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Synchronization in power-grid networks

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"Synchronisatie in stroomnetwerken"
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Abstract

Synchronization of power generators is important in order to have a normal operating power grid. Synchronized generators ensure a reliable energy supply and may prevent power outages in a stable operating power grid. In pursuance of power grid stability, many models have been published. In this thesis two different models are discussed. Both models focuses on the stability of the synchronous state of power-grid networks. The results of using these different models with respect to a nine-bus test system are presented and discussed. Moreover, information regarding the nine-bus system, such as optimal parameters, structural properties and phase differences is provided. This information is obtained by simulation on matlab software. Finally, this thesis discusses a comparison between the two models with respect to a nine-bus system, leading to quite different swing curves for both models.

Contents

Chapter 1

Introduction

Power-grid networks have to operate all day and are under constant threat of disturbances such as faults. These disturbances may lead to an unstable power grid in which power outages can arise. The key problem in ensuring stability of a power grid is desynchronization. This is a process causing an absence of synchronization, by which we mean that there is an element in the network that doesn't oscillate at the same frequency as the rest of the network. In order to maintain stability it is necessary for the network to be in a synchronous state. We characterize a synchronous state of a network with n generators by:

$$\dot{\delta_1} = \dot{\delta_2} = \ldots = \dot{\delta_n} \tag{1.1}$$

where $\delta_i(t)$ is the angular displacement of the i-th generators rotor from a synchronously rotating reference frame. In this context we consider the stability of synchronous power-grid networks which are subjected to a small disturbance. Disturbances we call small are for instance variations in loadings and gradual changes in rotor speeds. On the other hand we have large disturbances such as three phase faults or a loss of a large load which won't be treated in this thesis.
In the case of a small disturbance, we consider an effective network model. The effective network model reduces the network to a smaller one (the effective network) in which the non-generator nodes are eliminated. In this effective network model, loads are respresented as constant impedances. This is in contrast to the second model which will be treated, called an oscillator model in which only the buses are eliminated and loads are represented as second-order oscillators. Further comparisons between the two models are treated in Chapters 4 and 5. The results attained from simulating both models are shown in tables as well as graphically.

The main focus of this research is to find and compare stability results regarding the two different models in the case of a small disturbance. This stability study gives insight on the problem of maintaning a stable synchronous state and can therefore be helpfull in optimizing stability in power-grid networks.

1.1 The Dynamics

Recall that a power grid is a network consisting of producers, distributors and consumers. The producers are the generators which supply electricity to the consumers or distributive components in the grid. The consumers are the loads which draw electric power from the grid. Remark that these three components of a power grid are constantly changing due to variations in supplies and demands, and due to disturbances that may occur. In order to describe this behaviour of the network it is essential to have an equation of motion governing the dynamics of the network. Such an equation which describes the dynamics of generator i in an n-generator network is given by [2]:

$$\frac{2H_i}{\omega_R}\frac{d^2\delta_i}{dt^2} + \frac{D_i}{\omega_R}\frac{d\delta_i}{dt} = A_i - \sum_{j=1,j\neq i}^{n} K_{ij} sin(\delta_i - \delta_j - \gamma_{ij}) \qquad i = 1, \dots, n \tag{1.2}$$

and is called the Swing-equation. Here represents H_i the inertia constant of the i-th machine, ω_R the reference frequency of the network, δ_i the angular displacement of the i-th machine's rotor with respect to a synchronously rotating reference frame, D_i the damping of the i-th machine accounting for mechanical friction as well as the electrical effect of the generator's damper windings, A_i the effective power input to machine i and the summation on the right-hand side represents the effective power output of machine i, where K_{ij} is a coupling term. This equation along with closely related equations will play a central role in this study and will be often referred to as swing equations.

1.2 Structure of this report

The thesis is organized as follows. In Chapter 2 is an effective network model (EN-model) presented in which a bus impedance matrix is used to describe a power-grid network. Chapter 3 treats a sycnhronous motor model (SM-model) along with its dynamics and applies it to an example. In Chapter 4 we perform a comparative analysis between the EN-model and SM-model. In Chapter 5, we apply both models to a nine-bus test system. Chapter 6 presents and discusses the overall results regarding the nine-bus test system. Finally, the Appendix contains the matlab codes concerning the examples which are presented in the previous chapters.

Chapter 2

An effective network model

This chapter explains an effective network model in which the network structures is contained in a so-called admittance matrix. For the most part we'll follow the approach given by Adilson E. Motter, Seth A. Myers, Marian Anghel and Takashi Nishikawa in their paper "Spontaneous synchrony in power-grid networks" [1]. This model concentrates primarily on the network nodes that represent the generators (the effective network). These generators are assumed to be represented by constant voltages behind transient reactances. In contrast to the synchronous motor model, we assume that the loads are constant impedances. These assumptions, along with the other assumptions of the classical model are the fundamentals of the effective network model [4].

2.1 Admittance matrix

The structure of a network is of importance in order to deduce a stability criterium. This network structure can be represented by a so called admittance matrix $\mathbf{Y_0}$ whose elements $y_{0,ij}$ are admittances between network nodes i and j. Such a nodal admittance is closely related to a complex resistance called an impedance, in the sense that the nodal admittance is the reciprocal of the impedance: $y_{0,ij} = 1/z_{0,ij}$, where $z_{0,ij}$ is the impedance between nodes i and j. This impedance $z_{0,ij}$ has real part $r_{0,ij}$ called a resistance and an imaginairy part $x_{0,ij}$ called a reactance such that $z_{0,ij} = r_{0,ij} + jx_{0,ij}$. Note that here we use the letter j as imaginary unit. The complex form of the admittance is usually denoted by $y_{0,ij} = g_{0,ij} + jb_{0,ij}$.
Because we represent an impedance by a complex number and thus also admittances, we have an angle corresponding to each admittance. We denote the angle of admittance matrix element $y_{0,ij}$ by α_{ij} such that $y_{0,ij} = |y_{0,ij}|e^{j\alpha_{ij}}$. The fact that an admittance is represented as a complex number may seem a bit ambiguous. But since we're working with networks in which the current $\mathbf{I}$ alternates we can also represent the current as complex number. The same holds for the voltage $\mathbf{V} = |\mathbf{V}|\mathbf{e}^{\mathbf{j}\mu}$, and thus by Ohm's law we have: $\mathbf{V} = \mathbf{IZ}$ or $\mathbf{I} = \mathbf{YV}$, which makes it probably more clear.
In this report we mention a few types of admittances corresponding to particular network components. For example, we have line admittances corresponding to the admittances of the transmission lines and equivalent shunt admittances for the loads corresponding to the admittances of the loads. Sometimes, a particular network component has a pure imaginary number as admittance, the reciprocal of the reactance. This is for instance the case, when we are talking about the transient reactance of a generator. In this Chapter, we won't go into any deeper details concerning this transient reactance. But it is helpful to keep in mind, that this transient reactance occurs when we're representing a generator in a certain way.

In order to make the structure of the admittance matrix clearer, we introduce a small example. In this example we consider a small network with only one load called load A and two generators indicated by the numbers 1 and 2 given inside red circles. This network is illustrated in figure 2.1: The numbers 1,2 and 3 inside the white colored circles indicate bus numbers.

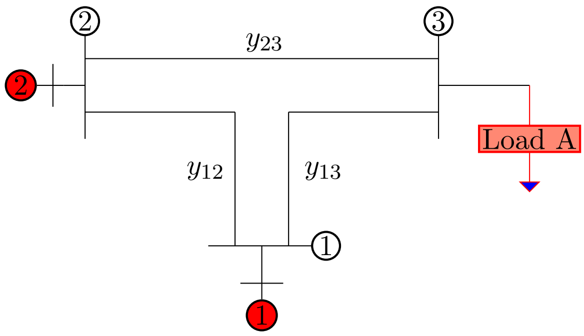


Figure 2.1: two generator and one consumer network

The letters y_{12}, y_{13} and y_{23} indicate line admittances between their corresponding buses. Because we'll see a few different admittance matrices in this thesis, we start-off with the simplest one which we call the physical admittance matrix $\mathbf{Y_0}$. This matrix takes only the line admittances into account and is a symmetrical matrix with off-diagonal elements $\mathbf{Y_{0,ij}}$ the negative of the line admittance between buses i and j, where $i \neq j$. The diagonal elements $\mathbf{Y_{0,ii}}$ are the self-admittances which are the sum of all line admittances connected to bus i. If there is no connection between two buses then the corresponding matrix elements takes the value zero. Thus, the admittance matrix looks like

$$\mathbf{Y_0} = \begin{bmatrix} y_{12}+y_{13} & -y_{12} & -y_{13} \\ -y_{12} & y_{12}+y_{23} & -y_{23} \\ -y_{13} & -y_{23} & y_{13}+y_{23} \end{bmatrix}$$

2.2 The effective network

With a view to making use of the structure of a network, we represent a network by a so-called admittance matrix as we've seen in the previous section. For a m-node network we define the admittance matrix which takes only the transmission lines into account by $\mathbf{Y_0}$. If we add the equivalent shunt admittances of the loads and the transient reactances of the generators to the corresponding components we get $\mathbf{Y_0'}$, where $\mathbf{Y_0'}$ is related to the node-voltage and node-current through $\mathbf{I} = \mathbf{Y_0'} \cdot \mathbf{V}$ [4]. This results in an admittance matrix $\mathbf{Y_0'}$ with $\mathbf{Y_{0ij}'}$ the negative of the admittances between nodes i and $j \neq i$ and $\mathbf{Y_{oii}'}$ the sum of all admittances connected to node i. In order to obtain an effective admittance matrix $\mathbf{Y^{EN}}$ in which all non-generator nodes are eliminated we apply a Kron-reduction scheme. This Kron reduction scheme makes use of the assumption that all non-generator nodes are considered to be constant impedances. This leads to the fact that all r non-generator nodes have zero injection currents, resulting in [1]

$$\begin{pmatrix} \mathbf{I_n} \\ \mathbf{0} \end{pmatrix} = \begin{pmatrix} \mathbf{Y_{0,n\times n}'} & \mathbf{Y_{0,n\times r}'} \\ \mathbf{Y_{0,r\times n}'} & \mathbf{Y_{0,r\times r}'} \end{pmatrix} \begin{pmatrix} \mathbf{V_n} \\ \mathbf{V_r} \end{pmatrix}$$

Eliminating $\mathbf{V_r}$ gives the effective admittance matrix:

$$\mathbf{Y^{EN}} = \mathbf{Y_{0,n\times n}'} - \mathbf{Y_{0,n\times r}'} \cdot \mathbf{Y_{0,r\times r}^{-1'}} \cdot \mathbf{Y_{0,r\times n}'} \tag{2.1}$$

2.3 Linearization

In the effective network model, we describe the the dynamics of the generators by the Swing equation of the form:

$$\frac{2H_i}{\omega_R}\frac{d^2\delta_i}{dt^2}+\frac{D_i}{\omega_R}\frac{d\delta}{dt}=P_{mi}-P_{ei} \tag{2.2}$$

This swing equation can written in the form given in 1.2 as can be seen in Chapter 4. The equation given in 2.2 is a nonlinear second-order differential equation that can't be solved analytically. Therefore, we linearize this equation around a synchronous state with a view to study the network under small perturbations. Let $P_{ei}^*, P_{mi}^*, \delta_i^*, \omega_i^*$ denote the mechanical power, electrical power, phase angle and frequency associated with the equilibrium synchronous state respectively. Further, assume that $\delta_i = \delta_i^* + \delta_i'$, $P_{ei} = P_{ei}^* + P_{ei}'$ and $P_{mi} = P_{mi}^* + P_{mi}'$. Then the left-hand side and right-hand side of 2.2 reads:

$$\tfrac{2H_i}{\omega_R}\tfrac{d^2\delta_i}{dt^2}+\tfrac{D_i}{\omega_R}\tfrac{d\delta}{dt}=\tfrac{2H_i}{\omega_R}\tfrac{d^2\delta_i^{'}}{dt^2}+\tfrac{D_i}{\omega_R}\tfrac{d\delta_i^{'}}{dt}$$

$$P_{mi}(\omega_i)-P_{ei}(\delta_1,\ldots,\delta_n)\approx P_{mi}(\omega_i^*)+\tfrac{\partial P_{mi}(\omega_i^*)}{\partial\omega_i}(\omega_i-\omega_i^*)-$$
$$\Big(P_{ei}(\delta_i^*,\ldots,\delta_n^*)+\tfrac{\partial P_{ei}(\omega_i^*,\delta_1^*,\ldots,\delta_n^*)}{\partial\delta_i}(\delta_i-\delta_i^*)+\ldots+\tfrac{\partial P_{ei}(\omega_i^*,\delta_1^*,\ldots,\delta_n^*)}{\partial\delta_n}(\delta_n-\delta_n^*)\Big)$$

Thus, the linearized differential equation is:

$$\tfrac{2H_i}{\omega_R}\tfrac{d^2\delta_i^{'}}{dt^2}+\tfrac{D_i}{\omega_R}\tfrac{d\delta_i^{'}}{dt}=\tfrac{\partial P_{mi}(\omega_i^*)}{\partial\omega_i}(\omega_i-\omega_i^*)-\textstyle\sum_{j=1}^n\tfrac{\partial P_{ei}(\omega_i^*,\delta_1^*,\ldots,\delta_n^*)}{\partial\delta_j}(\delta_j-\delta_j^*)$$

$$\tfrac{2H_i}{\omega_R}\tfrac{d^2\delta_i^{'}}{dt^2}+\tfrac{D_i}{\omega_R}\tfrac{d\delta_i^{'}}{dt}=\tfrac{\partial P_{mi}(\omega_i^*)}{\partial\omega_i}(\omega_i^{'})-\textstyle\sum_{j=1}^n\tfrac{\partial P_{ei}(\omega_i^*,\delta_1^*,\ldots,\delta_n^*)}{\partial\delta_j}(\delta_j^{'})$$

$$\tfrac{2H_i}{\omega_R}\tfrac{d^2\delta_i^{'}}{dt^2}+\tfrac{D_i}{\omega_R}\tfrac{d\delta_i^{'}}{dt}=\tfrac{\partial P_{mi}}{\partial\omega_i}\omega_i^{'}-\textstyle\sum_{j=1}^n\tfrac{\partial P_{ei}}{\partial\delta_j}\delta_j^{'}$$

We assume that the droop equation $\frac{\partial P_{mi}}{\partial\omega_i}=\frac{-1}{\omega_R R_i}$ holds with $R_i > 0$ and the electrical output is given by

$$P_{ei}'=\sum_{j=1}^n E_iE_j\left(B_{ij}cos(\delta_{ij}^*)-G_{ij}sin(\delta_{ij}^*)\right)\delta_{ij}'$$

where $\delta_{ij}^*=\delta_i^*-\delta_j^*$, $\delta_{ij}'=\delta_i'-\delta_j'$, E_i the internal-voltage magnitude, G_{ij} the real part of Y_{ij} and B_{ij} the imaginary part of Y_{ij}. Substituting these equations gives rise to

$$\tfrac{2H_i}{\omega_R}\tfrac{d^2\delta_i^{'}}{dt^2}+\tfrac{D_i}{\omega_R}\tfrac{d\delta_i^{'}}{dt}=\tfrac{-1}{\omega_R R_i}\tfrac{d\delta_i^{'}}{dt}-\textstyle\sum_{j=1}^n\tfrac{\partial P_{ei}}{\partial\delta_j}\delta_j^{'}$$

$$\tfrac{d^2\delta_i^{'}}{dt^2}=-\left(\tfrac{D_i+\frac{1}{R_i}}{2H_i}\right)\tfrac{d\delta_i^{'}}{dt}-\tfrac{\omega_R}{2H_i}\textstyle\sum_{j=1}^n\tfrac{\partial P_{ei}}{\partial\delta_j}\delta_j^{'}$$

The first term on the right-hand side forms $\mathbf{BX_2}$, where $\mathbf{X_2}$ is a n-dimensional vector with elements $\dot{\delta_i'}$ and $\mathbf{B}$ is a diagonal matrix with elements

$$\beta_i=\left(\frac{D_i+\frac{1}{R_i}}{2H_i}\right)$$

The second term on the right-hand side becomes

$$\frac{\omega_R}{2H_i}\sum_{j=1}^{n}\frac{\partial P_{ei}}{\partial \delta_j}\delta_j^{'} = \frac{\omega_R}{2H_i}\sum_{j=1}^{n}\frac{\partial P_{ei}^{'}}{\partial \delta_j}\delta_j^{'}$$

$$= \frac{\omega_R}{2H_i}\sum_{j=1}^{n}\frac{\partial}{\partial \delta_j}\left[\sum_{k=1}^{n} E_iE_k\left(B_{ik}cos(\delta_{ik}^{*}) - G_{ik}sin(\delta_{ik}^{*})\right)(\delta_i^{'} - \delta_k^{'})\right]\delta_j^{'} \tag{2.3}$$

Observe that 2.3 equals to zero if $j \neq i$ and $j \neq k$ or if $k = j = i$. In case $k = j \neq i$:

$$= \frac{\omega_R}{2H_i}\sum_{j=1}^{n}\frac{\partial}{\partial \delta_j}\left[E_iE_j\left(B_{ij}cos(\delta_{ij}^{*}) - G_{ij}sin(\delta_{ij}^{*})\right)(\delta_i^{'} - \delta_j^{'})\right]\delta_j^{'}$$

$$= \frac{\omega_R}{2H_i}\sum_{j=1}^{n}\frac{\partial}{\partial \delta_j}\left[E_iE_j\left(B_{ij}cos(\delta_{ij}^{*}) - G_{ij}sin(\delta_{ij}^{*})\right)(-\delta_j^{'})\right]\delta_j^{'}$$

$$= \frac{\omega_R}{2H_i}\sum_{j=1}^{n}\left(E_iE_j\left(G_{ij}sin(\delta_{ij}^{*}) - B_{ij}cos(\delta_{ij}^{*})\right)\right)\delta_j^{'}$$

In the case of $j \neq i$ we form a matrix $\mathbf{P}$ with off diagonal elements given by:

$$P_{ij} = \frac{\omega_R}{2H_i}E_iE_j\left(G_{ij}sin(\delta_{ij}^{*}) - B_{ij}cos(\delta_{ij}^{*})\right)$$

If $k \neq j = i$ than it follows from 2.3 that:

$$= \frac{\omega_R}{2H_i}\frac{\partial}{\partial \delta_i}\left[\sum_{\substack{k=1\\k\neq j=i}}^{n} E_iE_k\left(B_{ik}cos(\delta_{ik}^{*}) - G_{ik}sin(\delta_{ik}^{*})\right)(\delta_i^{'} - \delta_k^{'})\right]\delta_j^{'}$$

$$= \frac{\omega_R}{2H_i}\left[\sum_{\substack{k=1\\k\neq j=i}}^{n}\frac{\partial}{\partial \delta_i}\left[E_iE_k\left(B_{ik}cos(\delta_{ik}^{*}) - G_{ik}sin(\delta_{ik}^{*})\right)(\delta_i^{'} - \delta_k^{'})\right]\right]\delta_j^{'}$$

$$= \frac{\omega_R}{2H_i}\left[\sum_{\substack{k=1\\k\neq j=i}}^{n} E_iE_k\left(B_{ik}cos(\delta_{ik}^{*}) - G_{ik}sin(\delta_{ik}^{*})\right)\right]\delta_j^{'}$$

$$= -\frac{\omega_R}{2H_i}\left[\sum_{\substack{k=1\\k\neq j=i}}^{n} E_iE_k\left(G_{ik}sin(\delta_{ik}^{*}) - B_{ik}cos(\delta_{ik}^{*})\right)\right]\delta_j^{'} = -\sum_{k\neq i} P_{ik}$$

Thus, from $k \neq j = i$ it follows that

$$P_{ij} = -\sum_{k\neq i} P_{ik}$$

Hence,

$$P_{ij} = \begin{cases} \frac{\omega_R E_i E_j}{2H_i}\left(G_{ij}sin\delta_{ij}^{*} - B_{ij}cos\delta_{ij}^{*}\right) & \text{als } i \neq j \\ -\sum_{k\neq i} P_{ik} & \text{als } i = j \end{cases} \tag{2.4}$$

Let $\mathbf{X_1}$ a vector with elements $\delta_i^{'}$. This gives the following set of $2n$ first-order differential equations:

$$\dot{\mathbf{X_1}} = \mathbf{X_2}$$

$$\dot{\mathbf{X_2}} = -\mathbf{P}\mathbf{X_1} - \mathbf{B}\mathbf{X_2}$$

2.4 Uncoupling system of ODE's

We have the following coupled system of first-order differential equations

$$\dot{\mathbf{X_1}} = \mathbf{X_2}$$
$$\dot{\mathbf{X_2}} = -\mathbf{P}\mathbf{X_1} - \mathbf{B}\mathbf{X_2}$$

With a view to uncoupling this system of differential equations we let $\mathbf{H}$ be a diagonal matrix with elements H_i so that $\mathbf{P} = \mathbf{H^{-1}P'}$. Where

$$P'_{ij} = \begin{cases} \frac{\omega_R E_i E_j}{2}\left(G_{ij}sin\delta^*_{ij} - B_{ij}cos\delta^*_{ij}\right) & \text{als } i \neq j \\ -\sum_{k\neq i} P'_{ik} & \text{als } i = j \end{cases}$$

Notice that $det(H^{-1}P' - \lambda I) = det(H^{\frac{-1}{2}}P'H^{\frac{-1}{2}} - \lambda I)$, which means that the eigenvalues of the two matrices given by $\mathbf{P} = \mathbf{H^{-1}P'}$ and $\mathbf{P''} = \mathbf{H^{\frac{-1}{2}}P'H^{\frac{-1}{2}}}$ are equal.
This results in a partially symmetrized matrix given by:

$$P''_{ij} = \begin{cases} \frac{\omega_R E_i E_j}{2\sqrt{H_i H_j}}\left(G_{ij}sin\delta^*_{ij} - B_{ij}cos\delta^*_{ij}\right) & \text{als } i \neq j \\ -\sum_{k\neq i} P''_{ik} & \text{als } i = j \end{cases}$$

We now argue that the antisymmetric part $\frac{1}{2}\left(\mathbf{P''} - \mathbf{P''^T}\right)$ of $\mathbf{P''}$ is small enough to be able to diagonalize $\mathbf{P''}$. On the other hand we assume that β_i is the same for all generator nodes and we'll drop the subscript for readability. We diagonalize $\mathbf{P}$ as in $\mathbf{P} = \mathbf{QJQ^{-1}}$ with $\mathbf{Q}$ a matrix of eigenvalues of $\mathbf{P}$ and $\mathbf{J}$ a matrix of eigenvalues of $\mathbf{P}$. This leads to

$$\begin{cases} \dot{\mathbf{X_1}} = \mathbf{X_2} \\ \mathbf{Q^{-1}}\dot{\mathbf{X_2}} = -\mathbf{JQ^{-1}X_1} - \mathbf{Q^{-1}BX_2} \end{cases}$$

Take $\mathbf{Z_1} = \mathbf{Q^{-1}X_1}$ and $\mathbf{Z_2} = \mathbf{Q^{-1}X_2}$ then,

$$\begin{cases} \dot{\mathbf{Z_1}} = \mathbf{Z_2} \\ \dot{\mathbf{Z_2}} = -\mathbf{JZ_1} - \beta\mathbf{Z_2} \end{cases}$$

As noticed earlier, the matrix $\mathbf{J}$ is a diagonal matrix with elements α_j, the eigenvalues corresponding to $\mathbf{P}$, with $1 \leq j \leq n$. This leads to n partially decoupled 2 dimensional systems in the form of:

$$\begin{pmatrix} \dot{Z_{1j}} \\ \dot{Z_{2j}} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\alpha_j & -\beta \end{pmatrix} \begin{pmatrix} Z_{1j} \\ Z_{2j} \end{pmatrix} \tag{2.5}$$

In order to determine the stability of the system we compute the eigenvalues of the matrix given in (3.2). The characteristic polynomial of the matrix given in (3.2) is:

$$p(\lambda) = \begin{vmatrix} -\lambda_j & 1 \\ -\alpha_j & -\beta - \lambda_j \end{vmatrix} = -\lambda_j(-\beta - \lambda_j) + \alpha_j = \lambda_j^2 + \beta\lambda_j + \alpha_j$$

Leading to the eigenvalues:

$$\lambda_{j\pm}(\alpha_j, \beta) = \frac{-\beta}{2} \pm \frac{1}{2}\sqrt{\beta^2 - 4\alpha_j}$$

These eigenvalues are used to determine the stability of the synchronous state. The synchronous state is stable if and only if

$$\Re(\lambda_{j\pm}(\alpha_j, \beta)) \leq 0$$

where the algebraic multiplicity and geometric multiplicity corresponding to eigenvalue 0 are equal.

2.5 Improvement of synchronization stability

In the previous section we found that the stability of the synchronous state is determined by:

$$\Lambda_\beta(\alpha) := max_\pm \Re\left(\frac{-\beta}{2} \pm \frac{1}{2}\sqrt{\beta^2 - 4\alpha_j}\right) \leq 0 \tag{2.6}$$

Notice that in the above notation we take β fixed. In the case that $\alpha, \beta \in \mathbb{R}$, we have:

1. $\Lambda_\beta(\alpha) = 0$ correspondes to $\alpha = 0$
2. $\Lambda_\beta(\alpha) < 0$ corresponds to $\alpha > 0$
3. $\Lambda_\beta(\alpha) = -\frac{\beta}{2}$ correspondes to $\alpha \geq \frac{\beta^2}{4}$

Because $\Lambda_\beta(\alpha)$ is a decreasing function for fixed β we see that the stability of the synchronous state depends on the smallest nonzero eigenvalue α_2 of $\mathbf{P}$. Now, it's easy to see that $\Lambda_\beta(\alpha)$ as a function of β attains its minimum at

$$\beta_{opt} = 2\sqrt{\alpha_2}$$

Since β_{opt} is the optimal value for achieving stability of the synchronous state, we set $\beta_i = \beta_{opt}$. This leads to

$$R_i = \frac{1}{4H_i\sqrt{\alpha_2} - D_i} \qquad\qquad D_i = 4H_i\sqrt{\alpha_2} - \frac{1}{R_i} \tag{2.7}$$

Thus by adjusting the droop parameter R_i and the damping coefficient D_i we can improve the stability of the synchronous state.

Chapter 3

A synchronous motor model

We have seen a model of the kind called an effective network model [1]. With this model we can succesfully deduce the stability of a certain network based on its topology. In this chapter, another model is presented, which we'll refer to as synchronous motor model or SM-model. For the most part we'll follow the approach given by Martin Rohden, Andreas Sorge, Dirk Witthaut and Marc Timme in their paper "Impact of network topology on synchrony of oscillatory power grids" [3]. The model depends on a closely related swing equation governing the dynamics of each element in the network. The application of the model allows us to study the effect of the network structure on the synchrony of this network.

3.1 The dynamics

In the synchronous motor model we consider a grid as a network consisting of coupled oscillators. Every oscillator in this network represents either a load or a generator. The difference between the modelling of a generator and a load is that the power flow of a load is in the opposite direction to that of a generator. This means that a load convertes electrical power into mechanical power and can therefore be seen as a motor. To make a distinction between these two network elements we make a sign convention by denoting consumed power by $P < 0$ and generated power by $P > 0$. In the Introduction we characterized a synchronous state of a network by means of the change in the phase difference δ from a synchronously rotating reference frame. This phase difference, which we'll denote with θ in the context of the synchronous motor model, lies at the basis of the synchronous motor model. Mathematically, we can relate this phase difference with the actual phase angle $\phi(t)$ of a generators rotor by

$$\phi(t) = \Omega t + \theta(t) \tag{3.1}$$

Where Ω denotes the frequency of the synchronously rotating reference frame and t denotes the time variable. This relation between $\phi(t)$ and $\theta(t)$ enables us to derive the equation of motion for θ by using the principle of energy conservation. This principle leads to the following formula:

$$P_i^{mech} = P_i^{diss} + P_i^{acc} + P_i^{elec} \tag{3.2}$$

where P_i^{mech} denotes the generated power, P_i^{diss} the dissipated power, P_i^{acc} the accumulation power and P_i^{elec} the electric power transmitted to the rest of the network. Recall that the kinetic energy of a rotating body with moment of inertia I is given by $E_{kin} = I\frac{(\dot{\phi})^2}{2}$ which leads to $P_i^{acc} = \frac{dE_{kin,i}}{dt} = \frac{1}{2}I_i\frac{d}{dt}(\dot{\phi}_i)^2$. The power dissipated by friction of each element is given by $P_i^{diss} = \widetilde{D}_i(\dot{\phi})^2$.

Using these as well as 3.1 leads to:

$$P_i^{acc} = \frac{1}{2} I_i \frac{d}{dt} \left(\Omega + \dot{\theta}_i\right)^2 = \frac{1}{2} I_i \left(2\Omega\ddot{\theta}_i + \ddot{\theta}_i\right)$$

$$P_i^{diss} = \widetilde{D_i} \left(\Omega + \dot{\theta}_i\right)^2 = \widetilde{D_i} \left(\Omega^2 + 2\Omega\dot{\theta}_i + \dot{\theta}_i^2\right)$$

Substituting these two equation in 3.2 and assuming slow phase changes compared to the reference frequency, that is $|\dot{\theta}_i| << \Omega$, gives

$$I_i\Omega\ddot{\theta}_i + \frac{1}{2} I_i \ddot{\theta}_i = P_i^{mech} - \widetilde{D_i} \left(\Omega^2 + 2\Omega\dot{\theta}_i + \dot{\theta}_i^2\right) - P_i^{elec}$$

$$I_i\Omega\ddot{\theta}_i = P_i^{mech} - \widetilde{D_i}\Omega^2 - 2\widetilde{D_i}\Omega\dot{\theta}_i - P_i^{elec} \tag{3.3}$$

The electric power P_i^{elec} transmitted to the neighbouring nodes of node i is given by the active power, i.e. [4][5]

$$P_i^{elec} = \Re(S_i) = \Re\Big[E_i\overline{I_i}\Big] = \Re\left[E_i\left(\sum_{k=1}^{n} \overline{Y_{ik}^{SM} E_k}\right)\right] = \Re\left[\sum_{k=1}^{n} |E_i||E_k| e^{j(\theta_i - \theta_k)} |Y_{ik}^{SM}| e^{-j\alpha_{ik}^{SM}}\right]$$

$$= \sum_{k=1}^{n} |E_i E_k Y_{ik}^{SM}| cos(\theta_i - \theta_k - \alpha_{ik}^{SM}) = \sum_{k=1}^{n} |E_i E_k Y_{ik}^{SM}| sin(\theta_i - \theta_k - \alpha_{ik}^{SM} + \frac{\pi}{2}) \tag{3.4}$$

Where α_{ik}^{SM} is the angle of the admitance Y_{ik} and $E_i = |E_i|e^{j\theta_i}$ the internal voltage of the i-th machine. Substituting 3.4 in 3.3 leads to:

$$I_i\Omega\ddot{\theta}_i = P_i^{mech} - \widetilde{D_i}\Omega^2 - 2\widetilde{D_i}\Omega\dot{\theta}_i - \sum_{k=1}^{n} |E_i E_k Y_{ik}^{SM}| sin(\theta_i - \theta_k - \alpha_{ik}^{SM} + \frac{\pi}{2}) \tag{3.5}$$

In [3] ohmic losses are neglected such that the admittance is purely imaginary, $Y_{ik}^{SM} = jB_{ik}^{SM}$. This assumption simplifies 3.4 as follows:

$$P_i^{elec} = \sum_{k=1}^{n} |E_i E_k Y_{ik}^{SM}| sin(\theta_i - \theta_k - \alpha_{ik}^{SM} + \frac{\pi}{2}) = \sum_{k=1}^{n} P_{ik}^{max} sin(\theta_i - \theta_k) \tag{3.6}$$

where P_{ik}^{max} denotes the maximum real power transferred between any two nodes, which is given by

$$P_{ik}^{max} = |E_i||E_k|B_{ik}^{SM}$$

If there is no transmission line between two network elements, we have $P_{ik}^{max} = 0$. Substituting 3.6 in 3.3 leads to:

$$I_i\Omega\ddot{\theta}_i = P_i^{mech} - \widetilde{D_i}\Omega^2 - 2\widetilde{D_i}\Omega\dot{\theta}_i - \sum_{k=1}^{n} P_{ik}^{max} sin(\theta_i - \theta_k) \tag{3.7}$$

Defining: $K_{ik} = \frac{P_{ik}^{max}}{I_i\Omega}$, $\alpha = \frac{2\widetilde{D_i}}{I_i}$ and $P_i = \frac{P_i^{mech} - D_i\Omega^2}{I_i\Omega}$ results in equation (4) of [3]:

$$\ddot{\theta}_i = P_i - \alpha_i\dot{\theta}_i + \sum_{k=1}^{n} K_{ik} sin(\theta_k - \theta_i) \tag{3.8}$$

3.2 Energy Conservation

Notice that the second-order differential equation given in 3.8 is closely related to the swing equation given in the Introduction. Actually, the swing equation given in 3.8 can be written in the form of equation 1.2. This derivation is given in Chapter 4 for equation 3.5 where ohmic losses aren't neglected. Besides this interesting relation there is a property which may need some clarification. That is, in every network within a synchronous state: the sum of the consumed power equals the sum of the generated power. We clarify this property in the following derivation: Take $\alpha_i = \alpha\ \forall i$ and rescale the time: $s = \alpha t$, thus $t = \frac{s}{\alpha}$, this gives

$$\frac{d\theta_i}{ds} = \frac{d\theta_i}{dt}\frac{dt}{ds} = \frac{d\theta_i}{dt}\frac{1}{\alpha} \qquad \Longrightarrow \qquad \frac{d\theta_i}{dt} = \alpha\frac{d\theta_i}{ds}$$

$$\frac{d^2\theta_i}{ds^2} = \frac{d}{ds}\left(\frac{d\theta_i}{ds}\right) = \frac{d}{ds}\left(\frac{d\theta_i}{dt}\frac{dt}{ds}\right) = \frac{d^2\theta_i}{dt^2}\frac{dt}{ds}\frac{dt}{ds} + \frac{d\theta_i}{dt}\frac{d^2}{ds^2} = \frac{d^2\theta_i}{dt^2}\left(\frac{1}{\alpha}\right)^2 + \frac{d\theta_i}{dt}\frac{d}{ds}\left(\frac{1}{\alpha}\right) = \frac{d^2\theta_i}{dt^2}\left(\frac{1}{\alpha}\right)^2$$

Thus

$$\alpha^2\frac{d^2\theta_i}{ds^2} = P_i - \alpha^2\frac{d\theta_i}{ds} + \sum_j k_{ij} sin\left(\theta_j - \theta_i\right)$$

$$\frac{d^2\theta_i}{ds^2} = \frac{P_i}{\alpha^2} - \frac{d\theta_i}{ds} + \sum_j \frac{k_{ij}}{\alpha^2} sin\left(\theta_j - \theta_i\right)$$

$$\frac{d^2\theta_i}{ds^2} = \widetilde{P_i} - \frac{d\theta_i}{ds} + \sum_j \widetilde{k_{ij}} sin\left(\theta_j - \theta_i\right)$$

Remark that in a synchronous steady-state: $\frac{d\theta_i}{dt} = \frac{d^2\theta_i}{dt^2} = 0$, thus

$$\widetilde{P_i} + \sum_j \widetilde{k_{ij}} sin\left(\theta_j - \theta_i\right) = 0$$

$$-\sum_i \widetilde{P_i} = \sum_i\sum_j \widetilde{k_{ij}} sin\left(\theta_j - \theta_i\right) = \sum_{i=1}^{n}\sum_{j=1}^{i-1} \widetilde{k_{ij}} sin\left(\theta_j - \theta_i\right) + \sum_{i=1}^{n}\sum_{j=i+1}^{n} \widetilde{k_{ij}} sin\left(\theta_j - \theta_i\right)$$

By using the fact that $k_{jj} sin\left(\theta_j - \theta_j\right) = 0$ and $k_{ij} = k_{ji}$. With $sin(-x) = -sin(x)$ we obtain the following:

$$-\sum_i \widetilde{P_i} = \sum_{i<j} \widetilde{k_{ij}} sin\left(\theta_j - \theta_i\right) + \sum_{i>j} \widetilde{k_{ij}} sin\left(\theta_j - \theta_i\right) = 0$$

3.3 Example

In order to get a grip on the model we introduce a small example. In this example we consider a generator with phase angle θ_1 connected to a consumer with phase angle θ_2 by a transmission line with capacity $K = k_{12} = k_{21}$. The network is given in figure 3.1.

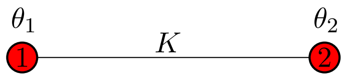


Figure 3.1: one consumer network

For simplicity we neglect ohmic losses, which enables us to use 3.8. Using this leads to the following system of differential equations:

$$\begin{cases} \frac{d^2\theta_1}{dt^2} = P_1 - \alpha_1 \frac{d\theta_1}{dt} + k_{12} sin\left(\theta_2 - \theta_1\right) \\ \frac{d^2\theta_2}{dt^2} = P_2 - \alpha_2 \frac{d\theta_2}{dt} + k_{21} sin\left(\theta_1 - \theta_2\right) \end{cases} \tag{3.9}$$

Recall that the system is synchronized if $\dot{\theta}_1 = \dot{\theta}_2$. Therefore, we'll focus on the difference in frequencies. Defining $\Delta P = P_2 - P_1$ and $\Delta\chi = \Delta\dot{\theta}$ where $\Delta\theta = \theta_2 - \theta_1$ leads to.

$$\frac{d^2\theta_2}{dt^2} - \frac{d^2\theta_1}{dt^2} = P_2 - P_1 - \alpha_2 \frac{d\theta_2}{dt} + \alpha_1 \frac{d\theta_1}{dt} + k_{21} sin\left(\theta_1 - \theta_2\right) - k_{12} sin\left(\theta_2 - \theta_1\right)$$
$$\Delta\ddot{\theta} = \Delta P - \alpha\Delta\dot{\theta} - k_{12} sin\left(\theta_2 - \theta_1\right) - k_{12} sin\left(\theta_2 - \theta_1\right)$$
$$\Delta\dot{\chi} = \Delta P - \alpha\Delta\chi - 2K sin(\Delta\theta)$$

Rewriting this second-order differential equation to a system of first-order differential equations, leads to:

$$\begin{cases} \Delta\dot{\chi} = \Delta P - \alpha\Delta\chi - 2K sin(\Delta\theta) \\ \Delta\dot{\theta} = \Delta\chi \end{cases} \tag{3.10}$$

Now, we calculate the equilibrium points of the system which automatically correspond to a synchronized equilibrium of the system.

$$\Delta P - 2K sin\left(\Delta\theta\right) = 0$$
$$sin\left(\Delta\theta\right) = sin\left(\pi - \Delta\theta\right) = \frac{\Delta P}{2K}$$

Thus, we have equilibrium points:

$$T_1 := \begin{pmatrix} \Delta\chi^* \\ \Delta\theta^* \end{pmatrix} = \begin{pmatrix} 0 \\ arcsin\left(\frac{\Delta P}{2K}\right) \end{pmatrix} \qquad T_2 := \begin{pmatrix} \Delta\chi^* \\ \Delta\theta^* \end{pmatrix} = \begin{pmatrix} 0 \\ \pi - arcsin\left(\frac{\Delta P}{2K}\right) \end{pmatrix} \tag{3.11}$$

Because $arcsin(x)$ is only defined on $-1 \le x \le 1$ there doesn't exist an equilibrium point if the load exceeds the capacity of the line, i.e. $\Delta P > 2K$. This means that the coupling strength must be higher than the critical coupling strength $K_c = \frac{\Delta P}{2}$. In the case that $\Delta P < 2K$ we have two equilibrium points and if $\Delta P = 2K$ there is only one equilibrium point given by

$$T_3 := \begin{pmatrix} \Delta\chi^* \\ \Delta\theta^* \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{\pi}{2} \end{pmatrix}$$

In order to determine the stability of the equilibrium points we calculate the Jacobi matrix:

$$J = \begin{pmatrix} \frac{\partial f_1}{\partial \Delta\chi} & \frac{\partial f_1}{\partial \Delta\theta} \\ \frac{\partial f_2}{\partial \Delta\chi} & \frac{\partial f_2}{\partial \Delta\theta} \end{pmatrix} = \begin{pmatrix} -\alpha & -2Kcos(\Delta\theta) \\ 1 & 0 \end{pmatrix}$$

Then we have the following Jacobian for equilibrium point T_1

$$J(T_1) = \begin{vmatrix} -\alpha - \lambda_1 & -2Kcos\left(arcsin\left(\frac{\Delta P}{2K}\right)\right) \\ 1 & 0 - \lambda_1 \end{vmatrix} = (-\alpha - \lambda_1)(-\lambda_1) + 2Kcos\left(arcsin\left(\frac{\Delta P}{2k}\right)\right)$$

$$= \lambda_1^2 + \alpha\lambda_1 + 2K\sqrt{1 - \left(\frac{\Delta P}{2K}\right)^2}$$

With eigenvalues given by

$$\lambda_1 = \frac{-\alpha \pm \sqrt{\alpha^2 - 4\left(2K\sqrt{1 - \left(\frac{\Delta P}{2K}\right)^2}\right)}}{2} = \frac{-\alpha}{2} \pm \sqrt{\left(\frac{\alpha}{2}\right)^2 - \sqrt{4K^2 - \Delta P^2}}$$

Similarly, we find for equilibrium point T_2:

$$J(T_2) = \begin{vmatrix} -\alpha - \lambda_2 & -2Kcos\left(\pi - arcsin\left(\frac{\Delta P}{2K}\right)\right) \\ 1 & 0 - \lambda_2 \end{vmatrix} = \lambda_2^2 + \alpha\lambda_2 - 2K\sqrt{1 - \left(\frac{\Delta P}{2K}\right)^2}$$

$$\lambda_2 = \frac{-\alpha \pm \sqrt{\alpha^2 - 4\left(-2K\sqrt{1 - \left(\frac{\Delta P}{2K}\right)^2}\right)}}{2} = \frac{-\alpha}{2} \pm \sqrt{\left(\frac{\alpha}{2}\right)^2 + \sqrt{4K^2 - \Delta P^2}}$$

Notice that one eigenvalue of λ_2 is always a positive real number while both eigenvalues of λ_1 always have a negative real part. Consequently, T_2 is an unstable equilibrium point and T_1 a stable equilibrium point.

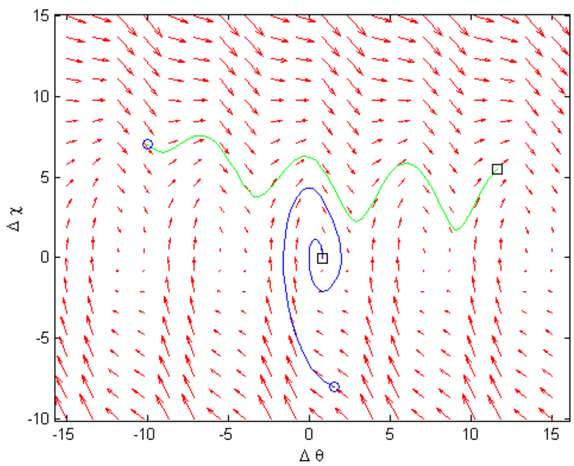
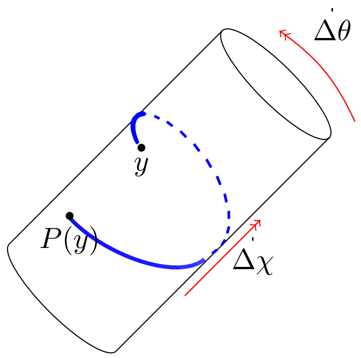


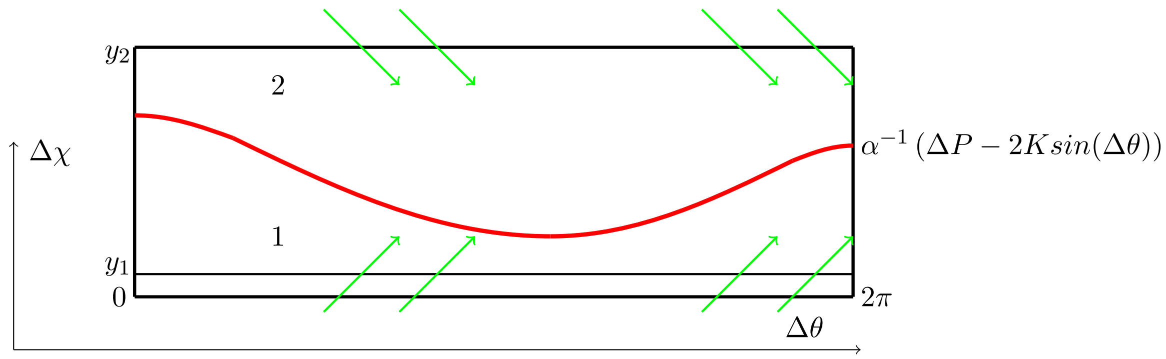
Figure 3.2: phase plane

In figure 3.2 we see the phase plane corresponding to system 3.10 with parameter values $\Delta P = 4, \alpha = 1$ and $K = 4$. The blue colored graph represents a trajectory of the solution for initial values $(\Delta\theta, \Delta\chi) = (\frac{\pi}{2}, -8)$ on time-domain $[0, 5]$. The green graph represents another trajectory for initial values $(-10, 7)$ on the same time-domain.

From the above analysis we know that there is no stable equilibrium point to our system if the load exceeds the capacity of the transmission line. In this case we have a power outage and all trajectories converge to a limit cycle. To show this we'll find a closed area R in which there exists a solution that stays in this area. Let $\alpha > 0$ and C be a solution of 3.10, then we create a rectangle R of width w and height h that encloses C. Because system 3.10 is invariant under addition of 2π, we define $w = 2\pi$ and observe that the phase space is a rolled up rectangle with undefined height h. This phase space is illustrated in the figure below where the blue line represents C.



In the figure above the trajectory C starts at $(0, y)$ and ends at $(2\pi, P(y))$ where P indicates the Poincaré map. We observe that at $\Delta\chi = 0$ mod 2π the points y and $P(y)$ don't necessarily coincide. In order to show that they actually do coincide, we show that there exists a fixed point y^* of P, i.e. $P(y^*) = y^*$. But before we do so, we enroll the above rectangle and we partition R in two regions partitioned by the nullcline $\Delta\chi = \alpha^{-1}(\Delta P - 2Ksin(\Delta\theta))$ as shown in the figure below.



In region 1 we have $\Delta\chi < \alpha^{-1}(\Delta P - 2Ksin(\Delta\theta))$, which leads to:

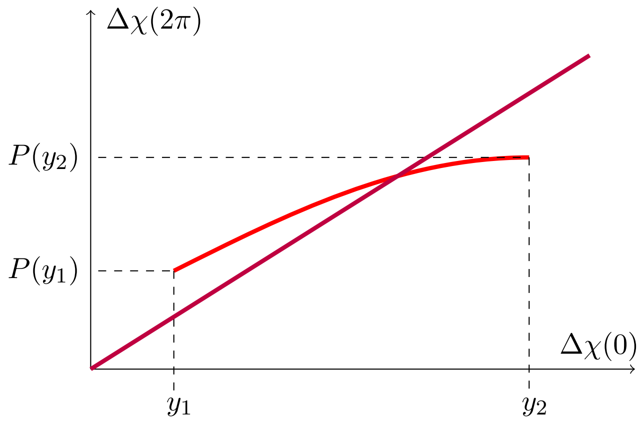
$$\Delta\dot{\chi} = \Delta P - \alpha\Delta\chi - 2Ksin(\Delta\theta) > \Delta P - \Delta P + 2Ksin(\Delta\theta) - 2Ksin(\Delta\theta) > 0$$

In region 2 we have $\Delta\chi > \alpha^{-1}(\Delta P - 2Ksin(\Delta\theta))$ leading to:

$$\Delta\dot{\chi} = \Delta P - \alpha\Delta\chi - 2Ksin(\Delta\theta) < 0$$

Let y_1 and y_2 fixed such that $0 < y_1 < \alpha^{-1}(\Delta P - 2K)$ and $y_2 > \alpha^{-1}(\Delta P + 2K)$. Let C be in the rectangle with height $h = y_2 - y_1$ and width $w = 2\pi$.

Now it still remains to be shown that there exists a fixed point y^* such that $P(y^*) = y^*$. To show that such a fixed point exists we need to know a little bit more about the graph $P(y)$. Notice that if we start at the point $(\Delta\theta, \Delta\chi) = (0, y_1)$ we have a strictly upward flow that won't return back to $\Delta\chi = y_1$ which means that $P(y_1) > y_1$. The same argument holds for $(\Delta\theta, \Delta\chi) = (0, y_2)$ where we have a strictly downward flow leading to $P(y_2) < y_2$. Further, we have that $P(y)$ is continuous by the theorem that solutions of differential equations depend continuously on the values of the initial conditions. Because in addition $P(y)$ is monotonic (otherwise trajectories would cross each other) we have by the intermediate value theorem that there must exist a fixed point as illustrated in the figure below. The intersection of the purple line with the red graph represents our fixed point.



Chapter 4

Comparative analysis of the EN-model and the SM-model

Both models can succesfully be applied to power-grids as we shall see later on in Chapter 5. But, before we apply both models to a test system we first need to perform a comparative analysis of both models. This will enable us to compare the results obtained by using these two models. This comparison is already done by Takashi Nishikawa and Adilson E. Motter in their paper "Comparative analysis of existing models for power-grid synchronization" [2]. Therefore, we'll follow their approach in comparing the EN-model with the SM-model. Because the effective network model and the synchronous motor model have their own kind of swing equation, it is a necessity to know how these two equations are related. From [2] it is known that these two swing equations are related to each other in the sense that they can be written in the form:

$$\frac{2H_i}{\omega_R}\ddot{\delta}_i + \frac{D_i}{\omega_R}\dot{\delta} = A_i - \sum_{j=1,j\neq i} K_{ij} sin(\delta_i - \delta_j - \gamma_{ij}) \tag{4.1}$$

where $K_{ij} > 0$ represents the strength of the dynamical coupling between nodes i and j, γ_{ij} the phase shift in the coupling between nodes i and j and A_{ij} along with D_i determine the inherent frequency.

The below derivation of the relation between the equation given in 4.1 and the equation used in the effective network model starts off with the general swing equation of the ith generator in a network with n-genarators:

$$\frac{2H_i}{\omega_R}\ddot{\delta}_i + \frac{D_i}{\omega_R}\dot{\delta}_i = P_{mi} - P_{ei} \tag{4.2}$$

Because the frequency of a generator is under active control, we assume that all generators are initially synchronized at the reference frequency in a steady state [2]. This leads to $\ddot{\delta}_i = \dot{\delta}_i = 0$ and $P_{mi} = P_{ei}$. From now on we'll denote the mechanical power input of a generator in a steady state power-grid by P_{gi}, thus $P_{gi} = P_{mi} = P_{ei}$ in a steady-state. This value P_{gi} is then held constant in studying stability under the assumptions of the classical model [4] and leads to the similar swing equation:

$$\frac{2H_i}{\omega_R}\ddot{\delta}_i + \frac{D_i}{\omega_R}\dot{\delta}_i = P_{gi} - P_{ei}$$

Recall, that the electrical power output is given by $P_i = Re\left(E_i\overline{I_i}\right)$ and notice that the effective admittance can be written as $Y_{ik}^{EN} = G_{ik}^{EN} + jB_{ik}^{EN}$, which leads to [4]:

$$P_i + jQ_i = E_i\overline{I_i} = E_i\left(\sum_{k=1}^{n}\overline{Y_{ik}^{EN}}\,\overline{E_k}\right) = |E_i|^2\overline{Y_{ii}^{EN}} + \sum_{k=1,k\neq i}^{n}|E_i||E_k|e^{j\delta_{ik}}|Y_{ik}^{EN}|e^{-j\alpha_{ik}^{EN}}$$

$$P_i = |E_i|^2G_{ii}^{EN} + \sum_{k=1,k\neq i}^{n}|E_iE_kY_{ik}^{EN}|cos(\delta_k - \delta_i + \alpha_{ik}^{EN})$$

Consequently,

$$\frac{2H_i}{\omega_R}\ddot{\delta}_i + \frac{D_i}{\omega_R}\dot{\delta}_i = P_{gi} - |E_i|^2G_{ii}^{EN} - \sum_{k=1,k\neq i}^{n}|E_iE_kY_{ik}^{EN}|cos(\delta_i - \delta_k - \alpha_{ik}^{EN})$$

or

$$\frac{2H_i}{\omega_R}\ddot{\delta}_i + \frac{D_i}{\omega_R}\dot{\delta}_i = P_{gi} - |E_i|^2G_{ii}^{EN} - \sum_{k=1,k\neq i}^{n}|E_iE_kY_{ik}^{EN}|sin(\delta_i - \delta_k - \alpha_{ik}^{EN} + \frac{\pi}{2})$$

Defining $A_i^{EN} = P_{g,i} - |E_i|^2G_{ii}^{EN}$, $K_{ij}^{EN} = |E_iE_jY_{ij}^{EN}|$ and $\gamma_{ij}^{EN} = \alpha_{ij}^{EN} - \frac{\pi}{2}$ leads to the swing equation given in 4.1.

In the SM-model we introduced the swing equation with respect to the j-th generator in a network with n-generators by:

$$\ddot{\theta}_j = P_j - \alpha_j\dot{\theta}_j + \sum_{k=1}^{n}K_{jk}sin(\theta_k - \theta_j) \tag{4.3}$$

or if we don't neglect ohmic losses (see 3.5):

$$I_i\Omega\ddot{\theta}_i = P_i^{mech} - \widetilde{D}_i\Omega^2 - 2\widetilde{D}_i\Omega\dot{\theta}_i - \sum_{k=1}^{n}|E_iE_kY_{ik}^{SM}|sin(\theta_i - \theta_k - \alpha_{ik}^{SM} + \frac{\pi}{2}) \tag{4.4}$$

We'll write equation 4.4 in the form given in 4.1. Remark, that the reference frequency in the SM-model is given by Ω which is equivalent to the reference frequency of the EN-model which is given by ω_R. As in the case of the EN-model, we assume that the power-grid is initially synchronized at the reference frequency ω_R in a steady-state. This gives, $\ddot{\theta} = \dot{\theta} = 0$ and

$$P_{g,i} = P_i^{mech} - \widetilde{D}_i\Omega^2 = P_i^{elec} = \sum_{k=1}^{n}|E_iE_kY_{ik}^{SM}|sin(\theta_i - \theta_k - \alpha_{ik}^{SM} + \frac{\pi}{2})$$

Now we rewrite 4.4:

$$I_i\omega_R\ddot{\theta}_i + 2\widetilde{D}_i\omega_R\dot{\theta}_i = P_{g,i} - \sum_{k=1}^{n}|E_iE_kY_{ik}^{SM}|sin(\theta_i - \theta_k - \alpha_{ik}^{SM} + \frac{\pi}{2}) \tag{4.5}$$

The inertia constant H is defined as the stored kinetic energy in megajoules at synchronous speed divided by the machine rating in mega-volt ampere (ref). Mathematically, this reads:

$$H = \frac{\frac{1}{2}I\omega_R^2}{S_{mach}}$$

Where S_{mach} is the rating of the machine in MVA (ref). Defining $D_i = \frac{2\widetilde{D_i}\omega_R^2}{S_{mach}}$ as damping term we obtain:

$$\frac{2H_i}{\omega_R}\ddot{\theta}_i + \frac{D_i}{\omega_R}\dot{\theta}_i = \frac{1}{S_{mach}}P_{g,i} - \frac{1}{S_{mach}}\sum_{k=1}^{n}|E_iE_kY_{ik}^{SM}|sin(\theta_i - \theta_k - \alpha_{ik}^{SM} + \frac{\pi}{2}) \qquad (4.6)$$

This last equation is normalized to a common MVA base which is commonly used in power systems studies and doesn't modify the structure of the equation [2]. This simply means that both terms on the right-hand side are in per unit (p.u) quantities and we can just write the equation as

$$\frac{2H_i}{\omega_R}\ddot{\theta}_i + \frac{D_i}{\omega_R}\dot{\theta}_i = P_{g,i} - \sum_{k=1}^{n}|E_iE_kY_{ik}^{SM}|sin(\theta_i - \theta_k - \alpha_{ik}^{SM} + \frac{\pi}{2}) \qquad (p.u) \quad (4.7)$$

or

$$\frac{2H_i}{\omega_R}\ddot{\theta}_i + \frac{D_i}{\omega_R}\dot{\theta}_i = P_{gi} - |E_i|^2G_{ii}^{SM} - \sum_{k=1,k\neq i}^{n}|E_iE_kY_{ik}^{SM}|sin(\theta_i - \theta_k - \alpha_{ik}^{SM} + \frac{\pi}{2}) \qquad (4.8)$$

Defining $A_i^{SM} = P_{g,i} - |E_i|^2G_{ii}^{SM}$, $K_{ik}^{SM} = |E_iE_kY_{ik}^{SM}|$ and $\gamma_{ik}^{SM} = \alpha_{ik}^{SM} - \frac{\pi}{2}$ leads to the swing equation given in 4.1.

Chapter 5

The nine-bus test system

In this chapter we apply both the synchronous motor model as well as the effective network model to a nine-bus test system. The physical network representation of the nine-bus test system along with its bus voltages and voltage angles is given in figure 5.1.

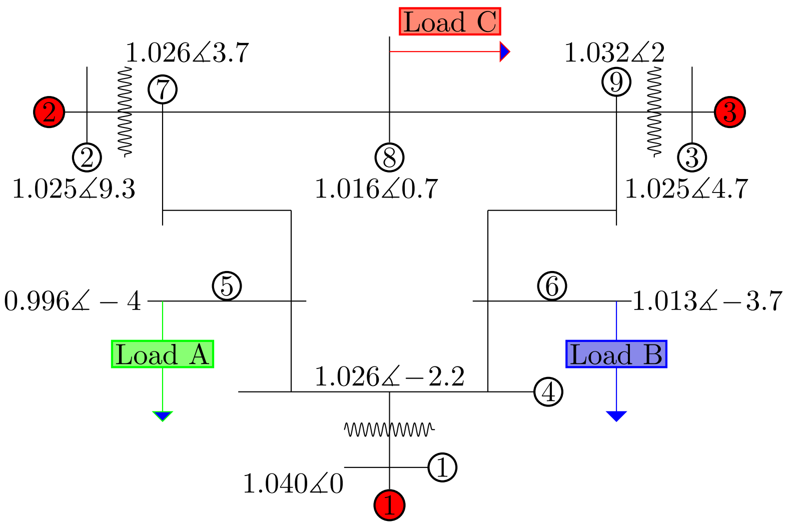


Figure 5.1: nine-bus test system

This network contains 3 generators indicated by red dots, 3 loads represented by colored arrows, 9 buses indicated by lines with numbers aside and 3 transformers indicated by wavy lines. The data concerning the three generators is given in table 5.1 and is taken from [4]. In this table, H denotes the stored energy at rated speed in MW·s and $x_d^{'}$ the transient reactance in p.u. on a 100-MVA base. This transient reactance could be used to estimate the electrical current during a fault condition when the circuit breaker interrupts this fault current. However, we shall only see this term reappear in the admittance matrices and the representation of the network machines.

parameters	Generator 1	Generator 2	Generator 3
type	hydro	steam	steam
$x_d^{'}$	0.0608	0.1198	0.1813
H	2364	640	301

Table 5.1: Generator Data

The initial conditions concerning the load-flow and impedances are given in the tables below and are also taken from [4]. Notice that the impedances are given by $Z_{ik} = R_{ik} + jX_{ik}$ where R_{ik} is the resistance and X_{ik} the reactance between elements i and k. The admittance can then be deduced from the impedance by $Y_{ik} = G_{ik} + jB_{ik} = 1/Z_{ik}$. On the other hand we have shunt admittances denoted by $B/2$ which is a result of transmission line modelling and will be treated in the section about transmission lines below. The power is given in complex form by $S = P + jQ$ where P denotes the Active/Real power and Q the reactive power.

Gen.No.	P_{gen}	Q_{gen}
1	71.6	27
2	163	6.7
3	85	-10.9

Load No.	P_{load}	Q_{load}
A	125	50
B	90	30
C	100	35

From Bus	To Bus	R	X	B/2	From Bus	To Bus	R	X	B/2
1	4	0	0.0576	0	2	7	0	0.0625	0
4	5	0.010	0.085	0.088	3	9	0	0.0586	0
4	6	0.017	0.092	0.079	7	8	0.0085	0.072	0.0745
5	7	0.032	0.161	0.153	9	8	0.0119	0.1008	0.1045
6	9	0.039	0.170	0.179					

Table 5.2: Impedance and load-flow data

5.0.1 Transmission lines

The links in a network represent transmission lines. For the network in figure 5.1 we use a so-called π-model [6]. In this π-model, two nodes are connected by an impedance. Each end of this impedance is then connected to the ground by a capacitor, both of equal capacitance. This π-model is illustrated in figure 5.2.

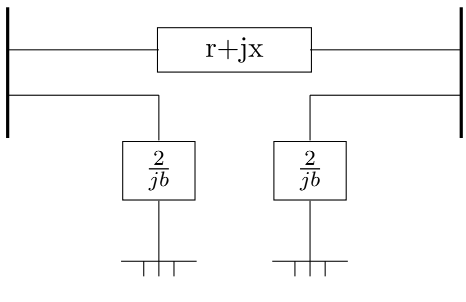


Figure 5.2: π-model

The equivalent admittance of the impedance and the shunt capacitors are given by

$$y_{imp} = g + jb = \frac{1}{r+jx} \qquad\qquad y_{shunt} = \frac{jb}{2}$$

respectively. Notice that the values of the shunt capacitors for the nine bus system are given in table 5.2. Putting this all together, we have the following admittance matrix representation of a transmission line:

$$\mathbf{Y}_{\textbf{transmission line}} = \begin{bmatrix} \frac{1}{r+jx} + \frac{jb}{2} & \frac{-1}{r+jx} \\ \frac{-1}{r+jx} & \frac{1}{r+jx} + \frac{jb}{2} \end{bmatrix}$$

In the following two sections we will deduce some matrices by applying both models to their corresponding network structures. This will enable us to model the generators and will lead to some results with respect to the phase coherence which can then be compared and analysed.

5.1 The effective network model applied

In both models we make use of a matrix to represent the network. Recall that the matrix used to represent the network in an effective network model is an effective admittance matrix. Before we'll derive this matrix we first need a matrix representing the physical network which we'll denote by $\mathbf{Y_0}$. In order to get this matrix we need a model for both the transmission lines and the loads. For the transmission lines we take the π-model which was treated on the previous page.

5.1.1 Loads

Recall that in the effective network model, the loads are represented by constant impedances. This means that the admittances of the loads are given by [5]:

$$Y_L = \frac{P_L - jQ_L}{|V_L|^2}$$

Where P_L is the active power of the load, Q_L the reactive power of the load and $|V_L|$ the magnitude of the corresponding bus voltage. These admittances are then added to the corresponding bus admittances in the admittance matrix $\mathbf{Y_0}$ of the network.

5.1.2 Admittance matrix of the nine-bus system

We are now able to compute the admittance matrix of the physical network $\mathbf{Y_0}$ and the adjusted admittance matrix $\mathbf{Y_0'}$ which takes the equivalent shunt admittances of the loads and the direct axis transient reactances of the generators into account. Here, we denote with y_{ij} the line admittance connecting bus i with bus j, $B_{ik}/2$ the pure imaginairy shunt capacitor admittance corresponding to the transmission line between buses i and k and we'll denote with $Y_{L,k}$ the equivalent load admittance conected with bus k. Notice that some of the diagonal elements are denoted by captial letters Y_{ik} with a value given below the matrix. The admittance matrix of the physical network is then given by:

$$\mathbf{Y_0} = \begin{bmatrix} y_{14} & 0 & 0 & -y_{14} & 0 & 0 & 0 & 0 & 0 \\ 0 & y_{27} & 0 & 0 & 0 & 0 & -y_{27} & 0 & 0 \\ 0 & 0 & y_{39} & 0 & 0 & 0 & 0 & 0 & -y_{39} \\ -y_{41} & 0 & 0 & Y_{44} & -y_{45} & -y_{46} & 0 & 0 & 0 \\ 0 & 0 & 0 & -y_{54} & Y_{55} & 0 & -y_{57} & 0 & 0 \\ 0 & 0 & 0 & -y_{64} & 0 & Y_{66} & 0 & 0 & -y_{69} \\ 0 & -y_{72} & 0 & 0 & -y_{75} & 0 & Y_{77} & -y_{78} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -y_{87} & Y_{88} & -y_{89} \\ 0 & 0 & -y_{93} & 0 & 0 & -y_{96} & 0 & -y_{98} & Y_{99} \end{bmatrix}$$

$$Y_{44} = y_{14} + y_{54} + y_{64} + j\tfrac{B_{45}}{2} + j\tfrac{B_{46}}{2} \qquad Y_{77} = y_{27} + y_{57} + y_{87} + j\tfrac{B_{57}}{2} + j\tfrac{B_{78}}{2}$$
$$Y_{55} = y_{45} + y_{75} + jB_{57} + j\tfrac{B_{45}}{2} \qquad Y_{88} = y_{78} + y_{98} + j\tfrac{B_{78}}{2} + j\tfrac{B_{89}}{2}$$
$$Y_{66} = y_{46} + y_{96} + jB_{46} + j\tfrac{B_{69}}{2} \qquad Y_{99} = y_{39} + y_{89} + y_{69} + j\tfrac{B_{89}}{2} + j\tfrac{B_{69}}{2}$$

5.1.3 Adjusted admittance matrix in the EN-model

The admittance matrix $\mathbf{Y_0}$ only includes the transmission lines. It measures how easy the network allows a current to flow from one bus to another as can be seen by Kirchoff's current law. Using the values given in table 5.2 leads to the physical admittance matrix given in the Appendix. The physical admittance matrix $\mathbf{Y_0}$ can also be abstractly denoted by [2]:

$$\mathbf{Y_0} = \begin{bmatrix} \mathbf{Y_0^{gg}} & \mathbf{Y_0^{gl}} \\ \mathbf{Y_0^{lg}} & \mathbf{Y_0^{ll}} \end{bmatrix} \tag{5.1}$$

where the matrix is seperated in four blocks with $\mathbf{Y_0^{gg}}$ corresponding to a block with dimensions $n_g \times n_g$ where n_g is the number of generator nodes in the system. This representation will help us to denote the adjusted matrix in terms of the physical admittance matrix along with the transient reactances and the equivalent impedances.
Before we compute the adjusted admittance matrix $\mathbf{Y_0'}$, we give a new representation of the nine bus system. Recall that in the effective network model, a generator is represented by a constant voltage source behind transient reactance. This means that the voltage source is of constant magnitude with variable phae δ which is connected to a terminal through a transient reactance $x'_{d,i}$. With this representation in mind we view the new nine bus network as in figure 5.3, where we renumbered the nodes.

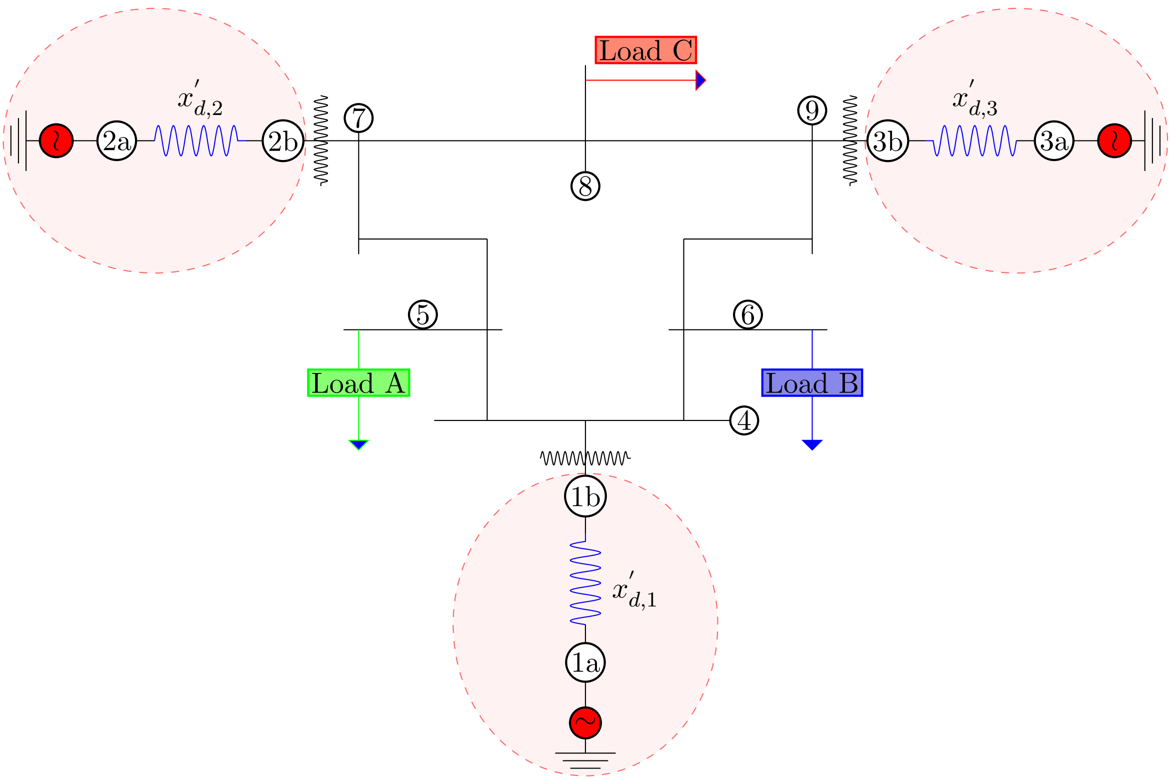


Figure 5.3: nine-bus test system regarding the EN-model

Observe that nodes 1a, 2a and 3a denote the internal nodes of the corresponding generators at which we have a constant voltage magnitude $|E_i|$ and variable phase δ_i. Nodes 1b, 2b and 3b denote terminal nodes with voltage magnitude $|V_i|$ and phase μ_i. The internal nodes are connected by a coil representing the transient reactances $x'_{d,i}$ corresponding to each generator. The representation of the nine bus system gven in figure 5.3 allows us to derive the adjusted admittance matrix $\mathbf{Y_0'}$ as in [2].

The adjusted admittance matrix has dimensions $N \times N$ where N is the sum of the internal nodes, terminal nodes and the number of buses $N = n_{int} + n_{term} + n_{bus}$. In case there's a load connected to a bus we count only the bus in n_{bus}. This means that $\mathbf{Y_0'}$ has dimensions 12×12. Where we define the adjusted admittance matrix $\mathbf{Y_0'}$ in abstract manner by [2]:

$$\begin{bmatrix} \mathbf{Y_d} & -\mathbf{Y_d} & \mathbf{0} \\ -\mathbf{Y_d} & \mathbf{Y_0^{gg}} + \mathbf{Y_d} & \mathbf{Y_0^{gl}} \\ \mathbf{0} & \mathbf{Y_0^{lg}} & \mathbf{Y_0^{ll}} + \widetilde{\mathbf{Y_L}} \end{bmatrix}$$

The matrix in the lower right corner denoted by $\widetilde{\mathbf{Y_L}}$ is a diagonal matrix with equivalent load admittances $Y_{L,i}$ at element $\widetilde{\mathbf{Y_{L,ii}}}$ if there is a load connected to bus i, otherwise $\widetilde{\mathbf{Y_{L,ii}}} = \mathbf{0}$. The matrix $\mathbf{Y_d}$ is a $n_g \times n_g$ diagonal matrix with transient reactance $x'_{d,i}$ at element $\mathbf{Y_{d,ii}}$. Now it's time to compute this adjusted admittance matrix $\mathbf{Y_0'}$ for the nine bus system. This leads to the matrix:

$$\begin{array}{cccccccccccc} \text{(1a)} & \text{(2a)} & \text{(3a)} & \text{(1b)} & \text{(2b)} & \text{(3b)} & \text{(4)} & \text{(5)} & \text{(6)} & \text{(7)} & \text{(8)} & \text{(9)} \end{array}$$

$$\begin{bmatrix}
\frac{1}{jx'_{d,1}} & 0 & 0 & \frac{-1}{jx'_{d,1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{jx'_{d,2}} & 0 & 0 & \frac{-1}{jx'_{d,2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{jx'_{d,3}} & 0 & 0 & \frac{-1}{jx'_{d,3}} & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{-1}{jx'_{d,1}} & 0 & 0 & \frac{-j}{x_{14}} + \frac{1}{jx'_{d,1}} & 0 & 0 & \frac{j}{x_{14}} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{-1}{jx'_{d,2}} & 0 & 0 & \frac{-j}{x_{27}} + \frac{1}{jx'_{d,2}} & 0 & 0 & 0 & 0 & \frac{j}{x_{27}} & 0 & 0 \\
0 & 0 & \frac{-1}{jx'_{d,3}} & 0 & 0 & \frac{-j}{x_{39}} + \frac{1}{jx'_{d,3}} & 0 & 0 & 0 & 0 & 0 & \frac{j}{x_{39}} \\
0 & 0 & 0 & \frac{j}{x_{14}} & 0 & 0 & Y_{77} & \frac{-1}{r_{45}+jx_{45}} & \frac{-1}{r_{46}+jx_{46}} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{-1}{r_{45}+jx_{45}} & Y_{88} & 0 & \frac{-1}{r_{57}+jx_{57}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{-1}{r_{46}+jx_{46}} & 0 & Y_{99} & 0 & 0 & \frac{-1}{r_{69}+jx_{69}} \\
0 & 0 & 0 & 0 & \frac{j}{x_{27}} & 0 & 0 & \frac{-1}{r_{75}+jx_{75}} & 0 & Y_{10} & \frac{-1}{r_{78}+jx_{78}} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-1}{r_{78}+jx_{78}} & Y_{11} & \frac{-1}{r_{89}+jx_{89}} \\
0 & 0 & 0 & 0 & 0 & \frac{j}{x_{39}} & 0 & 0 & \frac{-1}{r_{69}+jx_{69}} & 0 & \frac{-1}{r_{89}+jx_{89}} & Y_{12}
\end{bmatrix}$$

$$Y_{77} = \frac{B_{45}}{2} + \frac{B_{46}}{2} + \frac{1}{r_{45}+jx_{45}} + \frac{1}{r_{46}+jx_{46}} - \frac{j}{x_{14}} \qquad Y_{10} = \frac{B_{57}}{2} + \frac{B_{78}}{2} + \frac{1}{r_{75}+jx_{75}} + \frac{1}{r_{78}+jx_{78}} - \frac{j}{x_{27}}$$

$$Y_{88} = \frac{B_{57}}{2} + \frac{B_{45}}{2} + \frac{1}{r_{45}+jx_{45}} + \frac{1}{r_{57}+jx_{57}} + Y_{L,5} \qquad Y_{11} = \frac{B_{78}}{2} + \frac{B_{89}}{2} + \frac{1}{r_{78}+jx_{78}} + \frac{1}{r_{89}+jx_{89}} + Y_{L,8}$$

$$Y_{99} = \frac{B_{46}}{2} + \frac{B_{69}}{2} + \frac{1}{r_{46}+jx_{46}} + \frac{1}{r_{69}+jx_{69}} + Y_{L,6} \qquad Y_{12} = \frac{B_{89}}{2} + \frac{B_{69}}{2} + \frac{1}{r_{69}+jx_{69}} + \frac{1}{r_{89}+jx_{89}} - \frac{j}{x_{39}}$$

Notice, that the node numbers of the nine bus system given figure 5.3 are shown above the adjusted admittance matrix $\mathbf{Y_0'}$. These node numbers correspond to the columns underneath them and can be computed in the same order for the rows. By indexing the adjusted admittance matrix in this way we can easily compute his elements. For example, the admittance between terminal node $1b$ and bus 4 is given by $\mathbf{Y'_{0,1b,4}} = \mathbf{Y'_{0,4,1b}} = \frac{\mathbf{j}}{\mathbf{x_{14}}}$, that is the admittance of the transformator. On the other hand there is no direct connection between 1b and bus 5, thus $\mathbf{Y'_{0,1b,5}} = \mathbf{Y'_{0,5,1b}} = \mathbf{0}$

In order to compute the matrix $\mathbf{Y_0'}$ we first need to compute the equivalent admittances of the loads. These can be calculated using the bus voltages given in figure 5.1 and the load-flow data from table 5.2. The bus voltages in figure 5.1 are stated as a magnitude $|V|$ with corresponding phase μ in degrees aside them. We'll define them as $V = |V|e^{j\mu}$ with μ in radians. These equivalent admittances of the loads take the values:

$$Y_{L,5} = 1.2601 - 0.5040j \qquad Y_{L,6} = 0.8770 - 0.2923j \qquad Y_{L,8} = 0.9688 - 0.3391j$$

Using this information we compute the matrix $\mathbf{Y_0'}$ which is given in the Appendix. Now we use a Kron reduction in order to eliminate all non-generator nodes. For this procedure we partition the adjusted admittance matrix $\mathbf{Y_0'}$ in four blocks as can be seen in the matrices on the previous page. Let the green block be denoted by $\mathbf{Y'_{0,3\times3}}$, the blue block by $\mathbf{Y'_{0,3\times9}}$, the yellow block by $\mathbf{Y'_{0,9\times3}}$ and the red block by $\mathbf{Y_{0,9\times9}}$. Then, the effective admittance matrix is computed with the formula:

$$\mathbf{Y^{EN}} = \mathbf{Y'_{0,3\times3}} - \mathbf{Y'_{0,3\times9}} \cdot \mathbf{Y^{-1'}_{0,9\times9}} \cdot \mathbf{Y'_{0,9\times3}}$$

This effective network admittance matrix is also given in the Appendix and will be used to obtain the matrix $\mathbf{P}$ given in 2.4. Before we're able to compute the $\mathbf{P}$ matrix we need to calculate the internal voltages of the generators, that is $E_i = |E_i|e^{j\delta_i}$. These are the voltages whose phase angle coincides with the rotor angle of the corresponding generator by assumption of the classical model. These internal voltages change over time, but since we know the initial conditions of the nine-bus system we're able to calculate the initial internal voltages with the formulas [5]:

$$E_i = V_i + jx_{d,i} \cdot I_i \tag{5.2}$$

$$I_i = \frac{P_i - jQ_i}{\overline{V_i}} \tag{5.3}$$

This leads to:

$$E_1 = 1.0558 + 0.0419j \qquad E_2 = 0.9885 + 0.3549j \qquad E_3 = 0.9900 + 0.2322j$$

Now, we're able to compute the $\mathbf{P}$ matrix and thus its eigenvalues, where the smallest nonzero eigenvalue is given by:

$$\alpha_2 = 75.5117$$

From which we obtain $\beta_{opt} = 17.3795$. Using this, we can specify optimal droop parameters R_i and damping coefficients D_i for each generator if one of those are already given. In the case of no damping we have optimal droop parameters:

$$R_1 = 0.0012 \qquad R_2 = 0.0045 \qquad R_3 = 0.0096$$

Some other parameters concerning the nine-bus system are given in the table below. Notice that the 2-norm of the symmetric and antisymmetric parts justify our reasoning in the diagonalizing part of the $\mathbf{P''}$ matrix.

phase difference	effective network		physical network	
mean $\lvert\delta_{ij}\rvert/\pi$	mean $\lvert G_{ij}\rvert$	mean $\lvert B_{ij}\rvert$	mean$\lvert G_{ij}\rvert$	mean $\lvert B_{ij}\rvert$
0.0647	0.2365	1.2756	0.9499	11.9540

2-norm $\mathbf{P''}$	
Symmetric	Antisymmetric
125.1476	2.0741

Table 5.3: Network data

5.2 The Synchronous motor model applied

In the synchronous motor model we represent the nine-bus system in the same manner as in the effective network model. Thus, we still represent the physical network of the nine-bus system by the physical admittance matrix $\mathbf{Y_0}$. However, the adjusted matrix $\mathbf{Y_0'}$ will change since the loads are modelled as synchronous motors rather than constant impedances. This means that the loads are modelled by the same equation of motion as that used for generators, that is equation 3.5 or 3.8. The complication that arises by choosing this modelling method is that we don't exactly know the values of certain parameters as the inertia constants H_i and the transient reactances $x'_{d,i}$ of the motors. But before we dive in to some of the specifics of this modelling problem, we start with giving an alternative representation of the nine-bus system in which the loads are replaced by synchronous motors:

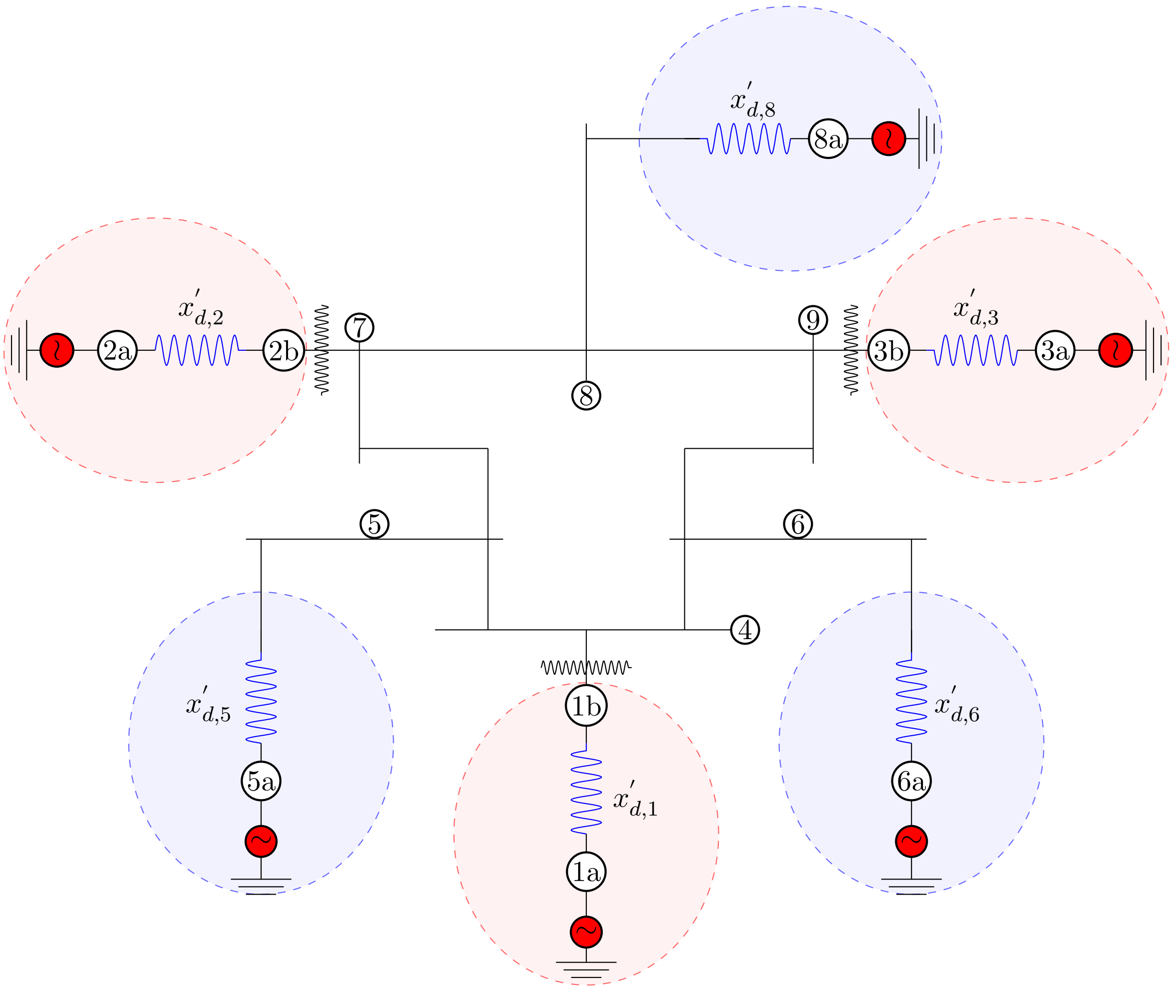


Figure 5.4: nine-bus test system regarding the SM-model

Notice, that we've added some nodes and numbered them. This way of representing the nine-bus system leads to a model with six internal nodes and transient reactances. In addition we've three machines who are already synchronized. These are the machines representing the loads. This isn't necessarily the case but for the sake of simplicity we take this as an assumption.

5.2.1 Adjusted admittance matrix in the SM-model

The above representation allows us to derive an adjusted admittance matrix of the nine-bus system with respect to the synchronous motor model. This adjusted matrix that will be denoted by $\mathbf{Y_0''}$ will be a $N' \times N'$ matrix where $N' = n'_{int} + n'_{term} + n_{bus}$ with n'_{int} the number of internal nodes, n'_{term} the number of terminal nodes an n_{bus} the number of buses. This means that the nine-bus system has dimensions 15×15 corresponding to all network nodes given in figure 5.4. Abstractly, we can denote this adjusted admittance matrix by:

$$\mathbf{Y_0''} = \begin{bmatrix} \mathbf{Y_d} & \mathbf{0} & -\mathbf{Y_d} & \mathbf{0} \\ \mathbf{0} & \mathbf{Y_d'} & \mathbf{0} & -\widetilde{\mathbf{Y_d}} \\ -\mathbf{Y_d} & \mathbf{0} & -\mathbf{T_0'} + \mathbf{Y_d} & \mathbf{T_0} \\ \mathbf{0} & -\widetilde{\mathbf{Y_d^T}} & \mathbf{T_0^T} & \mathbf{Y_0^{ll}} + \mathbf{Y_T} \end{bmatrix} \tag{5.4}$$

where $\mathbf{Y_d'}$ is defined as diagonal matrix with the transient reactances of motors on its diagonal. Because of the size of the adjusted matrix, we'll just compute clored blocks used in the abstract notation of $\mathbf{Y_0''}$ along with corresponding column and row indexing. Notice that the matrix $\widetilde{\mathbf{Y_d^T}}$ is the transpose of $\widetilde{\mathbf{Y_d}}$.

$$\mathbf{T_0} = \begin{array}{c} \begin{array}{cccccc} \textcircled{4} & \textcircled{5} & \textcircled{6} & \textcircled{7} & \textcircled{8} & \textcircled{9} \end{array} \\ \begin{bmatrix} \frac{j}{x_{14}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{j}{x_{27}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{j}{x_{39}} \end{bmatrix} \begin{array}{c} \textcircled{1b} \\ \textcircled{2b} \\ \textcircled{3b} \end{array} \end{array} \qquad \mathbf{Y_d'} = \begin{array}{c} \begin{array}{ccc} \textcircled{5a} & \textcircled{6a} & \textcircled{8a} \end{array} \\ \begin{bmatrix} \frac{1}{x'_{d,5}} & 0 & 0 \\ 0 & \frac{1}{x'_{d,6}} & 0 \\ 0 & 0 & \frac{1}{x'_{d,8}} \end{bmatrix} \begin{array}{c} \textcircled{5a} \\ \textcircled{6a} \\ \textcircled{8a} \end{array} \end{array}$$

$$\mathbf{Y_T} = \begin{array}{c} \begin{array}{cccccc} \textcircled{4} & \textcircled{5} & \textcircled{6} & \textcircled{7} & \textcircled{8} & \textcircled{9} \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{jx'_{d,5}} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{jx'_{d,6}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{jx'_{d,8}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{array}{c} \textcircled{4} \\ \textcircled{5} \\ \textcircled{6} \\ \textcircled{7} \\ \textcircled{8} \\ \textcircled{9} \end{array} \end{array} \qquad \mathbf{T_0'} = \begin{array}{c} \begin{array}{ccc} \textcircled{1b} & \textcircled{2b} & \textcircled{3b} \end{array} \\ \begin{bmatrix} \frac{j}{x_{14}} & 0 & 0 \\ 0 & \frac{j}{x_{27}} & 0 \\ 0 & 0 & \frac{j}{x_{39}} \end{bmatrix} \begin{array}{c} \textcircled{1b} \\ \textcircled{2b} \\ \textcircled{3b} \end{array} \end{array}$$

$$\widetilde{\mathbf{Y_d}} = \begin{array}{c} \begin{array}{cccccc} \textcircled{4} & \textcircled{5} & \textcircled{6} & \textcircled{7} & \textcircled{8} & \textcircled{9} \end{array} \\ \begin{bmatrix} 0 & \frac{1}{jx_{d,5}} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{jx_{d,6}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{jx_{d,8}} & 0 \end{bmatrix} \begin{array}{c} \textcircled{5a} \\ \textcircled{6a} \\ \textcircled{8a} \end{array} \end{array}$$

Since the matrix $\mathbf{Y_0^{ll}}$ is part of the physical admittance matrix given in 5.1, we're able to compute the adjusted admittance matrix if we know the values of the transient reactances $x'_{d,5}, x'_{d,6}, x'_{d,8}$. However, from [2] we know that the transient reactances are estimated by the formula $x'_{d,i} = (92.8) \cdot P_i^{-1.3}$. Thus:

$$x'_{d,5} = 0.2914 \qquad x'_{d,6} = 0.2098 \qquad x'_{d,8} = 0.2331$$

Using these values allows us to compute the adjusted admttance matrix $\mathbf{Y_0''}$ which is given in the Appendix. Now, we use a Kron reduction in order to eliminate all nodes except the six internal nodes corresponding to the generators and loads. This means that we use formula 2.1 by partitioning the matrix given in 5.4 as illustrated in the matrix below:

$$\begin{pmatrix} \mathbf{Y_d} & \mathbf{0} & -\mathbf{Y_d} & \mathbf{0} \\ \mathbf{0} & \mathbf{Y_d'} & \mathbf{0} & -\widetilde{\mathbf{Y_d}} \\ -\mathbf{Y_d} & \mathbf{0} & -\mathbf{T_0'} + \mathbf{Y_d} & \mathbf{T_0} \\ \mathbf{0} & -\widetilde{\mathbf{Y_d^T}} & \mathbf{T_0^T} & \mathbf{Y_0^{ll}} + \mathbf{Y_T} \end{pmatrix} \tag{5.5}$$

The resulting effective admittance matrix will be denoted by $\mathbf{Y^{SM}}$ and is given in the Appendix. Notice that this effective admittance matrix isn't enough for modelling the swing equation, because we're still missing the values of the inertia constants H_i and the internal voltage magnitudes with corresponding angles of the synchronous motors. However, the inertia constants can be determined by the formula $H_i = 0.04 \cdot P_i$ from [2]. This leads to

$$H_5 = 0.0500 \qquad H_6 = 0.0360 \qquad H_8 = 0.0400$$

Now, by using formulas 5.2 and 5.3 we're able to compute the internal voltages corresponding to the synchronous motors:

$$E_5 = 1.1650 + 0.2851i \qquad E_6 = 1.0849 + 0.1166i \qquad E_8 = 1.0934 + 0.2428i$$

At this moment we have computed all parameters which are needed in order to model the nine-bus system. The modeling of the nine bus system will be discussed in the next section.

5.3 Modeling of the nine-bus system

In this section we'll model the nine-bus system with both the EN-model as well as the SM-model. Recall that the network is represented differently in both models. In the case of the effective network model we have an effective network with 3 oscillators. And in the case of the sychronous motor model we have a network consisting of 6 oscillators. The red nodes in figure 5.5 indicate the generators and the black nodes the loads. Because all transient reactances $x'_{d,i}$ are positive, we have a complete graph in both models. This is a result which follows from a property of Kron reduction and can be found in [2].

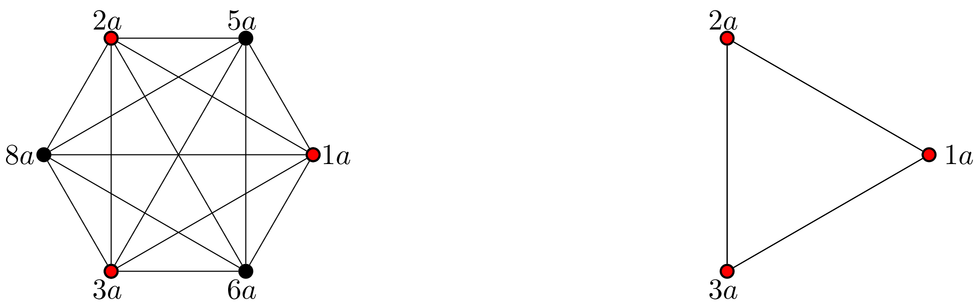


Figure 5.5: network in case of the SM model (left) and the EN model (right)

In Chapter 4 we showed that the swing equations of both models can be rewritten in a general form given by [2]:

$$\frac{2H_i}{\omega_R}\ddot{\delta}_i + \frac{D_i}{\omega_R}\dot{\delta} = A_i - \sum_{j=1, j\neq i} K_{ij} sin(\delta_i - \delta_j - \gamma_{ij})$$

We'll use this swing equation under the assumption of no-damping in order to model the nine-bus system, that is

$$\frac{2H_i}{\omega_R}\ddot{\delta}_i = A_i - \sum_{j=1, j\neq i} K_{ij} sin(\delta_i - \delta_j - \gamma_{ij}) \tag{5.6}$$

From Chapter 4 we know that the parameters A_i, K_{ij}, Y_{ij} and γ_{ij} are different in both models as can also be seen in the table below:

Parameters.	EN-model	SM-model
K_{ij}	$K_{ij}^{EN} = \lvert E_i E_j Y_{ij}^{EN}\rvert$	$K_{ij}^{SM} = \lvert E_i E_j Y_{ij}^{SM}\rvert$
γ_{ij}	$\gamma_{ij}^{EN} = \alpha_{ij}^{EN} - \frac{\pi}{2}$	$\gamma_{ij}^{SM} = \alpha_{ij}^{SM} - \frac{\pi}{2}$
Y_{ij}	$Y_{ij}^{EN} = \lvert Y_{ij}^{EN}\rvert exp(j\alpha_{ij}^{EN})$	$Y_{ij}^{SM} = \lvert Y_{ij}^{SM}\rvert exp(j\alpha_{ij}^{SM})$
A_i generators	$A_i^{EN} = P_i - \lvert E_i\rvert^2 G_{ii}^{EN}$	$A_i^{SM} = P_i - \lvert E_i\rvert^2 G_{ii}^{SM}$
A_i loads	-	$A_i^{SM} = -P_i - \lvert E_i\rvert^2 G_{ii}^{SM}$

Notice that there is no value given in the EN-model for the loads A_i since they are modeled as equivalent impedances. The parameters as stated in the table above are determined with matlab and given in the Appendix.

Using the parameters of the Swing equation as stated in the Appendix, we're able to solve the Swing equation given in 5.6 numerically. Before doing this, we rewrite 5.6 as a system of two first-order differential equations:

$$\frac{2H_i}{\omega_R}\dot{\omega}_i = A_i - \sum_{j=1, j\neq i} K_{ij} sin(\delta_i - \delta_j - \gamma_{ij})$$

$$\dot{\delta}_i = \omega_i - \omega_R$$

With ω_R the reference frequency of the network. This system is solved in matlab with ode45 which is based on the Runge-Kutta method. The initial angles are the angles corresponding to the internal voltages E_1, E_2 and E_3, i.e. the angles in degrees:

$$\measuredangle E1 = 2.2704 \qquad \measuredangle E2 = 19.7510 \qquad \measuredangle E3 = 13.2023$$

The initial frequency is taken equal to the reference frequency of the network, which is $60Hz$ or in radians per second $60 \cdot 2\pi$.
The graphs of the angles corresponding to the generators are shown in figures 5.6 and 5.7 below. These graphs are called Swing curves and show how the angles measured from a synchronously rotating reference frame behave over time. The time interval chosen is $[0, 2]$ as shown on the x-axis of the plots. The angles are measured in degrees and are shown on the y-axis.

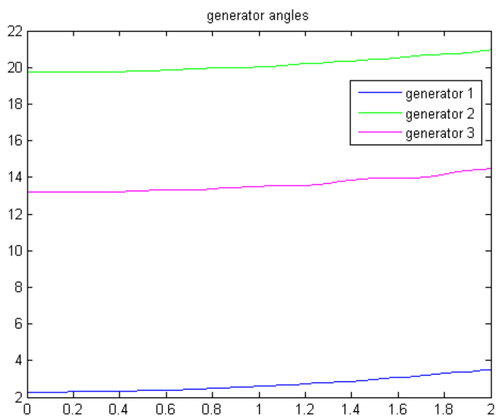


Figure 5.6: EN-model

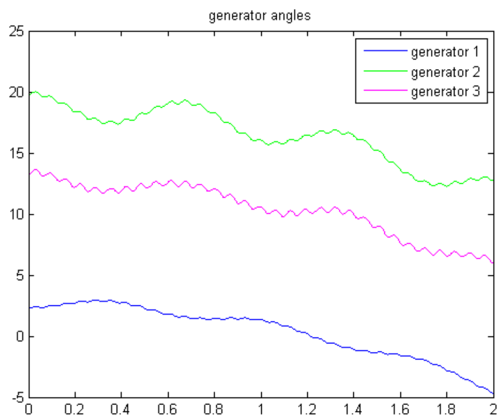
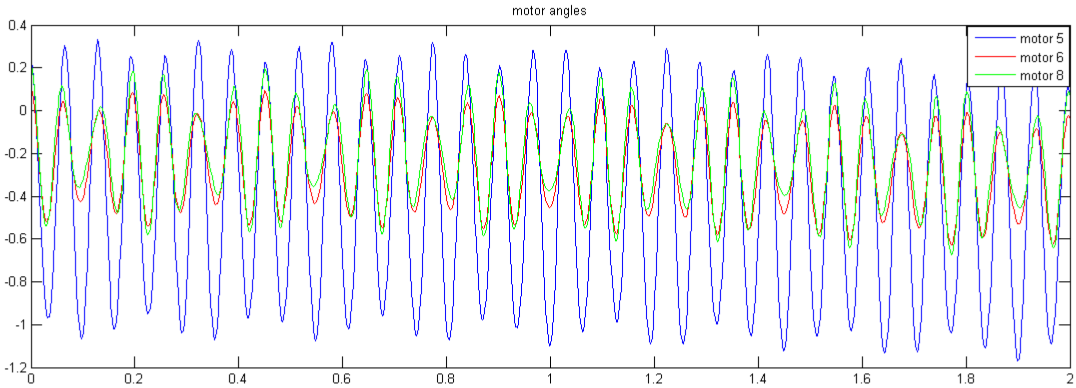


Figure 5.7: SM-model

From the figures above we see that the angles in the EN-model follow an slightly upward trend while the angles in the SM model follow a downward trend. This trend can also be observed when we extend the time interval. These differences in trends can be a consequence of the difference in modeling of the loads. We know that in the EN-model the loads are considered to be constant impedances. Thus the demand of electrical power in the network doesn't change. On the other hand, the loads in the SM-model are considered to be synchronous motors which means that the demand is changing over time.

Because the loads are modeled as synchronous motors in the SM-model we're able to compute their angles. The figure below shows the Swing curves for the synchronous motors. We



observe that the angles of the motors changes almost the same over time. Thus the motors are indeed synchronized. We can also observe that the phase difference is sometimes positive and sometimes negative. This behavior corresponds to an acceleration of the rotor if the phase difference increases or a deceleration of the rotor if the phase difference decreases.

5.4 Comparison between the EN-model and SM-model

In order to compare both models we introduce the concept of the order parameter. The order parameter is defined by [3]:

$$r(t) = \frac{1}{N} \sum_k e^{j\theta_k(t)} \tag{5.7}$$

With $\theta_k(t)$ the phase difference of oscillator k at time t and N the number of oscillators in the network. The phase of the order parameter is the average phase of all oscillators in the network. Hence, the order parameter r gives an indication on the phase cohesiveness. If the order parameter takes on the value 0 than we have an incoherent network and if $r = 1$ than the network is said to be coherent. We can illustrate this order parameter by a unit circle in which a line takes the angle of r with respect to the positive x-axis and the length of the line represents the modulus of r. Because the order parameter depends on the time we will just show it for particular times. Figures 5.8 and 5.9 give the order parameter at $t = 1$ for both models.

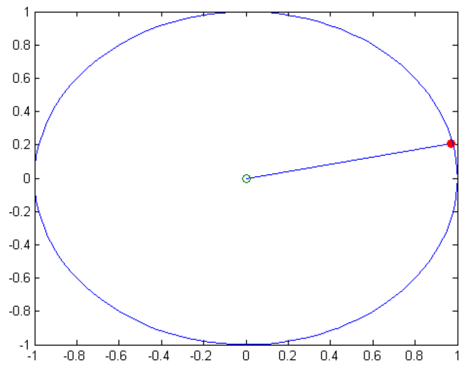


Figure 5.8: EN-model

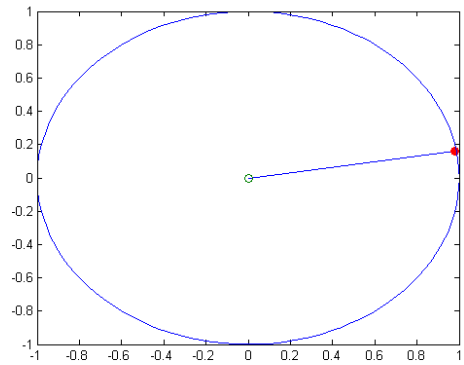


Figure 5.9: SM-model

It is probably more interesting to see the modulus of the order parameter against the time. This is illustrated in the figure below for both models.

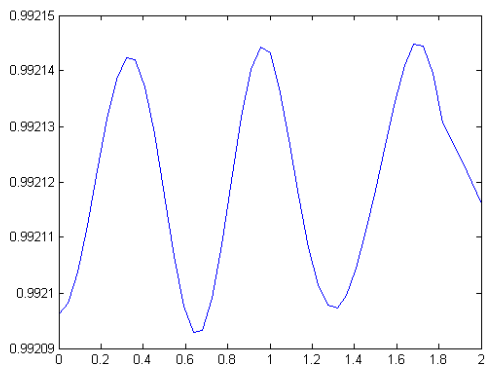


Figure 5.10: EN-model

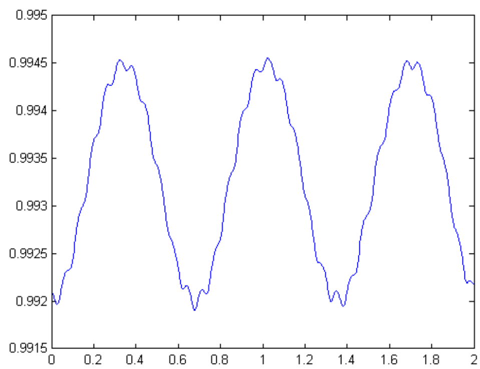


Figure 5.11: SM-model

Notice that figures 5.8 and 5.9 are almost the same. Figures 5.10 and 5.11 give a better view on the order parameter and we see that the phase coherence doesn't change much in the time interval $[0, 2]$. In both models are the generators coherent since the modulus is near 1.

Chapter 6

Concluding remarks

The results of this study show that the droop parameter and the damping coefficient can be adjusted so that the stability of the synchronous state can be improved. On the other hand, we've shown that the governing equation of motion for both models can be rewritten in a general form. The parameters for this general second-order differential equation are determined with Matlab for both models and stated in the Appendix. The effects of modeling the loads as synchronous motors and as constant impedances were shown in the Swing curves obtained by solving both swing equations. Because of the differences between the swing curves of the EN-model and SM-model we can conclude that the modeling of the loads has a substantial impact on the behavior of the generators in the nine-bus system. At the end of the thesis we considered the order parameter and evaluated it at different times. By programming the order parameter for the nine-bus system in Matlab it could be seen that the oscillators in both models are coherent during the time interval $[0, 2]$.

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Chapter 7

Appendix A

7.1 Computations regarding the nine bus system

In chapter 4 we saw a few matrices with respect to the nine bus test system. The matrices regarding the physical network, the adjusted network, the matrix **P** and the effective network are given below and are computed with Matlab.

The physical network is given by $\mathbf{Y_0}$ and reads:

.

$$\begin{pmatrix}
-17.3611j & 0 & 0 & 17.3611j & 0 & 0 & 0 & 0 & 0 \\
0 & -16j & 0 & 0 & 0 & 0 & 16j & 0 & 0 \\
0 & 0 & -17.0648j & 0 & 0 & 0 & 0 & 0 & 17.0648j \\
17.3611j & 0 & 0 & 3.3074-39.3089j & -1.3652+11.6041j & -1.9422+10.5107j & 0 & 0 & 0 \\
0 & 0 & 0 & -1.3652+11.6041j & 2.5528-17.3382j & 0 & -1.1876+5.9751j & 0 & 0 \\
0 & 0 & 0 & -1.9422+10.5107j & 0 & 3.2242-15.8409j & 0 & 0 & -1.2820+5.5882j \\
0 & 16j & 0 & 0 & -1.1876+5.9751j & 0 & 2.8047-35.4456j & -1.6171+13.6980j & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1.6171+13.6980j & 2.7722-23.3032j & -1.1551+9.7843j \\
0 & 0 & 17.0648j & 0 & 0 & -1.2820+5.5882j & 0 & -1.1551+9.7843j & 2.4371-32.1539j
\end{pmatrix}$$

The adjusted admittance matrix $\mathbf{Y_0'}$ regarding the effective network model is given by:

.

$$\begin{pmatrix}
-16.4474j & 0 & 0 & 16.4474j & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -8.3472j & 0 & 0 & 8.3472j & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -5.5157j & 0 & 0 & 5.5157j & 0 & 0 & 0 & 0 & 0 & 0 \\
16.4474j & 0 & 0 & -33.8085j & 0 & 0 & 17.3611j & 0 & 0 & 0 & 0 & 0 \\
0 & 8.3472j & 0 & 0 & -24.3472j & 0 & 0 & 0 & 0 & 16j & 0 & 0 \\
0 & 0 & 5.5157j & 0 & 0 & -22.5806j & 0 & 0 & 0 & 0 & 0 & 17.0648j \\
0 & 0 & 0 & 17.3611j & 0 & 0 & 3.3074-39.3089j & -1.3652+11.6041j & -1.9422+10.5107j & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1.3652+11.6041j & 3.8129-17.8423j & 0 & -1.1876+5.9751j & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1.9422+10.5107j & 0 & 4.1012-16.1333j & 0 & 0 & -1.2820+5.5882j \\
0 & 0 & 0 & 0 & 16j & 0 & 0 & -1.1876+5.9751j & 0 & 2.8047-35.4456j & -1.6171+13.6980j & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1.6171+13.6980j & 3.7410-23.6423j & -1.1551+9.7843j \\
0 & 0 & 0 & 0 & 0 & 17.0648j & 0 & 0 & -1.2820+5.5882j & 0 & -1.1551+9.7843j & 2.4371-32.1539j
\end{pmatrix}$$

The effective admittance matrix $\mathbf{Y^{EN}}$:

.

$$\begin{pmatrix}
0.8452-2.9880j & 0.2869+1.5131j & 0.2095+1.2257j \\
0.2869+1.5131j & 0.4199-2.7238j & 0.2132+1.0880j \\
0.2095+1.2257j & 0.2132+1.0880j & 0.2769-2.3681j
\end{pmatrix}$$

The matrix $\mathbf{P}$: .

$$\begin{pmatrix} 24.1842 & -13.5330 & -10.6512 \\ -44.3533 & 77.5886 & -33.2353 \\ -78.3067 & -73.9188 & 152.2254 \end{pmatrix}$$

The adjusted admittance matrix $\mathbf{Y_0''}$ regarding the synchronous motor model is given by:
.

$$\begin{pmatrix}
-16.4474j & 0 & 0 & 0 & 0 & 0 & 16.4474j & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -8.3472j & 0 & 0 & 0 & 0 & 0 & 8.3472j & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -5.5157j & 0 & 0 & 0 & 0 & 0 & 5.5157j & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -3.4320j & 0 & 0 & 0 & 0 & 0 & 0 & 3.4320j & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -4.7666j & 0 & 0 & 0 & 0 & 0 & 0 & 4.7666j & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -4.2899j & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4.2899j & 0 \\
16.4474j & 0 & 0 & 0 & 0 & 0 & -33.8085j & 0 & 0 & 17.3611j & 0 & 0 & 0 & 0 & 0 \\
0 & 8.3472j & 0 & 0 & 0 & 0 & 0 & -24.3472j & 0 & 0 & 0 & 0 & 16j & 0 & 0 \\
0 & 0 & 5.5157j & 0 & 0 & 0 & 0 & 0 & -22.5806j & 0 & 0 & -1.9422{+}10.5107j & 0 & 0 & 17.0648j \\
0 & 0 & 0 & 0 & 0 & 0 & 17.3611j & 0 & 0 & 3.3074{-}39.3089j & -1.3652{+}11.6041j & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 3.4320j & 0 & 0 & 0 & 0 & 0 & -1.3652{+}11.6041j & 2.5528{-}20.7702j & 0 & -1.1876{+}5.9751j & 0 & 0 \\
0 & 0 & 0 & 0 & 4.7666j & 0 & 0 & 0 & 0 & -1.9422{+}10.5107j & 0 & 3.2242{-}20.6075j & 0 & 0 & -1.2820{+}5.5882j \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 16j & 0 & 0 & -1.1876{+}5.9751j & 0 & 2.8047{-}35.4456j & -1.6171{+}13.6980j & 0 \\
0 & 0 & 0 & 0 & 0 & 4.2899j & 0 & 0 & 0 & 0 & 0 & 0 & -1.6171{+}13.6980j & 2.7722{-}27.5932j & -1.1551{+}9.7843j \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 17.0648j & 0 & 0 & -1.2820{+}5.5882j & 0 & -1.1551{+}9.7843j & 2.4371{-}32.1539j
\end{pmatrix}$$

The effective admittance matrix $\mathbf{Y^{SM}}$:

.

$$\begin{pmatrix}
0.1914{-}4.2014j & -0.0738{+}0.8162j & -0.0695{+}0.6377j & 0.0240{+}1.1100j & -0.0143{+}1.4270j & -0.0637{+}0.5496j \\
-0.0738{+}0.8162j & 0.1202{-}3.3067j & -0.0052{+}0.6460j & -0.0233{+}0.5826j & -0.0437{+}0.4378j & 0.0260{+}1.0843j \\
-0.0695{+}0.6377j & -0.0052{+}0.6460j & 0.1042{-}2.7351j & -0.0246{+}0.2620j & -0.0241{+}0.6355j & 0.0226{+}0.7758j \\
0.0240{+}1.1100j & -0.0233{+}0.5826j & -0.0246{+}0.2620j & 0.0634{-}2.5160j & -0.0154{+}0.4027j & -0/0237{+}0.3221j \\
-0.0143{+}1.4270j & -0.0437{+}0.4378j & -0.0241{+}0.6355j & -0.0154{+}0.4027j & 0.1313{-}3.0793j & -0.0328{+}0.4021j \\
-0.0637{+}0.5496j & 0.0260{+}1.0843j & 0.0226{+}0.7758j & -0.0237{+}0.3221j & -0.0328{+}0.4021j & 0.0725{-}2.9240j
\end{pmatrix}$$

The parameters of the Swing equation for both the EN-model as well as the SM-model are given in the table below:

Parameters.	EN-model	SM-model		
K_{ij}	$K^{EN}_{1a,2a} = 1.7091$	$K^{SM}_{1a,2a} = 0.9095$	$K^{SM}_{1a,5a} = 1.4071$	$K^{SM}_{2a,5a} = 0.7345$
	$K^{EN}_{1a,3a} = 1.3361$	$K^{SM}_{1a,3a} = 0.6893$	$K^{SM}_{1a,8a} = 0.6548$	$K^{SM}_{2a,6a} = 0.5042$
	$K^{EN}_{2a,3a} = 1.1841$	$K^{SM}_{2a,3a} = 0.69$	$K^{SM}_{1a,6a} = 1.6454$	$K^{SM}_{2a,8a} = 1.2759$
		$K^{SM}_{3a,5a} = 0.3210$	$K^{SM}_{3a,8a} = 0.8840$	$K^{SM}_{3a,6a} = 0.7057$
		$K^{SM}_{5a,6a} = 0.5274$	$K^{SM}_{5a,8a} = 0.4338$	$K^{SM}_{6a,8a} = 0.5420$
γ_{ij}	$\gamma^{EN}_{1a,2a} = -0.1874$	$\gamma^{SM}_{1a,2a} = 0.0902$	$\gamma^{SM}_{1a,5a} = -0.0217$	$\gamma^{SM}_{2a,5a} = 0.0399$
	$\gamma^{EN}_{1a,3a} = -0.1693$	$\gamma^{SM}_{1a,3a} = 0.1085$	$\gamma^{SM}_{1a,8a} = 0.1153$	$\gamma^{SM}_{2a,6a} = 0.0995$
	$\gamma^{EN}_{2a,3a} = -0.1935$	$\gamma^{SM}_{2a,3a} = 0.0080$	$\gamma^{SM}_{1a,6a} = 0.0100$	$\gamma^{SM}_{2a,8a} = -0.0240$
		$\gamma^{SM}_{3a,5a} = 0.0937$	$\gamma^{SM}_{3a,8a} = -0.0291$	$\gamma^{SM}_{3a,6a} = 0.0380$
		$\gamma^{SM}_{5a,6a} = 0.0382$	$\gamma^{SM}_{5a,8a} = 0.0733$	$\gamma^{SM}_{6a,8a} = 0.0814$
A_i	$A^{EN}_{1a} = -0.2276$	$A^{SM}_{1a} = 0.5023$	$A^{SM}_{2a} = 1.4974$	$A^{SM}_{3a} = 0.7422$
	$A^{EN}_{2a} = 1.1669$	$A^{SM}_{5a} = -1.3411$	$A^{SM}_{6a} = -1.0563$	$A^{SM}_{8a} = -1.0909$
	$A^{EN}_{3a} = 0.5636$			

7.2 Appendix B

The first code is used to model the nine-bus system with the EN-model

```
%EN-model
clear all; clc;

%generator data
base=100;
x_dacc1=0.0608;      x_dacc2=0.1198;    x_dacc3=0.1813;               %in pu
H1=2364/base;        H2=640/base;       H3=301/base;

%Line and transformer data, all values in pu on 220kV, 100MVA base
X14=0.0576;       %transformer reactance without transient reactance x_dacc
X27=0.0625;       %transformer reactance without transient reactance x_dacc
X39=0.0586;       %transformer reactance without transient reactance x_dacc
X45=0.0850;          R45=0.01;          halfYB45=0.088;
X46=0.092;           R46=0.0170;        halfYB46=0.079;
X57=0.161;           R57=0.032;         halfYB57=0.153;
X69=0.17;            R69=0.039;         halfYB69=0.179;
X78=0.072;           R78=0.0085;        halfYB78=0.0745;
X89=0.1008;          R89=0.0119;        halfYB89=0.1045;

%Active power P and Reactive power Q
P1gen=71.6/base;     Q1gen=27/base;      P1load=0;        Q1load=0;
P2gen=163/base;      Q2gen=6.7/base;     P2load=0;        Q2load=0;
P3gen=85/base;       Q3gen=-10.9/base;   P3load=0;        Q3load=0;
P5gen=0;             Q5gen=0;            P5load=125/base; Q5load=50/base;
P6gen=0;             Q6gen=0;            P6load=90/base;  Q6load=30/base;
P8gen=0;             Q8gen=0;            P8load=100/base; Q8load=35/base;

%Bus voltages and angles(degrees)
E1=1.040;            angleE1=0;          %slack bus
E2=1.025;            angleE2=9.3;        E22=E2*exp(1i*angleE2*(pi/180));
E3=1.025;            angleE3=4.7;        E33=E3*exp(1i*angleE3*(pi/180));
E4=1.026;            angleE4=-2.2;       E44=E4*exp(1i*angleE4*(pi/180));
E5=0.996;            angleE5=-4;         E55=E5*exp(1i*angleE5*(pi/180));
E6=1.013;            angleE6=-3.7;       E66=E6*exp(1i*angleE6*(pi/180));
E7=1.026;            angleE7=3.7;        E77=E7*exp(1i*angleE7*(pi/180));
E8=1.016;            angleE8=0.7;        E88=E8*exp(1i*angleE8*(pi/180));
E9=1.032;            angleE9=2.0;        E99=E9*exp(1i*angleE9*(pi/180));

%Current into the network at buses 1,2 & 3; transient internal voltages,
%and equivalent shunt admittances are given for the loads at buses 5,6 & 8.
I1=(P1gen-1i*Q1gen)/(E1);
I2=(P2gen-1i*Q2gen)/(conj(E22));
I3=(P3gen-1i*Q3gen)/(conj(E33));
E1acc=E1+1i*x_dacc1*I1;
E2acc=E22+1i*x_dacc2*I2;
E3acc=E33+1i*x_dacc3*I3;
```

```
Y_L5=((P5load-1i*Q5load)/(E5^2));
Y_L6=((P6load-1i*Q6load)/(E6^2));
Y_L8=((P8load-1i*Q8load)/(E8^2));

%Impedance matrix elements
Z14=X14*1i;          Z11=Z14;
Z27=X27*1i;          Z22=Z27;
Z39=X39*1i;          Z33=Z39;
Z41=X14*1i;          Z45=R45+1i*X45;  Z46=R46+1i*X46;  Z44=Z41+Z45+Z46;
Z54=R45+1i*X45;      Z57=R57+1i*X57;  Z55=Z54+Z57;
Z64=R46+1i*X46;      Z69=R69+1i*X69;  Z66=Z64+Z69;
Z75=R57+1i*X57;      Z72=X27*1i;      Z78=R78+X78*1i;  Z77=Z75+Z72+Z78;
Z87=R78+X78*1i;      Z89=R89+X89*1i;  Z88=Z87+Z89;
Z93=X39*1i;          Z98=R89+X89*1i;  Z96=R69+X69*1i;  Z99=Z93+Z98+Z96;

%admittance matrix
Y14=1/(Z14);   Y41=Y14;    Y11=Y14;
Y27=1/(Z27);   Y72=Y27;    Y22=Y27;
Y39=1/(Z39);   Y93=Y39;    Y33=Y39;
Y45=1/Z45;                Y54=Y45;    Y46=1/Z46;  Y64=Y46;
Y57=1/Z57;                Y75=Y57;    Y69=1/Z69;  Y96=Y69;
Y78=1/Z78;                Y87=Y78;    Y98=1/Z98;  Y89=Y98;
Y44=Y14+Y54+Y64+halfYB45*1i+halfYB46*1i;
Y55=Y45+Y75+halfYB57*1i+halfYB45*1i;
Y66=Y46+Y96+halfYB46*1i+halfYB69*1i;
Y77=Y27+Y57+Y87+halfYB57*1i+halfYB78*1i;
Y88=Y78+Y98+halfYB78*1i+halfYB89*1i;
Y99=Y39+Y89+Y69+halfYB89*1i+halfYB69*1i;


Y=[Y11,0,0,-Y14,0,0,0,0,0; 0,Y22,0,0,0,0,-Y27,0,0; 0,0,Y33,0,0,0,0,0,-Y39;...
   -Y41,0,0,Y44,-Y45,-Y46,0,0,0; 0,0,0,-Y54,Y55,0,-Y57,0,0;...
   0,0,0,-Y64,0,Y66,0,0,-Y69; 0,-Y72,0,0,-Y75,0,Y77,-Y78,0;...
   0,0,0,0,0,0,-Y87,Y88,-Y89;0,0,-Y93,0,0,-Y96,0,-Y98,Y99];

Yd = [1/(x_dacc1*1i),0,0; 0,1/(x_dacc2*1i),0; 0,0,1/(x_dacc3*1i)];
Y0accent([1:3],[1:3]) = Yd;
Y0accent([1:3],[4:6]) = -Yd;
Y0accent([1:3],[7:12]) = zeros(3,6);
Y0accent([4:6],[1:3]) = -Yd;
Y0accent([4:6],[4:6]) = Y([1:3],[1:3])+Yd;
Y0accent([4:6],[7:12]) = Y([1:3],[4:9]);
Y0accent([7:12],[1:3]) = zeros(6,3);
Y0accent([7:12],[4:6]) = transpose(Y0accent([4:6],[7:12]));
Y0accent([7:12],[7:12]) =Y([4:9],[4:9]);
Y0accent(8,8)=Y0accent(8,8)+Y_L5;
Y0accent(9,9)=Y0accent(9,9)+Y_L6;
Y0accent(11,11)=Y0accent(11,11)+Y_L8;
```

```
%Applying Kron reduction as in the nature physics paper
Ylinksboven = Y0accent([1:3],[1:3]);
Yrechtsboven = Y0accent([1:3],[4:12]);
Ylinksonder = Y0accent([4:12],[1:3]);
Yrechtsonder = Y0accent([4:12],[4:12]);
Ynieuw = Ylinksboven-Yrechtsboven*inv(Yrechtsonder)*Ylinksonder;  %or Y^EN

%{
%%%%%%%%%%%Also computes Ynieuw/Y^EN
Yd = [1/(x_dacc1*1i),0,0; 0,1/(x_dacc2*1i),0; 0,0,1/(x_dacc3*1i)];
Yt = [(-1/(X14))*1i,0,0; 0,(-1/(X27))*1i,0; 0,0,(-1/(X39))*1i];
Ygl = zeros(3,6);  Ygl(1,1)=(1/X14)*1i; Ygl(2,4)=(1/X27)*1i; Ygl(3,6)=(1/X39)*1i;
Y0accent([1:3],[1:3]) = Yd;
Y0accent([1:3],[4:6]) = -Yd;
Y0accent([1:3],[7:12]) = zeros(3,6);
Y0accent([4:6],[1:3]) = -Yd;
Y0accent([4:6],[4:6]) = Yt+Yd;
Y0accent([4:6],[7:12]) = Ygl;
Y0accent([7:12],[1:3]) = zeros(6,3);
Y0accent([7:12],[4:6]) = transpose(Ygl);
Y0accent([7:12],[7:12]) = [halfYB45*1i+halfYB46*1i+Y45+Y46+Y14, -Y45, -Y46,0,0,0;...
                           -Y45,halfYB57*1i+halfYB45*1i+Y45+Y57+Y_L5,0,-Y57,0,0;...
                           -Y46,0,halfYB46*1i+halfYB69*1i+Y46+Y69+Y_L6,0,0,-Y69;...
                           0,-Y57,0,halfYB57*1i+halfYB78*1i+Y57+Y78+Y27,-Y78,0;...
                           0,0,0,-Y78,halfYB78*1i+halfYB89*1i+Y78+Y89+Y_L8,-Y89;...
                           0,0,-Y69,0,-Y89,halfYB89*1i+halfYB69*1i+Y69+Y89+Y39];


%Applying Kron reduction as in the nature physics paper
Ylinksboven = Y0accent([1:3],[1:3]);
Yrechtsboven = Y0accent([1:3],[4:12]);
Ylinksonder = Y0accent([4:12],[1:3]);
Yrechtsonder = Y0accent([4:12],[4:12]);
Ynieuw = Ylinksboven-Yrechtsboven*inv(Yrechtsonder)*Ylinksonder;
%}

%parameters
K1a2a=abs(E1acc*E2acc*Ynieuw(1,2))
K1a3a=abs(E1acc*E3acc*Ynieuw(1,3))
K2a3a=abs(E2acc*E3acc*Ynieuw(2,3))
gamma1a2a=angle(Ynieuw(1,2))-(pi/2)
gamma1a3a=angle(Ynieuw(1,3))-(pi/2)
gamma2a3a=angle(Ynieuw(2,3))-(pi/2)
A1a=P1gen-(abs(E1acc)^2)*real(Ynieuw(1,1))
A2a=P2gen-(abs(E2acc)^2)*real(Ynieuw(2,2))
A3a=P3gen-(abs(E3acc)^2)*real(Ynieuw(3,3))
```

```
%Power angle equations
Pe1c=@(delta1,delta2,delta3)  (abs(E1acc))^2*real(Ynieuw(1,1))+...
    abs(E1acc)*abs(E2acc)*abs(Ynieuw(1,2))*cos(delta1-delta2-angle(Ynieuw(1,2)))+...
    abs(E1acc)*abs(E2acc)*abs(Ynieuw(1,3))*cos(delta1-delta3-angle(Ynieuw(1,3)));
Pe2c=@(delta1,delta2,delta3)  (abs(E2acc))^2*real(Ynieuw(2,2))+...
    abs(E2acc)*abs(E1acc)*abs(Ynieuw(2,1))*cos(delta2-delta1-angle(Ynieuw(2,1)))...
    +abs(E2acc)*abs(E3acc)*abs(Ynieuw(2,3))*cos(delta2-delta3-angle(Ynieuw(2,3)));
Pe3c=@(delta1,delta2,delta3)  (abs(E3acc))^2*real(Ynieuw(3,3))...
    +abs(E3acc)*abs(E1acc)*abs(Ynieuw(3,1))*cos(delta3-delta1-angle(Ynieuw(3,1)))...
    +abs(E3acc)*abs(E2acc)*abs(Ynieuw(3,2))*cos(delta3-delta2-angle(Ynieuw(3,2)));

omegaR=60*2*pi;
ff =@(tt,deltaa)[deltaa(4)-omegaR; deltaa(5)-omegaR; deltaa(6)-omegaR;...
    (omegaR/(2*H1))*(P1gen-Pe1c(deltaa(1),deltaa(2),deltaa(3)));...
    (omegaR/(2*H2))*(P2gen-Pe2c(deltaa(1),deltaa(2),deltaa(3)));...
    (omegaR/(2*H3))*(P3gen-Pe3c(deltaa(1),deltaa(2),deltaa(3))) ];
[tt,xaa] = ode45(ff,[0 2],[angle(E1acc) angle(E2acc) angle(E3acc) omegaR omegaR omegaR]);

figure % create new figure
subplot(2,1,1) % first subplot
plot(tt,xaa(:,1)*(180/pi),'b')
hold on
plot(tt,xaa(:,2)*(180/pi),'g')
hold on
plot(tt,xaa(:,3)*(180/pi),'m')
title('generator angles')
subplot(2,1,2) % second subplot
plot(tt,(xaa(:,4)));
hold on
plot(tt,(xaa(:,5)));
hold on
plot(tt,(xaa(:,6)));
title('frequencies')
figure % create new figure
plot(tt,(xaa(:,2)-xaa(:,1))*(180/pi))
hold on
plot(tt,(xaa(:,3)-xaa(:,1))*(180/pi),'c')
title('Angle differences')

%{
%order parameter with unit circle
xy = linspace(0,2*pi,120);
magn = 21;%length(xaa);
%order parameter with unit circle
r=(1/3)*(exp(1i*xaa(magn,1))+exp(1i*xaa(magn,2))+exp(1i*xaa(magn,3)))
r1 =real(r); r2 =imag(r);
figure
plot([0,r1],[0,r2])
hold on
```

```
scatter(0,0)
hold on
scatter(r1,r2,'filled')
hold on
plot(cos(xy), sin(xy))
%}
r=(1/3)*(exp(1i*xaa(:,1))+exp(1i*xaa(:,2))+exp(1i*xaa(:,3)))
modulus=abs(r);
xy = linspace(0,2,45);
figure
plot(xy,modulus)
set(gca, 'YTickLabel', get(gca,'YTick'))

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Nature Physics; Spontaneous Synchrony in power-grid networks

%Values given in Supplementary Table S1
%Physical network values
meanabsGphysical=sum(sum(abs(real(triu(Y,1)))))/9
meanabsBphysical=sum(sum(abs(imag(triu(Y,1)))))/9
%effective network values
meanabsGeffective=sum(sum(abs(real(triu(Ynieuw,1)))))/3
meanabsBeffective=sum(sum(abs(imag(triu(Ynieuw,1)))))/3
%mean absolute value angles divided by pi
meanphasedifferences=(abs(angle(E1acc)-angle(E2acc))...
+abs(angle(E2acc)-angle(E1acc))+abs(angle(E1acc)-angle(E3acc))...
+abs(angle(E3acc)-angle(E1acc))+abs(angle(E2acc)-angle(E3acc))...
+abs(angle(E3acc)-angle(E2acc)))/(6*pi)


%Values given in Table 1:Structural properties and synchronization
%Stability of the Systems considered.
P= zeros(3,3);
P(1,2)= (omegaR*abs(E1acc)*abs(E2acc))/(2*H1)*(real(Ynieuw(1,2))...
    *sin(angle(E1acc)-angle(E2acc))-imag(Ynieuw(1,2))*cos(angle(E1acc)-angle(E2acc)));
P(1,3)= (omegaR*abs(E1acc)*abs(E3acc))/(2*H1)*(real(Ynieuw(1,3))...
    *sin(angle(E1acc)-angle(E3acc))-imag(Ynieuw(1,3))*cos(angle(E1acc)-angle(E3acc)));
P(1,1)=-P(1,2)-P(1,3);
P(2,1)= (omegaR*abs(E2acc)*abs(E1acc))/(2*H2)*(real(Ynieuw(2,1))...
    *sin(angle(E2acc)-angle(E1acc))-imag(Ynieuw(2,1))*cos(angle(E2acc)-angle(E1acc)));
P(2,3)= (omegaR*abs(E2acc)*abs(E3acc))/(2*H2)*(real(Ynieuw(2,3))...
    *sin(angle(E2acc)-angle(E3acc))-imag(Ynieuw(2,3))*cos(angle(E2acc)-angle(E3acc)));
P(2,2)=-P(2,1)-P(2,3);
P(3,1)= (omegaR*abs(E3acc)*abs(E1acc))/(2*H3)*(real(Ynieuw(3,1))...
    *sin(angle(E3acc)-angle(E1acc))-imag(Ynieuw(3,1))*cos(angle(E3acc)-angle(E1acc)));
P(3,2)= (omegaR*abs(E3acc)*abs(E2acc))/(2*H3)*(real(Ynieuw(3,2))...
    *sin(angle(E3acc)-angle(E2acc))-imag(Ynieuw(3,2))*cos(angle(E3acc)-angle(E2acc)));
P(3,3)=-P(3,1)-P(3,2);
```

```
eigenvaluesP=eig(P)
%We see that all eigenvalues of P are positive real numbers.
%So the real part of lambda attains its minimum at -beta/2
%The eigenvalues for the optimal system are given in lambda1=lambda2
beta1=2*sqrt(eigenvaluesP(1));
beta2=2*sqrt(eigenvaluesP(2));
beta3=2*sqrt(eigenvaluesP(3));
lambda1=real(-(beta2/2)+(1/2)*sqrt(beta2^2-4*eigenvaluesP(2)))
lambda2=real(-(beta2/2)-(1/2)*sqrt(beta2^2-4*eigenvaluesP(2)))
%deduce optimal droop parameters in the case of no damping D=0;
R1 = (1)/(4*H1*sqrt(eigenvaluesP(2)))
R2 = (1)/(4*H2*sqrt(eigenvaluesP(2)))
R3 = (1)/(4*H3*sqrt(eigenvaluesP(2)))
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%system heterogeneity
B=angle(Ynieuw(:))*(180/pi);
C=[B(4),B(7),B(8)];
C1=[B(1),B(4),B(5),B(7),B(8),B(9)];
SystemhetrY=std(C)/mean(C)
D=angle(Y(:))*(180/pi);
E=[D(10),D(19),D(20),D(28),D(29),D(30),D(37),D(38),D(39),D(40),...
    D(46),D(47),D(48),D(49),D(50),D(55),D(56),D(57),D(58),D(59),D(60),...
    D(64),D(65),D(66),D(67),D(68),D(69),D(70),D(73),D(74),D(75),D(76),...
    D(77),D(78),D(79),D(80)];
F=[E(4),E(10),E(14),E(17),E(20),E(28),E(31),E(34),E(36)];
SystemhetrY0=std(E)/mean(F)
%two norm P''
Paccacc= zeros(3,3);
Paccacc(1,2)= (omegaR*abs(E1acc)*abs(E2acc))/(2*sqrt(H1*H2))*(real(Ynieuw(1,2))...
    *sin(angle(E1acc)-angle(E2acc))-imag(Ynieuw(1,2))*cos(angle(E1acc)-angle(E2acc)));
Paccacc(1,3)= (omegaR*abs(E1acc)*abs(E3acc))/(2*sqrt(H1*H3))*(real(Ynieuw(1,3))...
    *sin(angle(E1acc)-angle(E3acc))-imag(Ynieuw(1,3))*cos(angle(E1acc)-angle(E3acc)));
Paccacc(1,1)=-Paccacc(1,2)-Paccacc(1,3);
Paccacc(2,1)= (omegaR*abs(E2acc)*abs(E1acc))/(2*sqrt(H2*H1))*(real(Ynieuw(2,1))...
    *sin(angle(E2acc)-angle(E1acc))-imag(Ynieuw(2,1))*cos(angle(E2acc)-angle(E1acc)));
Paccacc(2,3)= (omegaR*abs(E2acc)*abs(E3acc))/(2*sqrt(H2*H3))*(real(Ynieuw(2,3))...
    *sin(angle(E2acc)-angle(E3acc))-imag(Ynieuw(2,3))*cos(angle(E2acc)-angle(E3acc)));
Paccacc(2,2)=-Paccacc(2,1)-Paccacc(2,3);
Paccacc(3,1)= (omegaR*abs(E3acc)*abs(E1acc))/(2*sqrt(H3*H1))*(real(Ynieuw(3,1))...
    *sin(angle(E3acc)-angle(E1acc))-imag(Ynieuw(3,1))*cos(angle(E3acc)-angle(E1acc)));
Paccacc(3,2)= (omegaR*abs(E3acc)*abs(E2acc))/(2*sqrt(H3*H2))*(real(Ynieuw(3,2))...
    *sin(angle(E3acc)-angle(E2acc))-imag(Ynieuw(3,2))*cos(angle(E3acc)-angle(E2acc)));
Paccacc(3,3)=-Paccacc(3,1)-Paccacc(3,2);
symmpart=0.5*(Paccacc+transpose(Paccacc))
antisymmpart=0.5*(Paccacc-transpose(Paccacc))
norm(symmpart)
norm(antisymmpart)
```

The code given below is used to model the nine-bus system with the SM-model

```
clear all; clc;
%generator data
base=100;
x_dacc1=0.0608;      x_dacc2=0.1198;   x_dacc3=0.1813;              %in pu
H1=2364/base;        H2=640/base;      H3=301/base;

%Line and transformer data, all values in pu on 220kV, 100MVA base
X14=0.0576;
X27=0.0625;
X39=0.0586;
X45=0.0850;          R45=0.01;         halfYB45=0.088;
X46=0.092;           R46=0.0170;       halfYB46=0.079;
X57=0.161;           R57=0.032;        halfYB57=0.153;
X69=0.17;            R69=0.039;        halfYB69=0.179;
X78=0.072;           R78=0.0085;       halfYB78=0.0745;
X89=0.1008;          R89=0.0119;       halfYB89=0.1045;

%Active power P and Reactive power Q
P1gen=71.6/base;     Q1gen=27/base;     P1load=0;        Q1load=0;
P2gen=163/base;      Q2gen=6.7/base;    P2load=0;        Q2load=0;
P3gen=85/base;       Q3gen=-10.9/base;  P3load=0;        Q3load=0;
P5gen=0;             Q5gen=0;           P5load=125/base; Q5load=50/base;
P6gen=0;             Q6gen=0;           P6load=90/base;  Q6load=30/base;
P8gen=0;             Q8gen=0;           P8load=100/base; Q8load=35/base;

%inertia constants of synchronous motors
H5=0.04*P5load;
H6=0.04*P6load;
H8=0.04*P8load;

%transient reactances of synchronous motors
x_dacc5=min(92.8*P5load*100.^(-1.3), 1);
x_dacc6=min(92.8*P6load*100.^(-1.3), 1);
x_dacc8=min(92.8*P8load*100.^(-1.3), 1);

%Bus voltages and angles(degrees)
E1=1.040;            angleE1=0;         %slack bus
E2=1.025;            angleE2=9.3;       E22=E2*exp(1i*angleE2*(pi/180));
E3=1.025;            angleE3=4.7;       E33=E3*exp(1i*angleE3*(pi/180));
E4=1.026;            angleE4=-2.2;      E44=E4*exp(1i*angleE4*(pi/180));
E5=0.996;            angleE5=-4;        E55=E5*exp(1i*angleE5*(pi/180));
E6=1.013;            angleE6=-3.7;      E66=E6*exp(1i*angleE6*(pi/180));
E7=1.026;            angleE7=3.7;       E77=E7*exp(1i*angleE7*(pi/180));
E8=1.016;            angleE8=0.7;       E88=E8*exp(1i*angleE8*(pi/180));
E9=1.032;            angleE9=2.0;       E99=E9*exp(1i*angleE9*(pi/180));

%Current into the network at buses 1,2,3,5,6 & 8; transient internal voltages,
I1=(P1gen-1i*Q1gen)/(E1);
```

```
I2=(P2gen-1i*Q2gen)/(conj(E22));
I3=(P3gen-1i*Q3gen)/(conj(E33));
I5=(P5load-1i*Q5load)/(conj(E55));
I6=(P6load-1i*Q6load)/(conj(E66));
I8=(P8load-1i*Q8load)/(conj(E88));
E1acc=E1+1i*x_dacc1*I1;
E2acc=E22+1i*x_dacc2*I2;
E3acc=E33+1i*x_dacc3*I3;
E5acc=E55+1i*x_dacc5*I5;
E6acc=E66+1i*x_dacc6*I6;
E8acc=E88+1i*x_dacc8*I8;

%Impedance matrix elements
Z14=X14*1i;          Z11=Z14;
Z27=X27*1i;          Z22=Z27;
Z39=X39*1i;          Z33=Z39;
Z41=X14*1i;          Z45=R45+1i*X45;  Z46=R46+1i*X46;  Z44=Z41+Z45+Z46;
Z54=R45+1i*X45;      Z57=R57+1i*X57;  Z55=Z54+Z57;
Z64=R46+1i*X46;      Z69=R69+1i*X69;  Z66=Z64+Z69;
Z75=R57+1i*X57;      Z72=X27*1i;      Z78=R78+X78*1i;  Z77=Z75+Z72+Z78;
Z87=R78+X78*1i;      Z89=R89+X89*1i;  Z88=Z87+Z89;
Z93=X39*1i;          Z98=R89+X89*1i;  Z96=R69+X69*1i;  Z99=Z93+Z98+Z96;

%admittance matrix
Y14=1/(Z14);   Y41=Y14;    Y11=Y14;
Y27=1/(Z27);   Y72=Y27;    Y22=Y27;
Y39=1/(Z39);   Y93=Y39;    Y33=Y39;
Y45=1/Z45;               Y54=Y45;    Y46=1/Z46;  Y64=Y46;
Y57=1/Z57;               Y75=Y57;    Y69=1/Z69;  Y96=Y69;
Y78=1/Z78;               Y87=Y78;    Y98=1/Z98;  Y89=Y98;
Y44=Y14+Y54+Y64+halfYB45*1i+halfYB46*1i;
Y55=Y45+Y75+halfYB57*1i+halfYB45*1i;
Y66=Y46+Y96+halfYB46*1i+halfYB69*1i;
Y77=Y27+Y57+Y87+halfYB57*1i+halfYB78*1i;
Y88=Y78+Y98+halfYB78*1i+halfYB89*1i;
Y99=Y39+Y89+Y69+halfYB89*1i+halfYB69*1i;

%%%%%%%%%%%%%equivalent shunt admittances for the loads
Y_L5=((P5load-1i*Q5load)/(E5^2));
Y_L6=((P6load-1i*Q6load)/(E6^2));
Y_L8=((P8load-1i*Q8load)/(E8^2));

Y=[Y11,0,0,-Y14,0,0,0,0,0; 0,Y22,0,0,0,0,-Y27,0,0; 0,0,Y33,0,0,0,0,0,-Y39;...
   -Y41,0,0,Y44,-Y45,-Y46,0,0,0; 0,0,0,-Y54,Y55,0,-Y57,0,0;...
   0,0,0,-Y64,0,Y66,0,0,-Y69; 0,-Y72,0,0,-Y75,0,Y77,-Y78,0;...
   0,0,0,0,0,0,-Y87,Y88,-Y89;0,0,-Y93,0,0,-Y96,0,-Y98,Y99];

Yd = [1/(x_dacc1*1i),0,0; 0,1/(x_dacc2*1i),0; 0,0,1/(x_dacc3*1i)];
Ydaccent = [1/(x_dacc5*1i),0,0; 0,1/(x_dacc6*1i),0; 0,0,1/(x_dacc8*1i)];
```

```
Ydwiggle = [0,((1)/(1i*x_dacc5)),0,0,0,0;...
            0,0,((1)/(1i*x_dacc6)),0,0,0;...
            0,0,0,0,((1)/(1i*x_dacc8)),0];
T0 = [(1i/X14),0,0,0,0,0;...
       0,0,0,(1i/X27),0,0;...
       0,0,0,0,0,(1i/X39)];
T0accent = [(1i/X14),0,0; 0,(1i/X27),0; 0,0,(1i/X39)];
YT = [0,0,0,0,0,0; 0,((1)/(1i*x_dacc5)),0,0,0,0;...
      0,0,((1)/(1i*x_dacc6)),0,0,0; 0,0,0,0,0,0;...
      0,0,0,0,((1)/(1i*x_dacc8)),0; 0,0,0,0,0,0];
Ydoubleaccent([1:3],[1:3]) = Yd;
Ydoubleaccent([1:3],[4:6]) = zeros(3,3);
Ydoubleaccent([1:3],[7:9]) = -Yd;
Ydoubleaccent([1:3],[10:15]) = zeros(3,6);
Ydoubleaccent([4:6],[1:3]) = zeros(3,3);
Ydoubleaccent([4:6],[4:6]) = Ydaccent;%
Ydoubleaccent([4:6],[7:9]) = zeros(3,3);
Ydoubleaccent([4:6],[10:15]) = -Ydwiggle;%
Ydoubleaccent([7:9],[1:3]) = -Yd;
Ydoubleaccent([7:9],[4:6]) = zeros(3,3);
Ydoubleaccent([7:9],[7:9]) = -T0accent+Yd;
Ydoubleaccent([7:9],[10:15]) = T0;
Ydoubleaccent([10:15],[1:3]) = zeros(6,3);
Ydoubleaccent([10:15],[4:6]) = -transpose(Ydwiggle);%
Ydoubleaccent([10:15],[7:9]) = transpose(T0);
Ydoubleaccent([10:15],[10:15]) = Y([4:9],[4:9])+YT;

%Inspect if Y0accent is symmetric
issym = @(xy) isequal(xy,xy.')
symmetricisone=issym(Ydoubleaccent)

%Applying Kron reduction as in the nature physics paper
Ylinksboven = Ydoubleaccent([1:6],[1:6]);
Yrechtsboven = Ydoubleaccent([1:6],[7:15]);
Ylinksonder = Ydoubleaccent([7:15],[1:6]);
Yrechtsonder = Ydoubleaccent([7:15],[7:15]);
Ynieuw = Ylinksboven-Yrechtsboven*inv(Yrechtsonder)*Ylinksonder;  %or Y^SM


%Parameters
K1a2a =abs(E1acc*E2acc*Ynieuw(1,2))
K1a5a =abs(E1acc*E5acc*Ynieuw(1,4))
K2a5a =abs(E2acc*E5acc*Ynieuw(2,4))
K1a3a =abs(E1acc*E3acc*Ynieuw(1,3))
K1a8a =abs(E1acc*E8acc*Ynieuw(1,6))
K2a6a =abs(E2acc*E6acc*Ynieuw(2,5))
K2a3a =abs(E2acc*E3acc*Ynieuw(2,3))
K1a6a =abs(E1acc*E6acc*Ynieuw(1,5))
K2a8a =abs(E2acc*E8acc*Ynieuw(2,6))
```

```
K3a5a =abs(E3acc*E5acc*Ynieuw(3,4))
K3a8a =abs(E3acc*E8acc*Ynieuw(3,6))
K3a6a =abs(E3acc*E6acc*Ynieuw(3,5))
K5a6a =abs(E5acc*E6acc*Ynieuw(4,5))
K5a8a =abs(E5acc*E8acc*Ynieuw(4,6))
K6a8a =abs(E5acc*E8acc*Ynieuw(5,6))
gamma1a2a = angle(Ynieuw(1,2))-(pi/2)
gamma1a5a = angle(Ynieuw(1,4))-(pi/2)
gamma2a5a = angle(Ynieuw(2,4))-(pi/2)
gamma1a3a = angle(Ynieuw(1,3))-(pi/2)
gamma1a8a = angle(Ynieuw(1,6))-(pi/2)
gamma2a6a = angle(Ynieuw(2,5))-(pi/2)
gamma2a3a = angle(Ynieuw(2,3))-(pi/2)
gamma1a6a = angle(Ynieuw(1,5))-(pi/2)
gamma2a8a = angle(Ynieuw(2,6))-(pi/2)
gamma3a5a = angle(Ynieuw(3,4))-(pi/2)
gamma3a8a = angle(Ynieuw(3,6))-(pi/2)
gamma3a6a = angle(Ynieuw(3,5))-(pi/2)
gamma5a6a = angle(Ynieuw(4,5))-(pi/2)
gamma5a8a = angle(Ynieuw(4,6))-(pi/2)
gamma6a8a = angle(Ynieuw(5,6))-(pi/2)
A1=P1gen-(abs(E1acc)^(2))*(real(Ynieuw(1,1)))
A2=P2gen-(abs(E2acc)^(2))*(real(Ynieuw(2,2)))
A3=P3gen-(abs(E3acc)^(2))*(real(Ynieuw(3,3)))
A5=-P5load-(abs(E5acc)^(2))*(real(Ynieuw(4,4)))
A6=-P6load-(abs(E6acc)^(2))*(real(Ynieuw(5,5)))
A8=-P8load-(abs(E8acc)^(2))*(real(Ynieuw(6,6)))



%%%%%%%%%%%%%%The code below isn't necessary%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%Rewriting Ydoubleaccent to Y0accent from EN-MODEL%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Yaccent([1:3],[1:3]) = Ydoubleaccent([1:3],[1:3]);
Yaccent([1:3],[4:6]) = Ydoubleaccent([1:3],[7:9]);
Yaccent([1:3],[7:12])= Ydoubleaccent([1:3],[10:15]);
Yaccent([4:6],[1:3]) = Ydoubleaccent([7:9],[1:3]);
Yaccent([4:6],[4:6]) = Ydoubleaccent([7:9],[7:9]);
Yaccent([4:6],[7:12]) = Ydoubleaccent([7:9],[10:15]);
Yaccent([7:12],[1:3]) = Ydoubleaccent([10:15],[1:3]);
Yaccent([7:12],[4:6]) = Ydoubleaccent([10:15],[7:9]);
Yaccent([7:12],[7:12]) = Y([4:9],[4:9]);
Yaccent(8,8)=Yaccent(8,8)+Y_L5;
Yaccent(9,9)=Yaccent(9,9)+Y_L6;
Yaccent(11,11)=Yaccent(11,11)+Y_L8;

%Applying Kron reduction as in the nature physics paper
Ylinksboven2 = Yaccent([1:3],[1:3]);
```

```
Yrechtsboven2 = Yaccent([1:3],[4:12]);
Ylinksonder2 = Yaccent([4:12],[1:3]);
Yrechtsonder2 = Yaccent([4:12],[4:12]);
Ynieuw2 = Ylinksboven2-Yrechtsboven2*inv(Yrechtsonder2)*Ylinksonder2;  %or Y^EN


%Yd = [1/(x_dacc1*1i),0,0; 0,1/(x_dacc2*1i),0; 0,0,1/(x_dacc3*1i)];
Y0accent([1:3],[1:3]) = Yd;
Y0accent([1:3],[4:6]) = -Yd;
Y0accent([1:3],[7:12]) = zeros(3,6);
Y0accent([4:6],[1:3]) = -Yd;
Y0accent([4:6],[4:6]) = Y([1:3],[1:3])+Yd;
Y0accent([4:6],[7:12]) = Y([1:3],[4:9]);
Y0accent([7:12],[1:3]) = zeros(6,3);
Y0accent([7:12],[4:6]) = transpose(Y0accent([4:6],[7:12]));
Y0accent([7:12],[7:12]) =Y([4:9],[4:9]);
Y0accent(8,8)=Y0accent(8,8)+Y_L5;
Y0accent(9,9)=Y0accent(9,9)+Y_L6;
Y0accent(11,11)=Y0accent(11,11)+Y_L8;

%Applying Kron reduction as in the nature physics paper
Ylinksboven3 = Y0accent([1:3],[1:3]);
Yrechtsboven3 = Y0accent([1:3],[4:12]);
Ylinksonder3 = Y0accent([4:12],[1:3]);
Yrechtsonder3 = Y0accent([4:12],[4:12]);
Ynieuw3 = Ylinksboven3-Yrechtsboven3*inv(Yrechtsonder3)*Ylinksonder3;  %or Y^EN
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%From here it's getting important%%%%%%%%%%%%%%%%%%%%


Pe1=@(theta1,theta2,theta3,theta5,theta6,theta8)  (abs(E1acc))^2*real(Ynieuw(1,1))+...
    abs(E1acc*E2acc*Ynieuw(1,2))*sin(theta1-theta2-angle(Ynieuw(1,2))+(pi/2))+...
    abs(E1acc*E3acc*Ynieuw(1,3))*sin(theta1-theta3-angle(Ynieuw(1,3))+(pi/2))+...
    abs(E1acc*E5acc*Ynieuw(1,4))*sin(theta1-theta5-angle(Ynieuw(1,4))+(pi/2))+...
    abs(E1acc*E6acc*Ynieuw(1,5))*sin(theta1-theta6-angle(Ynieuw(1,5))+(pi/2))+...
    abs(E1acc*E8acc*Ynieuw(1,6))*sin(theta1-theta8-angle(Ynieuw(1,6))+(pi/2));
Pe2=@(theta1,theta2,theta3,theta5,theta6,theta8)  (abs(E2acc))^2*real(Ynieuw(2,2))+...
    abs(E2acc*E1acc*Ynieuw(2,1))*sin(theta2-theta1-angle(Ynieuw(2,1))+(pi/2))+...
    abs(E2acc*E3acc*Ynieuw(2,3))*sin(theta2-theta3-angle(Ynieuw(2,3))+(pi/2))+...
    abs(E2acc*E5acc*Ynieuw(2,4))*sin(theta2-theta5-angle(Ynieuw(2,4))+(pi/2))+...
    abs(E2acc*E6acc*Ynieuw(2,5))*sin(theta2-theta6-angle(Ynieuw(2,5))+(pi/2))+...
    abs(E2acc*E8acc*Ynieuw(2,6))*sin(theta2-theta8-angle(Ynieuw(2,6))+(pi/2));
Pe3=@(theta1,theta2,theta3,theta5,theta6,theta8)  (abs(E3acc))^2*real(Ynieuw(3,3))+...
    abs(E3acc*E1acc*Ynieuw(3,1))*sin(theta3-theta1-angle(Ynieuw(3,1))+(pi/2))+...
    abs(E3acc*E2acc*Ynieuw(3,2))*sin(theta3-theta2-angle(Ynieuw(3,2))+(pi/2))+...
    abs(E3acc*E5acc*Ynieuw(3,4))*sin(theta3-theta5-angle(Ynieuw(3,4))+(pi/2))+...
    abs(E3acc*E6acc*Ynieuw(3,5))*sin(theta3-theta6-angle(Ynieuw(3,5))+(pi/2))+...
```

```
    abs(E3acc*E8acc*Ynieuw(3,6))*sin(theta3-theta8-angle(Ynieuw(3,6))+(pi/2));
Pe5=@(theta1,theta2,theta3,theta5,theta6,theta8)  (abs(E5acc))^2*real(Ynieuw(4,4))+...
    abs(E5acc*E1acc*Ynieuw(4,1))*sin(theta5-theta1-angle(Ynieuw(4,1))+(pi/2))+...
    abs(E5acc*E2acc*Ynieuw(4,2))*sin(theta5-theta2-angle(Ynieuw(4,2))+(pi/2))+...
    abs(E5acc*E3acc*Ynieuw(4,3))*sin(theta5-theta3-angle(Ynieuw(4,3))+(pi/2))+...
    abs(E5acc*E6acc*Ynieuw(4,5))*sin(theta5-theta6-angle(Ynieuw(4,5))+(pi/2))+...
    abs(E5acc*E8acc*Ynieuw(4,6))*sin(theta5-theta8-angle(Ynieuw(4,6))+(pi/2));
Pe6=@(theta1,theta2,theta3,theta5,theta6,theta8)  (abs(E6acc))^2*real(Ynieuw(5,5))+...
    abs(E6acc*E1acc*Ynieuw(5,1))*sin(theta6-theta1-angle(Ynieuw(5,1))+(pi/2))+...
    abs(E6acc*E2acc*Ynieuw(5,2))*sin(theta6-theta2-angle(Ynieuw(5,2))+(pi/2))+...
    abs(E6acc*E3acc*Ynieuw(5,3))*sin(theta6-theta3-angle(Ynieuw(5,3))+(pi/2))+...
    abs(E6acc*E5acc*Ynieuw(5,4))*sin(theta6-theta5-angle(Ynieuw(5,4))+(pi/2))+...
    abs(E6acc*E8acc*Ynieuw(5,6))*sin(theta6-theta8-angle(Ynieuw(5,6))+(pi/2));
Pe8=@(theta1,theta2,theta3,theta5,theta6,theta8)  (abs(E8acc))^2*real(Ynieuw(6,6))+...
    abs(E8acc*E1acc*Ynieuw(6,1))*sin(theta8-theta1-angle(Ynieuw(6,1))+(pi/2))+...
    abs(E8acc*E2acc*Ynieuw(6,2))*sin(theta8-theta2-angle(Ynieuw(6,2))+(pi/2))+...
    abs(E8acc*E3acc*Ynieuw(6,3))*sin(theta8-theta3-angle(Ynieuw(6,3))+(pi/2))+...
    abs(E8acc*E5acc*Ynieuw(6,4))*sin(theta8-theta5-angle(Ynieuw(6,4))+(pi/2))+...
    abs(E8acc*E6acc*Ynieuw(6,5))*sin(theta8-theta6-angle(Ynieuw(6,5))+(pi/2));

    theta5 =angle(E5acc);
    theta6= angle(E6acc);
    theta8= angle(E8acc);
omegaR=60*2*pi;
f =@(t,theta)[theta(7)-omegaR; theta(8)-omegaR; theta(9)-omegaR;...
    theta(10)-omegaR;theta(11)-omegaR;theta(12)-omegaR;...
    (omegaR/(2*H1))*(P1gen-Pe1(theta(1),theta(2),theta(3),theta(4),theta(5),theta(6)));..
    (omegaR/(2*H2))*(P2gen-Pe2(theta(1),theta(2),theta(3),theta(4),theta(5),theta(6)));..
    (omegaR/(2*H3))*(P3gen-Pe3(theta(1),theta(2),theta(3),theta(4),theta(5),theta(6)));..
    (omegaR/(2*H5))*(-P5load-Pe5(theta(1),theta(2),theta(3),theta(4),theta(5),theta(6)));
    (omegaR/(2*H6))*(-P6load-Pe6(theta(1),theta(2),theta(3),theta(4),theta(5),theta(6)));
    (omegaR/(2*H8))*(-P8load-Pe8(theta(1),theta(2),theta(3),theta(4),theta(5),theta(6)))]
[t,xa] = ode45(f,[0 2],[angle(E1acc) angle(E2acc) angle(E3acc) angle(E5acc)...
    angle(E6acc) angle(E8acc) omegaR omegaR omegaR omegaR omegaR omegaR]);

figure % create new figure
subplot(3,1,1) % first subplot
plot(t,xa(:,1)*(180/pi))
hold on
plot(t,xa(:,2)*(180/pi),'g')
hold on
plot(t,xa(:,3)*(180/pi),'m')
title('generator angles')
subplot(3,1,2) % second subplot
plot(t,(xa(:,4)),'b');
hold on
plot(t,(xa(:,5)),'r');
hold on
plot(t,(xa(:,6)),'g');
```

```
title('motor angles')

subplot(3,1,3) % third subplot
plot(t,(xa(:,2)-xa(:,1))*(180/pi),'g');
hold on
plot(t,(xa(:,3)-xa(:,1))*(180/pi),'m');
title('Angle differences')

%{
%order parameter with unit circle
xy = linspace(0,2*pi,120);
magn = 528;%length(xa);
%order parameter with unit circle
r=(1/3)*(exp(1i*xa(magn,1))+exp(1i*xa(magn,2))+exp(1i*xa(magn,3)))
%r=(1/6)*(exp(1i*xa(magn,1))+exp(1i*xa(magn,2))+exp(1i*xa(magn,3))...
%+exp(1i*xa(magn,4))+exp(1i*xa(magn,5))+exp(1i*xa(magn,6)))
r1 =real(r); r2 =imag(r);
figure
plot([0,r1],[0,r2])
hold on
scatter(0,0)
hold on
scatter(r1,r2,'filled')
hold on
plot(cos(xy), sin(xy))
%}


r=(1/3)*(exp(1i*xa(:,1))+exp(1i*xa(:,2))+exp(1i*xa(:,3)))
modulus=abs(r);
xy = linspace(0,2,1057);
figure
plot(xy,modulus)
```