to what extent the reflection phenomenon influences the test results for $a_{yy}(\rho LBD)^{-1}$ and $b_{yy}(\rho LBD)^{-1}(B/g)^{\frac{1}{2}}$: anyhow, the test results for $\omega(B/g)^{\frac{1}{2}} < 0.08$ have to be considered with some reserve (see Section B3.1).

Application of the strip theory implies a neglect of the mutual interactions between the various cross-sections of the ship (model); the influence of this on the hydrodynamic coefficients derived theoretically finds expression mainly in the low frequency range, because there the length of the waves generated by the oscillations is relatively great with respect to the size of the body. As explained before, the influence of the ends of the hull ('bow' and 'stern') on the hydrodynamic phenomena then may become relatively important.

For the low frequency range the viscous effects in themselves come into play rather than the free surface effects and may present an increasing influence on the hydrodynamic phenomena as the frequency decreases. For further details about the effect of the neglect of viscosity see Section C2.2.

For $\frac{h}{D}$ = 1.333 the agreement between theory and experiment in case of swaying turns out to be slightly better than for $\frac{h}{D}$ = 1.167. This is caused by the fact that for $\frac{h}{D}$ = 1.167 the keel clearance underneath the ship is smaller than for $\frac{h}{D}$ = 1.333. In case of a smaller keel clearance the circulation effect is stronger. For more details see Section C2.3.

C3.2: Comparison of theoretical and experimental results in case of pure yawing

The theoretical results for the hydrodynamic coefficients in case of pure yawing are plotted as solid lines in the figs. 15 through 18. As in the case of swaying the theoretical results (as presented in Section A3.4.2) are restricted to the frequency range considered in the experiments.

Even when regard is paid to the fact that - as a result of the uneven bottom in the experiments - the (real) values of $a_{\psi\psi}$ and $b_{\psi\psi}$ generally will be (slightly) greater than the measured values (see Section B3.1), the theoretically derived and the experimentally determined results for the hydrodynamic coefficients in case of yawing do not agree, at least as far as the considered frequency range is concerned; notably this holds good for $a_{\psi\psi}(\frac{1}{12}L^2\rho LBD)^{-1}$, in case of $b_{\psi\psi}(\frac{1}{12}L^2\rho LBD)^{-1}(B/g)^{\frac{1}{2}}$ the theoretical and experimental results present the same trend. The effect of the neglect of viscosity yields no satisfactory explanation. Therefore it is obvious to think of an explanation in terms of the effect of the strip theory in combination with the so-called end effects.

Essentially the strip theory is two-dimensional; consequently the solution for the three-dimensional yawing is obtained by hypothesizing that locally this rotational motion is equivalent to a transverse translatory motion of angle times the distance from the axis of rotation. Besides, in the strip theory the mutual interactions between the various cross-sections are neglected, while another complicating aspect is formed by the phase relation of the motions of the various cross-sections. For a discussion of the respective consequences of using the strip theory is referred to Section C2.1. In addition to the effects of the strip theory another complicating factor is formed by the fact that the strip theory cannot account for (the side force and) the yaw moment associated with a small keel clearance, because in this case circulation (i.e. the lengthwise motion of fluid, passing around the ends of the ship) belongs to the eventualities. For more details see Section C2.3.

It will be obvious by now that the effects of the strip theory together with the circulation effect are responsible for the general disagreement between the theoretically derived and the experimentally determined results for the hydrodynamic coefficients in case of yawing.

Section C4: Conclusions

For swaying the system ship-fluid can be considered as linear in case of small oscillations, at least for the frequency range examined. In case of very small oscillations also for yawing the system ship-fluid may be considered as linear.

Application of the strip theory with neglect of viscous effects is adequate for swaying on shallow water in case of small/moderate to high frequencies. In the frequency range considered the strip theory is not satisfactory for yawing on shallow water.

Further research into the hydrodynamic coefficients in case of swaying and yawing on shallow water has to be focused on three-dimensional conditions for low frequencies.

REFERENCES

[1] F. Biésel, F. Suquet,
'Les appareils générateurs de houle en laboratoire', La Houille Blanche, vol. 6, 1951,
no. II mars-avril pp. 147-165 (1re partie),
no. IV juillet-aout pp. 475-496 (2e partie),
no. V sept-oct pp. 723-737 (3e partie).

[2] M.A. Abkowitz,
'Lectures on ship hydrodynamics, steering and manoeverability', Report No. HY-5, May 1964, Hydrodynamics Department Hydro- og Aerodynamisk Laboratorium, Lyngby, Denmark.

[3] J.H. Vugts,
'The hydrodynamic forces and ship motions in waves', Thesis, Uitgeverij Waltman, Delft, 1970.

[4] W.E. Cummins,
'The impulse response function and ship motions', Schiffstechnik, 9, Heft 47, 1962, pp. 101-109.

[5] T.F. Ogilvie,
'Recent progress toward the understanding and prediction of ship motions', Proc. of the fifth Symposium on Naval Hydrodynamics, Bergen, Norway, sept. 1964.

[6] H.J. Zunderdorp, M. Buitenhek,
'Oscillator-techniques at the Shipbuilding Laboratory', Report no. 111, Nov. 1963, Shipbuilding Laboratory, Department of Naval Architecture, Delft University of Technology.

APPENDICES

Appendix I: Outline of solution of mixed boundary-value problem

According to Sections A2 and A3.1 the governing equation, the boundary conditions and the supplementary condition for the (first order) velocity potential $\Phi(\bar{y},\bar{z},\bar{t})$ are:

(I-1) $$\nabla^2\Phi = \frac{\partial^2\Phi}{\partial\bar{y}^2} + \frac{\partial^2\Phi}{\partial\bar{z}^2} = 0 \quad \text{in} \quad R\,, \tag{1}$$

(I-2) $$\frac{\partial^2\Phi}{\partial t^2} - g\,\frac{\partial\Phi}{\partial\bar{z}} = 0 \quad \text{on} \quad \bar{y} \geqq \frac{1}{2}B, \quad \bar{z} = 0\,, \tag{2}$$

(I-3) $$\frac{\partial\Phi}{\partial\bar{y}} = \omega\Big[\hat{a}\{U(\bar{z}) - U(\bar{z} - h + k)\} + \frac{\hat{v}_{kc}}{\omega}\, e^{-i\theta}\{U(\bar{z} - h + k) + {} - U(\bar{z} - h^+)\}\Big]e^{i\omega t} \quad \text{on} \quad \bar{y} = \frac{1}{2}B\,, \tag{3'}$$

(I-4) $$\frac{\partial\Phi}{\partial\bar{z}} = 0 \quad \text{on} \quad \bar{y} \geqq \frac{1}{2}B, \quad \bar{z} = h\,, \tag{4}$$

(I-5) $$\Phi(\bar{y},\bar{z},t)\Big|_{\bar{y} \to +\infty} \longrightarrow \text{outgoing dispersive wave}\,, \tag{5}$$

(I-6) $$\Phi(\bar{y},\bar{z},t)\,, \quad \Phi'(\bar{y},\bar{z},t) \quad \text{being finite in } R\,, \tag{6}$$

where

$$R = (\text{fluid})\ \text{region} \begin{cases} \bar{y} > \frac{1}{2}B, & 0 \leqq \bar{z} \leqq h \\ \bar{y} = \frac{1}{2}B, & 0 \leqq \bar{z} < (h - k)\,, \\ & (h - k) \leqq \bar{z} \leqq h\,, \end{cases}$$

$$U(\bar{z}) = \text{unity step function} = \begin{cases} 0 & \text{on} \quad \bar{z} < 0 \\ 1 & \text{on} \quad \bar{z} \geq 0 \end{cases} \quad ,$$

$$v_{fl} = \frac{\partial \Phi}{\partial \bar{y}} , \quad w_{fl} = \frac{\partial \Phi}{\partial \bar{z}} \quad .$$

Because of linearity of the problem the velocity potential $\Phi(\bar{y},\bar{z},t)$ can be conceived of as being composed of the velocity potential $\Phi_s(\bar{y},\bar{z},t)$ resulting from the motion of the ship only, and the velocity potential $\Phi_{kc}(\bar{y},\bar{z},t)$ resulting from the motion of the mass of water underneath the ship:

(I-7) $$\Phi(\bar{y},\bar{z},t) = \Phi_s(\bar{y},\bar{z},t) + \Phi_{kc}(\bar{y},\bar{z},t) \quad ;$$

each single velocity potential must satisfy the Laplace equation and relevant boundary conditions.
$\Phi(\bar{y},\bar{z},t)$, and consequently $\Phi_s(\bar{y},\bar{z},t)$ and $\Phi_{kc}(\bar{y},\bar{z},t)$, are simple-harmonic functions of time t. Therefore the assumed sinusoidal time dependence may be factored out:

(I-8) $$\Phi_{s,kc}(\bar{y},\bar{z},t) = \phi_{s,kc}(\bar{y},\bar{z}) \, T_{s,kc}(t) \; ,$$

where $\phi_{s,kc}(\bar{y},\bar{z})$ = harmonic (in $\bar{y}$ and $\bar{z}$) function,

$$T_s(t) = e^{i\omega t} \; , \quad T_{kc}(t) = e^{i(\omega t - \theta)} \quad .$$

Substitution of $\Phi_{s,kc}(\bar{y},\bar{z},t)$ as stated by eq. (I-8) into the eqs. (I-1), (I-2), (I-4) and (I-6) yields

(I-1') $$\nabla^2 \phi_{s,kc} = 0 \quad \text{in} \quad R \; ,$$

(I-2') $$\phi_{s,kc} + \frac{g}{\omega^2} \frac{\partial \phi_{s,kc}}{\partial \bar{z}} = 0 \quad \text{on} \quad \bar{y} \geq \frac{1}{2} B, \; \bar{z} = 0 \; ,$$

(I-4') $$\frac{\partial \phi_{s,kc}}{\partial \bar{z}} = 0 \quad \text{on} \quad \bar{y} \geqq \frac{1}{2} B, \quad \bar{z} = h \; ,$$

(I-6') $$\phi_{s,kc} \; , \; \phi'_{s,kc} \quad \text{being finite in R} \; ,$$

respectively.

Solution of the Laplacian (I-1') will be found by means of separation of variables:

(I-9) $$\phi_{s,kc}(\bar{y},\bar{z}) = Y_{s,kc}(\bar{y}) \, Z_{s,kc}(\bar{z}) \; ,$$

where $Y_{s,kc}(\bar{y})$ = function of $\bar{y}$ only, and $Z_{s,kc}(\bar{z})$ = function of $\bar{z}$ only. With this expression for $\phi_{s,kc}(\bar{y},\bar{z})$ eq. (I-1') gives:

$$Y''_{s,kc}(\bar{y})/Y_{s,kc}(\bar{y}) = - Z''_{s,kc}(\bar{z})/Z_{s,kc}(\bar{z}) = \alpha^2 \; ,$$

where α^2 = constant of separation; the double prime used as superscript means 'ordinary second derivative'. The solution of these equations is:

$$Y_{s,kc}(\bar{y}) = Ae^{+\alpha\bar{y}} + Be^{-\alpha\bar{y}} \quad \text{and} \quad Z_{s,kc}(\bar{z}) = Ce^{+i\alpha\bar{z}} + De^{-i\alpha\bar{z}} \; ,$$

where A, B, C en D are constants of integration.
Starting from the general assumption that α is a constant complex quantity, written as $\alpha = \alpha_r + i\alpha_i$ with $Re(\alpha) = \alpha_r$, $Im(\alpha) = \alpha_i$ (α_r and α_i real), it can be proved that merely the three following cases have to be considered:

1^o: $\alpha = 0$ $(\alpha_r = \alpha_i = 0)$, 2^o: $\alpha = \alpha_r$ $(\alpha_i = 0)$, 3^o: $\alpha = i\alpha_i$ $(\alpha_r = 0)$.

Using $\alpha = \alpha_r + i\alpha_i$ the expressions for $Y_{s,kc}(\bar{y})$ and $Z_{s,kc}(\bar{z})$ can be written as:

$$Y_{s,kc}(\bar{y}) = (Ae^{+\alpha_r\bar{y}} + Be^{-\alpha_r\bar{y}}) \cos(\alpha_i\bar{y}) + i(Ae^{+\alpha_r\bar{y}} + - Be^{-\alpha_r\bar{y}}) \sin(\alpha_i\bar{y}) \; ,$$

and

$$Z_{s,kc}(\bar{z}) = (De^{+\alpha_i \bar{z}} + Ce^{-\alpha_i \bar{z}}) \cos(\alpha_r \bar{z}) - i(De^{+\alpha_i \bar{z}} + - Ce^{-\alpha_i \bar{z}}) \sin(\alpha_r \bar{z}) .$$

Introduction of the supplementary condition (I-6') then yields the results

$$Z_{s,kc}(\bar{z}), \quad Z'_{s,kc}(\bar{z}) \quad \text{being finite on} \quad 0 \leqq \bar{z} \leqq h ,$$

and

$$Y_{s,kc}(\bar{y}), \quad Y'_{s,kc}(\bar{y}) \quad \text{being finite on} \quad \bar{y} = \tfrac{1}{2} B, \quad \text{if}$$

$$\left.\begin{array}{l} \text{either } \alpha_r = 0 \text{ (cases } 1^o \text{ and } 3^o\text{, respectively)} , \\ \text{or, both } \alpha_r > 0 \text{ and } A = 0 \text{ (case } 2^o) , \\ \text{or, both } \alpha_r < 0 \text{ and } B = 0 \text{ (case } 2^o) . \end{array}\right\} ;$$

the prime used as superscript means 'ordinary first derivative'.

Introducing the boundary conditions (I-4') and (I-2') into the three above-mentioned cases successively and using eq. (I-9) it is obtained:

1^o: for $\alpha = 0$: $\phi_{s,kc}(\bar{y},\bar{z}) = 0$ (the zero-solution);

2^o: for $\alpha = \alpha_r$: if $\alpha_r > 0$, $\phi_{s,kc}(\bar{y},\bar{z}) = Ke^{-\alpha_r \bar{y}} \cos\{\alpha_r(h - \bar{z})\}$

with $\omega^2 = - g\alpha_r \tan(\alpha_r h)$,

and, if $\alpha_r < 0$, $\phi_{s,kc}(\bar{y},\bar{z}) = K'e^{+\alpha_r \bar{y}} \cos\{\alpha_r(h - \bar{z})\}$

with $\omega^2 = - g\alpha_r \tan(\alpha_r h)$;

3^o: for $\alpha = i\alpha_i$: $\phi_{s,kc}(\bar{y},\bar{z}) = \{E \cos(\alpha_i \bar{y}) + i F \sin(\alpha_i \bar{y})\}.$

$. \cosh\{\alpha_i(h - \bar{z})\}$ with $\omega^2 = g \alpha_i \tanh(\alpha_i h)$,

where K, K', E and F are constants of integration.
With regard to case 2^o it has to be noted that the equation $\omega^2 = -g\,\alpha_r \tan(\alpha_r h)$ contains two series of real roots for α_r with the same absolute values but opposite signs:

$$\left.\begin{aligned} &\text{for } \alpha_r > 0 \text{ it holds: } \alpha_r = +m_1, +m_2, \ldots\ldots, +m_n, \ldots\ldots \\ &\text{for } \alpha_r < 0 \text{ it holds: } \alpha_r = -m_1, -m_2, \ldots\ldots, -m_n, \ldots\ldots \end{aligned}\right\}$$

with $m_n > 0$, arranged in order of increasing magnitude. The complete solution for case 2^o is composed of linear combinations of $K\,e^{-\alpha_r \bar{y}} \cos\{\alpha_r(h - \bar{z})\}$ and $K'e^{+\alpha_r \bar{y}} \cos\{\alpha_r(h - \bar{z})\}$ for the respective values of α_r, and can be written as:

$$\phi_{s,kc}(\bar{y},\bar{z}) = \sum_{n=1}^{\infty} C_n\, e^{-m_n \bar{y}} \cos\{m_n(h - \bar{z})\} ,$$

where C_n = constant of integration ,

(I-10[b]) m_n = positive roots of $\omega^2 = -\,g m_n \tan(m_n h)$ $(n = 1,2,\ldots;$

$m_1 < m_2 < \ldots < m_n < \ldots.)$. (8[b])

Similarly, with regard to case 3^o, the equation $\omega^2 = g\alpha_i \tanh(\alpha_i h)$ contains two real roots with the same absolute values but opposite signs: $\alpha_i = \pm\, m_0$, $m_0 > 0$. The complete solution for case 3^o is composed of a linear combination of $\{E \cos(\alpha_i \bar{y}) + i\,F \sin(\alpha_i \bar{y})\} \cosh\{\alpha_i(h - \bar{z})\}$ for the respective values of α_i, and can be written as:

$$\phi_{s,kc}(\bar{y},\bar{z}) = \{A \cos(m_0\bar{y}) + i\,B \sin(m_0\bar{y})\} \cosh\{m_0(h - \bar{z})\} ,$$

where A, B = constants of integration ,

(I-10[a]) m_0 = positive root of $\omega^2 = g\, m_0 \tanh(m_0 h)$. (8[a])

(I-10[a]) and (I-10[b]) are the relationships for the wave numbers: m_0 is the usual wave number and the m_n's satisfy $(n - \frac{1}{2})\pi < m_n h < n\pi$.

The solution for $\phi_{s,kc}(\bar{y},\bar{z})$ which satisfies the Laplacian (I-1'), the boundary conditions (I-2') and (I-4') and the supplementary condition

(I-6'), is composed of a linear combination of the solutions for the respective cases 1°, 2° and 3°:

$$\phi_{s,kc}(\bar{y},\bar{z}) = \{A\cos(m_0\bar{y}) + iB\sin(m_0\bar{y})\}\cosh\{m_0(h-\bar{z})\} + \sum_{n=1}^{\infty} C_n e^{-m_n\bar{y}}\cos\{m_n(h-\bar{z})\} .$$

Because of eq. (I-8) then it can be written:

$$\Phi_{s,kc}(\bar{y},\bar{z},t) = \left[\{A\cos(m_0\bar{y}) + iB\sin(m_0\bar{y})\}\cosh\{m_0(h-\bar{z})\} + \sum_{n=1}^{\infty} C_n e^{-m_n\bar{y}}\cos\{m_n(h-\bar{z})\}\right] T_{s,kc}(t) .$$

Introduction of boundary condition (I-5) into this expression yields:

$$\Phi_{s,kc}(\bar{y},\bar{z},t) = \left[C_0\cosh\{m_0(h-\bar{z})\}e^{-im_0\bar{y}} + \sum_{n=1}^{\infty} C_n e^{-m_n\bar{y}}\cos\{m_n(h-\bar{z})\}\right] T_{s,kc}(t) ,$$

where C_0 = constant of integration.

According to eq. (I-7) $\Phi(\bar{y},\bar{z},t)$ is composed of a linear combination of $\Phi_s(\bar{y},\bar{z},t)$ and $\Phi_{kc}(\bar{y},\bar{z},t)$. By means of eq. (I-8) the velocity potential $\Phi(\bar{y},\bar{z},t)$, satisfying the equation of Laplace (I-1), the boundary conditions (I-2), (I-4) and (I-5) and the supplementary condition (I-6), now becomes:

$$\text{(I-11)} \qquad \Phi(\bar{y},\bar{z},t) = i\frac{\omega}{m_0}(A_0 + B_0 e^{-i\theta})\cosh\{m_0(h-\bar{z})\}e^{i(\omega t - m_0\bar{y} + \frac{m_0 B}{2})} + - \sum_{n=1}^{\infty}\frac{\omega}{m_n}(A_n + B_n e^{-i\theta})e^{-m_n\bar{y} + \frac{m_n B}{2}}\cos\{m_n(h-\bar{z})\}e^{i\omega t} , \qquad (7)$$

where A_0, B_0, A_n, B_n = (so far) unknown constants of integration.

Substitution of $\Phi(\bar{y},\bar{z},t)$ as formulated by eq. (I-11) into boundary condition (I-3) gives the result (supposing that $\omega \neq 0$):

$$\text{(I-12)} \qquad (A_0 + B_0 e^{-i\theta}) \cosh\{m_0(h - \bar{z})\} + \sum_{n=1}^{\infty} (A_n + B_n e^{-i\theta}) \cos\{m_n(h - \bar{z})\} =$$

$$= \hat{a}\{U(\bar{z}) - U(\bar{z} - h + k)\} + \frac{\hat{v}_{kc}}{\omega} e^{-i\theta}\{U(\bar{z} - h + k) - U(\bar{z} - h^+)\} .$$

On account of Weierstrass's theorem the series

$$\sum_{n=1}^{\infty} (A_n + B_n e^{-i\theta})\cos\{m_n(h - \bar{z})\}$$

is supposed to be uniformly convergent on the closed interval $0 \leq \bar{z} \leq h$; as a consequence, this series can be differentiated and integrated term by term. The factors $(A_0 + B_0 e^{-i\theta})$ and $(A_n + B_n e^{-i\theta})$ in eq. (I-12) may be considered as the coefficients of $\cosh\{m_0(h - \bar{z})\}$ and $\cos\{m_n(h - \bar{z})\}$, respectively. Resolution of the right-hand side of eq. (I-12) into terms conformably to the left-hand side of this equation on $0 \leq \bar{z} \leq h$ is, generally, only possible if the row of even functions

$$\text{(I-13)} \qquad \cosh\{m_0(h - \bar{z})\},\ \cos\{m_1(h - \bar{z})\},\ \cos\{m_2(h - \bar{z})\}, \ldots\ldots,$$

$$\cos\{m_n(h - \bar{z})\}, \ldots\ldots$$

is complete on this interval. Concerning to this point the line will be taken that the functions (I-13) form a complete row (basis) on $0 \leq \bar{z} \leq h$. Besides the functions (I-13) are orthogonal an $0 \leq \bar{z} \leq h$. The so far unknown constants of integration A_0, B_0, A_n and B_n in eq. (I-11) then can be determined by applying the principle of orthogonality to eq. (I-12) on $0 \leq \bar{z} \leq h$. To that end eq. (I-12) is multiplied by $\cosh\{m_0(h - \bar{z})\}$ and $\cos\{m_j(h - \bar{z})\}$ $(j = 1,2,\ldots\ldots, n,\ldots\ldots)$, respectively and subsequently integrated over the interval in question, yielding:

$$(A_0 + B_0 e^{-i\theta}) \int_0^h \cosh^2\{m_0(h - \bar{z})\}d\bar{z} +$$

$$+ \sum_{n=1}^{\infty} (A_n + B_n e^{-i\theta}) \int_0^h \cosh\{m_0(h - \bar{z})\} \cos\{m_n(h - \bar{z})\}\, d\bar{z} =$$

$$= \hat{a} \int_0^h \{U(\bar{z}) - U(\bar{z} - h + k)\} \cosh\{m_0(h - \bar{z})\}\, d\bar{z} \; +$$

$$+ \frac{\hat{v}_{kc}}{\omega} e^{-i\theta} \int_0^h \{U(\bar{z} - h + k) - U(\bar{z} - h^+)\} \cosh\{m_0(h - \bar{z})\}\, d\bar{z}$$

and

$$(A_0 + B_0 e^{-i\theta}) \int_0^h \cosh\{m_0(h - \bar{z})\}\cos\{m_j(h - \bar{z})\}\, d\bar{z} \; +$$

$$+ \sum_{n=1}^{\infty} (A_n + B_n e^{-i\theta}) \int_0^h \cos\{m_j(h - \bar{z})\} \cos\{m_n(h - \bar{z})\}\, d\bar{z} =$$

$$= \hat{a} \int_0^h \{U(\bar{z}) - U(\bar{z} - h + k)\} \cos\{m_j(h - \bar{z})\}\, d\bar{z} \; +$$

$$+ \frac{\hat{v}_{kc}}{\omega} e^{-i\theta} \int_0^h \{U(\bar{z} - h + k) - U(\bar{z} - h^+)\} \cos\{m_j(h - \bar{z})\}\, d\bar{z} \; .$$

Using eqs. (I-10^a) and (I-10^b) it can be verified that

$$\int_0^h \cosh\{m_0(h - \bar{z})\}\cos\{m_n(h - \bar{z})\}d\bar{z} = 0 \quad \text{for all } n,$$

and

$$\int_0^h \cos\{m_j(h - \bar{z})\}\cos\{m_n(h - \bar{z})\}\, d\bar{z} \begin{cases} = 0 & \text{if} \quad n \neq j \\ \neq 0 & \text{if} \quad n = j \; . \end{cases}$$

By means of these relationships finally it can be derived for A_0, B_0, A_n and B_n:

(I-14^a) $$\frac{A_0}{\hat{a}} = A_0' = \frac{2\{\sinh(m_0 h) - \sinh(m_0 k)\}}{m_0 h + \sinh(m_0 h)\cosh(m_0 h)} \quad , \qquad (9^a)$$

(I-14^b) $$\frac{A_n}{\hat{a}} = A_n' = \frac{2\{\sin(m_n h) - \sin(m_n k)\}}{m_n h + \sin(m_n h)\cos(m_n h)} \quad , \qquad (9^b)$$

(I-15^a) $$\frac{B_0 \omega}{\hat{v}_{kc}} = B_0' = \frac{2\sinh(m_0 k)}{m_0 h + \sinh(m_0 h)\cosh(m_0 h)} \quad , \qquad (10^a)$$

(I-15^b) $$\frac{B_n \omega}{\hat{v}_{kc}} = B_n' = \frac{2\sin(m_n k)}{m_n h + \sin(m_n h)\cos(m_n h)} \quad . \qquad (10^b)$$

The velocity potential $\Phi(\bar{y},\bar{z},t)$ has been fully defined by now: $\Phi(\bar{y},\bar{z},t)$ as formulated by eq. (I-11) with constants of integration A_0, B_0, A_n and B_n, represented by eqs. (I-14^{a,b}) and (I-15^{a,b}), satisfies Laplace's equation (I-1) plus the set of boundary conditions (I-2), (I-3), (I-4) and (I-5) and the supplementary condition (I-6).

Appendix II: Basic formulae for dynamic tests

II.1: General introduction

In this Appendix II it will be explained in which way the hydrodynamic coefficients for pure swaying and yawing with zero speed of advance can be determined from the measurement of the exciting forces.

If, in the horizontal plane, a harmonically oscillating motion is imposed on the ship model, then the reactive forces of the fluid as well as the exciting forces on the ship model will be harmonic with the same period (supposing at least that merely first harmonic components of the forces are considered). By the linearity assumed the magnitude of the reactive forces of the fluid is directly proportional to the motion amplitudes.

In behalf of a description of the dynamic tests reference is made to fig. II.a; the notation is as explained in Section A3.3. The quantities relating to the fastening points of the struts of the oscillator at fore-part and hind-part of the ship model will be indicated by the subscripts 1 and 2, respectively. The distance from a fastening point to the centre of gravity G is denoted by l.

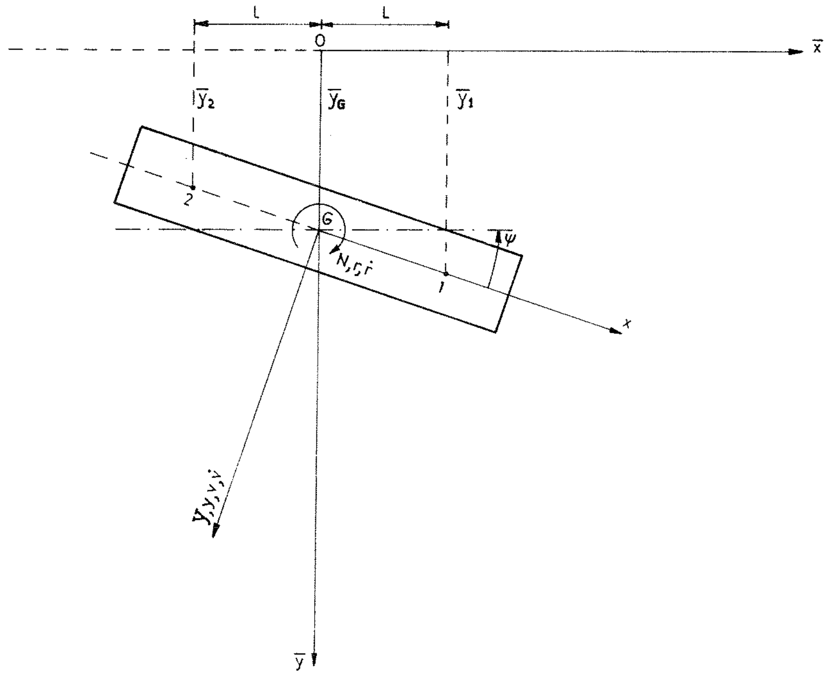

fig. II.a – Definition of symbols in dynamic tests.

The motion of the two struts of the horizontal oscillator may be represented by:

$$\bar{y}_1 = - \hat{a} \sin(\omega t + \delta), \quad \bar{y}_2 = - \hat{a} \sin(\omega t - \delta) ,$$

where $\hat{a}$ = amplitude (excentricity) of the motion of the struts,
ω = circular frequency of the motion (of the struts),
δ = phase difference between the periodic motion of both the fastening points and the motion of G.

As the forward speed of the ship model is zero and the longitudinal surge motion is neglected, the coordinates of G are given by:

$$\bar{x}_G = \text{constant} , \quad \bar{y}_G = (\bar{y}_1 + \bar{y}_2)/2 = - \hat{a} \cos(\delta)\sin(\omega t).$$

If the phase difference δ is supposed to be a time independent quantity, then the translational velocity v and the translational acceleration $\dot{v}$ of G relative to the undisturbed fluid can be written as:

$$v = \dot{\bar{y}}_G = - \hat{a}\, \omega \cos(\delta)\cos(\omega t) , \quad \dot{v} = \ddot{\bar{y}}_G = \hat{a}\, \omega^2 \cos(\delta)\sin(\omega t). \qquad (II\text{-}1^{a,b})$$

The rotation of the ship model with respect to the centre of gravity can be found as follows (see fig. II.b). The geometrical relations during yawing tests are:

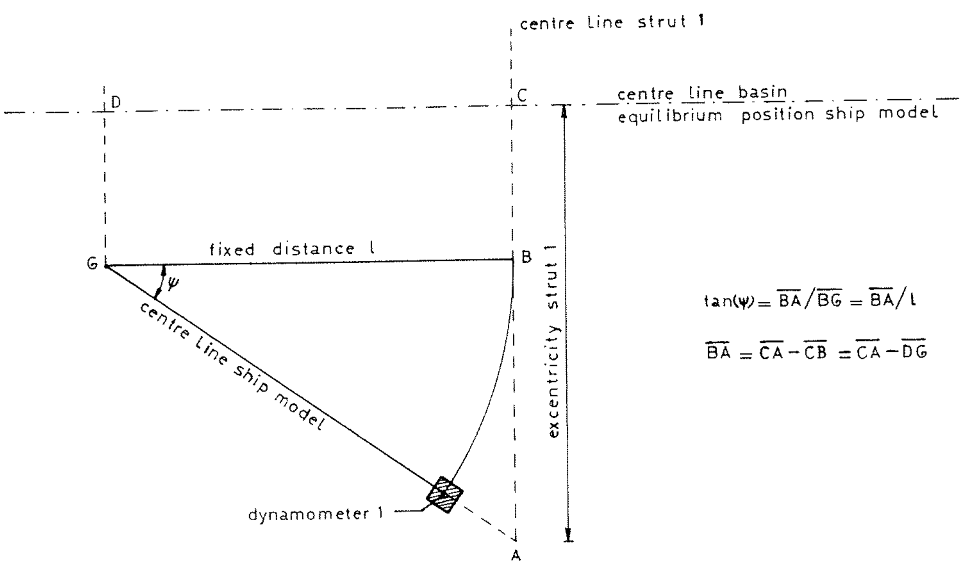

fig. II.b – Geometrical relations during yawing tests.

$$\tan(\psi) = \overline{BA}/\overline{BG} = \overline{BA}/1 \quad \text{and}$$

$$\overline{BA} = \overline{CA} - \overline{CB} = \overline{CA} - \overline{DG} = - \hat{a}\sin(\omega t + \delta) +$$

$$+ \hat{a}\cos(\delta)\sin(\omega t) = - \hat{a}\sin(\delta)\cos(\omega t) \ ;$$

consequently it holds good that:

$$\tan(\psi) = - \frac{\hat{a}}{1}\sin(\delta)\cos(\omega t) \ , \quad \text{or } \psi = \text{atan}\,\{- \frac{\hat{a}}{1}\sin(\delta)\cos(\omega t)\}.$$

Because of the fact that $|\frac{\hat{a}}{1}\sin(\delta)\cos(\omega t)| \leq 1$ ψ can be developed into a series:

$$\psi = - \frac{\hat{a}}{1}\sin(\delta)\cos(\omega t) + \frac{1}{3}\{\frac{\hat{a}}{1}\sin(\delta)\}^3\cos^3(\omega t) +$$

$$- \frac{1}{5}\{\frac{\hat{a}}{1}\sin(\delta)\}^5\cos^5(\omega t) + \frac{1}{7}\{\frac{\hat{a}}{1}\sin(\delta)\}^7\cos^7(\omega t) - \ldots.,$$

where $\cos^3(\omega t) = \frac{1}{4}\{3\cos(\omega t) + \cos(3\omega t)\},$

$$\cos^5(\omega t) = \frac{1}{16}\{10\cos(\omega t) + 5\cos(3\omega t) + \cos(5\omega t)\},$$

$$\cos^7(\omega t) = \frac{1}{64}\{35\cos(\omega t) + 21\cos(3\omega t) + 7\cos(5\omega t) +$$

$$+ \cos(7\omega t)\}.$$

This yields for the first harmonic component of ψ, denoted by $\psi^{(1)}_{harm.}$:

$$\psi^{(1)}_{harm.} = - \frac{\hat{a}}{1}\sin(\delta)\ [1 - \frac{1}{4}\{\frac{\hat{a}}{1}\sin(\delta)\}^2 + \frac{1}{8}\{\frac{\hat{a}}{1}\sin(\delta)\}^4 +$$

$$- \frac{5}{64}\{\frac{\hat{a}}{1}\sin(\delta)\}^6 + \ldots\ldots\ ]\cos(\omega t) =$$

$$= \psi^{(1)}_{0,harm.}\cos(\omega t)\ .$$

As merely first harmonic components are considered, $\psi^{(1)}_{harm.}$ will be indicated further by ψ:

(II-2^a) $$\psi = \psi_0 \cos(\omega t) \ ,$$

where $$\psi_0 = -\frac{\hat{a}}{l}\sin(\delta)\ [1 - \frac{1}{4}\{\frac{\hat{a}}{l}\sin(\delta)\}^2 + \frac{1}{8}\{\frac{\hat{a}}{l}\sin(\delta)\}^4 + - \frac{5}{64}\{\frac{\hat{a}}{l}\sin(\delta)\}^6 + \ldots\ldots] \ .$$

For the yaw angular velocity r and the yaw angular acceleration $\dot{r}$ then one obtains:

(II-3^{a,b}) $$r = \dot{\psi} = -\psi_0\ \omega \sin(\omega t) \ , \quad \dot{r} = \ddot{\psi} = -\psi_0\ \omega^2 \cos(\omega t) \ .$$

II.2: Pure swaying tests with zero speed of advance

Swaying is characterized by $\delta = 0$; this means that, on account of the eqs. (II-2^a) and (II-3^{a,b}):

$$\psi = 0 \ , \quad \dot{\psi} = r = 0 \ , \quad \ddot{\psi} = \dot{r} = 0 \ .$$

Eq. (II-1^{a,b}) then becomes:

(II-1^{c,d}) $$v = -\hat{a}\ \omega \cos(\omega t) \ , \quad \dot{v} = \hat{a}\ \omega^2 \sin(\omega t) \ .$$

The measured lateral forces are split up electronically into the following components by means of a Fourier analysis:

$$Y_1 = Y_{1,const.} + Y_{1,si} \sin(\omega t) + Y_{1,co} \cos(\omega t) \ ,$$

$$Y_2 = Y_{2,const.} + Y_{2,si} \sin(\omega t) + Y_{2,co} \cos(\omega t) \ ,$$

where the added subscripts const., si and co indicate a constant force component, a sinus force component and a cosinus force component, respectively. The resulting exciting force, acting in the centre of gravity, is a force oscillating harmonically:

(II-4) $$Y_{osc.} = Y_1 + Y_2 = (Y_{1,const.} + Y_{2,const.}) + $$
$$+ (Y_{1,si} + Y_{2,si}) \sin(\omega t) + (Y_{1,co} + Y_{2,co}) \cos(\omega t) .$$

The resulting exciting moment about the centre of gravity then is:

(II-5) $$N_{osc.} = l(Y_1 - Y_2) = l(Y_{1,const.} - Y_{2,const.}) +$$
$$+ l(Y_{1,si} - Y_{2,si}) \sin(\omega t) + l(Y_{1,co} - Y_{2,co}) \cos(\omega t) .$$

According to Section A3.3 the 'equations of motion' in case of pure yawing are:

(II-6) $$\left.\begin{aligned} (m + a_{yy})\,\ddot{y} + b_{yy}\,\dot{y} &= Y_{osc.} \\ a_{\psi y}\,\ddot{y} + b_{\psi y}\,\dot{y} &= N_{osc.} \end{aligned}\right\} \text{ pure swaying.} \qquad (17)$$

Substitution of eqs. (II-1^{c,d}), (II-4) and (II-5) into eq. (II-6) - bearing in mind thereby that in eq. (II-6) $\dot{y} = v$ and $\ddot{y} = \dot{v}$ - and successive equalization of the coefficients of the corresponding terms with $\sin(\omega t)$ and $\cos(\omega t)$, respectively, then yields:

(II-7) $$Y_{1,const.} = Y_{2,const.} = 0 ,$$

(II-8) $$m + a_{yy} = \frac{Y_{1,si} + Y_{2,si}}{\hat{a}\,\omega^2} ,$$

(II-9) $$b_{yy} = \frac{Y_{1,co} + Y_{2,co}}{-\hat{a}\,\omega} ,$$

(II-10) $$a_{\psi y} = \frac{l(Y_{1,si} - Y_{2,si})}{\hat{a}\ \omega^2} \quad ,$$

(II-11) $$b_{\psi y} = \frac{l(Y_{1,co} - Y_{2,co})}{-\ \hat{a}\ \omega} \quad .$$

II.3: Pure yawing tests with zero speed of advance

Yawing is characterized by $\delta = \frac{\pi}{2}$; this means that, on account of eq. (II-1^{a,b}):

$$v = 0 \ , \quad \dot{v} = 0 \quad .$$

Eq. (II-2^{a}) then can be written as:

(II-2^{b}) $$\psi = \psi_0 \cos(\omega t) \ , \quad \text{where} \quad \psi_0 = -\ \frac{\hat{a}}{l}\ \{1 - \frac{1}{4}\ (\frac{\hat{a}}{l})^2 + \frac{1}{8}\ (\frac{\hat{a}}{l})^4 - \frac{5}{64}\ (\frac{\hat{a}}{l})^6 + \ldots\ldots\} \quad .$$

According to Section A3.3 the 'equations of motion' in case of pure yawing are:

(II-12) $$\left.\begin{aligned} a_{y\psi}\ \ddot{\psi} + b_{y\psi}\ \dot{\psi} &= Y_{osc.} \\ (I_{zz} + a_{\psi\psi})\ddot{\psi} + b_{\psi\psi}\ \dot{\psi} &= N_{osc.} \end{aligned}\right\} \text{ pure yawing .} \qquad (18)$$

Substitution of eqs. (II-3^{a,b}), (II-4) and (II-5) into eq. (II-12) and successive equalization of the coefficients of the corresponding terms with sin(ωt) and cos(ωt), respectively, then yields:

(II-7) $$Y_{1,const.} = Y_{2,const.} = 0 \quad ,$$

(II-13) $$a_{y\psi} = \frac{Y_{1,co} + Y_{2,co}}{-\ \psi_0\ \omega^2} \quad ,$$

(II-14) $$b_{y\psi} = \frac{Y_{1,si} + Y_{2,si}}{-\psi_0\ \omega} \quad ,$$

(II-15) $$I_{zz} + a_{\psi\psi} = \frac{l(Y_{1,co} - Y_{2,co})}{-\psi_0\ \omega^2} \quad ,$$

(II-16) $$b_{\psi\psi} = \frac{l(Y_{1,si} - Y_{2,si})}{-\psi_0\ \omega} \quad .$$

Only small amplitudes of motion will be considered, i.e. - in this linearized case - amplitudes for which it holds good that $(\frac{\hat{a}}{l})^n << 1$ if $n \geq 2$; eq. (II-2^b) then changes into:

(II-2^c) $$\psi = \psi_0 \cos(\omega t), \qquad \text{where} \qquad \psi_0 = -\frac{\hat{a}}{l} \quad .$$

By the results derived above it can be seen that the mass forces (moments) are in phase and the damping forces (moments) are out of phase with the sway (yaw) motion.

Appendix III: Rough estimation of hydrodynamic coefficients for low frequencies

In this Appendix III a highly approximative method will be provided for estimating the hydrodynamic coefficients in case of (pure) swaying on shallow water.

This method - originally developped by P.A. Kolkman - starts from the principles of the long wave theory; consequently its validity is restricted to very low circular frequencies. Just as in part A of this report the determination of the hydrodynamic coefficients will occur in a simple stripwise manner. Therefore, besides the restrictions being inherent to the application of the long wave theory one has also to do in this method with the drawbacks associated with the strip theory (see Section C2.1); likewise no account is taken of the so-called end effects (see Section C2.3).

The hydrodynamic coefficients will be derived for the same ship (model) as in the report and, as much as possible, the same notation will be used. For a definition sketch see fig. III.a.

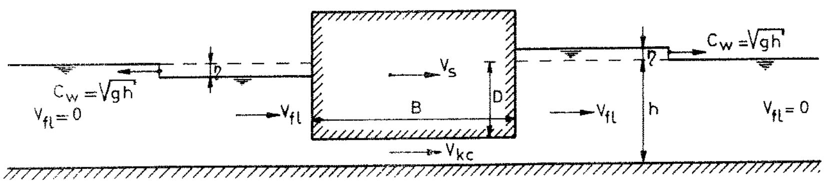

fig. III.a – Definition sketch.

The harmonic oscillations of the ship take place in exactly the same way and under similar conditions as outlined in Sections A2 and A3.1. The horizontal transverse translatory velocity of the ship is indicated by v_s. Let the height (or 'amplitude') of the generated long wave be η (see fig. III.a); η is supposed to be very small with respect to the water depth h: $\eta \ll h$. With neglect of friction effects, assuming that the wave propagates without distortion, the velocity of propagation can be represented by $c_w = \sqrt{gh}$. In conformity with the long wave theory the (horizontal) fluid velocities under the long wave (i.e. v_{fl}) are supposed to be uniformly distributed in the vertical plane; this also holds good at a (very) short distance from the ship's wall. The velocities in the undisturbed fluid region are equal to zero. Further it is supposed that the horizontal velocities in the keel clearance are distributed uniformly and can be represented by:

(III-1) $$v_{kc} = \hat{v}_{kc} \sin(\omega t) \quad .$$

Applying the law of conservation of momentum to the mass of water underneath the ship one obtains (per unit length):

$$\rho B(h - D) \frac{dv_{kc}}{dt} = - 2\rho g\eta(h - D) - \beta v_{kc} \quad , \tag{III-2}$$

where βv_{kc} represents a linearized friction force (β = proportionality constant > 0).

From the long wave theory it can be derived that:

$$v_{fl}h - \eta\sqrt{gh} = 0 \quad , \quad \text{or} \quad \eta = \frac{v_{fl}h}{\sqrt{gh}} \quad . \tag{III-3}$$

The equation of continuity reads as follows:

$$v_s D + v_{kc} (h - D) = v_{fl}h \ . \tag{III-4}$$

Elimination of η from eqs. (III-2) and (III-3) and substitution of v_{kc} yields for v_{fl}:

$$v_{fl} = - \frac{\omega B}{2\sqrt{gh}} \hat{v}_{kc} \cos(\omega t) - \frac{\beta}{2\rho(h - D)\sqrt{gh}} \hat{v}_{kc} \sin(\omega t) \ . \tag{III-5}$$

By substitution of the expressions for v_{fl} and v_{kc} into eq. (III-4) v_s becomes:

$$v_s = \hat{v}_{kc} \{C_1 \cos(\omega t) + C_2 \sin(\omega t)\} = \hat{v}_{kc} \sqrt{C_1^2 + C_2^2} \sin(\omega t + \zeta) \tag{III-6}$$

$$\text{with } \tan(\zeta) = \frac{C_1}{C_2} \quad ,$$

where

$$C_1 = - \frac{\omega B\sqrt{gh}}{2gD} \quad , \qquad C_2 = - \frac{h - D}{D} - \frac{\beta\sqrt{gh}}{2\rho g D(h - D)} \quad ; \tag{III-7^a}$$

the acceleration of the ship is:

$$\frac{dv_s}{dt} = \omega\hat{v}_{kc} \quad \{- C_1 \sin(\omega t) + C_2 \cos(\omega t)\} .$$

The (horizontal) hydrodynamic force acting on the ship per unit length is:

$$F_s = 2\rho g\eta D ;$$

Elimination of v_{fl} from eq. (III-3) by means of eq. (III-5) yields for η:

$$\eta = - \frac{\omega B}{2g} \hat{v}_{kc} \cos(\omega t) - \frac{\beta}{2\rho g(h - D)} \hat{v}_{kc} \sin(\omega t) .$$

Substituting η into the expression for the hydrodynamic force on the ship it can be derived for F_s:

(III-8[a]) $$F_s = \hat{v}_{kc} \{C_3 \cos(\omega t) + C_4 \sin(\omega t)\} ,$$

where

(III-9[a]) $$C_3 = - \rho\omega BD , \quad C_4 = - \frac{\beta D}{h - D} .$$

Eq. (III-8[a]) can be reduced to:

$$F_s = \omega\hat{v}_{kc} \frac{C_2 C_3 - C_1 C_4}{\omega(C_1^2 + C_2^2)} \{C_2 \cos(\omega t) - C_1 \sin(\omega t)\} +$$

$$+ \hat{v}_{kc} \frac{C_1 C_3 + C_2 C_4}{C_1^2 + C_2^2} \{C_1 \cos(\omega t) + C_2 \sin(\omega t)\} ,$$

which formula by means of the expression for $\frac{dv_s}{dt}$ and eq. (III-6) can be written as:

(III-8[b]) $$F_s = \frac{C_2 C_3 - C_1 C_4}{\omega(C_1^2 + C_2^2)} \frac{dv_s}{dt} + \frac{C_1 C_3 + C_2 C_4}{C_1^2 + C_2^2} v_s .$$

The sway added mass a'_{yy} and the sway damping force coefficient b'_{yy} (per unit length) can be determined from those respective parts of the hydrodynamic force which are directly proportional to the acceleration $\frac{dv_s}{dt}$ and the velocity v_s of the ship, therefore:

(III-10) $$a'_{yy} = \frac{C_2\,C_3 - C_1\,C_4}{\omega(C_1^2 + C_2^2)} ,$$

and

(III-11) $$b'_{yy} = \frac{C_1\,C_3 + C_2\,C_4}{C_1^2 + C_2^2} ,$$

where the prime used as superscript means 'per unit length'. For a prismatic body with length L it then holds good that:

(III-12) $$a_{yy} = L\,\frac{C_2\,C_3 - C_1\,C_4}{\omega(C_1^2 + C_2^2)} ,$$

(III-13) $$b_{yy} = L\,\frac{C_1\,C_3 + C_2\,C_4}{C_1^2 + C_2^2} .$$

From eq. (III-6) it follows that:

(III-14) $$\frac{\hat{v}_{kc}}{\omega\hat{a}} = \frac{1}{\sqrt{C_1^2 + C_2^2}} ,$$

where $\hat{a}$ = amplitude of (harmonically oscillating) ship motion. The phase difference ζ (i.e. identical with θ) between the fluid motion in the keel clearance underneath the ship and the ship motion becomes:

(III-15) $$\zeta = \text{atan}\,(C_1, C_2) .$$

If the friction in the keel clearance is neglected - which amounts to the supposition $\beta = 0$ - the respective coefficients C_1, C_2, C_3 and C_4 reduce to:

(III-7[b]) $$C_1 = -\frac{\omega B\sqrt{gh}}{2gD} , \qquad C_2 = -\frac{h - D}{D} ,$$

and

(III-9[b]) $$C_3 = -\rho\omega BD , \qquad C_4 = 0 .$$

Using these expressions for C_1, C_2, C_3 and C_4 the respective eqs. (III-12), (III-13), (III-14) and (III-15) can be written as:

(III-16) $$a_{yy} = L\frac{4\rho gBD^2(h - D)}{\omega^2B^2h + 4g(h - D)^2} ,$$

(III-17) $$b_{yy} = L\frac{2\rho\omega^2B^2D^2\sqrt{gh}}{\omega^2B^2h + 4g(h - D)^2} ,$$

(III-18) $$\frac{\hat{v}_{kc}}{\omega\hat{a}} = \frac{2D\sqrt{g}}{\sqrt{\omega^2B^2h + 4g(h - D)^2}}$$

and

(III-19) $$\zeta = \text{atan}\left\{\frac{-\omega B\sqrt{gh}}{-2g(h - D)}\right\} .$$

Taking the transition $\omega \to 0$ to the limit - with the restriction $(h - D) \neq 0$ - it is obtained from the respective eqs. (III-16), (III-17), (III-18) and (III-19):

(III-20) $$\lim_{\omega\to0}(a_{yy}) = \rho LB\frac{D^2}{h - D} ,$$

(III-21) $$\lim_{\omega\to0}(b_{yy}) = 0 ,$$

(III-22) $$\lim_{\omega\to0}\left(\frac{\hat{v}_{kc}}{\omega\hat{a}}\right) = \frac{D}{h - D}$$

and

(III-23) $$\lim_{\omega \to 0} (\zeta) = -\pi \ .$$

Taking the transition h → D to the limit the eqs. (III-16), (III-17), (III-18) and (III-19) change into:

(III-24) $$\lim_{h \to D} (a_{yy}) = 0 \ ,$$

(III-25) $$\lim_{h \to D} (b_{yy}) = 2\rho L h \sqrt{gh} \ ,$$

(III-26) $$\lim_{h \to D} \left(\frac{\hat{v}_{kc}}{\omega \hat{a}}\right) = \frac{2\sqrt{gh}}{\omega B}$$

and

(III-27) $$\lim_{h \to D} (\zeta) = -\frac{\pi}{2} \ ,$$

respectively. It has to be noted that the eqs. (III-24) and (III-25) are independent of the circular frequency ω.

In the above, expressions are derived for estimating the values of the hydrodynamic coefficients in case of (pure) swaying on shallow water. In order to compare the results of this highly approximative method with the theoretical and experimental results as presented in the respective parts A and B of this report, one has to start from the situation with zero friction in the keel clearance, i.e. a_{yy} and b_{yy} have to be calculated on account of eqs. (III-16) and (III-17), respectively. The values calculated in this way are plotted as dotted lines in the figs. 11 through 14. It can be stated that, generally, the agreement between the theoretical and experimental results as presented in Section C3.1 and the results based on the long wave approximation is quite reasonable, at least as far as the considered frequency range is concerned. For the influence of the effects of strip theory and neglect of viscosity as well as for the circulation effect on the calculational results is referred to Section C2.

Wanting to gain an insight into the influence of the friction effects in the keel clearance on a_{yy} and b_{yy}, one has to start from the respective eqs. (III-12) and (III-13) together with the expressions for C_1, C_2, C_3 and C_4 as presented in the eqs. (III-7^a) and (III-9^a). Substitution of C_1, C_2, C_3 and C_4 into the expressions for a_{yy} and b_{yy} yields:

(III-12') $$a_{yy} = L\frac{4\rho gBD^2(h - D)}{\{\omega^2B^2h + 4g(h - D)^2\} + W_1}$$

and

(III-13') $$b_{yy} = L\frac{2\rho\omega^2B^2D^2\sqrt{gh}\{1 + \frac{1}{\omega^2B^2h}(W_1 - W_2)\}}{\{\omega^2B^2h + 4g(h - D)^2\} + W_1},$$

where $$W_1 = \frac{4\beta\rho(h - D)^2\sqrt{gh} + \beta^2h}{\rho^2(h - D)^2} \quad \text{and} \quad W_2 = \frac{2\beta\sqrt{gh}}{\rho};$$

W_1 and W_2 indicate the influence of the friction effects in the keel clearance. The proportionality constant β in the linearized friction force, as introduced in eq. (III-2), is positive. On account of the eqs. (III-12') and (III-13') it will be obvious then that an increase of β will decrease a_{yy} and will increase b_{yy}, at least as far as the frequency range considered in the experiments is concerned. From calculations it appears that the influence of the friction effects in the keel clearance in case of a_{yy} is smaller than in case of b_{yy}.
It might be expected that the influence of the friction effects in the keel clearance finds expression in the experimental results, notably at larger amplitude of ship motion $\hat{a}$: for, the linearized friction force is directly proportional to the amplitude of motion. Indeed, for amplitudes of motion up to $\hat{a} = 0.05$ m the tendency is found that a_{yy} slightly decreases and b_{yy} increases when $\hat{a}$ increases; naturally this holds good only if $\omega > 0.4$ rad.s^{-1}. One thing and another confirms the supposition, made in Section B3.2.1, relating to the presence of friction effects in the fluid underneath the ship.

As a matter of fact, there can only be question of a pure long wave approximation if $\omega = 0$.
For very low circular frequencies a good agreement should be expected

between the theoretical results as derived in part A of this report and the results of the long wave approximation. However, the contrary is the case. As cause might be mentioned the supposition - inherent to the long wave approximation - that already in vertical planes at extremely short distances from the ship's walls the horizontal fluid velocities are uniformly distributed along the height, what is generally not the case in the theoretical approach from part A. In this latter case there will be only question of a uniform velocity distribution in the vertical planes very close to the ship's walls, if the keel clearance tends to zero. Therefore, the relevant results of the theory as presented in part A and the long wave approximation are only allowed to be compared if both ω tends to zero (i.e. pure long wave approximation) and h tends to D (i.e. zero keel clearance). Indeed it can be ascertained that the respective eqs. (29^a) and (III-24) as well as the respective eqs. (30^a) and (III-25) do agree. Likewise, the above explains why there exists in case of $\frac{h}{D} = 1.167$ a better agreement between the theoretical results from part A and the results of the long wave approximation than in case of $\frac{h}{D} = 1.333$.