Long-term Availability Analysis

Water Treatment Plants: Complex Repairable Systems with Deteriorating Characteristics

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Pumpstation Leiduin around 1900 (Krol, 2017).
Krol, H. (2017). Pompstation Leiduin omstreeks 1900 [photo]. Retrieved from https://ilibrariana.wordpress.com/2017/05/25/heemstede-op-stoom-stoomwerktuigen-in-heemstede/.

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The method and key findings presented in this report will lead to a scientific publication.

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List of Abbreviations

ANN	Artificial Neural Network
ARA	Arithmetic Reduction of Age
ARI	Arithmetic Reduction of Intensity
ARP	Alternating Renewal Process
CDF	Cumulative Distribution Function
CTMC	Continuous-Time Markov Chain
DBN	Dynamic Bayesian Network
DFT	Dynamic Fault Tree
DWTP	Drinking Water Treatment Plant
FTA	Fault Tree Analysis
HPP	Homogeneous Poisson Process
i.i.d.	independently and identically distributed
ITS	Inverse Transform Sampling
LR	Likelihood Ratio
MCS	Monte Carlo Simulation
MDP	Markov Decision Process
MLE	Maximum Likelihood Estimation
MTBF	Mean Time Between Failures
MTTF	Mean Time To Failure
MTTR	Mean Time To Repair
NHPP	Non-Homogeneous Poisson Process
PDF	Probability Density Function
PLP	Power Law Process
QRP	Quasi-Renewal Process
RAM	Reliability Availability Maintainability
RBD	Reliability Block Diagram
SMDP	Semi-Markov Decision Process
SMP	Semi-Markov Process
VAP	Virtual Age Process
WTP	Water Treatment Plant
WWTP	Wastewater Treatment Plant

Glossary

(Stochastic) Availability	The probability that an asset is functioning as required at a given time.
Deterioration	The continuous process of ageing of an asset, resulting in a decreasing performance in terms of reliability and/or availability.
Imperfect repair	A repair action that brings the concerned asset back to a condition state somewhere between “same as old” and “same as new”.
Maintenance	All actions regarding repairs, replacements and inspections.
Maintenance strategy	A set of decision rules with respect to repairs, replacements and inspections.
Minimal repair	A repair action that brings the concerned asset back to the same condition state as it was in just before failure (“same as old”).
One-unit system	A system that is represented without further consideration of the configuration of its components.
Perfect repair	A repair action that brings the concerned asset back to a “same as new” state.
Reliability	The probability that an asset will not fail within a specified period of time.
Repairable system	“A system which, after failing to perform one or more of its functions satisfactorily, can be restored to fully satisfactory performance by any method, other than replacement of the entire system” (Ascher & Feingold, 1984, p. 1)

Acknowledgement

This report is the final piece of our MSc in Civil Engineering, respectively the tracks Water Management (Jasper) and Construction Management and Engineering (Roland), at Delft University of Technology. It is our story about Water Treatment Plants, mixing the ingredients *complexity*, *repairability*, *deterioration* and molding them into an availability assessment.

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Summary

Water Treatment Plants (WTPs) play a key role in assuring water availability to citizens and businesses. Maintenance costs for these plants can take up to a quarter of the total operating budget. Therefore, the asset manager's goal is to optimise maintenance strategies with respect to quantity, quality and cost. In this asset management context *availability* is an important indicator upon which these maintenance strategies or system modifications are put to the test.

The complexity of WTPs and deterioration of their repairable assets are factors that influence the availability, making it not a constant measure. This research sets out to find a *method* for modelling the availability over time for a WTP with these three aspects (complex, repairable, and deteriorating assets).

A review of the state-of-the-art reveals that there is currently no known *modelling approach* for modelling the availability of complex systems with deteriorating, repairable assets. However, there are models that can handle one or more of these aspects. Thus, the solution to this gap in the literature is to combine multiple models in order to achieve the defined research objective.

The model approach proposed in this study consists of a two-level hierarchical model, which can be divided into the *system level* and the *component level*. At the system level there is one model: the Reliability Block Diagram (RBD). An RBD can deal with the complexity of a WTP by calculating the system availability based on the components' availabilities, in accordance with the system configuration. At the component level two different deterioration models can be used: the Non-Homogeneous Poisson Process (NHPP) model and the Semi-Markov Process (SMP) model. These component models are able to model the availability over time for assets that are repairable and do deteriorate. Each block in the system RBD corresponds to a component in the system. For each of these components it must be decided which of the two component models to use. To facilitate in this decision-making the flowchart in Figure 1 should be used.

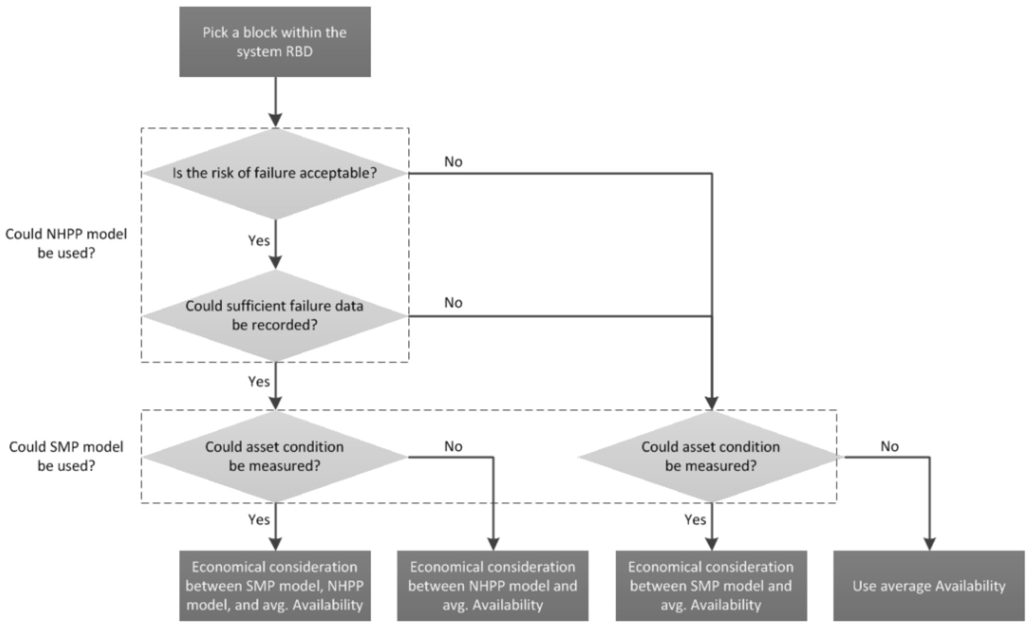

Figure 1: Flowchart for optimal model selection at component level

The proposed hierarchical model is accompanied with a method to assist asset managers in its application. The method consists of four steps:

1. Set up an RBD at system level
2. Apply the flowchart for choosing the appropriate component model for each block in the RBD
3. Model the components' availabilities with the component models
4. Use the RBD to model the system availability

The method is applied on a fictitious case study of a Drinking Water Treatment Plant (DWTP), named Leeward Dune. The availability for several treatment steps is modelled, which are combined to obtain the system availability function (see Figure 2).

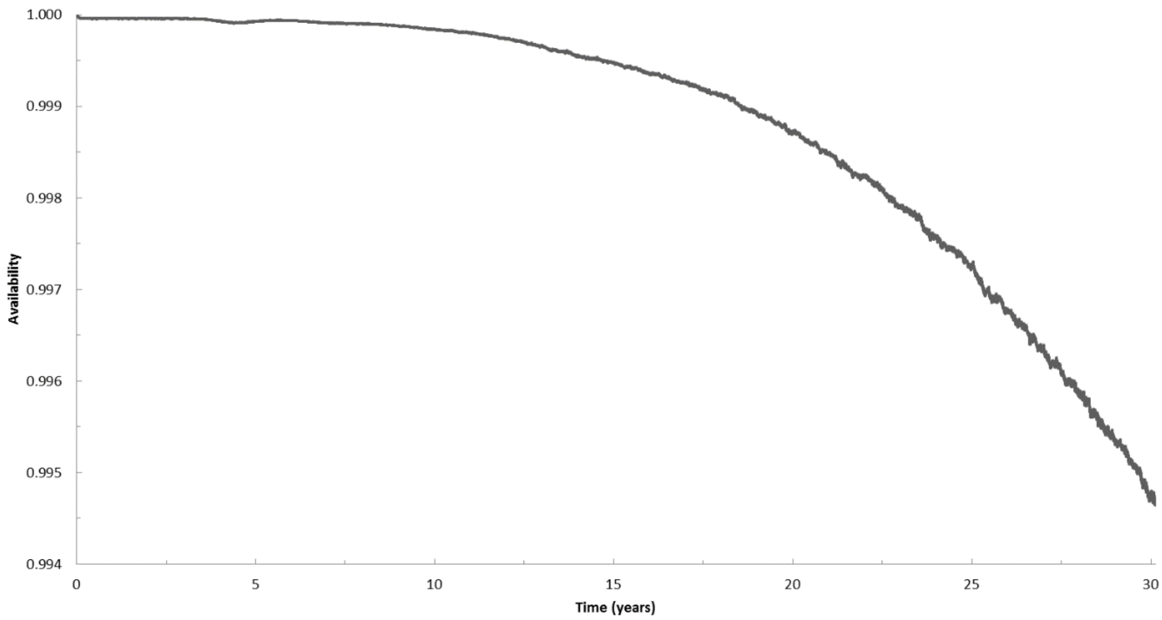

Figure 2: Availability over time for the case study's system 'Leeward Dune'

The conclusion of this research is that availability of complex, repairable systems with deteriorating assets can be modelled over time with a two-level hierarchical model, where the *system level model* is a Reliability Block Diagram (RBD) and the *component level model* is either a Non-Homogeneous Poisson Process (NHPP) or a Semi-Markov Process (SMP) model. This proposed modelling approach can facilitate asset managers in decision-making regarding system configuration and maintenance strategies. Another advantage is that this method can be used for modelling at different system scales. The case study shows how (part of) a WTP can be modelled with it, but it is also possible to model at the scale of one treatment step or even one asset. The system boundaries can be adjusted based on the desired level of detail in the RBD.

In the end, some directions for further research are suggested. First, further investigation into the dependencies between components is recommended. Second, the inclusion of costs is proposed as a research topic in continuation of this study, to improve the optimisation process of maintenance strategies and system configurations. Third, an extension to the SMP model could be the addition of intermediate values for the instantaneous availability, since in the proposed methodology this is assumed to be a binary value. A fourth consideration might be the use of virtual age processes instead of the NHPP, in order to relax the assumption that only minimal repairs are possible.

The method and key findings presented in this report will lead to a scientific publication.

1 Introduction

Water Treatment Plants (WTPs) play a key role in ensuring water availability to citizens and businesses. These plants are highly complex systems and need proper asset management in order to satisfy demands in quantity and quality of the water available. Maintenance, quantity, quality and costs need to be balanced in order to find an optimal asset management strategy for a WTP.

Maintenance of Water Treatment Plants

Maintenance of Wastewater Treatment Plants (WWTPs) takes up 7 – 26% of total operational costs and for Drinking Water Treatment Plants (DWTPs) 11 – 21%, see Table 1. Cost information from 2018 for total drinking water production at Waternet gives ~28% (~€8 mln) maintenance costs of the total exploitation costs (~€30 mln) for DWTPs and ~30% for WWTPs. Therefore, maintenance costs are an important part of the operational costs for both WWTPs and DWTPs.

Table 1: Maintenance costs as part of the total operational costs

WWTP	Percentage	DWTP	Percentage
Spain	7.4 (Hernández-Sancho & Sala-Garrido, 2009)	**Australia**	21 (NSW Department of Primary Industries, 2017)
	8.4 (Gómez, Gémar, Molinos-Senante, Sala-Garrido, & Caballero, 2017)	**Netherlands**	11 (Barrios, Siebel, van der Helm, Bosklopper, & Gijzen, 2008)
	10.1 (Molinos-Senante, Sala-Garrido, & Hernández-Sancho, 2016)		
	13.2 (Molinos-Senante, Hernandez-Sancho, & Sala-Garrido, 2014)		
	21 (Molinos-Senante, Hernández-Sancho, & Sala-Garrido, 2010)		
Australia	26 (NSW Department of Primary Industries, 2017)		
Germany	20 (Wendland, 2005)		
Turkey	9 (Turkmenlera & Aslanb, 2017)		

Maintenance strategies

Within asset management three different types of maintenance strategies are distinguished: corrective maintenance, preventive maintenance and maintenance improvement, see Figure 3. Predictive maintenance is a part of preventive maintenance and can be used to reduce unnecessary corrective and preventive (time-driven) maintenance (Mobley, 2002). It has been proven that predictive maintenance minimises the maintenance costs and improves the system performance (Grall, Bérenguer, & Dieulle, 2002; Tan & Raghavan, 2008). In the field of asset management there is a growing interest from asset owners as well as from the research community towards predictive maintenance (PwC & Mainnovation, 2017; Wu & Do, 2017). In order to be able to assess these maintenance strategies, the performance of the system under these regimes needs to be evaluated. Reliability, Availability and Maintainability (RAM) have been introduced as indicators of this performance.

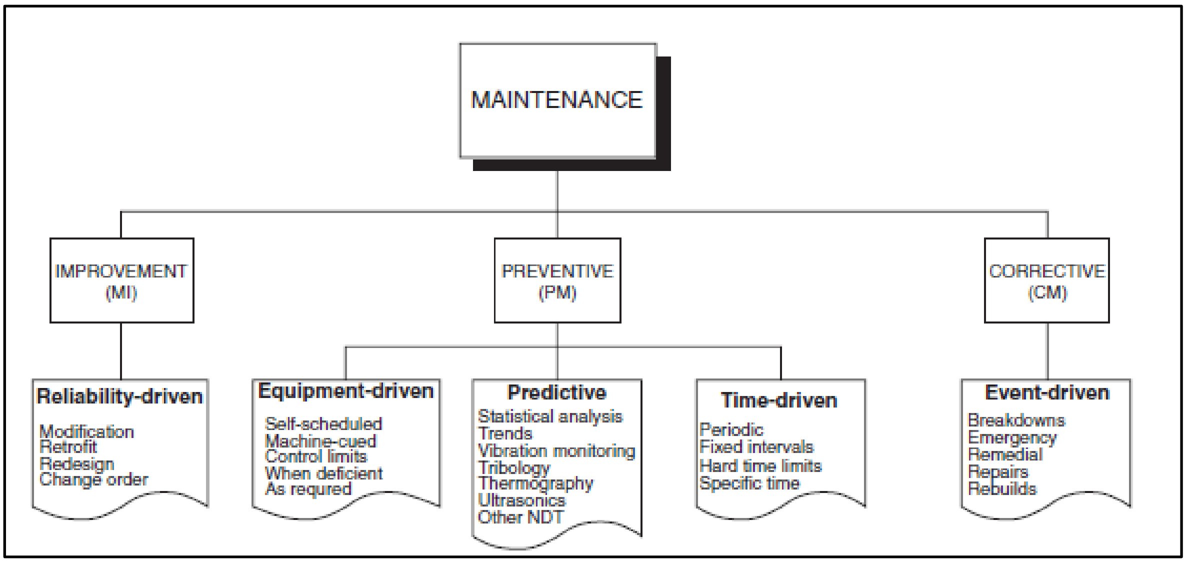

Figure 3: Different types of maintenance (Mobley, 2002)

Characterisations of Water Treatment Plants

In the context of asset management, WTPs have the following three characterisations: they are repairable, have a complex configuration and exhibit deteriorating characteristics.

1) Repairable

WTPs are repairable systems, since they *"can be restored to fully satisfactory performance by any method, other than replacement of the entire system"* (Ascher and Feingold, 1984, p. 1). In contrast, for non-repairable systems such as spaceships and disposable articles, one does not consider maintenance, but only replacement of the entire system to fulfil a certain function.

2) Complex configuration

The complexity of a WTP reveals itself in its configuration; it is an assembly of a multitude of structurally linked assets. Within an asset management context, one would like to know: 1) what the critical parts within a WTP are and 2) where and 3) when certain maintenance actions are necessary. This can only be understood if the configuration of a complex system is to be included in the assessment of maintenance strategies.

3) Deteriorating assets

Assets within WTPs are known to exhibit ageing characteristics (Rasmekomen & Parlikad, 2013); (Breysse, Vasconcelos, & Schoefs, 2007). This means that not only does performance of an asset change over time, it also deteriorates. Deterioration of assets thus needs to be incorporated within the assessment, in order to make a shift towards predictive maintenance possible (Baik, Jeong, & Abraham, 2006; Egger, Scheidegger, Reichert, & Maurer, 2013). In this research, deterioration is defined as the continuous process of ageing of an asset, resulting in a decreasing performance in terms of reliability and/or availability.

Availability modelling

This research focusses on the availability of the system. Reliability is of lesser importance than availability in predictive maintenance for complex, repairable systems: reliability only considers the frequency of interruptions, whereas availability also takes the impact of interruptions (i.e. downtime of the system) into account. Because WTPs are continuously operating systems, it is valuable for the asset manager to know what the probability is of a WTP being in an operating state (availability). It is of lesser value to know the probability of the number of times operation is interrupted or when the first interruption takes place (reliability). This research does consider literature on and the modelling of reliability, but only as a stepping stone towards the modelling of the availability.

The characterisations of the WTP invoke another necessity for the modelling method: the availability has to be considered as function *over time*. A WTP is a dynamic system and due to the ageing and repairable characterisations, the availability of the system does not necessarily stay the same over time. Therefore, as a fourth criterion for the modelling method, the change of availability *over time* will be considered as well.

It should be noted that the instantaneous availability is assumed to be a binary measure, being either available (1) or non-available (0). In modelling, the availability is a probabilistic quantity and is known as the *stochastic availability*[1], which is the probability of being available at a given point in time (Neil & Marquez, 2012; Qiu, Cui, & Gao, 2017):

$$A(t) = P(A(t) = 1)$$

Research goal

The research goal is defined as follows:

Developing a methodology for asset managers of Water Treatment Plants which makes predictions about the availability over time for a system that 1) is repairable, 2) has a complex configuration, and 3) consists of deteriorating assets.

[1] When *availability modelling* is mentioned in this report, this is in fact regarding the *stochastic availability*.

2 Literature review on reliability and availability modelling

Chapter 2 deals with the literature study on reliability and availability modelling. Table 2, page 28, shows the overview of the consulted references. The literature is evaluated based on the four research criteria as defined in the previous chapter:

- The proposed methodology must be applicable for systems that:
  - are repairable;
  - have a complex configuration;
  - consist of deteriorating assets;
- The proposed methodology must be capable of modelling the availability over time.

Combinatorial methods are discussed first in section 2.1, showing that deterioration is not included within these models. Therefore, in section 2.2 deterioration modelling is investigated and in sub-sections 2.2.1 - 2.2.3 different types of deterioration models are handled. Then, in section 2.3, hierarchical models are examined, which are based on combining multiple sub-models in order to reduce the model complexity. Section 2.4 provides the overview of the consulted literature for this chapter. Finally, in section 2.5 a conclusion is drawn on this literature review on reliability and availability modelling.

2.1 Combinatorial models

The relatively simple way of performing a reliability or availability analysis for a complex system is to use combinatorial models such as a Reliability Block Diagram (RBD) or Fault Tree Analysis (FTA). In many studies that use an RBD or FTA for modelling the reliability or availability, average values are assumed for the performance of components. When the goal is to obtain a quick and rough estimation of the system performance this approach can be sufficient. Two examples of studies applying this approach into the field of water treatment are the studies by Bourouni (2013) and Taheriyoun and Moradinejad (2015), but literature from other engineering fields is consulted as well.

The research by Bourouni (2013) is about the availability assessment of a Reverse Osmosis (RO) plant. Both the RBD and FTA method are presented in order to make the comparison between the two. Only average values are used as input for both methods, so just an estimation of the average availability for a certain time interval can be calculated this way.

Taheriyoun and Moradinejad (2015) performed a reliability analysis for a Wastewater Treatment Plant (WWTP) by means of an FTA. Reliability is defined here as the complementary probability of no failure occurring in a given period (one year in this case). Based on the estimated reliability values for the processes in the WWTP the system reliability is calculated using the fault tree. In the end, the results are validated via Monte Carlo simulation. However, the purpose of doing a Monte Carlo simulation is not clear, since only average values are considered in the FTA and no distributions are used. The paper implies the input data for the FTA is also used for the Monte Carlo simulation. When enough simulations are performed, with Monte Carlo simulation the outcome then converges to the same result as calculated with FTA, meaning the Monte Carlo simulation would always give the same results as the FTA. Moreover, it should be clear that reliability calculations in general only provide information about the time to the first failure and do not account for maintenance actions, which makes it a measure of performance for non-repairable systems. This makes the reliability not an appropriate measure for the assessment of complex repairable systems.

Choi and Chang (2016) applied FTA for the reliability and availability assessment of seabed storage tanks. The fault tree can be considered as complex due to its many AND and OR gates. The reliability of the system is plotted as a function of time, according to the formula $R(t) = e^{-\lambda t}$. However, all failure rates are assumed to be constant, so no deterioration is included. For the availability the average over a given period is calculated, similar to both studies described above.

Velásquez and Lara (2018) executed a Reliability, Availability and Maintainability (RAM) analysis for an electrical power system, consisting of seven components in series. No RBD or FTA is shown in this work, probably because the configuration simply consists of seven components in series, but the basic calculation rules for serial configurations are applied to compute the system results. The failure rate is taken as a measure for unreliability and the product of the failure rate and the average downtime per failure is said to be the (average) unavailability. A constant failure rate is assumed, meaning no deterioration is included. This study only provides average values for the reliability and availability of the components and thus the system values are computed as average values as well.

In the work of Rao et al. (2009) a Dynamic Fault Tree (DFT) is used for the availability analysis of an electrical power supply system of a nuclear power plant. A Markov chain approach is applied for a simple system and Monte Carlo simulation approach for a complex system. Time-averaged expressions for availability are given, but, as the authors remark themselves, these can only be used for exponentially distributed failure and repair times (no deterioration). The unavailability is computed as a function of time, but it is approached as the unavailability during a certain mission time ("*unavailability obtained is 4.8e-6 for a mission time of 10,000h with* 10^6 *simulations.*" (Rao et al., 2009, p. 879). Furthermore, the authors give failure probabilities, but use failure distributions in their calculations. It is unclear how these failure probabilities and distributions are derived. Implicitly an Alternating Renewal Process (ARP[2]) is applied, which suggests the system is 'same as new' after repair. However, this assumption is not tested in the presented work. Also a simulation tool was created in this research, but in the paper an elaborate description is lacking.

From the studies discussed above it can be concluded that ageing characteristics are not included into the modelling of the system's performance in these studies and the combinatorial methods give a simplistic view. Therefore, models that include ageing characteristics (deterioration models) or models that are more sophisticated in other aspects are discussed hereafter. These are: renewal processes (2.2.1.1), Poisson process and virtual age process models (2.2.1.2), Markov models (2.2.2), Bayesian networks (2.2.3) and hierarchical models (2.3).

2.2 Deterioration modelling

Deterioration modelling can be an important tool within reliability and availability analysis. If deterioration models are well calibrated and validated, they provide useful information for asset management decisions with respect to maintenance and modification strategies (Caradot et al., 2017; Yuan, 2017).

Commonly, three categories of deterioration models can be distinguished: physical models, statistical models, and artificial intelligence models (Ana & Bauwens, 2010; Caradot et al., 2017; Edirisinghe, Setunge, & Zhang, 2013; Rokstad & Ugarelli, 2015; Yuan, 2017):

- Physical models are based on understanding the physical processes causing deterioration of assets. They have not been widely applied in the water sector, mainly due to the complexity of the physical mechanisms behind the deterioration process.
- Statistical models are driven by probabilistic relationships between input (history of asset conditions) and output (predicted deterioration) (Egger et al., 2013). They are used most often for complex infrastructure systems, since they are commonly the best suitable type of model considering the type of data available.
- Artificial intelligence models (or machine learning models) can learn to identify relationships between input and output, based on data input. They are often seen as 'black box' models, which means there is no explicit relationship between input and output. These models require lots of data and a greater computational power compared to the other two model types.

Based on the above, the scope of this research is limited to the group of statistical models. These are used most frequently for deterioration modelling of (water) infrastructure, since the statistical models have a better applicability in this field compared with the other two model types mentioned above.

Yuan (2017) has introduced guidelines for the deterioration modelling of water and wastewater assets. He finds that *"Traditional approaches emphasised the mean deterioration trend and heeded too little the characterisation of uncertainty involved."* (Yuan, 2017, p. 33). His objective is to "*revert this trend and bring stochastic deterioration modelling back to focus*". He provides an eight-step procedure for a deterioration

[2] See sub-section 2.2.1.1 at page 16 for more detailed explanation of the ARP.

modelling process, from which the sixth step *("Select a deterioration model for detailed analyses*" (Yuan, 2017, p. 30)) will be examined more closely.

In this procedure a flowchart is introduced for the selection of a deterioration model, see Figure 4 at page 21. Data based deterioration models considered in this flowchart are multi-state deterioration models, continuous deterioration models and point process models. Furthermore, if no data is available the flowchart suggests using the asset's design life for the mean life of a lifetime distribution model.

Continuous deterioration models are excluded from this review, since they are better equipped for very specific deterioration processes such as crack growth (Guida & Pulcini, 2011), resistance of carbon-film resistors (Li, Cui, & Lin, 2017) or light intensity of LED lamps (Pan, Wei, Fang, & Yang, 2018). We are not interested in the specific deterioration processes taking place in a system, but only in the overarching deterioration of the performance as seen in the reliability or availability over time.

However, the other two groups of models mentioned in the flowchart by Yuan (2017) are extensively reviewed in this section. These are the multi-state deterioration models (Markov models) and point process models (renewal processes, Poisson process models, and VAP models). In addition, also Bayesian networks are discussed here, since they are also commonly used for deterioration modelling according to literature research. To summarise, the following types of models are addressed in this section:

- Point process models:
    - Renewal processes
    - Poisson process models
    - Virtual age process models
- Markov models
- Bayesian networks

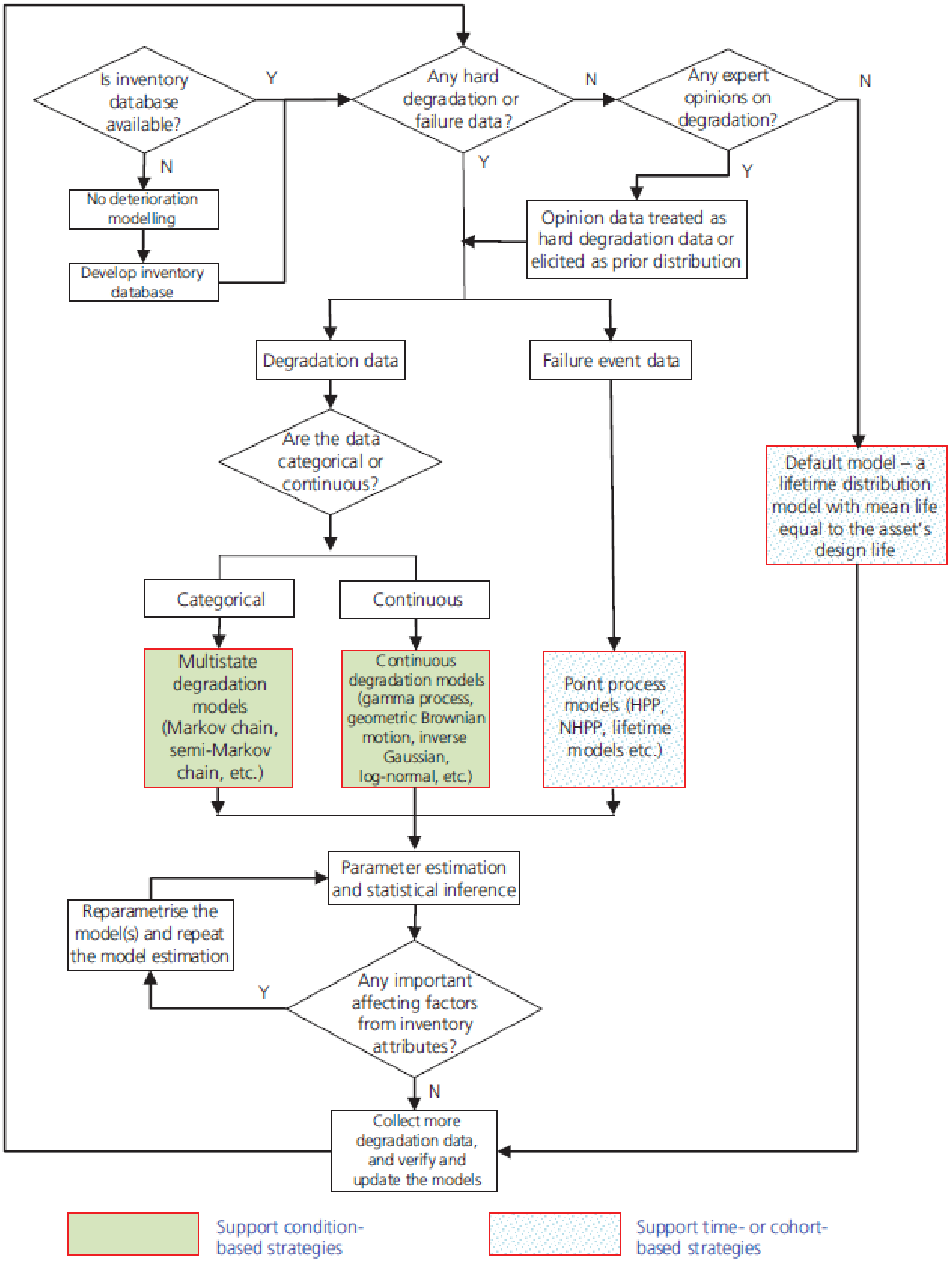

Figure 4: Flowchart from Yuan (2017) for deterioration model selection

2.2.1 Point process models

A stochastic point process is in fact a counting process. Stochastic point processes, and in particular the renewal processes, Poisson processes and virtual age processes, are well known within the field of reliability engineering for modelling repairable systems (Ascher & Feingold, 1984; Birolini, 2017; Cha & Finkelstein, 2018). These different types are discussed in this sub-section.

2.2.1.1 Renewal processes

The definition of a renewal process is *"a sequence of independent, identically distributed non-negative random variables* X_1*,* $X_2, \ldots,$ *which with probability 1 are not all zero*" (Ascher & Feingold, 1984, p. 33). The term renewal indicates that the system is restored to a "same as new" state after repair, so one cannot model deterioration with it[3]. However, certain types are definitely relevant for this study, due to the suggested modelling techniques. A renewal process runs through so-called 'renewal cycles', where one cycle consists of an uptime and a downtime[4]. Many different applications and types of renewal processes exist, of which the most relevant literature is discussed below.

Maciejewski and Caban (2008) apply an Alternating Renewal Process (ARP) to calculate the steady-state availability for a repairable system. There is no ageing in their model, since they use an exponential distribution for the 'time to failure'. Furthermore, there is no complexity in the considered system.

The ARP is applied by Van der Weide and Pandey (2015) as well, to model the unavailability of standby safety equipment. Their model does take ageing into account, but only ageing *within* a renewal cycle. Actually, this model can only be used if it is assumed that with every repair the whole system is renewed. This is a problematic assumption, since they are dealing with *"an assembly of many sub-systems or components*" (Van der Weide & Pandey, 2015, p. 97). Thus they do not include the complexity of the system into their model.

A Quasi-Renewal Process (QRP) is applied by Rehmert and Nachlas (2009) for modelling the availability of a repairable system. Different from strict renewal processes (such as the ARP above) the QRP does not restore the system to a "same as new" state after repair, hence the addition of "quasi". As with the ARP, the QRP by the authors runs through renewal cycles of an uptime followed by a downtime. The difference here is that the QRP *"provides a useful representation of the operation of a system for which successive operating intervals are stochastically smaller"* (Rehmert & Nachlas, 2009, p. 273), which means it can model ageing *over* cycles. However, the authors do not consider the complexity of the system and only model a one-unit system.

2.2.1.2 Poisson process and Virtual Age Process models

An often encountered method for availability and reliability modelling of repairable systems is the use of the (Non-) Homogeneous Poisson Process ((N)HPP) or Virtual Age Process (VAP) models. These point process models can incorporate deterioration through change in the failure intensity of the process over time. They can model minimal repair (NHPP), perfect repair (HPP) or imperfect repair (VAP). Parameter estimation methods for Poisson processes are well defined, which increases the applicability (Rigdon & Basu, 2000; Van Dyck & Verdonck, 2014). VAP models are specifically designed to model imperfect repair. They are based on other point processes, most notably on the Power Law Process (PLP), which is a specific form of the NHPP (De Toledo, Freitas, Colosimo, & Gilardoni, 2015; Doyen & Gaudoin, 2004, 2011; Ramírez & Utne, 2013).

Spinato, Tavner, Van Bussel, and Koutoulakos (2009) determine failure intensities for different subsystems of a wind power turbine system. They assume these failure intensities are following an NHPP type of process, called the Power Law Process (PLP). They consider different subsystems, but do not aggregate to the system level: all subsystems are treated as individual one-unit systems. Furthermore, the authors do not look into repair times and consequently do not consider availability calculations.

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[3] The Quasi-Renewal Process (QRP) is able to model deterioration and is considered in this research (section 4.3.5.2). However, technically it is not a renewal process, hence the name "quasi".

[4] Downtime might be considered zero, for modelling purposes. Then the renewal cycle consists of uptimes only.

Gonzalez, Torres, and Rios (2014) apply an NHPP with Monte Carlo simulation in order to assess the reliability and availability indices of a power distribution test system with failures attributed to adverse weather and ageing. Deterioration is modelled via the Power Law Process (PLP). They compare different cases with constant failure and repair rates, failures due to weather, failures due to ageing and failures due to weather and ageing. However, they consider average values for availability and reliability indices, thus the effects of the components' ageing are hidden on the system level. Moreover, they do not show how to compute system results from the obtained component results.

De Toledo et al. (2015) model the reliability of dump trucks in the mining industry via Arithmetic Reduction of Age (ARA) and Arithmetic Reduction of Intensity (ARI) models, which are VAP models. These models allow for the possibility of imperfect repair (as opposed to perfect or minimal repair) and determine the failure intensity over time. The authors do not consider a complex system or repair times and only calculate reliability to the first failure.

Kim and Singh (2010) incorporate ageing of subsystems into an NHPP model for calculating reliability indices for power systems. They show for one index the effect of one of the parameters, but it is not described how the system reliability index is calculated from the components' reliabilities. Furthermore, from 32 components they consider five to be ageing and the rest not. The authors show many average reliability indices and one instance of a reliability index changing over time. Moreover, repair times are not included and thus availability modelling is not discussed in this work.

2.2.2 Markov models

Many deterioration models make use of the binary-state assumption, meaning the considered component or system is either in a working state or in a failed state. However, this assumption may not always be appropriate for accurate modelling (Soro, Nourelfath, & Aït-Kadi, 2010). Markov models are multi-state models that enable the inclusion of different asset condition states. With a Markov process, the modelled asset moves from one deterioration state to the next one. Maintenance actions can bring the asset back to a better condition state. Markov models can be used as an assisting tool for optimising the maintenance strategy of deteriorating systems. With these models the asset performance for deteriorating repairable systems can be computed, although deterioration and maintenance actions are not always included in the consulted studies. Some studies only provide the steady-state solution for the underlying Markov process, while others do consider the development of the performance over time.

A disadvantage of this model type is the fact that for complex systems the total number of possible system states grows exponentially with the size of the model. This is called the state space explosion problem (Lanus, Yin, & Trivedi, 2003). Therefore, Markov models are only applied to a one-unit system or a simplistic system containing just a few components. For complex systems, consisting of multiple components, the use of a Markov model is inappropriate.

2.2.2.1 Markov process

In the study by Chan and Asgarpoor (2006) a Markov process is used to find the optimal value for the maintenance rate λ_m. The steady-state availability is computed as a function of λ_m. The optimal maintenance rate is assumed to be the one for which the steady-state availability is maximised. The availability cannot be modelled over time with this Markov process and the approach is applied to a one-unit system, so no complexity in system configuration is considered.

Tan and Raghavan (2008) developed a practical framework for predictive maintenance scheduling of multi-state systems. In their model a Markov process is used to analyse a small water pipe system, consisting of three elements (the first two elements are in parallel, followed by the third element in series). The modelled measure here is the *mean performance for an operation cycle*. The influence of maintenance operations on this mean performance is analysed in this study. The concerned performance index corresponds to the availability, but only mean values are computed (no modelling over time). Besides, the method cannot be applied to complex systems.

Zhu (2012) presents an availability analysis tool for large networking systems, with the main focus on configurations with redundancy. A Markov chain model is applied for steady-state availability calculations of redundant components. For non-redundant components the availability is computed based on the Mean Time Between Failures (MTBF) and the Mean Time To Repair (MTTR). With this method only steady-state results are computed with the Markov chain model. Also the inclusion of deterioration in this case is doubted. The concerned system is a *k*-out-of-*n* system, which deteriorates to the next state each time a component fails. This corresponds to deterioration at the system level, but deterioration at the component level is not included in this study.

Soro et al. (2010) suggest a Markov model for the performance evaluation of multi-state systems. A discrete number of condition states is considered, where each condition state corresponds to a certain performance rate. The approach is capable of modelling the availability over time for deteriorating, repairable systems. However, the concerned system is handled as a one-unit system. Any complex system configuration is not regarded.

2.2.2.2 Semi-Markov Process

With the Markov process the holding times are exponentially distributed, meaning the probability of moving to the next state is the same for each time step. The so-called Semi-Markov Process (SMP) can handle with this limitation, since its holding times follow non-exponential distributions. For the analysis of deteriorating repairable assets, the latter is often assumed to be more appropriate (Barbu, Karagrigoriou, & Makrides, 2017; Black, Brint, & Brailsford, 2005; Thomas & Sobanjo, 2016; Tomasevicz & Asgarpoor, 2009; Vinayak & Dharmaraja, 2012; Wang, Cui, Zhang, & Peng, 2016). This enables the inclusion of time-dependency for the transition probabilities: the probability of moving to the next condition state depends on how long the asset is already in its current condition state.

Kleiner (2001) developed an SMP based methodology for modelling asset deterioration. Four discrete deterioration states are assumed and the fifth state corresponds to failure. The approach is based on Weibull distributions for the holding times of the condition states, and results in a survival curve (reliability function *R(t)*) for a deteriorating asset. The work by Kleiner is well-known in the field of reliability engineering and his papers are referred to in many other studies (e.g. Black et al. (2005) and Kim, Choi, Suh, and Lee (2015)). However, maintenance is not included in the methodology, so it is limited to reliability calculations. Furthermore, it cannot be used for modelling of complex systems.

Yin, Fricks, and Trivedi (2002) describe an SMP model to get closed-form formulas for the reliability *R(t)* and point availability *A(t)*. Their method was applied to two types of power supply units, consisting of respectively three and four components. The SMP model can plot the availability as a function of time for repairable, deteriorating assets, but it cannot be applied for complex systems, due to the aforementioned state space explosion problem that occurs with multi-state models applied to complex systems. The authors also describe a Markov chain model for comparison and show this second model cannot be used to derive closed-form formulas. The reliability function is plotted over time for several failure rates, but for availability analysis only the steady-state solution can be computed. Also this Markov chain model cannot be used to analyse complex systems.

Vinayak and Dharmaraja (2012) use a semi-Markov modelling approach to model deterioration of a one-unit system. Five discrete condition states are assumed here. Steady-state calculations for the SMP are presented that provide the steady-state availability, maintainability and Mean Time To Failure (MTTF) (as a measure for reliability). This approach cannot be used for modelling availability over time and it cannot be used for the analysis of complex systems as well.

Kumar, Jain, and Gandhi (2013) perform an availability analysis for repairable mechanical systems based on the steady-state solution of an SMP. The considered system consists of two pumps in standby redundancy, each pump having three condition states: operating, failed, and standby. Combining the states of both pumps results in a total number of five system states. The steady-state availability is calculated analytically and Monte Carlo simulation is used to validate the results. The authors conclude that the results of both methods are matching.

However, the system considered here is very simplistic. The pumps are assumed to have only one operating state, instead of assuming several deteriorating states. Besides, only steady-state solutions are obtained.

2.2.2.3 (Semi-)Markov Decision Processes

The Markov Decision Process (MDP) and Semi-Markov Decision Process (SMDP) can be seen as an extension of the discussed Markov and semi-Markov processes. In addition to the underlying Markovian process, with decision processes also actions and rewards are included in the modelling. (S)MDPs are often used to model and assess different maintenance policies, in order to enable better grounded decision-making. (S)MDPs can be an important tool for optimisation problems within the field of asset management. Availability analysis is often part of this optimisation process.

Ahmed, Khan, and Raza (2014) use an MDP to estimate the availability of a treatment unit in a gas processing plant. The unit is divided in 14 sub processes, for which the availability is determined based on the steady-state solution of the underlying Markov process. In the end, an RBD is used to calculate the (steady-state) system availability, so the availability is not modelled over time. Average failure and repair rates are used to estimate the availabilities of the sub processes, so deterioration is not included. The combination of the MDP and RBD is in fact a hierarchical model, but the authors never mention this term.

Chen and Trivedi (2005) present the use of an SMDP to optimise condition-based maintenance. This approach is useful for repairable assets that deteriorate, but only calculations for the steady-state availability are provided. Moreover, the described case system is assumed as a one-unit system. The SMDP cannot cope with complex configurations.

Tomasevicz and Asgarpoor (2009) also apply an SMDP and solve for the best maintenance policy by using a policy iteration method. First, the steady-state probabilities are calculated with the underlying SMP. Based on this, the steady-state availability can be computed as a function of the maintenance rate λ_m. The optimal maintenance rate is then determined as being the value for which the steady-state availability complies with the requirements. A numerical example is provided for an electric circuit breaker, having three discrete states of deterioration. However, only average values are used for the transition rates in the process. No parameters are given for any non-exponential distribution of the holding times in the SMP, which is remarkable. Also no complex system configuration is considered here, since a one-unit system is considered in the example.

2.2.3 Bayesian networks

Another modelling method for availability and reliability modelling of complex repairable systems is the use of a Bayesian network. A Bayesian network has two parts: 1) a directed acyclic graph and 2) a set of conditional probability functions. In reliability context these parts together define a probability density function of the failure probability of a system or component.

Görkemli and Ulusoy (2010) present reliability and availability calculations for a production system by using a fuzzy Bayesian method. In contradiction with most studies, they do address the system structure. Failure times and repair times are assumed to be exponentially distributed and fuzzy Bayesian estimates for the failure rate λ and repair rate μ are used for calculations. With these estimations the interval reliability (the reliability $R(t^*)$ at t^* being the end of the interval) and availability for the different components can be determined. The last step is combining the component results according to the system configuration, by using the minimal path sets method[5]. In this paper, deterioration of the system is not considered in the analysis. Moreover, the reliability and availability are not modelled over time, since only the interval availability is calculated.

Cai et al. (2015) propose a two-stage Dynamic Bayesian Network (DBN) in order to evaluate the reliability of a subsea pipe ram blowout preventer system in real-time. The method couples root cause analysis, based on observable information from the system, and reliability evaluation. They examine the complex system's

[5] See Appendix 1

reliability to the first failure for several cases wherein different components fail at different times. Repair times are not considered here, and thus availability calculations cannot be provided.

Cai, Liu, Zhang, Fan, and Yu (2013) assess the reliability and availability for a subsea blowout preventer, which is a repairable system with multiple components. The reliability and availability are calculated on the basis of a DBN, which itself is based on a fault tree where every basic event has multiple states. Within a basic event deterioration takes place by moving from a working state through deterioration states to a failed state. Perfect and imperfect repair are possible from the failed state. Finally, they show point reliability and availability for the event: *"blowout, given a kick, while the drill string is running through the blowout preventer*" (Cai et al., 2013, p. 7551) under both perfect and imperfect repair. While this may look as meeting all criteria, there are some limitations to this study: (1) the transition rates between states are assumed to be constant and exponentially distributed, which simplifies the deterioration process, (2) the DBN is based on failure *events*, not on failing components, and (3) the model here is only able to consider simple configurations (i.e. combinations of serial or parallel configuration).

2.3 Hierarchical models

The third and last modelling method discussed in this chapter is the use of a hierarchical model. For the reliability and availability analysis of systems with complex configurations, a hierarchical model can be a practical solution. With these models, a hierarchical structure of sub-models is used in order to reduce the model complexity. This is a well-known approach to solve the state space explosion problem that occurs when Markov models are applied to complex systems (Ramezani, Latif-Shabgahi, Khajeie, & Aslansefat, 2016). In most cases the hierarchy consists of two levels: the component level (lower level) and the system level (upper level). The results of the component modelling form the input for the modelling at the system level. First, at the component level the availability is computed, and second, these component results are used as input for the model at the system level to obtain system results.

Smith, Trivedi, Tomek, and Ackaret (2008) suggest a two-level hierarchical model to perform the availability analysis of blade server systems. Markov sub-models are used to calculate the steady-state availabilities of subsystems at the lower level and an FTA is applied to make the step towards system level. With the Markov sub-models it is possible to include multiple condition states of a subsystem, so the method would be suitable for including deterioration. However, the failure and repair rates are assumed to be constant, which is not realistic in many real-life situations. This method is not capable of modelling the availability over time, since only steady-state solutions can be obtained.

Tang and Trivedi (2004) make use of a hierarchical Markov model for the interval availability evaluation of high-available computer systems. The interval availability is defined as the average availability in an interval. Also the interval failure rate is calculated. Small intervals are used so the development of the availability and failure rate over time can be analysed (transient analysis), which is impossible with only steady-state analysis. This is illustrated by comparing the interval results and steady-state results. The system in the simple example model consists of two subsystems, which are both described by a Markov chain model. Both sub-models are combined in another simple Markov model at the system level. However, in the sub-models constant failure and repair rates are assumed, so asset deterioration is not incorporated in this paper. Also for the analysis of complex configurations, consisting of many components, this hierarchical Markov model is inappropriate.

Lanus et al. (2003) describe a hierarchical modelling technique for the availability and performance analysis of complex systems. Markov sub-models are used to compute the steady-state availability for the different subsystems in one level of the hierarchy. The described technique is about aggregation of all upstates together and all downstates together, in order to derive an equivalent failure rate and repair rate. These measures are then used as input parameters for the next level in the hierarchy, which is modelled with another Markov model. Examples are provided where this procedure is applied for systems consisting of two or three different levels. This modelling approach does consider complex, repairable systems, but asset deterioration and availability modelling over time are not included.

Ramezani et al. (2016) propose a hierarchical model for the computation of a Dynamic Fault Tree (DFT). With this methodology, for each dynamic gate in the fault tree the Markov model is converted into its equivalent two-state Markov model. Based on this, the steady-state availability is computed for all gates. These results are then combined into a two-state Markov model for the total system in order to compute the system availability. Two examples are provided in the paper. One of them considers a cardiac assistant device, which also contains a pump section. This pump section is represented by a two-state Markov model with constant failure and repair rate, meaning no asset deterioration is included this way. Also the modelling of availability over time is not covered with this study.

Trivedi, Kim, and Ghosh (2013) present different case studies in their paper. Two of them show the application of hierarchical models, where an RBD is used at the upper level (to deal with the complexity) and Markov chains at the lower level (that can model repairable, deteriorating components). Nevertheless, only steady-state solutions are computed here.

The work by Wong et al. (2006) provides a methodology to enable the availability assessment of tunnel designs, based on the Markov process. The main point of the authors is to convert multi-component systems into their two-state equivalents by grouping all upstates together and grouping all downstates together, in order to reduce the model complexity for systems with many condition states. The method is applied to five complex subsystems, for which the steady-state availability is computed. In the end, the system availability is computed through a simple fault tree, where the five subsystems are connected with an OR gate (when one of the subsystems fails, the system fails). Similar to the aforementioned methodologies, this method is not capable of modelling availability over time.

2.4 Overview consulted literature

Table 2: Overview of the consulted literature regarding reliability and availability modelling

Year	Author(s)	Method	Calculated performance measure(s)			Type of calculations			Is the methodology able to deal with:				Complexity of concerned system	
			Availability	Reliability	Other	Steady-state	Average (Interval)	Point	Repairability		Deterioration		No. of components or subsystems	Configuration
									Yes	No	Yes	No		
2001	Kleiner	SMP		X				X		X	X		1	one-unit system
2002	Yin, Fricks, & Trivedi	SMP, CTMC	X	X		X		X	X		X		a) 3; b) 4	a) Serial; b) Serial, Standby
2003	Lanus, Yin, & Trivedi	Hierarchical model (Markov process)	X		Performability	X			X			X	20	Series, Parallel, Redundant
2004	Tang & Trivedi	Hierarchical model (Markov process)	X		Failure rate		X		X			X	2	Unknown
2005	Chen & Trivedi	SMDP	X			X			X		X		1	one-unit system
2006	Wong, Tsang, & Chung	Hierarchical model (Markov process + FTA)	X			X			X		X		5 (complex) subsystems	Series
2006	Chan & Asgarpoor	Markov process	X			X			X		X		1	one-unit system
2008	Maciejewski & Caban	ARP	X			X	X		X			X	1	one-unit system
2008	Smith, Trivedi, Tomak, & Ackaret	Hierarchical model (Markov process + FTA)	X			X			X		X		Many	Series, Parallel, Redundant
2008	Tan & Raghavan	Markov process			Performance Index		X		X		X		3	Series, Parallel
2008	Spinato, Tavner, Van Bussel, & Koutoulakos	NHPP			Failure intensity			X	X		X		1	one-unit system
2009	Durga Rao et al.	FTA	X				X		X			X	Many	Unknown
2009	Tomasevicz & Asgarpoor	SMDP	X				X		X		X		1	one-unit system
2009	Rehmert & Nachlas	QRP	X					X	X		X		1	one-unit system
2010	Soro, Nourelfath, & Ait-Kadi	Markov process	X	X	Production rate			X	X		X		1	one-unit system
2010	Görkemli & Ulusoy	Fuzzy Bayesian method	X	X			X		X			X	Many	Series, Parallel
2010	Kim & Singh	NHPP, MCS			Reliability Index		X	X	X		X		Many	Unknown
2012	Zhu	Markov process (+ RBD)	X			X	X		X			X	Unknown (18)	Series, Parallel, Redundant
2012	Vinayak & Dharmaraja	SMP	X	X		X			X		X		1	one-unit system
2013	Trivedi, Kim, & Ghosh	Hierarchical model (CTMC + RBD)	X			X			X		X		a) 7 b) 18	a) Serial, Redundant, Standby b) Series, Parallel, Redundant
2013	Cai, Liu, Zhang, Fan, & Yu	DBN	X	X				X	X		X		Many	Series, Parallel
2013	Kumar, Jain, & Gandhi	SMP	X			X			X		X		2	Standby
2013	Bourouni	RBD, FTA	X				X			X		X	Many	Series, Parallel, Redundant
2014	Ahmed, Khan, & Raza	MDP + RBD	X			X			X			X	18	Series, Parallel
2014	Gonzalez, Torres, & Rios	(N)HPP, MCS, (ARP)			Reliability Index & Availability Index		X		X		X		Many	Unknown
2015	Van der Weide & Pandey	ARP	X			X		X	X		X		1	one-unit system
2015	De Toledo, Freitas, Colosimo, & Gilardoni	VAP		X	Failure intensity			X	X		X		1	one-unit system
2015	Cai et al.	DBN		X				X		X	X		Many	Unknown
2015	Taheriyoun & Moradinejad	FTA		X			X			X		X	Many	Series, Parallel, Redundant
2016	Ramezani, Khajeie, Latif-Shabgahi, & Aslansefat	Hierarchical model (Markov process + DFT)	X			X			X			X	Many	DFT gates: AND, OR, SPARE, PAND, FDEP
2016	Choi & Chang	FTA	X	X			X (Availability)	X (Reliability)	X			X	Many	Series, Parallel
2018	Velasquez & Lara	Minimal path sets	X		Failure rate		X		X			X	7	Series

2.5 Conclusions literature review

For complex systems such as WTPs the availability assessment of the whole system is of main interest to the asset manager. However, many studies consulted for this literature review do not consider complex configurations, but stay on the component level or treat the considered system as a one-unit system (see Table 2). Furthermore, the goal is to model the availability over time (point availability), which is not done in most studies as well. Often the steady-state or interval performance is computed, not providing any information about the development of the performance over time due to deterioration. From all literature reviewed in this research, seven of the papers meet at least three of the four criteria for the modelling approach[6]. These are briefly discussed first. Subsequently, general conclusions are presented for the model types that are handled in this chapter.

2.5.1 Conclusions on consulted studies

The study by Yin et al. (2002) describes both a Markov chain model and a Semi-Markov-Process (SMP) model. With the SMP model the point availability for a repairable system with deteriorating assets can be computed, but it can only be applied to a small system consisting of a few components. The same limitation counts for the paper by Soro et al. (2010), where a Markov model is applied to a one-unit system, the paper by Rehmert and Nachlas (2009) about a Quasi-Renewal Process (QRP) model, and the work by Van der Weide and Pandey (2015) concerning an Alternating Renewal Process (ARP).

On the other hand, Kim and Singh (2010) do consider a complex configuration with their Non-Homogeneous Poisson Process (NHPP) model, but it is unclear how they aggregate the results on a component level to the system level. Furthermore, they provide mostly average figures of reliability indices and only one instance of a reliability index trend over time. More importantly, they do not consider repair times and thus their model is unable to model availability.

The hierarchical modelling approaches by Wong et al. (2006), Smith et al. (2008) and Trivedi et al. (2013) do involve repair times and availability analysis, but these models are limited to steady-state results. Moreover, in these studies constant failure and repair rates are assumed between the different condition states in the Markov processes that describe the different subsystems. For the modelling of deteriorating assets within a WTP this is not a realistic assumption.

The work of Cai et al. (2013) is the only consulted paper that meets all four criteria. However, there are some remarks on this paper which make it still unsuitable for the intended purposes of this research: 1) they simplify the deterioration process, 2) the method used is based on *events*, not on *components* and 3) it only considers *minor* complexity, i.e. combinations of serial and parallel configurations.

Therefore, the conclusion is that none of the aforementioned literature presents a modelling approach that can model the availability over time for systems that meet all three system criteria (repairable and deteriorating assets, complex configuration). Below, also conclusions are given per model type that is discussed in this chapter.

2.5.2 General conclusions on discussed modelling techniques

From the literature review it can be concluded that the use of only a combinatorial model in the form of an RBD or FTA is too simplistic for the aim of this research. This approach can provide a relatively quick and easy estimation of the system performance, but the calculations are based on average or steady-state values for the component measures. The influence from maintenance and asset deterioration on the system performance, and how this develops over time, cannot be modelled with only an RBD or FTA. The deterioration models described in section 2.2 are more appropriate for modelling dynamic system behaviour. However, problems

[6] See chapter 1 (Introduction) for the research criteria and Table 2 (at page 22) for an overview of the consulted literature and how this is evaluated.

occur when these models are applied to complex systems with a large number of components. Hierarchical models are not facing this limitation and can be a solution for the problems that appear with Markov models, point process models and Bayesian networks.

Markov models
The advantage of Markov models is that multiple condition states can be included in the model, instead of only one working state and one failed state (binary modelling). This makes that multiple asset deterioration states can be considered. Also the influence of maintenance actions can be taken into account with Markov models. Not only minimal or perfect repair can be modelled: due to the possibility of including multiple condition states, also the modelling of imperfect repair actions is enabled.

The main disadvantage is the so-called state space explosion problem: the number of states grows exponentially with the size of the model. For this reason Markov models are only applied to one-unit systems or systems that consist of a few components.

Point process models
The advantages of the Poisson process and VAP models are that they are (1) able to model deterioration, (2) are well known within the field of reliability engineering and consequently have well defined parameter estimation methods, and (3) they can model perfect (HPP, VAP models), imperfect (VAP models) and minimal repair (NHPP, VAP models). Furthermore, parameter estimation methods for Poisson processes are well defined.

However, when these models are applied to a complex system as a whole, also parameters need to be determined for the system as a whole. Especially for systems with high reliability and availability, such as WTPs, there is not enough failure data available for accurate modelling of the complete system with one point process model. Besides, applying one point process model to a complete complex system reduces the model accuracy. Also the critical components cannot be identified without considering the system configuration.

Bayesian networks
Bayesian networks could be a solution for modelling the reliability and availability of complex, repairable systems with deteriorating components. However, it is based on a fault tree which focusses on specific failure mechanisms. In order to include all relevant failure mechanisms related to deterioration for all components, the number of nodes in the Bayesian network explodes. This makes computation costly and time-consuming. Furthermore, fault tree approaches become harder to comprehend with increasing complexity and size of systems (see also sub-section 2.2.3).

Hierarchical models
Hierarchical models can be a practical solution for the problems that occur with the availability modelling of complex system configurations. However, in literature no hierarchical modelling approach was found that models the availability over time (in combination with the other three criteria). In many studies only steady-state solutions are provided. In the next chapter the authors' solution to this gap in literature is presented.

3 Solution: two-level hierarchical model

From the literature study it was concluded that using only the use of a combinatorial model (RBD or FTA) or one of the discussed deterioration models (Markov models, point process models, and Bayesian networks) is insufficient for achieving the research goal as defined in chapter 1. Combinatorial models are too simplistic and the deterioration models are not suitable for complex system configurations. Therefore, a hierarchical model can be a practical solution to deal with these problems. To summarise, the two main advantages of using a hierarchical model are (Tang & Trivedi, 2004):

1. Reducing the model complexity.
2. Enabling the identification of critical subsystems or components.

However, in literature no hierarchical modelling approach was found that could model the availability over time for complex, repairable systems with deteriorating assets. Therefore, the authors' solution for this gap in literature is about combining multiple modelling techniques.

First of all, a two-level hierarchical model is proposed, which can be divided into a *component level* (lower level) and a *system level* (upper level). The first step is constructing a combinatorial model, according to the configuration of the concerned system. For this step, typically an RBD or FTA is used (Distefano & Puliafito, 2007; Smith et al., 2008; Tanino & Fukazawa, 1988; Yin et al., 2002). The next step is the availability modelling of the individual components (modelling at the component level). Only computation of the steady-state availability is not sufficient here. Therefore, at the component level the availability should be modelled over time by means of deterioration modelling. Then, the last step is combining the obtained component results with the constructed combinatorial model, in order to model the availability of the whole system.

In this chapter first the modelling at system level is further described in section 3.1. The choice between using an RBD or an FTA is argued here. Thereafter, in section 3.2 the availability modelling at the component level is addressed. The theory behind the proposed deterioration models for availability modelling at the component level is further explained here. Finally, a flowchart is presented in section 3.3 which supports the asset manager's decision-making on which deterioration model is the optimal choice for which asset.

3.1 System level: combinatorial model

The RBD and FTA are the two most widely used combinatorial models (Distefano & Puliafito, 2007). As their names might imply, the Reliability Block Diagram (RBD) is a success-oriented approach and the Fault Tree Analysis (FTA) is a failure-oriented approach (Rausand & Høyland, 2004; Wang, 2014).

3.1.1 Reliability Block Diagram

The RBD is a graphical analysis tool which expresses the contribution of individual components to the total system performance, in terms of reliability or availability (Guo & Yang, 2007). An RBD is drawn as a structure of linked blocks, where each block represents a function or a component. The output of such a block is True (or 1) when it is working, and False (or 0) when it is failed. The system can fulfil its function if an uninterrupted path of success is present from the beginning to the end of the RBD (Distefano & Puliafito, 2007).

The two most common configurations are the serial and parallel configurations. With components in series, the system only works when all components are working. With a parallel configuration, the system works as long as at least one component is working (Kim, 2011; Nikolaidis, Ghiocel, & Singhal, 2004).

With the RBD method, the system availability can be calculated as a function of the component availabilities. This can be expressed as:

$$A(t) = \psi(A_1(t), A_2(t), \dots, A_n(t))$$

Where the structure function ψ depends upon the system configuration (Wang, 2014). For independent components, the system performance can be obtained by deriving the minimal path sets. Within an RBD, a minimal path set is the minimal set of components whose functioning ensures the system working. A minimal path set is only a path of success when all components within the path are working. In other words, when one component is removed from a minimal path set, the set is no longer a path set (Kim, 2011; Mejjaouli & Babiceanu, 2016; Rausand & Høyland, 2004). The minimal path sets method is further described in Appendix 1 .

For the most common configurations, general structure formulas are at hand for the computation of the system availability (Birolini, 2017; Wang, 2014). Among these are the serial and parallel configurations:

- Series: $A(t) = \prod_{i=1}^{n} A_i(t)$
- Parallel: $A(t) = 1 - \prod_{i=1}^{n}(1 - A_i(t))$

To illustrate what an RBD looks like, Figure 5 shows an example of an RBD for a so-called *bridge structure*, which is widely used in the field of reliability and availability engineering. In this structure, the minimal paths would be {A,C}, {B,D}, {A,E,D} and {B,E,C} (Kim, 2011). The complete derivation of the structure function for the bridge structure can be found in Appendix 1 .

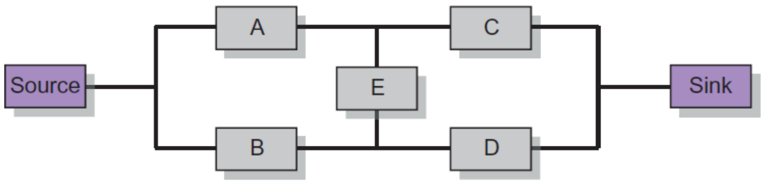

Figure 5: Reliability Block Diagram for a bridge structure, from Kim (2011)

3.1.2 Fault Tree Analysis

A fault tree is a top-down (deductive) graphical model that is used to analyse which combinations of component failures could cause system failure (Distefano & Puliafito, 2007). The occurrence of a system failure is known as the top event and is the starting point for the FTA. Fault trees consist of *events*, describing the failures of components in the system, and *logic gates*, describing the relationship between the different events (Belland & Wiseman, 2016; Birolini, 2017; Wang, 2014).

When an event in a fault tree is active, corresponding to a failure, the output of the event is True (or 1). On the other hand, when an event is non-active, the output is False (or 0) (Birolini, 2017). Based on the output of events and the connecting logic gates, the status of the top event can be determined. The two most well-known and widely used gates are the AND and OR gates, see Figure 6. The former results in a True output (failure) if all input events are True, and the latter gives a True output when at least one of its input events is True. Compared to the RBD method, the AND gate is equivalent to the parallel configuration and the OR gate is equivalent to the serial configuration in an RBD (Wang, 2014).

Similar to the RBD, the FTA method can be used to calculate the system availability as a function of the component availabilities, according to the system configuration. For fault trees, the system availability can be obtained by using the minimal cut sets, which are equivalent to the minimal path sets for an RBD. A minimal cut set is a minimal set of events that causes system failure. For the most common fault tree structures general structure formulas are known and present in literature (Birolini, 2017; Distefano & Puliafito, 2007; Mejjaouli & Babiceanu, 2016; Rausand & Høyland, 2004). Figure 7 shows an example of a fault tree for the same bridge structure as the RBD in Figure 5 was constructed for.

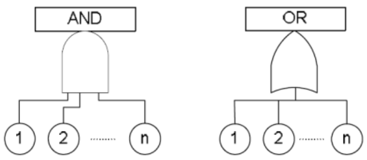

Figure 6: AND and OR gates as used in fault trees (Distefano & Puliafito, 2007)

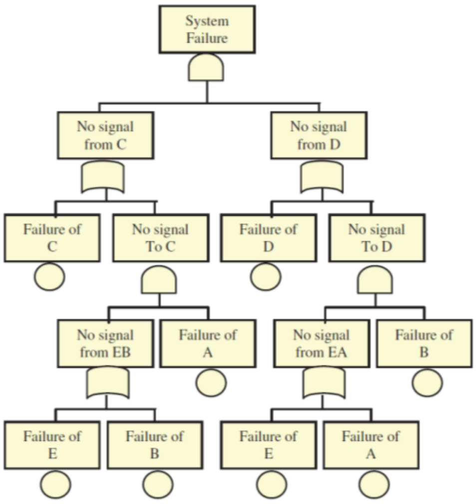

Figure 7: Fault Tree for a bridge structure (Kim, 2011)

3.1.3 Conclusion on combinatorial models

For the hierarchical modelling approach the choice must be made between using an RBD or FTA. Both are combinatorial models and for many systems, the RBD can even be converted into its equivalent fault tree, and vice versa (Distefano & Puliafito, 2007; Rausand & Høyland, 2004). When this is the case, the end results for both methods do not differ at all. However, both methods do have their own advantages and disadvantages.

With a fault tree, all potential causes of system failure can be identified. So when the aim of the analysis is a better understanding of how a system might fail, the FTA method is recommended (Rausand & Høyland, 2004). On the other hand, the main advantage of the RBD is a better readability (Distefano & Puliafito, 2007). This is well illustrated by the examples for a bridge structure in Figure 5 and Figure 7. This is only concerning a structure containing five components, but when the configuration grows more and more in size and complexity, it becomes harder to read the systems' fault tree and trace back the system structure.

In this research, the goal is to model the system availability as a function of the component availabilities. Identification of possible failure causes is of less interest here. Therefore, the use of an RBD is more suitable for the intended purposes and will be used for the modelling at the system level of the hierarchical model.

3.2 Component level: theory on proposed model types

At the lower level of the hierarchical model, each component corresponds to a block in the RBD. For each component the availability must be modelled over time, before system results can be obtained. Several deterioration models can be used for this, as described in section 2.2. As concluded in section 2.5, the Bayesian networks are less useful than the Markov models and point process models for modelling the availability of deteriorating components within complex, repairable systems. Therefore Bayesian networks will be left out of this research from here on. Renewal processes, because of their inability to model deterioration, are not considered hereafter as well. This leaves the Markov models and the Poisson process and VAP models. This section is about the underlying theory of these models and based on this it is decided which specific models to use.

3.2.1 Markov models

Markov models are based on Markovian processes. Several types can be distinguished, which are described in this section. The main distinction can be made between the Markov and semi-Markov process, which are discussed first. After that, also the (Semi-) Markov Decision Processes ((S)MDPs) are handled, which are extensions to the (semi-)Markov processes. At the end, it is argued which type of Markov model to use in the hierarchical model.

3.2.1.1 Markov process

The Markov process is characterised by its memoryless property (the Markov property), which means that the future state of a system or component only depends on the present state. What happened in the past has no influence on the probabilities of future condition states (Grabski, 2014; Ibe, 2013). Markov processes can have a discrete or continuous state space and a discrete or continuous time space (Ibe, 2013; Wong et al., 2006). Table 3 shows the four possible combinations of state space and time space. Moreover, a Markov process can be time-homogeneous or time-inhomogeneous (Ana & Bauwens, 2010).

Table 3: Different types of Markov processes (Wong, Tsang, & Chung, 2006)

Type	State space	Time space
1	Discrete	Discrete
2	Discrete	Continuous
3	Continuous	Discrete
4	Continuous	Continuous

Continuous-state Markov processes (type 3 and 4 in Table 3) are hardly used in the field of reliability engineering (Ramakumar, 1993). As mentioned before, continuous deterioration models are useful when the goal is to analyse very specific deterioration processes. However, for the aim of this research (availability modelling in order to optimise maintenance and system configuration) a discrete number of condition states is more appropriate. This also makes the modelling process more convenient. Therefore, only Markov processes with a discrete number of condition states are considered here. Continuous-state Markov processes are not considered here.

Markov chains

A Markov process with a discrete state space is referred to as a Markov chain (type 1 and 2 in Table 3). With Markov chains, the total time that the process stays in a particular state is called the sojourn time or holding time of that state (Ibe, 2013; Tomasevicz & Asgarpoor, 2009). Each state corresponds to a distribution of its holding times. This also counts for failed states. A key characteristic of the homogeneous Markov process is the fact that the holding times are independent and exponentially distributed. This means that the probability of going from one state to another does not change over time.

A Markov chain can be characterised as a jump process, that is a stochastic process making transitions between discrete states (Ibe, 2013). For continuous-time Markov chains these transitions can occur at any random time, while for discrete-time Markov chains the jumps only take place at fixed points in time.

Discrete-time Markov chains are fully described by their transition probability matrix, containing all transition probabilities. Such a transition probability $p_{ij}^{t,t+1}$ is the conditional probability of the process going from state *i* to state *j* in the time interval between *t* and *t* + 1 (Kleiner, 2001; Meyn & Tweedie, 1993). The transition probability matrix *P* is always a square *n* x *n* matrix, with *n* equal to the discrete number of condition states considered in the model.

$$P^{t,t+1} = \begin{bmatrix} p_{11}{}^{t,t+1} & p_{12}{}^{t,t+1} & \cdots & p_{1n}{}^{t,t+1} \\ p_{21}{}^{t,t+1} & p_{22}{}^{t,t+1} & \cdots & p_{2n}{}^{t,t+1} \\ \vdots & \vdots & \ddots & \vdots \\ p_{n1}{}^{t,t+1} & p_{n2}{}^{t,t+1} & \cdots & p_{nn}{}^{t,t+1} \end{bmatrix} \quad ; \quad p_{ij}{}^{t,t+1} \geq 0 \ (i = 1,2,\dots,n)$$

The sum of the probabilities in one row must be equal to 1, which makes it a stochastic matrix (Ibe, 2013).

$$\sum_{j=1}^{n} p_{ij}{}^{t,t+1} = 1 \quad ; \quad (i = 1,2, \ldots., n)$$

With a time-inhomogeneous Markov chain, the transition probabilities themselves are functions of time, so the transition matrix contains different values for each time step.
In case of a homogeneous Markov chain, the transition probabilities do not change over time, which makes $p_{ij}{}^{t,t+1} = p_{ij}$ for all discrete time steps (Ibe, 2013). This would simplify the matrix *P* to:

$$P = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{n1} & p_{n2} & \cdots & p_{nn} \end{bmatrix}$$

In order to visualise the theory, Figure 8 shows the state transition diagram of a homogeneous discrete-time Markov chain with three discrete condition states (*working, degraded* and *failure*). The process fully depends on the transition probabilities, with p_{ij} being the transition probability from state *i* to state *j*. Due to the homogeneousness, the transition probabilities are constant over time in this example.

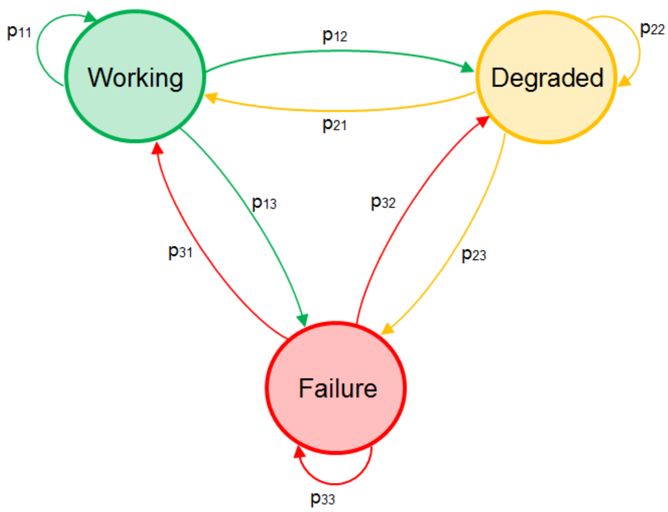

Figure 8: State transition diagram of a homogeneous discrete-time Markov chain

The probability of the process being in state *i* at time *t* is called a_i^t. The vector *A(t)* with dimensions *1* x *n* contains the probabilities of being in each individual state at a certain time *t* (Kleiner, 2001).

$$a_i^{t+1} = \sum_{j=1}^{n} a_j^t p_{ji}^{t,t+1} \quad ; \quad (i = 1,2, \ldots, n)$$

$$A(t) = \{a_1^t, a_2^t, \ldots, a_n^t\} \quad ; \quad \sum_{i=1}^{n} a_i^t = 1$$

$$A(t+1) = A(t)P^{t,t+1}$$

For homogeneous Markov chains the transition matrix *P* does not change over time, so the vector *A(t)* can be calculated as follows:

$$A(t) = A(0)P^t$$

where *A(0)* is the starting vector $[a_1^0, a_2^0, \ldots, a_n^0]$ with a_i^0 being the probability of the process being in the state *i* at *t* = 0 (Baik et al., 2006). After numerous time steps the vector *A(t)* converges to a steady-state situation, which means the influence of the starting vector *A(0)* decreases over time, and eventually becomes zero if the steady-state situation is reached.

With discrete-time Markov chains the state transitions can only occur at discrete points in time, while with continuous-time Markov chains these jumps can occur at any point in time. With a continuous time space, the holding time of a condition state can be any value, according to its exponential distribution. When the process enters a certain condition state *i*, a value for the holding time is generated based on its exponential distribution. When this time has passed by, the process enters the next state *j*, based on the transition matrix *P* (Ibe, 2013).

3.2.1.2 Semi-Markov Process

As mentioned before, with the Markov process (due to the Markov property) it is defined that the state at time *t + 1* only depends on the state at time *t* and the holding times are exponentially distributed. This means the transition probabilities are independent of how long the process is in a certain state (Black et al., 2005). However, for the description of a deterioration process a non-exponential distribution for the holding times is more appropriate (Barbu et al., 2017; Black et al., 2005; Thomas & Sobanjo, 2016; Tomasevicz & Asgarpoor, 2009; Vinayak & Dharmaraja, 2012; Wang et al., 2016). This enables the inclusion of time-dependency for the transition probabilities: the time of the process being in a certain condition state influences the probability of moving to a next state. For example, when an asset is in a certain condition state for a long time already, as a result of deterioration the probability of moving to the next deterioration state is likely to be higher compared to the situation where the asset is in that condition state only shortly. With a regular Markov process these transition probabilities are always constant in time, but with the so-called Semi-Markov Process (SMP) they are not. For this reason, the SMP is often assumed to be more realistic for deterioration modelling. With the SMP, the holding times are random variables following non-exponential distributions. The transitions (or jumps) between the different states still occur via the Markov process, according to the transition probabilities. However, due to the non-exponential distributed holding times, the Markov property is not valid anymore for the whole process, but only at the jump instants. For this reason it is called a *semi*-Markov process (Kleiner, 2001; Scheidegger, Hug, Rieckermann, & Maurer, 2011; Trivedi, Vaidyanathan, & Selvamuthu, 2015; Yuan, 2017).

The SMP shows some similarities with the non-homogeneous Markov process, because with both processes the transition probabilities change over time. However, with the non-homogeneous Markov process the transition probabilities are functions of the total process time, while with the semi-Markov process a transition probability is a function of the time the process is in a particular state already. Therefore, the Markov (memoryless) property is not valid for the SMP and this is the main difference with the non-homogeneous Markov process.

Similar to the regular Markov process, the SMP can have a discrete or a continuous time space. Figure 9 shows the state transition diagram of a continuous-time SMP with three condition states, with $Q_{ij}(t)$ being the conditional probability of moving from state *i* to state *j* with a holding time H_n no more than *t* (Ibe, 2013). This holds:

$$Q_{ij}(t) = P[X_{n+1} = j, H_n \leq t \mid X_n = i]$$

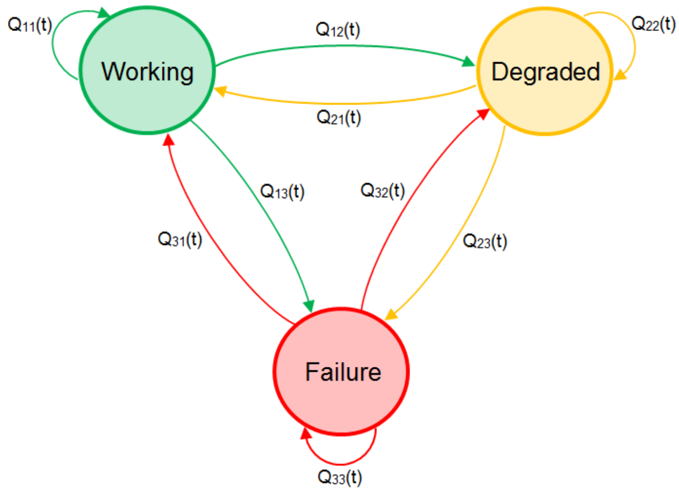

Figure 9: State transition diagram of a continuous-time Semi-Markov Process

In most studies, the two-parameter Weibull distribution is used to describe the holding times in an SMP. It has the characteristic that it can model a wide range of distributions, just by varying two parameters, the scale parameter and the shape parameter (Black et al., 2005). In the special case when the shape parameter = 1, the Weibull distribution equals an exponential distribution and the SMP is equal to a Markov process. More information about the two-parameter Weibull distribution can be found in Appendix 2.1.

A complaint that is often heard is the fact that there is insufficient data available for constructing realistic holding time distributions (Scheidegger et al., 2011). Nevertheless, if insufficient hard data is available, it might be possible to obtain data based on expert judgement (soft data) (Kleiner, 2001; Yuan, 2017). As stated by Yuan: “The lack of hard data is no excuse for not modelling the performance deterioration” (Yuan, 2017).

3.2.1.3 (Semi-)Markov Decision Processes

Markov Decision Processes (MDPs) and Semi-Markov Decision Processes (SMDPs) are used to solve dynamic decision-making problems under stochastic conditions, and can be seen as an extension of the Markov or semi-Markov process. (S)MDPs are used to model and asses different maintenance policies in order to enable better grounded decision-making. Therefore, these decision processes can be an important tool for optimisation problems within the field of asset management (Hu & Yue, 2007).

In addition to the regular Markovian processes, with (S)MDPs also actions and rewards are included in the model. When the process enters a certain state *i*, an action *a* (e.g. *do nothing, minimal repair, renewal*) from the action set *A(i)* is chosen. A set of actions (decision rules) define a policy (Wirahadikusumah, Abraham, & Castello, 1999). Rewards or costs are assigned depending on the action, state, and the time of the process being in that particular state (Hu & Yue, 2007). For example, if a component is in a working state and the chosen action is *doing nothing*, the reward function is fully based on the operating costs. If the component is in the failed state the reward function is based on the costs due to failure and lost opportunities (Srinivasan & Parlikad, 2014).

(S)MDPs consist of two main parts: first, the deterioration modelling based on the (semi-)Markov process, and second, modelling possible policies in order to find the optimal one (Amari, McLaughlin, & Pham, 2006; Black et al., 2005). The different policies can be compared by means of the objective value function *V* (Hu & Yue, 2007; Wirahadikusumah et al., 1999). This objective function is often expressed in rewards or costs over a defined time interval, or in costs or rewards per unit of time (White, 1993). Each policy results in a different value for the objective function and this way the different policies can be compared. However, in this research the first objective is to comply with the availability demands, so first of all the maintenance policies should be assessed based on the results of the availability modelling. After that, within the range of policies that meet the availability demands, the optimisation can be performed based on the objective value function *V*.

In practice, often the target availability is not necessarily the maximum availability, but there is a minimal threshold value that should be met. In that case, within the range of acceptable availabilities, the optimum policy can be found based on the objective function. This is often related to minimising the costs, while ensuring the demands are fulfilled (Tomasevicz & Asgarpoor, 2009).

3.2.1.4 Conclusions on Markov models at component level

Markov models are widely used for deterioration modelling. The distinction can be made between the Markov process and the semi-Markov process. With the Markov process the holding times are exponentially distributed, while the SMP is characterised by non-exponential holding time distributions. The former holds that the probability of moving to the next state is constant during the whole duration of the process being in a certain state. The latter means the transition probabilities are time-dependent: the time of the process being in a certain condition state determines the probability of moving to a next state.

It is concluded that non-exponential distributions for the holding times are more appropriate for real-life situations. For this reason, the choice is made to use the SMP for the deterioration modelling at component level. At last, the use of an (S)MDP is not considered here, since this research is all about availability modelling. Optimising maintenance strategies through including costs is not in the scope, making an (S)MDP not relevant here.

3.2.2 Poisson process and Virtual Age Process models

Poisson process models and the closely related Virtual Age Process (VAP) models are characterised as point process models. These types of processes are also called counting processes, as they count the occurrence of events over a period of time. In the field of reliability engineering they are used to describe the occurrence of failures of a repairable system.

The events described by a counting process, $N(t)$, can be represented by a sample path. This sample path shows how events of the counting process occur on a timescale. Figure 10 gives an example of a sample path, where five events occur over the period $t_0 - t_5$. The variable t_i describes at which point on the timescale an event takes place. The other variable that is introduced here, x_i, is the *interarrival time*, which is the time between two consecutive events. Interarrival times play a crucial role in counting processes for reliability engineering, since the parameters of these processes are estimated from them.

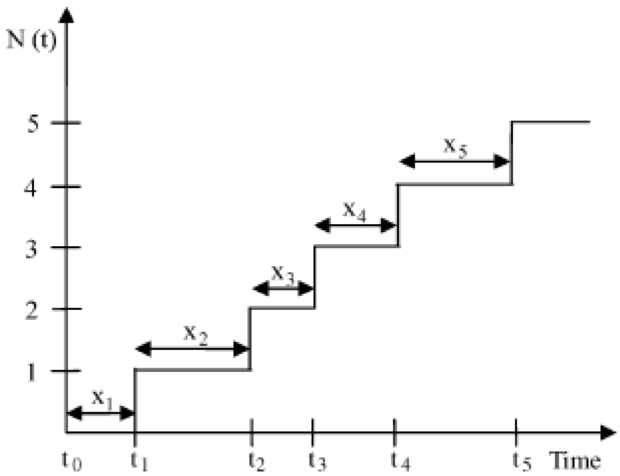

Figure 10: Sample path of a counting process, from Kim and Singh (2010)

In this sub-section the Poisson process and VAP models will be explained in more detail. Starting with general theory on Poisson Processes, going through the Homogeneous Poisson Process (HPP) and the Non-Homogeneous Poisson Process (NHPP) and following up with discussing the VAP models. Finally a conclusion on these point process models is given.

3.2.2.1 General theory on Poisson processes

The definition of a Poisson process is as follows: a counting process $N(t)$ is considered to be a Poisson Process if (Rigdon & Basu, 2000):

1. $N(0) = 0$
2. For any $a < b \leq c < d$, the random variables $N(a, b]$ and $N(c, d]$ are independent.
3. $$\lambda(t) = \lim_{\Delta t \to 0} \frac{P(N(t, t + \Delta t] = 1)}{\Delta t}$$
4. $$\lim_{\Delta t \to 0} \frac{P(N(t, t + \Delta t] \geq 2)}{\Delta t} = 0$$

The first condition (1) describes that the initial number of events of the counting process at time zero is zero. The meaning of the second condition (2) is that the counting process has the independent increments property. Basically, the independent increments property tells us that the number of events in two or more different intervals is independent of each other. Thirdly (3), there is a function, λ, called the intensity function, which describes the occurrence of events. The last condition (4) indicates that there can't be simultaneous events in the Poisson process.

The intensity function, $\lambda(t)$, also known as the Rate of Occurrence Of Failures (ROCOF)[7], is an important notion for the theory on Poisson processes. The intensity function is an absolute rate which is "*the time derivative of the expected number of failures during an interval*" (Ascher & Feingold, 1984, p. 23). The failure intensity is an indicator for the behaviour of a system. Basically, the failure intensity tells us what the expected number of failures in a certain period of time is.

3.2.2.2 Homogeneous Poisson Process

The Homogeneous Poisson Process (HPP) is the simplest Poisson process describing failures of a repairable system, since its failure intensity is constant: $\lambda(t) = 1/\theta$. A constant failure intensity means that the expected number of events for every interval of equal length stays the same. In terms of the interarrival times, x_i, from Figure 10: the times between events (failures) are independent, identical random variables from an exponential distribution with a mean of $1/\lambda$. The effect of all this is that the HPP can only describe components with random failures. It cannot describe components that deteriorate over time.

3.2.2.3 Non-Homogeneous Poisson Process

The Non-Homogeneous Poisson Process (NHPP) differs from the HPP in the form of its failure intensity. Whereas the HPP has a constant failure intensity, the NHPP has a non-constant failure intensity. This difference can be seen in Figure 11 and Figure 12. In Figure 11 there is an HPP with a failure intensity of $\lambda(t) = 1/\theta = 1/10$ and an NHPP with a failure intensity of:

$$\lambda(t) = \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1} = \frac{3}{10}\left(\frac{t}{10}\right)^{3-1} = 0.03t^2$$

It is clear from this figure that the NHPP's failure intensity increases with time. This corresponds to a deteriorating asset with decreasing interarrival times. Figure 12 displays two sample paths: one from an HPP and one from an NHPP. From this figure it follows that the expectation of the number of failures over time for

[7] Sometimes confusingly called "failure rate", see Ascher and Feingold (1984) for an elaborate explanation of the confusion due to this term in reliability engineering.

an HPP is linear and for an NHPP non-linear. For the latter the expected number of failures is increasing, indicating a deteriorating asset.

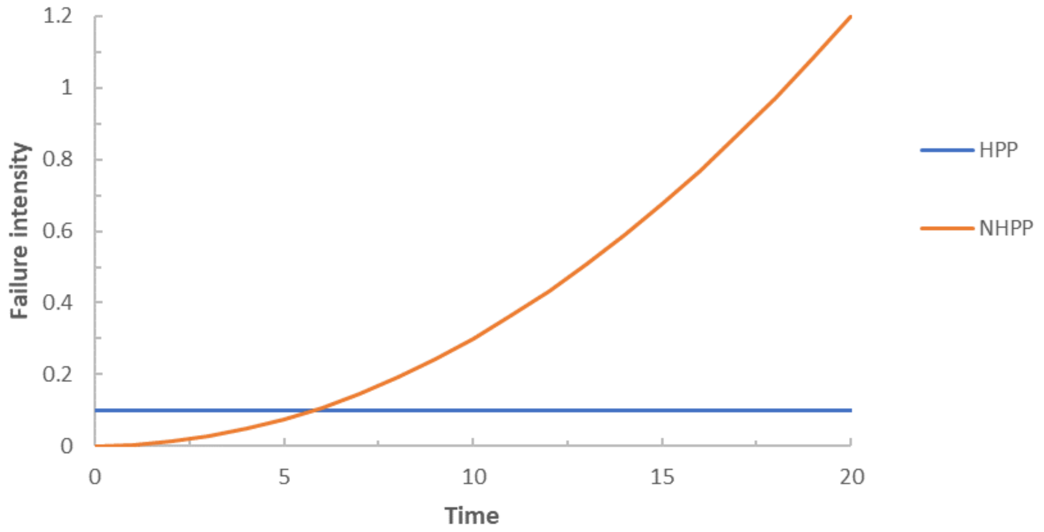

Figure 11: Failure intensity for the Homogeneous and Non-Homogeneous Poisson Process

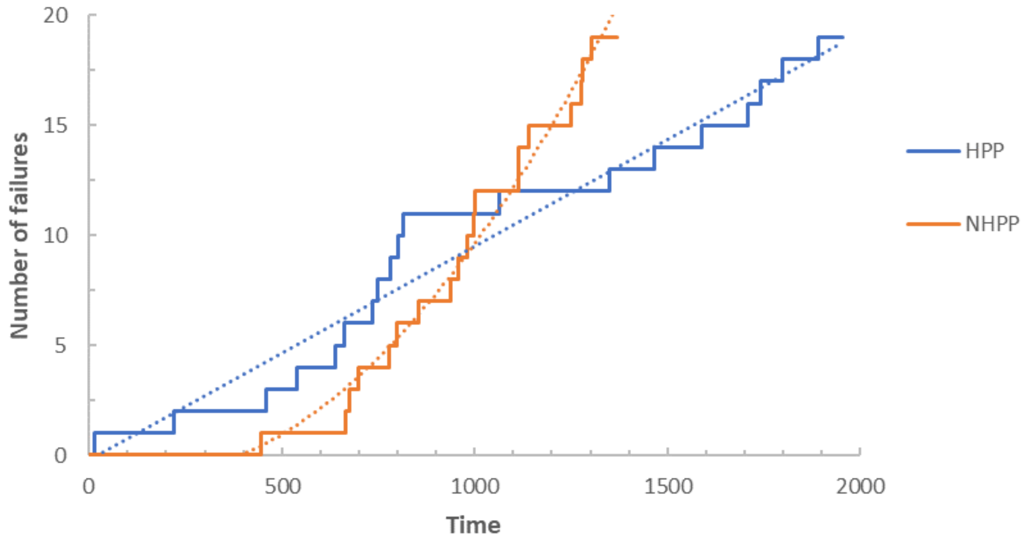

Figure 12: Sample paths for the Homogeneous and Non-Homogeneous Poisson Process

The Power Law Process

The Power Law Process (PLP)[8], a variant of the NHPP, is a popular model for modelling repairable systems, mainly because its estimation and prediction methods are simple (Pulcini, 2001) and its ability to model deterioration. It has been used in numerous papers, see for example Kim and Singh (2010), Korving, Clemens, and Van Noortwijk (2006), Block, Ahmadi, Tyrberg, and Kumar (2014), Gonzalez et al. (2014) and Kumar and Klefsjö (1992). Another commonly applied NHPP model is Log-Linear Process[9] (LLP) model (Ascher & Feingold, 1984, p. 83). These models are monotonic models, meaning they can only describe *either* increasing *or* decreasing intensity functions. From these two models the PLP model performs better (Korving et al., 2006, p.

[8] Also known under the name “Crow-AMSAA model” after its founders.

[9] Also known under the name “Cox-Lewis model” after its founders.

1082). Furthermore, the methods for parameter estimation and confidence intervals are well described (Rigdon & Basu, 2000)

The intensity function of the PLP takes the form of:

$$\lambda(t) = \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1} \qquad for\ \beta > 0\ ;\ \theta > 0$$

where:

β	=	the shape parameter
θ	=	the scale parameter

The shape parameter indicates whether the process has an increasing, decreasing or constant failure intensity, which has a different meaning in relation to the asset's performance, see Table 4.

Table 4: Values of the shape parameter of the Power Law Process and its consequences

Shape parameter value:	Failure intensity is...:	The asset's performance is...:
$\boldsymbol{\beta > 1}$	increasing	deteriorating
$\boldsymbol{\beta = 1}$	constant	neither deteriorating nor improving
$\boldsymbol{\beta < 1}$	decreasing	improving

3.2.2.4 Virtual Age Process models

VAP models are an answer to the problems that (N)HPP face: they can model imperfect repair. That means that conceptually they can account for repairs that bring a system to a state that is not "same as new" (HPP) or "same as old" (NHPP), but somewhere in between. They do this to the detriment of an extra parameter to be estimated. The general approach of these models is to reduce the *virtual age* of a system after repair. When a PLP is assumed this works as follows: the intensity function is manipulated in such a way that it resembles the intensity function at an earlier point on the timescale. This effect of the reduction of the intensity function of the VAP model can be seen in Figure 13. At time = 20, the failure intensity is reduced to the intensity at time 11.

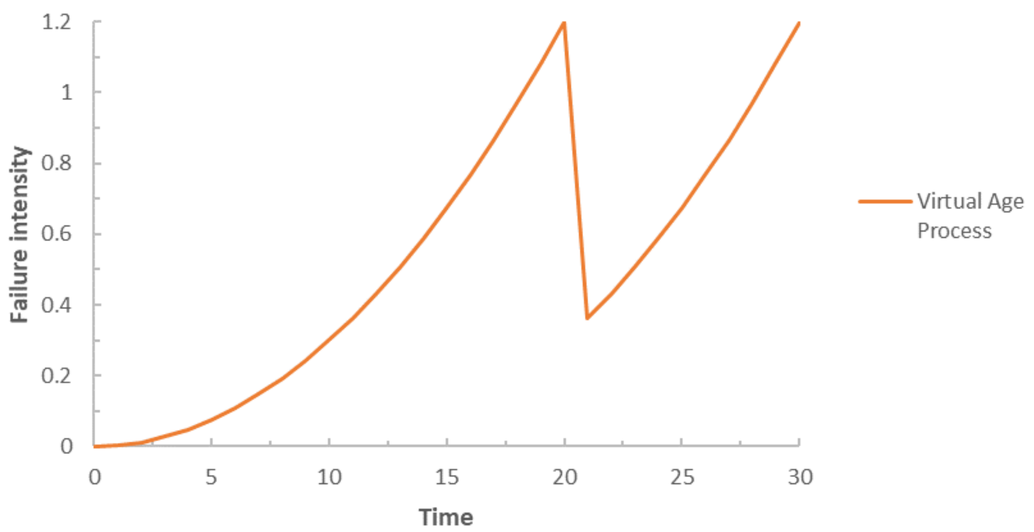

Figure 13: Failure intensity of a Virtual Age Process

Kijima has introduced imperfect repair models, known as Kijima I and Kijima II models (Kijima, 1989). The Kijima I model can restore the asset only to the state it was right after the last failure, thus the effect of the repair is minimally imperfect. The Kijima model II can restore the system to the virtual age of zero, meaning it can model perfect as well as imperfect repair. Doyen and Gaudoin built two types of VAP models, based on the PLP: Arithmetic Reduction of Intensity (ARI) and Arithmetic Reduction of Age (ARA) models (Doyen & Gaudoin, 2004). However, estimation methods for the repair efficiency (ρ) are not yet extended to the case of unknown parameters β and θ, which is a common occurrence in practice (Doyen, 2010).

3.2.2.5 Conclusions on NHPP and VAP models at component level

Three different types of Poisson processes are described in this sub-section. The HPP is excluded from the scope of the research, because it cannot model deteriorating components. The NHPP and the VAP models are both able to model deteriorating components. The VAP models are more complex than the NHPP, since they make use of an extra parameter in order to include the repair efficiency in the model. In theory this makes that the VAP models approach the components' real behaviour better than the NHPP. However, estimation of the repair efficiency parameter is not sufficiently described. The NHPP, and especially in the form of the PLP, have well established parameter estimation methods and have been applied in numerous cases. Therefore, the choice is made to apply the PLP as one of the models at the component level of the proposed two-level hierarchical model.

3.3 Flowchart for model selection at component level

As concluded in the previous section, the SMP model and the NHPP model are the two models proposed for availability modelling at component level of the two-level hierarchy. However, then the question arises which of these two models the asset manager should use for each block in the system RBD. To facilitate in this decision-making, the flowchart as shown in Figure 14 should be used. For each block in the RBD the flowchart should be used to decide which of the two models would be the optimal choice.

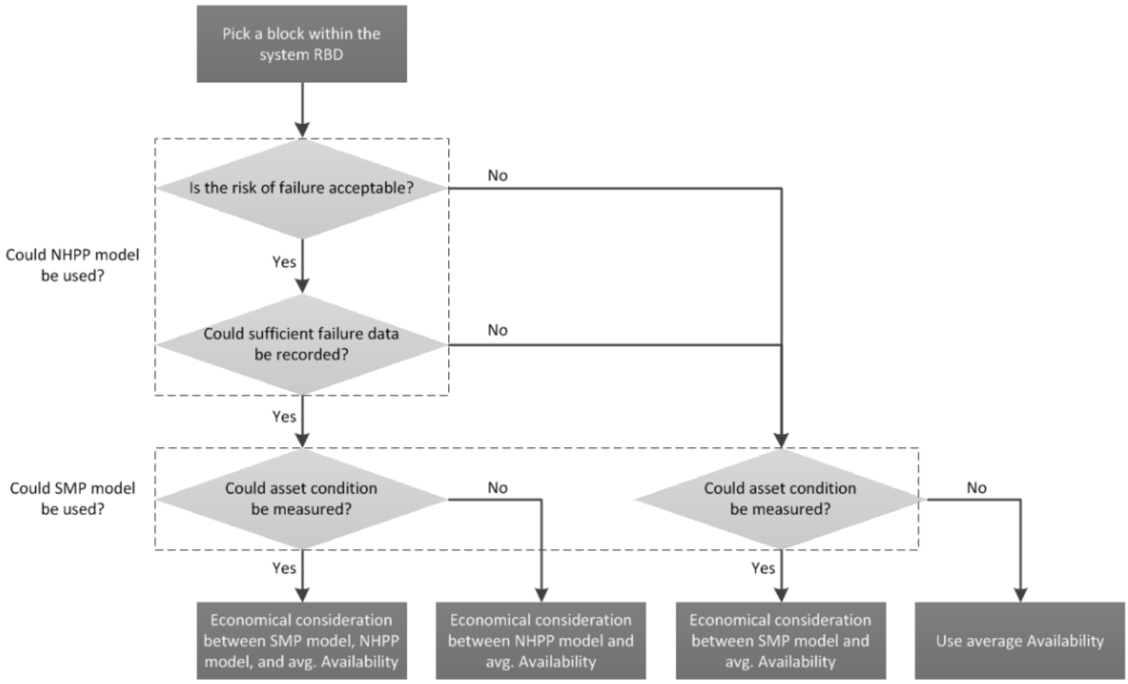

Figure 14: Flowchart for optimal model selection at component level

This flowchart is about the optimal model selection for a considered asset. The choice is mainly based on the asset characteristics and not only on the data that might be available. This latter is for instance the case for the flowchart provided by Yuan (2017). However, this way of reasoning does not always result in the optimal model choice. For example, it could be that for a particular asset only failure data is available and therefore an NHPP model is used to model the availability, although the SMP model is still the better choice for this asset and actually condition data should be registered.

The asset manager should not just make decisions based on the available data, since this does not always result in the optimal model choice. One should always wonder whether the right data is being recorded or not. The outcome might point out that another type of data should be recorded, in order to optimise the availability modelling of the component.

First of all, the asset manager should determine if the risk of failure is acceptable or not. This question is not necesserily required for determining the optimal model selection. However, discussions with asset managers at Waternet showed that from an asset manager's point of view this is often the first question that arises. Answering this question will be based on expert judgement. Within an organisation also risico matrices could be at hand to assist in this decision-making. When the risk of failure is said to be unacceptable, the NHPP model can directly be discarded. In that case failures due to deterioration are to be prevented by preventive maintenance, so there cannot be sufficient recorded failures.

In case the risk of failure is acceptable, the next question is whether sufficient failure events can be recorded for using the NHPP model or not. For answering this question a second flowchart is constructed, see Figure 15. For assets that do not experience failures, logically failure events cannot be recorded and the question can be answered with 'No'. If the asset does experience failure, the question arises whether these events have been recorded or not. If there are records of failure events, the asset manager must find out if there are *sufficient* records available. Rigdon and Basu (1989) give an indication for the number of failure events that should be sufficient for the parameter estimation of the PLP: *"if n<5, it is probably better to assume a Homogeneous Poisson Process than to assume a Power Law Process, since the Power Law Process involves two unknown parameters and the Homogeneous Poisson Process involves only one"* (Rigdon & Basu, 1989, p. 449). Thus if less than five failures events are recorded, more events should be recorded until there are sufficient recordings for using the NHPP model. If five or more failure events are recorded, the NHPP model can already be used. If there are no records of failure events, the asset manager can start to record them. However, one should take into account the time it takes to record sufficient events (n > 4). Therefore, if it is not reasonable to wait long enough to have sufficient events recorded, the NHPP model should not be chosen. Otherwise, the failure events should be recorded and the NHPP model can be used after recording sufficient failure events.

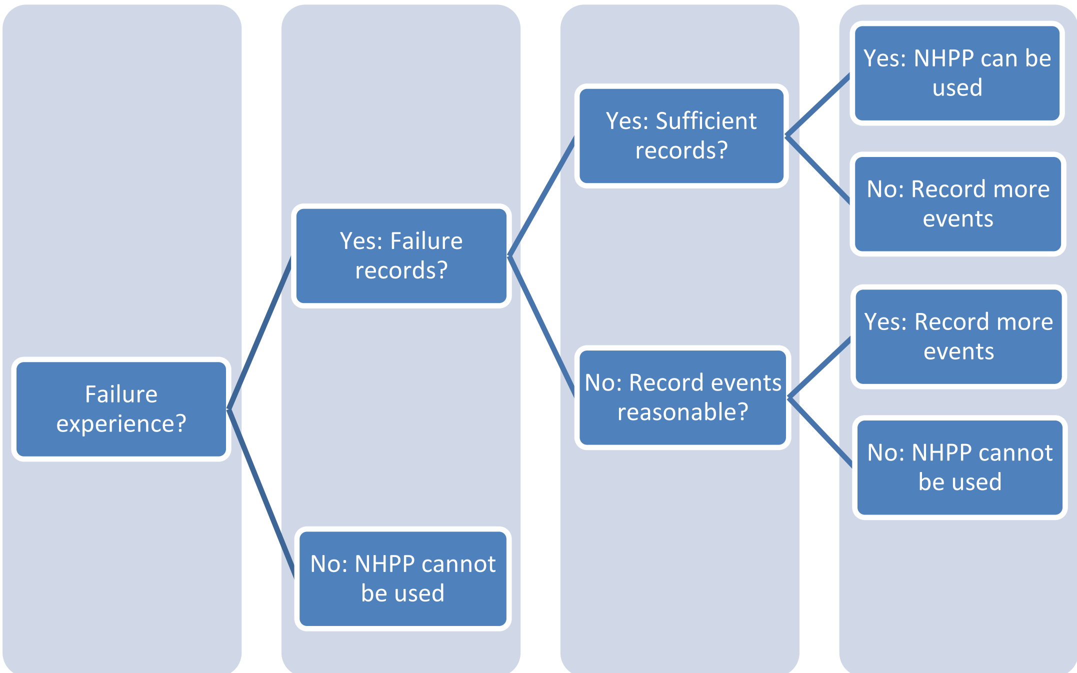

Figure 15: Flowchart for answering the question 'Could sufficient failure data be recorded?'

The last question in the main flowchart (Figure 14) is whether the condition of the asset can be determined or not. If condition monitoring is not possible, the SMP model cannot be used, since this model is based on condition data. On the other hand, if condition data can be recorded, the SMP model can be applied. However, there are some situations where this might not be valid. When the condition of an asset decreases very fast before a failure occurs, a failure cannot be predicted on time. This implies the holding times of the last deterioration states are much smaller than the holding times of the earlier deterioration states, which makes that a failure occurs very suddenly and cannot be forecasted on time. In such an exeptional situation the SMP model should not be used, even if condition monitoring is possible.

When both models cannot be applied, there is still input required for the hierarchical model. In that case the assumption can be made that the availability of the component is equal to the average availability at all times. For all other situations an economical consideration between the two or three possibilities is required, as shown in the flowchart. This means the costs for condition monitoring (SMP model) and failure event recording (NHPP model) should be investigated, to evaluate what is the best option. Although, it is unlikely that the use of average values is preferenced when the SMP model and/or the NHPP model can be used as well. This would only be the case when the application of these models would result in excessive high costs, such that the advantages do not outweigh these. In that case it could be preferred to use average values. However, when the required data is already available, the use of average values will not be considered.

4 Method description

As concluded in the previous chapter, it is proposed to use a two-level hierarchical model for the availability modelling of complex, repairable systems with deteriorating assets. The distinction is made between modelling at the system level, by means of a Reliability Block Diagram (RBD), and modelling at the component level, by using either the Semi-Markov Process (SMP) model or the Non-Homogeneous Poisson Process (NHPP) model. The methodical approach is step by step described in this chapter:

- Set up the RBD at system level (section 4.1)
- Apply flowchart for model choice at component level (section 4.2)
- Model availability at component level (section 4.3)
- Use RBD for system calculations (section 4.4)

4.1 Set up the RBD at system level

The first step of the hierarchical modelling approach is creating the RBD of the system that is concerned. It is important to define the level of detail in the RBD. Each block in the RBD corresponds to a component in the system, so it should be clear what type of asset can be seen as a component. When a Water Treatment Plant (WTP) is analysed, for instance a pump or sand filter can be seen as one block (component) within the system RBD.

When a higher level of detail is assumed, one could imagine that the RBD becomes too big to handle and too many model parameters are required as input. This would be the case if for example all parts of a pump within a WTP are regarded as components as well, and are all individually included in the system RBD. However, when the pump *itself* (or a number of pumps) is defined as the system to be modelled, regarding all parts of a pump as components can be justified. In that case the system scale is smaller, so the level of detail in the RBD is higher. On the other hand, when for example a complete treatment step of 12 pellet softening reactors is assumed as one single block in the system RBD, the level of detail is insufficient. This results in an RBD that does not reflect the complexity of the real system anymore and the modelling results provide too little information to the asset manager. To summarise, it is important to define the right level of detail within the RBD. This depends on the scale of the system the asset manager wants to apply the hierarchical model to.

When using an RBD, the common assumption is that no dependencies between components exist. However, in many real-life systems dependencies do occur, for example due to common cause failures. It should be kept in mind that the RBD as used in this research, cannot model such dependencies as described above.

4.2 Apply flowchart for model selection at component level

When the RBD for the system is created, the next step is the model selection at the component level, for each block in the RBD. Since two different deterioration models at the component level can be used, for each component it must be decided which one to apply. In section 3.3 a flowchart is provided (see Figure 14 at page 42) to assist in this decision making. The flowchart is about the optimal model choice for a considered asset. The choice is mainly based on the asset characteristics and not only on the data that might be available. For each component this flowchart should be used to come to the optimal model choice.

It could be that no data is available for applying a model, even though it seems to be the optimal option for the concerned component. In such a situation it is of course not possible to use the optimal model and other options have to be investigated. Maybe another type of data is sufficiently available, so the other of the two possible deterioration models (the SMP model or the NHPP model that is apparently not the optimal choice) can be used. Nevertheless, it must be decided whether to start gathering the required data for the optimal model or not. If so, it could be used when enough data has been obtained. However, it could also be argued that the advantages of the optimal model do not outweigh the extra costs of monitoring new data. In that case the asset manager could decide not to use the optimal model at all, but keep using the other model.

If no data is available at all, simply average values have to be assumed for that component until sufficient data has been recorded to use the optimal model. When more and more data is recorded, the model calibration can be updated and will become more accurate with an increasing database.

4.3 Model availability at component level

This section handles the use of the deterioration models at the component level of the two-level hierarchical model and explains the details of using them. The whole approach is illustrated in Figure 16 below. The three parts of the modelling approach are discussed in sub-sections 0 (Inverse Transform Sampling), 4.3.2 (constructing one iteration) and 4.3.3 (Monte Carlo simulation). In sub-sections 4.3.4 and 4.3.5 the SMP model and NHPP model are further explained and the distribution of the downtimes is presented in 4.3.6. Mathematical techniques regarding parameter estimation, trend tests and likelihood ratio tests can be found in Appendix 2 (SMP model), Appendix 3 (NHPP model) and Appendix 4 (lognormal distribution). Modelling can only be performed when these techniques are applied to the data in order to obtain the required model input.

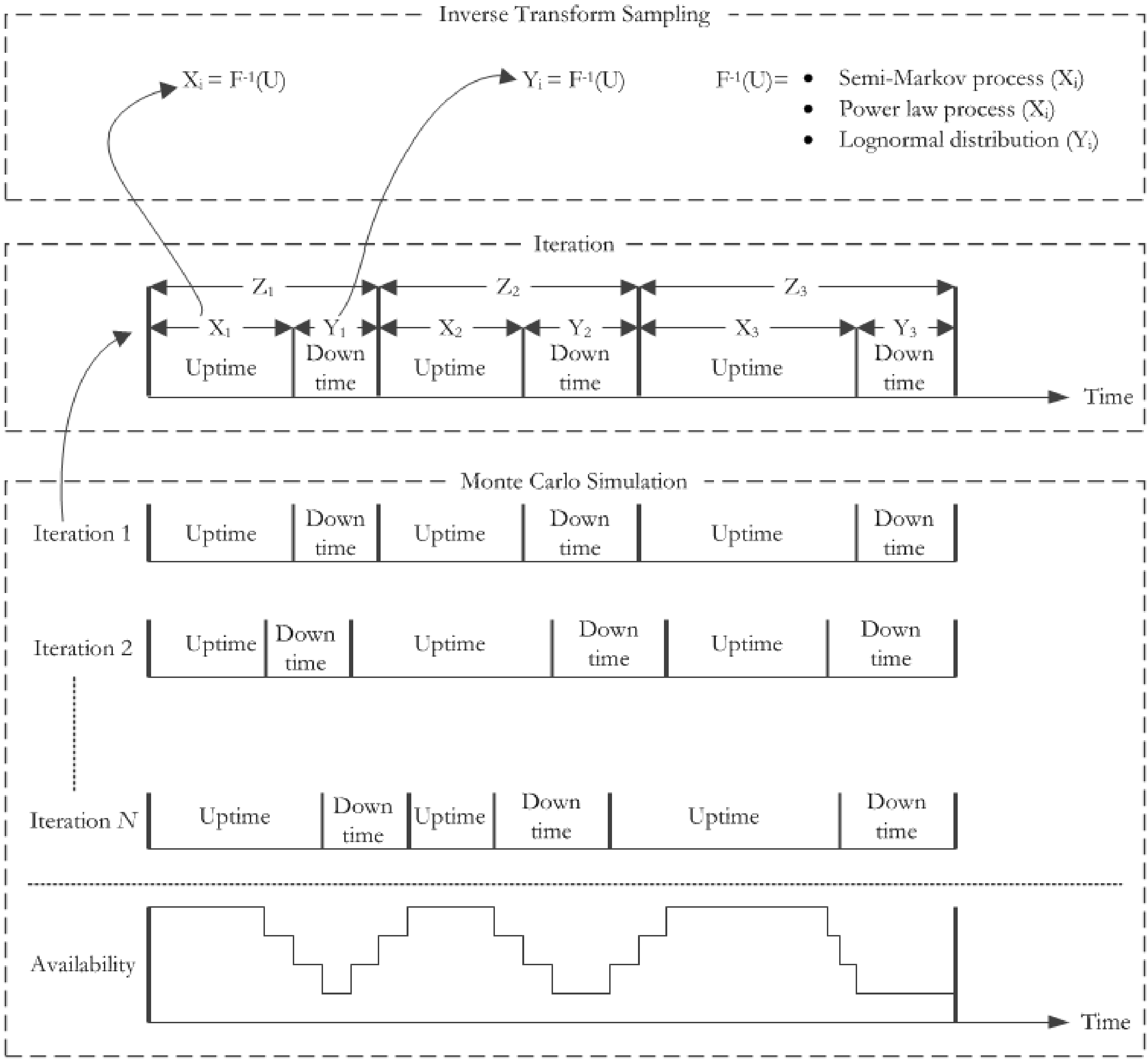

Figure 16: Method description of modelling the availability over time for components

4.3.1 Inverse Transform Sampling

Simulation of the individual uptimes and downtimes is done via Inverse Transform Sampling (ITS). The general approach of this method is to use the distribution function of a random variable, take its inverse Cumulative Distribution Function (CDF) and generate random numbers according to this inverse CDF by using the uniform distribution (Zio, 2013):

1. Let $F(x)$ be any invertible CDF of continuous random variable X
2. Take $F(x) = U$, where U is a random generated number from the continuous uniform distribution $U[0,1]$
3. Then: $X = F^{-1}(U)$

Example of the ITS method
As an example the exponential distribution is taken, defined by its PDF:

$$f(x) = \lambda e^{-\lambda x}$$

ITS is then applied as follows:

1. The CDF of the exponential distribution is given by:

$$F(x) = 1 - e^{-\lambda x}$$

2. $F(x) = U \ \rightarrow \ 1 - e^{-\lambda x} = U$
3. $X = F^{-1}(U) = \left(1 - e^{-\lambda*(U)}\right)^{-1} = -\frac{1}{\lambda}\ln(1-U) = -\frac{\ln(1-U)}{\lambda}$

In order to sample a random variable X from an exponential distribution, one has to take a λ and provide a random sample from the uniform distribution. The random sample from the uniform distribution can be provided by a pseudo random number generator, for example the *RAND()* function in MS Excel. The outcome X is thus a sample from the exponential distribution with parameter λ.

4.3.2 Modelling cycles of up- and downtimes: one iteration

Since there is no analytical or numerical solution to calculate the availability over time for the SMP or NHPP, a simulation method must be used to calculate the availability. The conceptual modelling approach that is used for both the SMP and NHPP model is an iteration, which consists of consecutive cycles of up- and downtimes as in Figure 17. During the uptime (X_i) the component is in an 'available' state and during the downtime (Y_i) in a 'non-available' state, thus the availability can be determined. Together, an uptime and a downtime form a cycle (Z_i). Without the inclusion of the downtimes it is not possible to model availability and thus only reliability computations can be performed.

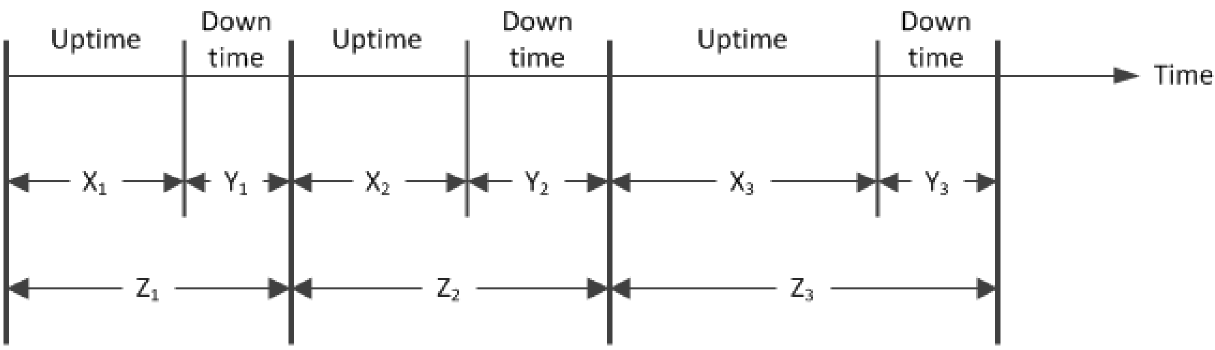

Figure 17: Conceptual example of an iteration

The SMP and NHPP model slightly differ in their approach of simulating iterations. For the SMP the uptime consists of several consecutive holding times for the different condition states, see Figure 19 at page 50. With the NHPP, one uptime consists of one single interarrival time. These uptimes for the SMP and NHPP models are further described in sub-sections 4.3.4 and 4.3.5. In the SMP model there is more freedom in how to build up the iteration. For example, the number of states composing an uptime can be different from component to component and from cycle to cycle within an iteration. Downtimes within an iteration can be different too for the SMP model. An elaboration on this can be found in sub-section 4.3.4.The downtimes for both models are described by the lognormal distribution which is discussed in sub-section 4.3.6.

4.3.3 Monte Carlo simulation

With the 'building blocks' from sub-sections 0 and 4.3.2, it is now possible to simulate one iteration. However, with the simulation of one iteration only the availability over time determined by that single iteration can be computed. What is needed is the combined result of many simulated iterations, which can be done via Monte Carlo simulation. In general, Monte Carlo simulation is a "*methodology for obtaining estimates of the solution of mathematical problems by means of random numbers*" (Zio, 2013, p. 1). For each iteration the input model parameters are randomly picked from their distributions. The model run is repeated many times, to ensure the distributions of the sampled model parameters converge to the real posterior distributions. This way, the whole range of possible input values is included.

A discrete time space is used for the modelling, so for each point in time the availability over n simulations is computed. The result approximates the stochastic availability of a component, which is defined as the probability of the component being available at time t:

$$A(t) = P(A(t) = 1) = \frac{\sum_{i=1}^{n} A_i(t)}{n}$$

This concept of modelling availability via Monte Carlo simulation is illustrated in Figure 18.

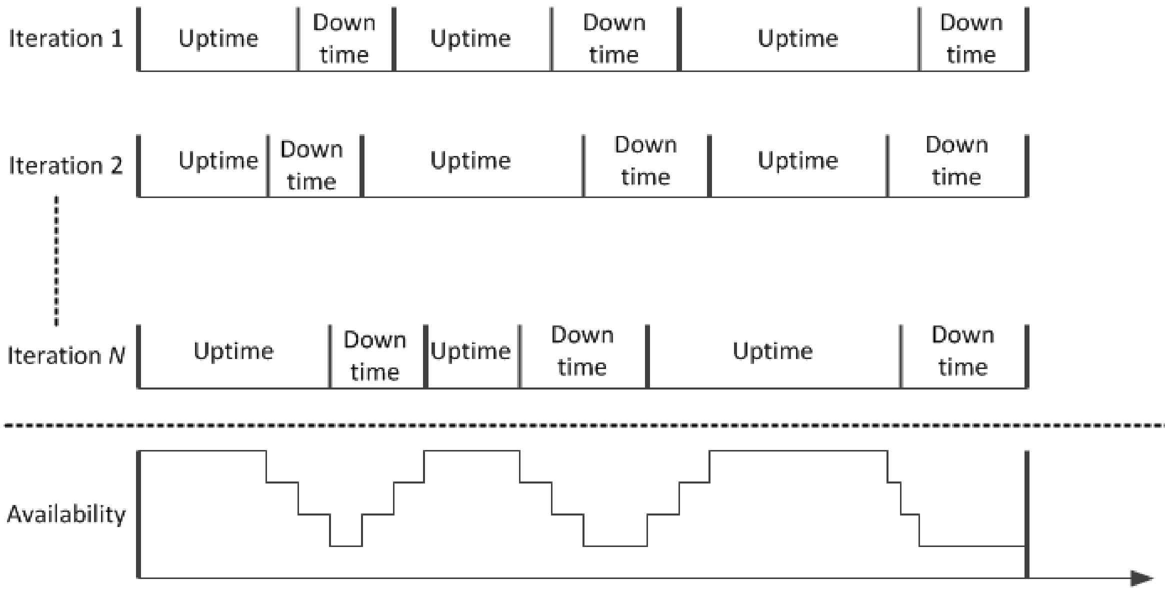

Figure 18: Conceptualised Monte Carlo simulation of for availability modelling at component level

4.3.4 Semi-Markov Process: availability modelling at component level

The Semi-Markov Process (SMP) can be used to model the deterioration of multi-state assets and is characterised by the non-exponential distributions of the holding times. When one SMP is used for the modelling of a complex system, the number of possible system states explodes and so does the complexity of the model (Lanus et al., 2003). However, for availability modelling at the component level this is no problem, so the SMP is suited. It can be used to model the (stochastic) availability over time for individual components. The performance of an SMP model strongly depends on the calibration of the holding time distributions, and calibration is often a major issue with deterioration models (Baik et al., 2006).

The SMP model developed in this research is based on the methodology by Kleiner (2001). In his work a semi-Markov modelling approach is described for a one-unit system. Five different condition states are assumed, where state 1 corresponds to *same as new* and state 5 to *failure*. For all condition states, the holding times follow a two-parameter Weibull distribution. Due to a lack of available data the parameters have been derived based on expert judgement. Furthermore, without repair or replacement the deterioration process is considered to be uni-directional. This means the process can only move from state i to j where $j \geq i$, implying a decreasing asset condition. Another assumption is that the system deteriorates just one state at the time. So for example, it cannot go directly from state 1 to state 3 within one time step.

The methodology developed by Kleiner is referred to in many other studies concerning deterioration modelling (e.g. the works by Black et al. (2005) Scheidegger et al. (2011) and Kim et al. (2015)) and it is also used as a starting point for this research. However, the model by Kleiner has its limitations. The approach does not consider any maintenance actions in the form of repairs or replacements, so the modelling only results in the reliability function (survival curve) for the one-unit system. Only the time until the first failure is described by Kleiner. This would only be sufficient for non-repairable systems because these are not used anymore after the first failure (a space shuttle for instance). However, repairable systems can undergo maintenance actions in case of failures in order to bring the system back to a working state. For this reason, the availability should be used as a performance index for repairable systems and therefore the model as described by Kleiner has to be extended for the goal of this study.

For availability modelling, the downtime as a result of failure and maintenance has to be included. Compared to methodologies that only aim at reliability modelling, not only the available states (uptime), but also the non-available states (downtime) should be included. For the non-available condition states (downstates), the holding times are defined by the time from the moment of failure until the moment the component is repaired or replaced. It is also possible that a component does not need to be in a working state directly after a maintenance action. When this is the case, the component remains non-active after the maintenance action is performed successfully. This is often relevant with parallel or redundant configurations. It should be clear that the holding time of a downstate ends at the moment the component is repaired or replaced, regardless of whether it is immediately taken into operation or not.

4.3.4.1 Inverse Transform Sampling (SMP model)

By using a semi-Markov modelling approach, multiple deterioration states can be included in the model, where each state has its own holding time distribution. The two-parameter Weibull distribution is used most often to describe the holding times of the upstates, since a wide range of distributions can be modelled with it by varying only two parameters (Black et al., 2005). When sufficient condition data is available the holding time distributions for the upstates can be estimated. In Appendix 2.1.1 the Maximum Likelihood Estimation (MLE) method is applied to the two-parameter Weibull distribution, so parameter estimations can be found there.

For reliability modelling only the upstates are considered. In Appendix 2.2 an example of reliability modelling with the SMP model is elaborated. However, for availability modelling also the downstates need to be included. As will be explained in sub-section 4.3.6, the downstates are assumed to be lognormal distributed. The number of downstates depends on the maintenance strategy and the holding time distributions for the downstates should be derived based on the data that is available with respect to maintenance. The parameter estimation method for the lognormal distribution is provided in Appendix 4.1.

It must be noted that only one maintenance strategy can be modelled at a time. To compare several maintenance policies, different model runs based on different strategies have to be executed.

To illustrate with a simple example, a decision rule as part of a maintenance policy could be:

> *When component X is observed to be in condition state 4, a repair is performed that brings the component back to state 2.*

This means that for this downtime the holding time is defined as the time from the moment the maintenance starts (and thus the component is unavailable), until the moment the repair is successfully performed and the component is in state 2 again.

The parameters for the holding time distributions of the uptimes and downtimes form the required input for the SMP model. The accuracy of the parameter estimation is directly related to the accuracy of the model (Tomasevicz & Asgarpoor, 2009). When the holding time distributions are all estimated, the ITS method can be applied. For the SMP, each cycle consists of several consecutive holding times, depending on the number of condition states and the maintenance strategy. This concept is illustrated in Figure 19.

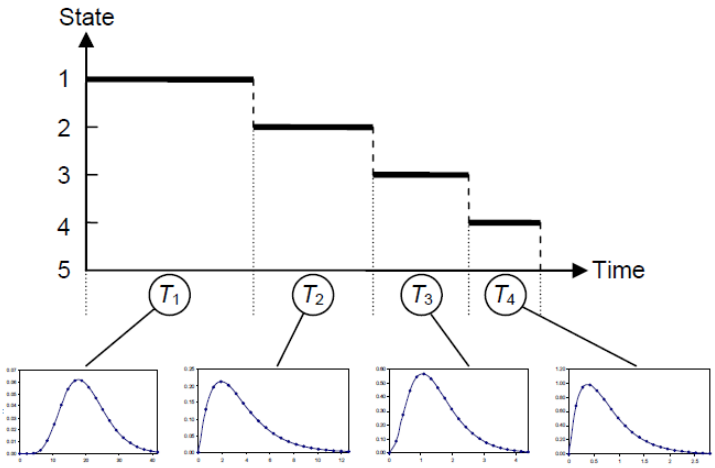

Figure 19: Inverse Transform Sampling for a Semi-Markov Process, from Welte (2008)

Each time the component enters a new state the holding time for that particular stay in that particular state needs to be determined. The holding times of the various condition states are assumed to be independently distributed random variables. The earlier described ITS method is used to pick a holding time from its distribution. For the two-parameter Weibull distribution this procedure is applied as follows:

1. Let *F(x)* be the CDF of the two-parameter Weibull distribution:

$$F(x) = 1 - e^{-\left(\frac{x}{\beta}\right)^{\alpha}} \quad ; \quad x > 0$$

   where:

x	=	holding time
β	=	Weibull scale parameter
α	=	Weibull shape parameter

2. *F(x)* = *U,* where *U* is a random generated number from the continuous uniform distribution *U*[0,1]

3. The inverse of the CDF is[10]:

$$x = -\beta(\ln(U))^{\frac{1}{\alpha}}$$

[10] See Appendix 6.1 at page 119 for complete derivation of the inverse CDF.

For the lognormal distribution this procedure is worked out in Appendix 4.2. With the outcomes of the ITS procedures now the holding times of the upstates and downstates can be generated. It should be noted that the SMP model is not limited to the Weibull and lognormal distributions that are concerned here. Other continuous distributions (e.g. gamma or normal distribution) can be used as well when it turns out they better resemble the data.

4.3.4.2 Monte Carlo simulation (SMP model)

An algorithm has been developed for performing the Monte Carlo simulation. A schematic overview of all the steps is shown in Figure 20. Hereafter a detailed description is given per step in the algorithm. Before running the model, the modeller should define the total iteration time T and the number of iterations that is desired.

T	=	Iteration time interval
n	=	Number of iterations

A discrete time space is used for modelling, so the time interval is divided in small time steps, denoted as Δt.

t	=	Discrete point in time, within T
Δt	=	Time step with length T/t

The algorithm contains the following steps:

1) With Monte Carlo simulation many iterations are repeated. This can be translated to programming code by using a loop, that is repeated n times. The algorithm starts with the first iteration, i= 1, where i defines the number of the iteration being executed.

2) It is assumed the component is same as new (state 1) at the start of the iteration time. So at the start of each iteration, the first holding time is generated with ITS. It should be noted that another beginning condition can be used as well if this is desired.

3) An iteration ends when iteration time interval T has passed (when $j = T$). For each time step j the condition state is determined. Therefore, a second loop is created that is repeated for every time step j within one iteration i.

4) Within this loop, the algorithm checks whether the generated holding time has already been passed. If this is not true, the component stays in the same condition state.

5) When the holding time has passed, the component makes a transition to the next condition state, depending on the maintenance strategy.

6) The ITS procedure is repeated and a holding time for the new state is generated.

7) In order to make the transition from condition state modelling to availability modelling, the following is assumed:
   - If the process is in an upstate, it is assumed to be available (availability = 1);
   - If the process is in a downstate it is said to be non-available (availability = 0).

   For each iteration this results in a one-dimensional array filled with the availabilities (being 0 or 1) for all t in the time interval T.

8) If $j < T$, the inner loop is repeated for the next time step j. When the iteration time interval T has passed (when $j = T$) the iteration is ended.

9) If $i < n$, the next iteration is performed. After the n^{th} iteration, when $i = n$, the Monte Carlo simulation stops.

10) For each t the stochastic availability is calculated as an average of all values (from all iterations) for the availability at that particular time t. This gives the stochastic availability:

$$A(t) = P(A(t) = 1) = \frac{\sum_{i=1}^{n} A_i(t)}{n}$$

This *A(t)* is computed for every *t*. The result is the stochastic availability as a function of time.

As mentioned before, the SMP model corresponds to a determined maintenance strategy. Therefore, it is important to check if the programming code corresponds to the policy that the modeller wants to use. When this is correct, and also the required input is given, the model can run. In 0 two examples are elaborated and the model results are provided. In the first example continuous condition monitoring is assumed, while in the second one it is presumed only periodic inspections are performed.

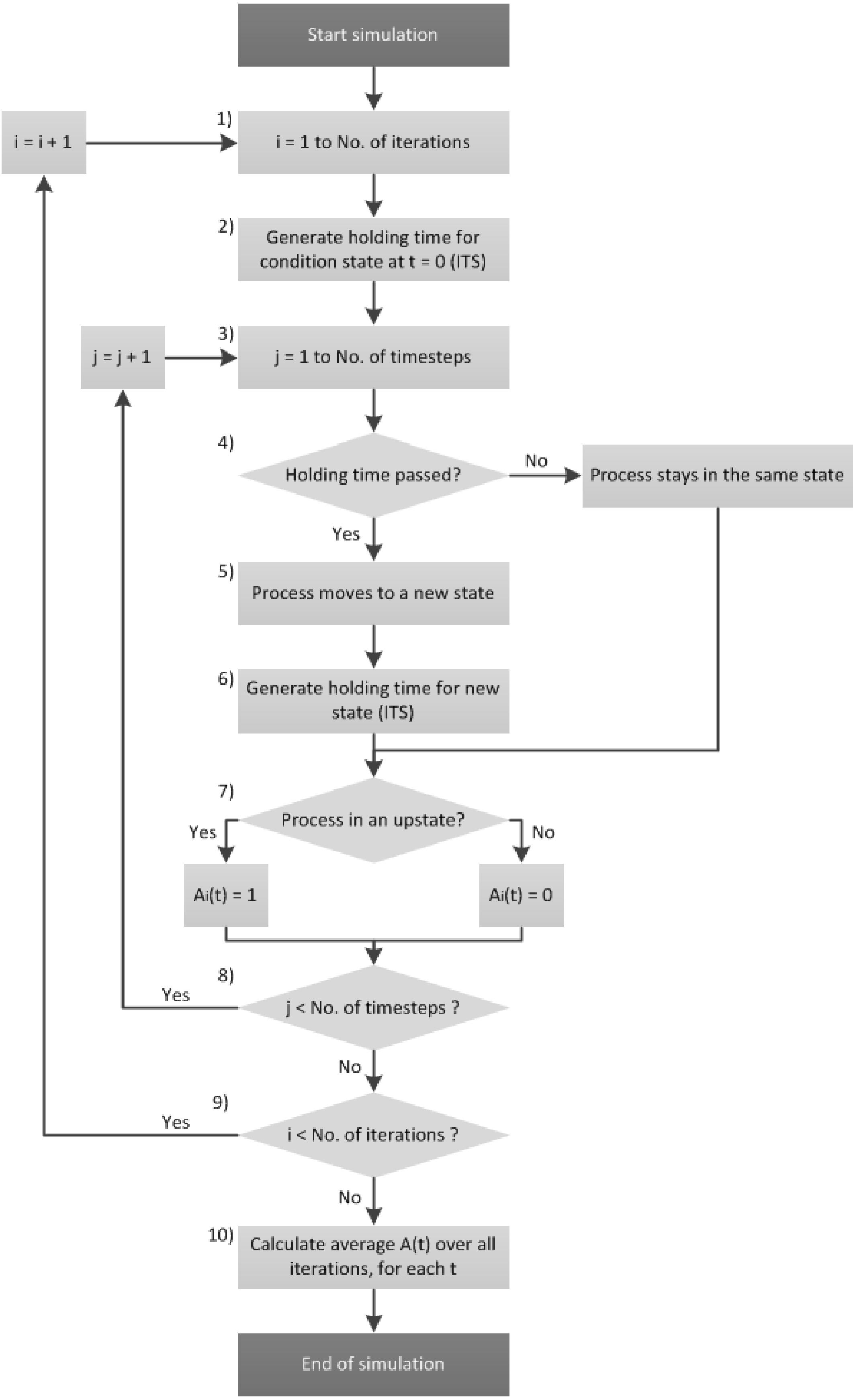

Figure 20: Algorithm for Monte Carlo simulation with the SMP model

4.3.5 NHPP: availability modelling at component level

4.3.5.1 Inverse Transform Sampling (NHPP model)

The NHPP model in this research is based on the paper of Kim and Singh (2010). The NHPP model of these authors is the NHPP variant called the Power Law Process (PLP). The method they use is called interval-by-interval, which is a form of ITS. The method describes the sampling of uptimes[11] of a PLP. Recalling from section 0, in order to sample the uptimes the inverse of the CDF has to be taken. Kim and Singh (2010) give the CDF as well as the inverse CDF for the PLP. In Appendix 3.1 the intensity function and the CDF, with all the parameters, are given. Parameter estimations for the shape parameter and scale parameter of the PLP can be found in Appendix 3.1.1.

The ITS procedure is applied as follows for the PLP:

1. Let $F_{t_{k+1}}(x)$ be the CDF of the PLP[12]:

$$F_{t_{k+1}}(x) = 1 - \exp\left(-\theta^{-\beta}\{(t_k + x_k)^\beta - (t_k)^\beta\}\right)$$

   where:

$$t_k = \left(\sum_{i=1}^{k-1} x_i\right)$$

2. *F(x)* = *U,* where *U* is a random generated number from the continuous uniform distribution *U*[0,1]

3. The inverse CDF is[13]:

$$x_k = \left(\left(\sum_{i=1}^{k-1} x_i\right)^\beta - \frac{\ln(U)}{\theta^{-\beta}}\right)^{\frac{1}{\beta}} - \sum_{i=1}^{k-1} x_i$$

The interval-by-interval method works by sampling the interarrival times x_k of the failure events. The sampled value of x_k can be obtained from the PLP inverse CDF $F_{t_{k+1}}(x)$ with parameters β and θ and a random number *U* from a uniform distribution *U*(0,1]. An example of the NHPP model for modelling reliability can be found in Appendix 3.4.

4.3.5.2 Monte Carlo simulation (NHPP model)

Recalling from sub-section 4.3.2, an iteration alternates between an uptime and a downtime. Furthermore, Monte Carlo simulation is used to obtain the availability over time. For this purpose, an algorithm is established that describes the simulation method for the NHPP model. The algorithm can be found in Figure 21 at page 56 and the explanation is given below. The idea of this simulation approach is based on the Quasi-Renewal Process (QRP) as used by Rehmert and Nachlas (2009). The QRP is a specific form of the Alternating Renewal Process (ARP), where the successive uptimes and/or downtimes increase or decrease stochastically[14]. In the NHPP model the uptimes decrease stochastically and the downtimes stay stochastically the same.

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[11] In PLP context these are called interarrival times.
[12] Kim and Singh use a different notation with λ instead of θ, see Appendix 6.2 at page 120 for the substitution of λ by θ.
[13] See Appendix 6.3 at page 121 for complete derivation of the inverse CDF.
[14] Stochastically decreasing (increasing) uptimes signify deterioration (improvement) of components; decreasing (increasing) downtimes signify lengthening (improvement) of repairs.

Algorithm for modelling availability over time

For the algorithm to be applicable, it is assumed that the parameters for the ITS of uptimes and downtimes are already known. Thus the parameters β and θ for the PLP distributed uptimes and the parameters μ and σ for the lognormal distributed downtimes are already estimated by the asset manager, see Appendix 3.1.1 (PLP) and Appendix 4.1 (lognormal distribution) for the estimation of these parameters. The algorithm consists of several loops: the QRP-loop (steps 2-10), the mini-loop (steps 7-9) and the iteration-loop (steps 1-4 and 11-12). Appendix 3.5 gives an example for modelling the availability with the NHPP model.

Start (step 1)

First, the asset manager has to determine the period covered by the Monte Carlo simulation. Thus, if one wants to calculate the availability over the next 10 years, parameter $T = 10$. Second, the number of Monte Carlo iterations has to be determined. If the asset manager chooses 1.000.000 iterations, $n = 1.000.000$. Furthermore, N signifies the current iteration number, i signifies the cycle number and $A(t)$ signifies the point availability at time t, where time runs from zero to T $(0 \leq t \leq T)$. At the start of the Monte Carlo simulation the stochastic availability for every point in time is equal to 1 $(A(t) = 1)$.

Step 2

First, an uptime and a downtime have to be sampled from the equation x_k, page 54, respectively equation y_k, page 57 and $x_{N,i} = x_k$ and $y_{N,i} = y_k$. Then, the cycle time $(t_{N,i})$ is determined by adding the downtime to the uptime:

$$t_{N,i} = x_{N,i} + y_{N,i}$$

Step 3

The next step is to determine the point in time where the last uptime ends (T_{QRP}). This is needed to determine in the next step if the QRP has already exceeded the specified time (T) for which the asset manager wants to calculate the availability. Thus by adding the latest sampled uptime to the total of all foregoing cycles in the current QRP, the end point of the last uptime is calculated $(T_{QRP} = t_j + x_{N,i})$.

Step 4

In this step a decision $(T_{QRP} \geq T\ ?)$ has to be made: *does end point of the last uptime exceed the specified end time* (T) *of the model?* If the answer is yes, the algorithm goes to step 11, where a decision is made to end or to continue the Monte Carlo simulation. If the answer is no, the algorithm goes to step 5.

Step 5

This step is where the total time of all cycles in the current iteration (t_j) is calculated:

$$t_j = \sum_{q=1}^{i} t_{N,q}$$

Step 6

Then, in the sixth step, the start of the downtime of the last sampled cycle (m) is calculated $(m = t_j - y_{N,i})$. The start of the downtime of the last sampled cycle calculated in this step is used to alter the availability at the whole period of time of the last sampled downtime. This is done in a mini-loop consisting of step 7, 8 and 9.

Step 7

The first step of the mini-loop described above consists of lowering the availability at time $t = m$, by the value of $1/n$. Recall from the *Start* that the availability is equal to 1 at all points in time $(A(t) = 1)$. For all downtimes this value is thus lowered by 1 divided by the total number of iterations.

Step 8

In the second step of the mini-loop a decision has to be made: *has the end of the last sampled downtime been reached* $(m \geq t_j\ ?)$. For the length of the last downtime the loop has to be ran through and stopped at the end of the last cycle (which is equal to the total time of all cycles in the iteration (t_j)). If the answer is yes, the algorithm goes to step 10. If not, the mini-loop is continued with step 9.

Step 9
The last step of the mini-loop is increasing variable m with one time step $(m = m + 1)$. After this step the mini-loop is started again at step 7.

Step 10
The last step of the QRP loop is the increase of the cycle number $(i = i + 1)$. With this step complete, a whole cycle (uptime + downtime) has been run through and the availability has been lowered at the period of downtime. Then the QRP loop starts again with step 2.

Step 11
In this step of the iteration loop a decision has to be made: *has the number of iterations reached the total number of iterations?* $(N \geq n\,?)$ If the answer is yes, the Monte Carlo simulation is ended at step 13. If the answer is no, the iteration loop is followed to step 12.

Step 12
The final step of the iteration loop resets the cycle number $(i = 1)$ and the total time of all cycles in the iteration $(t_j = 0)$. Furthermore, the iteration number is increased by one $(N = N + 1)$. After this step the iteration loop is started again with step 2.

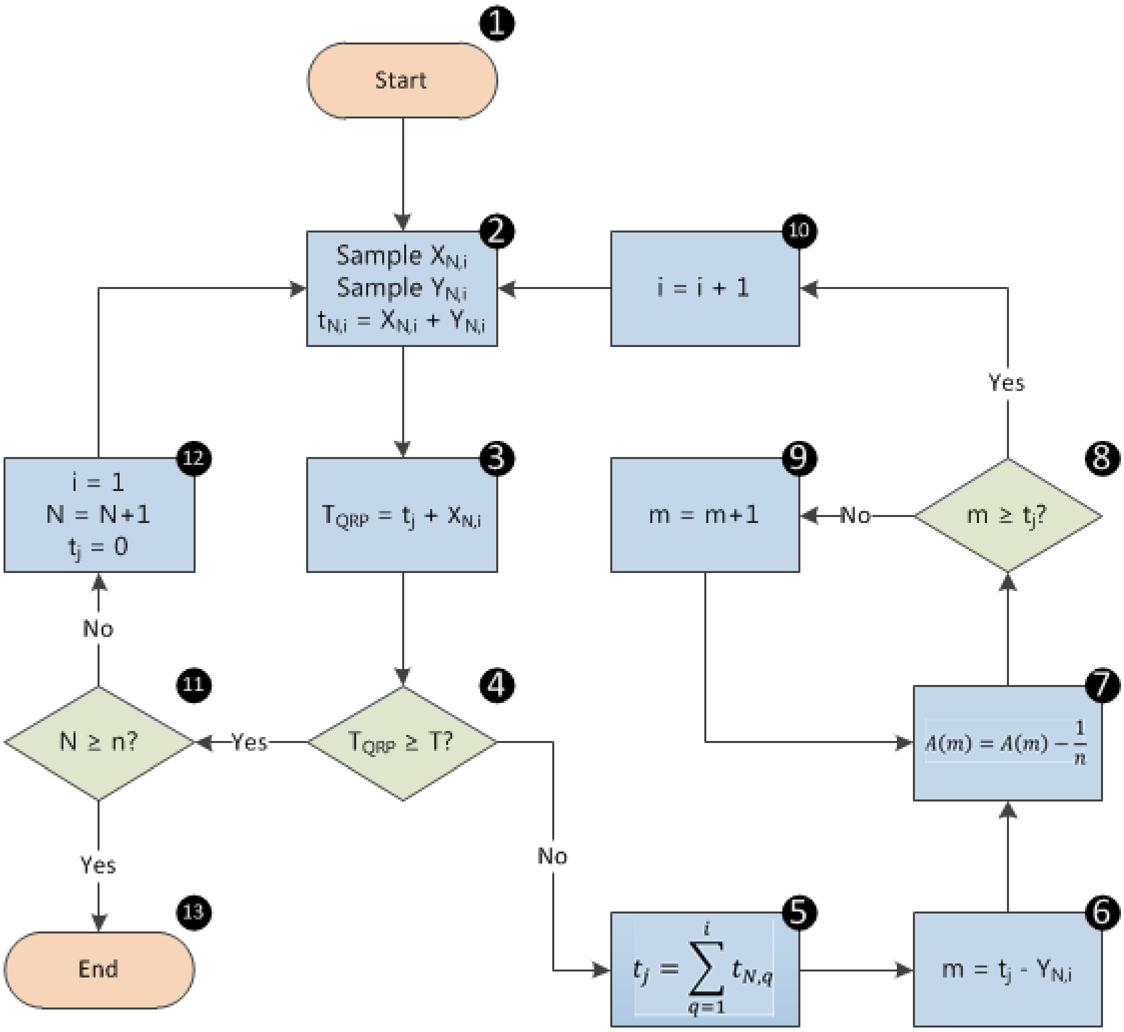

Figure 21: Algorithm for Monte Carlo simulation with the NHPP model

4.3.6 Downtime distribution: the lognormal distribution

The lognormal distribution is a commonly chosen distribution for downtimes (repair times) (Birolini, 2017, p. 464) and research finds that the lognormal distribution is the best fitting distribution in 2/3 of the repair time data sets (Kline, 1984). Therefore, the lognormal distribution is used for modelling the downtimes in both the SMP and the NHPP model. Other continuous distributions that have an inverse CDF, such as the exponential or Weibull distribution, can be used as well when the repair data resembles another distribution better than the lognormal distribution does. However, in this study it is assumed that all downtimes are lognormal distributed. The PDF of the lognormal distribution is given by:

$$f(x) = \frac{1}{\left(\sqrt{2\pi\sigma^2}\right)x} \exp\left(-\frac{(ln(x)-\mu)^2}{2\sigma^2}\right) \qquad with\ x > 0\ ;\ -\infty < \mu < \infty\ ;\ \sigma > 0$$

with:

σ	=	shape parameter (standard deviation of log(*x*))
μ	=	scale parameter (mean of log(*x*))

In Appendix 4.1 the parameter estimations according to the MLE method are given.

When the inverse CDF is implemented in the ITS procedure, a sample for the repair time is given by:

$$y_k = exp\left(\mu + \sigma\left(\Phi^{-1}(U)\right)\right)$$

with:

$\Phi^{-1}(U)$	=	inverse CDF of the normal distribution
U	=	random number from the uniform distribution *U*(0,1]
σ	=	shape parameter (standard deviation of log(*x*))
μ	=	scale parameter (mean of log(*x*))

The inverse CDF of the normal distribution (and thus the lognormal distribution) does not have a closed-form solution, thus approximations are being used. Many software applications, such as MS Excel, have built-in functions for approximating these inverse CDFs of the normal and lognormal distribution.

4.4 Use RBD for system calculations

To compute the system's availability as a function of the availability of its components, the minimal path sets method can be used, which is explained in Appendix 1 . In this section, four common RBD structures are addressed. The two simplest ones are the serial and parallel structures, which are described first. Also the *k*-out-of-*n* (redundant) configuration and the bridge structure are discussed. These four structures are the ones that are common ones to be present in a WTP. A wide range of RBDs for complex systems can be constructed with these four basic structures. For these configurations the structure functions are provided here. It should be noted that, when replacing *A(t)* with *R(t),* the same formulas can be used for calculations regarding the reliability.

Serial

In a serial configuration, the system fails when one of the components fails, and it is working as long as all components are working. A serial structure can therefore be named an *n*-out-of-*n* system. The RBD of a serial configuration is shown in Figure 22. Serial configurations can be found within all types of systems. Within a WTP, the different treatment steps are all connected in series. The water flows from one treatment step to the next one, so each treatment step can be represented by one component in the RBD in Figure 22.

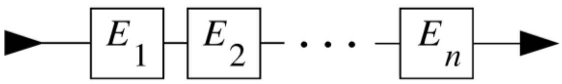

Figure 22: RBD for a serial configuration, from Birolini (2017)

The structure function for components in series is given by:

$$A_s(t) = \prod_{i=1}^{n} A_i\,(t)$$

Parallel

In a parallel configuration, the system fails when all the individual components are failed, and it is working as long as at least one of the components is working. A parallel configuration can thus be seen as a *1*-out-of-*n* system. An RBD for a parallel configuration is presented in Figure 23. An example of a parallel configuration within a WTP could be a pump set-up, which is operating when at least one pump is functioning.

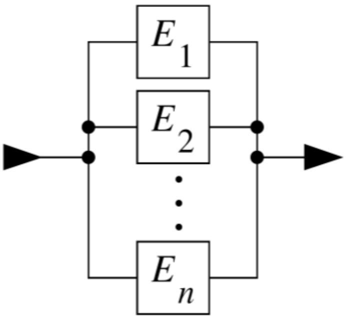

Figure 23: RBD for a parallel configuration, from Birolini (2017)

The availability for components in parallel can be computed with the structure function:

$$A_s(t) = 1 - \prod_{i=1}^{n} (1 - A_i(t))$$

***k*-out-of-*n* redundancy**

Including redundancy in the system design is a common method of improving the system reliability and availability. WTPs, and especially Drinking Water Treatment Plants (DWTPs), are known to contain a lot of redundancy, since a high availability is a strict requirement. Redundant configurations are often referred to as *k*-out-of-*n* structures, where *n* is the total number of components and *k* the number of components required to be working or failed in order to cause system working of failing. The distinction can be made between *k*-out-of-*n*:F and *k*-out-of-*n*:G configurations. The former means that the system fails if at least *k* components are failed (failure-based), while the latter means that the system works if at least *k* components are working (success-based). A *k*-out-of-*n*:F system can be treated as a (*n* – *k* + 1)-out-of-*n*:G system. The other way around, a *k*-out-of-*n*:G system is equal to a (*n* – *k* + 1)-out-of-*n*:F system (Wang, 2014).

Since an RBD is a success-based method, only the structure functions for the *k*-out-of-*n*:G configuration are included here.

In case of independently and identically distributed (i.i.d.) components the availability functions of all components are the same, which means the system availability is given by:

$$A_s(t) = \sum_{i=k}^{n} \binom{n}{i} [A_1(t)]^i [1 - A_1(t)]^{n-i} \quad , \quad if\ A_i(t) = A_1(t)$$

For non-i.i.d. *k*-out-of-*n*:G configurations the structure function changes to:

$$A_s(t) = \sum_{i=0}^{n-k} \sum_{j=1}^{\binom{n}{i}} \prod_{m=1}^{n} A_m(t)^{\delta_{m,j}} (1 - A_m(t))^{\bar{\delta}_{m,j}} \quad , \quad k = 1, \dots, n$$

where $\delta_{m,j}$ and $\bar{\delta}_{m,j}$ are complementary indicator functions for which

$$\sum_{m=1}^{n} \delta_{m,j} = k \quad ; \quad \sum_{m=1}^{n} \bar{\delta}_{m,j} = n - k$$

and

$$\delta_{m,j} = \begin{cases} 1\ if\ component\ m\ works \\ 0\ if\ component\ m\ is\ failed \end{cases}$$

Bridge structure

At last, the so-called bridge structure is described here. A bridge structure can be bi-directional (see Figure 24) and uni-directional. A bi-directional structure is assumed here, which means component E_5 can be included both in the route from E_1 to E_4 and the route from E_2 to E_3 (Birolini, 2017). Bridge structures are often present in WTPs in the form of conduits with valves, so that the water can be by-passed in multiple ways. In case of a failure somewhere in one of treatment lanes in the WTP, the bridge structures can be used to by-pass around that failure, instead of closing that whole treatment lane.

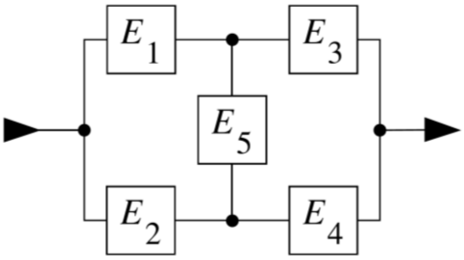

Figure 24: RBD for a bridge structure, from Birolini (2017)

For independent components, the system reliability of the bridge structure can be obtained by using minimal path or minimal cut sets (Wang, 2014). The availability of a bi-directional bridge structure with five elements can be derived with the minimal path sets method, as shown in Appendix 1 . The final structure function for the bridge structure is given by:

$$\begin{aligned} A_s(t) = A_1(t)A_3(t) + A_2(t)A_4(t) + A_1(t)A_5(t)A_4(t) + A_2(t)A_5(t)A_3(t) - A_1(t)A_2(t)A_5(t)A_3(t) \\ - A_1(t)A_2(t)A_5(t)A_4(t) - A_1(t)A_2(t)A_3(t)A_4(t) - A_1(t)A_5(t)A_3(t)A_4(t) \\ - A_2(t)A_5(t)A_3(t)A_4(t) + 2A_1(t)A_2(t)A_3(t)A_4(t)A_5(t) \end{aligned}$$

If the reliability functions $A_i(t)$ of the individual components are known, then for each time step the availability of the bridge structure can be calculated with the given formula.

5 Case Study

In this case study[15] on *Leeward Dune*, Watercity's Drinking Water Treatment Plant (DWTP), the method described in the previous chapter is applied. First the case study will be introduced in section 5.1. Second, the application of the method is given in section 5.2. For reasons of simplicity and comprehensibility only a part of Leeward Dune will be modelled. Several treatment steps are excluded to make the case fit-for-purpose: showing how to apply the method and models devised in the previous chapters 3 and 4.

5.1 System description

5.1.1 Leeward Dune: Watercity's Drinking Water Treatment Plant

The capital of the country of Waterland, Watercity, has 200,000 inhabitants. Its main DWTP is *Leeward Dune*. Leeward Dune's process for making potable water is presented in Figure 61 at page 79. The focus in this case study is on three consecutive process steps: rapid sand filtration, ozonation and softening. Pumps and valves that are of relevance to the availability of the system as a whole are included in the case study as well. The interest of Leeward Dune's asset managers lies in the development of the availability over a time for a period of 30 years.

Due to constraints in the pre-treatment process, Leeward Dune has a maximum production capacity of 2,400 m^3/hour. In this case study it is assumed that the availability can be defined as the '*probability that the system is able to meet the maximum production capacity*'. Furthermore, it is assumed that every component either is in an 'available state', meaning it is able to operate at its maximum production capacity, or in a 'non-available state' where it is not able to operate at all.

5.1.2 Rapid sand filtration

The rapid sand filtration process step consists of six filters. These filters take out suspended particles in the water that flow in from the dunes, see Figure 25. Every once in a while, these filters have to be backwashed: water and air are pumped in reverse direction, which unclogs the filters as in Figure 26.

Figure 25: Rapid sand filtration (own image)

Figure 26: Unclogging of the sand filter (Waternet, 2018c)

Each filter has a maximum capacity of 600 m^3/hour, making the maximum capacity of all filters combined 3,600 m^3/hour. Since Leeward Dune's overall production capacity is constrained to 2,400 m^3/hour, the rapid sand filtration needs four out of six filters to be working in order to meet maximum production capacity. From the sand filters the water flows through pipelines to the ozonation step.

[15] Data and numbers on production capacities in this case study are fictional and do not represent real-world values.

5.1.3 Ozonation

The next treatment step is the ozonation, which is a disinfection step that kills off bacteria, viruses and pesticides. Treatment takes place in a multi-chambered cellar, where the water is brought into contact with the ozone as in Figure 27.

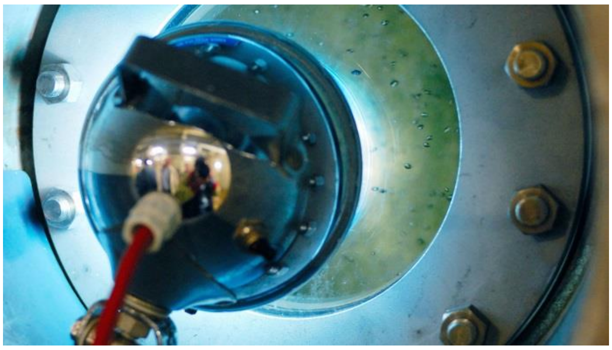

Figure 27: Outside view of the ozone treatment (Waternet, 2018b)

Leeward Dune has two so-called “ozone streets”, ozone street 1 and 2. These streets consist of an ozone generator, a cooling system, a multi-chambered cellar and an ozone dispensing system. One ozone street can handle 2,800 m^3/hour, thus one street is single-handedly able to meet the maximum production capacity. After the water is treated with ozone, it flows towards the parallel pump set-ups. Ozone street 1 is connected to pumps 1 and 2 and ozone street 2 is connected to pumps 3 and 4.

5.1.4 Pumps

After the ozonation, the water has to be pumped towards the softening process. This is done via two parallel pump set-ups, where pump 1 and 2 form one set-up and pump 3 and 4 the other set-up. One pump has a capacity of 2,500 m^3/hour, which is enough to meet the maximum production capacity.

Figure 28: Pumps 1 and 3 (own image)

5.1.5 Valves

Valves are used to direct flows of water through pipeline systems. In the case study’s system one important valve is included. The valve is located between the ozone streets and the pumps. Normally, water would flow from ozone street 1 to pumps 1 and 2 and from ozone street 2 to pumps 3 and 4. The valve makes it possible to direct (bypass) the water flow from ozone street 1 to pumps 3 and 4 and from ozone street 2 to pumps 1 and 2. The valve can handle flows of up to 3,600 m^3/hour in both directions.

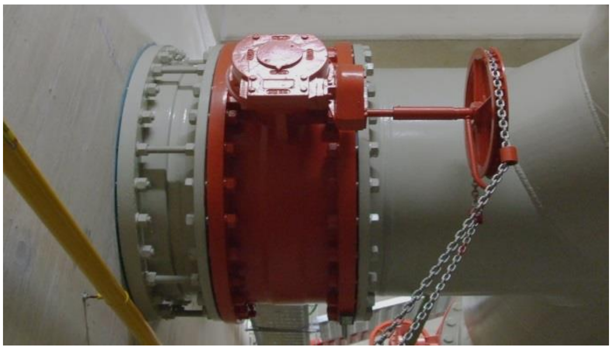

Figure 29: Valve (Waternet, 2018d)

Figure 30: Valve (Waternet, 2018e)

5.1.6 Softening

After the water is pumped to the softening, it enters the softening reactors. In the softening reactors the water comes into contact with sodium hydroxide (NaOH). The sodium hydroxide lets calcium precipitate on sand grains. When the grains increase in size, they sink to the bottom, where they are subsequently drained from the softening reactor.

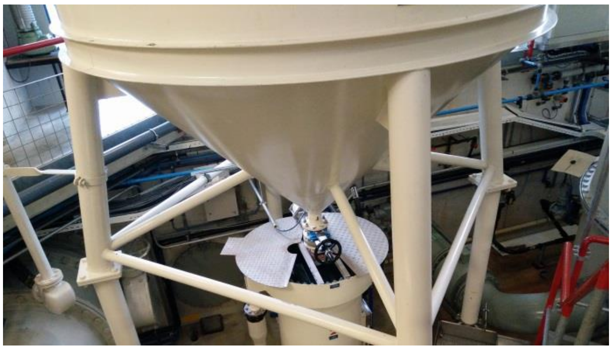

Figure 31: Softening reactor inlet (own image)

Figure 32: Softening reactors topsides (own image)

There are four softening reactors in total, each with a maximum production capacity of 800 m^3/hour. Thus, in order to meet the maximum production capacity of the total plant (2,400 m^3/hour), at least three of the four softening reactors need to be operating at full capacity. After this softening process the water flows towards the next treatment step, the carbon filtration, which is not included in this case study.

5.2 Application of the proposed methodology

5.2.1 Set up the RBD at system level

From the description of the system in section 5.1, a Reliability Block Diagram (RBD) can be constructed which includes all components and the system configuration, see Figure 33. The rapid sand filters are at the start of the RBD, in a *k*-out-of-*n* configuration, since four out of six are needed in order to meet maximum production capacity. After the rapid sand filtration the water flows through pipelines towards the two ozone streets. These pipelines are included in the RBD, but since it is assumed that these are always available they do not affect the system availability. Then after the ozonation the parallel pump set-ups are constructed in the RBD. They would form a series configuration with the ozone streets, were it not for the valve that is included in the system. Therefore the ozone streets, parallel pump set-ups and the valve form a so-called bridge structure, which is a well-known configuration type within the field of reliability engineering. The next component in the RBD is the cluster of pipelines from the pumps to the softening step, but as before it is assumed they do not affect the system availability. The softening step consists of four reactors in a *k*-out-of-*n* configuration, since three out of four are needed in order to meet maximum production capacity.

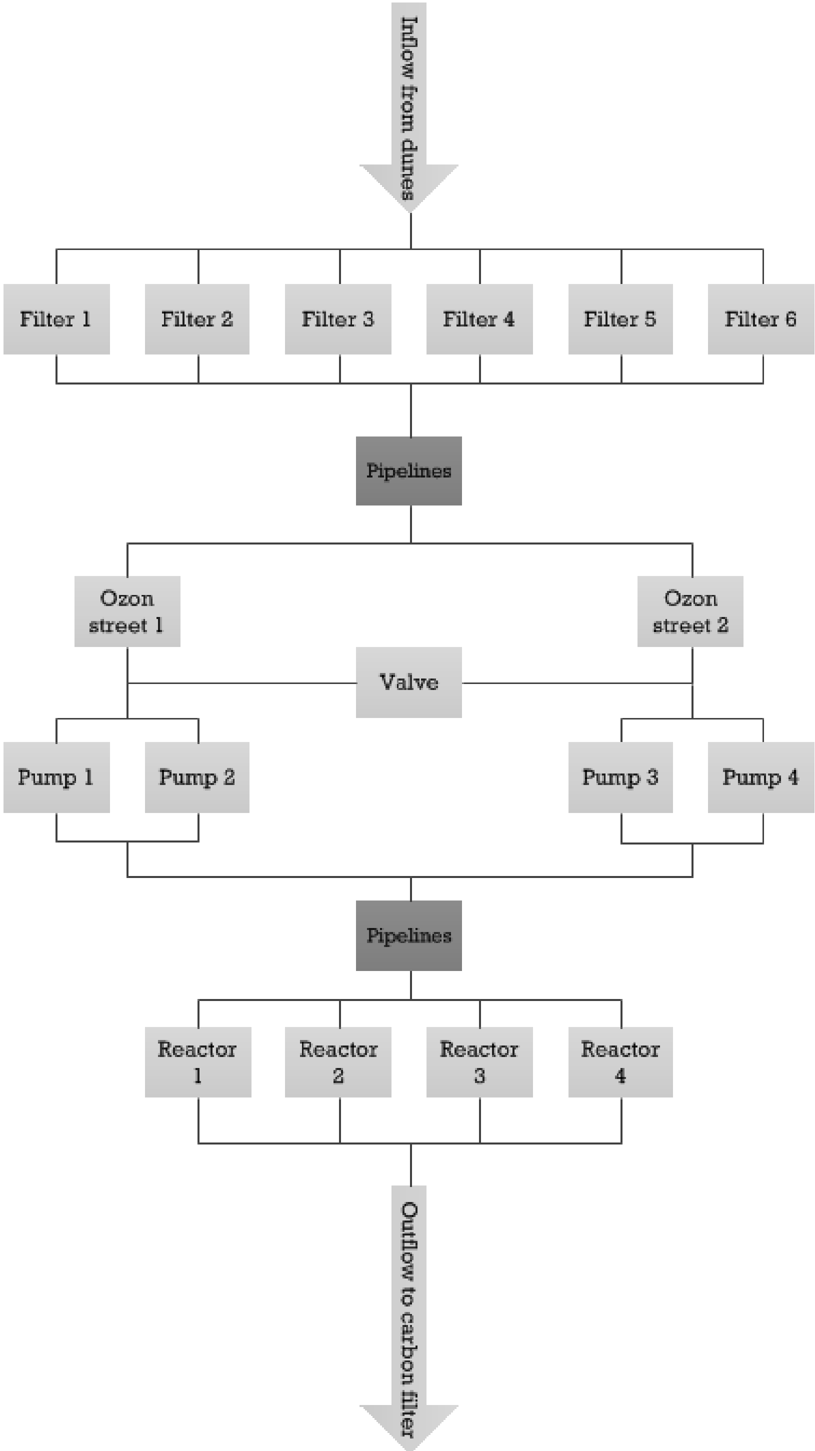

Figure 33: RBD of the case study system

5.2.2 Apply flowchart for model choice at component level

The next step in the proposed methodology is the use of the flowcharts shown in Figure 14 at page 42 and Figure 15 at page 43. In this section, the flowcharts are applied to the different assets within the described system, to assist in the decision-making with respect to the optimal model selection at the component level (SMP model or NHPP model). Each choice is clarified in figures by either a green (yes) or a red (no) decision shape[16].

5.2.2.1 Model selection: Rapid sand filters

The risk of failure of a rapid sand filter is considered to be acceptable, because of the redundancy in the configuration (four out of six sand filters need to be available). For the rapid sand filters sufficient failure events have been recorded over the past years (see Appendix 5.1.1), so according to the flowchart in Figure 35 the NHPP model could be used. According to the plant operators, the sand filtration is a relatively simple treatment step and the filters do not contain many mechanical parts, which makes there is no clear deterioration process observable and condition monitoring is hardly possible. Therefore, the SMP model cannot be applied for these assets. Following the flowchart outcome as shown in Figure 34, an economical consideration should be made between two possibilities: using the NHPP model and using average values. However, failures are already being recorded for the sand filters, so using the NHPP model will not lead to extra costs. Therefore, the NHPP model will be used to model the availability for the sand filters.

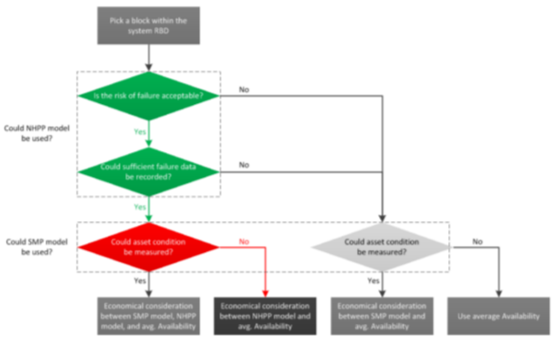

Figure 34: Flowchart application for rapid sand filters

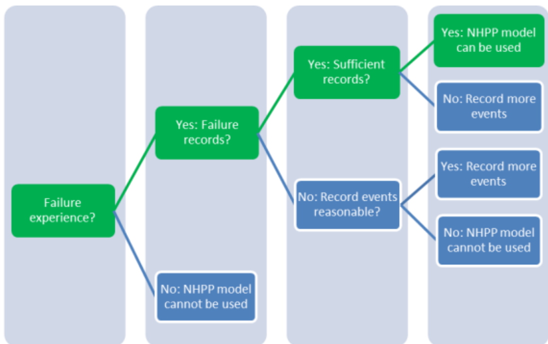

Figure 35: Second flowchart application for rapid sand filters

5.2.2.2 Model selection: Pumps

For the pumps the risk of failure is presumed to be unacceptable, so using the NHPP model can be discarded. The condition of the pumps can be monitored continuously, so according to the flowchart (Figure 36) an economical consideration should be made between using the SMP model or using average values. However, condition-based maintenance is desired here, so the SMP model is the optimal model choice for availability modelling of the pumps and there is no reason to choose for average values here.

[16] Figures might appear illegible in this sub-section, since they have been shrunk for reasons of space. However, colors indicate choices and legible versions of the flowcharts can be found at pages 37 and 38.

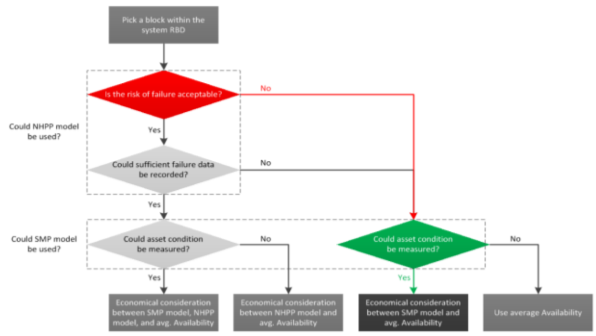

Figure 36: Flowchart application for pumps

5.2.2.3 Model selection: Valve

Operators and asset managers agree that risk of failing for the valve between the ozon streets and pump set-ups is acceptable. The valve has failed only once and has been repaired after this failure but there is no record of this failure availabe. Since the valve is already in operation since its construction 34 years ago, the recording of sufficient events in reasonable time is not expected and thus the NHPP model cannot be used (see the flowchart in Figure 38). Condition of the valve is currently unknown and operators notice that some other very weared valves at Leeward still function properly, while some newer valves experience failures. They indicate that it is not possible to monitor the condition of the valve and therefore the flowchart suggests using an average availability, see Figure 37. It is estimated that the repair of the valve took two days and with 34 years of operation the average availability becomes:

$$\frac{2 \cdot 24}{34 \cdot 365 \cdot 24} = 0.99984$$

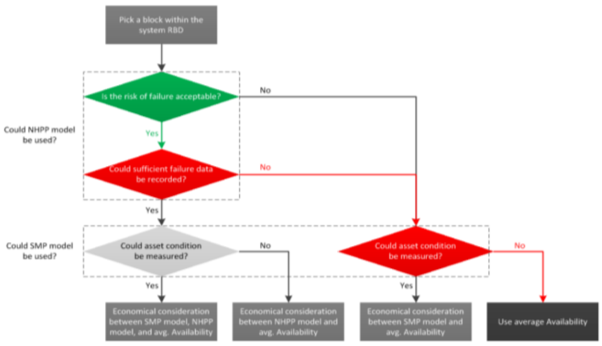

Figure 37: Flowchart application for valve

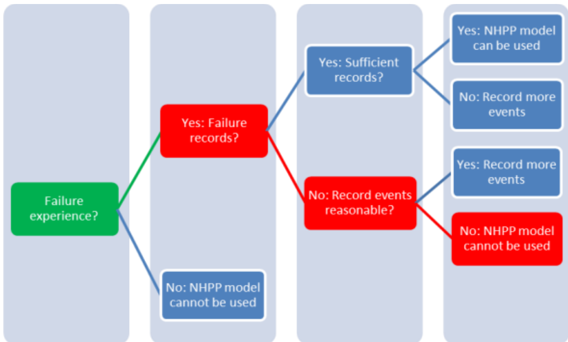

Figure 38: Second flowchart application for valve

5.2.2.4 Model selection: Ozonation

For the two ozone streets it is concluded by the asset managers that the risk of failure is unacceptable, so the NHPP model cannot be used. On the other hand, the condition can be monitored continuously and condition data has been recorded over the past years, so applying the SMP model is possible. This results in the economical consideration between using the SMP model and using average values, as indicated by the flowchart in Figure 39. However, since the risk of failure is unacceptable, condition-based maintenance is desired for the ozonation, making the SMP model the optimal choice. Simply using average values does not provide the right information to the asset manager.

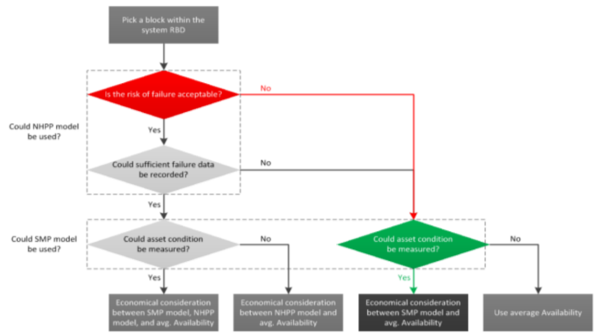

Figure 39: Flowchart application for ozonation

5.2.2.5 Model selection: Softening

For the softening reactors the risk of failure is considered as being acceptable, due to the redundancy of the softening reactors. Sufficient failure events have been recorded over the past years (see Appendix 5.4.1), so using the NHPP model is possible (see Figure 41). The condition of the softening reactors can be determined by inspection, so the SMP model can be used as well. Considering the outcome of the flowchart in Figure 40, eventually the economical consideration must be made between all three options: the NHPP model, the SMP model, and using average values. Because both failure data and condition data are already available for the softening reactors, using average values does not make any sense here. Nevertheless, using the SMP model will cost more compared to the NHPP model, due to the required condition monitoring. Besides, the risk of failure is assumed to be acceptable, so therefore it is argued that the NHPP model is the optimal model choice for availability modelling of the pumps.

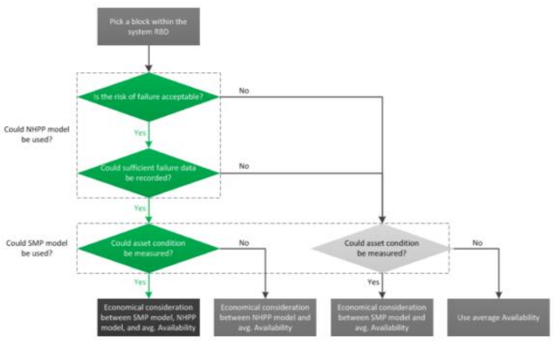

Figure 40: Flowchart application for softening

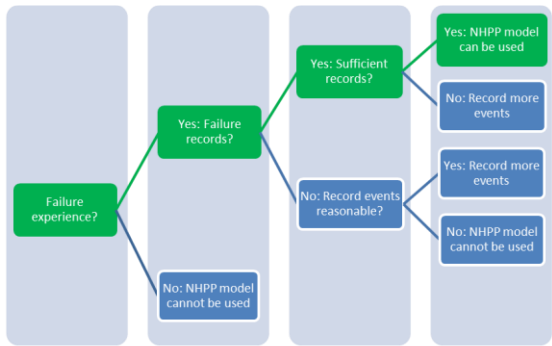

Figure 41: Second flowchart application for softening

5.2.3 Model availability at component level

In this sub-section the availability modelling for all described components is handled. Since the valve component is not modelled via either the NHPP or SMP model, but has an average availability, it will not be dealt with in this sub-section.

5.2.3.1 Modelling results: Rapid sand filters

In order to find out if there is a trend in the failure event data (which can be found in Appendix 5.1.1) suggesting deterioration, two separate trend tests with significance level $\alpha = 0.05$ are executed for every filter. The findings of these trend tests are found in Table 5. Application of the trend tests for the rapid sand filters can be found in Appendix 5.1.2. The trend tests suggest that there is no trend in the data of any of the filters. This is in accordance with the suspicion of the operators that there is no deterioration process taking place at the rapid sand filters. Therefore, it is concluded that these filters can be modelled by a Homogeneous Poisson Process (HPP). In order to find out if the data from all filters can be pooled together, a likelihood ratio test is

performed. This test measures the equality of the filters based on the failure event data. The outcomes of the test are given in
Table 6. In Appendix 3.3 it is described how this test is performed.

Table 5: Trend tests on data for rapid sand filters

Filter	Test statistic MIL-HDBK-189	Test statistic Laplace Trend Test	Outcome of tests (α = 0.05)
1	32.17	0.03	No trend in data
2	35.78	-0.97	No trend in data
3	45.55	-1.79	No trend in data
4	44.85	-0.91	No trend in data
5	44.85	0.36	No trend in data
6	44.07	-0.46	No trend in data

Table 6: Likelihood ratio test on data for rapid sand filters

Likelihood ratio statistic	Outcome of test (α = 0.05)
1.57	Identical components

The likelihood ratio test confirms that the filters are identical in their failure event pattern, thus a single parameter, applicable to all filters, can be estimated. Since the filters are modelled via an HPP, only one parameter (θ) has to be estimated, while the other (β) is already determined, see Table 7. Application of parameter estimation can be found in Appendix 3.1.1. The repair time data (see Appendix 5.1.1) is assumed to be identically and lognormal distributed for all filters. Parameters are estimated for the lognormal distribution[17], see Table 8, according to the method elaborated in Appendix 4.1.

Table 7: Parameter estimations for the uptimes of HPP model for rapid sand filters

Parameter	Value	Confidence bounds (95%)	
$\boldsymbol{\theta}$	683.12	571.04	831.95
$\boldsymbol{\beta}$	1	*No confidence bounds since HPP → β=1*	

Table 8: Parameter estimations for lognormal distributed downtimes for rapid sand filters

Parameter	Value
$\boldsymbol{\mu}$	4.89
$\boldsymbol{\sigma}$	0.98

These parameter estimations form the input for the NHPP model. The outcome of the model, the availability over time for a rapid sand filter, is shown in Figure 42. At first the rapid sand filter starts at an availability of 1 (initial condition), but soon descends to a steady availability of around 0.987. This steady availability is consistent with the HPP and its random failure behavior.

[17] Data has been altered from hours to days to make it comparable to the uptimes

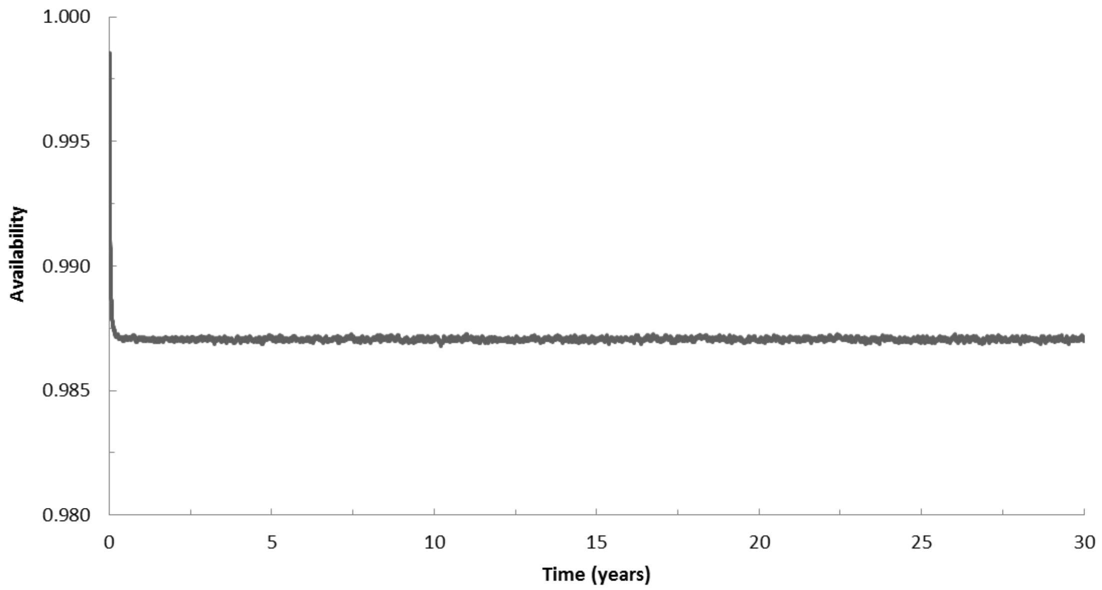

Figure 42: Availability over time for a single rapid sand filter

5.2.3.2 Modelling results: Pumps

The condition data and the repair time data for the pumps can be found in Appendix 5.2. The maintenance strategy that is assumed for the pumps consists of the following decision rules and assumptions:

- Four discrete upstates are assumed for the component, where state 1 = *same as new* and state 4 = *last deterioration state before failure*;
- For all four pumps the condition has been monitored over the last 16 years;
- There is no uncertainty in inspection results;
- When a pump is observed to be in state 4, a maintenance action is performed immediately;
- For the first five times this occurs to one pump, an imperfect repair is performed that brings the pump back to state 2;
- This downstate (imperfect repair from state 4 to state 2) is defined as state 5;
- When a pump enters state 4 for the 6th time, a perfect repair (renewal) is performed that brings the pump back to state 1 (same as new);
- This downstate (perfect repair) is defined as state 6;
- Thereafter, this cycle is repeated, so the 7th maintenance action is again an imperfect repair (state 5);
- When the component is in state 1, 2, 3 or 4 (the upstates), it is considered to be available (A = 1); when the component is in state 5 or 6 (the downstates), it is unavailable (A = 0);

Based on the available condition data, the parameters for the holding time distributions are estimated by applying the MLE method for the two-parameter Weibull distribution (as explained in Appendix 2.1.1). Based on the available condition data, the following Weibull parameter estimations for the upstates are obtained:

State 1:	α = 4.378
	β = 52.330
State 2:	α = 3.494
	β = 39.893
State 3:	α = 3.908
	β = 30.169

Figure 43 shows the estimated Weibull distributions for these three upstates. It should be noted that the holding time distribution for state 4 cannot be estimated due to the active maintenance strategy. When a pump enters state 4, maintenance is performed immediately, so the holding time is cut off directly. In the SMP model the holding time for state 4 is always equal to one day, which makes that the holding time distribution for state 4 does not influence the model outcome and it is not problematic that these parameter estimations are lacking.

The holding time distributions for the two downstates can be estimated based on the repair data. For the imperfect repair (state 5) the downtime has been recorded 40 times, and for the perfect repair (state 6) the downtime has been recorded seven times. For both downstates the parameters for the lognormal distribution can be estimated with the MLE method, as described in Appendix 4.1. This results in the following parameter estimations for the downstates:

State 5: $\mu = 0.350$
$\sigma = 0.561$
State 6: $\mu = 3.034$
$\sigma = 0.076$

Figure 44 shows the estimated distributions for both downstates.

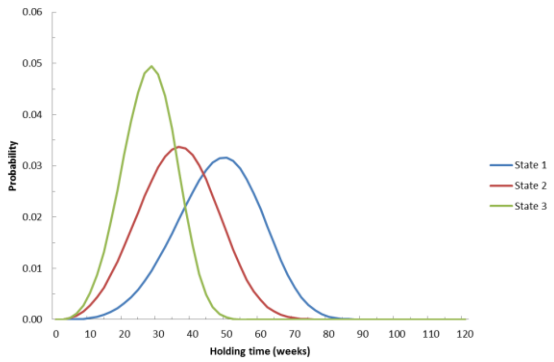

Figure 43: Estimated holding time distributions for the upstates of the softening reactors

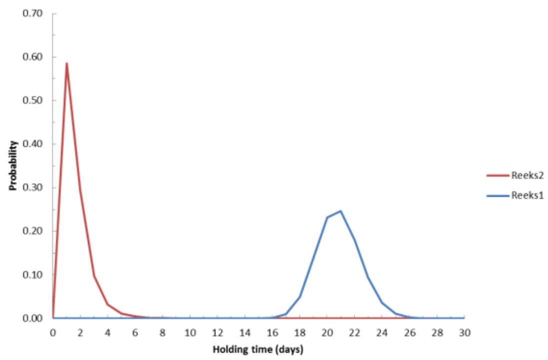

Figure 44: Estimated holding time distributions for the downstates of the softening reactors

Based on the maintenance strategy and the estimated holding time distributions the SMP model provides the availability over time, see Figure 45. The influence of the different maintenance actions is clearly visible in the graph. Until t = 7 years the availability converges towards a steady-state situation, where the intervals with a negative slope represent deterioration and the intervals with a positive slope show the condition improvement due to maintenance actions. The peaks around t = 9, 18 and 26 years are the result of the perfect repairs (each 6^{th} maintenance action for a pump).

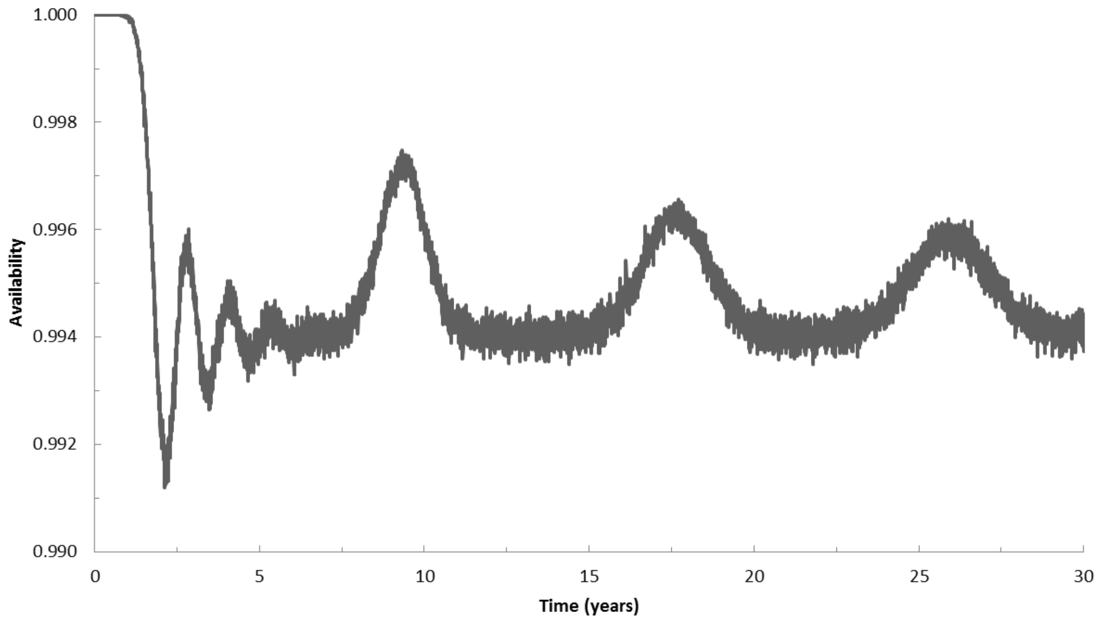

Figure 45: Availability over time for a single pump

5.2.3.3 Modelling results: Ozonation

The condition data and the repair time data for the two ozone streets are listed in Appendix 5.3. The maintenance strategy that is assumed for the softening reactors consists of the following decision rules and boundary conditions:

- Four discrete upstates are assumed for the component, where state 1 = *same as new* and state 4 = *last deterioration state before failure*;
- For both ozone streets, the condition can be monitored continuously;
- There is no uncertainty in inspection results;
- When an ozone street is monitored to be in condition state 4, a maintenance action is performed;
- The influence of the maintenance action for an ozone street decreases in time:
    - Repair number 1 – 3 bring the ozone street back to state 2;
    - Repair number 4 – 6 bring the ozone street back to state 3;
    - The 7^{th} repair action is a perfect repair (or renewal), bringing the ozone street back to state 1 (same as new);
    - Thereafter, this cycle is repeated, so the 8^{th} repair is considered as the first repair of a new cycle;
- All repair actions are corresponding to the same downstate, named state 5;
- When the component is in state 1, 2, 3 or 4 (the upstates), it is considered to be available (A = 1); when the component is in state 5 (the downstate), it is unavailable (A = 0);

One of the assumptions is that only one downstate is considered, with one distribution for all downtimes. It might seem more likely that the 7^{th} repair action of the cycle follows a different distribution, because it is a perfect repair (or replacement) instead of an imperfect repair, implying this action might take more time to perform by the maintenance crew. However, insufficient data is available regarding this perfect repair action, since it has been performed only a few times. For this reason, the downtime caused by the perfect repair is assumed to follow the same distribution as the downtime corresponding to the imperfect repair actions.

Based on the available condition data, the parameters for the holding time distributions can be estimated by using the Maximum Likelihood Estimation (MLE) method. For the upstates this MLE method is used to estimate the parameters of the two-parameter Weibull distribution, as described in Appendix 2.1.1. For the downstate the MLE method is used to derive the parameter estimations for the lognormal distribution, see Appendix 4.1.

Based on the available condition data, the following Weibull parameter estimations for the upstates are obtained:

State 1: $\alpha = 11.154$
$\beta = 100.460$
State 2: $\alpha = 5.468$
$\beta = 80.763$
State 3: $\alpha = 2.355$
$\beta = 72.775$

Similar as for the pumps, the holding time distribution for state 4 cannot be estimated as a result of the active maintenance strategy. However, as argued in sub-section 5.2.3.2, this does not influence the model outcome. For the downstate (state 5) the parameters for the lognormal distribution can also be estimated with the MLE method:

State 5: $\mu = 1.019$
$\sigma = 0.294$

The estimated holding time distributions are shown in Figure 46 and Figure 47.

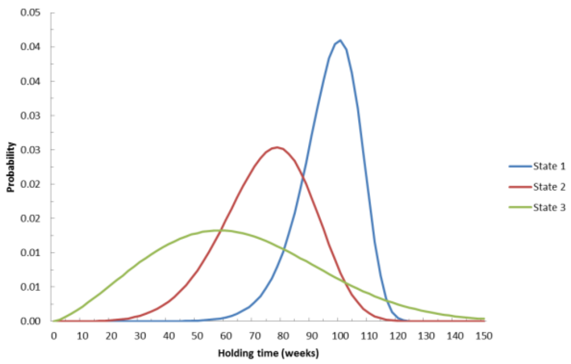

Figure 46: Estimated holding time distributions for the upstates of the ozone streets

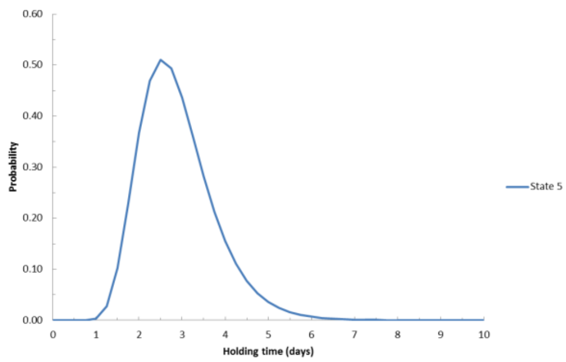

Figure 47: Estimated holding time distribution for the downstate of the ozone streets

The obtained parameter estimations are used as input for the SMP model, which results in the availability over time for one ozone street, see Figure 48. The several maintenance actions can be recognised in the graph. In the first years the availability is decreasing due to deterioration. Around t = 5 years the influence of the first repair can be seen, causing an increase in the availability. Until t = 10 years the function is fluctuating around a steady-state value. Thereafter, a clear decrease in the availability can be seen until t = 14 years, which can be explained by the fact that the fourth repair brings the ozone street back to state 3 instead of state 2. Because the influence of maintenance changes in time, the function does not converge towards a steady-state situation here.

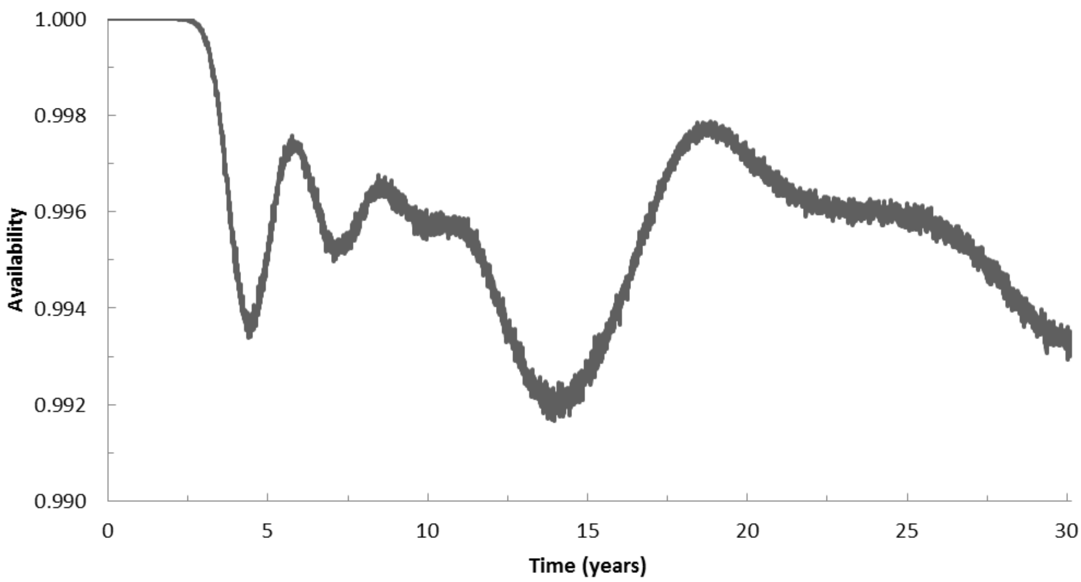

Figure 48: Availability over time for a single ozone street

5.2.3.4 Modelling results: Softening

In Appendix 5.4.1 the available failure data for the four softening reactors is provided. In order to find out if there is a trend in the failure data suggesting deterioration, two separate trend tests with significance level α = 0.05 are executed for every softening reactor. The findings of these trend tests are found in Table 9. More detailed outcomes of the trend tests for the softening reactors can be found in Appendix 5.4.2.

Table 9: Trend tests on data for softening reactors

Pump	Test statistic MIL-HDBK-189 Test	Test statistic Laplace Trend	Outcome of tests (α = 0.05)
1	11.47	6.81	Trend in data exists
2	15.35	3.95	Trend in data exists
3	3.87	12.60	Trend in data exists
4	12.33	8.24	Trend in data exists

The trend tests suggest that a trend in the data exists, but it is yet unknown if this is a deteriorating or improving trend. Nevertheless, the reactors are best modelled through a Non-Homogeneous Poisson Process (NHPP). In order to find out if the data from all reactors can be pooled together, a likelihood ratio test is performed. This test measures the equality of the reactors based on the failure event data. The outcomes of the test are given in Table 10. The likelihood ratio test is described in Appendix 3.3.

Table 10: Likelihood ratio test on data for softening reactors

Likelihood ratio statistic	Outcome of test (α = 0.05)
6.78	Identical components

The likelihood ratio test confirms that the softening reactors are identical in their failure event pattern, thus a single parameter, applicable to all reactors, can be estimated. Since the reactors are modelled via a NHPP, two parameters have to be estimated (β and θ), see Table 11. The confidence bounds (95%) for the shape parameter (β) confirm that the assumption of a NHPP (instead of a HPP) is justified, since the value of β = 1 is not included in the interval. Application of parameter estimation and calculations can be found in Appendix

3.1.1. The repair time data (see Appendix 5.4.1) is assumed to be identically and lognormal distributed for all pumps. Parameters are estimated for the lognormal distribution[18], see Table 12.

Table 11: Parameter estimations for the uptimes of the NHPP model for softening reactors

Parameter	Value	Confidence bounds (95%)	
β	2.84	2.13	3.54
θ	4748.71	1547.46	14572.43

Table 12: Parameter estimations for the downtimes of the NHPP model for softening reactors

Parameter	Value
μ	4.00
σ	1.81

These parameter estimations are used for the NHPP model. The outcome of the model, the availability over time for a softening reactor, is given in Figure 49. At first the availability is equal to 1 and then starts to descend. First slowly, but accelerating in speed of decrease and does so until the end of the simulation at t = 30 years. This behavior is consistent with a deteriorating NHPP: due to its minimal repair the ‘uptimes’ are stochastically decreasing with increasing time, which leads to an ever decreasing availability (assuming ‘downtimes’ stay stochastically the same).

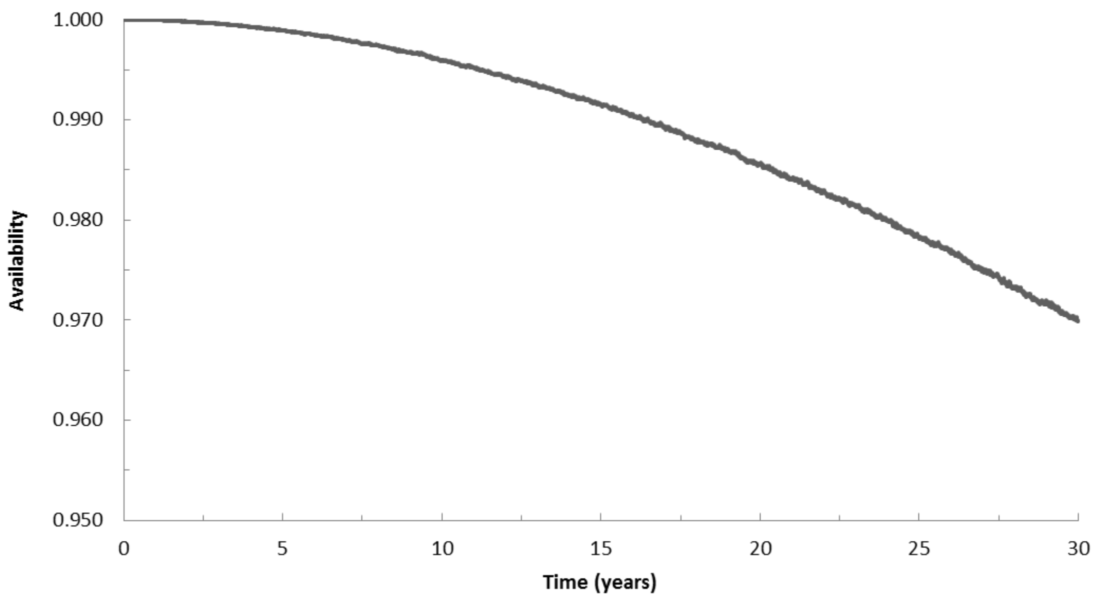

Figure 49: Availability over time for a single softening reactor

5.2.4 Use RBD for system calculations

With the RBD provided in sub-section 5.2.1 and the availability results at component level the system’s availability can be calculated. In order to do so, the proper calculations have to be made with the RBD. First the order of calculations is determined. At the highest level in the RBD one main configuration can be distinguished: a serial configuration (Ser) of the rapid sand filters, the bridge structure (containing the ozone streets, valve and pumps) and the softening reactors, see Figure 50.

[18] Data has been altered from hours to days to make it comparable to the uptimes

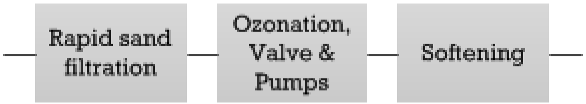

Figure 50: Leeward Dune system simple RBD

In this series configuration the first and last step can be directly calculated, using the *k*-out-of-*n* calculation from section 4.3.6. The central step, the bridge structure, cannot be calculated directly, since it contains other configurations itself: the two parallel configurations of the pumps. So before the bridge structure can be calculated, the availability of the parallel configurations has to be calculated. This parallel configuration is transformed in a single block "Pump set-up", see Figure 51 and Figure 52. The accompanying structure function for the calculation is:

$$A_s(t) = 1 - \prod_{i=1}^{n} (1 - A_i(t))$$

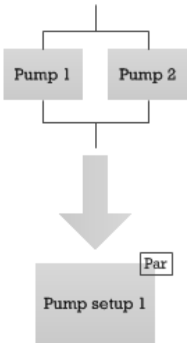

Figure 51: Representation of pump set-up 1 by a single block

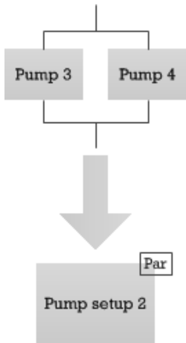

Figure 52: Representation of pump set-up 2 by a single block

Now the availability function for the pump set-up is known, the bridge structure can be considered as well. The structure function for the bridge structure is given by: (see section 4.4)

$$\begin{aligned} A_s(t) = {} & A_1(t)A_3(t) + A_2(t)A_4(t) + A_1(t)A_5(t)A_4(t) + A_2(t)A_5(t)A_3(t) - A_1(t)A_2(t)A_5(t)A_3(t) \\ & - A_1(t)A_2(t)A_5(t)A_4(t) - A_1(t)A_2(t)A_3(t)A_4(t) - A_1(t)A_5(t)A_3(t)A_4(t) \\ & - A_2(t)A_5(t)A_3(t)A_4(t) + 2A_1(t)A_2(t)A_3(t)A_4(t)A_5(t) \end{aligned}$$

where:

$A_1(t)$	=	Availability of ozone street 2
$A_2(t)$	=	Availability of ozone street 1
$A_3(t)$	=	Availability of pump set-up 2
$A_4(t)$	=	Availability of pump set-up 1
$A_5(t)$	=	Availability of the valve

This calculation transforms the bridge structure into a single block called "Ozonation, Valve and Pumps", see Figure 53.

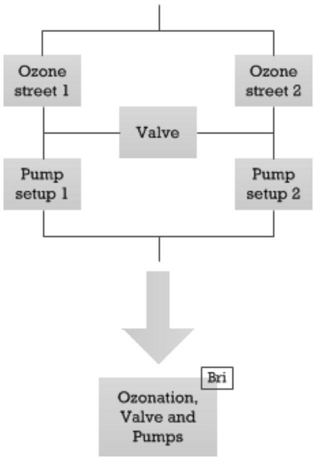

Figure 53: Representation of the bridge structure by a single block

The *k*-out-of-*n* calculations (3-out-of-4 and 4-out-of-6) for the first and last step of the series configuration result in the forming of two single blocks. One for the softening, see Figure 54, and one for the rapid sand filtration, see Figure 55.

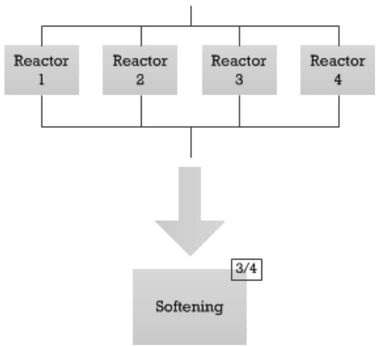

Figure 54: Representation of softening reactors by a single block

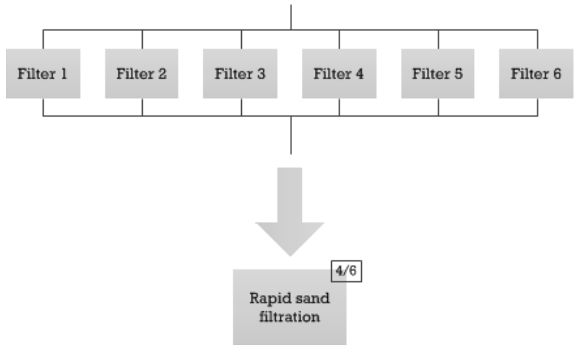

Figure 55: Representation of rapid sand filters by a single block

The structure function for a *k*-out-of-*n* configuration is: (see section 4.4)

$$A_s(t) = \sum_{i=k}^{n} \binom{n}{i} [A_1(t)]^i [1 - A_1(t)]^{n-i} \quad , \quad if\ A_i(t) = A_1(t)$$

The RBD now has the form as shown in Figure 50 and the final serial configuration calculation can be made. This transforms the serial configuration into a single block representing the whole system of rapid sand filters, ozone streets, valve, pumps and softening reactors, see Figure 56.

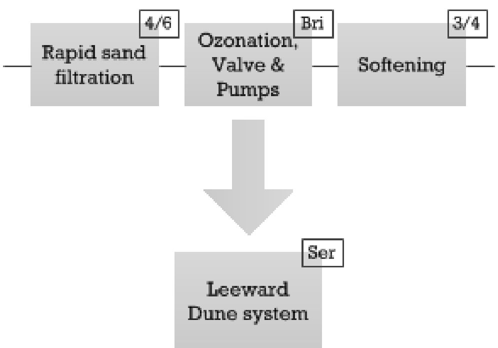

Figure 56: Representation of the case study system by a single block

The formula for calculating the availability of the serial configuration is:

$$A_s(t) = \prod_{i=1}^{n} A_i\,(t)$$

Via a software program (such as MS Excel) this calculation can be repeated for every time step *t* that has been specified. In the final step the availability for the system is determined. The availability over time for the concerned system in this case study is given in Figure 57.

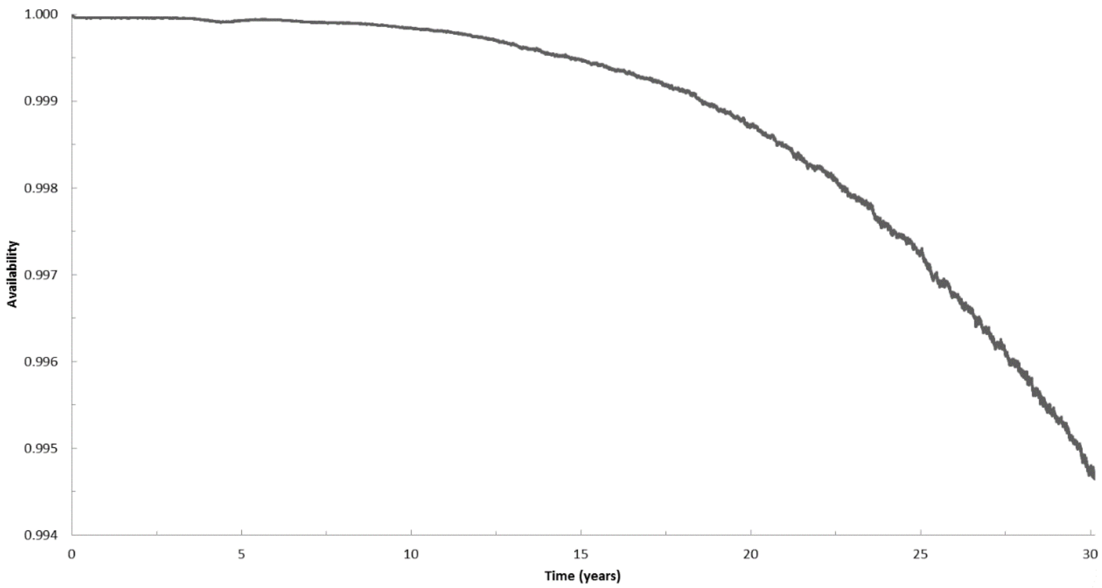

Figure 57: System availability over time for the case study system

The availability over time for Leeward Dune clearly shows a deteriorating trend in the availability. The main cause for this trend is the influence of the availability of the softening reactors: if the softening reactors are removed from the RBD computations, the system availability (Figure 58) follows a very different pattern. Without the softening, the availability resembles the course of the availability that the ozone streets follow (see Figure 48, page 72). In order to give an idea of the differences, if these were average availabilities, the inclusion of the softening means an increase in the average yearly *unavailability* from 0.35 – 0.96 hours to more than 40 hours (year 30).

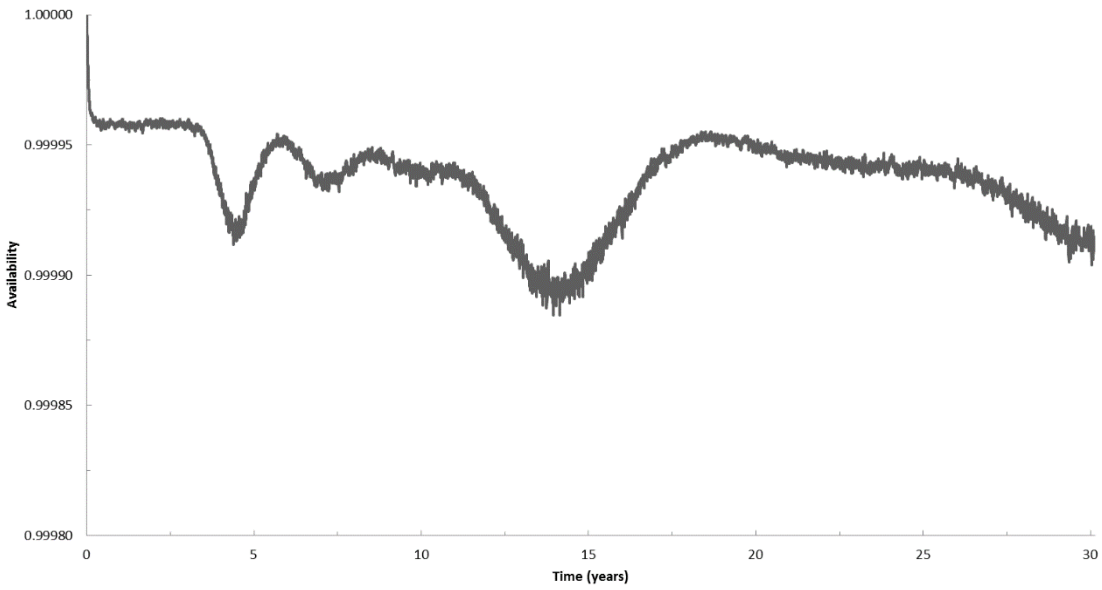

Figure 58: System availability over time with exclusion of the softening

When the ozone streets are taken out of the RBD computations as well (by making their availability equal to 1), the effect is that the system availability (Figure 59) resembles the availability of the rapid sand filters (Figure 42, page 68). It can be concluded from this figure that the availability of the pumps (Figure 45, page 70) is not influential on the system's availability. This has probably to do with the fact that only one of the four pumps has to function, in order to be able to meet maximum production capacity.

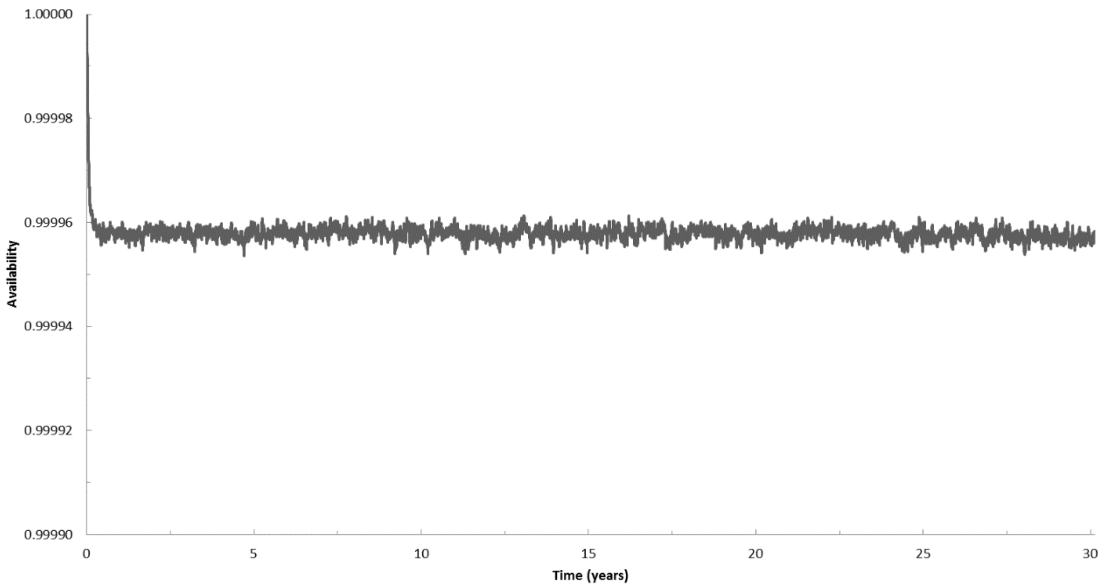

Figure 59: System availability over time with exclusion of the softening and ozonation

An asset manager might be interested in improving the long-term availability of the system and therefore in finding out the effect of modifications to the system. It has already been discovered that the softening reactors are the most influential in the system's availability. Thus the inclusion of an extra softening reactor and its effect on the system's availability is investigated.

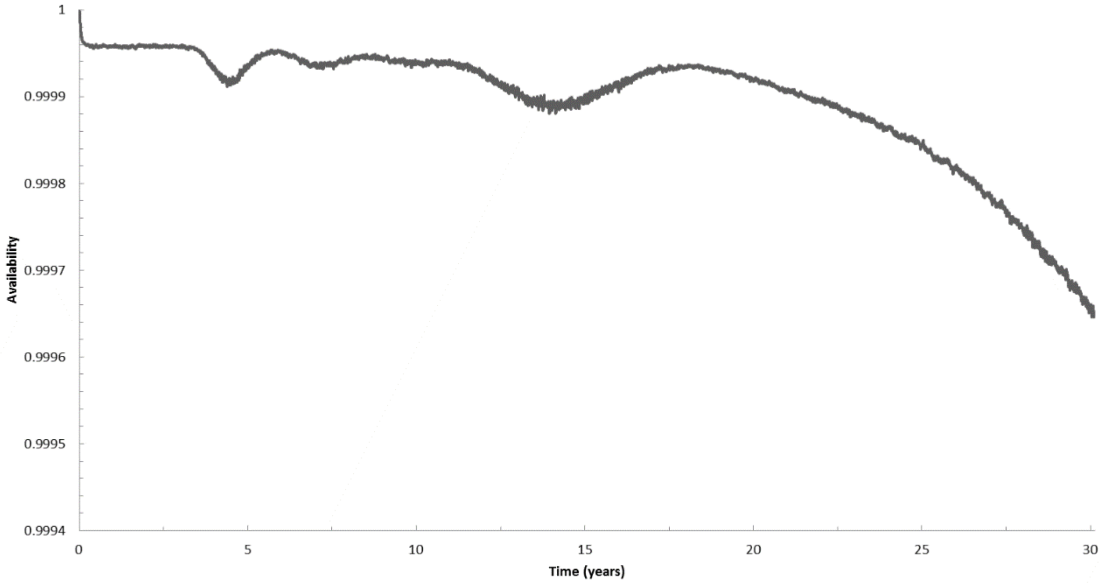

Figure 60: System availability with inclusion of an extra softening reactor

The inclusion of an extra softening reactor means that the configuration of the softening process step goes from a 3-out-of-4 to a 3-out-of-5 configuration. The result to the system's availability can be found in Figure 60. It shows that the system's availability main influence is a mixture of both the ozone streets and the softening reactors. For the first 15-20 years the availability is mainly dependent on the availability of the ozone streets, thereafter the deteriorating trend of the softening takes over. Comparing Figure 60 with Figure 57, one can see clearly that the inclusion of an extra softening reactor is beneficial to the availability of the system.

5.3 Conclusions on case study

This case study shows the application of the method that has been described in chapter 4. It shows how an asset manager can go from a description of the system's configuration and data on asset condition, failure events and repair times to the availability of the system. By means of excluding (or including) components from (or in) the calculations, their effect on the system's availability can be clarified. In this case study it is found that the softening reactors have a large influence on the system's availability. If the projected system's availability is not meeting the requirements, the model can be used to investigate the effects of altering (parts of) the system. In the case study, an extra softening reactor and its effect on the system's availability are modelled this way. Another change to the system, that has not been included in the case study, could be the alteration of maintenance strategies. Especially in the case of the SMP model, where different strategies can easily be incorporated in the model (see Appendix 2.3.1). Also, influence of decreasing repair times through altering the parameters of the downtime distributions, can be used to investigate for example shorter reaction times on failures or larger inventories.

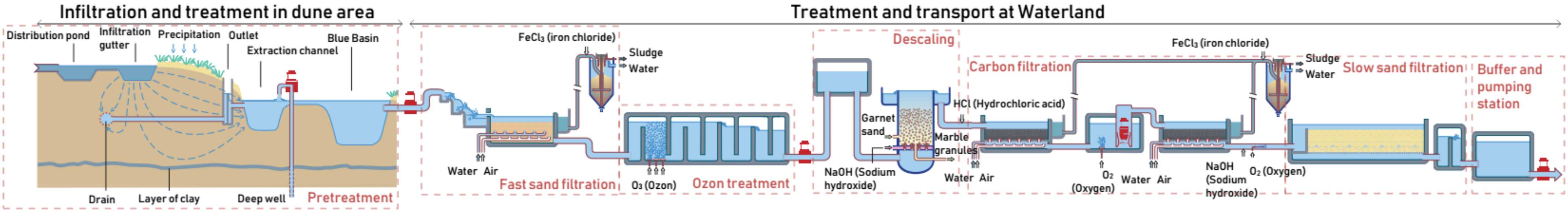

Figure 61: Schematic overview of the treatment steps at Leeward Dune, from Waternet (2018a)

6 Conclusions

This research sets out to find a method for modelling the availability over time for a water treatment plant, which has the following characteristics:

- Complex configuration
- Deteriorating assets
- Repairable assets

The models found in the state-of-the-art do not meet all these criteria. However, there are models that fulfil several of these criteria and therefore the proposed solution in this research is based on combining multiple modelling techniques. The overall model is a two-level hierarchical model, which can cope with complex configurations at the higher *system level* and with deteriorating and repairable assets at the lower *component level*.

At the component level two models can be chosen by the asset manager, the Semi-Markov Process (SMP) model and the Non-Homogeneous Poisson Process (NHPP) model. The main distinction between these two is being a *condition based* respectively a *failure based* type of model. To assist the asset manager in deciding what model is the optimal one to choose, a flowchart is constructed. This decision-making is based on risk analysis, data availability, predictability of failures and economic considerations. The use of Inverse Transform Sampling in a Monte Carlo simulation method provides the stochastic availability over time at the component level.

At the system level a Reliability Block Diagram (RBD) deals with the complex configurations within a water treatment plant. Availabilities of different components are aggregated to obtain the system availability, by taking into account their configuration within the system. Common configurations for water treatment plants, such as redundancy and bridge structures, can be dealt with in this way.

The models and their application are incorporated in a method which consists of four steps:

1. Set up an RBD at system level
2. Apply the flowchart for choosing the appropriate component model for each block in the RBD
3. Model the components' availabilities with the component models
4. Use the RBD to model the system availability

The application of the method to a case study with sampled, but realistic, data shows how the system's availability is computed. The benefits of the method for asset managers are gaining insight into the contribution of components to the system's availability and the ability to alter the system configuration or maintenance strategy in order to optimise these. Another advantage is that the model can be used for modelling at different levels. The case study shows how (part of) a WTP can be modelled with it, but it is also possible to model at the level of one treatment step or even one asset. The system boundaries can be adjusted based on the desired level of detail in the RBD.

7 Discussion

From an asset manager's point of view, the aim is to optimise the system configuration and maintenance strategy in order to satisfy the availability demands in the most cost-efficient way. This challenge has caused a shift from preventive and corrective maintenance towards predictive, condition-based maintenance. To evaluate changes to system configuration and maintenance strategies, it is important to gain insights about the development of the system availability over time, influenced by asset deterioration and maintenance actions. The proposed methodology in this report has been developed to facilitate in this.

An important feature of the methodology is that it is data driven and thus the adage "garbage in, garbage out" applies here. The SMP model is based on condition data, so it is important to know if and how the condition of assets can be determined. For some assets it is possible to monitor the condition continuously, which would be the ideal situation. For other assets the condition can only be determined through periodic inspections. Aiming for uniform condition monitoring, the NEN-2767 standard could serve as a guideline to asset managers, since it provides a methodology for consistent condition monitoring and registration for relevant assets. The NHPP model, on the other hand, is driven by failure data. It must be well defined when an asset has failed, and the recording of failures needs to be accurate and uniform. Likewise, the information regarding the performed maintenance actions must be well registered. What is the active maintenance strategy? What is the influence of a maintenance action on the asset condition? How long does it take to perform the maintenance action? This type of information is of much importance for accurate availability modelling. Without a proper definition and collection of condition data, failure data and maintenance actions, adoption of the methodology by asset managers and other users might prove difficult. Subsequently, the benefit of the proposed methodology is that it can be used as a demonstration of the usefulness of collecting data, since it produces tangible insights into future behaviour of assets. But, although proper data management is essential, not for all real-life systems sufficient hard condition and failure data will be available. Still, in that case methods are known for estimation of the required model parameters, such as expert judgement or estimation from zero-failure data. The modelling results will be less accurate, but until sufficient hard data is collected such methods can be used.

Finally, some considerations for further research are suggested. First, in the proposed methodology it is assumed that individual components deteriorate and/or fail independent of each other, which is inevitable with an RBD. Although, dependencies between components can exist in real-life systems. A solution might be the addition of extra components in the RBD, representing the processes that cause these dependencies. However, further investigation into this topic is needed to enable more informed conclusions.

Second, it should be noted that this study focusses on availability modelling, which is a first step in the optimisation process of maintenance strategies and system configurations. The second step would be the inclusion of costs, in order to find the optimal balance between ensuring a high availability on the one hand, and reducing costs on the other hand. Thus, the inclusion of costs is recommended as an extension to this study. Several options can be thought of: the Semi-Markov Decision Process (SMDP) as an extension of the SMP model could be used to find optimal maintenance strategies, and Life Cycle Costing (LCC) techniques such as using the net present value could be applied for discounting future revenue and expenses of maintenance strategies in both the NHPP and SMP model.

Third, with the SMP model multiple condition states can be included, where each condition state is corresponding to a value for the instantaneous availability of the concerned component. In the proposed methodology this instantaneous availability is assumed to be a binary measure (either 1 or 0). Therefore, an extension to the SMP model could be the inclusion of intermediate values for the instantaneous availability, ranging from fully available to completely failed, which is for instance regarded in the work by Soro et al. (2010). This addition would allow the assumption of components being partly available as a result of deterioration. For example, within a WTP this could be relevant for different types of filters that experience clogging.

Fourth, inclusion of other types of models at the component level could relax some assumptions. For the NHPP model the application of imperfect repair models, such as Virtual Age Process (VAP) models are recommended to be researched. They relax the assumption of minimal for the NHPP model (or perfect repair for the HPP model). VAP models such as the Arithmetic Reduction of Intensity (ARI) and the Arithmetic Reduction of Age

(ARA), introduced by Doyen and Gaudoin (2004) fit well with the Power Law Process used in the NHPP model. One should keep in mind, however, that with the generalisation to imperfect repair models, demands on data become more challenging: in addition to failures and repair times, the effect of repairs on the asset has to be estimated.

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Appendix 1 Minimal path sets

An RBD or fault tree can be used to derive the system performance. To do so, the so-called structure function must be derived according to the system configuration. With this structure function, the system performance can be calculated as a function of the performances of the individual components (Marichal, 2016; Mejjaouli & Babiceanu, 2016).

The structure function of any configuration can always be derived by identifying all possible path sets (so all possible combinations of working and non-working components that do result in a functioning system) (Marichal, 2016):

$$\phi(x) = \sum_{A \subseteq C} \phi(A) \prod_{i \in A} x_i \prod_{i \in C \setminus A} (1 - x_i)$$

where:

x	=	the computed performance measure
ϕ(*x*)	=	the structure function for performance measure *x*
A	=	a path set
C	=	set of all components in the system
i	=	# component

However, when the complexity of the system increases, the number of possible path sets increases and it becomes too complex to derive the structure function this way. In order to reduce this complexity, minimal path sets (for an RBD) or minimal cut sets (for FTA) can be used.

A minimal path set is the minimal set of components whose functioning ensures the system working. A minimal path set is only a path of success when all components within the path are working. In other words, when one component is removed from a minimal path set, the set is no longer a path set (Kim, 2011; Rausand & Høyland, 2004).
For a fault tree, the system solution is obtained by using the minimal cut sets, which are equivalent to the minimal path sets in an RBD. A minimal cut set is a minimal set of events in the fault tree that causes system failure (Birolini, 2017; Distefano & Puliafito, 2007; Rausand & Høyland, 2004).

In this research the choice is made to use an RBD for the modelling at the system level (the upper level in the hierarchical model). Therefore, only the derivation of the system solution for an RBD is described here. Although, it should be noted that the minimal cut sets method is equivalent to the minimal path sets method.

Solution
Every system can be displayed as a parallel structure of its minimal path sets. When at least one minimal path set is working, the system as a whole is functioning (Rausand & Høyland, 2004). Based on this, the structure function can be derived (Marichal, 2016):

$$\phi(x) = 1 - \prod_{j=1}^{r} \left(1 - \prod_{i \in P_j} x_i\right) \qquad (Equation\ 1)$$

where:

P	=	the set of minimal path sets
r	=	total number of minimal path sets
j	=	# minimal path set

To apply this formula the right way, it should be noted that $x_i^k = x_i$ for all *i* and *k*. This can be explained by the fact that a block in an RBD can only be successive once at the time (Rausand & Høyland, 2004). The application of the minimal path sets method will be illustrated with two examples.

Example 1

Assume the bridge structure in Figure 62. The minimal path sets for this configuration are {1,4}, {2,5}, {1,3,5} and {2,3,4}.

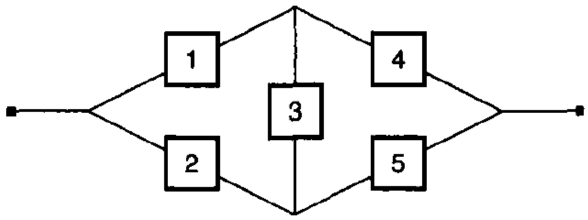

Figure 62: RBD for a bridge structure, from Rausand and Høyland (2004)

Then, the bridge structure can be seen as a parallel structure of its minimal path sets, see Figure 63.

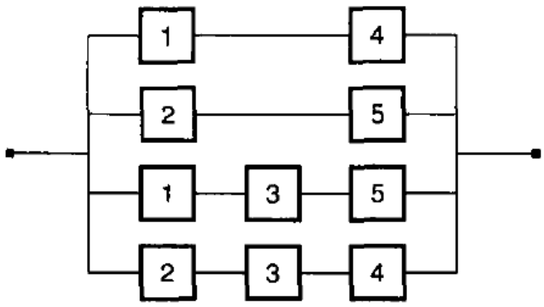

Figure 63: Alternative representation of the bridge structure's RBD, from Rausand and Høyland (2004)

By applying Equation 1 the structure function is derived:

$$\phi(x) = 1 - (1 - x_1x_4)(1 - x_2x_5)(1 - x_1x_3x_5)(1 - x_2x_3x_4)$$

$$= 1 - (1 - x_1x_4 - x_2x_5 + x_1x_2x_4x_5)(1 - x_1x_3x_5 - x_2x_3x_4 + x_1x_2x_3^2x_4x_5)$$

$$= 1 - (1 - x_1x_3x_5 - x_2x_3x_4 + x_1x_2x_3^2x_4x_5 + x_1x_4 + x_1^2x_3x_4x_5 + x_1x_2x_3x_4^2 - x_1^2x_2x_3^2x_4^2x_5 - x_2x_5 + x_1x_2x_3x_5^2 + x_2^2x_3x_4x_5^2 - x_1x_2^2x_3^2x_4x_5^2 + x_1x_2x_4x_5 - x_1^2x_2x_3x_4x_5^2 - x_1x_2^2x_3x_4^2x_5 + x_1^2x_2^2x_3^2x_4^2x_5^2)$$

Presuming $x_i^k = x_i$ then reduces the solution to:

$$\phi(x) = x_1x_3x_5 + x_2x_3x_4 + x_1x_4 + x_2x_5 - x_1x_2x_3x_4x_5 - x_1x_3x_4x_5 - x_1x_2x_3x_4 + x_1x_2x_3x_4x_5 - x_1x_2x_3x_5 - x_2x_3x_4x_5 + x_1x_2x_3x_4x_5 - x_1x_2x_4x_5 + x_1x_2x_3x_4x_5 + x_1x_2x_3x_4x_5 - x_2x_3x_4x_5$$

$$= x_1x_4 + x_2x_5 + x_1x_3x_5 + x_2x_3x_4 - x_1x_2x_3x_4 - x_1x_2x_3x_5 - x_1x_2x_4x_5 - x_1x_3x_4x_5 - x_2x_3x_4x_5 + 2x_1x_2x_3x_4x_5$$

Now the structure function for the bridge structure is derived.

Example 2
Assume the 3-out-of-4 system as illustrated in Figure 64. The minimal path sets for this redundant system are {1,2,3}, {1,2,4}, {1,3,4} and {2,3,4}.

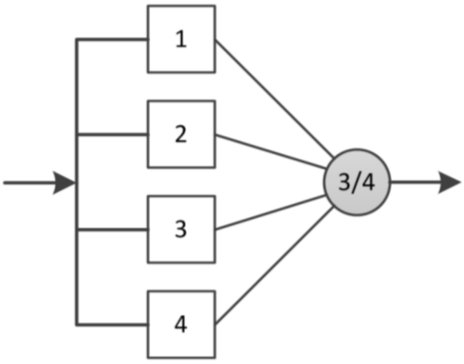

Figure 64: RBD for a 3-out-of-4 configuration

The 3-out-of-4 system can also be displayed as a parallel structure of its minimal path sets, see Figure 65.

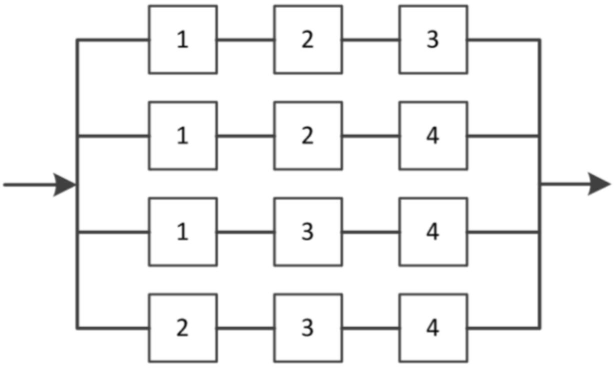

Figure 65: Alternative representation of the RBD for the 3-out-of-4 configuration

Applying Equation 1 results in the structure function for this configuration:

$$\phi(x) = 1 - (1 - x_1x_2x_3)(1 - x_1x_2x_4)(1 - x_1x_3x_4)(1 - x_2x_3x_4)$$

Solving this the same way as in Example 1 results in:

$$\phi(x) = x_1x_2x_3 + x_1x_2x_4 + x_1x_3x_4 + x_2x_3x_4 - 3x_1x_2x_3x_4$$

which is the structure function for a 3-out-of-4 system.

Appendix 2 SMP model

Appendix 2.1 Two-parameter Weibull distribution

For the description of the holding times in multi-state models, in many studies the two-parameter Weibull distribution is used due to its flexibility (Dolas, Jaybhaye, & Deshmukh, 2014; Fung & Jardine, 1982; Kim et al., 2015; Kleiner, 2001; Seki & Yokoyama, 1996; Thomas & Sobanjo, 2016). The Weibull distribution has the ability to model a wide range of shapes by varying just two parameters, the shape parameter and the scale parameter (Black et al., 2005; Cohen, 1965). The Cumulative Density Function (CDF) and Probability Density Function (PDF) of the two-parameter Weibull distribution are given by (Birolini, 2017, p. 458):

CDF:

$$F(x) = 1 - e^{-\left(\frac{x}{\beta}\right)^{\alpha}} \qquad ; \quad x > 0$$

PDF:

$$f(x) = \frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1} e^{-\left(\frac{x}{\beta}\right)^{\alpha}} \qquad ; \quad x > 0$$

where:

x	=	the variable that is described (the holding time)
α	=	the shape parameter
β	=	the scale parameter

In this research the shape parameter is named *alpha* and the scale parameter is named *beta* ($\alpha > 0$; $\beta > 0$). Although, in literature there is no consistency in the naming of these parameters, so it is important to keep in mind which one is the shape parameter and which one is the scale parameter in order to avoid confusion and calculation mistakes.

The shape parameter alpha is dimensionless and the scale parameter beta has the same units as the random variable, which is the holding in this case (Phan & McCool, 2009).
When the shape parameter alpha = 1, this yields the exponential distribution (so a constant hazard rate). For alpha > 1, the hazard rate is strictly increasing and for alpha < 1, the hazard rate is strictly decreasing (Birolini, 2017; Dolas et al., 2014; Yang, Xie, & Wong, 2007).

Appendix 2.1.1 Parameter estimation: Weibull distribution

The Maximum Likelihood Estimation (MLE) is one of the most applied analytical methods for estimating the parameters of the Weibull distribution (Barbu et al., 2017; Dolas et al., 2014; Seki & Yokoyama, 1996). MLE is relatively insensitive to scatter in the data and only a short computer program is required to perform the estimations. The MLE method is based on the maximisation of the likelihood function, under the condition that both parameters are larger than zero (Qiao & Tsokos, 1994; Ross, 1996). Since the log-likelihood function is better to use for mathematical calculations than the likelihood function itself, the natural logarithm of the likelihood is derived and used to solve the problem. The likelihood and log-likelihood functions are defined as followed:

- Likelihood function:

  $$L(\alpha, \beta) = \prod_{i=1}^{n} f(x_i)$$

  with $f(x_i)$ being the PDF of the two-parameter Weibull distribution and *n* the total number of data samples.

- Log-likelihood function:

$$LL\,(\alpha,\beta) = \sum_{i=1}^{n} ln\big(f(x_i)\big)$$

$$= \sum_{i=1}^{n} [ln(\alpha) - ln(\beta) + (\alpha-1)\,ln(x_i) - (\alpha-1)\,ln(\beta) - \left(\frac{x_i}{\beta}\right)^{\alpha}]$$

$$= n[ln(\alpha) - \alpha\,ln(\beta)] + (\alpha-1)\sum_{i=1}^{n} ln(x_i) - \sum_{i=1}^{n}\left(\frac{x_i}{\beta}\right)^{\alpha}$$

The parameter values that maximise the log-likelihood function (LL) also maximise the likelihood function (L) (McCool, 2012). Therefore, the parameter values for which the maximum value of the log-likelihood function is obtained are considered as the maximum likelihood estimators for alpha and beta (Ross, 1996). In order to solve this optimisation problem, one must obtain the values of alpha and beta for which the two partial derivatives of the log-likelihood function are equal to zero (McCool, 2012; Qiao & Tsokos, 1994). This means the following system of equations needs no be solved:

1) $\frac{\partial \mathrm{LL}}{\partial \alpha} = \frac{n}{\alpha} - n\ln(\beta) + \sum_{i=1}^{n}\ln(x_i) - \sum_{i=1}^{n}\ln\left(\frac{x_i}{\beta}\right)\left(\frac{x_i}{\beta}\right)^{\alpha} = 0$
2) $\frac{\partial \mathrm{LL}}{\partial \beta} = -\frac{n\alpha}{\beta} + \alpha\sum_{i=1}^{n} x_i^{\alpha}\,\frac{1}{\beta^{\alpha+1}} \;= 0$

From the second equation it follows:

$$-\frac{n\alpha}{\beta} + \frac{\alpha}{\beta}\sum_{i=1}^{n} x_i^{\alpha}\,\frac{1}{\beta^{\alpha}} \;= 0$$

$$-1 + \frac{1}{n}\sum_{i=1}^{n} x_i^{\alpha}\,\frac{1}{\beta^{\alpha}} \;= 0$$

$$\frac{1}{n}\sum_{i=1}^{n} x_i^{\alpha} \;= \beta^{\alpha}$$

So the expression for the maximum likelihood estimator for the scale parameter beta is:

$$\hat{\beta} = \left(\frac{1}{n}\sum_{i=1}^{n} x_i^{\hat{\alpha}}\right)^{\frac{1}{\hat{\alpha}}}$$

Filling in this expression for beta in the first equation gives the maximum likelihood estimator for the shape parameter alpha: (Qiao & Tsokos, 1994; Ross, 1996; Srinivasan & Wharton, 1975)

$$\hat{\alpha} = \left[\frac{\sum_{i=1}^{n} x_i^{\hat{\alpha}}\ln(x_i)}{\sum_{i=1}^{n} x_i^{\hat{\alpha}}} - \frac{1}{n}\sum_{i=1}^{n}\ln(x_i)\right]^{-1}$$

This equation cannot be solved analytically: in order to find the estimator for the shape parameter alpha, an iterative numerical approach is required (McCool, 2012; Phan & McCool, 2009). A genetic algorithm can be used, for example the Newton-Raphson method (Heo, Salas, & Kim, 2001; Qiao & Tsokos, 1994; Ross, 1996). Software tools like MS Excel, Matlab or R can be used for this.

Bias and censoring

For large data sets the values for the maximum likelihood estimators converge to the true values and become normally distributed. However, for small data sets the shape parameter is known to be biased (McCool, 2012; Phan & McCool, 2009). The bias of the scale parameter is often negligible (Ross, 1996). The bias of the shape parameter depends on the sample size and the degree of censoring (Yang et al., 2007). If there are no censored observations, the data set is called uncensored or complete (Ross, 1996).

In the field of safety and reliability engineering, right censored data samples are a common occurrence. A sample is said to be right censored if it is known that the value is greater than a certain lower boundary value, but the exact value is unknown (McCool, 2012). For example, if ten years of condition data for an asset is available, one data sample equals one single holding time. At the end of those ten years the asset is in a certain state *i*, but because the data ends it is not known when the asset will enter the next condition state *j*. Therefore, this last data sample is only known to be greater than a certain value, but the exact value is not known.

To overcome the bias for small data sets, bias correction methods can be used to obtain a more reliable parameter estimation. Two research papers that provide formulas for the unbiased estimation of the shape parameter are the works by Ross (1996) and Hirose (1999).

According to Ross (1996) the unbiased estimation of the shape parameter is given by:

$$\hat{\alpha}_U = \frac{\hat{\alpha}}{1 + \frac{1.37}{r - 1.92}\sqrt{\frac{n}{r}}}$$

where:

n	=	the number of data samples (observations)
r	=	the number of failures

This formula for the unbiased shape parameter can be used for datasets that include right censored data, since both *n* and *r* are included here. In case there are no censored observations in the data set, the number of failures is equal to the number of observations ($n = r$).

The formula provided by Hirose (1999) can only be used for a situation where there is no right censored data:

$$\hat{\alpha}_U = \frac{\hat{\alpha}}{1.0115 + \frac{1.278}{r} + \frac{2.001}{r^2} + \frac{20.35}{r^3} - \frac{46.98}{r^4}}$$

In this case *r* is equal to *n.*

Appendix 2.2 Reliability modelling with SMP model: example

As mentioned before, with the methodology developed by Kleiner (2001) the reliability function of a component can be determined. The first step is to estimate the distributions (the PDFs) of the holding times for each condition state. Due to a lack of historical data, the same conditon states and holding time distributions as used by Kleiner (2001) are assumed in this example. This means four upstates are used, which are described by two-parameter Weibull distributions. In Table 13 the Weibull parameters from the study by Kleiner are listed, where alpha is the shape parameter and beta is the scale parameter. Figure 66 shows the corresponding Weibull distributions.

Table 13: Weibull parameters for the holding times used by Kleiner (2001)

State	Alpha	Beta
1	2.350	17.544
2	3.568	27.778
3	1.732	12.346
4	2.961	11.364

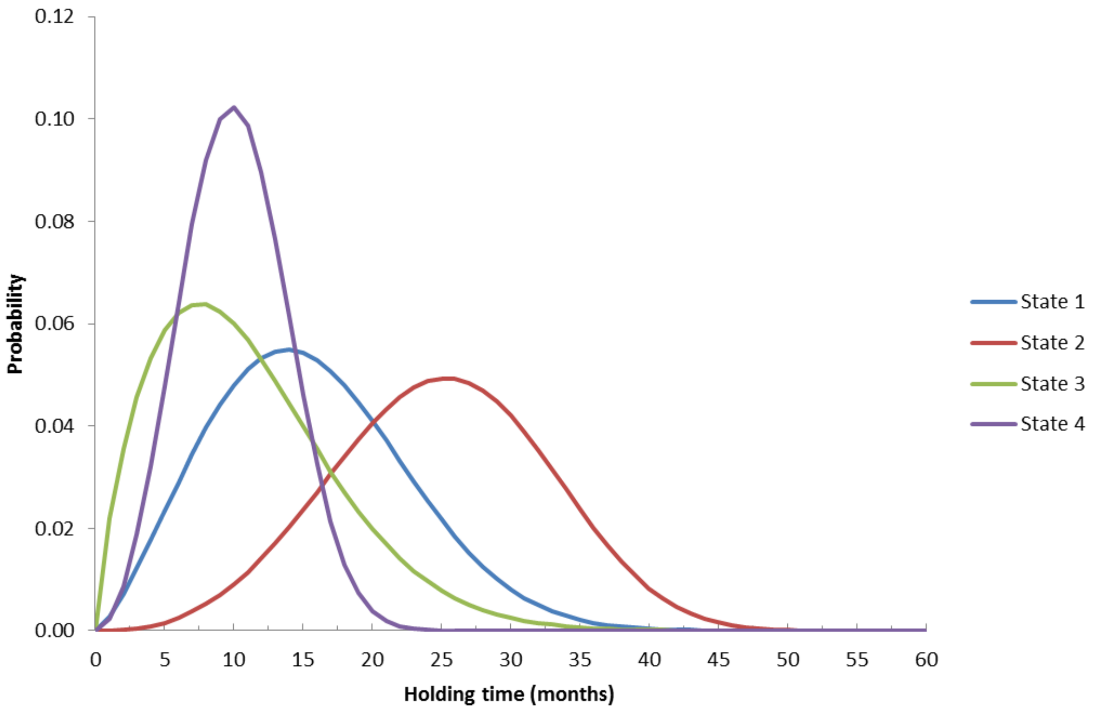

Figure 66: Weibull distributions for the holding times of the four upstates

For reliability modelling no maintenance actions are included, so only the time until the first failure is considered. When the component leaves state 4, it is assumed to be failed. Via Monte Carlo simulation, based on the ITS method, the PDFs of the cumulative holding times can be constructed, see Figure 67.The PDF of the cumulative holding times for states *1+2+3+4* is equal to the distribution of the time to the first failure.

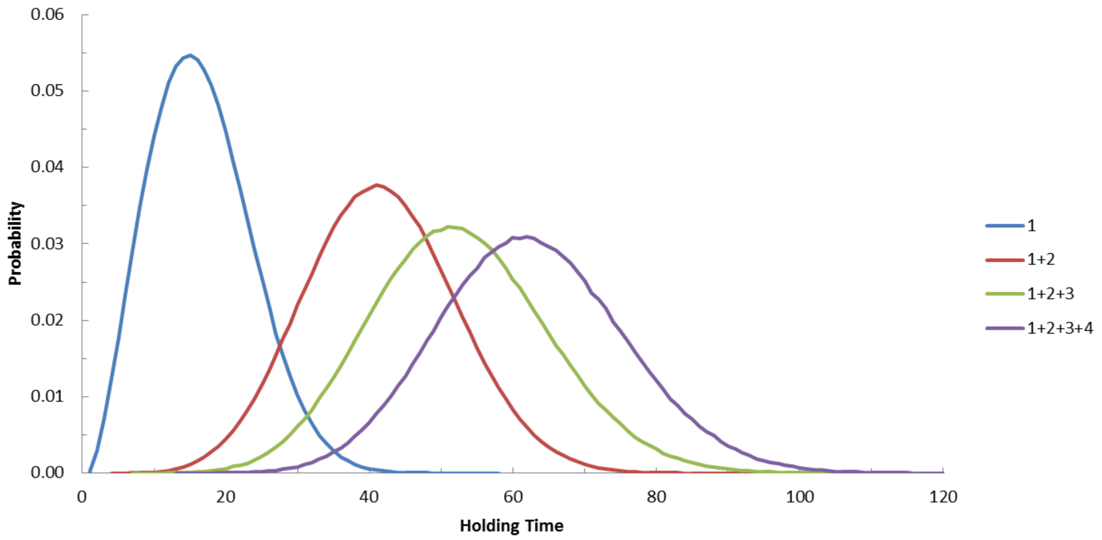

Figure 67: Distributions of the cumulative holding times

By making the transition from PDFs to CDFs for the cumulative holding times, the survival curves for the cumulative holding times are obtained (Figure 68). If a CDF is defined as *F(t)*, the reliability function *R(t)* equals:

$$R(t) = 1 - F(t)$$

In fact, the survival curve for states *1+2+3+4* is equal to the reliability function *R(t)* for the component, with respect to the first failure.

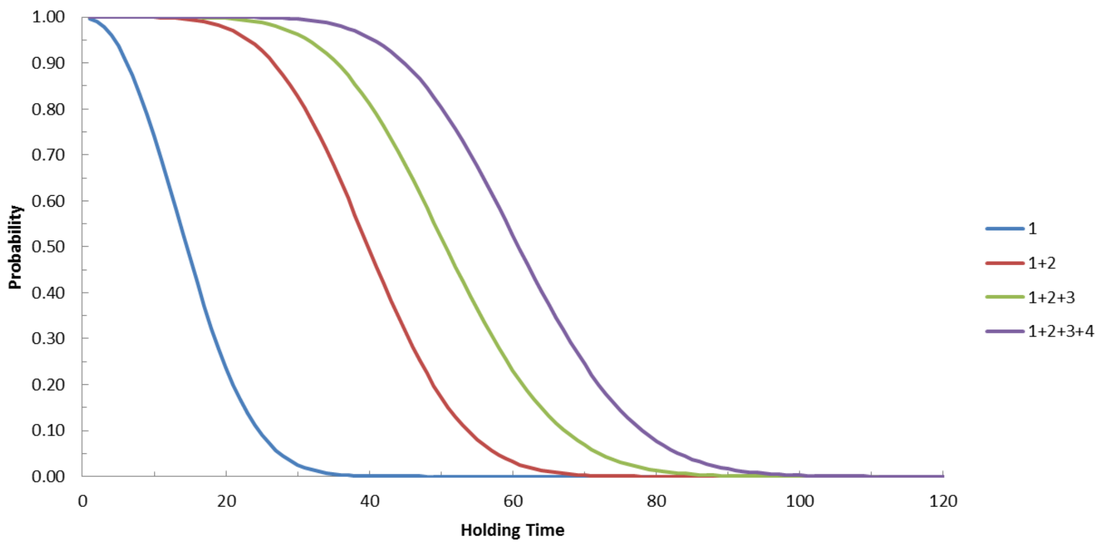

Figure 68: Survival curves for the cumulative holding times

Appendix 2.3 Availability modelling with SMP model: examples

Appendix 2.3.1 Example 1: continuous condition monitoring

In this example the outcomes of the SMP model for two different maintenance strategies are compared. The first strategy is about condition-based maintenance, while the second one is about corrective maintenance.

For both situations the following starting points and boundary conditions are presumed:

- Four discrete upstates are assumed for the component, where state 1 = *same as new* and state 4 = *last deterioration state before failure*;
- For the upstates, the Weibull parameters alpha and beta are listed in Table 14;
- The PDFs of the holding times for the upstates are given in Figure 69;
- The holding times of the upstates are multiplied by 7 to convert the units from weeks to days;
- The condition can be monitored continuously;
- There is no uncertainty in monitoring results;
- At t = 0 the component is in condition state 1;
- When the component is in an upstate, it is considered to be available (A = 1); when the component is in a downstate, it is unavailable (A = 0).

Maintenance strategy 1 is defined as follows:

- When component enters state 4, an imperfect repair is performed immediately that brings the component back to state 2;
- Thus, the holding time of state 4 is always one time unit (= 1 day), because the process moves to a downstate immediately;
- The downstate representing the repair from state 4 to state 2 is defined as state 5;
- For state 5, the parameters mu and sigma for the lognormal distribution are listed in Table 15;
- The PDF of the holding times for state 5 is given in Figure 70.

Maintenance strategy 2 is defined as follows:

- When the component fails, a perfect repair (or renewal) is performed that brings it back to state 1 (same as new);
- The downstate representing this perfect repair is defined as state 6;
- For state 6, the parameters mu and sigma for the lognormal distribution are listed in Table 15;
- The PDF of the holding times for state 6 is given in Figure 70.

The modelling results for both strategies are shown in Figure 71 and Figure 72. It is observed that both availability functions converge to a steady-state, although the function of strategy 1 converges quicker. On average, maintenance strategy 1 leads to a bit higher availability. This example shows how the SMP model can be used to compare the influence of different maintenance strategies onto the availability function.

Table 14: Weibull parameters for the holding time distributions of the uptimes (Kleiner, 2001)

State	Alpha	Beta
1	2.350	17.544
2	3.568	27.778
3	1.732	12.346
4	2.961	11.364

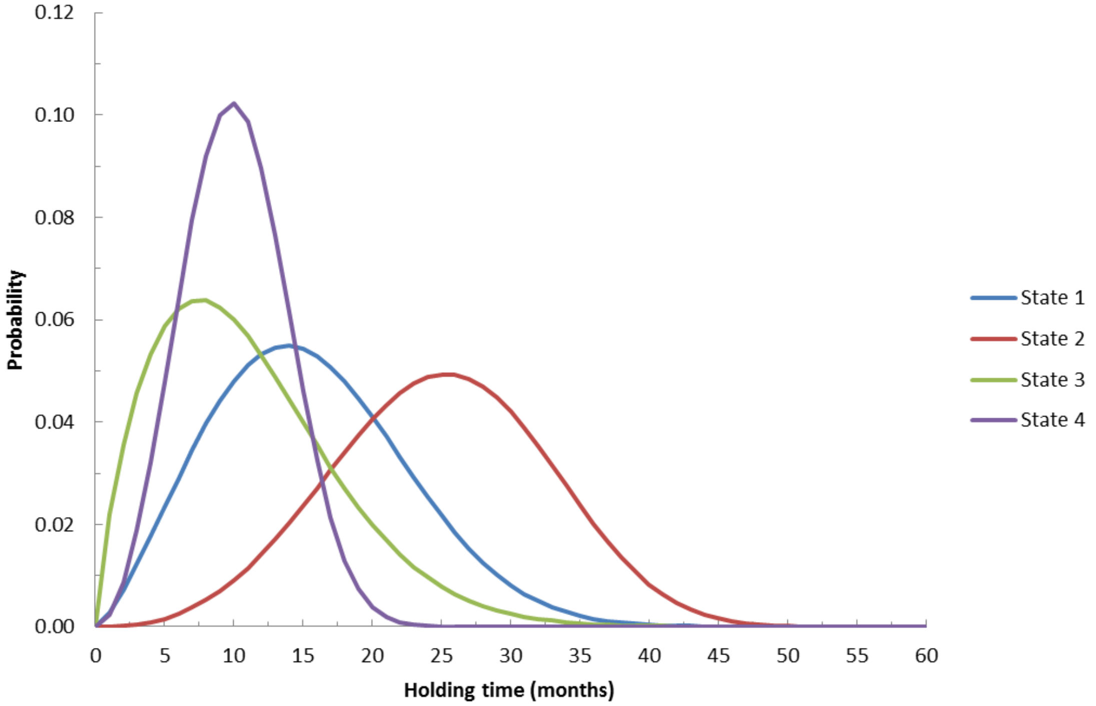

Figure 69: Holding time distributions for the upstates

Table 15: Lognormal parameters for the holding time distributions of the downtimes

State	Mu	Sigma
5	2.000	0.300
6	3.000	0.400

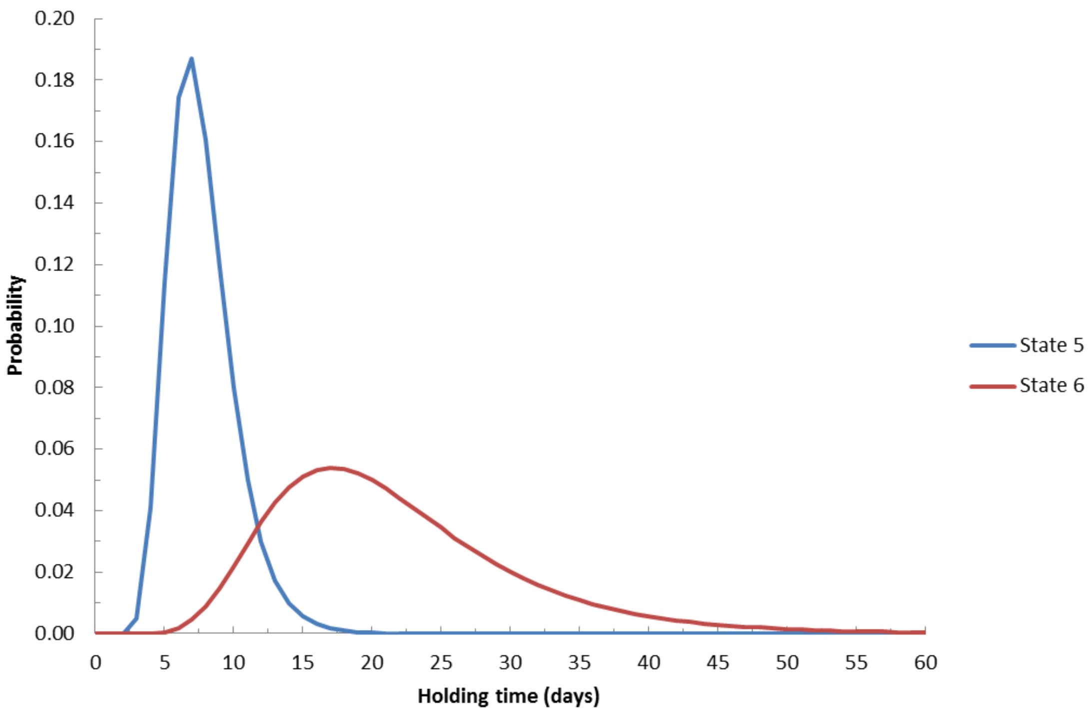

Figure 70: Holding time distributions for the downstates

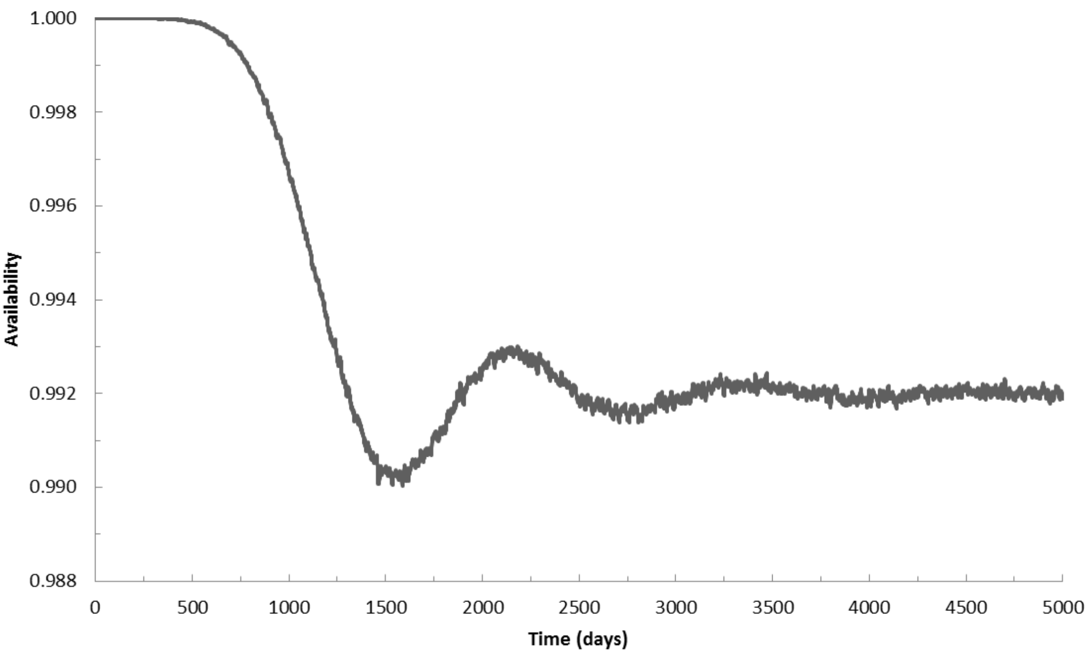

Figure 71: Availability over time for maintenance strategy 1

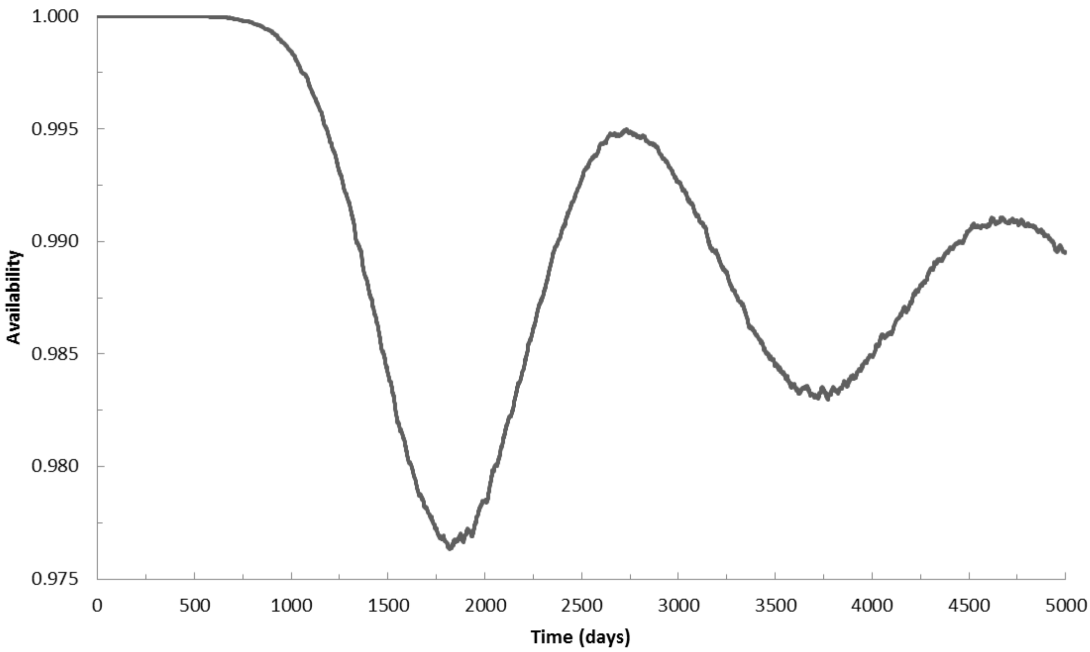

Figure 72: Availability over time for maintenance strategy 2

Appendix 2.3.2 Example 2: Periodic inspections

In Example 1 it is assumed the condition state of the component is monitored continuously. However, this would be the ideal situation and is not always true. Therefore, it is also possible to make use of inspection intervals in the SMP model. This means the condition is only determined periodic, where the average time between two inspections is the so-called *inspection interval*. Although, an inspection is not always performed exactly on the same day or time. For instance, a yearly inspection will not always take place at the first of January. Therefore, it is assumed that an inspection takes place at a random day somewhere in the inspection interval. Based on the outcome of the inspection it is decided which action to perform (e.g. *do nothing*, *repair*, *renewal*). An example is given here to illustrate the effect of using inspection intervals in availability modelling with the SMP model.

In this example the following starting points and boundary conditions are used:

- Four discrete upstates are assumed for the component, where state 1 = *same as new* and state 4 = *last deterioration state before failure*;
- The inspection interval is 200 days, so an inspection is performed at a random day within a period of 200 days;
- It is assumed that the component is available during inspection;
- There is no uncertainty in inspection results;
- At t = 0 the component is in condition state 1;
- When an inspection takes place and the component is observed to be in state 4, an imperfect repair is performed that brings the component back to state 2;
- This downstate (imperfect repair from state 4 to state 2) is defined as state 5;
- When the component fails before it is observed to be in state 4, a perfect repair (or renewal) is necessary, bringing the component back to state 1;
- This downstate (perfect repair) is defined as state 6;
- When the component is in state 1, 2, 3 or 4 (the upstates), it is considered to be available (A = 1); when the component is in state 5 or 6 (the downstates), it is unavailable (A = 0);
- The PDFs of the holding times for the upstates are given in Figure 69 (similar to Example 1);
- For the upstates, the Weibull parameters alpha and beta are listed in Table 14 (similar to Example 1);
- The holding times of the upstates are multiplied by 30 to convert the units from months to days;
- The PDFs of the holding times for the downstates are given in Figure 70 (similar to Example 1);
- For the downstates, the parameters mu and sigma for the lognormal distribution are given in Table 15 (similar to Example 1).

Figure 73 provides a visualisation of the maintenance strategy and the transitions between the different condition states. Figure 74 shows the availability function.

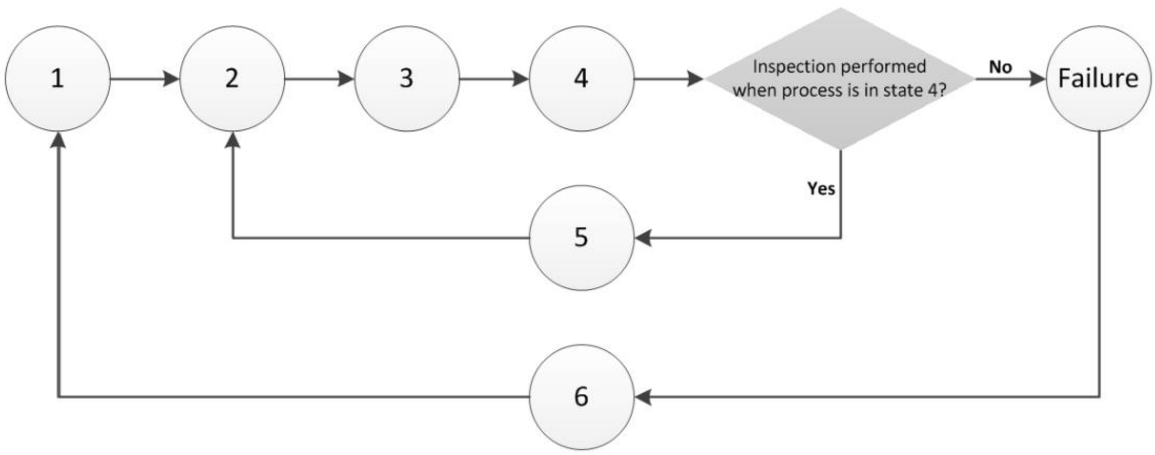

Figure 73: Visualisation of the maintenance strategy

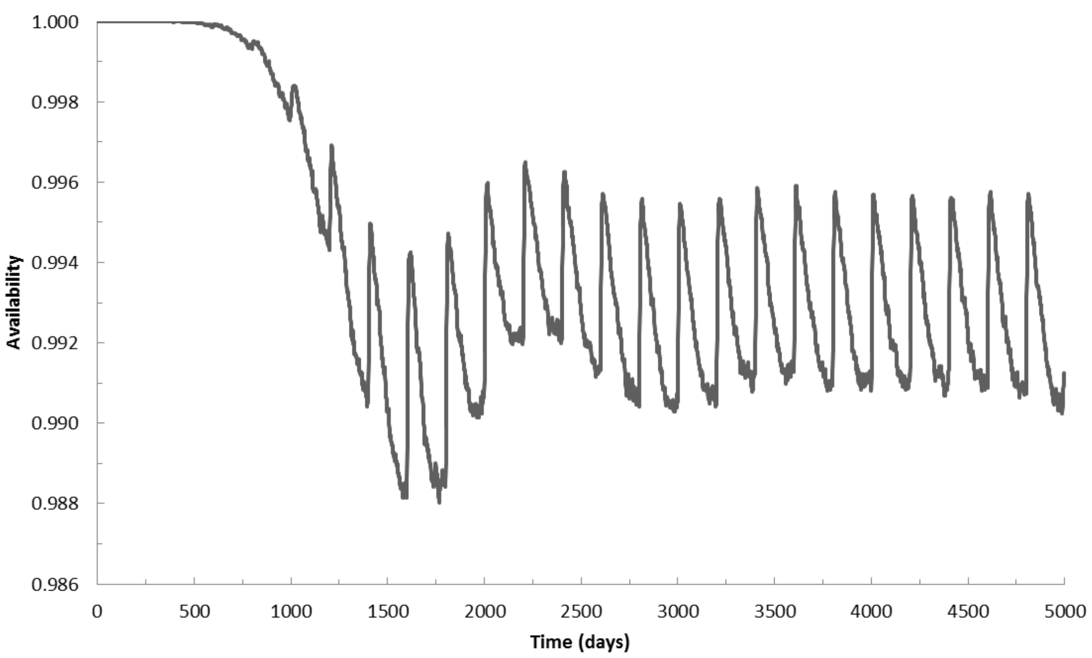

Figure 74: Availability over time for periodic inspections

Appendix 2.3.3 Discussion on availability modelling with SMP model

It the two examples it is assumed that the condition monitoring or the result of an inspection is fully correct at all times. However, the outcome of an inspection contains uncertainty, mainly due to human errors. When data is available with respect to the variation in inspection observations, the uncertainty could be included in the modelling.

Furthermore, the effect of maintenance actions could be uncertain as well. In Example 1, it is assumed a repair action brings the component back to condition state 2, at all times. However, the result of a maintenance action is likely to contain a certain variety as well. When the different transition probabilities for the semi-Markov process can be derived from available data, this variability in maintenance results can be included in the availability modelling as well.

With the SMP model the modeller can include many factors or processes that influence the component availability, aiming at a more realistic model. However, a more realistic model also comes with a price: more complexity means more required input data. The asset manager should find a balance between the model complexity on one hand, and the amount of required data on the other hand.

Appendix 3 NHPP model

Appendix 3.1 The Power Law Process

The Cumulative Distribution Function (CDF) for the kth + 1 failure of the Power Law Process (PLP) model is derived from the intensity function. The intensity function is given by[19]:

$$\lambda(t) = \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1}$$

where:

β	=	the shape parameter
α	=	the scale parameter

And the CDF for the kth + 1 failure is:

$$F_{t_{k+1}}(x) = 1 - \exp\left(-\theta^{-\beta}\{(qt_k + x_k)^{\beta} - (qt_k)^{\beta}\}\right)$$

with:

$$t_k = \sum_{i=1}^{k-1} x_i$$

where:

q	=	the repair adjustment factor
t_k	=	the time of the k^{th} failure
x_i	=	the i^{th} interarrival time

Theoretically the repair adjustment factor q determines the state of the system after repair: $q = 0$ for a "same as new" repair, which translates the PLP model into a renewal process; $q = 1$ for a "same as old" or "minimal repair", which means the reliability is exactly the same as before the repair; and $0 > q > 1$ for an "imperfect repair", which means the system is repaired to a certain virtual age.

For both reliability and availability modelling the assumption that $q = 1$ is used, implying minimal repair. This has two reasons:

1. $q = 0$ is excluded. This would mean that no deterioration, but renewal is modelled.
2. $0 > q > 1$ is excluded, since this would mean that another parameter has to be estimated (q), which leads to an increase in the data needed for this model.[20]

Thus, for $q = 1$, $F_{t_{k+1}}(x)$ becomes:

$$F_{t_{k+1}}(x) = 1 - exp\left(-\theta^{-\beta}\{(t_k + x_k)^{\beta} - (t_k)^{\beta}\}\right)$$

[19] Kim and Singh (2010) use slightly different notations, see for comparison Appendix 6.2 at page 120.

[20] However, if a sufficient amount of data with a good enough quality is available, 0>q>1 might be incorporated in the model as well.

Appendix 3.1.1 Parameter estimation: Power Law Process

The PLP parameters β and θ can be estimated with the Maximum Likelihood Estimation (MLE) method.

In case of failure truncated data these estimators are:

$$\hat{\beta} = \frac{n}{\sum_{i=1}^{n-1} ln\left(\frac{t_n}{t_i}\right)}$$

$$\hat{\theta} = \frac{t_n}{n^{1/\hat{\beta}}}$$

In case of time truncated data the estimators are slightly different:

$$\hat{\beta} = \frac{N}{\sum_{i=1}^{N} ln\left(\frac{t}{t_i}\right)}$$

$$\hat{\theta} = \frac{t}{N^{1/\hat{\beta}}}$$

The information necessary for calculating the parameters is for the two cases is given in Table 16 below.

Table 16: Information necessary for parameter estimation of the Power Law Process

Failure truncated	Time truncated	Description
$\boldsymbol{t_i}$	t_i	Time at which the i[th] failure event takes place
$\boldsymbol{t_n}$		Time of the n[th] failure event at which recording of failure events is ceased
	t	Time at which recording of failure events is ceased
$\boldsymbol{n}$		Number of failure events at which recording is ceased
	N	Number of failure events from the start of recording until the cessation of the recording at time t

The confidence intervals for the shape parameter are important in order to assess whether the NHPP reduces to a HPP or not. This happens when $\beta = 1$ is included within the confidence interval. The confidence interval for the shape parameter can be constructed as follows:

$$\frac{\chi^2_{1-\alpha/2}\big(2(n-1)\big)\hat{\beta}}{2n} < \beta < \frac{\chi^2_{\alpha/2}\big(2(n-1)\big)\hat{\beta}}{2n}$$

where:

$$\chi^2_{1-\alpha/2}\big(2(n-1)\big)$$

is the chi-square distribution with $\big(2(n-1)\big)$ degrees of freedom and confidence level $1-\alpha/2$ or $\alpha/2$.
with:

α = confidence level parameter

Appendix 3.2 Trend tests

The trend tests use as input chronologically ordered arrival times $T_1, T_2, \dots, T_i$. Interarrival times can be translated to arrival times by taking their cumulative, i.e.: $T_1 = X_1, T_2 = T_1 + X_2, \dots, T_i = T_{i-1} + X_i$.

There are two types of data collection possible, either *failure truncated* or *time truncated*. The collection of data in *failure truncated data collection* stops at a specified number of failures, where the last failure is $T_i = T_n$. Whereas the collection of data in *time truncated data collection* stops at a specified time T after the start of the data collection, where the last failure is $T_i = T_N$.

Appendix 3.2.1 Laplace Trend Test

The Laplace Trend Test is a test to find out if a trend exists in the data. More specifically, it considers if the data supports an HPP or an NHPP(Wang & Coit, 2005). The hypotheses are:

H_0: the data follow an HPP
H_1: the data follow an NHPP

The hypothesis H_0 is rejected if the test statistic $U_L$is larger or smaller than the right tail or the left tail of the standard normal distribution (μ = 0, σ = 1):

$$U_L < -z_{\alpha/2} \quad or \quad U_L > z_{\alpha/2}$$

where:

α = the desired significance level

with the interarrival times the Laplace Trend Test statistic U_L can be calculated:

$$U_L = \frac{\left[\frac{1}{k}\sum_{i=1}^{k} T_i\right] - \frac{1}{2}T^*}{T^*\sqrt{\frac{1}{12k}}}$$

where:

$k = m - 1$ for failure truncated data;
$k = m$ for time truncated data;
$T^* = T_n$ for failure truncated data;
$T^* = T$ for time truncated data.

with:

m = the number of failures
T = the time at which data collection is stopped

Appendix 3.2.2 Military Handbook Test (MIL-HDBK 189)

The Military Handbook Test is a test to find out if a trend exists in the data. Specifically the MIL-HDBK 189 trend test considers if the data follow no trend versus following the PLP model (whereas the Laplace considers NHPP in general). The hypotheses are:

H_0: the data do not follow a trend (HPP)
H_1: the data follow a trend (PLP)

The hypothesis H_0 is rejected if the test statistic X_j^2 is larger than the right tail or smaller than the left tail of the chi-square distribution at significance level α:

$$X_{2j}^2 < X_{1-\alpha/2}^2(2(j)) \quad or \quad X_{2j}^2 > X_{\alpha/2}^2(2(j))$$

with:

$$X^2_{2(j)} = \frac{2N}{\hat{\beta}}$$

where:

$$\hat{\beta} = \frac{N}{\sum_{i=1}^{N} ln\left(\frac{T}{T_i}\right)}$$

and $j = N$, for time truncated data

$$\hat{\beta} = \frac{n}{\sum_{i=1}^{n-1} ln\left(\frac{T_n}{T_i}\right)}$$

and $j = N - 1$, for failure truncated data

α	=	the desired significance level
T	=	the time at which data collection is stopped

Appendix 3.3 Likelihood Ratio test

This is a test for finding out if the components are equal, based on the comparison of the shape parameter (β). The complete derivation can be found in Rigdon and Basu (2000)

For time truncated data the Likelihood Ratio (LR) can be calculated as:

$$\mathrm{LR} = M\log\beta^* - \sum_{i=1}^{k} m_i \log\hat{\beta}_i$$

where:

$$\beta^* = \frac{M}{\sum_{i=1}^{k} \frac{m_i}{\hat{\beta}_i}}$$

with:

$\hat{\beta}_i$	=	the MLE estimation of the shape parameter for component *i* (see Appendix 3.1.1)
m_i	=	number of failures for component *i*
β^*	=	weighted harmonic mean of all $\hat{\beta}_i$'s
M	=	total number of failures from all components
k	=	number of components

The test statistic for this *likelihood ratio test* is:

$$-\frac{2 \times \mathrm{LR}}{\gamma}$$

where:

$$\gamma = 1 + \frac{1}{6(k-1)}\left(\sum_{i=1}^{k} \frac{1}{m_i} - \frac{1}{M}\right)$$

The test statistic is approximately chi-square distributed with $k - 1$ degrees of freedom. The null hypothesis (All shape parameters of all components are equal) is rejected when the test statistic is larger than the chi-square distribution with $k - 1$ degrees of freedom and a confidence level of α:

$$-\frac{2 \times \mathrm{LR}}{\gamma} > \chi^2_\alpha(k-1)$$

Appendix 3.4 Reliability modelling with NHPP model

Calculating reliability from the interarrival times

The algorithm for determining the reliability to the k[th] failure is as follows (see also Figure 75):

1. Take n = the number of Monte Carlo simulations

   p = number of failures of which one wants to know the reliability

   $N = 1$
2. Sample x_k from equation x_k at page 54 for $k = 1 \rightarrow p$
3. Take the sum t_N of all x_i for simulation N:

$$t_N = \sum_{i=1}^{p} x_i$$

4. If $N \leq n$, then $R(N) = t_N$ and $N = N + 1$. Then go to step 2
5. End simulation and give the distribution of R

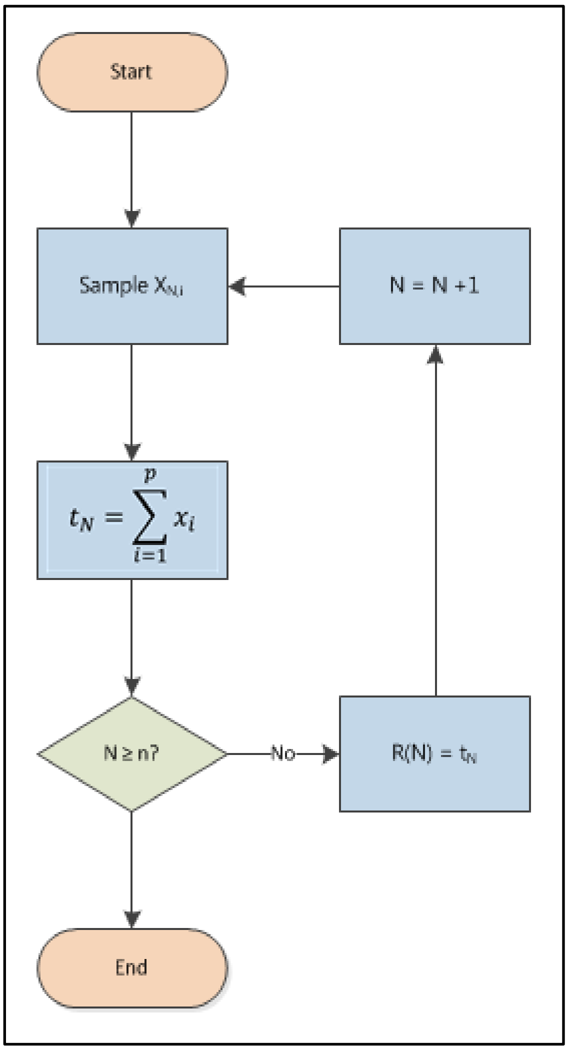

Figure 75: Algorithm for reliability modelling with the NHPP model

Appendix 3.4.1 Reliability modelling with NHPP model: example

To determine the reliability over time for an asset the following was set up as an example:

- β = 3,83
- θ = 2506,81 (λ = 9,58E-14)
- k = 1 (this refers to the reliability to the first failure)
- Z = random number from a uniform distribution $U(0,1]$

With these parameters the interarrival times of failures can be sampled according to the formula above:

$$x_k = \left(\left(\sum_{i=1}^{k-1} x_i\right)^{\beta} - \frac{\ln(Z)}{\theta^{-\beta}}\right)^{\frac{1}{\beta}} - \sum_{i=1}^{k-1} x_i$$

Table 17 table below gives two simulations of twenty sampled interarrival times (x_k) and the cumulative time of the component in operation (t_i):

Table 17: Two simulations of 20 sampled interarrival times

Interarrival time number k	Simulation 1 x_k	t_i	Simulation 2 x_k	t_i
1	2372	2372	2125	2125
2	448	2820	467	2592
3	54	2874	172	2764
4	111	2986	302	3065
5	27	3012	233	3299
6	559	3572	92	3391
7	38	3610	3	3394
8	433	4043	24	3418
9	397	4440	580	3999
10	13	4453	172	4170
11	43	4496	249	4419
12	77	4573	35	4454
13	160	4733	10	4464
14	132	4865	205	4668
15	258	5122	16	4684
16	129	5251	168	4853
17	20	5271	88	4941
18	158	5429	105	5045
19	107	5536	27	5073
20	75	5611	24	5096

A sample path of the first simulation is given in Figure 76, where $N(t)$ denotes the cumulative number of failures. As time passes by, one can clearly see the failures succeeding each other more rapidly.

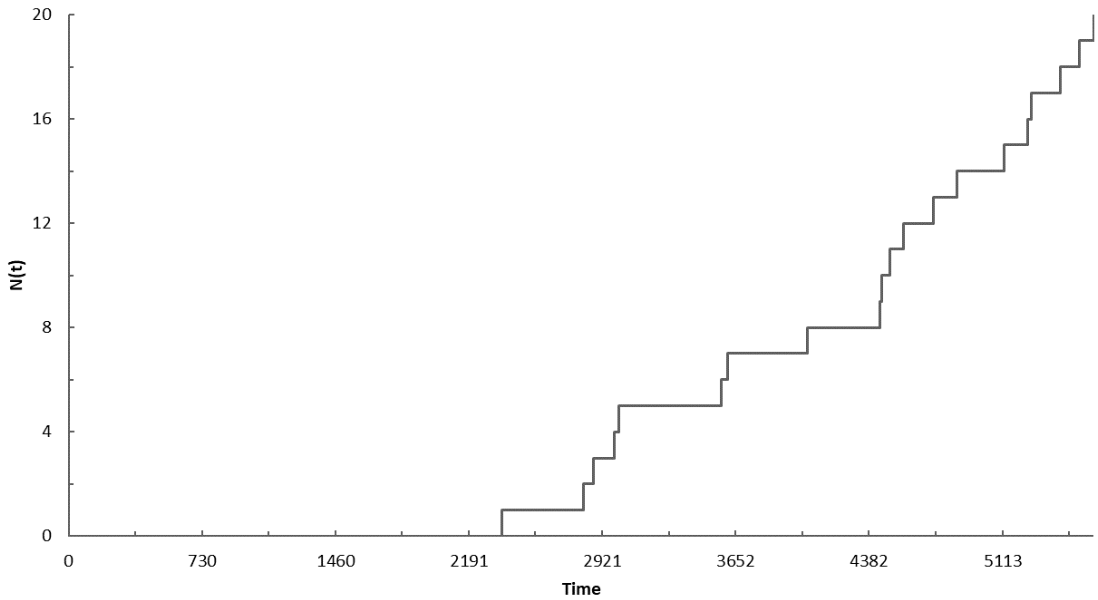

Figure 76: Sample path for simulation 1 of the reliability example

But, how to get from a single sample path to the reliability of an asset? First, 1,000,000 of these sample paths are simulated. Second, the **first failure** of every sample path is taken and these are grouped in a histogram. This creates a Probability Distribution Function (PDF) of the interarrival time to the **first failure**, see Figure 77. Third, this PDF is then modified into a Cumulative Distribution Function (CDF) and survival curve (inverted CDF), see Figure 78. Reliability is here defined as *"the probability of no failure"* and is thus equal to one minus the probability of the **first failure**, which is equal to the survival curve, see Figure 79.

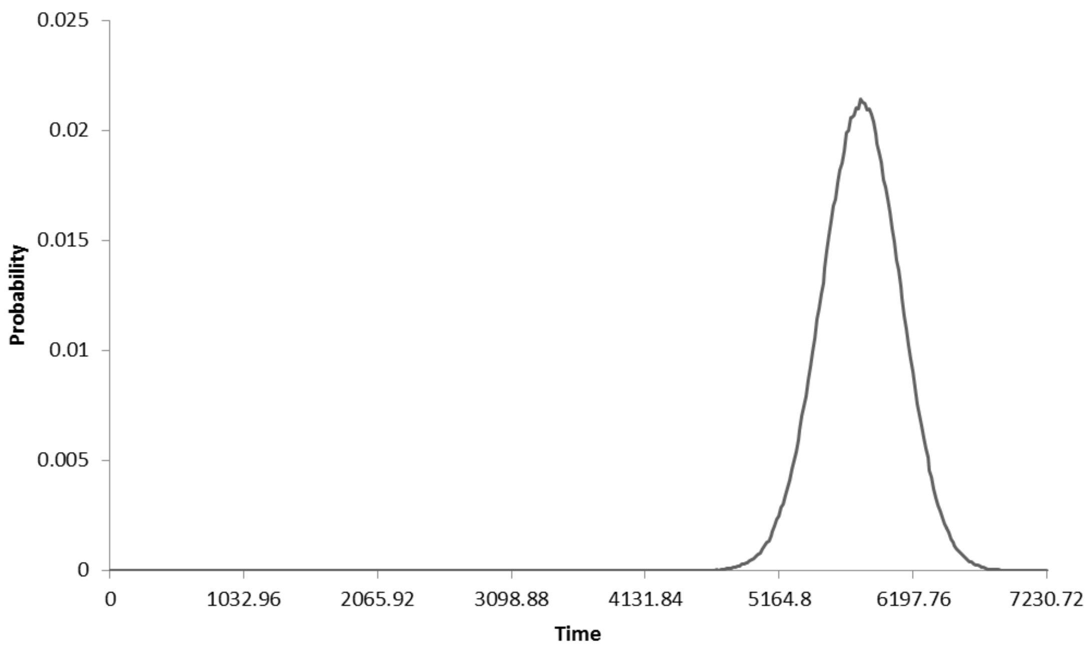

Figure 77: Probability Density Function of the interarrival time to the first failure (β = 3.83, λ = 9.58E-14)

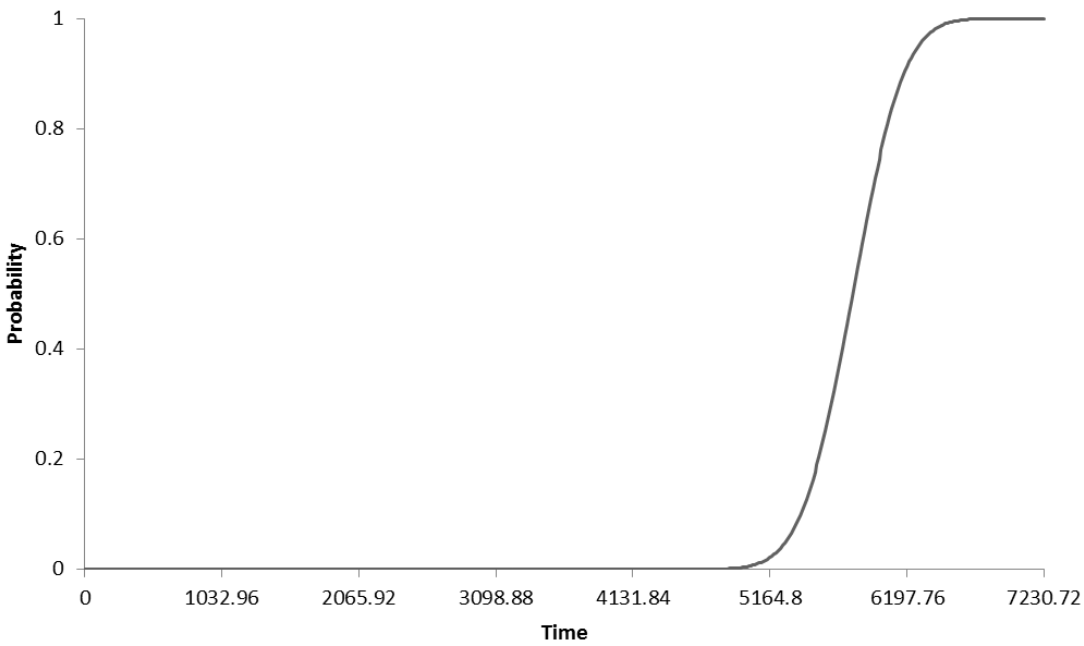

Figure 78: Cumulative Distribution Function of the interarrival time to the first failure (β = 3.83, λ = 9.58E-14)

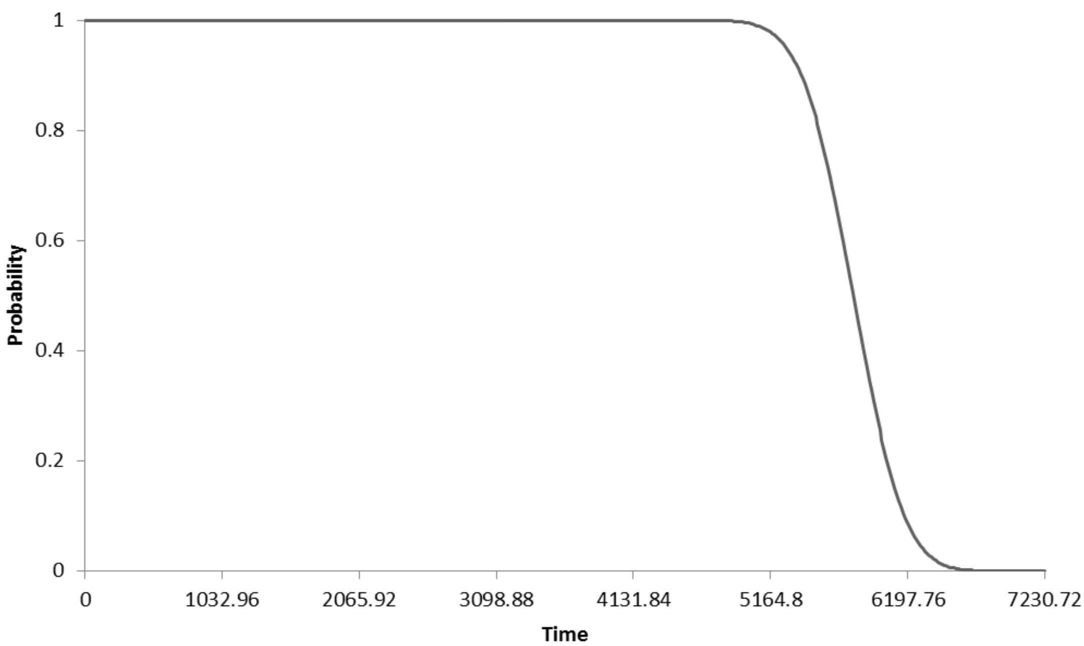

Figure 79: Reliability to the first failure (β = 3.83, λ = 9.58E-14)

Appendix 3.5 Availability modelling with NHPP model: example

To determine the availability over time for an asset the following was set up as an example[21]:

- Power Law Process parameters:
  - β = 3.83
  - θ = 2506.81
- Parameters for the lognormal distribution of the downtime:
  - μ = 2.73
  - σ = 0.73
- Z = random number from a uniform distribution $U(0,1]$
- $T = 2135.9$
- $n = 1$

With these parameters the up- and downtimes can be sampled according to the algorithm described in sub-section 4.3.5.

Table 18 gives a Monte Carlo simulation of one iteration with twenty sampled cycles with uptimes (x_k) and downtimes (y_k). All sampled times are in days. To illustrate the alternation of the process, the start and end times of the up- and downtimes are also given in Table 18. Furthermore, the availability of this simulation is illustrated in Figure 80. The starting date of the simulation process is March 28, 2018 0:00 hour. The first failure takes place exactly 410 days after the start at 0:00 hour, May 12, 2019. After being under repair for 18.6 days, the component is available again at 14:20 hour, May 30, 2019. As expected with deterioration, the uptimes of the component become shorter as time passes.

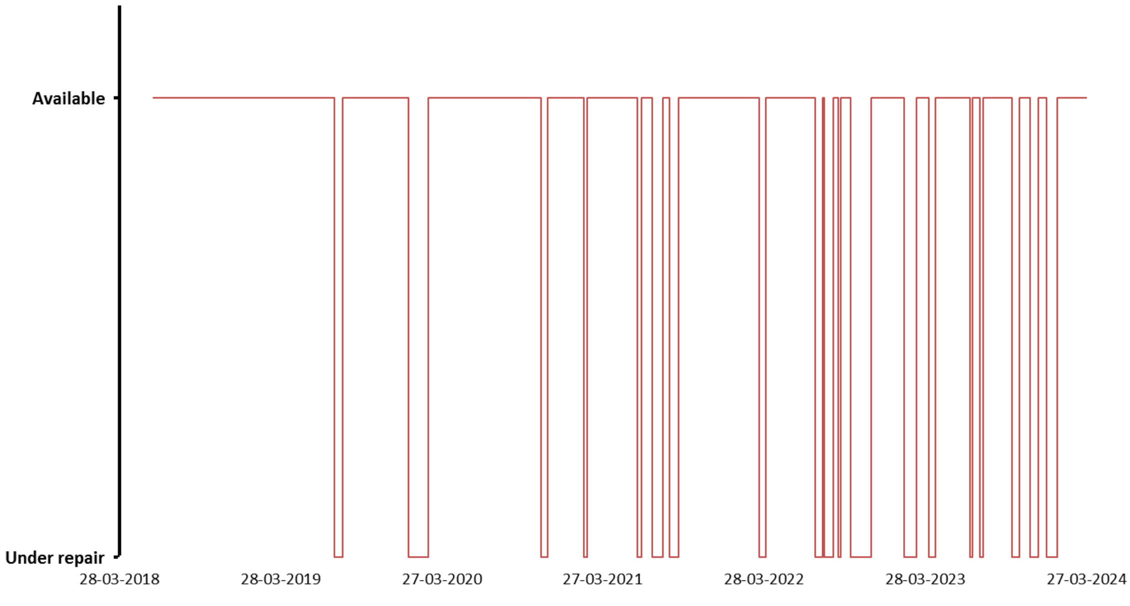

Figure 80: Availability over time for a single component in one simulation

[21] These parameters have been chosen in such a way that it gives a clear impression of the availability in Figure 80.

Table 18: A simulation of 20 cycles for the availability of a single component

Monte Carlo Simulation with one iteration (all times in days)						
Cycle	Sample times		Uptime		Downtime	
k	x_k	y_k	Start time	End time	Start time	End time
1	410.0	18.6	0	410.0	410.0	428.6
2	150.0	44.7	428.6	578.6	578.6	623.3
3	256.7	14.7	623.3	880.0	880.0	894.8
4	82.2	6.4	894.8	977.0	977.0	983.4
5	115.0	7.8	983.4	1098.4	1098.4	1106.2
6	24.9	25.1	1106.2	1131.1	1131.1	1156.2
7	14.3	20.1	1156.2	1170.5	1170.5	1190.6
8	183.0	15.6	1190.6	1373.6	1373.6	1389.2
9	111.5	17.2	1389.2	1500.7	1500.7	1517.8
10	2.8	21.5	1517.8	1520.7	1520.7	1542.1
11	11.1	4.7	1542.1	1553.3	1553.3	1557.9
12	22.4	48.0	1557.9	1580.3	1580.3	1628.3
13	72.9	28.3	1628.3	1701.2	1701.2	1729.5
14	27.8	15.6	1729.5	1757.3	1757.3	1772.9
15	79.2	4.7	1772.9	1852.1	1852.1	1856.7
16	17.8	6.3	1856.7	1874.5	1874.5	1880.8
17	66.3	17.0	1880.8	1947.1	1947.1	1964.0
18	23.9	17.5	1964.0	1987.9	1987.9	2005.4
19	20.0	23.0	2005.4	2025.4	2025.4	2048.4
20	69.2	18.2	2048.4	2117.6	2117.6	2135.9

Appendix 4 Lognormal distribution

The two-parameter lognormal distribution is described by its PDF:

$$f(x) = \frac{1}{(\sqrt{2\pi\sigma^2})x} \exp\left(-\frac{(ln(x)-\mu)^2}{2\sigma^2}\right) \qquad with\; x > 0\,;\; -\infty < \mu < \infty\,;\; \sigma > 0$$

with:

σ	=	shape parameter (standard deviation of log(*x*))
μ	=	scale parameter (mean of log(*x*))

Appendix 4.1 Parameter estimation: lognormal distribution

The derivation of the maximum likelihood estimators for μ and σ^2 is given below. The full derivation can be found in the work by Ginos (2009).

The likelihood function *L* of the lognormal distribution is defined as:

$$L(\mu,\sigma^2) = \prod_{i=1}^{n} f(x_i)$$

with $f(x_i)$ being the PDF of the lognormal distribution and *n* the total number of data samples. This can be elaborated:

$$L(\mu,\sigma^2) = \prod_{i=1}^{n}\left(\frac{1}{(\sqrt{2\pi\sigma^2})x_i}\exp\left(-\frac{(ln(x_i)-\mu)^2}{2\sigma^2}\right)\right)$$

$$= (2\pi\sigma^2)^{-n/2}\prod_{i=1}^{n}\left(\frac{1}{x_i}\exp\left(\sum_{i=1}^{n}-\frac{(ln(x_i)-\mu)^2}{2\sigma^2}\right)\right)$$

The log-likelihood function (*LL*) is then derived by taking the logarithm of the likelihood function *L*, resulting in:

$$LL(\mu,\sigma^2) = -\frac{n}{2}\ln(2\pi\sigma^2) - \sum_{i=1}^{n}\ln(x_i) - \frac{\sum_{i=1}^{n}\ln(x_i)^2}{2\sigma^2} + \frac{\sum_{i=1}^{n}\ln(x_i)\,\mu}{\sigma^2} - \frac{n\mu^2}{2\sigma^2}$$

The estimators for μ and σ^2 are the values that maximise the log-likelihood function. To find these, one must take the partial derivatives of *LL* with respect to μ and σ^2 and set them equal to 0. In the end, this results in the maximum likelihood estimators for the lognormal distribution:

$$\hat{\mu} = \frac{\sum_{i=1}^{n}\ln(x_i)}{n}$$

$$\hat{\sigma}^2 = \frac{\sum_{i=1}^{n}\left(\ln(x_i) - \frac{\sum_{i=1}^{n}\ln(x_i)}{n}\right)^2}{n}$$

Appendix 4.2 Inverse Transform Sampling: lognormal distribution

The CDF of the lognormal distribution is given by:

$$F_x(X) = \Phi\left(\frac{ln(x) - \mu}{\sigma}\right)$$

with:

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{t^2}{2}} dt$$

being the CDF of the normal distribution

The repair time sampling follows the Inverse Transform Sampling (ITS) method introduced in section 0:
The ITS method consists of three steps:

1. Let $F(x)$ be any invertible CDF of continuous random variable X
2. Take $F(x) = U$, where U is a continuous uniform distribution on $(0,1]$.
3. Then: $X = F^{-1}(U)$

   where:

X	=	sampled repair time
U	=	random number from uniform distribution $U(0,1]$
$F^{-1}(x)$	=	Inverse CDF

For the lognormal distribution this procedure results in the following:

1.
$$F_x(X) = \Phi\left(\frac{ln(x) - \mu}{\sigma}\right)$$

   with:

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{t^2}{2}} dt$$

2. Take $F(x) = U(0,1]$.

3. The inverse CDF of the lognormal distribution is given by:

$$F_x^{-1}(x) = exp\left(\mu + \sigma\left(\Phi^{-1}(x)\right)\right)$$

   with:

$$\Phi^{-1}(y) = \mu + \sigma\sqrt{2}erf^{-1}(2y - 1)$$

   being the inverse CDF of the normal distribution, where:

$$erf^{-1}(z) = \sum_{k=0}^{\infty} \frac{c_k}{2k + 1}\left(\frac{\sqrt{\pi}}{2} z\right)^{2k+1}$$

   is the inverse error function, with:

$$c_k = \sum_{m=0}^{k-1} \frac{c_m c_{k-1-m}}{(m + 1)(2m + 1)}$$

   Thus, a lognormal distributed repair time can be sampled as follows:

$$X = F_x^{-1}(U) = exp\left(\mu + \sigma\left(\Phi^{-1}(U)\right)\right)$$

Appendix 5 Case study: data and calculations

Appendix 5.1 Rapid sand filters: data and calculations

Appendix 5.1.1 Rapid sand filters: data

Table 19: Failure data for rapid sand filters (in days)

Filter number	1	2	3	4	5	6
Failure number						
1	23-03-1984	02-10-1985	14-05-1985	13-12-1984	11-10-1984	18-08-1983
2	10-01-1985	23-06-1986	07-08-1987	23-10-1987	14-05-1985	17-10-1983
3	16-07-1985	02-09-1986	19-08-1987	20-12-1987	11-11-1989	12-10-1984
4	18-07-1992	19-01-1988	05-12-1987	08-05-1991	27-11-1989	18-01-1987
5	04-02-1993	04-11-1992	19-06-1988	27-05-1991	02-07-1991	09-11-1990
6	25-07-1994	14-12-1996	26-11-1989	30-10-1991	22-11-1992	24-02-1996
7	02-07-1998	05-01-1997	29-01-1991	14-11-1991	16-06-1994	23-02-1998
8	14-12-2005	02-07-1998	12-10-1992	08-01-1992	27-09-1995	01-08-1998
9	01-02-2007	02-02-1999	12-10-1993	22-06-1992	19-04-2000	15-12-1998
10	09-10-2007	09-09-1999	23-05-1994	29-06-1994	22-09-2000	08-08-1999
11	29-01-2008	22-02-2002	01-12-1995	02-10-1994	16-10-2001	04-05-2003
12	20-10-2009	26-09-2002	25-11-1996	29-04-1996	01-04-2006	02-12-2003
13	26-03-2010	05-03-2004	15-04-1998	14-07-2000	13-12-2006	03-02-2006
14	13-10-2010	28-01-2005	29-07-1999	08-07-2002	09-07-2010	03-02-2007
15	31-07-2012	20-10-2005	17-09-1999	03-09-2002	27-05-2011	30-04-2013
16		10-03-2006	05-01-2003	02-04-2003	13-06-2012	06-10-2014
17		22-12-2008	29-06-2009	03-11-2006	25-03-2014	14-08-2016
18			03-05-2012	09-11-2006	13-04-2015	
19			23-03-2015	15-07-2008	09-11-2016	
20				27-09-2010		
21				04-10-2010		
22				23-03-2015		

Table 20: Downtime data for rapid sand filters (in hours)

Filter number	1	2	3	4	5	6
Failure number						
1	1	2	3	4	5	6
2	91	41	14	177	242	53
3	61	42	1633	66	89	113
4	174	314	333	61	171	50
5	104	165	98	264	156	68
6	384	241	122	48	286	343
7	147	362	845	58	353	55
8	154	93	606	63	354	98
9	398	263	65	99	389	72
10	29	49	98	54	224	106
11	944	181	224	41	81	50
12	903	128	18	61	26	503
13	24	223	210	368	575	67
14	61	89	91	247	729	170
15	800	71	80	70	119	45
16	459	58	314	818	72	471
17		71	18	201	1184	279

18		191	25	98	41	94
19			115	151	191	
20			92	776	38	
21				59		
22				104		

Appendix 5.1.2 Rapid sand filters: trend tests

Two trend tests are considered, the MIL-HDBK-189 and Laplace Trend Test. The calculation methods can be found in Appendix 3.2. The confidence bounds for both tests, given a confidence level of $\alpha = 0.05$, are given in Table 21. The test statistic that is calculated, and whether it falls inside or outside the confidence bounds, is given in Table 22. If the test statistic falls within the confidence bounds, the null hypothesis (no trend in data) cannot be rejected. In the case of the rapid sand filter, both tests show that for all filters the null hypothesis cannot be rejected (with confidence level = 95%). Thus, it is assumed that there is no trend in the data.

Table 21: Confidence bounds for trend tests on data for rapid sand filters

Filter	Confidence bounds MIL-HDBK-189 (α = 0.05)		Confidence bounds Laplace Trend Test (α = 0.05)	
	Lower	Upper	Lower	Upper
1	16.79	46.98	-1.96	1.96
2	19.81	51.97		
3	22.88	56.90		
4	27.57	64.20		
5	22.88	56.90		
6	19.81	51.97		

Table 22: Test statistics for trend tests on data for rapid sand filters

Filter	Test statistic MIL-HDBK-189	Inside or outside confidence bounds?	Test statistic Laplace Trend Test	Inside or outside confidence bounds?
1	32.17	Inside	0.03	Inside
2	35.78	Inside	-0.97	Inside
3	45.55	Inside	-1.79	Inside
4	44.85	Inside	-0.91	Inside
5	35.09	Inside	0.36	Inside
6	44.07	Inside	-0.46	Inside

Appendix 5.2 Pumps: data

Table 23: Condition data for pump 1

State 1		State 2		State 3		State 4		State 5		State 6	
holding time	end date	holding time	end date	holding time	end date	holding time	end date	holding time	end date	holding time	end date
[weeks]	dd-mm-yyyy	[weeks]	dd-mm-yyyy	[weeks]	dd-mm-yyyy	[days]	dd-mm-yyyy	[days]	dd-mm-yyyy	[days]	dd-mm-yyyy
54	11-01-2001	48	16-12-2001	21	09-05-2002	1	10-05-2002	1.4	12-05-2002	20.7	24-05-2008
54	26-11-2009	24	28-10-2002	24	17-04-2003	1	18-04-2003	1.9	20-04-2003		
		24	05-10-2003	21	29-02-2004	1	01-03-2004	1.5	02-03-2004		
		34	25-10-2004	27	01-05-2005	1	02-05-2005	2.3	05-05-2005		
		37	16-01-2006	33	01-09-2006	1	02-09-2006	2.4	04-09-2006		
		47	28-07-2007	40	02-05-2008	1	03-05-2008	1.7	29-07-2010		
		25	26-11-2009	35	27-07-2010	1	28-07-2010	1.2	21-09-2011		
		38	24-04-2011	21	19-09-2011	1	20-09-2011	1.8	03-11-2012		
		42	14-07-2012	16	31-10-2012	1	01-11-2012	2.9	02-04-2014		
		33	21-06-2013	40	29-03-2014	1	30-03-2014	3.0	20-09-2015		
		39	03-01-2015	37	16-09-2015	1	17-09-2015				
		59	10-11-2016								

Table 24: Condition data for pump 2

State 1		State 2		State 3		State 4		State 5		State 6	
holding time	end date	holding time	end date	holding time	end date	holding time	end date	holding time	end date	holding time	end date
[weeks]	dd-mm-yyyy	[weeks]	dd-mm-yyyy	[weeks]	dd-mm-yyyy	[days]	dd-mm-yyyy	[days]	dd-mm-yyyy	[days]	dd-mm-yyyy
29	22-07-2000	28	06-02-2001	25	27-07-2001	1	28-07-2001	0.7	29-07-2001	23.9	08-09-2007
43	25-02-2009	33	18-03-2002	35	16-11-2002	1	17-11-2002	2.0	19-11-2002	18.2	02-06-2015
		40	23-08-2003	43	18-06-2004	1	19-06-2004	0.9	20-06-2004		
		37	07-03-2005	18	13-07-2005	1	14-07-2005	1.3	16-07-2005		
		31	16-02-2006	27	24-08-2006	1	25-08-2006	2.4	28-08-2006		
		21	18-01-2007	30	14-08-2007	1	15-08-2007	0.9	05-06-2009		
		33	25-02-2009	14	03-06-2009	1	04-06-2009	1.5	10-07-2010		
		27	10-12-2009	30	07-07-2010	1	08-07-2010	0.7	12-09-2011		
		30	07-02-2011	31	11-09-2011	1	12-09-2011	1.3	11-10-2012		
		23	23-02-2012	33	09-10-2012	1	10-10-2012	1.7	15-04-2014		
		56	11-11-2013	22	12-04-2014	1	13-04-2014				
		30	14-11-2014	26	14-05-2015	1	15-05-2015				

Table 25: Condition data for pump 3

State 1		State 2		State 3		State 4		State 5		State 6	
holding time	end date	holding time	end date	holding time	end date	holding time	end date	holding time	end date	holding time	end date
[weeks]	dd-mm-yyyy	[weeks]	dd-mm-yyyy	[weeks]	dd-mm-yyyy	[days]	dd-mm-yyyy	[days]	dd-mm-yyyy	[days]	dd-mm-yyyy
44	03-11-2000	43	28-08-2001	33	19-04-2002	1	20-04-2002	2.9	23-04-2002	19.9	22-08-2008
58	16-04-2010	33	10-12-2002	19	19-04-2003	1	20-04-2003	2.9	22-04-2003	20.6	19-11-2016
		30	20-11-2003	23	28-04-2004	1	29-04-2004	1.1	30-04-2004		
		51	23-04-2005	30	20-11-2005	1	21-11-2005	1.7	23-11-2005		
		26	21-05-2006	32	28-12-2006	1	29-12-2006	2.1	31-12-2006		
		57	01-02-2008	26	01-08-2008	1	02-08-2008	0.8	03-11-2010		
		28	16-04-2010	28	01-11-2010	1	02-11-2010	1.1	04-02-2012		
		36	13-07-2011	29	02-02-2012	1	03-02-2012	1.0	12-07-2013		
		46	24-12-2012	28	10-07-2013	1	11-07-2013	1.4	02-07-2014		
		29	03-02-2014	21	29-06-2014	1	30-06-2014	0.9	05-11-2015		
		36	10-03-2015	34	03-11-2015	1	04-11-2015				
		27	11-05-2016	24	29-10-2016	1	30-10-2016				

Table 26: Condition data for pump 4

State 1		State 2		State 3		State 4		State 5		State 6	
holding time	end date	holding time	end date	holding time	end date	holding time	end date	holding time	end date	holding time	end date
[weeks]	dd-mm-yyyy	[weeks]	dd-mm-yyyy	[weeks]	dd-mm-yyyy	[days]	dd-mm-yyyy	[days]	dd-mm-yyyy	[days]	dd-mm-yyyy
39	29-09-2000	38	19-06-2001	32	26-01-2002	1	27-01-2002	1.9	29-01-2002	21.7	03-12-2008
64	09-07-2010	26	28-07-2002	25	17-01-2003	1	18-01-2003	0.8	18-01-2003	20.9	30-08-2016
		42	05-11-2003	28	17-05-2004	1	18-05-2004	6.3	24-05-2004		
		55	14-06-2005	42	05-04-2006	1	06-04-2006	1.0	07-04-2006		
		52	09-04-2007	32	19-11-2007	1	20-11-2007	0.2	20-11-2007		
		32	03-07-2008	19	10-11-2008	1	11-11-2008	1.0	04-09-2010		
		20	09-07-2010	8	02-09-2010	1	03-09-2010	2.3	20-01-2012		
		44	06-07-2011	28	17-01-2012	1	18-01-2012	1.1	04-05-2013		
		29	10-08-2012	38	02-05-2013	1	03-05-2013	1.0	03-09-2014		
		54	18-05-2014	15	01-09-2014	1	02-09-2014	1.0	15-11-2015		
		36	13-05-2015	26	13-11-2015	1	14-11-2015				
		16	06-03-2016	22	08-08-2016	1	09-08-2016				

Appendix 5.3 Ozonation: data

Table 27: Condition data for ozone street 1

State 1		State 2		State 3		State 4		State 5	
holding time	end date	holding time	end date	holding time	end date	holding time	end date	holding time	end date
107	18-05-1985	84	28-12-1986	36	08-09-1987	1	09-09-1987	3.0	12-09-1987
102	12-04-2002	71	18-01-1989	84	29-08-1990	1	30-08-1990	1.3	31-08-1990
		80	13-03-1992	45	25-01-1993	1	26-01-1993	5.7	01-02-1993
		93	17-11-1994	86	13-07-1996	1	14-07-1996	2.2	16-07-1996
		78	11-10-2003	44	19-05-1997	1	20-05-1997	4.7	25-05-1997
		51	09-02-2007	65	22-08-1998	1	23-08-1998	2.7	26-08-1998
		61	05-09-2010	87	25-04-2000	1	26-04-2000	3.1	30-04-2000
		74	30-08-2013	123	16-02-2006	1	17-02-2006	3.3	20-02-2006
				124	29-06-2009	1	30-06-2009	4.1	04-07-2009
				81	24-03-2012	1	25-03-2012	3.6	29-03-2012
				37	19-05-2014	1	20-05-2014	2.6	23-05-2014
				58	30-06-2015	1	01-07-2015	3.1	04-07-2015
				31	06-02-2016	1	07-02-2016	2.1	09-02-2016

Table 28: Condition data for ozone street 2

State 1		State 2		State 3		State 4		State 5	
holding time	end date	holding time	end date	holding time	end date	holding time	end date	holding time	end date
98	18-03-1985	98	29-01-1987	42	22-11-1987	1	23-11-1987	2.2	25-11-1987
91	29-12-2001	59	10-01-1989	92	15-10-1990	1	16-10-1990	2.5	19-10-1990
88	23-06-2016	86	11-06-1992	91	08-03-1994	1	09-03-1994	2.8	12-03-1994
		70	17-07-1995	53	22-07-1996	1	23-07-1996	3.0	26-07-1996
		97	11-11-2003	39	27-04-1997	1	28-04-1997	1.9	30-04-1997
		68	29-12-2006	75	09-10-1998	1	10-10-1998	2.7	13-10-1998
		56	17-08-2008	76	26-03-2000	1	27-03-2000	2.6	29-03-2000
		73	04-02-2011	95	03-09-2005	1	04-09-2005	3.5	07-09-2005
				29	20-07-2007	1	21-07-2007	1.8	22-07-2007
				55	09-09-2009	1	10-09-2009	2.7	12-09-2009
				15	23-05-2011	1	24-05-2011	2.6	26-05-2011
				69	18-09-2012	1	19-09-2012	2.5	21-09-2012
				75	03-03-2014	1	04-03-2014	3.4	08-03-2014
				31	14-10-2014	1	15-10-2014	2.3	17-10-2014

Appendix 5.4 Softening reactors: data and calculations

Appendix 5.4.1 Softening reactors: data

Table 29: Failure data for softening reactors (in days)

Pump number	1	2	3	4
Failure number				
1	10-02-1992	07-11-1989	05-08-2005	30-04-1989
2	31-03-1998	16-02-1995	14-05-2008	25-11-1997
3	02-10-2000	11-05-1998	06-12-2008	21-12-1998
4	19-11-2001	24-04-2000	11-12-2009	30-05-2000
5	10-06-2003	10-05-2006	25-07-2012	26-05-2001
6	07-11-2006	23-06-2006	16-08-2013	27-07-2005
7	06-04-2008	22-10-2006	20-06-2014	15-09-2009
8	06-03-2011	07-07-2007	04-12-2014	12-09-2010
9	12-08-2011	26-02-2008	16-12-2014	19-09-2011
10	27-09-2012	25-06-2009	12-04-2015	21-02-2013
11	01-08-2013	17-12-2010	25-07-2015	02-03-2014
12	18-08-2013	06-10-2011	30-08-2016	20-10-2014
13	12-11-2013	02-05-2012		07-10-2016
14	09-06-2015	15-06-2012		19-04-2017
15	21-08-2016	10-09-2012		
16	01-11-2016	26-11-2012		
17		03-03-2013		
18		16-07-2015		
19		19-05-2016		

Table 30: Downtime data for softening reactors (in hours)

Pump number	1	2	3	4
Failure number				
1	63	812	558	146
2	10	134	15	12
3	2	385	445	116
4	2146	35	119	31
5	238	285	70	4
6	9	11	4	10
7	34	23	11	1051
8	18	23	32	1538
9	8	9	241	6
10	30	1753	158	28
11	66	40	179	104
12	32	21	2	2141
13	300	194		207
14	244	104		62
15	9	429		
16	55	30		
17		13		
18		1		
19		9		

Appendix 5.4.2 Softening reactors: trend tests

Two trend tests are considered, the MIL-HDBK-189 and Laplace Trend Test. The calculation methods can be found in Appendix 3.2. The confidence bounds for both tests, given a confidence level of α = 0.05, are given in Table 31. The test statistic that is calculated, and if it falls inside or outside the confidence bounds, is given in Table 32. If the test statistic falls within the confidence bounds, the null hypothesis (no trend in data) cannot be rejected. In the case of the pump, both tests show that for all pumps the null hypothesis can be rejected (with confidence level = 95%). Thus it is assumed a trend in the data exists here.

Table 31: Confidence bounds for trend tests on data for softening reactors

Pump	Confidence bounds MIL-HDBK-189 (α = 0.05)		Confidence bounds Laplace Trend Test (α = 0.05)	
	Lower	Upper	Lower	Upper
1	18.29	49.48	-1.96	1.96
2	22.88	56.90		
3	12.40	39.36		
4	15.31	44.46		

Table 32: Test statistics for trend tests on data for softening reactors

Pump	Test statistic MIL-HDBK-189	Inside or outside confidence bounds?	Test statistic Laplace Trend Test	Inside or outside confidence bounds?
1	11.47	Outside	6.81	Outside
2	15.35	Outside	3.95	Outside
3	3.87	Outside	12.60	Outside
4	12.33	Outside	8.24	Outside

Appendix 6 Mathematical additions

Appendix 6.1 Inverse Transform Sampling: Weibull distribution

For the two-parameter Weibull distribution, the inverse of the CDF is derived as follows:

$$e^{-\left(\frac{x}{\beta}\right)^{\alpha}} = -(F(x) - 1)$$

$$-\left(\frac{x}{\beta}\right)^{\alpha} = \ln(-F(x) + 1)$$

$$-\frac{x}{\beta} = (\ln(-F(x) + 1))^{\frac{1}{\alpha}}$$

$$x = -\beta(\ln(-F(x) + 1))^{\frac{1}{\alpha}}$$

F(x) is equal to a random generated number U from the continuous uniform distribution *U*(0,1].

$$x = -\beta(\ln(-U + 1))^{\frac{1}{\alpha}}$$

(-*U* + 1) is also a random number in [0,1], so (–*U* + 1) can be replaced by U.

$$x = -\beta(\ln(U))^{\frac{1}{\alpha}}$$

U may not be exactly 0 because the natural logarithm of 0 is minus infinity, so the notation *U*(0,1] should be used instead of *U*[0,1] to point out that the random generated number cannot be exactly 0. In terms of modelling, when the random generated number is exactly 0, simply a new random number is generated.

Appendix 6.2 Intensity function: Power Law Process

Kim and Singh use the following notation for the intensity function of the Power Law Process (Kim & Singh, 2010):

$$\lambda(t) \ = \lambda\beta(t)^{\beta-1}$$

This notation leads to some confusing between the intensity function symbol $\lambda(t)$ and the scale parameter λ. Therefore, we use the more commonly used notation with θ instead of λ:

$$\lambda(t) \ = \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1}$$

Results are the same for:

$$\theta = \lambda^{-\frac{1}{\beta}}$$

Proof:

$$\lambda(t) = \lambda\beta(t)^{\beta-1} = \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1}$$

Thus:

$$\lambda\beta(t)^{\beta-1} = \beta\theta^{-1}\left(\frac{t}{\theta}\right)^{\beta}\left(\frac{t}{\theta}\right)^{-1}$$

$$\lambda(t)^{\beta-1} = \theta^{-1}\left(\frac{t}{\theta}\right)^{\beta}\left(\frac{t}{\theta}\right)^{-1}$$

$$\lambda t^{\beta}t^{-1} = \theta^{-1}t^{\beta}\left(\frac{1}{\theta}\right)^{\beta} t^{-1}\left(\frac{1}{\theta}\right)^{-1}$$

$$\lambda = \theta^{-1}\left(\frac{1}{\theta}\right)^{\beta}\left(\frac{1}{\theta}\right)^{-1}$$

$$\lambda = \left(\frac{1}{\theta}\right)^{\beta} = \theta^{-\beta}$$

And thus:

$$\theta = \lambda^{-\frac{1}{\beta}}$$

Furthermore:

$$\lambda = \theta^{-\beta}$$

Appendix 6.3 Inverse Transform Sampling: Power Law Process

$$U = Pr(x \leq U) = F(x)$$

So that:

$$x = F^{-1}(U)$$

Thus:

$$U = F_{t_{k+1}}(x) = 1 - exp\left(-\theta^{-\beta}\left\{(t_k + x_k)^{\beta} - (t_k)^{\beta}\right\}\right)$$

And since U has the same probability distribution as $1 - U$:

$$U = exp\left(-\theta^{-\beta}\left\{(t_k + x_k)^{\beta} - (t_k)^{\beta}\right\}\right)$$

$$\ln(U) = -\theta^{-\beta}\left\{(t_k + x_k)^{\beta} - (t_k)^{\beta}\right\}$$

$$-\frac{\ln(U)}{\theta^{-\beta}} = (t_k + x_k)^{\beta} - (t_k)^{\beta}$$

$$-\frac{\ln(U)}{\theta^{-\beta}} + (t_k)^{\beta} = (t_k + x_k)^{\beta}$$

$$\left((t_k)^{\beta} - \frac{\ln(U)}{\theta^{-\beta}}\right)^{\frac{1}{\beta}} = t_k + x_k$$

$$\left((t_k)^{\beta} - \frac{\ln(U)}{\theta^{-\beta}}\right)^{\frac{1}{\beta}} - t_k = x_k$$

with:

$$t_k = \sum_{i=1}^{k-1} x_i$$

Thus:

$$x_k = \left(\left(\sum_{i=1}^{k-1} x_i\right)^{\beta} - \frac{\ln(U)}{\theta^{-\beta}}\right)^{\frac{1}{\beta}} - \sum_{i=1}^{k-1} x_i$$