Design of a compact gravity equilibrator with an unlimited range of motion

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DESIGN OF A COMPACT GRAVITY EQUILIBRATOR WITH AN UNLIMITED RANGE OF MOTION

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ABSTRACT

A gravity equilibrator is a statically balanced system which is designed to counterbalance a mass such that any preferred position is eliminated and thereby the required operating effort to move the mass is greatly reduced. Current spring-to-mass gravity equilibrators are limited in their range of motion as a result of design limitations. An increment of the range of motion is desired to expand the field of applications. The goal of this paper is to develop a compact one degree of freedom mechanical gravity equilibrator that can statically balance a rotating pendulum over an unlimited range of motion. Static balance over an unlimited range of motion is achieved by a coaxial gear train that uses non circular gears. These gears convert the continuous rotation of the pendulum into a reciprocating rotation of the torsion bars. The shape of these gears are specifically designed to balance a rotating pendulum. Our gear train design and the method to calculate the parameters and the shape of the non circular gears are presented. A prototype is designed and built to validate that the presented method can balance a pendulum over an unlimited range of motion. Experimental results show that a work reduction of 86.8 % is achieved compared to an unbalanced pendulum and the hysteresis in the mechanism is 36.3 %.

Keywords: unlimited range of motion, non circular gears, static balancing, torsion bars

1 INTRODUCTION

A gravity equilibrator is designed to counterbalance a mass [1]. The advantage of such a system is that the operating effort to move the mass is greatly reduced [2]. This property is based on the principle of constant potential energy. Normally, a system tends to converge to a state where the potential energy is at a relative minimum. When a system exhibits constant potential energy, there is no such state. The result is that any preferred position is eliminated. In order to achieve a constant potential energy a countermass or a spring can be added to the system. A spring is preferred over a countermass in terms of weight and inertia forces [3].

A disadvantage of current spring-to-mass balancers is that helical springs are used. The spring crosses the working space of the mass and a zero-free-length spring is required to achieve perfect balance [4]. These springs are not readily available.

Investigation by InteSpring has resulted in a one degree of freedom (DoF) bottom hinged gravity equilibrator where a cluster of torsion bars is placed near the pivot point of the rotating mass, such that the working space of the mass is free of components [5,6]. Perfect balance is achieved by a non linear transmission between the cluster and the mass. This transmission is established by a Double Cam Transmission (DCT) which consists of two cams with varying radius that are placed side by side. The main disadvantage of the state of the art is that the rotating mass is limited in its range of motion which is $0 \leq \theta_{in} \leq \frac{\pi}{2}$ rad. This limit results in less suitable applications. Causes of this limit are:

- For a larger range, the radii of the cams are converging to the extrema; one radius becomes zero and one radius becomes maximal. This will result in extreme forces on the cams.
- When the cluster of torsion bars is rotated too much the torsion bars touch each other and prevent further rotation.
- The shear stress is exceeding the maximum allowable shear stress τ_{max} when the torsion bars are rotated further than the allowed angle.

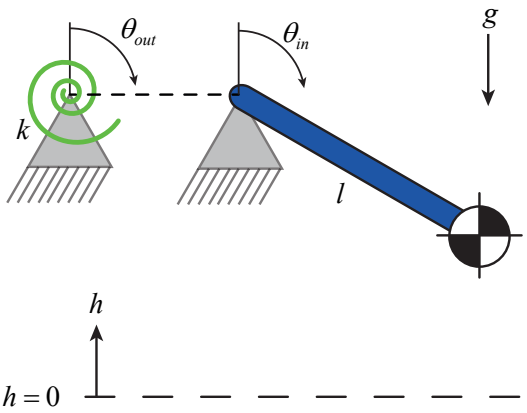


FIGURE 1. An exploded view of the different components to distinguish the parameters of the gravity equilibrator. On the left a torsion spring which angle of rotation is defined by θ_{out} and on the right a rotating mass on an arm, that is defined by the angle θ_{in}.

Another disadvantage of introducing the DCT is that the rotation axis of the mass and the rotation axis of the cluster of torsion bars are not coaxial. Not only does this lead to an increase in required space, it also causes an unbalance of forces. The result of this unbalance is that the structure around the mechanism is subjected to resultant forces, causing the whole structure to deform. This deformation is undesired in terms of alignment and energy storage.

The current mechanism is not suitable for a top hinged rotating mass, such as a top cover in a kitchen cabinet which has a range of motion of $\frac{\pi}{2} \leq \theta_{mass} \leq \pi$ rad, measured from the same reference point. Another example where this limited range of motion is a decisive factor to reject the current mechanism is in the case of a robot arm that should be able to rotate up to or more than 2π rad. The mechanism would restrict the maximum angle of rotation of the robot arm.

The goal of this paper is to develop a new gravity equilibrator that is able to statically balance a rotating mass over an unlimited range of motion which can be made more compact than the state of the art. The research has only been focused on mechanical transmissions where gears are used to transmit a torque or motion [7].

This paper is structured according to the following format. In the section "Method" it is explained how the research is performed and what design requirements are drawn and how these are established. In the section "Conceptual design" the principle of the coaxial gear train with non circular gears is explained, which is further treated in the section "Dimensional design". The section "Prototype" covers the design of the prototype and the evaluation of this prototype. The results are discussed and conclusions are drawn in the last two sections.

2 METHOD

The gravity equilibrator from Fig. 1 contains a mass and a torsion bar that is visualized by a spiral spring. The mass m rotates at a distance L from the pivot point and is connected to the input axis. The position of the mass is defined by the angle θ_{in}. The height h of the mass is measured from the datum plane $h = 0$. This reference point can be chosen arbitrary. The torsion bar has a stiffness k and the rotation of the spring is defined by the angle θ_{out}.

The principle of constant potential energy is used to design the gravity equilibrator which states that the sum of the potential energies in the system must be constant for all configurations:

$$V_g(\theta_{in}) + V_e(\theta_{out}) = V_{pot} = C \tag{1}$$

where V_g is the potential energy due to gravitational forces and V_e is the potential energy due to elastic forces.
The potential energy of the mass is calculated by

$$V_g(\theta_{in}) = mgh = mgL(1 + cos(\theta_{in})) \tag{2}$$

The potential energy that the torsion bar stores depends on the stiffness k, the angle θ_{out} and the initial angle θ_0 and is derived by

$$V_e(\theta_{out}) = \frac{1}{2}k(\theta_{out} + \theta_0)^2 \tag{3}$$

The stiffness depends on the cross section, the length and the material properties of the torsion bar. The elastic energy is assumed to be zero when the torsion bar is not rotated. Internal tensions in the material are not taken into account. The angle θ_0 is therefore equal to 0. Eqns. 2 and 3 are substituted in Eqn. 1 which yields

$$mgL(1 + cos(\theta_{in})) + \frac{1}{2}k\theta_{out}^2 = C \tag{4}$$

The constant value C is determined by setting θ_{in} and θ_{out} equal to zero. The constant value is equal to $2mgL$. Rearranging Eqn. 4 gives θ_{out} as a function of θ_{in}

$$\theta_{out} = \sqrt{\frac{2mgL}{k}} \cdot \sqrt{1 - cos(\theta_{in})} \tag{5}$$

Eqn. 5 gives the relation between θ_{in} and θ_{out} such that the rotating mass is balanced for every angle, see Fig. 2. For increasing θ_{in}, θ_{out} first increases to a maximum and after that decreases to its initial value. It can be stated that in order to achieve a perfectly balanced system, the input axis has to make a continuous

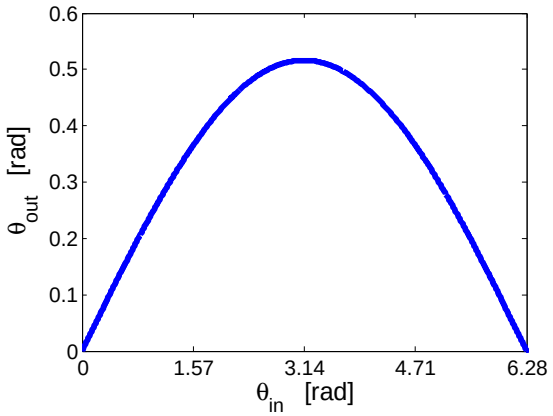


FIGURE 2. The relation between θ_{in} and θ_{out} from Eqn. 5 that is required to obtain a statically balanced system. It can be seen that for continuous increasing θ_{in}, θ_{out} first increases and then decreases again.

rotation while the output axis has to make a reciprocating rotation. Because θ_{out} from Eqn. 5 will give perfect balance, this angle is referred to as $\theta_{out,ref}$ which is used in the optimization process.

2.1 Design requirements and limitations

Multiple design requirements and limitations are determined in order to specify the research. The first three points are requirements that the mechanism has to fulfil and the last four points are limitations in the design.

- The pendulum is not restricted in its range of motion. This means that the angle θ_{in} can take on any real value.
- The transmission mechanism converts a continuous rotation of the input into a reciprocating rotation of the output as depicted in Fig. 2.
- The pendulum is statically balanced over its entire range of motion.
- The position of the pendulum is defined by a single degree of freedom, angle θ_{in}. This is visualized in Fig. 1.
- The system is analysed in a quasi-static state. The system is moving with negligible velocities. The change in kinetic energy and dynamic forces can be omitted.
- Only mechanisms with a mechanical transmission using gears are investigated [7].
- The mechanism is separated into three subsystems; *the load element*, *the transmission element* and the *energy storage element*. It is assumed that non of these subsystems will be combined. Energy storing elements with a specific shape that replace the transmission element are outside the scope of this research.

2.2 Evaluation criteria

The mechanism is analysed by two different criteria. These criteria are a measure of the performance of the mechanism.

- The first criterion is the balancing quality. This is defined by the work W_{bal} that is required to rotate the balanced pendulum over the work W_{unbal} to rotate the unbalanced pendulum. The work is determined by taking Riemann sum of the absolute values of the moment M_{bal} and M_{unbal}. The criterion is expressed in the efficiency η_{bal} and is determined by

$$\eta_{bal} = \left(1 - \frac{W_{bal}}{W_{unbal}}\right) \cdot 100\% \tag{6}$$

- The second criterion is the hysteresis. A quantification of this hysteresis is given by the ratio of the amplitude of the moment M_{hys} that is lost due to hysteresis over the amplitude of the moment M_{unbal} that the unbalanced pendulum exerts.

$$R_{hys} = \frac{M_{hys}}{M_{unbal}} \cdot 100\% \tag{7}$$

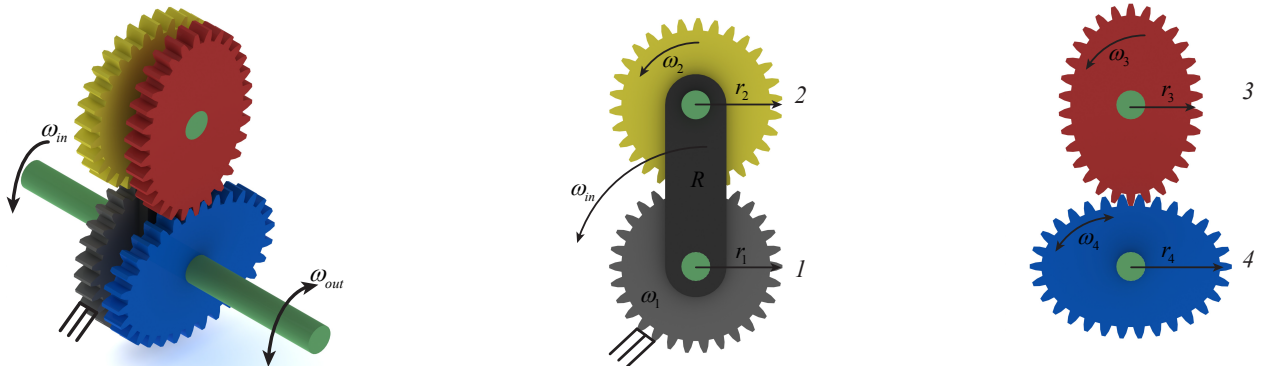
3 CONCEPTUAL DESIGN

In order to achieve perfect balance over an unlimited range a transmission must be introduced. This transmission must convert a continuous rotation of the input axis into a reciprocating rotation of the output axis as depicted by Fig. 2. Fig. 3 is a schematic representation of a transmission that satisfies the first two requirements stated in section 2.1 and is adapted from [8,9]. This transmission consists of two gear sets. Gear set 1, see Fig. 3(b), is built up from two gears with constant radii. Gear set 2, see Fig. 3(c), is built up from two gears with changing radii, called non circular gears. They are represented by oval gears. It does not matter which gear set uses non circular gears, the ratio between θ_{out} and θ_{in} will be equal. The relation between angle θ_{in} of the input axis and angle θ_{out} of this mechanism is derived in two steps, the elaboration of gear set # 1 and # 2.

3.1 Gear set # 1

The first gear set contains two gears which are both circular. There are three angular velocities to be distinguished, ω_{in}, ω_1 and ω_2, which are the angular velocities of the rotating mass, gear 1 and gear 2 respectively. Gear 1 is rigidly connected to the world, so it is stated that ω_1 is 0 rad/s. Gear 2 is connected to the arm R which rotates with angular velocity ω_{in} which is defined positive CCW. Gear 2 meshes with gear 1 and because the latter is fixed, gear 2 is forced to rotate. The angular velocity of gear 2 is given by:

$$\omega_2 = \left(1 + \frac{r_1}{r_2}\right) \cdot \omega_{in} \tag{8}$$



(a) (b) (c)

FIGURE 3. A schematic representation of the gear train that is adapted from [8]: (a) A three dimensional representation of the gear train to visualize the topology, (b) the first gear set which consists of two circular gears, (c) the second gear set which consists of two non circular gears, visualized by oval gears.

where r_1 and r_2 are the radii of gear 1 and gear 2 respectively. These radii are both constant and positive. Because the angular velocity ω_{in} is also constant and positive, it is concluded that the angular velocity ω_2 is also constant and positive.

3.2 Gear set # 2

The connection between the first and second gear set is established by gear 2 and 3 which are rigidly connected such that

$$\omega_3 = \omega_2 \tag{9}$$

Gear 4 is connected to the output axis. The angular velocity ω_{out} of gear 4 is a weighted combination of ω_{in} and ω_3. The factor of ω_{in} is derived in the same way as was done for the configuration of the first gear set. The factor of ω_3 is derived by the ratio r_3/r_4 between the two gears. The derivation of ω_{out} is

$$\omega_{out} = \left(1 + \frac{r_3}{r_4}\right) \cdot \omega_{in} - \frac{r_3}{r_4} \cdot \omega_3 \tag{10}$$

By substituting Eqns. 8 and 9 in Eqn. 10 ω_{out} is derived and is equal to

$$\omega_{out} = \left(1 - \frac{r_1 \cdot r_3}{r_2 \cdot r_4}\right) \cdot \omega_{in} \tag{11}$$

which gives the relation between ω_{in} and ω_{out} of the gear mechanism shown in Fig. 3. By integrating Eqn. 11 with respect to time, the output angle θ_{out} is derived

$$\theta_{out} = \int_0^t \omega_{out}\, \mathrm{d}t \tag{12}$$

The transmission ratio is defined by the ratio between ω_{out} and ω_{in}. The parameters r_i in Eqn. 11 determine this transmission ratio. The combination can be chosen in such a way that ω_{out}/ω_{in} is either positive, negative or equal to zero. With a positive transmission ratio, the combination of radii is smaller than one and θ_{out} increases for increasing θ_{in}. A negative transmission ratio is obtained when the combination of radii is larger than one and θ_{out} decreases for increasing θ_{in}. When the combination of radii is equal to one, the output angle is stationary for increasing θ_{in}.

In this mechanism the second gear set consists of two non circular gears. The radii r_3 and r_4 are variable. With changing radii the transmission ratio also changes. The radii are changed such that the output angle θ_{out} is tuned. To achieve perfect balance, θ_{out} first have to increase from zero to a certain angle and then decreases to zero as depicted in Fig. 2. A torsion bar attached to the output axis would store energy when θ_{out} increases and release this energy when θ_{out} decreases. When this behaviour is controlled and tuned, the elastic energy of the spring is tuned such that constant potential energy is achieved.

3.3 Concept variations

The presented concept transforms a continuous rotation of the input axis into a reciprocating rotation of the output axis. The two axes are coaxial such that the mechanism is smaller than the state of the art. In the presented configuration this last statement is not valid. From Fig. 3(b) it is seen how gear 2 rotates around the fixed gear 1. The minimal required diameter D_{min} that the mechanism covers is determined by the radii of the gears and is equal to

$$D_{min} = 2 \cdot (r_1 + 2r_2) = 2r_1 + 4r_2 \tag{13}$$

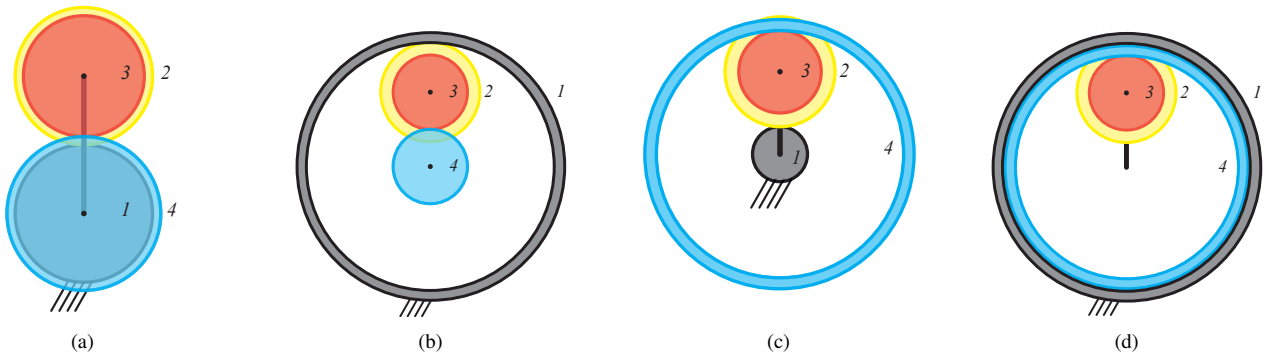


FIGURE 4. The four possible variations based on the original concept: (a) the original concept where all four gears have external toothing, (b) the concept where gear 1 is replaced by a gear with internal toothing, (c) the concept where gear 4 is replaced by a gear with internal toothing, (d) the concept where both gear 1 and 4 are replaced by gears with internal toothing.

TABLE 1. Transmission ratio of the four concept variations given in Fig. 4 where gear 3 and 4 are non circular gears. Concept 4(b) and 4(c) will always have a positive transmission ratio. These concepts will not be able to achieve a desired input-output relation. Concept 4(a) and 4(d) can achieve both a positive and a negative transmission ratio.

Concept	Transmission ratio $\frac{\omega_{out}}{\omega_{in}}$	Suitable
4(a)	$1-\frac{r_1 \cdot r_3}{r_2 \cdot r_4}$	Yes
4(b)	$1+\frac{r_1 \cdot r_3}{r_2 \cdot r_4}$	No
4(c)	$1+\frac{r_1 \cdot r_3}{r_2 \cdot r_4}$	No
4(d)	$1-\frac{r_1 \cdot r_3}{r_2 \cdot r_4}$	Yes

which results in an even larger mechanism because the diameter of gear 2 has to be added twice.

To obtain a smaller mechanism, different concept variations are shown in Fig. 4. These four concepts have the same topology but use different components. In Fig. 4(a) only external gears are used. In Fig. 4(b) gear 1 is replaced by a gear with internal teeth. The advantage of the internal gear is that gear 2 can rotate inside gear 1 which results in a smaller diameter of the mechanism. In Fig. 4(c) this idea is used for gear 4, gear 3 can rotate inside gear 4. Fig. 4(d) shows the variation where both gear 1 and gear 4 are replaced by internal gears. The transmission ratio of the four variations are given in Tab. 1 and are derived in the same way as was done for the original concept [App. C.1]. The second and third concept variation both have a transmission ratio which cannot be negative, regardless of the chosen radii. Therefore these variations are excluded from the feasible concept variations.

The fourth variation has a transmission ratio which can be positive and negative and uses two internal gears. The minimal required diameter that this mechanism covers is determined by

$$D_{min} = 2r_1 = 4r_2 \tag{14}$$

Because the diameter of this configuration is smaller than the diameter of the original concept given in Eqn. 13, this concept is chosen for further elaboration.

4 DIMENSIONAL DESIGN

With concept 4(d) chosen, the design can be worked out in detail. This process is divided in the calculation of pitch curves of gear set # 1 and gear set # 2, the extension from the two dimensional plane to the three dimensional space, the calculations of the exact teeth profiles and the calculations on the torsion bars.

4.1 Pitch curves of gear set # 1

Gear set # 1 uses circular gears. The internal gear 1 is fixed and the external gear 2 rotates inside gear 1. In order to achieve a periodic rotation, the transmission ratio between the input and gear 2 must be equal to

$$\frac{\omega_2}{\omega_{in}} = \left(1-\frac{r_1}{r_2}\right) \tag{15}$$

where r_1/r_2 has to be an integer and $r_1 > r_2$. The choice of the radii is arbitrary. The ratio of between these two radii are chosen to be equal to 2. In this way the angular velocity ω_2 is as small as possible and therefore angle θ_2 too. The radii of gear 1 and 2 are visualized in Fig. 5.

4.2 Pitch curves of gear set # 2

The pitch curves of non circular gears 3 and 4 are derived with an optimization method. This problem is identified as an

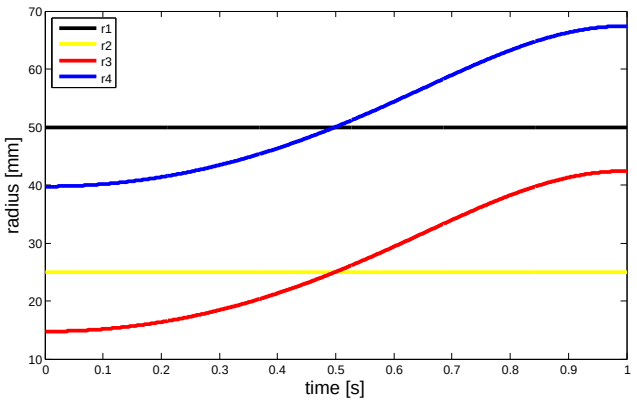


FIGURE 5. The radii of the four gears in the time domain. r_1 and r_2 are constant and r_3 and r_4 are variable.

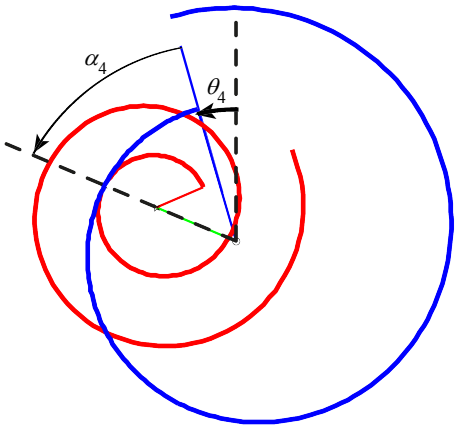


FIGURE 6. A visualization of the relative angles α_i and the absolute angles θ_i. The relative angles define the points where the gears mesh with each other. They are derived in a local polar coordinate system fixed to the gear whereas the angles θ_i are defined in the global polar coordinate system.

unconstrained optimization problem and a gradient based optimization is used. The objective is to minimize the energy difference between the reference energy $V_{e,ref}$ and the actual energy V_e. These can be derived by $\theta_{out,ref}$ and θ_{out} from Eqns. 5 and 12 respectively. The objective function $f(\mathbf{x})$ is normalized such that the range of the objective is $[0,1]$.

We define a range in the time domain from 0 to 1 seconds where the pendulum makes a rotation of 2π rad with constant angular velocity. At specified points in time we determine the radii r_3 and r_4 where the contact between the gears occur. To have a reasonable amount of optimization variables we define p control points equally spaced in the time range. The control points are

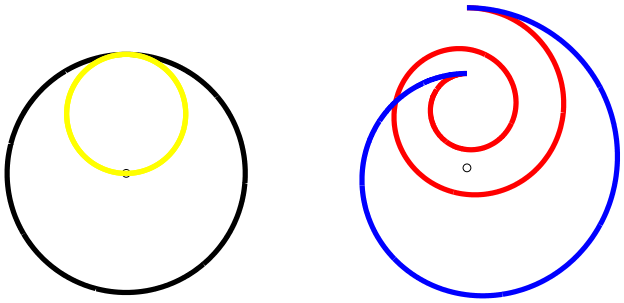


FIGURE 7. The four pitch curves of the four gears: (a) Fixed internal gear 1 is represented by the black curve, gear 2 is represented by the yellow curve, (b) gear 3 is represented by the red curve, internal gear 4 is represented by the blue curve.

the variables. By a spline function these p points are interpolated to increase the amount of samples n in the time range.

It is on these sample points that the energies $V_{e,ref}$ and V_e are determined and their difference is minimized by

$$f(\mathbf{x}) = \min_{\mathbf{x}} \frac{|\, V_{e,ref} - V_e \,|}{|\, V_{e,ref} \,|} \tag{16}$$

where the design vector $\mathbf{x}$ is defined as

$$\mathbf{x} = \left[r_{3,j}, r_{3,j+1} \ldots r_{3,p} \right] \tag{17}$$

By adjusting the variables the output angle θ_{out} is changed such that the objective function is minimized. The initial guess of the design variables can be chosen arbitrary. With the optimization, the minimum f($\mathbf{x}^*$) and the minimizer $\mathbf{x}^*$ are obtained. The minimum is in the order of magnitude of 10^{-6}. With a larger number of design variables, the minimum is closer to zero, but will not result in a significantly different pitch curve. Because the range of the objective function is $[0,1]$ it is assumed that the minimum is reached.

The optimization gives the radii of gear 3 and 4 as a function of time, see Fig. 5. Introducing the angles α_i which are defined as the polar coordinate local to every gear where the contact point between the gears occurs, see Fig. 6, the found r_3 and r_4 are projected into the polar coordinates (r_i, α_i) which describes the pitch curves of the gears, see Fig. 7. The radius of gear 1 is twice the radius of gear 2. The radius of gear 3 ranges between two extrema as well as the radius of gear 4.

Fig. 8 visualises the relative positions of the four gears for one rotation of the input axis. Six positions are shown and these correspond to the six references in Fig. 8(a). Fig. 8(b) shows the initial position, θ_{in} (blue line) and θ_{out} (green line) are both zero rad. Fig. 8(c) and 8(d) show the positions where the pendulum is rotating. In Fig. 8(e) the pendulum is at its lowest point and gear 4 is rotated to its maximal angle. While the input continues to rotate, θ_{out} decreases again. Fig. 8(g) shows the position where θ_{in} has made a rotation of 2π rad en θ_{out} has made a reciprocating rotation.

The factor $\sqrt{2MgL/k}$ from Eqn. 5 influences the magnitude of the radii of gear 3 and 4. When the factor is chosen too large, the magnitude becomes too large. This is due to the fact that a large factor corresponds to a small stiffness of the torsion bars compared to the mass. With a small stiffness, the torsion bars have to be rotated over a larger angle θ_{out} to achieve the required elastic energy to compensate for the energy of the mass. To achieve this larger angle, the combination of radii from Eqn. 11 has to change over a larger range. This can only be achieved by expanding the range of radii r_3 and r_4.
The opposite holds for a factor that is chosen too small. The stiffness of the torsion bars is very large compared to the mass and the maximum of θ_{out} can remain very small to obtain the required energy. When this factor is too small, the mechanism becomes more sensitive to design errors, e.g. design tolerances or backlash. The factor is chosen such that the best compromise is made. Iterative calculations result in a factor where $k = 15 \cdot MgL$. With this factor the maximal radius of gear 3 is not too large and the angle of rotation of the torsion bars is not too small.

4.3 Three dimensional design

From Fig. 7(b) it is seen that the pitch curve of gear 3 makes a rotation of 4π rad. If the pitch curve is designed in the two dimensional plane, the pitch curves of gear 3 and gear 4 intersect and the gear train cannot function. Therefore it is necessary to build the gears in the three dimensional space with a pitch that is equal or larger than the width of the teeth. Interference of the pitch curves is then avoided.

4.4 Generation of teeth profile

The teeth profiles are derived by the rack-cutter method [10, 11] [App. C.3]. This method generates the teeth profile by using a rack that unrolls around the pitch curve. Because the pitch curve of gear 1 and 2 have a constant radius, every tooth is equal.

The generation of the teeth on the non circular gears differs from the circular gears. The pitch curves have changing radii and the shape of the teeth depends on the curvature of the pitch curve. Therefore each tooth is different. In Fig. 9 the teeth are shown on the four pitch curves.

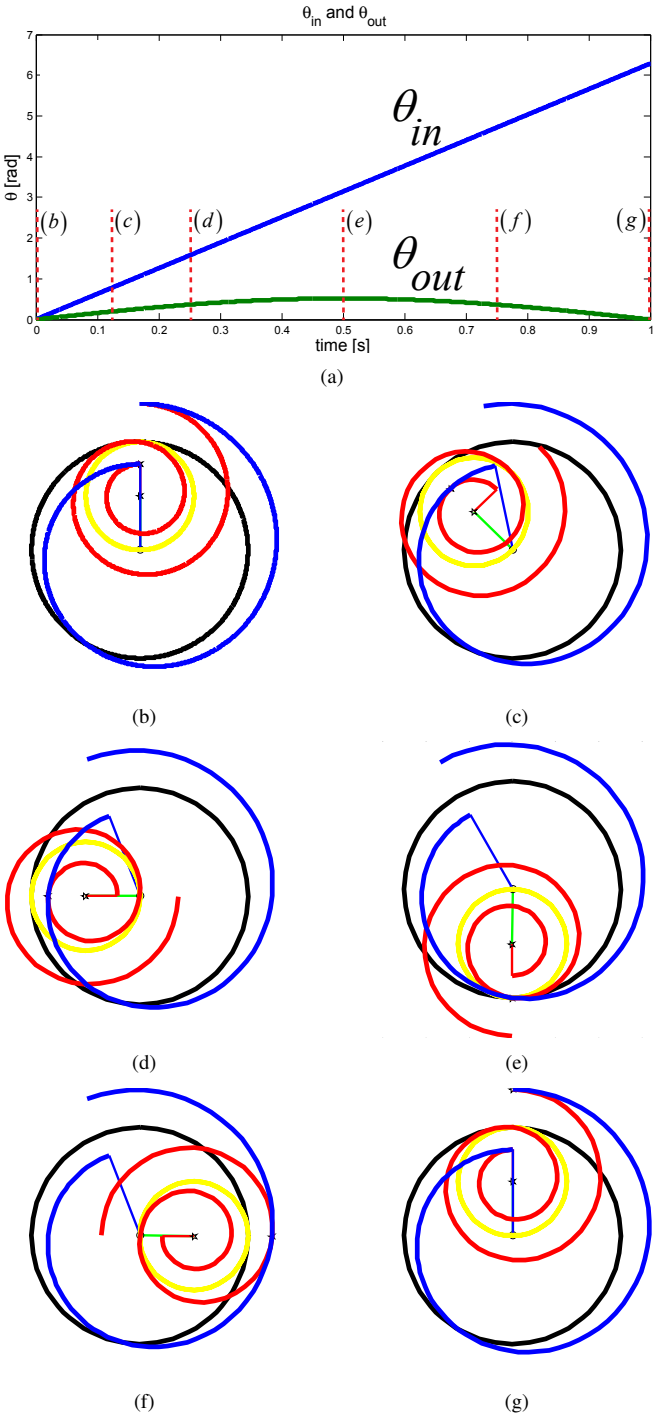


FIGURE 8. A visualization of the positions of the gears at different moments. These moments are referred to in (a): (a) the graph where the angle θ_{in} and θ_{out} are plotted, (b) the initial position where θ_{in} and θ_{out} are zero, (c), (d) other moments where both θ_{in} and θ_{out} are increasing, (e) at this moment θ_{out} is maximal, (f) a moment where θ_{in} increases and θ_{out} decreases, (g) the initial position which is equal to (a).

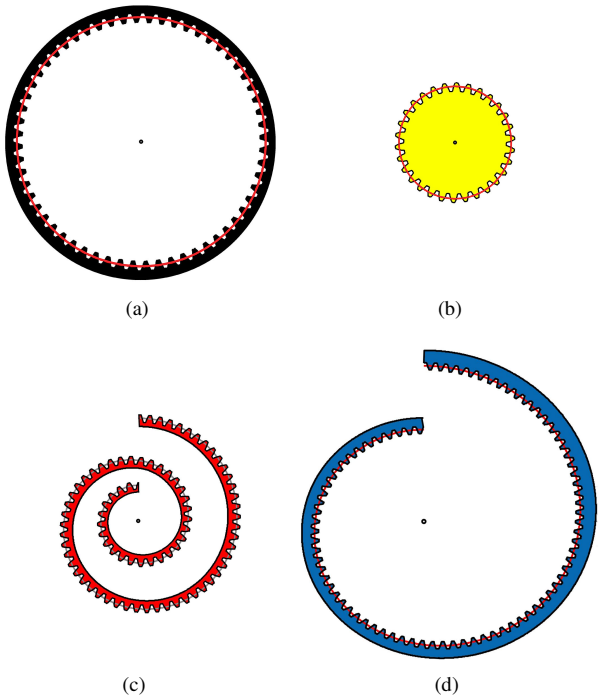


FIGURE 9. The pitch curves of the four gears with the teeth: (a) the pitch curve of gear 1 and the internal teeth, (b) the pitch curve of gear 2 and the external teeth, (c) the pitch curve of gear 3 and the external teeth, (d) the pitch curve of gear 4 and the external internal teeth.

4.5 Torsion bars

A torsion bar is chosen as energy storage element because of the advantageous dimensions. The material is hardened and tempered spring steel, type C75S [App A.3]. The bar has only one large dimension which is the length. Torsion bars have a high energy to volume ratio [12]. A rectangular cross section is chosen because this shape can be manufactured from sheet metal which is commonly available. The rectangular torsion bar can be clamped to create a form closed connection, such that a torque can be applied without the need of additional measures to prevent slip. The stiffness is [13, 14]

$$k = \frac{n \cdot \beta \cdot G \cdot b \cdot t^3}{l} \tag{18}$$

where n is the number of torsion bars, β is the shape coefficient, G is the modulus of elasticity in shear, b,t and l are the width, thickness and length of the torsion bar respectively. The shear stress τ in the torsion bars is calculated by

$$\tau = \frac{b \cdot G \cdot \theta}{l} \tag{19}$$

TABLE 2. The parameters of the torsion bars that are required to balance a mass.

Parameter	Symbol	Value
Number of torsion bars	n	5 [-]
Shape coefficient	β	0.333 [-]
Shear modulus	G	78 [GPa]
Width	b	18 [mm]
Thickness	t	1 [mm]
Minimal active length	l	54 [mm]

where τ cannot exceed the maximal allowed shear stress τ_{max} which is derived by

$$\tau_{max} = 0.58 \cdot \sigma_y \tag{20}$$

where σ_y is the yield stress of the material which is equal to 1.3 *GPa*.

The parameters that are varied in this design are n,b,t and l. The thickness t can take on a certain number of values because there is a limited number of sheet thicknesses available. The choice of the width and the length are free. The parameters are chosen such that the spring is able to absorb enough energy to balance the mass but does not exceed the maximal shear stress. The parameters are given in Tab. 2. The values of these parameters are derived in [App. A.1].

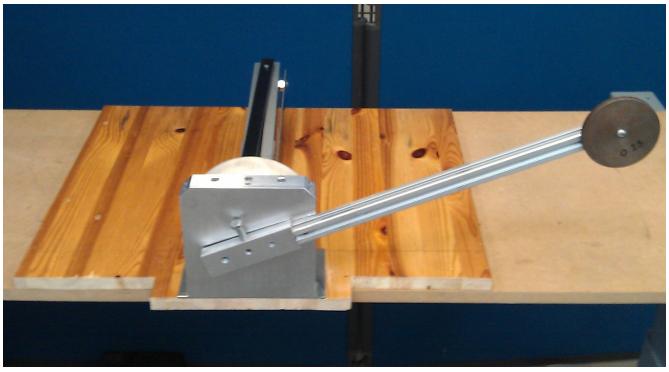
5 PROTOTYPE

A prototype has been built in order to validate the design of the gravity equilibrator with an unlimited range. An overview of the prototype together with the main components is shown in Fig. 10. The design of the prototype and the evaluation of this prototype are treated.

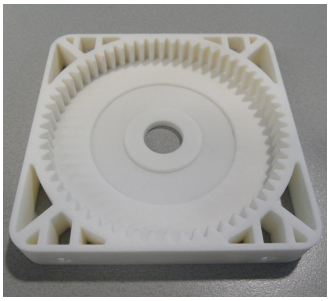
5.1 Design

The gears are made of polyamide which is used for fully functional prototypes with high mechanical qualities and are constructed with a rapid prototyping technique called *Selective Laser Sintering (SLS)*. This technique builds up the gears layer by layer. Successive powder layers are spread on top of each other. A laser beam scans the surface and selectively binds together the powder particles of the corresponding cross section of the product [15].

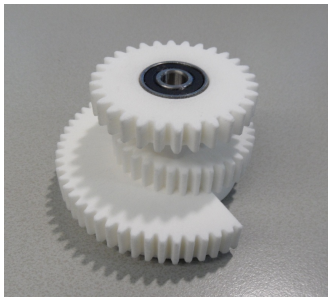
Fig. 10(a) gives the full picture of the prototype. The mass is visible that is connected to the arm. On the back the torsion



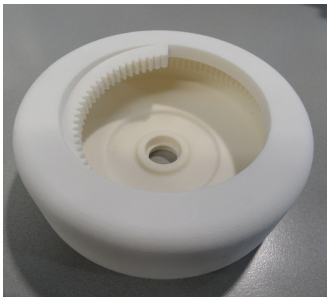
(a)



(b)



(c)



(d)



(e)

FIGURE 10. Pictures of the prototype: (a) a general overview, (b) fixed internal gear 1, (c) gear 2 and gear 3 combined, (d) output gear 4, (e) the structure that connects the multiple parts.

bars are visible. Fig. 10(b) shows internal gear 1 which is rigidly connected to the world. In Fig. 10(c) gears 2 and 3 are shown which are made up from one single part. Fig. 10(d) shows the gear that has internal toothing and is connected with the torsion bars. Fig. 10(e) shows the structure that constraints gear 2 and 3 and supports gear 4.

5.2 Measurement results

The prototype is tested on a universal testing machine M250-2.5 CT of Testometric. The testing machine can make a vertical translation and measure the pulling force. A pulley is

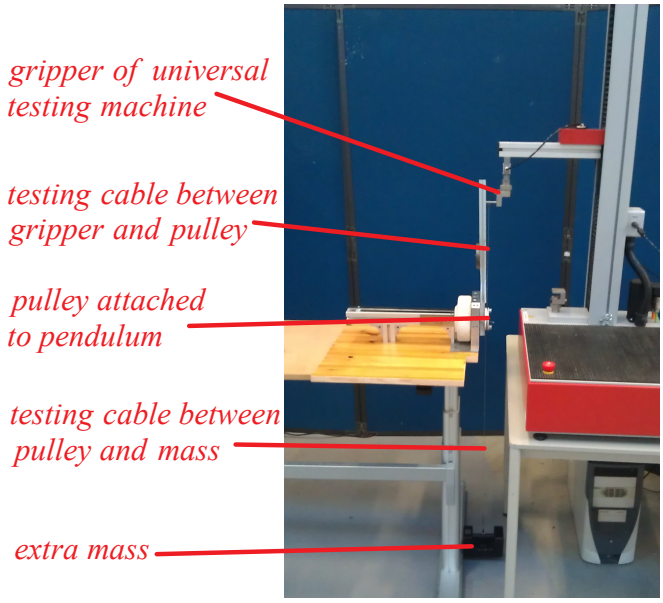


FIGURE 11. A schematic overview of the measurement setup. The pendulum is attached to the pulley. Two different cables are attached to this pulley. One of these cables is connected to the testing machine and one cable is connected to a counterweight of 5 kg.

placed at the pivot point such that the testing machine can exert a moment on the pendulum. When the pendulum is perfectly balanced, the required moment to move the pendulum is fluctuating around 0 Nm. Because the cable between the pulley and the testing machine can only be subjected to a pulling force a counterweight is added to the mechanism. This counterweight is a mass of 5 kg that is connected via a cable to the pulley. A constant moment on the input axis is added. A schematic overview is shown in Fig. 11.

Three consecutive measures are performed of the total system. The measurements are performed by rotating the pendulum 4π rad CCW, and 4π rad CW. The constant moment exerted by the counterweight is subtracted, such that the measurements fluctuate around 0 Nm. The results of these measurements are shown in Fig. 12 where the angle θ_{in} is located on the horizontal axis and the moment is located on the vertical axis. The measurement data of the gear train is shown in Fig. 13. This measurement is performed without the pendulum and torsion bars but with the counterweight which is subtracted from the measurement data. The data is smoothed in order to filter out the peaks that occur due to the high measurement frequency. The average of the three measurements is shown in red, which is the hysteresis loop. The mean value between the upper and lower part of this hysteresis loop is plotted in black. This black line gives an estimation of the required moment in absence of friction. The assumption is made that the magnitude of the friction for the way up and way

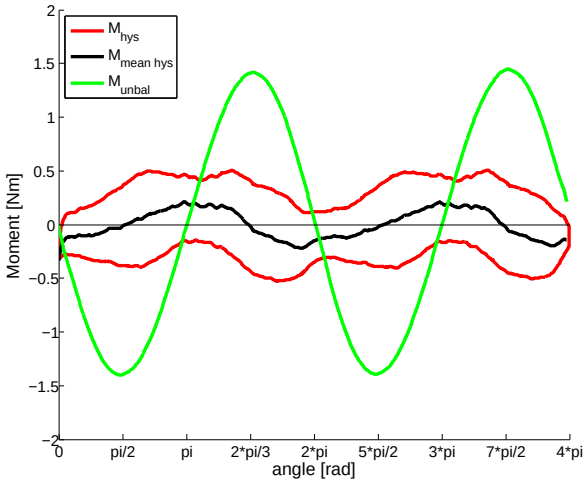


FIGURE 12. The measurement results of three consecutive measurements from 0 to 4π rad and back. The hysteresis loop is shown in red, the mean value of the hysteresis loop is shown in black. The mean value of the moment exerted by the unbalanced pendulum is shown in green.

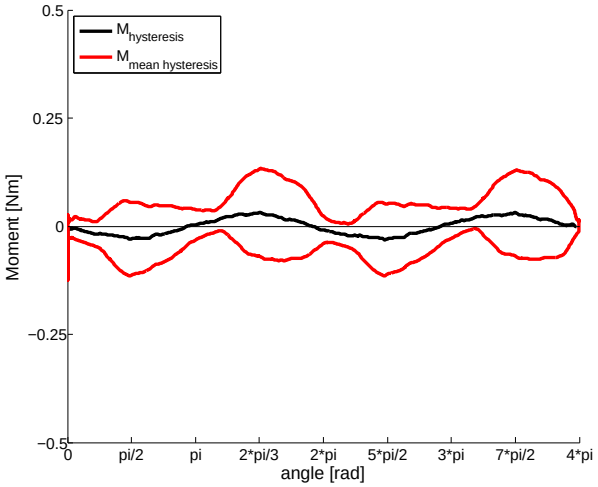


FIGURE 13. The hysteresis in the gear train which is not loaded by the torsion bars. Only the counterweight is added to the mechanism for the measurements. The hysteresis in the unloaded system is around 0.

back is equal at every position. In Fig. 12 the green line represents the mean value of the moment of the unbalanced pendulum, which is used to determine the efficiency. The work done by the balanced pendulum is derived by determining the area under the graph of the absolute mean moment that is required to rotate the mass over 4π rad. This area is calculated with the Riemann sum which takes the sum of areas of approximating rectangles with width $\triangle\theta_{in}$ and height M_{res} [16]

$$W_{bal} = \sum_{i=1}^{n} |M_{bal,i}| \cdot \triangle\theta_{in} \tag{21}$$

where $\triangle\theta_{in}$ is 0.01 rad and n is equal to $4\pi/0.01$.
The work done by the unbalanced pendulum is derived in the same manner

$$W_{unbal} = \sum_{i=1}^{n} |M_{unbal,i}| \cdot \triangle\theta_{in} \tag{22}$$

Using Fig. 12 the results are

$$W_{bal} = 1.49\ J$$
$$W_{unbal} = 11.3\ J$$

Substituting the obtained values in Eqn. 6 the efficiency is derived

$$\eta_{bal} = \left(1 - \frac{1.49\ J}{11.3\ J}\right) \cdot 100\% = 86.8\% \tag{23}$$

The hysteresis loop in Fig. 12 shows that a certain amount of losses are present in the system. To give a quantitative measure to this hysteresis the peak to peak amplitude of M_{hys} and the peak to peak amplitude of M_{unbal} are determined. From 0 to 4π rad the peak moment to move the balanced pendulum is 0.50 Nm. On the way back from 4π to 0 rad the peak moment is -0.52 Nm. The hysteresis is therefore 1.02 Nm. The peak to peak amplitude of the moment from the unbalanced pendulum is equal to 2.80 Nm. With Eqn. 7 the ratio of the hysteresis is derived

$$R_{hys} = \frac{1.02}{2.81} \cdot 100\% = 36.3\% \tag{24}$$

6 DISCUSSION

During assembly, problems were encountered regarding the alignment and tolerances of several components. The component from Fig. 10(e) is manufactured by bending a steel plate. The two axles are welded on the flanges of this bended plate. Because of the inaccuracy of the bending and the deformations due to the welding the two axles are not perfectly aligned. The misalignment is more than 1 mm and also they are not parallel to each other. This resulted in undesired friction. The component is supported by two bearings located at the two axles. The two bearings are aligned with an accuracy of less than 0.1 mm. The influence of the misalignment of axles on the component was of such magnitude that the friction on the bearings was very large. This problem is solved by a rather cumbersome but effective

method. The new component is manufactured from a solid bar. With proper turning and milling the component is reproduced but now with exact alignment of the two axles. The friction on the bearings is significant less than before [App. C.3].

The gears are manufactured by selective laser sintering. The accuracy of this manufacturing method is $\pm$ 0.3 *mm*. During the design of the gears, the tolerance was not taken into account. This resulted in gears that were slightly different than intended. The teeth of the gears did not mesh as was supposed. When the gears were assembled, they were pressed against each other causing high friction between the gears. For now, this problem is solved by milling a small amount from each tooth such that the gears meshed better. This had a positive effect on the friction.
A possible solution to overcome this problem in the future is to account for the accuracy of this method by scaling the gears. In this case the gears are too large, so when the gears are scaled down, the teeth have less friction when assembled.
The accuracy of other rapid prototyping manufacturing methods are in the same order of magnitude as selective laser sintering, so this is not an option to decrease the friction between the teeth.

The hysteresis in the system is considered to be large when compared to the maximal moment that the unbalanced pendulum exerts in the pivot point. It is assumed that this hysteresis is mainly caused by the friction in the system itself because only one extra pulley and one cable are added to make a connection between the system and the testing machine. In order to determine which components cause this friction, the components have been measured separately. The average hysteresis measured in Fig. 13 in the unloaded gear train is approximately 0.07 *Nm* but peaks up to 0.2 *Nm* are measured. This friction is caused by the teeth that mesh with each other. The points where the friction is larger, the teeth press harder against each other.
A measurement with the gear train and the pendulum gave the same hysteresis as when the pendulum was not attached, so it is concluded that the pendulum itself does not account for a significant amount for hysteresis.
The friction force from the ball bearings is negligible since the friction coefficient μ is 0.0015 [17]. Here is it assumed that the ball bearings are perfectly aligned.

At the moment that the balanced pendulum is rotated, the hysteresis is 0.66 *Nm* as calculated in Eqn. 24. The extra hysteresis of 0.59 *Nm* can probably be assigned to the teeth that are now loaded, causing the teeth to press harder against each other. When taking the gear train apart, it is seen that the component from Fig. 10(e) slides against gear 1 and 4. This sliding causes friction that accounts for a part of the 0.59 *Nm*.
At the moment that the pendulum is at $\pi/2$ rad, gear 4 slides against the housing of gear 1. This sliding accounts also for a part of the hysteresis. This is also visible in Fig. 12 where the hysteresis loop is wider. The hysteresis in the system is caused by dimensional problems such as the sliding of the components against each other, but also by the friction between the teeth.

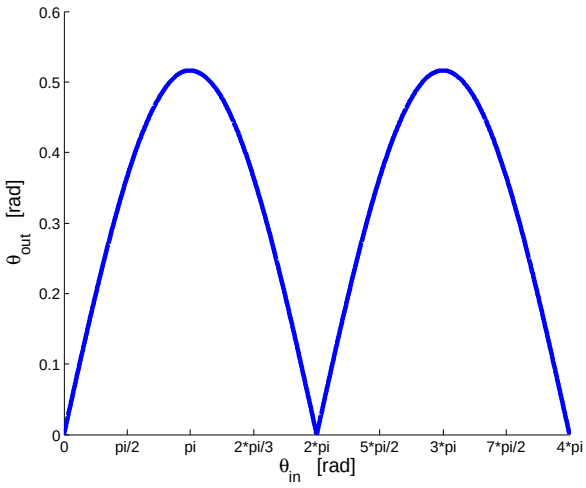


FIGURE 14. Angle θ_{out} changes discontinuously when the pendulum is in the upright position. This results in a jump in the pitch curves of the non circular gears 3 and 4. This discontinuity is caused by the required energy that the spring has to absorb to achieve perfect balance.

The dimensional design has to be improved such that this friction will diminish, but the friction between the teeth is a fundamental problem. The solution may lie in the type of manufacturing method. Now the gears are made from polyamide. This material is relative flexible when compared to materials that are normally used for gears. The elongation at break is around 20 %. This flexibility can account for a relative large amount of hysteresis. We think that metallic gears will have a positive results on the hysteresis because they are less flexible.

During the dimensional design the pitch curves of the non circular gears were obtained. We noticed that the pitch curves of these gears have a jump at the moment that the pendulum is in the upright position. It was not possible to smooth this jump such that the pitch curves would be continuous again. The reason why this jump is unavoidable can be explained by expanding Fig. 2, see Fig. 14. Now it is seen how angle θ_{out} changes discontinuously when θ_{in} is $2k\pi$ rad, with $k = \mathbb{Z}$.
This discontinuity in θ_{out} is caused by $\sqrt{1-cos(\theta_{in})}$ in Eqn. 5. θ_{out} can never become negative, but has largest steepness at the moment that $\sqrt{1-cos(\theta_{in})}$ is zero. Therefore, θ_{out} changes discontinuously in this point.

One may say that θ_{out} can also be negative, because the same energy is achieved when the torsion bars are rotated in the other direction. When this is implemented, the pitch curves of gear 3 and 4 become indeed continuous, see Fig. 15. The problem that now occurs is that there are four loops present in gear 3 and two loops in gear 4. It becomes simply impossible to create gears that can function without intersecting.

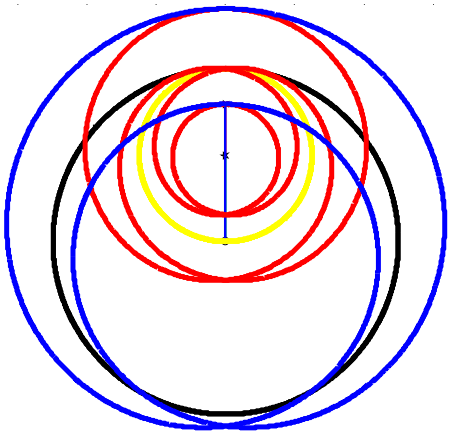


FIGURE 15. The pitch curves when the angle θ_{out} can also becomes negative. It is seen that the pitch curves of gear 3 (red) and gear 4 (blue) are now continuous but have multiple loops. This configuration of gears is impossible without having intersecting gears.

7 CONCLUSION

- A gravity equilibrator is designed that can theoretically balance a rotating pendulum with a single degree of freedom over an unlimited range of motion using torsion bars.
- A general method is presented to calculate the shape of the pitch curves of the non circular gears such that perfect balance is achieved. The shape can be adjusted by choosing different parameters for the mass m, the length of the arm l, and the stiffness of the torsion bars k.
- Different masses can be balanced by changing the active length of the torsion bars or the number of torsion bars.
- A work reduction of 86.8 % is achieved over a rotation of 4π rad when compared to an unbalanced pendulum.
- The hysteresis in the mechanism is 36.3 %, other manufacturing methods and improvements in the prototype are needed to reduce the hysteresis in the system.

ACKNOWLEDGEMENTS

We want to thank everybody from the InteSpring team for their effort and enthusiasm, Noud Werter in particular for his support and assistance in creating the theoretical model of the gear train.

NOMENCLATURE

b	Width of spring [m]
c	Clearance between two meshing teeth [mm]
C	Constant value of potential energy [J]
D_{min}	Minimal required diameter of the system [mm]
G	Shear modulus [Pa]
$f(\mathbf{x})$	Objective function [J]
F_{res}	Resultant force required to move the mass [N]
g	Gravity constant [m/s^2]
G	Shear modulus [MPa]
h	Height mass measured from datum plane [m]
k	Stiffness of torsion spring [Nm/rad]
l	Active length of torsion bars [m]
L	Length of lever [m]
m	Modulus of teeth [mm]
m	Mass [kg]
M_{hys}	Required moment to overcome hysteresis [Nm]
M_{unbal}	Moment caused by unbalanced pendulum [Nm]
M_{bal}	Moment caused by balanced pendulum [Nm]
n	Number of torsion bars [-]
N_{teeth}	Number of teeth [-]
p	Number of control points [-]
r_i	Radius of gear i [mm]
$r_{3,i}$	Radius of design variable i of gear 3 [mm]
R	Ratio of hysteresis [%]
t	Time [s]
t	Thickness of torsion bar [m]
V_e	Elastic energy spring [J]
$V_{e,ref}$	Reference elastic energy spring [J]
V_g	Potential energy mass [J]
V_{tot}	Total potential energy in system [J]
W_{bal}	Required work to rotate the balanced pendulum [J]
W_{unbal}	Required work to rotate the unbalanced pendulum [J]
$\mathbf{x}$	Design variables [mm]
α	Pressure angle teeth [deg]
α_i	Angle where gears mesh with each other [rad]
β	Shape coefficient [-]
η_{bal}	efficiency of the system [%]
θ_{in}	Angle of rotation of mass [rad]
θ_0	Initial angle of rotation of torsion bars [rad]
θ_i	Angle of rotation of gear i [rad]
θ_{out}	Angle of rotation of torsion bars [rad]
$\theta_{out,ref}$	Reference angle rotation of torsion bars [rad]
μ	Friction coefficient ball bearings [-]
σ_{tau}	Yield stress of material [MPa]
τ	Shear stress [MPa]
τ_{max}	Maximal allowable shear stress [MPa]
ω_{in}	Angular velocity of rotating mass, i.e. the input [rad/s]
ω_i	Angular velocity of gear 1 [rad/s]
ω_{out}	Angular velocity of gear 4, i.e. the output [rad/s]

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Appendix A

Torsion bars calculations

A.1 Dimensional design

In Fig. A.1 the shape of the slender torsion bar is shown. This bar is subjected to a torque T. The torque is derived by

$$T = \frac{GJ\theta}{l} \tag{A.1}$$

where G is the shear modulus, J is the torsional constant, θ is the angle of rotation and l is the effective length of the torsion bar. Note that $G \cdot J$ is the stiffness of one single bar.

$$k_1 = GJ \tag{A.2}$$

$$T = \frac{k_1\theta}{l} \tag{A.3}$$

The torsional constant J of a slender bar is

$$J = \beta b t^3 \tag{A.4}$$

where β is the shape coefficient, b is the width and t is the thickness of the torsion bar. The value of β depends on the ratio b/t. When this factor is large ($b \gg t$), then β is equal to

$$\beta = \frac{1}{3} \tag{A.5}$$

The stiffness k_1 is derived by

$$k_1 = \frac{\beta \cdot G \cdot b \cdot t^3}{l} \tag{A.6}$$

When n torsion bars used in the design, the stiffness of the total system is equal to

$$k_{total} = n \cdot k_1 \tag{A.7}$$

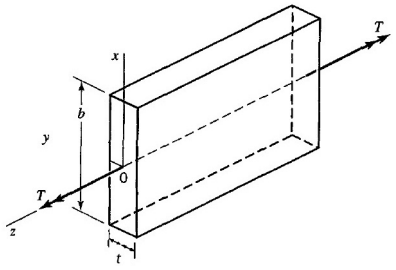


Figure A.1: Shape of a slender the torsion bar which is loaded with a torque T.

Calculation of torsion bars with rectangular cross section						
Parameter	**Symbol**	**Value**	**Unit**			
Width	**a**	18	mm	0,018	**m**	
Thickness	**b**	1	mm	0,001	**m**	
Mass weight	**m**	1	kg			
Arm	**l**	270	mm	0,27	**m**	
Shear modulus	**G**	78	GPa	7,80E+10	**Pa**	
Number of torsion bars	**n**	5	-			
rotation angle	**θ**	30	degrees	0,524	**rad**	
a/b is:	-	18	-			
Shape coefficient	**β**	0,333	-			
Torsion constant	**J**	5,994E-12	mm^4			J = β * a * b^3
Minimal active length	**l_{min}**	54,2	mm	0,054	**m**	l_{min} = b * G * θ / τ_{max}
Actve length	**l**	60	mm	0,06	**m**	
Torque	**T**	20,40	Nm			T = n * J * G * θ / l
Shear stress	**τ**	680,7	MPa	6,81E+08	**Pa**	τ = b * G * θ / l
Yield stress	**σ_{yield}**	1300,0	MPa	1,30E+09	**Pa**	
Factor shear/yield	**-**	0,58	-			
Maximal shear stress	**τ_{max}**	754,0	MPa	7,54E+08	**Pa**	
Safety factor	-	9,72%	%			τ / τ_{max} * 100%
Stiffness torsion bar	**k**	38,96	Nm/rad			k =n * β * G * a * b^3 / l
Maximum elastic energy	**V_e**	5,340689682	J			0,5 * k * θ^2
Maximum potential energy	**V_p**	5,2974	J			2 * m *g * h

Figure A.2: The excel sheet which is used to derive the required parameters of the torsion bars such that the required amount of energy is stored in the springs.

In Fig. A.5 the different parameters and equations are substituted in an excel file and by iterations the right dimensions are found for the width, thickness, minimal active length and mass of the system.
The shear stress τ in the torsion bars cannot exceed the maximal allowable shear stress τ_{max}. The maximum allowable shear stress is calculated by

$$\tau_{max} = 0.58\sigma_{yield} \tag{A.8}$$

where σ_{yield} is the yield stress of the material. With a yield stress of $1,3GPa$ the maximum allowable shear stress is $750MPa$. The shear stress in the torsion bar is calculated with

$$\tau = \frac{bG\theta}{l} \tag{A.9}$$

A.2 Construction problems

During the design process in SolidWorks the idea arose to flip the torsion bars such that they are clamped at the same place as where they are attached to the gear. In this way the structure around the mechanism is not subjected to a torque and it is not required to have a stiff structure.

During construction there were some big problems when the torsion bars were rotated. They are not loaded purely on torsion, but also on bending. Because of the lateral movement of the torsion bars, other forces than torsion were introduced in the torsion bars, which bended them out of the slots in the gear after some rotations of the pendulum. Because the torsion bars were clamped to the world far from the center of rotation, the initial bending was large. The gear was subjected to a serious amount of force, such that the teeth did not mesh good any more. Multiple configurations have been considered which are shown in the figures below.

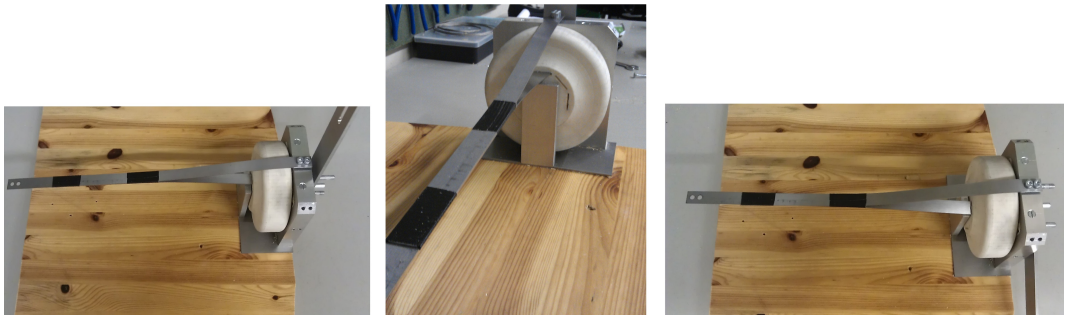


Figure A.3: The first concept where the torsion bars where clamped together at the end and were connected by means of a rigid connection. There was not lateral movement possible between the two torsion bars. This constraint resulted in bending forces on the torsion bars which pressed them outward and out of the gear.

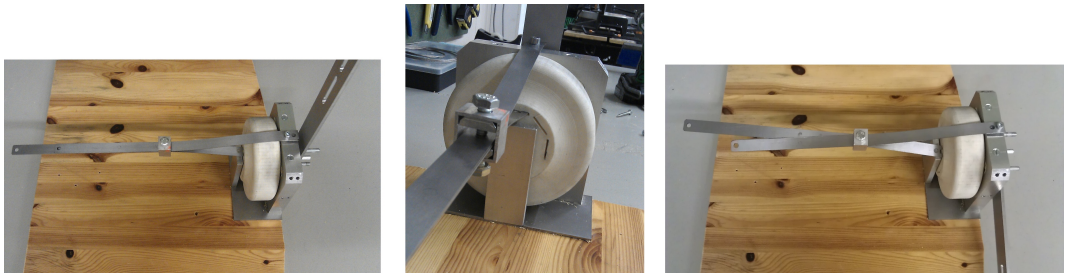


Figure A.4: The second concept where the torsion bars were pinned together at a specific point. Lateral movement is possible but relative rotation is prevented such that torsion can be applied to both torsion bars. The initial bending was still present, and the result was that the gear did not mesh with the other gear.

Finally the best solution turned out to go back to the original way; do not flip the torsion bars, but clamp them at the very end. In this way the torsion bars are not initially bended and the forces on the gears are mostly torsion. The lateral movement is small enough because the torsion bars did not bended out of the gear.

A.3 Material certificate

QR013 **Precision Metals** ac

Stockist en versnijder van band en draad.
Koper-, nikkel- en titaanlegeringen.

Roestvrij, zacht en koolstofstaal.
Gegloeid of hardgewalst, hardbaar of gehard.

MATERIAALCERTIFICAAT: EN 10204/3.1

Roveron Verenfabriek B.V. 181

Uw referentie: 3673

Smeltnummer:	Batch nr	Onze referentie:	Test rapport nr	Bron nr
765457	MIO-10-0537/2A	VO-11-0025	22406	587

MATERIAALOMSCHRIJVING

Veerstaal C75S Gehard en getemperd - Grijs blauw
EN 10132-4 EN 10140

MECHANISCHE EIGENSCHAPPEN + AFMETINGEN

Dikte/Dia (mm)	Breedte (mm)	Lengte (mm)	Aantal
2	305	2000	144,5

Rekgrens (Rp 0,2 %: N/mm² - MPa)	Treksterkte (N/mm²)	Korrelgrootte
1300 - 1325	1330 - 1360	

Rek (%)	Hardheid	Ruwheid
A80: 2	HRc 42 - 43	

CHEMISCHE EIGENSCHAPPEN (%)

Ag	Al 0,006	As	B	Be	Bi	C 0,7	Ca	Cd	Ce
Co	Cr 0,23	Cu	Fe	H	Li	Mg	Mn 0,62	Mo	N
Na	Nb/Cb	Ni	O	P 0,007	Pb	Pd	S 0,0006	Sb	Se
Si 0,23	Sn	Ta	Te	Ti	V	W	Y	Zn	Zr

Andere:

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Verenbandstaal C75

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Figure A.5: The material certificate of the torsion bars that are used in the design.

Appendix B

Proof of concept

In this appendix the proof of concept is shown. This model is built to test the influence of the offset in the non circular gears. This model is based on the concept that is adapted from [3]. This model is modified by changing the shape of the non circular gears that are required to achieve perfect balance. Perfect balance can only be achieved when the non circular gears have an offset. The reason that this offset is unavoidable can be explained by using Fig. 2. From this graph it can be seen that when θ_{in} is equal to 0 rad, the derivative of $\theta_{out,ref}$ is positive and maximal. In order to achieve this maximal positive angular velocity of the output axis, the ratio of radii as was given in Eqn. 9 has to be as small as possible. This is done by making r_3 large and r_4 small.
Now when θ_{in} approaches 2π rad, the angular velocity of the output axis is negative and maximal. This maximal negative angular velocity is achieved when r_3 is minimal and r_4 is maximal. Because the input axis has a periodicity of 2π, the point where θ_{in} is 0 rad is equal to the point where θ_{in} is 2π rad. The angular velocity has to change discontinuous from maximal positive to maximal negative. This explains the offset in the non circular gears





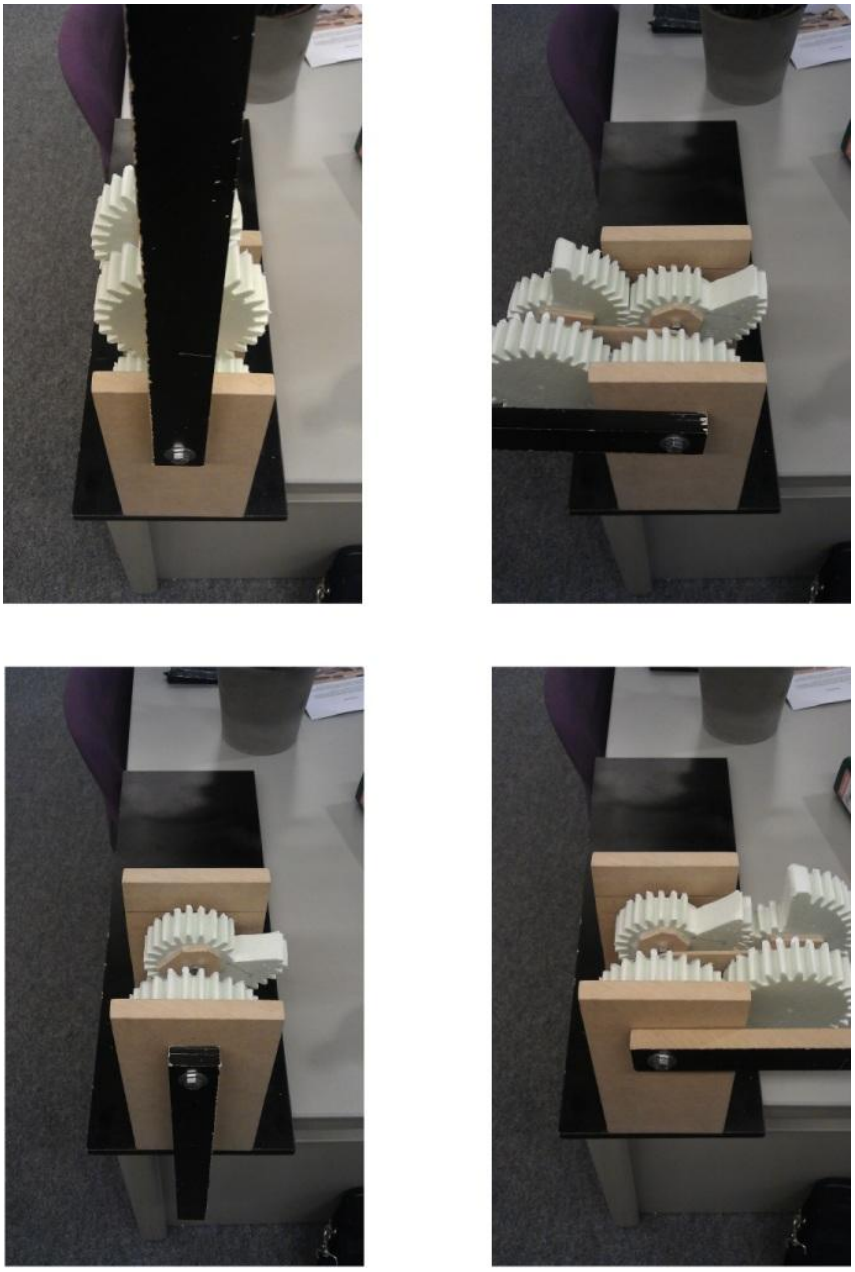


Figure B.1: The four positions of the arm. The offset in gear 3 and gear 4 is clearly visible. It can be seen that some modifications are made to the teeth at the place of this offset.

It can be seen that some modifications have to be made to the teeth that are located near the offset of the non circular gears. These modifications prevent the teeth from intersecting each other. When the gears are in contact at one point at a certain radius from the rotation point, the tangential velocity of a point at another radius is different according to

$$\Delta v = \omega \cdot \Delta r \tag{B.1}$$

Because these different velocities, the teeth also move with different velocities. When no modifications would be made, the teeth would prevent any rotation. With the current modification, the teeth will not intersect.

Appendix C

Prototype

C.1 Concept variations

The four concepts that are treated have different transmissions ratios. The difference between the transmissions ratios of concepts 4(b) and 4(c) with the concepts 4(a) and 4(d) are explained below by the derivation of the transmission ratios.

Concept 4(a)

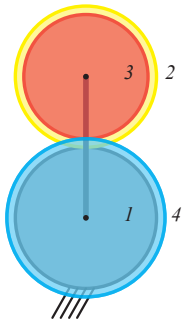


Figure C.1: Concept 4(a)

$$\omega_2 = \left(1 + \frac{r_1}{r_2}\right) \cdot \omega_{in}$$
$$\omega_3 = \omega_2$$
$$\omega_{out} = -\frac{r_3}{r_4} \cdot \omega_3 + \left(1 + \frac{r_3}{r_4}\right) \cdot \omega_{in}$$
$$\omega_{out} = -\frac{r_3}{r_4} \cdot \left(1 + \frac{r_1}{r_2}\right) \cdot \omega_{in} + \left(1 + \frac{r_3}{r_4}\right) \cdot \omega_{in}$$
$$\omega_{out} = -\frac{r_3}{r_4} \cdot \omega_{in} - \frac{r_1 \cdot r_3}{r_2 \cdot r_4} \cdot \omega_{in} + \omega_{in} + \frac{r_3}{r_4} \cdot \omega_{in}$$
$$\omega_{out} = \left(1 - \frac{r_1 \cdot r_3}{r_2 \cdot r_4}\right) \cdot \omega_{in}$$
$$\frac{\omega_{out}}{\omega_{in}} = 1 - \frac{r_1 \cdot r_3}{r_2 \cdot r_4}$$

Concept 4(b)

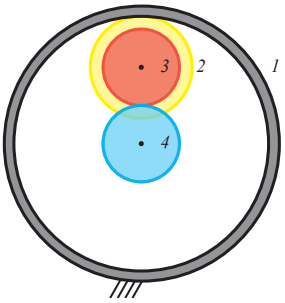


Figure C.2: Concept 4(b)

$$
\begin{aligned}
\omega_2 &= \left(1 - \frac{r_1}{r_2}\right) \cdot \omega_{in} \\
\omega_3 &= \omega_2 \\
\omega_{out} &= -\frac{r_3}{r_4} \cdot \omega_3 + \left(1 + \frac{r_3}{r_4}\right) \cdot \omega_{in} \\
\omega_{out} &= -\frac{r_3}{r_4} \cdot \left(1 - \frac{r_1}{r_2}\right) \cdot \omega_{in} + \left(1 + \frac{r_3}{r_4}\right) \cdot \omega_{in} \\
\omega_{out} &= -\frac{r_3}{r_4} \cdot \omega_{in} + \frac{r_1 \cdot r_3}{r_2 \cdot r_4} \cdot \omega_{in} + \omega_{in} + \frac{r_3}{r_4} \cdot \omega_{in} \\
\omega_{out} &= \left(1 + \frac{r_1 \cdot r_3}{r_2 \cdot r_4}\right) \cdot \omega_{in} \\
\frac{\omega_{out}}{\omega_{in}} &= 1 + \frac{r_1 \cdot r_3}{r_2 \cdot r_4}
\end{aligned}
$$

Concept 4(c)

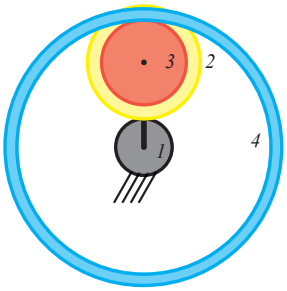


Figure C.3: Concept 4(c)

$$
\begin{aligned}
\omega_2 &= \left(1 + \frac{r_1}{r_2}\right) \cdot \omega_{in} \\
\omega_3 &= \omega_2 \\
\omega_{out} &= \frac{r_3}{r_4} \cdot \omega_3 + \left(1 - \frac{r_3}{r_4}\right) \cdot \omega_{in} \\
\omega_{out} &= \frac{r_3}{r_4} \cdot \left(1 + \frac{r_1}{r_2}\right) \cdot \omega_{in} + \left(1 - \frac{r_3}{r_4}\right) \cdot \omega_{in} \\
\omega_{out} &= \frac{r_3}{r_4} \cdot \omega_{in} - \frac{r_1 \cdot r_3}{r_2 \cdot r_4} \cdot \omega_{in} + \omega_{in} - \frac{r_3}{r_4} \cdot \omega_{in} \\
\omega_{out} &= \left(1 + \frac{r_1 \cdot r_3}{r_2 \cdot r_4}\right) \cdot \omega_{in} \\
\frac{\omega_{out}}{\omega_{in}} &= 1 + \frac{r_1 \cdot r_3}{r_2 \cdot r_4}
\end{aligned}
$$

Concept 4(d)

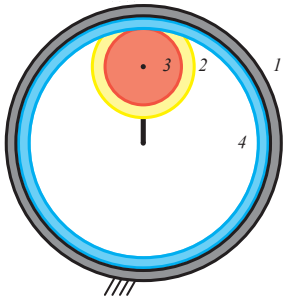


Figure C.4: Concept 4(d)

$$
\begin{aligned}
\omega_2 &= \left(1 - \frac{r_1}{r_2}\right) \cdot \omega_{in} \\
\omega_3 &= \omega_2 \\
\omega_{out} &= \frac{r_3}{r_4} \cdot \omega_3 + \left(1 - \frac{r_3}{r_4}\right) \cdot \omega_{in} \\
\omega_{out} &= \frac{r_3}{r_4} \cdot \left(1 - \frac{r_1}{r_2}\right) \cdot \omega_{in} + \left(1 - \frac{r_3}{r_4}\right) \cdot \omega_{in} \\
\omega_{out} &= \frac{r_3}{r_4} \cdot \omega_{in} - \frac{r_1 \cdot r_3}{r_2 \cdot r_4} \cdot \omega_{in} + \omega_{in} - \frac{r_3}{r_4} \cdot \omega_{in} \\
\omega_{out} &= \left(1 - \frac{r_1 \cdot r_3}{r_2 \cdot r_4}\right) \cdot \omega_{in} \\
\frac{\omega_{out}}{\omega_{in}} &= 1 - \frac{r_1 \cdot r_3}{r_2 \cdot r_4}
\end{aligned}
$$

C.2 Generation of the teeth profile

The generation of the teeth profile is performed by unrolling a predefined rack over each four pitch curves. The parameters of the rack determine the size of the teeth. This rack is visualised in Fig. C.5. The parameters that define the rack are given in Tab. C.1. The size of the rack that has been used is defined by the module m, the height h of the tooth, the clearance c between two meshing teeth and the profile angle α. The module is defined by

$$m = \frac{l_{curve}}{N_{teeth} \cdot \pi} \tag{C.1}$$

where l_{curve} is the total length of the pitch curve and N_{teeth} is the number of teeth on this pitch curve which be chosen arbitrary.

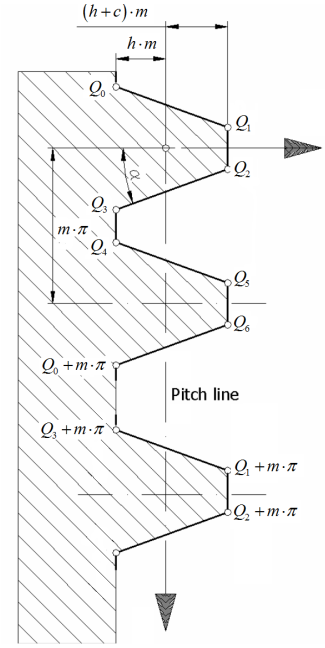


Figure C.5: The basic rack that is used for the generation of the teeth.

Table C.1: Parameters of the rack for the generation of the teeth of the non circular gears.

Parameter	Symbol	Value
Clearence between two meshing teeth	c [mm]	0.25
Height tooth	h [mm]	1
Module	m [mm]	1.34
Number of teeth	N_{teeth} [-]	80
Profile angle	α [rad]	0.35

The pitch line of the rack is placed on the pitch curve and is rotated such that the pitch line is tangent to the pitch curve. The common point where the pitch line and pitch curve touch is moved about the pitch curve, while the rack keeps tangent to the pitch curve. The successive discrete positions of the rack are shown in Fig. C.6. The intersection points of the lines from the rack define the outer shape of the teeth. By repeating this method over the pitch curve all teeth of the gears are generated and the result of the teeth of gear 3 is visible in the last figure of C.6.

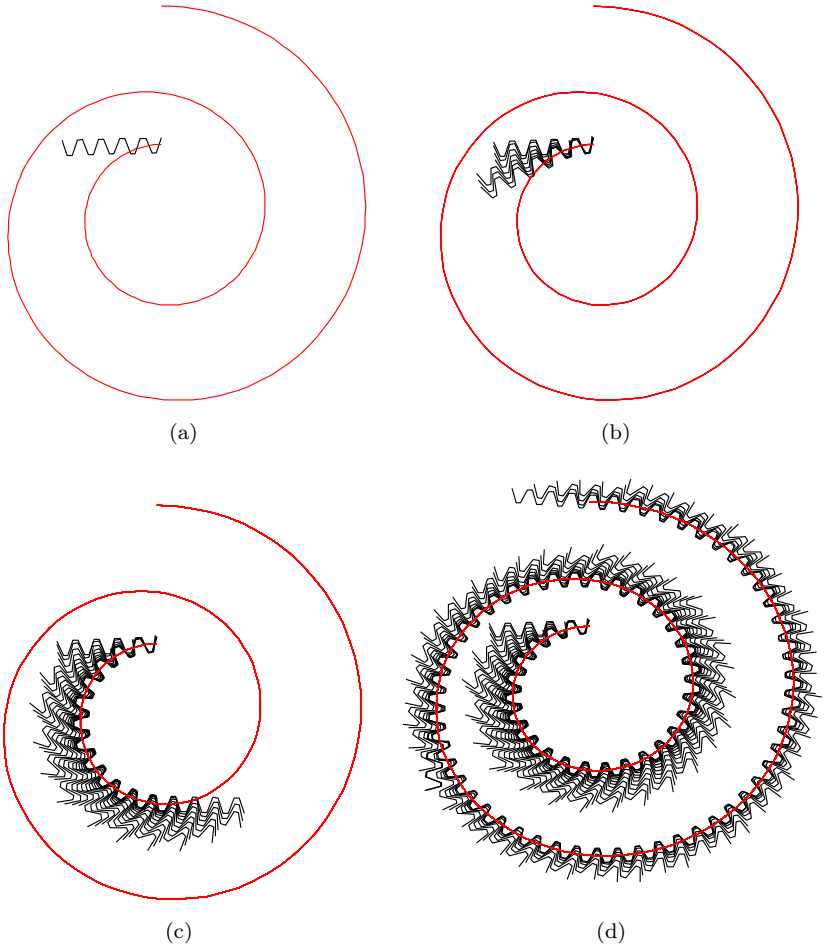


Figure C.6: Four figures of the generation of the teeth profile with the rack-cutter method: (a) the rack is placed at the beginning of the pitch curve and is rotated such that the pitch line of the rack is tangent to the pitch curve, (b) the rack unrolls around the pitch curve and the preceding locations of the rack are stored, (c) the first teeth on the pitch curve become visible, (d) the complete teeth profile of gear 3 is shown. The shape of the teeth can now be extracted from this figure.

C.3 Alignment problems

During assembly, problems were encountered regarding the alignment of the two axles. The component consist of three parts that are connected by means of two welds. The two axles are pressed into a bended plate, and to increase the strength of the connection, the part and the plate are welded together.
The problems of the seperate parts is that there is always a certain accuracy between the alignment of the two axles. When the holes are not perfectly aligned there is an offset between the center of the holes. It can also happen that the axles are under an angle, such that they are not parallel.
The problem of the misalignment was clearly present. A lot of friction was introduced and it was hard to rotate the component when it was in place. We decided to manufacture the component out of a single bar which has a diameter of 80 mm. This cumbersome method proofed to be effective. Because there are no welds or drilling holes, the axles are perfectly aligned.

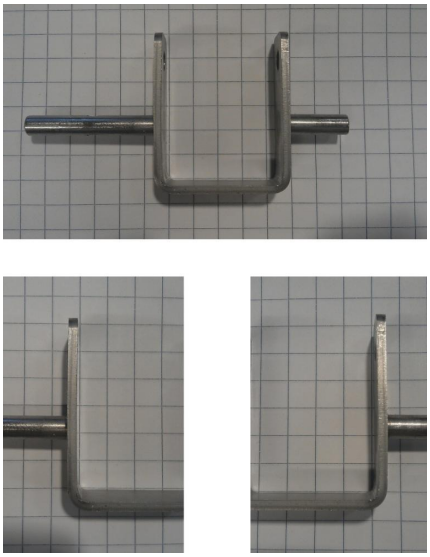


Figure C.7: The old component which is build out of three separate parts. The two axles are welded onto the bended plate. The result of welding is that the plate is deformed due to the heat transfer into the plate. It is not sure that the two axles are perfectly alligned.

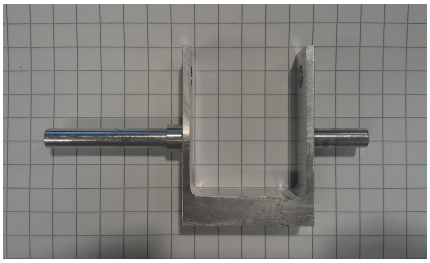


Figure C.8: The new component which is manufactured out of one single part. There is no deformation of the flanges. It can be stated that the two axles are perfectly alligned.

C.4 Drawings

C.4.1 Selective laser sintering parts

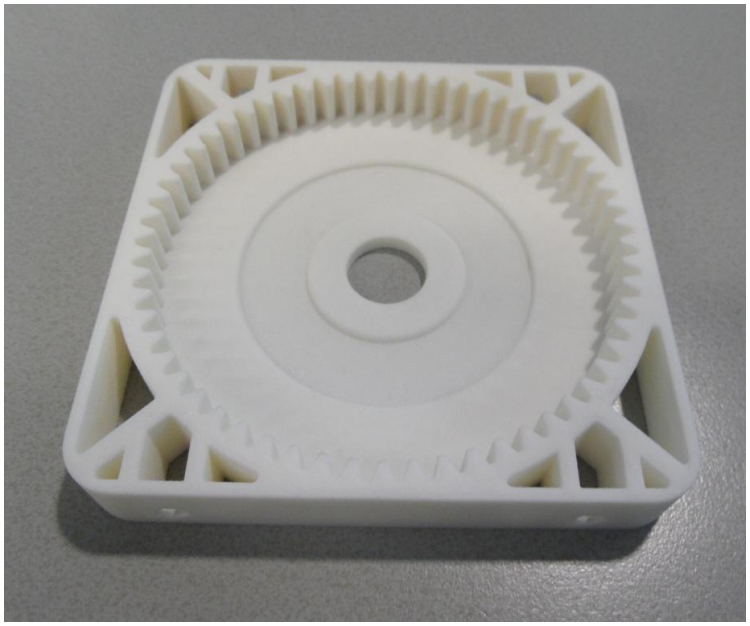


Figure C.9: Gear 1 is an internal gear that is connected to the world such that it does not rotate.

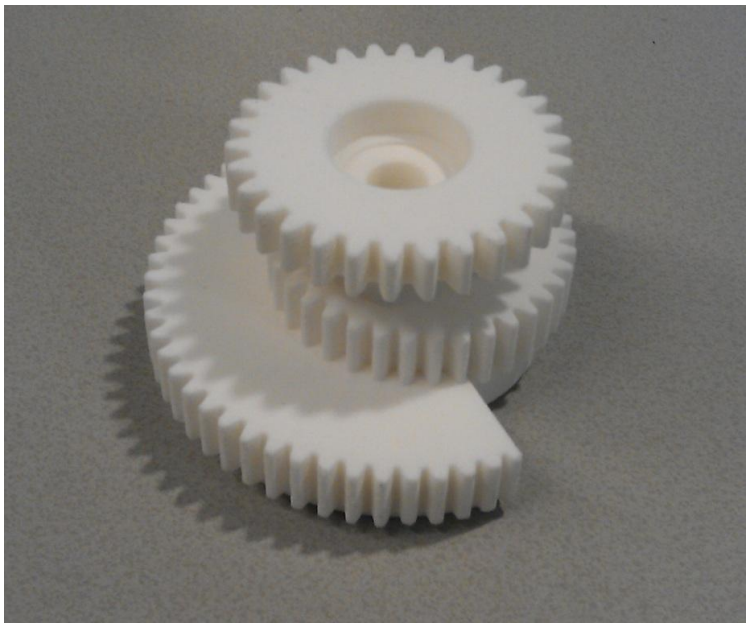


Figure C.10: Gear 2 and 3 external gears and connected to each other because their angular velocities are equal. Gear 2 meshes with gear 1 and therefore is forced to rotate.

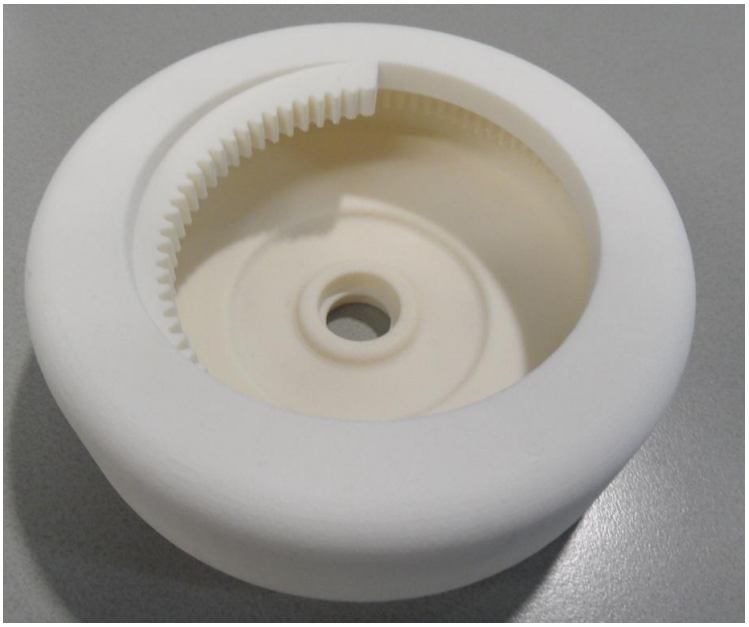
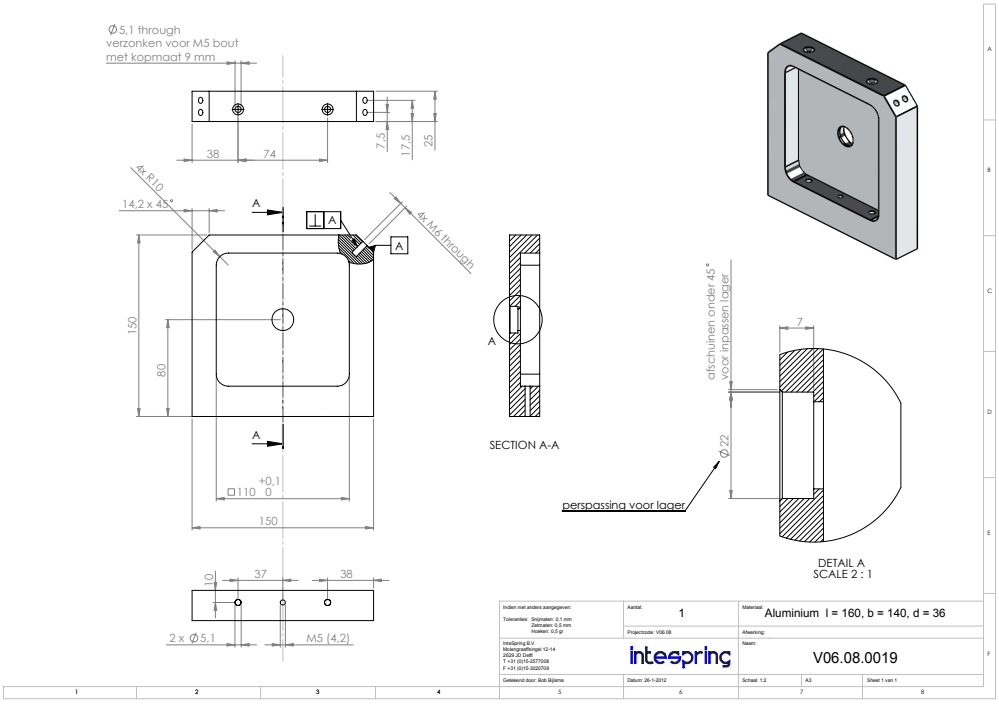
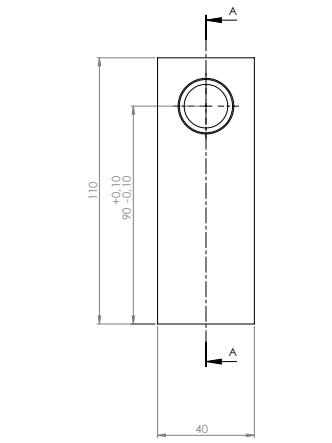
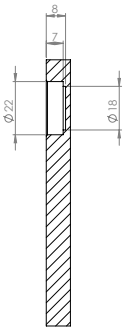


Figure C.11: Gear 4 is an internal gear with a non circular pitch curve. This gear meshes with gear 4 and makes a repricocating rotation.

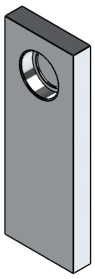
C.4.2 Milling and turning parts

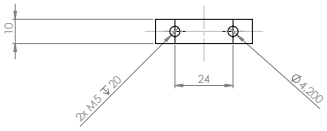




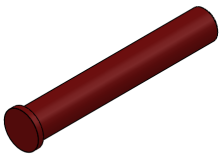


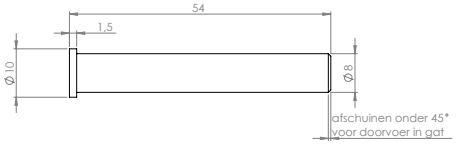
SECTION A-A





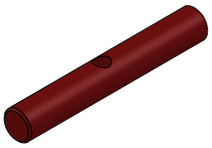
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Getekend door: Bob Bijlsma	Datum: 19-1-2012	Schaal: 1:1	A3 Sheet 1 van 1

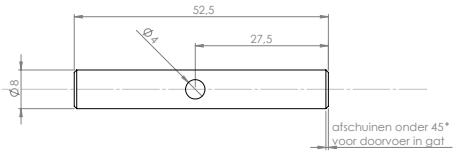




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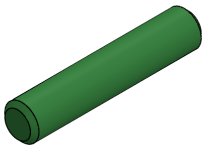
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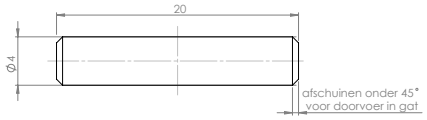




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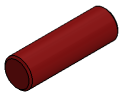
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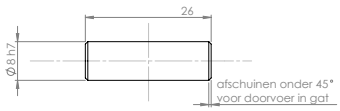




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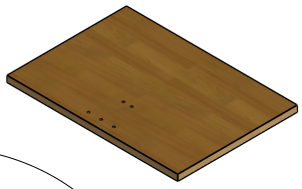
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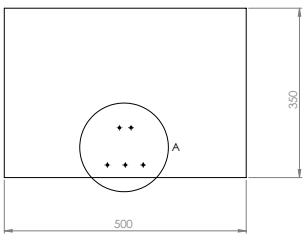


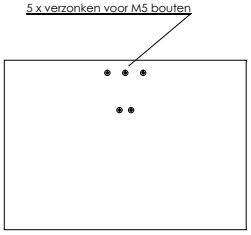


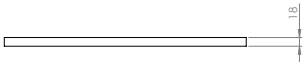
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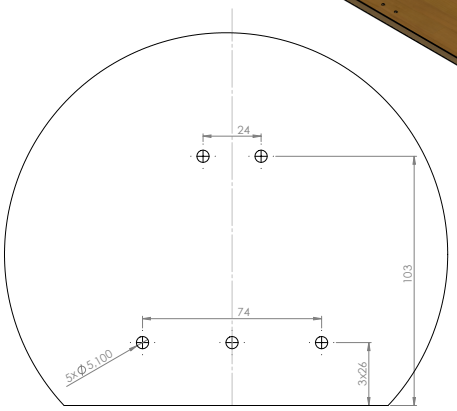
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Getekend door: Bob Bijlsma	Datum: 26-1-2012	Schaal: 2:1 | A3 | Sheet 1 van 1





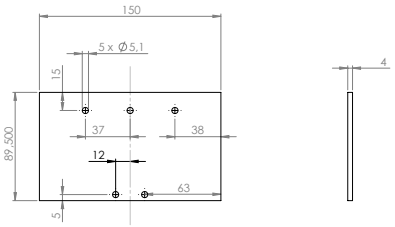


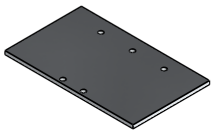




DETAIL A
SCALE 1 : 1

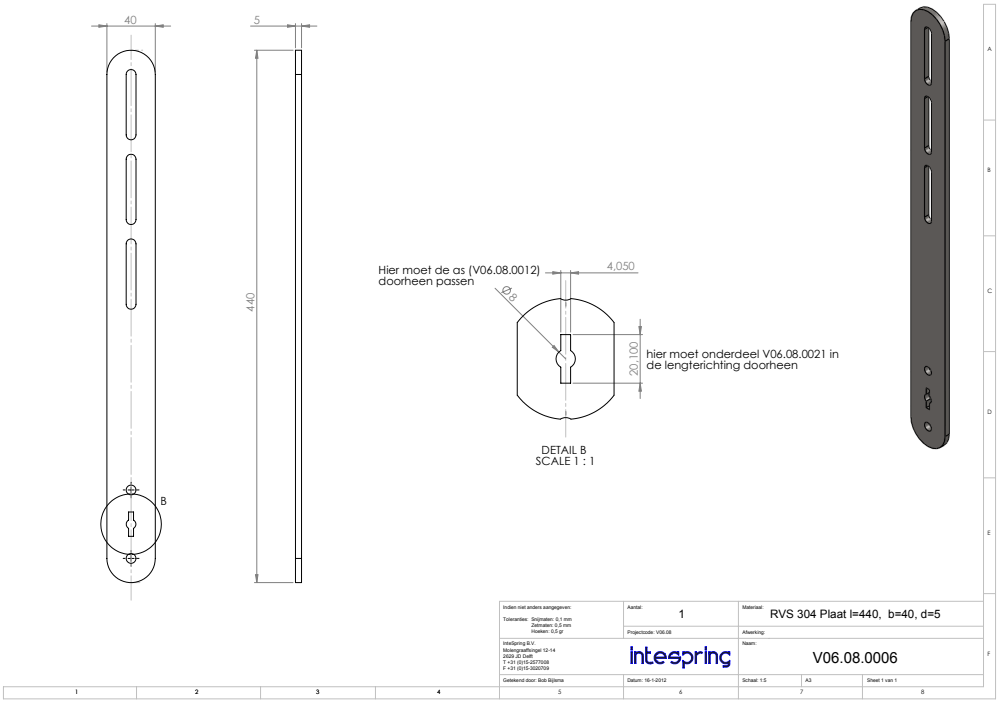
Indien niet anders aangegeven: Toleranties: Snijmaten: 0.1 mm, Zetmaten: 0.5 mm, Hoeken: 0.5 gr	Aantal:	Materiaal:
	Projectcode:	Afwerking:
IntieSpring B.V. Mekelweg/Rijnvgel 12-14 2629 JD Delft T +31 (0)15-2577008 F +31 (0)15-3020709	intespring	Naam: V06.08.0031
Getekend door:	Datum: 10-1-2012	Schaal: 1:5 | A3 | Sheet 1 van 1

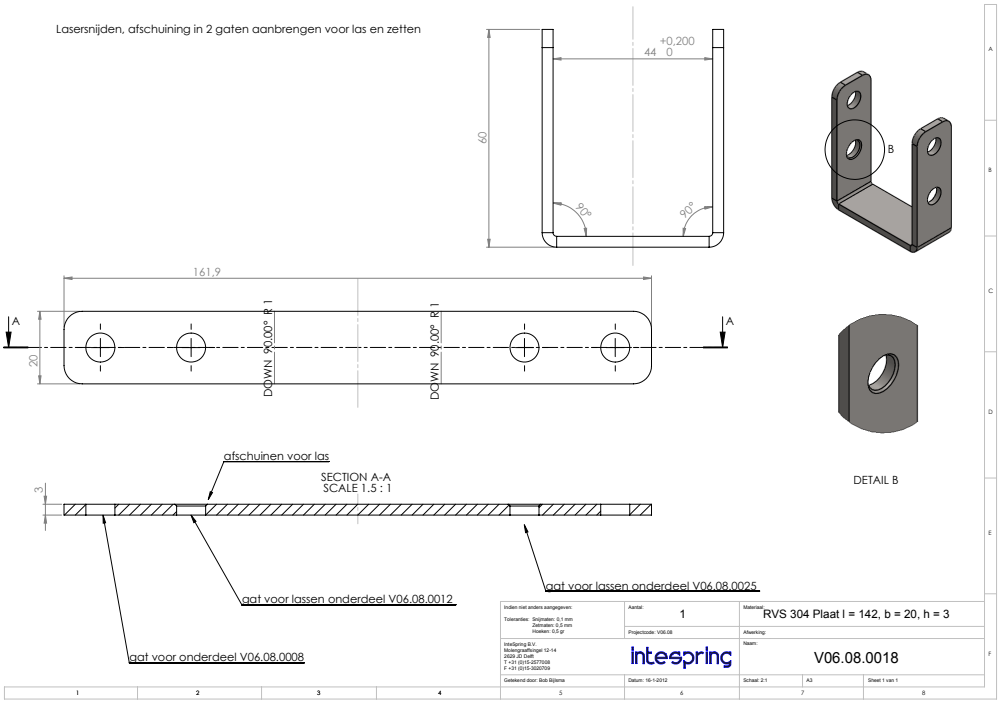




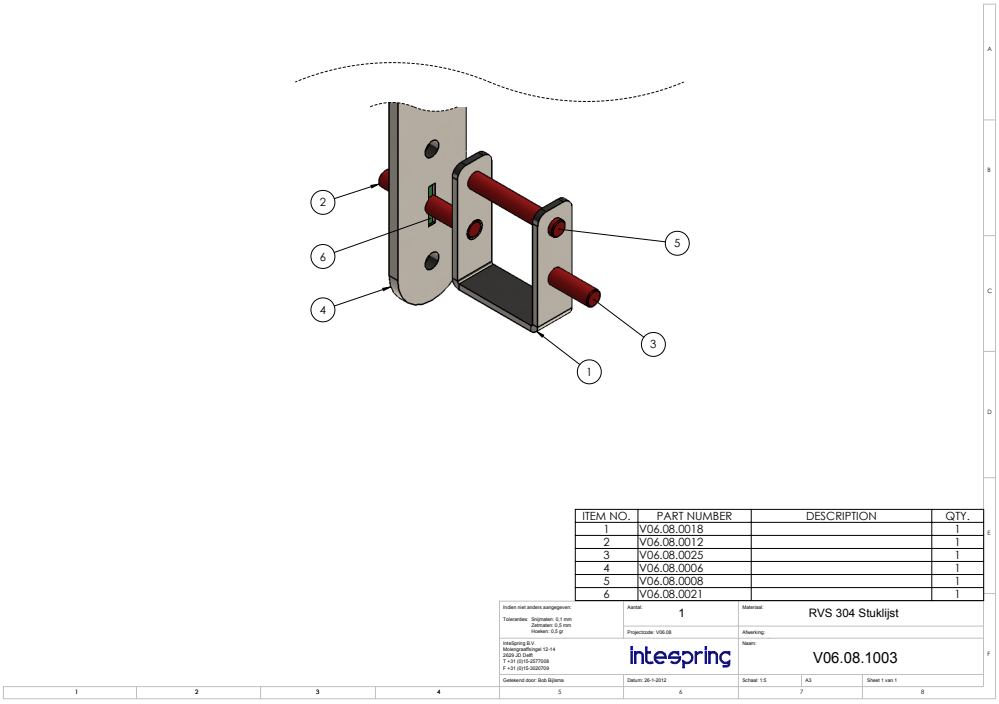
Indien niet anders aangegeven: Toleranties: Snijmaten: 0,1 mm Zetmaten: 0,5 mm Hoeken: 0,5 gr	Aantal:	Materiaal:	
	Projectcode:	Afwerking:	
IntesSpring B.V. Nikkerpadfforgat 12-14 2629 JD Delft T +31 (0)15-2577008 F +31 (0)15-3620709	intespring	Naam: V06.08.0044	
Getekend door:	Datum: 26-1-2012	Schaal: 1:2 / A3	Sheet 1 van 1

C.4.3 Lasercutting parts





C.4.4 Assembly drawings



ITEM NO.	PART NUMBER	DESCRIPTION	QTY.
1	V06.08.0018		1
2	V06.08.0012		1
3	V06.08.0025		1
4	V06.08.0006		1
5	V06.08.0008		1
6	V06.08.0021		1

Indien niet anders aangegeven:
Toleranties: Snijmaten: 0.1 mm
Zetmaten: 0.5 mm
Hoeken: 0.5 gr

Aantal: 1

Materiaal: RVS 304 Stuklijst

Projectcode: V06.08

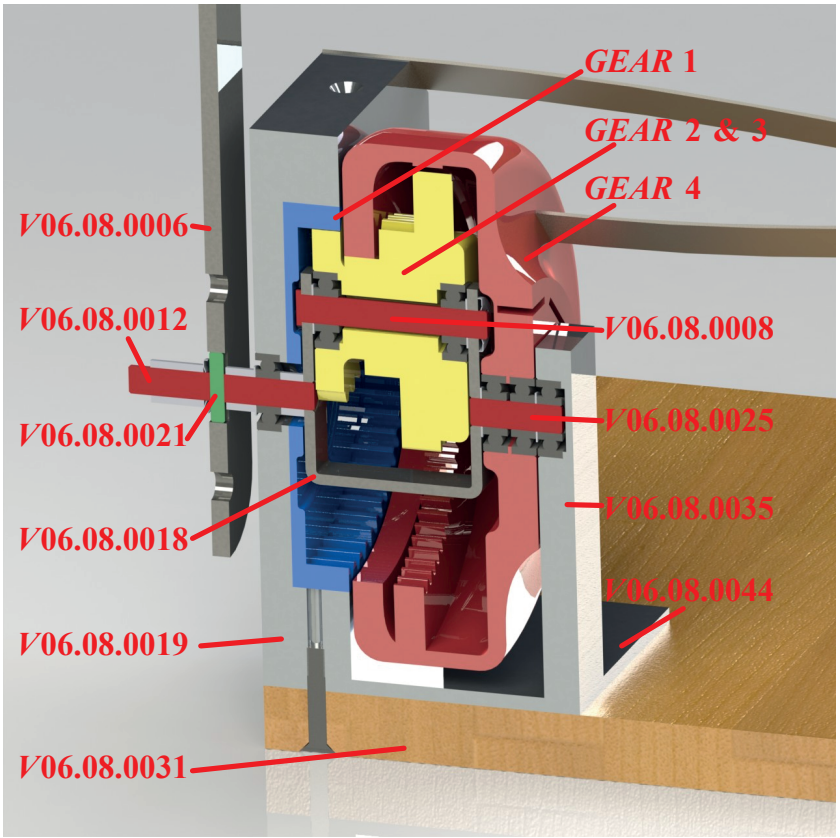
Afwerking:

IntesSpring B.V.
Horsterweg Zuidoost 12-14
3833 JC Duth
T +31 (0)15-2577066
F +31 (0)15-3822709

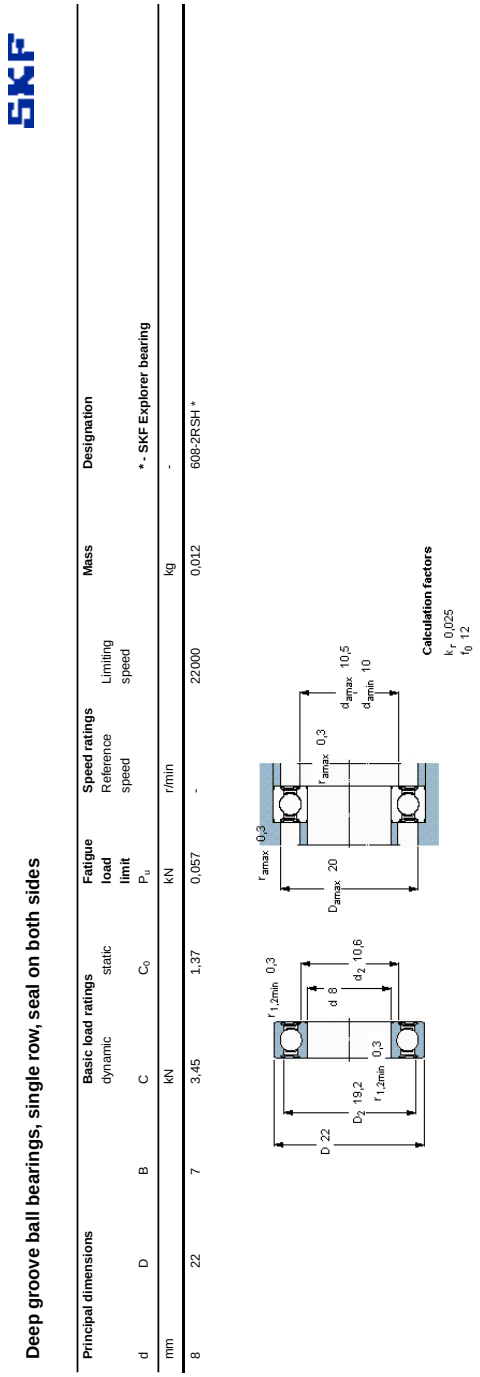
intespring

Naam: V06.08.1003

Getekend door: Bob Bijlsma | Datum: 26-1-2012 | Schaal: 1:5 | A3 | Sheet 1 van 1



C.5 Ball bearings



Deep groove ball bearings, single row, seal on both sides

Principal dimensions			Basic load ratings		Fatigue load limit	Speed ratings		Mass	Designation
			dynamic	static		Reference speed	Limiting speed		
d	D	B	C	C_0	P_u				* - SKF Explorer bearing
mm			kN		kN	r/min		kg	-
8	22	7	3,45	1,37	0,057	-	22000	0,012	608-2RSH *

Figure C.12: http://docs-europe.electrocomponents.com/webdocs/0f8e/0900766b80f8e833.pdf

C.6 Laser Sintering material properties

Laser Sintering Material Properties

Materialise
driving your innovations

Details Laser Sintering

Standard lead time: From 4 working days depending on part size, number of components and finishing degrees.
From 2 working days for smaller parts.

Standard accuracy: ± 0.3% (with lower limit of ± 0.3 mm).

Surface finish: Laser Sintering parts typically have a grainy surface but all kinds of (very) fine finishing are possible. They can be sandblasted, coloured (impregnated), painted, covered, coated, …

Maximum part dimensions: Dimensions are unlimited when the parts may be composed of several sub-parts. The build area is 700x380x580mm.

Minimum wall thickness: 1 mm, but living hinges are possible at 0.3 mm

Capacity: 2 Sinterstation 2500 HS HiQ with a build volume of 360x310x450mm
2 EOS P380 machine with a build volume of 340x340x620mm
5 EOSINT P730 machines with a build volume: 700x380x580mm
1 EOS Formiga P100 machine with a build volume: 200x250x300 mm
7 DTM machines with a build volume: 360x310x400 mm

Datasheet					
Laser Sintering					
	Units	Condition	**PA**	**PA-GF**	**Alumide**
Tensile Modulus	MPa	DIN EN ISO 527	1650+/- 150	3200 +/- 200	3800+/- 150
Tensile Strength	MPa	DIN EN ISO 527	48 +/- 3	51 +/- 3	48 +/- 3
Elongation at Break	%	DIN EN ISO 527	20 +/- 5	6 +/- 3	3.5 +/- 1
Flexural Modulus	N/mm²	DIN EN ISO 178	1500 +/- 130	2900 +/- 150	3600 +/- 150
Charpy – Impact strength	MPa	DIN EN ISO 179	53 +/- 3.8	35 +/- 6	29 +/- 2
Charpy – Notched Impact Strength	MPa	DIN EN ISO 179	4.8 +/- 0.3	5.4 +/- 0.6	4.6 +/- 0.3
Izod – Impact Strength	J/m²	DIN EN ISO 180	32.8 +/- 3.4	21.3 +/- 1.7	NA
Izod - Notched Impact Strength	J/m²	DIN EN ISO 180	4.4 +/- 0.4	4.2 +/- 0.3	NA
Ball Indentation Hardness		DIN EN ISO 2039	77.6 +/- 2	98	NA
Shore D-hardness		DIN 53505	75 +/- 2	80 +/- 2	76 +/- 2
Heat Deflection t°	°C	ASTM D648 (1.82MPa)	86	110	130
Vicat Softening Temperature B/50	°C	DIN EN ISO 306	163	163	169
Vicat Softening Temperature A/50	°C	DIN EN ISO 306	181	179	NA

The information and values included in these datasheets, although based on Materialise's knowledge and experience and thus presented in good faith and believed to be accurate, is provided for your guidance only. This information does not release a third party from conducting his own procedures and tests to determine suitability. All guarantees with respect to the information contained herein are explicitly denied.
Last update: 6/07/2011

Figure C.13: http://www.materialise.com/laser-sintering-materials

Appendix D

Measurement setup

The measurements were carried out with a M250-2.5 CT universal testing machine of Testometric, see Fig. D.1. The specifications of this machine are shown in Tab. D.1.



Figure D.1: M250-2.5 CT universal testing machine of Testometric

Table D.1: Specifications of the M250-2.5 CT universal testing machine of Testometric

Description	Specification
Machine capacity	2.5 kN
Speed range	0.001 to 1000 mm/min (in steps of 0.001 mm/min)
Crosshead travel (excluding grips)	1000 mm
Throat	200 mm

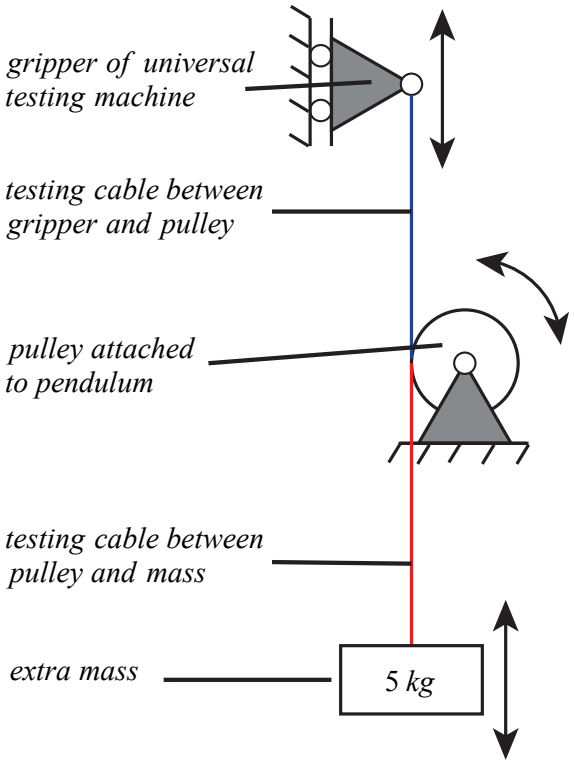


Figure D.2: Schemetic overview of the measurement setup

In Fig. D.2 the schematic representation of the measurement setup is given. Fig. D.3 shows the schematic setup which is used for the measurements. The cables that are used have a diameter of 1 mm. Multiple consecutive measurements have been executed. The measurements include the following

- Consecutive measurements of the whole system including the pendulum, the torsion bars, the gear train and the extra mass
- Consecutive measurements of the system without the torsion bars.
- Consecutive measurements of the system without the pendulum and the torsion bars. This measurement only include the gear train and the extra mass.
- One measurement of the mass only. This measurement should be subtracted from the other measurements.

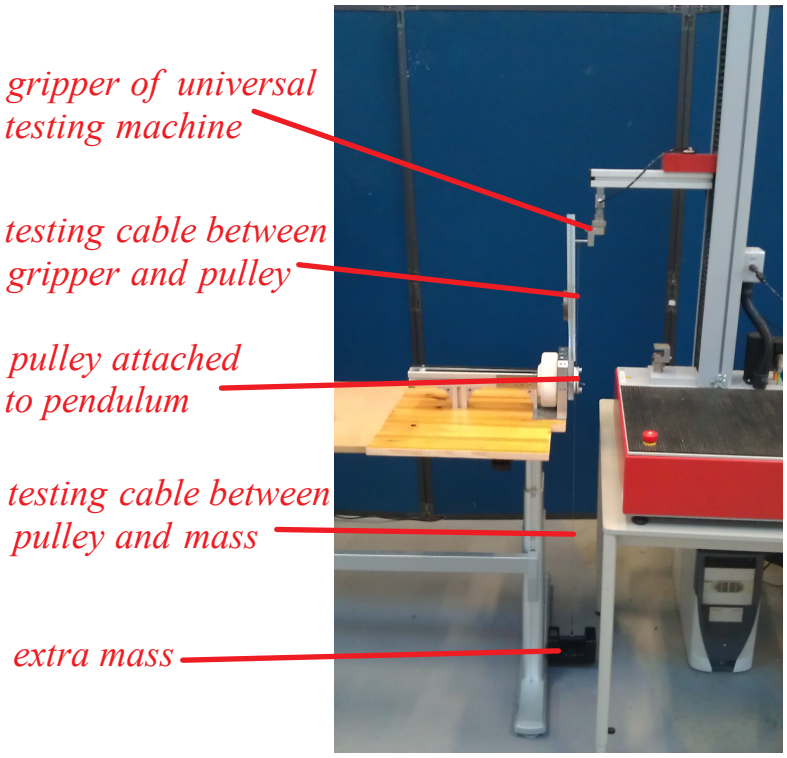


Figure D.3: Overview of the setup that is used in for the measurements.

Appendix E

Matlab code

The following matlab files are used to define the gears and the teeth profile on the gears.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% Run File, generation of non circular gears for creating         %%%
%%% perfect balance over an unlimitid range of motion               %%%
%%% MSc Assignment Bob Bijlsma, bobbijlsma@hotmail.com, 06-27560364 %%%
%%% 2011 - 2012                                                     %%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

clc;
close all;
clear all;

global t K m g l R tzeros r1 r2 x0 y0
% load file Constants.m
Constants
% load file Optimization.m
Optimization
% load file Calculations.m
Calculations
% load file Teeth.m   [adapted method]
Teeth
% load file Plots.m
Plots
% load file Print.m
Print
```

```
%%%%%%%%%%%%%%%%%%
%%% Constants %%%
%%%%%%%%%%%%%%%%%%

Ntand  = 80;                                % number of teeth [-]
step   = Ntand*30;                          % number of steps [-]
par    = 10;                                % number of design points [-]
t      = linspace(0,1,step);                % time span [s]
tzeros = 0*t;                               % zero vector [-]
m      = 1;                                 % mass [kg]
g      = 9.81;                              % gravitation constant [m/s2]
l      = 0.3;                               % arm [mm]
K      = 15*m*g*l;                          % stiffness of bars [N/mm]
R      = 25;                                % length arm [mm]
r1     = tzeros+(2*R);                      % radius r1 [mm]
r2     = r1-R;                              % radius r2 [mm]
count  = ones(1,40);
Switch = [0 0 0 1 0 1];
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% Initial guess curve %%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%
x0 = linspace(0,1,par);                                    % defined points in time
y0 = 10*[0.9 ones(1,par-1)];

%%%%%%%%%%%%%%%%%%%%%
%%% optimization %%%
%%%%%%%%%%%%%%%%%%%%%
options    = optimset('TolX',1e-10,'TolFun',1e-10,...
                      'MaxFunEvals',10000*step,'MaxIter',10000);  % Options
[yopt,afw] = fminunc(@fun,y0,options)                              % Optimization
```

```
function [Obj] = fun(x)
global  t K R tzeros r1 r2 m g l x0 y0

% Constants
y0 = x;
r3 = spline(x0,y0,t);                                % radius r3 defined by spline
r4 = r3+R;                                           % radius r4

% Omega's
omegaI = tzeros+2*pi;                                % ang. velocity input  [rad/s]
omega4 = (1-((r1.*r3)./(r2.*r4))).*omegaI;           % ang. velocity gear 4 [rad/s]

% Theta's
thetaI = cumtrapz(t,omegaI);                         % angle input  [rad]
theta4 = cumtrapz(t,omega4);                         % angle gear 4 [rad]

% Energies
Emass   = m*g*l*cos(thetaI)+m*g*l;                   % Pot. energy of mass [J]
Espring = 0.5*K*theta4.^2;                           % Pot. energy of spring [J]
Eref    = 2*m*g*l-Emass;                             % Reference energy [J]

% Objective
Obj  = sqrt(sum((Espring-Eref).^2)/length(y0)); % Objective function
```

```
%%%%%%%%%%%%%%%%%%%%%
%%% Calculations %%%
%%%%%%%%%%%%%%%%%%%%%

%Radii
r3=spline(x0,yopt,t);                                % radius r3 defined by spline
r4=r3+R;                                             % radius r4

%Omega's
omegaI=tzeros+2*pi;                                  % ang. vel. input  [rad/s]
omega1=tzeros;                                       % ang. vel. gear 1 [rad/s]
omega2=omegaI.*(1-r1/r2);                            % ang. vel. gear 2 [rad/s]
omega3=omega2;                                       % ang. vel. gear 3 [rad/s]
omega4=(1-((r1.*r3)./(r2.*r4))).*omegaI;             % ang. vel. gear 4 [rad/s]

%theta's                                             INTEGRATING OMEGA
thetaI=cumtrapz(t,omegaI);                           % angle input  [rad]
theta1=cumtrapz(t,omega1);                           % angle gear 1 [rad]
theta2=cumtrapz(t,omega2);                           % angle gear 2 [rad]
theta3=cumtrapz(t,omega3);                           % angle gear 3 [rad]
theta4=cumtrapz(t,omega4);                           % angle gear 4 [rad]

%alpha's       ANGLES BETWEEN REFERENCE POINT AND CONTACT POINT BETWEEN GEARS
alpha1=thetaI+theta1;                                % relative angle gear 1 [rad]
alpha2=thetaI-theta2+pi;                             % relative angle gear 2 [rad]
alpha3=thetaI-theta3;                                % relative angle gear 3 [rad]
```

```
alpha4=thetaI-theta4;                                % relative angle gear 4 [rad]

%shapes                                              SHAPES
curve = zeros(2,step,4);
curve(:,:,1)=[r1.*cos(alpha1);r1.*sin(alpha1)]; % shape of gear 1
curve(:,:,2)=[r2.*cos(alpha2);r2.*sin(alpha2)]; % shape of gear 2
curve(:,:,3)=[r3.*cos(alpha3);r3.*sin(alpha3)]; % shape of gear 3
curve(:,:,4)=[r4.*cos(alpha4);r4.*sin(alpha4)]; % shape of gear 4

% Energies
Emass   = m*g*l*(1+cos(thetaI));                     % pot. energy of mass [J]
Espring = 0.5*K*theta4.^2;                           % pot. energy of spring [J]
Etot    = Espring+Emass;                             % total energy in system [J]
Eref    = 2*m*g*l-Emass;                             % reference energy [J]
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% Teeth on non circular gears %%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
for k = 1:1
    % Calculate length of pitch curve
    lbaan = 0;
    for i=2:step
        dx = abs(curve(1,i,k) - curve(1,i-1,k));
        dy = abs(curve(2,i,k) - curve(2,i-1,k));
        lbaan = lbaan+sqrt(dx^2+dy^2);
    end

    % Parameters of rack
    h = 1;                                           % Height of tooth [mm]
    mo=lbaan/Ntand/pi;                               % Modulus [mm]
    a = 20;                                          % Pressure angle [ ]
    c = 0.25;                                        % Clearance [mm]
    clear Q                                          % Clear rack
    Q(1,1) = mo*(-pi/4-h*tan(a*(pi/180)));       Q(1,2) = -h*mo;
    Q(2,1) = mo*(-pi/4+(h+c)*tan(a*(pi/180)));   Q(2,2) = (h+c)*mo;
    Q(3,1) = mo*(pi/4-(h+c)*tan(a*(pi/180)));    Q(3,2) = (h+c)*mo;
    Q(4,1) = mo*(pi/4+h*tan(a*(pi/180)));        Q(4,2) = -h*mo;
    Q(5,1) = Q(1,1)+mo*pi;                           Q(5,2) = Q(1,2);
    Q(6,1) = Q(2,1)+mo*pi;                           Q(6,2) = Q(2,2);
    Q(:,1) = Q(:,1)-Q(1,1);                          % Define rack

    hold on;
    for i = 1:step
        if i==1                                                  % FIRST STEP
            u12 = [curve(1,i+1,k)-curve(1,i,k);curve(2,i+1,k)-curve(2,i,k)];
            n12 = [u12(2);-u12(1)]'/norm(u12);
            th2 = acos(-n12(2));
            th(i) = acos(-n12(2));  % hoek waarom rack geroteerd moet worden
            rot = [cos(th(i)) -sin(th(i)); sin(th(i)) cos(th(i))];
            arcl = 0;
            T1(:,1) = Q(:,1)-arcl;
            T1(:,2) = Q(:,2);
            for j=1:size(Q,1)
                R1(j,:) = rot*(T1(j,:))';
            end
            T2(:,1) =  R1(:,1) + curve(1,i,k);
            T2(:,2) =  R1(:,2) + curve(2,i,k);
            T2old = T2;
            xA = T2(1,1);                          % x coordinate first point
            yA = T2(1,2);                          % y coordinate first point
        elseif i==step                                           % LAST STEP
            uend = [curve(1,i,k)-curve(1,i-1,k);curve(2,i,k)-curve(2,i-1,k)];
            nend = [uend(2);-uend(1)]'/norm(uend);
            th(i) = acos(-nend(2));
            if(th(i)<th2)
                th(i)=-th(i);
```

```
52              end
53              rot = [cos(th(i)) -sin(th(i)); sin(th(i)) cos(th(i))];
54              dx = abs(curve(1,i,k) - curve(1,i-1,k));
55              dy = abs(curve(2,i,k) - curve(2,i-1,k));
56              arcl = arcl+sqrt(dx^2+dy^2);
57              T1(:,1) = Q(:,1)-arcl;
58              T1(:,2) = Q(:,2);
59              for j=1:size(Q,1)
60                  R1(j,:) = rot*(T1(j,:))';
61              end
62              T2(:,1) =  R1(:,1) + curve(1,i,k);
63              T2(:,2) =  R1(:,2) + curve(2,i,k);
64              xold43(count(10))=xold(4);          yold43(count(10))=yold(4);
65              xold53(count(10))=xold(5);          yold53(count(10))=yold(5);
66              xold3 = T2(3,1);                    yold3 = T2(3,2);
67              xold4 = T2(4,1);                    yold4 = T2(4,2);
68
69              [x7t y7t] = polyxpoly([xold4_2;xold5_2], [yold4_2;yold5_2],...
70                                    [xold3;xold4], [yold3;yold4]);
71              if ¬isempty(x7t)
72                  x7(count(9))=x7t;               y7(count(9))=y7t;
73                  count(9)=count(9)+1;
74              end
75          else                                                    % OTHER STEPS
76              u = [curve(1,i+1,k)-curve(1,i-1,k);curve(2,i+1,k)-curve(2,i-1,k)];
77              n = [u(2);-u(1)]'/norm(u);
78              th(i) = acos(-n(2));
79              if(th(i)<th2)
80                  th(i)=-th(i);
81              end
82              th2=acos(-n(2));
83
84              rot = [cos(th(i)) -sin(th(i)); sin(th(i)) cos(th(i))];
85              dx = abs(curve(1,i,k) - curve(1,i-1,k));
86              dy = abs(curve(2,i,k) - curve(2,i-1,k));
87              arcl = arcl+sqrt(dx^2+dy^2);
88              if arcl>mo*pi
89                  arcl = arcl-mo*pi;
90                  count(5) = count(5)+1;
91                  Switch(1)=0;
92                  Switch(2)=1;
93                  Switch(4) = 1;
94                  Switch(5) = 1;
95              end
96              T1(:,1) = Q(:,1)-arcl;
97              T1(:,2) = Q(:,2);
98              for j=1:size(Q,1)
99                  R1(j,:) = rot*(T1(j,:))';
100             end
101             T2(:,1) =  R1(:,1) + curve(1,i,k);
102             T2(:,2) =  R1(:,2) + curve(2,i,k);
103
104
105             % Filter out the wrong points
106             xold = [T2old(:,1)];        yold = [T2old(:,2)];
107             xnew = [T2(:,1)];           ynew = [T2(:,2)];
108             [x1t y1t] = polyxpoly(xold(1:2), yold(1:2), xnew(1:2), ynew(1:2));
109             if ¬isempty(x1t)
110                 xB(count(1))=x1t;       yB(count(1))=y1t;
111                 count(1)=count(1)+1;
112             end
113             [x2t y2t] = polyxpoly(xold(3:4), yold(3:4), xnew(3:4), ynew(3:4));
114             if ¬isempty(x2t)
115                 x2(count(2))=x2t;       y2(count(2))=y2t;
116                 count(2)=count(2)+1;
117                 Switch(6) = 1;
118                 count(2);
```

```
119             elseif isempty(x2t) && Switch(6) == 1
120                 xold3 = xold(3);        yold3 = yold(3);
121                 xold4 = xold(4);        yold4 = yold(4);
122                 Switch(6) = 0;
123                 count(2);
124             end
125             [x3t y3t] = polyxpoly(xold(2:3), yold(2:3), xnew(2:3), ynew(2:3));
126             if isempty(x3t) && Switch(1)==0 && Switch(2)==0
127                 xhoek5(count(4))=xold(3);   yhoek5(count(4))=yold(3);
128                 xhoek6(count(4))=xnew(3);   yhoek6(count(4))=ynew(3);
129                 count(4)=count(4)+1;
130             elseif isempty(x3t) && Switch(1)==1 && Switch(2)==0
131                 xhoek5(count(4))=xold(2);   yhoek5(count(4))=yold(2);
132                 xhoek6(count(4))=xnew(2);   yhoek6(count(4))=ynew(2);
133                 count(4)=count(4)+1;
134             end
135             if ¬isempty(x3t)
136                 x3(count(3))=x3t;
137                 y3(count(3))=y3t;
138                 xhoek1(count(3))=xold(2);    yhoek1(count(3))=yold(2);
139                 xhoek2(count(3))=xold(3);    yhoek2(count(3))=yold(3);
140                 xhoek3(count(3))=xnew(2);    yhoek3(count(3))=ynew(2);
141                 xhoek4(count(3))=xnew(3);    yhoek4(count(3))=ynew(3);
142                 count(3)=count(3)+1;
143                 Switch(1)=1;
144             end
145             T2old = T2;
146             Switch(2) = 0;
147             [x4t y4t] = polyxpoly(xold(4:5), yold(4:5), xnew(4:5), ynew(4:5));
148             if ¬isempty(x4t) && Switch(4) == 1;
149                 x4(count(6))=x4t;
150                 y4(count(6))=y4t;
151                 count(6)=count(6)+1;
152                 xold4_2=xold(4);            yold4_2=yold(4);
153                 xold5_2=xold(5);            yold5_2=yold(5);
154                 Switch(4) = 0;              Switch(3) = 1;
155                 count(6);
156             elseif ¬isempty(x4t)
157                 x4(count(6))=x4t;           y4(count(6))=y4t;
158                 count(6)=count(6)+1;
159             end
160             if Switch(5) == 1
161                 xold43(count(10))=xold(4);  yold43(count(10))=yold(4);
162                 xold53(count(10))=xold(5);  yold53(count(10))=yold(5);
163                 count(10)=count(10)+1;
164                 Switch(5) = 0;
165             end
166             [x5t y5t] = polyxpoly(xold(5:6), yold(5:6), xnew(5:6), ynew(5:6));
167             if ¬isempty(x5t)
168                 x5(count(7))=x5t;           y5(count(7))=y5t;
169                 count(7)=count(7)+1;
170             end
171             [x6t y6t] = polyxpoly(xold(5:6), yold(5:6), xnew(1:2), ynew(1:2));
172             if ¬isempty(x6t)
173                 x6(count(8))=x6t;           y6(count(8))=y6t;
174                 count(8)=count(8)+1;
175             end
176             if exist('xold4_2') && exist('xold5_2') && exist('yold4_2') &&...
177                     exist('yold5_2') && exist('xold4') && exist('xold3') && ...
178                     exist('yold4') && exist('yold3')
179                 [x7t y7t] = polyxpoly([xold4_2;xold5_2], [yold4_2;yold5_2],...
180                     [xold3;xold4], [yold3;yold4]);
181                 if ¬isempty(x7t)
182                     x7(count(9))=x7t;       y7(count(9))=y7t;
183                     count(9)=count(9)+1;
184                 end
185             end
```

```
186         end
187     end
188
189     for i=1:length(xold43)
190         [x8t y8t] = polyxpoly([xold43(i);xold53(i)], [yold43(i);yold53(i)],...
191             x5, y5);
192         if ¬isempty(x8t)
193             if length(x8t)==2
194                 x8(count(11))=x8t(2);
195                 y8(count(11))=y8t(2);
196                 count(11)=count(11)+1;
197             else
198                 x8(count(11))=x8t;
199                 y8(count(11))=y8t;
200                 count(11)=count(11)+1;
201             end
202         end
203     end
204
205     %% Cut vectors
206     numtand = 1;
207     for i=1:length(xB)
208         if i≠1
209             x1diff = abs(xB(i)−xB(i−1));
210             y1diff = abs(yB(i)−yB(i−1));
211             if x1diff > 0.5*mo*pi || y1diff > 0.5*mo*pi
212                 count(12) = 1;
213                 numtand = numtand+1;
214                 x1cut(count(12),numtand) = xB(count(13));
215                 y1cut(count(12),numtand) = yB(count(13));
216                 count(12) = count(12)+1;            count(13) = count(13)+1;
217             else
218                 x1cut(count(12),numtand) = xB(count(13));
219                 y1cut(count(12),numtand) = yB(count(13));
220                 count(12) = count(12)+1;            count(13) = count(13)+1;
221             end
222         else
223             x1cut(count(12),numtand) = xB(count(12));
224             y1cut(count(12),numtand) = yB(count(12));
225             count(12) = count(12)+1;                count(13) = count(13)+1;
226         end
227     end
228
229     numtand = 1;
230     for i=1:length(xhoek6)
231         if i≠1
232             xhoek6diff = abs(xhoek6(i)−xhoek6(i−1));
233             yhoek6diff = abs(yhoek6(i)−yhoek6(i−1));
234             rhoek6diff = sqrt(xhoek6diff^2+yhoek6diff^2);
235             if rhoek6diff > (Q(3,1)−Q(2,1))/2
236                 count(16) = 1;
237                 numtand = numtand+1;
238                 xhoek6cut(count(16),numtand) = xhoek6(count(17));
239                 yhoek6cut(count(16),numtand) = yhoek6(count(17));
240                 count(16) = count(16)+1;            count(17) = count(17)+1;
241             else
242                 xhoek6cut(count(16),numtand) = xhoek6(count(17));
243                 yhoek6cut(count(16),numtand) = yhoek6(count(17));
244                 count(16) = count(16)+1;            count(17) = count(17)+1;
245             end
246         else
247             xhoek6cut(count(16),numtand) = xhoek6(count(16));
248             yhoek6cut(count(16),numtand) = yhoek6(count(16));
249             count(16) = count(16)+1;                count(17) = count(17)+1;
250         end
251     end
252
```

```
253     numtand = 1;
254     for i=1:length(xhoek5)
255         if i≠1
256             xhoek5diff = abs(xhoek5(i)−xhoek5(i−1));
257             yhoek5diff = abs(yhoek5(i)−yhoek5(i−1));
258             rhoek5diff = sqrt(xhoek5diff^2+yhoek5diff^2);
259             if rhoek5diff > (Q(3,1)−Q(2,1))/2
260                 count(18) = 1;
261                 numtand = numtand+1;
262                 xhoek5cut(count(18),numtand) = xhoek5(count(19));
263                 yhoek5cut(count(18),numtand) = yhoek5(count(19));
264                 count(18) = count(18)+1;            count(19) = count(19)+1;
265             else
266                 xhoek5cut(count(18),numtand) = xhoek5(count(19));
267                 yhoek5cut(count(18),numtand) = yhoek5(count(19));
268                 count(18) = count(18)+1;            count(19) = count(19)+1;
269             end
270         else
271             xhoek5cut(count(18),numtand) = xhoek5(count(18));
272             yhoek5cut(count(18),numtand) = yhoek5(count(18));
273             count(18) = count(18)+1;        count(19) = count(19)+1;
274         end
275     end
276
277     numtand = 1;
278     for i=1:length(xhoek3)
279         if i≠1
280             xhoek3diff = abs(xhoek3(i)−xhoek3(i−1));
281             yhoek3diff = abs(yhoek3(i)−yhoek3(i−1));
282             if xhoek3diff > 0.5*mo*pi || yhoek3diff > 0.5*mo*pi
283                 count(20) = 1;
284                 numtand = numtand+1;
285                 xhoek3cut(count(20),numtand) = xhoek3(count(21));
286                 yhoek3cut(count(20),numtand) = yhoek3(count(21));
287                 count(20) = count(20)+1;            count(21) = count(21)+1;
288             else
289                 xhoek3cut(count(20),numtand) = xhoek3(count(21));
290                 yhoek3cut(count(20),numtand) = yhoek3(count(21));
291                 count(20) = count(20)+1;            count(21) = count(21)+1;
292             end
293         else
294             xhoek3cut(count(20),numtand) = xhoek3(count(20));
295             yhoek3cut(count(20),numtand) = yhoek3(count(20));
296             count(20) = count(20)+1;                count(21) = count(21)+1;
297         end
298     end
299
300     numtand = 1;
301     for i=1:length(xhoek1)
302         if i≠1
303             xhoek1diff = abs(xhoek1(i)−xhoek1(i−1));
304             yhoek1diff = abs(yhoek1(i)−yhoek1(i−1));
305             if xhoek1diff > 0.5*mo*pi || yhoek1diff > 0.5*mo*pi
306                 count(22) = 1;
307                 numtand = numtand+1;
308                 xhoek1cut(count(22),numtand) = xhoek1(count(23));
309                 yhoek1cut(count(22),numtand) = yhoek1(count(23));
310                 count(22) = count(22)+1;
311                 count(23) = count(23)+1;
312             else
313                 xhoek1cut(count(22),numtand) = xhoek1(count(23));
314                 yhoek1cut(count(22),numtand) = yhoek1(count(23));
315                 count(22) = count(22)+1;
316                 count(23) = count(23)+1;
317             end
318         else
319             xhoek1cut(count(22),numtand) = xhoek1(count(22));
```

```
            yhoek1cut(count(22),numtand) = yhoek1(count(22));
            count(22) = count(22)+1;
            count(23) = count(23)+1;
        end
    end

    numtand = 1;
    for i=1:length(x3)
        if i≠1
            x3diff = abs(x3(i)−x3(i−1));
            y3diff = abs(y3(i)−y3(i−1));
            if x3diff > 0.5*mo*pi || y3diff > 0.5*mo*pi
                count(24) = 1;
                numtand = numtand+1;
                x3cut(count(24),numtand) = x3(count(25));
                y3cut(count(24),numtand) = y3(count(25));
                count(24) = count(24) +1;
                count(25) = count(25)+1;
            else
                x3cut(count(24),numtand) = x3(count(25));
                y3cut(count(24),numtand) = y3(count(25));
                count(24) = count(24)+1;
                count(25) = count(25)+1;
            end
        else
            x3cut(count(24),numtand) = x3(count(24));
            y3cut(count(24),numtand) = y3(count(24));
            count(24) = count(24)+1;
            count(25) = count(25)+1;
        end
    end

    numtand = 1;
    for i=1:length(xhoek4)
        if i≠1
            xhoek4diff = abs(xhoek4(i)−xhoek4(i−1));
            yhoek4diff = abs(yhoek4(i)−yhoek4(i−1));
            if xhoek4diff > 0.5*mo*pi || yhoek4diff > 0.5*mo*pi
                count(26) = 1;
                numtand = numtand+1;
                xhoek4cut(count(26),numtand) = xhoek4(count(27));
                yhoek4cut(count(26),numtand) = yhoek4(count(27));
                count(26) = count(26)+1;
                count(27) = count(27)+1;
            else
                xhoek4cut(count(26),numtand) = xhoek4(count(27));
                yhoek4cut(count(26),numtand) = yhoek4(count(27));
                count(26) = count(26)+1;
                count(27) = count(27)+1;
            end
        else
            xhoek4cut(count(26),numtand) = xhoek4(count(26));
            yhoek4cut(count(26),numtand) = yhoek4(count(26));
            count(26) = count(26)+1;
            count(27) = count(27)+1;
        end
    end

    numtand = 1;
    for i=1:length(x2)
        if i≠1
            x2diff = abs(x2(i)−x2(i−1));
            y2diff = abs(y2(i)−y2(i−1));
            if x2diff > 0.5*mo*pi || y2diff > 0.5*mo*pi
                count(14) = 1;
                numtand = numtand+1;
                x2cut(count(14),numtand) = x2(count(15));
```

```
                y2cut(count(14),numtand) = y2(count(15));
                count(14) = count(14) +1;
                count(15) = count(15)+1;
            else
                x2cut(count(14),numtand) = x2(count(15));
                y2cut(count(14),numtand) = y2(count(15));
                count(14) = count(14)+1;
                count(15) = count(15)+1;
            end
        else
            x2cut(count(14),numtand) = x2(count(14));
            y2cut(count(14),numtand) = y2(count(14));
            count(14) = count(14)+1;
            count(15) = count(15)+1;
        end
    end

    numtand = 1;
    for i=1:length(x7)
        if i≠1
            x7diff = abs(x7(i)-x7(i-1));
            y7diff = abs(y7(i)-y7(i-1));
            if x7diff > 0.5*mo*pi || y7diff > 0.5*mo*pi
                count(28) = 1;
                numtand = numtand+1;
                x7cut(count(28),numtand) = x7(count(29));
                y7cut(count(28),numtand) = y7(count(29));
                count(28) = count(28) +1;
                count(29) = count(29)+1;
            else
                x7cut(count(28),numtand) = x7(count(29));
                y7cut(count(28),numtand) = y7(count(29));
                count(28) = count(28)+1;
                count(29) = count(29)+1;
            end
        else
            x7cut(count(28),numtand) = x7(count(28));
            y7cut(count(28),numtand) = y7(count(28));
            count(28) = count(28)+1;
            count(29) = count(29)+1;
        end
    end

    numtand = 1;
    for i=1:length(x4)
        if i≠1
            x4diff = abs(x4(i)-x4(i-1));
            y4diff = abs(y4(i)-y4(i-1));
            if x4diff > 0.5*mo*pi || y4diff > 0.5*mo*pi
                count(30) = 1;
                numtand = numtand+1;
                x4cut(count(30),numtand) = x4(count(31));
                y4cut(count(30),numtand) = y4(count(31));
                count(30) = count(30) +1;
                count(31) = count(31)+1;
            else
                x4cut(count(30),numtand) = x4(count(31));
                y4cut(count(30),numtand) = y4(count(31));
                count(30) = count(30)+1;
                count(31) = count(31)+1;
            end
        else
            x4cut(count(30),numtand) = x4(count(30));
            y4cut(count(30),numtand) = y4(count(30));
            count(30) = count(30)+1;
            count(31) = count(31)+1;
        end
```

```
454     end
455
456     numtand = 1;
457     for i=1:length(x8)
458         if i≠1
459             x8diff = abs(x8(i)-x8(i-1));
460             y8diff = abs(y8(i)-y8(i-1));
461             if x8diff > 0.5*mo*pi || y8diff > 0.5*mo*pi
462                 count(32) = 1;
463                 numtand = numtand+1;
464                 x8cut(count(32),numtand) = x8(count(33));
465                 y8cut(count(32),numtand) = y8(count(33));
466                 count(32) = count(32) +1;
467                 count(33) = count(33)+1;
468             else
469                 x8cut(count(32),numtand) = x8(count(33));
470                 y8cut(count(32),numtand) = y8(count(33));
471                 count(32) = count(32)+1;
472                 count(33) = count(33)+1;
473             end
474         else
475             x8cut(count(32),numtand) = x8(count(32));
476             y8cut(count(32),numtand) = y8(count(32));
477             count(32) = count(32)+1;
478             count(33) = count(33)+1;
479         end
480     end
481
482     numtand = 1;
483     for i=1:length(x5)
484         if i≠1
485             x5diff = abs(x5(i)-x5(i-1));
486             y5diff = abs(y5(i)-y5(i-1));
487             if x5diff > 0.5*mo*pi || y5diff > 0.5*mo*pi
488                 count(34) = 1;
489                 numtand = numtand+1;
490                 x5cut(count(34),numtand) = x5(count(35));
491                 y5cut(count(34),numtand) = y5(count(35));
492                 count(34) = count(34) +1;
493                 count(35) = count(35)+1;
494             else
495                 x5cut(count(34),numtand) = x5(count(35));
496                 y5cut(count(34),numtand) = y5(count(35));
497                 count(34) = count(34)+1;
498                 count(35) = count(35)+1;
499             end
500         else
501             x5cut(count(34),numtand) = x5(count(34));
502             y5cut(count(34),numtand) = y5(count(34));
503             count(34) = count(34)+1;
504             count(35) = count(35)+1;
505         end
506     end
507
508     numtand = 1;
509     for i=1:length(x6)
510         if i≠1
511             x6diff = abs(x6(i)-x6(i-1));
512             y6diff = abs(y6(i)-y6(i-1));
513             if x6diff > 0.5*mo*pi || y6diff > 0.5*mo*pi
514                 count(36) = 1;
515                 numtand = numtand+1;
516                 x6cut(count(36),numtand) = x6(count(37));
517                 y6cut(count(36),numtand) = y6(count(37));
518                 count(36) = count(36) +1;
519                 count(37) = count(37)+1;
520             else
```

```
                x6cut(count(36),numtand) = x6(count(37));
                y6cut(count(36),numtand) = y6(count(37));
                count(36) = count(36)+1;
                count(37) = count(37)+1;
            end
        else
            x6cut(count(36),numtand) = x6(count(36));
            y6cut(count(36),numtand) = y6(count(36));
            count(36) = count(36)+1;
            count(37) = count(37)+1;
        end
    end
    %% Orderen van punten en plotten van juiste curve
    ux = [];
    uy = [];
    for j =1:Ntand
        xB = x1cut(:,j);                    yB = y1cut(:,j);
        xC = flipud(xhoek6cut(:,2*j));      yC = flipud(yhoek6cut(:,2*j));
        xJ = flipud(xhoek6cut(:,2*j-1));    yJ = flipud(yhoek6cut(:,2*j-1));
        xD = flipud(xhoek5cut(:,2*j));      yD = flipud(yhoek5cut(:,2*j));
        xK = flipud(xhoek5cut(:,2*j-1));    yK = flipud(yhoek5cut(:,2*j-1));
        xE = flipud(xhoek3cut(:,j));        yE = flipud(yhoek3cut(:,j));
        xF = flipud(xhoek1cut(:,j));        yF = flipud(yhoek1cut(:,j));
        xG = x3cut(:,j);                    yG = y3cut(:,j);
        xH = flipud(xhoek4cut(:,j));        yH = flipud(yhoek4cut(:,j));
        xL = x2cut(:,j);                    yL = y2cut(:,j);
        xM = x7cut(:,j);                    yM = y7cut(:,j);
        xM = unique(xM(find(xM)));          yM = unique(yM(find(yM)));
        xN1 = x4cut(:,j);                   yN1 = y4cut(:,j);
        xO = x8cut(:,j);                    yO = y8cut(:,j);

        [rxN1,cxN1,vxN1] = find(xN1);          [ryN1,cyN1,vyN1] = find(yN1);
        [b1xN1,m1xN1,n1xN1] = unique(vxN1);  [b1yN1,m1yN1,n1yN1] = unique(vyN1);
        m1N1 = [m1xN1;m1yN1]; m1N1 = unique(m1N1); m1N1 = sort(m1N1);
        xN1 = vxN1(m1N1);                   yN1 = vyN1(m1N1);
        xN = [];                            yN = [];
        xmean = (xM+xO)/2;                  ymean = (yM+yO)/2;
        rM = sqrt((xM-xmean)^2+(yM-ymean)^2);
        for i=1:length(xN1)
            rpunt1 = sqrt((xmean-xN1(i))^2+(ymean-yN1(i))^2);
            if rpunt1 < rM
                xN = [xN;xN1(i)];            yN = [yN;yN1(i)];
            end
        end

        xP1 = x5cut(:,j);                   yP1 = y5cut(:,j);
        xP = [];                            yP = [];
        for i=1:length(xP1)
            rO = sqrt(xO^2+yO^2);
            rP1 = sqrt(xP1(i)^2+yP1(i)^2);
            if (rP1<rO)
                xP = [xP;xP1(i)];            yP = [yP;yP1(i)];
            end
        end

        if j~=Ntand
            xQ = x6cut(:,j);                yQ = y6cut(:,j);
            XX = [xB;xC;xD;xE;xF;xG;xH; xJ;xK;xL;xM;xN;xO;xP;xQ];
            YY = [yB;yC;yD;yE;yF;yG;yH; yJ;yK;yL;yM;yN;yO;yP;yQ];
        else
            XX = [xB;xC;xD;xE;xF;xG;xH; xJ;xK;xL;xM;xN;xO];
            YY = [yB;yC;yD;yE;yF;yG;yH; yJ;yK;yL;yM;yN;yO];
        end

        if j==1
            XX=[xA;XX];                     YY=[yA;YY];
        end
```

```
588          [rx,cx,vx] = find(XX);                [ry,cy,vy] = find(YY);
589          [b1x, m1x, n1x] = unique(vx);         [b1y, m1y, n1y] = unique(vy);
590          m1 = [m1x;m1y]; m1 = unique(m1); m1 = sort(m1);
591          vx = vx(m1);                          vy = vy(m1);
592          ux = [ux;vx];                         uy = [uy;vy];
593      end
594      figure(1)
595      plot(curve(1,1:step,k),curve(2,1:step,k),'r',0,0,'ok',ux,uy,'k',...
596          'Linewidth',2);
597      hold off
598      axis square; axis([-50 50 -50 50])
599      count  = ones(1,40);
600      Switch = [0 0 0 1 0 1];
601      clear xmean ymean
602  end
```

```
1   %%%%%%%%%%%%%
2   %%% Plots %%%
3   %%%%%%%%%%%%%
4   figure()
5   subplot(1,3,1)
6       plot(t,r1,'k',t,r2,'m',t,r3,'r',t,r4,'b','LineWidth',2)
7       title('radii','Fontsize',15);
8       xlabel('time','Fontsize',15); ylabel('radius','Fontsize',15)
9       legend('r1','r2','r3','r4','Location','NorthWest')
10  subplot(1,3,2)
11      plot(t,thetaI,t,theta4,'LineWidth',2)
12      title('\theta_{in} and \theta_{4}','Fontsize',15);
13      xlabel('time','Fontsize',15); ylabel('angle','Fontsize',15)
14      legend('\theta_I','\theta_4','Location','NorthWest')
15  subplot(1,3,3)
16      plot(t,Emass,t,Eref,t,Etot,'LineWidth',2); ylim([0 8]);
17      title('Energy','Fontsize',15);
18      xlabel('Time','Fontsize',15); ylabel('Energy','Fontsize',15)
19      legend('Spring','Mass','Ref','Etot','Location','NorthWest')
20
21  figure()
22      plot(Eref - Espring)
23      title('Energy difference','Fontsize',15);
24      xlabel('Time','Fontsize',15); ylabel('Energy','Fontsize',15)
25
26  figure()
27      plot(thetaI,theta4,'LineWidth',4)
28      xlabel('\theta_{in}   [rad]','Fontsize',18);
29      ylabel('\theta_{out}   [rad]','Fontsize',18);
30      axis([0 2*pi 0 0.6])
31      set(gca,'XTick',0:0.5*pi:2*pi)
32      set(gca,'XTickLabel',{'0','1.57','3.14','4.71','6.28'},'FontSize',15)
33      set(gca,'YTick',0:0.1:0.6)
34      set(gca,'YTickLabel',{'0','0.1','0.2','0.3','0.4','0.5','0.6'},...
35          'FontSize',15)
36
37  figure()
38  for i=1:numel(t)
39      plot([0 R*cos(thetaI(i))],[0 R*sin(thetaI(i))],'g','LineWidth',2);
40      hold on
41      plot(0,0,'ok')
42      plot([R*cos(thetaI(i)) R*cos(thetaI(i))+min(r3)*cos(theta3(i))],...
43           [R*sin(thetaI(i)) R*sin(thetaI(i))+min(r3)*sin(theta3(i))],...
44           'r','LineWidth',2)
45      plot([R*cos(thetaI(i)) R*cos(thetaI(i))+r3(i)*cos(thetaI(i))],...
46           [R*sin(thetaI(i)) R*sin(thetaI(i))+r3(i)*sin(thetaI(i)) ],'pk')
47      plot([0 min(r4)*cos(theta4(i))],[0 min(r4)*sin(theta4(i))],...
48           'b','LineWidth',2)
49      rotmatrix1=[cos(theta1(i)) -sin(theta1(i)); sin(theta1(i)) cos(theta1(i))];
50      rotmatrix2=[cos(theta2(i)) -sin(theta2(i)); sin(theta2(i)) cos(theta2(i))];
```

```
    rotmatrix3=[cos(theta3(i)) -sin(theta3(i)); sin(theta3(i)) cos(theta3(i))];
    rotmatrix4=[cos(theta4(i)) -sin(theta4(i)); sin(theta4(i)) cos(theta4(i))];
    rotatedcurve1=rotmatrix1*curve(:,:,1);
    rotatedcurve2=rotmatrix2*curve(:,:,2);
    rotatedcurve3=rotmatrix3*curve(:,:,3);
    rotatedcurve4=rotmatrix4*curve(:,:,4);
    rotatedandshiftedcurve2=[R*cos(thetaI(i))+rotatedcurve2(1,:);...
                             R*sin(thetaI(i))+rotatedcurve2(2,:)];
    rotatedandshiftedcurve3=[R*cos(thetaI(i))+rotatedcurve3(1,:);...
                             R*sin(thetaI(i))+rotatedcurve3(2,:)];
    plot(rotatedcurve1(1,:),rotatedcurve1(2,:),'k','LineWidth',4)
    plot(rotatedandshiftedcurve2(1,:),rotatedandshiftedcurve2(2,:),...
         'y','LineWidth',4)
    plot(rotatedandshiftedcurve3(1,:),rotatedandshiftedcurve3(2,:),...
         'r','LineWidth',4)
    plot(rotatedcurve4(1,:),rotatedcurve4(2,:),'b','LineWidth',4)
    axis([-70 70 -70 70])
    axis square
    pause(0.1)
    hold off
    input('enter to go on ', 's');
end
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% Print to .txt file %%%
%%%%%%%%%%%%%%%%%%%%%%%%%%
fid1  = fopen('ring1.txt','wt');
fid2  = fopen('gear2.txt','wt');
fid3  = fopen('gear3mettand3D.txt','wt');
fid4  = fopen('gear4mettand3D.txt','wt');
speed = linspace(0,-20,length(ux));

for j = 1:step
    fprintf(fid1,'%f \t %f \t %d \n',curve(1,j,1)',curve(2,j,1)',0);
    fprintf(fid2,'%f \t %f \t %d \n',curve(1,j,2)',curve(2,j,2)',0);
end

for j = 1:length(ux)
    fprintf(fid3,'%f \t %f \t %d \n',ux(j),uy(j),speed(j));
    fprintf(fid4,'%f \t %f \t %d \n',ux(j),uy(j),speed(j));
end
fclose(fid1); fclose(fid2); fclose(fid3); fclose(fid4);
```