WoodWeaver

Fabricating curved objects without moulds or glue

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Abstract. *In this study, we propose a novel computational system called WoodWeaver for fabricating curved surfaces from conventional materials without using moulds. We use a cutting-based material bending method called Dukta. Our system enables a user to design a single free-form curved surface and to fabricate it based on the bending deformation characteristics of the material. The system also indicates an invalid shape; that is, one that will break upon bending deformation. The user can then design a valid shape with this information. We also develop an optimal Dukta pattern that is the smallest-sized gap pattern necessary to represent a user-designed shape. Given a valid shape, the system generates the optimal pattern and a machine cuts four modules with this pattern. Finally, the user assembles these modules to obtain the desired shape.*
Keywords. *Digital fabrication; personal fabrication; computational design; mould bending; interactive modelling.*

BACKGROUND

Recently, digital fabrication devices have found widespread application, and they are increasingly being used directly or indirectly by many users to fabricate customized products on their own. These users almost all use common fabrication machines and share the digital data used in fabrication on the website [1]. Customizability and manufacturability are important to these users.

Despite these advances, the fabrication of curved objects is laborious. Previous methods necessitated the use of a mould to produce certain curved objects and were applicable only for mass production, not for on-demand or adaptive production.

The main objective of this study was to create a system for manufacturing curved objects with-out moulds or glue. To achieve this, we expanded the use of Dukta, which is a cutting-based material bending method [2]. The Dukta method enables rigid materials to behave in a flexible and elastic manner if cut in certain patterns. This method is widely used around the world in the fields of furniture design and architectural design. The second objective of this study was to develop a formula for design and fabrication using the Dukta method.

Related research

The ZipShape: method can also be used to manufacture curved objects without a mould (Schindler, 2008; Schindler and Espinosa, 2011); however, it requires a 5-axis computer numerical control (CNC)
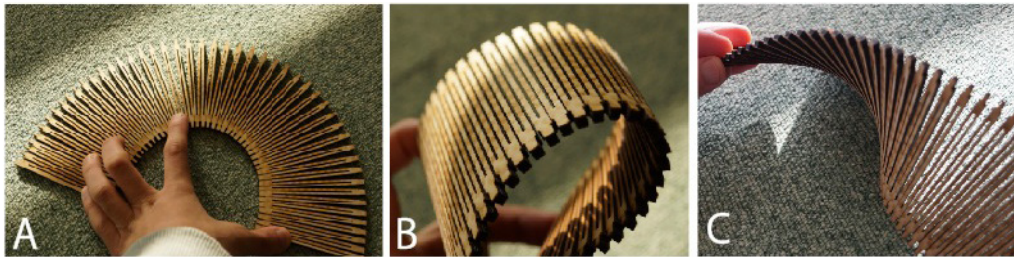
machine and a vacuum gluing machine. These machines are not commonly available, and the fabrication process is laborious. Furthermore, the modelling software used in the ZipShape method does not provide feedback when the designed bending shape is invalid. In contrast, our system requires only a general 2-axis CNC cutting machine, and it provides feedback that indicates an invalid shape. Moreover, our fabrication process is very easy to use because it involves only CNC cutting and user assembly of the fabricated parts to obtain the final shape. Using our system, the user can fabricate material-efficient, adaptive curved objects for furniture or architectural design.

Focus of the study
Based on the discussion above, we propose a computational framework for
- the design of a bending shape;
- the determination of an optimal Dukta pattern; and
- the development of an easy mouldless bending method.

WHAT IS DUKTA
Dukta is a material processing method that enables rigid materials to be made flexible and elastic with only a special cutting pattern. By changing the geometry of this pattern, a variety of elastic properties can be generated in the rigid materials. We classify Dukta deformations into three types: bending (A), folding (B), and twisting (C), as shown in Figure 1.

In this study, we focus on deformations A and B and propose a design and fabrication system called WoodWeaver for a single curved surface with these deformations. To develop this system, we formulate the relation between each Dukta pattern and the deformation shape generated by that pattern. Furthermore, we develop an optimal pattern A, which represents a given (user-designed) deformation shape. This pattern also shows a smaller-sized local gap pattern than does the usual pattern A (see details in the next section).

FORMULATION OF PATTERNS A AND B
Each pattern has three kinds of parameters:
- Pattern parameters, which are related to the flexibility of the Dukta-generated material and describes the pattern;
- An adaptive parameter, which is automatically generated by the system according to the designed deformation shape and describes the space in the pattern; this space is not related to the flexibility of Dukta; and
- A deformation parameter, which is the limit of local deformation in the pattern.

We will first focus on pattern B, because it is simpler to formulate than pattern A. The variables d_b, l_b and s_b denote the pattern parameters. The variable d_b represents the red line, l_b represents the green one, and s_b is the light blue one (Figure 2). The variable p_b represents the space between material, the purple line in Figure 2 and is the adaptive parameter; and the variable θ_b denotes the bending angle, or deformation parameter. The user needs to find appropriate pattern parameters that allow the material to be bent to the angle θ_b. By fabricating a small sample of experimental material and bending
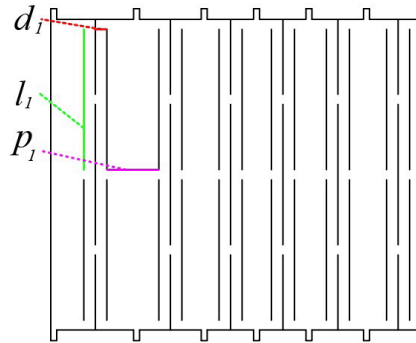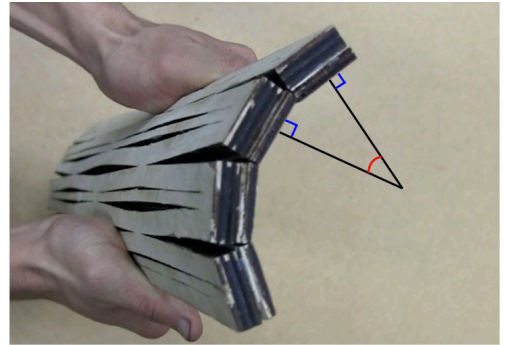
it within the elastic deformation range, we can determine the deformation parameter and obtain the appropriate pattern parameters that enable the material to be flexible without exceeding the deformation parameter (Figure 3).

Next, we propose an optimal pattern A, which is the heterogeneous pattern corresponding to the bending curvature. The local gaps of optimal pattern A are smaller than those of the usual homogeneous pattern A (Figures 4 and 5).

As shown in Figure 6, the variables l_a, s_a and d_a denote the pattern parameters; p_a and y_a denote the adaptive parameters; and y_{a_max} denotes the deformation parameter. For optimal pattern A, we can also obtain the deformation parameter and the appropriate pattern parameter by fabricating a small sample of experimental material and bending it within the range of elastic deformation. These pat-

tern parameters enable the material to be flexible without exceeding the deformation parameter.

MODEL OF DEFORMATION SHAPE REPRESENTED BY OPTIMAL PATTERN A

As shown in Figure 7, the shape ('shape A') represented by pattern A corresponds to a circular arc curve that preserves tangential continuity. We can define the valid shape A as a feasible region on the plane (r, θ) (Figures 8 and 9), where r and θ denote the inner angle and radius of each circular arc that constitutes the circular arc curve. To define this feasible region, we consider the local deformation in pattern A to be the deformation of a cantilever and apply the elastic curve equation. That enables us to get the limit of r and θ from predefined parameters, as shown in (1)–(3).

Gaps in deformation shape

$$y_a = \frac{w l_a^3}{3E} \tag{1}$$

$$\theta_a = \frac{w l_a^2}{E} \tag{2}$$

$$\theta_a = \frac{3 y_a}{l_a} \tag{3}$$

The arc that is beyond the feasible region is an invalid arc. This means that deformation corresponding to the invalid arc will lead to breakage. We explain the details of the feasible region below.

OVERVIEW OF THE WOODWEAVER SYSTEM

The proposed WoodWeaver system consists of four processes: First, the user inputs parameters and the system defines the feasible region. Second, the user designs the bending shape with the constraints defined in the plane (r, θ) that we call the feasible region. Third, the system generates a fabrication plan for CNCcutting. Fourth, the user edits the generated infeasible pattern into a feasible one. As a result of these processes, the user obtains the designed curved surface constructed with deformations A and B (Figure 8). The bending curvature is described by deformation A, so the shape that the user designs is a free-form circular arc curve, with a valid shape defined by the feasible region.

INPUT AND FEASIBLE REGION
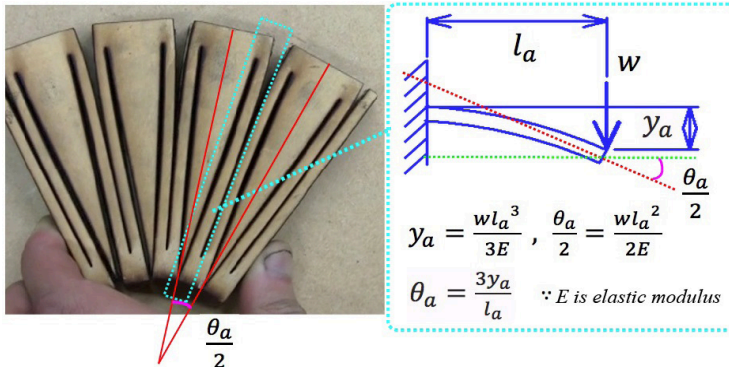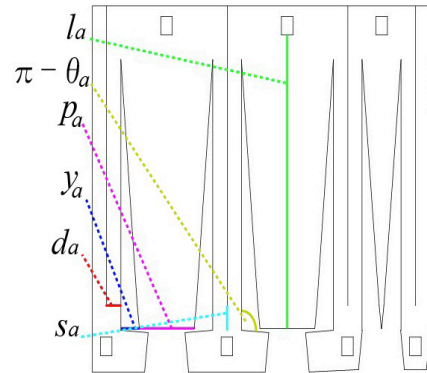
To define constraints, the user inputs the pattern pa-

Figure 6
Heterogeneous pattern A.

rameters and the deformation parameter for pattern A and pattern B. This feasible region is defined by (4) and (5).

$$\theta_a(r) = ArcTan\left(\frac{d_a}{r - l_a}\right) \tag{4}$$

$$\theta_{a\_max} = \frac{3 y_a}{2 l_a} \tag{5}$$

$$l_a + \frac{d_a}{\tan(\theta_{a\_max})} \leq r \leq \infty \tag{6}$$

$$\theta_a(r) \leq \theta \leq \theta_{max}$$
$$\because \theta_{max} = Min(\theta_{a\_max}, \theta_b) \tag{7}$$

Equation (4) can be derived from the length of the radius in pattern A. The relation between the radius and the inner angle of the arc is shown in Figure 10. The inverse function of (4) is (8).

Figure 7
Local deformation in pattern A. The load on the cantilever is w and is the deformation parameter (also the cantilever's displacement). The red line in the right photo correspond to the radius of the arc used in the designed curve.

Figure 8

Final shape of designed piece.
Flat sides are constructed us-
ing deformation A and curved
surfaces are constructed using
deformation B.

Figure 9

The feasible region. Equation
(4) denotes the red curve, and
θ_{max} denotes the blue line.
The green line corresponds to
θ_{a_max} or θ_b.
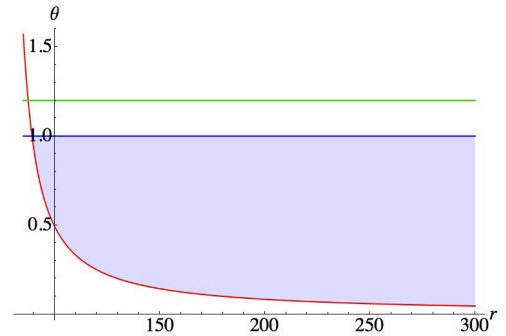
$$r_a(\theta_a) = l_a + \frac{d_a}{\tan\frac{\theta_a}{2}} \qquad (8)$$

Equation (5) is the same as (3). This feasible region is the possible bending shape set within the inputted deformation parameter.

INTERACTIVE MODELLING WITH CONSTRAINTS

The modelling process consists of three steps. The first step includes user modelling of the curve and discretization by the system. The user models the free-form bending shape by designing a parametric curve, and the system simultaneously discretizes this curve with finite circular arcs (Figure 11). The second step is the indication and suggestion of validity by the system. By judging whether each circular arc obtained is in a feasible region or not, the system visualizes the valid or invalid state of the discretized parametric curve. If an invalid shape is seen, the system suggests a valid shape. If this suggested shape does not satisfy the user's requirements, the user can edit the bending shape. We describe the details of this editing below. The third step involves generating a fabrication plan for cutting using CNC, and the fourth step is pattern editing, whereby the user can edit an infeasible pattern into a feasible one.

Modelling and discretization

The modelling and discretization process is described by three steps:

1. Designing the parametric curve.
2. Discretizing this curve using a Biarc algorithm and obtaining circular arcs $(r_d, \theta_d) \in A$, as shown in Figure 12. Biarc is an algorithm to discretize

Figure 10

Photo showing the relation of
the parameters for pattern A.
Each parameter is shown in
Figure6. We obtain the equa-
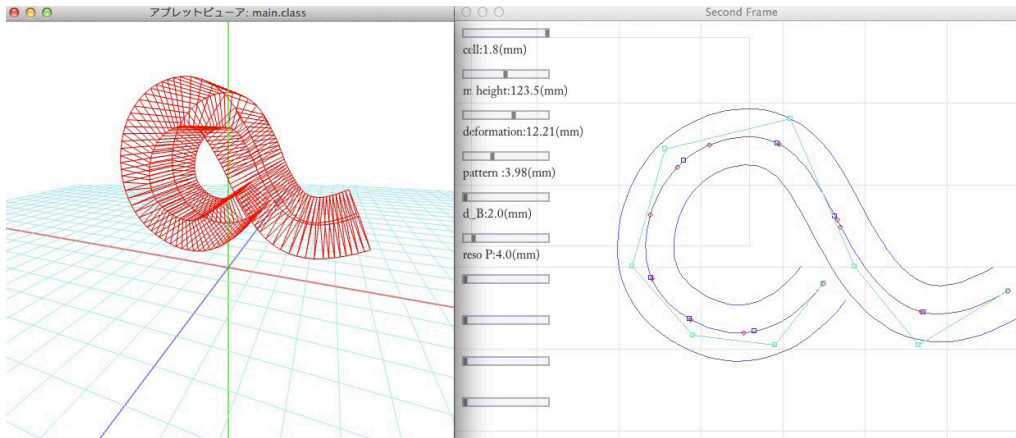tion from the length of the red
and green lines.
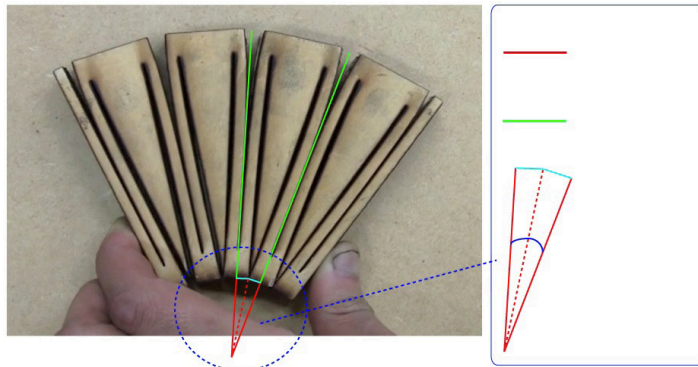
Figure 11
User-interface of WoodWeaver. The user designs the bending shape in the right-hand window and the final shape is visualized in the left-hand window.

a tangent continuous curve. This algorithm splits the curve into a monotonic curvature region and interpolates this region with two arcs. There is a one-parameter family of interpolation (i.e., parameter $p \in [a, b]$, $a, b \in \Re$). We initially use the parameter $p = (a + b)/2$. The details of the discretization process are described in (Jakubczyk, 2012).

3. Splitting all arcs further and defining new arcs $(r_e, \theta_e) \in A'$. A splitting algorithm is implemented with equation (4) above and the following equation.

$$\theta_e = \frac{\theta_d}{\left[\frac{\theta_d}{\theta_a}\right]}, r_e = r_d \qquad (9)$$

($\therefore \theta_a = \theta_a(r_d)$ where [] is the Gauss symbol)

With these processes, the user-modelled parametric curve is discretized to finite circular arcs ($r_e, \theta_e) \in A'$ simultaneously with modelling. This curve represents the bending shape represented by pattern A.

Indication and suggestion of validity
The system judges whether each obtained arc is in a feasible region or not, and the system indicates which parts are valid or invalid. As shown in Figure 13, the red and yellow arcs that are out of the feasible region are invalid parts, and the blue and green ones in the feasible region are valid parts. The curve in which there is at least one part that will break upon bending deformation is called an invalid shape. As mentioned above, we can easily obtain the validity of a shape; however, the user cannot always easily find a valid shape when doing so manually. Therefore, the system suggests a valid shape when it is difficult to determine it manually. The feasible region has two parameters, θ and r, which respectively denote the angle and radius of the arc.
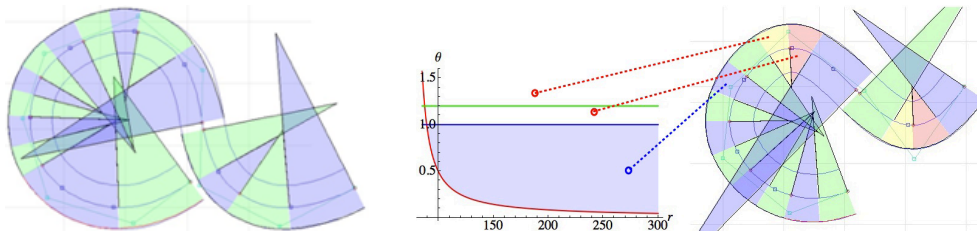

Figure 12
Discretized curve obtained using Biarc algorithm.

Figure 13
Correspondence between arcs and the feasible region.

Figure 14
Infeasible pattern. Connected
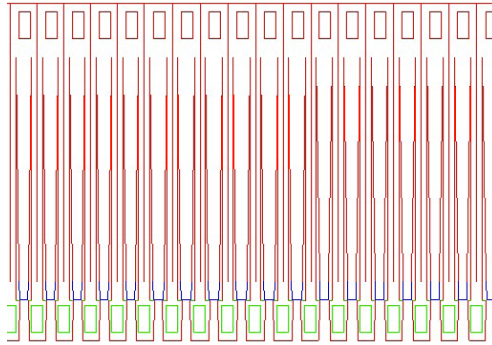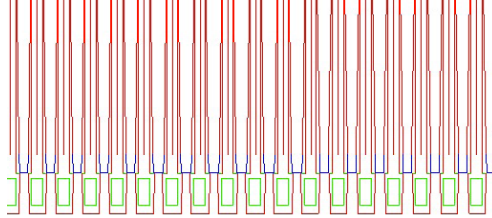parts are too thin to fabricate.

Figure 15
Feasible pattern. Connected
parts have been made larger
for easier fabrication.

If an invalid shape is invalid only because of an invalid r, this means that r is too small; in this case, it is easy to find a valid shape manually because it is obvious that the curve whose curvature is too sharp has to be more gradual. However, if an invalid shape consists of an invalid θ (i.e. its value is too large), it is difficult to find a valid shape; in this case, the system specifically suggests a valid shape by replacing an arc whose θ lies outside the feasible region with one whose θ lies within it. In replacing the arc, we need to preserve the tangent continuity between each arc.

Generating a fabrication plan

If the user can obtain a valid desired shape, the system generates 2-D CNC cutting patterns. For pattern A, the given parameters are l_a, y_{a_max}, d_a, s_a, and θ_e, and the system calculates the adaptive parameters p_a and y_a based on the given parameters.

$$y_a = \frac{\theta_e l_a}{3} \qquad (10)$$

$$p_a = \cos\left(\frac{\theta_e}{2}\right)\{(r_a - l_a)\tan\left(\frac{\theta_e}{2}\right) - d_a\} \qquad (11)$$

Equation (10) is derived from (3). Equation (11) is deduced to be similar to (4) by considering the relation between the radius of the arc and each parameter. Given the parameters used in pattern A, the system draws pattern A as shown in Figure 6.

For pattern B, the given parameters are l_b, d_b, s_b, θ_{ei} and θ_{ei+1}, the system calculates an adaptive parameter p_b using (12).
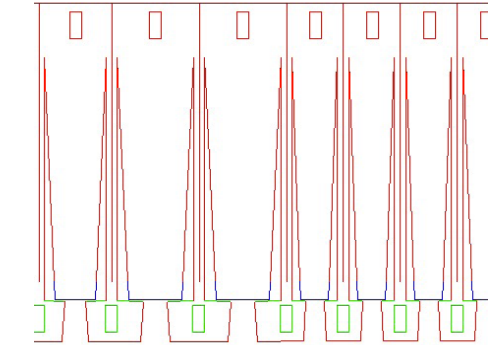
$$p_b = r_{ei}\sin\frac{\theta_{ei}}{2} + r_{ei+1}\sin\frac{\theta_{ei+1}}{2} + 2d_b \qquad (12)$$

∴ i is the number of arc and i and i +1 are adjacent arcs.

Given the parameters used in pattern B, the system draws pattern B as shown in Figure 2.

Pattern editing

Even if an obtained pattern is geometrically valid, an infeasible pattern may be generated. This occurs when patterns have parts that may be too thin to fabricate; in this case, the user can modify the pattern by reducing the number of splits and thus the number of arcs (r_d, θ_d) ∈ A. This increases the size of each arc and makes the pattern coarse. Because the number of arcs used in the designed curve corresponds to the number of pattern elements, reducing the number of splits means modifying the infeasible pattern into a feasible one (Figures 7, 14 and 15).

We describe the new splitting algorithm for pattern modification below.

1. Discretize this curve by using a Biarc and obtaining circular arcs (r_d, θ_d) ∈ A.
    (The first step here is the same as the second step described in the *Modelling and discretization* section above.)

$$\theta_e = \frac{\theta_d}{\left\lceil \frac{\theta_d}{\frac{\theta_{a\_max}}{k}} \right\rceil}$$

2. If θ_{a_max} / 2 ≤ θ_d, then
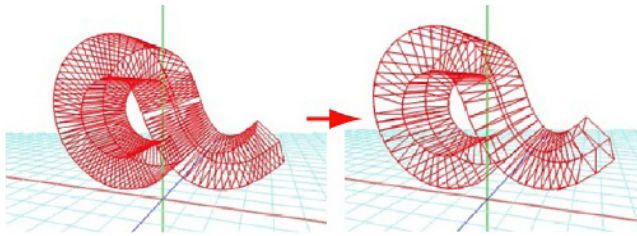   However, if θ_a (r) ≤ θ_d ≤ θ_{a_max} / 2, then $\theta_e = \theta_d$.

Figure 16
Visualized final shape will have a coarser appearance when the pattern is changed to a coarser one.

∴ k is a granularity parameter of discretization.

3. Apply this step to all arcs (r_d, θ_d).

FABRICATION

By inputting the pattern and deformation parameters in each pattern, the user models the bending shape and edited pattern (Figure 16). Finally, the WoodWeaver system generates a fabrication plan as a PDF file of vector data for CNC cutting. With CNC cutting, we obtain two pattern A modules and two pattern B modules. The pattern A modules will form the sides of the final shape, and the pattern B modules will form the curved surfaces (Figures 8 and 17-19). The user obtains the final shape by assembling the four modules. The connecting parts of patterns A and B interlock and specify the designed curvature. We made a real scale chair with this system, using two pieces of plywood whose width, height, and thickness were about 1800 × 900 × 15 mm, respectively. We were able to use panels efficiently and get various final shapes because all cut modules are rectangles in our system. The CNCcutting took 3.5 h and assembly took 0.4 h.

EXAMPLES OF PIECES CONSTRUCTED USING WOODWEAVER SYSTEM

We constructed the items shown in Figures 20–22 using the WoodWeaver system with a laser cutter.

VALIDITY AND LIMITATIONS

In this study, we proposed a model for designing deformation shapes based on patterns A and B. In particular, the relation between pattern A and the deformation shape resulting from it is not obvious. We formulated this relation and developed an optimal pattern A. The validity of our deformation shape model was verified because the connection parts of pattern A were tightly fitting (Figure 23). The angle of the connection parts in pattern A (Figure 6) was deduced with the local deformation model for pattern A. This system enabled us to design and fabricate free-form bending shapes from patterns A and B. In this study, we focused on the geometric properties of Dukta deformation. The physical properties of Dukta were not considered and the elastic phe-
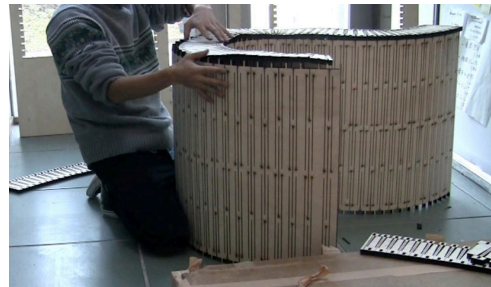
Figure 17
Intermediate stage of assembly showing one pattern A and one pattern piece.

Figure 18
Last stage of module assembly.

Figure 19
Final shape of assembled module.

Figure 20
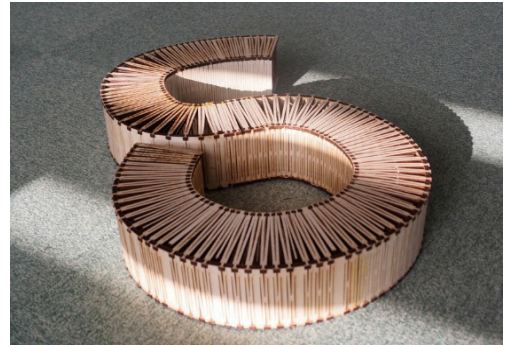Fabricated sample not using optimal pattern.

Figure 21
Fabricated sample used as a chair.

Figure 22
Fabricated sample with optimal pattern.

nomena and structural stiffness of Dukta were not taken into account. With our approach, we can design and obtain the deformation shape required to prevent materials from breaking during the forming process.

CONCLUSIONS

We proposed complete system called WoodWeaver for designing and fabricating a curved surface using the Dukta method. Our system includes a method for modelling free-form circular arc curves with geometric constraints. This method is applicable not only to Dukta, but also to other structures in architecture or furniture (Figure 24). The WoodWeaver system can therefore be applied to other kinds of fabrication methods. In addition, this study developed formulas specifically for design and fabrication using the Dukta method. In particular, we focus on the geometric property of Dukta deformation and do not take into account physical properties of Dukta method (i.e..elastic phenomena and structural stiffness). The physical properties of materials used in the Dukta method need to be analysed further for practical application.

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