Residual Stresses In Injection Molded Products

K.M.B. Jansen

Delft University of Technology,
faculty of Industrial Design Engineering
Landbergstraat 15, 2628 DE Delft,
The Netherlands
k.m.b.jansen@tudelft.nl

Abstract. During the molding process residual stresses are formed due to thermal contraction during cooling as well as the local pressure history during solidification. In this paper a simple analytical model is reviewed which relates residual stresses, product shrinkage as well as warpage to the temperature and pressure histories during molding. Precise excimer laser layer removal measurements were performed to verify the predicted residual stress distributions. In addition, detailed shrinkage and warpage measurements on a large series of polymers and for different molding conditions were performed and are shown to compare well with the model predictions.

1. INTRODUCTION

Injection molded polymer products always have residual stresses that may adversely affect the product performance and these stresses are directly related to the processing conditions like holding pressure, injection velocity, melt and mold temperature. Engineers therefore often use numerical tools to simulate the molding process and determine the residual stress state after molding and related parameters such as product shrinkage and warpage.

In this paper we present a simple thermo-elastic model which results in a series of closed-form equations which relate residual stress distributions and shape distortions to the local cavity pressure and temperature histories. The main assumption in this thermo-elastic model is that we neglect all viscoelastic effects during the liquid to solid transition. In that way the governing equations simplify considerably which allows us to track how the temperatures and pressures determine our product properties. Such a way of modelling was successfully used in literature before to predict cooling stresses in free quenched products [2, 15]. Titomanlio, Brucato and Kamal [1] however showed that the stress situation in injection molding was not only determined by the temperature but also by the pressure history. In the derivation below we extend the thermo-elastic model for free quenching to include pressure effects and specialize our theory for the injection molding process.

2. THEORY

The best way to understand how the pressure gets frozen-in is to consider the stress history of a layer that cools through the solidification temperature, denoted as T_s. At the starting time $t=0$ the pressure is assumed to be zero. The stress history then consists of a fluid part where the stress is equal to minus the melt pressure and a solid part:

$$\sigma_i = -P|_0^{t_{sz}} + \int_{t_{sz}}^{t} \dot{\sigma}^d dt \qquad i = x,y \tag{1}$$

where t_{sz} denotes the time that layer z solidifies and $\sigma_i{}^d$ is the deviatoric stress

$$\sigma_i^d = \frac{E}{1-\nu}[\varepsilon_i - (\epsilon^T + \epsilon^{cr} + \epsilon^{cure} + \epsilon^P)] \tag{2}$$

in which ε_i is the observable strain and ϵ^j are the volumetric strains:

$\epsilon^T = \alpha\Delta T$		Thermal expansion	(3)
$\epsilon^{cr} = C^{cr}\Delta\xi$	$C^{cr} = \dfrac{\rho_{cr}-\rho_{am}}{3\rho_{cr}}$	Crystallization shrinkage	(4)
$\epsilon^{cure} = C^{cure}\Delta\zeta$	$C^{cure} = \dfrac{\Delta\rho_{cure}}{3\rho_{cured}}$	Reaction shrinkage	(5)
$\epsilon^P = -\beta\Delta P$	$\beta = \dfrac{1-2\nu}{E}$	Compressive strain	(6)

and variables T, ξ and ζ represent the temperature, degree of crystallization and the degree of cure, respectively. Because the crystallization and cure effects are completely analogous to the thermal contribution, we will continue this analysis only with the thermal and pressure induced strain terms. In e.g. [14] it is shown how to include crystallization effects. Note that in the above equations all parameters are assumed to be constant and independent of temperature.

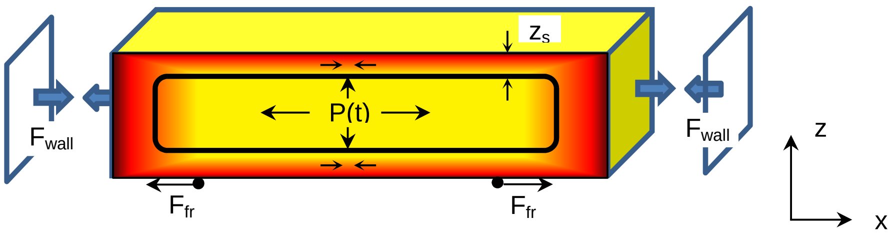

FIGURE 1. Schematic view of stress state in partly solidified product part

In order to determine the product shrinkage term ε_i we need to consider the overall force balance. The physical picture that we have is that of a thin solidified elastic membrane with thickness z_s which is stretched by the melt pressure in the core until it touches the side walls. This stretching is hindered by friction forces F_{fr} (see Fig.1).

The local dimensional change of a product in time step dt, $d\varepsilon_i$ is determined from the force balance which states that the average of the in-plane stresses over the thickness direction is balanced by the sum F_i of the reaction forces to the mold wall and the in-plane friction of the product with the mold:

$$\int_0^{z_s} \dot{\sigma}^d dz = \frac{E}{1-\nu}\int_0^{z_s} [\dot{\varepsilon}_i - (\alpha\dot{T} - \beta\dot{P})]\, dz = \dot{F}_i \tag{7}$$

In this equation only the temperature varies over the thickness. The averaging process therefore results in

$$\dot{\varepsilon}_i = \alpha\dot{\bar{T}} - \beta\dot{P} + \frac{1-\nu}{z_s E}\dot{F}_i \tag{8}$$

in which the dots denote differentiation with respect to time and the bar stands for the average over the solid layer thickness z_s. Inserting this in Eq.1 and using $P(t_{sz}) = P_s(z)$ and $T(t_{sz}) = T_s$ then gives [6]

$$\sigma_i = -P_s(z) + \frac{E}{1-\nu}\left[\, \varepsilon_i^{free} - \alpha(T - T_s) - \beta(\overline{P_s} - P)\right] + \int_{t_{sz}}^{t} \frac{\dot{F_i}}{z_s} dt \tag{9}$$

where ε_i^{free} is the unhindered shrinkage in absence of pressure and friction forces (see Eq.10 below). It is obvious that this equation largely simplifies in cases where either the external friction forces and the pressure are absent or if the shrinkage is hindered and ε_i itself vanishes. In the sections below we will discuss some limiting cases.

Case 1: Free quenching (polymer extrusion)

This case corresponds to the unconstrained solidification of a heated product, such as the cooling of a hot polymer slab or the solidification of an extruded product but also to the solidification of glass products. With vanishing pressure and friction contribution Eq.8 reduces to the first ter only and integrating this over the time interval at which the layers are in the solid state (as in Eq.1) then results in

$$\varepsilon_i^{free}(t) = \alpha \int_0^t \bar{\dot{T}} dt \tag{10}$$

$$\sigma_i^{free}(z,t) = \frac{E}{1-\nu}\left[\varepsilon_i^{free}\Big|_{t_{sz}}^{t} - \alpha(T - T_s)\right] dt \tag{11}$$

This solution has been known since long [15] and is incorrectly thought to be valid for injection molding as well. For a cooling slab with thickness $2L$ in a medium with heat transfer coefficient H from initial temperature T_0 to T_∞ the temperature is given by [2]

$$T(z,t) = T_\infty + (T_s - T_\infty) \exp\left[\frac{-\alpha_1^2 at}{L^2}\right] \cos\left[\frac{\alpha_1 z}{L}\right] / \cos\alpha_1 \tag{12}$$

in which α_1 is the root of $\alpha_1 \tan\alpha_1 = Bi$ which can be approximated as [thesis Jansen]

$$\alpha_1 = \frac{\pi}{2}\frac{Bi}{1+Bi} + O(Bi^{-2}) \tag{13}$$

and $Bi = HL/k$ is the dimensionless Biot number. More detailed solutions for this free quenching case are discussed in refs. [2,6]. In essence the resulting stress contribution has a parabolic profile and with tensile stresses in the core and compressive stresses on the surface. Because of these compressive stresses on the surface, free quenched products are less sensitive to small scratches and cracks at the surface.

Case 2: Constrained cooling (Injection Molding)

The boundary conditions for injection molding are quite different from those of the free quenching case discussed above. In injection molding the shrinkage during solidification is prevented during most of the molding process for several reasons. First of all, because of the melt pressure in core, the solidified membrane is stretched pressed to the mold walls, as is shown in Fig.1. Secondly, at higher pressures the frictional forces with the mold surfaces also prevent the product from shrinking and, thirdly, geometrical constraints like ribs and bosses are quite effective in preventing shrinkage, even if the internal pressure vanishes. Therefore the strain ε_i vanishes and Eqs.1 and 2 reduce to the in-plane stress equation before ejection:

$$\sigma_i^{IM} = -P(t) + \frac{E}{1-\nu}[0 - \alpha(T - T_s) + \beta(P(t) - P_s(z))] \tag{14}$$

After ejection the product shrinks proportional to the average stress at the instant of ejection and we get for the stresses during the cooling process after ejection:

$$\sigma_i^{IM}(z,t) = \frac{E}{1-\nu}\left[\alpha\left(\overline{T}(t) - T(z,t)\right) + \beta(\overline{P_s} - P_s(z))\right] \tag{15}$$

These stresses still consist of two parts: the thermal stresses and the pressure induced stresses. Important to realize is that after cooling to a uniform temperature, T_∞, the thermal part vanishes, resulting in the final expressions for the as-molded residual stresses:

$$\sigma_i^{IM}(z) = \frac{1-2\nu}{1-\nu}[\overline{P_s} - P_s(z)] \tag{16}$$

and shrinkage:

$$\varepsilon_i^{IM} = -\alpha(T_s - T_\infty) + \beta\overline{P_s} \tag{17}$$

Case 3: Unequal cooling during injection molding

In practice mold geometries of injection molded parts tend to be quite complicated and the two mold walls can often not be kept at exactly the same temperature. Such situations in particular occur near corners in thin walled products and are the main cause of overall shape distortions and warpage. The result is that the solid layer at the hottest side will grow slower than that of the cold side, resulting in an unequal final stress distribution. In [12] we showed how warpage of flat plates and corner products due to unequal cooling can be calculated. The derivation starts with Eq.16 above. The moment due to the now asymmetric stress distribution is given as

$$M_i = \int_{-L/2}^{L/2} z\, \sigma_i^{IM}(z) dz \tag{16}$$

and is balanced by the bending moment $M_b = h^3 E\kappa_i / 12(1-\nu)$, resulting in a prediction for the warpage curvature of a flat plate [12]:

$$\kappa_i = \frac{12(1-2\nu)}{L^3} \int_{-L/2}^{L/2} z\, P_s(z) dz \tag{17}$$

A similar expression for the warpage of a corner product was obtained [12].

3. EXPERIMENTAL VALIDATION

In section 2 above we derived a series of closed form expressions relating the residual stresses and shape deformations due to processing to the local temperature and pressure profiles. In this section we will compare the predictions using the equations with both measured or numerically calculated stress and deformation values, thereby focusing on the injection molding process (Cases 2 and 3). Note that standard simulation programs tend to overpredict the peak pressure as well as the pressure decay during cooling [10]. Therefore, in all validation studies we used *measured* instead of calculated cavity pressure histories. In that way we could have a more straight forward validation of the idea that residual stresses and shrinkages in injection molded products are directly related to the local pressure history.

Case 1: Validation of stresses after injection molding

It is not simple to actually obtain reliable quantitative measurement data for the through thickness stresses in polymer products. X-ray techniques that are popular for metals have a far too low resolution and optical methods using e.g. birefringence do not work since they reflect bot molecular orientation and residual stresses. Moreover, by making through thickness cross sectional slices, the residual stress part tends to vanish. The most suitable method appears to be the Layer Removal method of Treuting and Read [17]. In this method small layers of a flat plate are removed by carefully milling, resulting in changes in the stress distribution and a corresponding warpage. By analyzing the warpage changes as a function of the removed layer depth the original stress distribution can be calculated. However, when applied to polymers the heat introduced by milling process is expected to affect the residual stresses. Therefore in [8] we modified the layer removal process by using a KrF excimer laser and showed that we could then remove layers without such heating effects.

We used this technique on an injection molded polycarbonate plate to compare residual stress predictions with measurements. As is shown in Fig.2, the stresses in an injection molded product are of the order of ±5 MPa and are tensile in the core and show a compressive minimum just below the surface. The predicted stress profile (full lines) compares well with the measured stresses (lines with symbols), both regarding the magnitude and the shape. Similar results were obtained for experiments with a different material (not shown) and with a much lower applied pressure (Fig.2, right). For the experiment at lower holding pressure the cavity pressure was observed to drop to zero during the solidification process such that the core layer vitrified at zero pressure. In Fig.2-right this shows up as a constant core stress level, both for the measure and predicted curves. Note that in this special case the boundary conditions changed from a constrained shrinkage case to a free shrinking case with zero pressure. How this change in boundary affects the stress and shrinkage predictions is explained in more detail in refs. [6,7,9].

Further notice that in the low holding pressure case the surface is in a state of tensile stress. As is well know, this has a negative effect on the product durability and it is therefore recommended to avoid low molding pressures where possible.

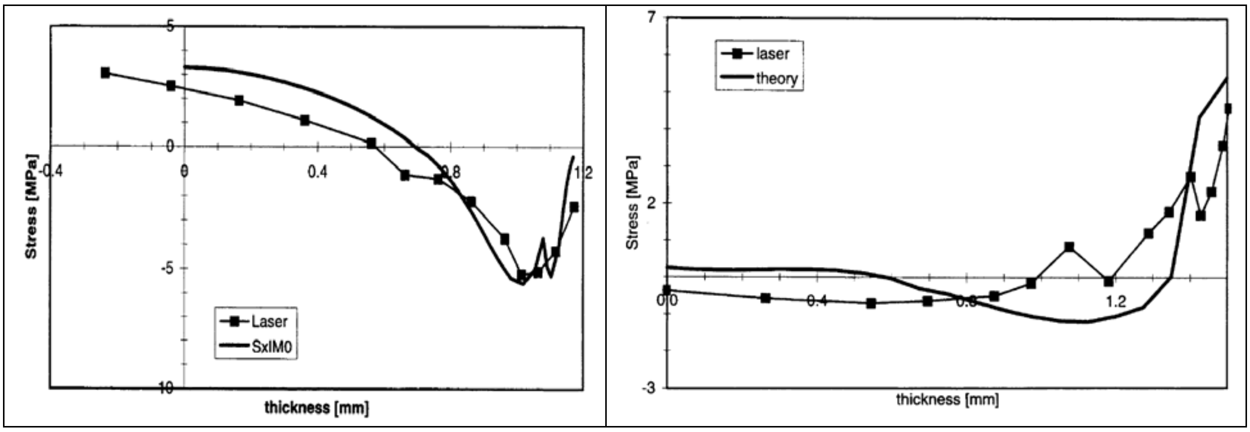

FIGURE 2. Comparison of measured (squares) and predicted (full line, Eq.16) residual stresses for a polycarbonate plate. Left molded at 65 MPa, right molded at 17 MPa [8].

Regarding residual stresses we can thus conclude that the local cavity pressure indeed has a direct effect on the as-molded stress distribution and that our simple thermo-elastic theory is able to predict this behavior correctly.

Case 2: Validation of shrinkage predictions

In [11] a systematic study on the effect of processing conditions on shrinkage was performed for seven common thermoplastic polymers (ABS, PC, PS, HIPS, PBT, PBT-GF30 and HDPE). The processing parameter studied were holding pressure, injection velocity, melt temperature and mold temperature. Fig.3 shows results of this study for ABS, PC and PS. The shrinkage values for these amorphous polymers vary between 0.4% and 1% and decrease with increasing holding pressure. The other processing conditions (injection velocity, mold and melt temperature) had less effect on the shrinkage than the holding pressure. In all cases the predictions according to the simple thermo-elastic model (Eq.17) were seen to perfectly agree with all measured shrinkage values (Fig.3 in [11]). For semi crystalline materials a small difference between length and width shrinkage was observed which was attributed to the orientation of the crystallites during flow. A more in-depth study on this phenomena was performed in [14]. In that paper the anisotropy in the modulus, thermal expansion and compressibility was first measured as a function of the flow path and subsequently used to predict the shrinkage anisotropy. One of these results for polypropylene is shown in Fig.3 (right, below). Here the model slightly over predicts the shrinkage, which was attributed to the fact that part of the crystallization shrinkage occurred already in the fluid state and therefore not effectively contributed to the shrinkage and stress build-up [14].

Note that in order to include anisotropic effects the above thermo-elastic theory was modified by replacing the in plane stress equation (Eq.2) by its anisotropic variant [13,14]:

$$\sigma_i^d = Q_{ij}\left[\varepsilon_j - (\epsilon_j^T + \epsilon_j^C + \epsilon_j^R + \epsilon^P)\right] \tag{18}$$

Here Q_{ij} represents the in-plane stiffness coefficient matrix. For more details we refer to refs. [13,14].

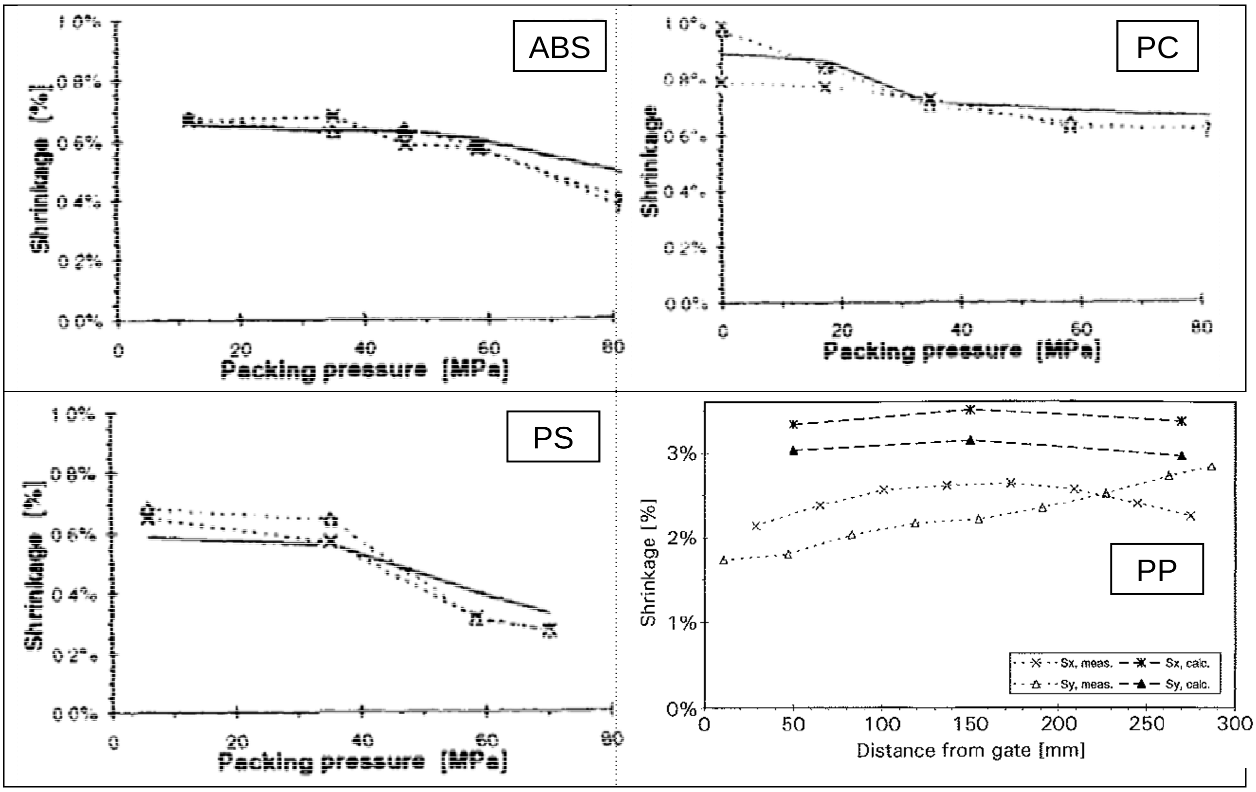

FIGURE 3. Comparison of measured (symbols) and predicted (full lines, Eq.17) residual stresses for ABS, PC and PS as a function of the packing pressure [11]. Right-below the shrinkage for polypropylene is shown as a function of the flow path. Here the long dashed lines are the model predictions [14].

All in all we can conclude that the proposed model turns out to be able to predict injection molding shrinkage quite accurately.

Case 3: Unequal cooling during injection molding

Warpage of molded parts is one of the largest problems in injection molding and was always attributed to the temperature difference between the two mold halves. The typical way to reduce warpage was therefore to adjust the cooling system such that the temperature difference was minimal. Based on the above ideas, however, it appeared quite likely that not only the temperature but also the holding pressure should be a key variable to control the warpage process. In [12] we conducted therefore an experimental warpage study on flat plates and corner products in which we both varied the mold temperature difference and the holding pressure. In order to obtained accurate values for the local temperatures in the curved mold surfaces we paid special attention to integrate fast response thermocouples flush with the mold surface [12].

The results of this study for flat plates are shown in Fig.4-left. Decreasing the temperature difference indeed decreased the warpage, as expected. However the effect of pressure was also quite noticeable. At low pressure the plate curved towards the hot side, whereas at higher pressures it curved towards the cold mold side. These trends are all captured by our thermo-elastic model predictions (Eq.17). The absolute warpage values however are under predicted which was attributed to the large sensitivity of warpage predictions to model parameters like the solidification temperature [12].

For corner products similar observations were made. Fig.4-right in fact shows that spring forward (angles larger than 90°) or spring backward (angles smaller than 90°) could be obtained by varying both the temperature difference or the holding pressure. The study further showed that large radius corner products showed considerable more warpage than products with an outer radius of 4 mm or less.

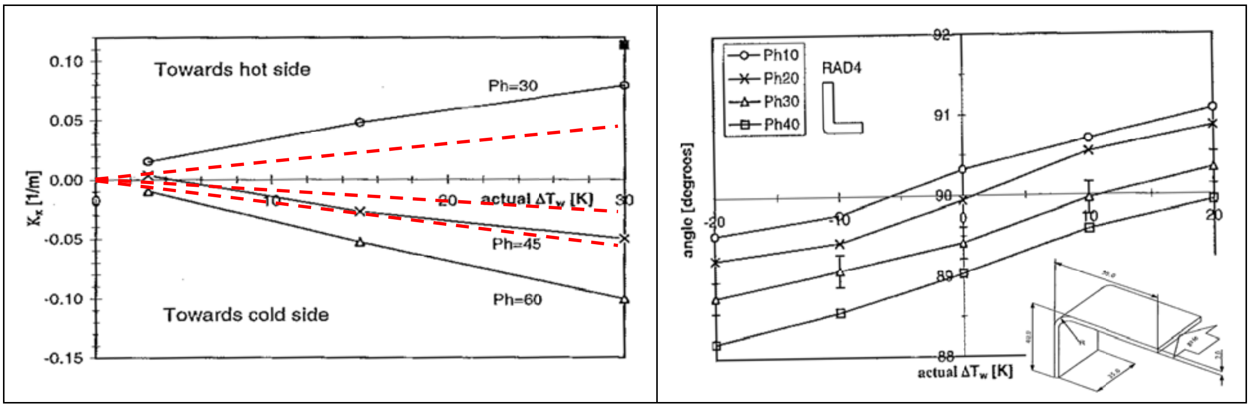

FIGURE 4, LEFT: Measured (symbols) and predicted (dashed lines) warpage of asymmetrically cooled flat plate [12]; **RIGHT:** Measured warpage in corner product as function of temperature difference and holding pressure [12]

4. CONCLUSIONS

In the present paper we summarized the series of studies performed on the relation between molding parameters and residual stresses, shrinkage and warpage of injection molded products. We first summarized the basic theory and resulting closed form equations and then systematically compared these predictions with experimental validation evidence.

We can conclude that the simplistic approach of neglecting all viscoelastic effects and treating the solidification process as an instantaneous change from the melt to the solid state is quite effective in describing all residual stress related issues in the molding process. This study also showed that the local cavity pressure should be considered as the most important parameter which determined both residual stresses, shrinkage and warpage in molded products. The main effect of temperature is that it dictates the growth of the solidified layer and thus the speed at which the pressure is frozen-in.

5. REFERENCES

1. G. Titomanlio, V. Brucato and M.R. Kamal, Int Polym Process **1**, 55 (1987)
2. L.C.E. Struik, *Internal stresses, dimensional stabilities and molecular orientations in plastics* (Wiley and sons, Brisbane, 1990)
3. F.P.T. Baaijens, Rheol Acta **30**, 284 (1991)
4. K.M.B. Jansen, Int Polym process **9**, 82-89 (1994)
5. K.M.B. Jansen and G. Titomanlio, Polym Eng Sci **36**, 2029-2040 (1996)
6. G. Titomanlio and K.M.B. Jansen, Polym Eng Sci **36**, 2041-2049 (1996)
7. R. Pantani, K.M.B. Jansen and G. Titomanlio, Int Polym process **12**, 369-402 (1997)
8. K.M.B. Jansen, J.J.W. Orij, C.Z. Meijer and D.J. van Dijk, Polym Eng Sci **39**, 2030-2041 (1999)
9. K.M.B. Jansen, R. Pantani and G. Titomanlio, Polym Eng Sci **38**, 254-264 (1998)

10. K.M.B. Jansen, D.J. van Dijk and E.V. Burgers, Int Polym Process **13**, 99-104 (1998)
11. K.M.B. Jansen, D.J. van Dijk and M.H. Husselman, Polym Eng Sci **38**, 838-846 (1998)
12. K.M.B. Jansen, D.J. van Dijk and P.J. Keizer, Int Polym Process **13**, 417-424 (1998)
13. K.M.B. Jansen, D.J. van Dijk and M.J.A. Freriksen, Polym Compos **19**, 325-334 (1998)
14. K.M.B. Jansen, Int Polym Process **13**, 309-317 (1998)
15. B.D. Aggarwala and E. Saibel, Phys Chem Glasses **2**, 137 (1961)
16. K.M.B. Jansen, *Calculation and Control of Heat Transfer in injection molding*, PhD thesis, TU Delft (1993)
17. R.G. Treuting and W.T. Read, J Appl Phys **22**, 130 (1951)