MOTION OF QUANTUM VORTEX LINES NEAR REALISTIC ROUGH BOUNDARIES

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Abstract We numerically solve the three–dimensional, time–dependent Gross–Pitaevskii equation to model a superfluid flowing over a realistic rough boundary. Our model for the boundary is based on the atom–force microscope image of a NbTi vibrating wire used to generate turbulence in actual experiments. We find that near the boundary a dense region of quantum vortices is created, which forms a kind of 'superfluid boundary layer'.

INTRODUCTION

This work is concerned with turbulence in superfluid liquid helium ^{4}He and ^{3}He-B. From the point of view of the hydrodynamics, these quantum fluids are characterised by two properties: absence of viscosity (hence the name 'superfluid') and quantized vorticity. The latter, a consequence of the existence and the uniqueness of a complex quantum wavefunction Ψ, implies that vorticity exists only in the form of vortex lines of fixed circulation h/m where h is Planck's constant and m the mass of the relevant boson. Recent studies of homogeneous isotropic turbulence away from boundaries have revealed similarities between superfluid turbulence and ordinary turbulence (e.g. the same Kolmogorov energy spectrum [1]) as well as differences (e.g. power–law rather than Gaussian velocity statistics [2]).

Less attention has been paid to superfluid turbulence near boundaries, but new visualisation techniques [3] motivate the study of superfluid velocity profiles in pipes and channels, raising the theoretical issue of what should be the correct boundary conditions. The superfluid boundary condition is simple, at least in principle. At the boundary, the velocity component which is perpendicular to the boundary must be zero; the tangential components are arbitrary (being inviscid, the superfluid can slip along the boundary).

On second thoughts, the following difficulty arises. The walls of the typical experimental cell which contains liquid helium are made of metal, and are unlikely to be smooth at the very small length scale of the vortex core radius a_0 (approximately equal to 10^{-10} m in ^{4}He and 10^{-8} m in ^{3}He-B). Numerical simulations of the motion of thin vortex filaments based on the Biot–Savart law [4] have showed that a vortex line which slides along a flat plane may become trapped onto a small hemispherical bump located on that plane. This result suggests that in the experiments vortex lines may become pinned at the roughness of the walls. Moreover, the potential flow around a particularly sharp corner of a rough boundary may exceed the critical velocity for generation of new vortex lines [5], which may become pinned too. Thus, no matter whether vortex lines are pre–existent or generated at the boundary itself, the region near the boundary may be very different from the bulk of the flow. The aim of this work is to shed light into this possibility using a model based on actual experiments.

METHOD

Our model is the Gross–Pitaevskii equation for the complex wavefunction $\Psi(\mathbf{r}, t)$:

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + g|\Psi|^2\Psi - \mu\Psi + V\Psi, \tag{1}$$

where $\mathbf{r}$ is space, t time, $\hbar = h/(2\pi)$, g the interaction parameter, and μ the chemical potential. The confining potential $V(x, y)$ is a step function which is zero within the fluid and takes a prescribed value V_0 outside it. Using the Madelung transformation, it can be shown that equation (1) implies the continuity equation, and, at length scales larger than the healing length $\xi = \hbar/\sqrt{2m\mu}$, the (compressible) Euler equation. Quantum vortex lines are solutions of equation (1) such that, firstly, $\Psi \to 0$ on the axis of the vortex, and, secondly, the phase of Ψ changes by 2π going around the axis. The radius a_0 of the vortex core region (where the density $|\Psi|^2$ changes from zero on the axis to its bulk value at infinity) is of the order of the healing length ξ.

We solve equation (1) for a prescribed velocity v along the x direction. The potential $V(x, y)$, see figure (1), is the height of the surface of a niobium–titanium wire, as measured by an atom–force microscope; the wire was used by Richard Haley and Chris Lawson to generate turbulence at Lancaster University. It is apparent from figure (1) that this typical surface is rough, consisting of 'mountains' and 'valleys', probably created by the etching process while making the wire. We perform numerical simulations at different values of v, changing the horizontal and vertical scale of the rough boundary represented by the observed surface roughness.

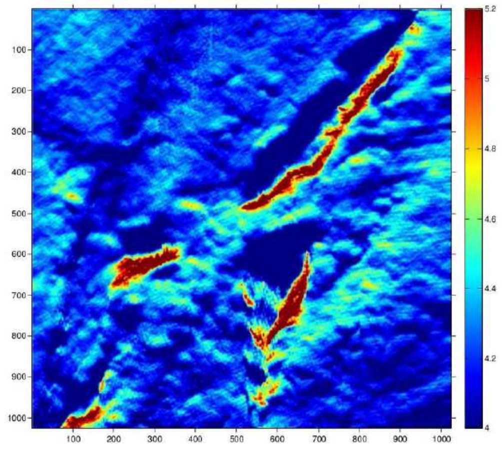

Figure 1. Atom-force microscope image of a small region of the surface of a vibrating NbTi wire used by Rich Haley and Chris Lawson (Lancaster University) to generate superfluid turbulence, as measured by atom–force microscope. The region extends $1\ \mu\mathrm{m}$ in both x and y directions; the maximum height of the surface is $16.1\ \mathrm{nm}$ in the z direction.

RESULTS

At the beginning of the numerical simulation there are no vortices. Since the potential flow around the highest 'mountain' exceeds the critical velocity, vortex lines are nucleated. The vortices interact with each other and with their images across the boundary. Sometimes vortices slide along the boundary, sometimes they become pinned. Occasionally, vortices break away in the form of vortex rings. We find that after the initial transient, a kind of 'superfluid boundary layer' forms, where the length of vortex line saturates, see figure (2). The height of this layer corresponds to the highest 'mountain'. Work is in progress to determine the vortex line density in the boundary layer as a function of the parameters of the problem.

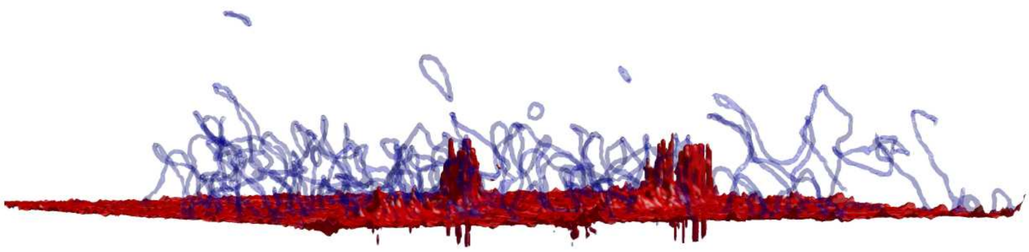

Figure 2. Snapshot of the region of quantum vortex lines which forms near the rough boundary for $V_0 = 20\mu$. The flow is from right to left. What is plotted are isosurfaces of the density $|\Psi|^2$. The blue tubes correspond to vortex lines, the red surface to the rough boundary.

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