PHENOMENOLOGICAL THEORY OF RADAR TARGETS

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PROEFSCHRIFT

TER VERKRIJGING VAN DE GRAAD VAN DOCTOR IN
DE TECHNISCHE WETENSCHAPPEN AAN DE TECHNISCHE
HOGESCHOOL DELFT, OP GEZAG VAN DE
RECTOR MAGNIFICUS IR. H.R. VAN NAUTA LEMKE,
HOOGLERAAR IN DE AFDELING DER ELEKTROTECHNIEK,
VOOR EEN COMMISSIE UIT DE SENAAT TE
VERDEDIGEN OP WOENSDAG 16 DECEMBER 1970
TE 14 UUR

DOOR

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ELEKTROTECHNISCH INGENIEUR
GEBOREN TE BATAVIA

1970
DRUKKERIJ BRONDER-OFFSET N.V.
ROTTERDAM

DIT PROEFSCHRIFT IS GOEDGEKEURD DOOR DE PROMOTOR
PROF. DR. IR. J. P. SCHOUTEN.

Aan Etty,

Mariska en Marc

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1 INTRODUCTION

1. Approach

In this work, we consider the radar as a source of electromagnetic radiation which illuminates an object. The radar target may be a single object in space such as an airplane or a metallized balloon, or a multiplicity of objects such as a flock of birds, a meteor trail, or raindrops distributed within a rain cloud. The radar target may also be an extended target such as a type of terrain, a type of water surface, a shoreline, or a mountain range, or even the sun, another planet, or the moon. In fact, any type of conceivable natural or manmade target may be considered a possible candidate for radar illumination.

Single targets are defined in contrast to time-varying targets, such as when an airborne radar flies over terrain, the sea surface, or forests. Also, a ground radar might observe moving chaff or dipole clouds, rain clouds, or clouds of dust particles. These targets are called "distributed" radar targets.

The intent of the present work is to deal with all these targets, at least potentially, and hence the phenomenological character of the investigation. By this, we mean simply that the radar target appears to us as an object for investigation, through the processes of radar illumination.

We soon find that an enormous field of investigation is thus staked out – not only because of the variety of targets but also because of the variety of illuminations, such as the fundamental frequencies and polarizations, pulse shapes, antenna gain patterns, and waveforms that are available. These illuminations may have a static character or they may vary with time in a systematic or random manner.

Each illumination of the target causes a reflection or radar return, which is sampled by the radar receiver. The receiver may have its own antenna gain pattern and may be tuned to a set of preferred frequencies, polarizations, or wave forms. The radar receiver may be situated at a different location from the radar transmitter, as in the bistatic case; for instance, the transmitter may be on the ground and the receiver may be airborne, or the receiver may be in the proximity of the transmitter, as with "monostatic" radar (which is the case for most large ground radar observation stations).

Each of these possibilities for radar application may become the foundation for a complete technology with its own specific intentional character and resulting problematics. The introduction of modern computers as part of the radar system has even further expanded the scope, to the extent that branches of science and engineering have been developed under such names as signal processing theory, radar information theory, radar detection theory, and optimal decision theory, to indicate a thrust of technological advancement in these areas. For the most part, the intent of these branches is to optimize a systems performance parameter or set of parameters.

In this work, we will not deal with system optimizations, since this would entail delving into each system separately and analyzing its performance – a truly gigantic engineering task for each major system, as exemplified by such a field as air traffic control by radar.

Our primary task will be to keep the target in (radar) focus and to try to delineate some basic characteristics that may be of interest to a radar systems engineer, but will not necessarily be. It may seem strange to say that the behavior of a radar target as an object is treated with utmost indifference in most radar systems applications; however, a search through the literature soon convinces us that this is indeed the case. The single object is usually considered as a "point source" of scattering and an extended target as a distribution of point sources [1]. Although this method of differentiation has certain merits, it clearly cannot be an adequate model for a radar target as a scatterer of electromagnetic waves. This may give us a clue to the above-mentioned negligence. The solution of electromagnetic scattering problems of arbitrary objects poses notoriously difficult mathematical problems. Hence, the practical engineer tends to shy away from these solutions and he substitutes a simpler model which he can understand and apply to his problem, although the model itself has inadequacies. Recent efforts using high-speed computers which can handle electromagnetic scattering problems seem to move in the direction of closing the radar target technology gap. The present work is intended as a contribution in that direction.

Again, we will restrict ourselves to consideration of a time harmonic source only, but we will insist on diversity of transmitter and receiver polarizations. This is to allow for complete electromagnetic scattering characteristics of the radar target. We will avoid all model building of targets, which has applicability only in special cases; instead, we will attempt to focus on phenomenological aspects of target behavior that are true for all radar targets, such as the basic properties of reciprocity and linearity and the geometric properties of symmetry and asymmetry.

With these rudimentary building blocks, already a substantial structure can be erected which is made the subject of the first part of this work, Chapters 2, 3, and 4. In the second part, Chapters 5 through 8, we focus on distributed targets, which are targets characterized by certain statistical properties, i.e., distributions, random variables, and averages. An important task in this context is to define precisely the concept of statistically independent (mutually incoherent) targets. In the whole work, the attempt will be to focus attention on radar targets within a phenomenological framework; i.e., a radar object is considered as an entity, independent of the specific state of radar illumination or reception. The fruits of this type of investigation will clearly have important applications to problem areas concerning object selection, discrimination and sorting, and specifically target identification [2].

The phenomenological approach leads to consideration of a radar target when its scattering matrix T is known. The first part deals primarily with the properties of the scattering matrix representation. In the second part, the analog representation for distributed targets is given by the stokes reflection matrix R. The stokes matrix for distributed targets has 9 degrees of freedom, whereas the scattering matrix is given by six independent parameters. Hence, it follows that not every distributed target can be represented by an average single target. The question is then raised whether a distributed target may be viewed as an average single target and some type of target noise. A large part of Chapter 7 is devoted to these problems, and the question is answered affirmatively. The main decomposition theorem which follows is then applied in Chapter 8 to terrain targets, using a Kirchhoff integration method published recently by Fung [3, 4, 5] and reported also by Beckmann [6]. An important special case of orientation-independent terrain target model was introduced recently by Williams, Cooper, and Huynen [7] and is used as an illustration of the developed concepts.

The decomposition theorem also may be applied to radar observation of single objects. It provides an answer to a classical problem: to determine the average radar target that an object represents when it is observed from a range of aspect angles or at a range of frequencies. The solution to this problem is given by the mean single target that arises from the decomposition of the average over the observed range of aspects or frequencies.

2. General Literature Review

The present work grew out of developments over a period of 20 years, 1950 to 1970.

The early work was inspired by Sinclair [8]. A series of papers by Booker [9], Rumsey [10], Deschamps [11], Kales [12], and Bohnert [13] on the subject of polarization with reference to radar antennas provided a foundation for future work. From this period stem early studies by Huynen et al. [14] on radar return from ground targets and rain for fixed observation directions using a polarization scanning radar. Important pioneering work on the theory of radar targets scattering was reported by Gent [15] and Kennaugh [16, 17]. The latter introduced the concept of characteristic null polarizations of a radar target. An early attempt at single radar target classification using the received complex voltage with rotating linear polarization illumination was developed by Copeland [18]. Graves [19] gave a method for computing the total power of the backscattered wave of a single target. Several research laboratories reported studies on polarization characteristics of symmetric radar targets, by Crispin [20], Bechtel and Ross

[21], and Huynen [22, 23]. A significant summary of the state-of-the-art of radar measurements was presented at the Radar Reflectivity Measurements Symposium in 1964: Huynen [24], Landry [25], and Webb and Allen [26]. Subsequently, in a special IEEE issue on radar reflectivity, Lowenschuss [27] and Huynen [28] discussed theory and measurement techniques for target scattering matrices, which included asymmetric objects.

The subject of time-varying distributed targets was developed mostly independent of the above-cited literature on single targets. The early work of Gent [15] is exceptional because he also discusses distributions of single targets. Statistical models for terrain are given by Spetner and Katz [29]. The question of whether reciprocity is valid for rough surface scattering is studied by Ament [30]. Ko [31] presents an introduction with application to partially polarized scattering. A classical work on scattering from rough surfaces, treated mostly by scalar theory, was published in 1963 by Beckmann and Spizzichino [32]. We refer to the extensive literature documented in this work. Other work by Beckmann [33, 34], Parks [35], and Renau and Collinson [36] deals with theory and measurement of various rough surface models. A basic reference work, Born and Wolf [37], on optical scattering and diffraction appeared in 1966. In this volume the theory of partial coherence is presented. An impressive literature is referenced in this book. The work of Fung [3, 4, 5] on vector scattering theory considers also depolarization of electromagnetic waves. Krishen, Koepsel, and Durrani [38] measured the cross-polarization from rough surface models. A summary work on polarization of radar signals appeared in Russian: Kanareykin, Pavlov, and Potekhin [39]. The book contains several translated early publications of this writer. Also noteworthy in this time period are the book on radar astronomy edited by Evans and Hagfors [40], Beckmann's book on depolarization of electromagnetic waves [6], and Transactions of the IEEE special issue on partial coherence [41]. Beckmann's book summarizes much of current literature on depolarization, including Fung's results. Recent work by Stogryn [42] treats the complete electromagnetic scattering from rough surfaces by a Kirchhoff approximation technique. An excellent introduction to high-frequency scattering is given in the early work of Van de Hulst [43]. Recent very good reference works have been written by Crispen et al. [48] and Ruck et al. [49].

Most of the literature mentioned treats scattering from radar targets by constructing specific statistical and geometrical models, thus restricting its applicability to those cases. The phenomenological theory presented here applies equally well to all radar targets. It uses only elementary properties of electromagnetic scattering, such as linearity and reciprocity; elementary statistical concepts, such as statistical independence of targets; and elementary geometry – the properties of symmetry and asymmetry and of convex and nonconvex shapes.

This brief literature survey could easily be extended to include several hundred sources of past and current work on electromagnetic scattering, theory, and measured results which have potentially or definitely some relationship to radar targets. For further information, we refer the reader to the literature lists provided in the works mentioned.

2 POLARIZATION OF WAVES AND ANTENNAS

3. Elliptically Polarized Waves and Antennas

For the purpose of this report, a radar transmitter is defined as a source of electromagnetic (em) plane waves. The plane waves are considered at a single fixed frequency $f = (\omega/2\pi)$. The propagating electric and magnetic fields $\underline{E}_t$ and $\underline{H}_t$ are both directed transverse to the direction of propagation $\underline{k}$ of the plane wave. It is sufficient to specify the electric $\underline{E}_t$ field, since for far-field em propagation the direction of the magnetic field $\underline{H}_t$ is perpendicular to the direction of $\underline{E}_t$ and its magnitude is proportional to that of $\underline{E}_t$.

The electric field that describes the plane wave in general has two components, E_x and E_y, in perpendicular directions transverse to $\underline{k}$. Hence, the plane wave is determined by a two-dimensional time harmonic electric vector:

$$\underline{E}_t = \begin{bmatrix} E_x \\ E_y \end{bmatrix}_t = \begin{bmatrix} a_x \cos(\omega t - kz + \alpha_x) \\ a_y \cos(\omega t - kz + \alpha_y) \end{bmatrix} \tag{3.1}$$

Here $k = (2\pi f/c)$, where c is the free space wave propagation velocity and z is taken in the $\underline{k}$ direction. For mathematical convenience, k is often made the magnitude of the propagation vector $\underline{k}$; a_x and a_y are the field component magnitudes, and α_x and α_y are their phases. We then write (3.1) as follows:

$$\underline{E}_t = \mathrm{Re} \begin{bmatrix} a_x e^{i\alpha_x} \\ a_y e^{i\alpha_y} \end{bmatrix} e^{i(\omega t - kz)} \tag{3.2}$$

where Re stands for "real part of." As is customary with time harmonic problems, we drop the exponential propagation factor and Re in (3.2). Hence, the plane wave is fully determined by two complex valued components, E_x and E_y:

$$\underline{E} = \begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} a_x e^{i\alpha_x} \\ a_y e^{i\alpha_y} \end{bmatrix} \tag{3.3}$$

As we will show shortly, equation (3.3) describes an elliptically polarized plane wave. Since the wave is produced by the radar transmit antenna, the same expression may be used to characterize the transmit antenna; we define

$$\underline{a} = \begin{bmatrix} a_x e^{i\alpha_x} \\ a_y e^{i\alpha_y} \end{bmatrix} = \begin{bmatrix} \sqrt{g_x} \\ \sqrt{g_y}\, e^{i\delta_\alpha} \end{bmatrix} e^{i\alpha_x} \tag{3.4}$$

where $g_x = a_x^2$ and $g_y = a_y^2$ are called the antenna gain functions in the x- and y-channels. The total antenna gain $g_o = g_x + g_y$ is a measure of antenna radiation efficiency in a given direction of illumination. The term δ_α is the phase difference between the x and y channels of the antenna.

An antenna may also be used with a radar receiver. In that case, we associate with that antenna the antenna gain and phase characteristic it would have if it were used as a transmit antenna.

The antenna gain patterns are often used for target illumination efficiency calculations; however, for a discussion of properties of radar targets, equation (3.4) is inconvenient, since the gain functions are tied to the fixed (x, y, z) antenna coordinate frame and the targets are independent of this frame. A more natural geometrical form is sought to express the elliptically polarized (ep) wave that the antenna produces. We find this through a discussion of geometric variables of the ep wave.

We observe how the $\underline{E}_t$ vector of an ep wave propagates along the z-axis (Fig. 1). First, we move with the tip of the $\underline{E}_t$ vector through space, describing an elliptical spiral about the z-axis. We may also observe how the tip of the $\underline{E}_t$ vector moves as the wave passes through an (x, y) plane at fixed position on the z-axis. The latter situation is depicted by Fig. 2. Note that the direction of rotation about the positive z-axis is opposite with the two methods of observation. This fact has led to considerable confusion and ambiguity in the definition of sense of circular polarization.

We now proceed to determine the ep wave in terms of geometric parameters. In Fig. 2 is shown the locus that the tip of the $\underline{E}_t$ describes as the plane wave passes through the fixed (x-y) plane. The locus is a tilted ellipse; its geometry is given by the axial ratio, $r = \tan \tau$, where τ is the ellipticity angle shown in Fig. 2. The orientation of the ellipse with respect to the z-axis is determined by orientation angle ϕ; its size is given by the magnitude a. The sense in which the ellipse is traversed is shown in the negative direction, as was discussed above. The geometric parameters that determine the ep wave are thus the ellipticity angle τ, the orientation ϕ, the magnitude a, and the sense. We will find shortly that the sense can be incorporated with the sign of τ.

We now introduce coordinates (x', y') such that effectively $\phi = 0$ for the ellipse in these coordinates. We find for this case:

$$\underline{E}_t(a, \tau) = \begin{bmatrix} a \cos \tau \cos (\omega t - kz + \alpha) \\ - a \sin \tau \sin (\omega t - kz + \alpha) \end{bmatrix} = \mathrm{Re} \begin{bmatrix} a \cos \tau \\ i\, a \sin \tau \end{bmatrix} e^{i(\omega t - kz + \alpha)} \tag{3.5}$$

or, in complex notation:

$$\underline{E}(a, \tau) = a \begin{bmatrix} \cos \tau \\ i \sin \tau \end{bmatrix} e^{i\alpha} \tag{3.6}$$

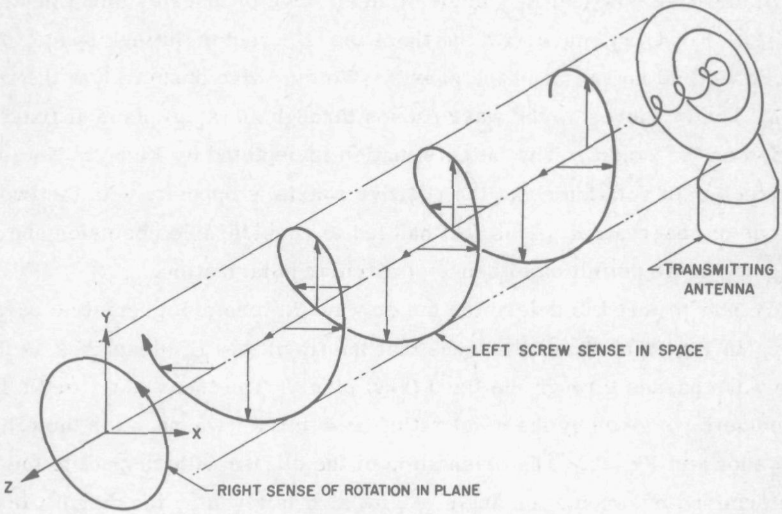


Fig. 1 Left-Sensed Transmitted Polarized Wave

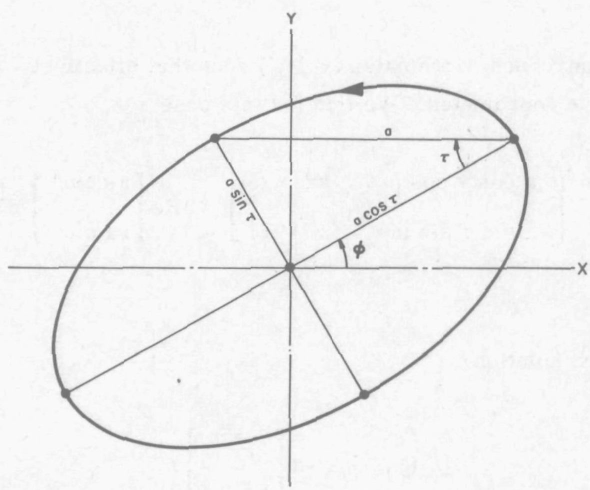


Fig. 2 Left-Sensed Polarization Ellipse in Fixed Plane

We note that a change of the sign of τ in (3.6) changes the direction of sense of polarization. The general expression for an ep wave with ellipse oriented at angles ϕ is obtained from (3.6) through application of a rotation matrix:

$$\underline{E}(a, \phi, \tau) = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \underline{E}(a, \tau) \tag{3.7}$$

The range of ellipticity angle τ is $-45° \leq \tau \leq +45°$. For linear polarization, $\tau = 0°$; for right circular polarization, $\tau = +45°$; for left circular polarization, $\tau = -45°$.

For antennas we derive a similar expression:

$$\underline{a}(a, \alpha, \phi, \tau) = a \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \cos\tau \\ i\sin\tau \end{bmatrix} e^{i\alpha} \tag{3.8}$$

Form (3.8) is equivalent to (3.4), except that now it expresses the ep antenna by geometric parameters. It is possible to determine the set (a, α, ϕ, τ) from the set $(a_x, a_y, \alpha_x, \alpha_y)$ and conversely. To derive these equations, an algebra specially adapted to polarization calculations is developed first in Sec. 4.

The angle α in (3.8) is called the "absolute phase" of the antenna; it determines the phase reference of the antenna at time $t = 0$. For many practical applications, the absolute phase of the antenna may be ignored. Note that the absolute phase of the ep wave transmitted by the antenna changes if the antenna is moved in the direction of propagation. Similarly, the orientation of the transmitted ep wave is changed if the antenna is rotated about the wave propagation direction. There is a close analogy between wave concepts related to absolute phase and those related to wave orientation. For example, the measurement of power of an ep wave eliminates the absolute phase of the wave, and similarly a measurement of power with a circularly polarized receiver antenna (which has no orientation preference) eliminates the effects of wave orientation.

For single targets, some interesting observations can be made in this context which pertain to independence of absolute phase and orientation of the illuminating plane wave. The absolute phase of the wave that illuminates the target can be changed by simply moving the target along the observation direction $\underline{k}$, leaving its position otherwise unaltered. Obviously, this translation does not otherwise alter the illumination of the target, and hence it follows that target scattering properties which relate to the target as a physical object are independent of absolute phase changes. All power scatter measurements satisfy this requirement – but many other quantities not containing absolute phase can be found.

The second property refers to effective changes of orientation angle ϕ of the illuminating ep wave. This can be accomplished simply by rotating the target a fixed angle about the $\underline{k}$ axis. Again, we observe that no change in physical properties and exposure of the target results from the change of target orientation. Hence, it follows that target scattering parameters which relate to the target as a physical object are independent of wave orientation ϕ. Measurements with a circularly polarized receiver (either right- or left-sensed) will produce parameters that satisfy this requirement, but many others can be found, as will be shown later. The last property is referred to as "orientation independent target parameters."

The above-mentioned analogies between target properties related to absolute phase and to wave orientation also are applied later to achieve a better physical understanding of em scattering from rough surfaces.

4. Polarization Matrix Algebra and Special Polarizations

In this section, some algebraic properties will be summarized which will prove helpful in simplifying calculations with polarization vectors that might otherwise be tedious and cumbersome.

We start with the general equation (3.8) of an ep antenna expressed in geometrical parameters:

$$\underline{a}(a, \alpha, \phi, \tau) = a \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \cos\tau \\ i\sin\tau \end{bmatrix} e^{i\alpha} = e^{\phi \mathbf{J}} \; \underline{a}(a, \alpha, \tau) \tag{4.1}$$

where we put

$$e^{\phi \mathbf{J}} = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} = \cos\phi \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \sin\phi \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \tag{4.2}$$

or

$$e^{\phi \mathbf{J}} = \cos\phi \; \mathbf{I} + \sin\phi \; \mathbf{J} \tag{4.3}$$

where $\mathbf{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is the unit matrix and $\mathbf{J} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ is a spatial 90° rotation matrix, for which $\mathbf{J}^2 = -\mathbf{I}$. It is easy to verify from these properties that

$$e^{\phi \mathbf{J}} = \sum_{n=o}^{\infty} \frac{\phi^n}{n!} \mathbf{J}^n = \cos\phi \; \mathbf{I} + \sin\phi \; \mathbf{J} \tag{4.4}$$

The part $\underline{a}(\tau)$ with $a = 1$ and $\alpha = 0$ may be written

$$\underline{a}(\tau) = \begin{bmatrix} \cos\tau \\ i\sin\tau \end{bmatrix} = \begin{bmatrix} \cos\tau & i\sin\tau \\ i\sin\tau & \cos\tau \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = e^{\tau \mathbf{K}} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{4.5}$$

where

$$e^{\tau \mathbf{K}} = \cos\tau \; \mathbf{I} + \sin\tau \; \mathbf{K} \tag{4.6}$$

$$\mathbf{K} = \begin{bmatrix} o & i \\ i & o \end{bmatrix} \quad ; \quad \mathbf{K}^2 = -\mathbf{I} \tag{4.7}$$

Finally, we introduce

$$\mathbf{L} = \mathbf{J}\,\mathbf{K} = -\mathbf{K}\,\mathbf{J} = \begin{bmatrix} -i & o \\ o & i \end{bmatrix} \quad ; \quad \mathbf{L}^2 = -\mathbf{I} \tag{4.8}$$

$$e^{\nu \mathbf{L}} = \cos \nu \;\mathbf{I} + \sin \nu \;\mathbf{L} = \begin{bmatrix} e^{-i\nu} & o \\ o & e^{+i\nu} \end{bmatrix} \tag{4.9}$$

The matrices $\mathbf{I}$, $\mathbf{J}$, $\mathbf{K}$, $\mathbf{L}$ are a representation of the quaternion group with the following multiplication table:

	I	J	K	L
I	I	J	K	L
J	J	-I	L	-K
K	K	-L	-I	J
L	L	K	-J	-I

(4.10)

In textbooks on quantum mechanics, $i\mathbf{J}$, $i\mathbf{K}$, $i\mathbf{L}$ are called Pauli-spinmatrices. We will show shortly their relevance to rotations of the so-called Poincaré sphere.

If $\mathbf{J}$ stands for $\mathbf{J}$, $\mathbf{K}$, or $\mathbf{L}$ we have the following useful rule:

$$e^{(\alpha+\beta)\,\mathbf{J}} = e^{\alpha\,\mathbf{J}} \cdot e^{\beta\,\mathbf{J}} = e^{\beta\,\mathbf{J}} \cdot e^{\alpha\,\mathbf{J}} \tag{4.11}$$

which follows from commutativity and (4.4). Another interesting and useful rule is:

$$\mathbf{K}\, e^{\alpha \mathbf{J}} = \mathbf{K}\,(\cos\alpha\ \mathbf{I} + \sin\alpha\ \mathbf{J}) = (\cos\alpha\ \mathbf{I} - \sin\alpha\ \mathbf{J})\,\mathbf{K} = e^{-\alpha \mathbf{J}}\,\mathbf{K} \tag{4.12}$$

Similar relationships hold if we replace $\mathbf{K}$ and $\mathbf{J}$ by two nonidentical members of $(\mathbf{J}, \mathbf{K}, \mathbf{L})$.

Using these rules, we can easily show that $e^{\alpha \mathbf{J}}\, e^{\beta \mathbf{K}} \neq e^{\beta \mathbf{K}}\, e^{\alpha \mathbf{J}}$. In fact,

$$e^{\alpha \mathbf{J}}\, e^{\beta \mathbf{K}} - e^{\beta \mathbf{K}}\, e^{\alpha \mathbf{J}} = 2 \sin\alpha \sin\beta\ \mathbf{L} \tag{4.13}$$

since

$$e^{\alpha \mathbf{J}}\, e^{\beta \mathbf{K}} = e^{\alpha \mathbf{J}}\,(\cos\beta\ \mathbf{I} + \sin\beta\ \mathbf{K}) = e^{\alpha \mathbf{J}} \cos\beta + \sin\beta\, \mathbf{K}\, e^{-\alpha \mathbf{J}}$$

$$= e^{\alpha \mathbf{J}} \cos\beta + \sin\beta\ \mathbf{K} \cos\alpha + \sin\beta \sin\alpha\ \mathbf{L}$$

and

$$e^{\beta \mathbf{K}}\, e^{\alpha \mathbf{J}} = (\cos\beta\, \mathbf{I} + \sin\beta\, \mathbf{K})\, e^{\alpha \mathbf{J}} = e^{\alpha \mathbf{J}} \cos\beta + \sin\beta\ \mathbf{K}\, e^{\alpha \mathbf{J}}$$

$$= e^{\alpha \mathbf{J}} \cos\beta + \sin\beta\ \mathbf{K} \cos\alpha - \sin\beta \sin\alpha\ \mathbf{L}$$

These algebraic rules are used efficiently to combine angles. Frequently, we have to compute matrices of the form

$$
\begin{aligned}
e^{\alpha_1 \mathbf{J}} e^{\beta \mathbf{K}} e^{\alpha_2 \mathbf{J}} &= e^{\alpha_1 \mathbf{J}} (\cos\beta\, \mathbf{I} + \sin\beta\, \mathbf{K})\, e^{\alpha_2 \mathbf{J}} \\
&= \cos\beta\, e^{(\alpha_1+\alpha_2)\mathbf{J}} + \sin\beta\, \mathbf{K}\, e^{(\alpha_2-\alpha_1)\mathbf{J}} \\
&= \cos\beta \cos(\alpha_1+\alpha_2)\, \mathbf{I} + \cos\beta \sin(\alpha_1+\alpha_2)\, \mathbf{J} \\
&\quad + \sin\beta \cos(\alpha_1-\alpha_2)\, \mathbf{K} + \sin\beta \sin(\alpha_1-\alpha_2)\, \mathbf{L}
\end{aligned}
\tag{4.14}
$$

We notice sum and difference of angles α_1 and α_2 appearing naturally, a result that would be difficult to anticipate by using matrix multiplication and trigonometric identities only. We now return to expression (4.1) to show how the above matrices appear in the expression for the ep antenna. From (4.1) and using (4.5), we find that

$$
\underline{a}\,(a\,,\ \alpha\,,\ \phi\,,\ \tau) = a\, e^{i\alpha}\, e^{\phi\, \mathbf{J}}\, e^{\tau\, \mathbf{K}} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{4.15}
$$

Equation (4.15) expresses the ep antenna completely with exponential matrices in terms of geometrical parameters a, α, ϕ, and τ.

In the next section, we introduce the dot or scalar product between polarization vectors. The dot product is used to calculate the received voltage at the terminals of the receiver antenna. For the present, we are interested only in algebraic properties connected with the scalar product. Given two polarization vectors $\underline{a}$ and $\underline{b}$, we define

$$
\underline{a} \bullet \underline{b} = a_x e^{i\alpha_x} b_x e^{i\beta_x} + a_y e^{i\alpha_y} b_y e^{i\beta_y} = \underline{b} \cdot \underline{a} = a'\, b = b'\, a \tag{4.16}
$$

where the prime indicates a transposed (row) vector and the last two forms are matrix multiplications.

If A is a 2×2 matrix which transforms $\underline{a}$ to $(A\underline{a})$, we have, by applying (4.16), the following important rule:

$$(A\underline{a}) \cdot \underline{b} = (A\underline{a})' \, \underline{b} = \underline{a}' A' \underline{b} = \underline{a}' (A'\underline{b}) = \underline{a} \cdot A'\underline{b} \tag{4.17}$$

where A' is the transposed matrix A (obtained by reflections of A about the main diagonal). These rules are useful for verifying the following result. If $\underline{a}$ is given by (4.15), we show that

$$\underline{a} \cdot \underline{a}^* = |a_x|^2 + |a_y|^2 = |\underline{a}|^2 = a^2 \tag{4.18}$$

By repeated application of (4.11) and (4.17), we have:

$$\begin{aligned} \underline{a} \cdot \underline{a}^* &= a\, e^{i\alpha} e^{\phi \mathbf{J}} e^{\tau \mathbf{K}} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot a\, e^{-i\alpha} e^{\phi \mathbf{J}} e^{-\tau \mathbf{K}} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ &= a^2 e^{-\phi \mathbf{J}} e^{\phi \mathbf{J}} e^{\tau \mathbf{K}} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot e^{-\tau \mathbf{K}} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ &= a^2 e^{-\tau \mathbf{K}} e^{\tau \mathbf{K}} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = a^2 \end{aligned}$$

From equation (4.18) we also find that $\underline{a} \cdot \underline{a}^* = g_x + g_y = g_o$; hence, $g_o = a^2$ is the total antenna gain.

With each antenna polarization $\underline{a}$, there are polarizations $\underline{a}_\perp$ which are termed orthogonal to $\underline{a}$. For orthogonal polarizations, we have:

$$\underline{a} \cdot \underline{a}_\perp^* = 0 \tag{4.19}$$

It is easy to show that if $\underline{a} = \underline{a}(\theta, \tau)$, the orthogonal polarization is found as: $\underline{a}_\perp = \underline{a}(\theta + \pi/2, -\tau)$. Orthogonal polarizations are not unique, since they may have an arbitrary amplitude $a_\perp$ and absolute phase $\alpha_\perp$:

$$\underline{a}_\perp(\phi, \tau) = \underline{a}_\perp e^{i\alpha_\perp} e^{\left(\phi+\frac{\pi}{2}\right) \mathbf{J}} e^{-\tau \mathbf{K}} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{4.20}$$

To show (4.19) is straightforward:

$$\underline{a} \cdot \underline{a}_\perp^* = a e^{i\alpha} e^{\phi \mathbf{J}} e^{\tau \mathbf{K}} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot a_\perp e^{-i\alpha_\perp} e^{\left(\phi+\frac{\pi}{2}\right) \mathbf{J}} e^{\tau \mathbf{K}} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= a a_\perp e^{i(\alpha-\alpha_\perp)} e^{-\frac{\pi}{2} \mathbf{J}} e^{\tau \mathbf{K}} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot e^{\tau \mathbf{K}} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= -a a_\perp e^{i(\alpha-\alpha_\perp)} e^{\tau \mathbf{K}} \mathbf{J} e^{\tau \mathbf{K}} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= -a a_\perp e^{i(\alpha-\alpha_\perp)} \mathbf{J} e^{-\tau \mathbf{K}} e^{+\tau \mathbf{K}} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 0$$

Other special polarizations, useful for later reference, are if $\underline{a} = \underline{a}(a, \alpha, \phi, \tau)$:

"Receiver" or transverse polarization: $\underline{a}_R = \underline{a}(a, -\alpha, -\phi, \tau)$

Conjugate polarization: $\underline{a}_C = \underline{a}(a, -\alpha, \phi, -\tau)$

Symmetric polarization: $\underline{a}_S = \underline{a}(a, \alpha, -\phi, -\tau)$

Figure 3 shows some of these polarizations with reference to $\underline{a}(\phi, \tau)$.

The "receiver" polarization $\underline{a}_R$ has the property that an antenna which transmits this polarization transverse to $\underline{a}$ gives maximum reception at the receiver with polarization $\underline{a}$; i.e., maximum reception is not received in general by using identical antennas for transmitting and receiving. Conversely, an incoming wave $\underline{a}$ is maximally received by antenna $\underline{a}_R$. This concept is easily verified by using, for example, a linearly polarized antenna at 45° orientation for transmitting and receiving. The antennas will face each other in orthogonal positions; hence, no transmission of power can occur between these identical antennas.

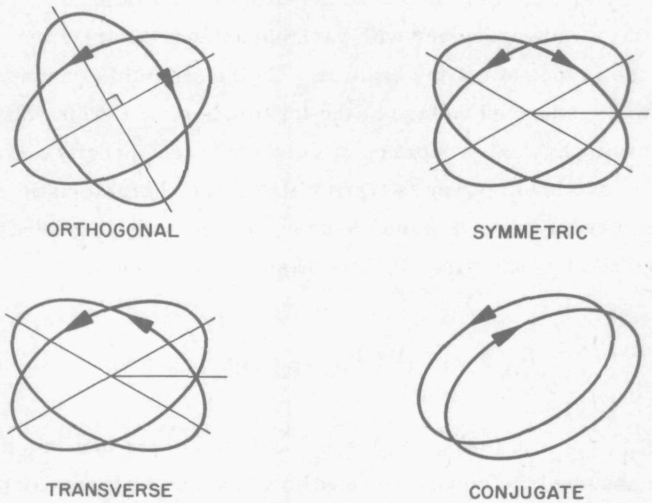


Fig. 3 Four Definitions of Polarization Pairs

The symmetrical polarization occurs frequently in the theory of symmetrical targets to be discussed later. The following relationship concerning $\underline{a}_S$ is used in the next section for the derivation of reciprocity:

$$i\,\mathbf{L}\,\underline{a} = ia\,e^{i\alpha}\,\mathbf{L}\,e^{\phi\mathbf{J}}\,e^{\tau\mathbf{K}}\begin{bmatrix}1\\0\end{bmatrix} = ia\,e^{i\alpha}\,e^{-\phi\mathbf{J}}\,e^{-\tau\mathbf{K}}\begin{bmatrix}-i\\0\end{bmatrix} = \underline{a}_S\,(\phi,\tau) \tag{4.21}$$

5. Determination of Antenna Polarization

Consider a radar transmit antenna with polarization $\underline{a}(\phi_A, \tau_A)$ which transmits an ep wave $\underline{E}_A(\phi_A, \tau_A)$. We wish to determine the polarization parameters a, ϕ_A, τ_A by radar measurements on the field $\underline{E}_A$. We have at our disposal a radar receiver with variable antenna polarization $\underline{b}(\phi_B, \tau_B)$ which we place in the path of the beam $\underline{E}_A$ in the far field of antenna $\underline{a}$. We first compute the received voltage at the terminals of $\underline{b}$. This voltage will satisfy two basic physical properties of em theory: reciprocity and linearity. Furthermore, maximum power is transmitted if the characteristic receiver polarization is received. We intend to show that the following form for the voltage received at $\underline{b}$ satisfies all three basic requirements:

$$V = \underline{E}_A(\phi_A, \tau_A) \bullet \underline{b}_R^*(\phi_B, \tau_B) = \underline{a} \bullet \underline{b}_R^* \tag{5.1}$$

where $\underline{b}_R^*(\phi_B, \tau_B) = \underline{b}(-\phi_B, -\tau_B) = \underline{b}_S$ and the dot product was defined in Sec. 4. We assumed the voltage to be calibrated such that we may put $\underline{E}_A = \underline{a}$ in (5.1).

The form (5.1) is clearly linear; we now show reciprocity. The reciprocity theorem states that if one uses the receiver antenna as transmitter and one receives with the transmitting antenna, the resulting voltages received in the two cases are the same. The fact that:

$$\begin{aligned} V &= \underline{a} \bullet \underline{b}_R^* = \underline{a} \bullet \underline{b}_S = \underline{a} \bullet i\,\mathsf{L}\,\underline{b} \\ &= i\,\mathsf{L}\,\underline{a} \bullet \underline{b} = \underline{a}_S \bullet \underline{b} = \underline{a}_R^* \bullet \underline{b} = \underline{b} \bullet \underline{a}_R^* \end{aligned} \tag{5.2}$$

proves that reciprocity is satisfied. Now, if antenna $\underline{b}$ receives and has unit receiver polarization $\underline{b}_R^{(1)}$, maximum reception occurs if $\underline{a} = a\,\underline{b}_R^{(1)}$ since then $|V| = a \left| \underline{b}_R^{(1)} \cdot \underline{b}_R^* \right| = ab$; conversely, if $\underline{a}$ receives, maximum reception is achieved if $\underline{b} = b\underline{a}_R^{(1)}$. Hence $|V| = b\,|\underline{a}_R^{(1)} \cdot \underline{a}_R^*| = ab$, which satisfies the reciprocity.

Equation (5.1) is the basic equation for reception which we will use later for the study of reception from radar targets. At present, (5.1) is used to measure by means of a set of receiver antenna polarizations $\underline{b}$, the polarization properties of antenna $\underline{a}$. We first evaluate:

$$V = a\, e^{i\alpha}\, e^{\phi_A \mathbf{J}}\, e^{\tau_A \mathbf{K}} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot b\, e^{i\beta}\, e^{-\phi_B \mathbf{J}}\, e^{-\tau_B \mathbf{K}} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= ab\, e^{i(\alpha+\beta)}\, e^{-\tau_B \mathbf{K}}\, e^{(\phi_A+\phi_B)\mathbf{J}}\, e^{\tau_A \mathbf{K}} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= ab\, e^{i(\alpha+\beta)}\, e^{-\tau_B \mathbf{K}} \left[\cos(\phi_A+\phi_B)\,\mathbf{I} + \sin(\phi_A+\phi_B)\,\mathbf{J}\right] e^{\tau_A \mathbf{K}} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= ab\, e^{i(\alpha+\beta)} \left[\cos(\phi_A+\phi_B)\, e^{+(\tau_A-\tau_B)\mathbf{K}} + \mathbf{J} \sin(\phi_A+\phi_B)\, e^{(\tau_A+\tau_B)\mathbf{K}}\right] \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= ab\, e^{i(\alpha+\beta)} \Big\{ \cos(\phi_A+\phi_B) \left[\cos(\tau_A-\tau_B)\,\mathbf{I} + \sin(\tau_A-\tau_B)\,\mathbf{K}\right]$$

$$+ \sin(\phi_A+\phi_B)\,\mathbf{J} \left[\cos(\tau_A+\tau_B)\,\mathbf{I} + \sin(\tau_A+\tau_B)\,\mathbf{K}\right]\Big\} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= ab\, e^{i(\alpha+\beta)} \left[\cos(\phi_A+\phi_B)\cos(\tau_A-\tau_B) - i\sin(\phi_A+\phi_B)\sin(\tau_A+\tau_B)\right] \tag{5.3}$$

From this, the power received at $\underline{b}$, $P(\phi_B, \tau_B) = |V|^2$ is easily found:

$$P(\phi_B, \tau_B) = \frac{a^2 b^2}{2} \left[1 + \sin 2\tau_A \sin 2\tau_B + \cos 2(\phi_A + \phi_B) \cos 2\tau_A \cos 2\tau_B\right] \tag{5.4}$$

Notice the symmetry between a and b antenna parameters, which agrees with reciprocity. Equation (5.4) is of fundamental significance, since it shows which parameters of a are measured directly by a set of polarizations of receiver antenna b. Equation (5.4) agrees with the fact that, given a and b, maximum reception is achieved if $\tau_B = \tau_A$, and $\phi_A = -\phi_B$; then $P_{max} = a^2 b^2$.

An obvious expansion of the cos 2 $(\phi_A + \phi_B)$ term in (5.4) leads to a linear expression for power received, with four unknown terms of a. Hence, a set of four independent measurements with receiver b will solve for the antenna parameters of a (except absolute phase). The set usually chosen for the receiver polarizations are power measurements with "horizontal" ($\phi_B = 0$, $\tau_B = 0$), "vertical" ($\phi_B = 90°$, $\tau_B = 0$), 45° linear ($\phi_B = 45°$, $\tau_B = 0$), and right-circular ($\tau_B = +45°$) receiver polarizations (Williams, Cooper, and Huynen [7]). Equation (5.4) has further important properties which are discussed in the next section.

6. Stokes Parameters, Polarization Sphere, Chart, and Space

The basic equation for power received, for transmission between two antennas a and b, was found in equation (5.4). We rewrite this as:

$$P = \frac{a^2 b^2}{2} \left[1 + \sin 2\tau_A \sin 2\tau_B + \cos 2\tau_A \cos 2\phi_A \cos 2\tau_B \cos 2\phi_B - \cos 2\tau_A \sin 2\phi_A \cos 2\tau_B \sin 2\phi_B\right] \tag{6.1}$$

We now define the stokes parameters of ep antenna a (a, ϕ, τ):

$$\left.\begin{aligned} g_0 &= a^2 \\ g_1 &= a^2 \sin 2\tau \\ g_2 &= a^2 \cos 2\tau \cos 2\phi \\ g_3 &= a^2 \cos 2\tau \sin 2\phi \end{aligned}\right\} \tag{6.2}$$

Using the g notation for stokes parameters of antenna $\underline{a}$ (a, ϕ_A, τ_A) and h for antenna $\underline{b}$ (b, ϕ_B, τ_B), equation (6.1) is reduced to a particularly simple form:

$$P = \frac{1}{2}(g_0 h_0 + g_1 h_1 + g_2 h_2 - g_3 h_3) \tag{6.3}$$

We notice that the stokes vector;

$$g = (g_0, g_1, g_2, g_3) = (g_0, \underline{g}) \tag{6.4}$$

has four real components, which are not independent, since

$$g_0 = \sqrt{g_1^2 + g_2^2 + g_3^2} = |\underline{g}| \tag{6.5}$$

From (6.2), it follows that the three-vector $\underline{g}$ is given simply by a point on a sphere, with polar angles 2ϕ and 2τ and radius $g_0 = a^2$, as shown in Fig. 4. We recall that g_0 represents the total antenna gain of $\underline{a}$. The spherical presentation of polarization is called the Poincaré sphere. We will also use the nomenclature polarization sphere, and the polarization plane for projections of the spherical representation on a plane surface.

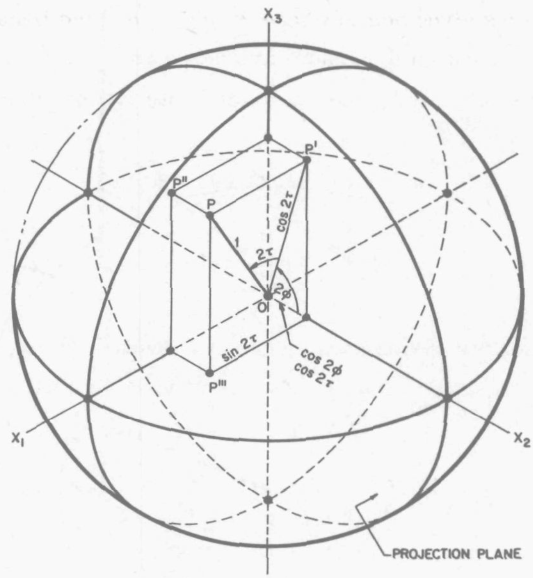


Fig. 4 Polarization Sphere

There exists an extensive literature on the use of spherical geometry and its related projections to solve various engineering problems concerning polarization (Deschamps [11], Knittel [44], Bolinder [45]). We recall also the close analogy between impedance concepts and polarization phenomena. From this, we may conclude that the widely used Smith chart is a spherical projection of an "impedance sphere" representation of impedance; conversely, the geometry of the Smith chart has been used for polarization calculations [11]. In this work we will make extensive use of the polarization sphere concept and also of an orthogonal projection of the sphere on a plane. This so-called polarization chart is shown in Fig. 5.

A short discussion of the polarization sphere and chart follows. Notice that the polar angles are determined by 2τ and 2ϕ. Since negative values of ellipticity angle τ represent left-sensed polarization, one-half hemisphere (corresponding to positive τ) gives the right-sensed polarizations, the other half (with negative τ) the left-sensed ones. The great circle which separates the two hemispheres gives points where $\tau = 0$, i.e., the linear polarizations. Other interesting points on the sphere are those where $\tau = \pm 45°$, ($2\tau = \pm 90°$); these are the "poles" which indicate the circular polarizations.

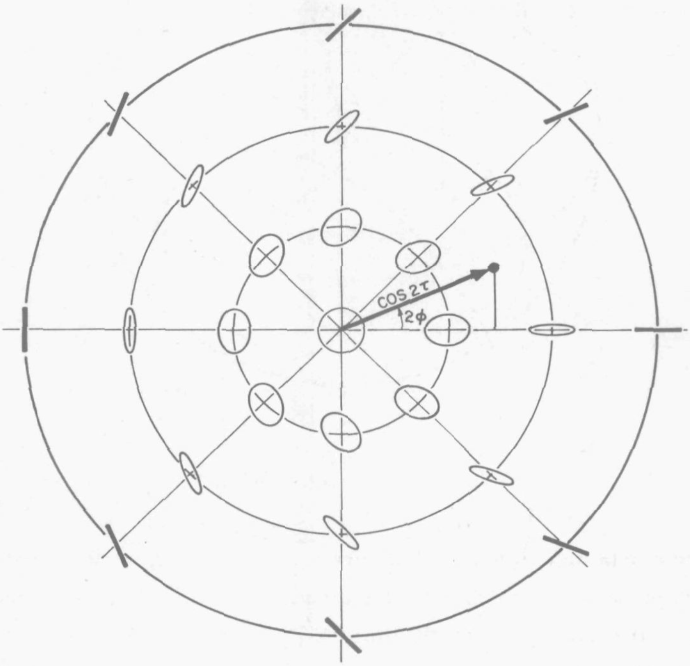


Fig. 5 Polarization Chart

It follows that we need two charts, each representing one hemisphere, to map the whole sphere on a plane. Figure 5 shows such a circular polarization chart, which maps all positive or right-sensed polarizations. ("Right sensed" means following the screw sense of a helical antenna which would produce the wave.) The circumference of the circular chart gives all linear polarizations, the center right circular. Notice the effect of 2ϕ on points of the chart such that "horizontal" polarization ($\phi = 0°$) is mapped on the extreme right-hand side of the chart while "vertical" polarization ($2\phi = 180°$) is mapped on the extreme left-hand side. All points on the vertical axis through the center of the chart represent polarizations with orientation $\phi = 45°$. Note the interesting fact that the radial distance of a point on the polarization chart is measured by $\cos 2\tau$.

Figure 6 shows antenna polarization $\underline{a}$, $\underline{a}_{\perp}$, $\underline{a}_R$, $\underline{a}_C$, and $\underline{a}_S$ in relationship to each other on the polarization sphere.

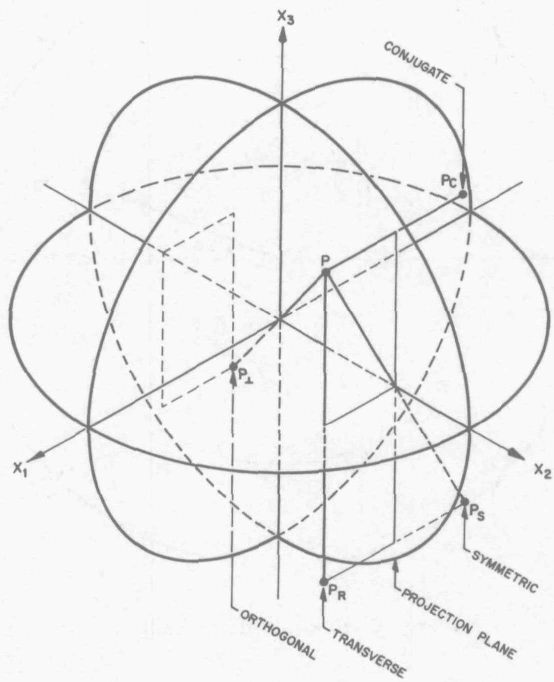


Fig. 6 Polarization Pairs on the Sphere

The sphere and chart are useful for representing states of polarization of an ep wave if one is not interested in its amplitude behavior, either (1) because it is held constant, which is the case for a wave produced by a radar transmitter whose output power is held fixed or (2) because the amplitude is irrelevant, which is the case for the so-called null polarizations of a radar target to be discussed later. However, the most general case of a varying ep wave, for instance, for the scattered return from an object as it varies its exposure with direction of illumination, cannot be mapped on a sphere. Instead (if we exclude absolute phase), it could be mapped as a point in 3-dimensional polarization space, where the distance to the origin represents the power of the ep wave (Lowenschuss [27], Huynen [28]). Very little attention has been paid thus far to the possibility of developing a differential geometry in 3-dimensional polarization space for possible application to radar targets. Although we do not intend to pursue these matters any farther here, an interesting and important algebraic property concerning the angle between two stokes vectors in polarization space will be derived later in this section.

We left the discussion of transmission of power between two antennas $\underline{a}$ and $\underline{b}$ with equation (6.1). This equation closely resembles a scalar product between the stokes vector representations g and h of $\underline{a}$ and $\underline{b}$, except for the minus sign in the last term. However, we may write (6.3) as:

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} g_0 \\ g_1 \\ g_2 \\ g_3 \end{bmatrix} \cdot \begin{bmatrix} h_0 \\ h_1 \\ h_2 \\ h_3 \end{bmatrix} = M\, g \cdot h \qquad (6.6)$$

The matrix M operating on g contains the minus sign which is due to a transformation of coordinates of antennas $\underline{a}$ and $\underline{b}$ facing each other, instead of being aligned in the same direction. We will find shortly that if the received wave is due to target scattering, M is a symmetric matrix called the stokes reflection matrix (also called Mueller matrix) for a single (nondistributed) target; this transforms the transmit polarization $\underline{a}$, whose equivalent stokes vector is g, into the return signal from the target, which is intercepted by receiver antenna $\underline{b}$ or equivalently h.

We close this section with some relations which are useful in computing the stokes parameters directly from a given ep antenna $\underline{a}$ (expressed in x- and y-coordinates as before). If $\underline{a} = \underline{a}\,(a, \phi, \tau)$, then $g(\underline{a}) = (g_0, g_1, g_2, g_3)$

where

$$\left.\begin{aligned} g_0 &= \underline{a} \cdot \underline{a}^* \\ g_1 &= i\,\mathbf{J}\,\underline{a} \cdot \underline{a}^* \\ g_2 &= i\,\mathbf{L}\,\underline{a} \cdot \underline{a}^* \\ g_3 &= -i\,\mathbf{K}\,\underline{a} \cdot \underline{a}^* \end{aligned}\right\} \tag{6.7}$$

We show the derivation for g_2:

$$g_2 = i\,\mathbf{L}\,\underline{a} \cdot a^* = i\,\mathbf{L}\, a\, e^{i\alpha}\, e^{\phi \mathbf{J}}\, e^{\tau \mathbf{K}} \begin{bmatrix}1\\0\end{bmatrix} \cdot a\, e^{-i\alpha}\, e^{\phi \mathbf{J}}\, e^{-\tau \mathbf{K}} \begin{bmatrix}1\\0\end{bmatrix}$$

$$= a^2\, e^{-\tau \mathbf{K}}\, e^{-\phi \mathbf{J}}\, i\,\mathbf{L}\, e^{\phi \mathbf{J}}\, e^{\tau \mathbf{K}} \begin{bmatrix}1\\0\end{bmatrix} \cdot \begin{bmatrix}1\\0\end{bmatrix}$$

$$= a^2\, e^{-\tau \mathbf{K}}\, e^{-2\phi \mathbf{J}}\, e^{-\tau \mathbf{K}}\, i\,\mathbf{L} \begin{bmatrix}1\\0\end{bmatrix} \cdot \begin{bmatrix}1\\0\end{bmatrix}$$

$$= a^2\, e^{-\tau \mathbf{K}} (\cos 2\phi\ \mathbf{I} - \sin 2\phi\ \mathbf{J})\, e^{-\tau \mathbf{K}} \begin{bmatrix}1\\0\end{bmatrix} \cdot \begin{bmatrix}1\\0\end{bmatrix}$$

$$= a^2\, (\cos 2\phi\, e^{-2\tau \mathbf{K}} - \sin 2\phi\ \mathbf{J}) \begin{bmatrix}1\\0\end{bmatrix} \cdot \begin{bmatrix}1\\0\end{bmatrix}$$

$$= a^2 \cos 2\phi \cos 2\tau$$

The converse problem, starting from the known stokes vector g, to find the antenna representation, is also easily found: From stokes vector g, we find the spherical coordinate parameters a^2, 2ϕ, and 2τ. These define a, ϕ, and τ, in which $\underline{a}$ is expressed by (4.15). We notice that absolute phase α is not determined by stokes vectors.

We wish to prove the following interesting property: Given two polarized antennas $\underline{a}$ and $\underline{b}$ and their corresponding stokes vectors g and h in polarization space, then:

$$|\underline{a} \bullet \underline{b}^*| = a\,b \cos \frac{1}{2} \left[\underline{g}\,(\underline{a}),\ \underline{h}\,(\underline{b})\right] \tag{6.8}$$

where a and b are the amplitudes of $\underline{a}$ and $\underline{b}$ as usual, and $\underline{g}$ and $\underline{h}$ the corresponding 3-dimensional stokes vectors.

This property bears an obvious resemblance to the scalar product law for two real vectors. Notice, however, the dependency on half-angle between corresponding stokes vectors! The proof of this remarkable property is straightforward. First, from ordinary (stokes) vectors in 3-space, we have:

$$\underline{g} \bullet \underline{h} = |\underline{g}|\ |\underline{h}| \cos (\underline{g}, \underline{h}) = g_0\, h_0 \cos (\underline{g}, h) = g_1\, h_1 + g_2\, h_2 + g_3\, h_3 \tag{6.9}$$

Now:

$$\left.\begin{aligned} g_1 &= g_0 \sin 2\,\tau_A \\ g_2 &= g_0 \cos 2\,\tau_A \cos 2\,\phi_A \\ g_3 &= g_0 \cos 2\,\tau_A \sin 2\,\phi_A \end{aligned}\right\} \tag{6.10}$$

and similarly for $\underline{b}$ and $\underline{h}$. Since $a^2 = g_0$, $b^2 = h_0$, we have from (6.9) and (6.10)

$$g_0\, h_0 \cos (\underline{g}, \underline{h}) = a^2 b^2 \left[\sin 2\tau_A \sin 2\tau_B + \cos 2\,(\phi_A - \phi_B) \cos 2\tau_A \cos 2\tau_B\right] \tag{6.11}$$

Next we compute:

$$\underline{a} \bullet \underline{b}^* = a\, e^{i\alpha} e^{\phi_A \mathbf{J}} e^{\tau_A \mathbf{K}} \begin{bmatrix}1\\0\end{bmatrix} \bullet b\, e^{-i\beta} e^{\phi_B \mathbf{J}} e^{-\tau_B \mathbf{K}} \begin{bmatrix}1\\0\end{bmatrix}$$

$$= ab\, e^{i(\alpha-\beta)} e^{-\tau_B \mathbf{K}} e^{(\phi_A-\phi_B)\mathbf{J}} e^{\tau_A \mathbf{K}} \begin{bmatrix}1\\0\end{bmatrix} \bullet \begin{bmatrix}1\\0\end{bmatrix}$$

$$= ab\, e^{i(\alpha-\beta)} e^{-\tau_B \mathbf{K}} \left[\cos(\phi_A - \phi_B)\, \mathbf{I} + \sin(\phi_A - \phi_B)\, \mathbf{J}\right] e^{\tau_A \mathbf{K}} \begin{bmatrix}1\\0\end{bmatrix} \bullet \begin{bmatrix}1\\0\end{bmatrix}$$

$$= ab\, e^{i(\alpha-\beta)} \left[\cos(\phi_A - \phi_B)\, e^{(\tau_A-\tau_B)\mathbf{K}} + \sin(\phi_A - \phi_B)\, \mathbf{J}\, e^{(\tau_A+\tau_B)\mathbf{K}}\right] \begin{bmatrix}1\\0\end{bmatrix} \bullet \begin{bmatrix}1\\0\end{bmatrix}$$

$$= ab\, e^{i(\alpha-\beta)} \left[\cos(\phi_A - \phi_B) \cos(\tau_A - \tau_B) - i \sin(\phi_A - \phi_B) \sin(\tau_A + \tau_B)\right] \tag{6.12}$$

Notice that this result also follows from (5.3), if we change the sign of β and ϕ_B. From (6.12), it follows that:

$$|\underline{a} \bullet \underline{b}^*|^2 = \frac{a^2 b^2}{2} \left[1 + \sin 2\tau_A \sin 2\tau_B + \cos 2(\phi_A - \phi_B) \cos 2\tau_A \cos 2\tau_B\right]$$

$$= \frac{a^2 b^2}{2} \left[1 + \cos(\underline{g}, \underline{h})\right] = a^2 b^2 \cos^2 \frac{1}{2}(\underline{g}, \underline{h}) \tag{6.13}$$

By taking the square root on both sides of (6.13), the proposition follows.

This rule (6.8) is now applied to the general problem of decomposition of antenna polarization $\underline{a}$ in terms of two orthogonal polarizations, $\underline{b}$ and $\underline{b}_\perp$, which for convenience are normalized: $b = |\underline{b}| = 1$, $b_\perp = |\underline{b}_\perp| = 1$. Let

$$\underline{a} = c_1 \underline{b} + c_2 \underline{b}_\perp \tag{6.14}$$

The complex coefficients c_1 and c_2 may be found through application of the rule (6.8) to equation (6.14) using the orthonormal properties of $\underline{b}$ and $\underline{b}_\perp$:

$$\left.\begin{aligned} |c_1| &= |\underline{a} \cdot \underline{b}^*| = a \cos \tfrac{1}{2} (\underline{g}, \underline{h}) = a \cos \delta \\ |c_2| &= |\underline{a} \cdot \underline{b}_\perp^*| = a \cos \tfrac{1}{2} (\underline{g}, -\underline{h}) = a \sin \delta \end{aligned}\right\} \tag{6.15}$$

Here, as usual, g is the stokes vector of $\underline{a}$ and h that of $\underline{b}$, and 2δ is the angle between $\underline{g}$ and $\underline{h}$. Notice (Fig. 6) that in polarization space the vector corresponding to $\underline{b}_\perp$ is $-\underline{h}$!

The phases β_1 and β_2 of c_1 and c_2 are still undetermined. We write: $\beta_1 = \alpha + \beta$, $\beta_2 = \alpha - \beta$. Substituting this into (6.14) gives:

$$\underline{a} = a\, e^{i\alpha} (\cos \delta\, e^{i\beta}\, \underline{b} + \sin \delta\, e^{-i\beta}\, \underline{b}_\perp)$$

The angle β may be associated with rotation 2β of vector $\underline{g}$ in polarization space about the fixed $\underline{h}$ axis, as shown in Fig. 7.

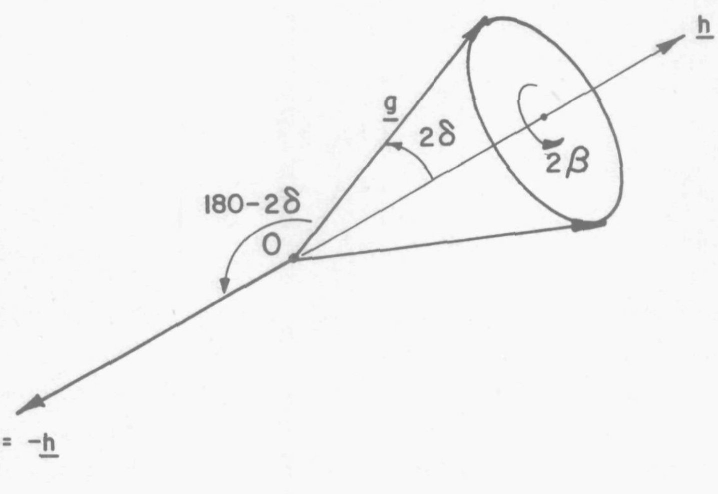


Fig. 7 Polarization Space Vectors

Equation (6.16) will be used later in the theory of null polarizations of radar targets. We have shown that the formal decomposition of $\underline{a}$ has important geometric significance for the corresponding stokes vector representation in polarization space.

7. Mixed Stokes Vectors and a Remarkable Theorem

We found in the previous section that an ep antenna with polarization $\underline{a}$ is represented by a 4-dimensional stokes vector g ($\underline{a}$), which determines $\underline{a}$ completely except for an absolute phase term. The four real components (g_0, g_1, g_2, g_3) of g were shown to be not independent since $g_0^2 = g_1^2 + g_2^2 + g_3^2$, but a 3-dimensional independent subspace determined by vectors $\underline{g} = (g_1, g_2, g_3)$ defined a so-called polarization space. For constant antenna gain $g_0 = a^2$, this space reduces to points on a 2-dimensional polarization sphere (Poincaré sphere) of constant radius g_0.

In this section we will develop a generalization of the stokes vector concept to include complex components which are defined by two antenna polarizations $\underline{a}_1$ and $\underline{a}_2$:

$$\left.\begin{aligned} g_0 &= \underline{a}_1 \cdot \underline{a}_2 \\ g_1 &= i\, \mathbf{J}\, \underline{a}_1 \cdot \underline{a}_2 \\ g_2 &= i\, \mathbf{L}\, \underline{a}_1 \cdot \underline{a}_2 \\ g_3 &= -i\, \mathbf{K}\, \underline{a}_1 \cdot \underline{a}_2 \end{aligned}\right\} \qquad (7.1)$$

The four-vector g, thus defined, is called a <u>mixed</u> stokes vector and is written g ($\underline{a}_1$, $\underline{a}_2$). Since $\underline{a}_1$ and $\underline{a}_2$ are arbitrary, the components of g are, in general, complex. The previously defined g ($\underline{a}$) with real components is shown as a special case, since g ($\underline{a}$) = g ($\underline{a}$, $\underline{a}^*$). The mixed stokes vectors appear naturally with a remarkable theorem, which we state as follows: Given two voltages, $V = \underline{a}_1 \cdot \underline{b}_2$ and $W = \underline{a}_2 \cdot \underline{b}_1$, and mixed stokes vectors g ($\underline{a}_1$, $\underline{a}_2$) and h ($\underline{b}_1$, $\underline{b}_2$) defined by (7.1), then the product is:

$$VW = (\underline{a}_1 \cdot \underline{b}_2)(\underline{a}_2 \cdot \underline{b}_1) = \frac{1}{2}(g_0 h_0 + g_1 h_1 + g_2 h_2 + g_3 h_3) = \frac{1}{2} g \cdot h \tag{7.2}$$

The proof of (7.2) is very simple; it is obtained by decomposing vectors $\underline{a}_1$, $\underline{a}_2$, $\underline{b}_1$, and $\underline{b}_2$ into x and y components on both sides of (7.2), using the definitions (7.1) and the definitions of $\mathbf{J}$, $\mathbf{K}$, and $\mathbf{L}$ from Sec. 4.

To illustrate the usefulness of (7.2), we give special examples:

Case 1

Let $\underline{a}_1 = \underline{a}$, $\underline{a}_2 = \underline{a}^*$, $\underline{b}_1 = \underline{b}$ and $\underline{b}_2 = \underline{b}^*$, $g = g(\underline{a}, \underline{a}^*) = g(\underline{a})$, and $h = h(\underline{b}, \underline{b}^*) = h(\underline{b})$. Then: $|\underline{a} \cdot \underline{b}^*|^2 = \frac{1}{2}(g_0 h_0 + \underline{g} \cdot \underline{h}) = \frac{1}{2} g_0 h_0$ $[1 + \cos(\underline{g}, \underline{h})] = \frac{1}{2} a^2 b^2 [1 + \cos(\underline{g}, \underline{h})] = a^2 b^2 \cos^2 \frac{1}{2}(\underline{g}, \underline{h})$

Hence:

$$|\underline{a} \cdot \underline{b}^*| = ab \cos \frac{1}{2}\left[\underline{g}(\underline{a}), \underline{h}(\underline{b})\right] \tag{7.3}$$

This is the scalar product rule derived by direct calculation in the previous section.

Case 2

Let $\underline{a}_1 = \underline{a}$, $\underline{a}_2 = \underline{a}^*$, $\underline{b}_1 = \underline{b}^*$, $\underline{b}_2 = \underline{b}$, $V = (\underline{a} \cdot \underline{b})$, $W = V^* =$ $(\underline{a}^* \cdot \underline{b}^*)$; $g = g(\underline{a}, \underline{a}^*) = g(\underline{a})$, $h_1 = h_1(\underline{b}^*, \underline{b})$. Since we prefer to write $h = h(\underline{b}, \underline{b}^*) = h(\underline{b})$ instead of $h_1(\underline{b}^*, \underline{b})$, we notice the following general rule, derived simply by observing the general definitions (7.1): If $s(\underline{a}, \underline{b}) =$ (s_0, s_1, s_2, s_3) is a mixed stokes vector, then $s_1(\underline{b}, \underline{a}) = (s_0, -s_1, s_2, s_3)$ is obtained from s by change of the sign of the second component of s. Applying this rule to $h_1(\underline{b}^*, \underline{b})$, we find:

$$|\underline{a} \cdot \underline{b}|^2 = \frac{1}{2}(g_0 h_0 - g_1 h_1 + g_2 h_2 + g_3 h_3) \tag{7.4}$$

where $g = g(\underline{a})$, $h = h(\underline{b})$. This equation is useful for calculations of power. For example, if we replace $\underline{b}$ by $\underline{b}_R^* = \underline{b}(b, -\phi_B, -\tau_B)$ such that $g'(\underline{b}_R^*) = (g_0, -g_1, g_2, -g_3)$, equation (7.4) becomes:

$$|\underline{a} \cdot \underline{b}_R^*|^2 = \frac{1}{2}(g_0 h_0 + g_1 h_1 + g_2 h_2 - g_3 h_3) \qquad (7.5)$$

This was the equation for transmission between two antennas $\underline{a}$ and $\underline{b}$ which was derived by direct computation in Sec. 6.

Case 3

Let $\underline{a}_1 = \underline{a}$, $\underline{a}_2 = \underline{b}$, $\underline{b}_1 = \underline{a}$, $\underline{b}_2 = \underline{b}$, $V = (\underline{a} \cdot \underline{b})$, $W = V = (\underline{b} \cdot \underline{a})$, $g = g(\underline{a}, \underline{b})$, and $h = h(\underline{a}, \underline{b}) = g(\underline{a}, \underline{b})$. Substitution into (7.2) gives:

$$(\underline{a} \cdot \underline{b})^2 = \frac{1}{2}\left(g_0^2 + g_1^2 + g_2^2 + g_3^2\right) \qquad (7.6)$$

However, from definition (7.1), $g_0 = (\underline{a} \cdot \underline{b})$ and hence by substitution into (7.6) we find if $g = g(\underline{a}, \underline{b})$:

$$g_0^2 = g_1^2 + g_2^2 + g_3^2 \qquad (7.7)$$

This shows that the rule for stokes vectors $g(\underline{a})$ with real components representing an ep wave $\underline{a}$ applies also to the mixed stokes vector $g(\underline{a}, \underline{b})$ with complex valued components.

Case 4

With $g = g(\underline{a}, \underline{b}) = (g_0, g_1, g_2, g_3)$, we associate a vector $g' = (g_0, -g_1, -g_2, -g_3)$ called the opposite of g. It is easy to verify that $g' = g(\mathbf{J}\,\underline{b}, \mathbf{J}\,\underline{a})$:

$$g_0' = \mathbf{J}\,\underline{b} \cdot \mathbf{J}\,\underline{a} = \underline{b} \cdot \underline{a} = \underline{a} \cdot \underline{b} = g_0$$

$$g_1' = i\,\mathbf{J}\,(\mathbf{J}\,\underline{b}) \cdot (\mathbf{J}\,\underline{a}) = i\,\mathbf{J}\,\underline{b} \cdot \underline{a} = -i\,\mathbf{J}\,\underline{a} \cdot b = -g_1$$

$$g_2' = i\,\mathbf{L}\,(\mathbf{J}\,\underline{b}) \cdot (\mathbf{J}\,\underline{a}) = -i\,\mathbf{J}\,\mathbf{L}\,\mathbf{J}\,\underline{b} \cdot \underline{a} = -i\,\mathbf{L}\,\underline{b} \cdot \underline{a} = -g_2$$

$$g_3' = -i\,\mathbf{K}\,(\mathbf{J}\,\underline{b}) \cdot (\mathbf{J}\,\underline{a}) = i\,\mathbf{J}\,\mathbf{K}\,\mathbf{J}\,\underline{b} \cdot \underline{a} = i\,\mathbf{K}\,\underline{b} \cdot \underline{a} = -g_3$$

We apply this rule to (7.2). Let $\underline{a}_1 = \underline{a}$, $\underline{a}_2 = \underline{b}$, $\underline{b}_1 = J\,\underline{b}$, $\underline{b}_2 = J\,\underline{a}$; then $V = (\underline{a} \bullet J\,\underline{a})$, $W = (\underline{b} \bullet J\,\underline{b})$, $g = g\,(\underline{a}, \underline{b})$, and $h = g' = g\,(J\,\underline{b}, J\,\underline{a})$.
Hence: $(\underline{a} \cdot J\,\underline{a})\,(\underline{b} \bullet J\,\underline{b}) = \frac{1}{2}\left(g_0^2 - g_1^2 - g_2^2 - g_3^2\right) = 0$;
since $V = (\underline{a} \bullet J\,\underline{a}) = \underline{a} \bullet \underline{a}_\perp^* = 0$. This gives the same result, $g_0^2 = g_1^2 + g_2^2 + g_3^2$ as was shown in Case 3.

Case 5

With $g = g\,(\underline{a}, \underline{b}) = (g_0, g_1, g_2, g_3)$, we associate the complex conjugate: $g^* = \left(g_0^*, g_1^*, g_2^*, g_3^*\right)$. It is easy to show from definitions (7.1) that $g^* = g\,(\underline{b}^*, \underline{a}^*)$. If we now apply to g^* the result of (7.4) for opposite stokes vector g', we have:

$$g^{*'} = g\,(J\,\underline{a}^*, J\,\underline{b}^*) = g\,(\underline{a}_\perp, \underline{b}_\perp) = \left(g_0^*, -g_1^*, -g_2^*, -g_3^*\right)$$

Now put in (7.2): $\underline{a}_1 = \underline{a}$, $\underline{a}_2 = \underline{b}$, $\underline{b}_1 = \underline{a}_\perp$, $\underline{b}_2 = \underline{b}_\perp$, $V = (\underline{a} \bullet \underline{b}_\perp)$, $W = (\underline{b} \bullet \underline{a}_\perp)$, $g = g\,(\underline{a}, \underline{b}) = (g_0, g_1, g_2, g_3)$, and $h = g^{*'} = g\,(\underline{a}_\perp, \underline{b}_\perp) = \left(g_0^*, -g_1^*, -g_2^*, -g_3^*\right)$.
Then:

$$(\underline{a} \bullet \underline{b}_\perp)\,(\underline{a}_\perp \bullet \underline{b}) = \frac{1}{2}\left(|g_0|^2 - |g_1|^2 - |g_2|^2 - |g_3|^2\right) \qquad (7.8)$$

Notice that the left-hand side of (7.8) in general is not zero, and hence the sum of squares of absolute values of complex components g_1, g_2, and g_3 of a mixed stokes vector g in general does not equal $|g_0|^2$!

Case 6

Let $\underline{a}_1 = \underline{a}$, $\underline{a}_2 = \underline{a}^*$, $\underline{b}_1 = \underline{b}_\perp$, $\underline{b}_2 = \underline{b}_\perp^*$; then $g = g\,(\underline{a}, \underline{a}^*) = g\,(\underline{a})$.
Let $h = h\,(\underline{b}) = (h_0, h_1, h_2, h_3)$; then $h^{*'} = h\left(\underline{b}_\perp, \underline{b}_\perp^*\right) = h\,(\underline{b}_\perp) = \left(h_0^*, -h_1^*, -h_2^*, -h_3^*\right) = (h_0, -h_1, -h_2, -h_3)$, since $h\,(\underline{b})$ components are real.
Now $V = \left(\underline{a} \bullet \underline{b}_\perp^*\right)$ and $W = \left(\underline{a}^* \bullet \underline{b}_\perp\right) = V^*$. Hence $|\underline{a} \bullet \underline{b}_\perp^*|^2 = \frac{1}{2}(g_0 h_0 - g_1 h_1 - g_2 h_2 - g_3 h_3) = \frac{1}{2}\left[g_0 h_0 - g_0 h_0 \cos\,(\underline{g}, \underline{h})\right] = a^2 b^2 \sin^2 \frac{1}{2}\,(\underline{g}, \underline{h})$

From this follows the interesting property

$$|\underline{a} \cdot \underline{b}_{\perp}^{*}| = a\,b \sin \frac{1}{2} (\underline{g}, \underline{h}) \tag{7.9}$$

This result will be applied in a later section to the theory of null polarization of radar targets.

Case 7

In addition to the properties related to single targets. the mixed stokes vectors and theorem (7.2) will be useful in the theory of distributed (time varying) targets and fields. Let $\underline{a}_1\,(t)$ and $\underline{a}_2\,(t)$ be time-varying fields (a precise definition is given in Chap. 5) and consider $\underline{b}_1 = \underline{b}_{2R}^{*}$, $\underline{b}_2 = \underline{b}_{1R}^{*}$ fixed receiver antennas for sampling each field which produce voltages $V_1\,(t) = \left[\underline{a}_1\,(t) \cdot \underline{b}_{1R}^{*}\right]$ and $V_2\,(t) = \left[\underline{a}_2\,(t) \cdot \underline{b}_{2R}^{*}\right]$, as was discussed in Sec. 5. Suppose one is interested in the same average $\left\langle V_1\,V_2 \right\rangle$; then by application of (7.2) we find:

$$\left\langle V_1\,(t)\,V_2\,(t) \right\rangle = \frac{1}{2} \left\langle g\left[\underline{a}_1\,(t),\,\underline{a}_2\,(t)\right] \right\rangle \cdot h\left(\underline{b}_{2R}^{*},\underline{b}_{1R}^{*}\right) \tag{7.10}$$

where we used the shorthand notation: $g \cdot h = g_0\,h_0 + g_1\,h_1 + g_2\,h_2 + g_3\,h_3$. The interesting property of (7.10) is that it separates the time-varying field components $\underline{a}_1\,(t)$ and $\underline{a}_2\,(t)$ from the fixed receiver antennas such that these can be studied independently. Further details will be given in Sec. 25.

3 SINGLE RADAR TARGETS

8. Introduction to Radar Target Scattering

Radar targets are usually characterized by their "patterns," which are plots of radar cross section (RCS) versus angle of observation at a given radar frequency and polarization of transmitter and receiver. Figure 8 shows a typical situation in which a ground radar station illuminates a target in space. The direction of illumination which determines the target's exposure is shown in the figure by the aspect direction. The target's coordinate frame is aligned with the target's axis. With reference to this coordinate frame, the target's aspect direction is given by roll angle and pattern angle (often also called aspect angle). The pattern angle gets its name from the fact that most "static" RCS patterns, obtained by radar range measurements, are measured as a function of that angle for fixed roll positions of the target.

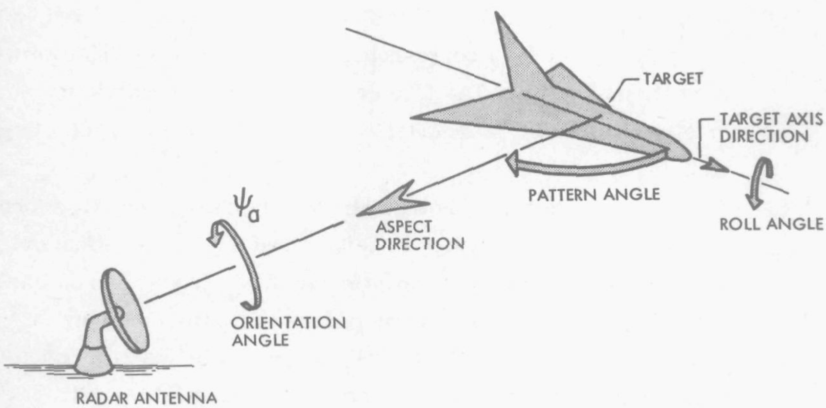


Fig. 8 Target Aspect Direction and Orientation Angle

With target exposure thus determined, the target's position in space is still not completely fixed. The target may still be oriented at an angle ψ_a about the line of sight direction, leaving the exposure of the target the same. The angle ψ_a is called the target axis orientation angle, since it determines the target orientation with reference to the fixed aspect direction. Whereas target aspect direction is important for target scattering, since it exposes different parts of the target's surface, the orientation angle ψ_a is a geometric motion parameter only, which orients the target in space with reference to a fixed ground station for a fixed target exposure. Hence, the target's scattering is in general also dependent on ψ_a, since ψ_a effectively changes the orientation of polarization of the illumination. If targets are sensitive to changes in polarization orientations – and we will see shortly that they are – the target backscatter will in general be dependent on ψ_a. However, from the point of view of the observer, the target axis orientation is purely a dynamical variable of the target motion, which does not affect target exposure and hence the scattering matrix T associated with that exposure. From this discussion, we may draw several important conclusions:

(1) The target exposure determines the target's backscatter properties; with it is associated a scattering matrix T.

(2) A change of target axis orientation ψ_a, with exposure fixed, may be accounted for by a corresponding orthogonal coordinate transformation of matrix T. The coordinate transformation is due to an effective change of orientation of polarization of target illumination.

(3) Although the scattering is thus affected by changes in target orientation, since the scattering is determined by T, the effect of ψ_a on the scattering can be accounted for if ψ_a is known from the target's position relative to the radar observation station.

(4) For studies that aim to link radar target scattering properties to basic body geometry and structure, it is essential to subtract the effect of ψ_a on the target scattering. This procedure leads to so-called orientation invariant target parameters [28].

(5) The above-mentioned procedure can be carried out successfully only if the scattering matrix T of the target is known from observations. Hence, most radars that observe targets with a single polarization produce target signatures that are obscured by the ψ_a parameter. It is unlikely that these target signatures obtained with single polarization radars can be shown to have significant correlations with target geometry and structure.

(6) A significant exception to the preceding discussion is the use of radars that have a single circular polarization. Since the circular polarization illumination itself is unbiased to target orientation, the target amplitude return signature in this case will also be orientation invariant.

(7) We will show shortly the rather suprising and important fact that the process of eliminating the effect of ψ_a on the scattering matrix T, and hence on target signature, does not require knowledge of ψ_a itself; only T has to be known from radar measurements.

The previous discussion has stressed the significance of target axis orientation ψ_a on radar scattering. The rather detailed digression was felt to be justified because a general lack of appreciation of these facts is found among investigators, and a host of misconceptions regarding radar signature analysis and procedures may result. Many efforts have been directed toward correlation of target signatures from conventional linearly polarized radars with target size, shape, and structure. Failure of such attempts have then led to the erroneous conclusion that radar target signatures are useless for such purposes. A phenomenological understanding of the target scattering process entailed might have averted such a conclusion.

The same ideas concerning target orientation may be applied with significant results to distributed targets. For a terrain, the "target axis" becomes the average surface normal. Local surface patches have normals that deviate in a random fashion from the average normal. The random orientation changes of the local normal will be shown to generate "target noise" with well-defined polarization characteristics.

9. Target Scattering Matrix and Operator

The scattering matrix (SM) of a radar target may be considered a generalization of the ordinary radar cross section (RCS) type of radar observable. While RCS is a measure of the intensity of target scattering for single polarization radar transmission and reception, the SM includes the target scattering for all polarization combinations of transmit antenna $\underline{a}$ and receiver antenna $\underline{b}$ (Fig. 9). Not only intensity but also the phase of the returned wave is supplied by the SM.

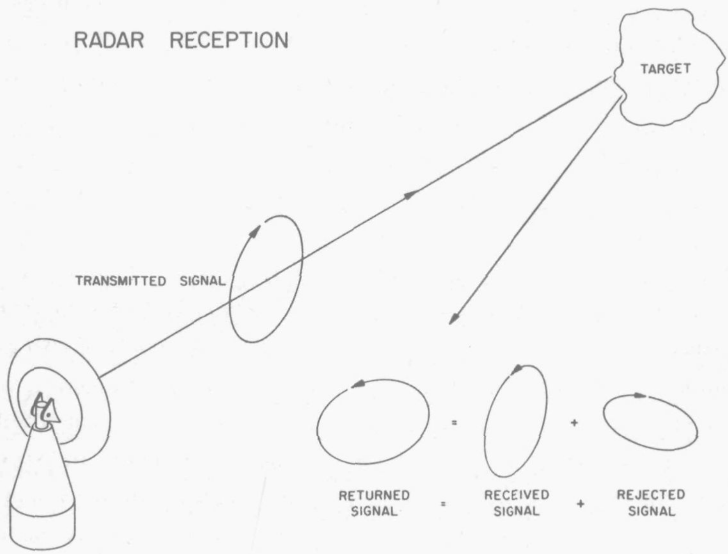


Fig. 9 Radar Target Reception

The SM transforms the transmit polarization $\underline{a}$ into the polarization of the scattered field $\underline{E}^S$, which in turn is sampled by the radar receiver $\underline{b}$. The vector identity $\underline{E}^S = T^S\underline{a}$ defines the target transformation T^S. We now use equation (5.1) to obtain the voltage received at antenna $\underline{b}$ for any incoming wave $\underline{E}^S$:

$$V = \underline{E}^S \cdot \underline{b}_R^*(b, \phi_B, \tau_B) = T^S \underline{a} \cdot i\mathbf{l}\, \underline{b}(b, \phi_B, \tau_B) =$$

$$= i\mathbf{L}\, T^S \underline{a} \cdot \underline{b} = T\underline{a} \cdot \underline{b} = \underline{a} \cdot T'\underline{b} \qquad (9.1)$$

where

$$T = i\mathbf{L}\, T^S = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} T^S = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \qquad (9.2)$$

This is the form we used for the target scattering matrix T. The term $\mathbf{L} = \begin{bmatrix} -i & 0 \\ 0 & i \end{bmatrix}$ was defined in section (4) as a member of the **I**, **J**, **K**, **L** group. From reciprocity, the same result is produced if antennas $\underline{a}$ and $\underline{b}$ are interchanged:

$$V = T\underline{a} \cdot \underline{b} = T\underline{b} \cdot \underline{a} = \underline{a} \cdot T\underline{b} \qquad (9.3)$$

Hence, comparing (9.1) with (9.3) we find $T = T'$ or $t_{12} = t_{21}$; i.e., T is a symmetrical matrix. *In these operations,* T' *means the transposed matrix* of T, obtained by reflection of T about the principal diagonal. From reciprocity, it thus follows that T is a 2×2 complex symmetric matrix.

The law of reciprocity does not hold if propagation effects related to the earth's magnetic field (Faraday rotation) are present. However, in that case, the asymmetry of T can be used to determine and to eliminate the effect of Faraday rotation on the target scattering [46]. For the present discussion, we assume that the Faraday rotation effect has been removed, so that the target is represented by a symmetric SM.

We observe in (9.3) that the antenna polarizations are given by $\underline{a}$ and $\underline{b}$ and T represents the radar target at a given radar frequency and fixed target exposure (or for fixed direction of illumination). Hence, a radar target is determined by T and conversely any symmetric T stands for some physical target at a given aspect direction and frequency. However, we do not claim uniqueness; many physical targets may have at some exposure the same T.

Also the same physical target in general is represented by many T's, since T changes with every direction of illumination and radar frequency.

The matrix T is defined in terms of polarization. We normally use in this work a pair of horizontally H and vertically V polarized unit vectors to define a basis. In this form, the matrix is defined as

$$T = \begin{bmatrix} H-H & H-V \\ V-H & V-V \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \tag{9.4}$$

Here H - V is a complex number, t_{12}, which is proportional to the received horizontally polarized component of the returned signal (in amplitude and phase), while the target is illuminated by a vertically polarized transmitter; and similarly for the other designations. The general scattering matrix is thus completely determined by the backscattered returns from a target for horizontally and vertically polarized target illuminations. Once T is determined by (9.4), the backscattered return from the target is known for any combination of polarized antennas $\underline{a}$ and $\underline{b}$.

It is equally possible to express vectors $\underline{a}$ and $\underline{b}$ and T in some other polarization reference frame; for example, we could have used a circular basis, with right- and left-circular (RC and LC) polarization vectors as basic orthogonal unit vectors. Later we will even make use of a nonorthogonal basis produced by so-called null polarizations. Obviously, in all these bases T will be numerically different matrices that can be converted into each other, and all these forms of T represent the same radar target.

This author has stressed in the past the use of the term "operator" to indicate the physically invariant property of T for representing a radar target [47]. The radar target scattering "operator" is defined as the set of all representations of T. The Russian literature [39] has followed the "operator" nomenclature introduced by this author; however, American authors prefer the term "scattering matrix" to describe the scattering properties of radar targets. As is often done in mathematics, the properties of the "operator" that are independent of special matrix representations are studied by solving a so-called eigenvalue problem, which is characteristic to the operator at hand. Solutions to the eigenvalue problems of T are expressed by eigenvalues and eigenvectors, and it is expected that properties of these may then be associated with physical properties of radar targets. These topics are the subject of following sections.

10. Eigenvalues and Eigenvectors for the Target Scattering Operator

In the previous section, it was noted that properties of the target scattering operator T may be studied through solutions to an eigenvalue problem. The characteristic eigenvalue problem of T is presented in the following form:

$$T\underline{x} = t\underline{x}^* \tag{10.1}$$

In general, there are two independent solutions to this equation:

$$\begin{aligned} T\underline{x}_1 &= t_1\underline{x}_1^* \\ T\underline{x}_2 &= t_2\underline{x}_2^* \end{aligned} \tag{10.2}$$

As usual, the asterisk denotes complex conjugation. The complex scalars t_1 and t_2 are called the eigenvalues; the two normalized vectors $\underline{x}_1$ and $\underline{x}_2$ are the corresponding eigenvectors, which are characteristic for the problem. Note that (10.1) is not of the form $A\underline{x} = a\underline{x}$ usually found in textbooks; for those problems, $\underline{x}_1$ is determined up to a phase factor (if $\underline{x}_1$ is a solution, $e^{i\alpha}\,\underline{x}_1$ also is); on the other hand, with (10.1), because of the conjugation sign on the right-hand side, $\underline{x}_1$ is phase determined with the phase of t_1.

First, we show that the t<u>w</u>o eigenvectors in (10.2) are orthogonal if the eigenvalues are not equal in magnitude, in which case the solution is called degenerate.

Since T is a symmetric operator, we have:

$$|T\underline{x}_1 \cdot \underline{x}_2| = |\underline{x}_1 \cdot T\underline{x}_2| \tag{10.3}$$

Substituting (10.2) into (10.3), we find:

$$|t_1|\,|\underline{x}_1^* \cdot \underline{x}_2| = |t_2|\,|\underline{x}_1 \cdot \underline{x}_2^*| \tag{10.4}$$

Hence, if $|t_1| \neq |t_2|$,

$$|\underline{x}_1 \cdot \underline{x}_2^*| = 0$$

from which follows:

$$\underline{x}_1 \cdot \underline{x}_2^* = 0 \tag{10.5}$$

This was the condition for orthogonality of vectors $\underline{x}_1$ and $\underline{x}_2$. Since they were also normalized,

$$\underline{x}_1 \cdot \underline{x}_1^* = 1 \quad ; \quad \underline{x}_2 \cdot \underline{x}_2^* = 1 \tag{10.6}$$

the vectors $\underline{x}_1$ and $\underline{x}_2$ form an orthonormal set. Following the usual procedure, it is now possible to construct a unitary transformation $U = [\underline{x}_1, \underline{x}_2]$ for which

$$U'U^* = \begin{bmatrix} \underline{x}_1 \cdot \underline{x}_1^* & \underline{x}_1 \cdot \underline{x}_2^* \\ \underline{x}_2 \cdot \underline{x}_1^* & \underline{x}_2 \cdot \underline{x}_2^* \end{bmatrix} = \mathrm{I} \tag{10.7}$$

On the right hand side is indicated the unit matrix I . We use U to bring the matrix T to diagonal form. First we have:

$$TU = \left[T\underline{x}_1, T\underline{x}_2\right] = \left[t_1\underline{x}_1^*, t_2\underline{x}_2^*\right]$$

Hence,

$$U'TU = \begin{bmatrix} \underline{x}_1 \cdot t_1\underline{x}_1^* & \underline{x}_1 \cdot t_2\underline{x}_2^* \\ \underline{x}_2 \cdot t_1\underline{x}_1^* & \underline{x}_2 \cdot t_2\underline{x}_2^* \end{bmatrix} = \begin{bmatrix} t_1 & 0 \\ 0 & t_2 \end{bmatrix} \tag{10.8}$$

We now define

$$T_d = \begin{bmatrix} t_1 & 0 \\ 0 & t_2 \end{bmatrix} \tag{10.9}$$

which is called the diagnoalized scattering matrix.

Using (10.8) and applying the unitary property (10.7), we now solve for the matrix T in terms of eigenvalues and eigenvectors:

$$T = U^* T_d U^{*\prime} \tag{10.10}$$

This decomposition of the target scattering matrix T in terms of eigenvalue parameters will be basic to all further developments in this report. It has many interesting geometric properties which we will develop later.

We next determine some properties of the total backscattered power. Consider the total power per unit area P_{tot} contained in the backscattered return:

$$P_{tot} = \underline{E}^S \cdot \underline{E}^{S*} = T\underline{a} \cdot (T\underline{a})^* \tag{10.11}$$

Let us decompose the transmit polarization $\underline{a}$ in terms of orthonormal eigenvectors $\underline{x}_1$ and $\underline{x}_2$ of T:

$$\underline{a} = a_1\underline{x}_1 + a_2\underline{x}_2 \tag{10.12}$$

Substitution of (10.12) into P_{tot} and using orthonormal properties of $\underline{x}_1$ and $\underline{x}_2$ gives:

$$\begin{aligned} P_{tot} &= (a_1 T\underline{x}_1 + a_2 T\underline{x}_2) \cdot (a_1^* T^* \underline{x}_1^* + a_2^* T^* \underline{x}_2^*) = \\ &= |a_1|^2 |t_1|^2 + |a_2|^2 |t_2|^2 = \\ &= a^2 |t_1|^2 - |a_2|^2 \left(|t_1|^2 - |t_2|^2\right) \end{aligned} \tag{10.13}$$

where $g_o = a^2 = |a_1|^2 + |a_2|^2$ is the total transmit antenna gain defined in Section 3. We may assume, without loss of generality, that $|t_1| \geq |t_2|$ since the order of the eigenvalues has not been fixed. Also, for convenience, the transmit antenna is normalized to have a unit gain: $a = 1$. We now wish to determine for which transmit antenna polarization $\underline{a}$, with fixed gain $a = 1$, the maximum power is returned from target T.

The only variable in (10.13) is $|a_2|$, and it is easily found from this equation that P_{tot} is a maximum if $|a_2| = 0$ when $P_{tot} = |t_1|^2$ and that this maximum is achieved by illuminating the target with a transmit antenna of unit gain and polarization: $\underline{a} = \underline{x}_1$.

The maximum power returned is a target characteristic, since it is given solely by parameters derived from the scattering matrix T. We designate a target variable m for the maximum return: $P_{tot}(\max) = |t_1|^2 = m^2$. The positive value m is called the radar target magnitude; it gives an overall electromagnetic measure of target size, similarly as wave magnitude a gives an overall measure of electromagnetic antenna size (gain).

With these results, we are now equipped to determine other target parameters. The eigenvector $\underline{x}_1$ now also has assumed physical significance; it is called the "maximum polarization" $\underline{x}_1 = \underline{m}$, which is defined by geometric variables ψ and τ_m as follows:

$$\underline{m}(\psi, \tau_m) = \begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{bmatrix} \begin{bmatrix} \cos\tau_m \\ i\sin\tau_m \end{bmatrix} \tag{10.14}$$

The two eigenvalues are written in the following form:

$$\left.\begin{aligned} t_1 &= m\, e^{2i(\nu+\rho)} \\ t_2 &= m \tan^2\gamma\, e^{-2i(\nu-\rho)} \end{aligned}\right\} \tag{10.15}$$

We note that this definition agrees with $|t_1| = m$ and $|t_1| \geq |t_2|$ if $0 \leq \gamma \leq 45°$, which is the range of γ. The significance of the angle γ will be clearer later; it is called the characteristic angle and it plays an important role as a target characteristic. The angle ν is sometimes called "relative phase" because it assumes that role in the diagonal matrix (10.9), but since this nomenclature may be ambiguous for general T we prefer the more descriptive term "target skip angle," since values of ν have some relationship to depolarization owing to the number of bounces of the reflected signal. The range of ν is: $-45° \le \nu \le +45°$.

The quantity ρ is called the absolute phase of the target; it disappears with power measurements, and it may be altered arbitrarily by moving the radar target along the line-of-sight direction, leaving the target's attitude otherwise unaltered. Hence, the absolute phase is a mixed target parameter; it is determined by the target surface geometry and composition as well as by the target's spatial position.

The two parameters ψ and τ_m which determine $\underline{m}$ also are important target parameters. The angle ψ, the target orientation angle, can be made zero simply by rotating the radar target about the line-of-sight axis, keeping target exposure otherwise unchanged. The angle τ_m is, of course, the ellipticity angle of the maximum polarization $\underline{m}$. However, it will play a significant role in determining target symmetry (when $\tau_m = 0$) or asymmetry (when $\tau_m \neq 0$). We will often call τ_m the target "helicity angle" for reasons which will be apparent later. Its range is $-45° \le \tau_m \le +45°$.

With these identifications, we now show how the six target parameters m, ρ, ψ, τ_m, ν, and γ determine the target scattering matrix T. To this end, first the unitary transformation U in (10.7) is found by substituting $\underline{x}_1 = \underline{m}$ and $\underline{x}_2 = \underline{m}_\perp$. Now $\underline{m}(\psi, \tau_m)$, given by (10,14), and $\underline{m}_\perp = \underline{m}(\psi + \pi/2, -\tau_m)$ may be written as:

$$\underline{m}(\psi, \tau_m) = e^{\psi \mathbf{J}} e^{\tau_m \mathbf{K}} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{10.16}$$

$$\underline{m}_\perp(\psi, \tau_m) = \mathbf{J} e^{\psi \mathbf{J}} e^{-\tau_m \mathbf{K}} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = e^{\psi \mathbf{J}} e^{\tau_m \mathbf{K}} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{10.17}$$

From this it is easily verified that $\underline{m} \bullet \underline{m}_\perp^* = 0$, since

$$\underline{m} \cdot \underline{m}_{\perp}^{*} = e^{\psi \mathbf{J}} e^{\tau_m \mathbf{K}} \begin{bmatrix}1\\0\end{bmatrix} \cdot e^{-\psi \mathbf{J}} e^{-\tau_m \mathbf{K}} \begin{bmatrix}0\\1\end{bmatrix} =$$

$$= e^{-\tau_m \mathbf{K}} (e^{-\psi \mathbf{J}} e^{\psi \mathbf{J}}) e^{\tau_m \mathbf{K}} \begin{bmatrix}1\\0\end{bmatrix} \cdot \begin{bmatrix}0\\1\end{bmatrix} = 0$$

Now:

$$U = \left[\underline{m}, \underline{m}_{\perp}\right] = e^{\psi \mathbf{J}} e^{\tau_m \mathbf{K}} \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = e^{\psi \mathbf{J}} e^{\tau_m \mathbf{K}} = U(\psi, \tau_m) \quad (10.18)$$

Substitution of (10.18) and (10.15) into (10.10) gives the desired result:

$$T = U^* T_d U^{*\prime} = e^{\psi \mathbf{J}} e^{-\tau_m \mathbf{K}} e^{-\nu \mathbf{L}} \begin{bmatrix}m & 0\\0 & m \tan^2 \gamma\end{bmatrix} e^{-\nu \mathbf{L}} e^{-\tau_m \mathbf{K}} e^{-\psi \mathbf{J}}$$

(10.19)

Next we define

$$U(\psi, \tau_m, \nu) = e^{\psi \mathbf{J}} e^{\tau_m \mathbf{K}} e^{\nu \mathbf{L}} \quad (10.20)$$

It is easily shown that $U(\psi, \tau_m, \nu)$ is also a unitary matrix:

$$U'(\psi, \tau_m, \nu)\, U^*(\psi, \tau_m, \nu) = \left(e^{\nu \mathbf{L}} e^{\tau_m \mathbf{K}} e^{-\psi \mathbf{J}}\right)\left(e^{\psi \mathbf{J}} e^{-\tau_m \mathbf{K}} e^{-\nu \mathbf{L}}\right) = \mathbf{I}$$

(10.21)

Hence, (10.19) may be rewritten as

$$T = U^*(\psi, \tau_m, \nu)\, m \begin{bmatrix}1 & 0\\0 & \tan^2 \gamma\end{bmatrix} U^{*\prime}(\psi, \tau_m, \nu) \quad (10.22)$$

This shows that T, diagonalized by a unitary transformation, has positive diagonal terms. Both forms (10.19) and (10.22) are useful for calculations. The latter is especially useful for obtaining geometric canonical representations of T on the polarization sphere, where (ψ, τ_m, ν) are rotation angles of the sphere about three orthogonal axes and γ assumes the role of sole remaining identifier of canonical targets.

Before proceeding further with these topics, we give some physical examples of target matrix representations.

11. Derivation of Received Backscattered Power

A basic relationship for received backscattered power P collected at the terminals of receiver antenna $\underline{b}$, due to the scattered return from target T, which is illuminated by a plane wave from transmit antenna $\underline{a}$, is presented in this section. The usual procedure is to start with the radar equation.* In our discussion on output power, direction gain and antenna losses (transmitting and receiving system constants) are included in the antenna magnitude factors a and b. Propagation losses and radar cross section are lumped into the target magnitude factor m. The target polarization effects are determined by the normalized (m = 1) scattering matrix T. The results obtained will be fundamental to all subsequent developments in the remainder of this work.

The power is expressed as a scalar product of stokes vectors of antenna $\underline{b}$ and returned wave, which in turn is derived from a stokes reflection matrix M applied to the stokes vector of antenna $\underline{a}$. The derivations are somewhat laborious but straightforward. Applications are given in later sections. We start with equation (9.1) for received voltage:

$$V = T(m, \rho, \gamma, \nu, \tau_m, \psi)\underline{a}(a, \phi_A, \tau_A) \bullet \underline{b}(b, \phi_B, \tau_B) \qquad (11.1)$$

*See Ruck et al. [49, p. 7], which includes a discussion of propagation and transmitting and receiving losses.

The target scattering matrix T is shown expressed in geometric parameters, as are the antennas $\underline{a}$ and $\underline{b}$. The received power $P = |V|^2$ is found from (11.1) through application of the scalar product rule (7.4) between stokes vectors. Let

$$p(T\underline{a}) = (p_0, p_1, p_2, p_3) \tag{11.2}$$

and

$$h(\underline{b}) = (h_0, h_1, h_2, h_3) \tag{11.3}$$

We find from (7.4) that

$$P = |T\underline{a} \cdot \underline{b}|^2 = \frac{1}{2}(p_0 h_0 - p_1 h_1 + p_2 h_2 + p_3 h_3) \tag{11.4}$$

Note the change of position of the minus sign in (11.4) compared with (6.3) owing to the definition of $h(\underline{b})$:

$$h(\underline{b}) = (b^2, b^2 \sin 2\tau_B, b^2 \cos 2\tau_B \cos 2\phi_B . b^2 \cos 2\tau_B \sin 2\phi_B) \tag{11.5}$$

The p-vector components are found from equation (11.2) by substituting $\underline{E} = T\underline{a}$ for $p(\underline{E})$:

$$\left.\begin{aligned} p_0 &= \underline{E} \cdot \underline{E}^* = T\underline{a} \cdot T^*\underline{a}^* = T^*T\underline{a} \cdot \underline{a}^* \\ p_1 &= i\mathbf{J}\,\underline{E} \cdot \underline{E}^* = i\mathbf{J}\,T\underline{a} \cdot T^*\underline{a}^* = iT^*\mathbf{J}\,T\underline{a} \cdot \underline{a}^* \\ p_2 &= i\mathbf{L}\,\underline{E} \cdot \underline{E}^* = iT^*\mathbf{L}\,T\underline{a} \cdot \underline{a}^* \\ p_3 &= -i\mathbf{K}\,\underline{E} \cdot \underline{E}^* = -iT^*\mathbf{K}\,T\underline{a} \cdot \underline{a}^* \end{aligned}\right\} \tag{11.6}$$

We notice that all p_i in (11.6) are of the form $p_j = H_j\underline{a} \cdot \underline{a}^*$, where $H_j = H_j^{*\prime}$ is a hermetian matrix. The term H_j may be expressed by four real numbers, m_{j0}, m_{j1}, m_{j2}, and m_{j3}, as follows:

$$H_j = \begin{bmatrix} m_{j0} + m_{j2} & m_{j3} - im_{j1} \\ m_{j3} + im_{j1} & m_{j0} - m_{j2} \end{bmatrix} = m_{j0}\mathbf{I} + im_{j1}\mathbf{J} + im_{j2}\mathbf{L} - im_{j3}\mathbf{K} \quad (11.7)$$

Hence $H_j\underline{a} \cdot \underline{a}^*$ is evaluated simply by

$$H_j\underline{a} \cdot \underline{a}^* = (m_{j0}\mathbf{I} + im_{j1}\mathbf{J} + im_{j2}\mathbf{L} - im_{j3}\mathbf{K})\underline{a} \cdot \underline{a}^* =$$

$$= m_{j0}g_0 + m_{j1}g_1 + m_{j2}g_2 + m_{j3}g_3 \quad (11.8)$$

where we used (11.4) with $p(\underline{E})$ replaced by: $g(\underline{a}) = (g_0, g_1, g_2, g_3)$. Hence, H_j is conveniently written in the form

$$H_j = (m_{j0}\mathbf{I} + m_{j1}i\mathbf{J} + m_{j2}i\mathbf{L} - m_{j3}i\mathbf{K}) = (m_{j0}; m_{j1}; m_{j2}; m_{j3}) \quad (11.9)$$

We now may define four hermetian matrices in (11.6) similarly:

$$\left.\begin{aligned} H_0 &= \tfrac{1}{2}T^*T &&= (m_{00}; m_{01}; m_{02}; m_{03}) \\ H_1 &= -\tfrac{i}{2}T^*\mathbf{J}T &&= (m_{10}; m_{11}; m_{12}; m_{13}) \\ H_2 &= \tfrac{i}{2}T^*\mathbf{L}T &&= (m_{20}; m_{21}; m_{22}; m_{23}) \\ H_3 &= -\tfrac{i}{2}T^*\mathbf{K}T &&= (m_{30}; m_{31}; m_{32}; m_{33}) \end{aligned}\right\} \quad (11.10)$$

Then from (11.4) and substituting (11.6) and (11.10), and finally using (11.8), we obtain:

$$P = \begin{bmatrix} H_0\underline{a} \cdot \underline{a}^* \\ H_1\underline{a} \cdot \underline{a}^* \\ H_2\underline{a} \cdot \underline{a}^* \\ H_3\underline{a} \cdot \underline{a}^* \end{bmatrix} \bullet \begin{bmatrix} h_0 \\ h_1 \\ h_2 \\ h_3 \end{bmatrix} = \begin{bmatrix} m_{00} & m_{01} & m_{02} & m_{03} \\ m_{10} & m_{11} & m_{12} & m_{13} \\ m_{20} & m_{21} & m_{22} & m_{23} \\ m_{30} & m_{31} & m_{32} & m_{33} \end{bmatrix} \begin{bmatrix} g_0 \\ g_1 \\ g_2 \\ g_3 \end{bmatrix} \bullet \begin{bmatrix} h_0 \\ h_1 \\ h_2 \\ h_3 \end{bmatrix}$$

or

$$P = Mg(\underline{a}) \bullet h(\underline{b}) \tag{11.11}$$

Hence, by computing the m_{ij} components from equations (11.10), it is possible to compute the stokes reflection matrix M in (11.11) in terms of target parameters derived from the target scattering matrix T. This is still a rather tedious process. To simplify matters, it is convenient to eliminate the target orientation ψ from the scattering matrix. This is easily done by the following device. Equation (11.1) for voltage is rewritten as:

$$V = T\underline{a} \bullet \underline{b} = e^{\psi \mathbf{J}}\, T_o\, e^{-\psi \mathbf{J}}\, \underline{a} \bullet \underline{b} = T_o\, (e^{-\psi \mathbf{J}}\, \underline{a}) \bullet (e^{-\psi \mathbf{J}}\, \underline{b}) = T_o \underline{a}_\psi \bullet \underline{b}_\psi \tag{11.12}$$

where $\underline{a}_\psi = \underline{a}(a, \phi_A - \psi, \tau_A)$.

The calculation for $P = |V|^2$ based upon (11.12) now leads to

$$P = Mg(\underline{a}) \bullet h(\underline{b}) = M_o g(\underline{a}_\psi) \bullet h(\underline{b}_\psi) \tag{11.13}$$

where

$$g(\underline{a}_\psi) = \left[a^2, a^2 \sin 2\tau_A, a^2 \cos 2\tau_A \cos 2(\phi_A - \psi), a^2 \cos 2\tau_A \sin 2(\phi_A - \psi)\right] \tag{11.14}$$

and $h(\underline{b}_\psi) = h[\underline{b}(\phi_B - \psi)]$ similarly.

The term M_o is the stokes reflection matrix corresponding to T_o, with $\psi = 0$. It is calculated from (11.10) by substituting T_o for T. For H_o we find by substituting $T_o = T(\psi = 0) = U^*T_dU^{*'}$ from (10.10) in equation (11.10):

$$H_o = \frac{1}{2} T_o^* T_o = \frac{1}{2} e^{\tau_m \mathbf{K}} T_d^* e^{\tau_m \mathbf{K}} e^{-\tau_m \mathbf{K}} T_d e^{-\tau_m \mathbf{K}} =$$

$$= \frac{1}{2} e^{\tau_m \mathbf{K}} e^{2\nu \mathbf{L}} T_d(\gamma) T_d(\gamma) e^{-2\nu \mathbf{L}} e^{-\tau_m \mathbf{K}} \tag{11.15}$$

For $T_d(\gamma)$ we write:

$$T_d(\gamma) = m \begin{bmatrix} 1 & 0 \\ 0 & \tan^2 \gamma \end{bmatrix} = \frac{m}{2 \cos^2 \gamma} \begin{bmatrix} 1 + \cos 2\gamma & 0 \\ 0 & 1 - \cos 2\gamma \end{bmatrix}$$

$$= \frac{m}{2 \cos^2 \gamma} (\mathbf{I} + i \mathbf{L} \cos 2\gamma) \tag{11.16}$$

Hence,

$$H_o = \frac{m^2}{8 \cos^4 \gamma} e^{\tau_m \mathbf{K}} e^{2\nu \mathbf{L}} [(1 + \cos^2 2\gamma) \mathbf{I} + 2i \cos 2\gamma \mathbf{L}] e^{-2\nu \mathbf{L}} e^{-\tau_m \mathbf{K}} =$$

$$= Q_o e^{\tau_m \mathbf{K}} [(1 + \cos^2 2\gamma) \mathbf{I} + 2i \cos 2\gamma \mathbf{L}] e^{-\tau_m \mathbf{K}} =$$

$$= Q_o \left[(1 + \cos^2 2\gamma) \mathbf{I} + 2i \cos 2\gamma \mathbf{L} e^{-2\tau_m \mathbf{K}} \right] =$$

$$= Q_o \left[(1 + \cos^2 2\gamma) \mathbf{I} + 2i \cos 2\gamma \sin 2\tau_m \mathbf{J} + 2i \cos 2\gamma \cos 2\tau_m \mathbf{L} \right] \tag{11.17}$$

where

$$Q_o = \frac{m^2}{8 \cos^4 \gamma}$$

Since $H_0 = (m_{00}; m_{01}; m_{02}; m_{03})$ by (11.10), we find:

$$\left.\begin{aligned} m_{00} &= Q_o(1 + \cos^2 2\gamma) \\ m_{01} &= 2Q_o \cos 2\gamma \sin 2\tau_m \\ m_{02} &= 2Q_o \cos 2\gamma \cos 2\tau_m \\ m_{03} &= 0 \end{aligned}\right\} \tag{11.18}$$

The derivation for H_1, H_2, and H_3 are left to the reader. We give the following result for the oriented stokes reflection matrix M_o:

$$M_o = \begin{bmatrix} A_o + B_o & F & C & H \\ F & -A_o + B_o & G & D \\ C & G & A_o + B & -E \\ H & D & -E & A_o - B \end{bmatrix} \tag{11.19}$$

where

$$\left.\begin{aligned} A_o &= Q_o f \cos^2 2\tau_m \\ B_o &= Q_o(1 + \cos^2 2\gamma - f \cos^2 2\tau_m) \\ B &= Q_o[1 + \cos^2 2\gamma - f(1 + \sin^2 2\tau_m)] \\ C &= 2Q_o \cos 2\gamma \cos 2\tau_m \\ F &= 2Q_o \cos 2\gamma \sin 2\tau_m \\ D &= Q_o \sin^2 2\gamma \sin 4\nu \cos 2\tau_m \\ E &= Q_o \sin^2 2\gamma \sin 4\nu \sin 2\tau_m \\ G &= Q_o f \sin 4\tau_m \\ H &= 0 \end{aligned}\right\} \tag{11.20}$$

with the definitions:

$$Q_o = \frac{m^2}{8 \cos^4 \gamma} \tag{11.21}$$

and

$$f = 1 - \sin^2 2\gamma \sin^2 2\nu \tag{11.22}$$

Substitution of (11.19) into (11.13) gives the basic result for received backscattered power. The next sections present special features and special cases of this important result.

12. Properties of Received Power From Radar Targets

The equation for received power P from radar targets was expressed in (11.13) by a scalar product of stokes vectors representing the returned wave Mg(a) and receiver antenna h(b), where M is the stokes reflection matrix and g(a) and h(b) are the stokes vector representations of antennas a and b. Written in full, the expression is reduced to:

$$P(a, \phi_A, \tau_A ; b, \phi_B, \tau_B) = M_o(m, \psi = 0, \tau_m, \nu, \gamma) g(a, \Phi_A, \tau_A) \bullet h(b, \Phi_B, \tau_B) =$$

$$= \begin{bmatrix} A_o + B_o & F & C & H \\ F & -A_o + B_o & G & D \\ C & G & A_o + B & -E \\ H & D & -E & A_o - B \end{bmatrix} \begin{bmatrix} a^2 \\ a^2 \sin 2\tau_A \\ a^2 \cos 2\tau_A \cos 2\Phi_A \\ a^2 \cos 2\tau_A \sin 2\Phi_A \end{bmatrix} \bullet$$

$$\bullet \begin{bmatrix} b^2 \\ b^2 \sin 2\tau_B \\ b^2 \cos 2\tau_B \cos 2\Phi_B \\ b^2 \cos 2\tau_B \sin 2\Phi_B \end{bmatrix} \tag{12.1}$$

where

$$\Phi_A = \phi_A - \psi \quad , \quad \Phi_B = \phi_B - \psi \tag{12.2}$$

The real quantities $A_o, B_o, B, C, D, E, F, G,$ and H which determine M are given by target parameters $m, \gamma, \nu, \tau_m,$ and ψ, which are derived from the target scattering matrix T. We found it more convenient to incorporate target orientation ψ with the antennas $\underline{a}$ and $\underline{b}$, which reduces M to $M_o = M(\psi = 0)$. A further simplification is achieved if the antenna gains $g_o = a^2$ and $h_o = b^2$ are incorporated with the multiplicative factor Q_o which appears in all the terms composing M. Let

$$Q = a^2 b^2 Q_o = \frac{a^2 b^2 m^2}{8 \cos^4 \gamma} \tag{12.3}$$

By simply replacing Q_o by Q in (11.20), leaving the notation otherwise the same, we now write (12.1) in full:

$$\begin{aligned}
&P(\phi_A, \tau_A ; \phi_B, \tau_B) = \\
&= A_o + B_o + (-A_o + B_o) \sin 2\tau_A \sin 2\tau_B + \\
&\quad + F (\sin 2\tau_A + \sin 2\tau_B) + A_o \cos 2(\Phi_A - \Phi_B) \cos 2\tau_A \cos 2\tau_B + \\
&\quad + C (\cos 2\Phi_A \cos 2\tau_A + \cos 2\Phi_B \cos 2\tau_B) + \\
&\quad + B \cos 2(\Phi_A + \Phi_B) \cos 2\tau_A \cos 2\tau_B + \\
&\quad - E \sin 2(\Phi_A + \Phi_B) \cos 2\tau_A \cos 2\tau_B + \\
&\quad + D (\sin 2\Phi_A \sin 2\tau_B \cos 2\tau_A + \sin 2\Phi_B \sin 2\tau_A \cos 2\tau_B) + \\
&\quad + G (\cos 2\Phi_A \sin 2\tau_B \cos 2\tau_A + \cos 2\Phi_B \sin 2\tau_A \cos 2\tau_B)
\end{aligned} \tag{12.4}$$

Both equivalent forms (12.1) and (12.4) have useful application. The form (12.4) was first published by Huynen [28]. Matrix M is also called the stokes reflection matrix, or the Mueller matrix, after Mueller's studies in optics. We reserve the notation R for the more general case of scattering from distributed targets. Hence, R reduces to M for a single radar target. Notice the general trace rule for M:

$$\text{trace } M = 2(A_o + B_o) \geq 0 \tag{12.5}$$

Also from (11.20), $A_o \geq 0$ and $B_o \geq 0$ are easily verified. Other interesting relationships between parameters of M which may be checked are:

$$\left.\begin{aligned} Q_1 &= B_o^2 - B^2 - (E^2 + F^2) = 0 \\ Q_2 &= 2A_o(B_o + B) - (C^2 + D^2) = 0 \\ Q_3 &= 2A_o(B_o - B) - (G^2 + H^2) = 0 \end{aligned}\right\} \tag{12.6}$$

Notice that M is a symmetric matrix $M = M'$; this follows from reciprocity, since P remains unchanged if the operation of transmit and receive antennas $g(\underline{a})$ and $h(\underline{b})$ is interchanged. We observe that A_o, B_o, B, C, D, E, F, and G in M_o are functions of m, γ, ν, and τ_m; hence, these parameters cannot all be independent. Four relationships have to be satisfied to make an independent set. We will show later (Chap. 6) that the following relationships between coefficients of M are sufficient to make an independent set (notice $H \neq 0$ in M if $\psi \neq 0$):

$$\left.\begin{aligned} 2A_o(B_o + B) &= C^2 + D^2 \\ 2A_o(B_o - B) &= G^2 + H^2 \\ 2A_oE &= DG - CH \\ 2A_oF &= CG + DH \end{aligned}\right\} \tag{12.7}$$

We will show later that $A_o \geq 0$, $B_o \geq 0$ and relationships (12.7) are necessary and sufficient conditions that M represents a single (nondistributed) radar target with scattering matrix T. Hence, it should be possible to "reconstruct" T from the target parameters A_o, B_o, B, C, D, E, F, G, and H if $A_o \geq 0$, $B_o \geq 0$ and conditions (12.7) are satisfied. This problem has practical significance, since the M-parameters are obtainable from radar measurements on a target by measuring amplitudes (power) only. The solution to this problem is postponed until Sec. 14. All but the absolute phase of T can thus be reconstructed. The following sections will treat some of these problems in more detail.

Notice the obvious but important fact that all parameters constituting M_o are independent of ψ, and hence they have the target axis orientation independent property referred to in Sec. 8.

Some intuitive concepts related to the physical significance of the parameters of M_o may be helpful. The sum $A_o + B_o$ may be considered roughly as a measure of total power in the returned wave from the target. [Strictly, it is total power only if transmit antenna $\underline{a}$ is linearly polarized with orientation $\phi_A = 45° + \psi$, or is uniformly random polarized: $g(\underline{a}) = (a^2, 0, 0, 0)$.] For sphere-type targets, A_o is the only nonzero parameter ($\gamma = 45°$, $\nu = 0°$, $\tau_m = 0°$) and hence A_o may be viewed roughly as the total return power from regular, smooth, convex parts of the scatterer, whereas B_o may be thought of as a measure of total power of the target's irregular, rough nonconvex depolarizing components. For symmetric targets ($\tau_m = 0$), E, and G are zero and $B_o = B$. In general, $B_o \geq |B|$ which also follows from (12.6), and thus $B_o - B$ is a measure of total nonsymmetric depolarization whereas $B_o + B$ may be associated with total symmetric depolarized power of the target return.

The parameter pairs C and D, E and F, and G and H, being off-diagonal terms, are associated generally with depolarization components of the scattered return; they may be positive, zero, or negative. The pair C and D is associated with depolarization components of symmetric targets, E and F with depolarization due to nonsymmetry. The pair G and H may be viewed as "coupling terms"; G "couples" the target's symmetric and nonsymmetric terms, while H is a measure of coupling of components due to target orientation misalignment; i.e., H can always be made zero by a proper orientation (rotation about the radar line-of-sight direction) of the target.

These heuristic concepts may be useful for associating physical target properties with radar parameters.

Some special cases of radar illumination and reception of practical significance are listed below. Many ground-based radars in the past have used a single linearly polarized antenna for transmission and for receiving. If we substitute $\tau_A = \tau_B = 0$ and $\Phi_A = \Phi_B = \Phi = \phi - \psi$ in (12.4), we obtain for received power from an arbitrary radar target in this case:

$$P_{\parallel}(\phi) = 2A_o + B_o + 2C \cos 2\Phi + B \cos 4\Phi - E \sin 4\Phi \qquad (12.8)$$

Similarly, for linear-orthogonal transmission and reception, we find for $\tau_A = \tau_B = 0$, $\Phi_A = \Phi = \phi - \psi$, $\Phi_B = \pi/2 + \Phi$:

$$P_{\perp}(\phi) = B_o - B \cos 4\Phi + E \sin 4\Phi \qquad (12.9)$$

Notice that the sum of $P_{\parallel}(\phi)$ and $P_{\perp}(\phi)$ gives the total backscattered power for linearly polarized illumination,

$$P_{tot}(\phi) = 2(A_o + B_o) + 2C \cos 2\Phi \qquad (12.10)$$

For circular polarization, we consider three cases designated by (RC - RC), (LC - LC), and (RC - LC), where RC is right circular and LC left circular polarization and the pair denotes states of transmitter and receiver antennas. Substituting $\tau = 45°$ for RC antennas and $\tau = -45$ for LC antennas, we find for the received power:

$$P(RC - RC) = 2(B_o + F) \qquad (12.11)$$

$$P(LC - LC) = 2(B_o - F) \qquad (12.12)$$

$$P(RC - LC) = 2A_o \qquad (12.13)$$

For symmetric targets $(\tau_m = 0)$, $F = 0$ and hence we find that same-sensed circularly polarized antennas measure B_o which we associated with total power of depolarizing components of the return scattering, while opposite-sensed circularly polarized antennas measure A_o, which we associated with total regular nondepolarized scattering. Hence, for the nondepolarizing sphere-type target, $P(RC - RC) = P(LC - LC) = 0$ and $P(RC - LC) = 2A_o$ give maximum return; whereas for a strongly depolarizing trough-type target, $P(RC - RC) = P(LC - LC) = 2B_o$ is maximum and $P(RC - LC) = 0$. In general, CP measurements separate the regular nondepolarizing target return components from the irregular depolarizing components.

For future reference, we need not only the oriented stokes matrix $M_o(m, \psi = 0, \tau_m, \nu, \gamma)$ but also the stokes matrix $M(m, \psi, \tau_m, \nu, \gamma)$. Considering equations (11.12) and (11.13) and taking account of the definitions of $g(\underline{a})$ and $h(\underline{b})$, we find the transformation from $g(\underline{a}_\psi)$ to $g(\underline{a})$:

$$g(\underline{a}_\psi) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos 2\psi & \sin 2\psi \\ 0 & 0 & -\sin 2\psi & \cos 2\psi \end{bmatrix} g(\underline{a}) \tag{12.14}$$

and similarly for $h(\underline{b})$ and $h(\underline{b}_\psi)$.

For the general stokes reflection matrix, we now find:

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos 2\psi & -\sin 2\psi \\ 0 & 0 & \sin 2\psi & \cos 2\psi \end{bmatrix} \begin{bmatrix} A_o + B_o & F & C & 0 \\ F & -A_o + B_o & G & D \\ C & G & A_o + B & -E \\ 0 & D & -E & A_o - B \end{bmatrix} \times$$

$$\times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos 2\psi & \sin 2\psi \\ 0 & 0 & -\sin 2\psi & \cos 2\psi \end{bmatrix}$$

$$M = \begin{bmatrix} A_o + B_o & F & C\cos 2\psi & C\sin 2\psi \\ F & -A_o + B_o & \begin{matrix} G\cos 2\psi + \\ -D\sin 2\psi \end{matrix} & \begin{matrix} G\sin 2\psi + \\ +D\cos 2\psi \end{matrix} \\ C\cos 2\psi & \begin{matrix} G\cos 2\psi + \\ -D\sin 2\psi \end{matrix} & \begin{matrix} A_o + B\cos 4\psi + \\ +E\sin 4\psi \end{matrix} & \begin{matrix} B\sin 4\psi + \\ -E\cos 4\psi \end{matrix} \\ C\sin 2\psi & \begin{matrix} G\sin 2\psi + \\ +D\cos 2\psi \end{matrix} & \begin{matrix} B\sin 4\psi + \\ -E\cos 4\psi \end{matrix} & \begin{matrix} A_o - B\cos 4\psi + \\ +E\sin 4\psi \end{matrix} \end{bmatrix} \tag{12.15}$$

We notice, first, that the matrix is symmetric. The results in (12.15) are more elegantly contained if we introduce:

$$\left.\begin{aligned} H_\psi &= C\sin 2\psi \\ C_\psi &= C\cos 2\psi \end{aligned}\right\} \tag{12.16}$$

$$\left.\begin{aligned} G_\psi &= G\cos 2\psi - D\sin 2\psi \\ D_\psi &= G\sin 2\psi + D\cos 2\psi \end{aligned}\right\} \tag{12.17}$$

$$\left.\begin{aligned} E_\psi &= E\cos 4\psi - B\sin 4\psi \\ B_\psi &= E\sin 4\psi + B\cos 4\psi \end{aligned}\right\} \tag{12.18}$$

Then (12.15) is:

$$M = \begin{bmatrix} A_o + B_o & F & C_\psi & H_\psi \\ F & -A_o + B_o & G_\psi & D_\psi \\ C_\psi & G_\psi & A_o + B_\psi & -E_\psi \\ H_\psi & D_\psi & -E_\psi & A_o - B_\psi \end{bmatrix} \tag{12.19}$$

This form is particularly convenient for later work.

We now summarize our findings: the received power from a single radar target has been shown to have the form

$$P = Mg(\underline{a}) \bullet h(\underline{b}) \tag{12.20}$$

where M is the stokes reflection matrix and $g(\underline{a})$, $h(\underline{b})$ represent the stokes vectors for the transmit and receiver antennas. Also we notice the curious trace-rule which M has to obey:

$$\text{trace } M = 2(A_o + B_o)$$

The nine remaining parameters that determine M are thus A_o, B_o, B_ψ, C_ψ, D_ψ, E_ψ, F, G_ψ, and H_ψ.

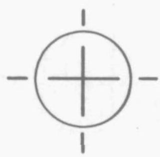
13. Special Radar Target Matrix Representations

Some special radar target scattering matrices are listed below. All targets are normalized to have magnitude $m = 1$. Target matrices that differ only by a phase term are considered to represent the same target. We distinguish between symmetric and nonsymmetric radar targets.

(1) Symmetric Targets

(a) A large sphere or flat plate at normal incidence made of any material is represented by the unit matrix. Target parameters are $\gamma = 45°$, $\nu = 0°$, $\tau_m = 0°$.

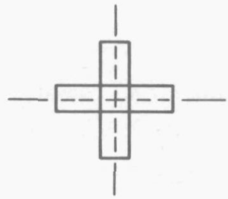
$$T_o = \mathbf{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; \text{ symbol}$$



Notice the obvious independence on orientation angle ψ of this target; we show later that it is the only symmetric target representation (with $\tau_m = 0$) that has this property.

(b) A large trough (two planes intersecting at 90°) oriented with axis (the plane's line of intersection) horizontal or vertical, is given by $\gamma = 45°$, $\nu = +45°$, $\tau_m = 0°$, $\psi = 0$, or $90°$.

$$T_T = \pm \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \pm i\,\mathrm{L}$$; symbol



This target (Fig. 10) is symmetric, although not roll-symmetric. The viewing angle is considered normal to the trough's open face.

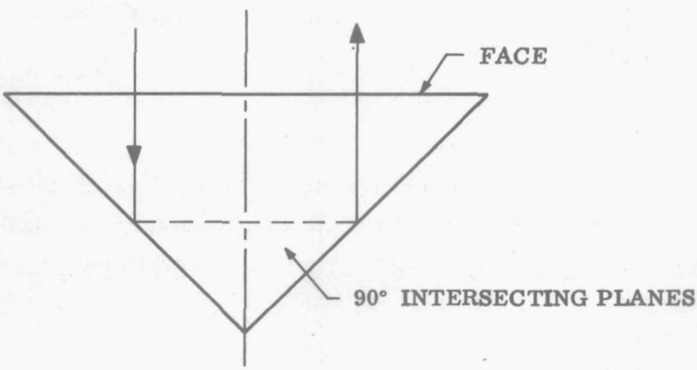
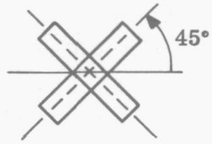


Fig. 10 Reflection From a Trough

This target has a two-bounce reflection characteristic, associated with maximum value of "skip angle" $|\nu|$. This target obviously is not orientation independent. Notice that $\psi = 0°$ or $90°$ changes only the absolute phase (±) of T_T; hence, we treat the two cases as the same target.

(c) A large trough as in (b), but with axis oriented at 45° or -45°, is given by $\gamma = 45°$, $\nu = +45°$, $\tau_m = 0°$, $\psi = \pm 45°$.

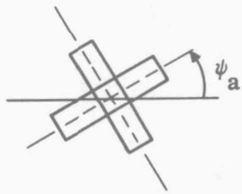
$$T_T(45°) = \pm \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \mp i\,\mathbf{K} \quad ; \text{ symbol}$$



Notice that this target completely depolarizes a horizontally or vertically polarized incoming illumination.

(d) A large trough as in (b) and (c), but with axis oriented an arbitrary angle ψ_a: $\gamma = 45°$, $\nu = +45°$, $\tau_m = 0°$, $\psi = \psi_a$.

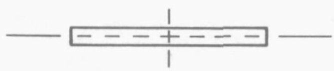
$$T_T(\psi_a) = \begin{bmatrix} \cos 2\psi_a & \sin 2\psi_a \\ \sin 2\psi_a & -\cos 2\psi_a \end{bmatrix} ; \text{ symbol}$$



Notice that targets (b), (c), and (d) have identical reflections for incoming circular polarization (CP), since CP is insensitive to target orientation.

(e) A horizontal line target (wire): $\gamma = 0°$, ν = arbitrary, $\tau_m = 0°$, $\psi = 0$.

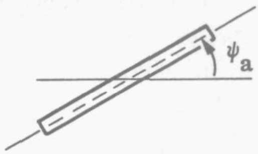
$$T_L = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} ; \text{ symbol}$$



This target's return is horizontally polarized, independent of any incoming polarization.

(f) A line target with axis oriented at angle ψ_a: $\gamma = 0°$, ν = arbitrary, $\tau_m = 0°$, $\psi = \psi_a$.

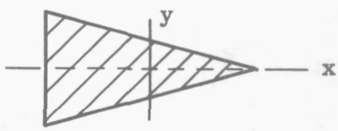
$$T_L(\psi_a) = \begin{bmatrix} \cos^2 \psi_a & \sin \psi_a \cos \psi_a \\ \sin \psi_a \cos \psi_a & \sin^2 \psi_a \end{bmatrix} \quad ; \text{ symbol}$$



Since $T_L(\psi_a) = 1/2\, T_o + 1/2\, T_T(\psi_a)$, we find that the line target may be considered as composed of a sphere and a trough with proper phasing relative to each other. This concept holds true for any symmetric radar target, as will be shown below.

(g) <u>A symmetric target with horizontal axis</u>; parameters: γ, ν, $\tau_m = 0$, $\psi = 0$.

$$T_S = \begin{bmatrix} e^{+2i\nu} & 0 \\ 0 & \tan^2 \gamma\, e^{-2i\nu} \end{bmatrix} \quad ; \text{ symbol}$$



This radar target covers a wide class of physical targets; in fact, until recently very little attention was paid to nonsymmetric targets. All targets having an axis of roll-symmetry are symmetric at <u>all</u> aspect angles. Such targets cover the well known shapes: cones, cylinders, ellipsoids, and combinations of these. Other targets which are basically nonsymmetric, such as corner reflectors, may have planes of symmetry through a line-of-sight direction and a target axis for which the target is symmetric. The fact that $\tau_m = 0$ is characteristic for all these targets is shown as follows. We recall that τ_m was the ellipticity angle of the maximum polarization $\underline{m}$. Figure 11 shows a general nondegenerate radar target symmetric with respect to the x-axis; it also shows the characteristic polarization $\underline{m}$. Because of the symmetry of the target, there would be another maximum polarization $\underline{m}'$ shown in the figure, obtained from $\underline{m}$ by reflection from the plane of symmetry. However, the eigenvalue theory for nondegenerate targets does not allow for two different maximum polarizations. Hence, $\underline{m}$ and $\underline{m}'$ have to be the same polarization.

There are only two solutions, as can easily be verified from Fig. 11. Either $\underline{m}$ is "horizontal" polarization (aligned with the x-axis) or "vertical" polarization (aligned with the y-axis). In both cases, $\tau_m = 0$ (linear polarization) and $\psi = 0°$ or $\psi = 90°$. This completes the proof of this remarkable property of symmetric targets.

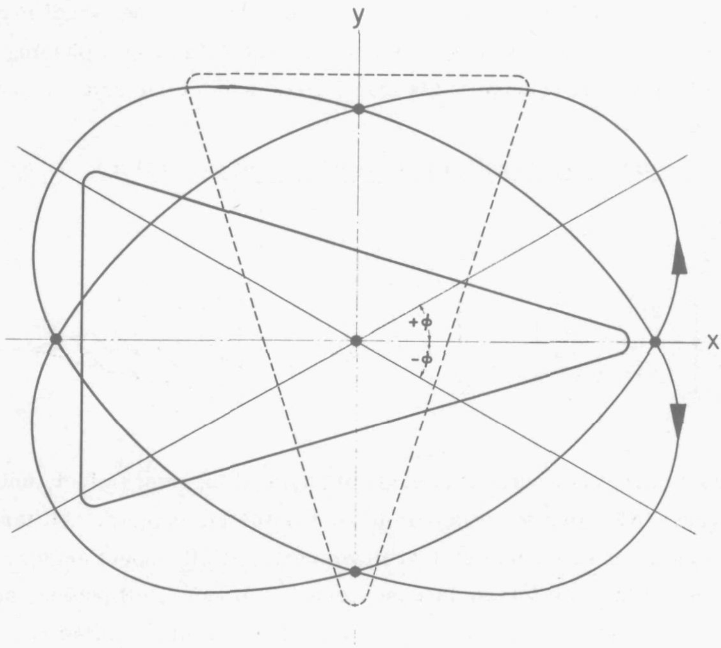
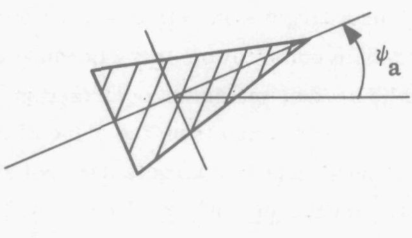


Fig. 11 Symmetric Target

(h) A symmetric target with arbitrary axis orientation angle ψ_a:



This is an obvious extension of the previous case. We restate the following important general rule: symmetric targets are characterized by $\tau_m = 0$ and $\psi = \psi_a$ or $\psi = \pi/2 + \psi_a$; i.e., the polarization which gives maximum return is linear and is either aligned with the symmetry axis or orthogonal to it. For the "line target" T_L, as we have seen, the maximum polarization is always aligned with the target axis ("horizontal" polarization), but for the general symmetric target the maximum (linear) polarization may switch from "horizontal" to "vertical," depending on aspect angle.

Since the dependence on target orientation ψ always includes the target axis orientation ψ_a, which is strictly determined by target dynamics, it is often desirable to decompose ψ as follows:

$$\psi = \psi_a + \phi_m \tag{13.1}$$

The angle ϕ_m is called the "relative target orientation," since it is the target orientation measured relative to the target reference axis. Hence, relative target orientation ϕ_m is independent of the target axis orientation.

From the discussions above, it follows that for all symmetric radar targets $\tau_m = 0$ and $\phi_m = 0°$ or $90°$. For general nonsymmetric targets, τ_m and ϕ_m may have arbitrary values within their respective ranges.

Notice that since for symmetric targets: $4\psi = 4\psi_a \pm 2n\pi$, measurement of ψ through radar observations provides a direct measure of the target's axis orientation. However, for nonsymmetric targets no such simple rule exists. Conversely, since the variation of target axis orientation ψ_a is a dynamic variable which obscures the target return signature for target identification purposes based upon em scattering, equation (13.1) provides a technique for eliminating ψ_a even if it cannot be measured separately. From radar measurements on T, we simply eliminate the effect of ψ from the target return signature! All these practically important procedures require *that orientation* ψ *be measurable from radar observations.* It is clear at this point that a single polarization radar (with one antenna polarization for transmitting and receiving) cannot provide this function.

We conclude the discussion on symmetric targets with a comment on target decompositions: case (g) shows T_S as a diagonal matrix, given by two parameters ν and γ and an arbitrary absolute phase. We also could have written:

$$T_S = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \tag{13.2}$$

where a and b are complex and $|a|^2 + |b|^2 = 1 + \tan^2\gamma$ provides the normalization. This form is convenient since we can introduce complex numbers c and d such that $a = c + d$, and $b = c - d$. It follows that the symmetric target T_S may be considered as the sum of a sphere target T_O and a trough T_T: $T_S = cT_O + dT_T$, with proper amplitudes and phases. The same argument holds true for the general oriented symmetric target $T_S(\psi_a)$, which is decomposed in a sum of T_O (independent of orientation) and an oriented trough $T_T(\psi_a)$.

(2) Nonsymmetric Targets

Nonsymmetric targets have so-called "helicity" since $\tau_m \neq 0$. The most typical example is that described in paragrah (i).

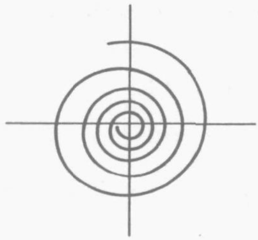
(i) A helix with right screw sense with parameters $\gamma = 0°$, ν = arbitrary, $\tau_m = 45°$, $\psi = 0$. Substituting these values into (10.19) gives the scattering matrix:

$$T_{HR} = \frac{1}{2}\begin{bmatrix} 1 & -i \\ -i & -1 \end{bmatrix}$$; symbol

The helix is considered viewed along its axis.

(j) A helix with left screw sense is determined similarly: $\gamma = 0°$, ν = arbitrary, $\tau_m = -45°$, $\psi = 0$.

$$T_{HL} = \frac{1}{2}\begin{bmatrix} 1 & i \\ i & -1 \end{bmatrix} ; \text{ symbol}$$



The effect of change of target orientation (rotation about the helix axis) is easily calculated. Since $T_{HR} = 1/2\,(i\,\mathbf{L} - \mathbf{K})$,

$$T_{HR}(\psi_a) = e^{\psi_a \mathbf{J}}\, T_{HR}\, e^{-\psi_a \mathbf{J}} = \frac{1}{2}(i\mathbf{L} - \mathbf{K})\, e^{-2\psi_a \mathbf{J}} =$$

$$= \frac{1}{2}(i\,\mathbf{L} \cos 2\psi_a - i\,\mathbf{K} \sin 2\psi_a - \mathbf{K} \cos 2\psi_a - \mathbf{L} \sin 2\psi_a) =$$

$$= e^{2i\psi_a}\, \frac{1}{2}(i\mathbf{L} - \mathbf{K}) = e^{2i\psi_a}\, T_{HR}$$

This agrees with the intuitive concept that a rotation about the helix axis changes only the phase of the returned signal (at twice the rotation angle).

(k) N-targets. A certain class of nonsymmetric targets appears naturally in the theory of distributed targets, which will be discussed extensively in later chapters. These "targets" are associated with nonsymmetric target-noise components which appear in the radar return scattering from nonsymmetric rough surfaces. We will call these "N-targets" (N standing for "nonsymmetric noise").

N-targets are defined by $\tau_m = \pm 45°$. Similarly, as an oriented (with $\psi = 0$) S target is given by $T_S = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$, an oriented N-target is given by

$$T_N = \begin{bmatrix} a & b \\ b & -a \end{bmatrix} \tag{13.3}$$

This is easily verified by substituting $\tau_m = 45°$ into (10.19).

A general N-target is obtained from the oriented one by a rotation ψ about the line-of-sight direction:

$$T_N(\psi) = e^{\psi \mathbf{J}} \begin{bmatrix} a & b \\ b & -a \end{bmatrix} e^{-\psi \mathbf{J}} = e^{\psi \mathbf{J}} (ia\,\mathbf{L} - ib\mathbf{K})\, e^{-\psi \mathbf{J}} =$$

$$= (ia\,\mathbf{L} - ib\,\mathbf{K})\, e^{-2\psi \mathbf{J}} = i\,\mathbf{L}\,(a \cos 2\psi + b \sin 2\psi) - i\,\mathbf{K}\,(b\cos 2\psi - a \sin 2\psi) =$$

$$= ia'\mathbf{L} - ib'\mathbf{K} = \begin{bmatrix} a' & b' \\ b' & -a' \end{bmatrix} \tag{13.4}$$

where

$$\left.\begin{aligned} a' &= a \cos 2\psi + b \sin 2\psi \\ b' &= -a \sin 2\psi + b \cos 2\psi \end{aligned}\right\} \tag{13.5}$$

Hence, the change of orientation with ψ leaves the form (13.3) unchanged. Notice that for two special cases, $a = 0$ and $b = 0$, form (13.3) reduces to the trough targets (c) and (d) discussed before; these, however, were classified there as symmetric targets ($\tau_m = 0$)! This shows that trough-targets may be considered also as a special type of N-target, a point of view which is developed further in Chap. 8. We will find there that "trough-noise" components appear naturally in the decomposition of the scattered return from symmetric rough surfaces.

14. Correspondences Between Scattering Matrix and Stokes Matrix

A close correspondence exists between the target scattering matrix T and target stokes reflection matrix M representations of radar targets. The correspondence can be made one to one if we exclude or ignore the absolute phase ρ of the target in T. To make this more precise, the concept of "scattering matrix with relative phase only" (SMR) was introduced [28]. We quote from this paper:

> There are two types of general scattering matrices, and a usage and measurement procedure for each. The first includes the absolute phase of the target, while the second determines only relative phases among matrix coefficients. The absolute phase of a target depends on its local position, aspect, surface structure, and the radar frequency. Relative phases, on the other hand, are phase differences among the individual scattering matrix coefficients where any one may act as phase reference for the others. The scattering matrix with absolute phase (SMA) is sensitive to target displacement along the line of sight, while the scattering matrix with relative phase (SMR) is not. One additional absolute phase measurement attached to the SMR determines SMA. From a measurements point of view, the difference between the two scattering matrix types is profound. Determination of the SMA requires the ability to measure absolute phase, while the SMR can be obtained by amplitude measurements only. The latter case leads to the indirect measurement of the scattering matrix.

The indirect measurements of amplitudes only, determine the M-matrix. Hence, the "inverse problem" consists of reconstructing the SMR from a knowledge of stokes matrix M. We will show shortly that there exists indeed a close resemblance between the set of unitary transformations U which diagonalizes T (Sec. 10) and a corresponding set of orthogonal transformations O which transform M. This fact could lead us to expect that also it will be possible to find the characteristic parameters of T simply by solving a corresponding eigenvalue problem of the form $Mg = \alpha g$ for the symmetric stokes matrix M. However, no such exact correspondence can be found, since M is not diagonalized after the orthogonal transformations involving ψ, τ_m, and ν are used to transform M to reduced form. This is verified easily from (11.20) by putting $\psi = \tau_m = \nu = 0$ in M, and we find that the nondiagonal term C of M does not vanish ($C = 2Q \cos 2\gamma$) ! This fact complicates the solution of the "inverse problem." However, it is possible to reduce M systematically by using known properties of the ψ, τ_m, and ν transformations and thus retrieve the values of ψ, τ_m, ν, and γ which determine the SMR. For most applications the stokes matrix M is not determined completely by amplitude measurements, since this would involve using a multitude of antenna transmit and receive polarization combinations, which in most cases is impractical. Instead, one proceeds with dual-polarized amplitude- and phase-sensitive antenna measurements, which determines the coefficients of T directly (Huynen [28]).

We discuss next the correspondence between unitary transformations on T and orthogonal transformations on M, which have important and interesting application to the classification of fixed radar targets.

(1) Unitary and Orthogonal Transformations

The following properties of orthogonal transformations on M are easily verified. From (10.22) we have:

$$T = e^{\psi \mathbf{J}} e^{-\tau_m \mathbf{K}} e^{-\nu \mathbf{L}} \begin{bmatrix} m & 0 \\ 0 & m \tan^2 \gamma \end{bmatrix} e^{-\nu \mathbf{L}} e^{-\tau_m \mathbf{K}} e^{-\psi \mathbf{J}} \tag{14.1}$$

The problem at hand is to associate with each unitary transformation $e^{\psi \mathbf{J}}$, $e^{-\tau_m \mathbf{K}}$. $e^{-\nu \mathbf{L}}$ a corresponding orthogonal transformation which reduces M to $M(2\gamma)$ such that:

$$M = O_1(2\psi)O_2(-2\tau_m)O_3(2\nu)M(2\gamma)O_3^{-1}(2\nu)O_2^{-1}(-2\tau_m)O_1^{-1}(2\psi) \tag{14.2}$$

where $O_i^{-1}(2\alpha) = O_i(-2\alpha)$. However, as we have seen, $M(2\gamma)$ is not of diagonal form. A simple calculation will show the following:

$$O_1(2\psi) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos 2\psi & -\sin 2\psi \\ 0 & 0 & \sin 2\psi & \cos 2\psi \end{bmatrix} \tag{14.3}$$

$$O_2(2\tau_m) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\tau_m & -\sin 2\tau_m & 0 \\ 0 & \sin 2\tau_m & \cos 2\tau_m & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{14.4}$$

$$O_3(2\nu) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\nu & 0 & \sin 2\nu \\ 0 & 0 & 1 & 0 \\ 0 & -\sin 2\nu & 0 & \cos 2\nu \end{bmatrix} \tag{14.5}$$

The orthogonal transformations are chosen such that they represent positive rotations about the three orthogonal axes in polarization space. (See Sec. 6.) A detailed listing of orthogonal transformations applied to M is given in Sec. 31.

For the reduced $M(2\gamma)$ we find, with $Q_o = m^2/2(1 + \cos 2\gamma)^2$ as usual:

$$M(2\gamma) = Q_o \begin{bmatrix} 1 + \cos^2 2\gamma & 0 & 2\cos 2\gamma & 0 \\ 0 & -\sin^2 2\gamma & 0 & 0 \\ 2\cos 2\gamma & 0 & 1 + \cos^2 2\gamma & 0 \\ 0 & 0 & 0 & \sin^2 2\gamma \end{bmatrix} \tag{14.6}$$

The orthogonal transformations O_1, O_2, O_3 in equation (14.2), which reduces M to $M(2\gamma)$, may be combined such that:

$$M = OM(2\gamma)O^{-1} \tag{14.7}$$

We now substitute M back into the equation for received power:

$$P = Mg \cdot h = OM(2\gamma)O^{-1}g \cdot h = M(2\gamma)(O^{-1}g) \cdot (O^{-1}h) = M(2\gamma)g_1 \cdot h_1 \tag{14.8}$$

This result has important consequences, as we will see shortly. The equation shows that the received power from a fixed radar target given by M and two antennas g and h is equal to the return from a "canonical target" given by $M(2\gamma)$ with antennas g_1 and h_1 which are obtained from g and h by an orthogonal transformation.

This property makes it possible to study the nature of scattering from all fixed radar targets (given by four independent variables, γ, ν, τ_m, and ψ) for arbitrary antenna polarizations g and h, through a study of canonical targets $M(2\gamma)$ which are determined by one variable γ only. The target and antenna magnitudes m, a, and b may be considered normalized to unity for this particular discussion. The above-mentioned property reveals a basic simple structure of the nature of all fixed radar target scattering as a function of polarization. It provides the basis for classifying fixed radar targets according to their canonical derivatives. Further discussion of these concepts is given in Chap. 4.

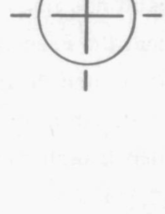
The correspondence between scattering matrix T and stokes matrix M representations may also be used to derive an effective target matrix $\overline{T}$ from M. The "effective" $\overline{T}$ is equal to T except for a multiplicative absolute phase factor. We can show explicitly how the effective scattering matrix $\overline{T}$ is constructed from the conditions for a single target M. Given M, we first find target parameter ψ from the orthogonal transformation $O_1(2\psi)$ on M such that $M' = O_1^{-1}(2\psi)MO_1(2\psi)$ which makes $H' = 0$. Subsequently, we perform the orthogonal transformation $O_2(2\tau_m)$ on M' (which leaves $H'' = 0$) such that $G'' = 0$. Then from (12.7), also $E'' = 0$ and $F'' = 0$. The final transformation $O_3(2\nu)$ is performed on the remaining matrix M'', which makes $D''' = 0$; then $A_o''' = m^2$ and $B_o''' = \cos^2 2\gamma$. Hence, the effective target matrix $\overline{T}(m, \gamma, \nu, \tau_m, \psi)$ is reconstructed from stokes matrix M.

(2) Listing of Corresponding T and M Representations of Various Targets

This listing follows the sequence and discussion of targets in Sec. 13. The targets are normalized as usual with $m = 1$:

(a) Sphere target: $\gamma = 45°$, $\nu = 0°$, $\tau_m = 0°$, $A_o = 1/2$, $B_o = B = 0$.

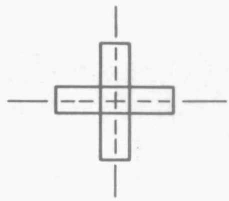
$$T_O = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \; ; \; M_O = \frac{1}{2}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



(b) <u>Horizontal or vertical trough:</u> $\gamma = 45°$, $\nu = 45°$, $\tau_m = 0°$, $\psi = 0°$ or $90°$

$A_o = 0$, $B_o = B = 1/2$.

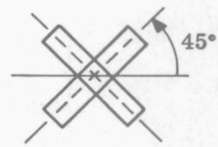
$$T_T = i\,\mathbf{L} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \; ; \; M_T = \frac{1}{2}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$



(c) <u>Troughs oriented at ±45°</u>: $\gamma = 45°$, $\nu = 45°$, $\tau_m = 0°$, $\psi = \pm 45°$,

$A_o = 0$, $B_o = 1/2$, $B = -1/2$.

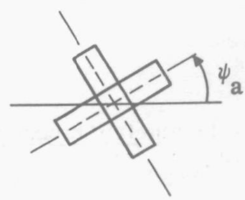
$$T_T(45°) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \; ; \; M_T(45°) = \frac{1}{2}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



(d) <u>Troughs oriented at ψ_a</u>: $\gamma = 45°$, $\nu = 45°$, $\tau_m = 0°$, $\psi = \psi_a$.

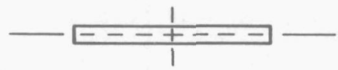
$$T_T(\psi_a) = \begin{bmatrix} \cos 2\psi_a & \sin 2\psi_a \\ \sin 2\psi_a & -\cos 2\psi_a \end{bmatrix} ;$$

$$M_T(\psi_a) = \frac{1}{2}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos 4\psi_a & \sin 4\psi_a \\ 0 & 0 & \sin 4\psi_a & -\cos 4\psi_a \end{bmatrix}$$



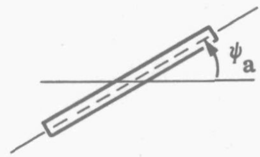
(e) <u>Long horizontal wire:</u> $\gamma = 0°$, $\tau_m = 0°$, $\psi = 0$, $A_o = 1/8$, $B_o = B = 1/8$, $C = 1/4$.

$$T_L = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} ; \quad M_L = \frac{1}{4} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$



(f) Long wire oriented at ψ_a: $\gamma = 0°, \ \tau_m = 0°, \ \psi = \psi_a$.

$$T_L(\nu_a) = \begin{bmatrix} \cos^2 \psi_a & \frac{1}{2} \sin 2\psi_a \\ \frac{1}{2} \sin 2\,\phi_a & \sin^2 \psi_a \end{bmatrix} ;$$



$$M_L(\psi_a) = \frac{1}{4} \begin{bmatrix} 1 & 0 & \cos 2\psi_a & \sin 2\psi_a \\ 0 & 0 & 0 & 0 \\ \cos 2\psi_a & 0 & \cos^2 2\psi_a & \frac{1}{2} \sin 4\psi_a \\ \sin 2\psi_a & 0 & \frac{1}{2} \sin 4\psi_a & \cos^2 2\psi_a \end{bmatrix}$$

(g) Symmetric target, $\psi_a = 0°$: $\tau_m = 0°, \ \psi = \phi_m = 0°$ or $90°$, $B_o = B, \ a = e^{+2i\nu}, \ b = e^{-2i\nu} \tan^2 \gamma, \ E = F = G = H = 0, \ 4A_oB_o = C^2 + D^2$.

$$T_S(0) = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} ; \quad M_S(0) = \begin{bmatrix} A_o + B_o & 0 & \pm C & 0 \\ 0 & -A_o + B_o & 0 & \pm D \\ \pm C & 0 & A_o + B_o & 0 \\ 0 & \pm D & 0 & A_o - B_o \end{bmatrix}$$

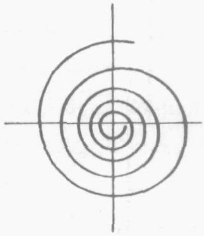
(h) Symmetric target oriented at ψ_a: $\tau_m = 0, \ \psi = \psi_a + \phi_m, \ a = e^{+2i\nu}$, $b = e^{-2i\nu} \tan^2 \gamma$.

$$T_S(\psi_a) = \frac{1}{2} \begin{bmatrix} (a + b) + (a - b) \cos 2\psi & (a - b) \sin 2\psi \\ (a - b) \sin 2\psi & (a + b) - (a - b) \cos 2\psi \end{bmatrix}$$

$$M_S(\psi_a) = \begin{bmatrix} A_o + B_o & 0 & C\cos 2\psi & C\sin 2\psi \\ 0 & -A_o + B_o & -D\sin 2\psi & D\cos 2\psi \\ C\cos 2\psi & -D\sin 2\psi & A_o + B\cos 4\psi & B\sin 4\psi \\ C\sin 2\psi & D\cos 2\psi & B\sin 4\psi & A_o - B\cos 4\psi \end{bmatrix}$$

(i) <u>Helix with right screw sense:</u> $\gamma = 0°$, $\tau_m = 45°$, ψ_a = arbitrary.

$$T_{HR} = \frac{1}{2}\begin{bmatrix} 1 & -i \\ -i & 1 \end{bmatrix} ; \quad M_{HR} = \frac{1}{4}\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$



(j) <u>Helix with left-screw sense:</u> $\gamma = 0°$, $\tau_m = -45°$, ψ_a = arbitrary.

$$T_{HL} = \frac{1}{2}\begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix} ; \quad M_{HL} = \frac{1}{4}\begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(k) <u>N-target:</u> $\tau_m = 45°$, $|a^2 + b^2| = \tan^2\gamma$, ψ = arbitrary.

$$T_N = \begin{bmatrix} a & b \\ b & -a \end{bmatrix} ; \quad M_N = \begin{bmatrix} B_o & F & 0 & 0 \\ F & B_o & 0 & 0 \\ 0 & 0 & B & -E \\ 0 & 0 & -E & -B \end{bmatrix}$$

Between B_o, B, E, and F we have: $B_o^2 = B^2 + E^2 + F^2$. The N-target plays a significant role in the theory of distributed nonsymmetric radar targets.

4 MEASUREMENT OF SINGLE RADAR TARGETS

15. Introduction to Radar Target Measurements

The subject of radar target measurements covers a wide range of activities, aimed at varied applications, which at times presents a confusing picture due to conflicting requirements. One limiting factor is economic. Radar measurements are expensive to obtain, and target models have to be constructed with great precision in shape and according to specified material properties. Radar test equipment is expensive, and requires periodic calibration and experienced personnel to operate and maintain. Radar range facilities suffer from notorious background interference problems. For economic reasons, usually only a minimal amount of information about the target is acquired. The radar system engineer is interested in obtaining radar cross-section (RCS) patterns which provide received amplitude, or power, in decibels as a function of a complete rotation in aspect angle of the target.

The RCS patterns are measured at various frequencies and with given polarization of radar transmitter and receiver. For non-roll-symmetric objects such as an airplane, the data will be repeated for various roll-positions of the target. Even with minimum requirements of, say, two RCS patterns for "horizontal" (H - H) and "vertical" polarization (V - V) , the snowballing effect of several frequencies, roll-cuts, and different target models soon presents a flood of RCS data to the investigator, who is then faced with the necessity to draw some useful conclusions. This presents a source of potential confusion: For complex shapes, the above-mentioned minimal RCS data are not nearly adequate to fully determine the radar target at fixed frequency aspect angle and roll-cut. The target scattering matrix for a general target at given frequency, aspect angle, and roll-position is determined by five independent parameters (excluding absolute phase). The

RCS data for (H - H) and (V - V) polarization provide only two pieces of information.

Even for roll-symmetric targets which are given by three parameters, an additional RCS pattern is required to complete the data set. Usually the (L45° - L45°) linear polarization RCS pattern is specified for extra pattern, but we showed that for symmetric targets this still leaves an ambiguity in the sign of the skip angle ν (Huynen [28]). The ambiguity is not important if only linear or circular polarization for radar transmitter and receiver are employed, but it does make a difference for RCS patterns with mixed combinations of circular and linear polarizations: (RC-L). Since most radars operate either with linear or with circular polarizations, the three linear RCS patterns (H - H), (V - V), and (L45° - L45°) provide sufficient data for radar target simulation of roll-symmetric objects. For those targets the helicity angle τ_m is zero. However, most targets such as an airplane are not roll-symmetric ($\tau_m \neq 0$) and the three linear RCS patterns are not adequate even to predict or simulate the radar return for linear polarization. It has been shown that five linear polarization patterns for 0°, 30°, 60°, 90°, and 120° orientation and one (RC-L) combination pattern will give adequate RCS prediction capability [28]. Since the systems engineer is primarily interested in radar return prediction techniques, the emphasis in radar target technology has traditionally been focused on these problems. In comparison, very little has been done to date in systematic exploration of radar parameters based upon target scattering characteristics. The difficulty lies with the choice of parameters and the expense of obtaining a complete set such that the data may be used to compute other parameters (Huynen [24]) If only certain parameters are determined [for instance, for oriented ($\psi_a = 0$) targets, the patterns (H - H) and (V - V)], it is difficult to use or compare these data with other sets of measurements i.e., in a dynamic situation the object might be oriented at different angles ψ_a with respect to the observer.

Radar target measurements aimed at determination of the stokes scattering matrix of objects for a given orientation have to be performed for different polarizations in such numbers that the elements of the scattering matrix or of the stokes matrix can be calculated.

To date, relatively few efforts to measure complete sets of data on special types of radar targets have been performed. Complete polarization maps of some fixed ground targets and rain clouds were measured during the years 1951 – 1954 by Huynen et al. [14]. During 1960, Copeland obtained complete scattering matrix data for nonmoving radar targets by measuring complex voltage return displayed by an ellipse on an oscilloscope [18]. Complete dynamic characteristic null-locus plots, representing the scattering matrix of symmetric targets, were determined by Huynen in 1960 – 1962 [22]. During the years 1962 – 1964, results of complete sets of target parameters versus aspect angle and roll angle of nonsymmetric radar targets were published (Huynen [24]). Related efforts on symmetric targets were reported by Lowenschuss [27]. Direct measurements of the scattering matrix were reported by Freeny [50], Crispin [20], and Webb and Allen [26]. Beckmann's current book [6] deals with depolarization of targets without emphasizing completeness of data. Recently, a Russian book by Kanareykin et al. [39] on polarization of radar signals appeared, which summarizes some of the earlier published results.

16. Target Matrix Restrictions

The concept of a matrix seems well established in mathematics. In applications, a matrix usually denotes some type of transformation of a vector quantity from one state to another. Since the vector has a physical significance (such as force, velocity, or field), it is customary to study the properties that are independent of a specific representation or coordinate system. In contrast, the matrix transformations usually are referenced to a particular coordinate system. To discuss the matrix concept for arbitrary coordinate systems, one speaks of a matrix operator. The theory of linear operators in various mathematical spaces is a well developed subject of modern mathematics. The vector spaces usually considered in these theories have very general properties such as those of continuity and differentiability. In Hilbert-space, there is defined a scalar product, etc.

Less frequently are discussed the transformation properties of subclasses of vector spaces that are due to restrictions. For example, instead of allowing vectors of any length and direction, we might consider vector transformations on a sphere where all vectors have the same length. These "restrictions" on matrix transformations usually are important from a practical standpoint. A certain type of equipment, such as a radar unit, may have practical limits of operation such that only certain type of measurements produce selected types of data. One wishes to know and understand what the measured data represent in terms of properties of the radar target that is being illuminated. Most radars operate within a narrow frequency range, thus restricting the scattering data to those frequencies. Hence, one speaks of radar cross-section (RCS) data at a given frequency range. Another restriction not so commonly understood is due to the use of a particular type of polarization of the radar transmitter and receiver antenna. One labels the RCS data according to the polarization used, such as "vertical" or "horizontal" or "circularly polarized."

In many practical situations, the transmitting and receiving antennas are independently linearly polarized (with orientations ϕ_T and ϕ_R). To accommodate this case, this writer has introduced the concept of the "linear restricted scattering matrix" (LSM) [28]. The LSM represents the target based upon information that can be gathered by a combination of linearly polarized transmitting and receiving antennas, the latter registering amplitudes (power) only. The concept of LSM is important for radar target simulations. Given a dynamic flight pattern of the target and the geometric location of the radar relative to the flight path, one wishes to simulate or reconstruct the dynamic received pattern based on "static" measured on computed RCS data from the target. In many cases the polarizations of interest for the dynamic simulation are those of linearly and circularly polarized radars.

Note that at a given viewing angle the target's exposure to the radar illumination may have a different orientation compared with the case in which the "static" patterns were obtained. The change of orientation for a given exposure of the target is equivalent to a change of orientation of the linear polarization of the radar. From this observation, it follows that for dynamic simulation with a linearly polarized radar one has to know the target scattering for all linear polarizations. This is exactly the information which the LSM provides. One may argue that the general SM also provides this data, but the LSM is more economical to obtain if one is interested only in linear polarization radars, as previously pointed out by this writer [28]:

> The LSM is measured by six linearly polarized amplitude patterns. Five of these patterns are for parallel reception with orientations $\phi = 0°, 30°, 60°, 90°$, and $120°$.
> The sixth pattern is for orthogonal reception. The advantage of the LSM procedure is that these measurements can be performed with simpler equipment and with greater precision than is possible with techniques, which require antennas other than the linear type or phase measurements. For roll-symmetric targets, the LSM is measured simply by three linearly polarized patterns for parallel reception at orientations 0°, 45°, and 90°.

As an extra bonus, the LSM also is adequate to predict the cross-circular (RC-LC) radar reception under simulated flight conditions, but it cannot in general provide the (RC-RC) or (LC-LC) patterns. A detailed analysis [28] further shows that the LSM is equal to the ordinary SM of a target, but with an ambiguity in the sign of target parameters ν and τ_m still unresolved.

17. Theory of Characteristic Null-Polarizations

In this section, we will develop the theory of null-polarizations, which is used to characterize a fixed radar target. This theory will be shown to have many useful applications. In previous sections, we found that the target scattering matrix T , given by five target parameters and an absolute phase, defined a general stationary radar target at a given fixed radar frequency. Two independent null-polarizations are determined by two points on the polarization sphere, i.e., by four independent parameters. One additional parameter, the target amplitude m , together with the two characteristic null-polarizations of a radar target, determines the target scattering matrix (excluding the absolute phase).

The concept of null-polarization of a radar target may be viewed as analogous to the roots or nulls of a polynomial function in the complex plane which are used to characterize that function. The null-polarization of a fixed radar target is that polarization $\underline{n}$ of identical radar transmitter and receiver antennas which produces zero voltage reception at the receiver terminals:

$$V = T\underline{n} \cdot \underline{n} = 0 \qquad (17.1)$$

In light of the definition of orthogonal vectors, $\underline{n}_\perp^* \cdot \underline{n} = 0$, we may consider $\underline{n}$ as the eigenvector solution of a type of eigenvalue problem, which is characteristic for the null-polarizations:

$$T\underline{n} = c\underline{n}_\perp^* \qquad (17.2)$$

This equation has, in general, two solutions: $\underline{n}^\pm$, $c^\pm$ and $\underline{n}_\perp^\pm$.

We first consider the corresponding 3-dimensional stokes vectors $\underline{p}(\underline{n})$ and $\underline{p}(\underline{n}_\perp)$ on the unit sphere in polarization space (Poincaré-sphere). Also *considered are the vectors* $\underline{p}(\underline{m})$ *and* $\underline{p}(\underline{m}_\perp)$, *where* $\underline{m}$ *is the maximum* polarizaton discussed in Sec. 10 and $\underline{m}_\perp$ the orthogonal maximum polarization which was defined by:

$$\left.\begin{aligned} T\underline{m} &= t_1\underline{m}^* \\ T\underline{m}_\perp &= t_2\underline{m}_\perp^* \end{aligned}\right\} \qquad (17.3)$$

where $t_1 = m\, e^{2i(\nu+\rho)}$ and $t_2 = m \tan^2\gamma\, e^{-2i(\nu-\rho)}$. We will find shortly the geometric significance of the angle γ .

Figure 12 shows the four polarization vectors (orthogonal field vectors map as opposite polarization vectors). The angle between $\underline{p}(\underline{m})$ and $\underline{p}(\underline{n}_\perp)$ is called 2α . We intend to show that $\alpha = \gamma$ for both $\underline{n}^+$ and $\underline{n}^-$. We use the half-angle "cosine rule" of Sec. 6 to find c :

$$\begin{aligned} |c\underline{n}_\perp^* \cdot \underline{m}| &= |c|\,|\underline{n}_\perp^* \cdot \underline{m}| = |c| \cos\alpha = \\ &= |T\underline{n} \cdot \underline{m}| = |\underline{n} \cdot T\underline{m}| = m\,|\underline{n} \cdot \underline{m}^*| = m \sin\alpha \end{aligned} \qquad (17.4)$$

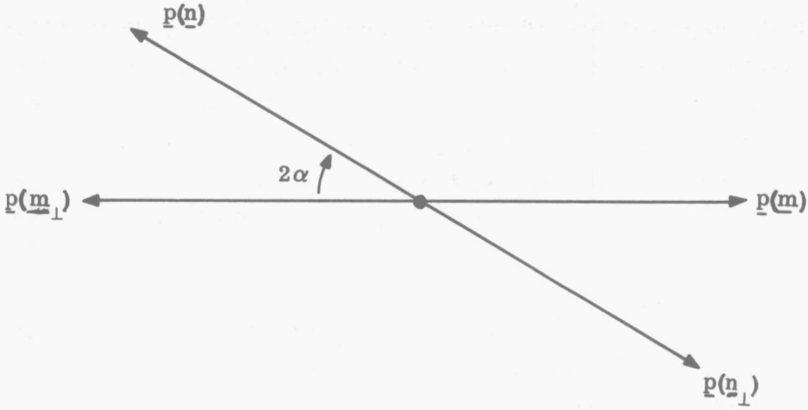


Fig. 12 Null- and Maximum Polarization

from which it follows that:

$$|c| = m \tan \alpha \tag{17.5}$$

Similarly, from (17.3) we have:

$$|c\underline{n}^*_\perp \cdot \underline{m}_\perp| = |c| \sin \alpha = |T\underline{n} \cdot \underline{m}_\perp| = |\underline{n} \cdot T\underline{m}_\perp| =$$
$$= m \tan^2 \gamma |\underline{n} \cdot \underline{m}^*_\perp| = m \tan^2 \gamma \cos \alpha$$

from which $\quad |c| \sin \alpha = m \tan^2 \gamma \cos \alpha \, . \qquad (17.6)$

From (17.5) and (17.6), it follows that $\alpha = \gamma$, where we have defined γ in the range $0 \le \gamma \le 45°$ as usual. Hence:

$$|c| = m \tan \gamma \tag{17.7}$$

This result shows that both null-polarization vectors lie on a conical surface, with $\underline{p}(\underline{m})$ and $\underline{p}(\underline{m}_\perp)$ as axis and 4γ as included cone angle. The only question remaining is what the positions of $\underline{n}^\pm$ are on the conical surface. We intend to show that $\underline{p}(\underline{n}^+)$, $\underline{p}(\underline{n}^-)$ and $\underline{p}(\underline{m})$ vectors are in a plane such that $\underline{p}(\underline{n}^+)$ and $\underline{p}(\underline{n}^-)$ have an included angle of 4γ . This establishes the polarization "fork" concept of unit vectors illustrated in Fig. 13.

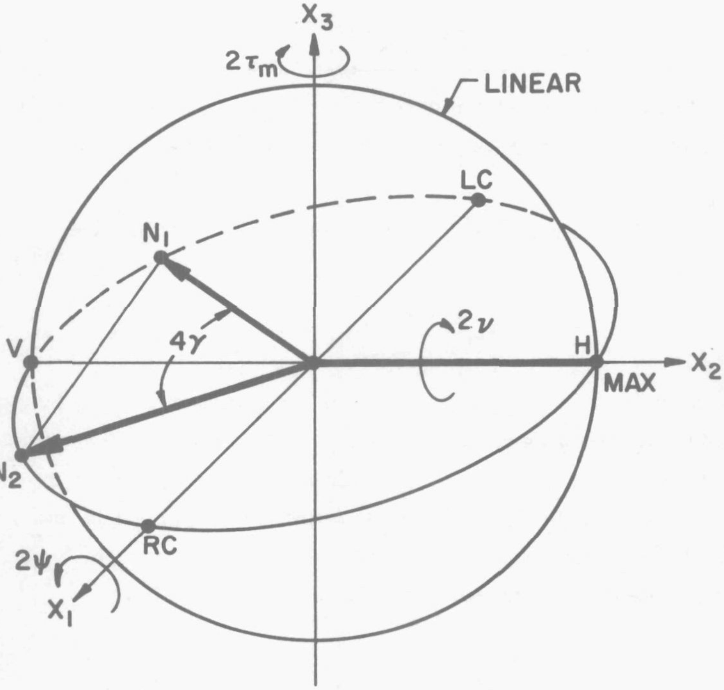


Fig. 13 Polarization Fork

What is necessary to prove the "fork" concept is to consider $\underline{n}$ decomposed into orthogonal components $\underline{m}$ and $\underline{m}_\perp$, similar to what was done in Sec. 6. We write:

$$\underline{n}^{\pm} = \sin\gamma\, e^{i\beta\pm}\, \underline{m} + \cos\gamma\, \underline{m}_\perp \tag{17.8}$$

which agrees with previous results: $|\underline{n}^{\pm} \cdot \underline{m}^*| = \sin\gamma$ and $|\underline{n}^{\pm} \cdot \underline{m}_\perp^*| = \cos\gamma$. The unknown angle β determines rotation with angle 2β about the $\underline{p}(\underline{m})$,

$\underline{p}(\underline{m}_\perp)$ axis in polarization space. (See Sec. 6.) Substitutions of (17.8) into (17.1) leads by a straightforward calculation to:

$$e^{i\beta\pm} = \mp i\, e^{-2i\nu} \tag{17.9}$$

Since the two null polarizations may have arbitrary phases, the solutions for the null-polarizations may be put in the following symmetric form:

$$\underline{n}^{+} = \sin\gamma\, e^{-i(45°+\nu)}\, \underline{m} + \cos\gamma\, e^{i(45°+\nu)}\, \underline{m}_\perp \tag{17.10}$$

$$\underline{n}^{-} = -\sin\gamma\, e^{-i(45°+\nu)}\, \underline{m} + \cos\gamma\, e^{i(45°+\nu)}\, \underline{m}_\perp \tag{17.11}$$

Notice that in this form $\underline{n}^{+}$ and $\underline{n}^{-}$ differ from each other only by a change of sign of γ (if, for this occasion, we allow other than positive angles for γ). Hence, if $\underline{p}(\underline{n}^{+})$ makes an angle 2γ with $\underline{p}(\underline{m}_\perp)$, $\underline{p}(\underline{n}^{-})$ lies in the same plane, making an angle -2γ with $\underline{p}(\underline{m}_\perp)$, and hence $\underline{p}(\underline{n}^{-})$ and $\underline{p}(\underline{n}^{+})$ enclose an angle of 4γ, which proves the "polarization fork" concept.

Some useful orthogonal null-polarizations are as follows:

$$\underline{n}^{\pm}_{\perp} = \cos\gamma\, e^{+i(45°-\nu)}\, \underline{m} \pm \sin\gamma\, e^{-i(45°-\nu)}\, \underline{m}_\perp \tag{17.12}$$

The significance of angle ν is also clear from (17.10), (17.11), and (17.12). It determines the rotation of the "prongs" of the fork concept about the $\underline{p}(\underline{m})$, $\underline{p}(\underline{m}_\perp)$ "handle" by an amount of 2ν.

In the following section, we derive the expressions for received voltage and power in terms of the characteristic target null-polarizations.

18. Voltage Reception in Terms of Two Target Polarizations

There exists an interesting and useful general relationship for received voltage if the target is characterized by two arbitrary but independent polarizations $\underline{c}^{(1)}$ and $\underline{c}^{(2)}$. These may be defined in terms of maximum polarization $\underline{m}$ and its orthogonal $\underline{m}_\perp$ by four complex constants, C_1, C_2, C_3 and C_4, as follows:

$$\left.\begin{aligned} \underline{c}^{(1)} &= C_1\underline{m}^* + C_2\underline{m}_\perp^* \\ \underline{c}^{(2)} &= C_3\underline{m}^* + C_4\underline{m}_\perp^* \end{aligned}\right\} \tag{18.1}$$

For convenience, we consider $\underline{c}^{(1)}$ and $\underline{c}^{(2)}$ normalized to unit vectors such that $|C_1|^2 + |C_2|^2 = 1$ and $|C_3|^2 + |C_4|^2 = 1$. The target scattering $T\underline{a}$, where T is the scattering matrix and $\underline{a}$ is the transmit antenna, may be decomposed similarly:

$$T\underline{a} = a_1\underline{m}^* + a_2\underline{m}_\perp^* \tag{18.2}$$

where

$$\left.\begin{aligned} a_1 &= T\underline{a} \bullet \underline{m} = \underline{a} \bullet T\underline{m} = t_1(\underline{a} \bullet \underline{m}^*) \\ a_2 &= T\underline{a} \bullet \underline{m}_\perp^* = \underline{a} \bullet T\underline{m}_\perp = t_2(\underline{a} \bullet \underline{m}_\perp^*) \end{aligned}\right\} \tag{18.3}$$

As usual, we used here the relationships: $T\underline{m}_1 = t_1\underline{m}^*$ and $T\underline{m}_2 = t_2\underline{m}_\perp^*$ as defined in Sec. 10.

We now solve from (18.1) for $\underline{m}^*$ and $\underline{m}_\perp^*$ in terms of $\underline{c}^{(1)}$ and $\underline{c}^{(2)}$:

$$\left.\begin{aligned} \underline{m}^* &= \frac{1}{\Delta}\left(C_4\underline{c}^{(1)} - C_2\underline{c}^{(2)}\right) \\ \underline{m}_\perp^* &= \frac{1}{\Delta}\left(- C_3\underline{c}^{(1)} + C_1\underline{c}^{(2)}\right) \end{aligned}\right\} \tag{18.4}$$

where

$$\Delta = C_1C_4 - C_2C_3 \neq 0 \tag{18.5}$$

Now let:

$$T\underline{a} = b_1\underline{c}^{(1)} + b_2\underline{c}^{(2)} \tag{18.6}$$

where

$$b_1 = \frac{1}{\Delta}\left(a_1 C_4 - a_2 C_3\right)$$
$$b_2 = \frac{1}{\Delta}\left(- a_1 C_2 + a_2 C_1\right) \qquad (18.7)$$

Substitution of (18.4) into (18.3) gives:

$$a_1 = \frac{t_1}{\Delta}\left[C_4 (\underline{c}^{(1)} \bullet \underline{a}) - C_2 (\underline{c}^{(2)} \bullet \underline{a})\right]$$
$$a_2 = \frac{t_2}{\Delta}\left[- C_3 (\underline{c}^{(1)} \bullet \underline{a}) + C_1 (\underline{c}^{(2)} \bullet \underline{a})\right] \qquad (18.8)$$

Substitution of (18.8) into (18.7) leads to:

$$b_1 = \frac{1}{\Delta^2}\left[\left(C_4^2 t_1 + C_3^2 t_2\right) (\underline{c}^{(1)} \bullet \underline{a}) - (C_2 C_4 t_1 + C_1 C_3 t_2) (\underline{c}^{(2)} \bullet \underline{a})\right]$$
$$b_2 = \frac{1}{\Delta^2}\left[- (C_2 C_4 t_1 + C_1 C_3 t_2) (\underline{c}^{(1)} \bullet a) + \left(C_2^2 t_1 + C_1^2 t_2\right) (\underline{c}^{(2)} \bullet \underline{a})\right] \qquad (18.9)$$

By substitution of (18.9) into (18.6), we find:

$$T\underline{a} = \left[d_1 (\underline{c}^{(1)} \bullet \underline{a}) + d_2 (\underline{c}^{(2)} \bullet \underline{a})\right] \underline{c}^{(1)} + \left[d_2 (\underline{c}^{(1)} \bullet \underline{a}) + d_3 (\underline{c}^{(2)} \bullet \underline{a})\right] \underline{c}^{(2)} \qquad (18.10)$$

where

$$d_1 = \frac{1}{\Delta^2}\left(C_4^2 t_1 + C_3^2 t_2\right)$$
$$d_2 = \frac{-1}{\Delta^2}\left(C_2 C_4 t_1 + C_1 C_3 t_2\right) \qquad (18.11)$$
$$d_3 = \frac{1}{\Delta^2}\left(C_2^2 t_1 + C_1^2 t_2\right)$$

The desired result $V = T\underline{a} \cdot b$, expressed in terms of arbitrary characteristic target polarization, is found from (18.10):

$$V = T\underline{a} \cdot \underline{b} = d_1 (\underline{c}^{(1)} \cdot \underline{a})(\underline{c}^{(1)} \cdot \underline{b}) + d_2 \Big[(\underline{c}^{(1)} \cdot \underline{a})(\underline{c}^{(2)} \cdot \underline{b}) + (\underline{c}^{(1)} \cdot \underline{b})(\underline{c}^{(2)} \cdot \underline{a})\Big] + d_3(\underline{c}^{(2)} \cdot \underline{a})(\underline{c}^{(2)} \cdot \underline{b}) \qquad (18.12)$$

This expression clearly is symmetric in $\underline{a}$ and $\underline{b}$ (due to reciprocity), and it is defined by target parameters t_1 and t_2 and the characteristic polarization $\underline{c}^{(1)}$ and $\underline{c}^{(2)}$ which are given by the constants C_1, C_2, C_3, and C_4.

We notice the curious fact that expression (18.12) has three terms with coefficients d_1, d_2, and d_3 which consist of products of scalar products, each of which may also be considered a type of voltage. Two special cases of (18.12) are of practical interest. The first is when the mixed term of d_2 is zero. The other case is when d_1 and d_2 are zero, which occurs for the null-polarizations, as we find shortly.

For the first case, consider $\underline{c}^{(1)} = \underline{m}^*$, $\underline{c}^{(2)} = \underline{m}_\perp^*$ and hence $C_1 = 1$, $C_2 = 0$, $C_3 = 0$, $C_4 = 1$, $\Delta = 1$, $d_1 = t_1$, $d_2 = 0$, $d_3 = t_2$ and thus:

$$V = T\underline{a} \cdot \underline{b} = t_1(\underline{m}^* \cdot \underline{a})(\underline{m}^* \cdot \underline{b}) + t_2(\underline{m}_\perp^* \cdot \underline{a})(\underline{m}_\perp^* \cdot \underline{b}) \qquad (18.13)$$

This identity could have been obtained directly, since

$$\underline{b} = (\underline{m}^* \cdot \underline{b})\underline{m} + (\underline{m}_\perp^* \cdot \underline{b})\underline{m}_\perp \qquad (18.14)$$

Then:

$$\begin{aligned} T\underline{a} \cdot \underline{b} &= (\underline{m} \cdot T\underline{a})(\underline{m}^* \cdot \underline{b}) + (\underline{m}_\perp \cdot T\underline{a})(\underline{m}_\perp^* \cdot \underline{b}) = \\ &= (T\underline{m} \cdot \underline{a})(\underline{m}^* \cdot \underline{b}) + (T\underline{m}_\perp \cdot \underline{a})(\underline{m}_\perp^* \cdot \underline{b}) = \\ &= t_1(\underline{m}^* \cdot \underline{a})(\underline{m}^* \cdot \underline{b}) + t_2(\underline{m}_\perp^* \cdot \underline{a})(\underline{m}_\perp^* \cdot \underline{b}) \end{aligned} \qquad (18.15)$$

For the second case, consider $\underline{c}^{(1)} = \underline{n}_{\perp}^{+*}$, $\underline{c}^{(2)} = \underline{n}_{\perp}^{-*}$, where the $\underline{n}^{\pm}$ terms are the target's characteristic null-polarizations discussed in Sec. 17. For $\underline{c}^{(1)}$ and $\underline{c}^{(2)}$, we thus have from (18.1):

$$\underline{c}^{(1)} = \underline{n}_{\perp}^{+*} = \cos\gamma\, e^{-i(45^\circ - \nu)}\underline{m}^* + \sin\gamma\, e^{i(45^\circ - \nu)}\underline{m}_{\perp}^*$$

$$\underline{c}^{(2)} = \underline{n}_{\perp}^{-*} = \cos\gamma\, e^{-i(45^\circ - \nu)}\underline{m}^* - \sin\gamma\, e^{i(45^\circ - \nu)}\underline{m}_{\perp}^* \qquad (18.16)$$

Here, $C_1 = \cos\gamma\, e^{-i(45^\circ - \nu)}$, $C_2 = \sin\gamma\, e^{i(45^\circ - \nu)}$, $C_3 = \cos\gamma\, e^{-i(45^\circ - \nu)}$ $C_4 = -\sin\gamma\, e^{i(45^\circ - \nu)}$, $\Delta = -\sin 2\gamma$, $d_1 = 0$, $d_2 = im/2\cos^2\gamma$ and $d_3 = 0$ and hence:

$$V = T\underline{a} \bullet \underline{b} = \frac{im}{2\cos^2\gamma}\left[(\underline{n}_{\perp}^{+*} \cdot \underline{a})(\underline{n}_{\perp}^{-*} \bullet \underline{b}) + (\underline{n}_{\perp}^{-*} \bullet \underline{a})(\underline{n}_{\perp}^{+*} \cdot \underline{b})\right] \qquad (18.17)$$

Of particular interest is the case for parallel reception: $\underline{a} = \underline{b}$. Then (18.17) reduces to:

$$V_{\parallel} = T\underline{a} \bullet \underline{a} = \frac{im}{\cos^2\gamma}(\underline{n}_{\perp}^{+*} \bullet \underline{a})(\underline{n}_{\perp}^{-*} \bullet \underline{a}) \qquad (18.18)$$

It is interesting to note that the voltage of (18.18) is composed of the product of two scalar products, each of which may also be considered a voltage. Equation (18.18) forms the basis for a theory developed by Copeland [18], where the complex voltage return from a target as a function of varying linear polarizations $\underline{a}$ is determined by a locus in the complex plane. The difference between our result (18.18) and Copeland's work is in the factor ($m/\cos^2\gamma$) which is not present in Copeland's derivation (which is based on heuristic arguments). The factor is important if we compare scattering from radar targets where γ is substantially different, i.e., the case of a convex surface (where $\gamma \simeq 45^\circ$) and a long wire (where $\gamma = 0^\circ$). The question of proper normalization of scattering from radar targets is one that is rarely discussed adequately in the literature.

The received power for parallel reception $P_{\parallel} = |V|^2$ is found easily from the general "cosine" formula derived in Sec. 7. Let $\underline{A} = \underline{A}(\underline{a}) = (A_1, A_2, A_3)$ be the 3-dimensional stokes vector representation of $\underline{a}$ and let $\underline{N}^{\pm} = \underline{N}^{\pm}(\underline{n}^{\pm}) = (N_1^{\pm}, N_2^{\pm}, N_3^{\pm})$ be the stokes vectors corresponding to the null-polarizations. Then we find for received power by application of the cosine rule to (18.18):

$$P_{\parallel} = \frac{m^2}{\cos^4 \gamma} \cos^2 \frac{1}{2}(-\underline{N}^{+}, \underline{A}) \cos^2 \frac{1}{2}(-\underline{N}^{-}, \underline{A})$$

or

$$P_{\parallel} = \frac{m^2}{\cos^4 \gamma} \sin^2 \frac{1}{2}(\underline{N}^{+}, \underline{A}) \sin^2 \frac{1}{2}(\underline{N}^{-}, \underline{A}) \qquad (18.19)$$

Of course, if $\underline{A} = \underline{N}^{\pm}$ we find from (18.19) that $P_{\parallel} = 0$, since then $V_{\parallel} = T\underline{n} \cdot \underline{n} = 0$. Equation (18.19) shows that if the two null-polarization $\underline{N}^{\pm}$ terms are known on the polarization sphere, then the target return is completely determined if also the maximum target amplitude m is known. The angle γ is determined, since 4γ is the included angle between the two null-polarization vectors (Sec. 17).

Equation (18.19) leads to a characterization of radar targets based on characteristic null-polarizations, which will be discussed in the following sections.

19. Canonical Representation of Single Targets; Gamma Target Maps

In the previous section, it was shown that the stokes matrix representation M of a fixed radar target can be reduced by orthogonal transformations to a so-called "gamma target" with stokes matrix $M(2\gamma)$, which is given by one parameter γ only (the magnitude m being normalized to unity). The orthogonal transformation on M was shown to be equivalent to an orthogonal transformation on the set of all antenna transmit polarizations $\underline{p}(\underline{a})$ and receiver polarizations $\underline{p}(\underline{b})$. Of practical significance is the case where antennas $\underline{a}$ and $\underline{b}$ are related, such as for radars with parallel reception when $\underline{b} = \underline{a}$, or orthogonal reception, where $\underline{b} = \underline{a}_{\perp}$. For those cases, we may associate with a point $\underline{p}(\underline{a})$ on the polarization sphere, the received power for radar polarization $\underline{a}$. By connection of all points $\underline{p}(\underline{a})$ on the sphere which have equal reception, a constant amplitude locus is obtained. The set of all loci which cover the polarization sphere is called a map of the fixed radar target.

Thus, we established the important fact that if for a particular gamma target, i.e., for a target at fixed position with a fixed value of γ and $\psi = \tau_m = \nu = 0$, the constant return amplitude map on the polarization sphere as a function of transmitter polarizations is given, we obtain the map of a general radar target with the same value of γ, but arbitrary ψ, τ_m, and ν, by a rotation of the gamma target map through angles 2ψ, $2\tau_m$, and 2ν with respect to the reference position on the sphere.

Since a rotation does not destroy the geometry of the map on the sphere, it is sufficient to study the maps of γ-targets for purposes of radar target classification. In other words, a "class" of fixed radar target maps is generated by a given γ-target map simply by the process of applying all possible rotations of the given γ-target map on the sphere with respect to a given projection plane. Thus, a one-parameter classification of all possible fixed radar targets is obtained. The γ-target map for parallel reception is called a gamma sphere. Figure 14 shows the gamma sphere for a target with $\gamma = 22\ 1/2°$. The contours for equal received power are labeled in decibels. We notice on the gamma sphere the two points where zero reception is obtained; these are the null-polarizations $\underline{p}(\underline{n}^{\pm})$ (points of ∞ dB). Also of importance is the single point for maximum reception, which identifies the maximum polarization $\underline{p}(\underline{m})$ (point of 0 dB). Notice the "polarization fork" construction between null-polarizations (the prongs) and the maximum polarization (the handle), which was discussed in Sec. 17.

We will next analyze the construction of the gamma-sphere representation and its polarization plane projection of gamma targets. The antenna polarization $\underline{p}(\underline{a}) = (p_1, p_2, p_3)$ is given as usual by the orthogonal coordinates:

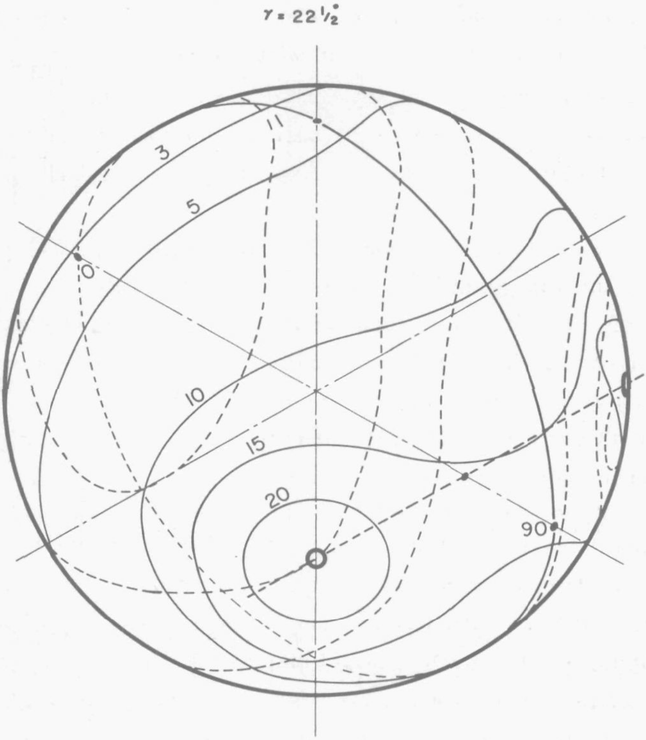


Fig. 14 Gamma Sphere

$$\left.\begin{aligned} p_1 &= \sin 2\tau \\ p_2 &= \cos 2\tau \cos 2\phi \\ p_3 &= \cos 2\tau \sin 2\phi \end{aligned}\right\} \qquad (19.1)$$

where τ is the ellipticity angle and ϕ the antenna orientation of elliptically polarized antenna $\underline{a}(\phi, \tau)$ as usual. (The amplitude a is normalized to unity.)

The normalized gamma-target returned power is given by (14.8):

$$P_{\|}(\gamma) = Q_0 \begin{bmatrix} 1+\cos^2 2\gamma & 0 & 2\cos 2\gamma & 0 \\ 0 & -\sin^2 2\gamma & 0 & 0 \\ 2\cos 2\gamma & 0 & 1+\cos^2 2\gamma & 0 \\ 0 & 0 & 0 & \sin^2 2\gamma \end{bmatrix} \begin{bmatrix} 1 \\ p_1 \\ p_2 \\ p_3 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ p_1 \\ p_2 \\ p_3 \end{bmatrix} \tag{19.2}$$

where

$$Q_0 = \frac{1}{8\cos^4 \gamma} \tag{19.3}$$

It is easy to verify that (19.2) is equivalent to:

$$P_{\|}(\gamma) = \frac{1}{4\cos^4 \gamma} \left[(\cos 2\gamma + p_2)^2 + \left(\sin^2 2\gamma\right) p_3^2 \right] \tag{19.4}$$

The $p_2 - p_3$ projection plane of all points $\underline{p} = (p_1, p_2, p_3)$ which constitute the polarization sphere (Poincaré sphere) is the circular polarization chart. In general, we need two charts, one for a hemisphere of all right-sensed $(\tau > 0)$ polarizations and the other for the left-sensed $(\tau < 0)$ polarizations, to cover all transmit polarizations. However, since (19.4) is independent of p_1, the contour maps on both charts coincide and hence one chart suffices to represent all transmit polarizations.

Also we notice from (19.4) the same value of $P_{\|}$ for positive and negative p_3, which indicates symmetry for the gamma-target maps about the horizontal $(p_3 = 0)$ axis. The two characteristic null-polarizations, where $P_{\|} = 0$, are easily identified from (19.4), since then $p_2 = -\cos 2\gamma$, $p_3 = 0$. $(P_{\|} = 1)$ is achieved if $p_2 = 1$, $p_3 = 0$, i.e., for horizontal polarization. One other interesting point is $\underline{m}_{\perp}$ at vertical polarization $(p_2 = -1,\ p_3 = 0)$ where $P_{\|}(\underline{m}_{\perp}) = \tan^4 \gamma$, as one could have expected. Notice also the characteristic "fork" construction of the gamma-sphere presentation between null-polarizations and maximum polarizations. The enclosed angle between null-polarization vectors was shown in Sec. 17 to be 4γ, which checks with the projection $p_2 = -\cos 2\gamma$.

Figures 15 through 18 give gamma-target maps for $\gamma = 0°$, 15°, 30°, and 45°. The case $\gamma = 0$ corresponds to the map for a horizontal line target. (See Sec. 14.) The other extreme, $\gamma = 45°$, represents a sphere target. For $\gamma = 45°$ and $\nu = 0$, the eigenvalues $t_1 = t_2$. and hence the sphere is a degenerate target, which is shown by a circle of maxima for all linear polarizations. For other values of γ, there are no simple physical equivalents for gamma targets. From a canonical gamma-target map and corresponding gamma spheres, the map of every other radar target is obtained by rotation of the corresponding gamma sphere and then projecting it back onto the $(p_2 - p_3)$ polarization charts. Notice also the obvious but important fact that the position of null-polarizations for gamma targets determines the value of γ and hence the complete map. Since the map is unaltered by rotations, we find the following general rule: The polarization chart map of constant amplitude loci for general radar targets is determined uniquely by the position of null-polarizations on the map and the level of maximum return.

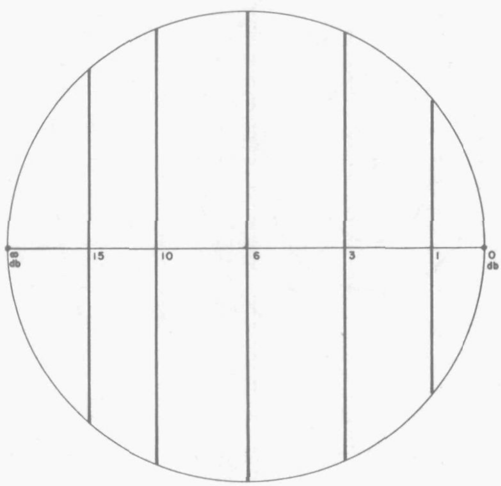


Fig. 15 Gamma Target: $\gamma = 0°$

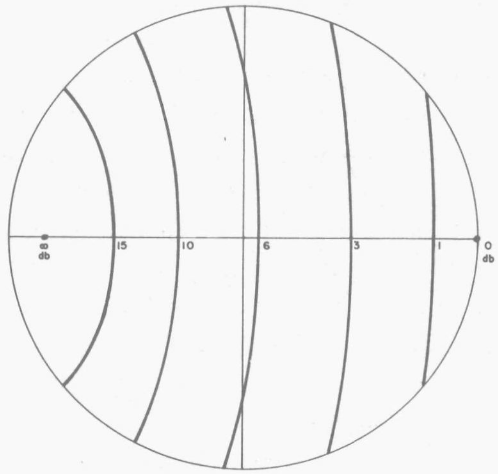


Fig. 16 Gamma Target: $\gamma = 15°$

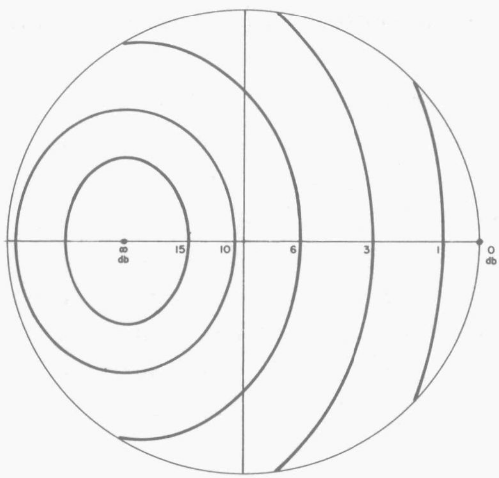


Fig. 17 Gamma Target: $\gamma = 30°$

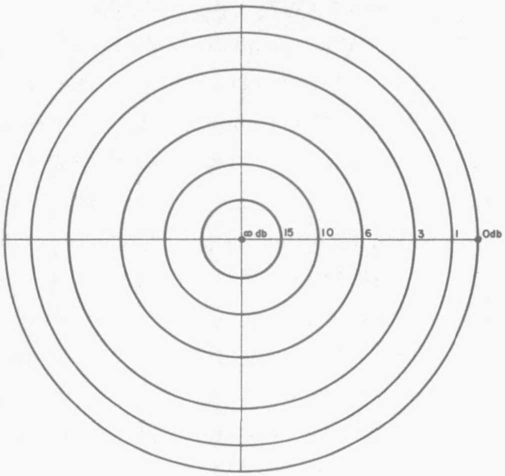


Fig. 18 Gamma Target: $\gamma = 45°$

20. Measurements of Radar Targets at Fixed Aspect Angles*

A radar which operated at K-band was installed on a movable truck for the purpose of performing measurements on targets at fixed aspects. The purpose of the measurements was to determine the amplitude return from a fixed target as a function of polarization of the radar transmitter. The aim of the program was to verify some of the theoretical predictions, such as the existence generally of two null-polarizations and one maximum polarization as part of the polarization sphere amplitude map. It was by no means certain that an actual measurement could be performed which would verify the predicted behavior due possibly to rapid fluctuations of target characteristics, or to effects of noise and multiple-bounce ray path interference.

The radar antenna was designed to produce arbitrary polarized waves, and the intensity of return was recorded either on a signal-strength indicator, or

*See Huynen, Thille and Thormahlen [14].

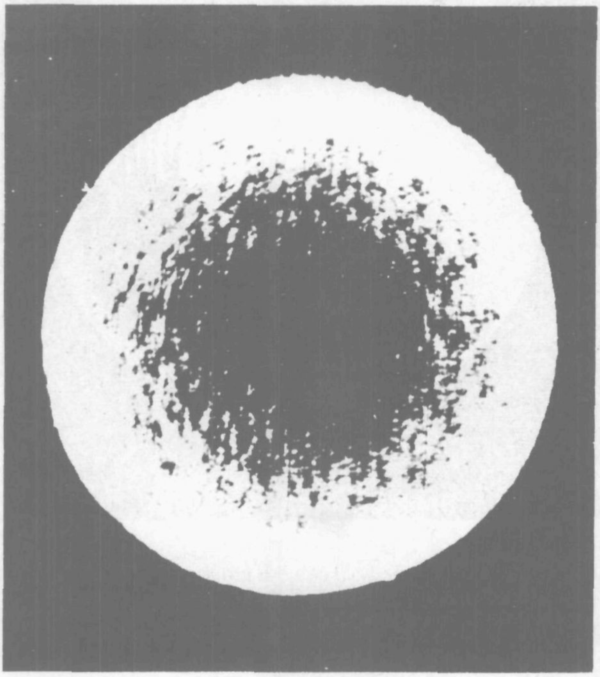
as a luminous dot on a circular scope, which simulated the polarization plane; i.e., each point on the circular scope represents a state of transmitted polarized wave. For each of the two different senses of polarization, a different polarization plane is necessary to complete the target map on the sphere.

Owing to the time lapse for completing a set of measurements, the target characteristics might have changed due to wind forces, movement of trees, movement of buoys on water surfaces, slight swaying of bridges, towers, and transmission cables, etc. Also the power output of the transmitter might change with time and the frequency might drift. Considerable effort went into the preparation of the equipment to assure a stable transmitted wave.

The K-band system was made to change polarizations automatically in such a fashion that the polarization chart was scanned completely starting with circular polarization at the center of the chart and progressing outward from the center in a spiral movement until the linear polarizations at the outside of the circular chart were obtained. The sense of polarization was then reversed and the spiraling movement proceeded inward to complete the total polarization coverage on the second chart for the opposite sense. The amplitude return intensity was modulated to the light intensity of the spiraling spot on the circular scope, and this was photographed on a photosensitive plate of the camera attached to the scope. It took one minute to complete cne spiral movement and thus two minutes to complete a full polarization map of a target at fixed aspect angle.

The following set of results was obtained by automatic scanning of polarization using a variable polarization antenna at K-band. Owing to the nonlinear behavior of the scope response characteristic, the variation of light intensity on the scope is not proportional to the amplitude intensity radar return. However, the position of nulls is clearly indicated on these photographs. Each point on the circular chart represents a state of transmitted polarization according to the geometry of the polarization chart, as was described previously. The center of the chart is circular polarization while the outside periphery represents the linear polarizations, the right-hand position being horizontal polarization and the left-hand side being vertical polarization.

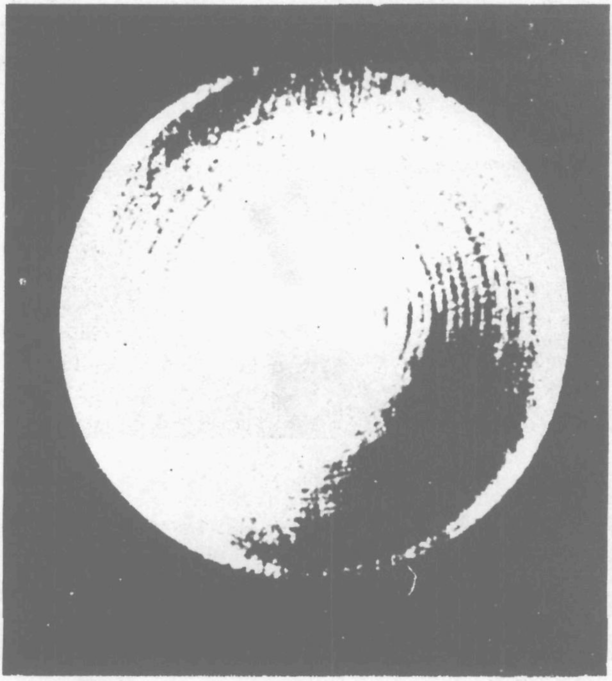
Figure 19 shows the result for an "isotropic" target which consisted of a stratus layer of clouds at about 2000 feet. The range was 1 mile; the frequency was K-band; the weather was windy and misty. The photograph shows the amplitude return characteristics for right-sensed polarizations. Notice the location of the null at circular polarization. The ragged appearance of the circular area around the deep null indicates a variation with time, as the cloud changed while the scanning operation proceeded. By different settings of the saturation level on the scope, the areas of nulls could be either expanded or narrowed according to requirement.



Subject: Cloud-stratus layer at about 2000 feet ceiling
Range: 1 miles Frequency band: K
Weather: Windy, misty Sense: RC

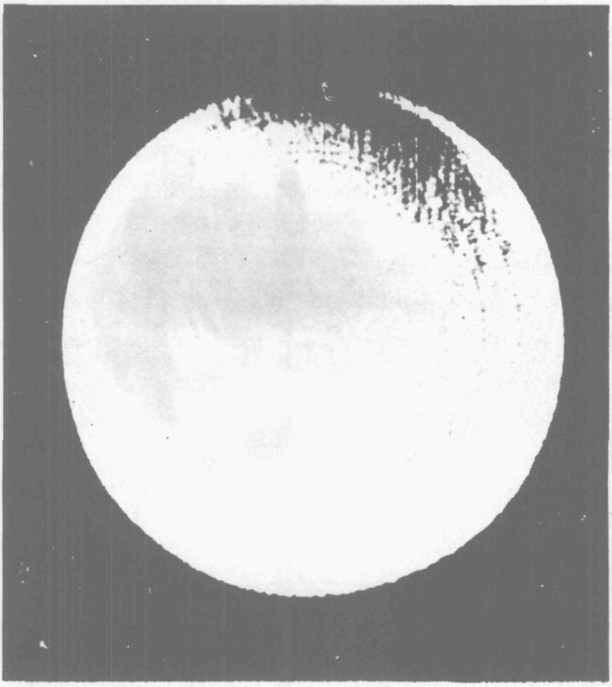
Fig. 19 Polarization Map for a Cloud

Figures 20 and 21 show the results obtained on a railroad bridge. Notice the two areas of nulls which are now displaced from the center. The first photograph shows the result for right-sensed polarization where one null is clearly visible, the other null being extended to the back half of the spherical representation, i.e., to the left-sensed polarizations.



Subject: Dumbarton Railroad Bridge
Range: 9 miles — Frequency Band: K
Weather: Light rain, windy — Sense: RC

Fig. 20 Polarization Map for a Bridge (RC)



Subject: Dumbarton Railroad Bridge
Range: 9 miles Frequency Band: K
Weather: Light rain, windy Sense: RC

Fig. 21 Polarization Map for a Bridge (LC)

21. Null-Locus Measurements on Symmetric Targets

The null-locus measurement technique was developed by the author in the early 1960's [47]. We review only some of the highlights here; for further details, see the previously published literature [22, 24]. This material also was incorporated in a recent Russian book on radar targets [39].

Symmetric targets are given by three independent parameters ($\tau_m = 0°$ and $\phi_m = 0°$ or $90°$). (See Sec. 13 and Fig. 11 for more details.) The two null-polarizations that determine the SM are no longer independent; in fact, one null-polarization determines the other. From a set of measurements, one plots through the use of the null-locator chart for each aspect angle the characteristic null-polarization of the target. Thus, a null-locus describes the target scattering behavior with changing aspect. Figure 22 shows a typical locus of null-polarizations for targets with horizontal plane of symmetry as a function of aspect angle. At fixed aspect angle, one null-polarization is assumed to be located in the upper half of the circular chart. Because of the symmetry, the corresponding second null-polarization will occur as the mirror image about the H-V axis. Thus, as a function of aspect angle, the first null-polarization locus has the corresponding second null-polarization locus as its mirror image about the H-V axes. These two null-loci are shown in Fig. 22. Observe that the polarizations have opposite senses. Details of the procedure follow.

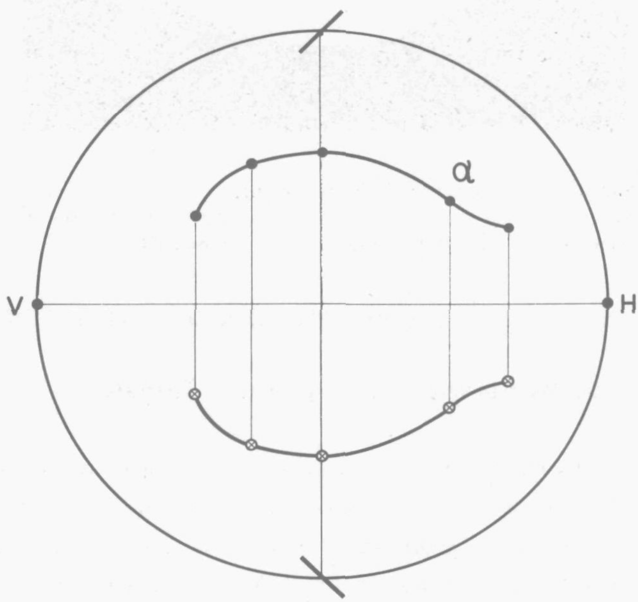


Fig. 22 Null-Locus for Symmetric Target

A target with horizontal symmetry was placed upon a foam tower and rotated along the vertical axis (Fig. 23). Four patterns of different polarizations were obtained and recorded on transparent paper. The four polarizations used were horizontal, vertical, linear at 45°, and right circular. The patterns as a function of rotation angle were superimposed upon each other, and decibel differences from three different patterns relative to the local maximum were obtained.

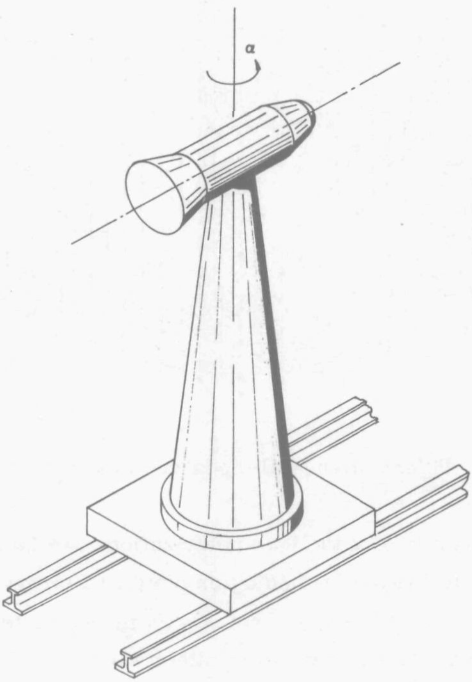


Fig. 23 Target Laboratory Position

A polarization null-locator chart (Fig. 24) was constructed using (18.18) and (18.19) such that knowledge of the decibel differences establishes the position of the characteristic target nulls on the circular polarization chart. The

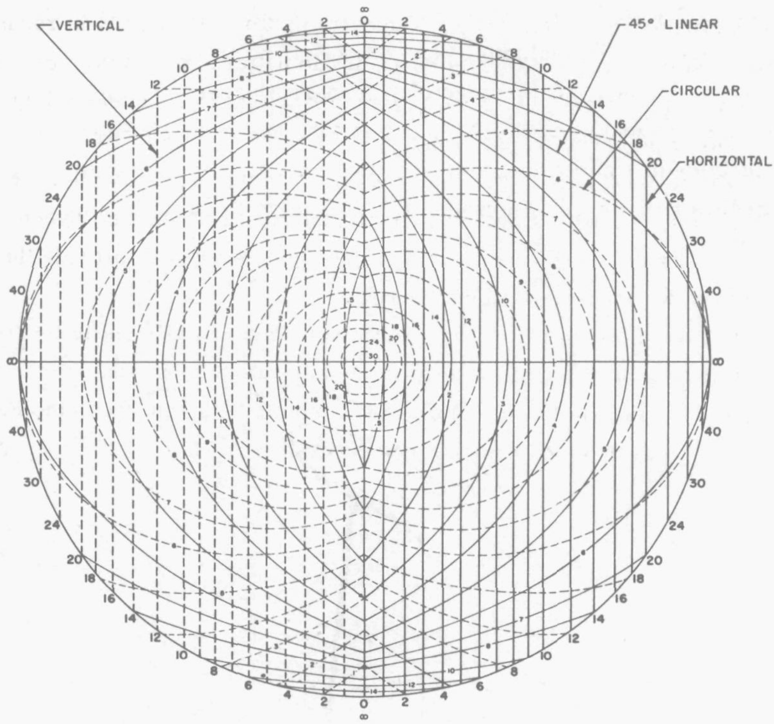


Fig. 24 Polarization Null-Locator Chart (45° Linear)

decibel-level lines for each of the four polarizations are indicated on the null-locator chart. The technique for finding or plotting a target polarization-null consists simply of reading from the four superimposed patterns the decibel differences relative to a local maximum (either H or V). The intersection of any two curves corresponding to the decibel difference values for the corresponding pattern gives the position of the right-sensed null on the polarization chart. For symmetric targets, the left-sensed null is then uniquely determined.

Notice that each point (polarization null) on the chart is obtained from intersections of three decibel-difference curves. This means that the method is overdetermined, since a point can be obtained from the intersection of two decibel-difference curves alone. The extra pattern gives built-in redundancy for the system of measurements; i.e., if one pattern disappears below the noise level, no information is lost, since the system guarantees a high return for the other polarization patterns.

In the Russian book [39], the redundancy in the original null-locator chart is removed by separation into two charts; the first and second Huynen charts. (See Figs. 25 and 26.) It is clear that the separated charts provide the same function as the combined original chart.

The four-polarization method works very fast and efficiently for hand-plotting target information obtained in the laboratory or on the pattern range. Observe that the null-locator chart is symmetric with respect to the horizontal axis. This makes an ambiguity check necessary.

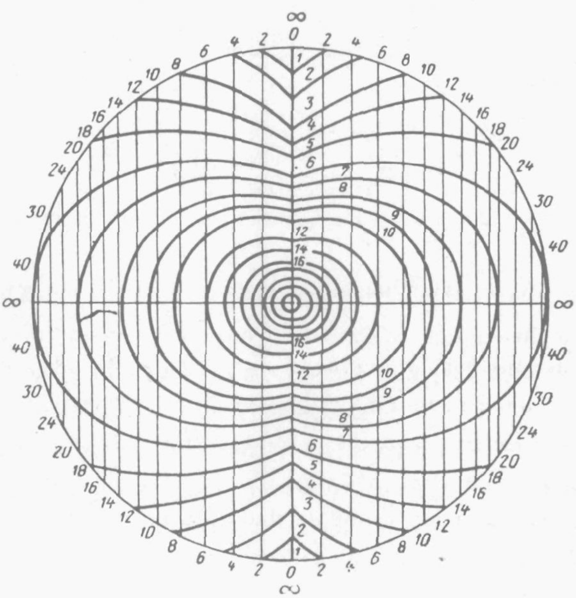


РИС. 9.7. Первая диаграмма Хьюнина.

Fig. 25 First Huynen Diagram (in Russian)

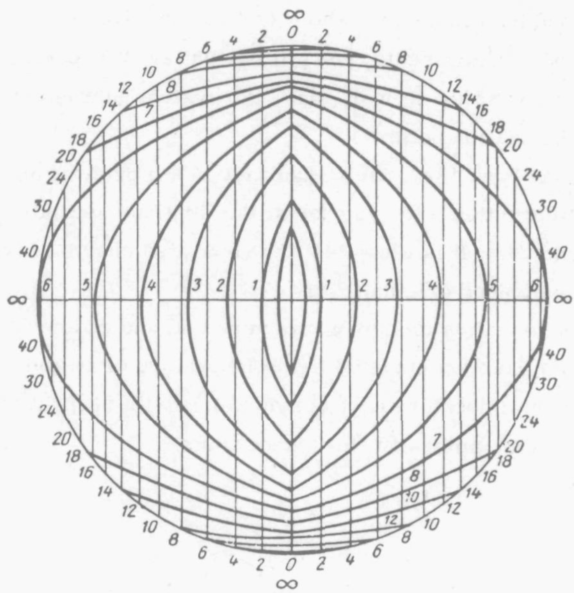


РИС. 9.8. Вторая диаграмма Хьюнина.

Fig. 26 Second Huynen Diagram (in Russian)

A new four-polarization system has been worked out which uses an elliptical polarization instead of the 45° linear one, and which eliminates the ambiguity. Figure 27 shows the null-locator chart designed for the improved method. Figure 28 actually illustrates the null determination for this case. The fourth pattern in the new case with elliptical polarization serves two purposes: (1) It resolves the ambiguity between two possible null-polarizations, and 2) it supplies built-in redundancy for the system. Thus, this technique supplies a minimum set of compatible data for complete radar target cross-section determination.

Experimental results are shown for two types of targets. The first measurements show the results for a pair of plates crossed at 90° with respect to each other. The intersecting axis was kept vertical during the measurement

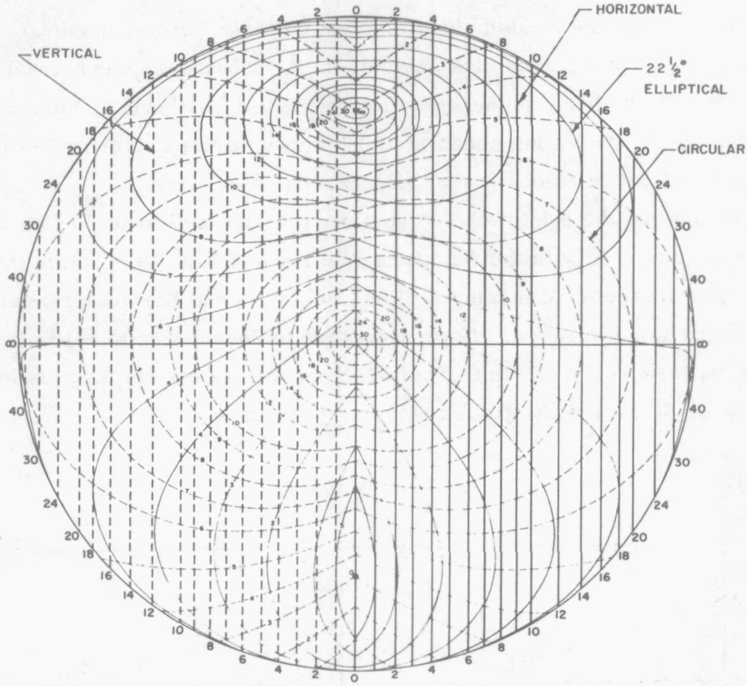


Fig. 27 Polarization Null-Locator Chart (Elliptical)

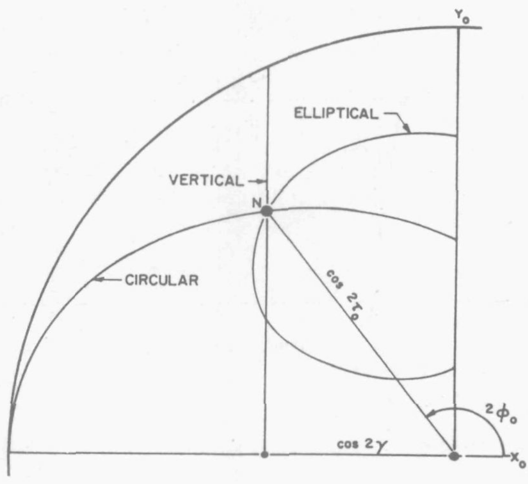


Fig. 28 Determination of Null

and the dihedral was rotated about the vertical axis with the angle α. Figure 29 shows the four patterns thus obtained separately and superimposed upon each other. The resulting null-plot for this target is shown in Fig. 30. We obtain a highly interesting question mark curve, formed by the null-locus for right-handed polarizations on the polarization chart.

One notices the remarkable regularity that the null-locus for the dihedral exhibits, compared with the fluctuating patterns of Fig. 29. Notice that on these patterns the local maximum is attained either at horizontal polarization or at vertical polarization. At no aspect angle does either the circular polarization pattern or the 45° linear polarization pattern exceed the maximum level. This result is characteristic for symmetric targets.

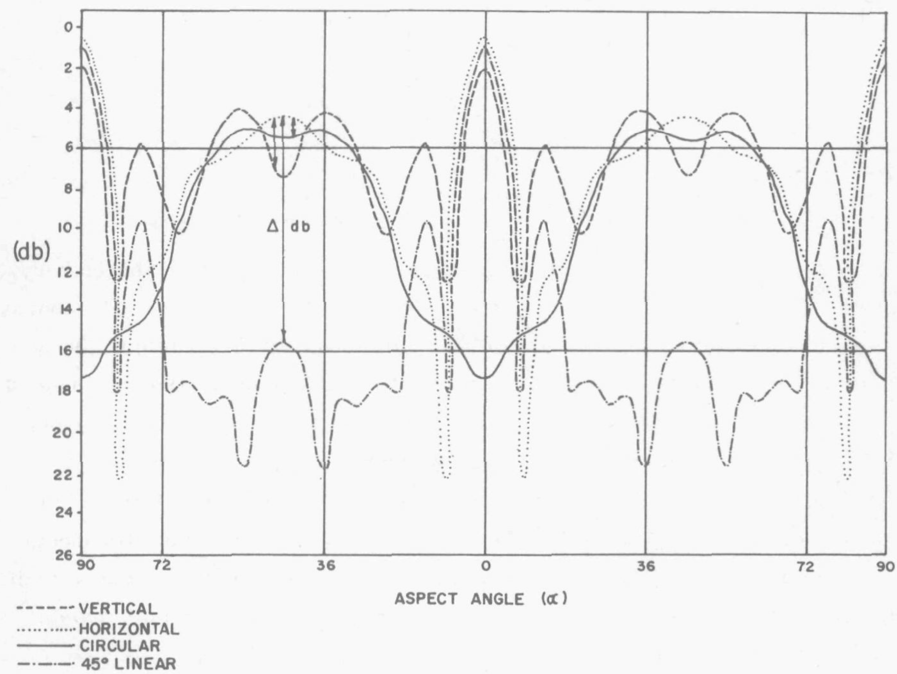


Fig. 29 Patterns for Dihedral

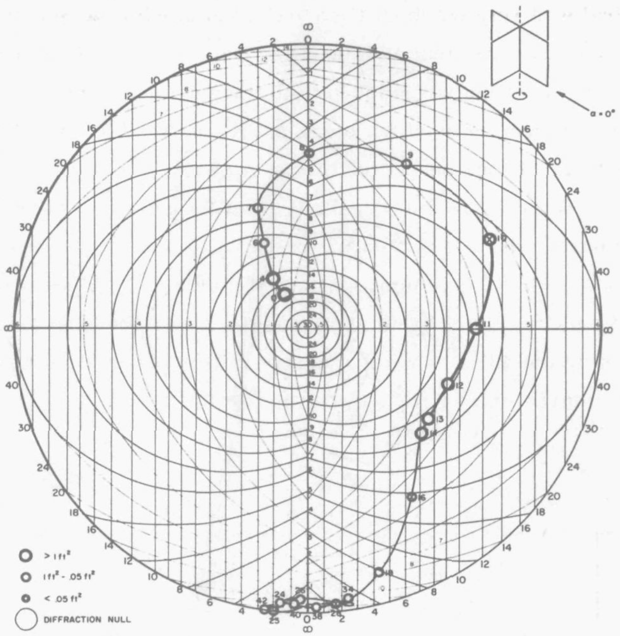


Fig. 30 Null Locus for Dihedral

The second experimental result was obtained from a convex-shaped body. Figure 31 shows the null-locus for the right-sensed polarizations. The convex-shaped object can be expected to exhibit a null-locus characteristic which is distributed around the center of the chart, at circular polarization, because of specular reflection. It is interesting to note, however, that deviations from the center position occur even for relatively strong signals. This indicates that for convex-shaped objects of a few wavelengths, the specular reflection considered as a local flat plate is no longer a valid assumption at all aspects.

We notice a peculiar phenomenon at $\alpha = 64°$: the maximum return itself disappears in the noise. At this angle, a so-called "diffraction null" exists for the target. At this angle no energy is returned to the radar; i.e., the target is "invisible" to the radar for any type of polarization.

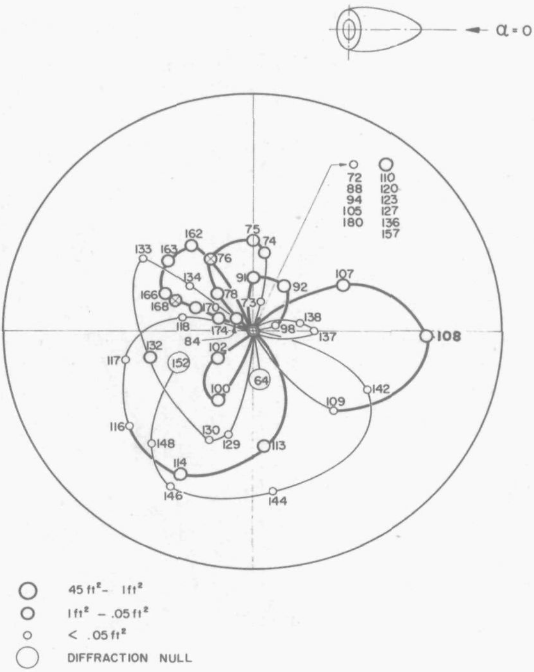


Fig. 31 Null Locus for Ogive

Figure 32 shows the null locus characteristic for a corner reflector. The behavior shown is a combination of the previous two cases. At $\alpha = 0$, the direction of incidence is normal to an interior face of the corner reflector. The null-behavior of the target will be mostly circular polarization. The wide circular loop starting from the center, and returning to it, shows the effect of specular scattering. At $\alpha = 35°$, the incident illumination direction becomes normal to the large triangular frontal facet of the corner reflector (which is the effective aperture for the three-bounce reflection). Again the characteristic null polarization is circular. The inner loop of the null-locus shows the specular character, and the smaller deviation from the center indicates the larger aperture. Finally, at $\alpha = 90°$ the wave is incident to the wedge (dihedral), which is made up of two interior faces of the corner reflector. The latter produces the strongly depolarizing two-bounce scattering which is shown by the tail of the null-locus.

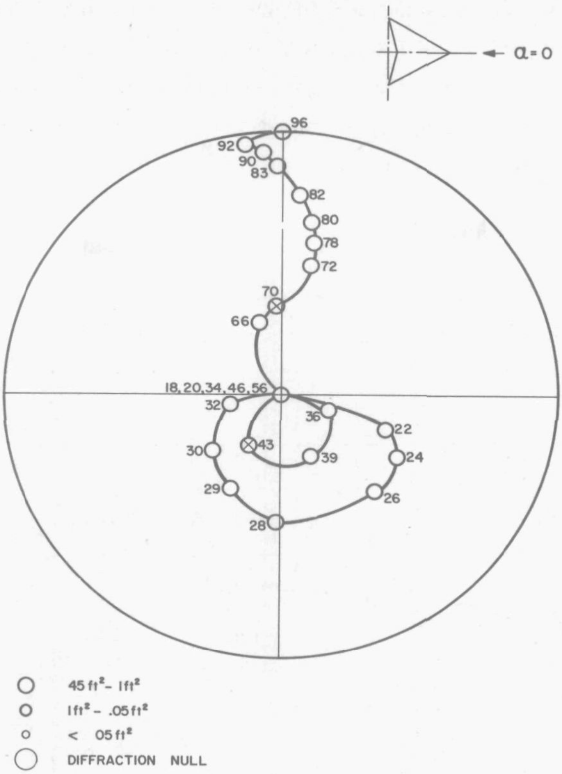


Fig. 32 Null Locus for Corner Reflector

22. Measurements on Nonsymmetric Targets

The null-plotting method, or other 2-dimensional display techniques [18], rapidly lose their attractiveness if more than two independent parameters have to be displayed on one map. A general target at fixed aspect is given by five parameters, and hence other methods of presentation have to be found. The simplest is of course to compute each parameter as a function of aspect angle, from a basic set of measured data, and to display each parameter separately.

Using a five linear polarization RCS measurement scheme, with orientations 0°, 30°, 60°, 90°, and 120°, it was possible to compute the helicity parameter $|\tau_m|$ and the relative target orientation ϕ_m for nonsymmetric targets. (For symmetric targets, we found: $\tau_m = 0$ and $\phi_m = 0°$ or $90°$). Figures 33 and 34 show the results for $4\phi_m$ and $|\sin 2\tau_m|$ as a function of aspect angle.

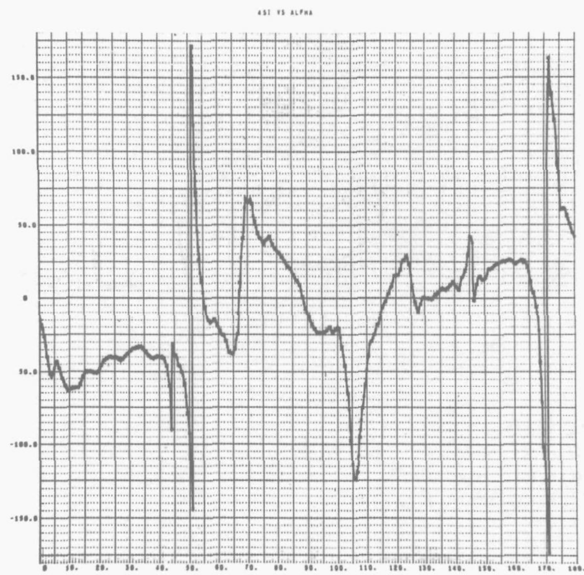


Fig. 33 Maximum Polarization Orientation Angle for Non-Symmetric Target

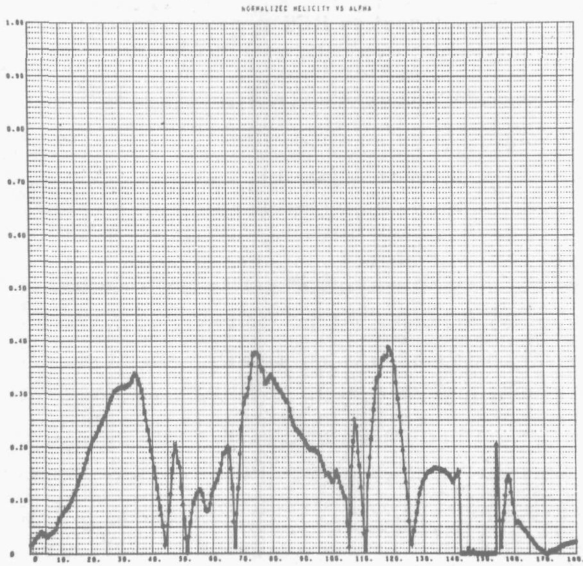


Fig. 34 Helicity Angle for Non-Symmetric Target

5 STATISTICALLY INDEPENDENT (MUTUALLY INCOHERENT) TARGETS

23. Introduction

The concept of "distributed target" arises from the fact that not all radar targets are stationary or fixed, but instead change with time. In fact, most natural targets vary with time to some degree owing to the flow of wind and stresses generated by temperature or pressure gradients. We may think of the motion of water surfaces, vegetated lands, and snow-covered grounds, not to mention obvious examples such as flocks of birds, clouds of water droplets, dust particles, and chaff. Aside from the natural movements of the target, the radar itself may be airborne or in space, moving with respect to the target and illuminating in time the different parts of an extended volume or surface.

The radar will receive in these cases the time-averaged samples of scattering from a set of different single targets. The set of single targets from which samples are obtained is called a "distributed radar target."

An important type of distributed target is that of an ensemble of targets generated by random processes. Each single target member of the ensemble is then a realization or sample event of the underlying random processes. In this manner an extended rough surface may be defined by the height profile $\zeta(x, y)$ as a function of position (x, y) on the plane. Since the height profile mainly affects the phase distribution of the components contributing to the surface radar scattering, studies related to this type of random process are often called scalar scattering theories. Similarly, the variation of direction of the surface normal may be considered a random process from which the expected average electromagnetic scattering from a rough surface is determined.

In addition to variations of local surface position and of local surface normal, which are geometric changes of object shape, one might consider variations in material properties due to such factors as local inhomogeneties and turbulence which create random processes affecting electrical surface reflection or volumetric scattering, which determine a type of distributed target.

The scattered return from a distributed radar target which is illuminated by a "monochromatic" plane wave with fixed frequency and polarization will in general be of the form of a partially polarized (pp) plane wave. This implies that the wave no longer has the coherent, monochromatic, completely polarized (cp) shape of an elliptically polarized (ep) wave; instead, it also includes incoherent randomly polarized components of polarization. The state of a pp wave is given by the so-called coherency matrix, the elements of which consist of the complex correlations $\langle E_x E_x^* \rangle$, $\langle E_y E_y^* \rangle$, $\langle E_x E_y^* \rangle$ of the electric $\underline{E}$ field components.

The coherency matrix is thus specified by four real numbers, representing average powers, which also determine the components of an equivalent stokes vector representation of the pp wave. It will be most convenient to use the stokes vector (representing average power) to describe electromagnetic scattering from distributed targets, whereas for single targets the scattering was most conveniently given by electric $\underline{E}$ field components. We showed that for single targets and cp waves there is in fact a one-to-one correspondence between the two methods of representation (if we exclude from consideration the absolute phase of the cp wave). However, for pp waves there is no one-to-one correspondence between $\underline{E}$ field and stokes vector representation owing to the randomly polarized components of the pp wave.

The properties of pp return scattering from distributed radar targets may be used to analyze these targets independently of the polarization of incoming illumination. For single targets the scattering matrix T served to define the target's far field backscattering of electric $\underline{E}$ field components for all incoming illuminations. Similarly, the pp return from distributed targets is determined by the stokes reflection matrix R, which transforms the stokes vector of the cp illumination into the stokes vector of the pp return. For single radar targets there is a one-to-one correspondence between matrix T and M (for single targets, we write R = M), as we found before. However, no such correspondence between T and R matrices exists for distributed radar targets.

This is easily demonstrated by the following argument. Both the 2×2 complex matrix T and 4×4 real matrix R are symmetric due to reciprocity. Hence, five real independent parameters and the absolute phase determine T, whereas it can be shown that nine real independent parameters determine R. Now, R is independent of the absolute phase since it is determined by average powers, and hence matrix R has 4 more degrees of freedom than T. Since the class of distributed radar targets is thus much larger than that of single targets, it is in general not possible to find an equivalent single target T matrix (field) representation for a distributed target, which is given by matrix R (based on average power).

Is it possible it may now be asked, to decompose a distributed radar target R into an averaged single target M and some type of target noise N? One of the main results in subsequent sections is the proof that this is indeed the case. The nature of the distributed target noise N is specified, and it is shown that N can be written as $N = M_{N1} + M_{N2}$ where M_{N1} and M_{N2} are "noise targets" which have equivalent scattering matrices T_{N1} and T_{N2}. It is interesting to note that the noise target N is "statistically independent" of the average single target M, which may be interpreted physically as follows: the return scattering components of N may be considered separately and independently of the scattering components of M, which implies that they all have positive powers and physically realizable stokes vector representations. This result could not be guaranteed if we simply and arbitrarily separated some M-target components from the total return scattering. The proof of physical realizability of the decomposition theorem constitutes one of the major results in this part of the work.

Another interesting and important result in this framework is that of a precise definition of "statistically independent targets" presented in a form that is independent of polarization of target illumination.

The general theory of distributed targets is applied to some special cases of practical significance. For radar scattering from rough surfaces R, the general decomposition theorem $R = M + N$ separates the scattered return into components which may be interpreted as due to a mean surface M and components associated with target scattering noise N.

For large extended surfaces, a Kirchhoff approximation method developed by Fung [3, 4, 5], can be applied for computing the return scattering. Here the decomposition has an interesting physical explanation: the mean surface M is associated for the most part with averages based upon the random phase component (due to random surface height distribution) of the scattered elemental returns. This has traditionally been an important field of investigation, the results of which we have labeled as scalar theories. The target noise components N, are due in general primarily to the distribution of orientation angle ψ of the local normal on the surface.

24. Partially Polarized Plane Waves

In Sec. 3 we discussed properties of monochromatic plane, elliptically polarized (ep) electromagnetic waves, such as might be observed in the far field scattering from a single stationary (fixed) radar target. In general, a different ep plane wave is produced by the radar transmitter, to illuminate the target. If both transmitter and target remain fixed, no change of the ep wave at a fixed receiver station in the far field of the target is observed. The far field scattering in that case is determined completely by the complex E_x and E_y components of the time-harmonic $\underline{E}$ field, which has amplitudes and phases in orthogonal x- and y-directions transverse to the fixed direction of illumination.

However, if transmitter, receiver, or target changes position, a time-varying ep wave with components $E_x(t)$, $E_y(t)$ is observed. Of particular interest are the time variations of the fields that can be attributed to random processes, underlying changes of the target, such as those for a sea state, or weather clouds, or in the case of airborne radar, owing to the fact that different parts of an extended terrain surface are illuminated in time. For uniformly random variations in (absolute) phase, $\langle E_x(t) \rangle$ and $\langle E_y(t) \rangle$ are zero; hence, it is customary (Born and Wolf [37, p. 545]) to define a time-varying, so-called partially polarized (pp) plane wave by the coherency matrix of the complex components $E_x(t)$ and $E_y(t)$ (see also Papas [57] for a simple introduction):

$$C_{xy} = \begin{bmatrix} \langle E_x E_x^* \rangle & \langle E_x E_y^* \rangle \\ \langle E_y E_x^* \rangle & \langle E_y E_y^* \rangle \end{bmatrix} \tag{24.1}$$

The time average < > is defined as usual:

$$<x(t)> = \lim_{T \to \infty} \frac{1}{T} \int_0^T x(t)\, dt \tag{24.2}$$

It is seen that C_{xy} is a hermetian matrix, defined by four real quantities. Using the notation $\underline{E}(a, \phi, \tau)$ of an ep wave in terms of its magnitude a, orientation ϕ, and ellipticity τ (see Sec. 3), the E_x and E_y components are easily found:

$$\begin{aligned} E_x &= a (\cos\phi \cos\tau - i \sin\phi \sin\tau) e^{i\alpha} \\ E_y &= a (\sin\phi \cos\tau + i \cos\phi \sin\tau) e^{i\alpha} \end{aligned} \tag{24.3}$$

The absolute phase α of the ep wave has no effect in the subsequent discussions of pp waves. From (24.3) the elements of the coherency matrix are found:

$$\langle E_x E_x^* \rangle = \left\langle \frac{a^2}{2} (1 + \cos 2\phi \cos 2\tau) \right\rangle \tag{24.4}$$

$$\langle E_y E_y^* \rangle = \left\langle \frac{a^2}{2} (1 - \cos 2\phi \cos 2\tau) \right\rangle \tag{24.5}$$

$$\langle E_x E_y^* \rangle = \left\langle \frac{a^2}{2} (\sin 2\phi \cos 2\tau - i \sin 2\tau) \right\rangle \tag{24.6}$$

This result shows that the four real quantities that define the pp wave are the components of the stokes vector $g = (g_0, g_1, g_2, g_3)$, where

$$
\begin{aligned}
g_0 &= \langle a^2 \rangle \\
g_1 &= \langle a^2 \sin 2\tau \rangle \\
g_2 &= \langle a^2 \cos 2\phi \cos 2\tau \rangle \\
g_3 &= \langle a^2 \sin 2\phi \cos 2\tau \rangle
\end{aligned}
\tag{24.7}
$$

In Sec. 6 we encountered the stokes vector $g(\underline{a})$ as an equivalent (except for absolute phase) representation of the ep electric field $\underline{E}$; however, no such correspondence between field and stokes vector representations exists for a pp wave. In fact, in many cases $\langle \underline{E} \rangle$ can be made zero by random fluctuations of absolute phase α which has no equivalent to the stokes vector representation.

For a pp wave given by stokes vector g, the following rule applies:

$$g_0^2 \geq g_1^2 + g_2^2 + g_3^2 \tag{24.8}$$

To prove this, we write $g = (g_0, \underline{g})$ where $\underline{g}$ is the vector (g_1, g_2, g_3). Hence, we have to show that $g_0^2 \geq \underline{g}^2 = \underline{g} \cdot \underline{g}$.

Let

$$g_0^2 = \lim_{T\to\infty} \left(\frac{1}{T} \int_0^T g_0(t)\, dt \right)^2 = \lim_{T\to\infty} \frac{1}{T^2} \int_0^T \int_0^T g_0(\tau)\, g_0(\sigma)\, d\tau\, d\sigma \tag{24.9}$$

$$|\underline{g}|^2 = \lim_{T\to\infty} \frac{1}{T}\int_0^T \underline{g}(t)\,dt \bullet \frac{1}{T}\int_0^T \underline{g}(t)\,dt$$

$$= \lim_{T\to\infty} \frac{1}{T^2}\int_0^T\int_0^T \underline{g}(\tau) \bullet \underline{g}(\sigma)\,d\tau\,d\sigma$$

$$= \lim_{T\to\infty} \frac{1}{T^2}\int_0^T\int_0^T |\underline{g}(\tau)|\;|\underline{g}(\sigma)|\cos[\underline{g}(\tau)\;,\;\underline{g}(\sigma)]\,d\tau\,d\sigma \tag{24.10}$$

Since for each time, in the integrand $g(t)$ represents an ep wave for which $g_0(t) = |\underline{g}(t)| = \left[g_1^2(t) + g_2^2(t) + g_3^2(t)\right]^{1/2}$ we have for (24.10):

$$|\underline{g}|^2 = \lim_{T\to\infty} \frac{1}{T^2}\int_0^T\int_0^T g_0(\tau)\,g_0(\sigma)\cos\delta\,d\tau\,d\sigma \tag{24.11}$$

Comparison of (24.9) with (24.11) shows that the result holds true for each finite integration time T. Hence, it follows from definition (24.2) that the results also must hold in the limit.

Condition (24.8) is often called "the condition for physical realizability" of a pp wave. This condition leads us to a physical decomposition $g = g_e + g_n$ of the pp wave $g = (g_0, g_1, g_2, g_3)$ in terms of a completely polarized (cp) wave $g_e = (|\underline{g}|, g_1, g_2, g_3)$ where $|\underline{g}| = \left(g_1^2 + g_2^2 + g_3^2\right)^{1/2}$ and a completely unpolarized part $g_n = (g_{on}, 0, 0, 0)$, such that $g_o = |\underline{g}| + g_{on}$ gives the total power of the pp wave g as the sum of its component powers.

The unpolarized part g_n has no preferred orientation or ellipticity; it represents uniformly random polarized scattering with reference to orientation angle ϕ and ellipticity angle τ. We associate it with completely unpolarized polarization noise. Obviously the decomposition $g = g_e + g_n$ is unique and physically realizable. These concepts also will play an important role, as we will demonstrate later in the discussion of scattering from distributed radar targets; however, for that case the decomposition theorem takes a more complex form.

It is easy to show that the sum of two stokes vectors is again a stokes vector. Let $g = g^A + g^B$ where $g^A = (g_0^A, \underline{g}^A)$ and $g^B = (g_0^B, \underline{g}^B)$ are given stokes vectors such that $g_0^A \geq |\underline{g}^A|$ and $g_0^B \geq |\underline{g}^B|$. Then, if $g = (g_0, \underline{g})$, we have:

$$\begin{aligned}
g_0^2 - \left(g_1^2 + g_2^2 + g_3^2\right) &= g_0^2 - (\underline{g} \bullet \underline{g}) \\
&= \left(g_0^A + g_0^B\right)^2 - (\underline{g}^A + \underline{g}^B) \bullet (\underline{g}^A + \underline{g}^B) \\
&= \left({g_0^A}^2 - \underline{g}^A \bullet \underline{g}^A\right) + \left({g_0^B}^2 - \underline{g}^B \bullet \underline{g}^B\right) \\
&\quad + 2g_0^A g_0^B - 2\underline{g}^A \bullet \underline{g}^B \\
&= \left({g_0^A}^2 - |\underline{g}^A|^2\right) + \left({g_0^B}^2 - |\underline{g}^B|^2\right) \\
&\quad + 2g_0^A g_0^B - 2|\underline{g}^A||\underline{g}^B| \cos(\underline{g}^A, \underline{g}^B) \geq 0
\end{aligned} \tag{24.12}$$

Equality in (24.12) holds only if g^A and g^B are cp stokes vectors which are parallel.

25. Statistically Independent Voltages

We consider in this section scattering from a set of independent radar targets. The precise definition of this concept will be our aim in this and the following section.

With each target T_i, we associate a voltage $V_i = T_i \underline{a} \cdot \underline{b}$ where $\underline{a}$ and $\underline{b}$ indicate the fixed radar transmitter and receiver antennas, and T_i is the target scattering matrix discussed in Sec. 9. The total voltage received from the set of targets is

$$V = V_0 + V_1 + V_2 + \ldots + V_n \tag{25.1}$$

The set of voltages is called statistically independent if the targets are statistically independent.

We start our discussion of statistically independent targets with some intuitive notions. Since the targets are independent of each other, their relative positions in the illumination direction are of no consequence, at least for "small" displacements of the order of a wavelength. Hence, it follows that the average return from statistically independent targets must be independent of the "absolute phase" (see Sec. 10 for definition) of the individual targets of the set. A simple and practically important example is to consider the absolute phase α_j of statistically independent voltages $V_j = A_j e^{i\alpha_j}$, $j = 1, 2, \ldots$ as a uniformly distributed random variable; the phase α_o of V_o may be arbitrary. Hence:

$$V = A_0 e^{i\alpha_0} + A_1 e^{i\alpha_1} + A_2 e^{i\alpha_2} + \cdots = \sum_{j=o}^{n} A_j e^{i\alpha_j} \tag{25.2}$$

and

$$P = VV^* = \sum_{j=o}^{n} |A_j|^2 + \sum_{\substack{o \\ j \neq k}}^{n}\sum A_j A_k^* e^{i(\alpha_j - \alpha_k)} \tag{25.3}$$

The average return is found simply:

$$\langle P \rangle = \sum_{j=o}^{n} |V_j|^2 = \sum_{j=o}^{n} P_j = P_0 + P_1 + P_2 + \cdots + P_n \tag{25.4}$$

Because of the random phases, the cross product terms in (25. 3) vanish in the average for a sum of statistically independent voltages. We find thus that the total average power is obtained by adding the individual powers of the component voltages. Hence, power additivity is a necessary condition for statistical independence of targets and voltages. We can make it sufficient by defining the statistical independence of targets and voltages as equivalent to having the property of power additivity.

It should be kept in mind that the foregoing result obtained by introducing voltages with uniformly random phases by no means exhausts the class of statistically independent voltages. Other random processes, for instance, based on variation of orientation angle ψ_j of individual target components T_j, may be a cause for statistically independent voltages. Because of the foregoing properties, statistically independent targets are often also called "phase independent." We now return to equation (25. 1) to find the general condition for phase independence:

$$\langle P \rangle = \langle VV^* \rangle = \sum_{j=o}^{n} \left\langle |V_j|^2 \right\rangle + \sum_{\substack{o \\ j \neq k}}^{n} \sum_{o}^{n} \left\langle V_j V_k^* \right\rangle \tag{25.5}$$

Hence,

$$\langle P \rangle = \sum_{j=o}^{n} \langle P_j \rangle$$

if for $j, k = 0, 1, 2, 3, \cdots n$ and $j \neq k$:

$$R_e \langle V_j V_k^* \rangle = 0 \tag{25.6}$$

This is the general condition for phase-independent voltages. For phase-independent targets, we may substitute $V_j = T_j \underline{a} \cdot \underline{b}$ into (25.6) to give:

$$R_e \left\langle (T_j \underline{a} \bullet \underline{b})(T_k^* \underline{a}^* \bullet \underline{b}^*) \right\rangle = 0 \qquad (25.7)$$

This condition is formally correct, but for statistically independent targets we prefer a statement which is dependent on T_j and T_k alone and which is independent of antenna polarizations $\underline{a}$ and $\underline{b}$. In the next section we show how this can be accomplished.

The property of power additivity (component powers are additive) for statistically independent targets has important consequences. First, it shows that target scattering is described more conveniently by the stokes reflection matrix R, which determines average backscattered power, rather than by the scattering matrix T which describes the target by instantaneous fields. The general decomposition theorem, to be proved later, which is applicable to R allows us to reconstruct corresponding independent targets with "effective" scattering matrices. The process of determining effective voltage from average power is well known from ordinary ac line voltage; we define the line voltage by the effective value derived from the averaged power generated in a 1-ohm resistor. This nomenclature carries over to signal voltages and corresponding target matrices.

To illustrate these concepts, let $<P>$ be the total average power return from a target, and suppose $<P>$ to be decomposed into a sum of $n + 1$ component powers:

$$<P> = <P_o> + <P_1> + \cdots + <P_n> \qquad (25.8)$$

Then, with each component power we associate an effective voltage $\overline{V}_i$ for which $<P_i> = |\overline{V}_i|^2$. Hence, we may write:

$$\overline{V} = \overline{V}_0 + \overline{V}_1 + \overline{V}_2 + \cdots + \overline{V}_n \qquad (25.9)$$

where $<P> = |\overline{V}|^2$ is a decomposition of the total effective voltage $\overline{V}$ into a sum of phase-independent (mutually incoherent) voltages $\overline{V}_i$. This formally correct approach will become physically meaningful, if to each independent component voltage can be assigned a physical significance.

For applications, we refer to Sec. 44, 45, and 46.

26. Statistically Independent Fields and Targets; Stokes Correlation Matrix

We found in Sec. 25 that two time-varying voltages $V_1(t)$ and $V_2(t)$ were called statistically independent if $R_e <V_1 V_2^*> = 0$. This condition guaranteed that the sum voltage $V = V_1 + V_2$ had average power equal to the sum of individual component powers. This concept is easily extended to fields and targets.

Consider two fields $\underline{E}_1(t)$ and $\underline{E}_2(t)$, the amplitudes and phases of which vary randomly with time, and two arbitrary fixed angennas $\underline{a}_1$ and $\underline{a}_2$ which are used to probe each field separately. The voltages registered at the antenna terminals will be $V_1(t) = \underline{E}_1(t) \bullet \underline{a}_{1R}^*$ and $V_2(t) = \underline{E}_2(t) \bullet \underline{a}_{2R}^*$. We now use the above property for statistically independent voltages to determine the statistical independence of fields. The fields $\underline{E}_1(t)$ and $\underline{E}_2(t)$ are called statistically independent if $R_e <V_1 V_2^*> = 0$. Now, in Sec. 7 it was shown that if $g = g[\underline{E}_1(t), \underline{E}_2^*(t)]$ and $h = h(\underline{a}_{2R}, \underline{a}_{1R}^*)$ are mixed stokes vectors, then:

$$R_e \left\langle V_1 V_2^* \right\rangle = \frac{1}{2} R_e \left\langle g\left[\underline{E}_1(t) \, , \, \underline{E}_2^*(t)\right] \right\rangle \bullet h\left(\underline{a}_{2R} \, , \, \underline{a}_{1R}^*\right) \qquad (26.1)$$

and since antennas $\underline{a}_1$ and $\underline{a}_2$ are arbitrary, $R_e <V_1 V_2^*> = 0$ only if

$$R_e \left\langle g\left[\underline{E}_1(t) \, , \, \underline{E}_2^*(t)\right] \right\rangle = 0 \qquad (26.2)$$

Hence, two fields, $\underline{E}_1(t)$ and $\underline{E}_2(t)$, are called statistically independent if the real part of the time-averaged components of the corresponding mixed stokes vector are zero.

The above development may be extended to distributed radar targets. Consider the fields $\underline{E}_1(t)$ and $\underline{E}_2(t)$ due to radar scattering from time-varying targets with scattering matrixes $T_1(t)$ and $T_2(t)$. If the transmitter and receiver antennas are $\underline{a}$ and $\underline{b}$ as usual, the voltages received from each target separately are $V_1 = T_1 \underline{a} \bullet \underline{b}$ and $V_2 = T_2 \underline{a} \bullet \underline{b}$. These voltages are statistically independent if $R_e < V_1 V_2^* > = 0$. This criterion is used to define statistical independence of target T_1 and T_2.

We wish to determine conditions on T_1 and T_2 which define statistical independence independently of antennas $\underline{a}$ and $\underline{b}$ which are used to probe the targets. To this goal, we define the mixed stokes vector $s(T_1 \underline{a}\ ,\ T_2^* \underline{a}^*) = (s_0, s_1, s_2, s_3)$ and $h(\underline{b}) = (h_0, h_1, h_2, h_3)$ the stokes vector of $\underline{b}$ as usual. The product role of Sec. 7 is now applied to voltages $V_1 = (T_1 \underline{a} \bullet \underline{b})$ and $V_2^* = (T_2^* \underline{a}^* \bullet \underline{b}^*)$:

$$V_1 V_2^* = \frac{1}{2}(s_0 h_0 - s_1 h_1 + s_2 h_2 + s_3 h_3) \tag{26.3}$$

where

$$\begin{aligned}
\frac{1}{2} s_0 &= \frac{1}{2} T_1 \underline{a} \bullet T_2^* \underline{a}^* = \frac{1}{2} T_2^* T_1 \underline{a} \bullet \underline{a}^* = W_0 \underline{a} \bullet \underline{a}^* \\
-\frac{1}{2} s_1 &= -\frac{i}{2} \mathbf{J}\, T_1 \underline{a} \bullet T_2^* \underline{a}^* = -\frac{i}{2} T_2^* \mathbf{J}\, T_1 \underline{a} \bullet \underline{a}^* = W_1 \underline{a} \bullet \underline{a}^* \\
\frac{1}{2} s_2 &= \frac{i}{2} \mathbf{L}\, T_1 \underline{a} \bullet T_2^* \underline{a}^* = \frac{i}{2} T_2^* \mathbf{L}\, T_1 \underline{a} \bullet \underline{a}^* = W_2 \underline{a} \bullet \underline{a}^* \\
\frac{1}{2} s_3 &= -\frac{i}{2} \mathbf{K}\, T_1 \underline{a} \bullet T_2^* \underline{a}^* = -\frac{i}{2} T_2^* \mathbf{K}\, T_2 \underline{a} \bullet \underline{a}^* = W_3 \underline{a} \bullet \underline{a}^*
\end{aligned} \tag{26.4}$$

Now let

$$W_j = w_{j0} \mathbf{I} + i w_{j1} \mathbf{J} + i w_{j2} \mathbf{L} - i w_{j3} \mathbf{K} \tag{26.5}$$

Then

$$W_j \underline{a} \bullet \underline{a}^* = w_{j0}\, g_0 + w_{j1}\, g_1 + w_{j2}\, g_2 + w_{j3}\, g_3 \tag{26.6}$$

where $g = g(\underline{a}) = (g_0, g_1, g_2, g_3)$, as usual. Now (26.3) can be written in concise form:

$$V_1 V_2^* = W\left(T_1, T_2^*\right) g(\underline{a}) \bullet h(\underline{b}) \tag{26.7}$$

and hence

$$R_e \left\langle V_1 V_2^* \right\rangle = R_e \langle W \rangle g(\underline{a}) \bullet h(b) \tag{26.8}$$

The condition for statistical independence of targets is now apparent; since $g(\underline{a})$ and $g(\underline{b})$ are arbitrary stokes vectors representing antennas $\underline{a}$ and $\underline{b}$ which are used to probe the targets, $R_e \langle V_1 V_2^* \rangle = 0$ only if

$$R_e \left\langle W\left(T_1, T_2^*\right)\right\rangle = 0 \tag{26.9}$$

This is the condition for statistical independence of time-varying radar targets T_1 and T_2, which is independent of the probing antennas. The matrix $W(T_1, T_2^*)$ is a generalization of the stokes-reflection matrix M (also called the Mueller matrix) which is obtained if $T_1 = T_2$. We will call W the mixed stokes matrix of the two targets. For distributed targets $\langle W \rangle$ is called the "stokes correlation matrix." The mixed stokes matrix has interesting properties. We show next that $W(T_1, T_2^*)$ can be written as a product of matrices which depend on T_1 and on T_2 only. To do this we write:

$$\begin{aligned} T_1 &= a_1 \mathbf{I} + b_1 \mathbf{K} + c_1 \mathbf{L} \\ T_2 &= a_2 \mathbf{I} + b_2 \mathbf{K} + c_2 \mathbf{L} \end{aligned} \tag{26.10}$$

where a_i, b_i, c_i, $i = 1, 2$ are complex coefficients and $\mathbf{J}$, $\mathbf{K}$, $\mathbf{L}$ are the rotation matrices as usual (defined in Sec. 4). Now, the matrix W is determined by the coefficients w_{jk} defined by (26.5) and (26.4). A straightforward calculation leads to:

$$W = \frac{1}{2}\begin{bmatrix} a_1a_2^* + b_1b_2^* + c_1c_2^* & +i\left(+c_1b_2^* - b_1c_2^*\right) & i\left(a_1c_2^* - c_1a_2^*\right) & i\left(b_1a_2^* - a_1b_2^*\right) \\ +i\left(c_1b_2^* - b_1c_2^*\right) & -a_1a_2^* + b_1b_2^* + c_1c_2^* & -\left(b_1a_2^* + a_1b_2^*\right) & -\left(a_1c_2^* + c_1a_2^*\right) \\ +i\left(a_1c_2^* - c_1a_2^*\right) & -\left(a_1b_2^* + b_1a_2^*\right) & a_1a_2^* - b_1b_2^* + c_1c_2^* & -\left(b_1c_2^* + c_1b_2^*\right) \\ i\left(b_1a_2^* - a_1b_2^*\right) & -\left(a_1c_2^* + c_ia_2^*\right) & -\left(b_1c_2^* + c_1b_2^*\right) & a_1a_2^* + b_1b_2^* - c_1c_2^* \end{bmatrix} \tag{26.11}$$

We find that W is a symmetric matrix. We next show that W may be composed as a product of matrices:

$$W = \frac{1}{2}\begin{bmatrix} a_1 & 0 & -ic_1 & ib_1 \\ 0 & -a_1 & -b_1 & -c_1 \\ -ic_1 & -b_1 & a_1 & 0 \\ ib_1 & -c_1 & 0 & a_1 \end{bmatrix} \hat{\mathbb{I}} \begin{bmatrix} a_2^* & 0 & ic_2^* & -ib_2^* \\ 0 & -a_2^* & -b_2^* & -c_2^* \\ ic_2^* & -b_2^* & a_2^* & 0 \\ -ib_2^* & -c_2^* & 0 & a_2^* \end{bmatrix} \tag{26.12}$$

or, written compactly:

$$W\left(T_1\ ,\ T_2^*\right) = \frac{1}{2}\,\hat{T}_1\ \hat{I}\ \hat{T}_2^* \tag{26.13}$$

where $\hat{T}_1$ stands for the first matrix in (26.12) and $\hat{T}_2$ contains the coefficients of T_2. The symbol $\hat{I}$ stands for the stokes matrix $2M_o$ (Sec. 14a), which is the M equivalent of the unit target scattering matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

The study of correlations between distributed radar targets would find their natural starting point with the above-developed concepts summarized by equation (26.13). However, in the remainder of this work, we will restrict our attention for the most part to the study of $R = \langle W(T\ ,\ T^*)\rangle$, which determines the average radar return of distributed targets.

Equation (26.7) has an interesting application, with the following important problem: we will have occasion (see Chap. 8) to consider the received voltage of a target as the sum of local voltages V_n, derived from local scattering matrices T_n on a surface S. Let

$$V = \int V_n \, dS = \int T_n \underline{a} \cdot \underline{b} \, dS \tag{26.14}$$

The total power received is, then, using (26.7):

$$P = VV^* = \iint V_{n1} V_{n2}^* \, dS_1 \, dS_2 = \iint W\left(T_{n1}\ ,\ T_{n2}^*\right) g(\underline{a}) \cdot h(\underline{b}) \, dS_1 \, dS_2 \tag{26.15}$$

For time-varying targets, the average power is found from

$$\langle P \rangle = \iint \left\langle W\left(T_{n1}\ ,\ T_{n2}^*\right)\right\rangle dS_1 \, dS_2 \, g(\underline{a}) \cdot h(\underline{b}) = R \, g(\underline{a}) \cdot h(\underline{b}) \tag{26.16}$$

This equation is useful for the calculation of average power and of the stokes reflection matrix R. An application to rough surface scattering is given in Sec. 46.

6 DISTRIBUTED RADAR TARGETS

27. Scattering From Distributed Targets

The last sections paved the way for a treatment of radar scattering from distributed (i.e., moving) targets, where the time-averaged measured power is used to characterize its electromagnetic (vector) properties. The averaged received power is obtained from (12.20) by a simple averaging process:

$$\langle P \rangle = \langle M \rangle g(\underline{a}) \cdot h(\underline{b}) \tag{27.1}$$

The averaging is applied to all matrix elements of the stokes reflection matrix. To avoid a too complicated notation, we now define in accordance with (12.15):

$$\left.\begin{aligned}
A_o &= \langle A_o \rangle \\
B_o &= \langle B_o \rangle \\
B_\psi &= \langle B \cos 4\psi \rangle + \langle E \sin 4\psi \rangle \\
C_\psi &= \langle C \cos 2\psi \rangle \\
D_\psi &= \langle D \cos 2\psi \rangle + \langle G \sin 2\psi \rangle \\
E_\psi &= \langle E \cos 4\psi \rangle - \langle B \sin 4\psi \rangle \\
F &= \langle F \rangle \\
G_\psi &= \langle G \cos 2\psi \rangle - \langle D \sin 2\psi \rangle \\
H_\psi &= \langle C \sin 2\psi \rangle
\end{aligned}\right\} \tag{27.2}$$

The averaged matrix $\langle M \rangle$ we give the symbol $R = \langle M \rangle$. Hence, we find:

$$R = \begin{bmatrix} A_o + B_o & F & C_\psi & H_\psi \\ F & -A_o + B_o & G_\psi & D_\psi \\ C_\psi & G_\psi & A_o + B_\psi & -E_\psi \\ H_\psi & D_\psi & -E_\psi & A_o - B_\psi \end{bmatrix} \tag{27.3}$$

We notice the trace property: trace $R = 2(A_o + B_o)$ is preserved for distributed targets.

In later discussions, we will frequently omit subscript ψ in (27.3) if no possible misunderstanding can arise; this will be done in most cases when the discussion concerns the most general treatment of asymmetric targets. However, for symmetric targets the distinction between, say, B and B_ψ will be preserved.

We come now to an important observation which points out a basic distinction between the radar cross-section behavior of distributed targets and that of single targets. We notice in (27.3) that nine independent target parameters determine the stokes reflection matrix for distributed targets, whereas it is known that five independent parameters determine the target scattering matrix. (We omitted the absolute phase as an independent parameter since it is excluded with power measurements.) This indicates that the class of averaged distributed targets is larger than that of single targets. Thus, it will not be surprising if examples are found of distributed targets that have no equivalent target-scattering matrix; i.e., these averaged distributed targets cannot be represented by an averaged single target. Such an example is that of a uniformly distributed (random) dipole cloud. This target is, on the average, symmetric and orientation independent, but it is known that there exists a cross-polarized return for linearly polarized illumination, as stated by Krishen et al. [38] and Long [56].

The only known symmetric orientation independent <u>single</u> target has the return characteristic of a large sphere (i.e., specular characteristic) for which it is known that no cross-polarized component exists. This proves that the dipole cloud does not have a single target equivalent. However, it is possible to think of the uniform dipole cloud return as due to an average spherical target plus a remainder component. The remainder component will then account for the average cross-polarized return component. We will show that these ideas may be generalized to apply to any type of distributed targets.

We now postulate, and will prove shortly, that the average return from a distributed target can be decomposed into a component due to an average single target and a remainder component. Since the remainder component arises from a statistical process (as a variation of changes from an averaged single target), it can be assumed that the remainder term will have a noisy behavior. We associate with the remainder component the term "target noise."

A large portion of the coming work consists of establishing general decomposition theorems which will separate the average target return of a distributed target into an average single target and remainder components. It will be most essential to show the physical realizability of the decompositions. Also, the question of uniqueness of the decomposition will be considered.

One of the immediate conclusions of the foregoing discussion is that if a distributed target represents a single target (determined by five components), four restrictions must be placed upon the nine stokes matrix parameters. It will be our first task to establish the nature of these restrictions. It will also be shown that the physical realizability condition imposed upon the stokes-return vector determines certain basic inequalities between the nine stokes parameters. All this will constitute topics for analysis and discussion in the following sections.

28. General Symmetric Distributed Targets

In this section, we consider the class of symmetric distributed radar targets. This class is most important for the usual examples encountered in applications, since with most applications the radar target on the average contains a plane of symmetry through the radarline-of-sight direction and an average target symmetry axis. For average flat terrain or sea state, the symmetry axis will be the vertical normal; for rough bodies of revolution, the axis is of course the roll axis of the body.

We found in Section 13 that symmetric single targets are characterized by a so-called maximum polarization $\underline{m}$ which is linear $(\tau_m = 0)$ and which is oriented either along the symmetry axis $(\phi_m = 0°)$ or orthogonal to it $(\phi_m = 90°)$. Hence, symmetric single targets have no helicity $(\tau_m = 0)$, and hence $B_o = B$ and $E = F = G = 0$; from $\phi_m = 0$ or $90°$ we also find $H = 0$ for a coordinate frame aligned with the targets axis $(\psi_a = 0)$. These properties may be generalized to symmetric distributed targets.

We consider a target with the following properties:

(1) The radar response for (RC-RC), right-circular polarization, is the same as for (LC-LC), left-circular polarization; i.e., the target is insensitive to the sense of circular polarization. From this we find easily that $F = 0$.

(2) The radar response for linear polarization, $[L(\phi) - L(\phi)]$, where ϕ is orientation measured from the target symmetry axis, is the same for positive or negative values of ϕ. From this property, it follows that $E = 0$ and $H = 0$.

(3) The radar response for (V-RC) is the same as for (V-LC), where V stands for linear (vertical) polarization oriented along the target's symmetry axis; i.e., $V = L(0)$. From this we find that $G = 0$.

A distributed target which has these properties will be called a general symmetric distributed target. It is characterized by $E = F = G = H = 0$, but $B_o \neq B$ in general. It is possible to consider symmetric distributed targets as composed of single targets with the following properties:

(1) They have a symmetric distribution of helicity $\tau_m = \bar{\tau}_m + \Delta\tau_m$ about a mean $\bar{\tau}_m = 0$.

(2) They have a symmetric distribution of orientations $\psi = \bar{\psi} + \Delta\psi$ about a mean $\bar{\psi} = 0$.

(3) The random variables $\Delta\psi$ and $\Delta\tau$ are statistically independent of each other and of the other single target parameters: m, γ, and ν.

We now apply these properties to the target parameters given by (27.2), omitting all terms containing averages of $\sin 2\Delta\psi$ and $\sin 2\Delta\tau$ and substituting $\bar{\tau}_m = 0$ and $\bar{\psi} = 0$.

Then:

$$A_o = \langle A_o \rangle = \left\langle Q_o f \cos^2 2\Delta\tau_m \right\rangle \tag{28.1}$$

$$B_o = \left\langle Q_o\left(1 + \cos^2 2\gamma - f \cos^2 2\Delta\tau_m\right)\right\rangle \tag{28.2}$$

$$B = \left\langle Q_o\left[1 + \cos^2 2\gamma - f\left(1 + \sin^2 2\Delta\tau_m\right)\right] \cos 4\Delta\psi\right\rangle \tag{28.3}$$

$$C = \left\langle 2Q_o \cos 2\gamma \cos 2\Delta\tau_m \cos 2\Delta\psi \right\rangle \tag{28.4}$$

$$D = \left\langle Q_o \sin^2 2\gamma \sin 4\nu \cos 2\Delta\tau_m \cos 2\Delta\psi\right\rangle \tag{28.5}$$

$$E = F = G = H = 0 \tag{28.6}$$

where

$$Q_o = m^2/8 \cos^4 \gamma \tag{28.7}$$

and

$$f = 1 - \sin^2 2\gamma \sin^2 2\nu \tag{28.8}$$

It is seen that these properties suffice to construct symmetric distributed targets as defined above. Notice from (28.2) and (28.3) that in general $B_o \neq B$ and Q_1, Q_2, and $Q_3 \neq 0$. Symmetric distributed targets cover a much larger class of targets then just the single symmetric targets. In many cases, no equivalent single target may be found, since Q_1, Q_2, Q_3 (12.6) are not zero. However, we will show in Chapter 7 that the return from a symmetric distributed target may be decomposed into scattering from an average symmetric single target plus a remainder component which has the character of target noise (N-target).

29. Derivation of Fundamental Inequality of Target Scattering

We found in Sec. 28 that the average electromagnetic return from a distributed target is characterized by the stokes matrix R applied to the stokes vector $g(\underline{a})$ of transmit antenna $\underline{a}$. We will be interested in necessary and

and sufficient conditions on R such that $s = Rg$ is a physically realizable stokes vector. The condition for physical realizability of s was found in (24.8):

$$s_0^2 \geq s_1^2 + s_2^2 + s_3^2 \tag{29.1}$$

If (29.1) is true for all values of $g(\underline{a})$, we say that the distributed target R is physically realizable. Substitution of R into (29.1) gives:

$$\begin{bmatrix} A_o + B_o & F & C & H \\ F & -A_o + B_o & G & D \\ C & G & A_o + B & -E \\ H & D & -E & A_o - B \end{bmatrix} \begin{bmatrix} g_0 \\ g_1 \\ g_2 \\ g_3 \end{bmatrix} = \begin{bmatrix} s_0 \\ s_1 \\ s_2 \\ s_3 \end{bmatrix} \tag{29.2}$$

The condition for the physical stokes vector results in the following:

$$[g_o(A_o + B_o) + g_1F + g_2C + g_3H]^2 - [g_oF + g_1(-A_o + B_o) + g_2G + g_3D]^2 +$$

$$- [g_oC + g_1G + g_2(A_o + B) - g_3E]^2 - [g_oH + g_1D - g_2E + g_3(A_o - B)]^2 \geq 0$$

$$\tag{29.3}$$

or

$$A_o^2\left(g_o^2 - g_1^2 - g_2^2 - g_3^2\right) + 2A_oB_o\left(g_o^2 + g_1^2\right) + 2A_oB\left(-g_2^2 + g_3^2\right) +$$

$$- F^2\left(g_o^2 - g_1^2\right) - C^2\left(g_o^2 - g_2^2\right) - H^2\left(g_o^2 - g_3^2\right) +$$

$$- E^2\left(g_2^2 + g_3^2\right) - D^2\left(g_1^2 + g_3^2\right) - G^2\left(g_1^2 + g_2^2\right) + B_o^2\left(g_o^2 - g_1^2\right) - B^2\left(g_2^2 + g_3^2\right) +$$

$$+ 2g_og_1[(A_o + B_o)F - (-A_o + B_o)F - CG - DH] +$$

$$+ 2g_o g_2[(A_o + B_o)C - FG - (A_o + B)C + EH] +$$

$$+ 2g_o g_3[(A_o + B_o)H - DF + CE - (A_o - B)H] +$$

$$+ 2g_1 g_2[CF - (-A_o + B_o)G - (A_o + B)G + DE] +$$

$$+ 2g_1 g_3[FH - (-A_o + B_o)D + EG - (A_o - B)D] +$$

$$+ 2g_2 g_3[CH - DG + (A_o + B)E + (A_o - B)E] \geq 0 \tag{29.4}$$

Since g is a monochromatic stokes vector,

$$g_o^2 = g_1^2 + g_2^2 + g_3^2 \tag{29.5}$$

Hence, we find:

$$\left(g_o^2 - g_1^2\right)\left[B_o^2 - B^2 - (E^2 + F^2)\right] + \left(g_o^2 - g_2^2\right)[2A_o(B_o + B) - (C^2 + D^2)] +$$

$$+ \left(g_o^2 - g_3^2\right)\left[2A_o(B_o - B) - (G^2 + H^2)\right] + 2g_o g_1[2A_o F - (CG + DH)] +$$

$$+ 2g_2 g_3[2A_o E + (CH - DG)] + 2g_o g_2[C(B_o - B) + (EH - FG)] +$$

$$+ 2g_1 g_3[-D(B_o - B) + (EG + FH)] + 2g_o g_3[H(B_o + B) - (DF - CE)] +$$

$$+ 2g_1 g_2[-G(B_o + B) + (CF + DE)] \geq 0 \tag{29.6}$$

We introduce the following quadratic quantities:

$$Q_1 = B_o^2 - B^2 - (E^2 + F^2) \tag{29.7}$$

$$Q_2 = 2A_o(B_o + B) - (C^2 + D^2) \tag{29.8}$$

$$Q_3 = 2A_o(B_o - B) - (G^2 + H^2) \tag{29.9}$$

Also let:

$$\left.\begin{aligned} g_o^2 - g_1^2 &= q_1 \\ g_o^2 - g_2^2 &= q_2 \\ g_o^2 - g_3^2 &= q_3 \end{aligned}\right\} \tag{29.10}$$

$$g_i g_j = q_{ij} \qquad (i \neq j;\ 0, 1, 2, 3) \tag{29.11}$$

$$Q_{01} = 2A_oF - (CG + DH) \tag{29.12}$$

$$Q_{23} = 2A_oE + (CH - DG) \tag{29.13}$$

$$Q_{02} = C(B_o - B) + (EH - FG) \tag{29.14}$$

$$Q_{13} = -D(B_o - B) + (EG + FH) \tag{29.15}$$

$$Q_{03} = H(B_o + B) - (DF - CE) \tag{29.16}$$

$$Q_{12} = -G(B_o + B) + (CF + DE) \tag{29.17}$$

Then:

$$\begin{aligned} &q_1Q_1 + q_2Q_2 + q_3Q_3 + 2q_{01}Q_{01} + 2q_{23}Q_{23} + \\ &+ 2q_{02}Q_{02} + 2q_{13}Q_{13} + 2q_{03}Q_{03} + 2q_{12}Q_{12} \geq 0 \end{aligned} \tag{29.18}$$

For q_{ij}, we find the useful relationships

$$\left.\begin{aligned} q_1 q_2 &= q_{03}^2 + q_{12}^2 \\ q_1 q_3 &= q_{02}^2 + q_{13}^2 \\ q_2 q_3 &= q_{01}^2 + q_{23}^2 \end{aligned}\right\} \tag{29.19}$$

$$\left.\begin{aligned} q_1^2 &= (q_2 + q_3)^2 - 4q_{01}^2 = (q_2 - q_3)^2 + 4q_{23}^2 \\ q_2^2 &= (q_1 + q_3)^2 - 4q_{02}^2 = (q_1 - q_3)^2 + 4q_{13}^2 \\ q_3^2 &= (q_1 + q_2)^2 - 4q_{03}^2 = (q_1 - q_2)^2 + 4q_{12}^2 \end{aligned}\right\} \tag{29.20}$$

Notice that the first three terms in (29.18) have square coefficients (they are positive) whereas the cross-product terms may be positive or negative, depending upon the sign of g_1, g_2 or g_3.

For a single target, only completely polarized returns are obtained; hence, the inequality in (29.18) becomes equality. It is easily verified that due to the changing sign possibility of the coupling terms $g_0 g_1$, etc., all six terms are zero for the single target case, and hence $Q_{ij} = 0$. From this it follows also that $Q_j = 0$ $(j = 1, 2, 3)$. We can verify that only four conditions of $Q_{ij} = 0$ and $Q_j = 0$ are independent.

The following four equations are characteristic for a general single target [compare with (12.7)]:

$$\left.\begin{aligned} 2A_o(B_o + B) &= C^2 + D^2 \\ 2A_o(B_o - B) &= G^2 + H^2 \\ 2A_o E &= DG - CH \\ 2A_o F &= CG + DH \end{aligned}\right\} \tag{29.21}$$

From these four equations, the other equalities are easily derived. For example:

$$2A_oEH = DGH - CH^2$$

$$2A_oFG = CG^2 + DGH$$

$$2A_o(EH - FG) = -C(G^2 + H^2) = -2A_oC(B_o - B)$$

$$Q_{02} = C(B_o - B) + (EH - FG) = 0$$

Similarly for $Q_{ij} = 0 \quad (i \neq j;\ 0, 1, 2, 3)$.

The single target case determined by a scattering matrix by five independent parameters thus has four restricting equations imposed upon the nine independent parameters of the general distributed target case (i.e., $5 = 9 - 4$). The basic problem of how to separate from the general distributed target the averaged single target components with additional "noise" terms, we will solve gradually in several steps. We will show that the decomposition theorem also supplies the desired necessary and sufficient condition for physical realizability of R.

30. Basic Proofs That $Q_i \geqq 0$ (i=1, 2, 3)

In these proofs (there will be three inequality proofs), we will use the basic stokes matrix-parameter inequality. The trick will be first to select only two of the six q_{ij} terms. This can always be done, as is easily verified from the relationships. For instance, if we wish to retain $q_{02} = g_0g_2$ and $q_{13} = g_1g_3$ as in the first proof, we choose a set of parameter coefficients g_0, g_1, g_2, g_3 with "positive" sign. ("Positive" is either positive or negative.) Next, we choose g_0 and g_2 "positive" and g_1, g_3 "negative" (the opposite of "positive"). Then we add the two results. It is easily checked that only q_{02} and q_{13} remain in the sum. The other trick used is to choose special values for q_1, q_2, q_3 and q_{02} and q_{13}. These values cannot be arbitrary but must satisfy $q_1, q_2, q_3 \geq 0$ and the relations (29.19) and (29.20). Physically, this procedure amounts to a special choice of transmit polarization.

(a) Proof That $Q_2 \geq 0$

Let

$$q_1 Q_1 + q_2 Q_2 + q_3 Q_3 + 2q_{02}[-C(B_o - B) + (FG - EH)] +$$

$$+ 2q_{13}[-D(B_o - B) + (EG + FH)] \geq 0 \qquad (30.1)$$

We choose

$$\left.\begin{aligned} q_{02} &= 2A_o C \\ q_{13} &= 2A_o D \end{aligned}\right\} \qquad (30.2)$$

We use the property

$$q_1 q_3 = q_{02}^2 + q_{13}^2 \qquad (30.3)$$

Hence,

$$q_1 q_3 = 4A_o^2(C^2 + D^2) \qquad (30.4)$$

Choose

$$q_1 = 4A_o^2 \qquad (30.5)$$

$$q_3 = C^2 + D^2 \qquad (30.6)$$

Substitution into (30.1) gives:

$$4A_o^2Q_1 + q_2Q_2 + (C^2 + D^2)\left[2A_o(B_o - B) - (G^2 + H^2)\right] +$$

$$+ 4A_oG(CF + DE) + 4A_oH(DF - CE) - 4A_o(B_o - B)(C^2 + D^2) =$$

$$= \left[4A_o^2\left(B_o^2 - B^2\right) - 2A_o(B_o - B)(C^2 + D^2) - 2A_o(B_o + B)(G^2 + H^2) +\right.$$

$$\left. + (C^2 + D^2)(G^2 + H^2)\right] - \left[+ 4A_o^2(E^2 + F^2) - 4A_oG(CF + DE) - 4A_oH(DF - CE) +\right.$$

$$\left. + (C^2 + D^2)(G^2 + H^2)\right] + 2A_o(B_o + B)(G^2 + H^2) +$$

$$- (C^2 + D^2)(G^2 + H^2) + q_2Q_2 \geq 0 \qquad (30.7)$$

The first term in brackets is recognized as Q_2Q_3. The second term will be shown to be nonnegative. Let

$$\begin{array}{lll} H = h \cos \alpha & D = d \cos \beta & F = f \cos \gamma \\ G = h \sin \alpha & C = d \sin \beta & E = f \sin \gamma \end{array} \qquad (30.8)$$

$$\Pi^2 = 4A_o^2(E^2 + F^2) - 4A_oG(CF + DE) - 4A_oH(DF - CE) + (C^2 + D^2)(G^2 + H^2) =$$

$$= 4A_o^2f^2 - 4A_o f\,dh \cos[\alpha - (\beta + \gamma)] + d^2h^2 \geq 0 \qquad (30.9)$$

The last statement in (30.9) is determined by the triangle-geometric equivalent, Π being associated with the side opposite the angle $\alpha - (\beta + \gamma)$ of the triangle with sides Π, $2A_of$, and dh; hence, $\Pi^2 \geq 0$.

We now can write for (30.7):

$$Q_2Q_3 - \Pi^2 + [q_2 + (G^2 + H^2)]Q_2 \geq 0$$

or:

$$Q_2[q_2 + 2A_o(B_o - B)] \geq \Pi^2 \geq 0 \qquad (30.10)$$

Since $q_2 \geq 0$ and $2A_o(B_o - B) \geq 0$ [see (12.6) and (27.2)], the term in brackets is nonnegative and hence $Q_2 \geq 0$.

The proof uses the basic inequality for the scattered stokes vector and the fact that equation (30.1) may be interpreted with specially chosen values of q_{ij} (which is equivalent to a special choice of transmitted polarization). For the proof, it was not even necessary to evaluate q_2, although this could easily be done since $q_2^2 = (q_1 + q_3)^2 - 4q_{02}^2$. We also notice that the same property, $Q_2 \geq 0$, holds for any transformation of the basic stokes vector coordinate system to another allowable coordinate system $\left(\text{i.e., a transformation which preserves the property } s_0^2 \geq s_1^2 + s_2^2 + s_3^2\right)$.

(b) <u>Proof That $Q_1 \geq 0$</u>

Let

$$q_1Q_1 + q_2Q_2 + q_3Q_3 + 2q_{01}[-2A_oF + (CG + DH)] + \\ + 2q_{23}[-2A_oE + (DG - CH] \geq 0 \qquad (30.11)$$

and

$$\left.\begin{aligned} q_{01} &= (B_o - B)F \\ q_{23} &= (B_o - B)E \end{aligned}\right\} \qquad (30.12)$$

Hence,

$$q_2q_3 = q_{23}^2 + q_{01}^2 = (B_o - B)^2(E^2 + F^2) \qquad (30.13)$$

Let

$$q_2 = (B_o - B)^2 \qquad (30.14)$$

$$q_3 = (E^2 + F^2) \qquad (30.15)$$

Substitution into (30.11) gives:

$$q_1Q_1 + (B_o - B)^2\left[2A_o(B_o + B) - (C^2 + D^2)\right] + (E^2 + F^2)\left[2A_o(B_o - B) + \right.$$
$$\left. - (G^2 + H^2)\right] - 4A_o(B_o - B)(E^2 + F^2) +$$
$$+ 2(B_o - B)G(CF + DE) + 2(B_o - B)H(DF - CE) \geq 0 \qquad (30.16)$$

or

$$\left[\left(B_o^2 - B^2\right)2A_o(B_o - B) - \left(B_o^2 - B^2\right)(G^2 + H^2) - 2A_o(B_o - B)(E^2 + F^2) + \right.$$
$$\left. + (E^2 + F^2)(G^2 + H^2)\right] - \left[(B_o - B)^2(C^2 + D^2) - 2(B_o - B)G(CF + DE) + \right.$$
$$\left. - 2(B_o - B)H(DF - CE) + (E^2 + F^2)(G^2 + H^2)\right] +$$
$$+ \left(B_o^2 - B^2\right)(G^2 + H^2) - (E^2 + F^2)(G^2 + H^2) + q_1Q_1 \geq 0 \qquad (30.17)$$

For the second term in brackets, we can write:

$$\Pi_1^2 = (B_o - B)^2d^2 - 2(B_o - B)dfh\cos[\alpha - (\beta + \gamma)] + d^2h^2 \geq 0 \qquad (30.18)$$

This follows from the triangle relationship where

$$\begin{array}{lll} H = h\cos\alpha & D = d\cos\beta & F = f\cos\gamma \\ G = h\sin\alpha & C = d\sin\beta & E = f\sin\gamma \end{array} \qquad (30.19)$$

The first term of (30.17) in brackets is recognized as Q_1Q_3; hence, (30.17) becomes

$$Q_1Q_3 + \left[q_1 + (G^2 + H^2)\right]Q_1 \geq \Pi_1^2 \geq 0$$

or

$$Q_1\left[q_1 + 2A_o(B_o - B)\right] \geq 0 \qquad (30.20)$$

Since the term in brackets is nonnegative, the property $Q_1 \geq 0$ follows.

(c) Proof That $Q_3 \geq 0$

From the general inequality, we choose only the following terms, the other being eliminated through a process of adding $\pm q_{ij}$ $(i \neq j)$:

$$q_1Q_1 + q_2Q_2 + q_3Q_3 + 2q_{03}[-H(B_o + B) + (DF - CE)] +$$
$$+ 2q_{12}[-G(B_o + B) + (CF + DE)] \geq 0 \qquad (30.21)$$

We may choose arbitrary values for q_{03} and q_{12}, but q_1, q_2 and $q_3 \geq 0$. Let

$$\left.\begin{aligned} q_{03} &= 2A_oH \\ q_{12} &= 2A_oG \end{aligned}\right\} \qquad (30.22)$$

Now since

$$q_1q_2 = q_{03}^2 + q_{12}^2 = 4A_o^2(G^2 + H^2) \qquad (30.23)$$

let

$$q_1 = 4A_o^2 \qquad (30.24)$$

$$q_2 = G^2 + H^2 \qquad (30.25)$$

We get:

$$4A_o^2Q_1 + (G^2 + H^2)\left[2A_o(B_o + B) - (C^2 + D^2)\right] + q_3Q_3 +$$

$$+ 4A_oG(CF + DE) + 4A_oH(DF - CE) - 4A_o(B_o + B)(G^2 + H^2) =$$

$$= \left[4A_o^2\left(B_o^2 - B^2\right) - 2A_o(B_o + B)(G^2 + H^2) - 2A_o(B_o - B)(C^2 + D^2) +\right.$$

$$\left.+ (C^2 + D^2)(G^2 + H^2)\right] - \left[4A_o^2(E^2 + F^2) - 4A_oG(CF + DE) +\right.$$

$$\left.- 4A_oH(DF - CE) + (C^2 + D^2)(G^2 + H^2)\right] + 2A_o(B_o - B)(C^2 + D^2) +$$

$$- (C^2 + D^2)(G^2 + H^2) + q_3Q_3 \geq 0 \tag{30.26}$$

Previously (30.9), it was shown that the second term in square brackets Π^2 was nonnegative. Hence:

$$Q_2Q_3 - \Pi^2 + \left[q_3 + .(C^2 + D^2)\right]Q_3 \geq 0$$

or

$$Q_3[q_3 + 2A_o(B_o + B)] \geq \Pi^2 \geq 0 \tag{30.27}$$

Since the term in brackets in (30.27) is positive, the proof $Q_3 \geq 0$ follows.

31. Orthogonal Transformation Properties

In the following sections, we summarize some properties of orthogonal transformations on the stokes scattering matrix R. These properties reveal a basic structure of distributed radar targets which result in some general theorems. These theorems state that certain nonnegative target parameters, in general, cannot vanish independently. Either all are nonzero, which results in a general distributed target, or they all vanish together, which results in a nondistributed target (either a single target or a zero target). Exceptions to these general rules are shown to exist. The exceptional cases will play an important role later with the decomposition theorems.

(a) ψ-Transformation Properties

$$\Omega \doteq \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \cos 2\psi & \sin 2\psi \\ 0 & 0 & -\sin 2\psi & \cos 2\psi \end{bmatrix} \begin{bmatrix} A_o + B_o & F & C & H \\ F & -A_o + B_o & G & D \\ C & G & A_o + B & -E \\ H & D & -E & A_o - B \end{bmatrix} =$$

$$= \begin{bmatrix} A_o + B_o & F & C & H \\ F & -A_o + B_o & G & D \\ \begin{matrix} C \cos 2\psi + \\ + H \sin 2\psi \end{matrix} & \begin{matrix} G \cos 2\psi + \\ + D \sin 2\psi \end{matrix} & \begin{matrix} (A_o + B) \cos 2\psi + \\ -E \sin 2\psi \end{matrix} & \begin{matrix} -E \cos 2\psi + \\ +(A_o - B) \sin 2\psi \end{matrix} \\ \begin{matrix} H \cos 2\psi + \\ - C \sin 2\psi \end{matrix} & \begin{matrix} D \cos 2\psi + \\ -G \sin 2\psi \end{matrix} & \begin{matrix} -(A_o + B) \sin 2\psi + \\ -E \cos 2\psi \end{matrix} & \begin{matrix} (A_o - B) \cos 2\psi + \\ +E \sin 2\psi \end{matrix} \end{bmatrix} \quad (31.1)$$

$$\Omega \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos 2\psi & -\sin 2\psi \\ 0 & 0 & \sin 2\psi & \cos 2\psi \end{bmatrix} =$$

$$= \begin{bmatrix} A_o + B_o & F & \begin{matrix} C \cos 2\psi + \\ +H \sin 2\psi \end{matrix} & \begin{matrix} H \cos 2\psi + \\ -C \sin 2\psi \end{matrix} \\ F & -A_o + B_o & \begin{matrix} G \cos 2\psi + \\ +D \sin 2\psi \end{matrix} & \begin{matrix} D \cos 2\psi + \\ -G \sin 2\psi \end{matrix} \\ \begin{matrix} C \cos 2\psi + \\ +H \sin 2\psi \end{matrix} & \begin{matrix} G \cos 2\psi + \\ +D \sin 2\psi \end{matrix} & \begin{matrix} A_o + B \cos 4\psi + \\ -E \sin 4\psi \end{matrix} & \begin{matrix} -E \cos 4\psi + \\ -B \sin 4\psi \end{matrix} \\ \begin{matrix} H \cos 2\psi + \\ -C \sin 2\psi \end{matrix} & \begin{matrix} D \cos 2\psi + \\ -G \sin 2\psi \end{matrix} & \begin{matrix} -E \cos 4\psi + \\ -B \sin 4\psi \end{matrix} & \begin{matrix} A_o - B \cos 4\psi + \\ +E \sin 4\psi \end{matrix} \end{bmatrix} \quad (31.2)$$

The ψ-Transformation results in the following:

Invariants: A_0, B_0, F, Q_1, Q_{01}, and $Q_2 + Q_3$ (31.3)

and

$$\left.\begin{aligned} C' &= C \cos 2\psi + H \sin 2\psi \\ H' &= -C \sin 2\psi + H \cos 2\psi \end{aligned}\right\} \tag{31.4}$$

$$\left.\begin{aligned} D' &= D \cos 2\psi - G \sin 2\psi \\ G' &= D \sin 2\psi + G \cos 2\psi \end{aligned}\right\} \tag{31.5}$$

$$\left.\begin{aligned} B' &= B \cos 4\psi - E \sin 4\psi \\ E' &= B \sin 4\psi + E \cos 4\psi \end{aligned}\right\} \tag{31.6}$$

$$\left.\begin{aligned} Q'_{02} &= Q_{02} \cos 2\psi + Q_{03} \sin 2\psi \\ Q'_{03} &= -Q_{02} \sin 2\psi + Q_{03} \cos 2\psi \end{aligned}\right\} \tag{31.7}$$

$$\left.\begin{aligned} Q'_{12} &= Q_{12} \cos 2\psi + Q_{13} \sin 2\psi \\ Q'_{13} &= -Q_{12} \sin 2\psi + Q_{13} \cos 2\psi \end{aligned}\right\} \tag{31.8}$$

$$\left.\begin{aligned} Q'_{23} &= Q_{23} \cos 4\psi + \frac{1}{2}(Q_2 - Q_3) \sin 4\psi \\ \frac{1}{2}(Q'_2 - Q'_3) &= -Q_{23} \sin 4\psi + \frac{1}{2}(Q_2 - Q_3) \cos 4\psi \end{aligned}\right\} \tag{31.9}$$

Two important relationships that can be derived from the above transformations are:

$$\left.\begin{aligned} B_o + B &= (B'_o + B')\cos^2 2\psi + (B'_o - B')\sin^2 2\psi + E'\sin 4\psi \\ B_o - B &= (B'_o - B')\cos^2 2\psi + (B'_o + B')\sin^2 2\psi - E'\sin 4\psi \end{aligned}\right\} \quad (31.10)$$

$$\left.\begin{aligned} Q_2 &= Q'_2 \cos^2 2\psi + Q'_3 \sin^2 2\psi + Q'_{23}\sin 4\psi \\ Q_3 &= Q'_3 \cos^2 2\psi + Q'_2 \sin^2 2\psi - Q'_{23}\sin 4\psi \end{aligned}\right\} \quad (31.11)$$

Since ψ is arbitrary, it is always possible to choose a special transformation ψ_1 such that $E' = 0$ in (31.6). We then find from (31.10) the following curious relationship:

$$\left.\begin{aligned} B_o + B &= (B'_o + B')\cos^2 2\psi_1 + (B'_o - B')\sin^2 2\psi_1 \\ B_o - B &= (B'_o - B')\cos^2 2\psi_1 + (B'_o + B')\sin^2 2\psi_1 \end{aligned}\right\} \quad (31.12)$$

From (31.12) it follows, since $B'_o \geq |B'|$ (this is a consequence of: B_o and Q_1 invariant), that if $B_o - B = 0$, then $B_o + B = 0$, and hence $B_o = B = 0$; i.e., the only time $B_o - B = 0$, in general, is when both B_o and B become zero!

A similar argument used later will show that then $A_o = 0$ also, but then $R = 0$ which represents the zero-target, which can be proved easily using (29.7), (29.8), and (29.9), taking into account that $Q_1 \geq 0$, $Q_2 \geq 0$, and $Q_3 \geq 0$. Stated another way, we arrive at the following theorem: <u>For a general distributed target A_o, $B_o - B$, and $B_o + B$ cannot be zero</u>!

A similar argument may be applied to relationship (31.11). We find a transformation ψ_2 such that $Q'_{23} = 0$ in (31.9). Under these conditions:

$$\left.\begin{aligned} Q_2 &= Q'_2 \cos^2 2\psi_2 + Q'_3 \sin^2 2\psi_2 \\ Q_3 &= Q'_3 \cos^2 2\psi_2 + Q'_2 \sin^2 2\psi_2 \end{aligned}\right\} \quad (31.13)$$

Now, since $Q_2' \geq 0$ and $Q_3' \geq 0$, we have, if $Q_2 = 0$ then $Q_2' = 0$ and $Q_3' = 0$ and hence $Q_3 = 0$. Later we show that then $Q_1 = 0$ also. Hence, the general rule: For distributed targets, Q_1, Q_2, and Q_3 cannot be independently zero. If one of them is zero, then all of them are, and this results in the case of a single (nondistributed) target.

Notice, however, that singular exceptions to these rules exist. The rule derived from (31.12) is based upon the assumption that $\sin 2\psi_1$ or $\cos 2\psi_1$ is not zero, or that $\sin 4\psi_1 \neq 0$. It is clear from (31.6) that if $E = 0$, the condition $E' = B \sin 4\psi_1 = 0$ can be satisfied, for general B, only if $\sin 4\psi_1 = 0$, and this causes the rule to fail. Hence, we find that $B_o - B$ and $B_o + B$ can be independently zero if $E = 0$! From $Q_1 = B_o^2 - B^2 - E^2 - F^2 \geq 0$, it then follows that also $F = 0$. Hence, we find the rule for distributed targets: $B_o - B$ and $B_o + B$ can be independently zero if $E = 0$ and $F = 0$. Later we derive similar rules for A_o and $B_o - B$, and A_o and $B_o + B$.

A similar exceptional case may be found for (31.13) if $\sin 4\psi_2 = 0$. This occurs if $Q_{23} = 0$ when we find from (31.9) that $Q_{23}' = 1/2\,(Q_2 - Q_3)$ $\sin 4\psi_2 = 0$ is satisfied for general $(Q_2 - Q_3)$ only if $\sin 4\psi_2 = 0$. Hence, Q_2 and Q_3 can be independently zero if $Q_{23} = 0$. Similar rules will be found between Q_1 and Q_2, and Q_1 and Q_3.

(b) τ-Transformation Properties

$$\Omega = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\tau & \sin 2\tau & 0 \\ 0 & -\sin 2\tau & \cos 2\tau & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_o + B_o & F & C & H \\ F & -A_o + B_o & G & D \\ C & G & A_o + B & -E \\ H & D & -E & A_o - B \end{bmatrix} =$$

$$= \begin{bmatrix} A_o + B_o & F & C & H \\ \begin{matrix} F\cos 2\tau + \\ +C\sin 2\tau \end{matrix} & \begin{matrix} (-A_o + B_o)\cos 2\tau \\ +G\sin 2\tau \end{matrix} & \begin{matrix} G\cos 2\tau + \\ +(A_o + B)\sin 2\tau \end{matrix} & \begin{matrix} D\cos 2\tau + \\ -E\sin 2\tau \end{matrix} \\ \begin{matrix} C\cos 2\tau + \\ -F\sin 2\tau \end{matrix} & \begin{matrix} (A_o - B_o)\sin 2\tau \\ +G\cos 2\tau \end{matrix} & \begin{matrix} (A_o + B)\cos 2\tau \\ -G\sin 2\tau \end{matrix} & \begin{matrix} -E\cos 2\tau + \\ -D\sin 2\tau \end{matrix} \\ H & D & -E & A_o - B \end{bmatrix} \tag{31.14}$$

$$\Omega \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\tau & -\sin 2\tau & 0 \\ 0 & \sin 2\tau & \cos 2\tau & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} A_o + B_o & \begin{matrix} F\cos 2\tau + \\ +C\sin 2\tau \end{matrix} & \begin{matrix} C\cos 2\tau + \\ -F\sin 2\tau \end{matrix} & H \\ \begin{matrix} F\cos 2\tau + \\ +C\sin 2\tau \end{matrix} & \begin{matrix} (-A_o + B_o)\cos^2 2\tau \\ +(A_o + B)\sin^2 2\tau \\ +G\sin 4\tau \end{matrix} & \begin{matrix} \frac{1}{2}(A_o - B_o)\sin 4\tau \\ +\frac{1}{2}(A_o - B)\sin 4\tau \\ +G\cos 4\tau \end{matrix} & \begin{matrix} D\cos 2\tau + \\ -E\sin 2\tau \end{matrix} \\ \begin{matrix} C\cos 2\tau + \\ -F\sin 2\tau \end{matrix} & \begin{matrix} \frac{1}{2}(A_o - B_o)\sin 4\tau \\ +\frac{1}{2}(A_o + B)\sin 4\tau \\ +G\cos 4\tau \end{matrix} & \begin{matrix} (-A_o + B_o)\sin^2 2\tau \\ +(A_o + B)\cos^2 2\tau \\ -G\sin 4\tau \end{matrix} & \begin{matrix} -E\cos 2\tau + \\ -D\sin 2\tau \end{matrix} \\ H & \begin{matrix} D\cos 2\tau + \\ -E\sin 2\tau \end{matrix} & \begin{matrix} -E\cos 2\tau + \\ -D\sin 2\tau \end{matrix} & A_o - B \end{bmatrix} \tag{31.15}$$

The τ-Transformation results in the following:

Invariants: $A_o + B_o$, $B_o + B$, H, Q_3, Q_{03}, and $Q_1 + Q_2$

also:

$$\left. \begin{aligned} C' &= C\cos 2\tau - F\sin 2\tau \\ F' &= C\sin 2\tau + F\cos 2\tau \end{aligned} \right\} \tag{31.16}$$

$$\left. \begin{aligned} D' &= D\cos 2\tau - E\sin 2\tau \\ E' &= D\sin 2\tau + E\cos 2\tau \end{aligned} \right\} \tag{31.17}$$

$$\left. \begin{aligned} G' &= G\cos 4\tau + \left[A_o - \frac{1}{2}(B_o - B)\right]\sin 4\tau \\ A_o' - \frac{1}{2}(B_o' - B') &= -G\sin 4\tau + \left[A_o - \frac{1}{2}(B_o - B)\right]\cos 4\tau \end{aligned} \right\} \tag{31.18}$$

$$\left.\begin{aligned} Q'_{01} &= Q_{01}\cos 2\tau + Q_{02}\sin 2\tau \\ Q'_{02} &= -Q_{01}\sin 2\tau + Q_{02}\cos 2\tau \end{aligned}\right\} \qquad (31.19)$$

$$\left.\begin{aligned} Q'_{13} &= Q_{13}\cos 2\tau + Q_{23}\sin 2\tau \\ Q'_{23} &= -Q_{13}\sin 2\tau + Q_{23}\cos 2\tau \end{aligned}\right\} \qquad (31.20)$$

$$\left.\begin{aligned} Q'_{12} &= Q_{12}\cos 4\tau + \frac{1}{2}(Q_1 - Q_2)\sin 4\tau \\ \frac{1}{2}(Q'_1 - Q'_2) &= -Q_{12}\sin 4\tau + \frac{1}{2}(Q_1 - Q_2)\cos 4\tau \end{aligned}\right\} \qquad (31.21)$$

Two important inverse relationships are:

$$\left.\begin{aligned} 2A_o &= 2A'_o\cos^2 2\tau + (B'_o - B')\sin^2 2\tau + G'\sin 4\tau \\ B_o - B &= (B'_o - B')\cos^2 2\tau + 2A'_o\sin^2 2\tau - G'\sin 4\tau \end{aligned}\right\} \qquad (31.22)$$

$$\left.\begin{aligned} Q_1 &= Q'_1\cos^2 2\tau + Q'_2\sin^2 2\tau + Q'_{12}\sin 4\tau \\ Q_2 &= Q'_2\cos^2 2\tau + Q'_1\sin^2 2\tau - Q'_{12}\sin 4\tau \end{aligned}\right\} \qquad (31.23)$$

Since τ is arbitrary, it will in general be possible to choose τ_1 such that $G' = 0$ in (31.18); since $A'_o \geq 0$ and $(B'_o - B') \geq 0$, it then follows that if $A_o = 0$, $B_o - B = 0$, and conversely if $B_o - B = 0$, $A_o = 0$. Previously we found that then $B_o + B = 0$ also and hence $R = 0$. Hence, the theorem: <u>For a distributed target, $A_o > 0$ and $B_o > |B|$</u>.

A similar argument applied to (31.23) leads to the general rule: Q_1, Q_2, Q_3 cannot be independently zero. If all are zero, we have a single (nondistributed) target.

Exceptions to these rules exist, however, if $\sin 4\tau_1 = 0$. This occurs in (31.22) when $G = 0$; then from (31.18), $G' = 0$ only if $\sin 4\tau_1 = 0$.

Hence, A_o and $B_o - B$ may be independently zero if $G = 0$. Since $Q_3 = 2A_o(B_o - B) - (G^2 + H^2) \geq 0$, this implies that $Q_3 = 0$ and $H = 0$ also. A similar exception applies to (31.23) if $Q_{12} = 0$; then, from (31.21), $Q'_{12} = 0$ implies $\sin 4\tau_2 = 0$. Hence, Q_1 and Q_2 can be independently zero if $Q_{12} = 0$.

(c) <u>ν-Transformation Properties</u>

$$\Omega = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\nu & 0 & \sin 2\nu \\ 0 & 0 & 1 & 0 \\ 0 & -\sin 2\nu & 0 & \cos 2\nu \end{bmatrix} \begin{bmatrix} A_o + B_o & F & C & H \\ F & -A_o + B_o & G & D \\ C & G & A_o + B & -E \\ H & D & -E & A_o - B \end{bmatrix} =$$

$$= \begin{bmatrix} A_o + B_o & F & C & H \\ \begin{matrix} F\cos 2\nu \\ +H\sin 2\nu \end{matrix} & \begin{matrix} (-A_o + B)\cos 2\nu \\ +D\sin 2\nu \end{matrix} & \begin{matrix} G\cos 2\nu \\ -E\sin 2\nu \end{matrix} & \begin{matrix} D\cos 2\nu + \\ +(A_o - B)\sin 2\nu \end{matrix} \\ C & G & A_o + B & -E \\ \begin{matrix} +H\cos 2\nu \\ -F\sin 2\nu \end{matrix} & \begin{matrix} D\cos 2\nu + \\ +(A_o - B)\sin 2\nu \end{matrix} & \begin{matrix} -E\cos 2\nu \\ -G\sin 2\nu \end{matrix} & \begin{matrix} (A_o - B)\cos 2\nu \\ -D\sin 2\nu \end{matrix} \end{bmatrix} \qquad (31.24)$$

$$\Omega \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ & \cos 2\nu & 0 & -\sin 2\nu \\ 0 & 0 & 1 & 0 \\ 0 & \sin 2\nu & 0 & \cos 2\nu \end{bmatrix} =$$

$$= \begin{bmatrix} A_o + B_o & \begin{matrix} F\cos 2\nu + \\ +H\sin 2\nu \end{matrix} & C & \begin{matrix} H\cos 2\nu + \\ -F\sin 2\nu \end{matrix} \\ \begin{matrix} F\cos 2\nu \\ +H\sin 2\nu \end{matrix} & \begin{matrix} (-A_o + B_o)\cos^2 2\nu \\ +(A_o - B)\sin^2 2\nu \\ +D\sin 4\nu \end{matrix} & \begin{matrix} G\cos 2\nu \\ -E\sin 2\nu \end{matrix} & \begin{matrix} \frac{1}{2}(A_o - B_o)\sin 4\nu \\ +\frac{1}{2}(A_o - B)\sin 4\nu \\ +D\cos 4\nu \end{matrix} \\ C & \begin{matrix} G\cos 2\nu + \\ -E\sin 2\nu \end{matrix} & A_o + B & \begin{matrix} -E\cos 2\nu + \\ -G\sin 2\nu \end{matrix} \\ \begin{matrix} H\cos 2\nu \\ -F\sin 2\nu \end{matrix} & \begin{matrix} \frac{1}{2}(A_o - B_o)\sin 4\nu \\ +\frac{1}{2}(A_o - B)\sin 4\nu \\ +D\cos 4\nu \end{matrix} & \begin{matrix} -E\cos 2\nu \\ -G\sin 2\nu \end{matrix} & \begin{matrix} -(A_o - B_o)\sin^2 2\nu \\ +(A_o - B)\cos^2 2\nu \\ -D\sin 4\nu \end{matrix} \end{bmatrix} \qquad (31.25)$$

The ν-Transformation results in the following:

Invariants: $A_o + B_o$, $B_o - B$, C, Q_2, Q_{02}, $Q_1 + Q_3$

$$\left.\begin{aligned} H' &= H \cos 2\nu - F \sin 2\nu \\ F' &= H \sin 2\nu + F \cos 2\nu \end{aligned}\right\} \qquad (31.26)$$

$$\left.\begin{aligned} E' &= E \cos 2\nu + G \sin 2\nu \\ G' &= -E \sin 2\nu + G \cos 2\nu \end{aligned}\right\} \qquad (31.27)$$

$$\begin{aligned} D' &= D \cos 4\nu + \left[A_o - \frac{1}{2}(B_o + B)\right] \sin 4\nu \\ A_o' - \frac{1}{2}(B_o' + B') &= -D \sin 4\nu + \left[A_o - \frac{1}{2}(B_o + B)\right] \cos 4\nu \end{aligned} \qquad (31.28)$$

$$\left.\begin{aligned} Q_{01}' &= Q_{01} \cos 2\nu + Q_{03} \sin 2\nu \\ Q_{03}' &= -Q_{01} \sin 2\nu + Q_{03} \cos 2\nu \end{aligned}\right\} \qquad (31.29)$$

$$\left.\begin{aligned} Q_{12}' &= Q_{12} \cos 2\nu + Q_{23} \sin 2\nu \\ Q_{23}' &= -Q_{12} \sin 2\nu + Q_{23} \cos 2\nu \end{aligned}\right\} \qquad (31.30)$$

$$\left.\begin{aligned} Q_{13}' &= Q_{13} \cos 4\nu - \frac{1}{2}(Q_1 - Q_3) \sin 4\nu \\ \frac{1}{2}(Q_1' - Q_3') &= Q_{13} \sin 4\nu + \frac{1}{2}(Q_1 - Q_3) \cos 4\nu \end{aligned}\right\} \qquad (31.31)$$

Two inverse relationships of interest are:

$$\left.\begin{aligned} 2A_o &= 2A_o' \cos^2 2\nu + (B_o' + B') \sin^2 2\nu + D' \sin 4\nu \\ B_o + B &= (B_o' + B') \cos^2 2\nu + 2A_o' \sin^2 2\nu - D' \sin 4\nu \end{aligned}\right\} \qquad (31.32)$$

$$\left.\begin{aligned} Q_1 &= Q_1' \cos^2 2\nu + Q_3' \sin^2 2\nu - Q_{13}' \sin 4\nu \\ Q_3 &= Q_3' \cos^2 2\nu + Q_1' \sin^2 2\nu + Q_{13}' \sin 4\nu \end{aligned}\right\} \tag{31.33}$$

Since ν is arbitrary, we may choose ν_1 such that $D' = 0$, from which it follows that A_o and $B_o + B$ cannot be independently zero unless $D = 0$. Then from $Q_2 \geq 0$, $C = 0$ also. Similarly, Q_1 and Q_3 can, in general, not be independently zero unless $Q_{13} = 0$.

32. Canonical Distributed Targets; N-Targets

The results thus far obtained have shown that distributed targets in general are determined by nine independent parameters: A_o, B_o, B, C, D, E, F, G, and H. Of these, a group of three parameters, A_o, $B_o + B$, and $B_o - B$, stood out for special consideration. While C, D, E, F, G, and H may have positive, negative, or zero values, the group A_o, $B_o + B$, and $B_o - B$ can have only nonnegative values, and if one of them becomes zero, all must be zero. The parameters also have to satisfy the three inequalities: $Q_1 \geq 0$, $Q_2 \geq 0$, and $Q_3 \geq 0$. We associate a general distributed target with the fact that A_o, $B_o + B$, and $B_o - B$ are not zero. The question can now be posed: Is it possible to find exceptional cases of distributed targets that are simpler in form and for which A_o, $B_o + B$, and $B_o - B$ may be zero individually?

We will show shortly that such targets indeed exist, and these targets are called "canonical targets" because of their simpler form and exceptional nature. Canonical targets, as we will see, play an important role in the so-called canonical decomposition, where a general distributed target R is decomposed into an averaged single target M (which has an equivalent scattering matrix) and a canonical target.

We now determine the nature of canonical targets. The exceptional conditions we found in Sec. 31, such that A_o, $B_o + B$, $B_o - B$ can be individually zero, are summarized as follows:

(1) $(B_o + B)$ and $(B_o - B)$ independently zero if $E = F = 0$ and $Q_1 = 0$
(2) A_o and $(B_o + B)$ independently zero if $C = D = 0$ and $Q_2 = 0$
(3) A_o and $(B_o - B)$ independently zero if $G = H = 0$ and $Q_3 = 0$

We also found a coresponding set of rules for the group of quadratic parameters Q_1, Q_2, and Q_3. These parameters in general only have nonnegative values, and if one of them becomes zero, all must be zero (which results in a single target). Exceptional cases were:

(1) Q_2 and Q_3 independently zero if $Q_{23} = 0$
(2) Q_1 and Q_3 independently zero if $Q_{13} = 0$
(3) Q_1 and Q_2 independently zero if $Q_{12} = 0$

From these conditions, we may construct three types of special distributed targets, depending upon whether $B_o - B = 0$, $B_o + B = 0$, or $A_o = 0$.

(a) Type I Symmetrical Canonical Target

For these targets, $B_o = B$ and $E = F = G = H = 0$. This target is determined by four parameters, A_o, B_o, C, and D. Also: $Q_1 = 0$, $Q_2 \neq 0$, $Q_3 = 0$, and $Q_{ij} = 0$ $(i \neq j)$. This target is a special case (since $B_o = B$) of the general case of symmetric distributed target (Sec. 28). This target may be conceived from a distribution of symmetrical single targets (for which $B_o = B$, $E = F = G = H = 0$).

(b) Type II Symmetrical Canonical Target

For these targets, $B_o = -B$, $C = D = E = F = 0$. This target is determined by A_o, B_o, G, and H. Also $Q_1 = 0$, $Q_2 = 0$, $Q_3 \neq 0$, and $Q_{ij} = 0$ $(i \neq j)$. This target also is a special case of general symmetric distributed target (where $B_o = -B$). It may be obtained from a Type I symmetrical canonical target by a $\pm 45°$ rotation of orientation angle ψ.

(c) Canonical N-(Noise) Targets

These canonical targets have $A_o = 0$ and $C = D = G = H = 0$. This target is given by values of B_o, B, E, and F. Here $Q_1 \neq 0$, $Q_2 = 0$, $Q_3 = 0$, $Q_{ij} = 0$ $(i \neq j)$. In Sec. 14, case (k), we discussed the case of the single N-target for which $A_o = 0$, $C = D = G = H = 0$, and also $Q_1 = Q_2 = Q_3 = 0$. The distributed N-target may be constructed from a distribution of single N-targets.

A glance through the orthogonal transformation tables will convince us that the three canonical targets listed above can be obtained from each other by orthogonal transformations. For example, the Type I symmetric target is obtained from an N-target by a rotation $\tau = \pm 45°$, the Type II symmetrical target by a rotation $\nu = \pm 45°$.

The distributed canonical N-target plays a fundamental role with the decomposition theorems which we consider shortly. They contribute to the remainder (target-noise) components of radar scattering from general distributed targets. We will show that the general decomposition of distributed targets is obtained from the canonical decomposition of distributed targets

$$R' = M' + N'$$

where M' is a mean single target and N' a distributed N-target, through an orthogonal transformation on R', M'. and N', such that $R = OR'O^{-1}$, $M = OM'O^{-1}$, and $R_N = ON'O^{-1}$. The most general decomposition thus has the form $R = M + R_N$, where R_N is the remainder noise component, obtained by an orthogonal transformation of N-targets. Thus, we may include the Type I and Type II canonical targets as special cases of the class of all orthogonal transformation of N-targets.

These arguments have led us to <u>consider the N-targets as the sole generators of all remaining noise components R_N</u>. The privileged selection of the distributed N-target for canonical decompositions of distributed targets R stems from the fact that they appear naturally with applications which we will discuss later (Chap. 8). The Type I and Type II canonical symmetric targets have no such natural appeal.

7 TARGET DECOMPOSITION THEOREMS

33. Fundamental Irreducibility of Single Targets

In previous sections we found a basic result: For distributed targets, the parameters A_0, $B_0 - B$, $B_0 + B$, Q_1, Q_2, and Q_3 are nonnegative. For single targets, however, $Q_1 = Q_2 = Q_3 = 0$. We will now show a fundamental inequality for the sum of two distributed targets I and II:

$$Q_i^{I+II} \geq Q_i^{I} + Q_i^{II} \quad (i = 1, 2, 3) \tag{33.1}$$

We present the proof for Q_1. Since:

$$Q_1 = B_0^2 - (B^2 + E^2 + F^2) = B_0^2 - \underline{B} \bullet \underline{B} \tag{33.2}$$

where we define a vector

$$\underline{B} = (B, E, F) \tag{33.3}$$

we have

$$Q_1^{I+II} = \left(B_0^{I+II}\right)^2 - \left(\underline{B}^{I+II} \bullet \underline{B}^{I+II}\right) = \left(B_0^{I}\right)^2 - \left(\underline{B}^{I} \bullet \underline{B}^{I}\right)$$

$$+ \left(B^{II}\right)^2 - \left(\underline{B}^{II} \bullet \underline{B}^{II}\right) + 2\, B_0^{I}\, B_0^{II} - 2\, \underline{B}^{I} \bullet \underline{B}^{II} \tag{33.4}$$

Now, since $Q_1 \geq 0$, $B_0 \geq |\underline{B}| = (\underline{B} \bullet \underline{B})^{1/2}$ and hence (33.4) shows

$$Q_1^{I+II} = Q_1^{I} + Q_1^{II} + 2\left(B_0^{I}\, B_0^{II} - |\underline{B}^{I}|\ |\underline{B}^{II}| \cos\delta\right) \tag{33.5}$$

which gives the desired result in (33.1) for $i = 1$. Similar proofs are easily provided for $i = 2$ and $i = 3$ by juxtaposition of variables.

From (33.5) we find that equality for the summation law is achieved only if $\delta = 0°$ and $B_0^I = |\underline{B}^I|$, $B_0^{II} = |\underline{B}^{II}|$. For two single (nondistributed) targets, $Q_1^I = 0$, $Q_1^{II} = 0$ and hence $B_0^I = |\underline{B}^I|$, $B_0^{II} = |\underline{B}^{II}|$, but $\delta = 0$ is satisfied only if $\underline{B}^I$ and $\underline{B}^{II}$ are parallel. The same arguments would apply to Q_2 and Q_3 for the two single targets, and hence we conclude that equality in (33.1) would be possible only if the targets had proportional scattering matrices, i.e., if they were the same except for the target amplitude term. One target could be a greater distance from the radar than the other but otherwise the same and viewed at the same aspect direction, hence, the sum of two single targets I and II produce in general $Q_i^{I+II} > 0$ and hence we have proved the irreducibility theorem: <u>A single radar target cannot be decomposed into two or more independent different (single) radar targets</u>.

This theorem, which could be considered basic to electromagnetic scattering problems, points to a fundamental limitation of traditional attempts at "sectionalizing" an object of complex shape, such as an airplane, into independent simpler shapes. Although at higher frequencies, when size to wavelength ratio is large, such methods for computing the radar cross section have had some success, the theorem shows the futility of such attempts at the lower frequencies.

34. Decomposition of Distributed N-Target Into Two Single N-Targets

In this section we will show that a distributed N-target is decomposable into the sum of two single N-targets:

$$N = M_{N1} + M_{N2} \tag{34.1}$$

The decomposition is useful, since the single N-targets M_{N1} and M_{N2} have equivalent scattering matrices, which may point to possible physical sources (voltages or fields) that contribute to N. The decomposition (34.1) is not unique, as we illustrate below.

For N we have the condition for physical realizability: $B_0^N \geq 0$ and $Q_1^N = B_0^{N\,2} - \underline{B}^N \bullet \underline{B}^N \geq 0$, where $\underline{B}^N = (B^N, E^N, F^N)$ and hence $B_0^N \geq |\underline{B}^N|$.

For the single N-targets we have: $B_0^{N1} \geq 0$, $Q_1^{N1} = 0$ and hence $B_0^{N1} = |\underline{B}^{N1}|$; similarly, $B_0^{N2} = |\underline{B}^{N2}|$. Furthermore:

$$\left.\begin{aligned} B_0^N &= B_0^{N1} + B_0^{N2} \\ \underline{B}^N &= \underline{B}^{N1} + \underline{B}^{N2} \end{aligned}\right\} \tag{34.2}$$

The geometric construction shown in Fig. 35 illustrates these properties. The construction clearly demonstrates that many solutions are possible in the plane of the figure, and many more in 3-dimensional space.

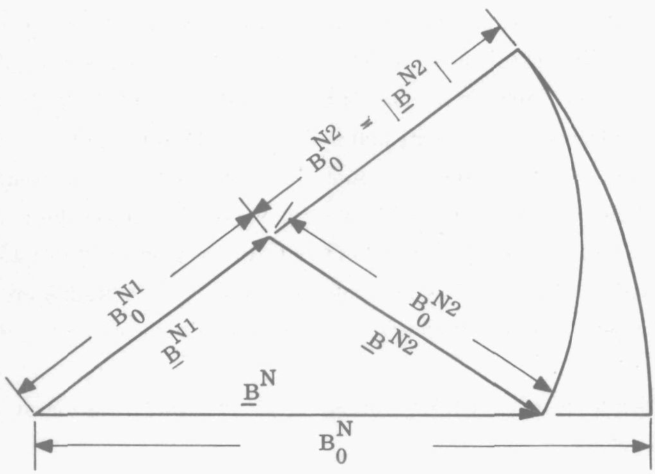


Fig. 35 Decomposition of Distributed Noise Target

We might mention incidentally that the same additive rule construction would apply if $(B_0, \underline{B})$ were considered as 4-dimensional stokes vectors.

The N-target decomposition theorem is used to prove the general decomposition theorem in Sec. 36. Furthermore, since single N-targets have equivalent scattering matrices, the decomposition theorem may be used to identify physical noise sources by voltages and fields which contribute to the distributed N-target noise scattering.

35. Canonical Decomposition of Symmetric Distributed Targets

The class of symmetric distribued targets includes many targets that are important for practical applications.

Not only symmetric distributions of symmetric targets fall within this class but also symmetric distributions of nonsymmetric targets with zero average helicity. One may well assume that all important practical distributed surfaces, such as symmetric distributed rough surfaces, symmetric distributed terrain surfaces, and sea profile models will fall within this class. The class is characterized by the existence at all times of a plane of symmetry through the radar line-of-sight direction and the average target axis of symmetry, or the average surface normal of the distributed radar target. If the target axis is "vertical," a vertical plane of symmetry through the line of sight exists. For the symmetric distributed target, we always assume a preferred coordinate system aligned with one "vertical" axis in the symmetry plane (in the case of terrain) and one (horizontal) axis perpendicular to the plane of symmetry. The so-called orientation angle ψ determines orientation about the radar line-of-sight direction as measured clockwise from the vertical direction (where $\psi = 0$).

For this lineup of coordinate systems, the stokes matrix for a symmetric distributed target is:

$$R_S = \begin{bmatrix} A_0 + B_0 & o & C & o \\ o & -A_0 + B_0 & o & D \\ C & o & A_0 + B & o \\ o & D & o & A_0 - B \end{bmatrix} \qquad (35.1)$$

The H-term is zero because of the preferred alignment of coordinate axis with the plane of symmetry $(\psi = 0)$.

We now proceed to decompose the symmetric distributed target into an average single target component and a remainder component. The remainder component will, in general, have the character of a type of noise called target noise, which has no average single target characteristic. Hence, the remainder terms in the stokes reflection matrix represent average target noise powers.

The average single target stokes matrix also represents average power (for different modes of transmit and receive polarizations). It has, furthermore, the important property that there exists an equivalent effective scattering matrix which is representative of an average single target (since any single target has a representative scattering matrix). However, the impression should not be gained that the single target representation itself may not be due to a noise source. This type of noise we associate with diffuse scattering. Since we are dealing with power averages, nothing is said by the present decomposition of the instantaneous character of the average single target.

A good illustration of this point is the radar return from a homogeneous dipole-cloud with independent elements. The "averaged single target" return for this case is representative of a large sphere or flat plate at normal incidence, but clearly this average sphere representation resulted from a statistical process, and not from a stationary " sphere" target.

We now proceed with the analysis leading to the so-called canonical decomposition theorem for symmetric targets. The most general case obtained from orthogonal transformations of the canonical case will be discussed later.

We show that the symmetric distributed target with stokes matrix M_S may be decomposed as follows:

Let:

$$R_S = \begin{bmatrix} A_0 + B_0^S & o & C & o \\ o & -A_0 + B_0^S & o & D \\ C & o & A_0 + B_0^S & o \\ o & D & o & A_0 - B_0^S \end{bmatrix} + \begin{bmatrix} B_0^N & o & o & o \\ o & B_0^N & o & o \\ o & o & B^N & o \\ o & o & o & -B^N \end{bmatrix} \tag{35.2}$$

In short,

$$R_S = M_S + N_S \tag{35.3}$$

For the symmetric single averaged target M_S, we have the two constraints

$$4\, A_0\, B_0^S = C^2 + D^2 \tag{35.4}$$

$$B_0^S = B^S \tag{35.5}$$

The remainder "noise" components are:

$$B_0^N = B_0 - B_0^S \tag{35.6}$$

$$B^N = B - B_0^S \tag{35.7}$$

We know from the general theory that $A_0 \geq 0$ and $B_0 \geq |B| \geq 0$. We now have to show that N_S is physically realizable.

First, we show that $B_0^N \geq 0$:

$$4\,A_0\,B_0^N = 4\,A_0\,B_0 - (C^2 + D^2) = Q_2 + 2A_0\,(B_0 - B) \geq 0 \qquad (35.8)$$

Hence, the "total noise power," B_0^N, is physically realizable. Next, we have to show that the other basic inequality for physical realizability of N_S: $Q_1^N \geq 0$ is also satisfied:

$$Q_1^N = B_0^{N2} - B^{N2} = B_0^2 - B^2 - 2\,B_0^S\,(B_0 - B) \qquad (35.9)$$

or

$$2\,A_0\,Q_1^N = \left[2\,A_0\,(B_0 + B) - (C^2 + D^2)\right](B_0 - B) \qquad (35.10)$$

The term in brackets is $Q_2 \geq 0$ and, since $A_0 \geq 0$ and $(B_0 - B) \geq 0$, the proposition $Q_1^N \geq 0$ follows. This shows that the decomposition (35.2) is realizable for any general symmetric distributed target, independent of a particular model, by which it may have been constructed.

The remainder noise component is determined by two target parameters, $B_0^N \geq 0$ and B^N, while $Q_1^N = \left(B_0^{N2} - B^{N2}\right) \geq 0$. Notice that the noise component is a special case of the general distributed N-target which is given by B_0^N, B^N, E^N, and F^N, and $Q_1^N \geq 0$, $B_0^N \geq 0$ (Sec. 34).

We may perform one more reduction on the special N-target. Consider the following decomposition:

$$N_S = \begin{bmatrix} B_0^N & & & \\ & B_0^N & & \\ & & B^N & \\ & & & -B^N \end{bmatrix} = \begin{bmatrix} B_1^N & & & \\ & B_1^N & & \\ & & B_1^N & \\ & & & -B_1^N \end{bmatrix} + \begin{bmatrix} B_2^N & & & \\ & B_2^N & & \\ & & -B_2^N & \\ & & & B_2^N \end{bmatrix}$$

(35.11)

where we have written $B_0^N = B_1^N + B_2^N$ and $B^N = B_1^N - B_2^N$.

It is easy to find, from $Q_1^N = 4B_1^N \cdot B_2^N \geq 0$ and $B_0^N = B_1^N + B_2^N \geq 0$, that both B_1^N and B_2^N are positive. The "B_1" stokes matrix represents a trough, with axis horizontal or vertical; the "B_2" stokes matrix represents a trough oriented with axis ±45°. (See Sec. 14.) Hence, we achieve a decomposition of the distributed N_S-target into two single targets which have the trough-like characterizations. The advantage of this procedure is that the stokes matrix single target representations have equivalent effective scattering matrices, which in turn relate to voltages and fields. This is important for applications where the physical target model usually is constructed in terms of voltages and fields. Since the target decompositions were based on additive properties of power, the equivalent voltages and fields must be mutually statistically independent. Hence, the physical model should include noise sources which generate the target-noise components of the scattering independently of the average single target return. We will give an important application of these ideas in Chap. 8 for the case of terrain scattering, where the model for the target noise sources is found with the distribution of orientation of local normal on the terrain surface.

However, we wish to emphasize that the decomposition (35.2) into averaged single target and target-noise components is of a general, phenomenological nature, based on properties of stokes matrices, which is not dependent on any particular physical target model.

36. Decomposition of Arbitrary Distributed Targets Into Three Single Targets

The aim of this and the following two sections is to prepare for the proof of the following target decomposition theorem: An arbitrary distributed target R may be decomposed into an averaged single target M and a remainder distributed N-target, noise component.

This theorem is called the "canonical" decomposition of R, since the remainder is of the simplest form: a canonical distributed N-target. A more general decomposition of R can always be constructed through an orthogonal transformation on R such that $R = OR'O^{-1}$ and by applying to R' the canonical decomposition $R' = M' + N'$, such that now $R = OM'O^{-1} + ON'O^{-1} = M + R_N$, from which it follows that R in the most general case is decomposable into a mean single target M and a distributed noise target R_N which is obtained from an N-target by an orthogonal transformation. In this case, the remainder R_N is of more complex structure than N itself.

The proof of the canonical decomposition of R proceeds in three stages:

(1) We show in this section that any R can be decomposed into three single targets:

$$R = M_0 + M_1 + M_2$$

(2) We show in the next section that the sum of any two single targets M_I and M_{II} is decomposable into a single target and a single N-target:

$$M_I + M_{II} = M + M_N$$

(3) Finally, from the two theorems, we prove the canonical decomposition of general distributed targets:

$$R = M + N$$

The analysis proceeds as follows: Let $R = M + N$ such that:

$$R = \begin{bmatrix} A_0 + B_0^T & F^T & C & H \\ F^T & -A_0 + B_0^T & G & D \\ C & G & A_0 + B^T & -E^T \\ H & D & -E^T & A_0 - B^T \end{bmatrix} + \begin{bmatrix} B_0^N & F^N & o & o \\ F^N & B_0^N & o & o \\ o & o & B^N & -E^N \\ o & o & -E^N & -B^N \end{bmatrix} \tag{36.1}$$

where

$$\left.\begin{aligned} B_0 &= B_0^T + B_0^N \\ B &= B^T + B^N \\ E &= E^T + E^N \\ F &= F^T + F^N \end{aligned}\right\} \qquad (36.2)$$

For the single target M, we have the four conditions (29.21):

$$\left.\begin{aligned} 2\,A_0\left(B_0^T + B^T\right) &= C^2 + D^2 \\ 2\,A_0\left(B_0^T - B^T\right) &= G^2 + H^2 \\ 2\,A_0\,E^T &= DG - CH \\ 2\,A_0\,F^T &= CG + DH \end{aligned}\right\} \qquad (36.3)$$

These relationships determine B_0^T, B^T, E^T and F^T of the M-target uniquely, and hence B_0^N, B^N, E^N and F^N of the N-target also are determined uniquely. Thus, the decomposition $R = M + N$ is unique if we can prove the physical realizability of the N-target.

For physical realizability, we have to show that $B_0^N \geq 0$ and $Q_1^N \geq 0$, since for the N-target the other conditions $A_0^N \geq 0$, $Q_2^N \geq 0$, $Q_3^N \geq 0$ are satisfied. The condition $B_0^N \geq 0$ is easily shown:

$$
\begin{aligned}
4\,A_0\,B_0^N &= 4\,A_0\,B_0 - 4\,A_0\,B_0^T \\
&= 4\,A_0\,B_0 - (C^2 + D^2 + G^2 + H^2) \\
&= \left[2\,A_0\,(B_0 + B) - (C^2 + D^2)\right] + \left[2\,A_0\,(B_0 - B) - (G^2 + H^2)\right] \\
&= Q_2 + Q_3 \geq 0 \qquad (36.4)
\end{aligned}
$$

Now, since $A_0 \geq 0$, $Q_2 \geq 0$ and $Q_3 \geq 0$, $B_0^N \geq 0$ follows.

Incidentally, if equality holds in (36.4), then $A_0 \neq 0$ gives $B_0^N = 0$ and hence for a single averaged target $(Q_1 = Q_2 = Q_3 = 0)$ the noise components disappear completely (if $B_0^N = 0$, from $Q_1^N \geq 0$ it follows that $B^N = E^N = F^N = 0)$, and this agrees with the previously stated (sec. 33) irreducibility of a single target.

Next our main task is to prove $Q_1^N \geq 0$:

$$
\begin{aligned}
Q_1^N &= {B_0^N}^2 - {B^N}^2 - {E^N}^2 - {F^N}^2 \\
&= \left(B_0 - B_o^T\right)^2 - \left(B - B^T\right)^2 - \left(E - E^T\right)^2 - \left(F - F^T\right)^2 \\
&= \left(B_0^2 - B^2 - E^2 - F^2\right) + \left({B_0^T}^2 - {B^T}^2 - {E^T}^2 - {F^T}^2\right) \\
&\qquad - 2\left(B_0\,B_o^T - B\,B^T - E\,E^T - F\,F^T\right) \qquad (36.5)
\end{aligned}
$$

Since for a single target $Q_1^T = 0$, the second term in (36.5) drops out and we have:

$$
Q_1^N = Q_1 - 2\left(B_0\,B_0^T - B\,B^T - E\,E^T - F\,F^T\right)
$$

or

$$2 A_0 Q_1^N = 2 A_0 Q_1 - (B_0 - B)(C^2 + D^2) - (B_0 + B)(G^2 + H^2)$$
$$+ 2 (CG + DH) F - 2 (CH - DG) E \qquad (36.6)$$

After some rearranging and using the expressions for Q_2, Q_3, Q_{01}, and Q_{23}, we can show that from (36.6) we find:

$$4 A_0^2 Q_1^N = Q_2 Q_3 - \left[Q_{01}^2 + Q_{23}^2\right] \qquad (36.7)$$

This is a basic result from which several conclusions can be drawn: Since $Q_2 \geq 0$ and $Q_3 \geq 0$ and $A_0 \geq 0$, expression (36.7) would have the desired result, $Q_1^N \geq 0$, if the term in brackets on the right would disappear. However, in general Q_{01} and Q_{23} are not zero and this presents a difficulty in the proof of the general canonical decomposition theorem. (For symmetric distributed targets, no such difficulty was encountered.) The difficulty may be resolved by first using equation (36.7) to prove the following theorem: A general distributed target R may be decomposed into the sum of three single targets.

The proof of this theorem based upon (36.7) is very simple. Starting with R, we perform first on R an orthogonal transformation $O(2\tau_0)$ such that $Q'_{01} = 0$ for the transformed $R' = O(2\tau_0) R O^{-1}(2\tau_0)$. This can always be done as shown by equation (31.10). Next we apply another orthogonal transformation $O(2\psi_0)$ on R' which results in $R_0 = O(2\psi_0) R' O^{-1}(2\psi_0)$. The angle ψ_0 is chosen such that $Q_{23} = 0$ holds for R_0, but since Q_{01} is invariant under a ψ-transformation, we also have for R_0: $Q_{01} = 0$. The strategy now becomes clear; on R_0 we apply the canonical decomposition which results in $R_0 = M_0 + N_0$ where N_0 is realizable, since $B_0^{N_0} \geq 0$ and from equation (36.7) we have for the R_0-decomposition:

$$4 \left(A_0^{R_0}\right)^2 Q_1^{N_0} = Q_2^{R_0} Q_3^{R_0} - \left[{Q_{01}^{R_0}}^2 + {Q_{23}^{R_0}}^2\right]$$
$$= Q_2^{R_0} Q_3^{R_0} \geq 0 \qquad (36.8)$$

since orthogonal transformation transforms a distributed target into another distributed target $\left(A_0^{R_0} \geq 0,\ Q_2^{R_0} \geq 0, \text{ and } Q_3^{R_0} \geq 0\right)$ and by transformation ψ_0 and τ_0 we made $Q_{01}^{R_0} = 0$ and $Q_{23}^{R_0} = 0$. Hence, we showed that the decomposition $R_0 = M_0 + N_0$ is realizable. We can always decompose the distributed N_0-target into two single N-targets: $N_0 = M_{N1} + M_{N2}$. (See Sec. 34.) Hence, we have $R_0 = M_0 + M_{N1} + M_{N2}$.

Finally we perform the inverse orthogonal transformations $O(2\psi_0)$ and $O(2\tau_0)$ on R_0 to reproduce the original R. Since orthogonal transformations on single targets M_0, M_{N1}, and M_{N2} produce other single targets, the proof of the theorem is completed and therefore:

$$R = M + M_1 + M_2 \tag{36.9}$$

This theorem is used as an intermediate step for the final proof that $Q_1^N \geq 0$ in equation (36.7) for general R. In the next section, we proceed with the second part of the proof, to show that the sum of two single targets may be decomposed into a single target and a single N-target.

37. Decomposition of Two Single Targets Into Single Target and Single N-Target

We wish to prove in this section the following property: Let $R = M_I + M_{II}$, where M_I and M_{II} are two single targets, then $R = M + M_N$, where M may be considered a mean single target and M_N is the single N-target remainder noise component. The proof of this theorem uses the derivation of Sec. 36 for general $R = M + N$ decomposition. We showed that the decomposition is unique if we can prove its physical realizability. There was no problem in the determination of M. For the N-target noise, we showed $B_0^N \geq 0$.

The second condition was to prove that $Q_1^N = B_0^{N^2} - B^{N^2} - \left(E^{N^2} + F^{N^2}\right) \geq 0$. The derivation produced (36.7):

$$4 A_0^2 Q_1^N = Q_2 Q_3 - \left(Q_{01}^2 + Q_{23}^2\right) \tag{37.1}$$

We wish to show that for the sum of two independent single targets $Q_1^N = 0$.

This is such an important result for the proof of the general target decomposition theorem that we present a complete proof here: For the right-hand side of (37.1), we can write:

$$
\begin{aligned}
Q_2 Q_3 - Q_{01}^2 - Q_{23}^2 &= \left[2 A_0 (B_0 + B) - (C^2 + D^2)\right]\left[2 A_0 (B_0 - B) - (G^2 + H^2)\right] \\
&\quad - \left[2 A_0 F - (CG + DH)\right]^2 - \left[2 A_0 E + (CH - DG)\right]^2 \\
&= 4 A_0^2 \left(B_0^2 - B^2\right) + (C^2 + D^2)(G^2 + H^2) - 2 A_0 (B_0 + B) \\
&\quad (G^2 + H^2) - 2 A_0 (B_0 - B)(C^2 + D^2) - 4 A_0^2 (E^2 + F^2) \\
&\quad - (CG + DH)^2 - (CH - DG)^2 + 4 A_0 F (CG + DH) \\
&\quad - 4 A_0 E (CH - DG) = \\
&= 2 A_0 \left[2 A_0 Q_1 + (B_0 - B) Q_2 + (B_0 + B) Q_3 - 4 A_0 (B_0^2 - B^2)\right. \\
&\quad \left. + 4 A_0 (E^2 + F^2) - 2 E Q_{23} - 2 F Q_{01}\right] \\
&= 2 A_0 \left[-2 A_0 Q_1 + (B_0 - B) Q_2 + (B_0 + B) Q_3 - 2 E Q_{23}\right. \\
&\quad \left. - 2 F Q_{01}\right]
\end{aligned}
\tag{37.2}
$$

Define:

$$Q_2 Q_3 - Q_{01}^2 - Q_{23}^2 = 2 A_0 \chi \tag{37.3}$$

where

$$\chi = -2 A_0 Q_1 + (B_0 - B) Q_2 + (B_0 + B) Q_3 - 2 E Q_{23} - 2 F Q_{01} \tag{37.4}$$

We shall see shortly that χ plays an important role in the theory of matrices of type R developed in Sec. 40. We wish to show that $\chi = 0$ if $R = M_I + M_{II}$. We have:

$$\left.\begin{aligned}
Q_1 &= 2\left(B_0^I B_0^{II} - B^I B^{II} - E^I E^{II} - F^I F^{II}\right) \\
Q_2 &= 2\left[A_0^I \left(B_0 + B\right)^{II} + A_0^{II} \left(B_0 + B\right)^I - C^I C^{II} - D^I D^{II}\right] \\
Q_3 &= 2\left[A_0^I \left(B_0 - B\right)^{II} + A_0^{II} \left(B_0 - B\right)^I - G^I G^{II} - H^I H^{II}\right] \\
Q_{01} &= 2 A_0^I F^{II} + 2 A_0^{II} F^I - (C^I G^{II} + D^I H^{II}) - (C^{II} G^I + D^{II} H^I) \\
Q_{23} &= 2 A_0^I E^{II} + 2 A_0^{II} E^I + (C^I H^{II} - D^I G^{II}) + (C^{II} H^I - D^{II} G^I)
\end{aligned}\right\} \quad (37.5)$$

since for single target M_I: $Q_i^I = 0$ $(i = 1, 2, 3)$ and $Q_{ij}^I = 0$ $(i \neq j\ ;\ 0, 1, 2, 3)$ and similarly for M_{II}. Substitution of these expressions into (37.4) gives after simple algebra the desired result $\chi = 0$. This completes the proof of the theorem.

Some interesting observations can be made at this point:

(1) The condition $\chi = 0$ may be applied to radar targets as a test for observing two independent single targets.

(2) The theorem just proved provides an insight into the nature of target noise sources: The single target noise is generated as the residue after combining two single targets into one average single target.

This fact throws new light on the general decomposition, $R = M + N$, which we will prove shortly: The general distributed target R is composed of a summation (a time average) of independent single targets $M_1, M_2, \ldots, M_n$, which results in an average single target and the distributed N-target residue component. The residue is obtained as the sum of all single noise target residue components, arising from the summation of single targets. All these facts indicate that the canonical decomposition will have important practical applications. Some illustrations are presented in Chap. 8.

38. Canonical Decomposition of General Distributed Targets

We wish to prove in this section that a general distributed target R may be decomposed into the sum of a general single target M and a remainder noise component N: $R = M + N$. The summation is understood in the phase-independent statistical sense (discussed in Chap. 5) of the components. Two conditions were necessary for the proof: The first was that the total noise power term was nonnegative: $B_0^N \geq 0$. The second condition was to show that: $Q_1^N = B_0^{N2} - B^{N2} - \left(E^{N2} + F^{N2}\right) \geq 0$, from which all the conditions for a physically realizable noise target N: $A_0^N = 0$, $B_0^N - B^N \geq 0$, $B_0^N + B^N \geq 0$, $Q_1^N \geq 0$, $Q_2^N = 0$, $Q_3^N = 0$ are satisfied. The condition $B_0^N \geq 0$ was shown without difficulty, but $Q_1^N \geq 0$ remains to be determined.

The procedure we will follow is an indirect proof of this property. First, we showed that if any distributed target matrix R consists of the sum of two independent single targets: $M_1 + M_2$, then it is decomposable into a single target M and a single noise target: M_N. After this is shown, the general decomposition theorem follows easily. In Sec. 36 we proved that any R-matrix is decomposable into the sum of three independent single targets: $R = M_0 + M_1 + M_2$. From the above theorem, we then have for the first two single targets: $M_0 + M_1 = M_I + M_{NI}$; hence $R = M_I + M_2 + M_{NI}$. Next we apply the same property to the first two single targets in R: $M_I + M_2 = M + M_{NII}$. Substitution into R gives the desired decomposition:

$$R = M + M_{NI} + M_{NII} = M + N \tag{38.1}$$

The last statement follows from the fact that the sum of two single N-target results in a distributed N-target.

This completes the proof of the general target decomposition theorem. From the decomposition theorem are derived necessary and sufficient conditions for R to be a distributed target matrix.

39. Necessary and Sufficient Conditions for Physical Realizability of the Stokes Matrix

We review briefly the procedure by which necessary conditions on the stokes matrix R were established. We started (Sec. 11) with single target parameters A_0, $B_0 + B$, $B_0 - B$ which were derived from the target scattering matrix T, to show that these were nonnegative. Through time averages, the corresponding distributed target parameters A_0, $B_0 - B$, $B_0 + B$ of R also were shown to be nonnegative. For R we found the trace rule, trace R $= 2\,(A_0 + B_0)$, to be universally valid. Starting from these properties, we then considered the time-averaged scattering: $s = Rg\,(\underline{a}) = (s_0, s_1, s_2, s_3)$ from the distributed target R which is illuminated by a fixed transmit antenna $\underline{a}$.

It was shown that s is a partially polarized stokes vector which satisfies the condition for physical realizability (Sec. 24), $s_0^2 \geq s_1^2 + s_2^2 + s_3^2$, and from this fact we could show (Sec. 30) that then the condition on the quadratic terms $Q_1 \geq 0$, $Q_2 \geq 0$ and $Q_3 \geq 0$ is necessary. It was then shown (Sec. 38) under which conditions R has a unique canonical decomposition $R = M + N$ into an average single target M and a remainder noise component N.

A careful study reveals that the seven conditions: trace $R = 2\,(A_0 + B_0)$, $A_0 \geq 0$, $B_0 - B \geq 0$, $B_0 + B \geq 0$, $Q_1 \geq 0$, $Q_2 \geq 0$ and $Q_3 \geq 0$ were necessary to prove the theorem. The question still unanswered is: Are these conditions also sufficient for R to be physically realizable? We recall that physical realizability of R implies that if $g\,(\underline{a})$ is a completely polarized antenna illumination, then $s = Rg$ is physically realizable as a stokes vector. We know that the distributed noise target N may be decomposed into the sum of two single noise targets M_N: $N = M_{N1} + M_{N2}$ (Sec. 34). Hence:

$$s = Rg = Mg + M_{N1}\,g + M_{N2}\,g \tag{39.1}$$

The right-hand side of (39.1) consists of a sum of three completely polarized stokes vectors, each of which is physically realizable. In Sec. 24 we showed that the sum of realizable stokes vectors again produces a physically realizable stokes vector.

This then completes the proof for physical realizability of R, and hence the seven conditions mentioned above are shown to be necessary and sufficient. A matrix which has these properties is called "of type R." The next section will have more discussion on matrices of type R.

40. Higher Order Matrices of Type R

We found in Sec. 31 a curious similarity between orthogonal transformations of the nine first-order target parameters, A_0, B_0, B, C, D, E, F, G, and H, and the nine quadratic terms, Q_1, Q_2, Q_3, and Q_{01}, Q_{02}, Q_{03}, Q_{12}, Q_{13}, Q_{23}. This correspondence is not accidental, as will be shown in this section by a simple observation.

Let $Rg = s$ be the backscattered stokes vector of target R. Then (Sec. 29):

$$q_1 Q_1 + q_2 Q_2 + q_3 Q_3 + \sum_{i \neq j} q_{ij} Q_{ij} = s_0^2 - s_1^2 - s_2^2 - s_3^2$$

$$= s' \bullet s = \begin{bmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{bmatrix} Rg \bullet Rg = R \begin{bmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{bmatrix} R g \cdot\bullet g \tag{40.1}$$

The last step is possible due to the symmetry of the R-matrix. Now q_1, q_2, q_3, and q_{ij} are determined solely by the transmit antenna $g(\underline{a})$, and Q_i, Q_{ij} are determined completely by the target parameters given by R. Since (40.1) shows a linear relationship on both sides of the equality signs, the Q-terms must be determined completely by a linear addition of coefficients of the matrix:

$$[Q] = R \begin{bmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{bmatrix} R \tag{40.2}$$

If we substitute R into (40.2) and compare coefficients of the resulting matrix with the definition of Q_1, Q_2, Q_3, and Q_{ij} in Sec. 29, we find a particularly simple result:

$$[Q] = \begin{bmatrix} Q_{00} & Q_{01} & Q_{02} & Q_{03} \\ Q_{10} & Q_{11} & Q_{12} & Q_{13} \\ Q_{20} & Q_{21} & Q_{22} & Q_{23} \\ Q_{30} & Q_{31} & Q_{32} & Q_{33} \end{bmatrix} \tag{40.3}$$

where $Q_{ij} = Q_{ji}$ $(i \neq j)$ are given by (29.12) through (29.17) and:

$$\left.\begin{aligned} Q_{00} &= \frac{Q_1}{2} + \frac{Q_2 + Q_3}{2} + \frac{Q_4}{2} \\ Q_{11} &= -\frac{Q_1}{2} + \frac{Q_2 + Q_3}{2} - \frac{Q_4}{2} \\ Q_{22} &= \frac{Q_1}{2} + \frac{-Q_2 + Q_3}{2} - \frac{Q_4}{2} \\ Q_{33} &= \frac{Q_1}{2} - \frac{-Q_2 + Q_3}{2} - \frac{Q_4}{2} \end{aligned}\right\} \tag{40.4}$$

Here Q_1, Q_2, and Q_3 were defined by (29.7), (29.8), and (29.9) and

$$Q_4 = 2\left(A_0 - B_0\right)^2 - \left(B_0^2 - B^2\right) - (C^2 - D^2) + (E^2 - F^2) + (G^2 - H^2) \tag{40.5}$$

The matrix (40.3) and the forms (40.4) suffice to explain the correspondence between orthogonal transformations on **R** and on [Q] coefficients.

First, we observe that an orthogonal transformation O on R such that $R' = ORO^{-1}$ leads by definition (40.2) to $[Q]' = O\,[Q]\,O^{-1}$, since the matrix

$$\begin{bmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{bmatrix}$$

is unaffected by the orthogonal transformation (of the ψ, τ, and ν rotation variety, as was considered in previous sections). We recall that

$$R = \begin{bmatrix} A_0 + B_0 & F & C & H \\ F & -A_0 + B_0 & G & D \\ C & G & A_0 + B & -E \\ H & D & -E & A_0 - B \end{bmatrix} \tag{40.6}$$

The correspondences under orthogonal transformations between coefficients of R in (40.6) and [Q] in (40.3) are now easily established; i.e., $F \to Q_{01}$, $C \to Q_{02}$, $H \to Q_{03}$, etc., and $A_0 \to \frac{Q_1}{2}$, $B_0 \to \frac{Q_2 + Q_3}{2}$, $B \to \frac{-Q_2 + Q_3}{2}$. We notice that the term Q_4 is invariant under all orthogonal transformations. The absence of a term equivalent to Q_4 in the R-matrix leads to the trace rule which is characteristic for R. We list the following interesting correspondences:

$$2\,A_0 \rightarrow Q_1$$

$$(B_0 - B) \rightarrow Q_2$$

$$(B_0 + B) \rightarrow Q_3$$

These results explain the observed similarities of the orthogonal transformation between first-order and second-order transformations of R.

The correspondence between [Q] and R matrices can be made complete if we subtract the Q_4 term, which has no R matrix equivalent, from [Q]. Let

$$Q_R = [Q] - \frac{1}{2} Q_4 \begin{bmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{bmatrix} \tag{40.7}$$

Then Q_R has the same properties as R, as we will prove shortly.

We already know that for the first-order terms in R: A_0, $B_0 + B$ and $B_0 - B \geq 0$. Furthermore, for the first-order terms in Q_R (second order for R) we showed: $Q_1, Q_2, Q_3 \geq 0$. From $[Q]\, g \bullet g \geq 0$, it follows that $Q_R\, g \bullet g \geq 0$ since the second term resulting from Q_4 in (40.1) vanishes. This, of course, is equivalent to $R\, g \bullet g \geq 0$, which gives the return from a distributed target for parallel reception.

The trace rule for R was: trace $R = 2\,(A_0 + B_0)$; now for Q_R we find: trace $Q_R = Q_1 + Q_2 + Q_3$, which is equivalent to $2\,(A_0 + B_0)$, since $2\,A_0 \rightarrow Q_1$ and $2\,B_0 \rightarrow Q_2 + Q_3$. What we like to show is that Q_R behaves in every respect like an R matrix, such that if $Q_R\, g = \mathbf{s}$, then $\mathbf{s}$ is a stokes vector for which $\mathbf{s}' \cdot \mathbf{s} \geq 0$ and, hence,

$$Q_R \begin{bmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{bmatrix} Q_R\, g \bullet g = [Q]^{(2)}\, g \bullet g \geq 0 \tag{40.8}$$

If this were the case, then we could find second-order relations for Q_R equivalent to the second-order relations for R:

$$\left.\begin{aligned} Q_1 &= \left(B_0^2 - B^2\right) - (E^2 + F^2) \geq 0 \\ Q_2 &= 2\,A_0\,(B_0 + B) - (C^2 + D^2) \geq 0 \\ Q_3 &= 2\,A_0\,(B_0 - B) - (G^2 + H^2) \geq 0 \end{aligned}\right\} \tag{40.9}$$

which for Q_R become:

$$\left.\begin{aligned} Q_1^{(2)} &= Q_2\,Q_3 - \left(Q_{23}^2 + Q_{01}^2\right) \geq 0 \\ Q_2^{(2)} &= Q_1\,Q_3 - \left(Q_{02}^2 + Q_{13}^2\right) \geq 0 \\ Q_3^{(3)} &= Q_1\,Q_2 - \left(Q_{12}^2 + Q_{03}^2\right) \geq 0 \end{aligned}\right\} \tag{40.10}$$

We will show that these relations are indeed satisfied.

The proof $Q_1^{(2)} \geq 0$ is shown immediately from the decomposition $R = M + N$. In (37.1) we found: $4\,A_0^2\,Q_1^N = Q_1^{(2)}$ and since the N-target was shown to be physically realizable, $Q_1^N \geq 0$ and hence $Q_1^{(2)} \geq 0$, since $A_0 \geq 0$. That also $Q_2^{(2)} \geq 0$ and $Q_3^{(2)} \geq 0$ can be shown easily by the following argument. First, we verify by direct computation the interesting results:

$$Q_1^{(2)} = Q_2\,Q_3 - \left(Q_{23}^2 + Q_{01}^2\right) = 2\,A_0\,\chi \tag{40.11}$$

$$Q_2^{(2)} = Q_1\,Q_3 - \left(Q_{02}^2 + Q_{13}^2\right) = (B_0 - B)\,\chi \tag{40.12}$$

$$Q_3^{(2)} = Q_1\,Q_2 - \left(Q_{12}^2 + Q_{03}^2\right) = (B_0 + B)\,\chi \tag{40.13}$$

where χ is as defined in (37.6). From the fact that $Q_1^{(2)} \geq 0$, it follows that $\chi \geq 0$, and also from (40.12) and (40.13) we have $Q_2^{(2)} \geq 0$ and $Q_3^{(2)} \geq 0$. Hence, the correspondence between R and Q_R matrices is complete.

Any matrix Q_R that behaves in every respect like an R-matrix is called "of type R." A matrix R is of type R if A_0, $B_0 + B$ and $B_0 - B \geq 0$, Q_1, Q_2, $Q_3 \geq 0$ and trace $R = 2(A_0 + B_0)$. Hence, Q_R is of type R if Q_1, Q_2, $Q_3 \geq 0$, and $Q_1^{(2)}$, $Q_2^{(2)}$ and $Q_3^{(2)} \geq 0$, and trace $Q_R = Q_1 + Q_2 + Q_3$.

Once this is established, higher order matrices of type R may be constructed.

Let

$$Q_R^{(2)} = Q_R \begin{bmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{bmatrix} Q_R - \frac{1}{2} Q_4^{(2)} \begin{bmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{bmatrix} \tag{40.14}$$

Then, by similar reasoning, we can show that $Q_R^{(2)}$ with first-order terms $Q_1^{(2)}, Q_2^{(2)}, Q_3^{(2)} \geq 0$ is of type R, and in general any higher order matrix $Q_R^{(n)}$ thus generated will be of type R. Two important applications are given in the next section.

41. Two Basic Criteria for Radar Target Classification

We will show that the concept of higher order matrices of type R has important application to radar target classification problems. Consider a case where a radar views a time-varying set of objects in space. The question is if the foregoing theory allows one to determine how many independent targets are observed. For two cases necessary criteria are presented:

(1) If a single object is observed, $R = M$ and then $Q_R = 0$.

(2) If R consists of the sum of two independent single targets: $R = M_I + M_{II}$, then $Q_R^{(2)} = 0$.

The first property is easily shown. Let $Rg = s$; since $R = M$, and g is completely polarized (cp) illumination, s is cp. Hence:

$$s_0^2 - s_1^2 - s_2^2 - s_3^2 = \begin{bmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{bmatrix} s \bullet s = 0$$

or

$$R \begin{bmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{bmatrix} R\, g \bullet g = [Q]\, g \bullet g = Q_R\, g \bullet g = 0$$

This is satisfied for all cp vectors g if $Q_R = 0$. The nine conditions thus specified for R to be a single target are not all independent, as was shown in Sec. 12. Only four independent conditions on R were necessary.

The second theorem follows from the fact that if $R = M_I + M_{II}$, then from Sec. 37: $Q_1^{(2)} = 0$, $Q_2^{(2)} = 0$ and $Q_3^{(2)} = 0$. Since the matrix $Q_R^{(2)}$ is of type R, and the first-order diagonal terms vanish for this matrix, all terms must vanish; hence, $Q_R^{(2)} = 0$. This completes the proof of the two theorems.

One may speculate at this point if perhaps for three independent single targets $Q_R^{(3)} = 0$, etc. However, it is easy to show that this is not the case. The higher order matrices of type R reveal a basic structure from which the above negative conclusion can be drawn. As before (37.4), we define:

$$\chi = -2\, A_0\, Q_1 + (B_0 - B)\, Q_2 + (B_0 + B)\, Q_3 - 2\, E\, Q_{23} - 2\, F\, Q_{01} \tag{41.1}$$

The following interesting connections for higher order matrices of type R are easily derived:

$$Q_R^{(2)} = \chi R \tag{41.2}$$

$$Q_R^{(3)} = \chi^2 Q_R \tag{41.3}$$

$$Q_R^{(4)} = \chi^5 R \tag{41.4}$$

$$Q_R^{(5)} = \chi^{10} Q_R \tag{41.5}$$

For two independent single targets, $\chi = 0$ and hence $Q_R^{(2)} = 0$ as was shown above. The condition $Q_R^{(3)} = 0$ would lead to the same result: $\chi = 0$ (assuming Q_R not zero; otherwise, this would indicate a single target) and hence $Q_R^{(2)} = 0$, which is the condition for two independent single targets. We thus conclude that the case of three independent single targets cannot be characterized by simple criteria. This negative result is not too surprising, since we showed that a general distributed target may be decomposed into three independent targets, and the general case has no simple criteria.

8 APPLICATIONS TO ROUGH SURFACE SCATTERING

42. Introduction

In a recently published book [6], Beckmann discusses electromagnetic scattering from rough extended surfaces, such as terrain or sea state. His discussion follows a formulation by Fung [5] of Kirchhoff integration over the surface, which includes the effect on the scattering of the local normal $\underline{n}$. Older treatments use an averaged normal $\bar{\underline{n}}$ on the surface, such that the effect of local normal $\underline{n}$ is averaged out at the initial stages of the calculation, instead of at the final stage as is done by Fung. The result of this older procedure was to exclude all local effects of depolarization on the scattering. This led to the widely held opinion that Kirchhoff integration techniques cannot predict the depolarization effects characteristic of rough surface scattering, except for depolarization effects associated with an infinite plane with average Fresnel reflection coefficients $\bar{R}^+$ and $\bar{R}^-$.

In these theories, the statistical nature of the rough surface is introduced as a random height displacement of local position $\underline{r}$ on the surface, which affects locally the phase $e^{i\underline{k}\cdot\underline{r}}$ relative to the direction $\underline{k}$ of incoming illumination. The emphasis is thus on the evaluation of the integral

$$\rho = \iint e^{2i\underline{k}\cdot\underline{r}}\, dS \tag{42.1}$$

which sums the phase contributions to scattered return, over the uniformly illuminated area on the surface. By averaging over an ensemble of such surfaces, the average return $<\rho>$ and average intensity $<\rho\rho^*>$ are determined [32]. Because the integrand in (42.1) is a scalar which does not include the polarization of the illuminating field, the procedures associated with the use of equation (42.1) are called "scalar" theories, although frequently a polarization-dependent average reflection coefficient is introduced as multiplier with integral (42.1). The first book by Beckmann and Spizzichino on scattering from rough surfaces [32] deals almost exclusively with the "scalar" theory of scattering as defined above.

It is interesting at this point to note the basic difference between the "scalar" theory, based on averaged surface normal $\underline{\bar{n}}$, and the electromagnetic approach, which takes into account the local normal $\underline{n}(\theta, \psi)$ where θ is the "local aspect angle," between the local normal and incoming illumination direction $\underline{k}$, and ψ is the angle which determines the rotation of $\underline{n}$ about $\underline{k}$ (see Fig. 36); ψ is called the local orientation angle of surface element dA.

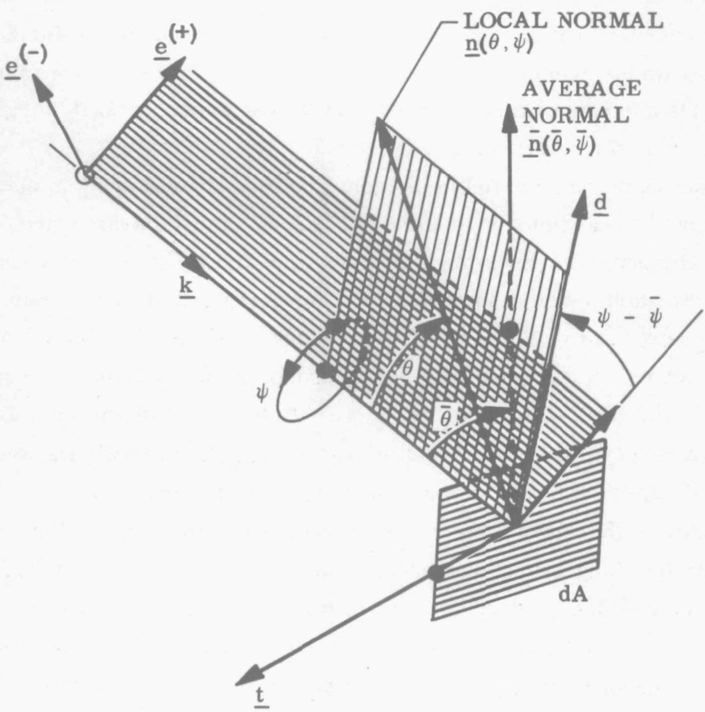


Fig. 36 Geometry of Local Surface Scattering

Hence, the em theory, which includes $\underline{n}(\theta, \psi)$, introduces the effects on scattering of the orientation angle ψ, whereas orientation ψ is absent in the scalar theories. The averaged $\bar{\underline{n}}$ has a fixed position with $\bar{\psi} = 0$ and average angle $\bar{\theta}$ relative to the $\underline{k}$ direction (which is held fixed during the integration over the surface elements). The angle $\bar{\theta}$ is the average aspect angle over all surface elements. With $\bar{\theta}$ are associated the average reflection coefficients on the surface $\bar{R}^{+}(\bar{\theta})$ and $\bar{R}^{-}(\bar{\theta})$. Since $\bar{\theta}$ changes, with change of aspect direction $\underline{k}$, the target scattering dependenee on average aspect angle is well documented in the literature [32]. However, the dependence on orientation $\bar{\psi}$ is largely ignored since the average orientation angle $\bar{\psi} = 0$ does not change with changes of $\underline{k}$.

We intend to emphasize in this study the ψ-dependence on local scattering of the total target scattering. We will show that due to the orientation angle ψ-dependence of local normal we may account for a type of depolarized target scattering whose existence has not been emphasized before. The orientation angle ψ is a random variable which contributes to depolarized "target noise" components in the return scattering.

In summary, the far field electromagnetic back scattering is obtained by integration of local fields over elements dS on the illuminated surface. The surface elements are determined by a position vector $\underline{r}$ and a local normal $\underline{n}(\theta, \psi)$. Random height displacements of surface position $\underline{r}$ have an effect on the phase $\exp(i\underline{k} \bullet \underline{r})$ of the return scattering, while the random changes in surface normal $\underline{n}$ are accounted for by independent variations in local aspect angle θ, and local orientation angle ψ. The average of random variations in θ leads to average surface reflection coefficient $\overline{R}(\bar{\theta})$, while the average of random changes in ψ is the source of target orientation noise.

Details of these concepts will be worked out in the next sections. We will start with the Fung formulation of Kirchhoff integration; then proceed to separate the independent scattering components, based upon the general decomposition theorems derived in previous sections. The distributed terrain target is thus decomposed into an averaged terrain target plus target noise. Hence, the averaged target for a rough flat terrain surface will be a smooth flat surface which is characterized by two averaged surface reflection coefficients.

We will also discuss the scattering from nonflat, rough surfaces, such as rolling hills or contoured terrain surfaces.

43. Radar Backscatter From Rough Extended Surfaces

In Beckmann's recent book [6], he discusses radar backscatter from extended rough surfaces based on a physical optics formulation, which includes the effects of material properties of the surface. On page 94 of his book, he arrives at expressions for parallel and cross-polarized components of the backscattered electric field (E_p viz. E_c) for a linearly polarized incident field:

$$E_p = C \iint \cos\theta \, [(R^+ - R^-) - (R^+ + R^-) \cos 2\psi] \, e^{-2i(\underline{k} \cdot \underline{r})} \, dS \qquad (43.1)$$

$$E_c = C \iint \cos\theta \, (R^+ + R^-) \sin 2\psi \, e^{-2i(\underline{k} \cdot \underline{r})} \, dS \qquad (43.2)$$

where

$$\left.\begin{aligned} \cos\theta &= -\underline{n} \cdot \underline{k} \\ \cos\psi &= \underline{a} \cdot \underline{t} \\ C &= \frac{ik}{2} e^{-2kR_o} / 4\pi R_o \end{aligned}\right\} \qquad (43.3)$$

Identifying quantities are:

(1) For the illumination:
Frequency $f = ck/2\pi$ (c is the light velocity)
Linear polarization vector $\underline{a}$ of incident field
Direction of incoming plane wave: $\underline{k}$
Far field source distance from origin: R_o

(2) For the Surface:

Local normal $\underline{n}$: average normal $\bar{\underline{n}}$

Local tangent $\underline{t} = \underline{n} \times \underline{k} / |\underline{n} \times \underline{k}|$

Local position of point on surface S : $\underline{r}$

Local complex relative permittivity: $\epsilon' = \epsilon - i\, 60\, \lambda\, \sigma$

(ϵ is dielectric constant, σ conductivity, $\lambda = 2\pi/k$)

The Fresnel reflection coefficients are:

$$R^{+} = \frac{\epsilon' \cos\theta - \sqrt{\epsilon' - \sin^2\theta}}{\epsilon' \cos\theta + \sqrt{\epsilon' - \sin^2\theta}} \tag{43.4}$$

$$R^{-} = \frac{\cos\theta - \sqrt{\epsilon' - \sin^2\theta}}{\cos\theta + \sqrt{\epsilon' - \sin^2\theta}} \tag{43.5}$$

For further details on coordinates and units and restrictions of applicability of physical optics methods, we refer to the referenced book and to Fig. 36 of this work.

Polarization "in the plane" of incoming direction $\underline{k}$ and average normal $\bar{\underline{n}}$ is designated by $\underline{e}^{(+)}$, "out of the plane" by $\underline{e}^{(-)}$. Locally, polarization "in the plane" of $\underline{k}$ and $\underline{n}$ is given by direction $\underline{d} = \underline{k} \times \underline{t}$, "out of the plane" by $\underline{t}$. For reasons which will become clear shortly, we prefer to define an angle ψ_n such that $\cos\psi_n = \underline{e}^{(+)} \cdot \underline{d} = \underline{e}^{(-)} \cdot \underline{t}$ where $\underline{e}^{(+)}$ is fixed as polarization "in the plane." The angle ψ_n is called the <u>local orientation</u> angle of surface normal $\underline{n}$ with respect to the direction of incoming illumination.

For the average "vertical" normal $\bar{\underline{n}}$ of a flat terrain, the orientation angle $\bar{\psi}_n$ is zero. Equations (43.1) and (43.2) can now be written for the three cases of linear parallel reception: (1) "in the plane": $E_p^{(+)}$; (2) "out of the plane": $E_p^{(-)}$; and (3) linear cross-polarized reception in or out of the plane (which makes no difference because of the reciprocity theorem):

$$\left.\begin{aligned}
E_p^{(+)} &= C\iint \cos\theta\left[(R^+ - R^-) + (R^+ + R^-)\cos 2\psi_n\right] e^{-2i(\underline{k}\cdot\underline{r})}\, dS \\
E_p^{(-)} &= C\iint \cos\theta\left[(R^+ - R^-) - (R^+ + R^-)\cos 2\psi_n\right] e^{-2i(\underline{k}\cdot\underline{r})}\, dS \\
E_c &= C\iint \cos\theta\,(R^+ + R^-)\sin 2\psi_n\, e^{-2i(\underline{k}\cdot\underline{r})}\, dS
\end{aligned}\right\}\quad (43.6)$$

These equations form the basis for our further investigations.

The first formulations of this kind, i.e., physical optics solution of radar backscatter with Fresnel coefficients including the local surface normal, were given by Fung [5]. Huynen, following Fung's work, derived equations (43.6) in a report [51] and extended these results [52], which led to the theory to be developed below.

Equations (43.6) simply express the (H-H), (V-V), and (H-V) = (V-H) components of the target scattering matrix, discussed in Sec. 8, where H stands for polarization "in the plane" (of $\underline{k}$ and $\bar{\underline{n}}$) and V is polarization "out of the plane":

$$T = \begin{bmatrix} E_p^{(+)} & E_c \\ E_c & E_p^{(-)} \end{bmatrix} = \iint T_n\, dS \qquad (43.7)$$

Here T_n is the so-called local target matrix (referred to the local normal $\underline{n}$) given by:

$$T_n = \frac{A_n}{2}\begin{bmatrix} R^+ - R^- + \cos 2\psi_n\,(R^+ + R^-) & \sin 2\psi_n\,(R^+ + R^-) \\ \sin 2\psi_n\,(R^+ + R^-) & R^+ - R^- - \cos 2\psi_n\,(R^+ + R^-) \end{bmatrix} \qquad (43.8)$$

with

$$A_n = \frac{ik}{4\pi R_o} e^{-2ik R_o - 2i \underline{k} \cdot \underline{r}} \cos \theta$$

The matrix T_n is easily shown to be equal to:

$$T_n = A_n \begin{bmatrix} \cos \psi_n & -\sin \psi_n \\ \sin \psi_n & \cos \psi_n \end{bmatrix} \begin{bmatrix} R^+ & 0 \\ 0 & -R^- \end{bmatrix} \begin{bmatrix} \cos \psi_n & \sin \psi_n \\ -\sin \psi_n & \cos \psi_n \end{bmatrix} \qquad (43.9)$$

The physical significance of the local target matrix T_n is thus revealed at once: It is simply the target reflection matrix for the local tangent plane, determined by local reflection coefficients $R^\pm$, which is rotated (reoriented) by an angle ψ_n with respect to the fixed antenna coordinate frame.

Several interesting comments can be made at this point. We notice the absence of "helicity" ($\tau_m = 0$) (Sec. 10) with the local target scattering matrix; hence, T_n represents locally a symmetric target (characterized by R^+ and R^-), which is oriented with the direction of the normal $\underline{n}$. In fact, it represents locally a roll-symmetric target, with $\underline{n}$ as roll-axis (a local flat tangent plane), since $R^\pm$ depends only on θ, not on rotation of incident direction about $\underline{n}$. For $\theta = 0$, $R^+ = -R^-$, such that $T_n = A_n' \, I$ becomes independent of ψ_n; it is then locally an orientation independent target (a flat plate at normal incidence). For a rough surface, the position vector $\underline{r}$, the direction of local normal $\underline{n}$, and the local surface material properties may be considered random variables. The position vector $\underline{r}$ influences the phase $e^{-2i(\underline{k} \cdot \underline{r})}$ in A_n, but appears nowhere else in T_n. For this reason, effects of scattering from rough surfaces due to random changes in position are called "scalar" effects and subsequent theories are called "scalar" theories.

In the past, scalar theories have received most attention from investigators. The random or noisy aspect of scalar scattering (due to random phase) is usually referred to as "diffuse" scattering. This term is used in contrast to "stationary phase" scattering, which is attributed to specular reflection components in the radar return. Beckmann's book on electromagnetic scattering from rough surfaces [32] is largely devoted to scalar scattering due to random variation of the height variable which affects the phase locally. With the scalar approach, the effects due to changes in position of the local normal $\underline{n}$ are usually considered to be averaged out at an early stage of the analysis. Instead, one considers the averaged fixed normal (vertical for flat terrain) $\underline{\bar{n}}$, with average angle of incidence given by $\bar{\theta}$. By this procedure, the effects of polarization for various types of terrain or sea state are accounted for within the frame work of the scalar theory by introduction of averaged reflection coefficients $\bar{R}^{\pm}(\bar{\theta})$.

With our approach, we show that the effect of changes of orientation ψ_n of the local normal gives rise to N-target-noise components in the target return scattering, with well-determined scattering characteristics. The composite effect of changes of the local normal $\underline{n}$ also results in random changes of local aspect angle θ. These changes are accounted for as contributing both to the averaged single target component and to the remainder-noise components in which the total scattering is decomposed. Details of these concepts will be analyzed in the following sections, which discuss scattering from flat rough surfaces and rough contoured surfaces (rolling hills).

44. Radar Scattering From Flat Rough Surfaces

A mean flat extended surface is a terrain or water surface which has an average fixed vertical normal. Most water surfaces are of this nature, and such "flat" terrain as deserts, cultivated vegetated fields, snow-covered grounds, grass land, and airport runways. The roughness of the terrain in terms of wavelengths is such that the physical optics formulation represents a reasonable approximation of the electromagnetic scattering problem. For detailed information on criteria for validity of physical optics methods, we refer the reader to the extensive literature on this subject which treats specific statistical models (Gaussian surfaces, etc.) of rough extended surfaces (Beckmann and Spizzichino [32] and Stogryn [42]).

Another distinction often made in structural models is between scattering from an ensemble average, which is an average over an ensemble of realizations of a rough surface given by random variables, and time averages of one particular realization (such as with samples obtained from terrain overflight). Statistical stationarity and ergodicity of random processes are concepts usually introduced at this stage. These distinctions can be entered into and should be considered with a more detailed analysis of statistical target models [41].

For our purposes, we need only rudimentary statistical concepts, the primary one introduced in Secs. 25 and 26 concerning statistical independence of voltages, fields, and targets. This distinction allows us to proceed from sums of independent voltages to corresponding sums of received power and vice versa. With this in mind, we now consider the received voltages at the terminals of antenna $\underline{b}$ from the extended target T which is illuminated by antenna $\underline{a}$. The gain patterns and polarization of antennas $\underline{a}$ and $\underline{b}$ are considered constant over the illuminated region on the surface. We find for received voltage:

$$V = T\,\underline{a} \cdot \underline{b} = \iint T_n\,\underline{a} \cdot \underline{b}\, dS = \iint V_n\, dS \tag{44.1}$$

where T_n is as defined by (43.8).

By this procedure, the analysis is reduced to a consideration of "local voltages" on the surface which resembles the classical scalar theory approach to scattering problems, but which is different in content, since polarization effects of the scattering are fully accounted for. However, many of the established mathematical techniques of the scalar theory now become applicable also to equation (44.1). The local voltage V_n is obtained by substituting T_n from (43.8) into (44.1):

$$V_n = \frac{A_n}{2}\begin{bmatrix} (R^+ - R^-) + \cos 2\psi_n (R^+ + R^-) & \sin 2\psi_n (R^+ + R^-) \\ \sin 2\psi_n (R^+ + R^-) & (R^+ - R^-) - \cos 2\psi_n (R^+ + R^-) \end{bmatrix} \underline{a} \bullet \underline{b} \tag{44.2}$$

We find that it is the sum of three voltages:

$$V_n = V_{n0} + V_{n1} + V_{n2} \tag{44.3}$$

where

$$\left.\begin{aligned}
V_{n0} &= \frac{A_n}{2}(R^+ - R^-)\, \underline{a} \bullet \underline{b} \\
V_{n1} &= \frac{A_n}{2}(R^+ + R^-) \cos 2\psi_n \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \underline{a} \bullet \underline{b} \\
V_{n2} &= \frac{A_n}{2}(R^+ + R^-) \sin 2\psi_n \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \underline{a} \bullet \underline{b}
\end{aligned}\right\} \tag{44.4}$$

This result is fundamental to our subsequent discussions.

The angle ψ_n, which is the orientation of the local normal $\underline{n}$, is now considered an independent random variable distributed symmetrically about a mean $\bar{\psi}_n = 0$, since for a flat terrain this is the orientation of the constant "vertical" normal. The symmetric distribution of ψ_n implies that there is no preference for normals pointing to the right or to the left of the plane of symmetry through direction of incidence $\underline{k}$ and average normal $\underline{\bar{n}}$, as shown in Fig. 37. Of course, the normal also is determined by local angle of incidence θ, which affects the reflection coefficients $R^{\pm}(\theta)$. Finally, the material properties of the surface which determine $R^{\pm}$ could be expressed by random variables with distributions on the surface. As indicated before, the local position vector $\underline{r}$ affects only the phase of the multiplication term A_n; it is accounted for by classical "scalar" methods.

With these scalar theories, the effects of changes of local normal $\underline{n}(\theta, \psi_n)$ and material properties are usually averaged out at the initial stages of the calculation and are absorbed by considering average reflection coefficients. Only heuristic justification is given for the validity of such an approach.

We intend to show that one may account for the electromagnetic scattering of rough surfaces by considering the variations of $\underline{r}$, which affects the

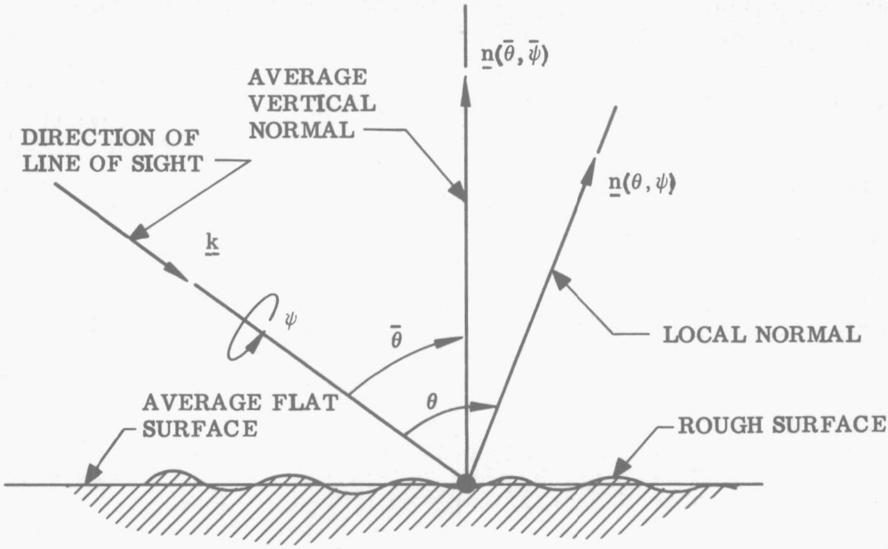


Fig. 37 Geometry of Flat Terrain

phase, and changes of local orientation ψ_n of the surface, while changes of local aspect direction θ and of material properties are considered to be averaged out and absorbed by average reflection coefficients $\overline{R}^{\pm}$. The proof of validity for this approach lies with the canonical decomposition: $R_S = M_S + N_S$ for symmetric rough surfaces derived in Sec. 35, which does not depend on any particular target model.

The term M_S represented a mean symmetric single target which has an effective scattering matrix $\overline{T}_S$, while the distributed noise target N_S could be decomposed uniquely into a sum of two single trough targets [see (35.11) and Sec. 14, cases b and c] with scattering matrices:

$$\left.\begin{aligned} \overline{T}_{T1} &= \sqrt{2\,B_1}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \\ \overline{T}_{T2} &= \sqrt{2\,B_2}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \end{aligned}\right\} \qquad (44.5)$$

Hence, the total effective voltage derived from the decomposition of total return power may be written as:

$$\bar{V} = \bar{V}^{S} + \bar{V}_1^{N} + \bar{V}_2^{N} \tag{44.6}$$

where

$$\bar{V}^{S} = m_o e^{i\alpha_o} \begin{bmatrix} \bar{R}^{+} & 0 \\ 0 & -\bar{R}^{-} \end{bmatrix} \underline{a} \cdot \underline{b} \tag{44.7}$$

$$\bar{V}_1^{N} = \sqrt{2 B_1}\, e^{i\beta_1} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \underline{a} \cdot \underline{b} \tag{44.8}$$

$$\bar{V}_2^{N} = \sqrt{2 B_2}\, e^{i\beta_2} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \underline{a} \cdot \underline{b} \tag{44.9}$$

The canonical decomposition theorem guarantees physical realizability and statistical independence of the effective component voltages. The phases β_1 and β_2 may be considered uniformly random variables.

We are interested in finding physical noise sources of the effective noise voltages $\bar{V}_1^{N}$ and $\bar{V}_2^{N}$ on the rough flat surface. The local voltages on the surface which contribute to the scattering were identified by (44.4). It is clear that all voltages V_{n2} contribute to $\bar{V}_2^{N}$; part of voltages V_{n1} contribute to $\bar{V}_1^{N}$, and the other part contributes also to $\bar{V}^{S}$. The main target noise source for $\bar{V}_2^{N}$ is clearly the term $\sin 2\psi_n$, whose average is zero. The same changes of local orientation ψ_n cause a splitting of the $\cos 2\psi_n$ term in V_{n1}, into two parts:

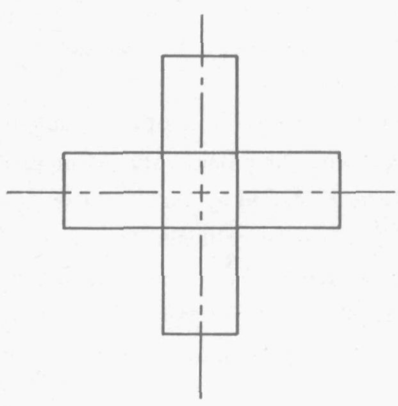
$$\cos 2\psi_n = \overline{\cos 2}\,\psi_n + \Delta(\cos 2\psi_n)$$

where $\bar{\Delta}(\cos 2\psi_n) = 0$.

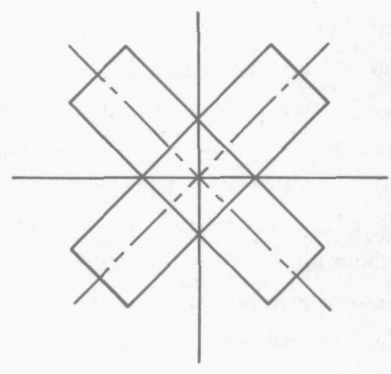
A heuristic argument can be made at this point, that all parts of V_{n1} with the $\overline{\cos 2\psi_n}$ term contribute to $\bar{V}^S$ while all parts with the $\Delta(\cos 2\psi_n)$ term act as a random noise source that contributes to $\bar{V}_1^N$. By these arguments, a physical interpretation is added to the formal result of the decomposition of symmetric targets.

Use of the term "trough noise" gives a phenomenological description of the target noise types and it serves as an aid in describing the noise behavior. For example, it is clear that under circularly polarized illumination (which is insensitive to target orientation) the two noise types behave in the same manner. The term "trough" should not be associated with a physical mechanism (double bounce) for scattering on the surface. It has been stated that the physical scattering behavior is explainable by the perturbations of the direction of the local normal from the averaged vertical normal on the surface.

The noise represented by type (b) troughs in Fig. 38 is described as cross-polarized target noise which was interpreted physically as due to perturbations of the orientation angle of the local normal.



(a)



(b)

Fig. 38 Geometries of Two Types of Trough Noise

The classical "scalar" scattering theory for rough surfaces considered only variation in surface height with average reflection coefficients which led to an average single target model. This theory could not adequately explain cross-polarization effects, observed during terrain over-flights with "vertical" polarization ("in the plane"), since oriented single symmetric targets do not produce cross-polarized components in the scattering, with linearly polarized illumination in the plane of symmetry. The present theory takes account of these cross-polarized terms.

45. Orientation Independent Targets

In the following sections, we develop several target models which may fit certain physical situations that occur frequently with radar target scattering from terrain or sea state surfaces. Different radar return characteristics are found, dependent upon the angle of incidence at which the target is observed, and the radar frequency. These effects include depolarization of the target and the determination of the averaged single target scattering and the remainder noise components of the return.

The key to these target models consists of assigning to the target statistical parameters such as variations in surface height, surface normal, and electrical properties which govern the target's scattering behavior. The standard classical so-called scalar or acoustical methods use exclusively the distribution of surface height which enters into the scattered (absolute) phase as a statistical parameter. Other variables such as changes of surface normal and electrical properties of the surface are considered only as averages. The scalar theory contributes to the "averaged single target" component of the scattered return. In contrast to these terms, we also have generators for the remainder-noise components of distributed target scattering.

One of the simplest target models is the class of orientation independent targets. We recall that target orientation was defined as rotation ψ about the radar line-of-sight direction. If the target scattering is independent of orientation ψ, the target is called orientation independent. Examples of orientation independent targets are homogeneous terrain and sea state surfaces at close to vertical incidence. Other examples include homogeneous clouds of rain, dust particles or chaff, and average scattering from rough spheres. It turns out that the scattering from orientation independent targets is uniquely determined by three target parameters: A_o, B_o, and F, where $F = 0$ for symmetric target distributions. These quantities may be determined by a simple set of measurements or by numerical calculation, for each physical situation. The details of the analysis are presented next.

The conditions for orientation independent scattering are easily derived from (27.2) by assuming the orientation angle ψ to be an independent random variable with uniform distribution. Then all terms of the stokes matrix containing ψ will vanish, and only A_o, B_o, and F remain. The most general form for the average returned power from an orientation independent target will thus be (12.4 and 27.2):

$$P(\phi_A, \tau_A ; \phi_B, \tau_B) = A_o + B_o + (-A_o + B_o) \sin 2\tau_A \sin 2\tau_B$$
$$+ F(\sin 2\tau_A + \sin 2\tau_B) + A_o \cos 2\tau_A \cos 2\tau_B \cos 2(\phi_A - \phi_B) \tag{45.1}$$

We recall from (11.20) and (27.2) that:

$$F = \left\langle 2 Q \cos 2\gamma \sin 2\tau_m \right\rangle \tag{45.2}$$

where τ_m is the helicity angle and $\tau_m = 0$ for symmetric single targets. For general symmetric target distribution, we assume τ_m to be symmetrically distributed about the mean $\bar{\tau}_m = 0$ such that (45.2) vanishes also for symmetric distributed targets. (An example of a nonsymmetric orientation independent target would be a bedspring surface, i.e., a surface covered with right- or left-wound helices.)

The physical significance of the components A_o and B_o is now examined. We seek a decomposition into target scattering due to an average single target and remainder noise components. It is easily recognized that the only symmetric average flat single target which is orientation independent is given by a flat facet at normal incidence with parameters $A_o^S \neq 0$, $C^S = D^S = 0$ and $B_o^S = B^S = 0$. Hence, A_o above is associated with the flat facet return, which accounts for the "specular" return of the terrain surface at normal incidence. For isotropic clouds of objects, A_o is representative of a sphere target; however, the return is no longer specular but rather has the character of Rayleigh noise due to amplitude and phase distributions of the single target "acoustic" scattering component.

For all cases, B_o is associated with target remainder noise. We may write $B_o^N = B_o = B_1 + B_2$. Since also $B^N = B_1 - B_2 = 0$, we find $B_1 = B_2 \neq 0$. Hence, the two types of "trough noise" into which the remainder noise may be decomposed are present in equal amounts. Each trough noise type may be considered as independent Rayleigh noises with noise powers proportional to B_1 and B_2. The effective voltages for single target and trough noise are thus:

$$\bar{V} = \bar{V}^S + \bar{V}_1^N + \bar{V}_2^N \tag{45.3}$$

where

$$\bar{V}^S = \sqrt{2\,A_o}\, e^{i\alpha_o}\, \underline{a} \cdot \underline{b} \tag{45.4}$$

$$\bar{V}_1^N = \sqrt{2\,B_1}\, e^{i\beta_1} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \underline{a} \cdot \underline{b} \tag{45.5}$$

$$\bar{V}_2^N = \sqrt{2\,B_2}\, e^{i\beta_2} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \underline{a} \cdot \underline{b} \tag{45.6}$$

In these equations, α_o is the phase distribution of the facet, or sphere, "acoustical" component, and β_1 and β_2 are uniformly random phases of the two trough components. The total effective scattering matrix for the orientation independent target is thus:

$$\begin{bmatrix} \sqrt{2\,A_o}\; e^{i\alpha_o} + \sqrt{2\,B_1}\; e^{i\beta_1} & \sqrt{2\,B_2}\; e^{i\beta_2} \\ \sqrt{2\,B_2}\; e^{i\beta_2} & \sqrt{2\,A_o}\; e^{i\alpha_o} - \sqrt{2\,B_1}\; e^{i\beta_1} \end{bmatrix} \qquad (45.7)$$

Note that in (45.5), (45.6), and (45.7), $B_1 = B_2$ since the noise powers were equal.

Notice also that in (45.7) the scattering matrix has random noise components with index "1" in the main diagonal. This is in contrast to the more conventional cross-polarized noise terms with index "2." We have thus shown that an effective scattering matrix with only one random noise component can not be a representative model for an orientation independent distributed target.

Figure 39 gives the return from the orientation independent target for vertical illumination.

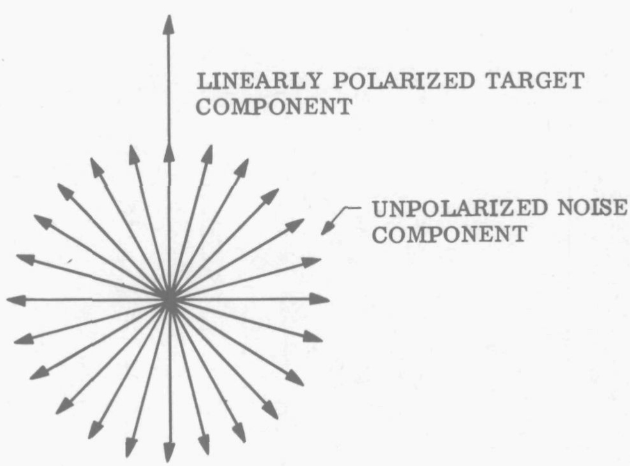


Fig. 39 Scattered Return From Orientation Independent Target

To account for the unpolarized noise component, shown in the figure, we recall that in this case the noise component stokes matrix is given by:

$$M_N = \begin{bmatrix} B_o & & & \\ & B_o & & \\ & & 0 & \\ & & & 0 \end{bmatrix} \tag{45.8}$$

For linearly polarized illumination, the scattered $s_N = M_N \, g\,(\underline{a})$ is found easily as:

$$s_N = \begin{bmatrix} B_o & & & \\ & B_o & & \\ & & 0 & \\ & & & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ \cos 2\,\phi_A \\ \sin 2\,\phi_A \end{bmatrix} = B_o \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{45.9}$$

Hence, s_N is completely unpolarized (Sec. 24).

The orientation independent target model was introduced by Huynen [7, 53] and applied to pulse return calculations of terrain for near vertical incidence.

46. Radar Scattering From Contoured Rough Surfaces

The discussion of the preceding section on scattering from average flat rough surfaces may be extended to contoured rough surfaces. This theory covers a large field of applications: radar scattering from sloping hills; mountains covered with rock, vegetation, or snow; and, in general, almost any type of terrain or water surface. Also scattering from single moving objects with rough or irregular surfaces may be treated by this theory. The decomposition $R = M + N$ of the rough surface with stokes matrix R produces an average, in general nonsymmetric, single object with stokes matrix M, with equivalent scattering matrix T, and the remainder N-type noise component scattering.

The analysis starts with the physical optics equation (44.1) for target scattering received voltage:

$$V = \left(\iint T_n \, dS\right) \underline{a} \bullet \underline{b} = T \, \underline{a} \bullet \underline{b} \tag{46.1}$$

where

$$T_n = \frac{A_n}{2} \begin{bmatrix} (R^+ - R^-) + \cos 2\psi_n (R^+ + R^-) & \sin 2\psi_n (R^+ + R^-) \\ \sin 2\psi_n (R^+ + R^-) & (R^+ - R^-) - \cos 2\psi_n (R^+ + R) \end{bmatrix} \tag{46.2}$$

For brevity, we may write:

$$T_n = a_n \, \mathbf{I} + b_n \cos 2\psi_n \, \mathbf{L} - b_n \sin 2\psi_n \, \mathbf{K} \tag{46.3}$$

where

$$\left.\begin{aligned} a_n &= \frac{A_n}{2} (R^+ - R^-) \\ b_n &= i \frac{A_n}{2} (R^+ + R^-) \end{aligned}\right\} \tag{46.4}$$

and $\mathbf{J}$, $\mathbf{K}$, and $\mathbf{L}$ defined in Sec. 4 as rotation matrices. For the flat surface case, we now consider the two random variables derived from position vector $\underline{r}$ and surface normal $\underline{n}$ which determine the geometry of the rough contoured surface. In contrast to the flat surface, the contoured rough surface has an average local normal $\underline{\bar{n}}(\bar{\theta}, \bar{\psi}_n)$ with local average aspect angle $\bar{\theta}$ and local average orientation $\bar{\psi}_n$. This is illustrated by Fig. 40.

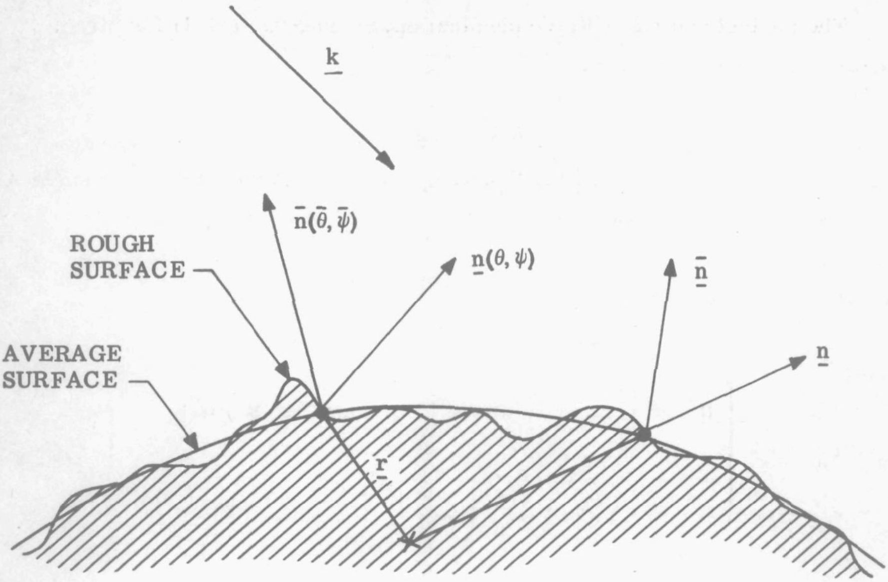


Fig. 40 Geometry of Contoured Terrain

The two random variables considered are the local surface heights ζ and the local orientation: $\psi_n = \bar{\psi}_n + \Delta\psi_n$. The local surface height displacement ζ affects only the multiplicative term in A_n, and leads to the "scalar" type of analysis of radar scattering from rough surfaces. The variation in local orientation $\Delta\psi_n$ *produces N-target noise components*, as we *show next*. Substitution of ψ_n into (46.3) gives:

$$T_n = a_n \mathbf{I} + \cos 2\,\Delta\psi_n\, b_n(\cos 2\,\bar{\psi}_n\, \mathbf{L} - \sin 2\,\bar{\psi}_n\, \mathbf{K}) + \qquad (46.5)$$
$$- \sin 2\,\Delta\psi_n\, b_n(\sin 2\,\bar{\psi}_n\, \mathbf{L} + \cos 2\,\bar{\psi}_n\, \mathbf{K})$$

Similarly to the treatment for the flat surface (where $\bar{\psi}_n = 0$ and $\bar{n}$ is the fixed vertical normal), we may consider four components of local voltage for the locally flat surface derived from (46.5):

$$V_n = \left(V_{n0}^T + V_{n1}^T\right) + \left(V_{n1}^N + V_{n2}^N\right) \tag{46.6}$$

Next, we write $\cos 2\,\Delta\psi_n = \overline{\cos 2\,\Delta\psi_n} + \Delta(\cos 2\,\Delta\psi_n)$ such that $\overline{\Delta}(\cos 2\,\Delta\psi_n) = 0$ for a symmetric distribution of $\Delta\psi_n$ about the orientation angle $\overline{\psi}_n$ of the local average normal. Then:

$$\begin{aligned}
V_{n0}^T &= \frac{A_n}{2}(R^+ - R^-)\,\underline{a} \bullet \underline{b} \\
V_{n1}^T &= i\frac{A_n}{2}(R^+ + R^-)\,\overline{\cos 2\Delta\psi_n}(\cos 2\overline{\psi}_n\,\mathbf{L} - \sin 2\overline{\psi}_n\,\mathbf{K})\,\underline{a} \bullet \underline{b} \\
V_{n1}^N &= i\frac{A_n}{2}(R^+ + R^-)\,\Delta(\cos 2\Delta\psi_n)\,(\cos 2\overline{\psi}_n\,\mathbf{L} - \sin 2\overline{\psi}_n\,\mathbf{K})\,\underline{a} \bullet \underline{b} \\
V_{n2}^N &= -i\frac{A_n}{2}(R^+ + R^-)\,\sin 2\Delta\psi_n\,(\sin 2\overline{\psi}_n\,\mathbf{L} + \cos 2\overline{\psi}_n\,\mathbf{K})\,\underline{a} \bullet \underline{b}
\end{aligned} \tag{46.7}$$

The first two terms contribute to the average single target, the last two to the N-target noise components. First, we notice that if $\overline{\psi}_n = 0$, the contoured surface becomes an average flat surface. The N-target noise components are generated by $\Delta(\cos 2\,\Delta\psi_n)$ and $\sin 2\,\Delta\psi_n$ voltage factors. Without the noise-generating terms, the stokes matrices of the N-target noise are:

$$T_{n1}^N = \frac{A_n}{2}(R^+ + R^-)\begin{bmatrix} \cos 2\overline{\psi}_n & \sin 2\overline{\psi}_n \\ \sin 2\overline{\psi}_n & -\cos 2\overline{\psi}_n \end{bmatrix} = b_n\, e^{\overline{\psi}_n \mathbf{J}}\,\mathbf{L}\,e^{-\overline{\psi}_n \mathbf{J}} \tag{46.8}$$

$$T_{n2}^N = \frac{A_n}{2}(R^+ + R^-)\begin{bmatrix} -\sin 2\overline{\psi}_n & \cos 2\overline{\psi}_n \\ \cos 2\overline{\psi}_n & \sin 2\overline{\psi}_n \end{bmatrix} = -b_n\, e^{\overline{\psi}_n \mathbf{J}}\,\mathbf{K}\,e^{-\overline{\psi}_n \mathbf{J}} \tag{46.9}$$

From the right-hand expression, we see clearly the physical significance of the T_{n1}^{N} and T_{n2}^{N} matrices: They represent the $\mathbf{L}$-type (with axis horizontal or vertical) and $\mathbf{K}$-type (with axis oriented +45° or -45°) trough-noise targets (Fig. 38) which were characteristic for the flat surface, but here are adjusted to the locally flat surface with local average orientation $\bar{\psi}_n$. Both noise targets are of the form

$$M_N = \begin{bmatrix} b & c \\ c & -b \end{bmatrix} = ib\,\mathbf{L} - ic\,\mathbf{K} \tag{46.10}$$

where b and c are complex, and hence they are special cases of general N-target noise (Sec. 14 k).

The effective voltage, which is registered at the antenna receiver terminals, is obtained from a time-average reception of power, which in turn is derived from the instantaneous power and voltage. Let the instantaneous voltage be given by (46.1). Then the derivation for time-average power is computed by (26.15):

$$\langle P \rangle = \iint \langle W(T_{nI}, T_{nII}^{*}) \rangle \, dS_I \, dS_{II} \; g(\underline{a}) \cdot h(\underline{b}) \tag{46.11}$$

or

$$\langle P \rangle = R \; g(\underline{a}) \cdot h(\underline{b}) \tag{46.12}$$

Hence, the stokes reflection matrix is determined for the contoured surface.

We now apply the general decomposition: $R = M + N$. The mean single target M has a general effective scattering matrix $\overline{T}$, whereas the N-target may be composed of two single N-targets with effective scattering matrices $\overline{T}_{N1}$ and $\overline{T}_{N2}$, each of which has the form given by (46.10). Hence, the total effective voltage $\overline{V}$ can be given as a sum of three statistically independent voltages:

$$\bar{V} = \bar{V}_0^T + \bar{V}_1^N + \bar{V}_2^N \tag{46.13}$$

where $\bar{V}_0^T$, $\bar{V}_1^N$ and $\bar{V}_2^N$ are of the form:

$$\begin{aligned}
\bar{V}_0^T &= (a_o\,\mathbf{I} + b_o\,\mathbf{L} + c_o\,\mathbf{K})\,\underline{a}\cdot\underline{b} \\
\bar{V}_1^N &= (b_1\,\mathbf{L} + c_1\,\mathbf{K})\,\underline{a}\bullet\underline{b} \\
\bar{V}_2^N &= (b_2\,\mathbf{L} + c_2\,\mathbf{K})\,\underline{a}\bullet\underline{b}
\end{aligned} \tag{46.14}$$

Our next task is to obtain a physical interpretation of the above-listed formal decomposition of voltages. In particular, we are interested in discovering plausible noise sources which may contribute to $\bar{V}_1^N$ and $\bar{V}_2^N$. The local surface voltages which contribute to the total scattering of the rough contoured surface were listed in (46.7). A glance at this list will convince us that the noise voltages V_{n1}^N and V_{n2}^N are the most likely candidates for a heuristic accounting of $\bar{V}_1^N$ and $\bar{V}_2^N$, since they have the required matrix form (46.10), and they can act as target noise sources (due to the assumed symmetry of the distribution of $\Delta\psi_n$ on the surface).

We now summarize the heuristic interpretation of our results. The surface roughness in this derivation was accounted for through introduction of two random variables: One giving variation of local position, which influences absolute phase of the scattering and which is treated classically by "scalar" theory; and the other a variation of normal from local average normal, which was analyzed through the orientation variable ψ_n, The effects of variation of local aspect angle θ and of material properties on the surface were absorbed by considering the scattering determined by average Fresnel reflection coefficients $\bar{R}^{\pm}(\bar{\theta})$ and an average local aspect angle $\bar{\theta}$. The justification for this approach may be found from the general phenomenological theory.

The main theorem found in Sec. 38 predicts that for any rough surface given by stokes matrix R, there exists a physically realizable decomposition: $R = M + N$, *into a mean single target* M and remainder N-type noise component scattering. Our previous approach provides such a model for contoured rough surfaces which fits the general (canonical) decomposition theorem.

The N-target components account for cross-polarized scattering with "vertical" incident polarization which, as has been observed for rough terrains, can be appreciable at close to vertical angles of incidence. The scalar theory alone can not account for these target-depolarization effects.

Recently, shadowing effects have also been included with the discussion of scattering from rough surfaces (Beckmann [54] and Sancer [55]).

The special case of scattering from contoured 2-dimensional rough surfaces is instructive to illustrate the theoretical results. The 2-dimensional profile of the surface is shown in Fig. 41.

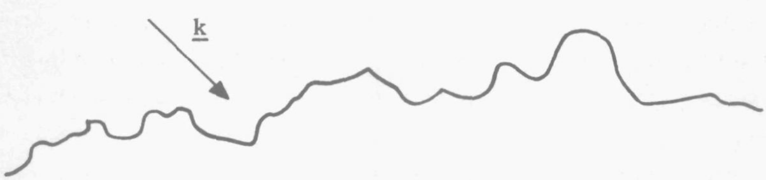


Fig. 41 Two-Dimensional Scattering From Contoured Rough Surfaces

We consider illumination in the direction $\underline{k}$ with $\underline{k}$ in the plane of the figure. Any surface normal will be parallel to the plane of the figure. It follows then that, for this illumination, the surface represents a symmetric distributed target for which the local scattering matrix is of the form:

$$T_n = a_n \mathbf{I} + b_n \mathbf{L}$$

The cross-polarized component $c_n \mathbf{K}$ has vanished, since each elemental surface patch is a symmetric target for which $B_{no} = B_n$ locally from (35.5) and (43.9) for $\psi_n = 0$. The averaged return also will have no cross polarized component; hence, for the total average $B_o = B$ and from equations (25.6) and (25.7), $B_o^N = B^N$, which shows that the decomposition results in an average single symmetric target and $\mathbf{L}$-type of remainder trough noise. We could have anticipated that the $\mathbf{K}$-type, cross-polarized, trough noise is zero. However, in this case, the $\mathbf{L}$-type of trough noise can not be explained as due to changes of orientation ψ_n of the local normal, since for all normals $\psi_n = 0$ and since all surface normals in Fig. 41 together with $\underline{k}$ are in the plane of the paper. The sources of $\mathbf{L}$-type of trough noise are found from the changes of aspect angle θ of local normal $\underline{n}$ and possibly the changes in surface material properties. This example illustrates the fact that N-type noise cannot be explained exclusively by the variations in local orientation ψ_n; changes in aspect angle θ and surface material properties will also contribute to the noise components.

LIST OF MAIN SYMBOLS

Vector Quantities:

$\underline{a}$	transmit antenna polarization
$\underline{b}$	polarization of receiver antenna used as transmitter electric field
$\underline{E}$	electric field
$g(\underline{a}) = (g_0, g_1, g_2, g_3) = (g_o, \underline{g})$	stokes vector of transmit antenna $\underline{a}$
$g(\underline{a}_1, \underline{a}_2)$	*mixed stokes vector of antennas* $\underline{a}_1$ *and* $\underline{a}_2$
$h(\underline{b}) = (h_0, h_1, h_2, h_3) = (h_o, \underline{h})$	stokes vector of receiving antenna $\underline{b}$
$\underline{k}$	direction of incoming target illumination
$\underline{m}$	polarization which gives maximum target return
$\underline{n}(\theta, \psi)$	surface normal in terms of aspect angle θ and orientation angle ψ
$\underline{n}^{\pm}$	characteristic target null-polarizations
$s = (s_o, \underline{s})$	stokes vector of scattered field

2×2 Matrices:

I, **J**, **K**, **L**	quaternion group matrices
T	target scattering matrix
$\overline{T}$	*effective target scattering matrix*
T_S	target scattering matrix of symmetric target
T_N	target scattering matrix of N- (noise) target
U	unitary transformation

4×4 Matrices:

M	stokes matrix of a single target
M_o	oriented single target stokes matrix ($\psi = 0$)

M_S	stokes matrix of single symmetric target
M_N	stokes matrix of single N- (noise) target
N	stokes matrix of remainder noise components of distributed target scattering
$O_1, O_2, O_3.$	orthogonal transformations of stokes matrix
$Q_R, Q_R^{(2)}, Q_R^{(3)}$	higher order matrices of type R
R	stokes matrix of distributed target
$R_N = ONO^{-1}$	general noise matrix
$\hat{T}, \hat{I}$	4×4 matrices derived from target matrix
$W(T_1, T_2)$	mixed stokes matrix
$\langle W \rangle$	stokes correlation matrix

<u>Arabic Lower-Case Letters</u>:

a, b	antenna magnitudes (also used to designate matrix elements
m	magnitude of targets
q_1, q_2, q_3, q_{ij}	quadratic components of stokes vector
t_1, t_2	eigenvalues of $T\underline{x} = t\underline{x}^*$

<u>Arabic Capital Letters</u>:

A_o	total power return of regular scattering
B_o	total power return of irregular scattering
$B_o + B$	power of symmetric depolarized scattering
$B_o - B$	power of nonsymmetric depolarized scattering
B_1, B_2	trough noise power components
C, D	symmetric components of stokes matrix
E, F	nonsymmetric components of stokes matrix
G, H	coupling components of stokes matrix
(H-V)	return for horizontally polarized transmission and vertically polarized reception; equivalently, (H-H) and (V-V)
P	received power at receiver antenna terminals
$\langle P \rangle$	average received power

Q, Q_o	multiplicative factors of power components
Q_1, Q_2, Q_3	nonnegative quadratic components of power
Q_{ij}	mixed quadratic components of power
Q_4	diagonal term not contained in matrix of type R
R^+, R^-	fresnel reflection coefficients (+ refers to "in the plane"; − refers to "out of the plane")
(RC-LC)	return for right-hand circular transmission and left-hand circular reception; equivalently, (RC-RC) and (LC-LC)
V	received voltage at receiver antenna terminals

Greek Letters:

α, β	absolute phases of transmit and receive antennas (also used to denote other angles)
γ	characteristic angle of a radar target (4γ is the angle between null-polarization vectors)
θ	aspect angle of incident illumination
ν	target skip angle (at a given aspect)
ρ	target absolute phase
τ	ellipticity angle of ep wave
τ_m	target helicity angle (ellipticity angle of $\underline{m}$)
ϕ	orientation angle of ep wave
ϕ_m	target orientation angle relative to target axis
χ	characteristic third-order function of stokes matrix parameters
ψ	target orientation angle (rotation about the line-of-sight direction)
ψ_a	orientation angle of target axis
$\Phi = \phi - \psi$	relative target orientation angle

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SUMMARY

The problem areas discussed in this work relate to the questions which arise when one attempts to characterize the objects that are observed by radar signals. Local reception of the scattered field from a radar target which is illuminated by a plane electromagnetic wave may in principle supply information about not only the position but also the nature of the target.

For the case where no specific physical target model is supplied, a general phenomenological description of the target is called for. By this we mean simply that the radar target appears to us as an object for investigation, through the processes of radar illumination.

In general, the far-field scattering from the target is a plane electromagnetic wave, which is linearly related to the illuminating field through a scattering matrix T. For any given situation, the characteristic target properties are represented by the target scattering matrix T.

To determine the matrix elements, a number of measurements are made with a polarization sensitive receiver for different polarizations of the illuminating field. A greater insight into the behavior of the scattered field for different polarizations and directions of illumination is acquired through the aid of so-called polarization charts, whereby the fields are represented by "stokes vectors." The stokes vector components in spherical coordinates are given by the intensity, the ellipticity, and the direction of principal axis of the polarization ellipse of the electric field. For constant intensity of the transmitted field, the stokes vectors are determined geometrically by points on the so-called Poincaré sphere.

The relationship between the stokes vectors of the illuminating and of the scattered fields is given by the stokes matrix M. The matrix elements of M may be expressed in terms of those of the scattering matrix T and vice versa. In Chapter 2 an algebra particularly suited to treat problems concerning matrix T is developed through matrices which represent the quaternion group.

In Chapter 3 the stokes vectors and the stokes matrix are defined in terms of the quaternion matrices. Through the measurement of different polarizations, the elements of the stokes matrix can be determined experimentally. Starting from an eigenvalue problem with specific eigenvectors for the scattering matrix T, a standard representation for this matrix is obtained. The corresponding stokes matrix M is then found, and expressions for the elements in terms of the characteristic parameters of the matrix T are given. It is shown that the matrices T and M have so-called null-polarizations and that radar target matrices can be deduced from a given target matrix by rotations of the Poincaré sphere. In Chapter 4 some experiments are described which illustrate and confirm these theoretical results.

In Chapter 5 the concept of a distributed target is introduced, which gives rise to random scattering processes; or, considered optically to partially polarized plane waves. Here we consider scattering from rough surfaces, such as terrain or sea state, clouds, chaff, etc. Only averaged power measurements are considered deterministic for the scattering process. The random processes cause a reduced coherence between the polarized components of the reflected fields. This leads in Chapter 6 to the definition of the stokes matrix R of distributed targets, which is determined by the average of the instantaneous values of the stokes matrix of single targets.

Some important inequalities which determine conditions for physical realizability of stokes vectors and stokes matrices for distributed targets are used to prove several central target decomposition theorems. This culminates in Chapter 7 in the so-called canonical decomposition theorem for general distributed targets, which states that a distributed target with stokes matrix R can be decomposed uniquely into a general single target with matrix M and a remainder noise-target component with matrix N, such that R = M + N, where M and N are physically realizable and statistically independent.

It is shown in Section 32 that the stokes matrix of a single target is irreducible; i.e., it cannot be split into other independent and realizable target components. It is also shown that the N-target noise matrix is a singular case of a general distributed target matrix.

In Chapter 8 these theorems are applied to the problem of radar scattering from rough surfaces. It is shown that the general decomposition theorems can be interpreted in detail within the framework of the physical rough surface target model.

SAMENVATTING

Het in dit proefschrift behandelde problemen complex heeft te maken met de moeilijkheden, die rijzen bij het karakteriseren van objecten door middel van radar-signalen.

Zendt men een gebundelde electromagnetische golf met vaste frequentie in de richting van een te detecteren object, dan zal plaatselijke ontvangst van de door het object verstrooide straling in beginsel informatie kunnen geven over de positie van het object, maar ook omtrent de aard daarvan.

Waar geen nadere omschrijving wordt gegeven aangaande het fysische model dat bepalend is voor het object, leidt deze beschouwingswijze vanzelf tot een algemene phenomenologische beschrijving. Hiermee wordt dan bedoeld dat het object alleen door de verstrooiings verschijnselen wordt gekenmerkt.
In het algemeen kan men stellen, dat het verstrooide electromagnetische veld, waargenomen op zodanige afstand van het object, dat het een vlakke-golf karakter heeft, door een verstrooiings-matrix (T) verbonden is met de invallende vlakke golfstraling.

Het is deze verstrooiings-matrix, die karakteristieke eigenschappen van het bestraalde object, in de gegeven situatie, bevat.
Om de in de matrix voorkomende elementen te leren kennen zijn een aantal metingen nodig, waarbij de polarisatie van het uitgezonden veld wordt gevarieerd, terwijl het verstrooide veld in polarisatie-gevoelige ontvangers wordt gemeten. Een goed inzicht in het gedrag van het verstrooide veld kan worden verkregen door gebruik te maken van diagrammen, waarbij de electromagnetische golven worden weergegeven door punten op de z.g. Poincaré-bol, welke punten worden vastgelegd door "stoke'se vektoren". De componenten voor de stoke'se vektor in bol-coordinaten worden door de intensiteit, de ellipticiteit en door de stand van de polarisatie ellips van het electrische veld bepaald.

Het verband tussen het uitgestraalde en het verstrooide veld wordt door een "stoke'se matrix" (M) vastgelegd. Met behulp van standaard matrices, die de quaternionen groep representeren wordt in hoofdstuk 2 een makkelijk hanteerbaar rekensysteem ontwikkeld, waarin ook de verstrooiingsmatrix wordt opgenomen. Hieruit wordt in hoofdstuk 3 een stoke'se matrix afgeleid. Door vermogens-metingen kunnen de elementen van deze matrix eenvoudig worden bepaald. Uitgaande van bijzondere "eigen-vektoren" van de verstrooiingsmatrix wordt een standaard-vorm voor deze matrix opgesteld. Er wordt bewezen, dat matrices T en M z. g. "nul-polarisaties" vertonen en vele objecten uit elkaar kunnen worden afgeleid door draaiingen van de Poincaré-bol. Verschillende verschijnselen dienaangaande zijn door proeven en metingen (hoofdstuk 4) bevestigd.

Diffracties aan "verdeelde objecten" geven in hoofdstuk 5 aanleiding tot de behandeling van "ruis verschijnselen" of meer optisch geformuleerd: tot "partieel gepolariseerde golven". Hierbij wordt gedacht aan verstrooiing over ruwe oppervlakken, zoals heuvelachtige en begroeide gebieden, wolken, zwermen van deeltjes etc.

Men komt dan tot een verminderde coherentie tussen de verschillende polarisatie toestanden van de teruggestraalde golf. Voor de metingen zijn slechts gemiddelde vermogens toegankelijk. Een en ander wordt uitgedrukt door hiermede statistisch samenhangende en in hoofdstuk 6 ingevoerde stoke'se matrices voor verdeelde objecten. Hiervan wordt in hoofdstuk 7 bewezen dat deze op canonische wijze kunnen worden gesplitst in een "enkelvoudig object" en "ruis", welke statistisch onafhankelijk van elkaar op eenduidige manier kunnen worden bepaald.

Er wordt (in sectie 33) bewezen, dat de stoke'se matrix van een enkel object niet meer is te splitsen in onafhankelijke componenten. De stoke'se ruis matrix treedt op als singulier geval van de algemene stoke'se matrix voor verdeelde objecten (zie sectie 32).

Aan de hand van de hiermede verworven inzichten kunnen in hoofdstuk 8 bekende berekeningen omtrent de diffractie aan ruwe oppervlakken worden geïnterpreteerd. Hiet blijkt dat de op algemene theoretische gronden uitgevoerde splitsingen ook in detail physisch kunnen worden verklaard.

LEVENSBERICHT

Jean Richard Huynen is in 1920 geboren te Batavia, voormalig Nederlands Indië. Hij volgde daar het lagere onderwijs tot zijn 12e jaar, alleen onderbroken door een jaar verblijf en reizen in de Verenigde Staten waar hij de 2e klasse lagere school voltooide in Akron, Ohio. Hij kwam daarna in Den Haag op de H. B. S. -B Stadhouderslaan waar hij de eerste twee schooljaren doorbracht. Daarna verhuisden zijn ouders naar Arnhem, waar hij op de Lorentz-H. B. S. - B in de Steenstraat het Middelbaar Onderwijs voltooide. Al vroeg toonde hij belangstelling voor toegepaste wiskunde en filosofie, zodat hij na een korte diensttijd in 1939 zich als student aan de Technische Hogeschool in Delft in de Afdeling der Werktuigbouwkunde liet inschrijven. De oorlogsjaren brachten een onderbreking in de studie, die in 1945 werd hervat. Na zijn propaedeutisch examen Werktuigbouwkunde, ging hij over naar de studierichting der Elektrotechniek. Hij was voor twee jaar assistent bij Prof. Dr. C. van Heel in het Optisch Laboratorium van Technische Physica. Hij studeerde af bij Prof. Dr. G. J. Elias. Na praktisch werk bij Philips in Eindhoven te hebben verricht verwierf hij in 1948 het diploma Elektrotechnisch Ingenieur. Kort daarop emigreerde hij met zijn echtgenote naar California, U. S. A., waar hij zich blijvend in de buurt van San Francisco vestigde. Hij werkte zeven jaar bij Dalmo Victor Co. o. a. aan het ontwerp van radar antennes. Sinds 1956 is hij verbonden aan de firma Lockheed Aircraft Corporation waar hij zich specializeerde in elektromagnetische diffractie problemen en daarvan speciaal de polarisatie eigenschappen van radar targets. Hij volgde in deze periode part-time wiskunde colleges aan de Stanford University, wat in 1961 besloten werd met een Master's degree van deze Universiteit.

STELLINGEN

I

Een moeilijk probleem bij het vergelijken van radarobjecten is de kwestie van normering. Hoe kunnen twee verschillende voorwerpen met elkaar in grootte worden vergeleken? In dit proefschrift wordt dit probleem op natuurlijke wijze opgelost.

II

Vaak wil men het gemiddelde weten van een reeks van stralingsmetingen aan een radarobject. Meestal wordt dit gemiddelde op heuristische gronden enigszins willekeurig bepaald. De in dit proefschrift beschreven canonische decompositie bepaalt op natuurlijke wijze een gemiddeld radarobject.

III

Hagfors beschrijft in zijn artikel ("A study of the depolarization of Lunar Radar Echoes" Radio Science Vol. 2 No. 5 May 1967) een "gedanken experiment" waarmee op grond van symmetrie eigenschappen van het ruwe maanoppervlak wordt bewezen, dat zekere elementen van de stokesmatrix (Muellermatrix) nul worden. Het resultaat $M_{34} = 0$ (blz. 447, vergl. 5), overeenkomende met $D=0$ in dit proefschrift, is echter onjuist voor symmetrische oppervlakken.

IV

Om geobserveerde depolarisatie in de radarontvangst gedurende overvlucht over een vlak terrein te verklaren hebben vele onderzoekers modellen gegeven welke depolarisatie kunnen veroorzaken. Meestal wordt hier gedacht aan distributies van dipolen over het ruwe oppervlak (zie bijv. Long [56]). Hoe dergelijke modellen ook fysisch verantwoord kunnen worden blijft meestal een open vraag.
In dit proefschrift wordt gesteld dat de depolarisatie op eenvoudige wijze verklaard kan worden als ruis ontstaan door de veranderingen van stand van de normaal op het ruwe oppervlak.

V

De in dit proefschrift voor radartargets ontwikkelde mathematische methoden zouden ook met vrucht in de netwerktheorie kunnen worden toegepast.

VI

Het elektromagnetische veld kan door quarternionen worden beschreven. Voor tijdharmonische velden voert men daarbij twee imaginaire eenheden in. Met behulp van de quarternionen algebra kunnen bekende omslachtige berekeningen met vectorvelden elegant worden afgeleid.

VII

Het scalaire gedeelte van het elektromagnetische quarternionen veld ontbreekt in de fysische wereld. Men kan aantonen dat de bronnen van dit onwerkelijke veld gevormd zijn door het scheppen of verdwijnen van elektrische of magnetische lading.

VIII

Het Amerikaanse juridisch systeem is in zijn huidige vorm niet meer te handhaven---(Chief Justice Burger in een speech voor de American Bar Association Convention op 10 Augustus 1970). Als oorzaken hiervoor kunnen worden genoemd de Liberalisering van de interpretatie van de grondwettelijke rechten van de mens en van de bevolkingsgroei.

IX

Filosofie kan betekenis hebben als uitspraak van een levensbeschouwing, of als methode tot inzicht in de werkelijkheid. Deze laatste ontwikkeling is langzamerhand overvleugeld door de groei der natuurwetenschappen. Niettemin volgt hieruit dat de filosofie werkelijke betekenis kan hebben bij het onderzoek naar grondslagen der moderne natuurwetenschappen en ook voor een kritiek daarop (Jaki: "The Relevance of Science", Univ. of Chicago Press, 1969).

X

De moderne sociale filosofie (zie Kwant "Sociale Filosofie" Aula-reeks nr. 132) is hoofdzakelijk gericht op het werk van Karl Marx. Hierbij wordt als hoofdthema uitgegaan van het begrip 'de mens als sociaal wezen'. Door deze veralgemenisering (objectivisering) verliest men het existentiele karakter van het sociaal bestaan.
Deze incomplete beschouwing leidt dan al gauw tot een conflictensituatie, waardoor ontstaan: het gevoel van verlies van identiteit, een gebrek aan individuele vrijheid en ontevredenheid met het maatschappelijke leven.

XI

De Kritiek van Staal ("Zinloze en zinvolle filosofie" De Gids Vol. 130 1966) op de fenomenologische school is een sprekend voorbeeld van het wederzijds gebrek aan communicatie tussen analytisch en fenomenologisch georienteerde filosofie. De analytisch geschoolde linguisten gaan ervan uit dat de taal als communicatie preciese betekenis moet hebben om 'zinvol' begrepen te worden. Het feit dat de taal ook een wordend karakter kan hebben (zie Kwant: "Fenomenologie van de taal" Aula-reeks nr. 131) waarin nog niet precies geformuleerde begrippen worden geuit, wordt als "zinloos' of als 'orakeltaal' bestempeld. Het is van groot belang dat studies worden ondernomen zodat deze scheidingen kunnen worden overbrugd. Dit zou kunnen gebeuren door een meer kritische taalbehandeling van de fenomenologisch georienteerden en ook door een verruiming van het zinvolle denkveld van de analytici.

J. Richard Huynen

Delft, 16 december 1970