

# Mission Planning for Sensor Network Deployment using a Fleet of Drones

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# Mission Planning for Sensor Network Deployment using a Fleet of Drones

MASTER OF SCIENCE THESIS

For obtaining the degree of Master of Science in Aerospace Engineering  
at Delft University of Technology

Pulkit Goyal

August 15, 2016



**Delft University of Technology**

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DELFT UNIVERSITY OF TECHNOLOGY  
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# Acronyms

<b>BF</b>	Best Fit
<b>BFD</b>	Best Fit Decreasing
<b>BPP</b>	Bin Packing Problem
<b>CMTAP</b>	Cooperative Multiple Task Assignment Problem
<b>CVRP</b>	Capacitated Vehicle Routing Problem
<b>FF</b>	First Fit
<b>FFD</b>	First Fit Decreasing
<b>LPT</b>	Longest Processing Time first
<b>MILP</b>	Mixed Integer Linear Programming
<b>MTVRP</b>	Multi-Trip Vehicle Routing Problem
<b>SPT</b>	Shortest Processing Time first
<b>TS</b>	Tabu Search
<b>UAVs</b>	Unmanned Aerial Vehicles
<b>UDP</b>	User Datagram Protocol
<b>VRP</b>	Vehicle Routing Problem





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## List of Symbols

$T_B$	Average flight-time for which power from the onboard battery is available, s
$T_H$	Time-horizon which establishes the maximum amount of time for which an operation can be carried out e.g., one working day, s
$V_{avg}$	Average airspeed of each drone in the fleet, m/s
$t_{drop}$	Dropping time for a drone, s
$t_d$	Service time at the depot for a drone, s
$V_K(i, j)$	Ground velocity with which a drone travels from vertex $i$ to $j$ , m/s
$C(n, k)$	Number of distinct combinations of $n$ objects taken $k$ at a time
$D(i, j)$	Euclidean distance between vertices $i$ and $j$ , m
$K$	Set of $m$ identical drones in a fleet
$N$	Set of $n$ customers with known locations
$Q$	Maximum capacity of each vehicle in the fleet
$R$	Set of all feasible routes
$m$	Number of drones in a fleet
$n$	Total number of customers
$q_i$	Demand of the customer $i$



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# Summary

Various methods for route planning of on-road vehicles to serve transportation requests have been developed in the literature in order to reduce transportation and operational costs. The applicability and thus development of these methods is primarily motivated by the field of application. This thesis deals with the mission planning for a fleet of drones which are conceived to complete the task of delivering geophones used in the seismic surveys. The drones are mainly advantageous in rough terrains as it helps to reduce the weight that the ground personnel have to carry and limit the number of times they have to climb up and down the terrain. Unlike conventional on-road vehicles used for delivery purposes, every drone in the fleet is constrained to make a frequent return trip back to the depot to pick up a new payload and restore its battery. As a result of this constraint, a centralized planner is proposed in this thesis. Although there have been many publications in the area of Multi-trip Vehicle Routing Problem (MTVRP), but there has been none to the best knowledge of the author dealing with the exact nature of the objective function and constraints arising out of the field of sensor network deployment by a fleet of drones. Therefore, this thesis is focused on the development of the methods aimed at finding solutions to the problems pertaining to the field of sensor network deployment by a fleet of drones. The problem of planning is decomposed into two phases: route formation and route scheduling. The first phase is handled using the extensive formulation of MTVRP aiming at minimizing the overall journey time. A heuristic method is also proposed for this phase which provides near-optimal solutions in a computationally efficient manner. The second phase of the planning algorithm deals with the unaddressed problem of depot congestion arising due to the frequent visits of each drone to the depot. This problem is formulated in the form of a Mixed-Integer Linear Program (MILP) that can be solved using available software. This phase is computationally intensive and comparatively slow, which restricts the usage of this mission planner in the re-planning phase to the cases involving longer journeys with limited number of routes. The results from a flight-test are also presented in order to demonstrate the mission planner. The methods developed in this thesis not only adapt the existing formulations and methods in the literature to fit to the field of application but also contribute to the growing research of MTVRP by dealing with the unaddressed problem of depot congestion.





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# Chapter 1

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## Technical Paper

This chapter contains the technical paper<sup>1</sup> delineating introduction, methodology, results and conclusions.

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<sup>1</sup>The technical paper has some overlap with the Appendices A, B and C in order to create a standalone version of the research carried out in this thesis.



# Mission Planning for Sensor Network Deployment using a Fleet of Drones

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Various methods for route planning of on-road vehicles to serve transportation requests have been developed in the literature in order to reduce transportation and operational costs. The applicability and thus development of these methods is primarily motivated by the field of application. This article deals with the mission planning for a fleet of drones to deploy sensors in a network. In particular, they are conceived to complete the task of delivering geophones in the seismic surveys. Unlike conventional on-road vehicles used for delivery purposes, every drone in the fleet is constrained to make a frequent return trip back to the depot to pick-up a new payload and restore its battery. A centralized planner is proposed in this article due to this constraint. The problem of planning is decomposed into two phases: route formation and route scheduling. The first phase is handled using the extensive formulation of Multi-Trip Vehicle Routing Problem (MTVRP) aiming at minimizing the overall journey time. A heuristic method is also proposed for this phase which provides near-optimal solutions in a computationally efficient manner. The second phase of the planning algorithm deals with the unaddressed problem of depot congestion arising due to the frequent visits of each drone to the depot. This problem is expressed in the form of a Mixed-Integer Linear Program (MILP) that can be solved using available software. This phase is computationally intensive and comparatively slow which restricts the usage of this mission planner in the re-planning phase to the cases involving longer journeys with limited number of routes. The results from a flight-test are also presented in order to demonstrate the mission planner.

## Nomenclature

$D(i, j)$	Euclidean distance between vertices $i$ and $j$ , m
$K$	Set of $m$ identical drones in a fleet
$m$	Number of drones in a fleet
$N$	Set of $n$ customers with known locations
$n$	Total number of customers
$P(n, k)$	$k$ -permutations of $n$
$Q$	Maximum capacity of each vehicle in the fleet
$R$	Set of feasible routes
$T_B$	Average flight-time for which the power from onboard battery is available, s
$t_d$	Service time at the depot for a drone, s
$T_H$	Time-horizon which establishes the maximum amount of time for which an operation can be carried out e.g., one working day, s
$t_{drop}$	Dropping time for a drone, s
$t_{ij}$	Time of travel from vertex $i$ to $j$ , s
$V_K(i, j)$	Ground velocity with which a drone travels from vertex $i$ to $j$ , m/s
$V_W$	Wind velocity, m/s
$V_{avg}$	Average airspeed of each drone in the fleet, m/s

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## I. Introduction

Drones are used for a wide range of purposes. Features such as planning and autonomous navigation while avoiding obstacles allow drones to do a variety of tasks. Their use is constantly being explored nowadays in farming, postal services, rescue operations, capturing crime scenes, and as monitoring platforms.<sup>1</sup> There are many instances where drones have been used for delivery purposes e.g., by Walmart,<sup>2</sup> Amazon,<sup>3</sup> DHL,<sup>4</sup> and Flirtey.<sup>5</sup> Drones are particularly useful for hard-to-reach places as they are safer, and more efficient, than sending people.<sup>6</sup> In the future, Unmanned Aerial Vehicles (UAVs) are expected to operate at a higher level of autonomy to carry out complex tasks while efficiently coordinating with the ground vehicles or personnel. They can be anticipated to work cooperatively and expedite the mission execution.

The research presented in this article is focused on the use of drones in yet another area - sensor network deployment. A sensor network comprises of spatially distributed sensors to monitor physical or environmental conditions.<sup>7</sup> A seismic survey is one of the areas of application in which a network of sensors is used to monitor rocks below ground. These surveys are spread over a large area of land, approximately 280 km<sup>2</sup>, and drones are anticipated to deploy and retrieve seismic sensors (often referred to as geophones). They will help in maintaining the pace of the survey and can assist the crew, especially in difficult terrain. Preliminary studies<sup>8</sup> list the cost savings from such an operation carried out with the help of drones. More details about the seismic surveys and benefits of using drones in managing operations are given in Section II.

This paper presents the method developed to plan routes for each drone in the fleet in order to complete a task in an “optimal” manner while respecting the constraints. The task is to deliver geophones from a depot to known locations. The aforementioned “optimal” criteria depend upon the problem being solved and results in the minimization of different attributes related to the problem e.g., the overall time in which the task is completed (the number of vehicles in the fleet, or their combination) and congestion at the depot. The constraints arise from different physical aspects related to the system e.g., the limited time for which the power from the onboard battery is available, the time-horizon in which the drones can be used (e.g., one working day), the maximum capacity of each drone, and the number of geophones required at each location.

Researchers have been working on the problem of optimizing the task of delivery of goods or services using vehicles on the road network for over five decades. However, the problem of pick-up and delivery using drones is relatively new. New regulations governing the flight of drones are required for the feasibility of pick-up and delivery in an urban environment but not in the secluded desert areas. These are the places where the use of drones is envisaged to accomplish the task of deployment of geophones in seismic surveys. There does not exist any literature to the best knowledge of the authors which directly deals with all problems associated with a delivery operation carried out using a fleet of autonomously flying drones. The drones have a very low payload capacity compared to the delivery trucks on the road. This feature entails frequent return of the drones in the fleet to the depot where operators have to mount new payload on the drones and restore their battery. This gives rise to the problem of depot congestion.

The research done in the area of problems related to the pick-up and delivery has always been motivated by its practical application. One of the most well known combinatorial optimization problem is Vehicle Routing Problem (VRP). It concerns with the determination of a set of vehicle routes to perform transportation requests with the given fleet at a minimum cost. Each route is associated with a corresponding cost and the total cost for all routes is generally minimized. This cost is specified using various parameters e.g., total distance traveled, total traveling time of all vehicles, total fuel consumption or traffic intensity depending upon the field of application of the method. There are many variants of VRP that exist in the literature depending upon different transportation requests, scheduling aspects, route constraints, fleet characteristics and optimization objectives. The overview of VRP, its variants, mathematical models and different exact and metaheuristics algorithms proposed in the literature are presented by Toth and Vigo.<sup>9</sup>

The problem dealt with in this article comes under one of the variants of VRP - Multi-Trip Vehicle Routing Problem (MTVRP). Each vehicle is allowed to perform at most one route in most of the studied models of VRP. In such cases, the planning is done assuming the number of vehicles to be unlimited. This assumption is not realistic for a fixed-size fleet containing vehicles with small payload capacity or when the planning time-horizon (e.g., one working day) is large with respect to the route duration. This is indeed the case for the present problem and MTVRP overcomes the mentioned limitations. Cattaruzza et al.<sup>10</sup> and Şen and Bülbül<sup>11</sup> present literature reviews on MTVRP. Although the first attempt to address MTVRP was made in 1990 by Fleischmann,<sup>12</sup> it has been actively pursued only in the last 15 years.

There exist many different formulations in literature for MTVRP and its extensions. They can be mainly categorized, as in the case of VRP, into “vehicle flow”<sup>10</sup> and “extensive”<sup>13,14</sup> formulations. The main



difference between the vehicle flow and extensive formulations is how increasing the size of the problem (in terms of number of customers) affects the problem. While there is a combinatorial increase in the number of constraints for vehicle flow formulations, the enumerated routes (and thus decision variables) increase in a combinatorial manner for extensive formulations with an increase in the problem size. One of the main advantages of extensive formulation over vehicle-flow formulations is that it allows the separation of costs and constraints governing the feasibility of a route from global constraints of the operation. Therefore, different intra-route constraints and complex cost functions can be incorporated easily into the problem.

The most widely studied version of MTRVP (also referred to as pure MTRVP<sup>10</sup>) deals with the transportation requests to deliver goods from a centralized depot to customers using a fixed fleet size. The goal of the MTRVP is to obtain vehicle routes which service all customers exactly once without violating vehicle capacity constraints. Koc and Karaoglan<sup>15</sup> propose a branch-and-cut exact algorithm which implements several valid inequalities taken from the literature and a heuristic algorithm, based on simulated annealing, to obtain upper bounds. Mingozzi et al.<sup>14</sup> propose an exact method to solve MTRVP based on two set-partitioning-like formulations which require a generation of all feasible trips and journeys respectively. Different valid inequalities and bounding procedures are used in the solution method. Loading times (at the depot) for the trips are not introduced in pure MTRVP but service times (at the customer's location) are sometimes included.<sup>16</sup>

The pure MTRVP embodies most of the constraints encountered in using a fleet of drones in seismic surveys such as the inclusion of multiple trips, limited vehicle capacity and visiting each customer only once. Various heuristic methods have been developed in the literature encompassing these constraints for on-road vehicles e.g., Taillard et al.<sup>16</sup> and Petch and Salhi<sup>17</sup> propose a two-phase and a three-phase heuristic algorithm respectively. In the successive work of Salhi and Petch,<sup>18</sup> a hybrid genetic algorithm is proposed. Cattaruzza et al.<sup>19</sup> propose another hybrid genetic algorithm for finding a solution to MTRVP. Brandão and Mercer<sup>20,21</sup> designed a tabu search (TS) algorithm to handle the MTRVP. Olivera et al.<sup>13</sup> proposed a TS algorithm with adaptive memory. The memory is composed of trips, a set of which is selected to form a VRP solution. It is then transformed into an MTRVP solution using a Bin Packing Problem (BPP) heuristic method.<sup>22</sup>

The constraint of limited trip duration and objective of minimizing the fleet size have been introduced separately (in the surveyed literature) addressing different extensions of MTRVP but not together in one method. For example, the limited time duration of each trip is considered in an extension of MTRVP dealing with *time windows*. Anaya-arenas et al.<sup>23</sup> take into consideration the trip duration limits and propose two constructive heuristics followed by a local search. Wang et al.<sup>24</sup> propose another heuristic, taking into account limited trip duration, which is based on an adaptive memory procedure.

The objective of minimizing the overall journey time has not been a part of the investigation in the MTRVP literature but is a topic of interest for the scientific community working on the Cooperative Multiple Task Assignment Problem (CMTAP)<sup>25,26</sup> or the load balancing problem. The CMTAP problem deals with the task assignment and coordination of a fleet of UAVs to perform 3 tasks of *classify, attack, and verify*, in order, on a specified targets on the ground and the load balancing problem is aimed at improving the distribution of workloads across multiple computing resources. The applicability of the CMTAP (or the load balancing problem) to the problem of sensor network deployment using a fleet of drones is limited to the extent that it provides insight into the formulation of objective function of minimizing the overall journey time.

For the present application, drones are required to make frequent visits to a depot due to their limited payload capacity and battery power. Drones can get their batteries changed and collect new payload at the depot. Frequent visits to the depot by drones in a fleet can create the problem of congestion at the depot. The problem of controlling the congestion at the depot is listed as one of the points in the future work by Olivera et al.<sup>27</sup> But there has not been any literature published dealing with this problem to the best knowledge of the authors. No algorithm has been found which minimizes the number of tasks finishing at the same time even in the job scheduling literature.<sup>28-31</sup>

In this article, we consider the route planning for multiple UAVs in the fleet to complete the task in minimum time while addressing the problem of congestion at the depot and thus, minimizing the resources to be used at the depot. The method proposed in this thesis is composed of two phases: route formation and route scheduling.

The outline of the article is as follows. First, the introduction to the seismic surveys is provided in Section II. Second, Section III will deal with the description of the methodology developed along with some

definitions. Sections IV and V are about two different phases of the planning algorithm including results. Sections VI and VII contain the overview of the mission planner and results from the flight demonstration respectively. Finally, in Section VIII, the article is concluded with recommendations for future research.

## II. Seismic Surveys

A seismic survey is a technique which is used to produce detailed images of various rock layers and their locations below ground. These surveys are used either to explore new oil and gas reservoirs or to monitor the existing fields. In a typical seismic survey, sound waves are bounced off underground rock formations and the reflecting waves are recorded by the sensors placed onto the ground. These sensors are generally laid down in a rectangular grid-like pattern as shown in Figure 1. These surveys cover a large area ( $\approx 280 \text{ km}^2$ ) and involve the displacement of 2-3 lines of geophones in one working day as depicted in Figure 1. The means used for pick-up and deployment of sensors depend upon the terrain e.g.  $4 \times 4$  vehicles can be used on rough terrain. The drones are now envisaged for use in this area of collecting and deploying geophones to save time and operational costs, especially in rough terrains.

The traditional geophones used in these surveys are not wireless, which is why they are laid down in lines. For rough terrain, wireless sensors are just now starting to be used. It should be noted that the use of wireless geophones lifts the restriction of placing the sensors in a line which can be of interest to geophysicists in exploring fields.

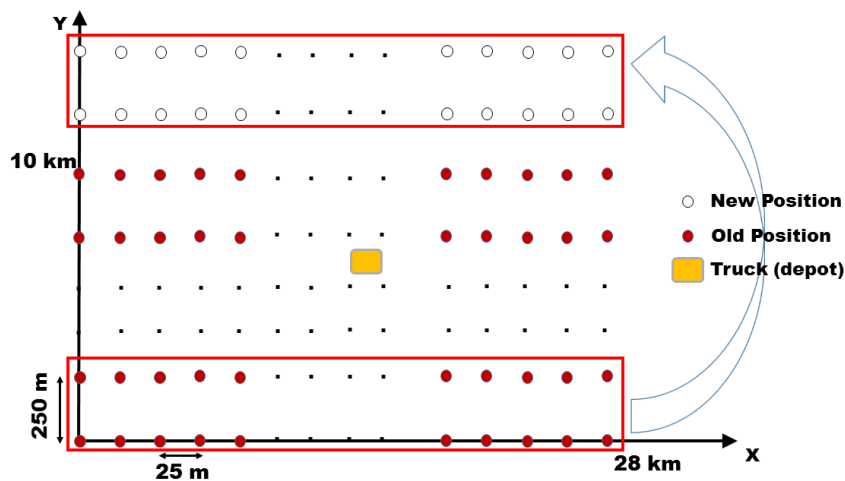


Figure 1: Typical field of seismic survey (not to scale)

The deployment and retrieval of geophones are carried out by personnel. It includes the time it takes for the personnel to travel using vehicles and carry equipment (and geophones) based on their strength and capacity to different sites. The collection or deployment of sensors can be time-consuming especially in difficult terrains such as sand dunes and steep rocky hills. This is because the ground vehicles cannot reach these places, and ground crew have to go on foot. Therefore, the use of drones for sensor deployment or retrieval operation in rough terrains can create a significant difference in maintaining the pace of the survey. Using drones to carry out the operation in difficult terrains not only saves the personnel from carrying an additional weight of the sensors but also limits the number of times they have to climb or walk down the terrain to their supporting vehicles. This ultimately allows them to work faster and maintain the pace of the survey while reducing the risk of injury. It is estimated that approximately 10% (300-400 in number) of nodes will be deployed using a fleet of drones in one day. A typical wireless geophone<sup>32</sup> weighs approximately 700 g.

For the purpose of sensor deployment operation, the drones to be used in the seismic survey are anticipated to carry the sensors from the truck or different pick-up locations and place them at the required sites. The ground personnel can then later put the sensor into the ground using the special equipment. At the pick-up

locations, the ground personnel are expected to dig out the sensor from the ground and be present there to mount it on the drone. Therefore the retrieval of the sensors from the pick-up locations will require a coordination with the ground personnel as opposed to the deployment operation.

This work focuses only on the problem of deployment of geophones. These scenarios exist in a real situation for small regions e.g., a region with difficult terrain, where geophones are either picked-up or delivered (but not both of them simultaneously) with a truck in the vicinity acting as a depot. Moreover, the sensors retrieved from the pick-up locations are needed to be brought to the depot/truck instead of directly dropping them at the customer locations (locations where sensors are needed to be placed). These sensors are needed to be recharged at the depot before their next deployment in the field. This characteristic, in combination with the consideration of the fact that the drones would be needed to fly huge distances with their limited battery if simultaneous pick-up and delivery are allowed, favors the idea of considering the problems of retrieval and deployment separately.

### III. Methodology

The overall task of deployment of geophones can be addressed by finding out a methodology for optimal route planning of a fleet of drones to deliver the seismic sensors to given locations. As stated in Section I, the proposed solution methodology consists of two phases: route formation and route scheduling. The first phase, route formation, consists of route enumeration, route selection followed by their assignment to different vehicles in the fleet resulting in the formation of journeys of each vehicle. This is handled using the extensive formulation of MTVRP. Unlike the problems dealt with in the MTVRP literature, the routes formed in this phase are aimed at minimizing the overall journey time while respecting the constraints of limited trip duration posed by the limited battery-time (among others).

Due to the limited payload capacity and battery power, each drone is bound to make frequent returns to the centralized depot in order to pick up new payload and restore their battery. For the high number of drones in a fleet, the simultaneous presence of multiple vehicles at the depot leads to the requirement of more resources (or persons) needed to cater to each vehicle at the depot. This is referred to as a problem of congestion at the depot. The routes within each vehicle's journey are scheduled in the second phase, referred to as route scheduling, which is aimed at minimizing the congestion at the depot.

Before exploring the methodology further, it is important to take a look at some important definitions and the framework in which they are applicable from the perspective of individual drones in the fleet. The following list enumerates the characteristics of each agent i.e., a drone in the fleet:

1. The autopilot onboard each drone is designed in such a way that it can autonomously fly from one location to another in a straight line.
2. The drones have take-off, landing and collision avoidance methods implemented on board.
3. The fleet is homogeneous i.e., all agents have the same mission and flying capabilities. It means that each drone in the fleet has the same maximum payload capacity, and average speed while flying in a straight line.

Some definitions which are referred to throughout the article are enumerated below:

**Dropping time** - the total time it takes for the drone at a customer's location to land, dismount the sensor and turn around to continue on its journey to the next destination. It is denoted as  $t_{drop}$ .

**Service time** - the total time it takes for the drone at the depot to land, collect a new sensor, and turn around to continue on its journey to the next destination. This time span also includes the time needed to change the battery of the drone. It is denoted as  $t_d$ .

**Trip/route**<sup>10</sup> - A sequence of customers preceded and followed by a visit to a depot and without any intermediate stop at the depot.

**Journey/tour**<sup>10</sup> - A sequence of trips/routes performed by the same vehicle.

Another important aspect of this problem is the drone capacity and customer's demand at each location. Instead of measuring the payload capacity of each drone in kg, it is expressed in terms of the number of

maximum geophones a drone can carry simultaneously during its trip. Each drone is expected to carry 2-3 geophones. Similarly, the demand of a customer is also expressed as the number of geophones needed to be delivered at each location. The customer's demand is always equal to one for this application.

## IV. Phase I - Route Formation

This section presents the formulation of the MTRVP aimed at achieving a minimum overall journey time. The solution to the resulting formulation is obtained using CPLEX optimization software<sup>33</sup> in MATLAB.

### A. Problem Statement

The problem consists of transportation requests to deliver goods from a *single* depot, denoted as point 0, to a given set of  $n$  customers, denoted as a set  $N = \{1, 2, \dots, n\}$ . Each customer's demand is equal to one. A fleet of  $m$  identical vehicles is based at the depot and is denoted by a set  $K = \{1, 2, \dots, m\}$ . Each vehicle in the fleet has a limited capacity  $Q$ . Time-horizon, denoted by  $T_H$ , establishes the duration of a working day.  $T_B$  denotes the amount of time for which power from the battery is available. It is assumed that the time spans ( $t_{drop}$ ,  $t_d$ ,  $T_H$ ,  $T_B$ ) and average speed of the vehicle are positive constants. It should also be noted that the Euclidean distances are considered in this problem.

Let  $G = (V, E)$  be a directed graph where  $V = \{0, 1, 2, \dots, n\}$  is the set of nodes and  $E = \{(i, j) \in V \times V : i \neq j\}$  is the set of edges. Each edge  $(i, j) \in E$  is characterized by a travel time  $t_{ij}$  which represents the time a drone takes to fly from node  $i$  to  $j$ . Note that the graph has asymmetric costs i.e.,  $t_{ij} \neq t_{ji}$  due to the presence of wind. The wind velocity is comparable to the average airspeed of the drone and hence, can significantly affect the travel time between the two customers. The time of travel is calculated using Equation 1.

$$t_{ij} = \frac{D(i, j)}{V_K(i, j)} \quad \forall i, j \in V \text{ and } i \neq j \quad (1)$$

where  $D(i, j)$  is the Euclidean distance between vertices  $i$  and  $j$ , and  $V_K(i, j)$  is the ground velocity with which a drone travels from vertex  $i$  to  $j$ .  $V_K(i, j)$  can be calculated using 2-D velocity triangle equations. The following variables are assumed to be known in these equations: wind velocity ( $V_W$ ), required direction of ground speed and magnitude of average airspeed ( $V_{avg}$ ) with which a drone travels while following a straight line trajectory. It is advised here that the subscript  $K$  in  $V_K(i, j)$  shouldn't be confused with the set of vehicles.  $K$  in the subsequent text refers to the set of vehicles present at the depot.

Onboard estimates of average airspeed and wind velocity can be used as input to the planning algorithm. In the absence of onboard estimates of airspeed and wind velocity, wind vanes can be used to find out the direction of the wind. The wind speed can then be obtained either by using anemometers or by flying a specified distance in and opposite to the direction of the wind. Note that the planning algorithm assumes that good estimates of the input parameters ( $T_H$ ,  $T_B$ ,  $t_{drop}$ ,  $t_d$ , wind velocity, average airspeed) are available.

Let  $R$  be a set of all feasible routes. Each route,  $r \in R$  is denoted by a sequence of customers  $(v_0, v_1, \dots, v_{n_r+1})$  where  $n_r$  denotes the number of customers visited on that route,  $v_0 = v_{n_r+1} = 0$  and  $v_1, \dots, v_{n_r} \in V \setminus \{0\}$ . The corresponding time associated with the route  $r$  is defined in Equation 2.

$$t_r = \sum_{i=0}^{n_r} t_{v_i, v_{i+1}} + n_r \times t_{drop} + t_d \quad \forall r \in R \quad (2)$$

where  $t_{drop}$  and  $t_d$  are dropping time and service time respectively. The demand en route  $r$  is denoted by  $q_r$  and is equal to the number of customers visited on that route,  $n_r$ .

A route,  $r$  (also referred to as trip) is called feasible if the following conditions are met:

1. the demand of customers being served on that route doesn't exceed the vehicle capacity i.e.,  $q_r \leq Q$ ,
2. the total travel time ( $\sum_{i=0}^{n_r} t_{v_i, v_{i+1}}$ ) on that route is less than  $T_B$ .

A solution is then a set of  $m$  sets of routes such that the maximum journey time is minimized. The solution set is given by  $s = (R^1, R^2, \dots, R^m)$  where each  $R^k \forall k \in K$  contains the set of routes assigned to a vehicle  $k$ . The solution to this problem should also satisfy the following general conditions of MTRVP:

1. each route starts and ends at the depot,
2. every customer is visited exactly once,
3. the sum of the demands of the customers in any trip doesn't exceed the vehicle capacity,  $Q$
4. the total duration of routes assigned to one vehicle doesn't exceed  $T_H$ .

## B. Problem Formulation

Given the statement in Subsection A, the extensive formulation of the problem is delineated below:

$$\min \left( \max_{k \in K} \sum_{r \in R} t_r x_r^k \right) \quad (3)$$

$$s.t. \quad \sum_{k \in K} \sum_{r \in R} a_{ir} x_r^k = 1 \quad \forall i \in V \setminus \{0\} \quad (4)$$

$$\sum_{r \in R} t_r x_r^k \leq T_H \quad \forall k \in K \quad (5)$$

$$x_r^k \in \{0, 1\} \quad \forall r \in R, k \in K \quad (6)$$

where  $x_r^k$  is a binary variable indicating whether the route  $r$  is selected and assigned to vehicle  $k$  ( $x_r^k = 1$ ) or not ( $x_r^k = 0$ ) and  $a_{ir} \in \{0, 1\}$  is the coefficient indicating whether the route  $r$  visits the customer  $i$  ( $a_{ir} = 1$ ) or not ( $a_{ir} = 0$ ). Equation 3 minimizes the journey time maximum among all vehicles, Equation 4 enforces the set partitioning constraints i.e., each customer is visited exactly once during the operation and Equation 5 enforces the time-horizon constraint. Note that the constraints due to limited payload capacity and battery-time are satisfied during enumeration of the set  $R$ . Each route starts and ends at the depot by default as it is ensured in the construction of routes.

## C. Solution Process

The process of obtaining a solution to the problem described in the previous subsections consists of two steps:

1. Route enumeration - During this phase, a set of all feasible routes,  $R$  is constructed.
2. Route selection - The selection and assignment of a few routes take place in this phase which results in the construction of subsets  $R^k \forall k \in K$ .

The set  $R$  consists of all possible permutations of customers. The cardinality (number of elements) of the set  $R$  is given by Equation 7 for a fleet consisting of vehicles with a maximum capacity 3.

$$|R| = \sum_{k=1}^Q P(n, k) \quad (7)$$

where  $P(n, k)$  denotes  $k$ -permutations of  $n$ . Here,  $k$  corresponds to the number of customers in a route. For vehicles with capacity 2, the particular order of the customers in a route  $r$  influences the associated time  $t_r$  due to the presence of a non-zero wind. Note that the size of the set  $R$  increases combinatorially with the number of customers  $n$ .

The route enumeration phase results in a construction of two variables: matrix  $\mathbf{A}$  of size  $n$  by  $|R|$  and a row vector  $\mathbf{t}$  of size 1 by  $|R|$ . The matrix  $\mathbf{A}$  and row-vector  $\mathbf{t}$  contain elements corresponding to  $a_{ir}$  and  $t_r$  in Equations 4 and 5 respectively.

## D. Exact Method

The solution methodology defined above is transformed into Mixed Integer Linear Programming (MILP) and solved using CPLEX optimization software. This methodology is referred to as exact solution as it guarantees to find the global-optimal solution. This method can provide a benchmark for other methods. However, due to some uncertainty in the input parameters ( $T_H$ ,  $T_B$ ,  $t_{drop}$ ,  $t_d$ ,  $V_W$ ,  $V_{avg}$ ) to the planning algorithm, the global-optimal solution may not be desirable for practical purposes. The following subsection delineates details about how the solution is obtained optimally *only up to a limit*.

### 1. Stopping Criteria

The following parameters are defined for premature termination of the MILP optimization in Phase-I:

1. Relative MILP gap tolerance<sup>33</sup> - It is set to be 0.01 and is a relative tolerance on the gap between the best integer objective and the objective of the best node remaining.
2. Absolute MILP gap tolerance<sup>33</sup> - It is set to be 30 s and is an absolute tolerance on the gap between the best integer objective and the objective of the best node remaining.

The optimization algorithm stops whenever any one among the above two conditions is reached. Since the solution obtained using these stopping criteria is not actually globally optimal but really close to it, this methodology is referred to as ‘near exact’ in the subsequent text.

### 2. Results

This section delineates the solution obtained for a simple scenario consisting of 41 customers separated 50 m apart as shown at the top in Figure 2. This scenario is referred to as SS-1 (**S**eismic **S**urvey scenario-1) in this article. The values of different parameters used in this simulation are listed in Table 1. Note that  $\sqrt{2}$  m/s magnitude wind is assumed to be flowing to the North-East direction (X and Y axes are aligned with east and north directions respectively).

Table 1: Values of different parameters for SS-1 problem instance

Parameter	Value
$V_{avg}$	15 m/s
$t_{drop}$	20 s
$t_d$	30 s
$T_B$	1200 s
$T_H$	1000 s
$Q$	2
$V_W$	[1, 1]

Table 2: Route planning after minimizing the maximum journey time for SS-1 problem instance

Vehicle	Journey
1	0 10 9 0 11 12 0 13 14 0 16 15 0 21 22 0 31 30 0
2	0 1 2 0 7 8 0 18 17 0 23 24 0 33 32 0
3	0 26 0 3 4 0 27 25 0 37 36 0 38 39 0
4	0 6 5 0 19 20 0 29 28 0 35 34 0 40 41 0

Table 2 shows the journey of each vehicle for the deployment of the seismic sensors (0 denotes the depot). It can be seen from the bottom of Figure 2 that the objective of minimizing the maximum journey time ensures less disparity in journey times of all vehicles (a characteristic similar to the load balancing problem). The amount of time spent at the depot for each route in the journey of a vehicle is shown in the red color. It can also be noted that interchanging the routes within a journey of a vehicle does not alter the total journey time of the vehicle whereas interchanging customers within a route can.

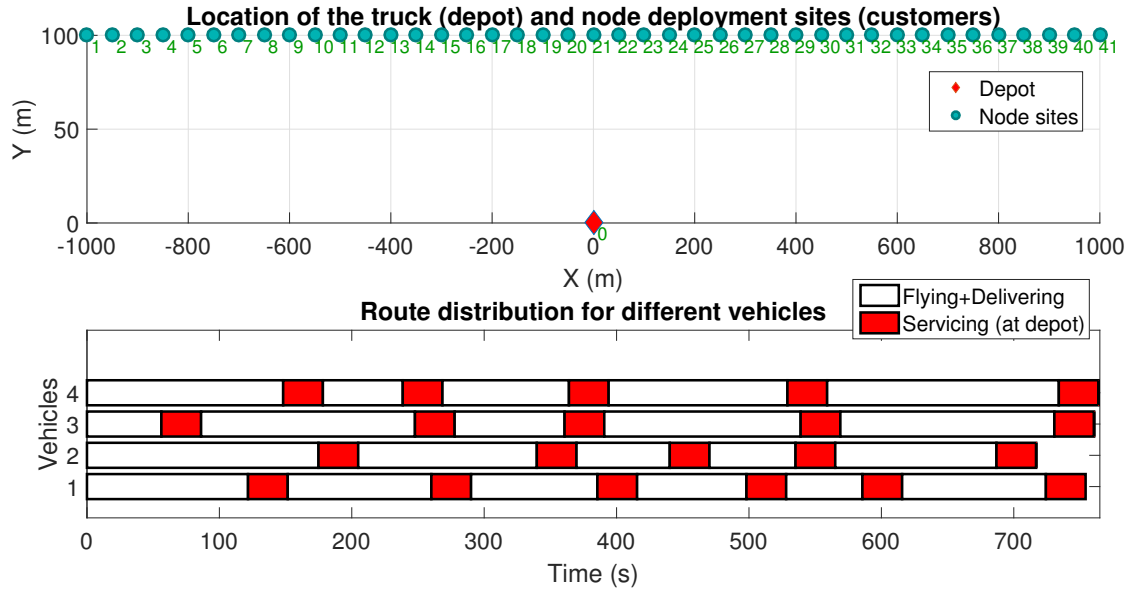


Figure 2: Route distribution after minimizing the maximum journey time for SS-1 problem instance (maximum journey time is 764.01 s)

## E. Heuristics

As stated in Section I, the problem of delivery or retrieval (VRP, MTRVP etc.) falls into the category of combinatorial optimization and MTRVP is an NP-hard problem.<sup>10,13</sup> Therefore, the memory requirements and time taken to compute an optimal solution may increase superpolynomially (perhaps exponentially) with the problem size. This subsection delineates the heuristics method proposed to find the route formations using reasonable memory and computation time for problems with a large number of customers. It restricts the enumeration of the set of feasible routes,  $R$  and makes use of the same MILP formulation (Section A) as an exact method. Note that, for a given fleet, the size of a problem can increase in two dimensions: number of customers ( $n$ ) and maximum capacity of each vehicle ( $Q$ ). For this application, the problem size is expected to increase only in  $n$  and is limited to a maximum value of 3 in the dimension of  $Q$ .

It can be observed from the journeys of different vehicles in Table 2 that the customers clustered together in one route lie in vicinity of each other in the Cartesian space. This “quality” of the solution emerges due to the low value of the upper bound on the number of customers that can be visited in one route. This upper bound is the main driving factor for the route formation phase and is either determined by the maximum payload capacity or the limited battery-time constraint. Note that this upper bound has comparatively high value for the on-road vehicles and thus, limits the applicability of the proposed heuristics method to the delivery problem involving on-road vehicles.

The proposed heuristics method makes use of this aspect of the global-optimum solution and limits the cardinality of the set of feasible routes. Instead of generating all feasible routes, the routes encompassing customers which lie in vicinity of each other are only included in the set  $R$ . By doing so, not only the memory requirements are lowered but also the computational time is saved for larger problems. Note that this means only the route enumeration step is changed while the route selection and assignment steps are carried out in the same manner as before. The heuristics algorithm for route enumeration is given as Algorithm 1.

### 1. Results

This subsection presents a comparison of the results obtained from the application of ‘near exact’ and heuristics methods on different problem instances given in literature. The simulations are conducted on a laptop with an Intel(R) Core(TM)i5-2430QM CPU 2.40 GHz processor and 4.00 GB RAM. The termination criteria delineated in Subsection 1 is used for both methods. The comparison of results obtained from the

**Data:** searchRadius, Increment, searchLimit, customerLocations  
**Result:** R (set of feasible routes)  
initialization;  
**for**  $i=1 : Q$  **do**  
    **for** each customer  $j=1 : n$  **do**  
        Find customers within searchRadius;  
        **while** number of customers within searchRadius is less than searchLimit **do**  
            increase searchRadius by Increment;  
            Find customers within searchRadius;  
        **end**  
        update R with unique permutations  $P(\text{customers}, i)$  which include customer  $j$   
    **end**  
**end**

**Algorithm 1:** Heuristics for route enumeration

‘near exact’ and heuristics methods for SS-1 problem instance is shown in Table 3. Cardinality of the set R, denoted by  $|R|$ , is an indirect measure of the memory requirements. It can be seen that the heuristics method has lower memory requirements than the exact method and provides solutions extremely close to the global-optimum solutions (computed with the predefined stopping criteria).

Table 3: Results for SS-1 problem instance using ‘near exact’ and heuristics method

Problem		‘Near exact’ method				Heuristics			
Name	Q	$ R $	Max. Journey Time (s)	Computation time (s)	Termination Limit	$ R $	Max. Journey Time (s)	Computation time (s)	Termination Limit
SS-1	2	1681	764.01	0.80	relative	211	767.02	0.30	relative
	3	65641	603.98	262.84	relative	1147	590.04	0.76	relative

Note that it is possible to obtain a slightly better solution using the heuristics method than the one obtained using ‘near exact’ method as is the case for  $Q = 3$  in Table 3. This is due to the property of branch-and-bound algorithm and the termination tolerances defined on it.

The results shown till now are obtained for the customers aligned in a straight line. The next logical question that comes to mind is whether the aspect of a solution used for the development of the heuristics method remains valid for the arbitrarily located customers or not. It so happens that the routes are formed from the customers lying in vicinity of each other even for the problem instances with arbitrary customer locations. The comparison of performances of the exact and heuristics methods on arbitrarily placed customers is assessed on benchmark problem instances<sup>34,35</sup> and is shown in Table 4.

Similar conclusions can be drawn from the results presented in Table 4 i.e., heuristics has lower memory requirements, and provides solutions (faster than the ‘near exact’ method), which are very close to the global-optimum solutions for all practical purposes. Note that the exact algorithm ran out of memory very quickly for  $Q = 3$  and  $n > 75$ .

Figure 3 shows customer and depot locations for different benchmark instances from the literature. The locations of the depot and customers are multiplied by a factor of 15 in order to have non-negligible flying-time for drones. Figure 3 is included in order to illustrate the different patterns of customers and depot locations for which the conclusions are drawn from the results shown in Table 4.

## V. Phase II - Route Scheduling

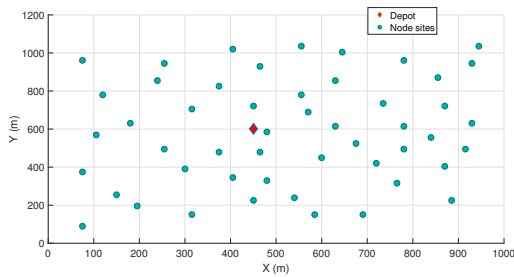
This section deals with the second phase of the planning algorithm - route scheduling. The routes within each vehicle’s journey are scheduled to minimize the maximum number of vehicles that are simultaneously present at the depot, referred to as the problem of controlling congestion at the depot. At this stage, it is assumed that a solution such as the one from Phase-I in Section IV is available i.e, routes have already been assigned to different vehicles aiming at a primary objective function such as minimizing the maximum



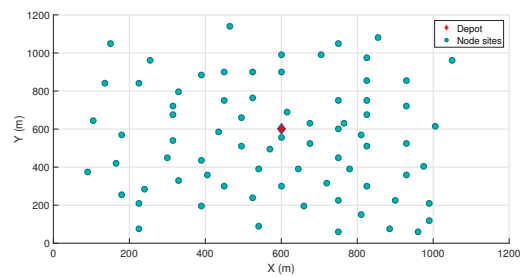
Table 4: Results for benchmark problem instances using ‘near exact’ and heuristics method

Problem		‘Near exact’ method				Heuristics			
Name	Q	$ R $	Max. Journey Time (s)	Computation time (s)	Termination Limit	$ R $	Max. Journey Time (s)	Computation time (s)	Termination Limit
CMT-1 (n=50)	2	2500	813.58	0.54	relative	316	804.52 (1.11% better)	0.20	relative
	3	120100	660.57	89.45	relative	2122	654.06 (0.98% better)	1.34	relative
CMT-2 (n=75)	2	5625	1202.80	2.26	relative	495	1203.30 (0.04% worse)	0.45	relative
	3	410775	959.22*		Out of Memory	3585	966.94 (0.80% worse)	2.33	relative
CMT-3 (n=100)	2	10000	1589.40	3.04	relative	708	1594.60 (0.33% worse)	0.23	relative
	3	980200			Out of Memory	5010	1271.50	3.19	relative
CMT-4 (n=149)	2	22201	2460.20	6.34	relative	1119	2456.40 (0.15% better)	1.13	relative
	3				Out of Memory	9213	1939.62	5.81	relative
CMT-5 (n=198)	2	39204	3293.75	8.36	relative	1508	3294.05 (0.01% worse)	1.51	relative
	3				Out of Memory	13148	2588.46	4.35	relative
CMT-11 (n=119)	2	14161	2894.22	4.17	relative	865	2933.61 (1.36% worse)	1.97	relative
	3	1657075*			Out of Memory	6223	2184.40	9.47	relative
CMT-12 (n=99)	2	9801	1778.17	4.56	relative	663	1826.19 (2.70% worse)	0.80	relative
	3	950895			Out of Memory	4125	1421.64	2.26	relative

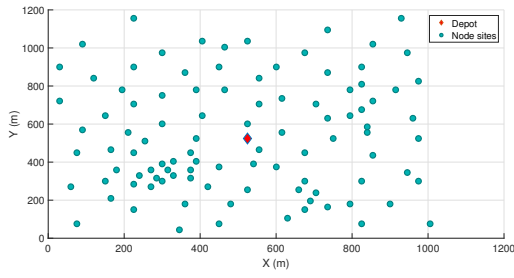
\* This is computed using a high-end computer for the purpose of comparison.



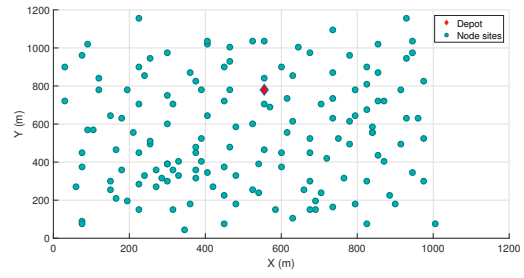
(a) CMT-1



(b) CMT-2



(c) CMT-3



(d) CMT-4

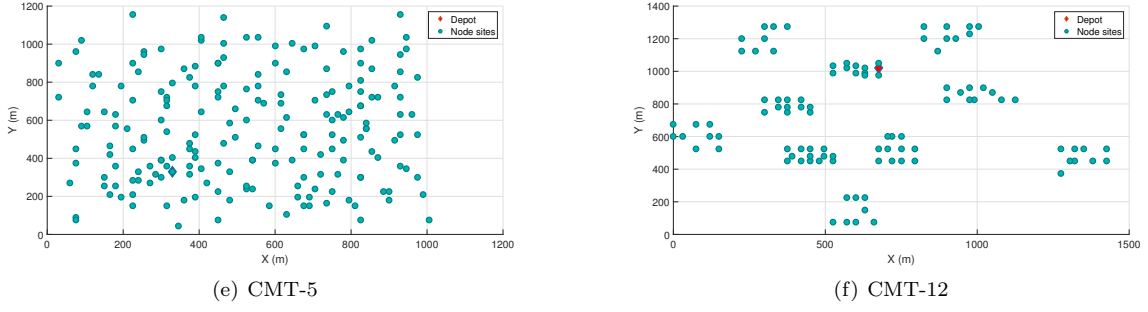


Figure 3: Benchmark instances<sup>34</sup> modified by multiplication with a factor of 15

journey time and/or fleet-size. This section presents the formulation of this problem aimed at achieving minimum congestion at the depot. The solution to the resulting MILP formulation is obtained using, once again, CPLEX optimization software in MATLAB.

It should be noted that, in general, the global optimum solution obtained by simultaneous optimization of the primary objective function and the congestion at the depot may result in a better solution than the one obtained using this methodology. Simultaneous optimization results in fixing upper bounds for both maximum journey time and number of routes that can be assigned to a vehicle. These can be taken as time-horizon  $T_H$  and number of customers  $n$  respectively but this generally results in a requirement of unfathomable amount of memory even for small problems.

There are three different ways, described in detail in Subsections A, B and C, by which the problem of minimizing the congestion at the depot can be handled: by shuffling the routes within each vehicle's journey, by introducing slacking time for each route or using both shuffling and slacking together. The slacking time corresponds to a waiting time for each vehicle in order to get serviced at the depot. Note that the congestion at the depot is required to be minimized only for the scenarios with fleet-size higher than the number of people (or service-points) available at the depot. If fleet-size is comparable to the number of service-points, the problem of congestion does not arise.

## A. Scheduling by Shuffling

The problem of minimizing the congestion at the depot is solved at this step by shuffling the routes within each vehicle's journey. Note that this step does not involve any change in assignment of customer locations or their order within each route. In other words, the routes in a vehicle's journey are ordered in time without making any change to the customers assigned to each route. Moreover, the value of the primary objective function remains unchanged with the shuffling of routes within the journey of a vehicle.

### 1. Problem Statement

The input to this problem consists of routes assigned to different vehicles. As before, consider a *single* depot, denoted as point 0, and a given set of  $n$  customers, denoted as a set  $N = \{1, 2, \dots, n\}$ . A fleet of  $m$  identical vehicles is based at the depot and is denoted by a set  $K = \{1, 2, \dots, m\}$ . A solution (e.g, from minimizing a primary objective function) is available. It is a set of  $m$  sets of routes and is given by  $s = (R^1, R^2, \dots, R^m)$  where each  $R^k$  contains the set of routes assigned to the vehicle  $k$ . Let the cardinality (number of elements) of each set  $R^k \forall k \in K$  be given by  $n^k = |R^k|$ . Note that  $n^k$  represents the number of routes assigned to a vehicle  $k$ .

Let us consider the journey time maximum among all the vehicles to be given by:

$$T = \left[ \max_{k \in K} \sum_{r \in R^k} t_r x_r^k \right]$$

It should be noted that the number of journeys is equal to the number of vehicles in the fleet. Let the length of different routes within a journey (or vehicle)  $k$  be denoted by a row vector  $t_k$  which is given as follows:

$$t^k = [t_1^k, t_2^k, \dots, t_{n^k}^k] \quad \forall k \in K \quad (8)$$

The time-axis is split into bins of size  $\Delta t$ . The solution to this problem then specifies the order of routes within each  $R^k \forall k \in K$  such that the maximum number of vehicles present simultaneously (at the depot) in these  $\lceil \frac{T}{\Delta t} \rceil$  time-bins is minimized. This order of routes is determined by assignment of  $n^k$  routes to  $n^k$  positions (the positions here correspond to the order of routes in the journey and not the customer locations) for each vehicle  $k$ .

## 2. Problem Formulation

Given the statement in Subsection 1, the formulation of the problem is delineated below:

$$\min \quad \bar{n} \quad (9)$$

$$s.t. \quad \sum_{r=1}^{n^k} x_{rj}^k = 1 \quad \forall k \in K, j \in \{1, 2, \dots, n^k\} \quad (10)$$

$$\sum_{j=1}^{n^k} x_{rj}^k = 1 \quad \forall k \in K, r \in \{1, 2, \dots, n^k\} \quad (11)$$

$$\begin{bmatrix} c_1^k \\ c_2^k \\ \vdots \\ c_{n^k}^k \end{bmatrix} = \begin{bmatrix} t_1^k & 0 & \dots & 0 & t_2^k & 0 & \dots & 0 & \dots & t_{n^k}^k & 0 & \dots & 0 \\ t_1^k & t_1^k & \dots & 0 & t_2^k & t_2^k & \dots & 0 & \dots & t_{n^k}^k & t_{n^k}^k & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & & \vdots & \vdots & \ddots & \vdots \\ t_1^k & t_1^k & \dots & t_1^k & t_2^k & t_2^k & \dots & t_2^k & \dots & t_{n^k}^k & t_{n^k}^k & \dots & t_{n^k}^k \end{bmatrix} \begin{bmatrix} x_{11}^k \\ x_{12}^k \\ \vdots \\ x_{1n^k}^k \\ x_{21}^k \\ x_{22}^k \\ \vdots \\ x_{2n^k}^k \\ \vdots \\ x_{n^k 1}^k \\ x_{n^k 2}^k \\ \vdots \\ x_{n^k n^k}^k \end{bmatrix} \quad \forall k \in K \quad (12)$$

$$\begin{aligned} (p-1) \cdot \Delta t - M \cdot (1 - y_{jp}^k) &\leq c_j^k < p \cdot \Delta t + M \cdot (1 - y_{jp}^k) + t_d \\ \forall p \in \{1, 2, \dots, \lceil \frac{T}{\Delta t} \rceil\}, j \in \{1, 2, \dots, n^k\}, k \in K \end{aligned} \quad (13)$$

$$\sum_{t=1}^{\lceil \frac{T}{\Delta t} \rceil} y_{jt}^k = 1 + \left\lfloor \frac{t_d}{\Delta t} \right\rfloor \quad \forall j \in \{1, 2, \dots, n^k\}, k \in K \quad (14)$$

$$\sum_{k=1}^m \sum_{j=1}^{n^k-1} y_{jt}^k \leq \bar{n} \quad \forall t \in \{1, 2, \dots, \lceil \frac{T}{\Delta t} \rceil\} \quad (15)$$

$$x_{rj}^k \in \{0, 1\}, \quad y_{jt}^k \in \{0, 1\} \quad \forall t \in \{1, 2, \dots, \lceil \frac{T}{\Delta t} \rceil\}, j \in \{1, 2, \dots, n^k\}, r \in \{1, 2, \dots, n^k\}, k \in K \quad (16)$$

where minimum possible value of  $\bar{n}$  denotes the maximum number of vehicles present at the depot in any time interval  $\Delta t$ ,  $x_{rj}^k$  is a binary variable indicating whether the route  $r$  of vehicle  $k$  is placed at the

location  $j$  ( $x_{rj}^k = 1$ ) or not ( $x_{rj}^k = 0$ ),  $c_j^k$  is the completion time of the route placed at the location  $j$  for a vehicle  $k$  i.e., the time instant at which the service of a vehicle at the depot has finished and it is ready to continue on its next route,  $t_d$  is the service time at the depot (as defined in Section III),  $y_{jt}^k$  is a binary variable and is equal to one if vehicle  $k$  is present at the depot and  $M$  is a positive number larger than all numbers encountered in the problem instance.

Equation 9 minimizes the maximum number of vehicles that are simultaneously present at the depot. It should be noted that the route finishing at the  $n^k$ -th location for a vehicle  $k$  is not included in the computation of the maximum number of vehicles present simultaneously at the depot, as evident from the upper bound of index  $j$  in Equation 15. This is because the last route in each vehicle's journey does not require any servicing at the depot. Equations 10 and 11 enforce that only one route can be placed at each location  $j$  and vice-versa. Equation 12 computes the completion time of each route in set  $s$ . Equations 13 and 14 are aimed at finding out the time-intervals in which a vehicle is present at the depot. It is computed by forcing the binary variable  $y_{jp}^k$  to take a value of 1 when the completion time of a route, placed at the position  $j$  for a vehicle  $k$ , falls between the time-interval  $(p - 1) \cdot \Delta t$  and  $p \cdot \Delta t$ . For that particular  $y_{jp}^k$ , the variables in the set  $\{y_{j(p-1)}^k, y_{j(p-2)}^k, \dots, y_{j(p-\lfloor \frac{t_d}{\Delta t} \rfloor)}^k\}$  are also set equal to one corresponding to the amount of time a vehicle is present at the depot.

The above set of equations is formulated taking into account the general time-interval bins of size  $\Delta t$  rather than the unit-size bins. This factor of  $\Delta t$  helps in reduction of the size of the problem by reducing the number of decision variables. The use of this factor becomes more pertinent for problems with a high journey time. The lower the value of  $\Delta t$ , the more accurate the calculation of the objective function. However, a very low value of  $\Delta t$  will not be possible for practical purposes and may not even be desirable. Taking the larger values of  $\Delta t$ , some 'end' cases may be overestimated. For example, consider a case shown in Figure 4 in which the difference between the time instant at which the service (at the depot) starts for a vehicle (vehicle 2 in this case) and time instant at which the service (at the depot) finishes for another vehicle (vehicle 1 in this case) is less than or equal to  $\Delta t$  but greater than or equal to 0. If the difference between these time instances lies within a bin of size  $\Delta t$  on the time axis, say between  $(n - 1) \cdot \Delta t$  and  $n \cdot \Delta t$ , the number of vehicles present at the depot between this time interval will be counted as two instead of one. In other words,  $\Delta t$  decides the resolution needed on the time-axis.

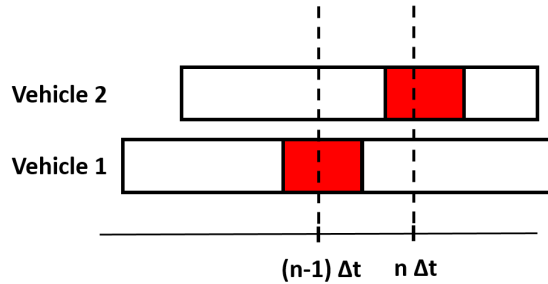


Figure 4: An 'end' case for the problem of controlling congestion at the depot

One way to take care of these end cases is to compute all possible global optimal solutions using the above formulation and then choose the solution which gives minimum value of congestion by further lowering the resolution on time-axis. However, it should be noted that the problem of enumerating all optimal solutions may be more difficult than finding a global optimum solution.<sup>36</sup> Another possible way, as stated in Section V, is to allow further independence in the scheduling of the routes by introducing slack time and is the topic of discussion in next subsection.

The formulation is valid for any  $\Delta t$  that ensures  $\frac{t_d}{\Delta t}$  is an integer and  $\Delta t \leq t_d$ . This restriction makes sure that the interval of size  $t_d$  falls exactly in  $(\frac{t_d}{\Delta t} + 1)$  number of bins. The higher the maximum journey time, the higher  $\Delta t$  should be to reduce the size of the resulting problem. For  $\Delta t > t_d$ , Equation 13 can be

replaced with Equation 17 taking into account only the completion time rather than the whole interval.

$$(p-1) \cdot \Delta t - M \cdot (1 - y_{jp}^k) \leq c_j^k < p \cdot \Delta t + M \cdot (1 - y_{jp}^k) \\ \forall p \in \{1, 2, \dots, \left\lceil \frac{T}{\Delta t} \right\rceil\}, j \in \{1, 2, \dots, n^k\}, k \in K \quad (17)$$

### 3. Results

This section presents the result obtained by minimizing the congestion at the depot using shuffling for SS-1 problem instance. The solution given in Table 2 obtained after minimizing the maximum journey time is used as an input to this problem. The interval size,  $\Delta t$  considered in this problem is 10 s.

Table 5: Routes scheduled by shuffling for SS-1 problem instance

Vehicle	Journey
1	0 11 12 0 13 14 0 31 30 0 10 9 0 16 15 0 21 22 0
2	0 1 2 0 33 32 0 7 8 0 18 17 0 23 24 0
3	0 27 25 0 3 4 0 38 39 0 26 0 37 36 0
4	0 35 34 0 29 28 0 6 5 0 40 41 0 19 20 0

Table 5 shows the journey of each vehicle obtained by minimizing the congestion at the depot (by shuffling). It can be seen from the bottom of Figures 5(a) and 5(b) that the number of vehicles present simultaneously at the depot has been reduced from three to two. It can also be noted that interchanging the routes within the journey of a vehicle did not alter its total journey time.

## B. Scheduling by Slacking

The problem of minimizing the congestion at the depot can be further solved by allowing slack time between all routes of each vehicle. This slack time may correspond to a waiting time in practice. Note that the addition of slack times will result in diminishing the quality of the solution w.r.t. the primary objective function of a minimum overall journey time.

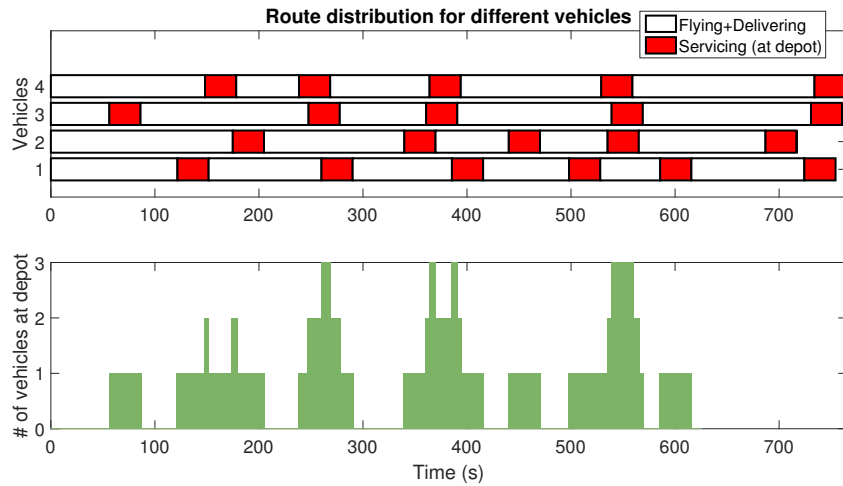
### 1. Problem Formulation

The input to this problem consists of routes assigned to different vehicles. This can either be a solution at the end of Phase-I or after clearing the congestion using shuffling. The problem statement is similar to the one given in Part 1 of Subsection A of Section V except that the definition of  $T$  is altered.  $T$  can be defined by taking into consideration the maximum amount of time that each vehicle's journey is allowed to slack.

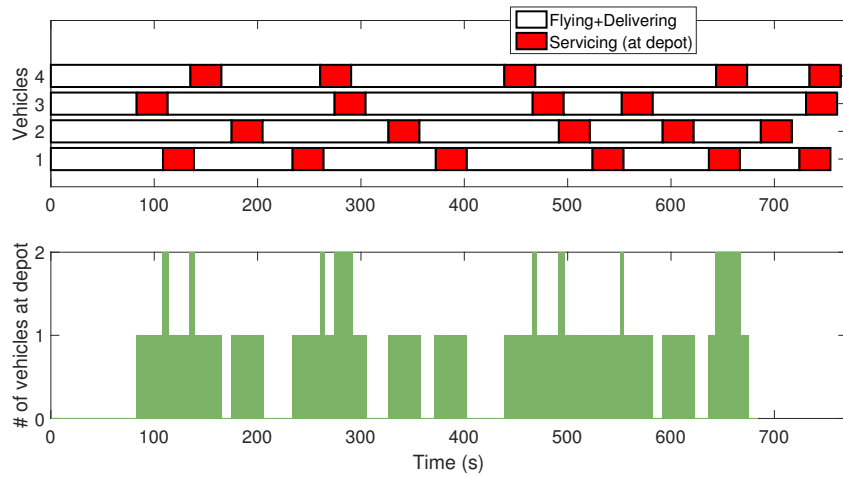
The time-axis is again split into bins of size  $\Delta t$ . The solution to this problem is the computation of slacking times,  $s_j^k$  of routes within each  $R^k \forall j \in \{1, 2, \dots, n^k\}$  and  $k \in K$  such that the maximum number of vehicles present simultaneously (at the depot) in these  $\left\lceil \frac{T}{\Delta t} \right\rceil$  time-bins is minimized. A second objective function of minimizing the completion time of the last route in each vehicle's journey is also added in order to reduce the value of overall journey time along with the congestion at the depot. The problem can be formulated in the form of MILP as shown below:

$$\min \quad w_{\bar{n}} \cdot \bar{n} + w_{\bar{c}} \cdot \bar{c} \quad (18) \\ s.t.$$

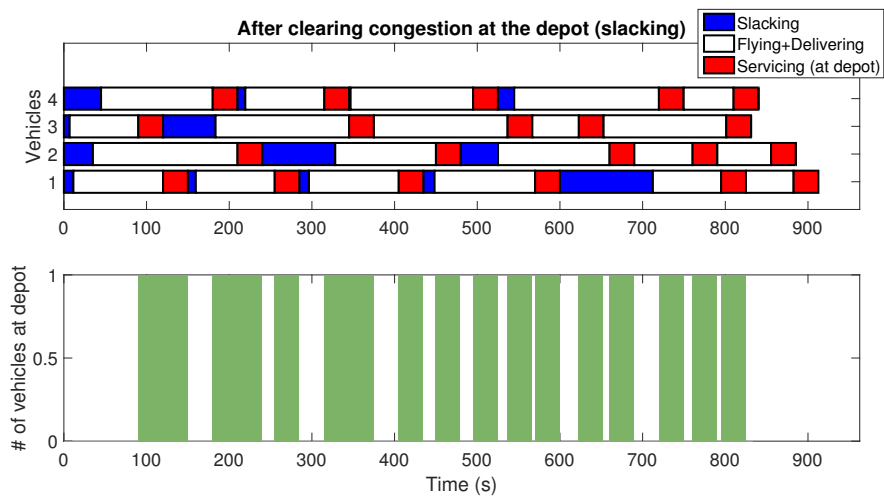
$$\begin{bmatrix} c_1^k \\ c_2^k \\ \vdots \\ c_{n^k}^k \end{bmatrix} = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 1 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \dots & 1 \end{bmatrix} \begin{bmatrix} t_1^k \\ t_2^k \\ \vdots \\ t_{n^k}^k \end{bmatrix} + \begin{bmatrix} 1 & 0 & \dots & 0 \\ 1 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \dots & 1 \end{bmatrix} \begin{bmatrix} s_1^k \\ s_2^k \\ \vdots \\ s_{n^k}^k \end{bmatrix} \quad \forall k \in K \quad (19)$$



(a) Before clearing the congestion at the depot (maximum journey time and computation time are 764.01 s and 0.8 s respectively)



(b) After clearing the congestion at the depot using shuffling (maximum journey time and computation time are 764.01 s and 5 s respectively)



(c) After clearing the congestion at the depot using shuffling followed by slacking (maximum journey time and computation time are 912.58 s and 400 s respectively)

Figure 5: Route distribution for SS-1 problem instance

$$(p-1) \cdot \Delta t - M \cdot (1 - y_{jp}^k) \leq c_j^k < p \cdot \Delta t + M \cdot (1 - y_{jp}^k) + t_d$$

$$\forall p \in \{1, 2, \dots, \left\lceil \frac{T}{\Delta t} \right\rceil\}, j \in \{1, 2, \dots, n^k\}, k \in K \quad (20)$$

$$\sum_{t=1}^{\left\lceil \frac{T}{\Delta t} \right\rceil} y_{jt}^k = 1 + \left\lfloor \frac{t_d}{\Delta t} \right\rfloor \quad \forall j \in \{1, 2, \dots, n^k\}, k \in K \quad (21)$$

$$\sum_{k=1}^m \sum_{j=1}^{n^k-1} y_{jt}^k \leq \bar{n} \quad \forall t \in \{1, 2, \dots, \left\lceil \frac{T}{\Delta t} \right\rceil\} \quad (22)$$

$$c_{n^k}^k \leq \bar{c} \quad \forall k \in K \quad (23)$$

$$y_{jt}^k \in \{0, 1\} \quad \forall t \in \{1, 2, \dots, \left\lceil \frac{T}{\Delta t} \right\rceil\}, j \in \{1, 2, \dots, n^k\}, k \in K \quad (24)$$

where minimum possible value of  $\bar{c}$  denotes the maximum journey time,  $w_{\bar{n}}$  and  $w_{\bar{c}}$  are the weights corresponding to two different objective functions and  $s_j^k$  is the slack time of the route placed at the position  $j$  within the journey of a vehicle  $k$ . Equation 18 minimizes the maximum number of vehicles that are simultaneously present at the depot along with the maximum journey time. Equation 19 computes the completion time of each route in set  $s$  after taking slack times into account. Note that the upper bound on  $\bar{n}$  is identified either to be the maximum number of vehicles (in the fleet or present simultaneously at the depot in the input solution to this problem) or maximum number of persons available at the depot for servicing the drones.

Equations 18 and 23 can also be replaced with Equations 25 and 26 in the formulation in order to minimize the congestion at the depot along with the maximum value of ‘sum of slacking times introduced for a vehicle.’

$$\min \quad w_{\bar{n}} \cdot \bar{n} + w_{\bar{s}} \cdot \bar{s} \quad (25)$$

$$\sum_{j=1}^{n^k} s_j^k \leq \bar{s} \quad \forall k \in K \quad (26)$$

## 2. Results

For SS-1 problem instance, the results obtained after minimizing the congestion at the depot using shuffling followed by slacking are shown in Figure 5(c). It can be seen that the maximum number of vehicles present at the depot has been reduced from two to one. Table 6 contains the amount of slack times introduced for different routes in each vehicle’s journey to achieve the optimum objective function. The maximum journey time obtained after application of this algorithm for SS-1 problem instance is 912.58 s.

Table 6: Slack times for SS-1 problem instance

Vehicle	Slack time for each route
1	11.54, 9.60, 11.43, 13.36, 112.40, 0
2	35.14, 88.2424, 45.11, 0, 0
3	7.00, 63.49, 0, 0, 0
4	44.99, 9.48, 1.81, 19.71, 0

It can be observed from the Table 6 that a non-zero slack-time is introduced for the first route of all four vehicles in the fleet. This has happened because of the occurrence of the ‘end’ case (as described in Subsection 2) for the second routes of vehicles 3 and 4. The minimum (7s for vehicle 3) among all vehicles can be removed from each vehicle’s journey to obtain a better solution.

It can be noted that a case in which slack variables are introduced before the start of the routes has as low an objective function value as other cases in which slack time is introduced at some other point during the journey (e.g., after delivering a geophone or before the time instant at which the service starts). This can be understood from the fact that the optimum solution for cases in which slack time is introduced at a point other than the beginning of the routes can be translated into an equivalent optimum solution for a case with slack times introduced at the beginning of the routes without moving the time-spans corresponding to the time spent by vehicles at the depot.

### C. Scheduling by Shuffling and Slacking

This subsection delineates the result obtained for minimizing the congestion at the depot using both shuffling and slacking together for SS-1 problem instance. Equations 12 and 19 can be combined together appropriately to compute completion time of routes incorporating the shuffling using assignment variables  $x_{rj}^k$  and slacking using slack time variables  $s_j^k$ . Figure 6 shows the resulting route distribution of each vehicle and the congestion at the depot after optimization. As expected, it can be seen that the overall journey time is slightly better than the one obtained using shuffling followed by slacking. The resulting formation of journeys has a maximum of one vehicle at a time requiring service at the depot. The order of routes obtained from shuffling and slacking together is different from the one obtained using shuffling followed by slacking as can be seen in Table 7. As in the previous case, non-zero slacking time gets introduced commonly across all the vehicles (at the beginning of journeys) to reduce the objective function value between the second routes of vehicles 2 and 3. This is due to the fact that the  $\Delta t = 10$  s and time instances at which the service at the depot finishes and starts for vehicles 2 and 3 respectively should be located in different time-bins in order to have a minimum value of  $\bar{n}$ .

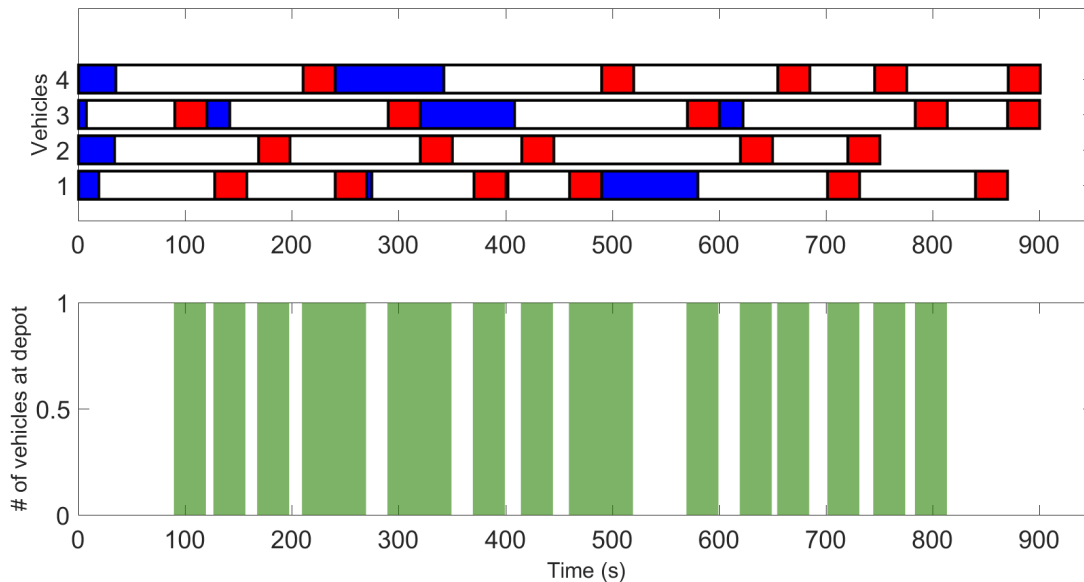


Figure 6: Journeys of each vehicle for SS-1 problem instance after clearing the congestion at the depot using both shuffling and slacking together (maximum journey time and computation time are 900.84 s and 600 s respectively)



Table 7: Journeys of different vehicles for SS-1 problem instance after minimization of congestion using both shuffling and slacking together

Vehicle	Journey
1	0 31 30 0 16 15 0 13 14 0 21 22 0 10 9 0 11 12 0
2	0 7 8 0 33 32 0 23 24 0 1 2 0 18 17 0
3	0 27 25 0 37 36 0 3 4 0 38 39 0 26 0
4	0 40 41 0 6 5 0 35 34 0 19 20 0 29 28 0

## VI. Mission Planning

This section is aimed at providing insight into how the results obtained will be used in practice. An optimized route for all drones in the fleet can be planned centrally with known customers' locations and good estimates of the input parameters ( $T_H$ ,  $T_B$ ,  $t_{drop}$ ,  $t_d$ ,  $V_W$ ,  $V_{avg}$ ). The constraint of making frequent visits by every drone in the fleet to the depot enables the update of the next route onboard each drone by the centralized planner.

During the execution of planned journeys, there can be a lead or lag in the schedule of a vehicle. This can be caused due to a number of factors such as a failure of a drone, or change in the value of one or more input parameters e.g., wind velocity etc. Frequent visits to the depot allow every drone to 'check-in' with the centralized computer. The lead or lag in the schedule can be easily detected by the centralized computer by comparing the scheduled arrival time to the actual time of arrival at the depot. If this time difference is more than an allowed threshold, the solution can be adjusted during the mission taking into account an update into the input parameters such as a new number of drones in the fleet and only the customer locations which are not yet served. After a delay is detected, the centralized computer can work on generating a new schedule within a pre-defined duration while the working fleet can be continued onto its previous schedule within that duration. A best optimal solution obtained within that amount of time (selected a priori) can be used for this purpose. The feature of MILP to get the best solution obtained within a specified amount of time supports the use of this methodology.

However, it should be noted that the solution in the first phase of the planning algorithm is obtained pretty fast using the heuristics method. But it takes time to compute the journeys of vehicles in the fleet during the second phase of the algorithm which is aimed at minimizing the congestion at the depot e.g., it took approximately 400 s to obtain the solution in the second phase (Figure 5(c)) using shuffling followed by slacking for the SS-1 problem instance. This is due to the fact that an actual MILP is still being solved using CPLEX to minimize the congestion at the depot. For cases in which journeys of the drones last for a few hours, both of these phases can be employed for the re-calculation of routes. But, for the cases involving short duration of journeys (e.g., less than an hour), a method aimed at handling each particular failure case can be used to re-plan the routes. For example, in case of a failure of a drone, distribution of the remaining routes of that drone can be done to the functional drones using the Longest Processing Time (LPT) algorithm.

## VII. Flight Test

This section presents the details of a flight demonstration carried out in order to illustrate the actual use of the planning algorithm and build the basic framework for its further development. The flight demonstration is carried out on three Parrot Bebop2 quadrotors running the Paparazzi open-source autopilot software. The mission planner is implemented in Simulink-MATLAB. The code is publicly available on Github<sup>a</sup>. The communication between the planner, drones and Ground Control Station (GCS) is carried out over a wireless network using User Datagram Protocol (UDP). The corresponding communication set-up is shown in Figure 7. Note that the communication between the drones and planner is only required to update the next route when they visit the depot to pick-up new payload and battery and is not required during their journeys. In the absence of any collision avoidance system on board, the concept of altitude layering (i.e.,

<sup>a</sup>Data is available at <https://github.com/kaku289/paparazzi/tree/planner> and <https://github.com/kaku289/plannerInMATLAB> [Accessed: August 9, 2016]

each drone is associated with an altitude at which it flies) is used in the planner to avoid collisions among the drones.

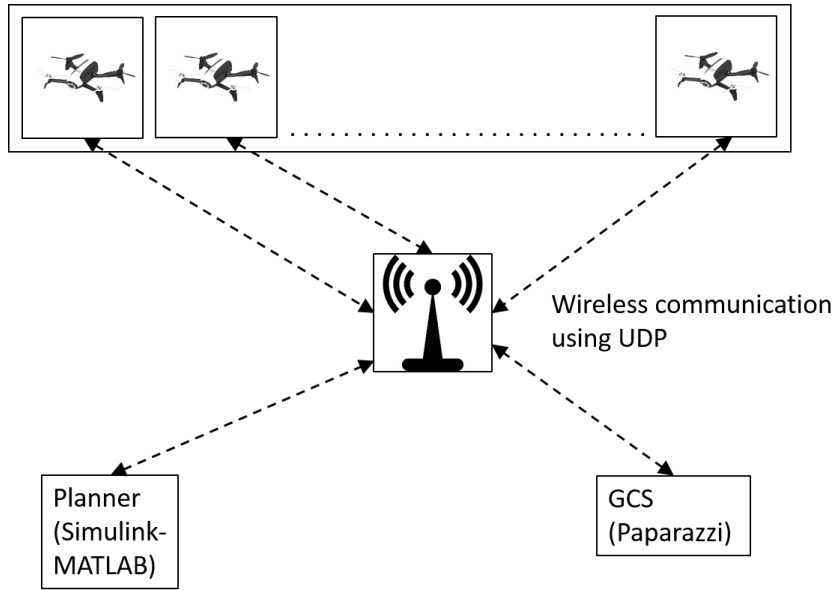


Figure 7: Communication set-up for the mission planner

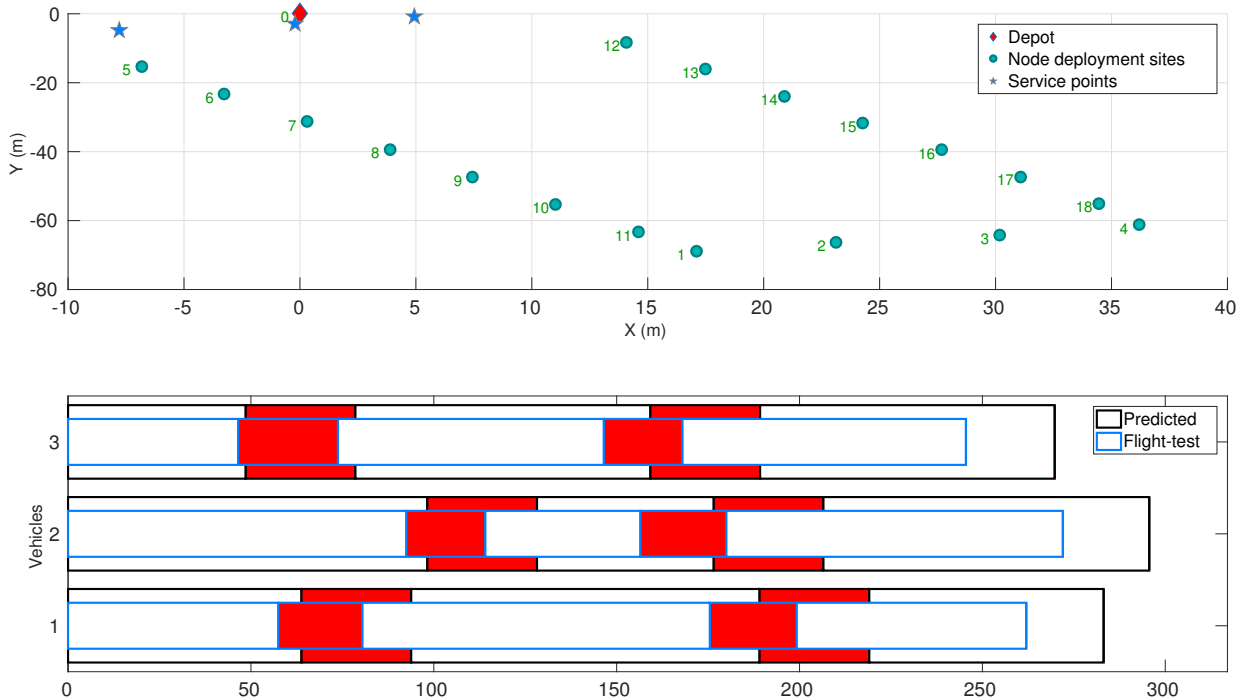
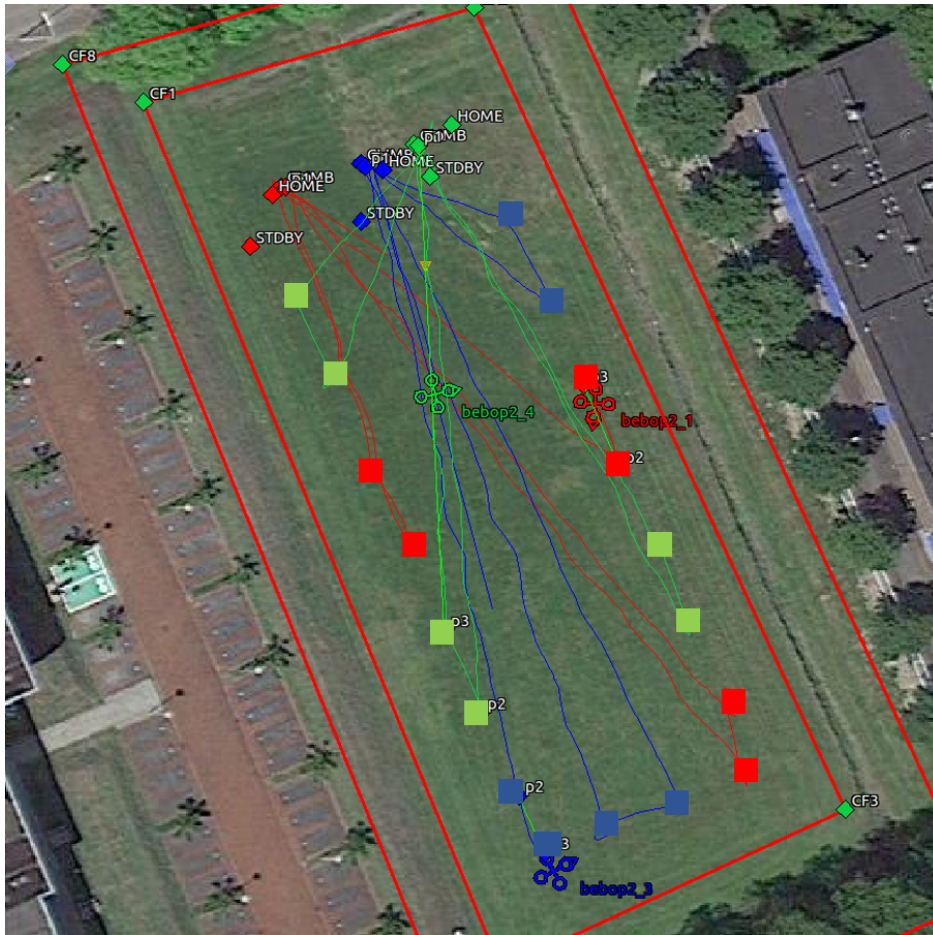


Figure 8: Scenario for flight demonstration and comparison of predicted and actually followed routes (time spent at the depot is shown in red color)



(a) Snapshot of drones during the flight demonstration



(b) Ground track of three drones in the fleet during the flight demonstration (1-Red, 2-Blue, 3-Green)

Figure 9: Drones during the flight demonstration (both figures are taken at different time instances during their journey)

The location of the depot and customers for the flight demonstration is shown at the top in Figure 8. The customers' locations are chosen manually based on the size of the field (in which the test is carried

out) while aiming for as much distance among them as possible to avoid any collisions among the drones during flight. Three separate service points are chosen near the depot where the drones will come back to the depot after visiting customers on each route. This flight demonstration did not involve any actual delivery of geophones. The delivery of geophones is simulated during the flight by hovering for 10 s at an altitude of 3 m above the destination point.

Table 8: Journeys of different vehicles for the flight demonstration

Vehicle	Journey
1	0, 8, 7, 0, 18, 4, 0, 15, 14, 0
2	0, 3, 2, 0, 13, 12, 0, 11, 1, 0
3	0, 6, 5, 0, 17, 16, 0, 10, 9, 0

Table 8 lists the journeys of each vehicle after the minimization of congestion using shuffling. The maximum number of drones present simultaneously at the depot is two. The estimated dropping ( $t_{drop}$ ) and service time ( $t_d$ ) are 12 s and 30 s respectively. Note that the service time doesn't include the time required to change the battery of the drone as the maximum duration of a journey (5 minutes) is less than the time for which a battery can last. Note that there is no need for further minimizing the congestion at the depot for this particular case as drones didn't require any service during the test. Considering the size of the field, the maximum speed at which a drone can fly is restricted to 2 m/s. Since the maximum average airspeed of the drone is  $\approx 20$  m/s, the control algorithm can easily follow the reference of 2 m/s despite what the prevailing wind conditions were during the test. Therefore, the wind conditions in the planner are taken to be 0 m/s.

The comparison of the predicted and actual routes followed is shown at the bottom of Figure 8. The deviation of actual flight time (part of the routes shown in white color) from predicted flight time ranges from 0.26% to 12.23%. The deviation of the time spent at the depot ranges from 9% to 28.3%. This is due to the fact that the landing procedure of the drone is based on the noisy sensor measurements and has to be made more robust. The snapshot of the drones during the flight demonstration is shown in Figure 9(a) and ground track, obtained from GCS in Paparazzi, is shown in Figure 9(b). It can be deduced that the planner can be used in practice with a tuning of the landing procedure.

## VIII. Conclusion

In this article we presented a centralised mission planner for a fleet of drones that can be used for sensor network deployment. The particular application considered in this article is the deployment of geophones in seismic surveys. If there are no associated time windows, it can also be used for a fleet of drones deployed for shipment deliveries or parcel services. The presented mission planner is composed of two phases - route formation and route scheduling. The route formation phase is aimed at minimizing the overall journey time and the proposed heuristics for this step is quite fast and accurate. Since the wind velocity can significantly affect the travel times of a drone between two points, it is also taken into account during this phase. The second phase - route scheduling - is aimed at minimizing the congestion at the depot and is solved in the form of MILP using CPLEX. This step is comparatively slow and restricts the usage of this planner in the re-planning phase to the cases involving longer journeys (of the order of a few hours) with limited number of routes. Re-planning may be required in case a drone fails or a deviation occurs in one of the input parameters to the planning algorithm such as wind velocity. A flight demonstration is carried out in order to illustrate the working of the mission planner.

Apart from the development of a heuristics method for the second phase of the planning algorithm to improve execution time, the application area of seismic surveys opens interesting avenues for future research such as optimization of the location of depot(s) along with the route formation, or finding out an optimal trajectory for the depot(s). This is possible because a depot can be a truck in this case as opposed to a warehouse in the conventional problems.

## Acknowledgments

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## References

- <sup>1</sup>Murphy, M., “15 best uses of drones in business,” 2016, Accessed: March 16, 2016.
- <sup>2</sup>Wahba, P., “Walmart seeks U.S. permission to test drones,” 2015, Accessed: April 14, 2016.
- <sup>3</sup>Amazon, “Amazon Prime Air,” 2016, Accessed: April 14, 2016.
- <sup>4</sup>Hern, A., “DHL launches first commercial drone ‘parcelcopter’ delivery service,” 2014, Accessed: April 14, 2016.
- <sup>5</sup>Moon, M., “Nevada gets first FAA-approved urban drone delivery,” 2016, Accessed: March 28, 2016.
- <sup>6</sup>Shell, “Eye in the sky,” 2016, Accessed: March 16, 2016.
- <sup>7</sup>Quaritsch, M., Kruggl, K., Wischounig-Strucl, D., Bhattacharya, S., Shah, M., and Rinner, B., “Networked UAVs as aerial sensor network for disaster management applications,” *e & i Elektrotechnik und Informationstechnik*, Vol. 127, No. 3, 2010, pp. 56–63.
- <sup>8</sup>Shell, “Drone Assisted Deployment and Retrieval of Seismic Nodes in Difficult Desert Terrain - Research Proposal with TU Delft,” 2015, Restricted Access, SR.11.23423.
- <sup>9</sup>Toth, P. and Vigo, D., *Vehicle Routing: Problems, Methods, and Applications, Second Edition*, SIAM, 2014.
- <sup>10</sup>Cattaruzza, D., Absi, N., and Feillet, D., “Vehicle routing problems with multiple trips,” *4OR*, 2016.
- <sup>11</sup>Şen, A. and Bülbül, K., “A survey on multi trip vehicle routing problem,” *In: VI. International Logistics & Supply Chain Congress*, 2008.
- <sup>12</sup>Fleischmann, B., “The vehicle routing problem with multiple use of vehicles,” *Technical Report, Fachbereich Wirtschaftswissenschaften*, 1990.
- <sup>13</sup>Olivera, A. and Viera, O., “Adaptive memory programming for the vehicle routing problem with multiple trips,” *Computers & OR*, Vol. 34, No. 1, 2007, pp. 28–47.
- <sup>14</sup>Mingozzi, A., Roberti, R., and Toth, P., “An Exact Algorithm for the Multitrip Vehicle Routing Problem,” *INFORMS Journal on Computing*, Vol. 25, No. 2, 2012, pp. 193–207.
- <sup>15</sup>Koc, C. and Karaoglan, I., “A Branch and Cut Algorithm for the Vehicle Routing Problem With Multiple Use of Vehicles,” *Proceedings of the 41st International Conference on Computers & Industrial Engineering*, 2011, pp. 554–559.
- <sup>16</sup>Taillard, É. D., Laporte, G., and Gendreau, M., “Vehicle Routing with Multiple Use of Vehicles,” *Journal of the Operational Research Society*, Vol. 47, No. 8, 1996, pp. 1065–1070.
- <sup>17</sup>Petch, R. J. and Salhi, S., “A multi-phase constructive heuristic for the vehicle routing problem with multiple trips,” *Discrete Applied Mathematics*, Vol. 133, No. 1-3, 2003, pp. 69–92.
- <sup>18</sup>Salhi, S. and Petch, R. J., “A GA based heuristic for the vehicle routing problem with multiple trips,” *Journal of Mathematical Modelling and Algorithms*, Vol. 6, No. 4, 2007, pp. 591–613.
- <sup>19</sup>Cattaruzza, D., Absi, N., Feillet, D., and Vidal, T., “A memetic algorithm for the Multi Trip Vehicle Routing Problem,” *European Journal of Operational Research*, Vol. 236, No. 3, 2013, pp. 833–848.
- <sup>20</sup>Brandão, J. and Mercer, A., “A tabu search algorithm for the multi-trip vehicle routing and scheduling problem,” *European Journal of Operational Research*, Vol. 100, No. 1, 1997, pp. 180–191.
- <sup>21</sup>Brandao, J. C. S. and Mercer, a., “The Multi-Trip Vehicle Routing Problem,” *The Journal of the Operational Research Society*, Vol. 49, No. 8, 1998, pp. 799.
- <sup>22</sup>Fukunaga, A. S. and Korf, R. E., “Bin-completion algorithms for multicontainer packing and covering problems,” *IJCAI International Joint Conference on Artificial Intelligence*, Vol. 28, 2005, pp. 117–124.
- <sup>23</sup>Anaya-arenas, A. M., Maria, A., Chabot, T., Renaud, J., and Ruiz, A., “Biomedical Sample Transportation : A Case Study Based on Québec ’s Healthcare Supply Chain Biomedical Sample Transportation : A Case Study Based on Québec ’s Healthcare Supply Chain,” , No. October, 2014.
- <sup>24</sup>Wang, Z., Liang, W., and Hu, X., “A metaheuristic based on a pool of routes for the vehicle routing problem with multiple trips and time windows,” *Journal of the Operational Research Society*, Vol. 65, No. 1, 2013, pp. 3.
- <sup>25</sup>Shima, T., Rasmussen, S. J., Sparks, A. G., and Passino, K. M., “Multiple task assignments for cooperating uninhabited aerial vehicles using genetic algorithms,” *Computers and Operations Research*, Vol. 33, No. 11, 2006, pp. 3252–3269.
- <sup>26</sup>Richards, A., Bellingham, J., Tillerson, M., and How, J., “Coordination and Control of Multiple {UAV}s,” *{AIAA} Conf. on Guidance, Navigation and Control*, , No. August, 2002, pp. 1–11.
- <sup>27</sup>Olivera, A. and Viera, O., “Solving the vehicle routing problem with multiple trips by adaptive memory programming,” *International Journal of Logistics Systems and Management*, Vol. 3, No. 4, 2007, pp. 440.
- <sup>28</sup>Peloquin, M., “A Comparison of Scheduling Algorithms for Multiprocessors,” 2010, pp. 1–17.
- <sup>29</sup>Leung, J., *Handbook of Scheduling: Algorithms, Models, and Performance Analysis*, Chapman & Hall/CRC Computer and Information Science Series, CRC Press, 2004.
- <sup>30</sup>Karger, D., Wien, J., and Stein, C., “Scheduling algorithms,” 2007, pp. 1–52.
- <sup>31</sup>Chapin, S. J., “Distributed and Multiprocessor Scheduling,” *ACM Comput. Surv.*, Vol. 28, No. 1, March 1996, pp. 233–235.
- <sup>32</sup>Innoseis, “TremorNet - Dramatically reduce land seismic acquisition costs with the industry’s lightest sensing system,” 2016, Accessed: April 14, 2016.
- <sup>33</sup>IBM, “User’s Manual for CPLEX,” 2009, ILOG CPLEX V12.1.
- <sup>34</sup>Christofides, N., Mingozzi, A., and Toth, P., “The Vehicle Routing Problem,” *Combinatorial Optimization*, 1979, pp. 315–338.

<sup>35</sup>Uchoa, E., Pecin, D., Pessoa, A., and Poggi, M., “New Benchmark Instances for the Capacitated Vehicle Routing Problem.” *Optimization-Online.Org*.

<sup>36</sup>IBM, “Using CPLEX to examine alternate optimal solutions,” 2010, Accessed: April 24, 2016.

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# Appendix A

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## Detailed Introduction

Drones are used for wide range of purposes. Their use is being explored constantly nowadays in farming, postal services, rescue operations, capturing crime scenes, and as monitoring platforms e.g., to monitor traffic accidents and to analyze maintenance and field worker distribution (Murphy, 2016). Features such as planning, and autonomous navigation while avoiding obstacles allow drones to do a variety of tasks. In future, Unmanned Aerial Vehicles (UAVs) are expected to operate at a higher level of autonomy to carry out complex tasks while efficiently coordinating with the ground personnel or vehicles. Additionally, they can be anticipated to work cooperatively and expedite the mission execution.

There are many instances where the drones have been used for the delivery purposes. Walmart has applied for drone licenses in the USA so that the customers can fly their shopping home (Wahba, 2015). Amazon is aiming to drop packages (weighing less than 5 lbs) flying through the sky at customers doors in half an hour through its Prime Air service (Amazon, 2016). The World's leading logistics firm, DHL, is also investigating the viability of drones for delivery purposes. It has successfully delivered to a pharmaceutical company in Germany using its parcelcopter (Hern, 2014). Flirtey, a start-up for real-time delivery by flying robots, recently did the first FAA-sanctioned drone delivery in a rural area in Nevada using its autonomous hexacopter (Moon, 2016).

Shell uses drones in some of Europe's largest energy plants, and is rolling them out in various oil and gas facilities. For example, inspection of the flaring stack at the Ormen Lange gas processing plant in Norway is carried out using drones in only few hours which could have taken two weeks forcing the plant to shut down had it been carried out by engineers. Inspections at the Bacton import terminal on the east coast of England using drones enable the gas supply sites to stay fully operational while checking the condition of the facility. Drones are particularly useful for hard-to-reach places as they are safer, and more efficient, than sending people (Shell, 2016).

The drones are envisaged to be used in yet another area - seismic surveys. These surveys are spread over a large area of land, approximately  $280 \text{ km}^2$ . The drones are anticipated to be used to deploy and retrieve seismic sensors, often referred to as geophones. They will

help in maintaining the pace of the survey and can assist crew, especially in difficult terrain. Preliminary studies (Shell, 2015) lists the cost savings from such an operation carried out with the help of drones. Further details about the seismic surveys and the benefits of using drones in managing operations are given in Chapter 1.

The task of realizing a fleet of drones to be used in the seismic surveys can be broadly classified into three categories: the development of each agent (i.e., the drone itself), the landing technique and the development of route planning for multiple agents in the fleet to complete the task in a certain optimal manner. The development of each agent encompasses various challenges in itself such as the type of drones (quadcopter, fixed-wing, hybrid etc.), onboard sensors, autopilot software, control modes etc. The focus of this research is on the development of a method to plan routes for each drone in the fleet. The route planning is done in an “optimal” manner while respecting constraints. The aforementioned “optimal” criteria depends upon the problem being solved and can result in minimization of different attributes related to the problem e.g., the overall time in which the task is completed, the number of vehicles being used to complete the task, or their combination. The constraints can arise from different physical aspects related to the system e.g., the limited time for which the power from the onboard battery is available, or the time-horizon in which the drones can be used (e.g., one working day).

The task is to deliver geophones from known locations. This task accomplishment can be viewed as a Vehicle Routing Problem (VRP). The problems subsumed under the term VRP corresponds to a large number of real-world applications as their solution yield substantial savings in overall transportation costs. The VRP problem was first introduced 57 years ago by Dantzig and Ramser (1959) as a real-world application concerning the delivery of gasoline to gas stations. Since then, the development of the mathematical modeling, boost in the computational power and integration of the information systems into the operations have made the wide-spread use of the optimization techniques possible.

There are many variants of VRP that exist in the literature depending upon different transportation requests, scheduling aspects, route constraints, fleet characteristics and optimization objectives. The overview of VRP, its variants, mathematical models and different exact and metaheuristics algorithms proposed in the literature are presented by Toth and Vigo (2014). The problem dealt in this thesis comes under one of the variants of VRP - Multi-Trip Vehicle Routing Problem (MTVRP). This variant concerns with the multiple use of vehicles which is needed due to the large ratio of number of customers (or locations) to the vehicle capacity. The solution obtained from VRP does not allow multiple trips of the vehicles. There has been an immense amount of literature published in the last five decades on problems concerning VRP. However the interest in MTVRP is relatively new with more scientists in the community focusing on it in the last 10-15 years (Cattaruzza, Absi, & Feillet, 2016).

The aim of the proposed research is not only to solve the problem related to the application of drones in carrying out the operations for seismic surveys, but also to contribute to the growing body of research in MTVRP. The proposed research is based on the features and challenges arising out of the fact that the drones have very low payload capacity as compared to the delivery trucks on the road. This feature entails frequent return of drones in the fleet to the depot, thus giving birth to the problem of depot congestion. The research in this thesis presents the methods developed and results obtained (for the deployment of geophones) from finding the optimal routes for a fleet of drones while minimizing the journey time and/or



fleet-size. The problem of depot congestion has also been formulated in this thesis in the form of Mixed Integer Linear Programming (MILP).

## A-1 Research Objective and Question

The aim of this research is to find/develop methods that can provide solutions to different challenges encountered in the route planning of a fleet of drones carrying out an operation of delivering geophones in seismic surveys. Based on the discussion so far, the research question can be formulated as follows: *How can the vehicle routes be determined while minimizing the overall journey time and/or fleet-size along-with the congestion at the depot for a fleet comprising of identical drones (or vehicles) with the limited payload capacity and battery power?*

The research question can be further split into sub-questions as follows:

1. What is the exact framework (assumptions and characteristics of seismic surveys) governing the methods to be used?
2. How can the problem be formulated to minimize the overall journey time with the imposition of following constraints: limited payload capacity of each vehicle, limited trip duration, single visitation to each customer?
3. How can the problem be formulated to minimize the fleet-size with the imposition of following constraints: limited payload capacity of each vehicle, limited trip duration, single visitation to each customer, finite time-horizon?
4. How can the problem be formulated to minimize the number of vehicles that are present simultaneously at the depot with the imposition of the constraints listed above?
5. How these problems can be solved e.g., using a heuristic or an exact method?

## A-2 Research Contribution

The research done in this thesis contributes to the existing fields of planning and MTRVP in the following ways:

1. It develops a 2-phase mission planner for a fleet of drones that can find applications in seismic surveys, shipment deliveries and parcel services.
2. During the first phase of the mission planner, it develops upon the formulations already present in the literature and fits them to solve the optimization problem related to the deployment of seismic sensors using a fleet of drones. This involves modification of the cost function, and inclusion of the bounded trip duration constraint which arises due to the limited battery power. A heuristics method is also presented which limits the combinatorial explosions during the route formation phase and provides near-optimal solutions very fast.

3. During the second phase of the mission planner, it deals with the unaddressed problem of controlling congestion at the depot by scheduling the routes using shuffling, slacking, or both.
4. It also presents the multi-objective optimization in the form of MILP for simultaneous minimization of (weighted combination of) maximum journey time, and maximum number of vehicles being used in the fleet.

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## Appendix B

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# Research Framework

The aim of this appendix is to present the details about the field of application and research framework for better understanding of the applicability of the methods being developed and the motivation behind them. This appendix outlines the wider scope of the research opportunity in the area of application which would funnel down to a narrow research topic. Section B-1 in this appendix briefly describes the area of application - seismic surveys - in order to understand the benefits of using drones in its operations which are presented in Section B-2. Section B-3 delineates the different challenges involved and areas of research possible in realisation of using drones to carry out the operations in the field and the framework of the subject of this thesis itself.

### B-1 Seismic Surveys

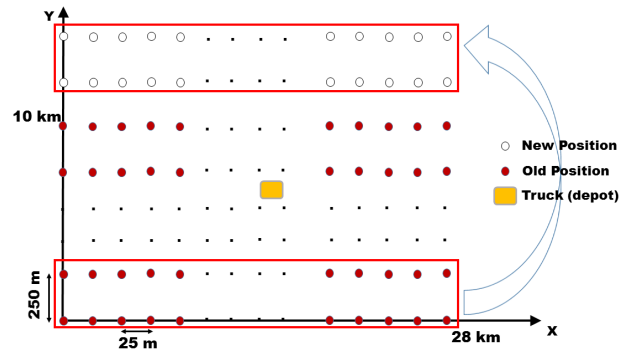
A seismic survey is a technique which is used to produce detailed images of various rock layers and their locations below ground. These surveys are used either to explore new oil and gas reservoirs or to monitor the existing fields. In a typical seismic survey, sound waves are bounced off underground rock formations and the reflecting waves are recorded by the sensors placed onto the ground. These sensors are generally laid down in a rectangular grid-like pattern as shown in Figure B-2. These surveys cover a huge ground ( $\approx 280 \text{ km}^2$ ) and involves displacement of 2-3 lines of geophones in one working day as depicted in Figure B-2. These sensors are usually picked-up and deployed by a ground crew using  $4 \times 4$  vehicles. But the drones are now envisaged in this area of collecting and deploying geophones to save time and operational costs especially in rough terrains.

The traditional geophones used in these surveys are not wireless, which is why they are laid down in lines. For rough terrain, wireless sensors are just now starting to be used. It should be noted that the use of wireless geophones lifts the restriction of placing the sensors in a line which can be of interest to geophysicists in exploring fields.

Typical configuration of a wireless geophone (Innoseis, 2016) is shown in Figure B-1 and it weighs approximately 700 *g*. One drone is expected to carry up to a maximum of 3 geophones.



**Figure B-1:** Wireless geophone (Innoseis, 2016)



**Figure B-2:** Typical field of seismic survey (not to scale)

## B-2 Drone assisted Deployment and Retrieval

As stated in the previous section, the deployment and retrieval of geophones is carried out by personnel. It includes the time it takes for the personnel to travel using vehicles and carry equipment (and geophones) based on their strength and capacity to different sites. The collection or deployment of sensors can be time consuming especially in difficult terrains such as sand dunes and steep rocky hills. This is because the ground vehicles can not reach these places, and ground crew have to go on foot. Therefore, the use of drones for sensor deployment or retrieval operation in rough terrains can create a significant difference in maintaining the pace of the survey. Using drones to carry out the operation in difficult terrains not only saves the personnel from carrying additional weight of the sensors but also limit the number of times they had to climb or walk down the terrain to their supporting vehicles. This ultimately allows them to work faster and maintain the pace of the survey while reducing the risk of injury. It is estimated that approximately 10% (300-400 in number) of nodes will be deployed using a fleet of drones in one day.

For the purpose of sensor deployment operation, the drones to be used in the seismic survey are anticipated to carry the sensors from the truck or different pick-up locations and place them at the required sites. The ground personnel can then later put the sensor into the ground using the special equipment. At the pick-up locations, the ground personnel are expected to dig out the sensor from the ground and be present there to mount it on the drone. Therefore the retrieval of the sensors from the pick-up locations will require a coordination with the ground personnel as opposed to the deployment operation.

## B-3 Research Focus

There are three main challenging areas in order to accomplish the sensor retrieval and deployment operation in seismic surveys using drones and they are identified below:

1. Development of each agent i.e., drone in the fleet - its type (quadrotor, fixed-wing, hybrid etc.), autopilot design, on-board control modes, hardware design to facilitate the mounting of geophones and to shun the sand in the deserts from affecting its parts.

2. Development of the method for safe landing of each agent at the depot and on terrains with different inclinations.
3. Development of the method to identify routes for a fleet of drones to accomplish the retrieval and deployment operation in an optimal manner.

As stated in Chapter A, this thesis is focused on the development of the route planning strategy for optimal deployment of geophones using a fleet of drones. Before diving into the relevant methods in the literature, it is apt to further concretize the framework of the problem in terms of the characteristics of each agent in the fleet and the whole operation.

### B-3-1 Characteristics of Each Agent in the Fleet

The following list enumerates the characteristics of each agent i.e., a drone in the fleet:

1. The autopilot onboard each drone is designed in such a way that it can autonomously fly from one location to another in a straight line.
2. The drones have take-off, landing and collision avoidance methods implemented on board.
3. The fleet is homogeneous i.e., all agents have same the mission and flying capabilities. It means that each drone in the fleet has the same maximum payload capacity, and average speed while flying in a straight line.

### B-3-2 Characteristics of the Operation

Certain defining characteristics of the operation which crucially identify the exact problem formulation are listed below:

1. The sensors retrieved from the pick-up locations are needed to be brought to the depot/truck instead of directly dropping them at the customer locations (locations where sensors are needed to be placed). These sensors are needed to be recharged at the depot before their next deployment in the field.
2. The sensor retrieval task requires coordination with the ground personnel as a person is needed to be present at the pick-up location to mount the sensor on the drone.
3. The sensor deployment task, on the other hand, does not require coordination with the ground personnel because the sensors can be placed automatically onto the ground at the destination locations without any human intervention. The exact placement of the sensors into the ground is then later carried out by the crew.

The problem of retrieval and deployment of geophones are treated as separate optimization problems in this thesis. These scenarios exist in a real situation for small regions e.g., a region with difficult terrain, where the geophones are either picked-up or delivered (but not both of

them simultaneously) with the truck in the vicinity acting as a depot. The first characteristic, in combination with the consideration of the fact that the drones would be needed to fly huge distances with their limited battery if simultaneous pick-up and delivery is allowed, favors the idea of considering these problems separately. It should be noted that the paths connecting customer locations to pick-up locations are not considered while considering these problems separately.

The second characteristic of the operation requiring coordination with the ground personnel for pick-up operation results in association of a time window at each pick-up location during which the ground personnel will be available to mount the sensor on the drone. This is not the case for the deployment operation. The association of this time-window draws distinction between the formulation of the pick-up and delivery problem. This aspect is further discussed in Chapter C.

Another important aspect of this problem is the drone capacity and customer's demand at each location. Instead of measuring the payload capacity of each drone in kg, it is expressed in terms of the number of maximum geophones a drone can carry simultaneously during its trip. Similarly, the demand of a customer is also expressed in the number of geophones needed to be delivered at each location. The customer's demand is always equal to one for this application.

### B-3-3 Problem Definitions

Before looking at the relevant literature, it is necessary to look at the problem definitions to be able to critically analyse the applicability and relevance of different methods developed in the literature. Based on the discussion so far, the problems to be solved can be phrased as follows:

1. To find out the routes of different vehicles in the fleet so as to obtain minimum overall journey time.
2. To find out the routes of different vehicles while minimizing the size of the fleet (number of drones being used).
3. To minimize the congestion at the depot.

Simultaneous presence of multiple vehicles at the depot leads to the requirement of more resources/persons needed to cater to each vehicle at the depot. This is referred to as a problem of congestion at the depot. All of the three problems listed above are directly relevant to the operation of deploying geophones whereas solution to the first problem is more relevant than the other two for collecting nodes. This is because the associated time windows in case of a pick up problem can determine to a great extent the challenge involved in finding out the optimal routes.

In addition to the above listed objective functions, the other important aspects involved in the route planning for a fleet of drones being used in collection and delivery operations in seismic surveys are as follows:

1. Inclusion of multiple trips of vehicles.

2. Inclusion of limited trip duration.

Both of the above listed constraints arise from the fact that the drones have limited payload capacity and battery power. The relevance of the literature is “evaluated” based on these two lists of components in Chapter C. It is to be noted that the lists presented here are not sufficient for a complete mathematical formulation of the problem and only contain elements that are required for the purpose of literature analysis.

The next appendix presents the methods developed in the relevant literature and a clear standpoint for the author to present the developments and results contributing to the field of MTRP.





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# Appendix C

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## Literature Analysis

Researchers have been working on the problem of optimizing the task of pick-up and delivery of goods or services using vehicles on the road network for over five decades. However the problem of pick-up and delivery using drones is relatively new. New regulations governing the flight of drones are required for the feasibility of pick-up and delivery in an urban environment but not in the secluded desert areas. These are the places where the use of drones is envisaged to accomplish the task of pick-up and delivery of geophones. There does not exist any literature to the best knowledge of the author which directly deals with all problems associated with a pick-up and delivery operation carried out using a fleet of autonomously flying drones.

The research done in the area of problems related to the pick-up and delivery has always been motivated by its practical application. This appendix presents the literature from different fields of application that are closely associated to the one at hand and expands, wherever necessary, on the methods developed in them depending upon the relevance and applicability to the problem described in Section B-3. The extent to which these methods are relevant is determined and their use is critically analyzed in each section. Depending upon the closely related field of applications, this appendix is further divided into five sections - third section on MTRVP being the most relevant one. Section C-1 encompasses some literature corresponding to the vehicle routing problem and its variants along-with their limitation for applicability to the problem at hand. Section C-2 lists the literature on MTRVP in detail and analyzes major concepts from the leading sources. Section C-3 describes the problem and relevant literature concerning coordination and control of multiple UAVs to perform multiple tasks on multiple targets. Section C-4 presents the literature concerning the problem of job scheduling e.g., on multiple processors. Job (analogous to routes in this case) scheduling is needed to tackle the problem of congestion at the depot.

### C-1 Vehicle Routing Problem (VRP) and its Variants

Since 1959 there have been many interesting contributions from researchers on VRP as a means to determine a set of vehicle routes to perform transportation requests with the given

fleet at a minimum cost. Each route is associated with a corresponding cost and the total cost for all routes is generally minimized. This cost is specified using various parameters e.g., total distance traveled, total traveling time of all vehicles, total fuel consumption or traffic intensity depending upon the field of application of the method.

There are many variants of VRP that exist in the literature and Capacitated Vehicle Routing Problem (CVRP) is the most studied version of the VRP. Detailed literature survey on different variants of VRP (not including MTRVP) is presented by Toth and Vigo (2014). The subsequent text in this section introduces the exact definition and formulation of the CVRP problem followed by a comparison of the problems in literature with the one at hand.

### C-1-1 Problem Statement

The CVRP consists of transportation requests to deliver goods from a *single* depot, denoted as point 0, to a given set of  $n$  customers with known locations, denoted as a set  $N = \{1, 2, \dots, n\}$ . The amount that has to be delivered to customer  $i \in N$  is usually called customer's demand and is given by  $q_i$ . A fleet of  $m$  identical vehicles is based at the depot and is denoted by a set  $K = \{1, 2, \dots, m\}$ . Each vehicle in the fleet has a limited capacity  $Q$ . The variables  $Q$  and  $q_i$  are assumed to be non-negative integers.

The given problem can be denoted in the form of a graph. Let  $G = (V, E)$  be a graph where  $V = \{0, 1, 2, \dots, n\}$  is the set of nodes and  $E \subseteq V \times V$  is the set of edges. Edges  $(i, j) \in E$  are characterized by the cost  $c_{ij}$  and travel time  $t_{ij}$ . Set  $V$  represents the depot and customers' locations and set  $E$  represents the connections among them.

The solution to the VRP problem then translates to finding out  $m$  number of routes, one for each vehicle, such that the following conditions are satisfied:

1. each customer is visited exactly once, and
2. the sum of the demands of customers visited in any route does not exceed the vehicle capacity,  $Q$ .

### C-1-2 Problem Formulation

There are different mathematical formulations that exist in the literature depending upon the variant of the problem in consideration and its practical application. Brief overview of different formulations is given below:

1. 2-index vehicle flow formulation for Capacitated Vehicle Routing Problem (CVRP) using an integer decision variable for each edge is introduced by Laporte, Mercure, and Nobert (1986).
2. 3-index vehicle flow formulation using a *binary* decision variable for each edge and each vehicle is proposed by Golden, Magnanti, and Nguyen (1977).
3. Balinski and Quandt (1964) introduced extensive formulation for the CVRP which is based on an extended set partitioning or set covering model.

As opposed to the vehicle flow formulations, the extensive flow formulation directly deals with the feasible routes and introduces 1-index binary decision variables deciding whether or not a particular route is included in the solution. This formulation involves enumeration of all feasible routes beforehand. The main difference between the vehicle flow and extensive formulations is how increasing the size of the problem (in terms of number of customers) affects the problem. While there is a combinatorial increase in the number of constraints for vehicle flow formulations, the enumerated routes (and thus decision variables) increase in a combinatorial manner for extensive formulation with an increase in the problem size.

Let  $R$  be the set of all feasible routes. A route (also referred to as trip) is called feasible if the demand of customers being served on that route does not exceed vehicle capacity,  $Q$ . A solution is a set of  $m$  routes such that the total cost of all routes is minimum. Each route,  $r \in R$  is denoted by a sequence of nodes  $(v_0, v_1, \dots, v_{n_r+1})$  where  $v_0 = v_{n_r+1} = 0$  and  $v_1, \dots, v_{n_r} \in V \setminus \{0\}$ . The corresponding cost of the route  $r$  can be defined as  $c_r = \sum_{i=0}^{n_r} c_{v_i, v_{i+1}}$ . The extensive formulation is then given below:

$$\text{minimize} \quad \sum_{r \in R} c_r \lambda_r \quad (\text{C-1})$$

$$\text{s.t.} \quad \sum_{r \in R} a_{ir} \lambda_r = 1 \quad \forall i \in N \quad (\text{C-2})$$

$$\sum_{r \in R} \lambda_r = m \quad (\text{C-3})$$

$$\lambda_r \in \{0, 1\} \quad \forall r \in R \quad (\text{C-4})$$

where  $\lambda_r$  is a binary variable indicating whether the route  $r$  is selected ( $\lambda_r = 1$ ) or not ( $\lambda_r = 0$ ) and  $a_{ir} \in \{0, 1\}$  is the coefficient indicating whether the route  $r$  visits the customer  $i$  ( $a_{ir} = 1$ ) or not ( $a_{ir} = 0$ ). Equation C-1 minimizes the total cost of the selected routes, Equation C-2 enforces the set partitioning constraints i.e., each customer is visited exactly once during the operation and Equation C-3 enforces the complete utilization of the fleet.

As stated by Toth and Vigo (2014), the set partitioning constraints can be converted to set covering constraints ( $\sum_{r \in R} a_{ir} \lambda_r \geq 1$ ) if the routing costs fulfill the triangle inequality i.e.,  $c_{ij} + c_{jk} \geq c_{ik}$ . The first major advantage of extensive formulation over compact (vehicle-flow formulations) is that better lower bounds can be obtained by solving its linear relaxation. Secondly, it allows the separation of costs and constraints governing the feasibility of a route from global constraints of the operation (e.g., Equations C-2 and C-3). Therefore, different intra-route constraints and complex cost functions can be incorporated easily into the problem.

There are many variants of VRP that exist in the literature depending upon their area of application e.g., different network characteristics (Toth & Vigo, 2014) and (Corbern & Laporte, 2013), transportation requests (Toth & Vigo, 2014), scheduling aspects (Solomon & Desrosiers, 1988), route constraints (Cattaruzza et al., 2016), fleet characteristics (Jiang, Ng, Poh, & Teo, 2014) and optimization objectives (Jozefowicz, Semet, & Talbi, 2008). The overview of VRP, its variants, mathematical models and different exact and heuristic algorithms (not including MTRP) proposed in the literature are presented by Toth and Vigo (2014).

The methods developed to solve classic VRP problem are not directly applicable to the problem that arises from the use of drones in seismic surveys as they do not allow vehicles to make multiple trips. But they are helpful to introduce and understand the most relevant variant of VRP which allows for vehicles in the fleet to perform multiple trips i.e., MTVRP. The aim of reviewing this literature was also to understand the solutions obtained from the methods solving VRP as they sometimes act as a first phase of a solution methodology developed to solve MTVRP (Taillard, Laporte, & Gendreau, 1996). MTVRP is the topic of discussion in the next section.

## C-2 Multi-Trip Vehicle Routing Problem (MTVRP)

As stated earlier, each vehicle is allowed to perform at most one route in most of the studied models of VRP. In such cases, the planning is done assuming the number of vehicles to be unlimited. These assumptions are not realistic for vehicles with small capacity or when the planning time-horizon (e.g., one working day) is large with respect to the route duration. This is indeed the case for the present problem and MTVRP overcomes the mentioned limitations. Cattaruzza et al. (2016) and Şen and Bülbül (2008) present the literature review on MTVRP. Although first attempt to address MTVRP was made in 1990 by Fleischmann (1990), it has been actively pursued only in the last 15 years.

There has been inconsistency throughout the literature in the naming of these problems. It has been called VRP with multiple use of vehicles, multitrip VRP, VRP with multiple depot returns, VRP with multiple routes etc. The aspect of returns to the depot has also been referred to as recycling of trucks or multiple traverses. This inconsistency in naming makes it harder to find the relevant literature. Similarly, there has been variability in the terminology used in the description of these problems. This problem is referred to as Multi-Trip Vehicle Routing Problem (MTVRP) in this text in concurrence with Cattaruzza et al. (2016). Following terminology is used in this review (Cattaruzza et al., 2016):

**Trip/route** - a sequence of visits to different customers preceded and followed by a visit to a depot. Note that it does not include any intermediate stop at the depot.

**Journey/tour** - A sequence of trips/routes performed by the same vehicle.

### C-2-1 Problem Statement

The MTVRP consists of transportation requests to deliver goods from a *single* depot, denoted as point 0, to a given set of  $n$  customers with known locations, denoted as a set  $N = \{1, 2, \dots, n\}$  (Olivera & Viera, 2007a). The amount that has to be delivered to customer  $i \in N$  is usually called customer's demand and is given by  $q_i$ . A fleet of  $m$  identical vehicles is based at the depot and is denoted by a set  $K = \{1, 2, \dots, m\}$ . Each vehicle in the fleet has a limited capacity  $Q$ . Time horizon, denoted by  $T_H$ , establishes the duration of a working day. Let  $G = (V, E)$  be a graph where  $V = \{0, 1, 2, \dots, n\}$  is the set of nodes and  $E \subseteq V \times V$  is the set of edges. Edges  $(i, j) \in E$  are characterized by the cost  $c_{ij}$  and travel time  $t_{ij}$ .

The MTVRP calls for the determination of a set of trips and an assignment of each trip to one vehicle to form its journey such that the total cost incurred is minimized and the following conditions are met:

1. each trip starts and ends at the depot,
2. each customer is visited by exactly one trip,
3. the sum of the demands of the customers in any trip does not exceed the vehicle capacity,  $Q$  and
4. the total duration of trips assigned to one vehicle does not exceed  $T_H$ .

The duration of a trip/route is defined by the sum of travel times on edges used in the trip. Loading times (at the depot) for the trips are not introduced in MTVRP but service times (at the customer's location) are sometimes included (Taillard et al., 1996). Note that each customer's demand in present scenario of seismic surveys is one since each location only requires 1 geophone.

### C-2-2 Problem Formulation

There exist many different formulations in literature for MTVRP and its extensions. They can be mainly categorized, as in the case of the VRP, into vehicle flow (Cattaruzza et al., 2016) and extensive (Olivera & Viera, 2007a), (Mingozi, Roberti, & Toth, 2012) formulations. These formulations are enumerated below:

1. 4-index vehicle flow formulation - This can be seen as an extended 3-index vehicle flow formulation for VRP (introduced in Subsection C-1). It is presented in more detail by Cattaruzza et al. (2016).
2. 3-index formulation with vehicle index (without trip index) Cattaruzza et al. (2016).
3. 3-index formulation with trip index (without vehicle index) Cattaruzza et al. (2016).
4. 2-index formulation (neither vehicle index nor trip index) Cattaruzza et al. (2016).
5. Extended formulation - This formulation can be seen as an extension of the extensive formulation of VRP with two indexes instead of one - the first one corresponding to each possible route and the other one corresponding to each vehicle in the fleet (Olivera & Viera, 2007a), (Mingozi et al., 2012).

As stated before, the extensive formulations involve enumeration of all possible routes which may or may not be beneficial depending upon the characteristics of a problem e.g. vehicle capacity, complexity of intra-route constraints etc. The methods developed to tackle MTVRP are benchmarked against the instances introduced by Taillard et al. (1996) in the literature.

The MTVRP is an NP-Hard problem. It is shortly proved by Olivera and Viera (2007a) as any VRP instance can be transformed to an equivalent MTVRP instance and VRP is an NP-Hard problem as shown by Lenstra and Kan (1981). It is also proved by Cattaruzza et al. (2016) by reduction to the VRP.

### C-2-3 Exact Methods

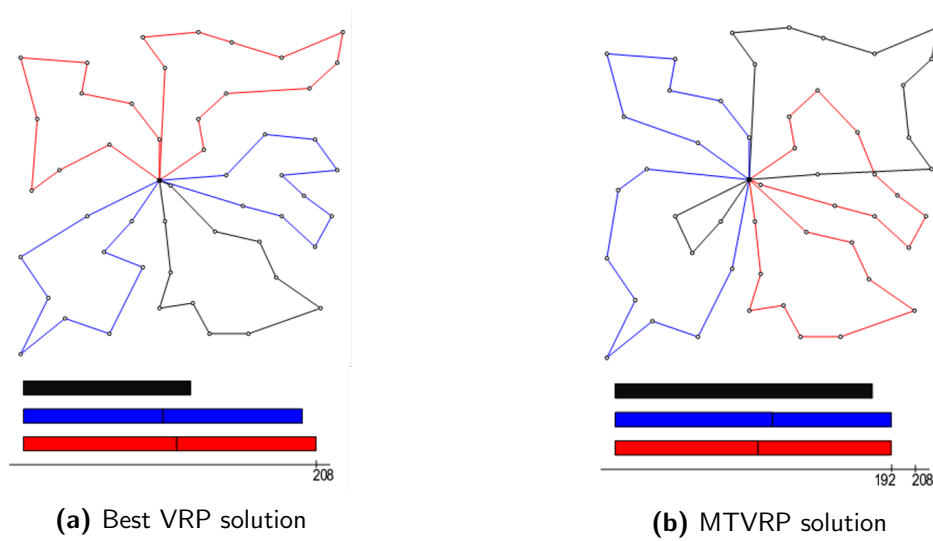
Koc and Karaoglan (2011) propose a branch-and-cut exact algorithm and it implements several valid inequalities taken from the literature and a heuristic algorithm based on simulated annealing to obtain upper bounds. Mingozzi et al. (2012) propose an exact method to solve MTVRP based on two set-partitioning-like formulations. The first and second formulations require generation of all feasible trips and journeys respectively. Four bounding procedures and different valid inequalities are used in the solution method.

### C-2-4 Heuristics

Taillard et al. (1996) proposed a two-phase heuristic approach for the MTVRP. In the first phase, several VRP solutions are obtained with an unspecified number of vehicles using a Tabu Search (TS) algorithm and these trips are inserted into a list. In the second phase, solutions to the MTVRP are constructed using a Bin Packing Problem (BPP) heuristic method (Fukunaga & Korf, 2005). Petch and Salhi (2003) propose a three-phase heuristic with an objective to minimize the maximal overtime. Savings-based heuristic is used to generate routes which are then combined to form complete solutions. The obtained MTVRP solutions are finally improved with an intensive local search heuristic. In the successive work of Salhi and Petch (2007), a hybrid genetic algorithm is proposed. Sequence of strictly increasing angles measured with respect to the depot are used as a non-binary chromosome representation. The chromosome is decoded resulting in a VRP solution which is then transformed into a MTVRP solution using a BPP heuristic. The solution is further improved using a local search procedure. Cattaruzza, Absi, Feillet, and Vidal (2013) propose another hybrid genetic algorithm for finding a solution to the MTVRP. It uses a local search method based on the combination of moving one customer to another trip and swaps between trips. Brandão and Mercer (1997) designed a TS algorithm which included MTVRP features among others. It was later simplified to handle the MTVRP (Brandao & Mercer, 1998). The moves in the algorithm are defined by moving a customer from one trip to another and by swapping two customers. Olivera et al. (Olivera & Viera, 2007a) proposed a TS algorithm with adaptive memory. The memory is composed of trips, a set of which is selected to form a VRP solution. It is then transformed into a MTVRP solution using a BPP heuristic.

The two phase approaches, in which routes are generated by solving VRP and assignment of routes to the vehicles is done using a BPP heuristics, can lead to infeasible solutions. The infeasibility can be caused by violation of constraints established by the time-horizon. One instance of such a case is shown in Figure C-1 by Olivera and Viera (2007a). Any packing of routes obtained from the best known VRP solution shown in Figure C-1a cannot honor the constraint of visiting all customers exactly once within the time-horizon of 200. However, re-computation of routes can lead to a feasible MTVRP solution shown in Figure C-1b. Note that the cost of re-computed routes for MTVRP is now higher than the total cost of routes for the best known VRP solution. This example illustrates the interdependency of formation of routes and their assignment to different vehicles in a fleet.

Heuristic methods generally do not guarantee global optimality. The main advantage of using heuristics is that they tend to give sub-optimal solutions quicker as compared to the exact solution methods. This advantage makes them a lucrative option in a real-world application. There has not been any algorithm in the literature that can obtain optimal solutions for all



**Figure C-1:** Solutions for VRP and VRPMT formulations over CMT-1 problem (Olivera & Viera, 2007a)

MTVRP benchmark instances in a reasonable time. The solution methods listed till now in this section generally minimize the sum of total journey costs. They do not embody the objective functions (described in Section B-3) and the constraint of limited trip duration desired for the problem at hand.

### C-2-5 Other Extensions of MTVRP

Though not in the case of pure MTVRP, limited time duration of each trip is considered in an extension of MTVRP dealing with *time windows*. Time windows (TW) indicate that each customer  $i$  is associated with a time interval  $[a_i, b_i]$  during which service should take place (Cattaruzza et al., 2016). The association of time windows results in introduction of various aspects into the problem e.g, each trip in the solution is now time stamped. Anaya-arenas, Maria, Chabot, Renaud, and Ruiz (2014) take into consideration the trip duration limits and propose two constructive heuristics followed by a local search. While the first heuristics creates trips which are then packed together to form journeys, the second heuristics directly create journeys. The trip length, in this case, is the elapsed time between the arrival at the first customer of the trip and the arrival at the depot during the same trip. Wang, Liang, and Hu (2013) propose another heuristic, taking into account limited trip duration, which is based on an adaptive memory procedure. It should be noted that the aspect of time windows is more relevant to the pick-up problem as stated in Appendix B and this thesis is focused on the deployment operation.

The objective of minimizing the fleet size is introduced by Juan Carlos Rivera, H. Murat Afsar (1997), Prins (2002), and Battarra, Monaci, and Vigo (2009). Juan Carlos Rivera, H. Murat Afsar (1997) study multitrip cumulative capacitated vehicle routing problem (mt-CCVRP) to minimize the sum of arrival times at required locations. The application of this problem can be found in disaster logistics as the arrival time of the relief supplies at each location is crucial.

Based on the discussion so far, it should be noted that the MTRVP allows vehicles to make multiple trips but it has to be developed upon in order to apply to the problem considered in this thesis (Sections C-5).

### C-3 Cooperative Multiple Task Assignment Problem (CMTAP)

All problems encountered in the literature till now are motivated from the use of vehicles on the road. The author has not encountered any literature that specifically deals with the pick-up and delivery task being carried out by a fleet of drones. However, there has been an area of application in which Unmanned Aerial Vehicles (UAVs) are used to perform multiple tasks on multiple targets for various military missions. A fleet of UAVs is required to perform 3 tasks of *classify, attack, and verify* on a specified targets on the ground. These tasks are needed to be accomplished in order i.e., classification is followed by attack and attack is followed by verification (Shima, Rasmussen, Sparks, & Passino, 2006). Coordination, timing constraints and flyable trajectories in addition to the task precedence constraints are needed to be taken into account to find a solution to this problem, referred to as Cooperative Multiple Task Assignment Problem (CMTAP) in the literature.

This problem can be reduced to VRP by simplifying some characteristics e.g., by relaxing task precedence constraints as only one task is needed to be performed at each target (analogous to a customer in the VRP). Hence, CMTAP is also an NP-Hard problem. Although this problem and any methods developed subsequently for its solution do not bear direct relevance to the problem at hand, it is still useful to look into its literature. This is because the trajectories and assignments in CMTAP are designed with an objective to minimize the overall mission completion time. This objective is same as the one identified for the problem at hand in Section B-3. Richards, Bellingham, Tillerson, and How (2002) formulate this problem in the form of Mixed Integer Linear Programming (MILP) and thus providing an insight into how aforementioned objective function can be framed.

### C-4 Job Scheduling Problem

For the present application, drones are required to make frequent visits to a depot due to their limited payload capacity and battery power. Drones can get their batteries changed and collect new payload at the depot. Frequent visits to the depot by drones in a fleet can create the problem of congestion at the depot. The problem of controlling the congestion at the depot is listed as one of the points in the future work by Olivera and Viera (2007b). But there has not been any literature published dealing with this problem to the best knowledge of the author. Therefore, it is deemed appropriate to review the literature in the next relevant field of application - job scheduling (Peloquin, 2010), (Leung, 2004), (Karger, Wien, & Stein, 2007), (Chapin, 1996). The scheduling problem on multiprocessors can be stated as a way of executing a set of tasks (analogous to routes in MTRVP) on a set of processors (analogous to vehicles in MTRVP) subject to some set of optimizing criteria. The examples of scheduling criteria include minimizing the expected runtime of a task set, minimizing communication delay, giving priority to certain users' processes etc. No algorithm has been found which min-



minimizes the number of tasks finishing at the same time. However, load balancing problem gives an insight into how the objective of minimizing maximum journey time can be constructed.

## C-5 Results and Analysis

From previous sections it can be deduced that there are several areas of application from which different characteristics can be integrated to form the list of all required aspects (delineated in Section B-3) defining the problem at hand. Ample literature exists to find the solution of VRP, MTVRP and their extensions but none that specifically deals with the combination of the desired characteristics. The solution to the VRP, MTVRP etc. translates to finding out routes of different vehicles in the fleet to perform transportation requests at a minimum cost. The methods developed for VRP does not allow vehicles to make multiple trips and therefore, not directly relevant to the problem of interest. They however provide a basis to understand different formulations and some solution methods developed for MTVRP. The MTVRP embodies most of the constraints encountered in using a fleet of drones in seismic surveys such as inclusion of multiple trips, and visiting each customer only once. The constraint of limited trip duration and the objective of minimizing the fleet size have been introduced separately (in the surveyed literature) addressing different extensions of MTVRP but not together in one method. The objective of minimizing the overall journey time has not been a part of the investigation in the MTVRP literature but is a topic of interest for the scientific community working on the CMTAP or load balancing. The applicability of the CMTAP (or load balancing) to the problem of interest is limited to the extent that it provides insight into the formulation of objective function of minimizing the overall journey time, referred to as overall mission completion time in the CMTAP literature. No relevant text in the literature has been found dealing with the problem of controlling the congestion at the depot. The research done in this MSc. thesis allows the route planning for multiple UAVs in the fleet to complete the task in minimum time and/or using minimum size of the fleet while addressing the problem of congestion at the depot and thus, minimizing the resources to be used at the depot.

## C-6 Conclusion

In this thesis topic, methods of route planning for a fleet of UAVs are to be developed to complete the task in minimum time and/or using minimum size of the fleet. The problem of controlling the congestion at the depot is also dealt with as higher fleet size and frequent returns to the depot can result in large resource demand at the depot. This literature survey presented texts from different fields of application relevant to the problem of interest. The numerous solution methods that are used to find solutions to different routing problems are also briefly presented. It also sheds light to the aspects in each application area that can be combined to formulate the problem of interest. The existing solution methods for MTVRP can then be adapted to find its solution. The problem of congestion at the depot has not been dealt with in the literature to the best knowledge of the author. The developed solution methodology will tackle with these two issues and can be used to carry out the operations of delivering geophones (seismic sensors) in seismic surveys. These methods can also be useful in cooperative mission planning for autonomously flying multiple UAVs in future.



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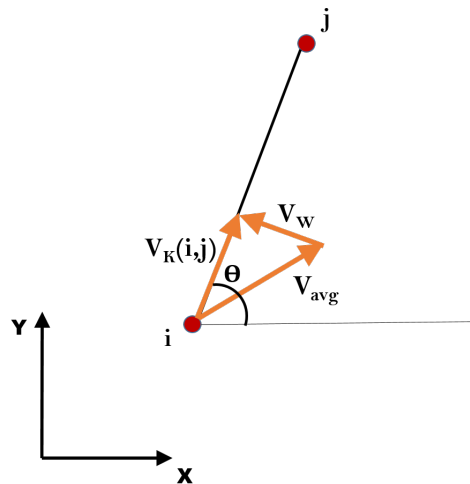
## Appendix D

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# Ground Velocity Calculations

This appendix is aimed at providing insight into the calculation of ground velocity  $V_K(i, j)$  with which a drone travels from vertex  $i$  to  $j$ .  $V_K(i, j)$  is used to compute the time of travel in Phase-I of the planning algorithm. It is calculated using 2-D velocity triangle equations. They are written in the matrix form in Equation D-1 and shown in Figure D-1.

$$|V_K(i, j)| \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix} = \begin{bmatrix} V_{Wx} \\ V_{Wy} \end{bmatrix} + \begin{bmatrix} \cos(\alpha) \\ \sin(\alpha) \end{bmatrix} V_{avg} \quad (\text{D-1})$$



**Figure D-1:** Representation of velocity triangle

where  $|V_K(i, j)|$  represents the magnitude of ground velocity, and  $\theta$  and  $\alpha$  are the angles made by  $V_K(i, j)$  and  $V_{avg}$  respectively with the  $x$ -axis.

The following variables are known in these equations: wind velocity components,  $V_{W_x}$  and  $V_{W_y}$ , required direction of the ground speed,  $\theta$  (calculated from known positions of customers  $i$  and  $j$ ) and magnitude of average airspeed,  $V_{avg}$  with which a drone travels while following a straight line trajectory.

Equation D-1 is reduced to Equation D-2 which is then solved for  $|V_K(i, j)|$  using symbolic representation in MATLAB.

$$(|V_K(i, j)| \cos(\theta) - V_{W_x})^2 + (|V_K(i, j)| \sin(\theta) - V_{W_y})^2 = V_{avg}^2 \quad (\text{D-2})$$

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# Appendix E

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## Implementation Details of Algorithms

This appendix is aimed at providing brief details of the way in which algorithms are implemented.

### E-1 Minimization of Maximum Journey Time

The problem formulated in Phase-I in Chapter 1 is converted into MILP and given as follows:

$$\min \bar{t} \tag{E-1}$$

$$s.t. \quad \sum_{k \in K} \sum_{r \in R} a_{ir} x_r^k = 1 \quad \forall i \in V \setminus \{0\} \tag{E-2}$$

$$\sum_{r \in R} t_r x_r^k \leq T_H \quad \forall k \in K \tag{E-3}$$

$$\sum_{r \in R} t_r x_r^k \leq \bar{t} \quad \forall k \in K \tag{E-4}$$

$$x_r^k \in \{0, 1\} \quad \forall r \in R, k \in K \tag{E-5}$$

This formulation is then converted into the following standard form for the purpose of solving it using CPLEX in MATLAB:

$$\min_x f^T x \text{ subject to } \begin{cases} A x \leq b \\ A_{eq} x = b_{eq} \\ lb \leq x \leq ub \\ \text{some variables in } x \text{ are integers} \end{cases} \tag{E-6}$$

where

$$x = [x_1^1, x_2^1, \dots, x_{|R|}^1, x_1^2, x_2^2, \dots, x_{|R|}^2, \dots, x_1^k, x_2^k, \dots, x_{|R|}^k, \dots, x_{|R|}^m, \bar{t}]$$

Matrices  $A$ ,  $A_{eq}$  and vectors  $f, b, b_{eq}, lb, ub$  are then suitably constructed.

## E-2 Minimization of Congestion at the Depot

The MILP formulation given in Phase-II (scheduling by shuffling) in Chapter 1 is converted into the standard form given in Equation E-6 for the purpose of solving it using CPLEX in MATLAB. The vector,  $x$  contains decision variables and is defined as follows:

$$x = \left[ x_{rj}^k, y_{jt}^k, c_j^k, \bar{n} \right] \quad \forall t \in \left\{ 1, 2, \dots, \left\lceil \frac{T}{\Delta t} \right\rceil \right\}, j \in \{1, 2, \dots, n^k\}, r \in \{1, 2, \dots, n^k\}, k \in K$$

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# Appendix F

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## Fleet-size Minimization

This appendix is aimed at delineating the formulation for minimizing the size of the fleet and thus planning the routes over full time-horizon. Section F-2 defines the formulation aimed at simultaneous minimization of maximum journey time and fleet-size.

### F-1 Minimization of Fleet-size

This section presents the formulation of the MTRVP aimed at achieving minimum fleet-size (number of drones in the fleet) required for the delivery of sensors to the specified customers' locations in the given time-horizon  $T_H$ .

This formulation requires the upper bound on the number of vehicles that can be used in the operation and can be easily taken to be equal to the number of customers i.e., fleet of  $n$  identical vehicles is now assumed to be based at the depot and is denoted by a set  $K = \{1, 2, \dots, n\}$ .

The problem can then be formulated in the form of MILP as given below:

$$\min \sum_{k=1}^n y^k \quad (\text{F-1})$$

$$s.t. \quad \sum_{k \in K} \sum_{r \in R} a_{ir} x_r^k = 1 \quad \forall i \in V \setminus \{0\} \quad (\text{F-2})$$

$$\sum_{r \in R} t_r x_r^k \leq T_H y^k \quad \forall k \in K \quad (\text{F-3})$$

$$x_r^k \in \{0, 1\}, \quad y^k \in \{0, 1\} \quad \forall r \in R, k \in K \quad (\text{F-4})$$

where  $y^k$  is a binary variable indicating whether any route is assigned to vehicle  $k$  ( $y^k = 1$ ) or not ( $y^k = 0$ ). This formulation is converted into the standard form given in Equation E-6 with the following decision variables:

$$x = [x_1^1, x_2^1, \dots, x_{|R|}^1, x_1^2, x_2^2, \dots, x_{|R|}^2, \dots, x_1^k, x_2^k, \dots, x_{|R|}^k, \dots, x_{|R|}^n, y^1, y^2, \dots, y^n]$$

## F-2 Simultaneous Minimization of Maximum Journey Time & Fleet-size

Both objectives of minimizing the overall journey time and fleet-size can be combined together and the resulting MILP formulation is given below:

$$\min \quad w_{\bar{t}} \bar{t} + \sum_{k=1}^n w_k y^k \quad (\text{F-5})$$

$$s.t. \quad \sum_{k \in K} \sum_{r \in R} a_{ir} x_r^k = 1 \quad \forall i \in V \setminus \{0\} \quad (\text{F-6})$$

$$\sum_{r \in R} t_r x_r^k \leq T_H y^k \quad \forall k \in K \quad (\text{F-7})$$

$$\sum_{r \in R} t_r x_r^k \leq \bar{t} \quad \forall k \in K \quad (\text{F-8})$$

$$x_r^k \in \{0, 1\}, \quad y^k \in \{0, 1\} \quad \forall r \in R, k \in K \quad (\text{F-9})$$

where  $w_{\bar{t}}$  and  $w_k$  are the weights corresponding to two different objective functions. The above MILP form is then translated into the standard form with the following  $x$  vector containing decision variables:

$$x = \left[ x_1^1, x_2^1, \dots, x_{|R|}^1, x_1^2, x_2^2, \dots, x_{|R|}^2, \dots, x_1^k, x_2^k, \dots, x_{|R|}^k, \dots, x_{|R|}^n, \bar{t}, y^1, y^2, \dots, y^n \right]$$

Note that both are conflicting objectives i.e., minimization of the fleet-size tends to allow for solutions having more overall journey time and vice-versa.



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## Appendix G

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# Minimization of Congestion at the Depot - using both Shuffling and Slacking

The MILP formulation for minimizing the congestion at the depot using both shuffling and slacking together is delineated in this appendix. The results from this formulation are included in Subsection C of Section V in Chapter 1.

$$\min \bar{n} \tag{G-1}$$

$$s.t. \sum_{r=1}^{n^k} x_{rj}^k = 1 \quad \forall k \in K, j \in \{1, 2, \dots, n^k\} \tag{G-2}$$

$$\sum_{j=1}^{n^k} x_{rj}^k = 1 \quad \forall k \in K, r \in \{1, 2, \dots, n^k\} \tag{G-3}$$

## 54 Minimization of Congestion at the Depot - using both Shuffling and Slacking

$$\begin{aligned}
 \begin{bmatrix} c_1^k \\ c_2^k \\ \vdots \\ c_{n^k}^k \end{bmatrix} &= \begin{bmatrix} t_1^k & 0 & \dots & 0 & t_2^k & 0 & \dots & 0 & \dots & t_{n^k}^k & 0 & \dots & 0 \\ t_1^k & t_1^k & \dots & 0 & t_2^k & t_2^k & \dots & 0 & \dots & t_{n^k}^k & t_{n^k}^k & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & & \vdots & \vdots & \ddots & \vdots \\ t_1^k & t_1^k & \dots & t_1^k & t_2^k & t_2^k & \dots & t_2^k & \dots & t_{n^k}^k & t_{n^k}^k & \dots & t_{n^k}^k \end{bmatrix} \begin{bmatrix} x_{11}^k \\ x_{12}^k \\ \vdots \\ x_{1n^k}^k \\ x_{21}^k \\ x_{22}^k \\ \vdots \\ x_{2n^k}^k \\ \vdots \\ x_{n^k 1}^k \\ x_{n^k 2}^k \\ \vdots \\ x_{n^k n^k}^k \end{bmatrix} \\
 &+ \begin{bmatrix} 1 & 0 & \dots & 0 \\ 1 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \dots & 1 \end{bmatrix} \begin{bmatrix} s_1^k \\ s_2^k \\ \vdots \\ s_{n^k}^k \end{bmatrix} \quad \forall k \in K \quad (\text{G-4})
 \end{aligned}$$

$$\begin{aligned}
 (p-1) \cdot \Delta t - M \cdot (1 - y_{jp}^k) &\leq c_j^k < p \cdot \Delta t + M \cdot (1 - y_{jp}^k) + t_d \\
 \forall p \in \{1, 2, \dots, \left\lceil \frac{T}{\Delta t} \right\rceil\}, j \in \{1, 2, \dots, n^k\}, k \in K & \quad (\text{G-5})
 \end{aligned}$$

$$\sum_{t=1}^{\left\lceil \frac{T}{\Delta t} \right\rceil} y_{jt}^k = 1 + \left\lfloor \frac{t_d}{\Delta t} \right\rfloor \quad \forall j \in \{1, 2, \dots, n^k\}, k \in K \quad (\text{G-6})$$

$$\sum_{k=1}^m \sum_{j=1}^{n^k-1} y_{jt}^k \leq \bar{n} \quad \forall t \in \{1, 2, \dots, \left\lceil \frac{T}{\Delta t} \right\rceil\} \quad (\text{G-7})$$

$$x_{rj}^k \in \{0, 1\}, \quad y_{jt}^k \in \{0, 1\} \quad \forall t \in \{1, 2, \dots, \left\lceil \frac{T}{\Delta t} \right\rceil\}, j \in \{1, 2, \dots, n^k\}, r \in \{1, 2, \dots, n^k\}, k \in K \quad (\text{G-8})$$

# Heuristics for Slacking

A heuristic algorithm for minimizing the congestion at the depot using slacking is proposed in this appendix. The details of the algorithm are as follows:

**Data:**  $R^k \forall k \in K$  (i.e., a solution either from Phase-1 or after minimization of congestion using shuffling), and  $c_j^k$

**Result:**  $s_j^k \forall j \in \{1, 2, \dots, n^k\}$  and  $k \in K$  (i.e., slack time for all routes)

initialize **currentRoutes** as empty, **maxPersons** as maximum no. of persons available at the depot for service;

**while**  $R^k \forall k \in K$  is not empty **do**

    insert one ‘earliest’ available route from each vehicle from  $R^k$  in **currentRoutes** ;

    Create time intervals of the form  $[a_i, b_i]$  corresponding to each route in **currentRoutes** corresponding to the amount of time spent by each vehicle at the depot;

    Find the interval with maximum number of intersections;

**if** *maximum number of intersections is less than maxPersons* **then**

        remove routes from  $R^k$  which it has in common with **currentRoutes** ;

        reset **currentRoutes** as empty;

**else**

        Sort the intervals in increasing order of their end points;

        Find the interval  $q$  (initialized as 0) in that sorted order, starting from first, such that all intervals before it has as many maximum number of intersections as **maxPersons**;

$p = \max(\maxPersons + 1, q)$ ;

        update  $s_j^k = b_{p-1} - a_i$  **if**  $b_{p-1} - a_i > 0 \forall i \in \{p, p + 1, \dots, m\}$  corresponding to routes in **currentRoutes** (there is an implicit mapping from index  $j$  and  $k$  to  $i$ );

        remove routes corresponding to  $i \in \{1, 2, \dots, p - 1\}$  from  $R^k$ ;

        reset **currentRoutes** as empty;

**end**

**end**

**Algorithm 1:** Heuristic algorithm for slacking

The performance and accuracy of this heuristic algorithm is yet to be compared against the MILP formulation given in Chapter 1.



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# Appendix I

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## Heuristics for Unit-capacity Drones

This appendix is aimed at providing insight into the reduction of MTVRP to a BPP for unit capacity drones, followed by the results obtained using BPP heuristics. For unit capacity drones in the fleet, the problem of route planning reduces to a bin packing problem. Since, each drone in such a case is bound to make a trip back to the depot after servicing one customer, the number of routes in set  $R$  equals to the number of customers,  $n$  i.e.,

$$|R| = C(n, 1) = n \quad (\text{I-1})$$

where  $C(n, k)$  denotes the number of distinct combinations of  $n$  objects taken  $k$  at a time and is given by  $\frac{n!}{(n-k)!k!}$ . Note that  $k = 1$  in the aforementioned formula and corresponds to the unit capacity of a drone. Each route  $r \in R$  is given as follows:

$$r_i = \{0, i, 0\} \quad \forall i \in V \setminus \{0\} \quad (\text{I-2})$$

The constraint of visiting each customer enforces every route to be a part of the solution and the problem of route planning for different objective functions can be phrased as follows:

1. Plan routes such that the task is completed as soon as possible.
2. Minimize the number of drones needed to accomplish the task such that each drone can work for a limited time (established by the time-horizon).
3. Optimize the location of the depot.

Each subsection below describes method(s) corresponding to each enumerated item in the above list. The above list also contains an objective of optimizing the depot location which can be interesting to solve as depot in this field of application can be a truck (instead of a conventional warehouse) on the ground whose location can be easily changed.

## I-1 Minimizing the Overall Journey Time

This objective function appeals to the use of BPP heuristics catering to the job scheduling problems on multiprocessors. Each processor is analogous to a drone in this scenario and each task corresponds to the task of delivering a seismic sensor to the required location. The goal is now to assign  $n$  routes to  $m$  vehicles in such a way that the demands of all customers is met as soon as possible. Each route  $i$  is associated with a weight  $w_i$  which is calculated as given in Equation I-3.

$$w_i = t_{drop} + t_d + \frac{D(0, i)}{V_K(0, i)} + \frac{D(0, i)}{V_K(i, 0)} \quad \forall i \in V \setminus \{0\} \quad (\text{I-3})$$

where  $t_{drop}$  and  $t_d$  are dropping time and service time respectively as defined in Chapter 1,  $D(i, j)$  is the Euclidean distance between vertices  $i$  and  $j$ , and  $V_K(0, i)$  is ground velocity with which a drone travels from the depot to customer  $i$ . Equation I-3 represents the aggregate sum of the total time it takes for a unit-capacity drone to travel on the route defined in Equation I-2 including dropping and service time.

The solution for such a problem can be given by different heuristics such as Longest Processing Time first (LPT) or Shortest Processing Time first (SPT) (Leung, 2004). In LPT, the tasks are sorted in decreasing order of their weights. This is followed by an assignment of each route (one at a time) to the vehicle that has the least total amount of time assigned to it. In case of a tie, it can either be broken arbitrarily or the route is assigned to the first vehicle that is tied.

Note that the constraint of limited time-horizon may not be satisfied for a solution obtained using these particular heuristics. This is because the presence of a fixed number of vehicles tends to increase the maximum completion time with increase in number of tasks and thus making it harder to satisfy the limited time-horizon constraints.

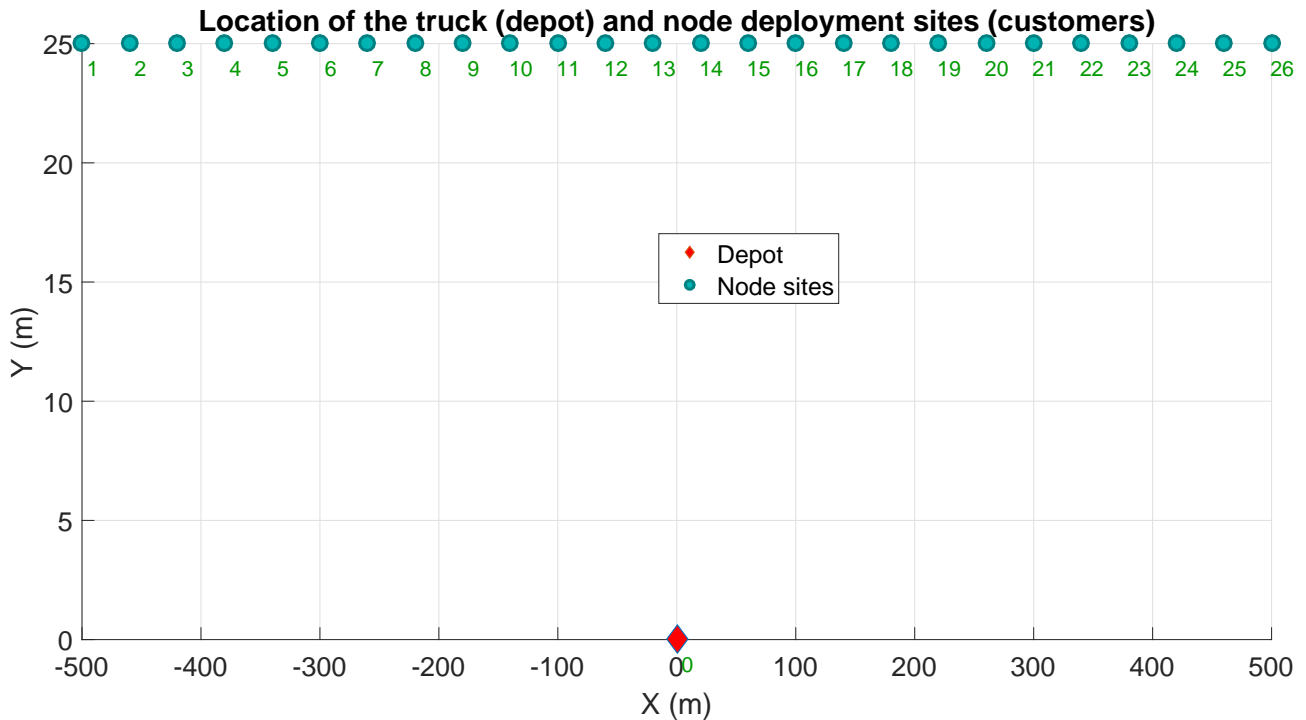
## I-2 Minimizing Fleet-size

This subsection deals with the problem of minimizing the fleet-size for unit capacity drones such that the following conditions are satisfied:

1. Every customer is visited and at most once,
2. Each drone can work for limited number of hours e.g. 8 hours in a working day. This time limit is established by introducing the constraints using time-horizon,  $T_H$ .

Since the capacity of each drone is exactly one, there are  $n$  routes each having the form given in Equation I-2. Note that the constraint of visiting each customer exactly once is inherent in the construction of routes. The weight associated with each route is exactly similar to the previous case and is given by Equation I-3.

The objective function of minimizing the fleet-size in such a case appeals to the use of BPP heuristics. The goal is now to pack  $n$  items, each having a weight  $w_i$ , in bins of capacity  $T_H$  in such a way that the total number of bins used in packing these items is minimized. A bin



**Figure I-1:** Distribution of customers' locations and depot

is analogous to a drone in this scenario and each item corresponds to the task of delivering a seismic sensor to the required location. It should be noted that the constraint ensuring a visit to every customer is taken care by packing all items.

The solution to the above defined problem can be obtained using different heuristics as described in different texts e.g., by Leung (2004). Four 'one-at-a-time' algorithms are described below:

**First Fit (FF)** - Starting with the first bin, each bin is checked one at a time until a bin is found that has the capacity to hold the item i.e., the remaining capacity of that bin is greater than or equal to the weight of the item.

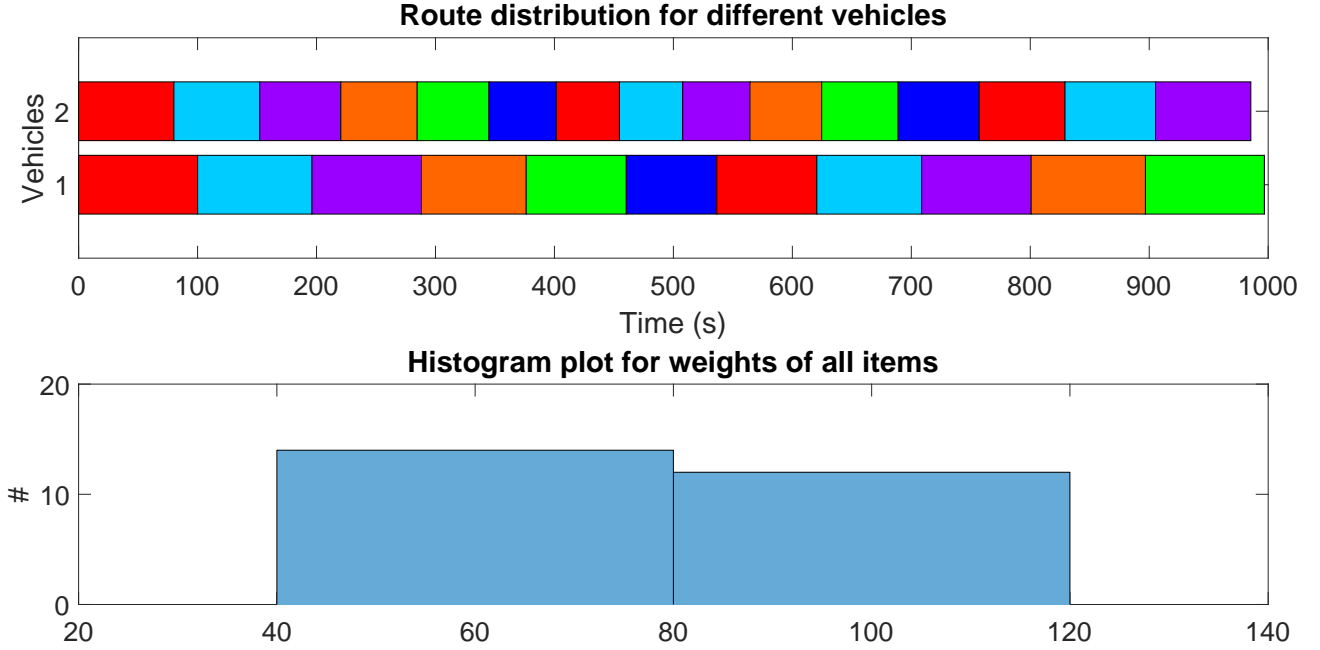
**Best Fit (BF)** - Considering all bins together, the item is packed into a bin that will have a least capacity left after packing it.

**First Fit Decreasing (FFD)** - The list of items is sorted in decreasing order of weights followed by the use of FF.

**BFD** - The list of items is sorted in decreasing order of weights before using BF.

The results obtained from the application of BFD to an instance of this problem are shown in Figure I-2. The distribution of customers' locations and the depot on a 2-D Cartesian plane is shown in Figure I-1.

Table I-1 shows the vertices that are assigned to each vehicle for the deployment of the seismic sensors. The values of different parameters (along-with their description) used in this



**Figure I-2:** Route distribution obtained from BFD for unit-capacity drones

**Table I-1:** Route planning for unit-capacity drones obtained from BFD

Vehicle	Vertices assigned
1	1, 2, 3, 4, 5, 7, 22, 23, 24, 25, 26
2	6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21

simulation is listed in Table I-2. Note that some of these values can be different for the actual operation of deploying sensors and can be set accordingly as an input to the code.

It can be seen that the minimum number of unit-capacity drones that are required to carry out the task in such a scenario is 2. From the route distribution shown at the top of Figure I-2 for these 2 vehicles, it can be concluded that the available time horizon of 1000s is packed tightly. The colors are used to differentiate different routes in each vehicle's journey. In fact, the solution given by the BFD algorithm in this case equals the lower bound  $m_{avg}$  computed using Equation I-4.  $m_{avg}$  represents the minimum number of drones that are needed to carry out this delivery task if routes are allowed to be completed in arbitrary fractions.

$$m_{avg} = \left\lceil \frac{\sum_{i=1}^n w_i}{T_H} \right\rceil \quad (\text{I-4})$$

The distribution of the weights in the list can determine whether the solution obtained using these algorithms will be optimal. From the histogram plot of the weights at the bottom of Figure I-2, it can be seen that there is only slight variation in the weights.



**Table I-2:** Value of different parameters in simulation

Parameter	Value
$V_{avg}$	20 m/s
$t_{drop}$	20 s
$t_d$	30 s
$T_B$	1200 s
$T_H$	1000 s
$V_W$	0 m/s

It should be noted that the solution is independent of the order in which the trips are carried out for each vehicle. In fact, the order of the trips within their respective journeys influences neither the total journey time of each vehicle nor the number of vehicles needed to carry out the operation.

The main advantage of these algorithms is that they are computationally inexpensive. However, the limitation of unit-capacity drones restricts their use in real scenarios and motivates the development of the methods that can take into account higher capacity of drones.

### I-3 Optimizing the Location of the Depot

A good placement of the depot can allow further reduction in the distances that the drones have to fly. The use of ground vehicle as a depot in this field of application allows for different placements and hence, presents an interesting scenario for optimizing the depot location. It should however be noted that the optimal computation of depot location is trivial in this case and is not dealt in this thesis for vehicles with general capacity.

The unit-capacity of drones makes the solution for optimal location of the depot trivial in this case as it enforces every drone to visit the depot after visiting one customer. If a depot is located at  $(x_0, y_0)$ , then the total distance covered by all drones is proportional to  $f(x_0, y_0)$  which is given as follows:

$$f(x_0, y_0) = \sum_{i=1}^n ((x_0 - x_i)^2 + (y_0 - y_i)^2) \quad (\text{I-5})$$

where  $(x_i, y_i)$  represents the location of the customer  $i$ . The optimal location can then be found using Jacobian and Hessian matrix computations of  $f(x_0, y_0)$  and is given by Equation I-6.

$$x_0^* = \frac{\sum_{i=1}^n x_i}{n} \quad y_0^* = \frac{\sum_{i=1}^n y_i}{n} \quad (\text{I-6})$$

It implies that the optimum location of the depot is the centroid of the demand points (customers' locations) for unit-capacity drones.

For drones with general capacity, the corresponding function which can be optimized for optimum depot location is given in Equation I-7.

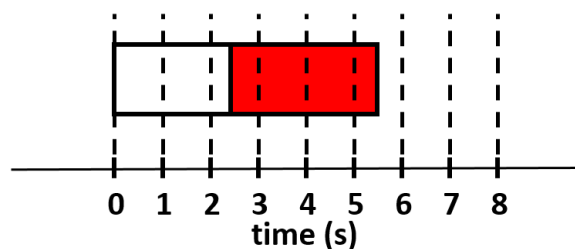
$$g(x_0, y_0) = \sum_{i=1}^n z_{0i} \times ((x_0 - x_i)^2 + (y_0 - y_i)^2) \quad (\text{I-7})$$

where  $z_{0i}$  is a binary variable and equals to one if the edge connecting depot (denoted by 0) and the customer  $i$  is a part of the journey of any vehicle. It can be easily seen that this objective function involves non-linear combination of decision variables for a problem aimed at simultaneous optimization of depot location and route planning. It can also be deduced that the optimum location of the depot in this case will lie within the convex polygon formed by the ‘outermost’ customers.

# Bin Count for an Arbitrary Interval using Linear Constraints

This appendix is aimed at explaining the set of constraints which are constructed in Equations G-5 and G-6 to find the time-intervals in which an arbitrary span of time spent at the depot lies. The aspect of arbitrariness in the timespan is caused by the implicit computation of completion time (i.e., it's not known beforehand) of each route placed at a position  $j$  in each vehicle's journey.

Consider an example in which the completion time of a route is 5.5 s and the time spent at the depot is 3 s. Let the time-axis is resolved in unit intervals i.e.,  $\Delta t = 1$  s. Maximum time,  $T$  is assumed to be 8 s. It implies that the vehicle spends time from 2.5 s to 5.5 s at the depot and the problem translates into finding out the time-intervals in which this time-span of 3 s lies (between 2.5 s and 5.5 s). This scenario is depicted in Figure J-1.



**Figure J-1:** An example of a route (time spent at the depot is shown in red color)

Let  $y_i \forall i \in \{1, 2, \dots, T\}$  is a binary variable and equals to one if a vehicle is present at the depot between the time-interval  $[(i - 1) \cdot \Delta t, i \cdot \Delta t)$ . Clearly, only variables  $y_3$ ,  $y_4$ ,  $y_5$ , and  $y_6$  take value 1 for the above stated example. The following set of constraints ensures this outcome:

$$0 - M \cdot (1 - y_1) \leq 5.5 < 1 + M \cdot (1 - y_1) + 3 \quad (\text{J-1})$$

$$1 - M \cdot (1 - y_2) \leq 5.5 < 2 + M \cdot (1 - y_2) + 3 \quad (\text{J-2})$$

$$2 - M \cdot (1 - y_3) \leq 5.5 < 3 + M \cdot (1 - y_3) + 3 \quad (\text{J-3})$$

$$3 - M \cdot (1 - y_4) \leq 5.5 < 4 + M \cdot (1 - y_4) + 3 \quad (\text{J-4})$$

$$4 - M \cdot (1 - y_5) \leq 5.5 < 5 + M \cdot (1 - y_5) + 3 \quad (\text{J-5})$$

$$5 - M \cdot (1 - y_6) \leq 5.5 < 6 + M \cdot (1 - y_6) + 3 \quad (\text{J-6})$$

$$6 - M \cdot (1 - y_7) \leq 5.5 < 7 + M \cdot (1 - y_7) + 3 \quad (\text{J-7})$$

$$7 - M \cdot (1 - y_8) \leq 5.5 < 8 + M \cdot (1 - y_8) + 3 \quad (\text{J-8})$$

$$y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 + y_8 = 3 + 1 \quad (\text{J-9})$$

where  $M$  is a very large number. The only solution to the above set of constraints is  $(y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8) = (0, 0, 1, 1, 1, 1, 0, 0)$ . Extrapolating this construction of constraints to include indexes of each route and time-interval, the resulting formulation given in Equations G-5 and G-6 can be obtained. The construction of these constraints is motivated from an example given by Hillier and Lieberman (2010) illustrating the choice of  $k$  out of  $n$  constraints.

This construction can be further generalized for an interval of arbitrary size by introducing variables,  $p$  and  $q$  such that

$$t_{i-1} - M \cdot (1 - y_i) - q \leq c < t_i + M \cdot (1 - y_i) + p \quad (\text{J-10})$$

$$\sum_{i=1}^T y_i = p + q + 1 \quad (\text{J-11})$$

This formulation results in choosing  $p$  bins before  $c$  and  $q$  bins after  $c$ . For the formulation presented in Appendix J,  $q = 0$  and  $p = t_d$ .

# Details of the Mission Planner

This appendix is aimed at providing brief details about the mission planner which is implemented in Simulink and MATLAB and works in conjunction with the Paparazzi open-source autopilot software.

Every drone in the fleet has  $Q + 1$  associated geo-referenced waypoints. The first waypoint corresponds to the location of the service point close to the depot and the rest of the  $Q$  waypoints correspond to each customer location en route. The location of these  $Q$  waypoints is updated (to reflect the next route on the journey) when a drone returns back to the service point. If the number of customers ( $n_r$ ) en route  $r$  is strictly less than  $Q$ , then, the location of last  $Q - n_r$  customers is set to be the same as the location of the last customer en route  $r$ .

The first step in the mission planner is to obtain the routes of each vehicle in the fleet. The solution obtained after either Phase-1 or Phase-2 of the planning algorithm can be used for this purpose. In second step, the obtained routes are then passed onto the real-time scheduler implemented for each drone in Simulink. The scheduler communicates with the GCS and drones by sending messages using User Datagram Protocol (UDP). It is composed of four stages. The brief details of each stage are delineated below:

**Stage 0** - At this stage, the scheduler is initialized. The inertial ground references are obtained over telemetry from each drone for proper initialization of the reference altitudes.

**Stage 1** - During this stage,  $Q + 1$  waypoints associated with the drone are moved to form the next route in its journey.

**Stage 2** - After receiving the confirmation from the previous stage, the *Start-Engine* command is sent to the drone in this stage. As soon as the drone receives this message, it continues its journey by moving onto the newly set route.

**Stage 3** - In this stage, the scheduler listens for the confirmation of completion of route by the drone. When the confirmation is received, the scheduler continues to its first stage for the next route. During this stage, the user can send an interrupt signal from the GCS in case the drone is not ready for continuing on its next route.



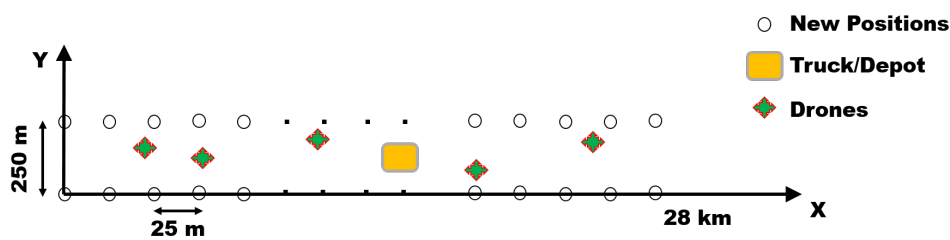
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# Appendix L

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## Possible Future Work

This appendix is aimed at providing insights into avenues that can be explored further. Consider an illustrative case comprising of a problem of delivery of geophones using a fleet of drones as shown in Figure L-1.



**Figure L-1:** An illustrative case for future research

For a fleet comprising of drones with payload capacity  $Q$ , the following research questions (in increasing order of complexity and generalization) can be formulated for future work:

1. What is the optimal location of the depot in order to optimally carry out the task of delivery?
2. In case of multiple depots, what will be the route taken by the drone(s) in the fleet and optimum location of the depot(s) in order to deliver the sensors in minimum time?
3. What will be the route taken by the drone(s) in the fleet to deliver the sensors in minimum time for given trajectories of the depot(s)?
4. What will be the route taken by the drone(s) in the fleet and optimum trajectories of the depot(s) to deliver the sensors in minimum time?





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## References

- Amazon. (2016). *Amazon prime air*. Retrieved from <http://www.amazon.com/b?node=8037720011/> (Accessed: April 14, 2016)
- Anaya-arenas, A. M., Maria, A., Chabot, T., Renaud, J., & Ruiz, A. (2014). Biomedical Sample Transportation : A Case Study Based on Québec ' s Healthcare Supply Chain Biomedical Sample Transportation : A Case Study Based on Québec ' s Healthcare Supply Chain. (October).
- Balinski, M. L., & Quandt, R. E. (1964, April). On an integer program for a delivery problem. *Oper. Res.*, *12*(2), 300–304. Retrieved from <http://dx.doi.org/10.1287/opre.12.2.300> doi: 10.1287/opre.12.2.300
- Battarra, M., Monaci, M., & Vigo, D. (2009). An adaptive guidance approach for the heuristic solution of a minimum multiple trip vehicle routing problem. *Computers and Operations Research*, *36*(11), 3041–3050. doi: 10.1016/j.cor.2009.02.008
- Brandão, J., & Mercer, A. (1997). A tabu search algorithm for the multi-trip vehicle routing and scheduling problem. *European Journal of Operational Research*, *100*(1), 180–191. Retrieved from <http://www.sciencedirect.com/science/article/pii/S0377221797000106> doi: 10.1016/S0377-2217(97)00010-6
- Brandao, J. C. S., & Mercer, a. (1998). The Multi-Trip Vehicle Routing Problem. *The Journal of the Operational Research Society*, *49*(8), 799. Retrieved from <http://www.jstor.org/stable/3009960?origin=crossref> doi: 10.2307/3009960
- Cattaruzza, D., Absi, N., & Feillet, D. (2016). Vehicle routing problems with multiple trips. *4OR*. Retrieved from <http://www.unibs.it/sites/default/files/ricerca/allegati/quadernowpdem3.pdf> doi: 10.1007/s10288-016-0306-2
- Cattaruzza, D., Absi, N., Feillet, D., & Vidal, T. (2013). A memetic algorithm for the Multi Trip Vehicle Routing Problem. *European Journal of Operational Research*, *236*(3), 833–848. Retrieved from <http://dx.doi.org/10.1016/j.ejor.2013.06.012> doi: 10.1016/j.ejor.2013.06.012
- Chapin, S. J. (1996, March). Distributed and multiprocessor scheduling. *ACM Comput. Surv.*, *28*(1), 233–235. Retrieved from <http://doi.acm.org/10.1145/234313.234410> doi: 10.1145/234313.234410

- Corbern, A., & Laporte, G. (2013). *Arc routing* (G. Laporte & ngel Corbern, Eds.). Philadelphia, PA: Society for Industrial and Applied Mathematics. Retrieved from <http://epubs.siam.org/doi/abs/10.1137/1.9781611973679> doi: 10.1137/1.9781611973679
- Şen, A., & Bülbül, K. (2008). A survey on multi trip vehicle routing problem. *In: VI. International Logistics & Supply Chain Congress.*
- Dantzig, G. B., & Ramser, J. H. (1959, October). The truck dispatching problem. *Manage. Sci.*, 6(1), 80–91. Retrieved from <http://dx.doi.org/10.1287/mnsc.6.1.80> doi: 10.1287/mnsc.6.1.80
- Fleischmann, B. (1990). The vehicle routing problem with multiple use of vehicles. *Technical Report, Fachbereich Wirtschaftswissenschaften.*
- Fukunaga, A. S., & Korf, R. E. (2005). Bin-completion algorithms for multicontainer packing and covering problems. *IJCAI International Joint Conference on Artificial Intelligence*, 28, 117–124. doi: 10.1613/jair.2106
- Golden, B. L., Magnanti, T. L., & Nguyen, H. Q. (1977). Implementing vehicle routing algorithms. *Networks*, 7(2), 113–148. Retrieved from <http://dx.doi.org/10.1002/net.3230070203> doi: 10.1002/net.3230070203
- Hern, A. (2014). *Dhl launches first commercial drone 'parcelcopter' delivery service.* Retrieved from <https://www.theguardian.com/technology/2014/sep/25/german-dhl-launches-first-commercial-drone-delivery-service> (Accessed: April 14, 2016)
- Hillier, F., & Lieberman, G. (2010). *Introduction to operations research.* McGraw-Hill. Retrieved from <https://books.google.nl/books?id=7nTHjgEACAAJ>
- Innoseis. (2016). *Tremornet - dramatically reduce land seismic acquisition costs with the industry's lightest sensing system.* Retrieved from <http://www.innoseis.com/tremornet/> (Accessed: April 14, 2016)
- Jiang, J., Ng, K. M., Poh, K. L., & Teo, K. M. (2014). Vehicle routing problem with a heterogeneous fleet and time windows. *Expert Systems with Applications*, 41(8), 3748 - 3760. Retrieved from <http://www.sciencedirect.com/science/article/pii/S0957417413009494>
- Jozefowicz, N., Semet, F., & Talbi, E.-G. (2008). The vehicle routing problem: Latest advances and new challenges. In B. Golden, S. Raghavan, & E. Wasil (Eds.), (pp. 445–471). Boston, MA: Springer US.
- Juan Carlos Rivera, H. Murat Afsar, C. P. (1997). Multistart Evolutionary Local Search for a Disaster Relief Problem. *Artificial Life*, 3(1), 51–61. doi: 10.1162/artl.1997.3.51
- Karger, D., Wien, J., & Stein, C. (2007). Scheduling algorithms. , 1–52. Retrieved from <http://link.springer.com/content/pdf/10.1007/978-3-540-24804-0.pdf>
- Koc, C., & Karaoglan, I. (2011). A Branch and Cut Algorithm for the Vehicle Routing Problem With Mltiple Use of Vehicles. *Proceedings of the 41st International Conference on Computers & Industrial Engineering*, 554–559.
- Laporte, G., Mercure, H., & Nobert, Y. (1986). An exact algorithm for the asymmetrical capacitated vehicle routing problem. *Networks*, 16(1), 33–46. Retrieved from <http://dx.doi.org/10.1002/net.3230160104> doi: 10.1002/net.3230160104
- Lenstra, J. K., & Kan, A. H. G. R. (1981). Complexity of vehicle routing and scheduling problems. *Networks*, 11(2), 221–227. Retrieved from <http://dblp.uni-trier.de/db/journals/networks/networks11.html#LenstraK81>
- Leung, J. (2004). *Handbook of scheduling: Algorithms, models, and performance analysis.* CRC Press. Retrieved from <https://books.google.nl/books?id=MAY1ZstmGPkC>

- Mingozzi, A., Roberti, R., & Toth, P. (2012). An Exact Algorithm for the Multitrip Vehicle Routing Problem. *INFORMS Journal on Computing*, 25(2), 193–207. Retrieved from <http://dx.doi.org/10.1287/ijoc.1110.0495> doi: 10.1287/ijoc.1110.0495
- Moon, M. (2016). *Nevada gets first FAA-approved urban drone delivery*. Retrieved from <http://www.engadget.com/2016/03/25/nevada-gets-first-faa-approved-urban-drone-delivery/> (Accessed: March 28, 2016)
- Murphy, M. (2016). *15 best uses of drones in business*. Retrieved from <http://www.techworld.com/picture-gallery/personal-tech/6-best-uses-of-drones-in-business-3605145/> (Accessed: March 16, 2016)
- Olivera, A., & Viera, O. (2007a). Adaptive memory programming for the vehicle routing problem with multiple trips. *Computers & OR*, 34(1), 28–47. Retrieved from <http://dx.doi.org/10.1016/j.cor.2005.02.044> doi: 10.1016/j.cor.2005.02.044
- Olivera, A., & Viera, O. (2007b). Solving the vehicle routing problem with multiple trips by adaptive memory programming. *International Journal of Logistics Systems and Management*, 3(4), 440. doi: 10.1504/IJLSM.2007.013213
- Peloquin, M. (2010). A Comparison of Scheduling Algorithms for Multiprocessors. , 1–17.
- Petch, R. J., & Salhi, S. (2003). A multi-phase constructive heuristic for the vehicle routing problem with multiple trips. *Discrete Applied Mathematics*, 133(1-3), 69–92. doi: 10.1016/S0166-218X(03)00434-7
- Prins, C. (2002). Efficient Heuristics for the Heterogeneous Fleet Multitrip VRP with Application to a Large-Scale Real Case. *Journal of Mathematical Modelling and Algorithms*, 1(2), 135–150. doi: 10.1023/A:1016516326823
- Richards, A., Bellingham, J., Tillerson, M., & How, J. (2002). Coordination and Control of Multiple {UAV}s. *{AIAA} Conf. on Guidance, Navigation and Control*(August), 1–11.
- Salhi, S., & Petch, R. J. (2007). A GA based heuristic for the vehicle routing problem with multiple trips. *Journal of Mathematical Modelling and Algorithms*, 6(4), 591–613. doi: 10.1007/s10852-007-9069-2
- Shell. (2015). *Drone assisted deployment and retrieval of seismic nodes in difficult desert terrain - research proposal with tu delft*. (Restricted Access, SR.11.23423)
- Shell. (2016). *Eye in the sky*. Retrieved from <http://www.shell.com/inside-energy/eye-in-the-sky.html> (Accessed: March 16, 2016)
- Shima, T., Rasmussen, S. J., Sparks, A. G., & Passino, K. M. (2006). Multiple task assignments for cooperating uninhabited aerial vehicles using genetic algorithms. *Computers and Operations Research*, 33(11), 3252–3269. doi: 10.1016/j.cor.2005.02.039
- Solomon, M. M., & Desrosiers, J. (1988). Time window constrained routing and scheduling problems. *Transportation Science*, 22(1), 1-13. Retrieved from <http://dx.doi.org/10.1287/trsc.22.1.1> doi: 10.1287/trsc.22.1.1
- Taillard, É. D., Laporte, G., & Gendreau, M. (1996). Vehicle Routeing with Multiple Use of Vehicles. *Journal of the Operational Research Society*, 47(8), 1065–1070. doi: 10.1057/palgrave.jors.0470810
- Toth, P., & Vigo, D. (2014). *Vehicle routing: Problems, methods, and applications, second edition*. SIAM.
- Wahba, P. (2015). *Walmart seeks U.S. permission to test drones*. Retrieved from <http://fortune.com/2015/10/26/walmart-drone-delivery/> (Accessed: April 14, 2016)
- Wang, Z., Liang, W., & Hu, X. (2013). A metaheuristic based on a pool of routes for the vehicle routing problem with multiple trips and time windows. *Journal of the Operational*

*Research Society*, 65(1), 3. Retrieved from <http://www.palgrave-journals.com/doi/10.1057/jors.2013.4> doi: 10.1057/jors.2013.4