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Bicycle headway modeling and its applications

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Bicycle headway modeling and its applications

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1 ABSTRACT

2 Little is known about microscopic operations of bicycle traffic. This lack of knowledge makes designing and
 3 analyzing facilities that are (also) used by cyclists, as well as assessing their efficiency and safety, very
 4 difficult. This paper puts forward a novel composite headway model for bicycle flows. Based on a working
 5 definition of a bicycle headway, the model assumes that the headway is the sum of the so-called empty zone
 6 (the minimum headway that a cyclist will have with respect to the bicycle he or she is following) and the free
 7 headway (the additional headway that cyclists maintain when they are not following). Both headways are
 8 stochastic variables, given the large heterogeneity in cyclists' behavior. Furthermore, we put forward a
 9 distribution-free model estimation approach that allows identifying the distributions of both the empty zone
 10 and the free headway without pre-specifying their functional form. The workings of the model and the
 11 estimation method are illustrated using data collected at a busy intersection in the Netherlands.

12 INTRODUCTION

13 The scientific and engineering knowledge on bicycle flow operations is limited. Navin (1) performed a series
 14 of experimental studies to determine the operating performance of a single bicycle, and the traditional traffic
 15 flow characteristics of a stream of bicycles and compared them to observed data. He concluded that under
 16 certain conditions, bicycle flows can be treated as vehicular flows, and concepts such as capacity of bicycle
 17 paths and their level of service could be sensibly defined. Other work predominantly looks at traffic
 18 engineering concepts as levels of service, but provides little insight into cycle behavior.

19 Basic information covers bicycle dimensions, with different dimensions found in different countries.
 20 According to Allen et al. (2) a typical U.S. bicycle is 1.75 m long and its handlebar is 0.60 m wide, while the
 21 Dutch design guidelines (3) indicate that 95% of all bicycles have a length less than 1.90 m and 100% of the
 22 handlebars are less than 0.75 m wide. A much earlier study by Radlicke (4) showed similar dimensions,
 23 implying that bicycles' dimensions have not changed considerably in the last 60 years. However, recently
 24 different types of more functional bikes have been developed, such as carrier (tri)cycles. Also, the share of
 25 scooters using the bicycle path has been increased. Related to the bicycle width, design guidelines give
 26 different values for the bicycle lane width. In general a minimal width for a one-way facility is 1.50 m (3,5).
 27 These norms are related to the necessary space for a single cyclist to move unconstrained in a lateral
 28 direction. According to Knoflacher (6) 50 % of the cyclists require at least 0.97 m for free movement in one-
 29 way traffic, whereas the 85%-value is 1.24 m. The values do not include the case when two or more cyclists
 30 are moving next to each other.

31 More research exists on fundamental diagrams for bicycle traffic. Critical density was found to be
 32 between 0.05 and 0.14 bicycles/m² (1,7-9), while more variation was found in jam density: between 0.27 and
 33 0.45 bicycles/m² (1,7-9). Most studies about the capacity of bicycle facilities come from Europe as the bicycle
 34 usage is much higher than in the United States and the bicycle infrastructure network is more developed.
 35 However, recently US and Canadian research groups have also performed several studies (14). The first
 36 earlier European studies by Smith reported capacities around 770 bicycles/h/m (10). Four years later,
 37 Homburger reported much higher capacities – 2,600 bicycles/h/m (7). Using headways in free flow
 38 conditions to estimate capacity, Botma (11) concluded that the capacity is between 3,800 and 4,600
 39 bicycles/h/m. However, this method is likely to slightly overestimate the capacity value (12). Navin
 40 performed an experiment with children following a lead bicyclist on an oval 2.5 m wide lane showing a
 41 capacity value of 4,000 bicycles/h/m (1). However, this experiment clearly does not represent real conditions.
 42 In Austrian guidelines (6), a distinction is made between for the capacity of one-way traffic (1,000
 43 bicycles/h/m) and two-way traffic (750-800 bicycles/h/m). This capacity distinction is further elaborated by
 44 Richard (13) showing the relation between capacity, bicycle lane width and type of traffic (one or two way).

45 Although some fundamental diagrams on bicycle traffic exist, most of them are either estimated on
 46 only the free flow branch (1,7,14-16) or based on simulation models (14). Both methods have major
 47 drawbacks. In the first case, no information is known for the congested branch of the fundamental diagram,
 48 so any fundamental diagram could be fit to the data. Secondly, simulation models do not necessarily cover
 49 cyclist behavior sufficiently accurate to estimate a fundamental diagram. This especially holds for cyclist
 50 behavior under congested conditions, as not much is known for this behavior.

51 The previous overview shows that indeed, only limited insight is available in bicycle traffic
 52 operations. As the mode share of bicycles increases, and the bicycle infrastructure network improves further,

bicycle traffic is in a transition towards an important mode, with not only issues in the interaction with vehicular and pedestrian traffic, but also capacities of bicycle paths are nowadays reached, especially in urban areas. Often, observations of congestions are not available, thus making it difficult if not impossible to accurately estimate capacity. In this paper we introduce a composite headway distribution model, distinguishing between constrained cyclists and free driving cyclists. We will show how we can use the so-called empty zone distribution to estimate the capacity.

This paper first introduces the composite headway distribution model, including the necessary definitions, followed by the description of the method to estimate this headway model.

HEADWAY MODELING

To gain more insight into the following behavior of cyclists, this behavior is analyzed by estimating a so-called composite headway distribution model, which distinguished between following (or constrained) and freely moving (unconstrained) cyclists. To do this, we first need to determine the leader-follower combinations, i.e. which cyclist j is being followed by following cyclist i .

Definition of a headway of a cyclist

Let t_i denote the time instant at which cyclist i passes the cross-section x . We will use the following simple procedure to determine the leader j : let $y_i(t_i)$ denote the lateral position of cyclist i at t_i (indicated by the white cross in FIGURE 1). The leader of i is the cyclist with the largest index $j < i$ for which:

$$|y_j(t_j) - y_i(t_i)| \leq \frac{1}{2}a \quad (1)$$

for some threshold value $a > 0$. Here, a typically equals the width of cyclist i 's handlebars plus additional shy-away distance on either side, in total corresponding to the free space ahead of the cyclist needed to cycle. The headway h_i of cyclist i is then defined by:

$$h_i = t_i - t_j \quad (2)$$

FIGURE 1 shows a graphical description of this headway definition.

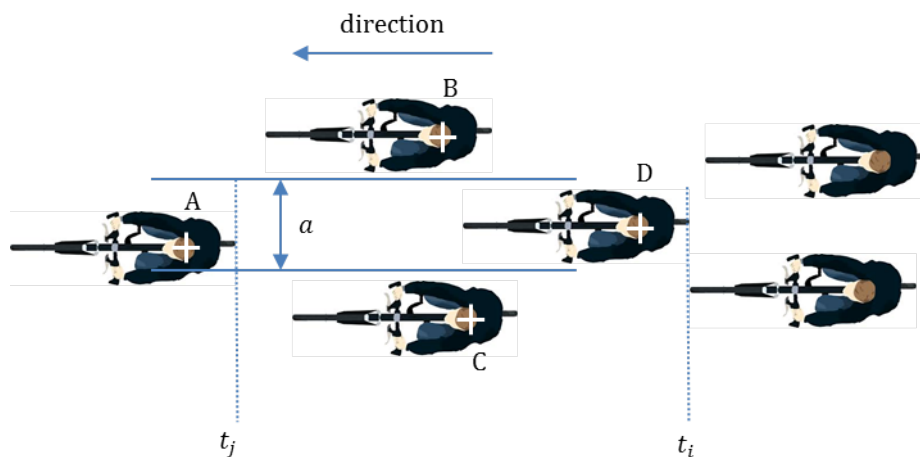


FIGURE 1 Headway definition using the width a as the key decision variable to determine which cyclist is being followed. In the figure, cyclist D is following cyclist A.

Composite headway modeling

To study the following behavior, we propose to use the semi-Poisson model first presented by Buckley (17). The model distinguished between constrained headways and unconstrained headways. An *unconstrained headway* is a headway when a cyclist is experiencing *no hindrance* from other cyclists ahead. The distribution of this free headway U is usually determined by drawing the analogy with a Poisson-point process (17). Consequently, the free headway is assumed to be exponentially distributed.

Constrained headways are headways of cyclists following the cyclist in front, while being unable or reluctant to overtake, even if the desired cycling speed is less than the speed of the cyclist that is followed. The headway will be equal to – or actually fluctuate around – a *desired minimum headway* (or *empty zone*). It is reasonable to assume that, due to the heterogeneity in cycling behavior, different cyclists will adopt different empty zones, and therefore the empty zone follows some random variable X . Causes for these interpersonal variations are, among other things, subjectiveness in what is perceived as comfortable and safe, differences in cycling purpose and skill. Also intrapersonal variations in the headway will exist since no cyclist will be able to maintain the same empty zone all the time.

Let $g(h)$ and $r(h)$ denote respectively the probability density functions of the empty zone and the free headway, and let ϕ denote the fraction of constrained cyclists, which are driving at their current minimum desired headway. The composite headway model of Buckley proposes that the probability density function $f(h)$ of the composite headway $H = X + U$ satisfies:

$$f(h) = \phi g(h) + (1 - \phi)r(h) \quad (3)$$

Furthermore, in (17) it is shown that the free headway density function satisfies:

$$r(h) = \frac{1}{A} \lambda \exp(-\lambda h) \int_0^h g(\tau) d\tau \quad (4)$$

where A is the so-called *normalization constant* defined by:

$$A = \int_0^\infty \lambda \exp(-\lambda \tau) g(\tau) d\tau \quad (5)$$

and where $\lambda \geq 0$ denotes the free headway arrival rate.

In the remainder of this section, we will briefly recall the approach from Wasielewski (18) to estimate the parameters of the model, i.e. the free headway arrival rate λ and the fraction of constrained cyclists ϕ , and – most importantly – the probability density function $g(h)$ of the empty zone. The approach put forward is *distribution free* in the sense that the functional form of the empty zone does not need to be specified beforehand. This allows us to study the inter- and intrapersonal variations in the cyclist minimum desired headways, but it also provides us with an approach to determine the capacity of the bicycle facility, even if there are no direct capacity measurements available. Finally, we are able to predict the headway distribution for different demand levels, and analyze the maximum flow for a crossing (car, pedestrian or bicycle) stream. Both applications will be briefly discussed below.

Capacity estimation

To determine the capacity of a bicycle facility using the composite headway model that has been identified using headway data, we will make the assumption that under capacity conditions, all cyclists are following. In other words, we have $\phi = 1$. Under this assumption, and given the choice we have made for the threshold value a to determine which bicycle is being followed, we can determine the capacity in *bicycles/h/m* by computing the inverse of the mean empty zone, i.e.:

$$C = \frac{1}{2aE(H)|_{\phi=1}} = \frac{1}{2aE(X)} \quad (6)$$

Gap acceptance modeling

Another application of the composite headway model is to use it to determine the (remaining) capacity of a non-prioritised crossing flow. For the case that we will describe in the ensuing of the paper, the crossing flow consists of cars that have to wait for a sufficiently large gap in the bicycle flow which they can pass. Which gap is acceptable depends on the critical gap of the car-driver. Given that the population of drivers is heterogeneous, it is reasonable to assume that the critical gaps follow some distribution.

To determine the (maximum) number of crossing manoeuvres of the cars, the critical gap distribution is confronted with the cycle headway distribution. Using simple simulation approaches that are based on

drawings from both distribution functions, it is relatively easy to determine numerical estimates for the crossing flow or capacity.

NON-PARAMETRIC COMPOSITE HEADWAY ESTIMATION

The section generalizes the distribution-free (or rather, *non-parametric*) estimation approach first proposed by Wasielewski (18) for freeway traffic, to bicycle flows. We are given a data set $\{h_i\}$ of headways observed at a certain cross section x .

The key assumption is now that there exists some value T^* (the *separation value*), such that $g(h) = 0$ for all $h > T^*$. That is, all observed headways larger than T^* stem from free moving cyclists. Given that this assumption is valid, we can identify a set of observed headways $\{h_i | h_i > T^*\}$. Since these headways follow an exponential distribution with mean arrival rate λ , we can determine the following estimate of the free headway arrival rate $\hat{\lambda}$:

$$\hat{\lambda} = \frac{1}{m} \sum_{i=1}^n \{h_i | h_i > T^*\} \quad (7)$$

where m is the number of elements in the set $\{h_i | h_i > T^*\}$, i.e. the number of headways larger than T^* . We can then show that an estimate for the normalization constant is given by the following expression (18):

$$\hat{A} = \frac{m}{n} \exp(\hat{\lambda} T^*) \quad (8)$$

where n is the total number of observed headways in the sample. Now, let $\hat{f}_n(h)$ denote an estimate for the composite headway distribution based on the sample of headways $\{h_i\}$. This can be a histogram, a Kernel estimate, or any approach to determine an estimate of the distribution. Now, let $r_1(h) = (1 - \phi)r(h)$. Wasielewski shows that by solving the following integral equation:

$$\hat{r}_1(h) = \frac{\hat{A} \hat{\lambda}}{\hat{\phi}} \exp(-\hat{\lambda} T^*) \int_0^h (\hat{f}_n(\tau) - \hat{r}_1(\tau)) d\tau \quad (9)$$

subject to:

$$\hat{\phi} = \int_0^\infty (\hat{f}_n(\tau) - \hat{r}_1(\tau)) d\tau \quad (10)$$

we can determine an estimate for the empty zone:

$$\hat{g}(h) = \frac{\hat{g}_1(h)}{\hat{\phi}} = \frac{\hat{f}_n(h) - \hat{r}_1(h)}{\hat{\phi}} \quad (11)$$

Note that the integral expression above can be solved iteratively. For more details on this, we refer to the original paper of Wasielewski (18), and more recent work of Hoogendoorn (19).

JAFFALAN CASE STUDY

In this section, we will illustrate the model and the proposed estimation method using data collected at a busy intersection. First, we will discuss the study site and the data collection approach taken in this study.

Data collection set-up

One of the intersections in the campus of the Delft University of Technology is well known for its large bicycle flows, see FIGURE 2a. This does not only lead to capacity issues for the bicycle traffic, but also safety is at stake, as the vehicle flows also need to pass this intersection during the peak hours. To quantify the safety and capacity issues, video images have been collected from a vantage point on a nearby building. The observations have been performed on weekdays from Wednesday, October 8, 2014 until Tuesday, October 14, 2014, from 7h30 (dawn) until 18h30 (dusk). An example of the footage is shown in FIGURE 2b.



a. Overview of intersection.

b. Counting line to count passing cyclists.

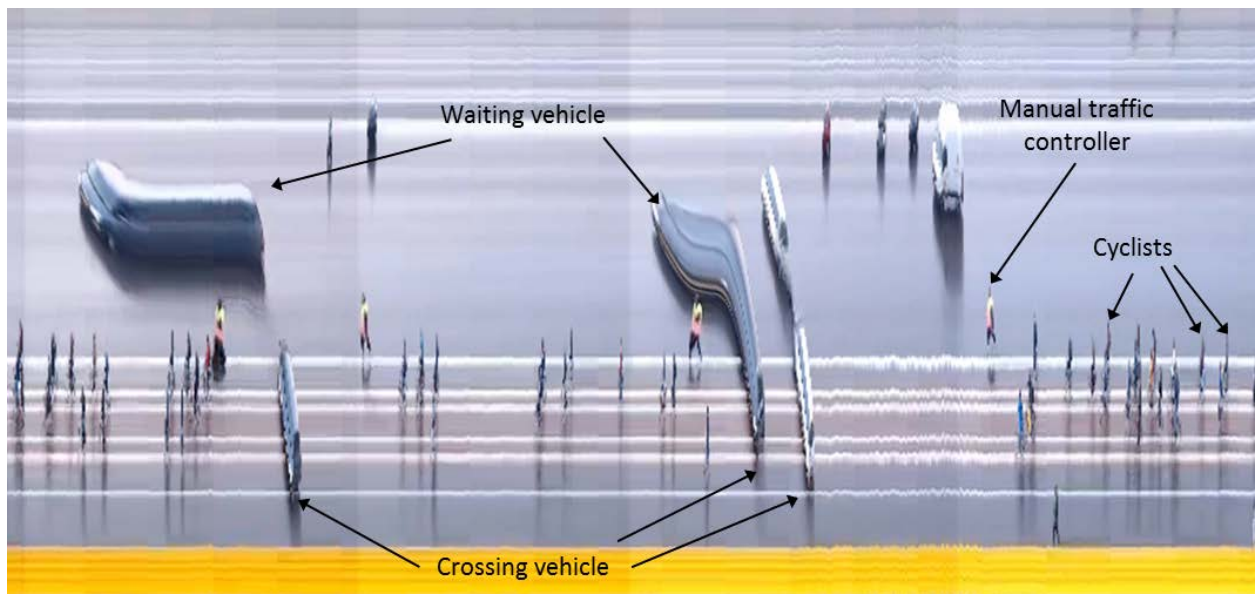
FIGURE 2 Overview of the intersection in the Delft University of Technology campus, where high bicycle flows pass an intersection (on the left). On the right the location of the cross-section for which the data are collected.

The counts have been based on so-called photofinish photos. For this technique, individual images are extracted from the video. With a frame rate of 10 fps, this leads to 10 frames or images per second. To compose the photofinish photo, we look at a single line of each image, the so-called counting line as indicated in FIGURE 2b. This counting line is extracted from each frame. In a separate image the counting lines from consecutive images are placed next to each other, thus forming the photofinish photo. The result is shown in FIGURE 3, where the vertical axis shows the intersection cross-section corresponding to the counting line and the horizontal axis represents the time. The vehicles and cyclists in the photofinish photo are clearly distorted, as their shape depends on their speed passing the cross-section: vehicles or cyclists with low speed (or standing still) are stretched (they are seen for a longer time period and thus in multiple images on the counting line), while fast moving cyclists are very small (they are only seen in a small amount of images).

We have made a photofinish photo for time periods of 20 minutes. On each photofinish photo, the positions and passing moments of the cyclists have been identified by identifying the centre of gravity of each cyclist. This results in the passing moments of cyclists along the counting line indicated in FIGURE 2b. We have assumed that the flow was unidirectional: towards the campus in the morning and towards the city centre in the evening.



a. Counting photo, with full height and 15min width. A zoom of the dashed area is shown in b.



b. Zoom of the dashed area marked in a.

FIGURE 3 Counting photos of Wednesday, October, 8 from 17h30 – 17h45.

Choice for estimation parameters

One important parameter in the estimation process is T^* , the headway value above which all cyclists are driving freely. In (19), a method is described to determine this separation value. The idea is simple: since all headways $h_i > T^*$ will follow an exponential distribution, the empirical survival function $\hat{S}_n(h) = 1 - \hat{F}_n(h)$ will be an exponential function. Plotting it with a logarithmic scale would yield a straight line for all values $h > T^*$. FIGURE 4 shows the result of the analysis using the data collected at the Jaffalaan. The graph shows a clear bend for headway values less than 2 seconds. To be on the safe side, we choose the separation value $T^* = 4s$.

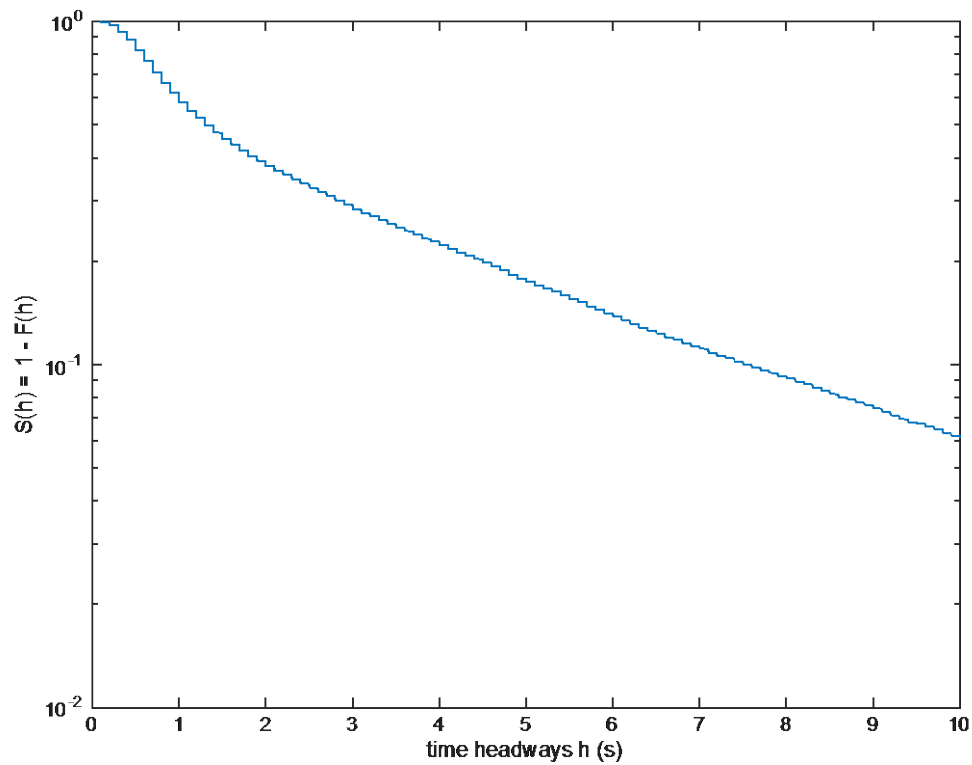


FIGURE 4 Determining the separation value T^* by plotting the survival function.

Estimation results

Using the data collected, we have identified the composite headway model presented earlier using the estimate method described in the previous section. FIGURE 5 shows the results of applying the procedure on the collected dataset. The figure shows the composite empirical headway distribution, the estimated distribution of the free headway (scaled with the estimated fraction of free driving cyclists $1 - \hat{\phi}$), and the estimated distribution of the empty zone (scaled with the estimated fraction of constrained cyclists $\hat{\phi}$).

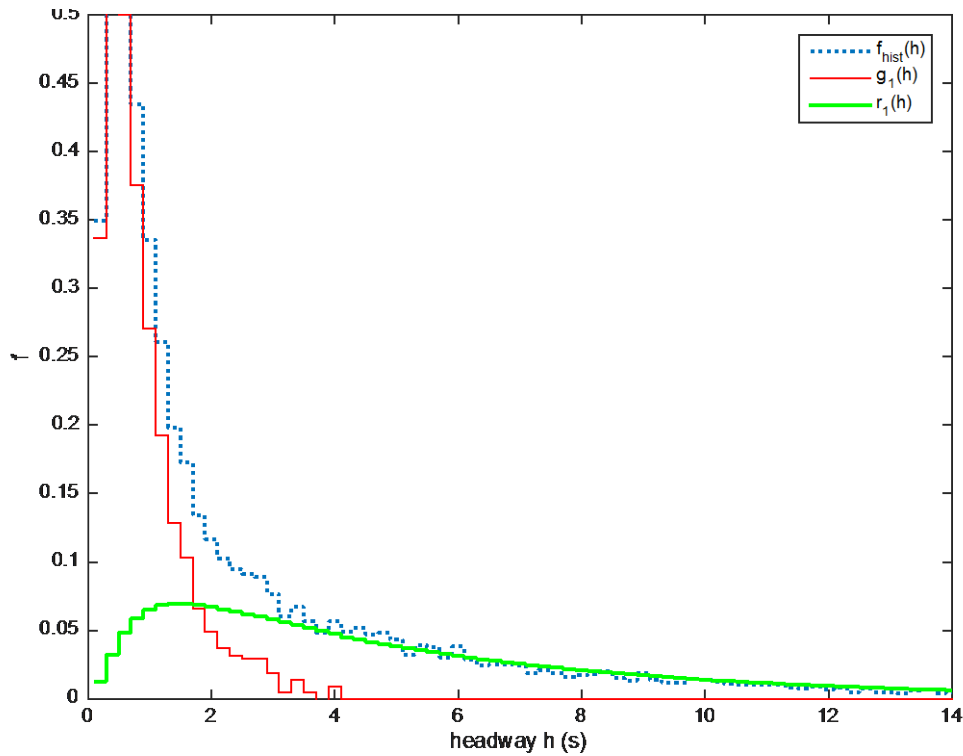


FIGURE 5 Headway estimation results for the Jaffalaan using $T^* = 4s$, showing the empty zone distribution (red line), the free headway distribution (green line), and the composite distribution (dashed, blue).

TABLE 1 gives an overview of the estimation results. The table shows that for this particular case, on average 54% of the cyclists are constrained. This is a relatively high number, showing that for this particular situation the demand is such that less than 50% of the cyclists (observed in the entire observation period) can cycle freely.

The cyclists that are constrained have an average empty zone of 0.784 seconds. As we can also see from FIGURE 5, the interpersonal (and intrapersonal) variation in the empty zones are substantial: the standard deviation of the empty zone is 0.660 seconds.

TABLE 1 Estimation Results for the Jaffalaan Data Set

$\hat{\lambda}$	$\hat{\phi}$	$E(X)$	$\sigma(X)$
0.202	54.1%	0.784	0.660

Note that the estimation results allow us to determine the (theoretical) capacity of the facility considered. Under the assumption that in capacity conditions, all cyclists are following, we can determine the capacity C in number of bicycles per:

$$C = \frac{3600}{0.784} = 4,593 \text{ bicycles/h} \quad (12)$$

Note that this number holds for the entire bicycle lane. The bicycle lane is 3 m wide, which would result in a capacity value of 1,531 bicycles/h/m. As the dominant flow is into one direction, not the full width is used, and the capacity per meter is underestimated. This can also be seen in the comparison with other international study results discussed in the introduction. This may be due to the way capacity is measured in some of the other studies, namely by measuring the queue discharge rate at a controlled intersection. The capacity value that we have determined stems from microscopic observations and can be interpreted as a (theoretical) free capacity estimate.

Additional insight into the following process of the cyclist can be gained by looking at the conditional probability $\theta(h)$ that a cyclist driving at headway h is following. According to Bayes rule (19):

$$\theta(h) = \phi \cdot \frac{g(h)}{f(h)} \quad (13)$$

For the considered case, this leads to the results shown in FIGURE 6. The figure shows that when driving at a small headway, it is almost certain that the cyclist is following. For larger headways, the probability is decreasing and eventually vanishes for headways larger than 5 seconds.

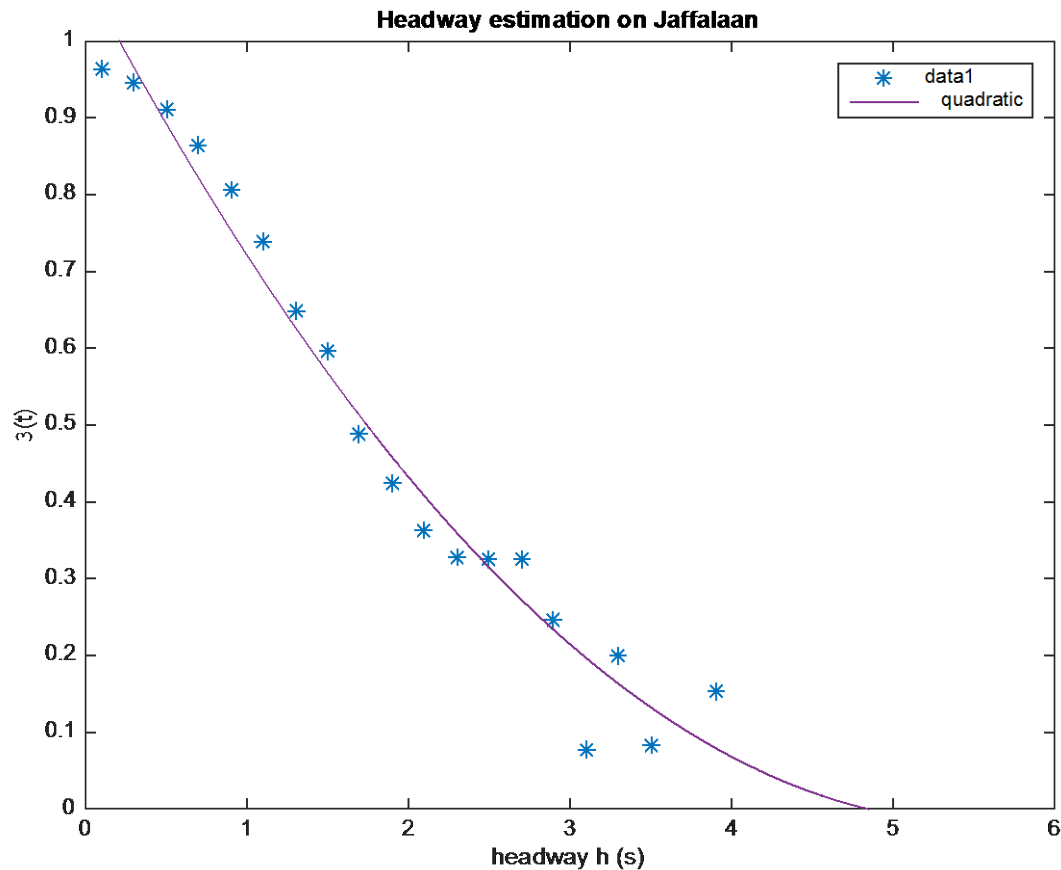


FIGURE 6 Conditional probability of following when cycling at a certain headway with respect to the predecessor.

CONCLUSIONS AND RECOMMENDATIONS

In this paper, we have put forward a novel composite headway distribution model for bicycle traffic flows. The model distinguishes between constrained and unconstrained cyclists, and is based on the semi-Poisson model of Buckley. The paper furthermore proposes an estimation approach that can be used to determine the key parameters of the model (arrival rates of the free headway, fraction of constrained cyclists), and the minimum headway (or the empty zone) distribution, using individual headway data. The latter contains important information about the following behavior of cyclists.

Using data collected from a crossing on the TU Delft campus, we have tested the estimation procedure and the model. The resulting estimates appear to be plausible, and can be used to determine some key parameters of the considered facility. For instance, capacity estimates can be determined from the model estimation outcomes. Furthermore, the share of constrained cyclists can be determined (and turned out to be quite high), which is an indicator of the level-of-service that the facility offers. Finally, the model can be used

to see how many gaps could be used by a crossing flow, and as such determine the crossing flow capacity. In the situation at hand, the question is whether the large bicycle flow goes together with the vehicle flow. To identify the possible gaps for the vehicle to pass, the so-called empty space distribution can be estimated, which is one of the outcomes of this method.

In this paper, the method has been applied to empirical data gathered on the TU campus. This implies that the majority of the cyclists consisted of students (young and in general fit adults), being experienced cyclists and used to crowded cycling conditions, as this is a daily occurring situation. Although this does not affect the applicability or the quality of the method, the resulting capacity should be handled with care. To come to a generically applicable capacity, it is therefore advised to collect data at other locations and compare the various obtained capacities.

Future work will deal with analyzing this process further. One of the key things to consider are the dependence of the free flow arrival rate and the fraction of constrained cyclists on the demand level q . In finding relations $\phi = \phi(q)$ and $\lambda = \lambda(q)$, we can determine the headway distribution for different demand levels. Confronting this with the critical gap distribution of the crossing flows allows for the determination of generic relations for the crossing flow capacity, which in turn can be used to assess the facility and decide which interventions are needed.

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