

A DOMAIN DECOMPOSITION APPROACH TO COMPUTE WAVE REFRACTION-DIFFRACTION (ECCOMAS CFD 2006)

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Abstract. *A methodology to simulate the unsteady wave transformation goes to the coast is presented. This methodology is based on a heterogeneous physical domain decomposition approach, in which, an inviscid-viscid interaction is investigated. Numerical results relative to wave refraction-diffraction problems are shown using two coupling strategies between the potential flow and the viscous flow.*

1 INTRODUCTION

A heterogeneous physical domain decomposition approach is followed to simulate the unsteady wave transformation goes to the shoreline. The motivation for this work stems from the difficulties in modelling the non-linear phenomena that appear at the wave transformation goes to the coast by elliptic models kind extended Berkhoff equations.

An inviscid-viscous interaction is investigated, by means of using a fluid viscous flow model in the free surface region to capture the atmosphere-ocean interface, and a potential flow approximation to describe the flow far from the interface.

Much of the literature dedicated to the subject is of the theoretical or experimental nature, the numerical investigation requiring too expensive techniques and highly refined grids to manage the complicated interface topologies, only in recent years, the discrete treatment of interfacial flows have received a considerable progress, and there are efficient numerical schemes that can be coupled with Navier-Stokes solvers¹

A strategy to incorporate the steep bathymetry is also presented in the case of potential flow. Two coupling strategies are investigated, differing in the transmission conditions. Both the adopted approaches make use of the velocity field due to the potential flow equation as boundary condition in the Navier-Stokes solution.

Numerical results with different non-breaking flows are simulated. The performance of the applied techniques of domain decomposition are compared with fully inviscid results,

relative to the extended Berkhoff equations, which are solved in the whole fluid domain, and with available experimental data. Finally, some of the physical phenomena observed experimentally in harbor resonance are reproduced.

2 GOVERNING EQUATIONS

The unsteady Navier-Stokes solver, coupled with a level set method² is used to describe dynamics at the interface and in the region near to the free surface (Ω^f).

$$\nabla \cdot \mathbf{u} = 0 \quad (1)$$

$$\frac{D\mathbf{u}}{Dt} = -\frac{1}{\rho}\nabla p + \mathbf{f} + \frac{1}{\rho}\nabla \cdot [\mu\nabla(\mathbf{u} + \mathbf{u}^T)] + \sigma\kappa\nu\delta(\mathbf{x} - \mathbf{x}_s) \quad (2)$$

$$\mathbf{u} = \vec{g} \quad \text{on external boundaries} \quad (3)$$

in which the two velocity components are prescribed at the inflow and outflow sections, like as in the upper part of domain. In the interior region (Ω^b) the flow is assumed to be inviscid and irrotational and a potential model which in terms of a velocity potential $\phi = \phi(x, y, z, t)$ satisfies the following equations:

$$\Delta\phi = 0 \quad (4)$$

$$\frac{\partial\phi}{\partial\mathbf{n}} = 0 \quad \text{on } \Gamma_b \quad (5)$$

$$\frac{\partial\phi}{\partial\mathbf{n}} = \mathcal{L}(\phi) \quad \text{on inflow - outflow boundary} \quad (6)$$

Here $\mathbf{u}$ is the fluid velocity, ρ and μ are the local values of density and dynamic viscosity, p is the pressure, $\mathbf{f}$ denotes the mass forces, σ is the surface tension coefficient, κ is the local curvature of the interface and ν is the unit normal vector at the interface oriented toward the air. In (2) the term $\delta(\mathbf{x} - \mathbf{x}_s)$ represents the Dirac function which is zero out the interface location $\mathbf{x}_s$. In (4) Γ_b is the rigid boundary at the bottom region, on which the equation (4) can be written as

$$\frac{\partial\phi}{\partial z} = -\frac{\partial h}{\partial t} - \frac{\partial h}{\partial x}\frac{\partial\phi}{\partial x} - \frac{\partial h}{\partial y}\frac{\partial\phi}{\partial y} \quad \text{on } z = -h(x, y) \quad (7)$$

3 VISCOUS-INVISCID COUPLING

Diferent strategies are adopted for the coupling between two models at the matching surface, among them the Neumann type coupling (NT) and the Dirichlet type coupling (DT). The Neumann type coupling is based on a Schwarz alternating method^{1,3}, while the Dirichlet type coupling is based on the solution of the dynamic problem corresponding to the potential flow.

3.1 The Neumann type coupling

On this kind of approach the two subdomains are overlapped, and the normal velocity component, obtained from Navier-Stokes solution is used as Neumann boundary condition in the potential flow problem (on Γ^B). Analogously, the solution of the potential problem gives back the velocity field which is used as boundary condition of the Navier-Stokes problem (on Γ^F). Then can be expressed formally:

$$\frac{\partial \phi}{\partial \mathbf{n}} = \mathbf{u}^F \cdot \mathbf{n} \quad \text{on } \Gamma^B \quad (8)$$

$$(u_1, u_2) = (u_1^B, u_2^B) \quad \text{on } \Gamma^F \quad (9)$$

where Γ^B is the upper part of the potential flow region, and Γ^F is the lower part of the Navier-Stokes region.

The Algorithm: Given a tolerance tol prescribed

Do $k = 1, \dots, N_{steps}$

Given $\mathbf{u}^B(t)$ solve the Navier-Stokes problem (1-3,9) at the time step $t + \Delta t$

Given $\mathbf{u}^F(t + \Delta t)$ solve the potential problem (4-6,8)

If $\| \mathbf{u}^B_k - \mathbf{u}^B_{(k-1)} \| < tol$, exit

enddo

3.2 The Dirichlet type coupling

This coupling procedure uses the normal stresses at the marching surface as a forcing pressure field. The boundary conditions on the marching surface $\Gamma^B = \Gamma^F$ reads then

$$(u_1, u_2) = (u_1^B, u_2^B) \quad \text{on } \Gamma^F \quad (10)$$

$$\frac{\partial \phi}{\partial t} = -\frac{p^F}{\rho} + \frac{2\mu}{\rho} \frac{\partial u_n}{\partial \mathbf{n}} - gz - \frac{|\mathbf{u}^F|^2}{2} \quad \text{on } \Gamma^B \quad (11)$$

for the upper and lower subdomains, respectively, g is the gravity acceleration.

The Algorithm: Given a tolerance tol prescribed

Do $k = 1, \dots, N_{steps}$

Given $\mathbf{u}^B(t)$ solve the Navier-Stokes problem (1-3,10) at the time step $t + \Delta t$

Given $\mathbf{u}^F(t + \Delta t)$ solve the potential problem (4-6) with the Dirichlet boundary condition along Γ^B derived by the solution of Eq. 11

If $\| \mathbf{u}^B_k - \mathbf{u}^B_{(k-1)} \| < tol$, exit

enddo

3.3 The Navier-Stokes/Euler Coupling

Another strategy examined is the Navier-Stokes/Euler Coupling^{4,5}. This coupling procedure is based on the zonal recognition technique proposed by Brezzi, Canuto and Russo

and named the Ψ -method⁴. The Ψ -method can be considered as a truncation technique that reduces the Navier-Stokes system to the Euler in the regions where the viscous term is small. This is done by replacing the viscous term $\Delta^2 u_1(\Delta^2 u_2)$ in the Navier-Stokes equation by a function $\Psi(\Delta^2 u_1)(\Psi(\Delta^2 u_2))$. This function coincides with $\Delta^2 u_1(\Delta^2 u_2)$ when the viscous term is large and equal to zero when its value is small, thus becoming the Euler equation. An adaptive zonal recognition procedure is implemented. The internal interface between the two regions uses Dirichlet/Neumann boundary conditions.

4 APPLICATIONS

The methodology presented here has been applied to describe several problems of water waves transformation, among them

- Water Waves Propagation over a horizontal bed
- Water Waves Propagation over an inclined bed
- Water Waves Propagation over an elliptical shoal

The numerical tests have been carried out employing the Finite Element Method with linear triangular elements.

5 CONCLUSIONS

- Viscous /inviscid interaction can be described by the methodology presented here.
- The internal interfaces between two regions uses Dirichlet/Neumann boundary conditions.
- For greater computational speed the viscous/inviscid coupling may also be thought of as two codes working within a single framework.
- Numerical results are promising although the coupling between viscous and inviscid regions still requires some adjustments.
- The using of enrichment method near to transition region can improve the precision of this methodology in cases of 3D flow domain.

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