Department of Precision and Microsystems Engineering

Synthesis of Inherently Moment Balanced Robotic Manipulators

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Synthesis of Inherently Moment Balanced Robotic Manipulators

A new method for synthesis of inherently dynamically balanced 1-DoF motion generation mechanisms

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Technische Universiteit Delft

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Preface

The topic of dynamic balancing was introduced to me in a guest lecture by Volkert van der Wijk as part of my Precision Mechanism Design class. This topic particularly grabbed my interest because the old theories of classical mechanics are used to create new solutions that prove to be useful in the high tech Mechanical Engineering field of today. In dynamic balancing, Kinematics and Dynamics are combined to create beautiful mechanisms that operate without vibrations of its base.

This guest lecture has led me to explore and contribute to the field of dynamic balancing for the past year. The result of this work is bundled in this thesis.

I would like to thank everyone that helped me with realizing this thesis. In particular, my supervisor Volkert van der Wijk, for his helpful comments, encouragement, and taking the time for insightful discussions that considerably helped me progress in my research. Furthermore, I would like to thank my parents and my brothers for their emotional support and helpful advice in the past year and the years before that.

Gijs Becht
The Hague, August 2020

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Introduction

In robotics, machine elements are accelerated in order for the machine to perform certain tasks, such as picking and placing objects. These accelerations result in inertia forces and inertia torques on the machine elements and on the base of the machine. These reaction forces and reaction torques on the base will lead to undesired vibrations at the base. The inertia forces on the base are called *shaking forces* and the inertia torques on the base of the machine are called *shaking moments*. A mechanism is called shaking force balanced when the resultant of the shaking forces on the base is zero. A mechanism is called shaking moment balanced when the resultant of the shaking moments on the base is zero. A mechanism is dynamically balanced when it is both shaking force and shaking moment balanced.

With the ever increasing demand in high production speeds and high precision of robotic manipulators, the problem of dynamic unbalance becomes prominent. When these mechanisms operate at high speeds or move large masses, shaking forces and shaking moments result in noise, vibration, wear and fatigue problems [2]. Dynamic unbalance exceptionally forms a problem when a mechanism is attached to a flexible base or is free-floating. Examples include cable robots [3], satellites [8], and humanoid robots [6]. Dynamic balancing eliminates shaking forces and shaking moments on the base, which results in low cycle times and high accuracy [1, 12]. While dynamic balancing eliminates the problems caused by shaking forces and shaking moments, dynamic balancing generally increases the masses, the moments of inertia, and the complexity of the mechanism [5]. This means that the method used for dynamic balancing must be critically selected such that these drawbacks are minimized.

The method of inherent dynamic balance aims at minimizing these drawbacks by considering the dynamic balance prior to the kinematic synthesis [10]. The main advantage of this procedure is that dynamic balance is not be determined by kinematical choices and balance solutions would not be limited beforehand. This results in balanced linkages that have a relatively low mass and inertia, which is advantageous for having low input torques, increased payload capabilities, and low bearing forces [11]. A dynamically balanced mechanism where all elements contribute to both the motion and the dynamic balance is considered an inherently dynamically balanced mechanism [10].

Inherently dynamically balanced mechanisms are constructed using the method of principal vectors. While the method of principal vectors has introduced a large number of new shaking force balanced mechanisms, the solutions found for shaking moment balanced mechanisms are still limited. Exceptionally, solutions with a nonlinear relation between link angular velocities are difficult to find with the current method, where shaking moment balanced solutions are extracted manually from the momentum equations [13]. Furthermore, with the current method, the designer is constrained to use specific motions that were not selected beforehand. The ideal order of operations would be to start the design process with a desired motion as input, and subsequently design an inherently balanced mechanism that reproduces this motion. This type of mechanism is called a motion generator. Currently, there is no design method that fulfils this qualification.

This thesis presents an overview of the current dynamic balancing methods and presents a new synthesis procedure for designing inherently dynamically balanced mechanisms, where a desired motion of the end effector is selected by the designer.

The main body of this thesis consists of the following three parts. The first two parts are presented in a paper format and can be read independently.

- The first part presents an overview of the current dynamic balancing methods with an emphasis on moment balancing. By discussing the working principles of different dynamic balancing methods this literature review aims to answer the following question. *How to synthesize dynamically balanced mechanisms?*
- The second part presents a new kinematic synthesis method for designing inherently dynamically balanced mechanisms, where a desired motion of the end effector is selected by the designer. This chapter will be submitted as a paper to the journal *Mechanism and Machine Theory*.
- The final part is an exploration of dynamically balanced multi degree of freedom mechanism. The goal of this chapter is to provide inspiration for new mechanisms and design methods.

The appendix provides additional background information for the second part. In this appendix, the derivation of the RR chain design equation is presented.

2

An overview of dynamic balancing methods with an emphasis on moment balancing

An overview of dynamic balancing methods with an emphasis on moment balancing

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1. Introduction

Unbalanced mechanisms generate shaking forces and shaking moments on its base. When these mechanisms operate at high speeds or move large masses, these shaking forces and shaking moments result in noise, vibration, wear and fatigue problems [1]. Furthermore, dynamic unbalance exceptionally forms a problem when a mechanism is attached to a flexible base or is free-floating. Examples include cable robots [2], satellites [3], and humanoid robots [4]. Dynamic balancing eliminates shaking forces and shaking moments on the base which allows for low cycle times and high accuracy [5, 6]. While dynamic balancing eliminates the problems caused by shaking forces and shaking moments, dynamic balancing generally increases the masses, the moments of inertia, and the complexity of the mechanism [7]. This means that the method used for dynamic balancing must be critically selected such that these drawbacks are minimized.

Van der Wijk has presented an inherent dynamic balancing approach, where the dynamic balance is considered prior the the kinematic synthesis [8]. The main advantage of this procedure is that dynamic balance is not be determined by kinematical choices and balance solutions would not be limited beforehand. This results in balanced linkages that have a relatively low mass and inertia, which is advantageous for having, among others, low input torques, increased payload capabilities and low bearing forces [9]. While this method provides new shaking force balanced mechanisms, the options for shaking moment balanced mechanisms are still limited. Inherent shaking force balanced mechanisms are moment balanced by selecting the correct relation between the moments of inertia and constraining the mechanism in such a way that the right relation of angular velocities between the links is achieved. These relations are found using the angular momentum equation. However, nonlinear relations between angular velocities of links are difficult to extract from the angular momentum equations. Solutions such as the dynamically balancing four-bar mechanisms are difficult to find using the angular momentum equation [10]. A new method for designing inherently moment balanced mechanisms is desired.

This literature review presents an overview of dynamic balancing methods with an emphasis on moment balancing. By discussing the working principles of different dynamic balancing methods this literature review aims to answer the following question. *How to synthesize dynamically balanced mechanisms?*

First, methods that are used for derivation of the dynamic balancing conditions are discussed in Section 2. These conditions are used to tune the design parameters of the mechanism links such that the mechanism is balanced. These parameters include masses, CoM locations, and moments of inertia. However, in the case of shaking moment balancing, most mechanisms are not balanceable by only tuning these parameters. For instance, a mechanism where all links rotate in the same direction can never be moment balanced. In these cases, additional kinematic synthesis steps are required, which are discussed in Section 3.

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2. Methods for derivation of the dynamic balancing conditions

Methods that only provide the conditions for shaking force balanced mechanisms use the condition that the CoM must be stationary. The methods that are used complete dynamic balancing involve the momentum equations. The dynamic balancing methods result in conditions that are used to determine the design parameters of a mechanism. These design parameters include link masses, link CoM locations, and link moments of inertia.

2.1. Shaking force balancing conditions

A mechanism is shaking force balanced when the common CoM of all links is stationary. Two methods that produce shaking force balanced mechanisms using this principle are the method of linearly independent vectors and the method of principal vectors.

2.1.1. Method of linearly independent vectors

The method of linearly independent vectors is proposed by Berkof and Lowen for the shaking force balancing of closed chain mechanisms [1]. Closed chain mechanisms satisfy the constraint that the chain of elements forms a closed loop, which called the loop closure equation. The loop closure equation is substituted into the equation for the common CoM. A mechanism is shaking force balanced when this equation is constant, which means that the time dependent terms must be set to zero. These time dependent terms are linearly independent and the conditions for shaking force balance can be obtained by equating the coefficients of the time dependent terms to zero.

2.1.2. Principal vectors

Principal vectors trace the common CoM of a mechanism. By making this common CoM stationary to the base a shaking force balanced mechanism is formed. Mechanisms synthesized using principal vectors are examples of inherently shaking force balanced mechanisms. These mechanisms can form the initial basis of synthesising mechanisms for certain tasks, such as grippers or bridges. In this way, the dynamic balance is considered prior to the kinematic synthesis. The main advantage of this procedure is that dynamic balance is not be determined by kinematical choices and balance solutions would not be limited beforehand. This results in balanced linkages that have a relatively low mass and inertia, which is advantageous for having, among others low input torques, increased payload capabilities and low bearing forces [9].

Principal vector linkages are not standard inherently moment balanced. However, they can be moment balanced by choosing the right link moments of inertia and constraining the system such that all DoFs involve counter rotations. This is further explained in section 3.2.

Figure 1 shows the principal vectors of a mechanism with two links AQ and AR, which are connected with a revolute joint in A. The CoM of link AQ is located in Q with mass m_A and the CoM of link AR is located in R with mass m_B. Principal point P_1 is found by projecting mass m_B in the principal joint A, and calculating the CoM of masses m_A and the projected m_B. The same can be done by projecting m_A onto A to find principal point P_2. By constructing a line parallel with AQ through P_2 and a line parallel with AR through P_1 the common CoM S is found as the intersection of these lines. Because distances P_1S and P_2S remain constant for all positions of the mechanisms, links can be added connecting the principal points. This linkage is called a pantograph and is depicted in Figure 2.

2.2. Shaking force and shaking moment balancing conditions from momentum equations

Methods that are used for either shaking force balancing or complete dynamic balancing involve the momentum equations. These methods result in a set of conditions that provide dynamically balanced mechanisms.

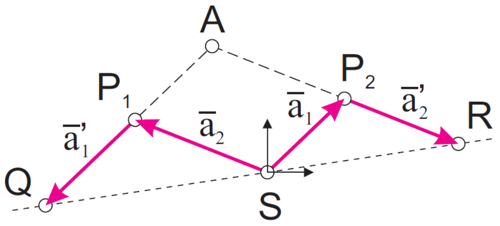


Figure 1: Principal vectors of link AQ with its CoM in Q and link AR with its CoM in R [8]

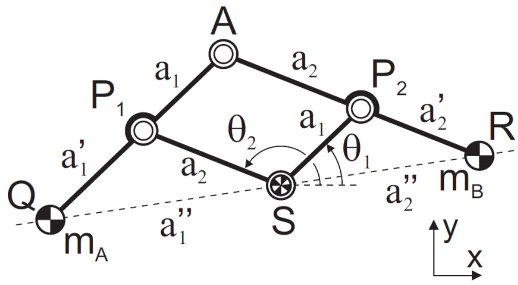


Figure 2: Pantograph mechanism with the common CoM in point S [8]

2.2.1. Momentum equations

Mechanisms are dynamically balanced when the sum of reaction forces and reaction moments on the base is zero. With other words, the total momentum of the system should remain constant for all motion. In practice, this is achieved by setting the momentum equations to zero. For shaking force balance, this means that the linear momentum should be zero which results in the common CoM of a mechanism being stationary. The condition for shaking force balance can be written as in [5]

$$\mathbf{p} = \sum_i m_i \dot{\mathbf{r}}_\mathbf{i} = 0 \tag{1}$$

where i is the link number and $\dot{\mathbf{r}}_i$ is the velocity vector of the link CoM with mass m_i. The condition for shaking moment balance for a planar mechanism can be written by equating the moment in z-direction about the point attached to the base o to zero [5]

$$\mathbf{h}_{o,z} = \sum_i I_i \dot{\alpha}_i + \mathbf{e}(\mathbf{r}_i \times m_i \dot{\mathbf{r}}_i) = 0 \tag{2}$$

where I_i is the moment of inertia of link i, $\dot{\alpha}_i$ is the absolute angular velocity of link i, and $\mathbf{e}$ is the unit vector in the z-direction $\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$.

2.2.2. Screw theory

Screw theory is an important tool in robot mechanics [11] and is recently introduced by de Jong for the application of dynamic balancing [12]. However, in his paper shaking moment balancing is only achieved instantaneously. This means that mechanisms are only moment balanced for certain poses. Thus, screw theory has not been used for complete dynamic balancing and this might be a promising direction for further research.

Through screw theory, momentum can be represented graphically by means of a momentum wrench. This graphical method is promising for synthesising moment balanced mechanisms, because it gives great insight in the dynamics of the mechanism. For instance:

- a mechanism with two moving rigid bodies is dynamically balanced when the momentum wrenches of the rigid bodies are equal, opposite, and lie on the same line for all positions of the mechanism;
- a mechanism with three moving rigid bodies is dynamically balanced when the momentum wrenches of the rigid bodies intersect in one point for all positions of the mechanism.

The momentum wrench is denoted in Plücker homogeneous coordinates [13] that describe a screw. A screw consists of two vectors, a vector $\mathbf{e}$ of direction and a vector $\mathbf{m}$ of moment. The general form of a screw is denoted as

$$\mathcal{S} = \begin{bmatrix} \mathbf{e} \\ \mathbf{m} \end{bmatrix} = s \begin{bmatrix} \hat{\mathbf{s}} \\ \mathbf{q} \times \hat{\mathbf{s}} + h\hat{\mathbf{s}} \end{bmatrix} \tag{3}$$

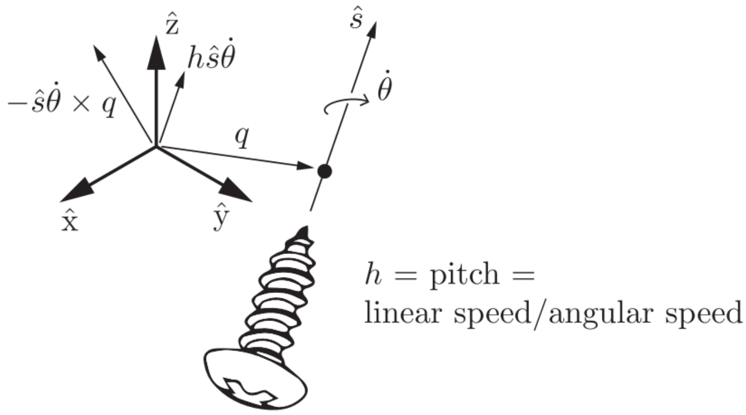


Figure 3: A screw axis $\mathcal{S}$ represented by a point $\mathbf{q}$, a unit direction $\hat{\mathbf{s}}$, and a pitch h [11]

where s is the magnitude, $\hat{\mathbf{s}}$ is the direction unit vector, $\mathbf{q}$ is the vector to any point on the screw axis, and h is the pitch of the screw. The use of screw notation in rigid body dynamics finds its roots in the fact that any rigid body motion can be produced by a translation along a line followed by a rotation about an axis parallel to that line. This means that rigid body motion can be described by a vector pair (or screw) containing the linear and angular velocity, denoted as $\mathbf{v}$ and $\boldsymbol{\omega}$ respectively, which is called the twist $\mathcal{V}$ and is denoted as

$$\mathcal{V} = \begin{bmatrix} \mathbf{e} \\ \mathbf{m} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\omega} \\ \mathbf{v} \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{s}}\dot{\theta} \\ -\hat{\mathbf{s}}\dot{\theta} \times \mathbf{q} + h\hat{\mathbf{s}}\dot{\theta} \end{bmatrix} \tag{4}$$

where $\dot{\theta}$ is the magnitude or angular speed. The graphical representation of a twist is depicted in Figure 3. Loading, such as force or momentum, can be represented in screw theory by means of a wrench. The momentum wrench $\mathbf{h}$ containing the linear and angular momentum, denoted as $\mathbf{p}$ and $\boldsymbol{\xi}$ respectively, and is denoted as

$$\mathbf{h} = \begin{bmatrix} \mathbf{m} \\ \mathbf{e} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\xi} \\ \mathbf{p} \end{bmatrix} = \begin{bmatrix} \mathbf{r}_h \times \mathbf{p} + \lambda_h \mathbf{p} \\ \mathbf{p} \end{bmatrix} \tag{5}$$

where $\mathbf{r}_h$ is a point on the screw and λ_h is the momentum pitch. Note that the direction of a wrench is determined by the linear part $\mathbf{p}$ as opposed to the twist, where the direction is determined by the angular part $\boldsymbol{\omega}$ [12]. For a point mass, the momentum wrench passes through that point mass. In the case of a body with an additional moment of inertia, the momentum wrench does not pass the CoM of that body. The momentum wrench passes through the point $\mathbf{r}_h$ which is calculated by

$$\mathbf{r}_h = \frac{\mathbf{p} \times \boldsymbol{\xi}}{|\mathbf{p}|^2} \tag{6}$$

An analysis of the dynamically balanced four-bar mechanisms shows the behaviour of momentum wrenches for dynamically balanced mechanisms. The known dynamically balanced mechanisms are the anti-parallelogram and the kite [14] and are depicted in Figures 4 and 5. The linear and angular momentum are calculated using the parameters provided in [14] and the kinematic relations provided in [15]. The momentum wrenches are drawn through the point $\mathbf{r}_h$ (Equation 6) and point in the direction of the linear momentum. This process is repeated for all positions of the mechanism using a MATLAB script. This shows that all momentum wrenches intersect in one point for all positions

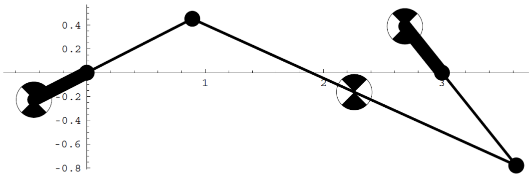


Figure 4: Inherently balanced crossed four bar mechanism [14]

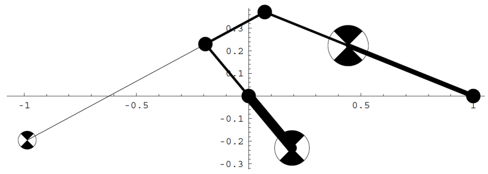


Figure 5: Inherently balanced four bar kite mechanism [14]

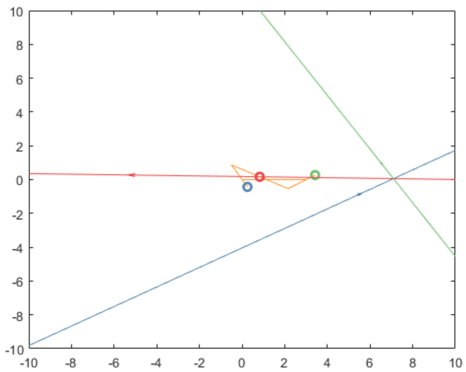


Figure 6: Dynamically balanced anti-parallelogram four-bar mechanism with intersecting screw lines

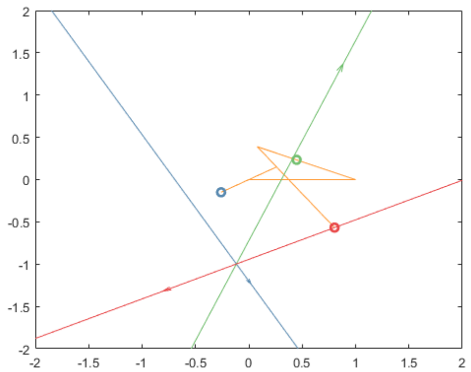


Figure 7: Dynamically balanced kite four-bar mechanism with intersecting screw lines

of the mechanism. However, this point is different for all positions. These results of a single pose are presented in Figure 6 and 7.

Another simulation of the dynamically balanced four-bar mechanisms is executed with equimomental systems. The principle of equimomental systems is discussed in Section 2.2.4. Because the CoM of the links is along the link, two point masses is sufficient for a dynamically equivalent model. Because the links are modelled as point masses without an extra moment of inertia, the momentum wrenches can be drawn through the point masses. These results are presented in Figure 8 and 9.

2.2.3. Reactionless path planning

Reactionless path planning is used in the design and control of free-floating space robots [16, 17], manipulators fixed to a flexible base [18], and humanoid robots [3]. Generally, in these cases no or minimal shaking forces and shaking moments are desired on the base of the robot. The main advantage of reactionless path planning is that robots can be operated in their complete workspace, or can be operated to move along a certain path that results in dynamic balance when needed.

Reactionless paths are derived from the reactionless null space (RNS) of a manipulator. The RNS is the set of manipulator motions that do not impose reaction forces and reaction moments on the base. The RNS can be computed by first composing the momentum equation of a manipulator

$$\mathbf{M}_b \mathcal{V}_b + \mathbf{M}_{bm} \dot{\boldsymbol{\theta}} = \mathcal{L}_b \tag{7}$$

where $\mathbf{M}_b$ is the inertia matrix with respect to the free floating base, $\mathcal{V}_b$ is the spatial velocity (or the twist), $\mathbf{M}_{bm}$ is the coupling inertia matrix, $\mathcal{L}_b$ is the momentum wrench, and $\dot{\boldsymbol{\theta}}$ is the vector containing the joint velocities. The goal is to calculate $\dot{\boldsymbol{\theta}}$ such that the momentum $\mathcal{L}_b$ is zero. This results in the following equation

$$\dot{\boldsymbol{\theta}} = -\mathbf{M}_{bm}^{-1} \mathbf{M}_b \mathcal{V}_b \tag{8}$$

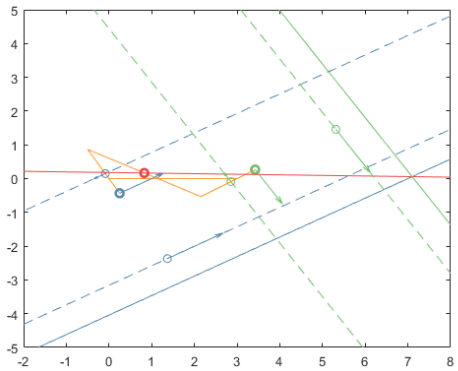


Figure 8: Dynamically balanced anti-parallelogram four-bar mechanism with equimomental system point masses and screw lines

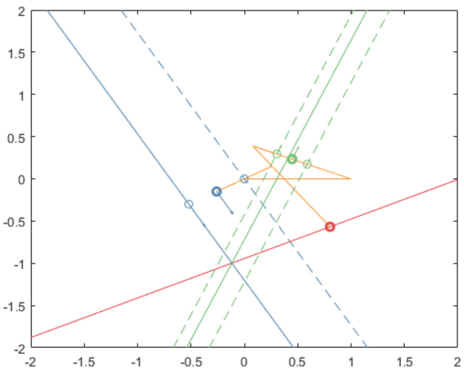


Figure 9: Dynamically balanced kite four-bar mechanism with with equimomental system point masses and screw lines

The number of reactionless paths can be increased by adding extra joints to the manipulator, increasing the dimension of $\dot{\boldsymbol{\theta}}$. For instance, the manipulator designed by Papadopoulos [19], which is depicted in Figure 10, can move in-plane with one redundant DoF. By calculating the joint velocities such that the momentum remains zero, a reactionless workspace is found spanned by the reactionless paths. This reactionless workspace is a subspace of the reachable workspace of the manipulator, which is depicted in Figure 11.

Reactionless paths can also be determined for shaking force balanced mechanisms using another method (van der Wijk, Unpublished). This is done by imposing a moment on a joint which is not connected to the base. Because the joint is not connected to the base and the CoM of the mechanism is stationary, the moment remains internal and no reaction moments are excited on the base. Due to the imposed moment, the mechanism will move and the end effector will trace a path which is considered a reactionless path. By tweaking the ratio of the moments of inertia of the links different reactionless paths are found. The mechanism will be shaking moment balanced when constrained such that it moves along these reactionless paths. By obtaining a large set of solutions for different moment of inertia ratios these paths can be graphed in a systematic way. A mechanism designer can search through these graphs to find a mechanism that follows the desired path.

2.2.4. Equimomental systems

An equimomental system is a model of a rigid body consisting of point masses that hold the same dynamic properties as the rigid body. This means that the mass, CoM location, and the moment of inertia are equal for the rigid body and the equimomental system. Any planar rigid body can be modelled with three point masses as depicted in Figure 12. A point mass is defined by its mass and its location in polar coordinates. Chaudhary and Saha have used equimomental systems to minimize shaking forces and shaking moments in mechanisms [20]. With this method, the momentum equations of the mechanism are written as functions of the parameters related to the point masses of the equimomental systems. Subsequently, the dynamic balancing is formulated as an optimization problem where the shaking forces and shaking moments are the objective function and the parameters of the point masses are the design variables.

Because the optimization procedure is executed by an algorithm, this method does not provide a lot of insight in design principles that lead to dynamic balance. However, equimomental systems allow for a graphical representation of the dynamics of a rigid body. The moment of inertia, which is difficult to represent graphically, no longer forms an issue as the equimomental system consists of only point masses. This is particularly interesting when used in combination with screw theory because the screw lines of moving point masses can be drawn through the point mass.

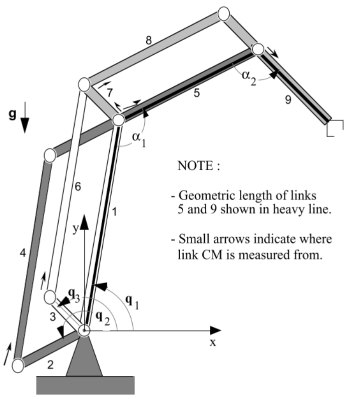


Figure 10: 3 DoF manipulator actuated at q_1 to q_3 [19]

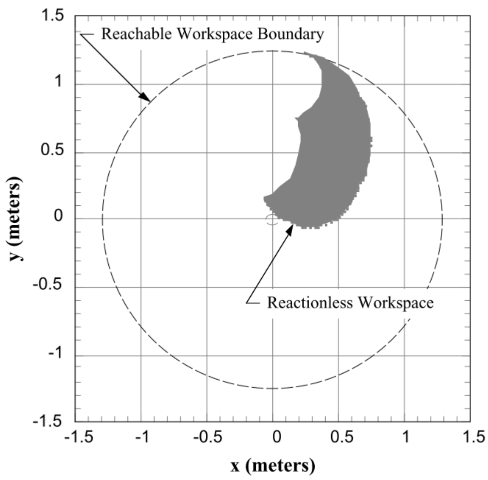


Figure 11: Complete workspace and reactionless workspace of the 3 DoF manipulator[19]

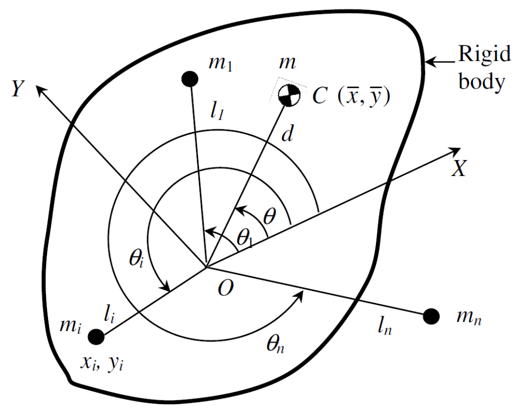


Figure 12: Dynamic equivalence of a planar rigid body [20]

2.2.5. Algebraic method

The dynamic balancing conditions of a four bar mechanism have been derived using the momentum equations in combination with the loop closure equations in [14]. However, this method is difficult to follow because of the long equations. Moore has used a new method where the problem is reformulated into an abstract algebra problem [21, 22]. This method is a systematic approach for dynamic balancing that results in a structured overview of the kinematic modes of a four bar mechanism and shows which modes are balanceable. However, the algorithm that is used for deducing these conditions is highly abstract and provides little insight in how balanceable mechanisms can be designed. Since this method involves a lot of new mathematical concepts to a reader with a mechanical engineering background, only a brief summary is presented here and the reader is referred to [21] for a detailed description of the method.

Moore has reformulated the problem of dynamically balancing a four-bar mechanism as a factorisation problem of Laurent polynomials. A Laurent polynomial is a linear combination of Laurent monomials. A Laurent monomial is defined as

$$z_1^{\beta_1} z_2^{\beta_2} \dots z_n^{\beta_n} \tag{9}$$

where $\beta_i \in \mathbb{Z}$. This means that the geometric constraint, also known as the loop closure relation, and the dynamic balancing conditions can also be written as a linear combination of Laurent polynomials by writing the vectors in their complex notation as

$$\mathbf{z}_1 = le^{\theta i} \tag{10}$$

where l is the magnitude and θ is the angle of the vector counter-clockwise from the x-axis. Moore uses a theorem, referred to as Theorem 3 in [21], that states that the following statements are equivalent:

1. When an irreducible Laurent polynomial G is zero, Laurent polynomial F is zero
2. A Laurent polynomial K exists such that $F = G \cdot K$

Since the dynamic balancing conditions can be written as a linear combination of Laurent polynomials, Theorem 3 can be used to derive the conditions or dynamic balancing. The geometric constraint is G, the shaking force balancing condition is F and the shaking moment balancing condition is K. Theorem 3 states that G must be irreducible, this condition leads to the kinematic modes in Figure 13. Furthermore, the dynamic balancing conditions of these kinematic modes are derived using a toric polynomial division algorithm. This results in three modes that are balanceable. These are the crossed parallelogam (case II, mode A), the kite (case III, mode A) and a mirrored kite (case IV, mode A).

3. Kinematic synthesis of moment balanced mechanisms

Not all mechanisms are shaking moment balanceable by adjusting the design parameters of the links. In these cases additional kinematic synthesis steps are required. First, mechanisms can be made balanceable by addition of extra elements that involve counter rotations. Second, mechanisms can be constrained such that all remaining DoFs involve counter rotations. These balancing methods are examples of dynamic balancing after the kinematic synthesis. Finally, mechanisms can be synthesised using building blocks that are already moment balanced. These building blocks are the families of inherently shaking moment balanced four-bar mechanisms.

3.1. Using additional elements

Mechanisms can be moment balanced by addition of new elements. These methods can provide insight and new ideas for inherent moment balancing. A selection of moment balancing solutions from [5, 6, 7, 23, 24] is treated and are categorized as follows.

- Direct balancing of the overall shaking moment

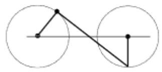
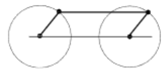
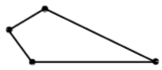
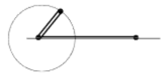
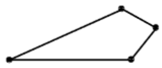
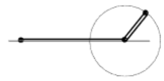
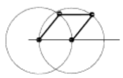
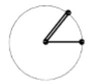
Case	Mode A	Mode B	Mode C
II Parallelogram $l_1 = l_2 \neq l_3 = d$	 $d\mathbf{z}_1\mathbf{z}_2 + l_1\mathbf{z}_1 - l_1\mathbf{z}_2 - d = 0$	 $\mathbf{z}_1 - \mathbf{z}_2 = 0$	
III Deltoid-1 $l_1 = l_3 \neq l_2 = d$	 $-d\mathbf{z}_1\mathbf{z}_2 - d\mathbf{z}_1 + l_1\mathbf{z}_2 + l_1\mathbf{z}_1^2 = 0$	 $\mathbf{z}_2 + 1 = 0$	
IV Deltoid-2 $l_1 = d \neq l_2 = l_3$	 $d\mathbf{z}_1\mathbf{z}_2 + l_2\mathbf{z}_1 - d\mathbf{z}_2 - l_2\mathbf{z}_2^2 = 0$	 $\mathbf{z}_1 - 1 = 0$	
V Rhomboid $l_1 = l_2 = l_3 = d$	 $\mathbf{z}_1 - \mathbf{z}_2 = 0$	 $\mathbf{z}_1 - 1 = 0$	 $\mathbf{z}_2 + 1 = 0$

Figure 13: Kinematic modes of the four bar [22]

- By addition of a single balancer element
  - ∗ Passive balancing unit [7, 23]
  - ∗ Active balancing unit [24, 6]
- By addition of an Assur Group [25]

• Direct balancing of individual members

- By addition of counter-rotary countermasses (CRCMs) [5, 23, 6]
- By addition of separate counter rotations
  - ∗ Planetary transmission [5, 7, 23, 6]
  - ∗ Parallelogram transmission [7, 23, 6] (or idler loop [5])

• By addition of duplicate mechanisms [5, 7]

By using these balancing methods, the motion freedom of the mechanism remains unchanged. However, the masses, inertias and complexity of the mechanism is increased significantly.

3.1.1. Direct balancing of the overall shaking moment

In the balancing of the overall shaking moment, the sum of the shaking moments of all elements is balanced by a single element or an by an Assur group.

Single element. When using a single element, this element can be actuated passively or actively. In the case of a passive element, the element is actuated by movement of the mechanism. Figure 14 shows the balancing of a four-bar mechanism using a passive element. Because the angular velocities of the four-bar links are not constant, a varying transmission ratio is required. An active dynamic balancing

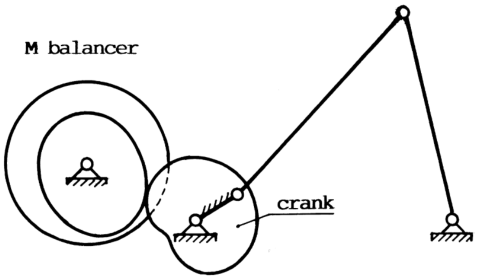


Figure 14: Balancing of the overall shaking moment using a counter-rotating mass with a varying transmission ratio [7]

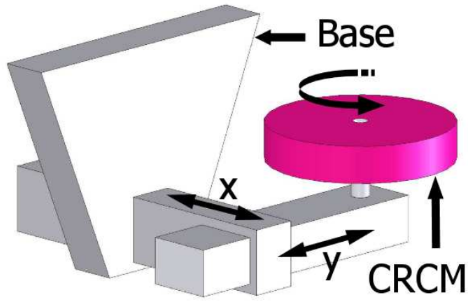


Figure 15: Balancing of the overall shaking moment using an active dynamic balancing unit [24]

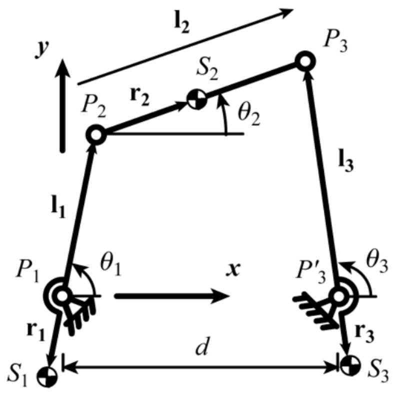


Figure 16: Dynamically unbalanced four-bar mechanism [25]

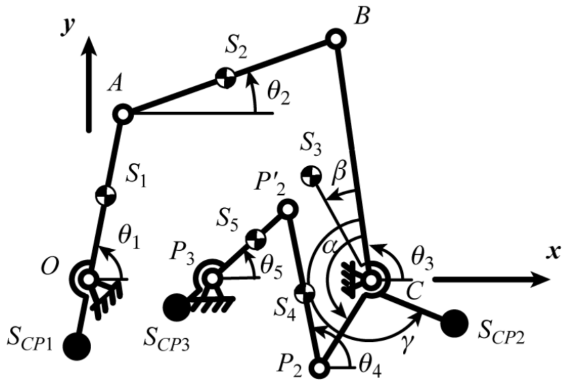


Figure 17: Dynamically balanced four-bar mechanism by means of an Assur group [25]

unit is shown in Figure 15. This unit is actuated separately from the mechanism and is controlled such that it delivers a moment which is equal and opposite of the moment resulting from the mechanism itself.

Assur group. An Assur group is a kinematic chain which does not add any supplementary degree of freedom into the mechanism [25]. Figure 17 shows how the four-bar mechanism depicted in Figure 16 is dynamically balanced by addition of an Assur group. This Assur group is the crossed four-bar mechanism which is balanceable by itself and is also used as a building block for dynamically balanced parallel mechanisms. The coupler link acts as a counter rotation. A crank-slider mechanisms can also be added to this four-bar mechnism instead of the crossed four-bar mechanism. The crank-slider is also balanceable be itself and also adds a counter rotation.

3.1.2. Direct balancing of individual members

A mechanism can be shaking moment balanced by counteracting the shaking moment resulting from each individual member. Counter-rotating countermasses (CRCM) act simultaneously as countermasses that provide shaking force balance and counter rotations that provide shaking moment balance. The balancing of a double pendulum using this principle is shown in Figure 20. Pure shaking moment balancing is achieved by attaching the counter-rotating masses to the base. The transmission

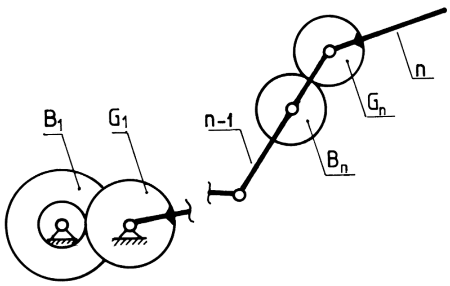


Figure 18: Balancing of individual members using planetary transmission along the links [7]

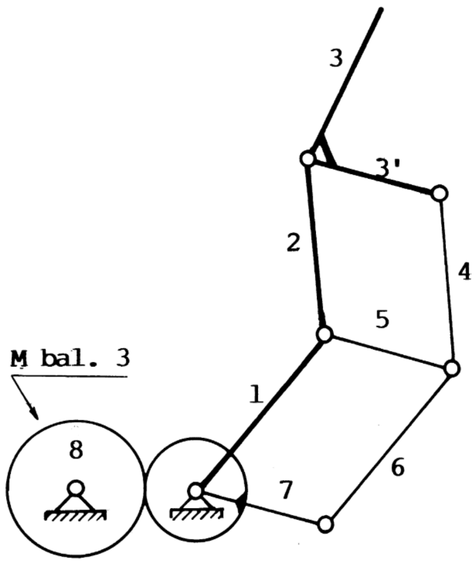


Figure 19: Balancing of individual members using parallelogram transmission [7]

from link to counter-rotating mass is established using either planetary transmissions (Figure 18) or parallelogram transmissions (Figure 19). Parallelogram transmissions play an important role in the synthesis of dynamically balanced linkages. This is evident from the fact that principal vector linkages are built up using parallelograms. Furthermore, other manipulators such as the 3 DoF manipulator in Figure 10 also consists of parallelograms. The reason for this is that a parallelogram can transfer a rotation between links without the coupler rotating. However, counter rotations are required for shaking moment balance. For this application the anti-parallelogram might offer a solution. The anti-parallelogram is shaking moment balanceable by itself and is used as a building block for multi-DoF parallel mechanisms.

3.1.3. Duplicate mechanisms

Another method to achieve simultaneous shaking force and shaking moment balance is by duplicating the mechanism. The duplicates of the mechanism provide mirrored motion which results in complete dynamic balance. Balancing using duplicate motion of a double pendulum is depicted in Figure 21.

3.2. Constraining multi-DoF linkages

Linkages with multiple DoFs can be shaking moment balanced by finding the conditions for shaking moment balancing and constraining the system such that these conditions are met. For example, Van der Wijk has shaking moment balanced the principal vector linkage in Figure 22 by constraining P_2 with a slider as depicted in Figure 23 [26]. With this additional constraint the mechanism is reduced to one DoF where $\dot{\theta}_1 = -\dot{\theta}_3$ and $\dot{\theta}_2 = 0$. When the moment of inertia of link one is equal to the moment of inertia of link three the mechanism is shaking moment balanced.

This condition for shaking moment balance is derived using the angular momentum equation presented in Section 2.2.1. This condition is formulated relatively compactly using the principal vector dimensions. Another method of deriving the shaking moment balance conditions is through reactionless path planning. A dynamically balanced mechanism is formed by constraining the mechanism to move along the reactionless path with a slider.

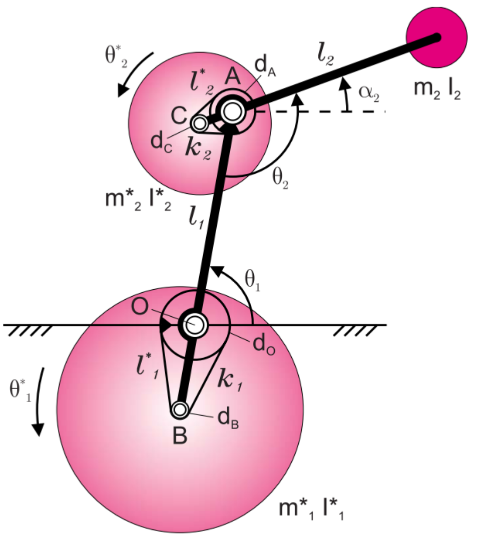


Figure 20: Balancing of individual members using counter-rotating countermasses [5]

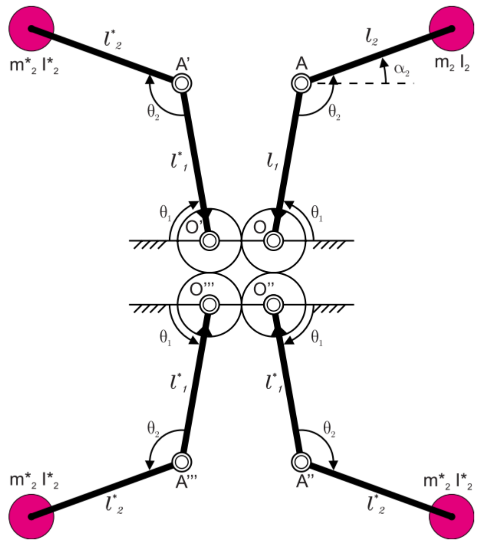


Figure 21: Balancing of the complete mechanism using duplicate mechanisms [5]

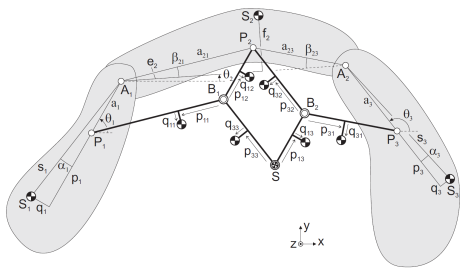


Figure 22: Principal vector linkage of a 3DoF mechanism with three principal elements in series [8]

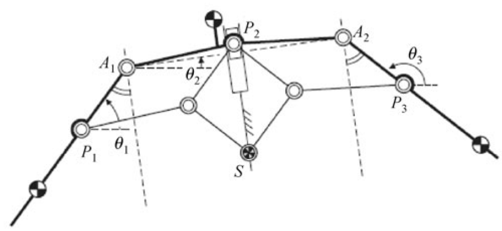


Figure 23: Principal vector linkage of three principal elements in series constrained with a slider in P_2 such that $\dot{\theta}_1$ = -$\dot{\theta}_3$ and $\dot{\theta}_2$ = 0 [26]

The disadvantage of this method is that the designer is constrained to use the balance conditions found after the kinematic synthesis. The reactionless motion can still be adjusted by changing the CoMs and the moments of inertia of the links. However, the ideal method would be to start with a desired path and design an inherently balanced mechanism such that is moves along this path.

3.3. Dynamically balanced four bar mechanisms as building blocks

In the synthesis of parallel mechanisms, the dynamically balanced four-bar mechanism can be used as legs of the parallel mechanism. Each leg is dynamically balanced by itself and drives a DoF of the end effector. Figure 24 shows a 3 DoF planar parallel mechanism consisting of three dynamically balanced crossed four bar mechanisms. The dynamically balanced mechanisms were first utilised as legs to design a dynamically balanced planar 3-DoF parallel manipulator [14]. Later, this was extended to a 6-DoF spatial parallel manipulator [27].

4. Discussion and Conclusions

In order to answer the question *How to synthesize dynamically balanced mechanisms?*, an overview was presented of dynamic balancing methods found in literature. First, the methods for derivation

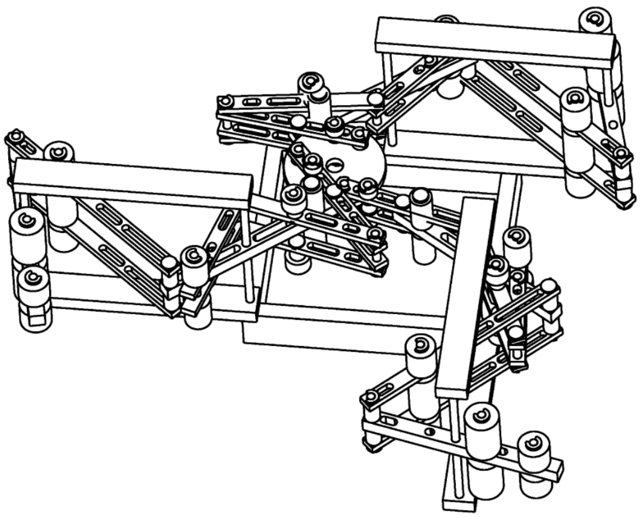


Figure 24: Dynamically balanced 3 DoF parallel mechanism consisting of three dynamically balanced four bar mechanisms [28]

of the dynamic balancing conditions were presented. These conditions can be used to tune design parameters of a mechanism to make it balanced. These design parameters include link masses, link CoM locations, and link moments of inertia. The methods for finding the dynamic balancing conditions are the following.

- Methods for derivation of shaking force balance conditions
  - Method of linearly independent vectors
  - Principal vectors
- Methods for shaking force and shaking moment balancing conditions
  - Momentum equations
  - Screw theory
  - Reactionless path planning
  - Equimomental systems
  - Algebraic method

The method of principal vectors allows for the kinematic synthesis of shaking force balanced mechanisms without lengthy calculations because the method relies on graphical derivations. In order to balance principal vector linkages for shaking moments, the correct relations between angular velocities and moments of inertia of the links have to be found. While these relations can be found using the angular momentum equations, it is difficult to find all possible relations with the nonlinear relations in particular. The dynamically balanced four-bar mechanisms prove that dynamically balanced mechanisms that have nonlinear angular velocity relations between the links exist. A systematic method that finds these relations for all types of mechanisms is desired. In order to avoid long equations, a graphical method is preferred. Furthermore, the method of principal vectors shows that graphical methods provide an intuitive understanding of the dynamics of a mechanism.

Screw theory allows for a graphical representation of linear and angular momentum using spatial coordinates. Using spatial coordinates, graphical conditions for shaking moment balance can be formulated which can form the basis of a design procedure for dynamically balanced mechanisms. For instance, the momentum wrenches of a mechanism with three rotating links should intersect in one point. An analysis of the dynamically balanced mechanisms supports this statement. Important to note here is that this should be the case for all positions of the mechanism. When the momentum wrenches intersect in one point for an arbitrary position of the mechanism it is not necessarily the case for all positions. Since momentum wrenches give a spatial representation of the momentum, screw theory is of particular interest for the synthesis of spatial mechanisms.

Equimomental systems allow for the dynamic modelling of a rigid body using only point masses. This allows for rigid bodies with moments of inertia to be, except for the mass, modelled graphically. This is exceptionally promising when used in combination with screw theory, because the location of the momentum wrench of a point mass is always known, as the momentum wrench intersects the point mass.

In reactionless path planning the spatial notation of screw theory is used to formulate the momentum equations. A systematic approach is used to determine the paths for which the mechanism is dynamically balanced. This method relies on the calculation of the null space of the coupling inertia matrix. This mathematical operation is useful to calculate the moment balancing conditions for an existing robot that is actively controlled and actuated at each DoF separately. However, if the intention is to design a mechanism that is dynamically balanced for all DoFs this operation provides little insight.

Finally, Moore's algebraic method of deriving dynamically balanced four-bar mechanisms is systematic and provides an overview for all possible solutions. Moore states that the method can likely be used to find dynamically balanced solutions of other mechanisms. However, the method is highly mathematically abstract which leaves little insight in the actual dynamics of the system.

In the case of shaking moment balancing, most mechanisms are not balanceable by only tuning these parameters. For instance, a mechanism where all links rotate in the same direction can never be moment balanced. In these cases, additional kinematic synthesis steps are required. These kinematic synthesis steps were discussed. The following methods for additional kinematic synthesis were discussed.

- Addition of new elements, such as counter rotating masses.
- Constraining multi-DoF mechanisms, such that the remaining DoFs involve counter rotations.
- Using moment balanced building blocks, such as the dynamically balanced four-bar mechanism.

By addition of new elements the motion freedom of the mechanism is maintained. However, this method adds a significant amount of masses, moments of inertia, and complexity to a mechanism. For this reason this method is of practical value in very limited situations [7].

Constraining multi-DoF linkages generally does not add a lot of mass and complexity to the mechanism since this can be achieved by the addition of simple elements such as a slider. However, the disadvantage is that the designer is constrained to the balance conditions found after the kinematic synthesis. While the reactionless motion can still be adjusted by changing the CoMs and the moments of inertia, the rough motion freedom is already determined. The ideal method would be to start with a desired path and design an inherently balanced mechanism that moves along this path.

The method of using dynamically balanced four bar mechanisms as building blocks shows promising results [6]. However, when the mechanism only consists of these building blocks, this method will lead to a limited set of possible outcomes, as only two types of building blocks are available. A more general method for designing dynamically balanced mechanism might lead to more building blocks suitable for the design of dynamically balanced parallel mechanisms.

Parallelogram transmissions are widely used in the dynamic balancing of mechanisms as can be seen in the principal vector linkage or in the balancing of individual members of a mechanism. The advantage of parallelograms is that they can produce a one to one transmission without the coupler rotating. The crossed parallelogram can transfer counter rotations which is promising for moment balancing. This is demonstrated by the use of the crossed four bar as an Assur group in the dynamic balancing of a four bar mechanism. A more general method of incorporating the crossed parallelogram into mechanisms might provide new dynamic balancing solutions.

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3

Synthesis method for inherently dynamically balanced 1-DoF motion generation mechanisms

Synthesis method for inherently dynamically balanced 1-DoF motion generation mechanisms

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Abstract

The method of inherent dynamic balancing aims at finding dynamically balanced mechanisms with a low mass and inertia. In this paper, a method is presented for the synthesis of inherently dynamically balanced 1-DoF pantographic linkages, where the desired motion of the end-effector is selected by the designer. This motion is defined as a set of precision positions. For this new method, the known method for RR chain synthesis is combined with the shaking moment balancing condition. For the special case where the relationship between link angular velocities is linear, the shaking moment balancing condition is substituted into the RR chain design equation. For the general case where the relation between link angular velocities is non-linear, the equation of motion is numerically solved for a range of possible solutions in order to find the solution which reproduces the desired motion.

Keywords: Dynamic balancing, Shaking force balancing, Shaking moment balancing, Kinematic synthesis, Pantograph, Motion generator

1. Introduction

Unbalanced mechanisms generate shaking forces and shaking moments on its base. When these mechanisms operate at high speeds or move large masses, these shaking forces and shaking moments result in noise, vibration, wear and fatigue problems [1]. Dynamic imbalance exceptionally forms a problem when a mechanism is attached to a flexible base or is free-floating. Examples include cable robots [2], satellites [3], and humanoid robots [4]. Dynamic balancing eliminates shaking forces and shaking moments on the base, which results in low cycle times and high accuracy [5, 6]. While dynamic balancing eliminates the problems caused by shaking forces and shaking moments, dynamic balancing generally increases the masses, the moments of inertia, and the complexity of the mechanism [7]. This means that the method used for dynamic balancing must be critically selected such that these drawbacks are minimized.

The method of inherent dynamic balance aims at minimizing these drawbacks by considering the dynamic balance prior to the kinematic synthesis [8]. The main advantage of this procedure is that dynamic balance is not be determined by kinematical choices and balance solutions would not be limited beforehand. This results in balanced linkages that have a relatively low mass and inertia, which is advantageous for having low input torques, increased payload capabilities, and low bearing forces [9]. A dynamically balanced mechanism where all elements contribute to both the motion and the dynamic balance is considered an inherently dynamically balanced mechanism [8].

The method of principal vectors is used for finding inherently dynamically balanced mechanisms. However, this method currently provides limited solutions for shaking moment balanced mechanisms. The shaking moment balanced solutions of principal vector linkages are formed by reducing the DoFs

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[10]. Similarly, the method of reactionless path planning is another dynamic balancing approach that uses the reduced motion freedom of a mechanism for it to move in a balanced way [3]. The disadvantage of these methods is that the designer is constrained to use specific motions that were not selected beforehand. The ideal order of operations would be to start the design process with a desired motion as input, and subsequently design an inherently balanced mechanism that reproduces this motion. Currently, there is no design method that fulfils this qualification.

This paper presents a synthesis procedure for designing inherently dynamically balanced mechanisms, where a desired motion of the end effector is selected by the designer. First, the general method for the synthesis procedure is discussed. Next, the synthesis method for the special case with a linear relationship between link angular velocities is explained. Finally, the synthesis method for the general case where this relationship is non-linear is discussed.

2. Synthesis method for dynamically balanced motion generators

The method presented in this paper uses the pantograph, a principal vector linkage, as a starting point. The pantograph is a shaking force balanced two DoF mechanism. For shaking moment balance, counter rotations are required. This means that the pantograph must be reduced to one DoF. As depicted in Figure 1, this is achieved by adding a slider to the right side of the parallelogram. Another way of constraining the mechanism is by adding a belt- or gear transmission between the first and second DoF of the pantograph.

The goal of the new method is to find the fixed- and moving pivot of the pantograph, denoted by $\mathbf{G}$ and $\mathbf{W}_1$ respectively, such that the end-link passes trough a set of predefined precision positions, as depicted in Figure 1. A precision position, denoted by $\mathrm{M_i}$, includes the (x,y) position and the angle of the end effector. This goal is achieved using the principle for crank design, also known as RR chain synthesis, from the well known four-bar mechanism synthesis procedure known as Burmester's theory [11]. By combining the RR chain design equation with the shaking moment balancing condition, dynamically balanced mechanisms are synthesized.

Figure 1 shows how an RR chain and a pantograph are related. A pantograph is formed by adding a parallelogram to the RR chain. The addition of the parallelogram does not change the kinematics of the mechanism, as the opposing links of a parallelogram have the same angular velocity. However, the parallelogram improves the stiffness, adds freedom in CoM distribution of the links, and provides additional options for actuation placement.

2.1. Dynamic balancing conditions

Mechanisms are dynamically balanced when the sum of reaction forces and the sum of the reaction moments on the base are zero. For shaking force balance, this means that the linear momentum should be zero, which results in the common CoM of all elements of a mechanism being stationary. The condition for shaking force balance is written as

$$\mathbf{p} = \sum_i m_i \dot{\mathbf{r}}_i = 0 \tag{1}$$

where $\mathbf{p}$ is the total linear momentum of the system, i is the link number and $\dot{\mathbf{r}}_i$ is the velocity vector of the link CoM with mass m_i [5]. The condition for shaking moment balance is that the angular momentum should be zero, which is written as

$$\mathbf{h}_{o,z} = \sum_i I_i \dot{\theta}_i + \mathbf{e}(\mathbf{r}_i \times m_i \dot{\mathbf{r}}_i) = 0 \tag{2}$$

where $\mathbf{h}_{o,z}$ is the angular momentum about point o and points in the out of plane direction z, I_i is the link moment of inertia about its CoM, $\dot{\theta}_i$ is the absolute angular speed of link i, and $\mathbf{e}$ is the unit

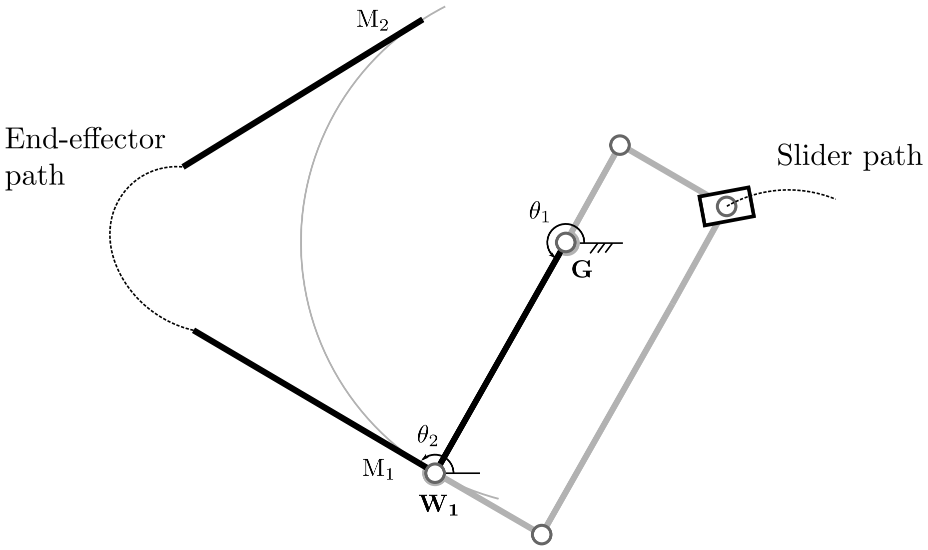


Figure 1: An RR chain (depicted in black) of which the end-link can move from precision position M_1 to M_2. The parallelogram (depicted in grey) combined with the RR chain forms a pantograph.

vector in the z-direction [5]. For the pantograph depicted in Figure 2, Equation 1 leads to the following conditions for shaking force balance

$$s_1 m_1 + l_1(m_2 + m_5) = l_3 m_4 + (s_3 - l_1)m_3 \tag{3}$$
$$s_2 m_2 + s_5 m_5 = s_4 m_4 + l_2 m_3 \tag{4}$$

Equation 2 results in the following condition for shaking moment balance

$$\mathbf{h}_{o,z} = (I_1 + I_3)\dot{\theta}_1 + (I_2 + I_4 + I_5)\dot{\theta}_2 + \sum_{i}^{5} \mathbf{e}(\mathbf{r}_i \times m_i \dot{\mathbf{r}}_i) = 0 \tag{5}$$

When the conditions $s_4 = 0$ and $s_3 = 0$ hold in conjunction with the shaking force balancing conditions, it is considered a special case. For this special case, Equation 5 leads to the following condition for shaking moment balance

$$\mathbf{h}_{o,z} = I_{1r}\dot{\theta}_1 + I_{2r}\dot{\theta}_2 = 0 \implies \frac{\dot{\theta}_1}{\dot{\theta}_2} = -\frac{I_{2r}}{I_{1r}} = k \tag{6}$$

where I_{ir} is the reduced inertia of link i, which can be obtained from the Equivalent Linear Momentum System (ELMS) of DoF i [8]. The reduced inertia I_{ir} is calculated as the inertia of the masses about the fixed pivot together with the link inertia. The reduced inertias of DoF 1 and 2 are written as

$$I_{1r} = (I_1 + I_3) + m_1 s_1^2 + m_3(s_3 - l_1)^2 \tag{7}$$
$$I_{2r} = (I_2 + I_4 + I_5) + m_2 s_2^2 + m_5 s_5^2 + m_4 s_4^2 + m_3 l_2^2 \tag{8}$$

The left side of Equation 6 is known as the invariant canonical form of the angular momentum equation [7, 12]. This form results in a linear relationship between the link angular velocities. This shaking moment balancing condition can be substituted into the RR chain design equation, which results in a design equation for dynamically balanced pantographic mechanisms. This procedure is treated in Section 3.

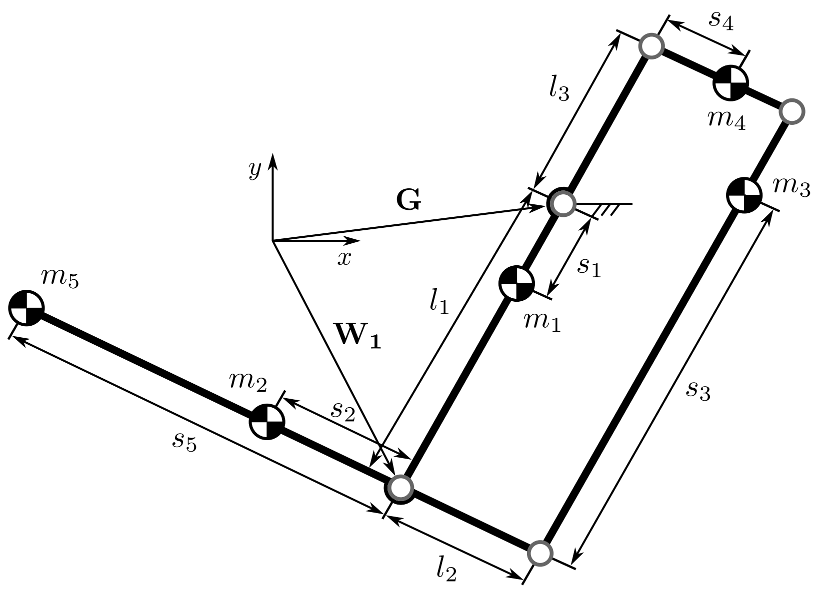


Figure 2: Pantographic mechanism

For the general case, where just the shaking force balancing conditions hold, the relation between the link angular velocities is not linear, and the value of k is different for each position of the mechanism. Equation 2 results in

$$\frac{\dot{\theta}_1}{\dot{\theta}_2} = k = f(\theta_1, \theta_2) \tag{9}$$

which is a non-linear differential equation and has to be solved numerically. This means that the kinematic synthesis for the general case requires a numerical approach, this is discussed in Section 4.

2.2. RR chain design method

The known methods for RR chain synthesis for up to five precision points are discussed by McCarthy in [11]. This includes graphical and algebraic methods. The algebraic method uses the RR chain design equation, which is a complex equation and is written as

$$(1 - e^{i\varphi_{1\mathrm{i}}})\mathbf{P}_{1\mathrm{i}} = (1 - e^{i\beta_{1\mathrm{i}}})\mathbf{G} + e^{i\beta_{1\mathrm{i}}}(1 - e^{i\alpha_{1\mathrm{i}}})\mathbf{W}_1 \qquad \mathrm{i} = 2, \ldots, n \tag{10}$$

where $\mathbf{P}_{1\mathrm{i}}$ is the pole of relative displacement of the end-link displacement from M_1 to M_i, $\mathbf{G}$ is the position of the fixed pivot, $\mathbf{W}_1$ is the position of the moving pivot, $\varphi_{1\mathrm{i}}$ is the angle of rotation about the pole of relative displacement, $\beta_{1\mathrm{i}}$ is the rotation angle of the first link about the fixed pivot, $\alpha_{1\mathrm{i}}$ is the angle of rotation of the end-link about the moving pivot ($\alpha_{1\mathrm{i}} = \varphi_{1\mathrm{i}} - \beta_{1\mathrm{i}}$), and n is the number of precision positions [11]. The vectors are in complex notation ($\mathbf{G} = x + iy$). Figure 3 gives an overview of the variables in the design equation. The location of the pole of relative displacement $\mathbf{P}_{1\mathrm{i}}$ is calculated as

$$\mathbf{P}_{1\mathrm{i}} = [\mathrm{I} - \mathrm{A}(\varphi_{1\mathrm{i}})]^{-1}(\mathbf{d}_\mathrm{i} - \mathrm{A}(\varphi_{1\mathrm{i}})\mathbf{d}_1) \tag{11}$$

where $[\mathrm{A}(\varphi_{1\mathrm{i}})]$ is the 2x2 rotation matrix with rotation angle $\varphi_{1\mathrm{i}}$, $\mathbf{d}_1$ is the vector from $\mathbf{G}$ to $\mathbf{W}_1$, and $\mathbf{d}_\mathrm{i}$ is the vector from $\mathbf{G}$ to $\mathbf{W}_\mathrm{i}$ [11].

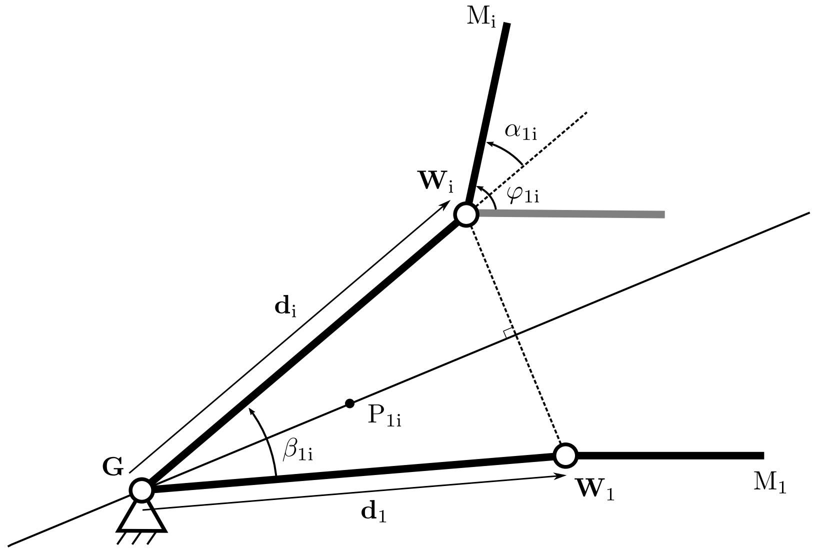


Figure 3: RR chain with the end-link moving from precision position M_1 to M_i, which is the rotation of angle $\varphi_{1\text{i}}$ about $\mathbf{P}_{1\text{i}}$, or the composite of a rotation of angle β_{1i} about $\mathbf{G}$ followed by a rotation $\alpha_{1\text{i}}$ about $\mathbf{W}_\text{i}$.

3. Motion generator synthesis of dynamically balanced mechanisms with a linear relationship between link angular velocities

The new synthesis method is based on the methods for RR chain synthesis discussed in [11]. These methods are extended by integrating the dynamic balancing condition. For the special case where the relation between link angular velocities is linear, the shaking moment balancing condition is algebraically substituted into the RR chain design equation. Because of the extra condition, the maximum number of precision positions drops from five to three.

3.1. Algebraic synthesis with up to three precision points

For the special case where $s_4 = 0$ and $s_3 = 0$, the value of k from Equation 6 is constant, resulting in the following conditions for the displacement angles

$$\beta_{1\text{i}} = k \cdot \varphi_{1\text{i}} \tag{12}$$

$$\alpha_{1\text{i}} = \varphi_{1\text{i}} - k \cdot \varphi_{1\text{i}} \tag{13}$$

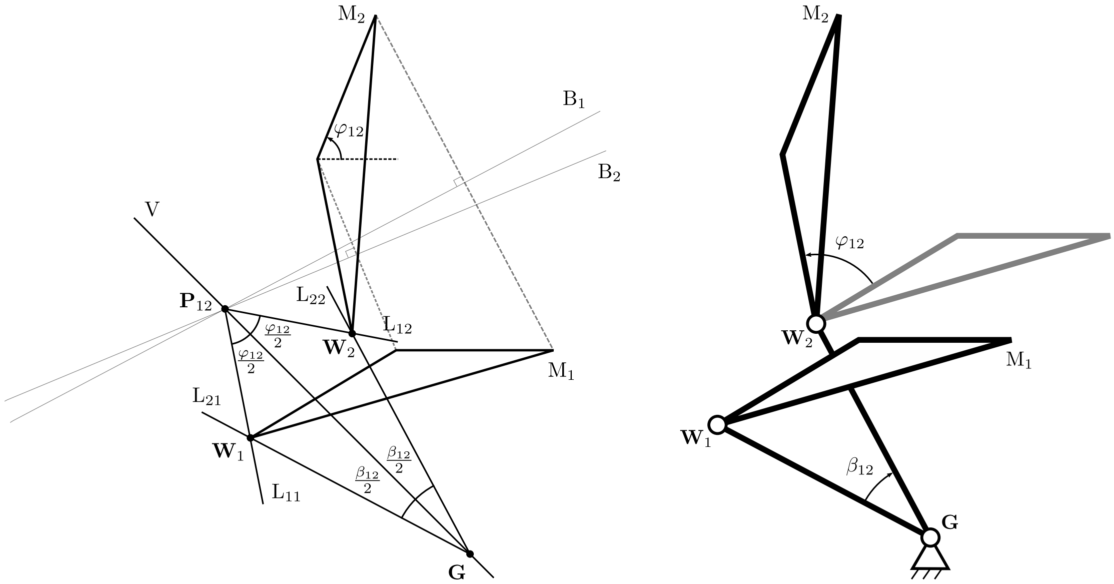
Substitution of Equation 12 and 13 into Equation 10 results in the following design equation for synthesis of dynamically balanced RR chains

$$\boxed{(1 - e^{i\varphi_{1\text{i}}})\mathbf{P}_{1\text{i}} = (1 - e^{ik\varphi_{1\text{i}}})\mathbf{G} + e^{ik\varphi_{1\text{i}}}(1 - e^{i\varphi_{1\text{i}}(1-k)})\mathbf{W}_1 \quad \text{i} = 2, \ldots, n} \tag{14}$$

Two precision positions means that $n = 2$ and results in a single complex equation with two complex unknowns ($\mathbf{G}$,$\mathbf{W}_1$). This means that the designer is free to select either the location of the fixed- or moving pivot and can compute the location of the remaining pivot. Three precision positions leads to two equations and two unknowns, meaning that from the system of two complex equations the two unknowns can be calculated.

3.2. Graphical synthesis with two precision positions

For two precision positions, the designer is free to select either the position of the fixed or the moving pivot. The construction for the location of the remaining pivot is presented.



(a) Graphical construction of the location of the moving pivot (b) Resulting linkage in position M_1 and M_2

Figure 4: Graphical synthesis of a dynamically balanced RR chain where the fixed pivot is freely selected

3.2.1. Select fixed pivot

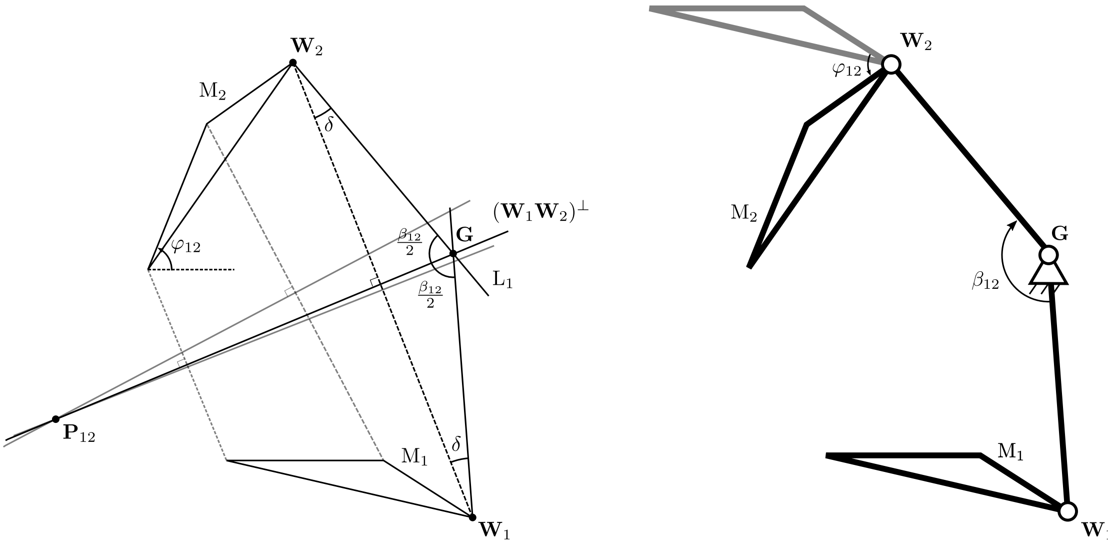
The graphical construction is illustrated in Figure 4 and performed by following these steps:

1. Construct pole $\mathbf{P}_{12}$ as the intersection of the perpendicular bisectors B_1 and B_2.
2. Measure the relative rotation angle φ_{12} between position M_1 and M_2.
3. Freely select any point as the fixed pivot $\mathbf{G}$.
4. Construct the line $V = \mathbf{GP}_{12}$.
5. Duplicate angle $\frac{\varphi_{12}}{2}$ on either side of V around $\mathbf{P}_{12}$ and construct the lines L_{11} and L_{12}.
6. Duplicate angle $\frac{\beta_{12}}{2} = \frac{k}{2} \cdot \varphi_{12}$ on either side of V around $\mathbf{G}$ and facing φ_{12} to construct the lines L_{21} and L_{22}.
7. The intersection of lines L_{11} and L_{21} determine the location of the moving pivot $\mathbf{W}_1$ and the intersection of lines L_{21} and L_{22} determine the location of the moving pivot $\mathbf{W}_2$.

3.2.2. Select moving pivot

The graphical construction is illustrated in Figure 5 and performed by following these steps:

1. Measure the relative rotation angle φ_{12} between position M_1 and M_2.
2. Freely select the location of the moving pivot $\mathbf{W}_1$ and determine the position of $\mathbf{W}_2$.
3. Construct the perpendicular bisector $(\mathbf{W}_1\mathbf{W}_2)^{\perp}$.
4. Calculate angle $\delta = \frac{\pi}{2} - \beta_{12}$.
5. Construct the line L_1 from $\mathbf{W}_1$ or $\mathbf{W}_2$ with angle δ from $(\mathbf{W}_1\mathbf{W}_2)^{\perp}$.
6. The intersection of $(\mathbf{W}_1\mathbf{W}_2)^{\perp}$ and L_1 is the fixed pivot $\mathbf{G}$.



(a) Graphical construction of the location of the fixed pivot (b) Resulting linkage in position M_1 and M_2

Figure 5: Graphical synthesis of a dynamically balanced RR chain where the moving pivot is freely selected

3.3. End effector path for mechanisms with a linear relationship between angular velocities

In order to know how to shape the path of the slider, or how the end-effector moves in-between precision positions, the path of the RR chain end effector must be known. The path traced by a dynamically balanced RR chain with a linear relation between angular velocities is called a hypotrochoid. A hypotrochoid is a curve described by a point P attached to a circle S rolling about the inside of a fixed circle C [13], as depicted in Figure 6. The parametric equations of the hypotrochoid are

$$\begin{cases} x = n\cos t + h\cos -\frac{n}{b}t \\ y = n\sin t - h\sin -\frac{n}{b}t \end{cases} \tag{15}$$

Where b is the radius of S, a is the radius of C, h is the distance from P to the centre of S and $n = a - b$. Equation 15 written as a function of the design parameters of an RR chain is

$$\begin{cases} x = l_1\cos\theta + s_5\cos k\theta \\ y = l_1\sin\theta - s_5\sin k\theta \end{cases} \tag{16}$$

Where l_1 and s_5 are the lengths of the first and second link respectively, and θ is the counter-clockwise angle of the first link with respect to the x-axis.

4. Motion generator synthesis of dynamically balanced linkages with a non-linear relationship between link angular velocities

For the general case where only the shaking force balancing conditions hold, the relation between the link angular velocities is not constant. This means that the shaking moment balancing condition cannot be substituted into to RR chain design equation. However, the shaking moment balancing condition leads to a non-linear EoM, which can be solved by a numerical solver. By solving the EoM for multiple designs, the design can be found for which the end-link position starts at the M_1, and ends in the desired end-link position.

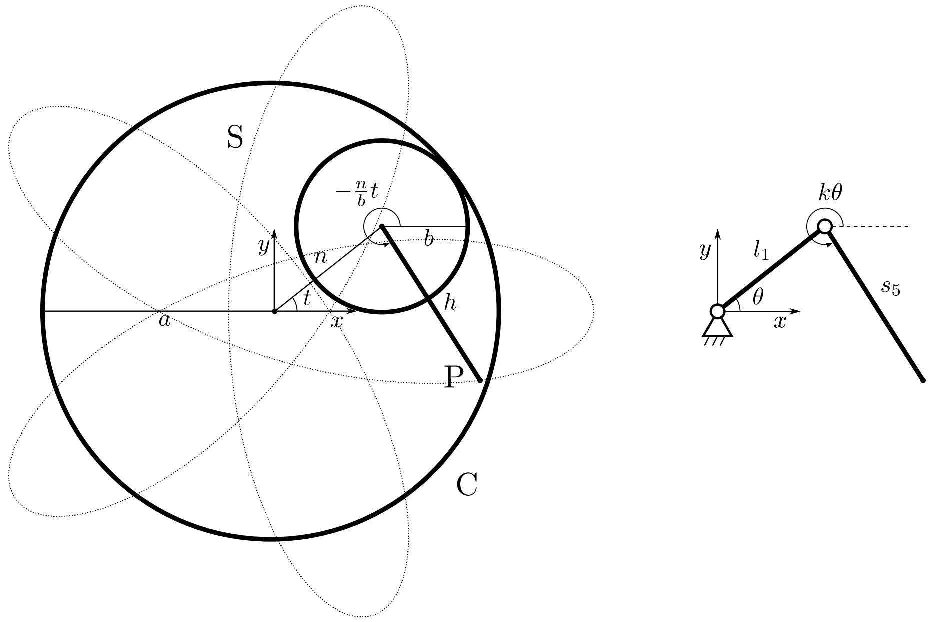


Figure 6: Point P attached to circle S rolling inside a circle C describes a hypotrochoid.

4.1. Equations of motion

By setting $\dot{\theta}_1 = 1$, Equation 9 leads to a set of EoMs, which is written as

$$\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\end{bmatrix} = \begin{bmatrix}1\\ -k(\theta_1,\theta_2,\mathbf{G},\mathbf{W}_1)\end{bmatrix} \tag{17}$$

where k is a function of θ_1, θ_2, $\mathbf{G}$, and $\mathbf{W}_1$. The expression for k is derived using Equation 2 and the Symbolic Math Toolbox in MATLAB. Length l_1 depends on the position of the fixed and moving pivot and is written as

$$l_1 = |\mathbf{W}_1 - \mathbf{G}| \tag{18}$$

Furthermore, in order to maintain shaking force balance, s_1 is written as a function of the other mass and length parameters using the first shaking force balancing condition (Equation 3)

$$s_1 = [l_3(m_4 + m_6) + (l_1 - s_3)m_3 - l_1(m_2 + m_5)]/m_1 \tag{19}$$

The other parameters must be chosen such that the second shaking force balancing condition is satisfied (Equation 4).

4.2. Numerical synthesis with up to two precision points

The shaking moment balancing condition cannot be substituted into the RR chain design equation. However, the RR chain design equation is used to determine a range of designs that start in M_1 and potentially end up in M_2. The use of the design equation ensures that the end-link starts in M_1, and that the moving pivot ends at the correct (x,y) location. However, the angle of the end-link at the second position is unknown, and has to be determined by solving the EoM. The difference between this result and the desired angle of the end-link is the error ε that has to be minimized. Figure 7 shows how possible locations of the fixed pivot result in different errors for the end-link angle in the end position. The design process is summarized in Figure 8 and follows the following steps

1. Define end-link starting position M_1 and ending position M_2

2. Freely select the moving pivot $\mathbf{W}_1$.
3. Determine the possible positions for the fixed pivot $\mathbf{G}$. This is the perpendicular bisector $(\mathbf{W}_1\mathbf{W}_2)^\perp$. The algebraic equation for this line is derived from the RR chain design equation and is written as

$$Ax + By = C \tag{20}$$

   where

$$A = (\cos\varphi_{12} - 1)(\lambda - p) - \sin\varphi_{12}(\mu - q) \tag{21}$$

$$B = \sin\varphi_{12}(\lambda - p) + (\cos\varphi_{12} - 1)(\mu - q) \tag{22}$$

$$C = (\cos\varphi_{12} - 1)(p(\lambda - p) + q(\mu - q)) - \sin\varphi_{12}(p\mu - q\lambda) \tag{23}$$

   where $\mathbf{W}_1 = [\lambda, \mu]^{\mathrm{T}}$ and $\mathbf{P}_{12} = [p, q]^{\mathrm{T}}$ [11].
4. Determine the timespan for the numerical integration. Since $\dot{\theta}_1 = 1$, the end time is equal to the angle of rotation of the first link β. The expression for β is written as

$$\beta = 2\arctan\frac{\frac{1}{2}|\mathbf{W}_2 - \mathbf{W}_1|}{|\mathbf{W}_1 - \mathbf{G}|} \tag{24}$$

   where

$$\mathbf{W}_2 = \mathbf{d}_2 + \mathrm{A}_{12}(\mathbf{d}_1 - \mathbf{W}_1) \tag{25}$$

   where $\mathbf{d}_1$ is an arbitrary position in M_1 and $\mathbf{d}_2$ is the same position in M_2.
5. Solve the EoM using a numerical solver and calculate the absolute error ε between the resulting angle φ_2' and the desired angle φ_2.
6. Repeat from step (2) if the absolute error ε does not meet the qualifications of the design.

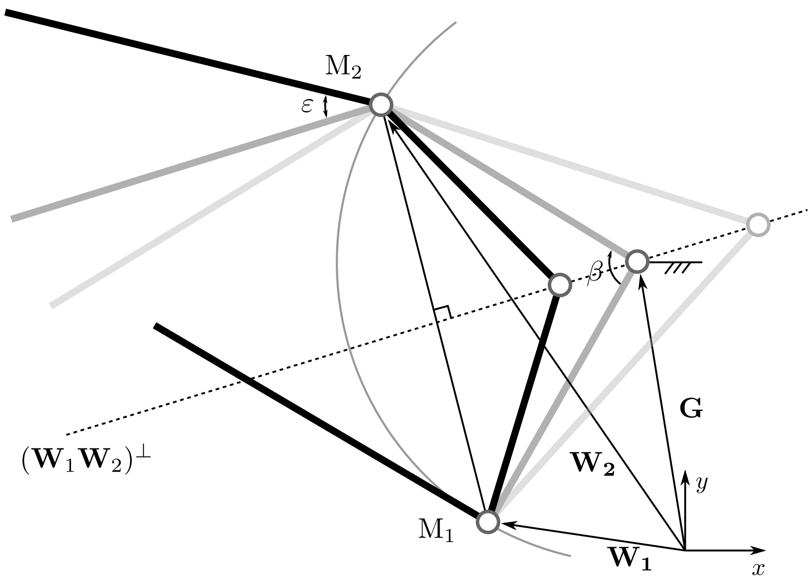


Figure 7: Different positions of the fixed pivot $\mathbf{G}$ along the perpendicular bisector $(\mathbf{W}_1\mathbf{W}_2)^\perp$ result in different values for ε.

These design steps can be executed using an optimization algorithm to find a solution where ε approaches zero. However, as this generally only finds a local optimum, mapping out the results for a range of inputs provides a better overview of the possible designs.

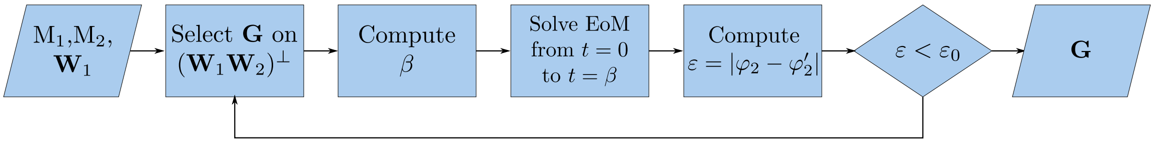


Figure 8: Design process

Table 1: Example of pantograph parameters from the DELA-1

Link lengths in mm	CoM placement along link in mm	Masses in g
$l_1 = \|\mathbf{W}_1 - \mathbf{G}\|$	s_1 = see Equation 19	$m_1 = 118.73$
$l_2 = 50$	$s_2 = 100.54$	$m_2 = 303.77$
$l_3 = 50$	$s_3 = 96.94$	$m_3 = 149.89$
	$s_4 = 24.48$	$m_4 = 24.22$
	$s_5 = 250$	$m_5 = 67.22$
		$m_6 = 784.68$

4.3. Validation example

For validation and demonstration purposes, a mapping is performed using the inputs of a dynamically balanced pantographic mechanism that was designed by Van der Wijk, the DELA-1. The same starting and ending position of the end-link were used. The other parameters for the link lengths, CoM locations and masses are copied from the DELA-1 and are summarized in Table 1. Except for the values of l_1 and s_1, which are dependent on the position of $\mathbf{G}$ and $\mathbf{W}_1$.

Following the steps from Section 4.2, the absolute error ε was calculated for a range of inputs. The x position of the fixed pivot was varied from $G_x = -0.03$ to $G_x = 0.06$, and R was varied from 0 to 0.8, where

$$R = \frac{I_1 + I_3}{I_2 + I_4 + I_5} \tag{26}$$

where I_i is the link moment of inertia of link i about its CoM, as depicted in Figure 2. For the input ranges of G_x and R, contour lines are plotted in Figure 9 for the value of ε. This plot shows a contour for which $\varepsilon \approx 0$, this means that the solutions along this line match with the desired precision positions. The solution that is in accordance with the DELA-1 can be found on this contour, this validates the method. Other solutions on the contour where $\varepsilon \approx 0$ are alternative designs. While these designs have slightly different end effector paths, they have in common that the end link moves from M_1 to M_2. Two solutions from the contour in Figure 2 are plotted in Figures 10 and 11 in their starting and ending position. Additionally, the end effector path is depicted by a red line and the moving pivot path is depicted by a blue line. These paths were found by solving the EoM of the mechanism. The red circles are data points of the end effector position obtained by Van der Wijk. Figure 10 shows that the path matches these data points, which validates the method. In Figure 11, an alternative solution is presented. While this solution has the same starting and ending position, the end effector path is slightly different.

5. Discussion

This research presents a new approach for the synthesis of inherently dynamically balanced mechanisms. However, because the method uses the pantograph as a starting point, the solutions are kinematically limited to pantographic mechanisms. Other inherently dynamically balanced mechanisms, such as the dynamically balanced four-bars [14], cannot be found using this method. Further research is required for finding design methods for alternative solutions. For multi-DoF solutions, the stacking of dynamically balanced 1-DoF pantographs, such as in [15, 16], might provide new solutions.

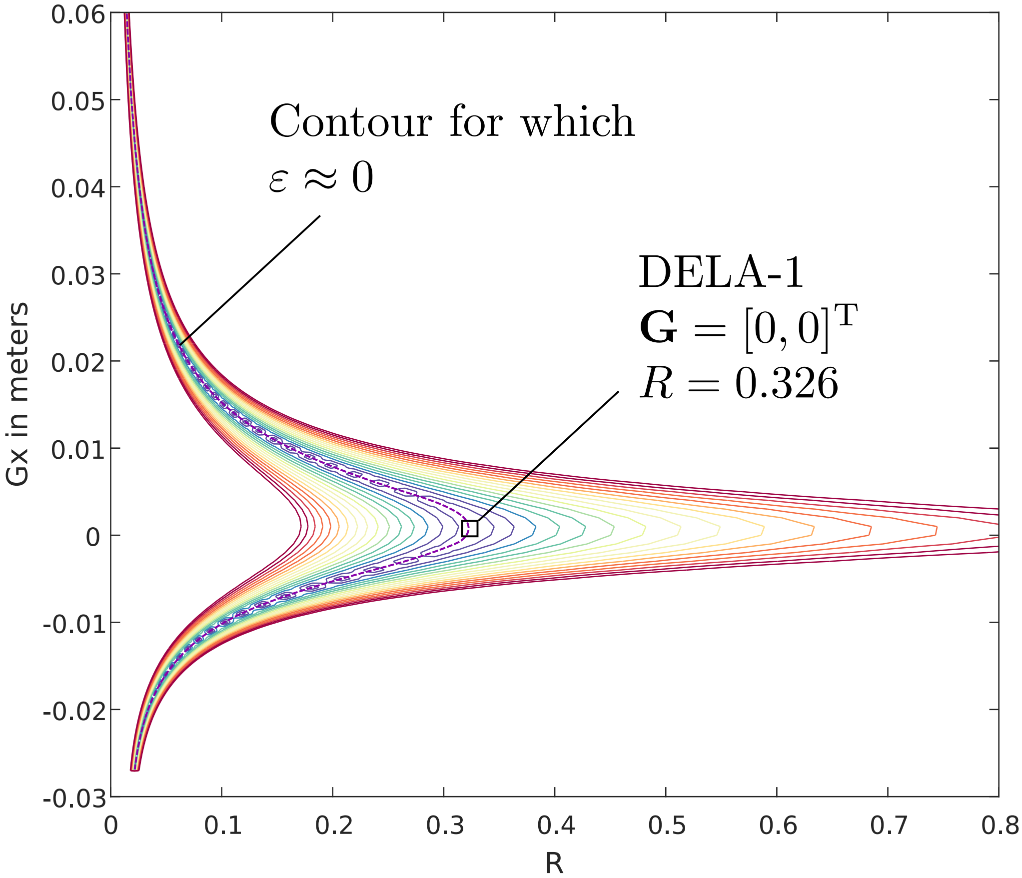


Figure 9: Contours depicting lines for constant values of ε, with the purple contour depicting the line for which $\varepsilon \approx 0$. Contours for which $\varepsilon > 0.08$ are not drawn. The inputs are the starting and ending positions of the DELA-1, and the parameters in Table 1.

The method discussed in Section 4.2 is suitable for two precision positions but can be extended to three and likely more. This is done by executing the method twice, first for M_1 to M_2, and next for M_2 to M_3. When a mapping is performed for these two instances, two contour lines can be drawn. The solution can be found at the intersection of these contours. Note that the selection procedure of the fixed- and moving pivot will have to be adjusted to the method for three precision positions discussed by McCarthy in [11]. This means that the moving pivot cannot be freely selected.

6. Conclusions

In this paper, a method was presented for the synthesis of a 1-DoF inherently dynamically balanced pantographic mechanism, where a desired motion of the end effector is selected by the designer. Using this method, a pantograph was designed that passes trough a set of predefined precision positions. The pantograph consists of an RR chain with an additional parallelogram. The parallelogram improves the stiffness, adds freedom to CoM distribution of the links, and provides additional options for actuator placement. The method presented in this paper finds its origin in the method of crank design from Burmester's theory. This method is extended by integrating the shaking moment balancing condition.

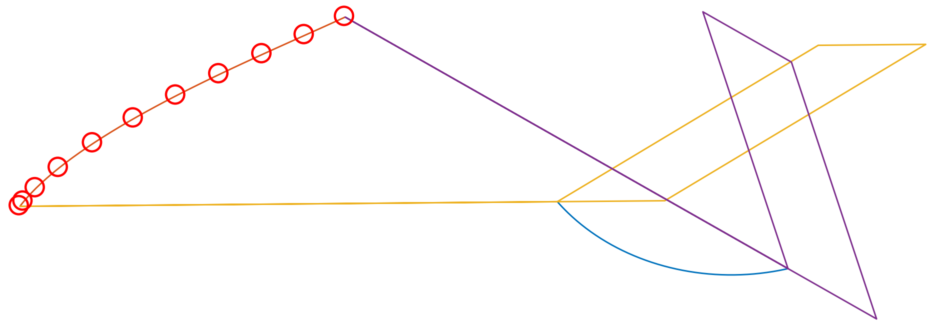


Figure 10: Beginning and ending positions of the dynamically balanced mechanism that matches the parameters of the DELA-1, where $G_x = 0$ and $R = 0.326$

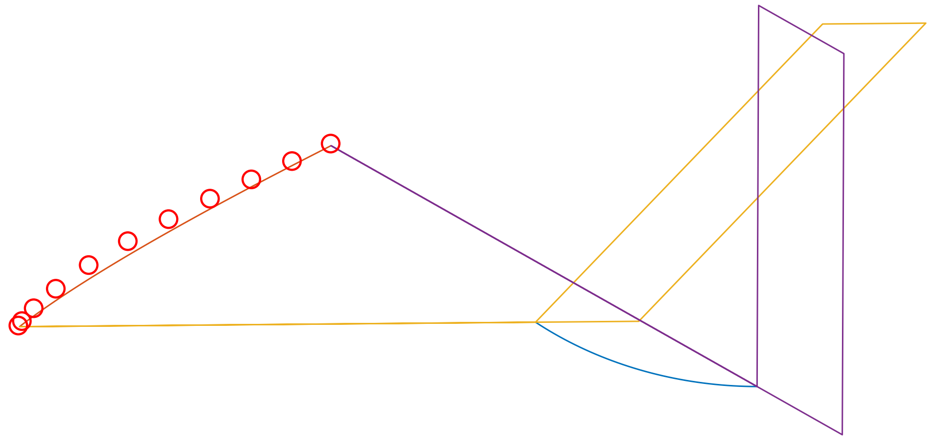


Figure 11: Beginning and ending positions of the dynamically balanced mechanism that has the same starting and ending position of the DELA-1, where $G_x = 0.027$ and $R = 0.045$

For the special case where the shaking moment balancing condition leads to a linear relationship between link angular velocities, the shaking moment balancing condition is easily substituted into the RR chain design equation. An algebraic method was presented for two or three precision positions for this special case. For two precision positions, the designer is free to select either the fixed- or moving pivot. Additionally, a graphical method was presented for two precision positions. For a linear relation between link angular velocities, the path traced by the end effector describes a hypotrochoid. The parametric equation for this path was presented.

For the general case where the relationship between link angular velocities is non-linear, a numerical procedure was presented. In this case, the shaking moment balancing condition leads to an EoM. The RR chain design equation is used to determine a range of solutions that start in the desired starting position and potentially end up in the desired ending position. The EoM is solved for this range of solutions in order to find the solutions that actually end up in the desired ending position. This method was used to design an example mechanism. A set of solutions was mapped for a range of fixed pivot locations and link moment of inertias. From this mapping, a mechanism that was synthesised by Van der Wijk, the DELA-1, was found. This validates the method.

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4

Towards inherently dynamically balanced 2-DoF mechanisms

The method discussed in Section 3 is limited to the synthesis of single DoF mechanisms. However, in situations where more versatile mechanisms are required, multi-DoF mechanisms are necessary. Dynamically balanced multi-DoF mechanisms can be synthesised by stacking multiple single DoF mechanisms. This method is used in [4, 14] by stacking dynamically balanced four-bar mechanisms for the synthesis of 3-DoF and 6-DoF parallel mechanisms.

The dynamically balanced RR chains from Section 3 can be stacked to synthesise multi-DoF mechanisms. In Figure 4.1, an example is presented of an inherently balanced 2-DoF mechanism resulting from RR chain stacking. The mechanism consists of a parallel chain of two RR chains connected with a horizontal link and an additional crank-slider on top of the horizontal link. The RR chains attached to the ground are placed under an angle with respect to each other such that the horizontal link can only translate vertically. These RR chains form a linear motion mechanisms known as the sarrus linkage. The crank-slider on top of the vertical link can move independently of the sarrus linkage. The fact that the sarrus linkage and the crank-slider can move independently mean that they are balanced seperately. The end effector can move freely in a rectangular area.

The mechanism presented in this chapter is just an example of how inherently dynamically balanced 2-DoF mechanisms can be formed. More research is required to explore additional solutions and to draft a synthesis procedure for balanced multi-DoF mechanisms. As discussed in Section 2, screw theory is a promising candidate to form the basis of this new method, especially for spatial mechanisms.

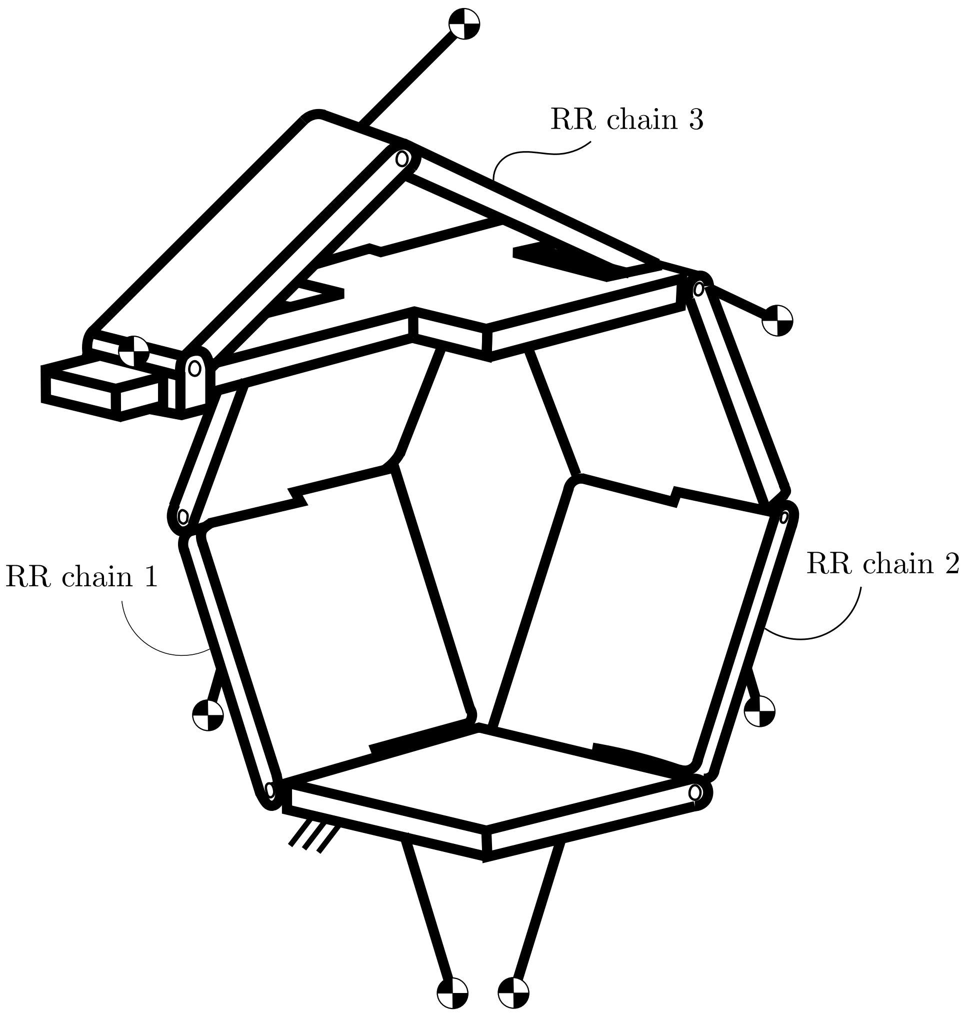

Figure 4.1: Inherently dynamically balanced 2 DoF mechanism, synthesised using multiple RR chains. RR chains one and two are connected parallel and form a sarrus linkage.

5

Discussion and Conclusions

In this thesis, a new method was presented for the synthesis of inherently dynamically balanced pantographic 1-DoF motion generation mechanisms. Leading up to this, the known methods for dynamic balancing were explored in a literature review. In this literature review, the following options for synthesis of shaking moment balanced mechanisms are discussed:

- Dynamic balancing after the kinematic synthesis by addition of new elements. The disadvantage of this method is that it increases the masses, inertia, and complexity of the mechanism.
- Constraining inherently shaking force balanced multi-DoF mechanisms such that the remaining DoFs are fully dynamically balanced. The disadvantage of this method is that the designer is constrained to use motions that were not selected beforehand.

Furthermore, screw theory was mentioned as a potential candidate to form the basis of a new design method. Using screw theory, the conditions for dynamic balancing can be depicted graphically with the momentum wrench. However, the instantaneous property of the momentum wrench makes it difficult to use for mechanism synthesis purposes. Nevertheless, screw theory is not ruled out for synthesis of dynamically balance mechanisms and might prove useful in further research. Screw theory might have potential for the dynamic balancing of spatial mechanisms in particular.

Next, the new method for synthesis of inherently dynamically balanced 1-DoF pantographic motion generators was presented. The method is based on the method of crank design from Burmester's theory. The method of crank design was extended by integrating the shaking moment balancing condition. Methods were discussed for the following mechanisms:

- Mechanisms with a linear relation between angular velocities, synthesised using:
  - Graphical synthesis with up to two precision positions, where the designer is free to select either the fixed- or moving pivot;
  - Algebraic synthesis with up to three precision positions. This method uses the modified RR chain design equation.
- Mechanisms with a nonlinear relation between angular velocities with up to two precision positions. This procedure uses numerical integration of the mechanism EoM.

Since the method uses the pantograph as a starting point, the solutions are kinematically limited to pantographic mechanisms. Other inherently dynamically balanced mechanisms, such as the dynamically balanced four-bars [9], cannot be found using this method. Further research is required for design methods for finding alternative solutions.

The method for synthesis of mechanisms with a nonlinear relationship between link angular velocities is suitable for two precision positions but can be extended to three and likely more. This is done by executing the method twice, first for M_1 to M_2, and next for M_2 to M_3. When a mapping is performed for these two instances, two contours for $\varepsilon \approx 0$ can be drawn. The solution can be found at the intersection of these contours. Note that the selection procedure of the fixed- and moving pivot will have to be adjusted to the method for three precision positions discussed by McCarthy in [7]. This means that the moving pivot cannot be freely selected.

Finally, a concept was presented for a 2-DoF inherently dynamically balanced mechanism. This mechanism was realized by stacking RR chains. The end effector of this mechanism can move in plane. The mechanism presented in this chapter is just an example of how inherently dynamically balanced 2-DoF mechanisms can be formed. More research is required to explore additional solutions and to draft a synthesis procedure for balanced multi-DoF mechanisms.

A

Appendix: Planar Kinematics

In order to algebraically design an RR chain that follows a set of predetermined precision positions a set of design equations will be formulated and solved. Solving these design equations results in the design parameters for the RR chain. The formulation of the design equation is a known procedure and is thoroughly described by McCarthy in [7]. Its most important concepts are discussed in this chapter.

As seen in Figure A.1, the end-link displacement of an RR chain is a composition of two planar displacements. The design equation originates from the triangle that is formed by the poles of the two individual displacements and the pole of their composite displacement.

Pole of relative displacement

A planar displacement [T] consists of a rotation and a translation and is denoted as

$$[\mathrm{T}] = [\mathrm{A}(\varphi), \mathbf{d}] \tag{A.1}$$

where $\mathrm{A}(\varphi)$ is the rotation matrix with rotation angle φ and $\mathbf{d}$ is the translation vector. A relative planar displacement between positions M_1 and M_2 as shown in Figure A.2 is denoted as

$$[\mathrm{T}_{12}] = [\mathrm{A}(\varphi_{12}), \mathbf{d}_2 - \mathrm{A}(\varphi_{12})\mathbf{d}_1] \tag{A.2}$$

where φ_{12} is the relative angle between M_1 and M_2, $\mathbf{d}_1$ is the position of the origin of M_1 and $\mathbf{d}_2$ is the position of the origin of M_2. A planar displacement can be expressed as a pure rotation about a pole $\mathbf{P}$, which means that the pole $\mathbf{P}$ remains unchanged by the planar displacement. This property leads to an expression for the pole $\mathbf{P}$ and the translation $\mathbf{d}$

$$\mathbf{P} = [\mathrm{T}]\mathbf{P} = \mathrm{A}(\varphi)\mathbf{P} + \mathbf{d} \implies \mathbf{P} = [\mathrm{I} - \mathrm{A}(\varphi)]^{-1}\mathbf{d}, \tag{A.3}$$

$$\mathbf{d} = [\mathrm{I} - \mathrm{A}(\varphi)]\mathbf{P} \tag{A.4}$$

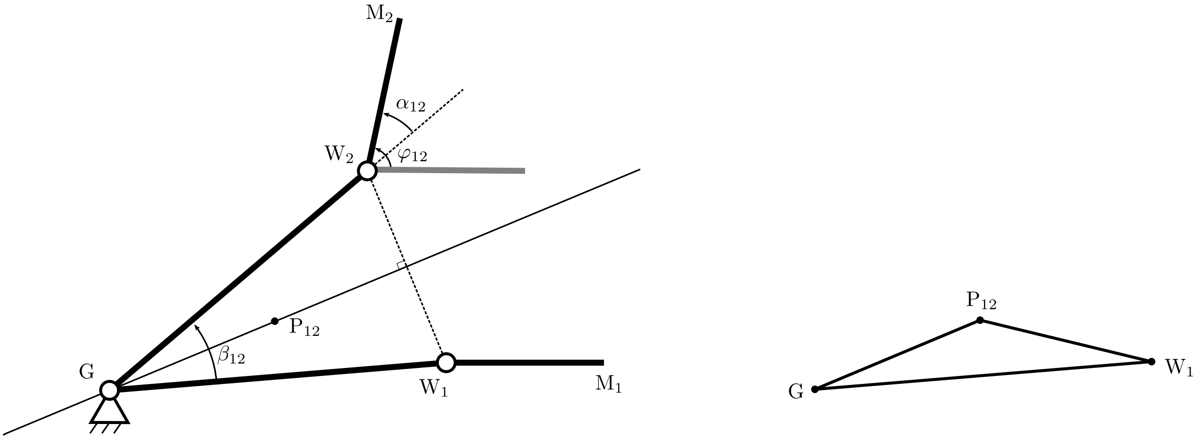

Figure A.1: An RR chain in two precision positions and its dyad triangle $\triangle\mathbf{GW}_1\mathbf{P}_{12}$

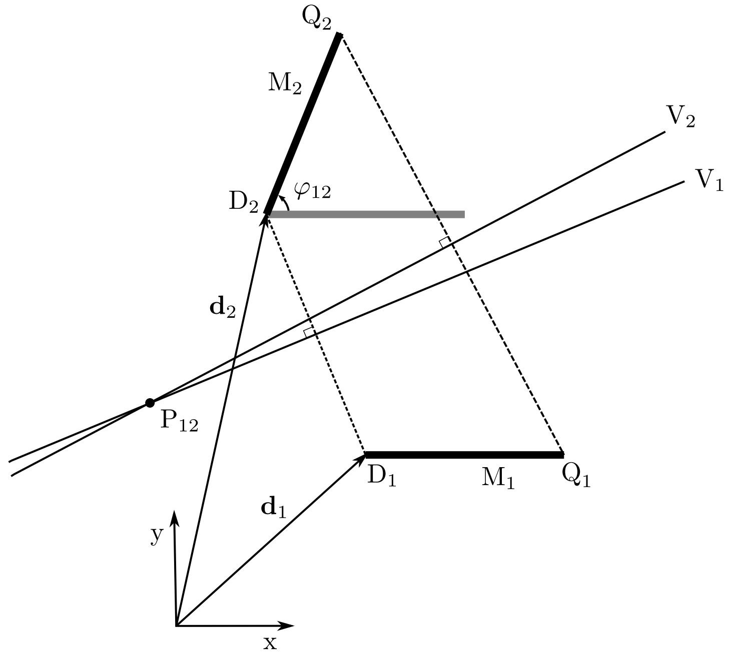

Figure A.2: Graphical construction of the pole $\mathbf{P}_{12}$ of the relative planar displacement $[\mathrm{T}_{12}]$

Combining Equations A.1 and A.4 results in an expression for a planar displacement $[\mathrm{T}(\varphi, \mathbf{P})]$ written in terms of its pole $\mathbf{P}$

$$[\mathrm{T}(\varphi, \mathbf{P})] = [\mathbf{A}(\varphi), [\mathrm{I} - \mathbf{A}(\varphi)\mathbf{P}]] \tag{A.5}$$

Combining Equations A.2 and A.3 results in an expression for the pole of a relative displacement

$$\mathbf{P}_{12} = [\mathrm{I} - \mathrm{A}(\varphi_{12})]^{-1}(\mathbf{d}_2 - \mathrm{A}(\varphi_{12})\mathbf{d}_1) \tag{A.6}$$

The pole of a relative displacement can also be constructed graphically with the known method of perpendicular bisectors. Figure A.2 shows that the pole is located at the intersection of perpendicular bisectors $\mathrm{V}_1 = (\mathrm{D}_1\mathrm{D}_2)^{\perp}$ and $\mathrm{V}_2 = (\mathrm{Q}_1\mathrm{Q}_2)^{\perp}$.

The RR chain design equation

An RR chain is a serial chain of two links with a fixed pivot **G** connecting the chain to the base and a moving pivot **W** connecting the two links. Figure A.1 shows that the displacement of the end-link from position M_1 to M_2 is the composite of a rotation of angle α_{12} about $\mathbf{W}_1$ followed by a rotation β_{1i} about $\mathbf{G}$. The result is a rotation φ_{12} about pole $\mathbf{P}_{12}$ [7]. The composition of two planar displacements is expressed as the product of the two, resulting in

$$[\mathrm{T}_{12}(\varphi_{12}, \mathbf{P}_{12})] = [\mathrm{T}(\beta_{12}, \mathbf{G})][\mathrm{T}(\alpha_{12}, \mathbf{W}_1)] \tag{A.7}$$

Writing Equation A.7 using the complex notation of Equation A.5 results in

$$\left[e^{i\varphi_{12}}, (1 - e^{i\varphi_{12}})\mathbf{P}_{12}\right] = [e^{i\beta_{12}}, (1 - e^{i\beta_{12}})\mathbf{G}][e^{i\alpha_{12}}, (1 - e^{i\alpha_{12}})\mathbf{W}_1] \tag{A.8}$$

where the complex notation of a planar displacement is $[\mathrm{T}(\varphi, \mathbf{P})] = [e^{i\varphi}, \mathbf{d}]$. Expanding Equation A.8 and equating the rotation and translation terms yields

$$e^{i\varphi_{12}} = e^{i\beta_{12}} e^{i\alpha_{12}} \implies \varphi_{12} = \beta_{12} + \alpha_{12} \tag{A.9}$$

$$(1 - e^{i\varphi_{12}})\mathbf{P}_{12} = (1 - e^{i\beta_{12}})\mathbf{G} + e^{i\beta_{12}}(1 - e^{i\alpha_{12}})\mathbf{W}_1 \tag{A.10}$$

Equation A.10 is the equation of the dyad triangle $\triangle\mathbf{G}\mathbf{W}_1\mathbf{P}_{12}$ and is the design equation of the RR chain. The dyad triangle is shown in Figure A.1.

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