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Black-box Online Aerodynamic Performance Optimization for a Seamless Wing with Distributed Morphing

Oscar Ruland*, Tigran Mkhoyan†, Roeland De Breuker‡, and Xuerui Wang§
*Delft University of Technology, Faculty of Aerospace Engineering,
Kluyverweg 1, 2629 HS Delft, The Netherlands*

Morphing is a promising bio-inspired technology, with the potential to make aircraft more economical and sustainable through adaptation of the wing shape for best efficiency at any flight condition. This paper proposes an online black-box performance optimization strategy for a seamless wing with distributed morphing control. Pursuing global performance, the presented method integrates a global radial basis function neural network (RBFNN) surrogate model with a derivative-free evolutionary optimization algorithm. The effectiveness of the optimization strategy was validated on a vortex lattice method (VLM) aerodynamic model of an over-actuated morphing wing augmented by wind tunnel experiment data. Simulations show that the proposed method is able to control the morphing shape and angle of attack to achieve various target lift coefficients with better aerodynamic efficiency than the unmorphed wing shape. The global nature of the on-board model allows the presented method to find shape solutions for a wide range of target lift coefficients without the need for additional model excitation maneuvers. Compared to the unmorphed shape, up to 14.6% of lift-to-drag ratio increase is achieved.

Nomenclature

A	=	amplitude
\mathcal{R}	=	aspect ratio
C_L	=	lift coefficient
C_{L_t}	=	target lift coefficient
C_D	=	drag coefficient
C_{D_0}	=	zero-lift-drag coefficient
C_J	=	cost penalty constant
c	=	chord
D	=	drag force
D_i	=	induced drag force
e	=	Oswald efficiency factor
f	=	frequency
J	=	cost
k_1	=	cost function singularity prevention quantity
k_2	=	cost function scaling factor
L	=	lift force
S	=	power spectral density
T_i	=	i^{th} Chebyshev polynomial
u_i	=	i^{th} virtual input

*M.Sc. Student, Department of Control and Operations, Faculty of Aerospace Engineering, Kluyverweg 1, 2629HS Delft, the Netherlands, O.L.Ruland@student.tudelft.nl.

†Ph.D. Candidate, Department of Aerospace Structures and Materials, Faculty of Aerospace Engineering, Kluyverweg 1, 2629HS Delft, the Netherlands, T.Mkhoyan@tudelft.nl, AIAA Student Member.

‡Associate Professor, Department of Aerospace Structures and Materials, Faculty of Aerospace Engineering, Kluyverweg 1, 2629HS Delft, the Netherlands, R.DeBreuker@tudelft.nl, AIAA Associate Fellow.

§Assistant Professor, Department of Aerospace Structures and Materials, and Department of Control and Operations, Faculty of Aerospace Engineering, Kluyverweg 1, 2629HS Delft, the Netherlands, X.Wang-6@tudelft.nl, AIAA Member.

W	=	neural net weights
X	=	buffer model inputs
x_{hinge}	=	hinge location as a fraction of chord length
x_0	=	initial solution point
Y	=	buffer model outputs
y	=	spanwise coordinate
Z	=	frequency domain signal
z_{te}	=	trailing-edge displacement
α	=	angle of attack
Δ_f	=	change in frequency
δ_f	=	flap deflection
θ	=	actuator angle
σ_0	=	initial standard deviation
ϕ	=	phase

Subscripts

i	=	iteration
m	=	measured
s	=	saturated

I. Introduction

Over the past century, aircraft have become increasingly more efficient. During the 1960s, improvements in engine technology and wing design lead to significant improvements in aircraft fuel economy. In recent years, this trend of increasing efficiency has started to stagnate. To further reduce both the cost of flying and environmental pollution, more radical departures from the conventional aircraft design are needed. One promising technology is active morphing, which enables shape transformation in-flight [1, 2]. The Wright Flyer, the first successful heavier-than-air powered aircraft, relied on twist morphing of its fabric-wrapped flexible wings to achieve roll control [3]. However, as aircraft flew with ever-increasing speeds, higher wing rigidity was required, which made morphing fade out in the 1940s-2000s. In recent years, morphing has again been made possible by advanced developments in material science such as shape memory alloys, compliant mechanisms, and piezoelectrics [1, 4].

The ability to reshape the wing in flight introduces the problem of determining what that shape should be for a wide range of operational conditions. The current method for cruise drag minimization is the scheduling of configuration settings through lookup tables as a function of gross weight, airspeed, and altitude. These lookup tables generally depend on analytical models, validated with wind tunnel or test flight data. However, different operating conditions, aircraft production variances, and repairs can result in uncertainties in the table-lookup method.

Online optimization has the potential to tailor the wing shape to any specific flight condition for achieving the best aerodynamic performance based on in-flight measurements. Much like birds, a smart morphing-wing aircraft could sense its environment and adapt its wings' shape to achieve the best performance in any condition, making it fully mission-adaptive. However, many challenges remain on the path towards operational smart morphing aircraft wings. To begin with, any online optimization method is reliant on the ability to accurately evaluate the aircraft's performance using on-board sensors. Furthermore, only a very limited amount of search space exploration could realistically be afforded on a typical commercial flight. Ideally, a global optimum in the optimization landscape should be found with limited and local explorations.

A real-time adaptive least-squares drag minimization approach has been proposed for the variable camber continuous trailing edge flap (VCCTEF) described in [5, 6]. This strategy uses a recursive least squares algorithm to estimate the derivatives of the aerodynamic coefficients with respect to the system inputs. The optimal wing shape and elevator deflection are then calculated from a constrained optimization problem using the Newton-Raphson method. Improvements to the model excitation method, on-board model, and optimization methods were demonstrated in wind tunnel experiments to achieve up to 9.4% drag reduction on the common research model (CRM) with the VCCTEF at off-design conditions at low subsonic speeds [7]. Simulations have also indicated that a 3.37% drag reduction is achievable on the CRM with a distributed mini-plain flap system at Mach 0.85 [8].

Whilst the coefficients of the linear-in-the-parameters multivariate polynomial model adopted in [7, 8] can be estimated with relatively low computational cost, the model is only valid in the local region around the trim condition. This means that in order to perform real-time drag minimization across the entire flight envelope, the model parameters need to be re-identified at every operational point. Moreover, the required model excitation maneuvers that comprise both angle-of-attack and flap deflection inputs would induce undesirable bumpiness, structural loads, and increased fuel consumption. Last but not least, the use of a local model together with a gradient-based optimization method makes the solution prone to converge onto a local optimum. By contrast, a global on-board model, while more difficult to identify online, could allow for continuous drag minimization throughout the flight envelope. Additionally, when paired with a global optimization method, global optima with even better performance could potentially be found.

The online performance optimization strategy proposed in this paper integrates an online trained global artificial neural network (ANN) surrogate model [9], also referred to as the on-board model, with an evolutionary optimization algorithm [10, 11]. The covariance matrix adaptation – evolutionary strategy (CMA–ES) black-box optimization method was adopted because of its robustness to noise, ability to optimize non-convex and multi-modal problems, and desirable global performance [12]. To reduce the time required for optimization and to effectively retain the knowledge gained from historical measurements, an on-board model is adapted online. For the online identification of this on-board model, radial basis function neural networks (RBFNNs) were employed because of their local sensitivity, robustness to noise, and effectiveness on scattered data [13, 14]. The integration of these methods allows for the optimization of the morphing wing’s shape based on scattered and noisy flight data in real time. A high-level overview of the online optimization framework is shown in Fig. 1.

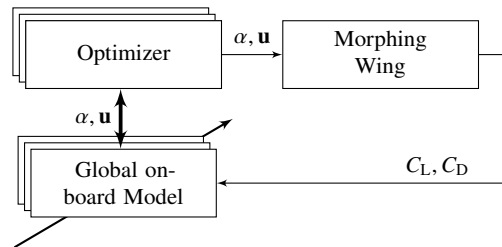


Fig. 1 High-level overview of the online shape optimization framework.

During each iteration of the online optimization method, the evolutionary optimizer uses the on-board model to evaluate the performance of populations of wing shape and angle-of-attack combinations α, \mathbf{u} to find the most promising input combinations. The most promising input combination is actuated and evaluated on the morphing wing system once per iteration. The resulting lift and drag coefficients C_L, C_D are then used to improve the on-board model for the next iteration. This online shape optimization strategy is evaluated on a simulation model of an over-actuated seamless active distributed morphing wing named SmartX-Alpha [15]. An overview of SmartX-Alpha is shown in Fig. 2a*.

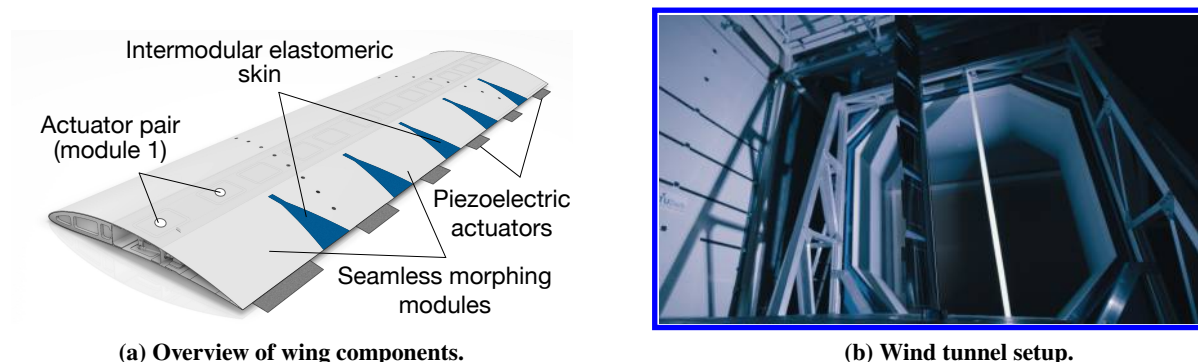


Fig. 2 Overview of the SmartX-Alpha wing demonstrator.

The $0.5 \text{ m} \times 1.8 \text{ m}$ rectangular half wing is made up of six translation induced camber (TRIC) morphing modules that comprise two actuators each and are seamlessly joined by a highly flexible elastomer skin. The TRIC morphing

*The project video can be found via <https://www.youtube.com/watch?v=SdagIiYRWyA&t=319s>

mechanism allows for both camber and twist morphing [16]. The ability to induce twisting of the trailing edge within each module by asymmetric actuator deflection allows for smoother morphing shapes than the distributed VCCTEFs. Control strategies for simultaneous gust and maneuver load alleviation have been demonstrated on SmartX-Alpha during wind tunnel experiments [17]. However, the in-flight drag minimization using the distributed morphing of SmartX-Alpha remains an open challenge, which will be addressed in this paper.

The main contributions of this paper are the first presentation and demonstration of a novel adaptable in-flight black-box performance optimization strategy for morphing wings. This data-driven approach is more adaptable and potentially able to realize higher performance than conventional shape scheduling by look-up tables. The morphed wing shape could be tailored in-flight to maximize the performance of the particular aircraft under consideration, rather than the performance of a model, built from previous test flight data on a similar aircraft. Moreover, compared to the state-of-the-art local gray-box methods which require additional model excitation maneuvers and re-identifications at each operational condition, the proposed approach retains the information learned in a global radial basis function neural network on-board model such that smooth and direct transitions to well-performing wing shapes can be achieved throughout the entire flight envelope. Furthermore, by integrating a derivative-free evolutionary optimization strategy with a global on-board model, global optima can be found. The proposed method has been validated on a vortex lattice method (VLM) model augmented by wind tunnel experiment data.

The structure of this paper is as follows. The morphing wing system is modeled in Sec. II. The optimization architecture is proposed in Sec. III. In Sec. IV, the simulation results are presented and discussed. Finally, the main conclusions are drawn in Sec. V.

II. System Modeling

A. Virtual inputs

The morphing wing system consists of 13 inputs: the deflections of the 12 actuators and the wing angle of attack. However, instead of using the actuator angles directly as system inputs, the optimizer and on-board model use a total of 5 virtual shape functions to describe the wing's shape. These virtual inputs u_1, \dots, u_5 scale the five basis shapes described by the first five Chebyshev polynomials of the first kind, re-scaled onto the $[0, 1.80]$ m domain, where 1.8 m is the half-wing span. The spanwise distribution of the local actuator deflection is a linear combination of the virtual inputs and the Chebyshev polynomials $T_i(y)$ as stated in Eq. (1).

$$\theta(y) = \sum_{i=1}^5 u_i T_i(y) \quad (1)$$

The virtual inputs and their contributions to the actuator deflection at each actuator location are shown in Fig. 3, where the triangular markers indicate the actuator positions. The translation-induced camber morphing mechanisms are modeled as a series of twistable plain flaps, whose local deflections vary linearly between the actuators. The deflection of each actuator is in turn dictated by the virtual inputs.

The virtual shapes reduce the 13-dimensional optimization domain for the real system to a six-dimensional one for the model, which significantly reduces the computational cost. Moreover, the basis shape functions enforce a certain degree of smoothness in the final morphed wing shape. Their use generally leads to smoother shapes than those resulting from 12 independent actuator deflections as they avoid shapes with large and frequent jumps in spanwise camber.

B. Aerodynamic model

The actuator deflections described by the virtual inputs are transformed to local flap deflections to produce the geometry that is to be evaluated by the aerodynamic model. First, the local vertical displacement of the trailing edge z_{te} is computed with Eq. (2), which was derived from digital image correlation measurements of symmetric morphing on SmartX-Alpha [15].

$$z_{te} = \theta k_{\theta} \quad (2)$$

where $k_{\theta} = 5.6 \times 10^{-4}$. The local plain flap deflection angle δ_f is then computed using Eq. (3), where x_{hinge} is the location of the flap hinge as a fraction of the chord length.

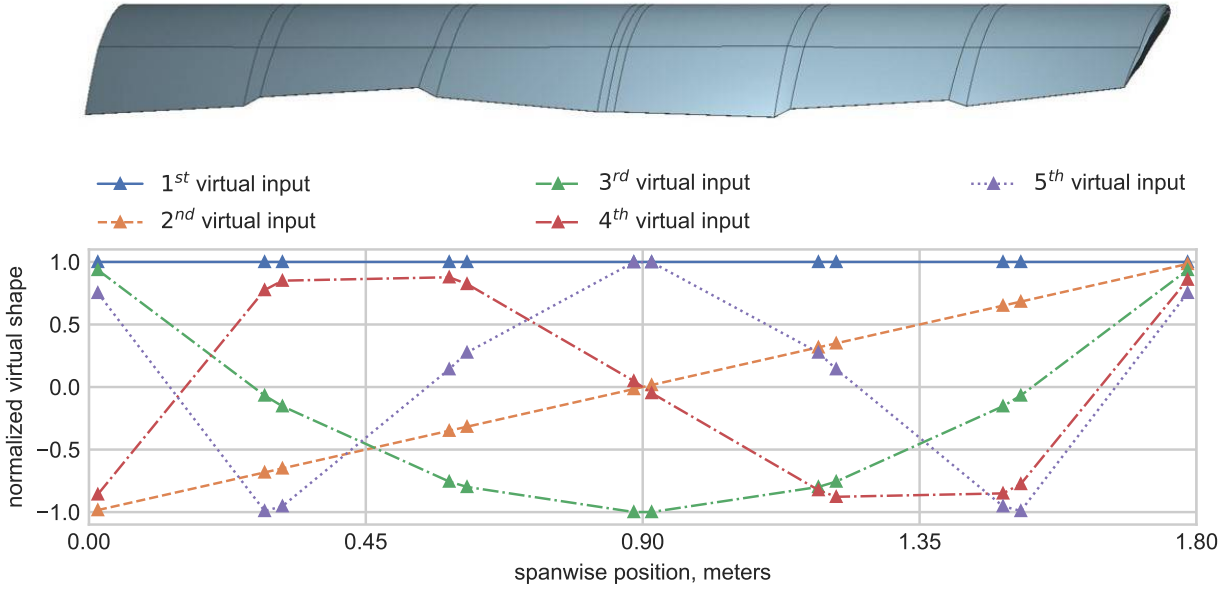


Fig. 3 Virtual shape functions that dictate the amount of camber morphing at each actuator location.

$$\delta_f = \sin^{-1} \left(\frac{z_{te}}{c \cdot (1 - x_{hinge})} \right) \quad (3)$$

Between the actuator locations, where the local flap angle is specified by the virtual inputs, the local flap angle varies linearly.

The aerodynamic performances of wing shape and angle-of-attack combinations are evaluated using a vortex lattice method (VLM) [18] model implemented in the Aerosandbox python package [19]. This method is used because of its high computational efficiency and scriptability. Since Aerosandbox is a relatively new open-source aerodynamic solver, and only one publication using this package exists in literature [20], its VLM implementation is verified against that of XFLR5 using the geometry of SmartX-Alpha. Figure 4 shows results from the Aerosandbox and XFLR5 VLM solvers, and wind tunnel measurements for constant spanwise actuator angle of -22 degrees. It can be observed from Fig. 4 that the outputs of Aerosandbox and XFLR5 VLM have a high consistency.

However, VLM neglects the effects of viscosity and thickness, and can only be used to estimate lift and induced drag. As a result, the models slightly overestimate the lift slope, although their lift predictions remain close to the wind tunnel measurements for the linear part of the lift curve. The drag on the other hand is consistently underestimated due to the lack of viscous drag effects in the model. Furthermore, while asymmetric flap deflections affect the lift-to-induced-drag ratio $\frac{L}{D_i}$ through reshaping the spanwise lift distribution, constant flap deflections along the wingspan do not affect $\frac{L}{D_i}$ at all. However, in order to optimize the morphing wings aerodynamic efficiency $\frac{L}{D}$, both the total drag and the effects of flap deflections on the lift-to-drag ratio should be modeled. Therefore, the model is augmented with an estimation of the zero-lift-drag coefficient C_{D_0} and a correction to the Oswald efficiency factor e based on data from a previous wind tunnel campaign with SmartX-Alpha. Furthermore, the use of the corrected model is restricted to the linear part of the lift curve, i.e., $-5.0 < \alpha < 10.0$ degrees. Wind tunnel measurements from seven angle-of-attack sweeps at different spanwise constant actuator angles were used to estimate C_{D_0} and e using the least-squares method and Eq. (4).

$$C_D = C_{D_0} + \frac{C_L^2}{\pi A e} \quad (4)$$

The estimates for C_{D_0} and e were interpolated by 1st and 2nd order polynomials respectively, as shown in Fig. 5. With these corrections and the induced drag from the Aerosandbox model, the total drag is estimated with Eq. (5), where $\bar{\delta}_f$ represents the mean flap angle. The efficiency factor of the constant deflection wing shape from Aerosandbox e_0 is estimated as 0.95.

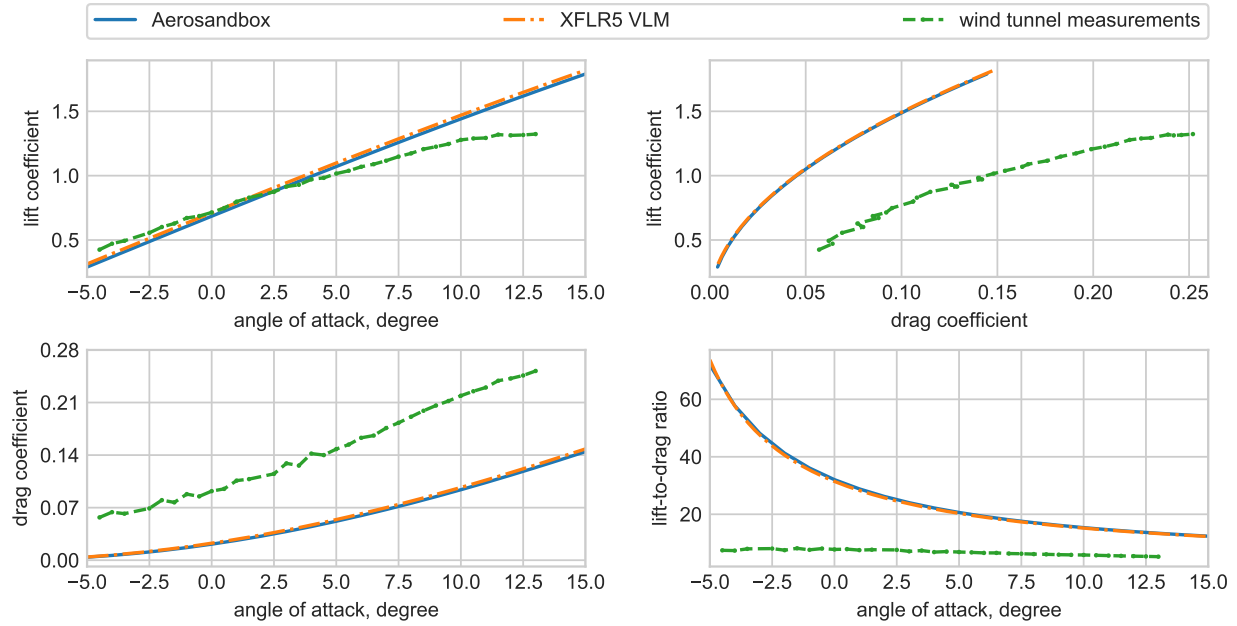


Fig. 4 Comparison of VLM solvers with wind tunnel measurements for a constant actuator angle of -22 degrees.

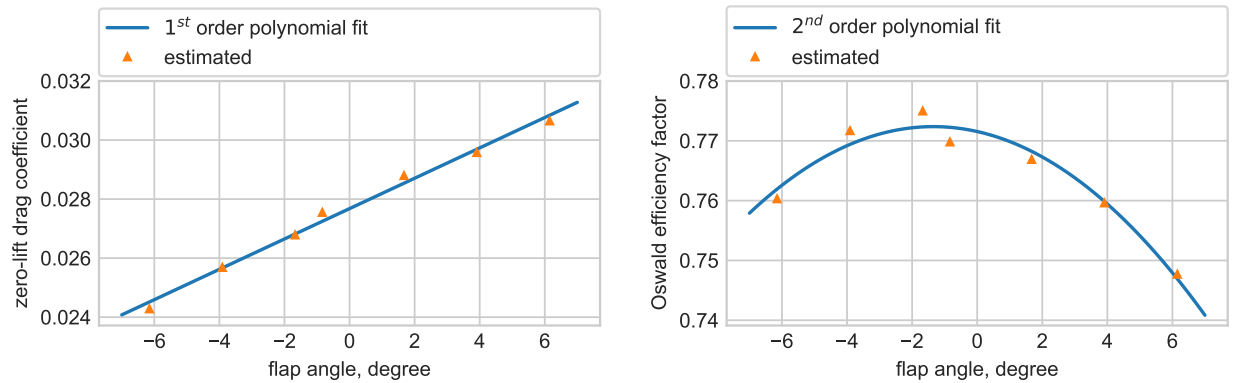


Fig. 5 Two correction functions estimated based on wind tunnel measurements.

$$C_D = C_{D_0}(\bar{\delta}_f) + C_{D_i} \cdot \frac{e_0}{e(\bar{\delta}_f)} \quad (5)$$

The effects of the corrections functions are shown in Fig. 6 for the case of a constant -22 degree actuator angle. Compared with the uncorrected drag polar from Fig. 4, the zero-lift-drag correction yields a result that is much closer to the wind tunnel measurements. However, the drag is still underestimated consistently. After correcting the drag predicted by Aerosandbox with both the zero-lift-drag and the Oswald efficiency corrections, the resulting drag polar closely matches the wind tunnel measurements.

Since the corrections were estimated using wind tunnel data, their validity is limited to the wing geometry and flow conditions that these measurements correspond to. In other words, the corrections shown in this section are only valid for the wing profile, the planform described above, and for an airspeed of 15 m/s at sea level air density. Nevertheless, the presented correction method is widely applicable to other cases.

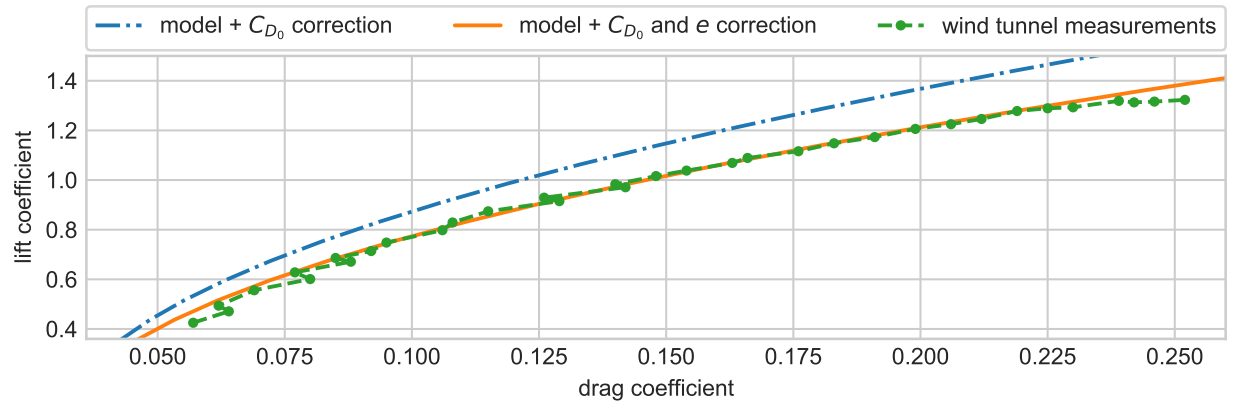


Fig. 6 Drag polar of the corrected aerodynamic model for a constant actuator angle of -22 degrees.

C. Secondary model

For future real-world operations, the use of white-box aerodynamic models such as the corrected model described above would be limited to training of the on-board model beforehand. In this manner, a priori knowledge about the system is transferred to the on-board model through the network weights. Although these will be adjusted during the online learning process, fewer adjustments are required than would be in the case of learning from scratch. In later stages of the technology, the network weights would hold the knowledge from previous flights, which is superior in quality compared to any model based predictions.

In order to demonstrate the ability of the online learning shape optimization procedure to adapt to a change in the system to be optimized, a secondary aerodynamic model is used in the simulation of the online shape optimization. The secondary model represents a comparable, but yet distinctly different morphing wing system.

The secondary model is comprised of the same wing planform as the nominal model, but with a NACA4312 airfoil instead (the SmartX-Alpha airfoil is NACA6510). As the VLM solver does not model the effects of airfoil thickness, only the maximum camber and location of maximum camber are different between the nominal and secondary models. Because equivalent wind tunnel data for this wing does not exist, the correction function estimation procedure cannot be repeated for the secondary wing model. Instead, the correction functions are altered directly. Therefore, the secondary model does not accurately model the aerodynamics of a known wing anymore. Instead, the secondary model represents the aerodynamics of an unknown wing, which are relatively close to those of the nominal model. The correction functions for both the nominal and secondary models are shown in Fig. 7.

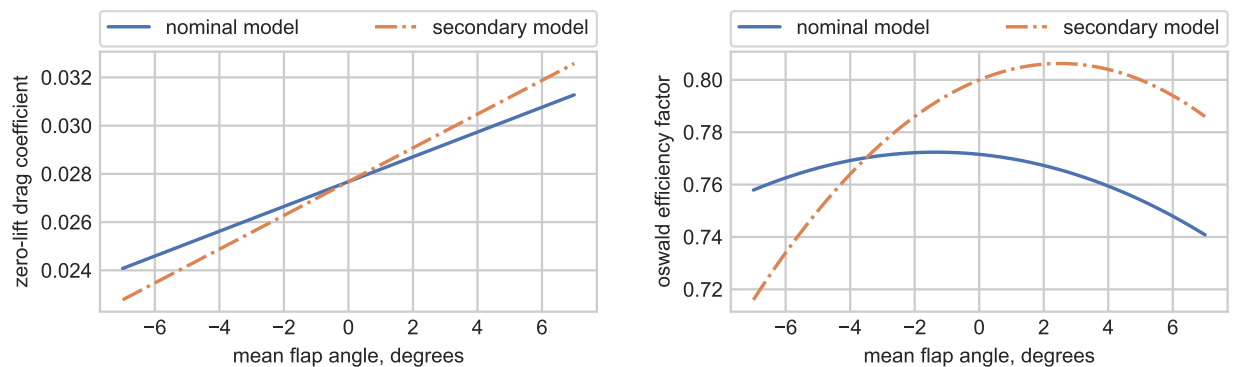


Fig. 7 Two correction functions for the nominal and secondary model.

D. Noise simulation and filtering

Real-world measurements were simulated by adding noise to the aerodynamic model outputs. The noise realizations used were derived from noise measurements from a previous wind tunnel experiment. The power spectral density (PSD) of the original noise signal, sampled at 1000 Hz was approximated by its periodogram. The PSD $\mathbf{S}(f_n)$ is sampled at n positive frequencies $f_n = [\Delta_f \ 2\Delta_f \ \dots \ n\Delta_f]^T$.

First, these power spectral densities are converted to amplitudes using $\mathbf{A}(f_n) = \sqrt{2\mathbf{S}(f_n)}$, where $\mathbf{A}(f_n)$ is the $n \times 1$ amplitude vector. Subsequently, the $n \times 1$ phase vector $\boldsymbol{\phi}(f_n)$ is built by assigning each spectral component a random phase between 0 and 2π radians. Next, a frequency domain signal $\mathbf{Z}(f_n)$ is constructed using Eq. (6).

$$\mathbf{Z}(f_n) = \mathbf{A}(f_n) \cdot e^{i\boldsymbol{\phi}(f_n)} \quad (6)$$

Second, the frequency domain signal is transformed to a time-domain signal using the inverse fast Fourier transform. The resulting time-domain signal and the original noise measurement are shown in Fig. 8. These noise realizations, although unique in the time domain, all are made of the same spectral components. As such, the power spectral densities of both signals are nearly identical.

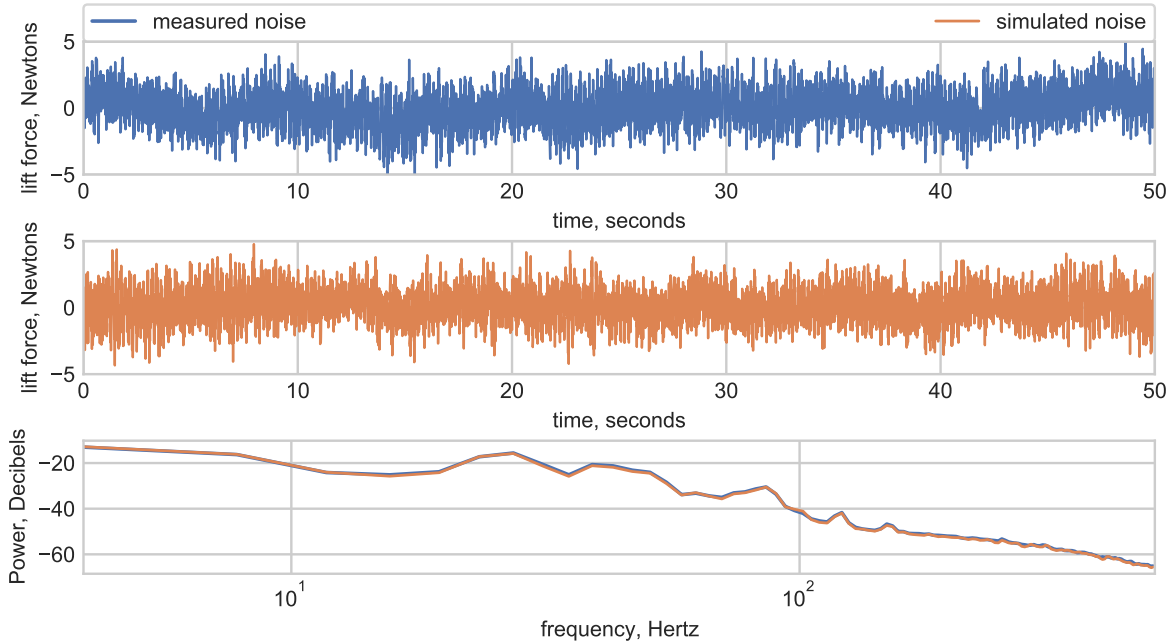


Fig. 8 Measured and simulated lift force noise signals.

Finally, the system output measurements are simulated by averaging over the 50-second noise realization for noise attenuation.

III. Optimization Architecture

In this section, the online shape optimization strategy and framework are proposed. First, an overview of the complete optimization architecture is presented. Then each of the individual components is elaborated upon in the following subsections. The architecture of the proposed online shape optimization framework is shown in Fig. 9.

The optimization procedure involves a fast and a slow loop. The optimizer, on-board model, and cost function work together in the fast loop, marked by the shaded arrows. The optimizer evaluates angle-of-attack (α) and wing shape combinations (\mathbf{u}) on the on-board model with a high frequency. The resulting lift and drag coefficients from the on-board model are valued with a cost function (J), which is also based on the target lift coefficient (C_{L_t}). These cost values are in turn used by the optimizer to produce a more promising set of inputs for the next iteration of the

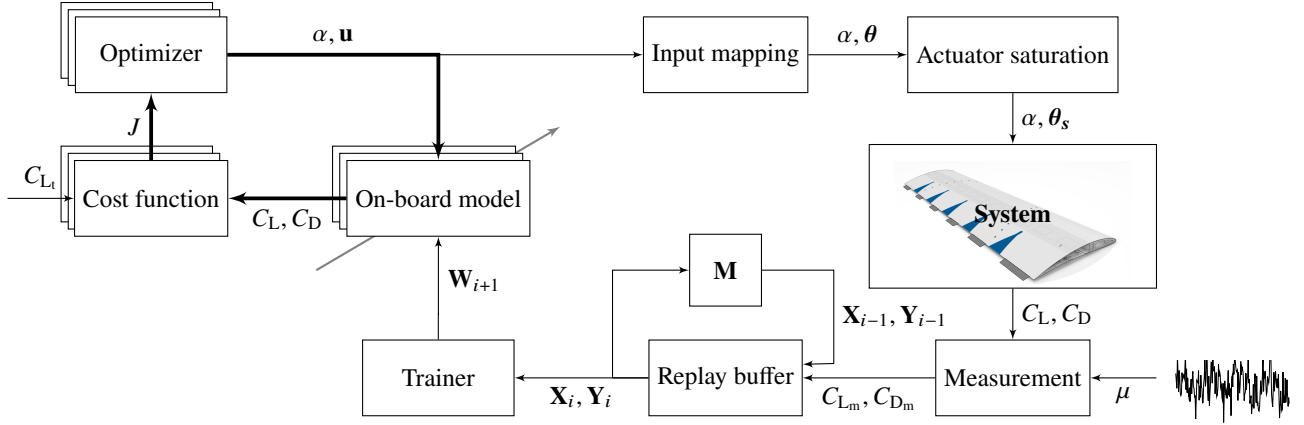


Fig. 9 Online shape optimization architecture.

optimization loop. Once the optimizer has converged onto the most promising set of inputs, they are evaluated on the system itself in the outer loop.

The on-board model and optimizer describe the wing shape in terms of 5 virtual inputs u_1, \dots, u_5 for wing shape smoothness and computational load reduction. However, since the shape of the morphing wing is controlled by 12 actuators $\theta_1, \dots, \theta_{12}$, the virtual input vector $\mathbf{u} \in \mathbb{R}^5$ needs to be mapped to the actuator input vector $\boldsymbol{\theta} \in \mathbb{R}^{12}$. Next, the actuator inputs are limited to their saturation limits of ± 25 degrees. Subsequently, the wing shape and angle of attack are actuated on the system. In this paper, the camber-morphing wing was simulated with an aerodynamic model of a wing with continuously distributed flaps. The resulting lift and drag coefficients are then contaminated with noise to simulate real-world measurements C_{L_m}, C_{D_m} . The inputs and outputs of the latest evaluation are added to the replay buffer, with a replacement strategy aimed at maintaining a global coverage of the input domain in memory. The model inputs \mathbf{X}_i and model outputs \mathbf{Y}_i in the buffer make up the training set that is used to train the on-board model. The training of the artificial neural networks that make up the on-board model results in new network weights \mathbf{W}_{i+1} . From here on, a new optimization cycle is initiated with an improved on-board model.

To evaluate the adaptability of the method, weights from previous training on a different wing, and no initial buffer data were used on the first iteration. In order to partly fill the empty buffer with data spread out over the input domain, the first 100 iterations were performed with quasi-random inputs instead of the optimizer-computed optima, which is known as the wandering phase.

As depicted in Fig. 9, the optimizer does not work with the system directly, but rather on the on-board surrogate model, which can be evaluated with much lower computational costs. The genetic optimization algorithm queries the on-board model with a population of inputs to be evaluated. The quality of these inputs is then determined from the model's outputs using a cost function. The optimizer in turn uses this information to generate a new group of candidate solutions. This loop is continued until the optimizer converges, after which this most promising input can be tested on the actual system.

The objective of the optimizer is to find the set of inputs α, u_1, \dots, u_5 that maximizes $\frac{C_L}{C_D}$ while meeting the target lift coefficient C_{L_t} without violating the angle-of-attack or actuator limits. The mathematical representation of this optimization problem is shown as

$$\max_{\alpha, \mathbf{u}} \frac{C_L(\alpha, \mathbf{u})}{C_D(\alpha, \mathbf{u})}, \quad \text{subject to } \alpha \in [\alpha_{\min}, \alpha_{\max}], \quad \theta_{\min} < \boldsymbol{\theta}(\mathbf{u}) < \theta_{\max}, \quad C_L(\alpha, \mathbf{u}) = C_{L_t} \quad (7)$$

This optimization problem is nonlinear and non-convex because C_L and C_D are nonlinear and non-convex functions of α and \mathbf{u} .

A. Cost function

As the optimizer queries the system with certain inputs, the corresponding outputs from the system need to be valued to in turn inform the optimizer how well the input performed. The inputs cannot simply be scored on their associated drag, as this would tempt the optimizer into minimizing the drag, by minimizing the lift produced. Instead, a

promising angle-of-attack and wing shape combination should result in both a low drag coefficient and a lift coefficient that is very close to the target lift coefficient. This is achieved with the cost function shown in Eq. (8).

$$J(C_L, C_D, C_{L_t}) = \underbrace{-\frac{C_L}{C_D}}_{\text{efficiency}} \cdot \underbrace{\frac{k_2}{k_1 + (C_L - C_{L_t})^2}}_{\text{deviation from lift target}} \quad (8)$$

The cost of any set of system outputs is dependent on the lift and drag coefficients, as well as on the target lift coefficient. The cost varies linearly with the aerodynamic efficiency $\frac{C_L}{C_D}$ and is inverse-quadratically related to the difference between the target and actual lift coefficients. A small quantity $k_1 = 1 \times 10^{-4}$ is added to prevent singularities for small error values. The parameter $k_2 = 2 \times 10^{-5}$ is used to scale the output to $[-1, 0]$. Two and three dimensional plots of the cost function for $C_{L_t} = 0.50$ are shown in Fig. 10. Note that the cost increases rapidly for any deviation from the target lift coefficient, while steps in the drag-coefficient axis generally result in smaller cost variations. In other words, a solution that provides low drag at a wrong lift coefficient is valued similarly to a solution that is associated with a higher drag at the right lift coefficient.

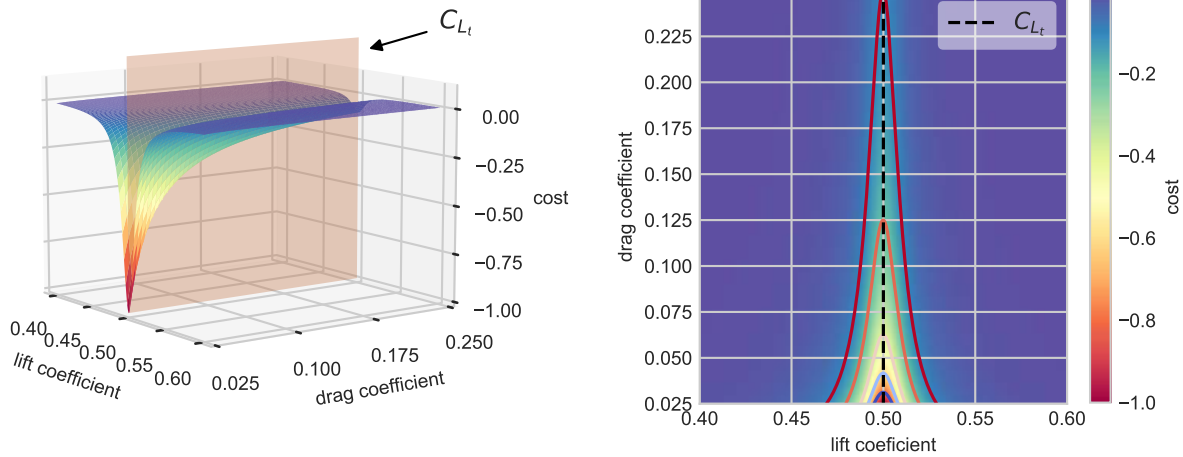


Fig. 10 Isometric (left) and top-down (right) view of the cost function for $C_{L_t} = 0.50$.

Additionally, the angle-of-attack and actuator constraints are also handled by the cost function. If a set of inputs violates any of these constraints, then its cost becomes as shown by Eq. (9).

$$J = (\alpha_i - \alpha^*)^2 + C_J, \quad J = (\theta_i - \theta^*)^2 + C_J \quad (9)$$

In the case that the angle of attack of a set of inputs to be evaluated is outside the bounds $[-2.5, 10.0]$ degrees, the associated cost will be the square of the difference between the angle of attack α and the middle of the domain $\alpha^* = 3.75$ degrees plus a large constant C_J . The valid range of θ is $[-25, 25]$ degrees so $\theta^* = 0$ and Eq. (9) reduces to $J = (\theta_i)^2 + C_J$. This cost penalty constant is set to $C_J = 10$ to ensure that the cost will always be higher than that of an input set that is not in violation of these constraints. The square term serves to provide a gradient towards the middle of the parameter domain.

B. Optimizer

The optimizer's goal is to find inputs to the on-board model that minimize the cost of the model outputs as determined by the cost function. This optimization is performed with the Covariance Matrix Adaptation Evolutionary Strategy (CMA-ES) algorithm [12]. CMA-ES is an evolutionary strategy for black-box optimization of non-linear, non-convex, and continuous problems. It can handle multi-modality and discontinuities in the function to be optimized and has desirable global performance. CMA-ES has been used for offline aero-structural optimization of a 3D morphing wing model in [21, 22].

In the framework proposed in this paper, CMA-ES operates by iteratively generating populations of inputs that are subsequently evaluated on the on-board model. Based on the returned costs of these candidate solutions, the mean and covariance matrix of the next generation's population are adapted. This process is repeated until the variation of the cost function converges to a threshold, selected as 1×10^{-6} . In the online shape optimization procedure, a population size of 150 was used. The middle of the input domain was used as the initial solution point \mathbf{x}_0 . Furthermore, the initial standard deviation σ_0 and the scaling of the input variables were selected such that $\mathbf{x}_0 \pm 2\sigma_0$ spanned the width of the inputs domains.

The total required number of function evaluations is dependent on the population size used, and also varies naturally due to the stochastic nature of the evolutionary strategy. Optimization with higher population sizes generally requires fewer optimizer iterations, but also requires more system evaluations per iteration. With a population size of 150, on average 180 optimizer iterations were needed with a total number of system evaluations of 27,000.

C. On-board model

The on-board model consists of two radial basis function artificial neural networks (RBFNNs) that model the mapping of the system inputs α, u_1, \dots, u_5 to the lift and drag coefficients. The C_L and C_D networks consist of a single hidden layer with 500 and 940 centers, respectively. More approximation power is needed for the C_D network than for the C_L network because of the higher degree of nonlinearity of the drag relation compared to the lift relation.

The training of the neural networks is done with mini-batch online training, with a batch size of 32. During training, the network weights are updated using the Adagrad algorithm proposed by Duchi et al. [23], with an initial learning rate of 0.01 and a mean squared error loss function.

The neural network models are not initialized with random weights, but rather with stored weights from a previous training session. In future applications, such a previous training session would be the online training performed during the most recent flight. For the simulations in this study, the starting weights for the online shape optimization will be weights from offline training on the nominal aerodynamic model. It is noteworthy that the simulated online optimization operates with the secondary model in the loop. Therefore, the initial weights serve only as a starting point and do not yet constitute a model that is representative of the system to be optimized.

For the initial offline training of the on-board model, a data set consisting of 261,360 wing shape and angle-of-attack combinations and their resulting lift and drag coefficients on the nominal model was used, with 10% of the data being reserved for validation. Both neural nets were trained from scratch for 2,000 epochs, which equated to roughly 23 hours of training time on a laptop (Intel® Core™ i7-4510U CPU, 8.00 GB RAM). Figure 11 shows the corresponding training and validation losses, converted to normalized root mean square errors (NRMSEs) for ease of comparison.

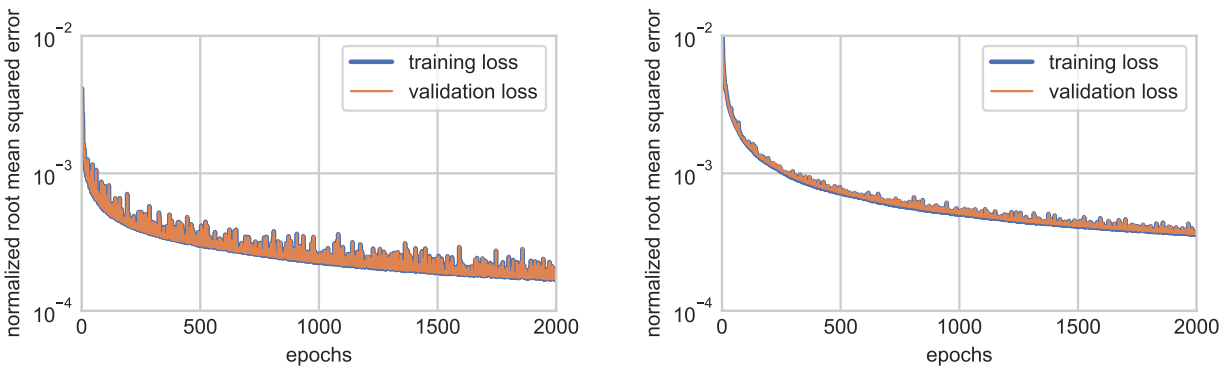


Fig. 11 Training and validation losses for the lift (left) and drag coefficient networks (right) in offline training.

Even with the higher approximation power of the C_D network, the NRMSE of the C_L network is lower because of the lower degree of nonlinearity in the lift relation. The loss curves of both networks still exhibit a decreasing trend towards the end of the training session. The training cut-off at 2,000 epochs is a trade-off between computational cost and starting point quality. The increased computational costs of further training yield an increasingly diminished return in accuracy, and the networks are only to serve as a starting point for the on-board model.

The main benefit of using the on-board model instead of direct system evaluations is the low computational cost. The CMA-ES optimizer typically requires thousands of function evaluations to converge on an optimum. On the neural

network models, hundreds of input combinations can be evaluated in less than one second, whereas on the aerodynamic model each evaluation takes 1.5 seconds on average. In other words, the indirect optimization using the on-board model is approximately 2500 times faster than the direct optimization on the aerodynamic model.

On a real-world aircraft, considerably more time would be required because of transients and noise filtering, making direct optimization unfeasible. Both direct optimization using the nominal aerodynamic model, and indirect optimization using the offline-trained on-board model were performed for a number of target lift coefficients. In order to make the computational time of the direct optimization more feasible, a population size of 9 was used for both. The resulting optimal shapes as computed by the CMA-ES optimizer are shown in Fig. 12.

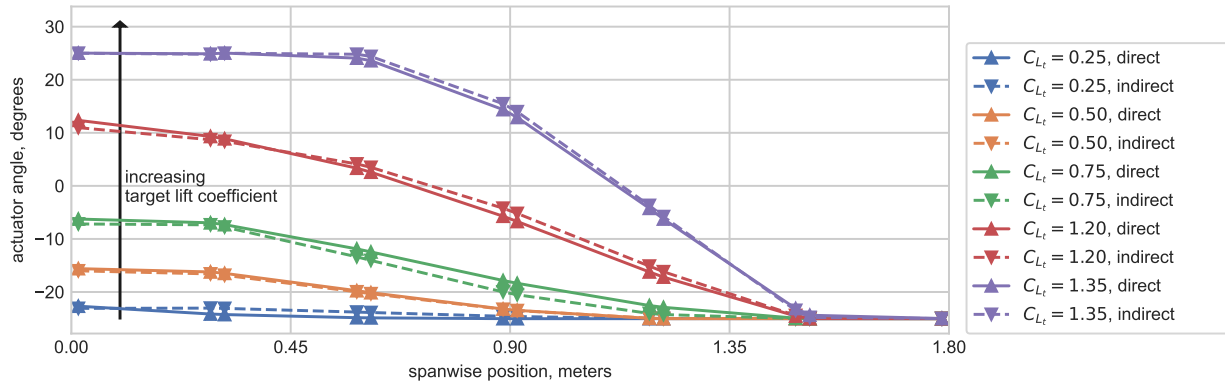


Fig. 12 Optimal wing shapes computed directly and indirectly on the system, for various target lift coefficients.

The optimal shapes computed by indirect optimization are very close to those computed using the system directly. On average, the direct optimization took 44.7 minutes per target lift coefficient, whereas the average computational time of the indirect optimization was only 3.9 seconds (about 688 times faster).

D. Replay buffer

During the online mini-batch training, the on-board model is trained on a set of training data kept in memory in the replay buffer. This buffer consists of a history of evaluated inputs and their corresponding lift and drag measurements. Since the on-board model is adjusted to adapt to this data, the contents of the buffer are of critical importance. If the training data set lacks data points in a region of the domain, then the neural nets will unlearn the previously learned from points in this region. This phenomenon, known as catastrophic forgetting was first described in [24]. Therefore, a simple first-in-first-out training set buffer will not be sufficient to learn and retain a globally accurate on-board model.

Instead, the replacing of old data points when the buffer is full is based on a nearest neighbor search on all points in the buffer inspired by the coverage maximization strategy described in [25]. The data point with the lowest mean euclidean distance to its 10 closest neighbors is replaced with the latest available data point. This replacement strategy aims to maximize the coverage domain of the training set by replacing the data points in regions of high data density and holding onto samples in data scarce regions of the domain.

During the relatively short simulations presented in this study, the maximum buffer size was not reached. In the future, the adaptation speed could be further improved by also prioritizing newer data points to older ones in the more densely populated areas of the domain.

IV. Results and Discussions

In this section, the results from two simulation experiments are presented. During the first simulation, the online optimization algorithm was run for 15 iterations with a fixed target lift coefficient of 0.75. During the second simulation, 275 iterations of online shape optimization were simulated with a target lift coefficient varying between 0.25 and 1.25. The aerodynamic efficiency of the resulting wing shapes was compared to that of the wing jig shape. The wing Jig shape is defined as the shape of the wing at rest, with all morphing actuators set to zero deflection. The wing jig shape does not have any pre-twist.

A. Single target lift coefficient

The online shape optimization framework was run for 115 iterations, of which the first 100 were performed in wandering mode and the rest in optimization mode. The inputs that were evaluated on the system are shown in Fig. 13, where the optimization phase is marked with a red background. As expected, both the angle of attack and the virtual inputs vary within their bounds with no recognizable pattern during the wandering phase. The cost associated with these pseudo-random inputs is generally high, with one notable exception at iteration 26, where the resulting lift coefficient was relatively close to the target lift coefficient by coincidence. Shortly after the algorithm enters the optimization phase at iteration 101, the inputs plateau. At iteration 102, a shape is tried that results in higher cost than the shape from the previous iteration. Subsequently the inputs move away from this location and, the associated cost falls down and converges.

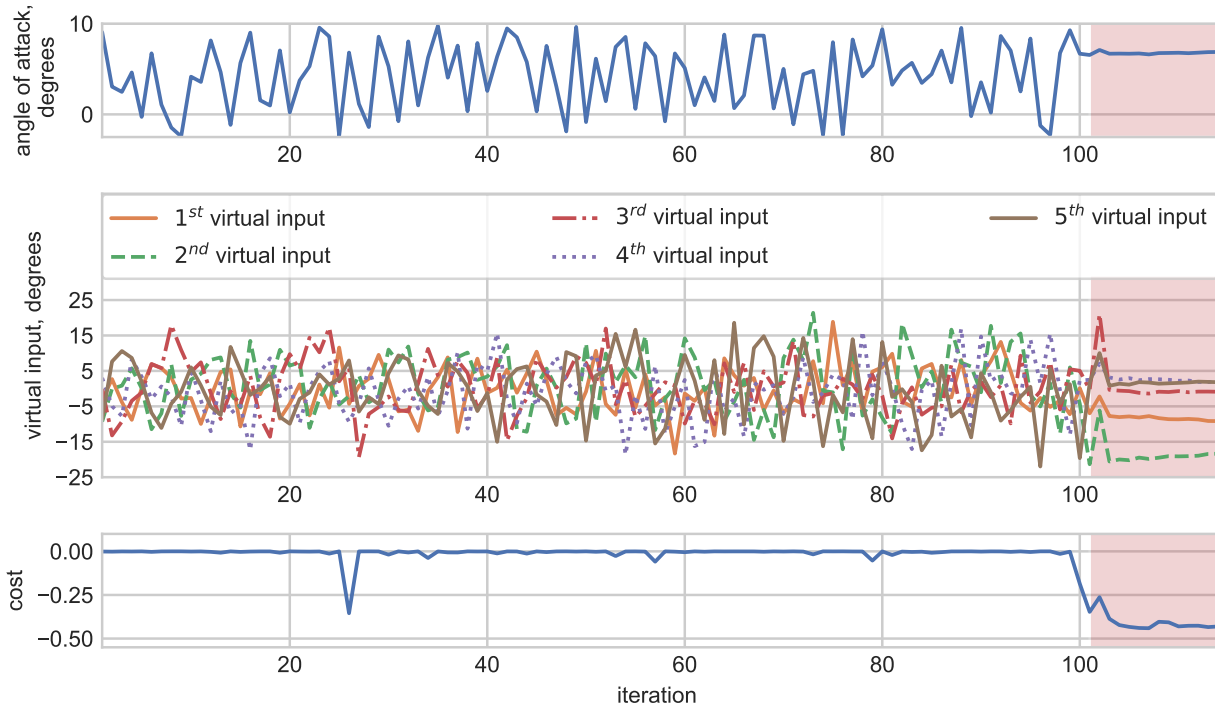


Fig. 13 Input history for wandering and optimization (red background) with $C_{L_t} = 0.75$.

More insight into the inner mechanisms of the optimization algorithm is provided by the optimal inputs as calculated by the optimizer, shown in Fig. 14. The optimal angle of attack and optimal virtual inputs remain unchanged for the first 32 iterations of the wandering phase. During this period, measurements are collected and the training buffer is partially filled. Training of the on-board model is only started after the size of the training set exceeds the batch size used for training.

At iteration 32, the online training is started and the algorithm's estimation of the optimal input changes with a sudden jump for the first time as the global minimum of the on-board model has shifted. Subsequently, the estimation of the optimal inputs changes repeatedly as the on-board model keeps training on an increasing number of data points and starts to represent the system more accurately. The fact that the optimal inputs only change slowly during the optimization phase, where estimated optimal inputs are evaluated on the system, indicates that the on-board model has captured the trends in the exploratory data quite well during the wandering phase. Two spikes in estimated optimal input can be observed at iterations 99 and 102. These two points correspond to an input that seemed promising based on the on-board model at the end of the wandering phase, but once tested on the system actually yielded a lower performance than expected. After evaluation on the system, this input combination does not show up in the optimal inputs in later iterations.

The wing shapes evaluated on the system during both phases are shown in Fig. 15. The pseudo-random shapes, shown in blue, span the full actuator domain. The optimal wing shape, shown in orange, starts out with only minor

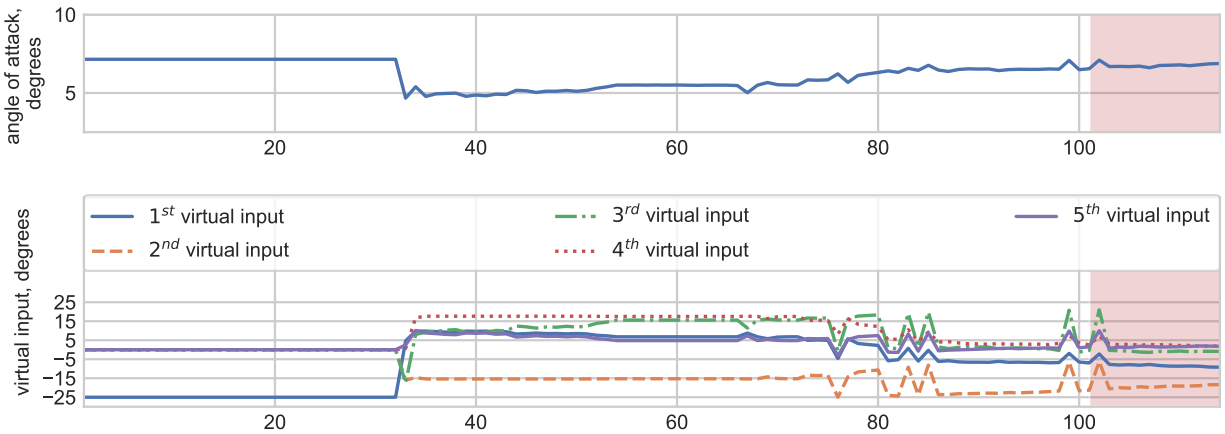


Fig. 14 Optimal inputs as calculated by the optimizer for a target lift coefficient of 0.75.

changes in camber near the wing root, as compared to the wing's jig shape. Towards the tip of the wing, the camber of the wing is decreased until the actuators in the tip module hit their maximum negative deflection angles of -25 degrees. This morphing shape brings the spanwise lift distribution of this zero-twist rectangular planform wing closer to the theoretically ideal elliptic lift distribution and thereby reduces the induced drag. One of the optimization phase shapes looks rather different from its counterparts. This is the shape that was tried on iteration 102 and resulted in an increase in cost compared to the previous iteration. In the following iterations, it was not repeated.

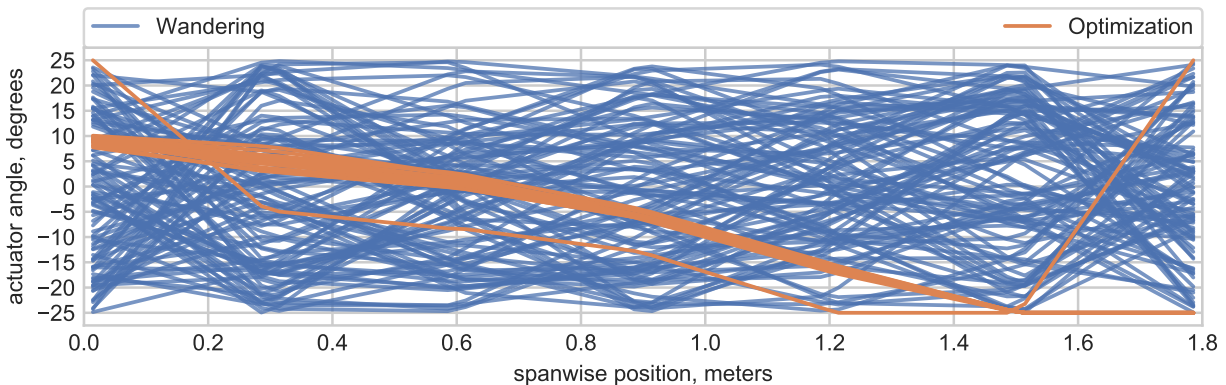


Fig. 15 Morphing shapes evaluated on the system in the wandering and optimization phases.

B. Various target lift coefficients

In order to investigate the ability of the online shape optimization algorithm to find optimal inputs for different target lift coefficients without repeated exploring, the optimization phase was extended to include two repeated series of steps and a window of gradual changes in the target lift coefficient as depicted in Fig. 16. The quality of the solutions actuated on the system was also evaluated by comparing their lift-to-drag ratios to those of the wing jig shape.

From iterations 100 to 160, the target lift coefficient is increased by 0.25 every 15 iterations. The steps in target lift coefficient are marked with dashed vertical lines. As a direct result of the steps in target lift coefficient, steps in the computed optimal angle of attack and virtual inputs can be seen at the corresponding iterations. For the duration of the steps, the optimal inputs are stable. The cost associated with the corresponding system outputs is also stable, although it is noisier due to the added measurement noise.

Between iterations 175 and 275 the target lift coefficient is decreased from 1.25 to 0.25 in steps of 0.01. As expected, the optimal angle of attack and mean camber of the optimal shape decreases as the target lift coefficient decreases. The

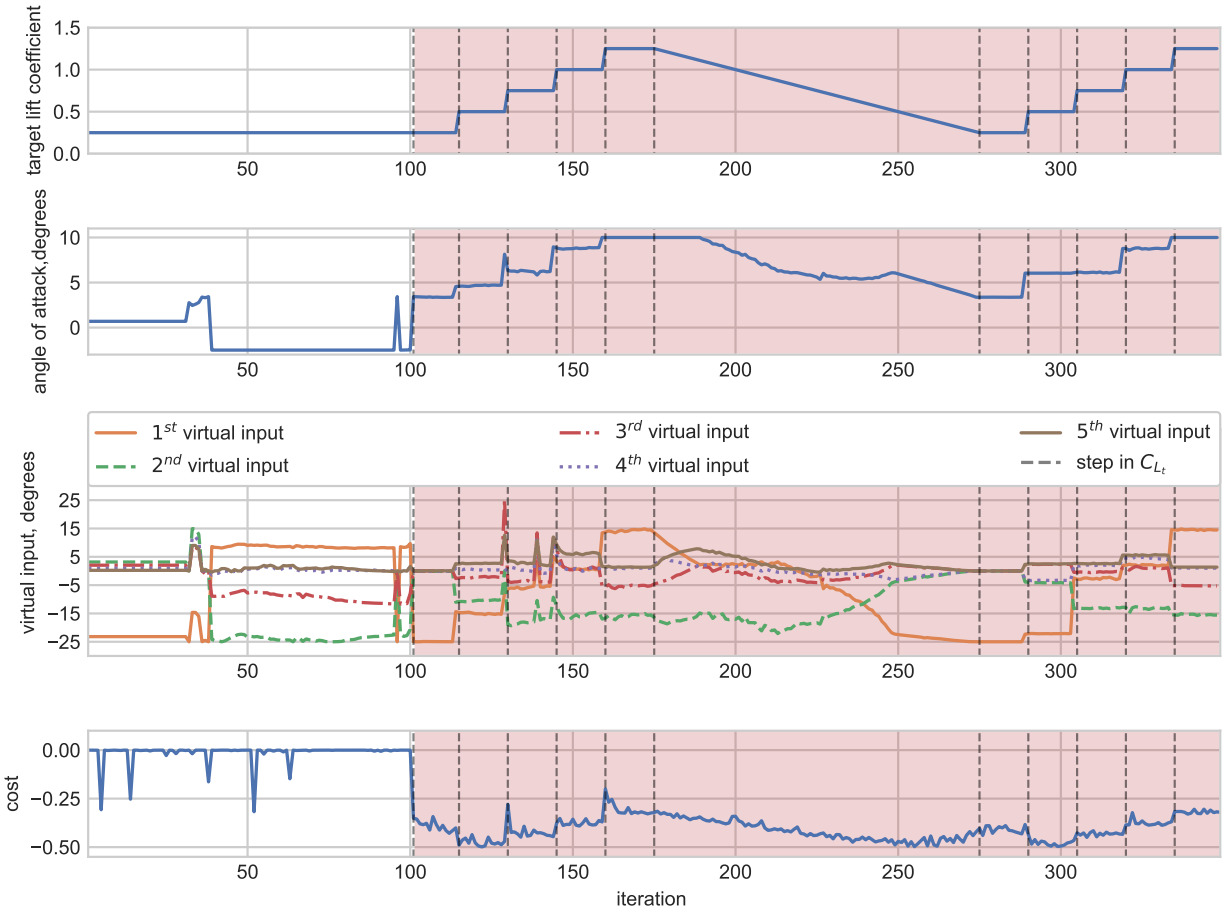


Fig. 16 Optimal inputs computed during the wandering and optimization (red background) phases.

first virtual input, which contributes a constant amount of camber morphing along the wingspan, decreases until it nears the negative actuator limit of -25 degrees between iterations 175 and 248. Meanwhile, the second virtual input, which represents a linear increase in spanwise camber morphing, becomes less negative. Here the optimizer increases the negative u_2 input because the lower u_1 input leaves less room for spanwise lift reduction before the actuators at the wingtip hit their maximum negative deflections. Between iterations 248 and 275, virtual inputs u_2 through u_5 are decreased to zero so that u_1 can all the way to the -25 degree actuator limit. In other words, for the target lift coefficient of 0.25, the optimizer sacrifices the increased lift induction efficiency of a more elliptical spanwise lift distribution for an overall less cambered airfoil. This makes sense since the airfoil already is relatively highly cambered, which is more efficient for producing higher lift coefficients.

After iteration 275, the same steps in target lift coefficient are repeated. The optimal inputs are almost the same between the runs, with the exception of $C_{L_t} = 0.50$ during iterations 290-305. Even though the inputs are different in this case, the costs are very similar. The average cost during iterations 115-130 is -0.475 with a standard deviation of 0.011, whereas the average cost during iterations 290-305 is -0.481 with a standard deviation of 0.018. Hence, on average the performance of the inputs evaluated during iterations 290-305 were slightly more desirable than those evaluated during iterations 115-130. Nevertheless, this again highlights the importance of accurate lift and drag estimations. Any combination of inputs can only be determined to be more efficient as long as the difference is measurable. In simulations without simulated measurement noise, the revisited target lift coefficients yielded the same inputs.

The lift coefficients and lift-to-drag ratios measured during the wandering and optimization phases are shown together with those of the jig shape in Fig. 17. As shown in Fig. 17, the quasi-random shapes from the wandering phase, shown in blue, produce lower lift-to-drag ratios than the jig shape, shown in green, in roughly 80% of the cases. Many possible shape variations exist that are aerodynamically inefficient, whereas only a smaller subset of shapes yield better

aerodynamic performance. By chance, some random inputs perform comparably or even better than the jig shape.

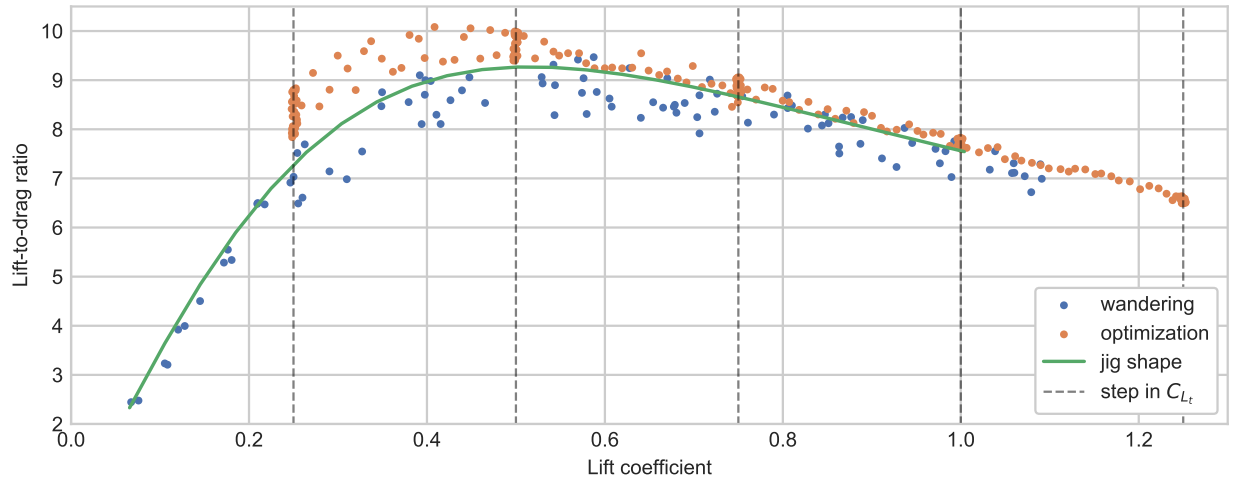


Fig. 17 Performance comparison of the jig shape and the online optimization shapes.

With the exception of only two data points, the optimization points, shown in orange, all outperform the jig shape in terms of aerodynamic efficiency. Although, for those two data points, the aerodynamic model output without simulated measurement noise does outperform the jig shape. Another effect of the measurement noise can be observed in the decreasing spread of the optimization point cloud with increasing lift coefficients. Naturally, as the lift and drag coefficients become larger, the lift-to-drag ratio becomes less sensitive to measurement noise. The clustering of optimization points at the target lift coefficients that were repeated for multiple iterations indicates that the optimizer is able to achieve the target lift coefficient very closely whilst also outperforming the jig shape.

An overview of the improvements in aerodynamic performance at various target lift coefficients achieved is shown in Tab. 1. As discussed before, the relatively highly cambered airfoil is naturally efficient at inducing higher lift coefficients. This is why the highest performance increase is observed for low lift coefficients (0.25-0.50). At $C_L = 0.25$ the lift-to-drag ratio is increased with approximately 14.6%. At higher target lift coefficients, less increase in aerodynamic efficiency can be gained from changing the average amount of camber. At $C_L = 1.00$ the lift-to-drag ratio is increased with approximately 2.5%. Due to the rectangular planform, and absence of twist in the jig shape, reshaping of the spanwise lift distribution closer to an elliptical distribution yields an aerodynamic performance increase at all target lift coefficients.

Table 1 Efficiency improvements of the optimized wing shapes compared to the jig shape.

C_{L_t} [-]	C_D [-]	$\frac{L}{D}$ [-]	$\frac{L}{D}$ increase [%]	C_D reduction [%]
0.25	0.02995	8.35	14.6	12.8
0.50	0.05108	9.79	5.6	5.3
0.75	0.08420	8.91	2.9	2.8
1.00	0.12906	7.75	2.5	2.4

V. Conclusions and Recommendations

In this paper, a novel online learning-based black-box approach to active morphing wing shape optimization was presented. Its objective is to maximize the steady-state lift-to-drag ratio for a given target lift coefficient using lift and drag measurements. The presented method integrates an online-trained artificial neural network (ANN) on-board model with an evolutionary optimization algorithm. This optimization strategy was tested in simulation on a model of a seamless camber morphing wing and its performance was compared to the performance of the wing jig shape. Before

optimizing, the algorithm was allowed to explore the optimization space with pseudo-random inputs in the wandering phase. Subsequently, in the optimization phase, the on-board model was used by the optimizer to find the optimal wing shape and angle of attack to achieve the target lift coefficient on the surrogate wing model.

During the wandering phase, the radial basis function neural networks were able to sufficiently learn the mapping between the angle of attack, wing shape, and the resulting aerodynamic forces to facilitate the optimizer to find wing shapes that outperformed the jig shape in terms of aerodynamic efficiency. Due to the global character of the neural network on-board model used, the presented optimization strategy was able to find wing shape and angle-of-attack combinations with lift-to-drag ratio increases of up to 14.6% for a wide range of target lift coefficients without requiring further exploration.

In the present case, the input space of the on-board model is comprised only of the wing shape and angle of attack. In actuality, the mapping of these parameters to the lift and drag coefficients is also influenced by the Reynolds number and Mach number. Nevertheless, due to the black-box nature of the neural network on-board model, future studies could, with little effort, incorporate the Reynolds and Mach numbers as additional inputs to expand its scope to the full flight envelope of any camber morphing platform.

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