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Free-surface Multiples and Full-waveform Inversion Spectral Resolution

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SUMMARY

Low frequencies play a crucial role in the convergence of full-waveform inversion to the correct model in most of its current implementations. However, the lower the frequencies, the bigger are the amplitudes of the surface waves, causing the inversion to be driven by the latter. If they are not blanked out or removed, this may lead to convergence problems. To analyze this situation, we consider the simplest case where surface waves are present: an acoustic layer over a halfspace. We earlier analyzed the contributions of various wave types to the wavenumber spectrum of a velocity perturbation above a reflecting
halfspace, without a free surface. Here, we extend this spectral sensitivity analysis to the case with a free surface, which generates multiples and ghosts. In this setting, the surface guided P-waves can be considered as a superposition of free-surface multiples. Our analysis shows that the conditioning of the linearized inverse problem, which is solved at each iteration of full-waveform inversion, becomes worse when multiples are taken into account. At the same time the inclusion of multiples increases the sensitivity to some low wavenumbers in the model spectrum, which should be beneficial for full-waveform inversion once a suitable preconditioner has been found.

Introduction

Full-waveform inversion (FWI) is known to suffer from factors that introduce additional non-linearities into the scattered wavefield. Free-surface related effects are one of those. Here, we consider the role of multiples and ghosts in a marine-type acquisition for the constant-density acoustic case. Multiples are usually classified as an additional factor of non-linearity, while ghosts are often considered as a modification of the wavelet. The simplest model for marine acquisition – a homogeneous layer over a homogeneous halfspace with a higher velocity – allows for an analytical derivation of its spectral coverage (Mora, 1989) and sensitivity patterns (Kazei et al., 2013). Here, we apply an extended version of the spectral analysis method, originally suggested by Devaney (1984) for diffraction tomography, to FWI analysis (Mora, 1989; Kazei et al., 2013).

Forward problem

A solution of the 2-D constant-density acoustic wave equation in a horizontally layered medium can be represented as (Brekhovskikh and Godin, 1998):

$$\hat{p}(\mathbf{r},t) = \int_{-\infty}^{\infty} dk_x e^{-ik_x x} \int_{-\infty}^{\infty} d\omega e^{i\omega t} p(k_x, z, \omega). \tag{1}$$

The pressure $\hat{p}(\mathbf{r},t)$ depends on position $\mathbf{r}$ and time t and the transformed pressure $p(k_x,z,\omega)$ on horizontal wavenumber k_x, depth z and angular frequency ω. Within a layer with velocity c_0, the solution of the wave equation is a linear combination of upgoing (+) and downgoing (−) wavefields of the form $\exp\left(\pm iz\sqrt{k_0^2-k_x^2}\right)$, with $k_0 = \omega/c_0$. Note that the chosen Fourier convention is the conjugate of the customary one. In the case of two halfspaces, where the upper one with sources and receivers and scatterers has a velocity c_0 and the deeper a velocity c_1, the wavefield for a point source at zero depth in the upper halfspace consists of an incident downgoing and a reflected upgoing wavefield:

$$p_0(k_x,z,\omega) = \frac{1}{-2i\sqrt{k_0^2-k_x^2}}\left(e^{-iz\sqrt{k_0^2-k_x^2}} + V(k_x)e^{+i(z-2H)\sqrt{k_0^2-k_x^2}}\right). \tag{2}$$

The wavefield in the deeper halfspace is downgoing (Brekhovskikh and Godin, 1998):

$$p_1 = [1+V(k_x)]\frac{e^{-iH\sqrt{k_0^2-k_x^2}}}{-2i\sqrt{k_0^2-k_x^2}} e^{-i(z-H)\sqrt{k_1^2-k_x^2}}, \quad V(k_x) = \frac{1-b}{1+b}, \quad b = \sqrt{\frac{k_1^2-k_x^2}{k_0^2-k_x^2}}. \tag{3}$$

Here, $V(k_x)$ is the reflection coefficient, H is the distance between source and the lower halfspace and $k_1 = \omega/c_1$ contains the velocity c_1 of the deeper halfspace.

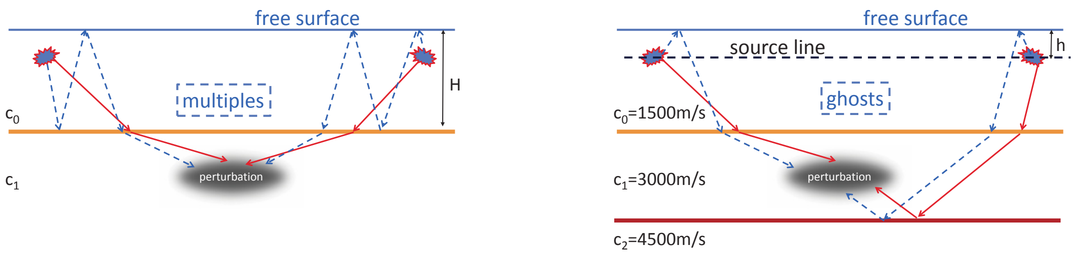

***Figure 1** Multiples and ghosts from the free surface in marine data.*

Next, we include a free surface as sketched in Figure 1. If all multiples are included, then the pressure p is changed to

$$p' = p\left(1 + \sum_{n=1}^{N}\left[-V(k_x)e^{-2iH\sqrt{k_0^2-k_x^2}}\right]^n\right) \xrightarrow{N\to\infty} \frac{p}{1+V(k_x)e^{-2iH\sqrt{k_0^2-k_x^2}}}. \tag{4}$$

The zeros of the denominator on the right-hand side correspond to normal modes or guided P-waves. To avoid convergence problems, we changed the free-surface reflection coefficient from -1 to -0.95. The fundamental mode, which is always present, corresponds to $k_x = k_0$. The existence and the number of higher-order modes depend on the layer thickness – the thicker the layer, the more zeros we have. Normal modes, however, do not penetrate to the deeper halfspace. They turn into inhomogeneous plane waves at the boundary and then decay exponentially. This means that to analyze the pressure in the deeper halfspace, we, in principle, neither need to include attenuation nor drop the higher-order multiples. Ghosts, as sketched in Figure 1, can be expressed with the same formula as the first-order multiples after replacement of H with h and taking $V(k_x) = 1$. If sources and receivers are very close to the free surface, we can simplify the expressions for the pressure by the following approximation, up to $O(h^2)$:

$$p'' = p' e^{ih\sqrt{k_0^2 - k_x^2}} \left(1 - e^{-i2h\sqrt{k_0^2 - k_x^2}}\right) = 2ip' \sin(h\sqrt{k_0^2 - k_x^2}) \simeq 2ihp' \sqrt{k_0^2 - k_x^2}. \quad (5)$$

Equation 5 shows that for small h, the ghost simply causes cancellation of the square roots in the denominators of equations (2) and (3), which simplifies our final expressions. Thus, the wavenumber spectra of the pressures are even more uniform than in the case without ghosts but reduced in absolute values in some parts, if h is small compared to the wavelength.

Inversion: theory

Given expression (4) for the wavefield, we can generalize Mora's (1989) technique of sensitivities. The scattered field is determined by the Born approximation. After Fourier transform in the horizontal source and receiver coordinates, the wavefield perturbation becomes

$$\delta u_{sr}(k_s, k_r) = \int k_0^2 \, \delta W(\mathbf{r}) G(k_s, \mathbf{r}) G(k_r, \mathbf{r}) \, d\mathbf{r}, \quad (6)$$

with $\delta W(\mathbf{r})$ a small perturbation of the background squared slowness. We earlier studied the case of two halfspaces without a free surface (Kazei et al., 2013). Updating the Green functions according to equations (4) and (5), we obtain a similar but slightly more complicated linear relation between the spatial perturbation spectrum and the transformed data in terms of sensitivities:

$$S(K_x, K_z) = \left| \frac{\delta u_{sr}\left(k_s(K_x, K_z), k_r(K_x, K_z)\right)}{\delta \widetilde{W}(K_x, K_z)} \right|. \quad (7)$$

Here K_x, K_z are coordinates in the spatial perturbation spectrum $\delta \widetilde{W}(K_x, K_z)$ (Wu and Toksöz, 1987, e.g.).

Sensitivities: results & discussion

First, Figure 2 shows the increase in the condition number of inversion at a single FWI iteration when inverting for a velocity perturbation in the *upper* layer. The criterion for existence of the first normal mode in case of a free layer over a halfspace is $H/\lambda \geq 1/(4\sqrt{1-(c_0/c_1)^2})$, where H is the layer thickness and $\lambda = 2\pi c_0/\omega$ is the wavelength in the upper layer (Brekhovskikh and Godin, 1998). For $c_1/c_0 = 2$, the critical ratio $H/\lambda \simeq 0.3$, meaning that the first mode has already appeared in the central panel of Figure 2, giving rise to peaks in several spots. Its interaction with the direct wave produces additional bright circular arcs.

Normal modes do not penetrate into deeper parts of the model. Sensitivity patterns become more uniformly illuminated by leaking modes (Roth et al., 1998) only. The lower part of the figures can only be reached by upgoing waves that were reflected at least once from the deeper parts (Kazei et al., 2013). If the perturbation is in the deeper halfspace and there are no deeper reflectors, we lack illumination by upgoing waves and any hope to restore more than one half of the perturbation spectrum is lost, as evident from Figure 3. Multiples are still helpful in inversion of low frequencies in this case. For Figure 4, we have added an additional deeper reflector, but we take only its first-order reflections into account. This

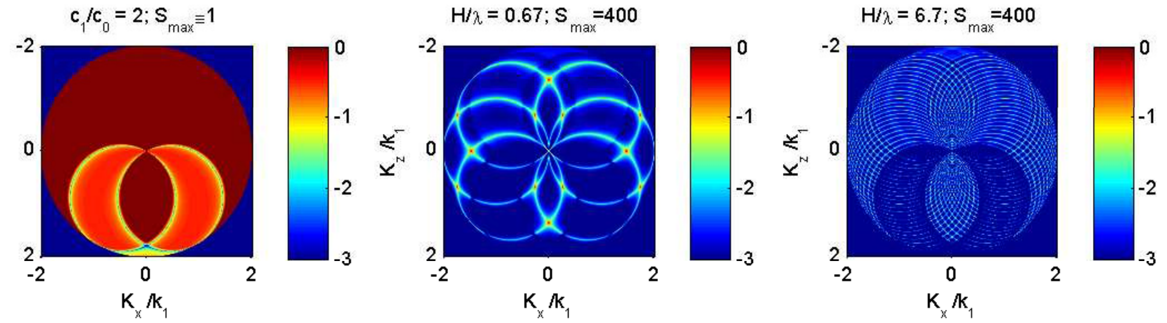

Figure 2 *Sensitivities in the upper layer for different frequencies. We plot* $\log_{10}(S/S_{\max})$ *to show the influence of inclusion of multiples on the condition number of the inverse problem.* $S_{\max}$ *is bigger whenever multiples are included, due to the increase of the maximum amplitude in the wavefields according to equation 4. The left panel shows the sensitivity pattern in the absence of a free surface. The central and right panels show sensitivities modified by the multiples of all orders. If the layer thickness is taken as 500 m, then the left panel corresponds to a quite low frequency of 3 Hz, while the right panel describes a high frequency of 30 Hz.*

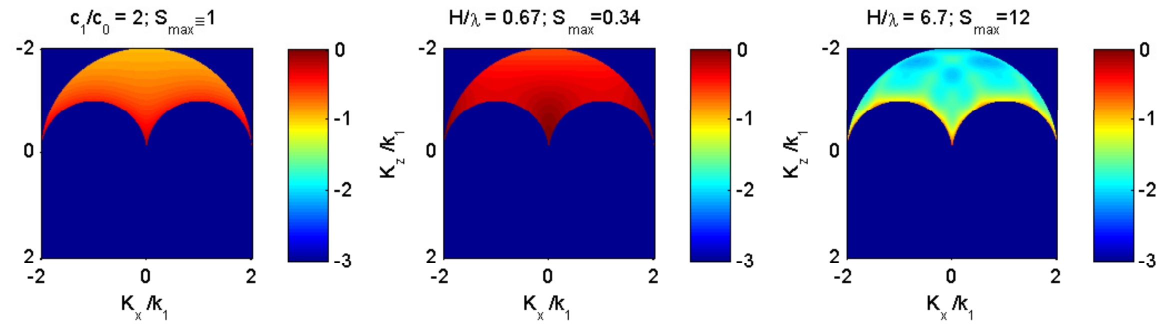

Figure 3 *As Figure 2, but for a perturbation in the deeper halfspace, sketched in the left panel of Figure 1. The lower half of the spectrum is not illuminated in this case and some low vertical wavenumbers are absent from the upper half (*$K_z > 0$*), since we have no upgoing waves at the depth of the perturbation.* $S_{\max}$ *is not always increased in this case since the phase shifts of multiples are determined by their paths in the upper halfspace (equation 4). These paths are not affected much by changes in the angle of refraction in the deeper halfspace, since the contrast is high and the incident angle has to be less than the critical one.*

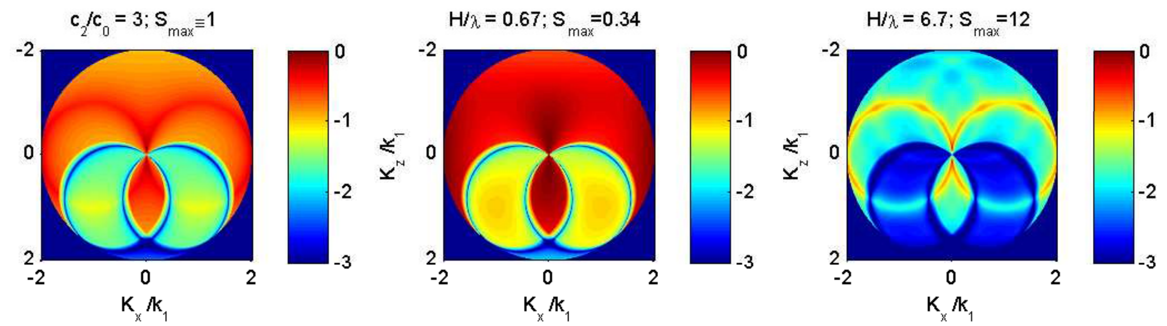

Figure 4 *As Figure 3, but with additional illumination from a deeper reflector. The reflector under the perturbation causes the whole spectrum to be illuminated. At wavelengths that are small compared to the upper-layer thickness, the full-wavefield becomes highly sensitive to a small subset of the wavenumber spectrum, making inversion for the other parts more difficult.*

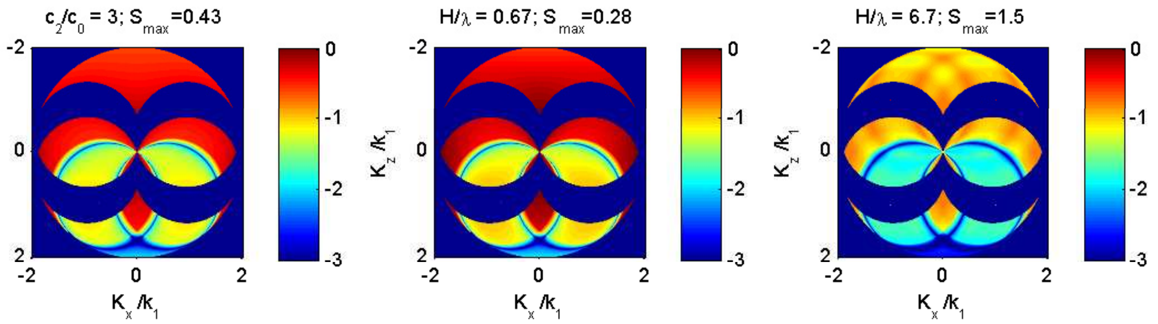

Figure 5 *As Figure 4, but without illumination from the long offsets.*

is sufficient to cover the whole spectrum in the idealized case when all offsets are present in the data. Figure 5 illustrates wavenumber coverage and intensity for the case when we lack refractions travelling almost horizontally at the perturbation depth (angles of propagation more than 20 degrees different from the horizontal). On the one hand, inversion of large-offset data gives rise to some low vertical wavenumbers in the perturbation spectrum. On the other hand, the convergence can be worse since the condition number is increased in the presence of large offsets, as can be seen by comparing Figures 4 and 5. Thus, skipping the large offsets is not crucial when very low frequencies are present in the data and, moreover, their inclusion into the inversion may decrease the convergence rate for inversion at some frequencies.

Conclusions

We have shown with the spectral sensitivities technique that first-order multiples in the data contain useful information for full-waveform inversion. If the free-surface multiples are caused by a reflector deeper than the target area for inversion, then normal modes affect the spectral relation, making single-frequency inversion more sensitive to a discrete subset of wavenumbers of the velocity model. Therefore, when multiples are included in FWI, the use of multiple frequencies should help because then the sensitivity patterns can be made more uniform by integration over frequencies. If the area of interest is below the reflector that creates the multiples, the condition number is even improved at some frequencies, which is good for full-waveform inversion convergence. A lack of large offsets in the data actually improves the convergence in the presence of multiples, in contrast to the case of diving-wave tomography. However, some wavenumbers can never be recovered in that case.

Acknowledgements

This project was proposed and supported by Fons ten Kroode of Shell Global Solutions International BV under CRDF grant RUG1-30020-ST-11. Additional support was provided by Saint-Petersburg State University under research grant 11.38.217.2014. The first author is grateful for helpful discussions to Jean Virieux and René-Edouard Plessix.

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