



Delft University of Technology

CT1000S Structural Mechanics Bridging course Structural Mechanics

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Home

Welcome to the bridging course Structural Mechanics!

This TeachBook contains the course schedule and material for the bridging course Structural Mechanics

Learning objectives, activities and assessment

Contents

- Learning objectives
- Learning activities
- Assessment

Learning objectives

At the end of this module, you show to be able to:

1. Model, evaluate and reflect on validity of support reactions, section forces and displacements in skeletal structures using differential equations.
2. Model, evaluate and reflect on validity of support reactions, section forces and displacements in statically indeterminate skeletal structures using the force- and hybrid method.
3. Model, evaluate and reflect on validity of the stress state on homogeneous planar elements for any coordinate system.
4. Model, evaluate and reflect on validity of buckling load of skeletal structures. This aligns with the learning objectives of Structural Mechanics 3 of the regular BSc programme of Civil Engineering.

As a prerequisite for the main objectives, you should be able to model, evaluate and reflect on forces, displacements and stresses in statically determinate structures. This is made specific in the following prerequisite learning objectives, you show to be able to:

- a. Model, evaluate and reflect on validity of shape (only for cables), support reactions and section forces in statically determinate skeletal structures and cables using equilibrium relations and virtual work (only on beam structures), required for learning objective 1, 2 and 3.

- b. Model, evaluate and reflect on validity of displacements of truss- and skeletal bending structures using Williot and forget-me-nots, required for learning objective 1, 2 and 4
- c. Model, evaluate and reflect on validity of normal- and shear stresses in thick- and thin-walled cross-sections due to normal forces, bending moments and torsional moments, required for learning objective 3

The learning objectives a-c align almost entirely with the learning objectives of Structural Mechanics 1 and 2 of the regular BSc programme of Civil Engineering. The learning objectives are summarized in three subjects:

- Statically indeterminate structures, including learning objective 1, 2, a and b
- Continuum mechanics, including learning objective 4, a, b and c
- Buckling, including learning objective 1, 4, a, b

This course replaces Structural Mechanics 1, 2 and 3 of the regular Bachelor programme as a preparation for the MSc Civil Engineering. It is expecting some prior knowledge on learning objective a – c to be able to finish the course on time.

Learning activities

The learning-line of structural mechanics as taught in the BSc programme of Civil Engineering is offered as separate modules (corresponding to the learning objectives 1-4) in a fixed order (1, 2, 3, 4). In class, each learning objective 1-4 will be introduced classically on a high-level. During this modules, there's enough time to work on the prerequisite modules (corresponding to learning objective a-c).

You start the course with an ungraded, self-assessed prerequisite test to identify shortcomings in their knowledge.

All of the separate modules are collected in this TeachBook, breaking down each module into separate parts of theory and skills including exercises. Assistance is provided to small student groups and individual students.

The support by lecturers and TAs in class comprises of:

- Classical introduction on learning objective 1-4
- Assistance on making a personal learning plan
- Additional explanation to small student groups on the literature,
- Guidance and feedback on exercises

Assessment

The learning will be assessed incrementally:

- Exam week period Q1, exam assignment on statically indeterminate structures
- Halfway Q2, exam assignments on statically indeterminate structures and continuum mechanics
- Exam week period Q2, exam assignments on buckling and continuum mechanics
- Exam week Q3, exam assignments on statically indeterminate structures, on buckling and on continuum mechanics

During the exam assignments, you're allowed to bring your books, notes, etc. and you're allowed to use your laptop including software like SymPy. You're not allowed to communicate during the exam session, which will be strictly monitored. Furthermore, you're not allowed to use structural analysis software.

For each exam sessions (consisting of one or more assignments) you'll have three hours.

You pass the courses when you scores sufficient (minimum 6.0) for an assignments on each of the subjects. Each subject is graded separately as a partial grade.

Course schedule



Click the dropdown blocks below to find a detailed schedule for the course activities per week. Clicking the links will take you to the relevant content pages.

Week 1 (September 2th) ▼

	Monday	Tuesday	Wednesday	Thursday	Friday	U h
Homework deadlines					Self-grade tests Prerequisites	W R e h e e P
Class activities	Introduction course and prerequisite test 1 Prerequisites		Prerequisites test Prerequisites		Evaluation test Prerequisites	



Week 2 (September 9th) ▼

	Monday	Tuesday	Wednesday	Thursday	Friday	
Homework deadlines			Read theory and make homework exercises Statics externally Prerequisites		Read theory and make homework exercises equilibrium relations and cables Prerequisites	
Class activities	Demonstration Differential equations New content		Guided practice on Statics externally Prerequisites		Guided practice on equilibrium relations and cables Prerequisites	



Week 3 (September 16th)



	Monday	Tuesday	Wednesday	Thursday	Friday	
Homework deadlines	Read theory and make homework exercises kinematic and constitutive relations Prerequisites		Read theory and make homework exercises SymPy New content			
Class activities	Guided practice on kinematic and constitutive relations Prerequisites		Guided practice on SymPy New content		Demonstration Force method extension New content	



Week 4 (September 23rd)



	Monday	Tuesday	Wednesday	Thursday	Friday	Up ho
Homework deadlines	Read theory and make homework exercises Extension Prerequisites		Read theory and make homework exercises Williot Prerequisites		Read theory and make homework exercises Force method extension New content	We Re a ho e : d k Pr
Class activities	Guided practice on extension Prerequisites		Guided practice on Williot Prerequisites		Guided practice on Force method extension New content	



Week 5 (September 30th)



	Monday	Tuesday	Wednesday	Thursday	Friday
Homework deadlines			Read theory and make homework exercises Section force diagrams Prerequisites		Read theory and make homework exercises Displacement bending Prerequisites
Class activities	Demonstration Force method bending New content		Guided practice on Section force diagrams Prerequisites		Guided practice on Displacement bending Prerequisites

◀
▶

Week 6 (October 7th)



	Monday	Tuesday	Wednesday	Thursday	Friday	U ho
Homework deadlines	Read theory and make homework exercises Force method bending New content		Read theory and make homework exercises Mechanisms and virtual work Prerequisites		Read theory and make homework exercises Force method frames New content	Wo Re a h e Dis N
Class activities	Guided practice on Force method bending New content		Guided practice on Mechanisms and virtual work Prerequisites		Read theory and make homework exercises Force method frames Prerequisites	



Week 7 (October 14th)



	Monday	Tuesday	Wednesday	Thursday	Friday
Homework deadlines			Read theory and make homework exercises Displacement method New content		
Class activities	Demonstration Displacement method New content		Guided practice on Displacement method New content		Demonstration stiffness discontinuities, support settlement and temperature influences New content



Week 8 (October 21st)



	Monday	Tuesday	Wednesday	Thursday	Friday	Upcoming
Homework deadlines			Read theory and make homework exercises Stiffness discontinuities and support settlement New content		Read theory and make homework exercises temperature influences New content	Friday 10: Pr for e assign Stati indeter struc New c Prereq
Class activities			Guided practice on Stiffness discontinuities and support settlement New content		Guided practice on temperature influences New content	



Week 10 (November 4th)



	Monday	Tuesday	Wednesday	Thursday	Friday	Upcoming homework
Homework deadlines					Prepare for exam assignment Statically indeterminate structures New content Prerequisites	Friday: theory material homework exercises cross-section properties Prerequisites
Class activities					Exam assignment 1 statically indeterminate structures New content Prerequisites	



Week 11 (November 11th)



	Monday	Tuesday	Wednesday	Thursday	Friday	
Homework deadlines					Read theory and make homework exercise cross-sectional properties Prerequisites	
Class activities	Review exam assignment 1 statically indeterminate structures New content Prerequisites Demonstration transforming stress- and strain tensors New content			Cancelled due to protest against cuts in higher education	Guided practice on cross-sectional properties Prerequisites	

Week 12 (November 18th)



	Monday	Tuesday	Wednesday	Thursday	Friday
Homework deadlines	Read theory and make homework exercise normal stresses Prerequisites	Read theory and make homework exercise shear stresses in bending elements Prerequisites			Read theory and make homework exercise core of cross-section Prerequisites
Class activities	Guided practice on normal stresses Prerequisites	Guided practice on shear stresses in bending elements Prerequisites			Guided practice on core of cross-section Prerequisites



Week 13 (November 25th)



	Monday	Tuesday	Wednesday	Thursday	Friday
Homework deadlines	Read theory and make homework exercise transforming tensors New content	Read theory and make homework exercise Circle of Mohr New content			Read theory and make homework exercise torsional internal forces Prerequisites
Class activities	Guided practice on transforming tensors New content	Guided practice on Circle of Mohr New content			Guided practice on torsional internal forces Prerequisites



Week 14 (December 2nd)



	Monday	Tuesday	Wednesday	Thursday	Friday
Homework deadlines	Read theory and make homework exercise shear stresses in torsional elements Prerequisites	Read theory and make homework exercise shear force centre Prerequisites			Read theory and make homework exercise failure criteria New content
Class activities	Guided practice on shear stresses in torsional elements Prerequisites	Guided practice on shear force centre Prerequisites			Guided practice on failure criteria New content



Week 15 (December 9th)



	Monday	Tuesday	Wednesday	Thursday	Friday	Upcoming
Homework deadlines		Prepare for exam assignments statically indeterminate structure and continuum mechanics New content Prerequisites				End of week home System with New c
Class activities		Exam assignment 2 statically indeterminate structure and 1 continuum mechanics New content Prerequisites		Review exam assignment 2 statically indeterminate structures and assignment 1 continuum mechanics New content Prerequisites		



Week 16 (December 16th)



	Monday	Tuesday	Wednesday	Thursday	Friday
Homework deadlines					End-of-week homework Systems with 1 dof New content
Class activities	Lecture New content	Lecture New content		Lecture New content	

Week 17 (December 23rd)

	Monday	Tuesday	Wednesday	Thursday	Friday
Homework deadlines					End-of-week homework Systems with 1 and infinite number dofs New content
Class activities	Lecture New content	Lecture New content		Lecture New content	

Week 18 (January 6th)

	Monday	Tuesday	Wednesday	Thursday	Friday
Homework deadlines					End-of-week homework Flexural buckling, Eulerian approach New content
Class activities	Lecture New content	Lecture New content		Lecture New content	



Week 20 (January 13th)



	Monday	Tuesday	Wednesday	Thursday	Friday	Upcoming homework
Homework deadlines					Prepare for exam assignments continuum mechanics and buckling New content Prerequisites	Monday week Prepare exam assignments static indeterminately structured continuum mechanics and buckling New content Prerequisites
Class activities					Exam assignment 2 continuum mechanics and 1 buckling New content Prerequisites	



Week 30 (April 14th)



	Monday	Tuesday	Wednesday	Thursday	Friday
Homework deadlines	Prepare for exam assignments statically indeterminate structure, continuum mechanics and buckling New content Prerequisites				
Class activities	Exam assignment 3 statically indeterminate structures, 3 continuum mechanics and 2 buckling New content Prerequisites				

How to use this TeachBook

Contents

- Linked literature and tools
- Interactive features
- Spot a mistake?
- Personalised book
- Version

This TeachBook combines the course schedule and content. The content is shown independently as well in the [Mechanics-BSc TeachBook](#). Specific version of this book are available on:

- Version only containing content for exam assignment 1:
<https://teachbooks.io/mechanics-BSc/2024-assignment1>
- Version only containing content for exam assignment 2:
<https://teachbooks.io/mechanics-BSc/2024-assignment2>
- Version only containing content for exam assignment 3:
<https://teachbooks.io/mechanics-BSc/2024-assignment3>

Linked literature and tools

This TeachBook links to relevant part of the other literature and only provides additional content when the other literature is insufficient.

- [Brightspace course page](#): this Brightspace page is only used for announcements.
- Engineering Mechanics: Volume 1: Equilibrium, by Hartsuijker and Welleman ([2006](#)).
 - eBook available for free via TU Delft at  Springer.
 - This book is also available in Dutch (only paid version) at **Boom**.
- Engineering Mechanics: Volume 2: Stresses, strains and displacements, by Hartsuijker and Welleman ([2007](#)).
 - eBook available for free via TU Delft at  Springer.
 - This book is also available in Dutch (only paid version) at **Boom**.

- **Mechanica: Statisch onbepaalde constructies en bezwijkanalyse**, by Hartsuijker and Welleman ([2007](#)).
 - Book available at **Boom**. This book is only available as a paid version in Dutch
- **Mechanica: Stabiliteit van het evenwicht**, by Hartsuijker and Welleman ([2023](#)).
 - Book available at **Boom**. This book is only available as a paid version in Dutch.
 - Summary available in Dutch [here](#)
 - Visual summary available in Dutch [here](#).
 - Answers available in Dutch [here](#).
 - Additional notes on example 3 of chpt 4 in Dutch [here](#)
 - Note on equivalent rotational spring stiffness [in Dutch here](#) and [in English here](#)
 - Python script to find buckling load, basic type 4 in English [here](#)
 - Python script to find buckling load, basic type 5 in English [here](#)
 - Python script to find numerical buckling load, example with different EI from slides in Dutch [here](#)
 - python script to find numerical buckling load, example with two forces from slides in Dutch [here](#)
 - Summary of formulas flexural buckling [in Dutch here](#) and [in English here](#).
 - Chapter 7, 12 and 13 are not part of this course.
- **Introduction to continuum mechanics**, by Hartsuijker and Welleman ([2007](#)).
 - Book available [here](#).
- **Niet-symmetrische en inhomogene doorsneden**, by C Hartsuijker and Welleman ([2009](#)).
 - Book available [here](#).
 - Answers available [here](#).
 - English version is part of Engineering Mechanics: Volume 2: Stresses, strains and displacements, by Hartsuijker and Welleman ([2007](#))
- Additional exercises in **ans* course** [Constructiemechanica BSc Civiele Techniek](#). This course contains all the practice assignments linked to the content of [Mechanics-BSc TeachBook](#). You can login into this course by [logging in into ANS - clicking](#)  [TU Delft](#) - clicking [Join Course](#) - clicking [Join for Constructiemechanica BSc Civiele Techniek](#). Afterwards, you can directly go to the course by clicking the course in [your personal ANS dashboard](#) or by clicking the provided links in the book.
- Self-assessed prerequisite tests and paper-based exams in **ans* course** [CT1000 s Structural Mechanics EXAM 24/25 \(Q1\)](#). In this course all the assignments specific for this year's course can be found. This includes the prerequisite tests and the exams. You are added to this course as soon as you participate in one of the tests or exams.

Interactive features

This TeachBook includes interactive features!

- Interactive quizzes: Some page include interactive 'H5p' quizzes. This allows you to check your understanding. Any interactions you have with this system are not stored, unless you login into H5p.
- Python interactivity: Click  -> [Live Code](#) on the top right corner of interactive page to start up a python-kernel in your browser! Any interactions you do here are not stored. You can also download the page as a notebook to apply the content on your own computer.

Spot a mistake?

If you spot any mistakes, you can click on  -> , login with a GitHub account and report your issue. It'll be solved soon!

Personalised book

If you'd like to make this TeachBook more personal by adding (private or public) annotations I can recommend the [Hypothesis extension](#). This is only for your own use, I won't monitor public post on this platform.

Version

This is the [2024-2025](#)-version of the TeachBook. Go to [/bridging_mechanics](#) to view the most recent version of this TeachBook, or adapt the year in [/bridging_mechanics/2024](#) to the year when you took the course.

Contact information

Contents

- Tom van Woudenberg
- Hans Welleman
- Teaching assistants
- TeachBook editing

This course is mainly taught by Tom van Woudenberg. Furthermore, Hans Welleman teaches the topic on buckling. Please contact us if you've any questions, feedback or when you've personal circumstances which we should know.

If you're stuck on a certain topic, you can also visit the teaching assistants during every lunch question hour in 6.41

Tom van Woudenberg

- Room 6.45
- 015-2789739
- T.R.vanWoudenberg@tudelft.nl



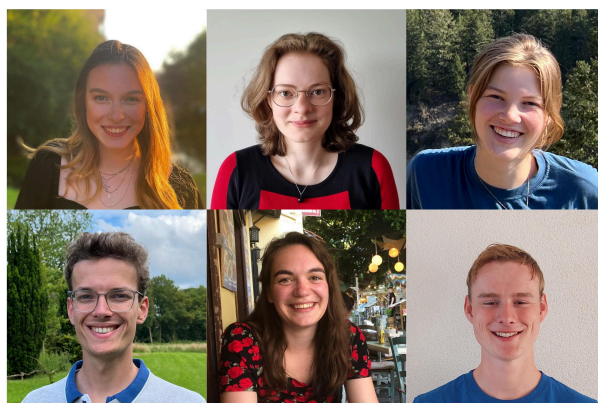
Hans Welleman

- Room 6.55
- 015-2784856
- J.W.Welleman@tudelft.nl



Teaching assistants

- Room 6.41
- Question hour during every lunch break
- 015-2783654
- SA-mech-CITG@tudelft.nl



- Gweneira van Koot
- Iris Oskam
- Jelke Jansen
- Laurens van Bavel
- Lisette de Langen
- Martijn Westerhof

TeachBook editing

This online TeachBooks has partly been edited by Bart Slingerland.

Week 1 (September 2nd)

	Monday	Tuesday	Wednesday	Thursday	Friday	Upcoming homework
Homework deadlines					Self-grade tests Prerequisites	Wednesday Read the and read homework exercise Stat external Prerequisites
Class activities	Introduction course and prerequisite test 1 Prerequisites		Prerequisites test Prerequisites		Evaluation test Prerequisites	



Lesson Monday September 2nd

Contents

- Course introduction
- Prerequisite test 1

Course introduction

During this lesson an overview will be given of the course as described in:

- [Course information](#)
- [Course schedule](#)
- [How to use this book](#)
- [Contact information](#)

Furthermore, we'll do a round of introduction.

Prerequisite test 1

During this first week you'll make two prerequisite tests. These will offer both an overview of the expected prerequisite knowledge and allows you to check whether you master those topics. This test won't count for your final grade of this course but is meant to give you insight in your learning.

Try to do this exercise on your own. If you cannot proceed, you're free to discuss the exercise with your fellow students and the teacher. If you do so, indicate how this helped you so that it's clear which topics you do or do not master.

If you missed the first lesson, you can find the assignment [↓ here](#). Please send [Tom](#) a scan of your filled in prerequisite test or bring the paper assignment to the next lesson.

Afterwards, your test will be scanned so that I can have some insights in your performance as well. However, you'll be asked to grade your test yourself on paper using the provided model-answer as a [preparation](#) for the lesson on Friday. During that lesson we'll discuss the results to identify which topics you should spend more time on during this quarter.

Lesson Wednesday September 4th

Contents

- Prerequisite test 2

Prerequisite test 2

Today we'll continue with the second prerequisite test. Again, this will offer both an overview of the expected prerequisite knowledge and allows you to check whether you master those topics. This test won't count for your final grade of this course but is meant to give you insight in your learning.

Try to do this exercise on your own. If you cannot proceed, you're free to discuss the exercise with your fellow students and the teacher. If you do so, indicate how this helped you so that it's clear which topics you do or do not master.

If you missed this lesson, you can find the assignment [↓ here](#). Please send [Tom](#) a scan of your filled in prerequisite test or bring the paper assignment to the next lesson.

Afterwards, your test will be scanned so that I can have some insights in your performance as well. However, you'll be asked to grade your test yourself on paper using the provided model-answer as a [preparation](#) for the lesson on Friday. During that lesson we'll discuss the results to identify which topics you should spend more time on during this quarter.

Homework deadline lesson

Friday September 6th

Contents

- Grade your test

As a preparation for [Friday's lesson](#) you'll be asked to grade your test yourself on paper using the provided model-answer. During that lesson we'll discuss the results to identify which topics you should spend more time on during this quarter.

Grade your test

You can view test including the model answer as soon as it is scanned at:

- **ans*** [prerequisite test 1](#)
- **ans*** [prerequisite test 2](#).

The model answer includes the levels to select for grading. You'll grade your prerequisite tests yourselves. Unfortunately that's not possible from within the ANS environment.

However, I'm curious to your own grading, so you can submit your grade via:

-  [form prerequisite test 1](#).
-  [form prerequisite test 2](#).

You can that directly after your prerequisite tests are scanned. After you submit your grading, please save the link provided in the summary screen to be able to view and edit your grading at a later stage.

Lesson Friday September 6th

Contents

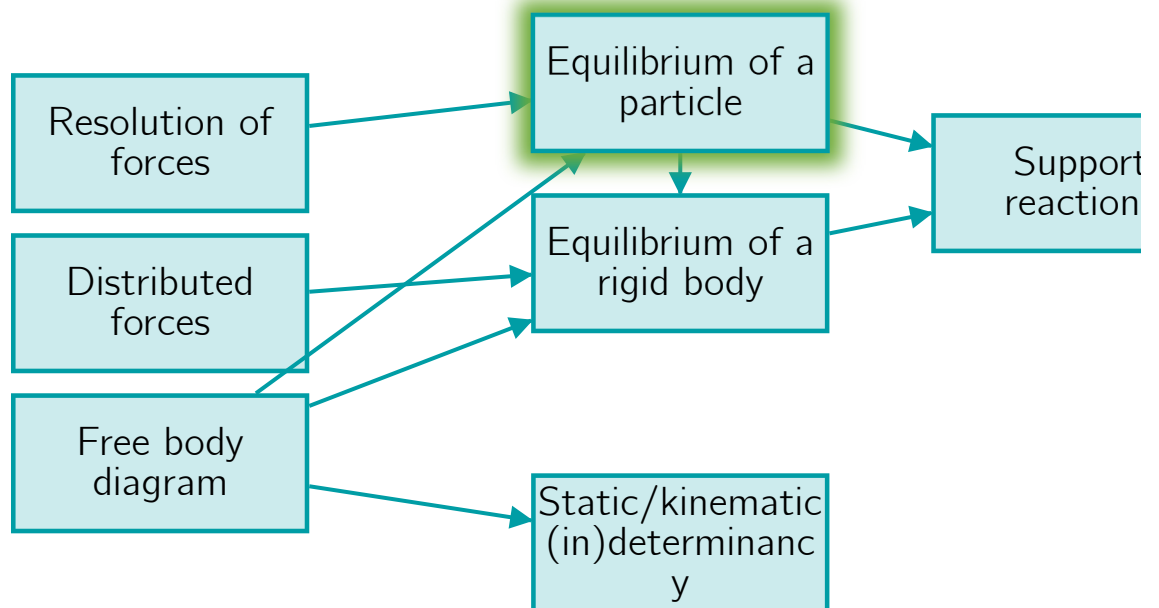
- Discuss prerequisite tests
- Start homework deadline for next week

As a preparation [you graded your own prerequisite test](#). Furthermore, I graded your work as well. Today we'll discuss the results. If there's time left, you can start working on next week homework deadline.

Discuss prerequisite tests

Today we'll discuss the outcomes of the prerequisite tests; we'll discuss how assignments were made by average and which mistakes are worth mentioning. If I've graded your work differently than you did yourself, there's time available to discuss in class.

Following, you'll be asked to make a knowledge graph of the topics which you encountered in the prerequisite tests, and how they link to the learning objectives of this course. An example is shown below:



Then, we'll discuss the structure of the prerequisite material as demonstrated in this book ordered by class-activities and the full overview of all the content on <https://teachbooks.github.io/mechanics-BSc>.

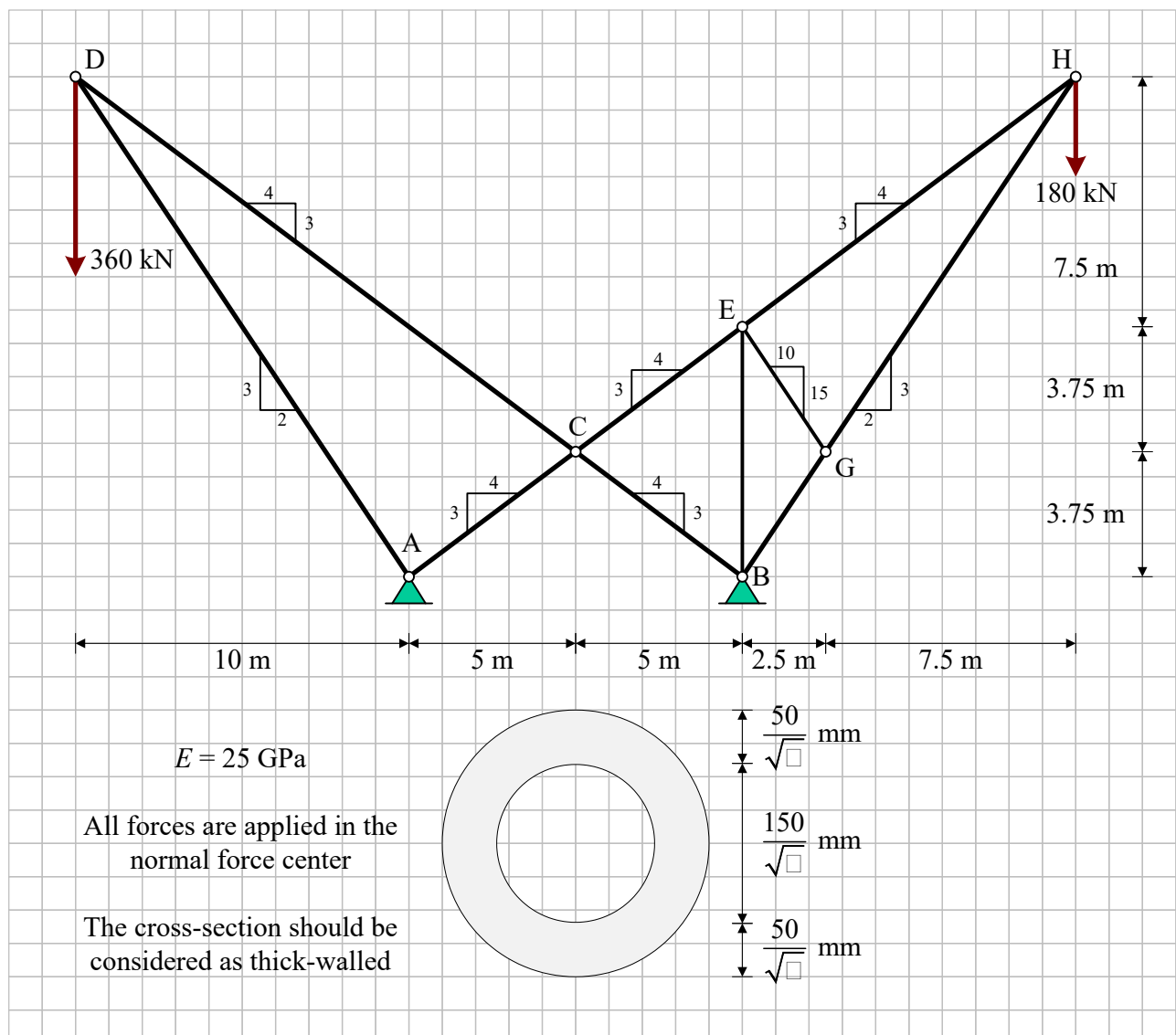
Start homework deadline for next week

If we've time left, you can start preparing for the first homework deadline on [Statics externally](#) which you'll need to apply in class on next Wednesday

Frequently made mistakes

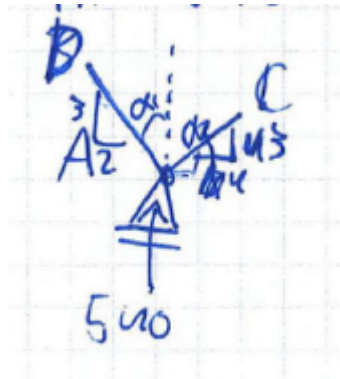
Contents

- 1.1 Determine the normal force in bar and
- Determine the displacement of node
- 2.1 Determinate the support reaction at
- 2.2 Determinate the bending moment in using virtual work
- 2.5 Draw the bending moment diagram
- 2.4 Draw the shear stress distribution just left of
- 2.5 Determine the displacement at ξ



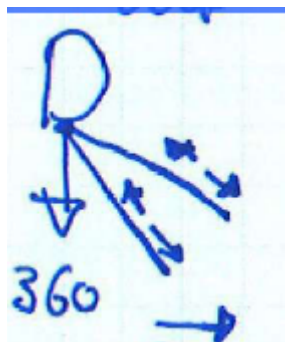
1.1 Determine the normal force in bar CD and AC

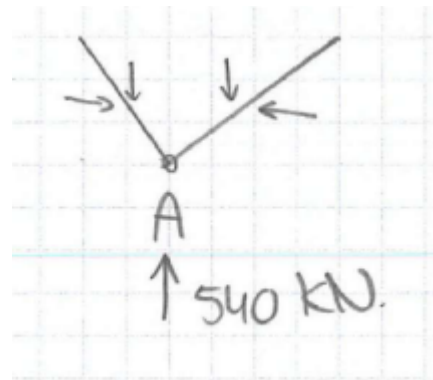
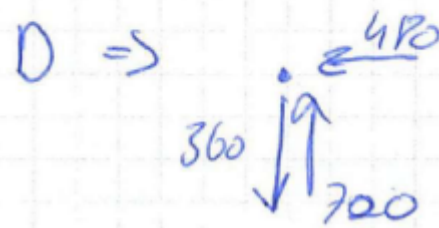
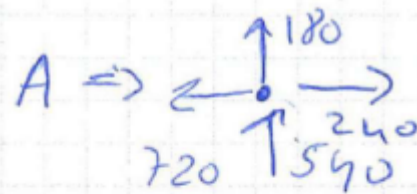
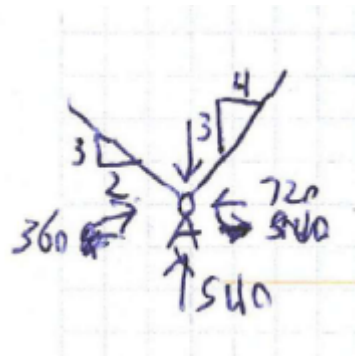
Horizontal support reactions

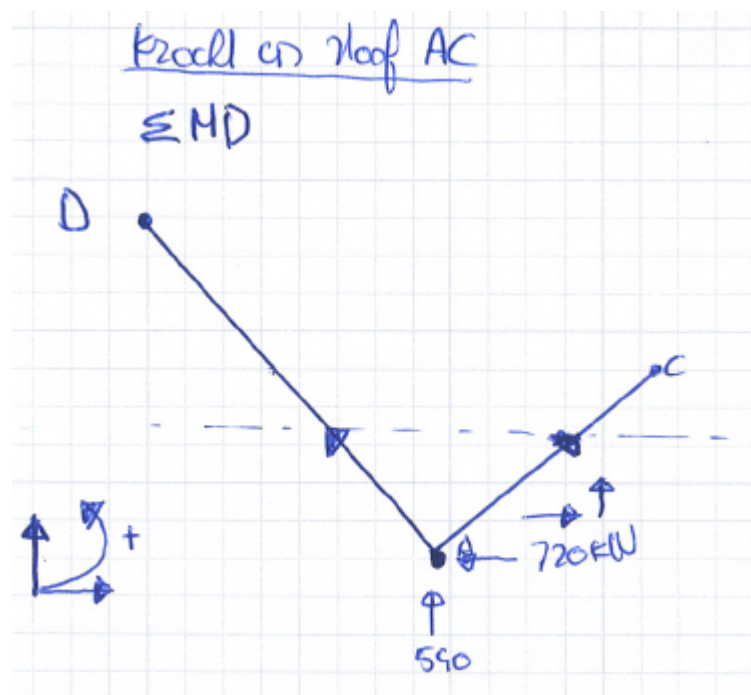
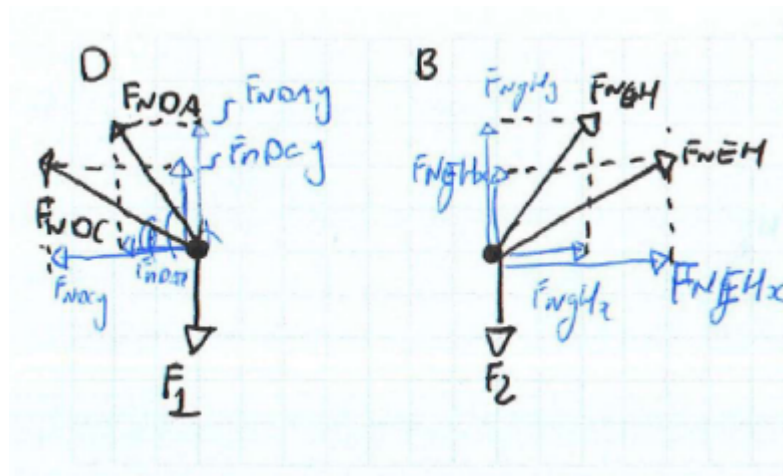
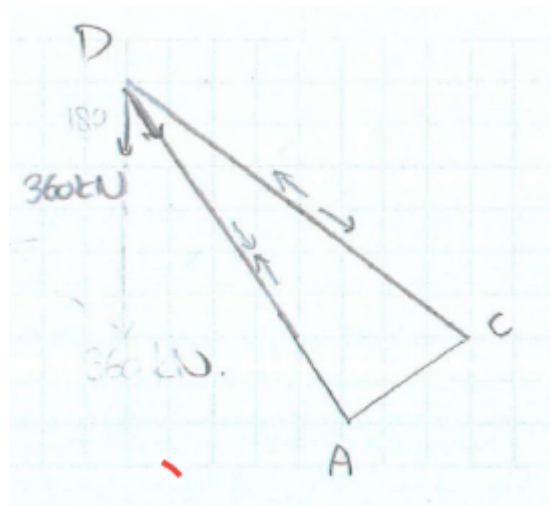


$$AD_H = AC_H$$
$$AD_H = 3 \Rightarrow AD = \frac{3AD_V}{4}$$

Free body diagrams







ontbinden

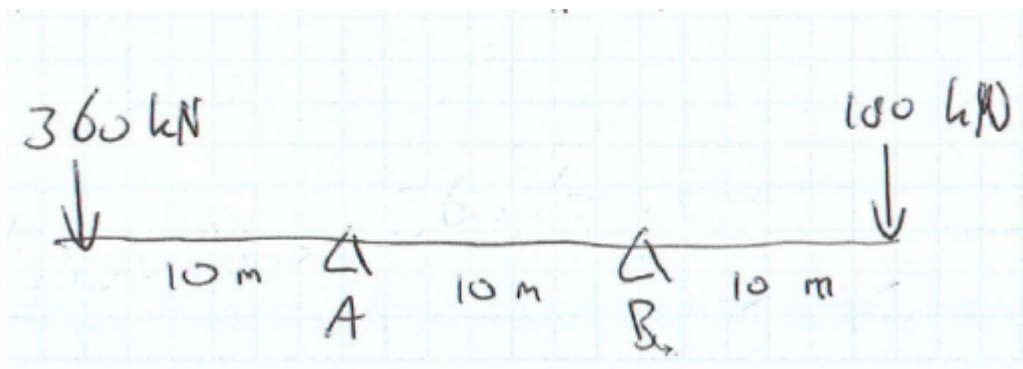
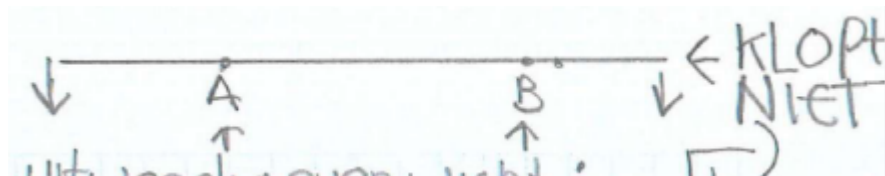
$$\text{length AC} \Rightarrow \sqrt{(3,75^2) + (5^2)} = 6,25$$

$$\frac{360}{15} = \frac{CD}{18,75}$$

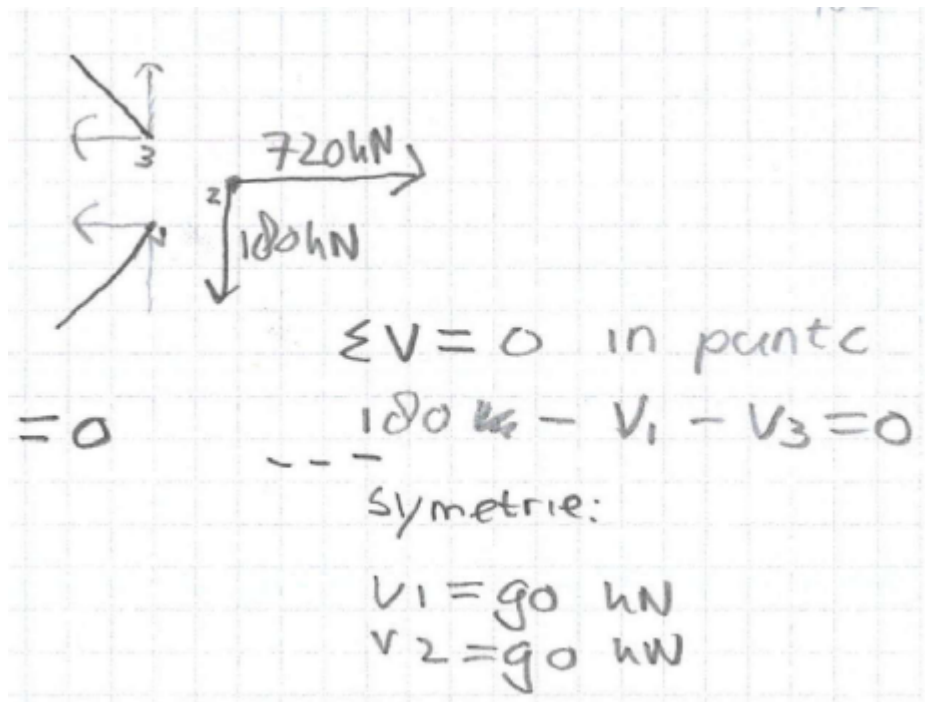
$$CD = \frac{360 \cdot 18,75}{15} = 450 \text{ kN}$$

$$CD = 450 \text{ kN}$$

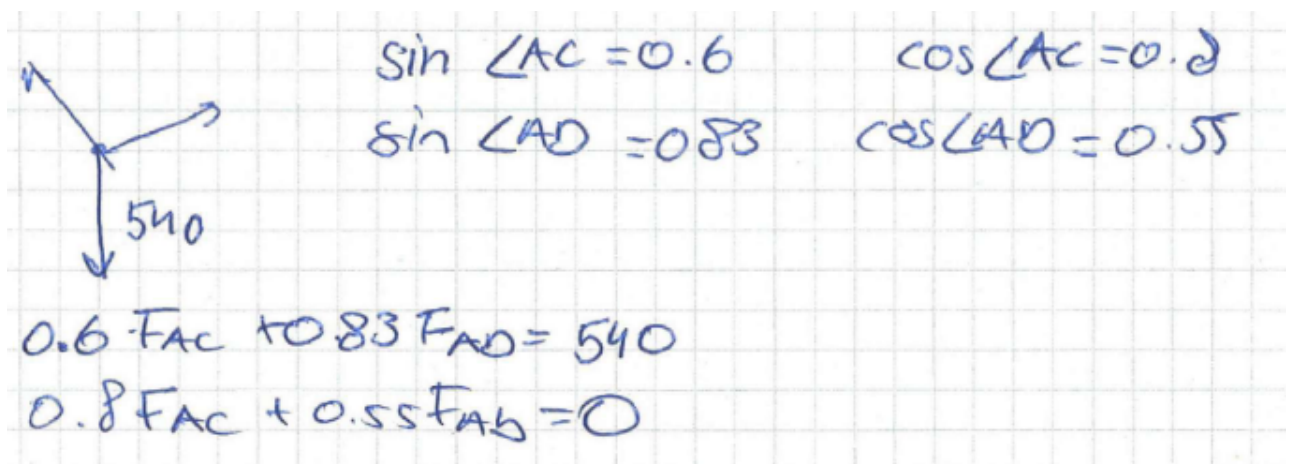
Change model

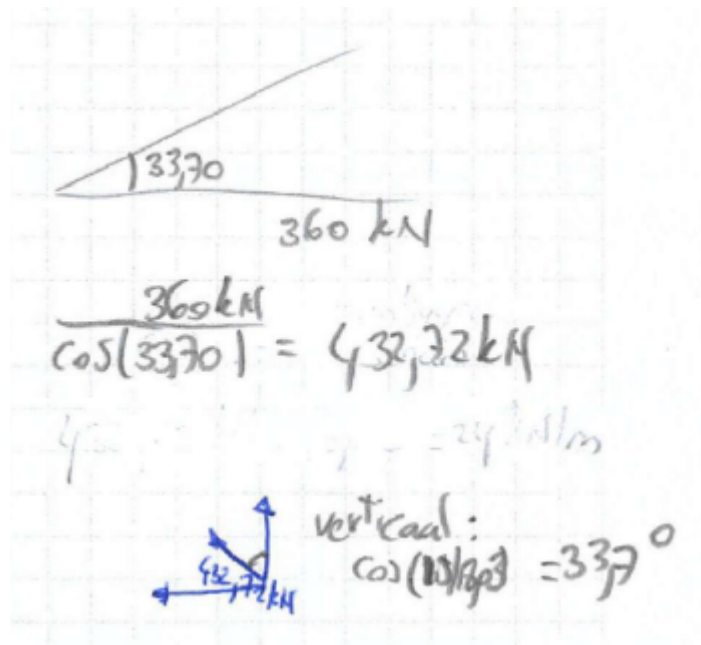


Symmetry



Angles

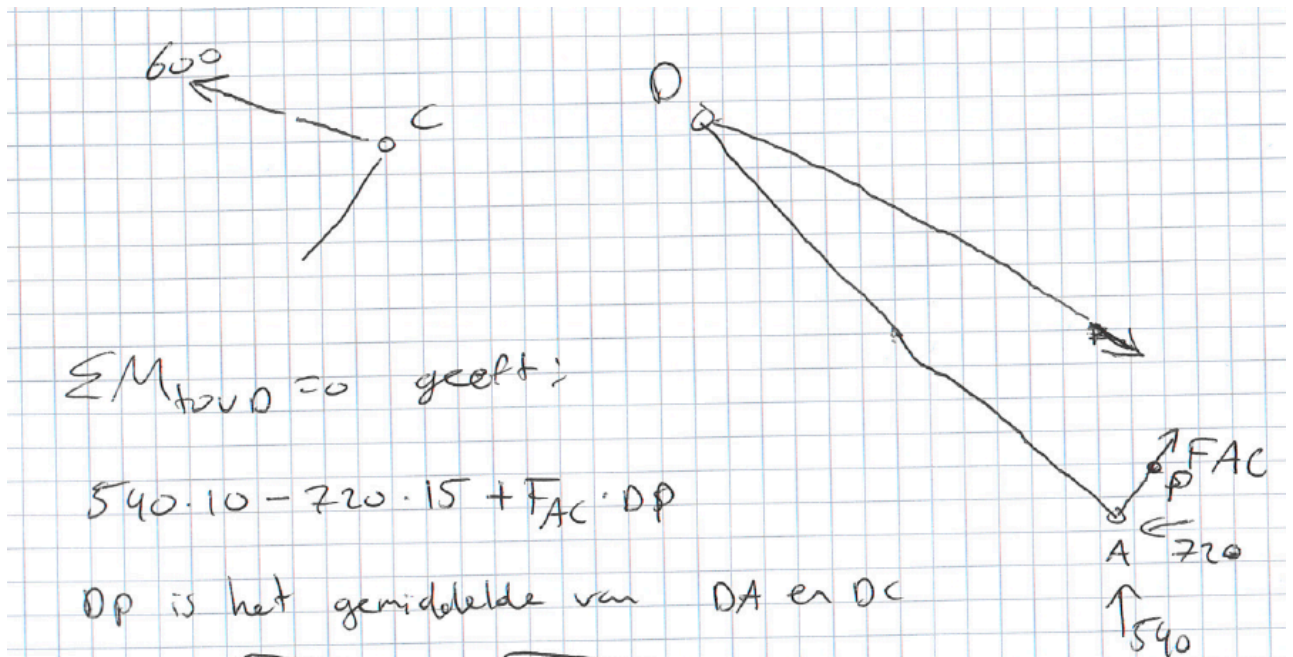




$$\angle ACD = 2 \cdot \tan^{-1}\left(\frac{3}{4}\right) \approx 73,7^\circ$$

$$AS = 6,25 \cdot \sin(73,7) = 6 \quad \text{ dus}$$

Moment equilibrium sloped force



Determine the displacement of node C

Area calculations

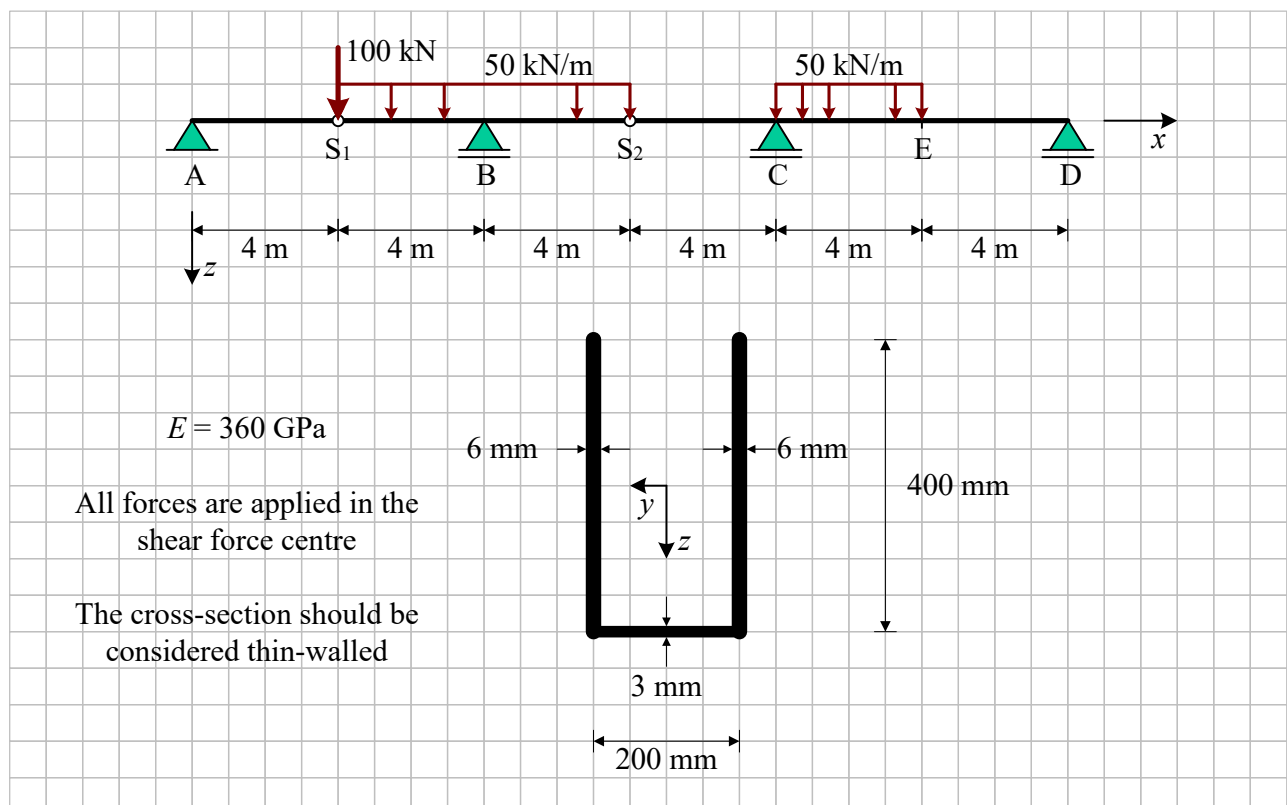
$$A = 2\pi r_1 - 2\pi r_2$$

Missing Williot

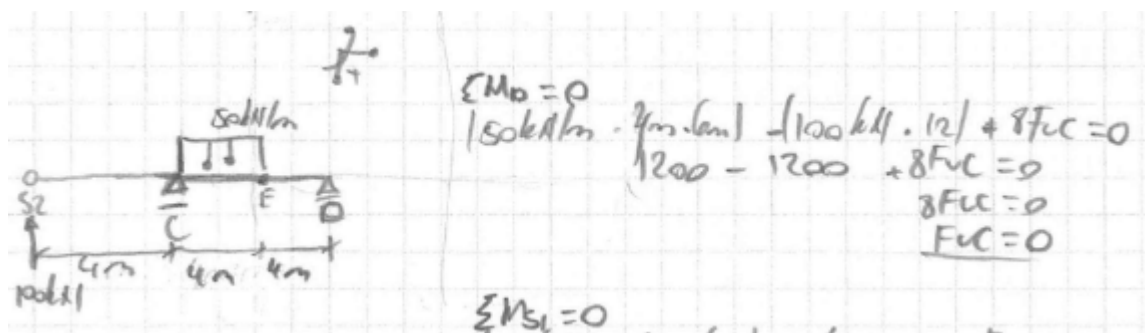
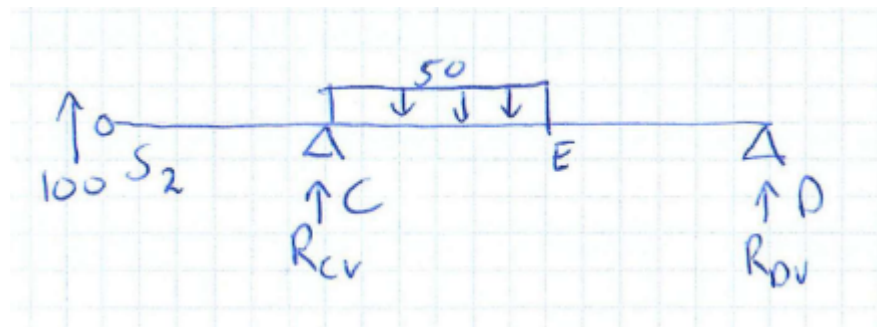
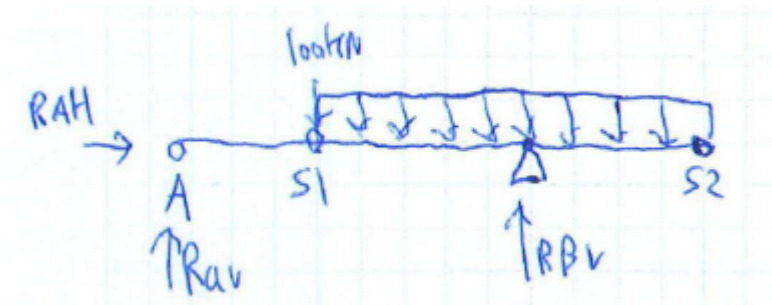
$$H = \frac{1}{5} \cdot 0,6 - \frac{4}{5} \cdot 0,15 = 0,36$$
$$V = \frac{3}{5} \cdot 0,6 - \frac{3}{5} \cdot 0,15 = 0,27$$

Verplaatsing C 0,36 m naar links en 0,27 m omhoog

2.1 Determinate the support reaction at D



FBD and supports



No free body diagram

$$\sum M(S_1) = 0 \Rightarrow A_U \cdot 4 = 0 \quad A_U = 0$$

$$\sum M(S_2) = 0 \Rightarrow 100 \cdot 8 + 50 \cdot 8 \cdot 4 - B_U \cdot 4 = 0$$

$$B_U = 600 \text{ kN}$$

$$\sum F_V = 0 \Rightarrow 700 = 600 + D_U$$

$$D_U = 100 \text{ kN}$$



$$4) \sum T(S_1A)_{S_1} = 0 \Rightarrow -V_A \cdot 4 = 0 \Rightarrow V_A = 0 \text{ kN}$$

$$5) \sum T(S_2A)_{S_2} = -0 \cdot 12 + 8 \cdot 100 + 400 \cdot 4 - 4 \cdot V_B = 0 \Rightarrow V_B = 600 \text{ kN} (\uparrow)$$

T... .. (9) ..

$$\sum M_{S_2} = 0 = -4 \cdot 400 + 4 \cdot B_V - 8 \cdot 100 + 0 = 0$$

$$-1600 - 800 + 4 B_V = 0, B_V = \frac{2400}{4} = 600 \text{ kN} \uparrow$$

Cut bar around force/hinge and forces in hinges

$$\sum M_A = 0$$

$$-100 \cdot 4 + S_V \cdot 4 = 0$$

$$S_V = 100 \text{ kN}$$

$$A_v = 0 \text{ kN}$$

$$B_v = 600 \text{ kN}$$

$$C_v = 0 \text{ kN}$$

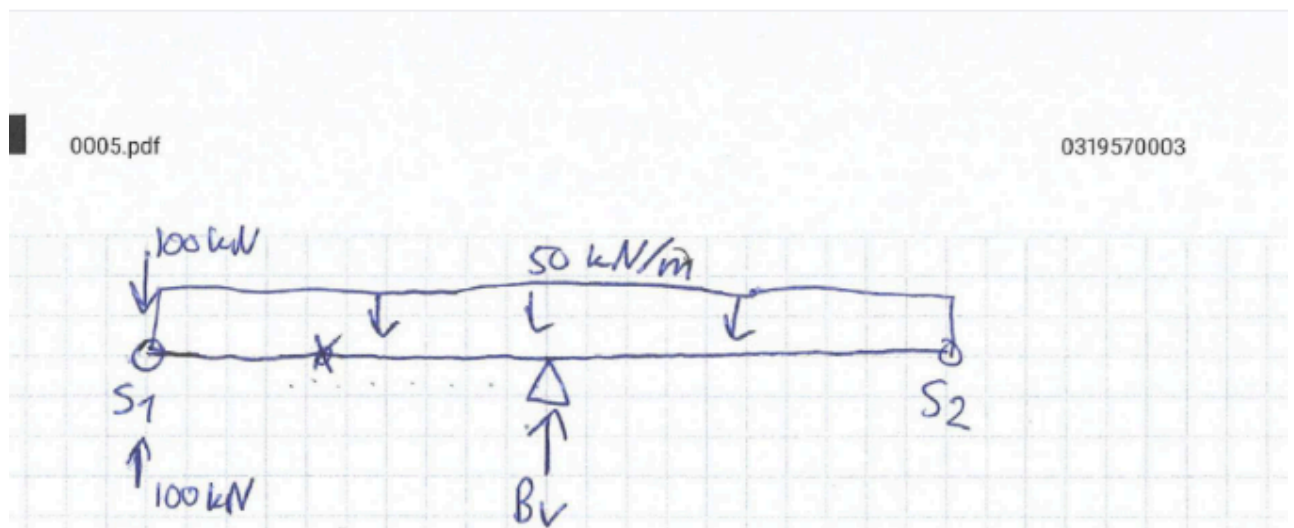
$$D_v = 100 \text{ kN}$$

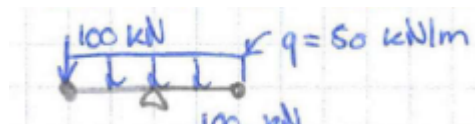
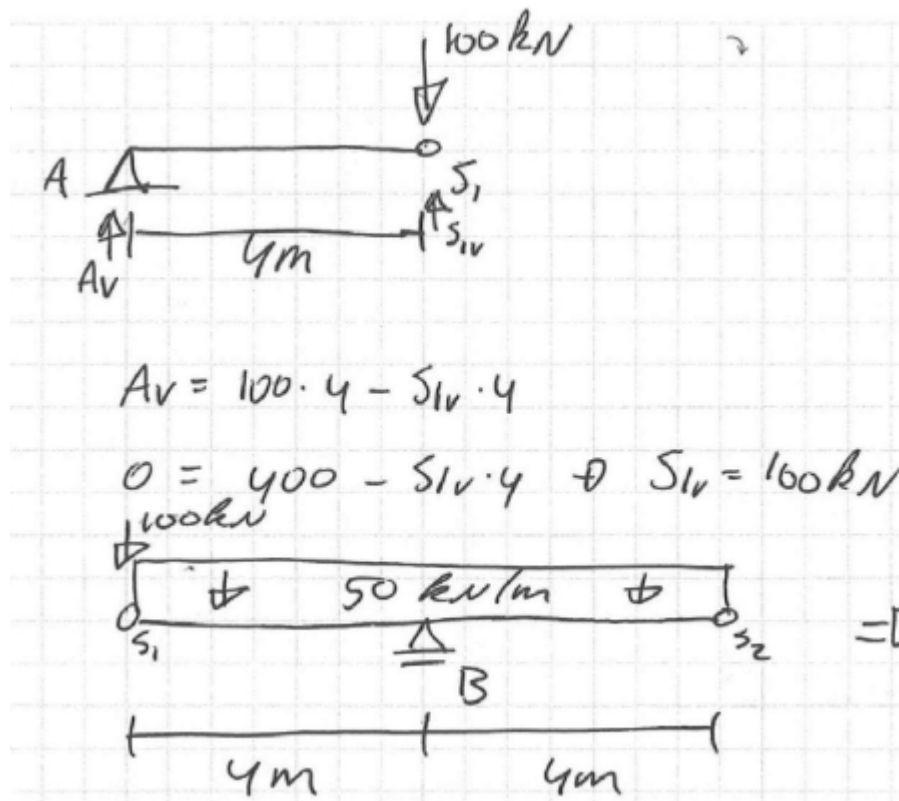
$$M_D = 400 \text{ kNm}$$

$$T_{\max} = 34 \text{ MPa}$$

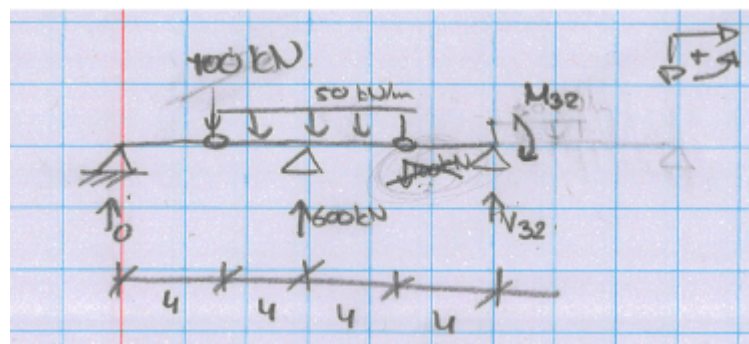
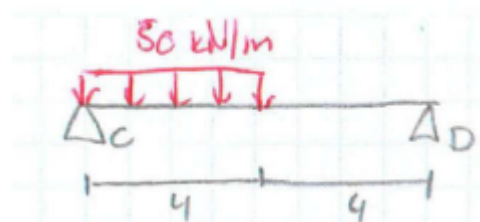
$$w_{s2} = 306 \text{ mm}$$

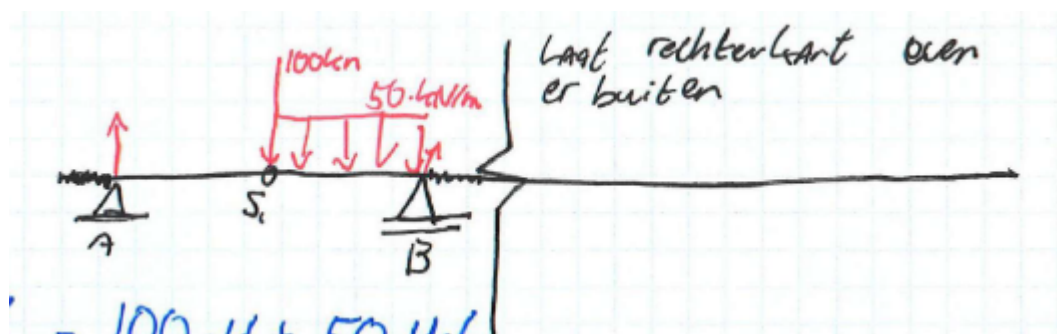
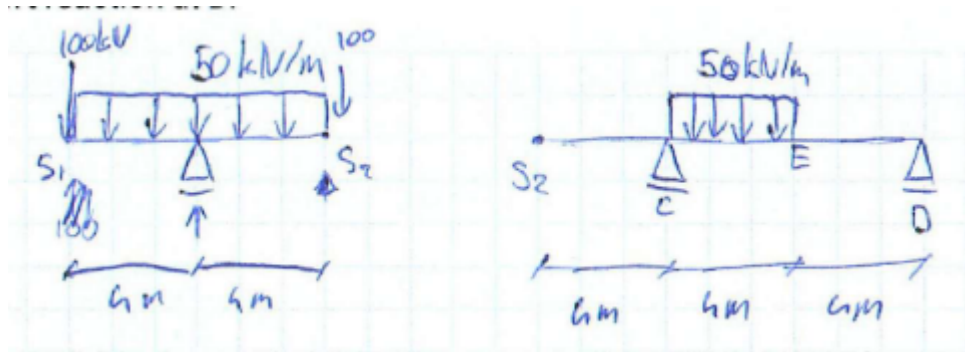
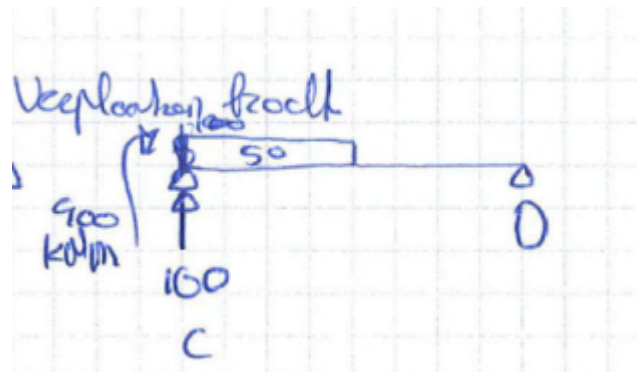
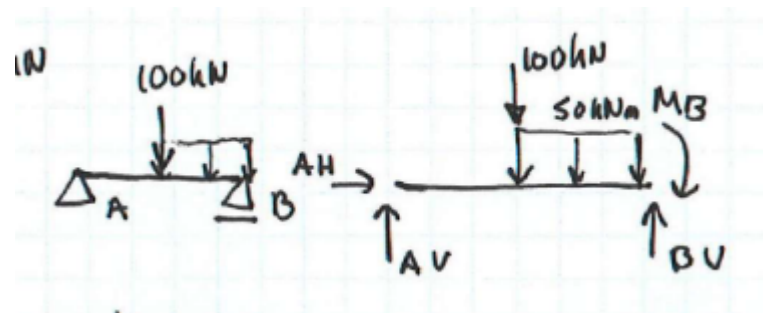
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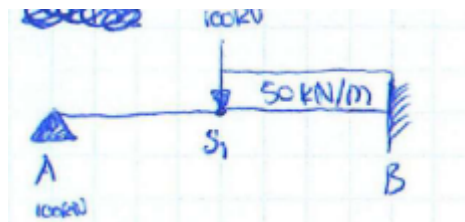
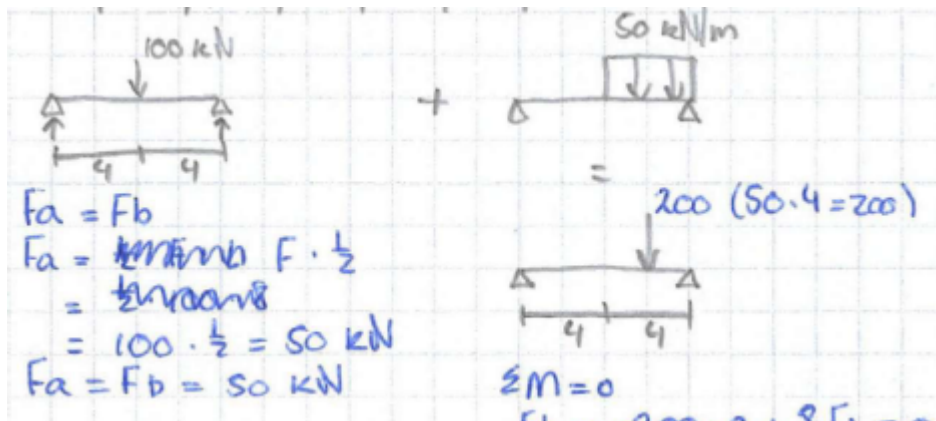


FBD of part of structure

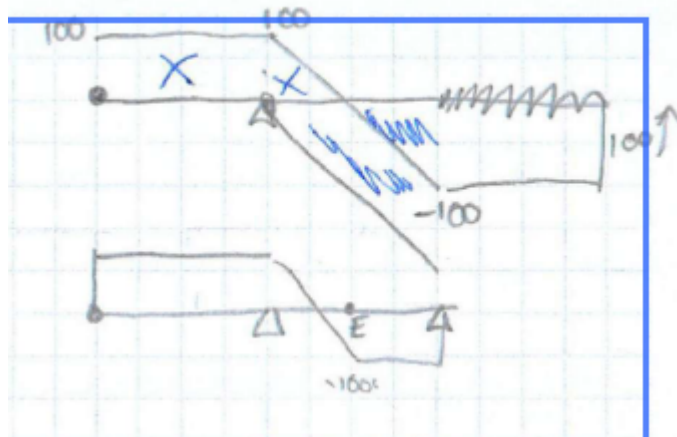


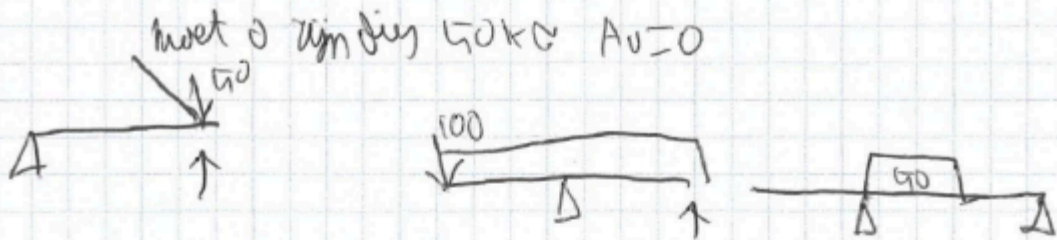


Wrong simplification



Starting with V-line





$$0 = 9 \cdot 0 - 100 \cdot 100 - 0.50 \cdot 4$$

$$S_2 = 100 \downarrow$$

$$V_B = \frac{2400}{4} = 600 \text{ kN}$$

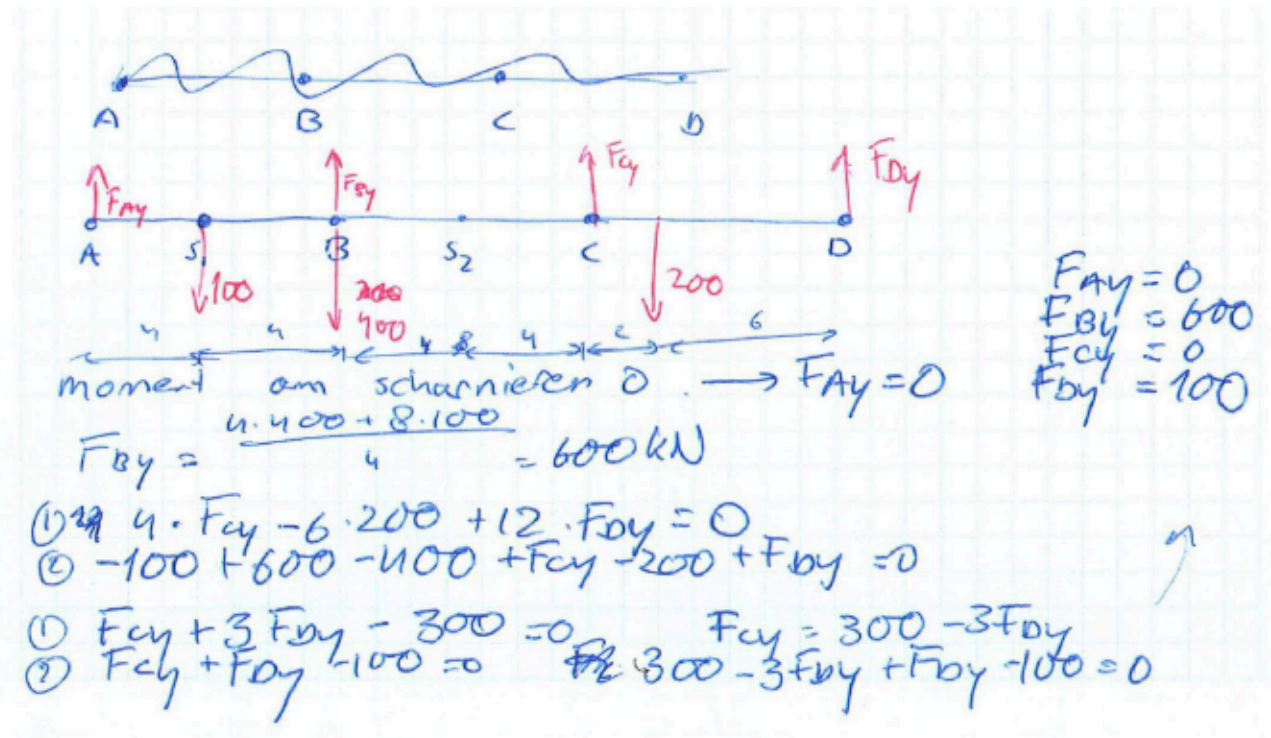


$$C \cdot 4 + 100 \cdot 4 - 50 \cdot 4 \cdot 6 = 0$$

$$C = \frac{2400}{4} = 600 \text{ kN} \quad V_C = 0$$

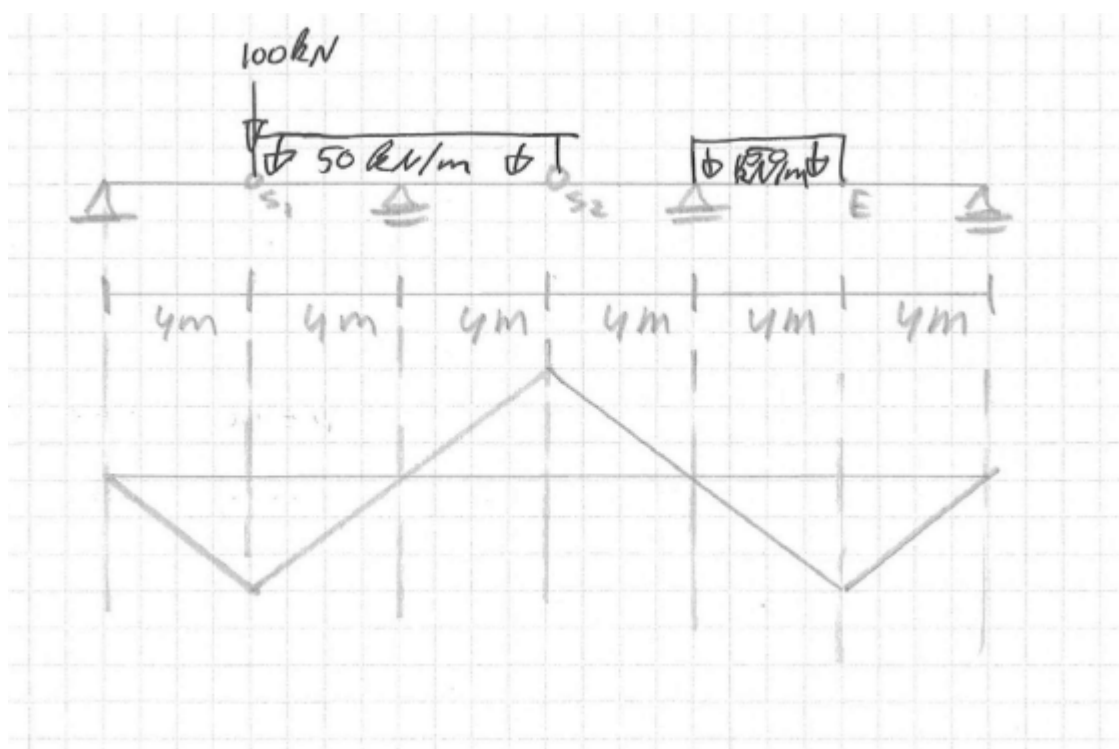
$$D_v = 100 \text{ kN}$$

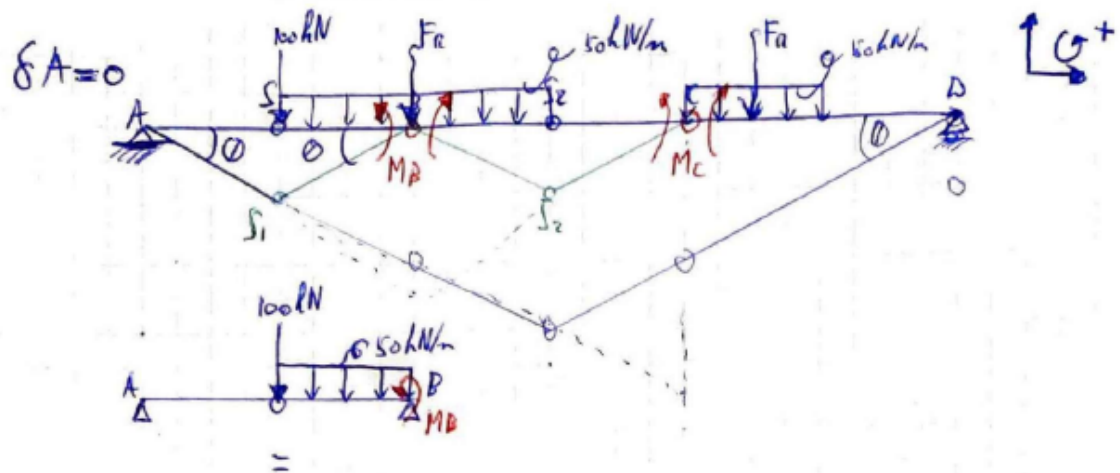
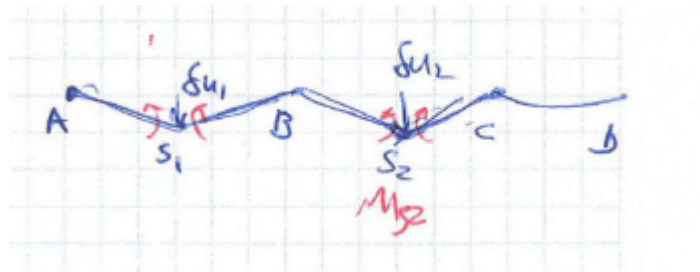
Missing equilibrium relations



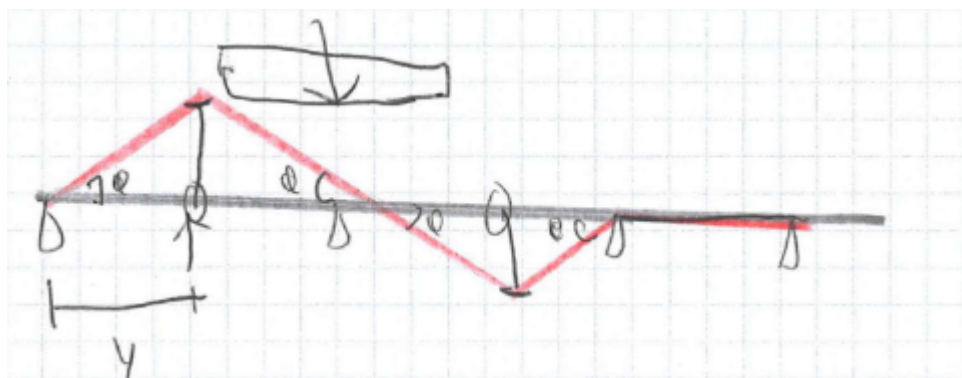
2.2 Determinate the bending moment in C using virtual work

Wrong mechanism

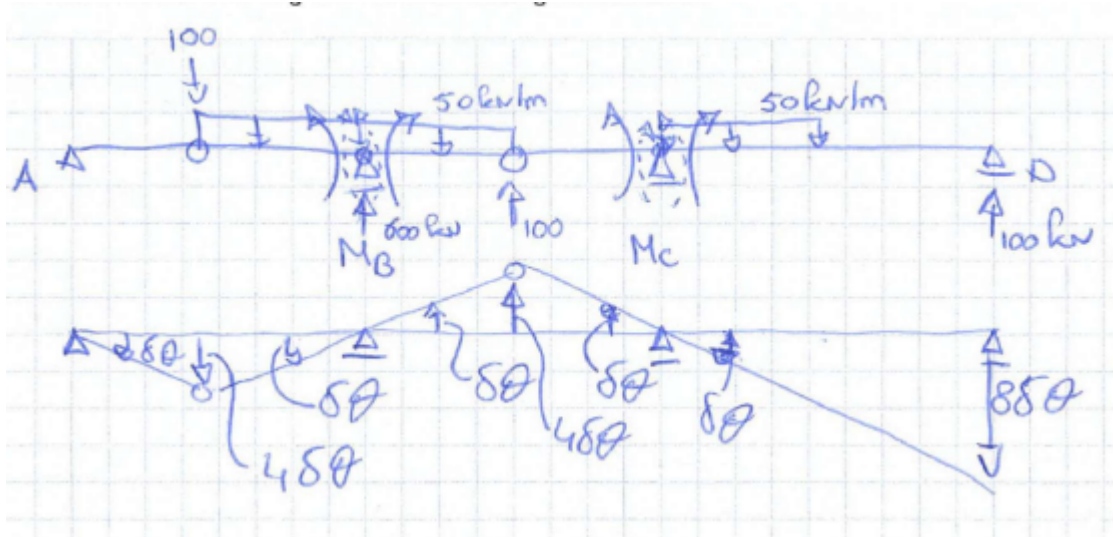




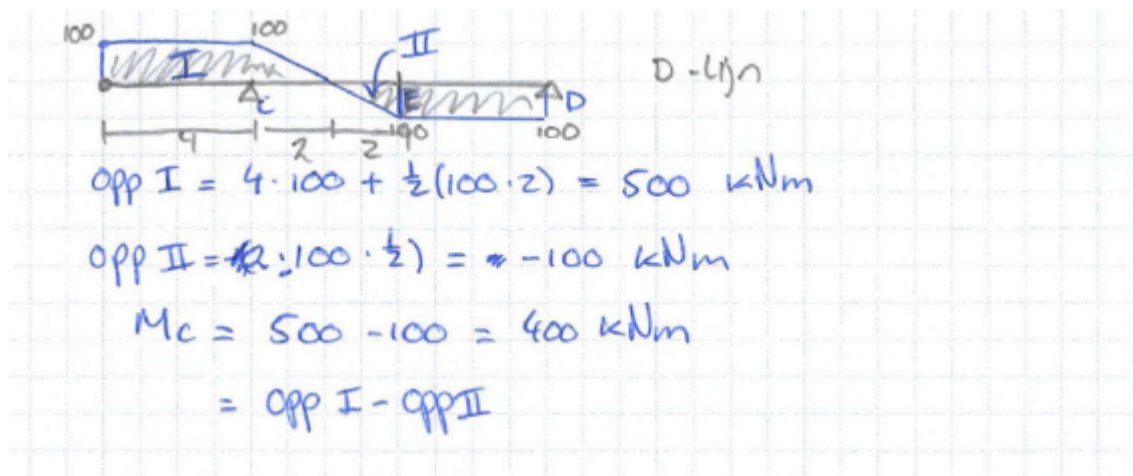
Mechanism incomplete

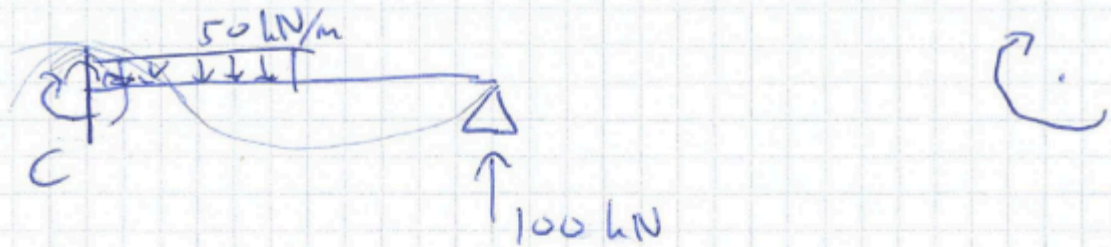


Displaced mechanism is not allowed



No virtual work





$$M_{\text{to } C} = 100 \cdot 8 - 50 \cdot 4 \cdot 2 = 400 \text{ kNm}$$

$$\sum \text{to } C = 0 \text{ check}$$

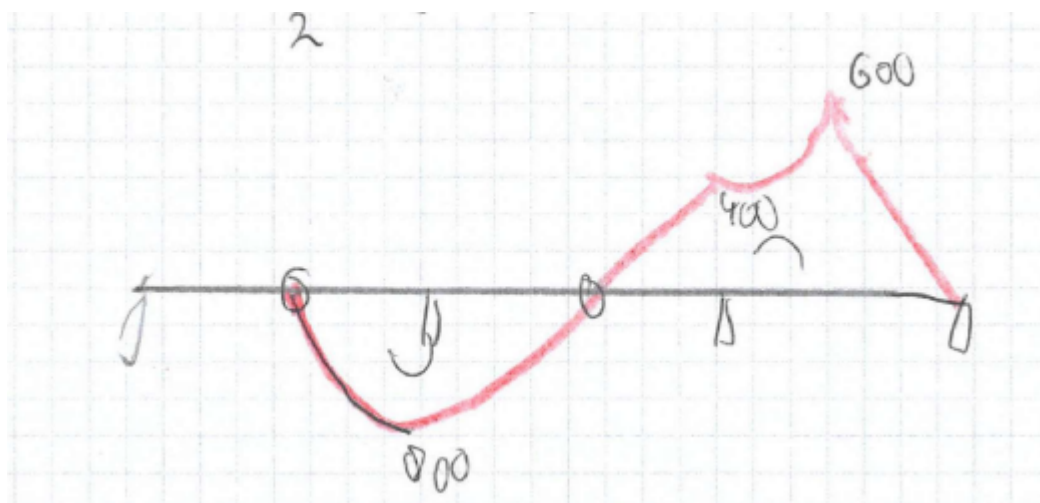
$$= 100 \cdot 8 - 50 \cdot 4 \cdot 2 - M_C = 0$$

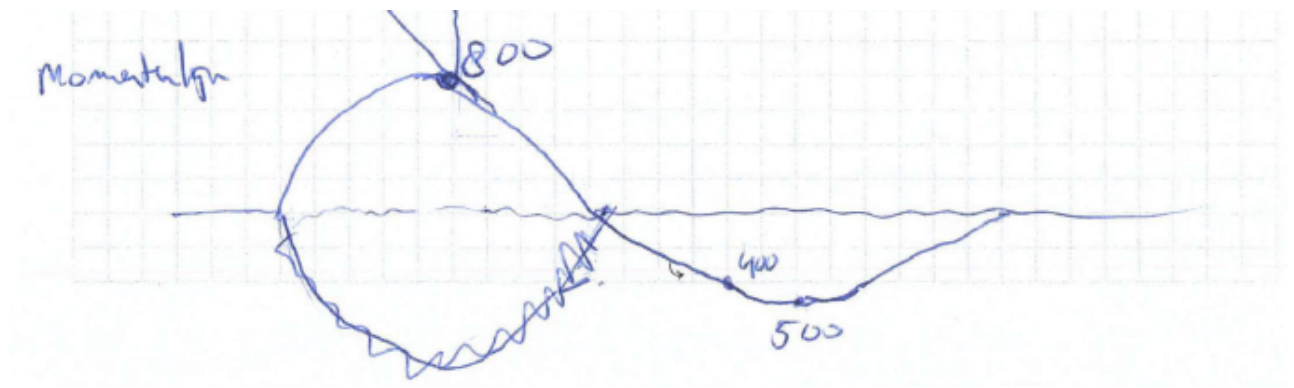
$$800 - 400 - M_C = 0$$

$$M_C = 400 \text{ kNm}$$

2.5 Draw the bending moment diagram

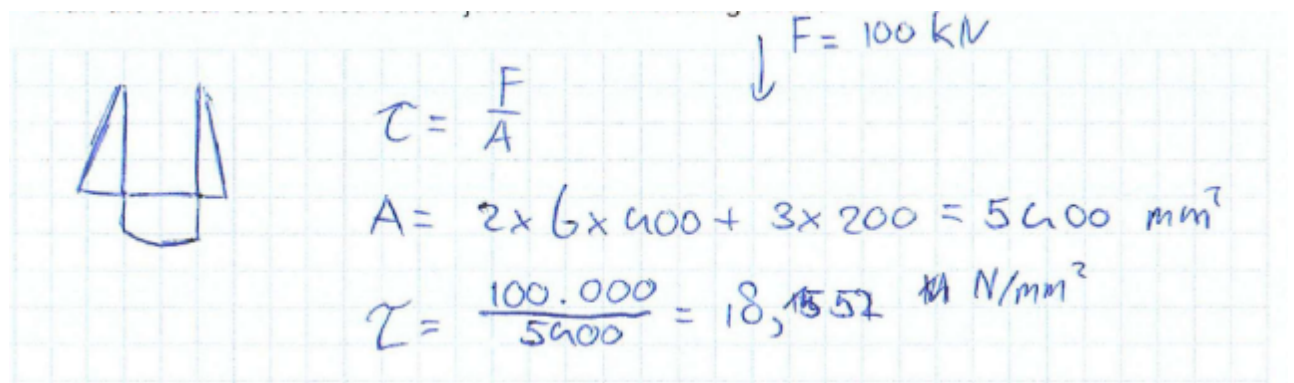
Bending moment diagram doesn't match loads





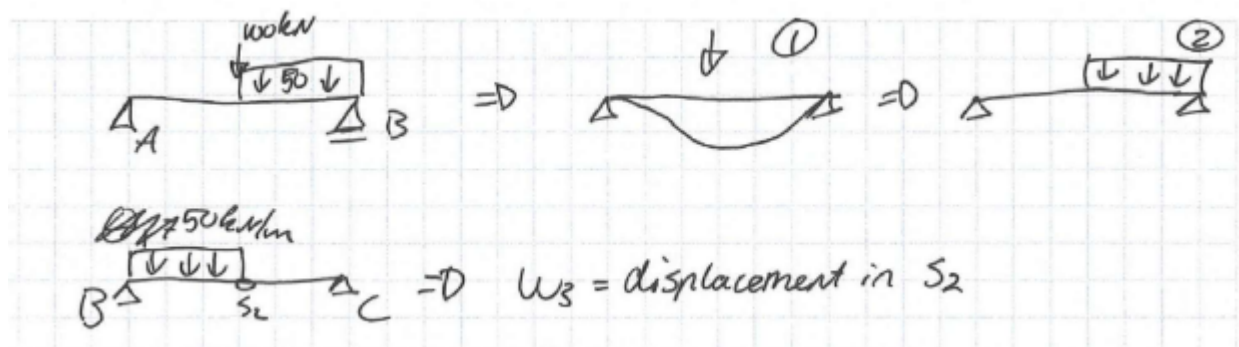
2.4 Draw the shear stress distribution just left of C

Wrong formula for τ



2.5 Determine the displacement at S_2

Forget-me-not does not apply



Week 2 (September 9th)

	Monday	Tuesday	Wednesday	Thursday	Friday	Upcoming homework
Homework deadlines			Read theory and make homework exercises Statics externally Prerequisites		Read theory and make homework exercises equilibrium relations and cables Prerequisites	More Read and homework exercises kinematics and statics Prerequisites
Class activities	Demonstration Differential equations New content		Guided practice on Statics externally Prerequisites		Guided practice on equilibrium relations and cables Prerequisites	

◀
▶

Lesson Monday September 9th

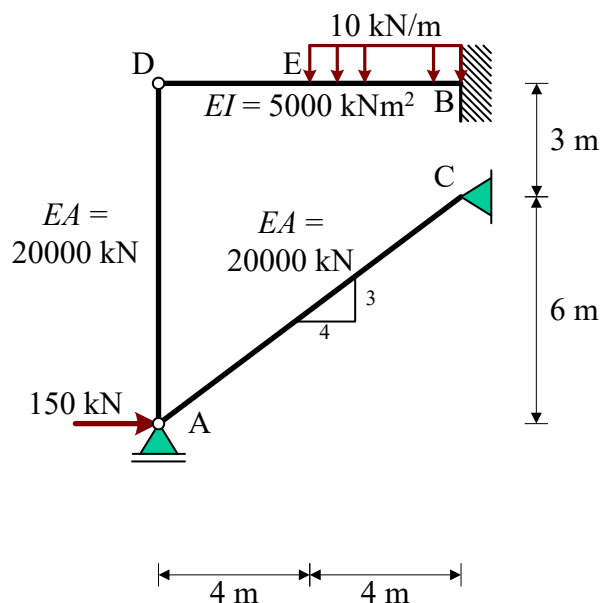
Contents

- Demonstration

During today's lesson it's demonstrated how you to use differential equations to solve structural problems.

Demonstration

Given a structure as shown below:



We can calculate the force distribution and displacements of this structure using differential equations.

First, let's define the segments with continuous loads. This requires splitting the top element DB into two because the distributed load causes a discontinuity. This leads to four segments:

-

-
-
- EB

Now, each segment can be drawn separately, indicating the loads, coordinate systems, dimensions and section forces at the edges of each segment.

Element AC

As element is hinged at both ends and is only loaded at its end, it's a [zero-force member](#). The free-body diagram therefore only includes normal forces:

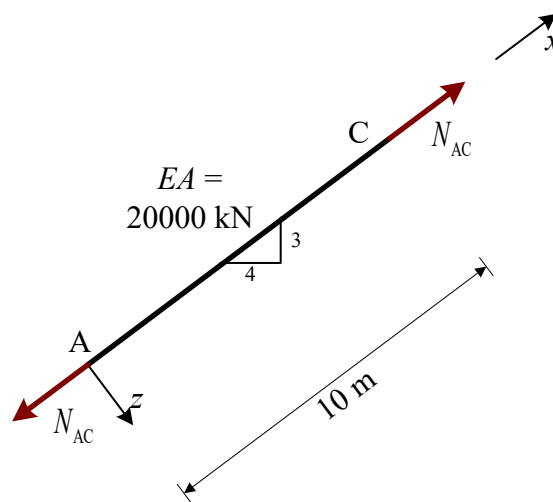


Fig. 1 Free body diagram element AC

As this element is unloaded, the loading equation for this element gives:

$$q_{x,AC} = 0$$

leading to the equations:

$$N_A(x) = C_1$$

$$u_A(x) = \frac{C_1 \cdot x + C_2}{20000}$$

Because of the support at C the first boundary condition is as follows:

$$u_A(10) = 0 \quad (1)$$

At C the normal force is unknown (equilibrium with support reactions).

At A the extension is unknown as A can move horizontally this leads to a potential displacement of A in the longitudinal direction of the element.

The normal force at A follows from the equilibrium of node A. It's free-body-diagram is:

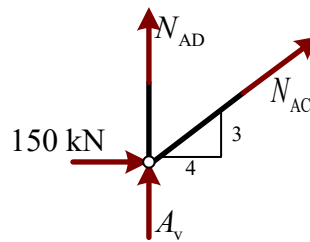


Fig. 2 Free body diagram node A

Horizontal equilibrium gives the second boundary condition:

$$\sum F_h|_A = 0 \rightarrow \frac{4}{5} \cdot N_{AC} - 150 = 0 \quad (2)$$

This element could be solved independently as there are two unknowns (two integration constants) and two equations ([\(1\)](#) and [\(2\)](#)).

Element AD

Again, this element is a [zero-force member](#). The free-body diagram therefore only includes normal forces:

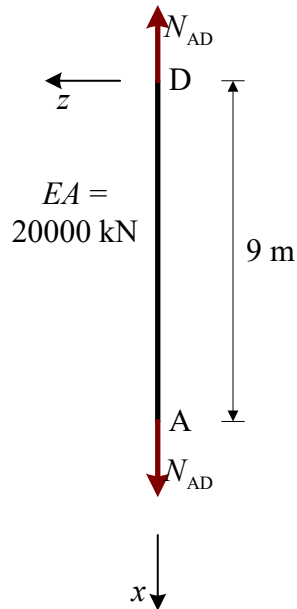


Fig. 3 Free body diagram element AD

As this element is unloaded too, the load equation for this element gives:

$$q_{x,AD} = 0$$

leading to the equations:

$$N_{AD}(x) = C_3$$

$$u_{AD}(x) = \frac{C_3 \cdot x + C_4}{20000}$$

Because of the support at A the first boundary condition is as follows:

$$u_{AD}(9) = 0 \quad (3)$$

At D another boundary condition can be identified from the consistency of displacements: the downwards displacement of the top of element must be the same as the downwards displacement of the left of element :

$$u_{AD}(0) = w_{DE}(0) \quad (4)$$

And finally, a boundary condition can be formulated as the normal force in D is in equilibrium with the forces from element . The free-body-diagram of D is:

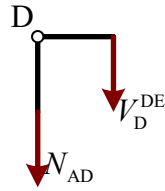


Fig. 4 Free body diagram node D

Vertical equilibrium gives:

$$\sum F_v|^\text{D} = 0 \rightarrow N_{AD} - V_D^{\text{DE}} = 0 \quad (5)$$

Element DE

The free-body diagram of this element is:

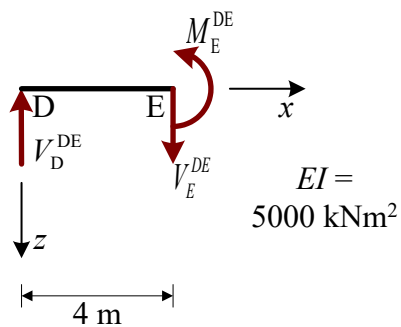


Fig. 5 Free body diagram element ED

As this element is unloaded, the load equation for this element gives:

$$q_{z,DE} = 0$$

leading to the equations:

$$V_D(x) = C_5$$

$$M_D(x) = C_5 \cdot x + C_6$$

$$\kappa_D(x) = \frac{C_5 \cdot x + C_6}{5000}$$

$$\varphi_{DE}(x) = \frac{\frac{1}{2} \cdot C_5 \cdot x^2 + C_6 \cdot x}{5000} + C_7$$

$$w_{DE}(x) = \frac{-\frac{1}{6} \cdot C_5 \cdot x^3 - \frac{1}{2} C_6 \cdot x^2}{5000} - C_7 \cdot x + C_8$$

Because of the hinge at D, a boundary condition can be formulated as:

$$M_{DE}(0) = 0 \quad (6)$$

At E another boundary condition can be identified from the consistency of displacements: both the downwards and rotational displacement of the right of element must be the same as the downwards and rotational displacement of the left of element :

$$w_{DE}(4) = w_{EB}(0) \quad (7)$$

$$\varphi_{DE}(4) = \varphi_{EB}(0) \quad (8)$$

Finally, there should be consistency with the section forces too, as shown in the free body diagram:

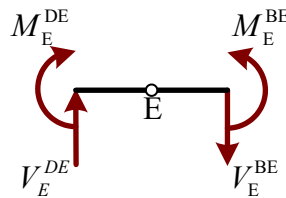


Fig. 6 Free body diagram node E

Vertical equilibrium gives:

$$\sum F_v|_E = 0 \rightarrow -V_E^{DE} + V_E^{BE} = 0 \quad (9)$$

Moment equilibrium gives:

$$\sum T|_E = 0 \rightarrow -M_E^{DE} + M_E^{BE} = 0 \quad (10)$$

Element BE

The free-body diagram of this element is:

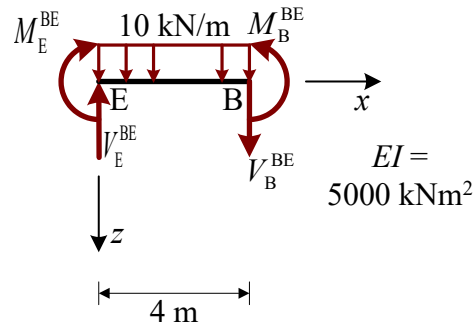


Fig. 7 Free body diagram element BE

As this element is loaded, the load equation for this element gives:

$$q_{z,BE} = 10$$

leading to the equations:

$$V_{BE}(x) = -10 \cdot x + C_9$$

$$M_{BE}(x) = -5 \cdot x^2 + C_9 \cdot x + C_{10}$$

$$\kappa_{BE}(x) = \frac{-5 \cdot x^2 + C_9 \cdot x + C_{10}}{5000}$$

$$\varphi_{BE}(x) = \frac{-\frac{5}{3} \cdot x^3 + \frac{1}{2} \cdot C_9 \cdot x^2 + C_{10} \cdot x}{5000} + C_{11}$$

$$w_{BE}(x) = \frac{\frac{5}{12} \cdot x^4 - \frac{1}{6} \cdot C_9 \cdot x^3 - \frac{1}{2} C_{10} \cdot x^2}{5000} - C_{11} \cdot x + C_{12}$$

Because of the support at D, two boundary conditions can be formulated directly:

$$\varphi_{BE}(4) = 0 \quad (11)$$

$$w_B(4) = 0$$

(12)

Solve system of equations

12 boundary conditions have been formulated with 12 integration constants. These can now be solved. You're advised to use your calculator or a symbolic programming tool to solve this.

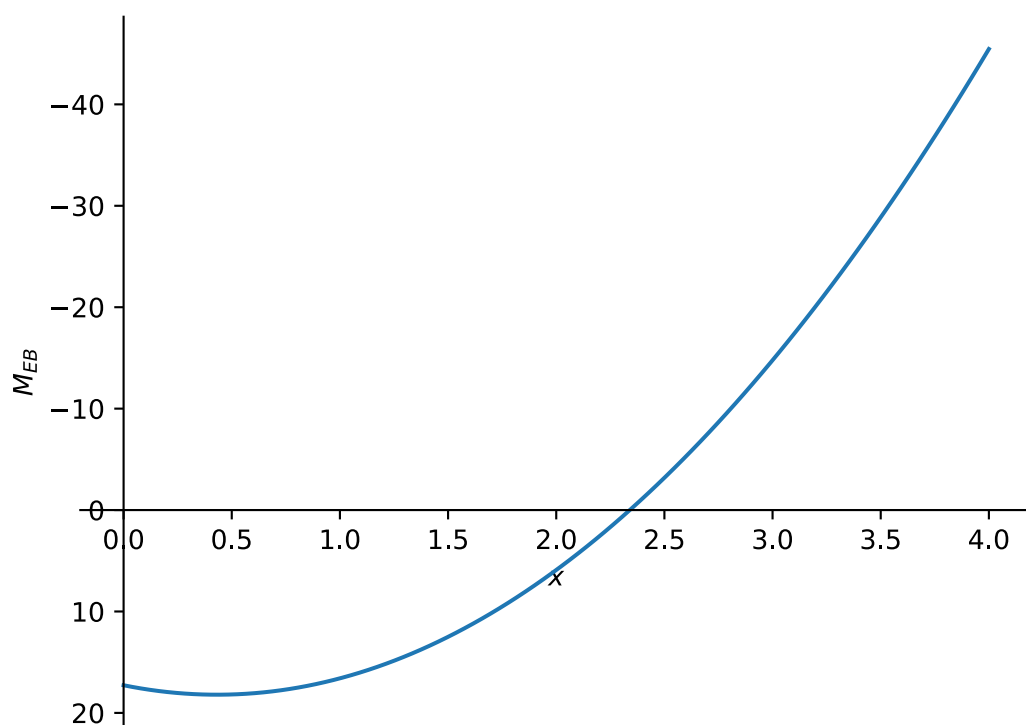
Filling in the boundary conditions gives:

$$\begin{aligned} \frac{C_1}{2000} + C_2 &= 0 \\ \frac{4C_1}{5} + 150 &= 0 \\ \frac{9C_3}{20000} + C_4 &= 0 \\ -C_4 + C_8 &= 0 \\ C_3 + C_5 &= 0 \\ C_6 &= 0 \\ C_{12} + \frac{4C_5}{1875} + \frac{C_6}{625} + 4C_7 - C_8 &= 0 \\ C_{11} - \frac{C_5}{625} - \frac{C_6}{1250} - C_7 &= 0 \\ -C_5 + C_9 &= 0 \\ C_{10} - 4C_5 - C_6 &= 0 \\ \frac{C_{10}}{1250} + C_{11} + \frac{C_9}{625} - \frac{8}{375} &= 0 \\ -\frac{C_{10}}{625} - 4C_{11} + C_{12} - \frac{4C_9}{1875} + \frac{8}{375} &= 0 \end{aligned}$$

Solving this gives:

$$\begin{aligned}
 C_1 &= -\frac{375}{2} \\
 C_2 &= \frac{3}{32} \\
 C_3 &= -\frac{1792}{415} \\
 C_4 &= \frac{504}{259375} \\
 C_5 &= \frac{1792}{415} \\
 C_6 &= 0 \\
 C_7 &= -\frac{4904}{778125} \\
 C_8 &= \frac{504}{259375} \\
 C_9 &= \frac{1792}{415} \\
 C_{10} &= \frac{7168}{415} \\
 C_{11} &= \frac{472}{778125} \\
 C_{12} &= \frac{2792}{155625}
 \end{aligned}$$

Now, the full force distribution and displacements of the structure has been solved. For example the moment distribution in is $-5 \cdot x^2 + \frac{1792}{415} \cdot x + \frac{7168}{415}$ which can be plotted as:



Solve using digital tools

To solve the system of equations using numerical tools, you can write down the equations as a matrix formulation $Ax = b$ which can easily be solved by ie. your graphical calculator or python.

$$\begin{bmatrix} 0 & 0 & \frac{1}{2000} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{4}{5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{9}{20000} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & \frac{3}{5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{4}{1875} & \frac{1}{625} & 4 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{625} & -\frac{1}{1250} & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -4 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{625} & \frac{1}{1250} & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \\ C_8 \\ C_{10} \\ C_{11} \\ C_{12} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

```
np.linalg.solve(A,b)
```

```
array([[ -1.87500000e+02],
       [  9.37500000e-02],
       [-4.31807229e+00],
       [ 1.94313253e-03],
       [ 4.31807229e+00],
       [ 0.00000000e+00],
       [-6.30232932e-03],
       [ 1.94313253e-03],
       [ 4.31807229e+00],
       [ 1.72722892e+01],
       [ 6.06586345e-04],
       [ 1.79405622e-02]])
```

Alternatively, a symbolic programming language can be used to solve the equations exactly. As an example the [sympy](#) package in [Python](#) is used:

```

import sympy as sym
sym.init_printing()

x = sym.symbols('x')

q = 10
EI = 5000
EA = 20000
F = 150

q_AC = 0
q_AD = 0
q_CD = 0
q_DE = 0
q_EB = q

C_1, C_2 = sym.symbols('C_1 C_2')
C_3, C_4 = sym.symbols('C_3 C_4')
C_5, C_6, C_7, C_8 = sym.symbols('C_5 C_6 C_7 C_8')
C_9, C_10, C_11, C_12 = sym.symbols('C_9 C_10 C_11 C_12')

N_AC = -sym.integrate(q_AC, x) + C_1
eps_AC = N_AC/EA
u_AC = sym.integrate(eps_AC, x) + C_2

N_AD = -sym.integrate(q_AD, x) + C_3
eps_AD = N_AD/EA
u_AD = sym.integrate(eps_AD, x) + C_4

V_DE = -sym.integrate(q_DE, x) + C_5
M_DE = sym.integrate(V_DE, x) + C_6
kappa_DE = M_DE/EI
phi_DE = sym.integrate(kappa_DE, x) + C_7
w_DE = -sym.integrate(phi_DE, x) + C_8

V_EB = -sym.integrate(q_EB, x) + C_9
M_EB = sym.integrate(V_EB, x) + C_10
kappa_EB = M_EB/EI
phi_EB = sym.integrate(kappa_EB, x) + C_11
w_EB = -sym.integrate(phi_EB, x) + C_12

eq1 = sym.Eq(u_AC.subs(x,10),0)
eq2 = sym.Eq(F + N_AC.subs(x,0)/5*4,0)
eq3 = sym.Eq(u_AD.subs(x,9),0)
eq4 = sym.Eq(w_DE.subs(x,0)-u_AD.subs(x,0),0)
eq5 = sym.Eq(V_DE.subs(x,0) + N_AD.subs(x,0),0)
eq6 = sym.Eq(M_DE.subs(x,0),0)
eq7 = sym.Eq(w_EB.subs(x,0)-w_DE.subs(x,4),0)
eq8 = sym.Eq(phi_EB.subs(x,0)-phi_DE.subs(x,4),0)
eq9 = sym.Eq(V_EB.subs(x,0) - V_DE.subs(x,4),0)
eq10 = sym.Eq(M_EB.subs(x,0) - M_DE.subs(x,4),0)
eq11 = sym.Eq(phi_EB.subs(x,4),0)
eq12 = sym.Eq(w_EB.subs(x,4),0)

sol = sym.solve([eq1,eq2,eq3,eq4,eq5,eq6,eq7,eq8,eq9,eq10,eq11,eq12],(C_1,C_2,C_3,C_4,C_5
display(sol)

```

$$\left\{ C_1 : -\frac{375}{2}, C_{10} : \frac{7168}{415}, C_{11} : \frac{472}{778125}, C_{12} : \frac{2792}{155625}, C_2 : \frac{3}{32}, C_3 : -\frac{1792}{415}, C_4 : \frac{504}{259375}, C_5 : \frac{1792}{415}, C_6 : 0, C_7 : -\frac{4904}{778125}, C_8 : \frac{504}{259375}, C_9 : \frac{1792}{415} \right\}$$

Homework deadline lesson

Wednesday September 11th

As a preparation for Wednesday's lesson you'll look into basic statics externally, consisting of the following topics:

Model

Elements

Coordinate system

Loads

Section forces

Supports

Hinges

Free body diagrams

Structures

Resolution of forces

Solve equilibrium

... of a planar particle

... of a 3D-particle

... of a body

... of a 3D-body

Solve support reactions for statically determinate structures

... using equilibrium

Solve section forces truss structure

... with zero-force members

... using equilibrium in a specific cross-section

Solve section forces frame structures

... using equilibrium in a specific cross-section

Please go through the topics which you can improve your skills on. Take at least a look at the examples and try a few exercises. During the lesson you'll make some advanced exercises.

Lesson Wednesday September 11th

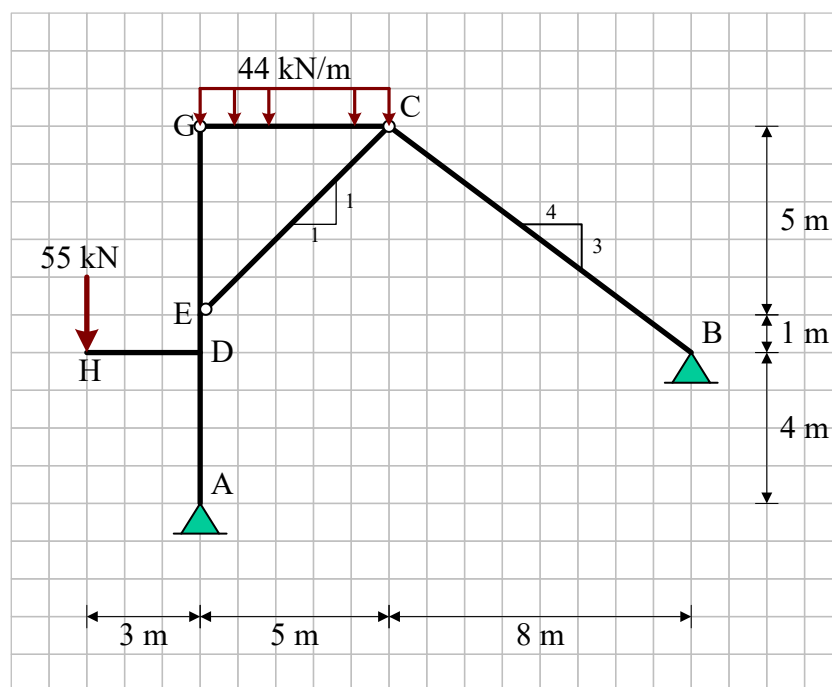
Contents

- Exercise Statics externally

During today's lesson you'll work on a complex exercise on the topic of 'Statics externally'. Please ask your questions regarding the [homework](#) as well!

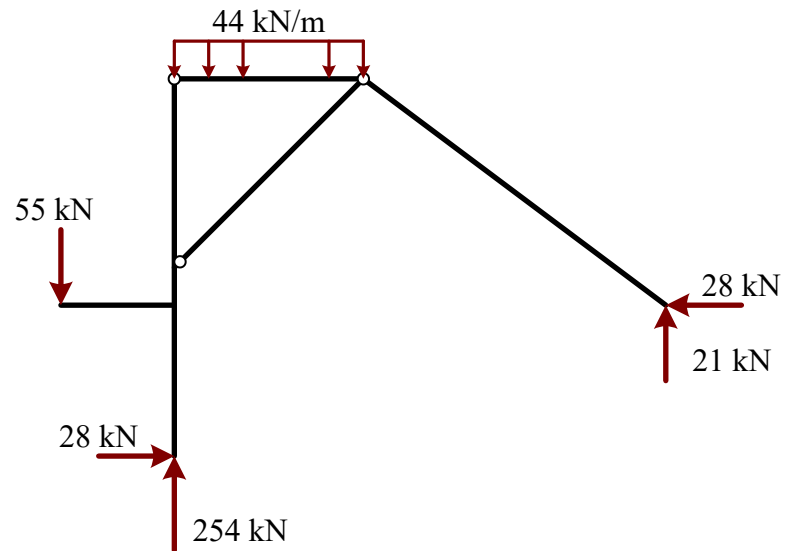
Exercise Statics externally

Given the following structure:



1. Find the support reactions
2. Find the bending moment, shear force and normal force just above D
3. Find the bending moment, shear force and normal force just left of C

💡 Solution assignment 1



💡 Solution assignment 2

- $M = 277 \text{ kNm}$ $\curvearrowright$
- $V = 28 \text{ kN}$ $\downarrow$
- $N = -199 \text{ kN}$

💡 Solution assignment 3

- $M = 0 \text{ kNm}$
- $V = 110 \text{ kN}$ $\uparrow$
- $N = +61 \text{ kN}$

Homework deadline lesson

Friday September 13th

As a preparation for Friday's lesson you'll look into equilibrium relations and cables, consisting of the following topics:

[Differential equations for equilibrium relations](#)

[... for extension](#)

[... for bending](#)

[... for cables](#)

[Solve section forces frame structures](#)

[... using differential equations](#)

[Solve section forces and shape cable](#)

[... using equilibrium in a specific cross-section](#)

[... using differential equations](#)

Please go through the topics which you can improve your skills on. Take at least a look at the examples and try a few exercises. During the lesson you'll make some advanced exercises.

Lesson Friday September 13th

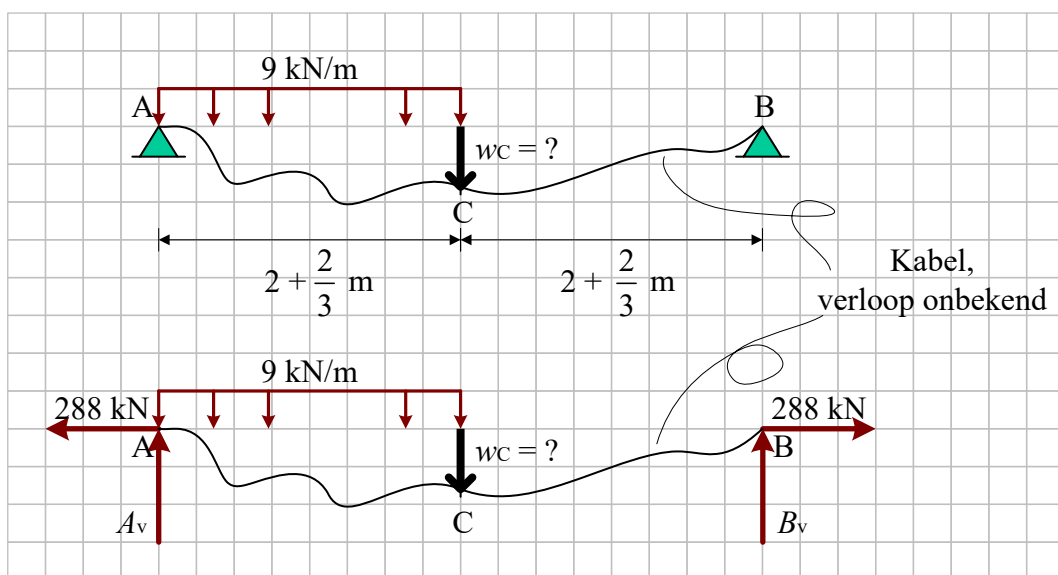
Contents

- Exercise Equilibrium relations and cables

During today's lesson you'll work on a complex exercise on the topic of Equilibrium relations and cables. Please ask your questions regarding the [homework](#) as well!

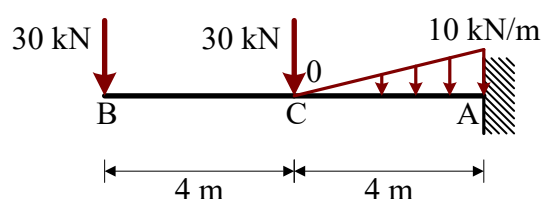
Exercise Equilibrium relations and cables

Given is the following structure:



1. Find the displacement halfway using equilibrium in a specific cross-section
2. Find the displacement halfway using the differential equations

Given is the following structure:

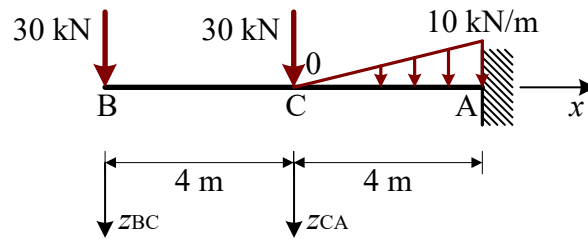


3. Find the moment distribution for the beam.

💡 **Solution assignment 1 and 2**

$$w_C = \frac{1}{18} \approx 0.0556 \text{ m} \downarrow$$

💡 **Solution assignment 3**



$$M_{BC} (x_{BC}) = -30x_{BC} \text{ (kNm)}$$

$$M_{CA} (x_{CA}) = -\frac{5}{12}x_{CA}^3 - 60.0x_{CA} - 120 \text{ (kNm)}$$

Week 3 (September 16th)

	Monday	Tuesday	Wednesday	Thursday	Friday	Upcoming homework
Homework deadlines	Read theory and make homework exercises kinematic and constitutive relations Prerequisites		Read theory and make homework exercises SymPy New content			Mon Read and r home exerc Exter Prereq
Class activities	Guided practice on kinematic and constitutive relations Prerequisites		Guided practice on SymPy New content		Demonstration Force method extension New content	



Homework deadline lesson

Monday September 16th

As a preparation for Monday's lesson you'll look into kinematic and constitutive relations, consisting of the following topics:

Model

Normal stress-strain relations

Fibre model

Strain diagram

... for extension

... for bending

Kinematic relations

... for extension

... for bending

Constitutive relations

... for extension

... for bending

Solve displacements truss structures

... using differential equations

Solve displacements frame structures

... using differential equations

Please go through the topics which you can improve your skills on. Take at least a look at the examples and try a few exercises. During the lesson you'll make some advanced exercises.

Lesson Monday September 16th

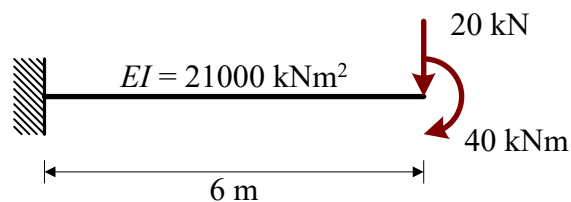
Contents

- Exercise Euler-Bernoulli

During today's lesson you'll work on a complex exercise on the topic of 'Euler-Bernoulli'. Please ask your questions regarding the [homework](#) as well!

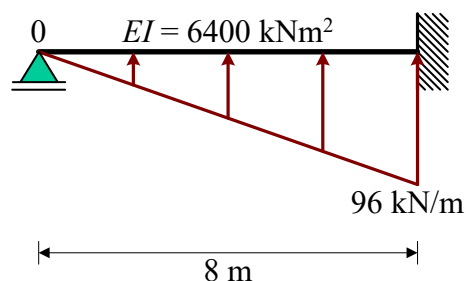
Exercise Euler-Bernoulli

Given the following structure.



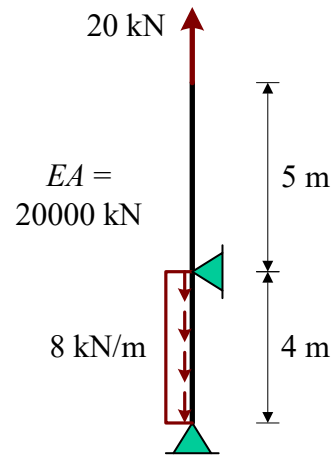
1. Use the Euler-Bernoulli equations to find the displacement halfway.

Given the following structure.



2. Use the Euler-Bernoulli equations to find the rotation at the left support

Given the following structure.



3. Use the Euler-Bernoulli equations to find the displacement of the top of the structure.

💡 **Solution assignment 1** ^

0.03 m ↓

💡 **Solution assignment 2** ^

0.064 rad ↻

💡 **Solution assignment 3** ^

0.005 m ↑

Homework deadline lesson

Wednesday September 18th

As a preparation for Wednesday's lesson you'll look into differential equations, consisting of the following topics:

[SymPy in general](#)

[SymPy for solving Euler-Bernoulli](#)

Please go through the topics which you can improve your skills on. Take at least a look at the examples and try a few exercises. During the lesson you'll make some advanced exercises.

Lesson Wednesday September 18th

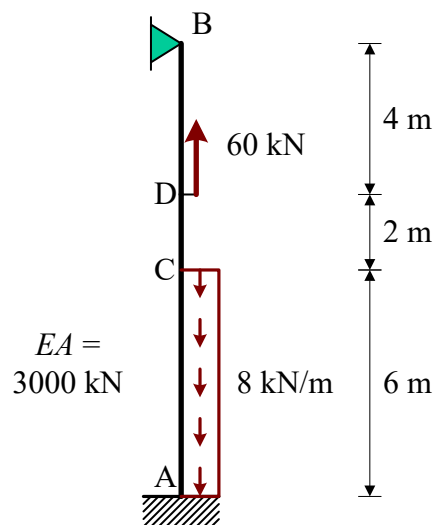
Contents

- Exercise SymPy

During today's lesson you'll work on a complex exercise on the topic of SymPy. Please ask your questions regarding the [homework](#) as well!

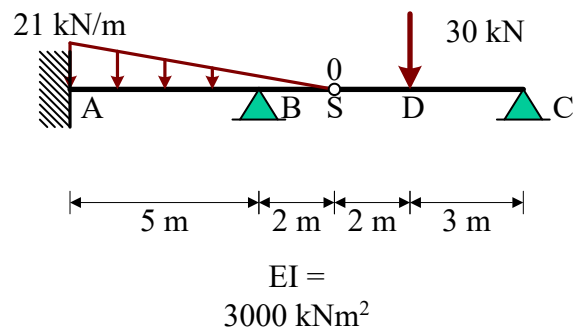
Exercise SymPy

Given the following structure.



1. Find the normal force distribution

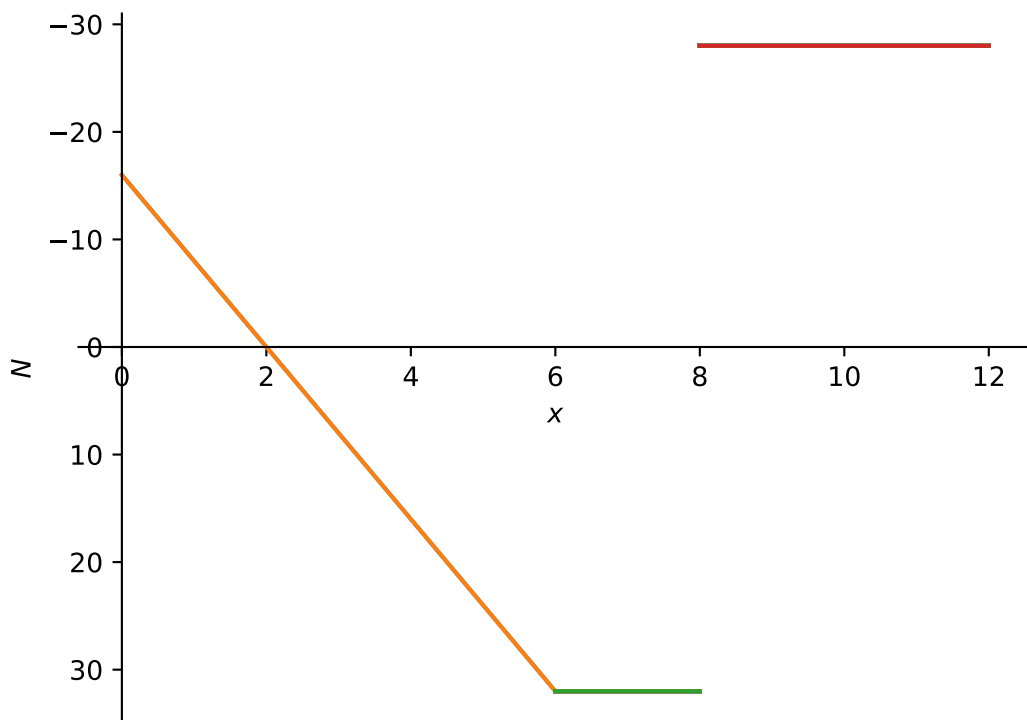
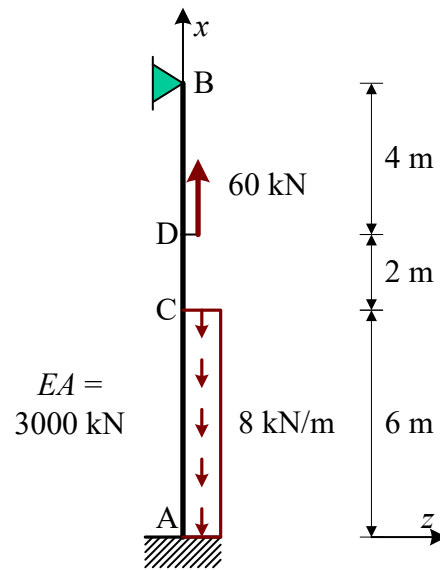
Given the following structure.



2. Find the bending moment distribution
3. Find the shear force distribution
4. Find the displacement distribution

💡 Solution assignment 1

Given the following coordinate system:



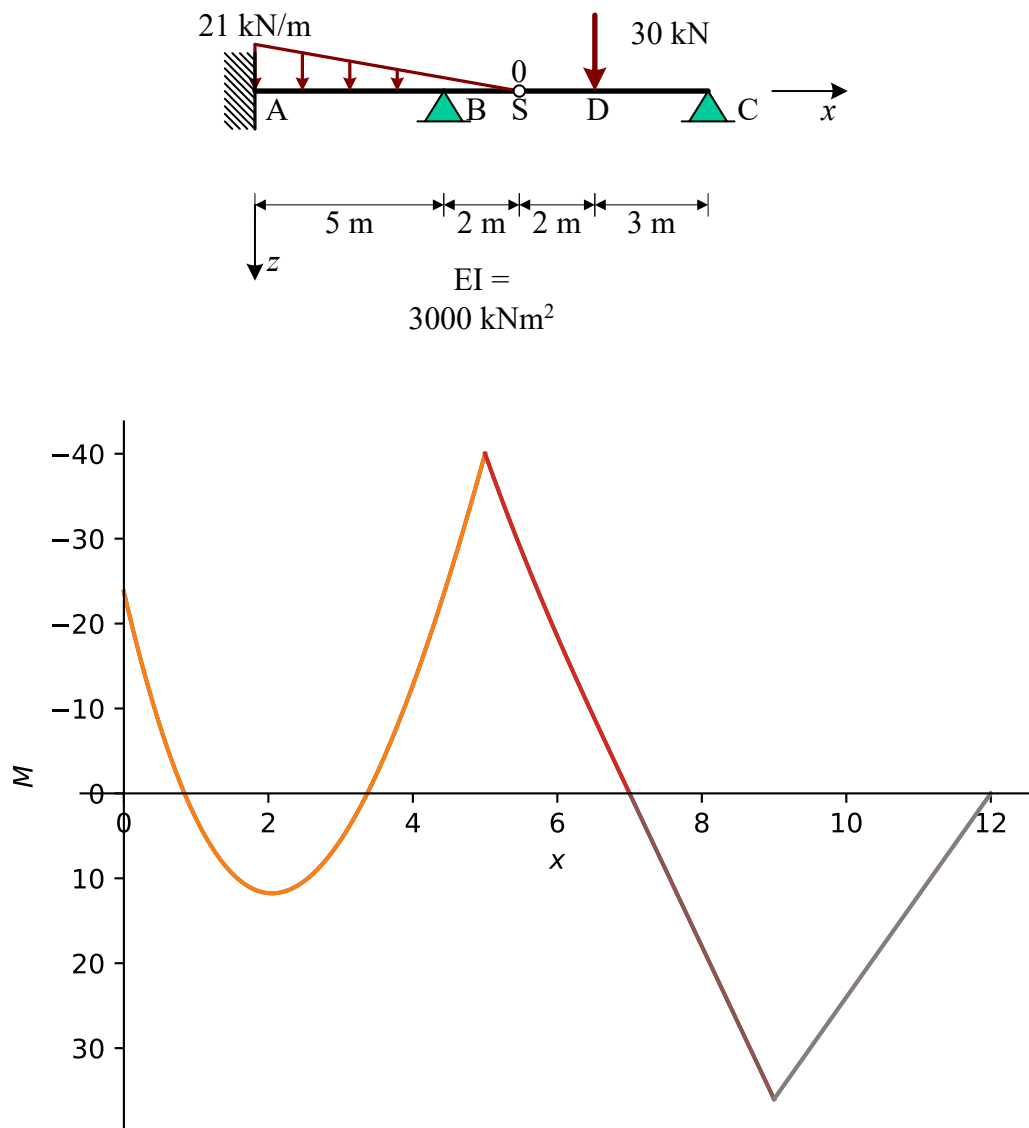
$$N_{AC} = 8x - 16 \text{ (kN)}$$

$$N_{CD} = 32 \text{ (kN)}$$

$$N_{DB} = -28 \text{ (kN)}$$

💡 Solution assignment 2

Given the following coordinate system:



$$M_{AB} = \frac{x^3}{2} - \frac{21x^2}{2} + \frac{147x}{4} - \frac{95}{4} \text{ (kNm)}$$

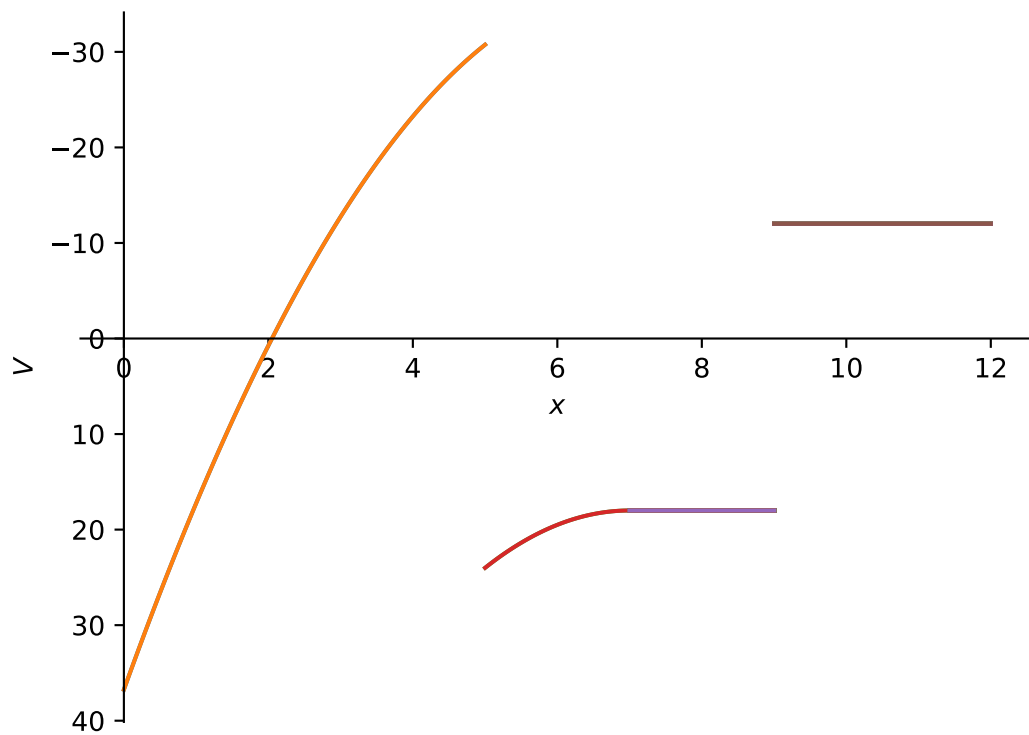
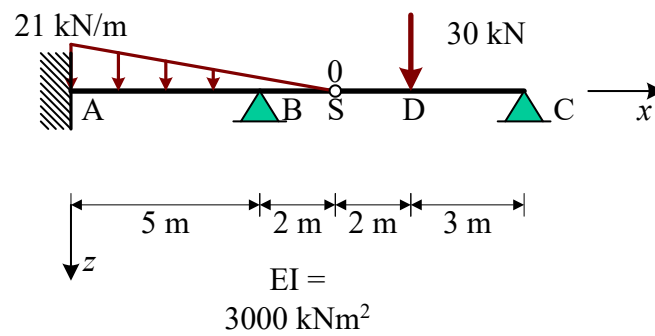
$$M_{BS} = \frac{x^3}{2} - \frac{21x^2}{2} + \frac{183x}{2} - \frac{595}{2} \text{ (kNm)}$$

$$M_{SD} = 18x - 126 \text{ (kNm)}$$

$$M_{DC} = 144 - 12x \text{ (kNm)}$$

💡 Solution assignment 3

Given the following coordinate system:



$$V_{AB} = \frac{3x^2}{2} - 21x + \frac{147}{4} \text{ (kN)}$$

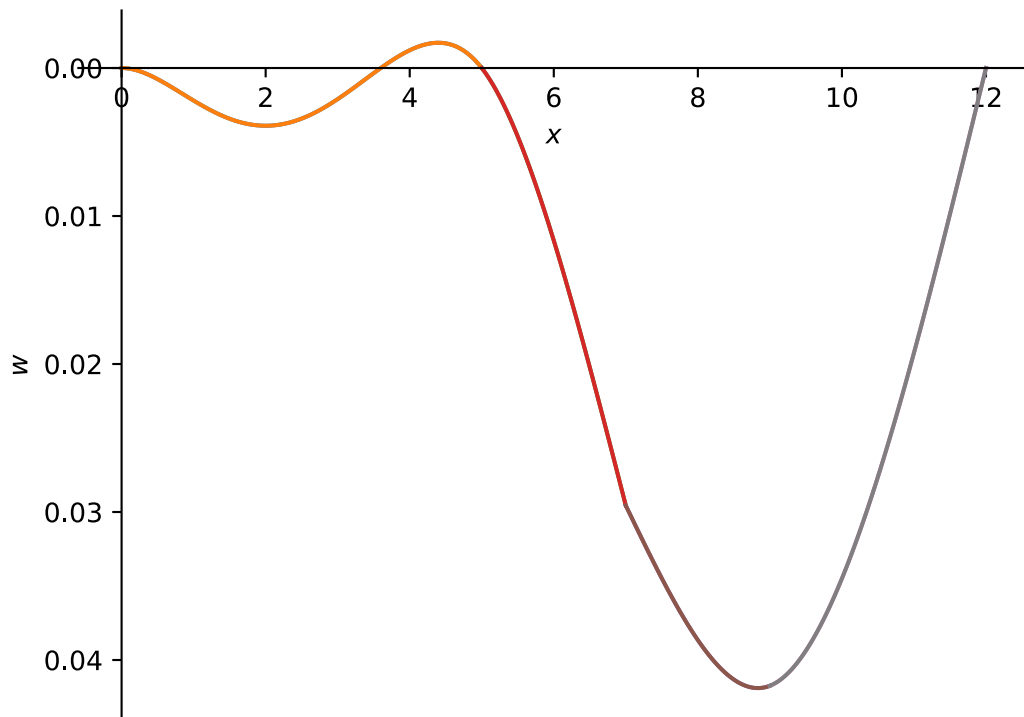
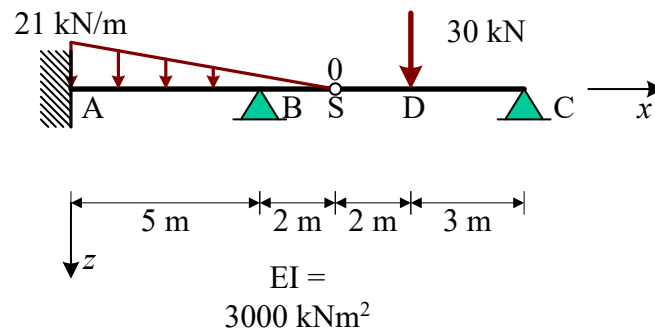
$$V_{BS} = \frac{3x^2}{2} - 21x + \frac{183}{2} \text{ (kN)}$$

$$V_{SD} = 18 \text{ (kN)}$$

$$V_{DC} = -12 \text{ (kN)}$$

💡 Solution assignment 4

Given the following coordinate system:



$$w_{AB} = -\frac{x^5}{120000} + \frac{7x^4}{24000} - \frac{49x^3}{24000} + \frac{19x^2}{4800} \text{ (m)}$$

$$w_{BS} = -\frac{x^5}{120000} + \frac{7x^4}{24000} - \frac{61x^3}{12000} + \frac{119x^2}{2400} - \frac{73x}{320} + \frac{73}{192} \text{ (m)}$$

$$w_{SD} = -\frac{x^3}{1000} + \frac{21x^2}{1000} - \frac{20537x}{150000} + \frac{7549}{25000} \text{ (m)}$$

$$w_{DC} = \frac{x^3}{1500} - \frac{3x^2}{125} + \frac{40213x}{150000} - \frac{11413}{12500} \text{ (m)}$$

If you don't have Python and SymPy installed, click  -> [Live Code](#) to activate live coding and use the cells below:

```
import sympy as sym
sym.init_printing()
```

```
x = sym.symbols('x')
C1, C2, C3, C4 = sym.symbols('C1 C2 C3 C4')
```

```
q =
```

```
V = sym.integrate(-q,x)+C1
M = sym.integrate(V,x)+C2
kappa = M / EI
phi = sym.integrate(kappa,x)+C3
w = sym.integrate(-phi,x)+C4
```

```
Eq1 = sym.Eq(
```

```
sol = sym.solve((Eq1,Eq2,Eq3,Eq4,Eq5,Eq6,Eq7,Eq8,Eq9,Eq10,Eq11,Eq12), (C1,C2,C3,C4,C5,C6,
display(sol)
```

```
M.subs(sol)
```

```

import numpy as np
import matplotlib.pyplot as plt

%config InlineBackend.figure_formats = ['svg']

def plot(w_list, x_range_list, ylabel):
    plt.figure()
    for i, w in enumerate(w_list):
        # check if x is the only symbol in the expression
        if len(w.free_symbols) > 1:
            raise ValueError('The expression must be a function of x only.')

        w_numpy = sym.lambdify(x, w)
        x_vals = np.linspace(x_range_list[i][0], x_range_list[i][1], 100)

        # if the expression is a constant, we need to make sure that it is broadcasted co
        if isinstance(w_numpy(x_vals), float) or isinstance(w_numpy(x_vals), int):
            w_numpy = np.vectorize(w_numpy)
            plt.plot([x_range_list[i][0], x_range_list[i][1]], [w_numpy(x_vals), w_numpy(x_
        else:
            plt.plot(x_vals, w_numpy(x_vals))

        plt.plot(x_vals, w_numpy(x_vals))
        plt.xlabel('$x$')
        plt.ylabel(ylabel)
    ax = plt.gca()
    ax.spines['right'].set_color('none')
    ax.spines['top'].set_color('none')
    ax.spines['bottom'].set_position('zero')
    ax.spines['left'].set_position('zero')
    ax.invert_yaxis()

```

plot(

Lesson Friday September 20th

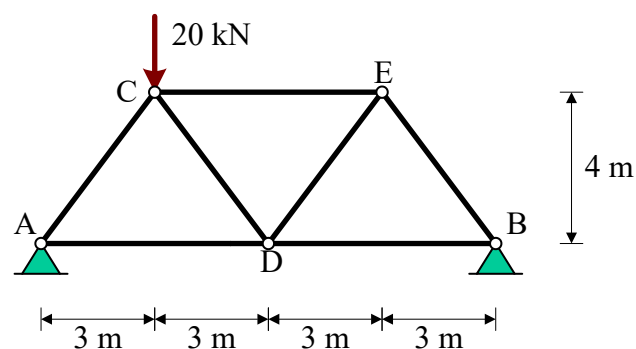
Contents

- Demonstration
- Define statically determinate structure with compatibility condition
- Solve displacements statically determinate structure in terms of statically indeterminate force
- Solve statically indeterminate structure with compatibility conditions

During today's lesson it's demonstrated how you to use the force method for statically indeterminate problems which only involve extension

Demonstration

Given a structure as shown below:



$$EA = 3750 \text{ kN}$$

Fig. 33 Structure

We can find the normal force distribution using the force method. Therefore, we need to know the degree of statical determinacy. The free-body-diagram of the full structure is as follows:

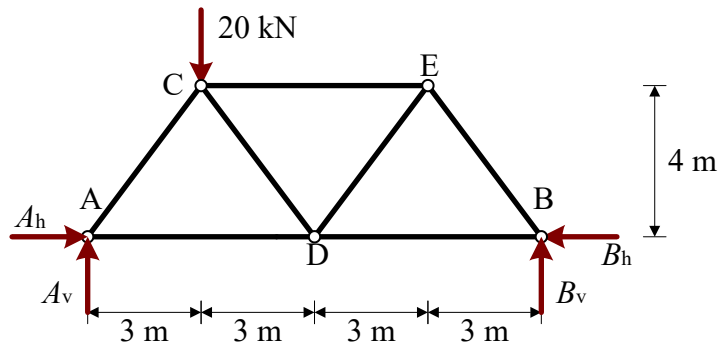


Fig. 34 Free-body-diagram full structure

There are 4 unknown support reactions. With only 3 equilibrium equations for this self-contained structure, it can be concluded that this structure is a first-order statically determinant structure.

To apply the force method we need to replace a part of the structure by a statically indeterminate force, which turns the structure into a statically determinate structure. To solve the statically indeterminate structure we need to include a compatibility condition.

Define statically determinate structure with compatibility condition

In this structure, we choose to replace the right support with a rolling hinged support. This leads to a horizontal statically indeterminate force B_v with the compatibility conditions $u_{h,B} = 0$:

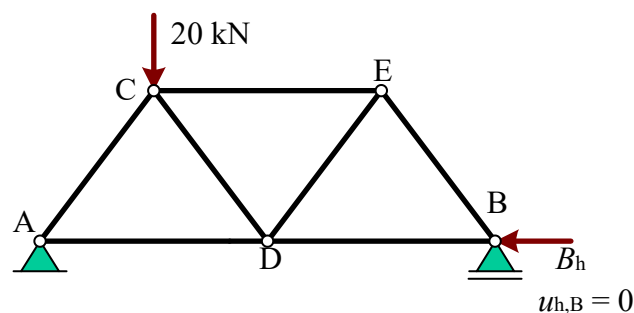


Fig. 35 Statically determinate structure with compatibility condition

Solve displacements statically determinate structure in terms of

statically indeterminate force

The force distribution and displacements in this statically determinate structure is now solved for in terms of B .

Support reactions

First, the support reactions are solved for

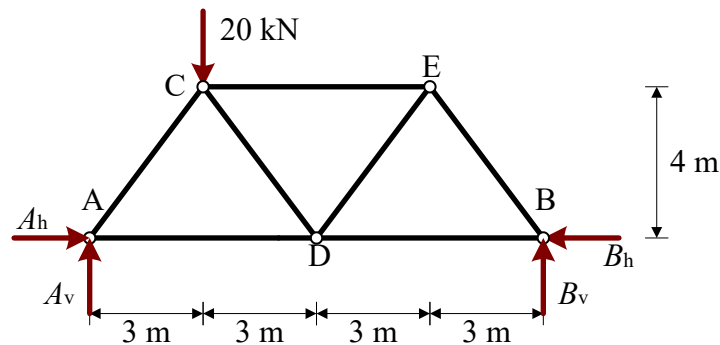


Fig. 36 Free-body-diagram full structure

$$\sum T|_A = 0 \rightarrow B_v = 5 \text{ kN}$$

$$\sum F_v = 0 \rightarrow A_v = 15 \text{ kN}$$

$$\sum F_h = 0 \rightarrow A_h = B_h$$

Leading to:

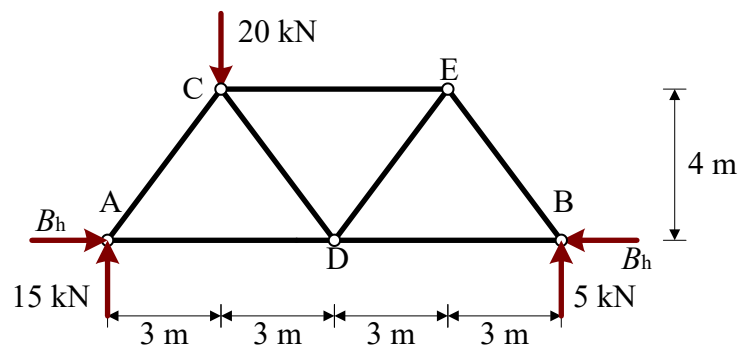


Fig. 37 Free-body-diagram full structure with resulting support reactions

Section forces

The section forces are solved for, starting with the forces in BE and BD:

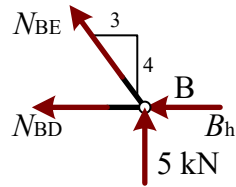


Fig. 38 Free-body-diagram joint B

$$\sum F_v = 0 \rightarrow N_{BE} = -6.25 \text{ kN}$$

$$\sum F_h = 0 \rightarrow N_{BD} = 3.75 - B_h$$

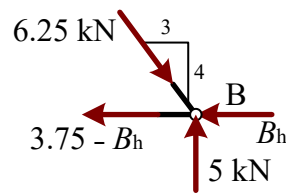


Fig. 39 Free-body-diagram joint B with resulting sections forces

Now, let's continue with a section through beams AD, CD and CE:

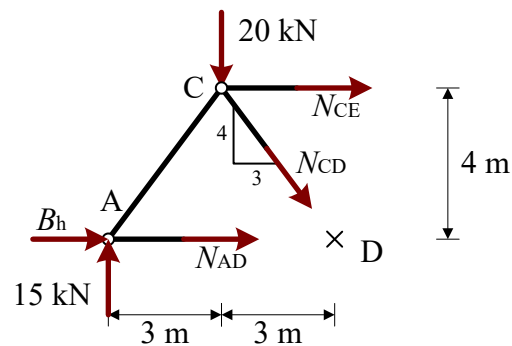


Fig. 40 Free-body-diagram part AC

$$\sum F_v = 0 \rightarrow N_{CD} = -6.25 \text{ kN}$$

$$\sum T|_D = 0 \rightarrow N_{CE} = -7.5 \text{ kN}$$

$$\sum F_h = 0 \rightarrow N_{AD} = 11.25 - B_h$$

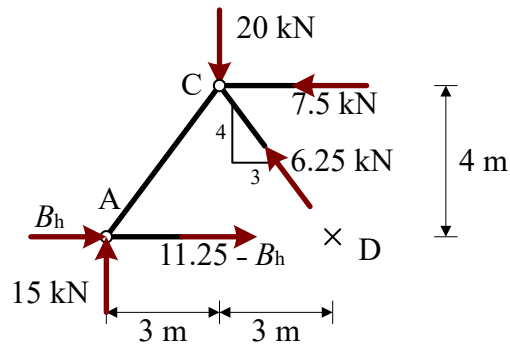


Fig. 41 Free-body-diagram part AC with resulting section forces

Thirdly, let's continue with the joint D:

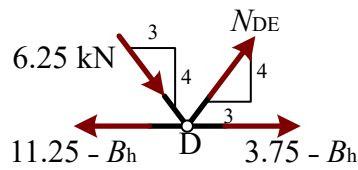


Fig. 42 Free-body-diagram joint D

$$\sum F_v = 0 \rightarrow N_{DE} = 6.25 \text{ kN}$$

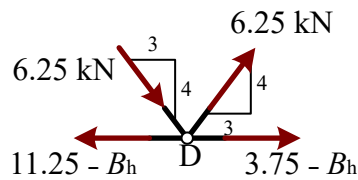


Fig. 43 Free-body-diagram joint D

And finally joint C:

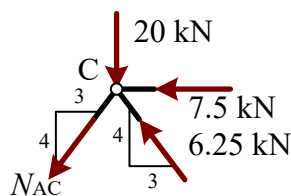


Fig. 44 Free-body-diagram joint D

$$\sum F_v = 0 \rightarrow N_{AC} = -18.75 \text{ kN}$$

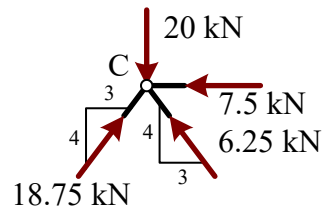


Fig. 45 Free-body-diagram joint D

Shortening/lengthening of elements

Now, for each element the shortening / lengthening can be calculated:

$$\Delta L = \frac{NL}{EA} \rightarrow$$

$$\begin{aligned}\Delta L_{AC} &= -0.025 \text{ m} \\ \Delta L_{CE} &= -0.012 \text{ m} \\ \Delta L_{BE} &= \frac{1}{120} \approx -0.00833 \text{ m} \\ \Delta L_{CD} &= \frac{1}{120} \approx -0.00833 \text{ m} \\ \Delta L_{DE} &= \frac{1}{120} \approx 0.00833 \text{ m} \\ \Delta L_{AD} &= 0.018 - \frac{1}{625} B_h = 0.018 - 0.0016 B_h \text{ m} \\ \Delta L_{DB} &= 0.006 - 0.0016 B_h \text{ m}\end{aligned}$$

Displacement structure due to 20 kN

For now we'll ignore the shortening/lengthening due to B :

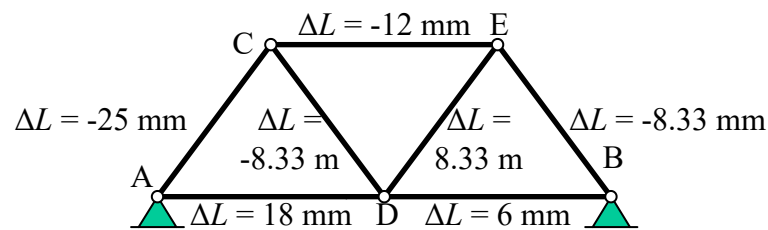


Fig. 46 Shortening/lengthening of elements

This gives the following Williot-diagram with a fixed AC:

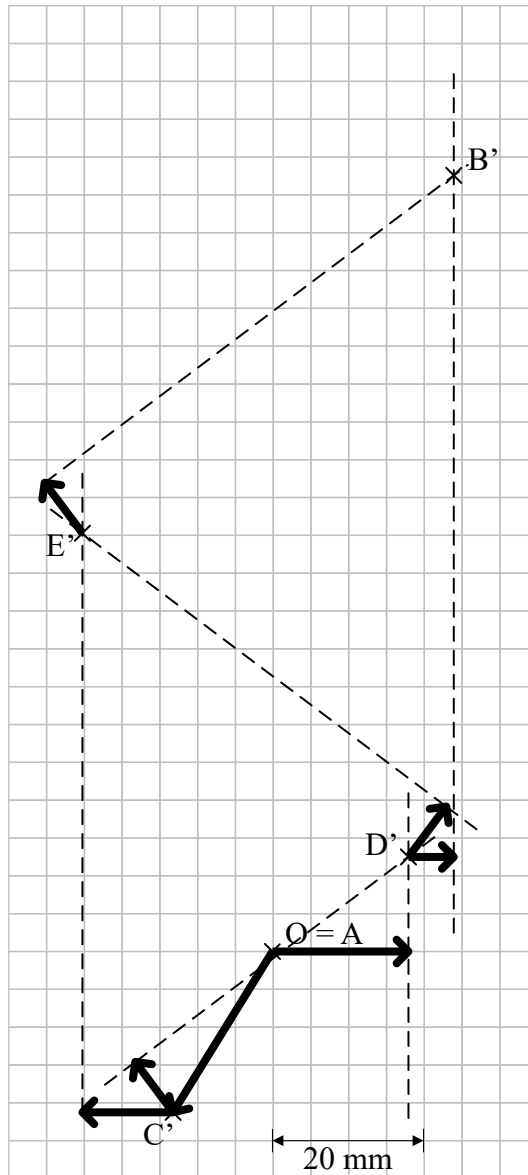


Fig. 47 Williot diagram with fixed AC

Leading to the following displacements if AC doesn't rotate:

joint	Displacement due to 20 with fixed AC in horizontal direction → (mm)	Displacement due to 20 with fixed AC = in vertical direction ↓ (mm)
A	0	0
C	−13	21
D	18	−12.5
E	−25	−55
B	24	−103

B shouldn't move vertically, so this structure has to be rotated back with

$$\theta \approx \frac{103}{12000} \approx 8.5476 \cdot 10^{-3} \text{ rad } \curvearrowright, \text{ leading to:}$$

joint	Displacement due to θ in horizontal direction $\rightarrow$ (mm)	Displacement due to θ in vertical direction $\downarrow$ (mm)
A	0	0
C	34	26
D	0	51
E	34	77
B	0	103

Resulting in total displacements of:

joint	Displacement due to in horizontal direction $\rightarrow$ (mm)	Displacement in vertical direction $\downarrow$ (mm)
A	0	0
C	21	47
D	18	39
E	9	22
B	24	0

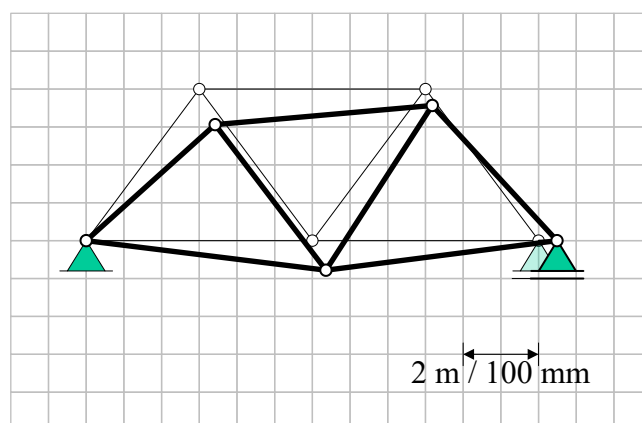


Fig. 48 Displaced structure

Displacement structure due to B_h

For the displacement due to B in kN, the following Williot-diagram with a fixed AD can be drawn:

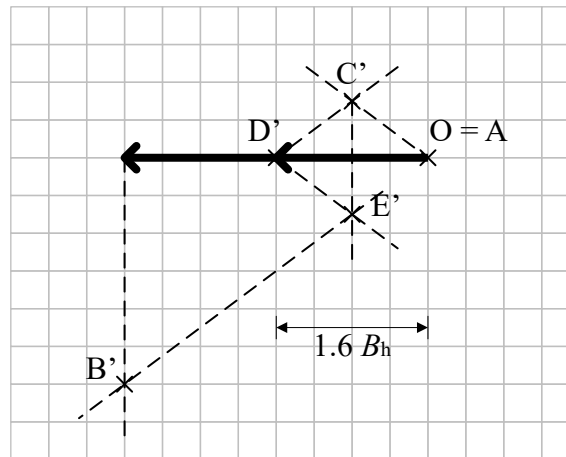


Fig. 49 Williot diagram with fixed AD

Leading to the following displacements if AD doesn't rotate:

joint	Displacement due to B in horizontal direction $\rightarrow$	Displacement due to B in vertical direction $\downarrow$
A	0	0
C	$-0.8B$	$-0.6B$
D	$-1.6B$	0
E	$-0.8B$	$0.6B$
B	$-3.2B$	$2.4B$

Again, B Shouldn't move vertically, so this structure has to be rotated back with

$$\theta = \frac{2.4B_h}{12000} = 0.0002B_h \text{ rad } \odot, \text{ leading to:}$$

joint	Displacement due to θ in horizontal direction $\rightarrow$ (mm)	Displacement due to θ in vertical direction $\downarrow$ (mm)
A	0	0
C	$-0.8B$	$-0.6B$
D	0	$-1.2B$
E	$-0.8B$	$-1.8B$
B	0	$-2.4B$

Resulting in total displacements of:

joint	Displacement due to in horizontal direction $\rightarrow$ (mm)	Displacement in vertical direction $\downarrow$ (mm)
A	0	0
C	$-1.6B$	$-1.2B$
D	$-1.6B$	$-1.2B$
E	$-1.6B$	$-1.2B$
B	$-3.2B$	0

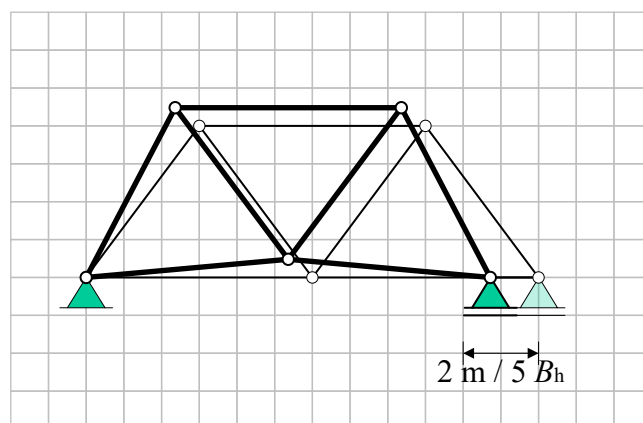


Fig. 50 Displaced structure

Solve statically indeterminate structure

with compatibility conditions

Now, we can fill in the compatibility conditions:

$$u_{B,h} = 0 \rightarrow 0.024 - 0.0032B_h = 0 \rightarrow B_h = 7.5 \text{ kN}$$

Section forces statically indeterminate structure

The section forces can be calculated by filling in the resulting B in our previous expressions:

Element	Normal force kN
AC	-18.75
CE	-7.5
BE	-6.25
CD	-6.25
DE	6.25
AD	3.75
DB	-3.75

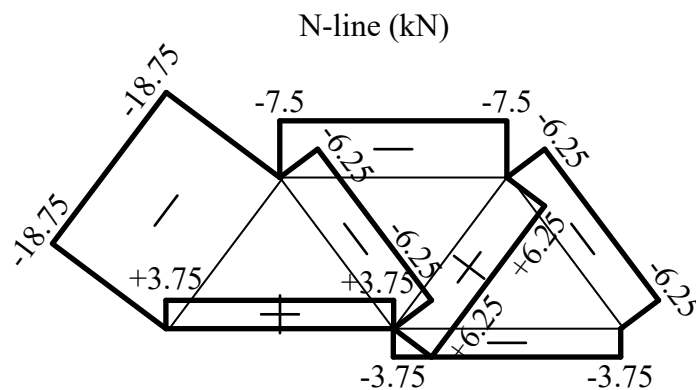


Fig. 51 Normal force distribution

Displacements statically indeterminate structure

Now, the displacements can be found as well by filling in the resulting B in our previous expressions:

joint	Displacement in horizontal direction → (mm)	Displacement in vertical direction ↓ (mm)
A	0	0
C	9	38
D	6	29.833
E	−3	12.66
B	0	0

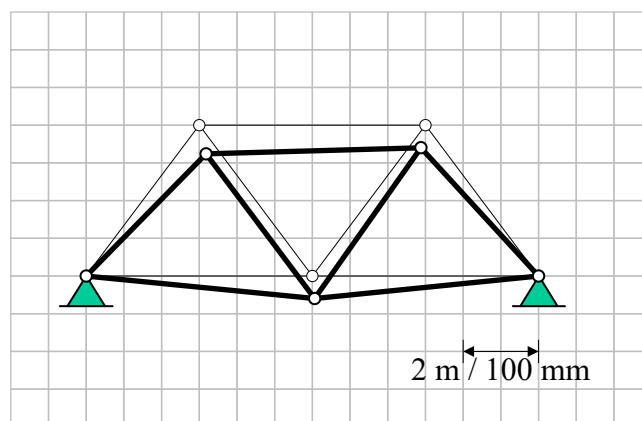


Fig. 52 Displaced structure

Week 4 (September 23th)

	Monday	Tuesday	Wednesday	Thursday	Friday	Upcoming homework
Homework deadlines	Read theory and make homework exercises Extension Prerequisites		Read theory and make homework exercises Williot Prerequisites		Read theory and make homework exercises Force method extension New content	Wednesday Read theory and make homework exercises Section force diagram bending Prerequisites
Class activities	Guided practice on extension Prerequisites		Guided practice on Williot Prerequisites		Guided practice on Force method extension New content	

Homework deadline lesson

Monday September 23th

As a preparation for Monday's lesson you'll look into displacement of extension structures:

[Model](#)

[Small rotations bar element](#)

[Static indeterminacy](#)

[Solve displacements truss structures](#)

[... using constitutive equation](#)

Please go through the topics which you can improve your skills on. Take at least a look at the examples and try a few exercises. During the lesson you'll make some advanced exercises.

Lesson Monday September 23th

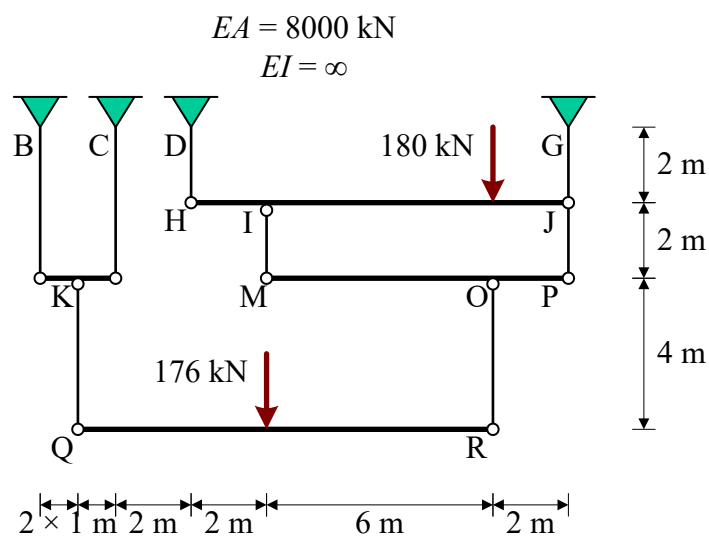
Contents

- Exercise Extension

During today's lesson you'll work on a complex exercise on the topic of Extension. Please ask your questions regarding the [homework](#) as well!

Exercise Extension

Given the following structure.



1. Find the displacement of R
2. Draw the displaced structure

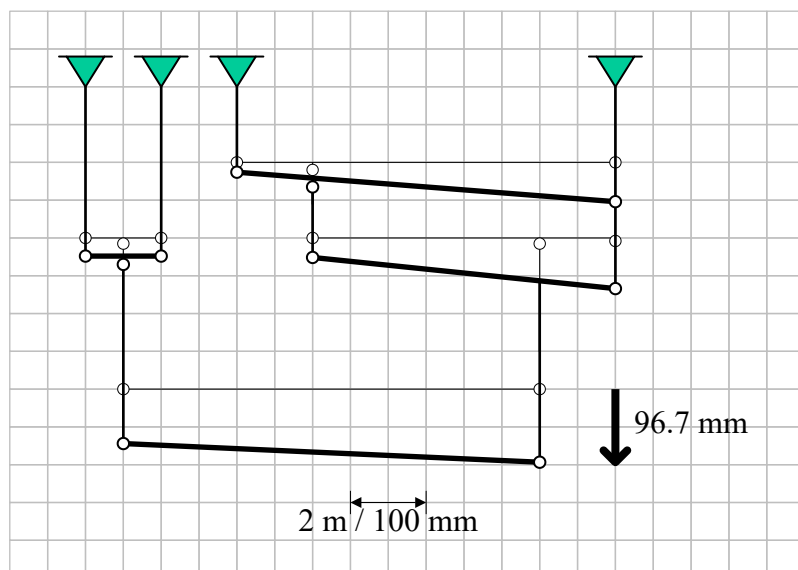
💡 Solution assignment 1

96.7 mm ↓

💡 Solution elongations cables

Cable	Elongation (mm)
BK	24
CK	24
DH	13
GJ	52
IM	5
JP	15
KQ	48
OR	40

💡 Solution assignment 2



Homework deadline lesson

Wednesday September 25th

As a preparation for Wednesday's lesson you'll look into displacement of truss structures using Williot:

[Solve displacements truss structures](#)

[... using Williot](#)

Please go through the topics which you can improve your skills on. Take at least a look at the examples and try a few exercises. During the lesson you'll make some advanced exercises.

Lesson Wednesday September 23th

Contents

- Exercise Williot

During today's lesson you'll work on a complex exercise on the topic of Williot. Please ask your questions regarding the [homework](#) as well!

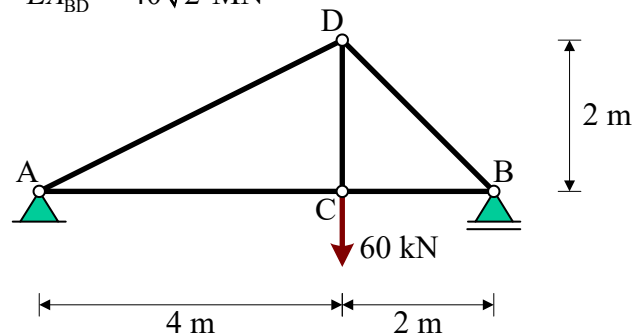
Exercise Williot

Given the following structure.

$$EA_{AC} = EA_{CB} = EA_{CD} = 40 \text{ MN}$$

$$EA_{AD} = 20\sqrt{5} \text{ MN}$$

$$EA_{BD} = 40\sqrt{2} \text{ MN}$$



1. Find the elongation/shrinkage of each elements
2. Find the displacement of all joints
3. Draw the displaced structure

Homework deadline lesson

Friday September 27th

As a preparation for Friday's lesson you'll look into the force method for truss structures:

[Apply force method](#)
[... for truss structures](#)

Please go through the topics which you can improve your skills on. Take at least a look at the examples and try a few exercises. During the lesson you'll make some advanced exercises.

Lesson Friday September 27th

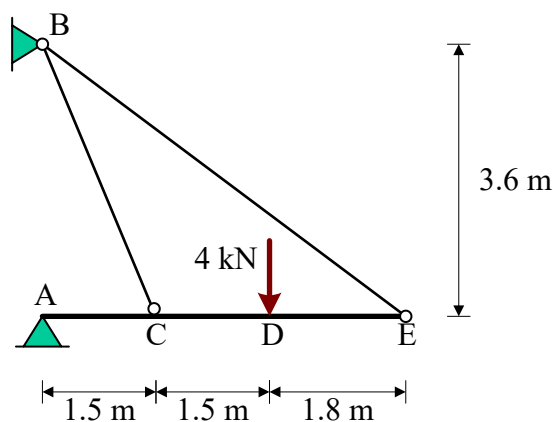
Contents

- Exercise force method for truss structures

During today's lesson you'll work on an exercise on the topic of the force method for truss structures. Please ask your questions regarding the [homework](#) as well!

Exercise force method for truss structures

Given the following structure.



$$E_{BC} = E_{BE} = 200 \text{ GPa}$$

$$A_{BC} = A_{BE} = 25 \text{ mm}^2$$

$$EI_{AE} = EA_{AE} = \infty$$

1. Find the normal force in element BC and BE
2. Find the displacement of joint E

 Solution assignment 1



$$N_{BE} \approx 3.074 \text{ kN}$$

$$N_{BC} \approx 2.273 \text{ kN}$$

 Solution assignment 2



$$w_E \approx 6.147 \text{ mm} \downarrow$$

Week 5 (September 30th)

	Monday	Tuesday	Wednesday	Thursday	Friday	Upcoming homework
Homework deadlines			Read theory and make homework exercises Section force diagrams Prerequisites		Read theory and make homework exercises Displacement bending Prerequisites	Monday Read theory and make homework exercises Prerequisites
Class activities	Demonstration Force method bending New content		Guided practice on Section force diagrams Prerequisites		Guided practice on Displacement bending Prerequisites	



Lesson Monday September 30th

Contents

- Demonstration

During today's lesson it's demonstrated how you to use the force method for statically indeterminate problems which only involve bending

Demonstration

Given a structure as shown below:

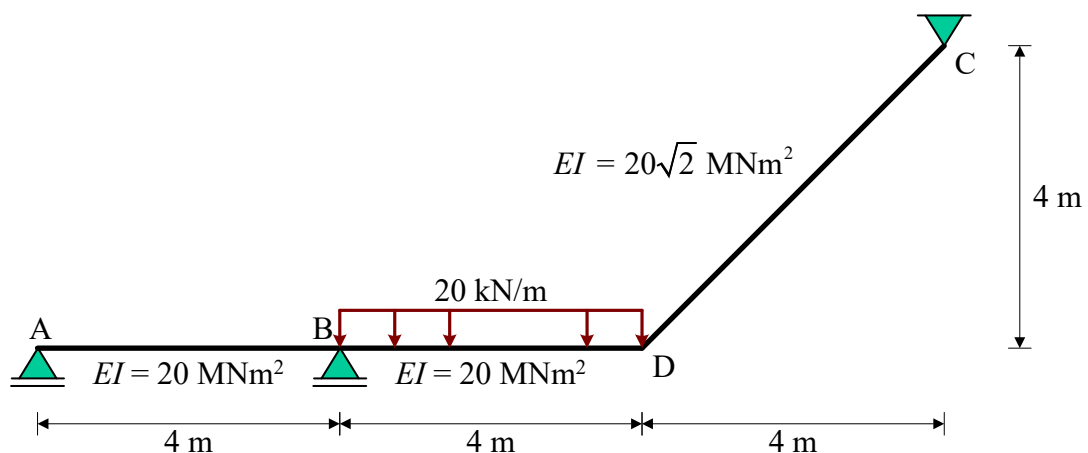


Fig. 56 Structure

We'll apply the force method, specifically, the 'hoekveranderingsvergelijkingen'.

Define mechanism compatibility conditions

To apply the 'hoekveranderingsvergelijkingen', we apply hinges, with statically indeterminate moments M_B and M_D and compatibility conditions $\varphi_B^{AB} = \varphi_B^{BD}$ and $\varphi_D^{BD} = \varphi_D^{DC}$

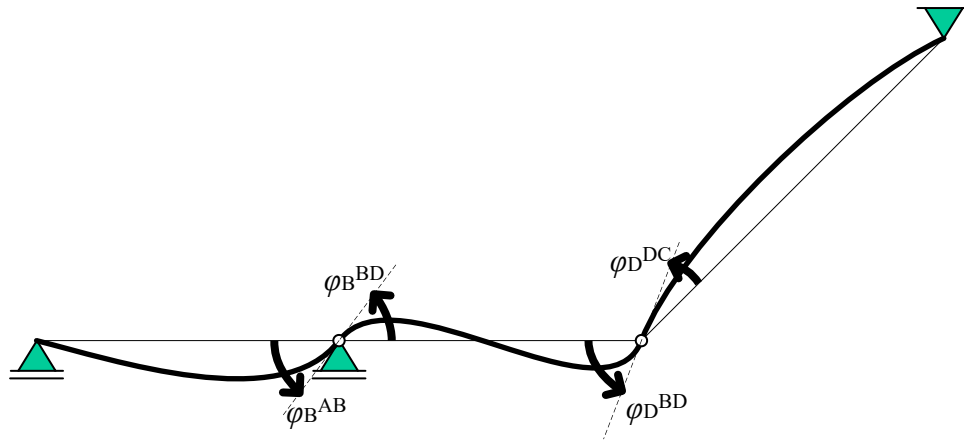


Fig. 59 Deformation due to bending

These rotations can be found using the forget-me-nots and the rotations of each of the elements due to the rigid body rotations:

- $\varphi_B^{AB} = \frac{M_B \cdot 4}{3 \cdot 20000} \rightarrow \varphi_B^{AB} = \frac{M_B \cdot 2}{30000}$
- $\varphi_B^{BD} = -\frac{M_B \cdot 4}{3 \cdot 20000} - \frac{M_D \cdot 4}{6 \cdot 20000} - \frac{20 \cdot 4^3}{24 \cdot 20000} - \theta \rightarrow \varphi_B^{BD} = -\frac{M_B \cdot 2}{30000} - \frac{M_D}{30000} - \frac{20}{30000} - \theta$
- $\varphi_D^{BD} = \frac{M_B \cdot 4}{6 \cdot 20000} + \frac{M_D \cdot 4}{3 \cdot 20000} + \frac{20 \cdot 4^3}{24 \cdot 20000} - \theta \rightarrow \varphi_D^{BD} = \frac{M_B}{30000} + \frac{M_D \cdot 2}{30000} - \frac{20}{30000} + \theta$
- $\varphi_D^{DC} = -\frac{M_D \cdot 4\sqrt{2}}{3 \cdot 20000\sqrt{2}} + \theta \rightarrow \varphi_D^{DC} = -\frac{M_D \cdot 2}{30000} + \theta$

Solve statically indeterminate structure with compatibility conditions and equilibrium

The statically indeterminate forces and unknown rigid body rotation θ can now be solved. However, with only two compatibility conditions an additional equation is needed. This follows from equilibrium, in which the virtual work formulation is most efficient. Therefore, the mechanisms can be reused:

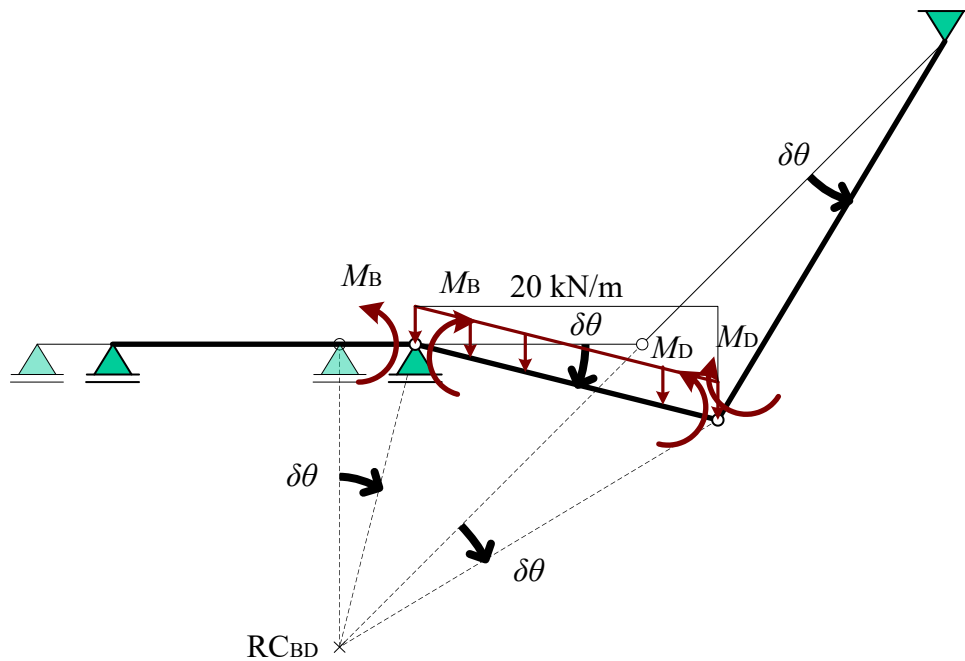


Fig. 60 Mechanism for virtual work calculation

This leads to the virtual work equations:

$$\delta A = 0 \rightarrow M_B \cdot \delta\theta + 20 \cdot 4 \cdot 2\delta\theta - M_D \cdot \delta\theta - M_D \cdot \delta\theta = 0 \rightarrow M_B + 160 - 2M_D =$$

The compatibility conditions are as follows:

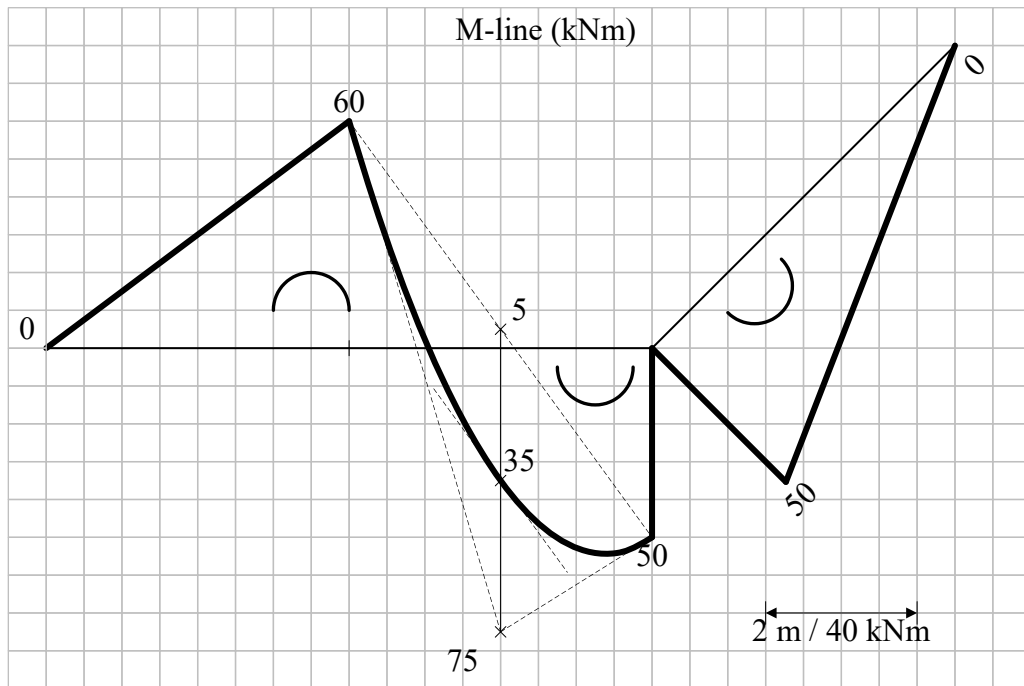
$$\begin{aligned} \bullet \varphi_B^{AB} &= \varphi_B^{BD} \rightarrow \frac{M_B \cdot 2}{30000} = -\frac{M_B \cdot 2}{30000} - \frac{M_D}{30000} - \frac{1}{375} - \theta \rightarrow \frac{M_B \cdot 4}{30000} + \frac{M_D}{30000} \\ \bullet \varphi_D^{BD} &= \varphi_D^{DC} \rightarrow \frac{M_B}{30000} + \frac{M_D \cdot 2}{30000} + \frac{1}{375} - \theta = -\frac{M_D \cdot 2}{30000} + \theta \rightarrow \frac{M_B}{30000} + \frac{M_D}{30000} \end{aligned}$$

Solving these 3 equations gives:

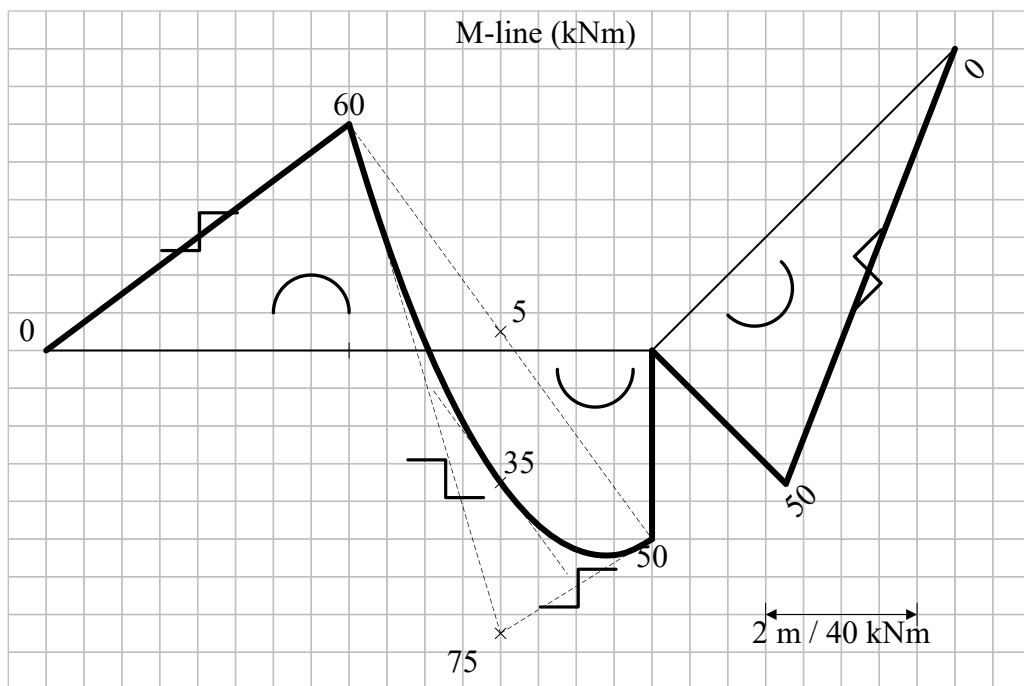
$$\begin{aligned} \bullet M_B &= -60 \text{ kNm} \\ \bullet M_D &= 50 \text{ kNm} \\ \bullet \theta &= \frac{11}{3000} \approx 0.0036667 \text{ rad} \end{aligned}$$

Section force diagrams

Given the solved moments, and $\frac{1}{8} \cdot 20 \cdot 4^2 = 40 \text{ kNm}$, the full moment diagram can be drawn:



The deformation signs for the shear forces follow from the M-line

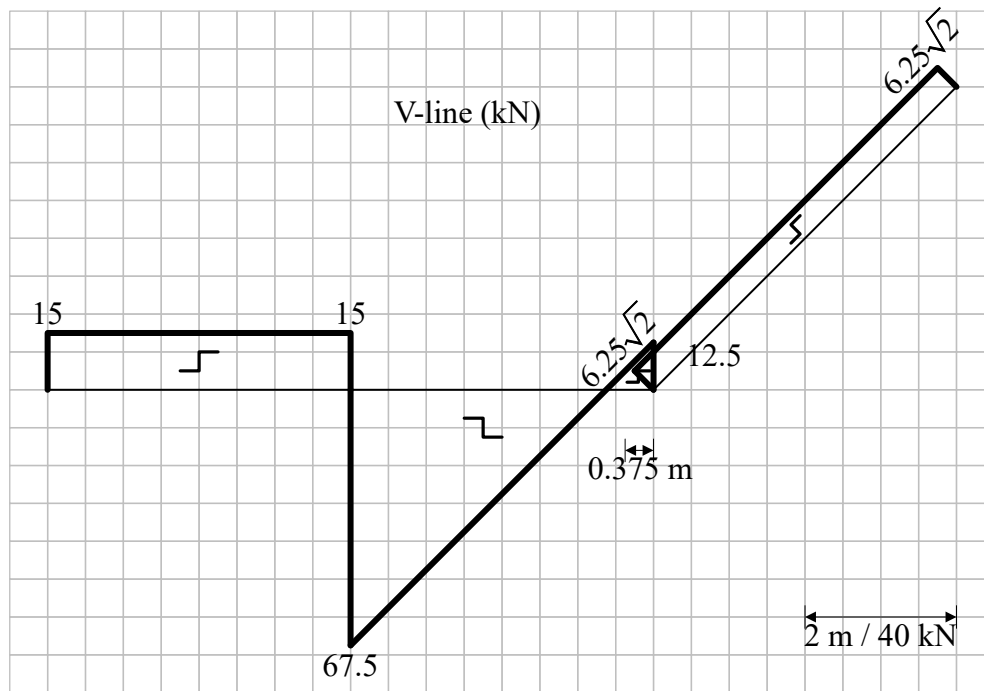


And the sloped give the shear forces themselves:

- $V_{AB} = \frac{60}{4} = 15 \text{ kN}$
- $V_B^{BD} = \frac{60 + 75}{2} = 67.5 \text{ kN}$
- $V_D = \frac{75 - 50}{2} = 12.5 \text{ kN}$
- $V_{DC} = \frac{50}{4\sqrt{2}} = 6.25\sqrt{2} \text{ kN}$

The intersection of the V-line with 0, which coincides with the maximum moment can be found with $\frac{7.5}{20} = 0.375$ m.

This leads to the following shear line:

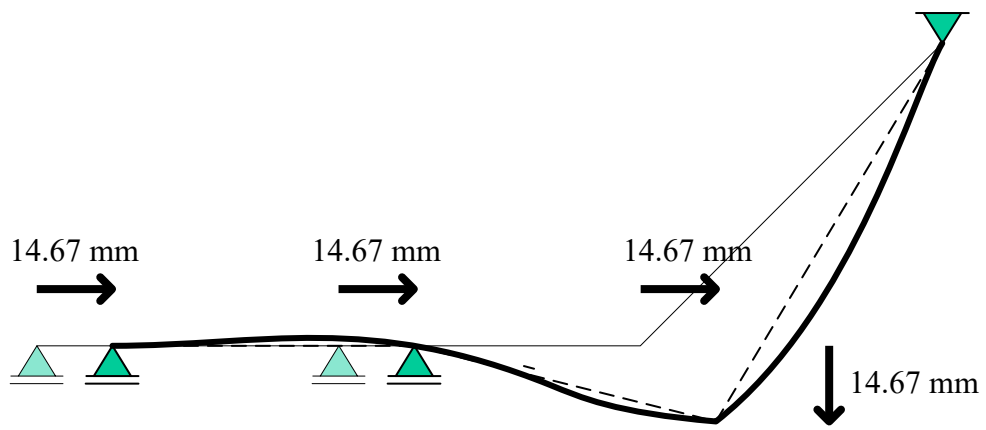


Displacements

The nodal displacement can be found using the rigid body deformations:

- $u_{x,A} = \frac{11}{3000} \cdot 4 = \frac{11}{750} \approx 0.01467$ m
- $u_{x,B} = \frac{11}{3000} \cdot 4 = \frac{11}{750} \approx 0.01467$ m
- $u_{x,D} = \frac{11}{3000} \cdot 4 = \frac{11}{750} \approx 0.01467$ m
- $u_{z,D} = \frac{11}{3000} \cdot 4 = \frac{11}{750} \approx 0.01467$ m

This, together with the given direction of curvatures from the M-line gives the following displaced structure:



Homework deadline lesson

Wednesday October 2nd

As a preparation for Wednesday's lesson you'll look into Section force diagrams:

[Solve section forces frame structures](#)

[Diagrams](#)

Please go through the topics which you can improve your skills on. Take at least a look at the examples and try a few exercises. During the lesson you'll make some advanced exercises.

Lesson October 2nd

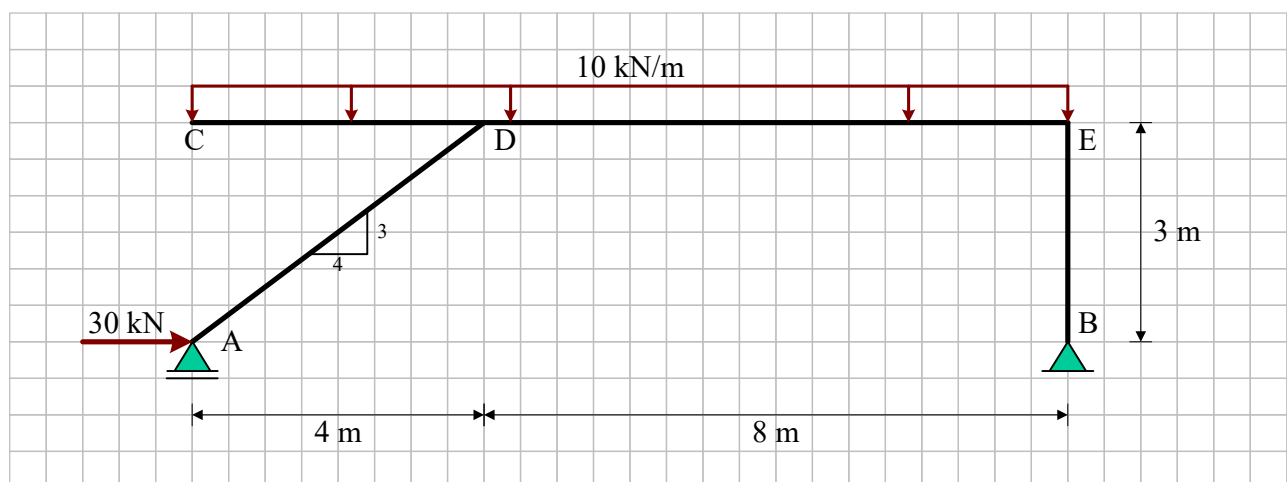
Contents

- Exercise Section force diagrams

During today's lesson you'll work on a complex exercise on the topic of Section force diagrams. Please ask your questions regarding the [homework](#) as well!

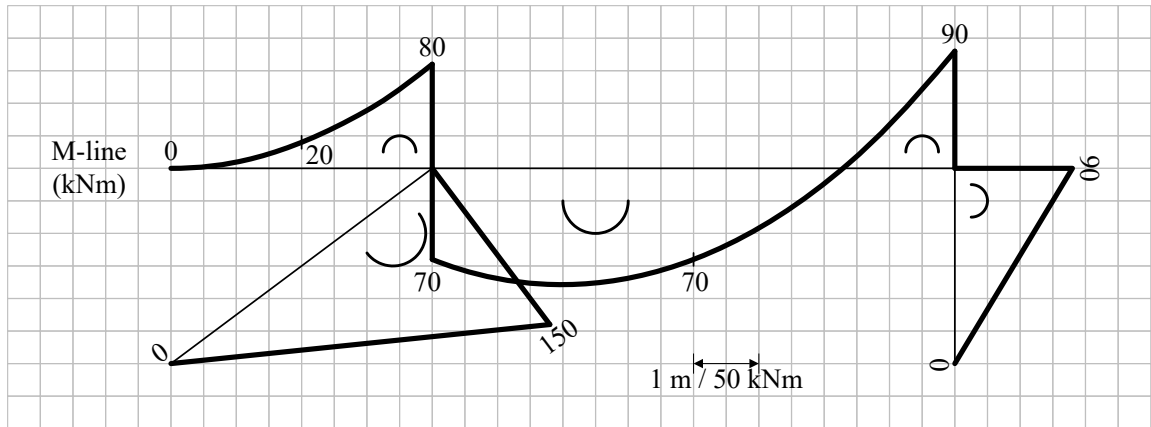
Exercise Section force diagrams

Given the following structure.

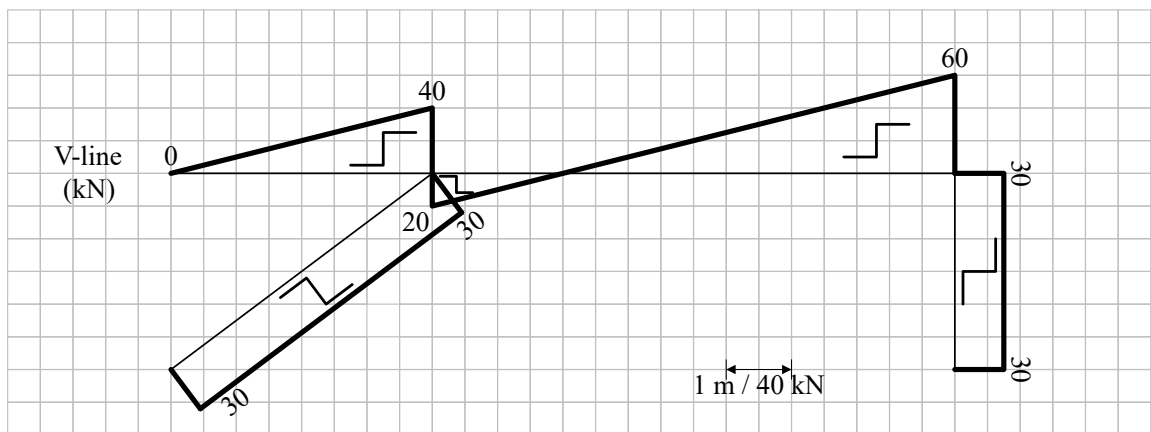


1. Find the bending moment distribution
2. Find the shear force distribution
3. Find the normal force distribution

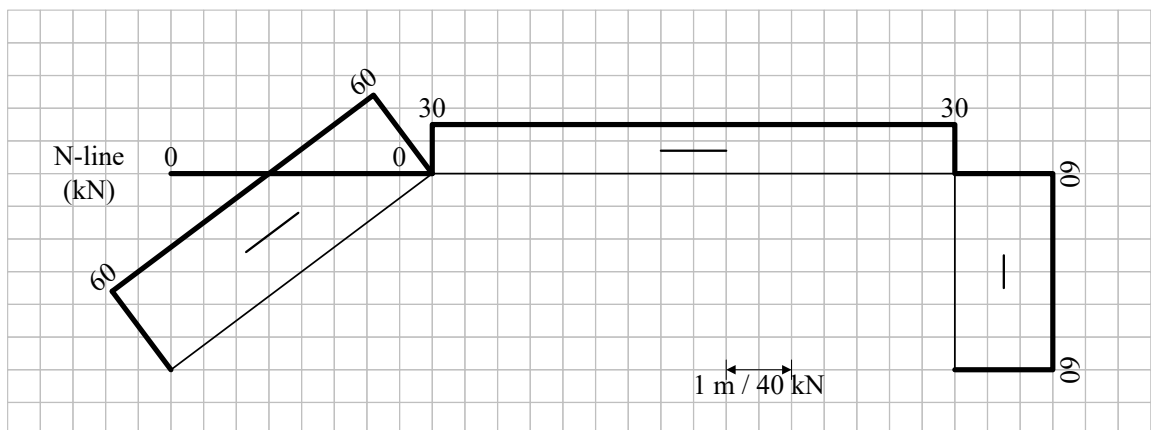
💡 Solution assignment 1



💡 Solution assignment 2



💡 Solution assignment 3



Homework deadline lesson Friday October 4th.

As a preparation for Friday's lesson you'll look into Displacements for bending:

[Solve displacements frame structures](#)

[... using forget-me-nots](#)

Please go through the topics which you can improve your skills on. Take at least a look at the examples and try a few exercises. During the lesson you'll make some advanced exercises.

Lesson Friday October 4th

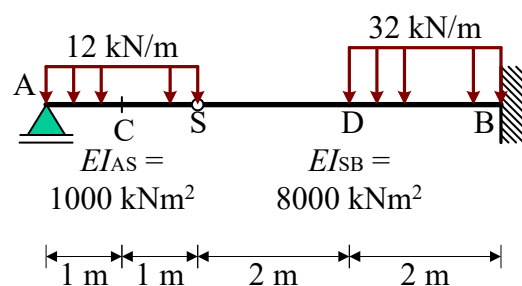
Contents

- Exercise Displacement for bending

During today's lesson you'll work on an exercise on the topic of the Displacement for bending. Please ask your questions regarding the [homework](#) as well!

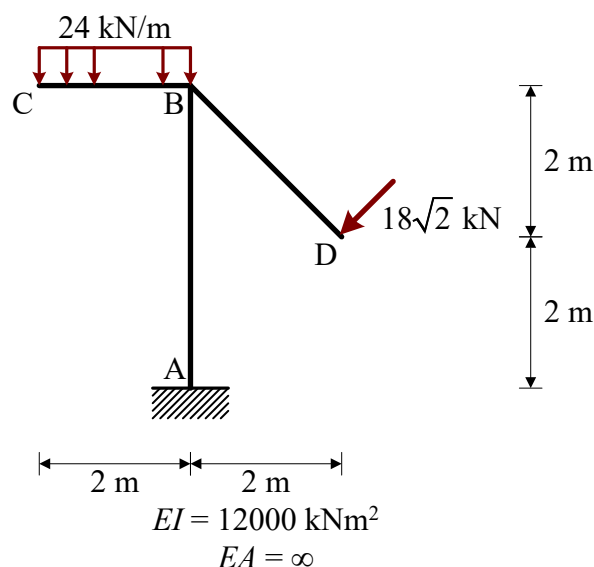
Exercise Displacement for bending

Given the following structure.



1. Find the displacement of C and D
2. Draw the displaced structure

Given the following structure.



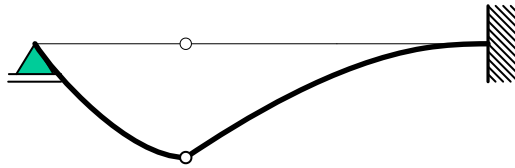
3. Find the displacement of C and D

4. Draw the displaced structure

 Solution assignment 1

- $w_C \approx 0.0278 \text{ m} \downarrow$
- $w_D = 0.018 \text{ m} \downarrow$

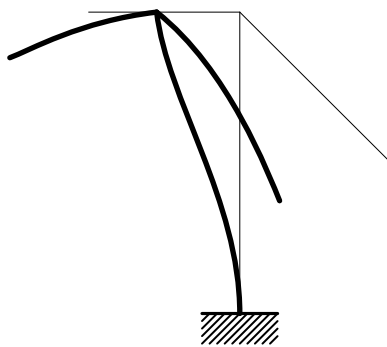
 Solution assignment 2



 Solution assignment 3

- $w_{v,C} = 0.012 \text{ m} \downarrow$
- $w_{h,C} = 0.016 \text{ m} \leftarrow$
- $w_{v,D} \approx 0.0033 \text{ m} \downarrow$
- $w_{h,D} \approx 0.019 \text{ m} \leftarrow$

 Solution assignment 4



Week 6 (October 7th)

	Monday	Tuesday	Wednesday	Thursday	Friday	Upcoming homework
Homework deadlines	Read theory and make homework exercises Force method bending New content		Read theory and make homework exercises Mechanisms and virtual work Prerequisites		Read theory and make homework exercises Force method frames New content	Wednesday Read theory and make homework exercises Displacement method New content
Class activities	Guided practice on Force method bending New content		Guided practice on Mechanisms and virtual work Prerequisites		Read theory and make homework exercises Force method frames Prerequisites	

◀
▶

Homework deadline lesson

Monday October 7th

As a preparation for Monday's lesson you'll look into Force method for bending:

[Apply force method](#)

[... for frame structures](#)

Please go through the topics which you can improve your skills on. Take at least a look at the examples and try a few exercises. During the lesson you'll make some advanced exercises.

Lesson October 7th

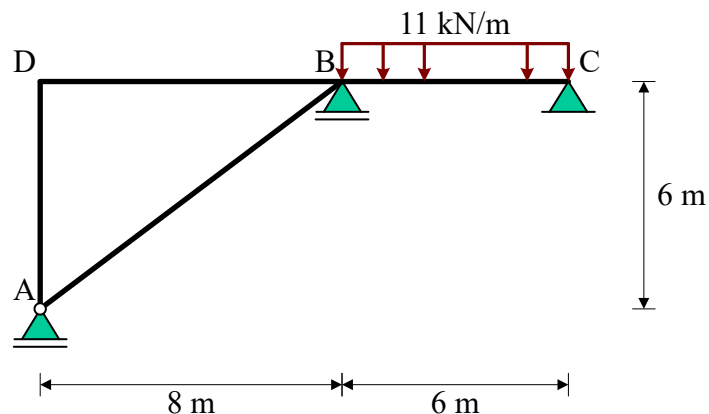
Contents

- Exercise Force method for frame structures

During today's lesson you'll work on a complex exercise on the topic of Force method for frame structures. Please ask your questions regarding the [homework](#) as well!

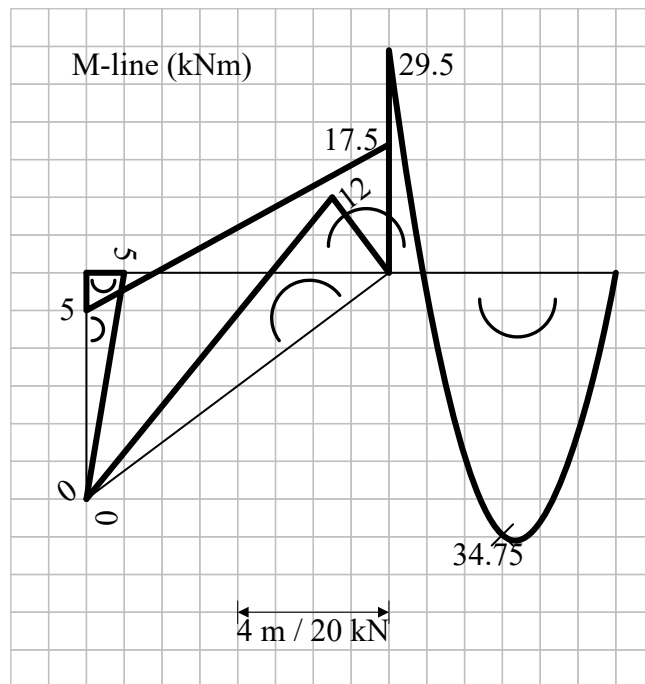
Exercise Force method for frame structures

Given the following structure.

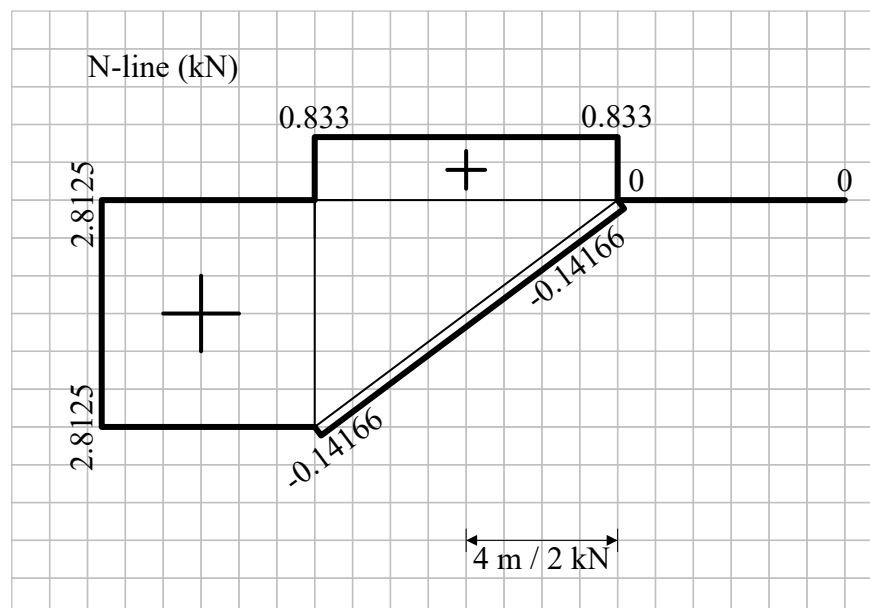


1. Find the bending moment distribution
2. Find the normal force distribution

💡 Solution assignment 1



💡 Solution assignment 2



Homework deadline lesson

Wednesday October 9th

As a preparation for Wednesday's lesson you'll look into Virtual work:

[Model](#)

[Displacement of rigid bodies in a mechanism](#)

[Virtual work](#)

[Solve structures using virtual work](#)

Please go through the topics which you can improve your skills on. Take at least a look at the examples and try a few exercises. During the lesson you'll make some advanced exercises.

Lesson October 9th

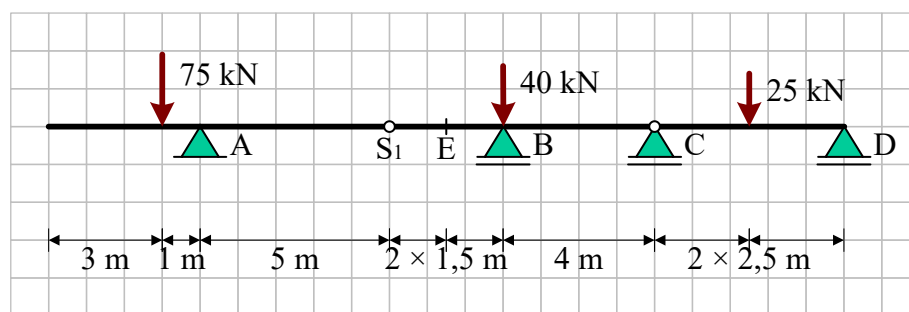
Contents

- Exercise Mechanisms and virtual work

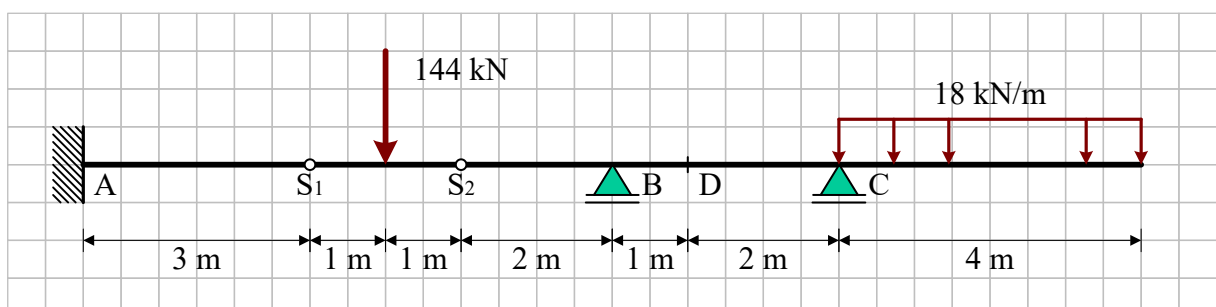
During today's lesson you'll work on a complex exercise on the topic of Mechanisms and Virtual work. Please ask your questions regarding the [homework](#) as well!

Exercise Mechanisms and virtual work

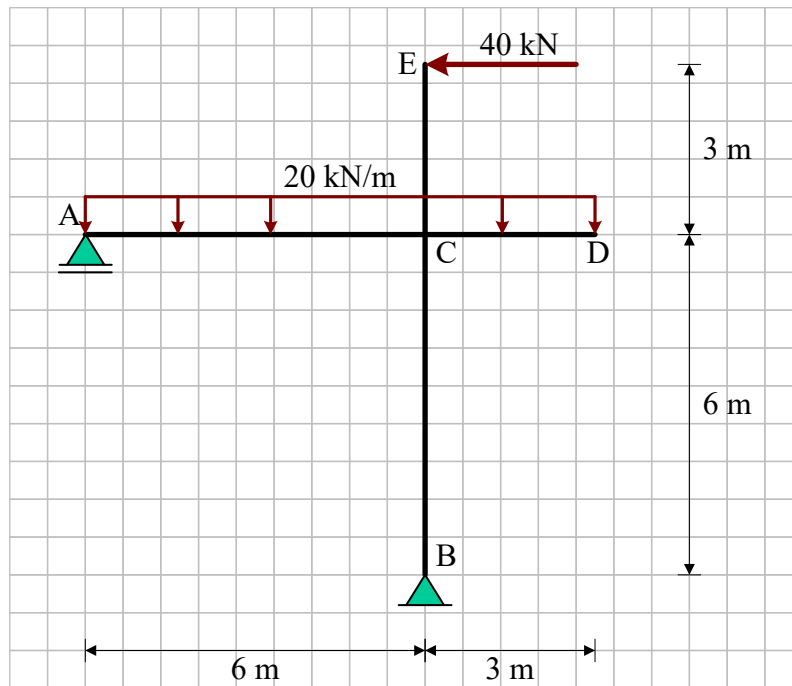
Given the following structure.



1. Find the vertical support reaction in C
2. Find the bending moment in E
3. Find the shear force just right of B



4. Find the vertical support reaction in B
5. Find the bending moment in D
6. Find the shear force just right of S_1



7. Find the bending moment just above C

8. Find the bending moment just below C

Solution assignment 1

$$C_v = 23.75 \text{ kN } (\uparrow)$$

Solution assignment 2

$$M_E = 22.5 \text{ kNm } (\cup)$$

Solution assignment 3

$$V_B^{BC} = 11.25 \text{ kN } (-|^-)$$

Solution assignment 4

$$B_v = 72 \text{ kN } (\uparrow)$$

Solution assignment 5

$$M_D = 144 \text{ kNm } (\cap)$$

 Solution assignment 6



$$V_{S_1} = 72 \text{ kN } (\neg|-)$$

 Solution assignment 7



$$M_C^{\text{CE}} = 120 \text{ kNm } (\supset)$$

 Solution assignment 8



$$M_C^{\text{BC}} = 240 \text{ kNm } (\subset)$$

Homework deadline lesson

Friday October 11th

As a preparation for Friday's lesson you'll look into the Force method for frames, or more specifically 'hoekveranderingsvergelijkingen' with moveable nodes:

[Apply force method](#)

[... for frame structures with movable nodes](#)

Please go through the topics which you can improve your skills on. Take at least a look at the examples and try a few exercises. During the lesson you'll make some advanced exercises.

Lesson October 11th

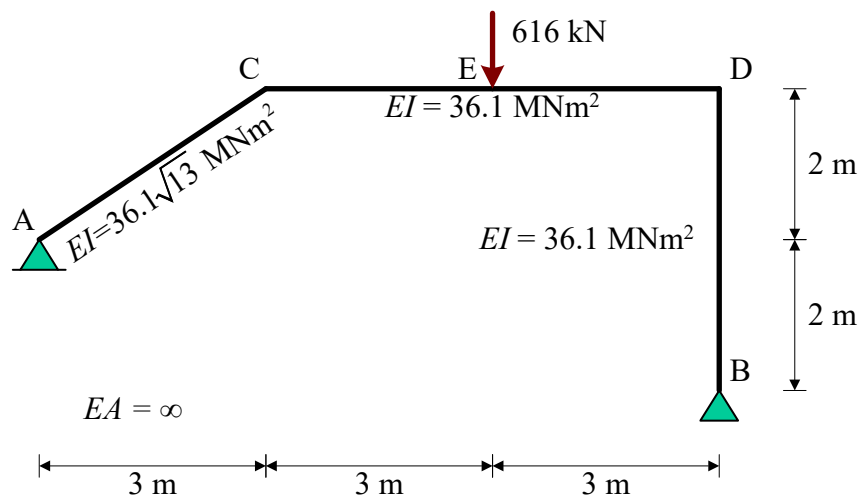
Contents

- Exercise 'Hoekveranderingsvergelijkingen' with moveable nodes

During today's lesson you'll work on a complex exercise on the topic of Force method for frames, or more specifically 'hoekveranderingsvergelijkingen' with moveable nodes. Please ask your questions regarding the [homework](#) as well!

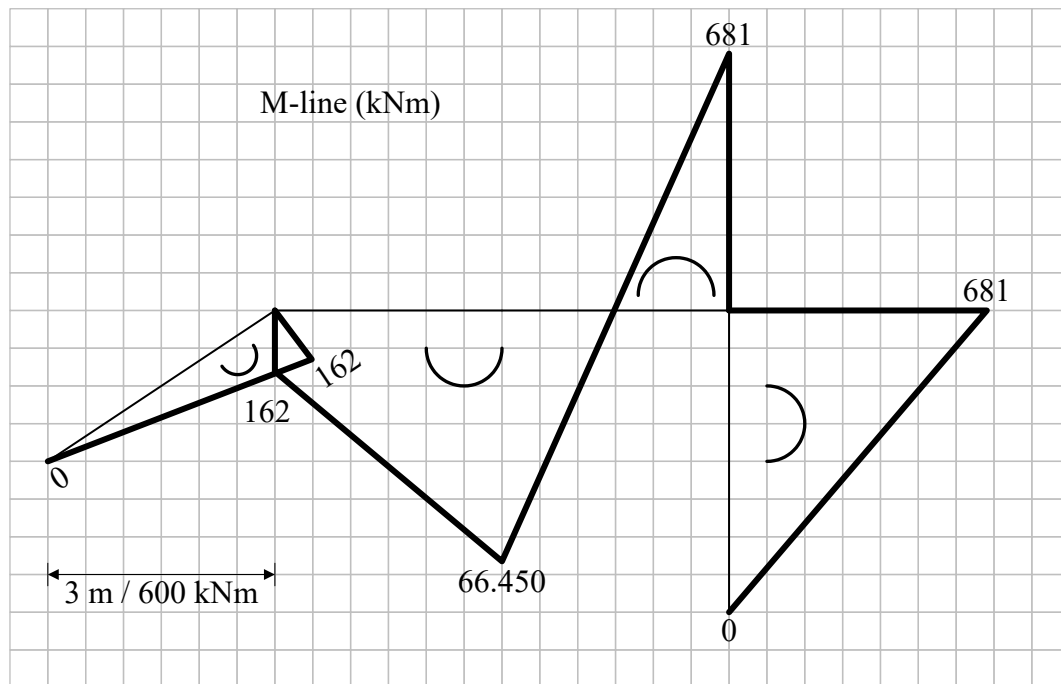
Exercise 'Hoekveranderingsvergelijkingen' with moveable nodes

Given the following structure.



1. Find the bending moment distribution
2. Find the displacement of E

💡 Solution assignment 1



💡 Solution assignment 2

- $u_{E,v} \approx 74.4391 \text{ mm } (\downarrow)$
- $u_{E,h} \approx 40 \text{ mm } (\rightarrow)$

Week 7 (October 14th)

	Monday	Tuesday	Wednesday	Thursday	Friday	Up to you
Homework deadlines			Read theory and make homework exercises Displacement method New content			We Re al ho e s disc an se Ne
Class activities	Demonstration Displacement method New content		Guided practice on Displacement method New content		Demonstration stiffness discontinuities, support settlement and temperature influences New content	



Lesson October 14th

Contents

- Demonstration extension
- Demonstration bending

During today's lesson it's demonstrated how you to use the displacement method.

Demonstration extension

Given a structure as shown below:

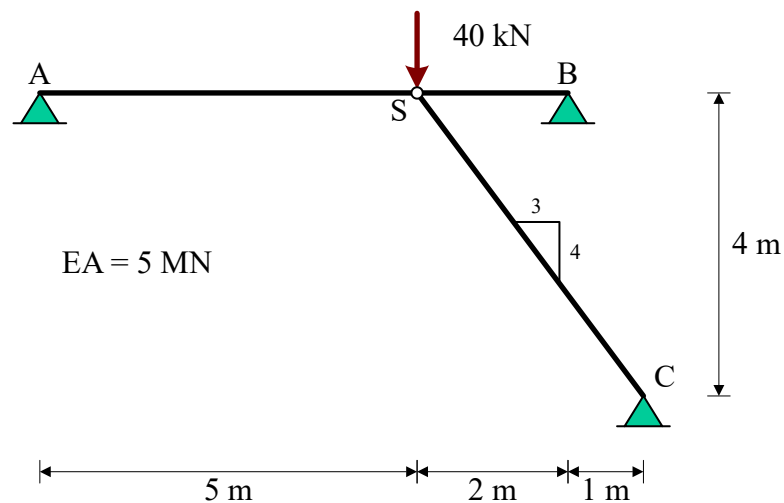


Fig. 66 Structure

For the displacement method, it's needed to choose one or multiple degrees of freedom which describe the displacement of the full structure. Then, using equilibrium, this displacement can be solved for. In this case, that can be the horizontal and vertical displacement of S. A positive displacement is arbitrarily chosen for both of them:

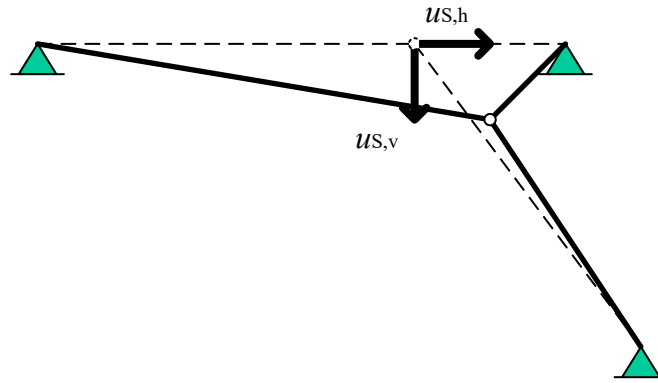


Fig. 67 Displaced structure with assumed $u_{B,h}$ and $u_{b,v}$

This leads to extension of the elements:

- $\Delta L_{AS} = +u_{S,h}$
- $\Delta L_{SB} = -u_{S,h}$

And for element SC in detail:

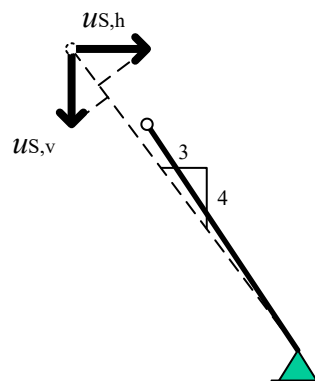


Fig. 68 Displaced SC

- $\Delta L_{SC} = -\frac{4}{5}u_{S,v} - \frac{3}{5}u_{S,h}$

Applying the constitutive and kinematic relation $N = EA \frac{\Delta L}{L}$ gives:

- $N_{AS} = 1000u_{S,h}$
- $N_{SB} = -2500u_{S,h}$
- $N_{SC} = -800u_{S,h} - 600u_{S,v}$

Now, the equilibrium of node S can be analysed:

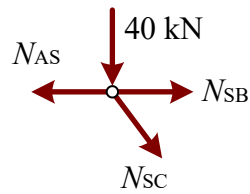


Fig. 69 Free-body-diagram node S

- $\Sigma F_h^S = 0 \rightarrow -N_{AS} + N_{SB} + \frac{3}{5}N_{SC} = 0$
- $\Sigma F_v^S = 0 \rightarrow 40 + \frac{4}{5}N_{SC} = 0$

This gives:

- $u_{S,h} = -\frac{3}{350} \text{ m} \approx -8.57 \text{ mm}$
- $u_{S,v} = \frac{193}{2800} \text{ m} \approx 68.9 \text{ mm}$

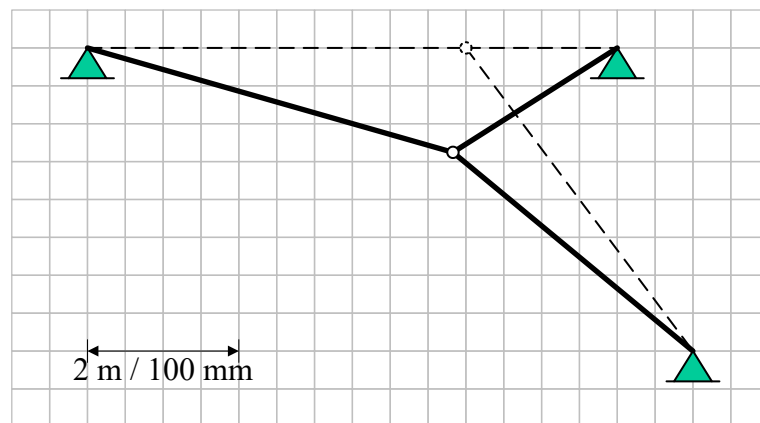


Fig. 70 Final displaced structure

Evaluating our previous expressions for the nodal forces gives:

- $N_{AS} \approx -8.57 \text{ kN}$
- $N_{SB} \approx 21.4 \text{ kN}$
- $N_{SC} = -50 \text{ kN}$

Demonstration bending

Given a structure as shown below:

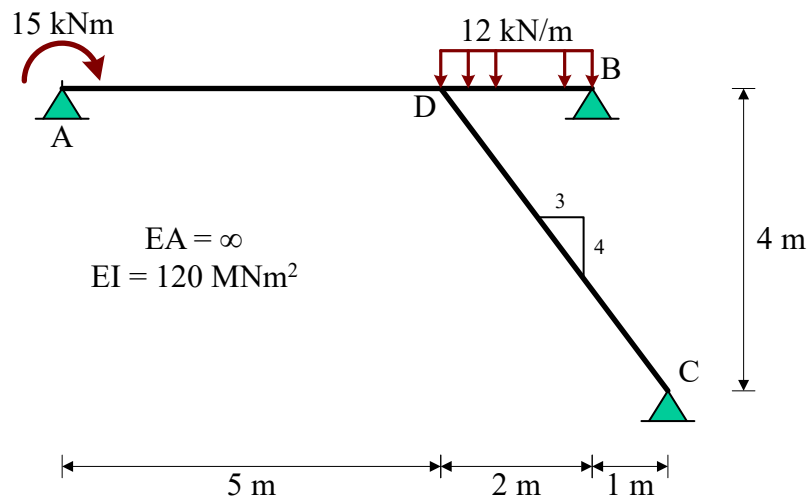


Fig. 71 Structure

Again, it's needed to choose one or multiple degrees of freedom which describe the displacement of the full structure. In this case, that can be the rotation of D. A positive displacement is arbitrarily chosen:

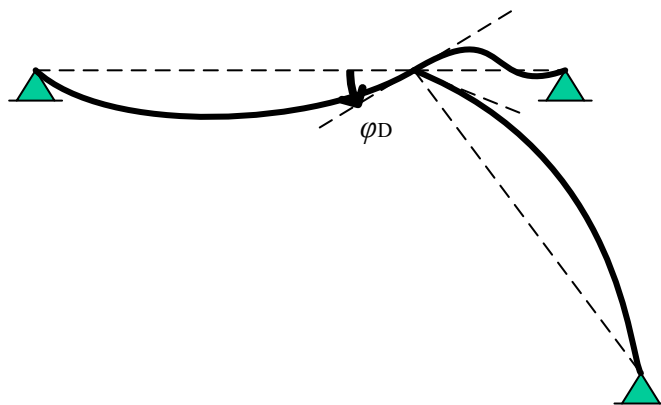


Fig. 72 Displaced structure with assumed φ_D

As the nodes will not displace, each element can be analysed using forget-me-nots. The directions of the bending moments are arbitrarily chosen:

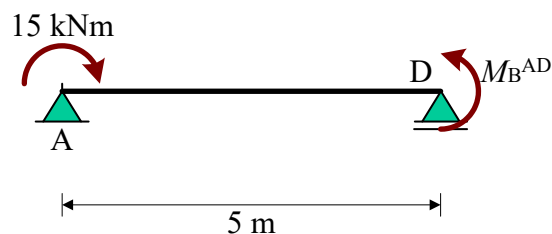


Fig. 73 Isolated element AD

The rotation φ can be calculated using forget-me-nots as: $\varphi_D = \frac{5M_D^{AD}}{3EI} + \frac{15 \cdot 5}{6EI}$ This can be rewritten as $M_D^{AD} = 72000\varphi_D - 7.5$

Similarly, for the other elements:

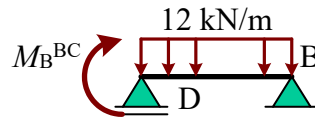


Fig. 74 Isolated element DB

$$M_D^{DB} = 180000\varphi_D - 6$$

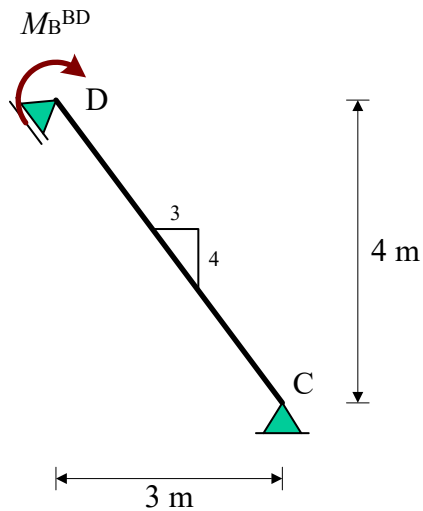


Fig. 75 Isolated element DC

$$M_D^{DC} = -72000\varphi_D$$

Now, the equilibrium of node D can be analysed:

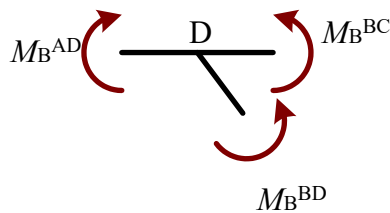


Fig. 76 Free-body-diagram node D only showing bending moments

$$\sum T = 0 \rightarrow -M_D^{AD} + M_D^{DB} + M_D^{DC} = 0$$

$$\text{This gives } \varphi = \frac{5}{9EI} = \frac{1}{216000} = 4.63 \cdot 10^{-6} \text{ rad}$$

Evaluating our previous expressions for the moments gives:

- $M_D^{AD} = -\frac{43}{6} \approx -7.17 \text{ kNm}$

- $M_D^{DB} = -\frac{41}{6} \approx -6.83 \text{ kNm}$
- $M_D^{DC} = -\frac{1}{3} \approx -0.333 \text{ kNm}$

This gives the following M-line:

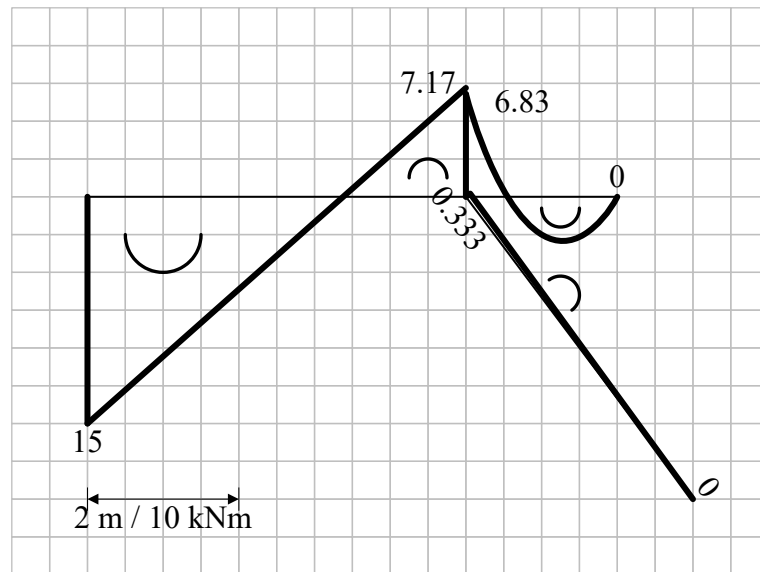


Fig. 77 Moment distribution

Homework deadline lesson

Wednesday October 16th

As a preparation for Wednesday's lesson you'll look into the Displacement method:

[Apply displacement method](#)

Please go through the topics which you can improve your skills on. Take at least a look at the examples and try a few exercises. During the lesson you'll make some advanced exercises.

Lesson October 16th

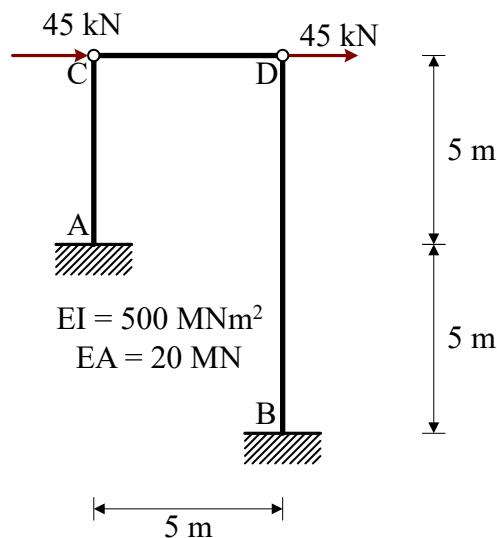
Contents

- Exercise Displacement method

During today's lesson you'll work on a complex exercise on the topic of the Displacement method. Please ask your questions regarding the [homework](#) as well!

Exercise Displacement method

Given the following structure. Use the horizontal displacement at D as the degree of freedom ($\rightarrow$).



1. Find the relation of the normal force in CD in terms of the degree of freedom w_D .
2. Solve the value of the degree of freedom.
3. Find the bending moment distribution.

💡 Solution assignment 1



In terms of the horizontal displacement at D:

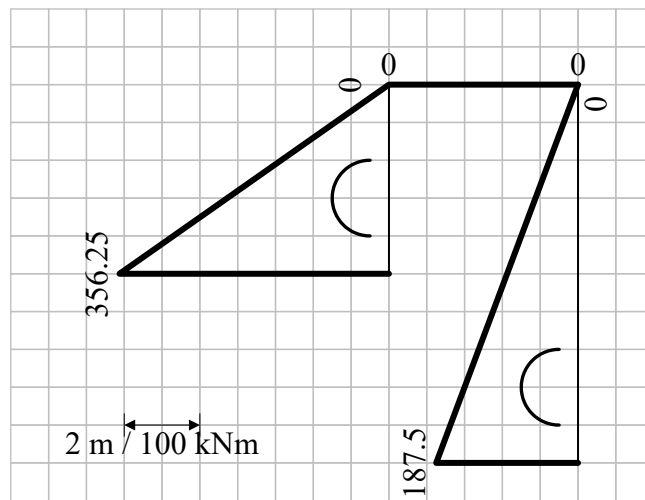
- $N_{CD} = 45 - 1500w_D$
- $N_{CD} = 3000w_D - 11.25$

💡 Solution assignment 2



$$w_D = 1.25 \text{ cm } (\rightarrow)$$

💡 Solution assignment 3



Lesson October 18th

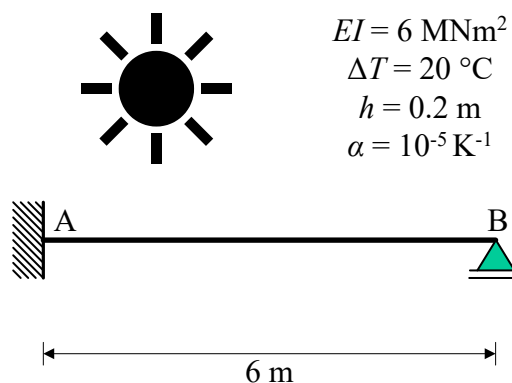
Contents

- Demonstration temperature influences
- Demonstration support settlement
- Stiffness influences

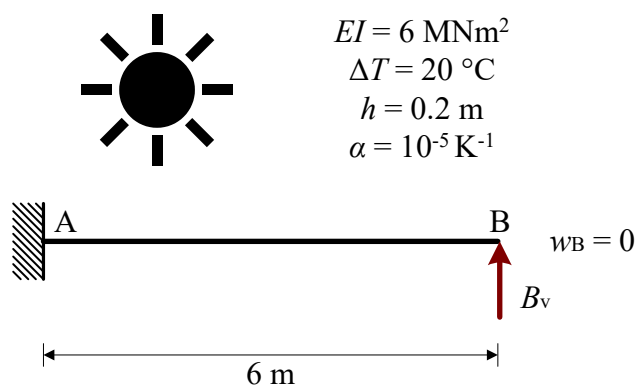
During today's lesson it's demonstrated how you to handle Temperature influences, stiffness discontinuities and support settlement for statically indeterminate structures.

Demonstration temperature influences

Given is the following structure:

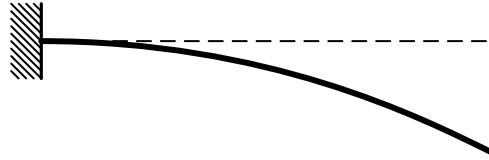


To look at the temperature influences, it's easier to look at a statically determinate structure. We could look at the statically determinate structure with the unknown statical indeterminate force B_v with the condition that the displacement at B is 0.

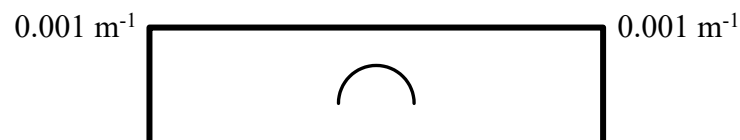


Displacement due to temperature influence

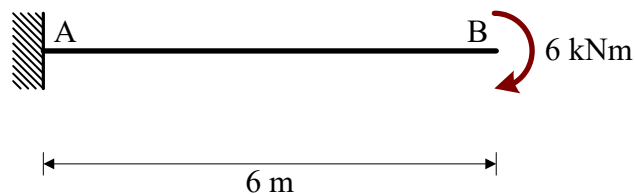
Let's focus on the displacement due to the temperature influence first. As the top is warmer than the bottom of the beam, the top of the beam will extend with respect to the bottom side. For the statically determinate beam this will lead to a deflection downwards:



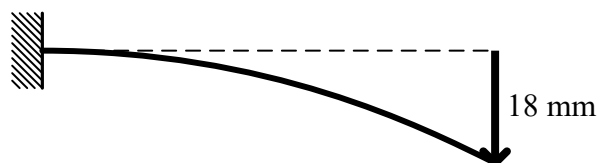
The corresponding curvature κ^T equals $\alpha \frac{\Delta T}{h} = 0.001 \frac{1}{m} (\curvearrowright)$, which is constant over the full beam:



The kinematically equivalent moment distribution has a couple at B of which the value follows from $M = \kappa \cdot EI = 6 \text{ kNm} (\curvearrowright)$

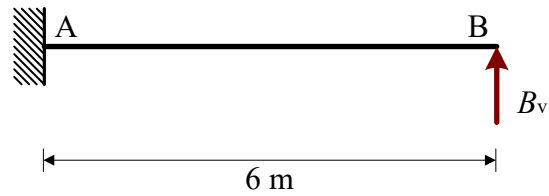


Using the forget-me-nots, this leads to a displacement at B of $\frac{M \cdot L^2}{2EI} = 18 \text{ mm} (\downarrow)$



Displacement due to statically indeterminate support reaction

If we only consider the statically indeterminate support reaction:



The displacement can be calculated as $w_B = \frac{B_v * L^3}{3EI} = \frac{3}{250} B_v (\uparrow)$

Solve for statically indeterminate support reaction

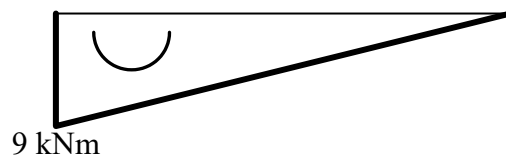
As the total displacement at B should be zero, B_v can be calculated:

$$18 = \frac{3}{250} B_v$$

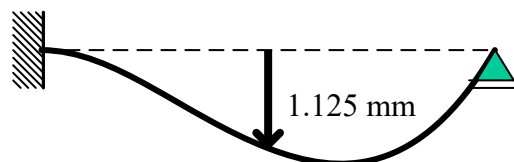
$$B_v = 1.5 \text{ kN } (\uparrow)$$

Find moment distribution and displacements

The curvature due to the temperature influence doesn't cause internal stresses. However, the support reactions does. This gives the following moment distribution:



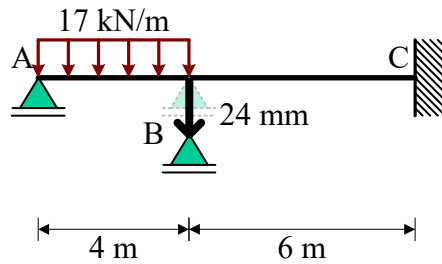
The full displacement can be found by applying the forget-me-nots, leading to:



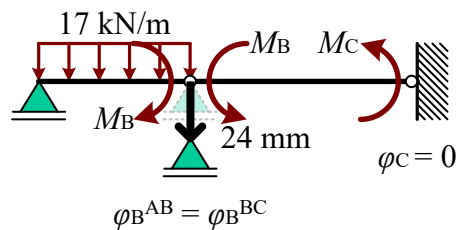
Demonstration support settlement

Given is the following structure:

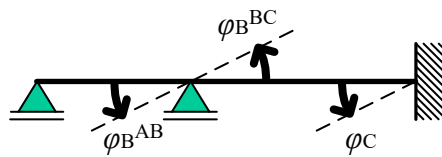
$$EI = 34000 \text{ kNm}^2$$



The displacement can be incorporated in the calculation using the force- or displacement method. This structure is solved using 'hoekveranderingsvergelijkingen', leading to the following statically determinate structure:



The positive rotations are defined as follows:



The first displacement condition can be filled in using forget-me-nots. The support settlement is taken into account as a (real) displacement and thus rotation:

$$\varphi_B^{AB} = \varphi_B^{BC}$$

$$\frac{-M_B \cdot 4}{3 \cdot EI} + \frac{17 \cdot 4^3}{24 \cdot EI} - \frac{w_B}{4} = \frac{M_B \cdot 6}{3 \cdot EI} - \frac{M_C \cdot 6}{6 \cdot EI} + \frac{w_B}{6}$$

The second displacement condition gives similarly:

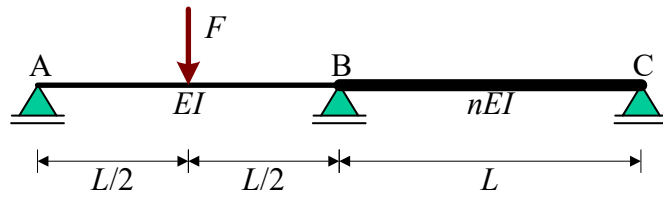
$$\varphi_C = 0$$

$$-\frac{M_B \cdot 6}{3 \cdot EI} = \frac{M_C \cdot 6}{6 \cdot EI} + \frac{w_B}{6} = 0$$

Solving this system of two equations gives: $M_B = -128 \text{ kNm}$ and $M_C = -132 \text{ kNm}$

Stiffness influences

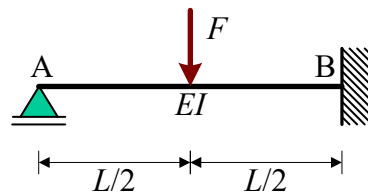
Given is the following structure:



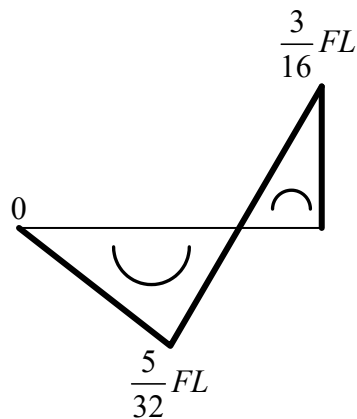
We'll investigate the influence of the stiffness of the right part.

High stiffness

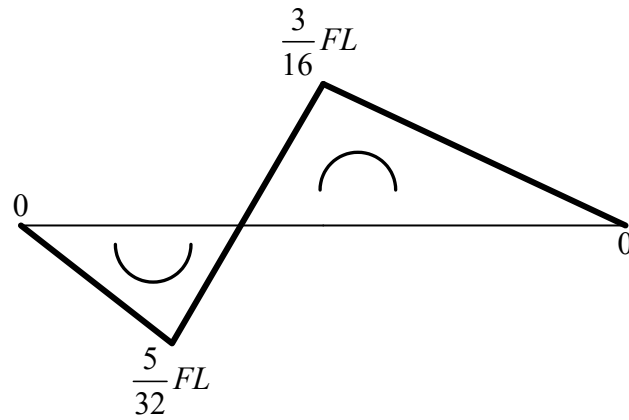
Let's start with a high stiffness, so $n \rightarrow \infty$. If beam BC is infinitely stiff, this part of the beam cannot bend. Therefore also the rotation in B is fixed, effectively simplifying the structure to:



Using the force or displacement method, this leads to a moment line:

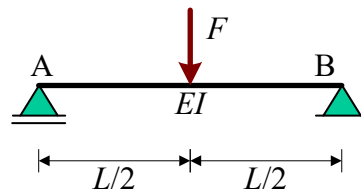


For the full structure, the moment line then leads to:

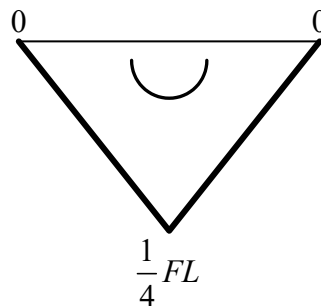


Low stiffness

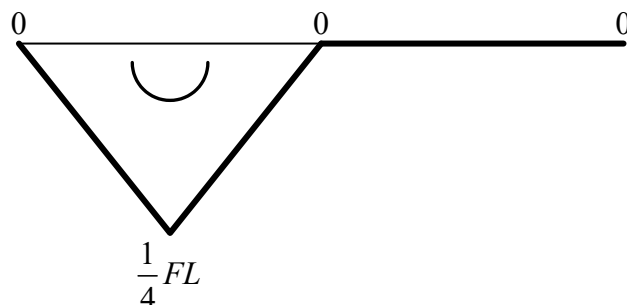
For a low stiffness, so $n \rightarrow 0$ the part BC of the beam can very easily bend, effectively simplifying the structure to:



For this statically determinate structure, the moment line is:

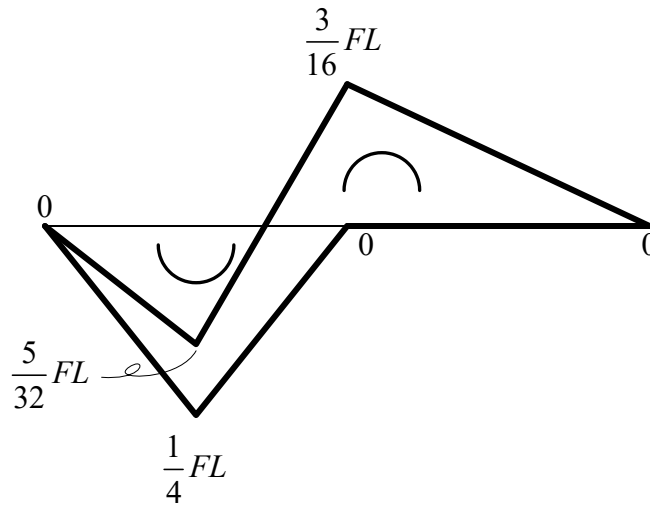


For the full structure, the moment line then leads to:



Envelope of moment line

Combining both solution, the full envelop of moment lines looks like:



In practise, the differences will not be as extreme as 0 or ∞ , leading to moments line within the envelope.

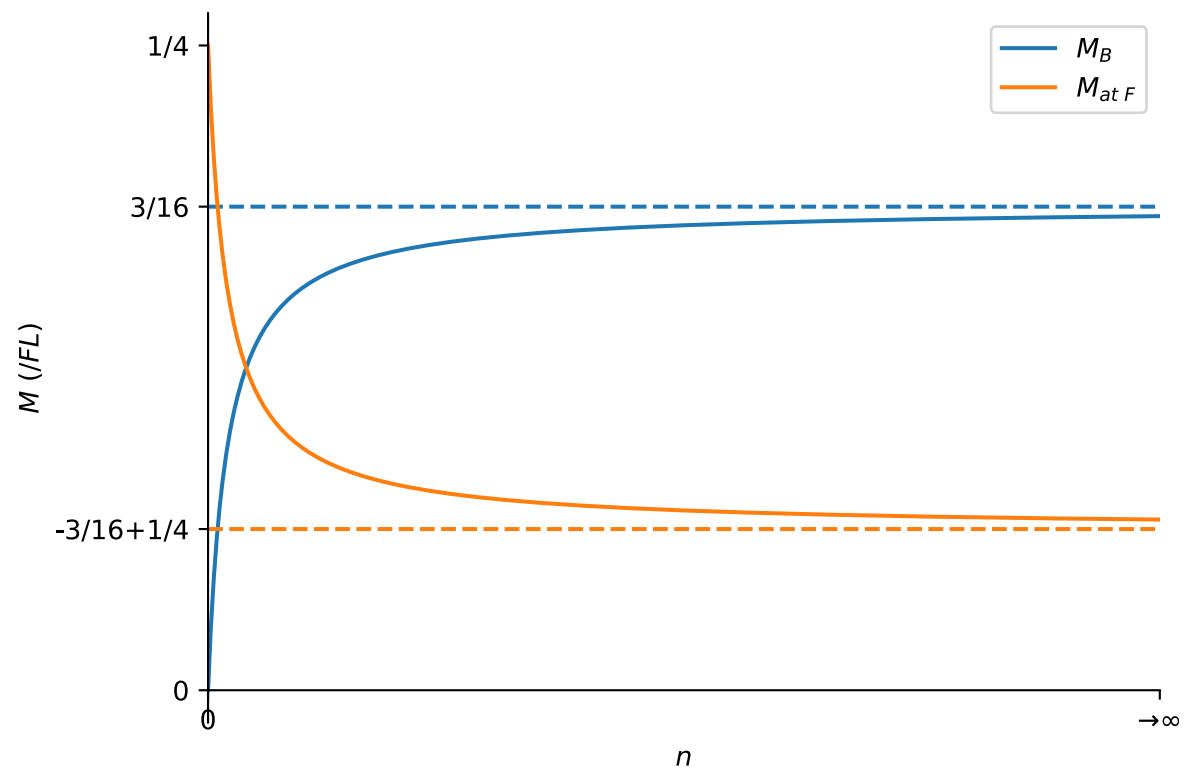
Exact calculation

Using the usual force or displacement method with an unknown stiffness of BC of nEI , the extreme moments can be found as:

$$M_B = \frac{n}{n+1} \frac{3}{16} FL$$

$$M_{at F} = \left(-\frac{n}{n+1} \frac{3}{16} + \frac{1}{4} \right) FL$$

This leads to the following diagrams:



Showing how for $n = 0$ and $n \rightarrow \infty$ the moments approach the extreme values obtained before.

Week 8 (October 21th)

	Monday	Tuesday	Wednesday	Thursday	Friday	Upcoming homework
Homework deadlines			Read theory and make homework exercises Stiffness discontinuities and support settlement New content		Read theory and make homework exercises temperature influences New content	Friday week 10: Preparation for exam assignment Statically indeterminate structures New content Prerequisites
Class activities			Guided practice on Stiffness discontinuities and support settlement New content		Guided practice on temperature influences New content	

◀
▶

Homework deadline lesson

Wednesday October 23th

As a preparation for Monday's lesson you'll look into the Support settlement and Stiffness influences:

[Solve structures subjected to support settlement](#)

[Solve structures subjected to stiffness influences](#)

Please go through the topics which you can improve your skills on. Take at least a look at the examples and try a few exercises. During the lesson you'll make some advanced exercises.

Lesson October 23th

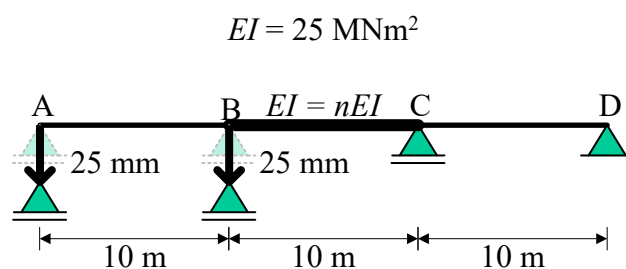
Contents

- Exercise Support settlement
- Exercise Stiffness discontinuities

During today's lesson you'll work on a complex exercise on the topic of Stiffness discontinuities and Support settlement. Please ask your questions regarding the [homework](#) as well!

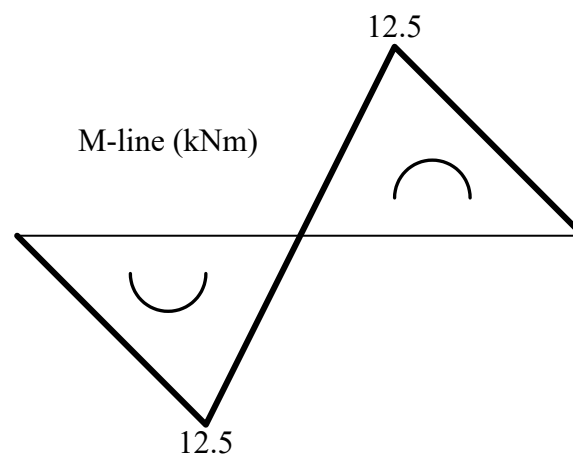
Exercise Support settlement

Given the following structure.



1. Find the moment distribution for $n = 1$

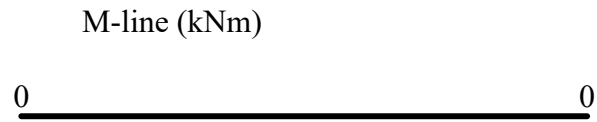
💡 Solution assignment 1



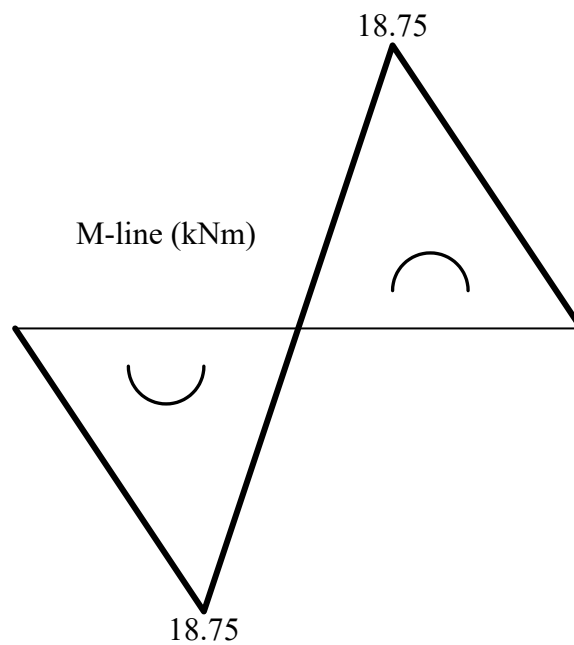
Exercise Stiffness discontinuities

2. Find the moment distribution for $n = 0$
3. Find the moment distribution for $n \rightarrow \infty$

💡 Solution assignment 2



💡 Solution assignment 3



Homework deadline lesson

Friday October 25th

As a preparation for Monday's lesson you'll look into the Temperature influences:

[Constitutive relations](#)

[... for temperature influences](#)

[Solve displacements statically determinate structures subjected to temperature influences](#)

[Solve statically indeterminate structures subjected to temperature influences](#)

Please go through the topics which you can improve your skills on. Take at least a look at the examples and try a few exercises. During the lesson you'll make some advanced exercises.

Lesson October 25th

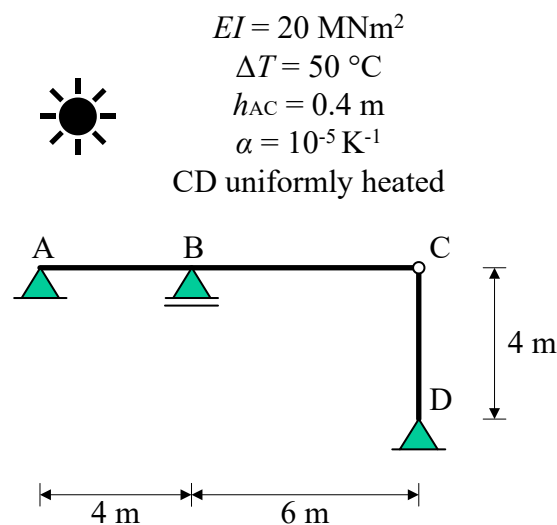
Contents

- Exercise Temperature influences

During today's lesson you'll work on a complex exercise on the topic of Temperature influences. Please ask your questions regarding the [homework](#) as well!

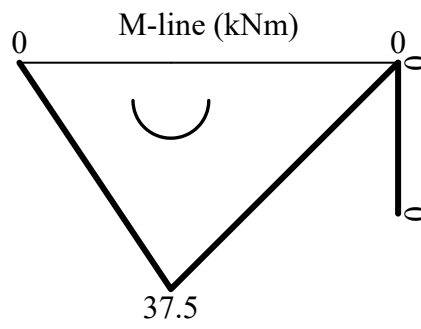
Exercise Temperature influences

Given the following structure.

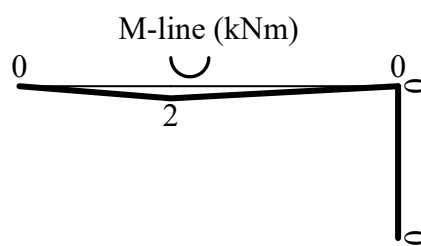


1. Find the moment line due to the temperature influence on AC
2. Find the moment line due to the temperature influence on CD
3. Find the total moment line.

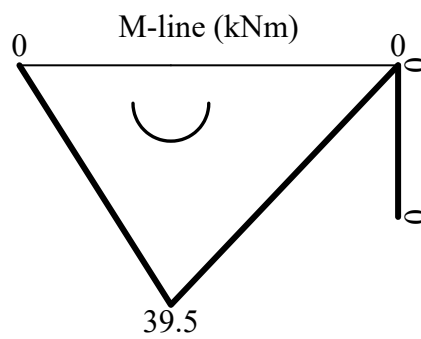
💡 Solution assignment 1



💡 Solution assignment 2



💡 Solution assignment 3



Week 10 (November 4th)

	Monday	Tuesday	Wednesday	Thursday	Friday	Upcoming homework
Homework deadlines					Prepare for exam assignment Statically indeterminate structures New content Prerequisites	Friday: Read theory and make homework exercise cross-sectional properties Prerequisites
Class activities					Exam assignment 1 statically indeterminate structures New content Prerequisites	



Preparation exam Friday November 8th

Contents

- Assignment on statically indeterminate structures

Assignment on statically indeterminate structures

As a preparation for the exam assignment on Statically indeterminate structures you can repeat all the topics of Statically indeterminate structures, including the prerequisites. This content can be viewed at:

- All content from Week 3 Friday to Week 8 Friday in this book
- The stripped version of the [Mechanics-BSc book](#) (url includes `/2024-assignment1`).
- The following chapters of the full [Mechanics-BSc book](#):
 - Statics: all topics as a prerequisite except for `Differential equations for equilibrium relations` and `Solve section forces ... using differential equations`.
 - Stresses and displacements: all topics as a prerequisite, except for `Cross-sectional properties` and `Solve displacements ... using differential equations`.
 - Statically indeterminate structures: all topics

Repeat those topics which you have difficulty with. If you've any questions along the way, please visit the TAs in 6.41 during lunch time.

Exam Friday November 8th

Contents

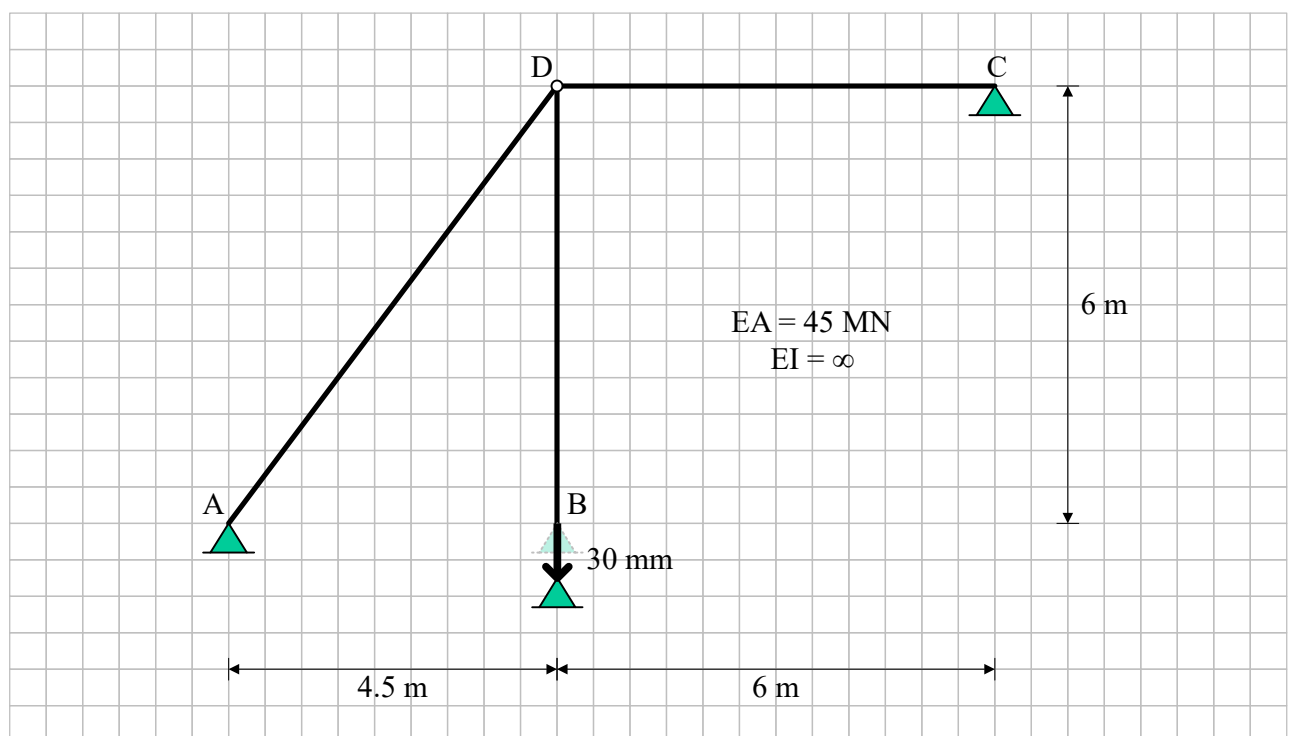
- Exam assignment

Today you'll make the first exam assignment covering Statically indeterminate structures including its prerequisites. For more information about the exam see [the assessment information in course information](#)

Exam assignment

Your own submission and its grading can be found here: [ans* exam assignment Statically indeterminate structures 1](#). The exam assignment was provided as follows:

Given is the structure as shown in the figure below.



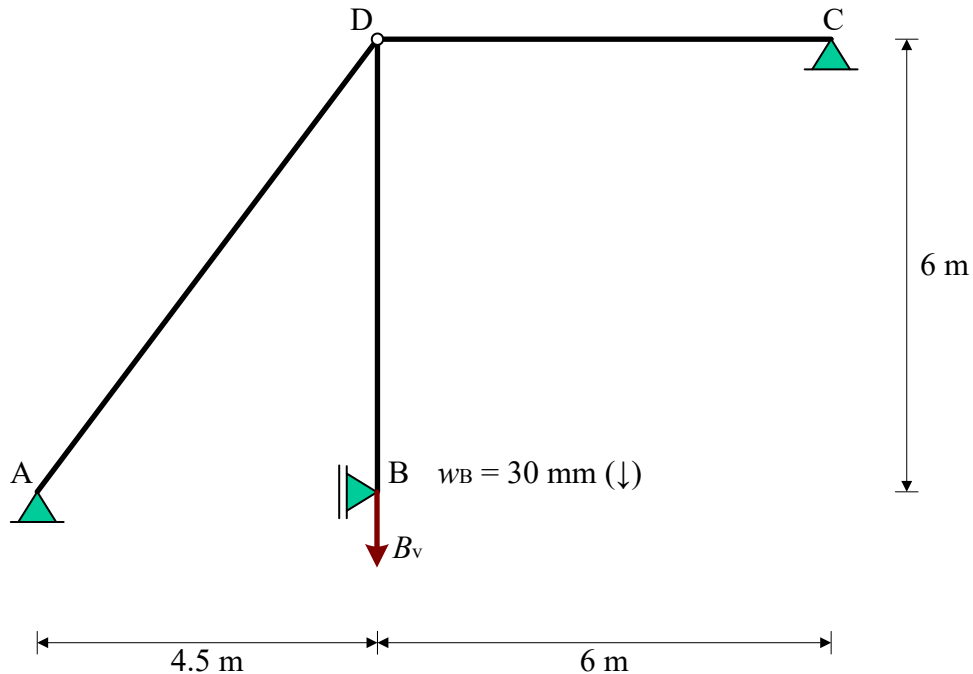
Calculate the displacement of D and the normal forces in all the members using a force-, displacement- or hybrid- ('hoekveranderingsvergelijkingen' with moveable nodes) method.

Solution assignment 1



Convert the structure into a statically determinate structure with a displacement or force condition.

For example when using the force method:



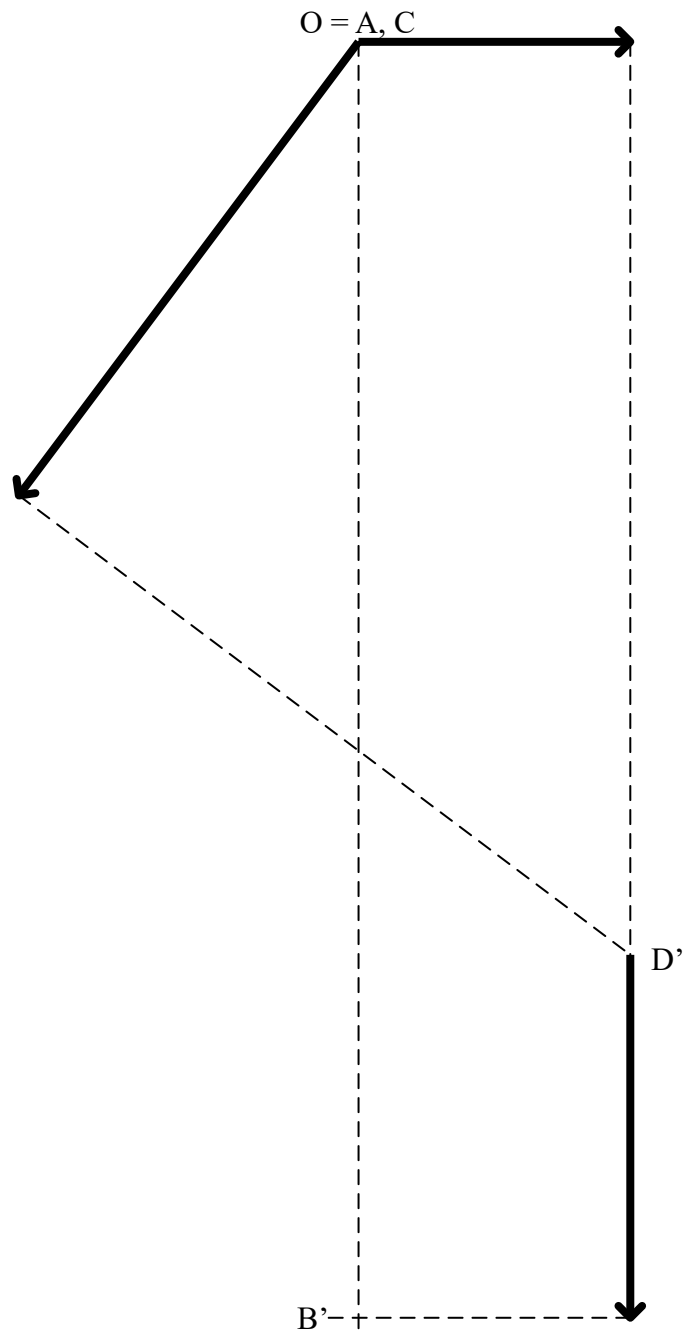
This gives:

- $N_{AD} = -\frac{4}{5}B_v$
- $N_{BD} = +B_v$
- $N_{CD} = -\frac{3}{4}B_v$

This gives elongations:

- $\Delta L_{BD} = \frac{B_v}{7500}$
- $\Delta L_{AD} = -\frac{B_v}{4800}$
- $\Delta L_{CD} = -\frac{B_v}{10000}$

Resulting in a williot like this (keeping the unknown value of B_v constant for all elongations):



This gives a vertical displacement of B of $\frac{3B_v}{6400}$.

With the requirements of 30 mm this leads to $B_v = 64$ kN, resulting in:

- $N_{BD} = +64$ kN
- $N_{AD} = -80$ kN
- $N_{CD} = -48$ kN
- $u_{D,h} = 6.4$ mm ($\rightarrow$)
- $u_{D,v} = 21.4667$ mm ($\downarrow$)

Week 11 (November 11th)

	Monday	Tuesday	Wednesday	Thursday	Friday	Upcoming home
Homework deadlines					Read theory and make homework exercise cross-sectional properties Prerequisites	Monday Read theory and make homework exercise cross-sectional properties Prerequisites
Class activities	Review exam assignment 1 statically indeterminate structures New content Prerequisites Demonstration transforming stress- and strain tensors New content			Cancelled due to protest against cuts in higher education	Guided practice on cross-sectional properties Prerequisites	



Lesson November 11th

Contents

- Results first exam assignment on Statically indeterminate structures
- Demonstration transforming stresses and Circle of Mohr

Today we'll discuss the results of the first [exam assignment on Statically indeterminate structures](#).

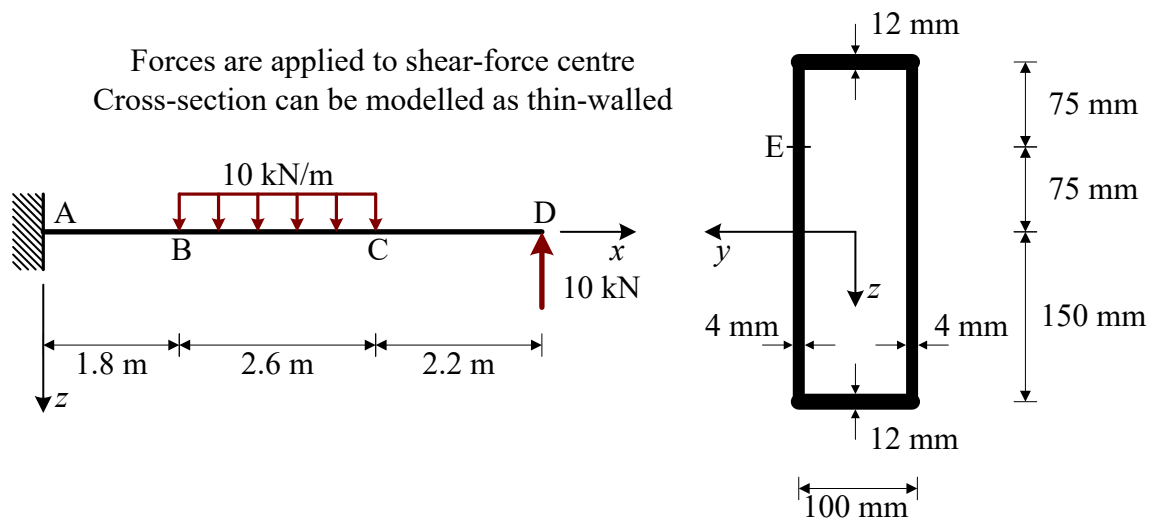
Results first exam assignment on Statically indeterminate structures

Approaches taken during the [exam assignment](#):

- Mostly applied displacement method with displacements at D
- One person applied force method for horizontal support reactions at C, another person for N_{BD} . Both of them abandoned that a
- Not explicitly clear which statically determinate structure is solved for
- Displacement of 30 mm used incorrectly as full total elongation of bar BD

Demonstration transforming stresses and Circle of Mohr

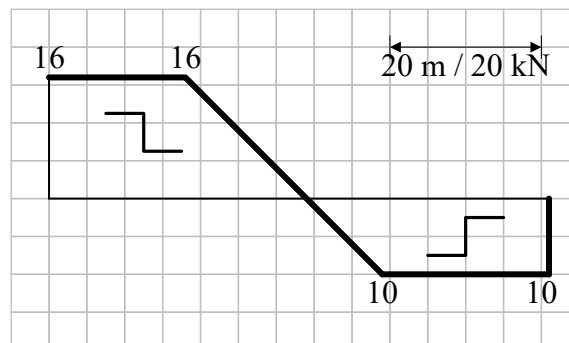
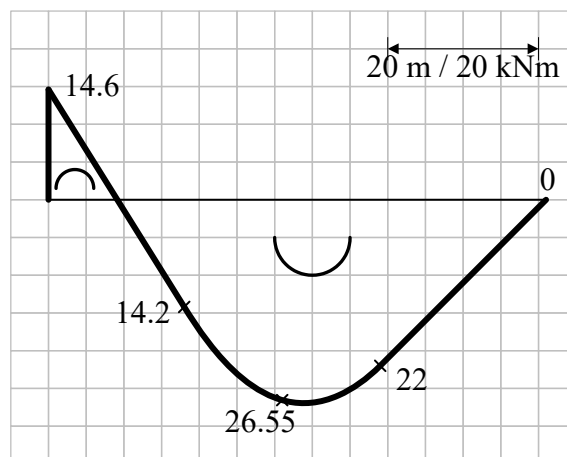
Given the following structure and cross section.



We'll find the maximum stresses in point E in cross-section A and its direction.

Internal forces

First, let's find the internal forces:



At cross-section A this gives a moment of -14.6 kNm and shear force of $+16$ kN.

Cross-sectional properties

For this thin-walled cross-section, the second moment of area of the cross-section can be calculated with:

$$I_{zz} = 2 \cdot \frac{1}{12} \cdot 4 \cdot 300^3 + 2 \cdot \left(\frac{1}{12} \cdot 100 \cdot 12^3 + 12 \cdot 100 \cdot 150^2 \right) = 72.0288 \cdot 10^6 \text{ mm}^4$$

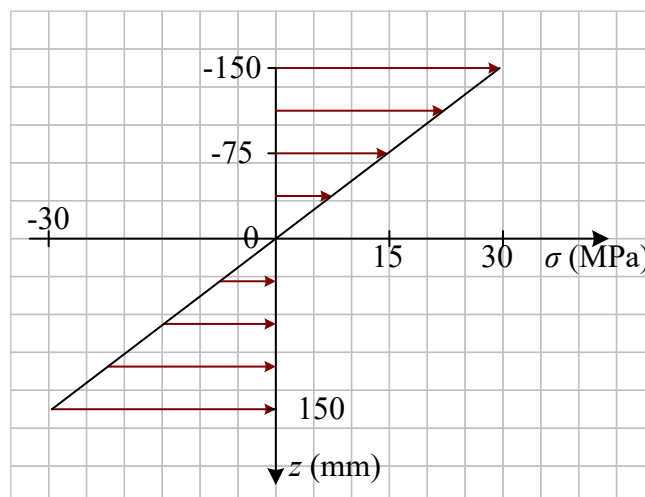
Normal and shear stresses

The normal stresses can be calculated as:

$$\sigma_E = \frac{-14600 \cdot -0.075}{72.0288 \cdot 10^{-6}} \approx +15 \cdot 10^6 \text{ Pa}$$

$$\sigma_{\max} = \frac{-14600 \cdot -0.150}{72.0288 \cdot 10^{-6}} \approx +30 \cdot 10^6 \text{ Pa}$$

Leading to the following diagram:



The shear forces can be calculated as:

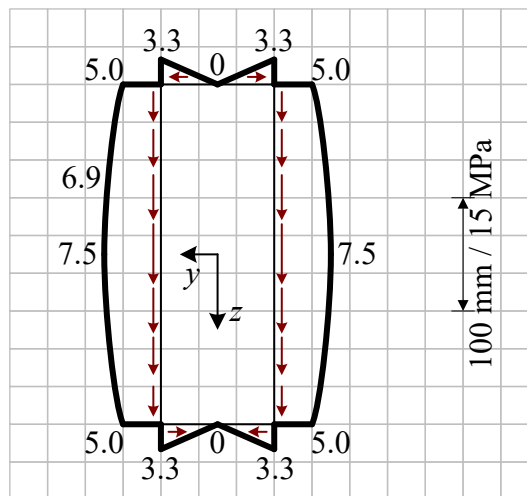
$$\sigma_{x\text{m,max flange}} = - \frac{16000 \cdot 12 \cdot 100 \cdot 150}{2 \cdot 12 \cdot 72.0288 \cdot 10^{-6}} \approx 1.7 \cdot 10^6 \text{ Pa}$$

$$\sigma_{xm,min\ web} = -\frac{16000 \cdot 12 \cdot 100 \cdot 150}{2 \cdot 4 \cdot 72.0288 \cdot 10^{-6}} \approx 5.0 \cdot 10^6 \text{ Pa}$$

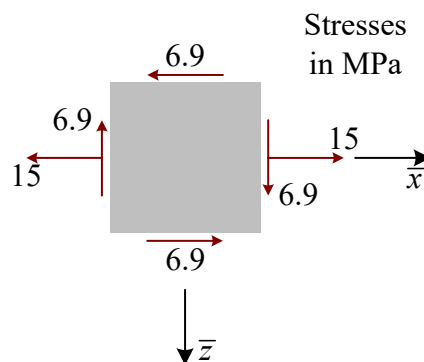
$$\sigma_{xm,E} = -\frac{16000 \cdot (12 \cdot 100 \cdot 150 + 2 \cdot 4 \cdot 75 \cdot 112.5)}{2 \cdot 4 \cdot 72.0288 \cdot 10^{-6}} \approx 6.9 \cdot 10^6 \text{ Pa}$$

$$\sigma_{xm,max\ web} = -\frac{16000 \cdot (12 \cdot 100 \cdot 150 + 2 \cdot 4 \cdot 150 \cdot 75)}{2 \cdot 4 \cdot 72.0288 \cdot 10^{-6}} \approx 7.5 \cdot 10^6 \text{ Pa}$$

Leading to the following diagram:

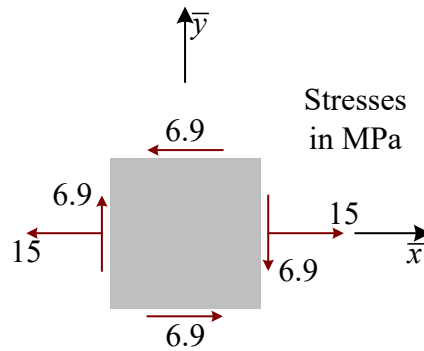


The stress state for a rectangular element at point E is therefore:



Find maximum stress

As the stress can be represented as a tensor. Therefore, a x, y -coordinate system is introduced:



Leading to the tensor σ :

$$\sigma = \begin{bmatrix} 15 & -6.9 \\ -6.9 & 0 \end{bmatrix} \text{ MPa}$$

The maximum stress can be found by applying the transformation rules:

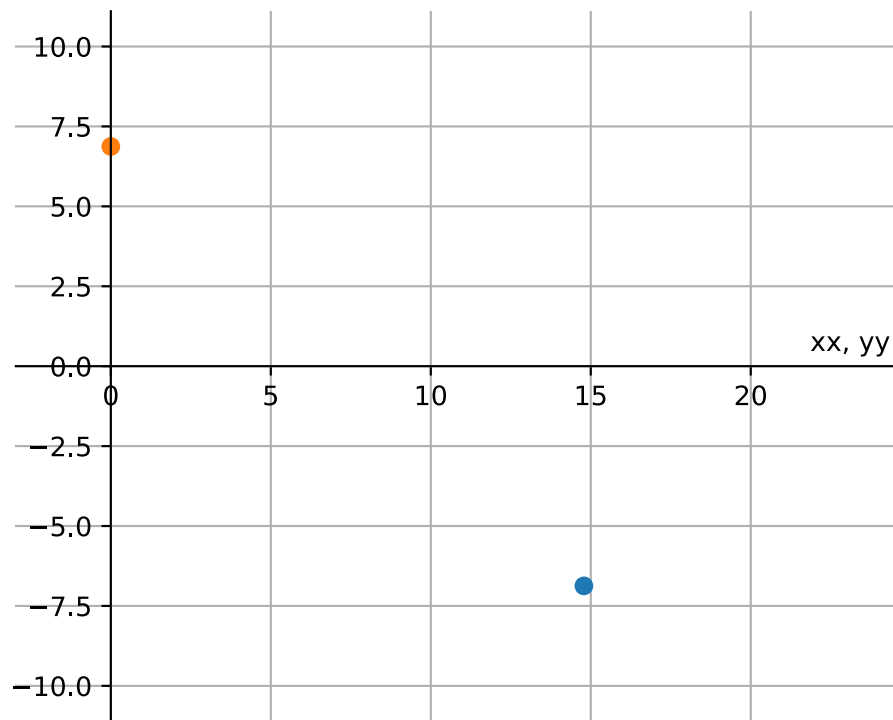
$$\sigma_1 = \frac{1}{2}(15 + 0) + \sqrt{\left(\frac{1}{2}(15 - 0)\right)^2 + (-6.9)^2} \approx 17 \text{ MPa}$$

With a corresponding angle α :

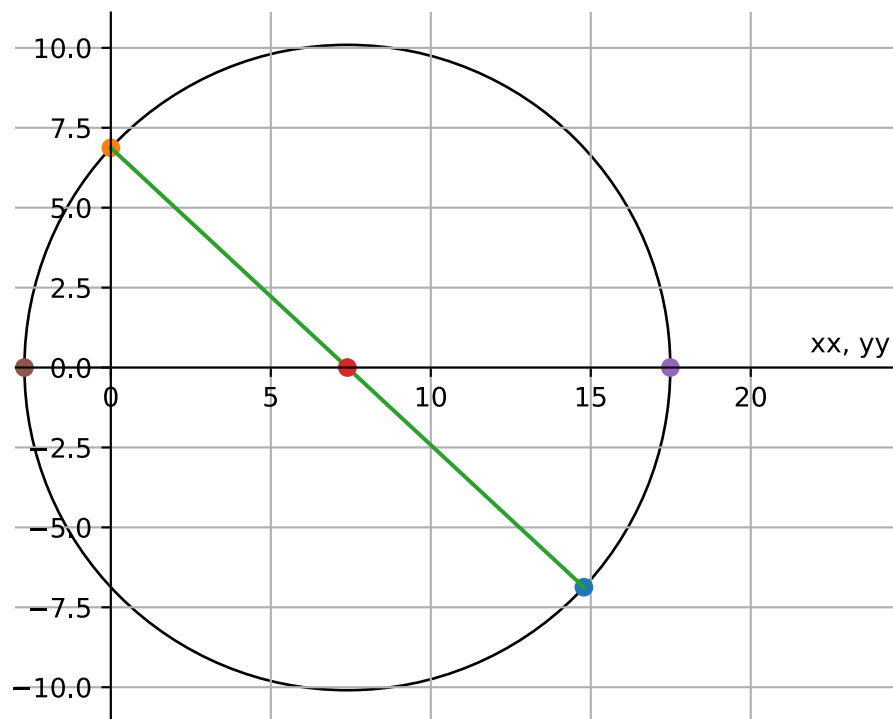
$$\alpha = \frac{1}{2} \arctan \left(\frac{-6.9}{\frac{1}{2}(15 - 0)} \right) \approx 21^\circ$$

This can also be found using a circle of Mohr:

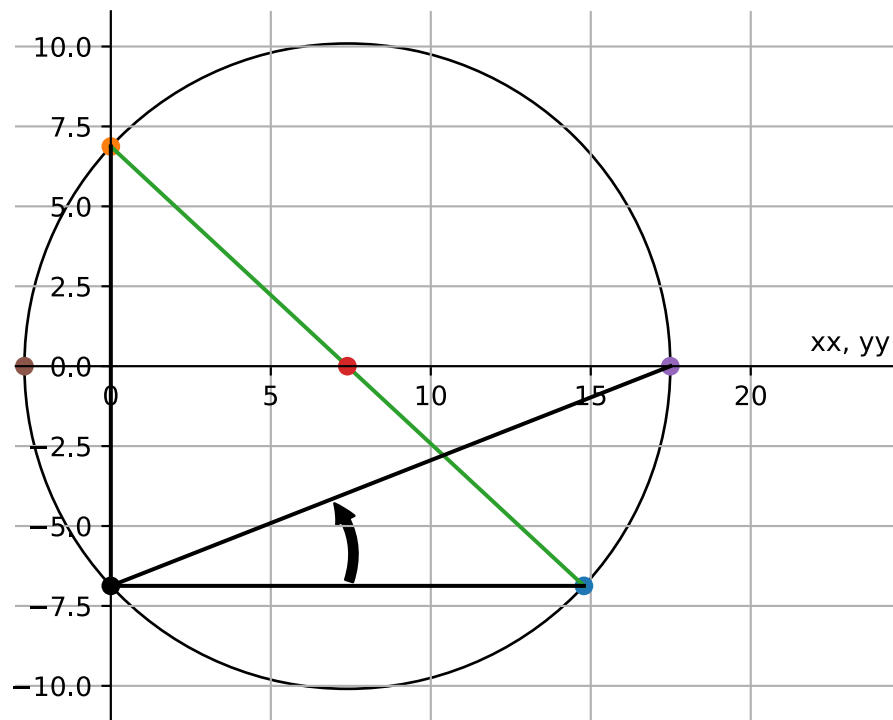
First, the two points $(15, -6.9)$ and $(0, -6.9)$ are added:



Then, Mohr's circle can be drawn



Given the same maximum of $\sigma_1 \approx 17$ MPa. Finally, the corresponding rotation can also be found:



Which gives the same $\alpha \approx 31^\circ$.

Homework deadline lesson

Friday November 15th

As a preparation for Friday's lesson you'll look into cross sectional properties, mainly the static moment of area and the moment of inertia.

[Static moment, centroid and normal force centre](#)

[Second moment of area and Section modulus](#)

Please go through the topics which you can improve your skills on. Take at least a look at the examples and try a few exercises. During the lesson you'll make some advanced exercises.

Lesson November 15th

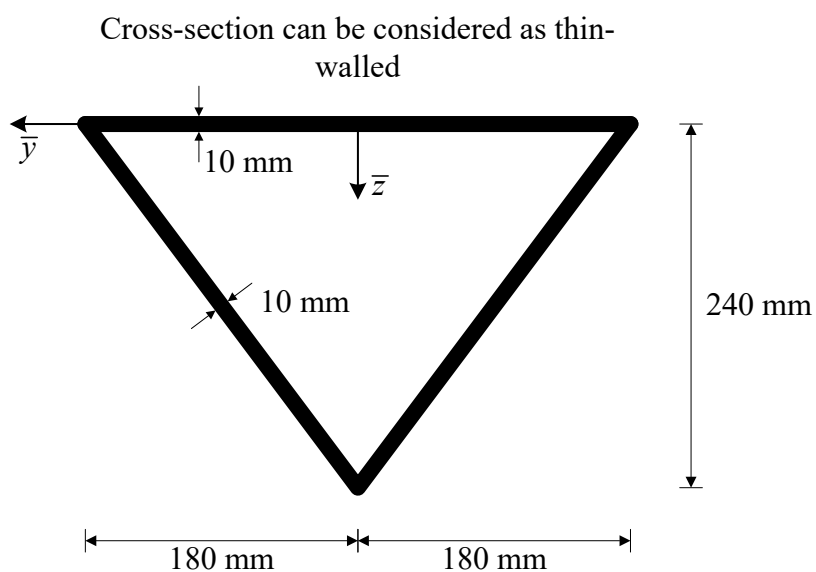
Contents

- Exercise Cross-sectional properties

During today's lesson you'll work on a complex exercise on the topic of Cross-sectional properties. Please ask your questions regarding the [homework](#) as well!

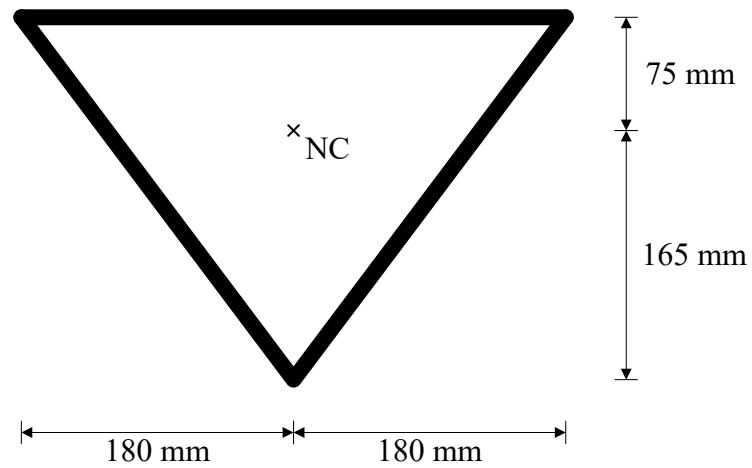
Exercise Cross-sectional properties

Given is the following cross-section:



1. Find the normal force centre
2. Find the second moment of area I_{zz}

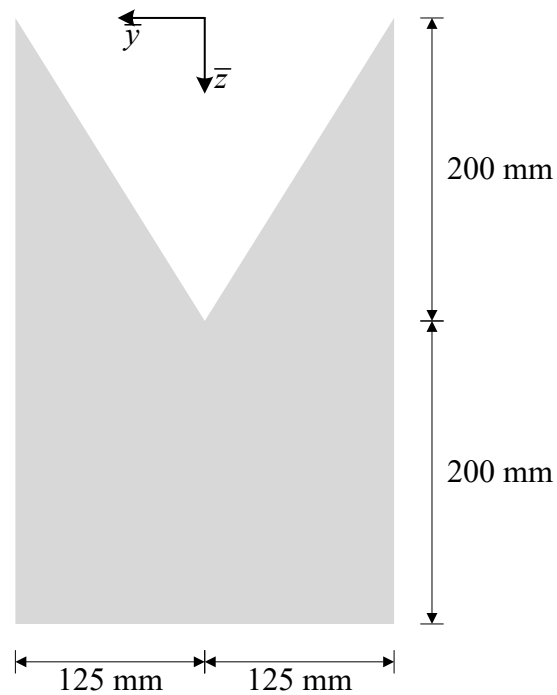
💡 Solution assignment 1



💡 Solution assignment 2

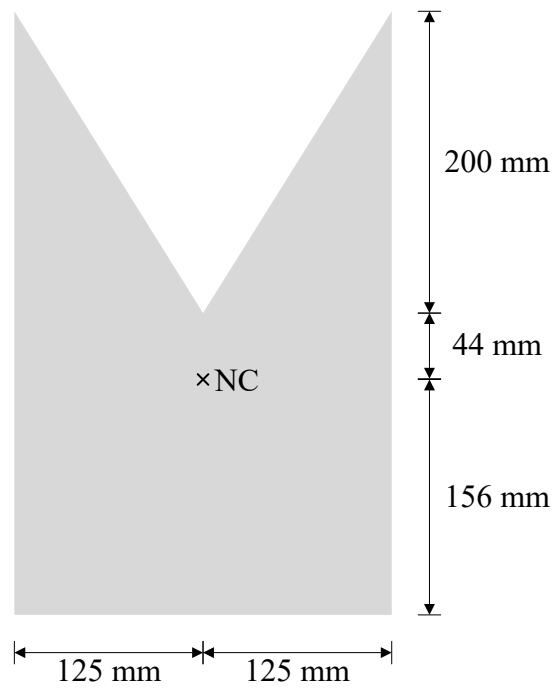
$$I_{zz} = 61.2 \cdot 10^6 \text{ mm}^4$$

Given is the following cross-section:



3. Find the normal force centre
4. Find the second moment of area I_{zz}

💡 Solution assignment 3

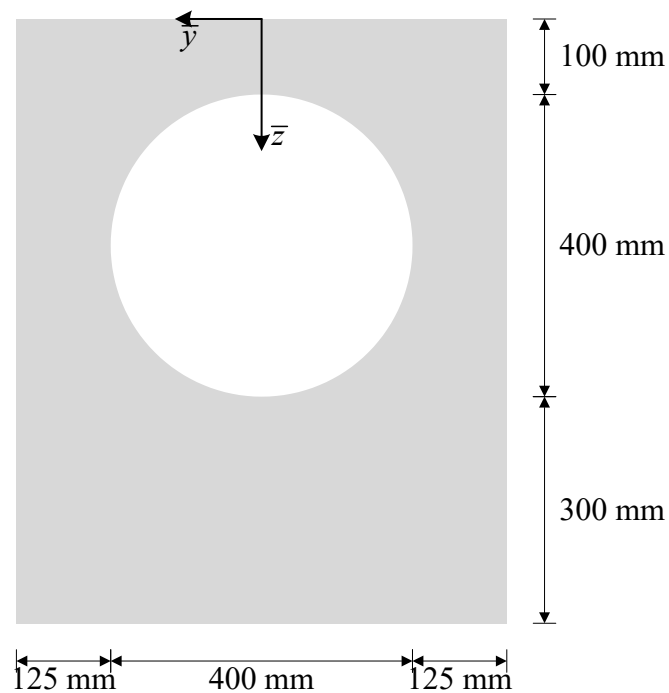


💡 Solution assignment 4



$$I_{zz} = 685.3 \cdot 10^6 \text{ mm}^4$$

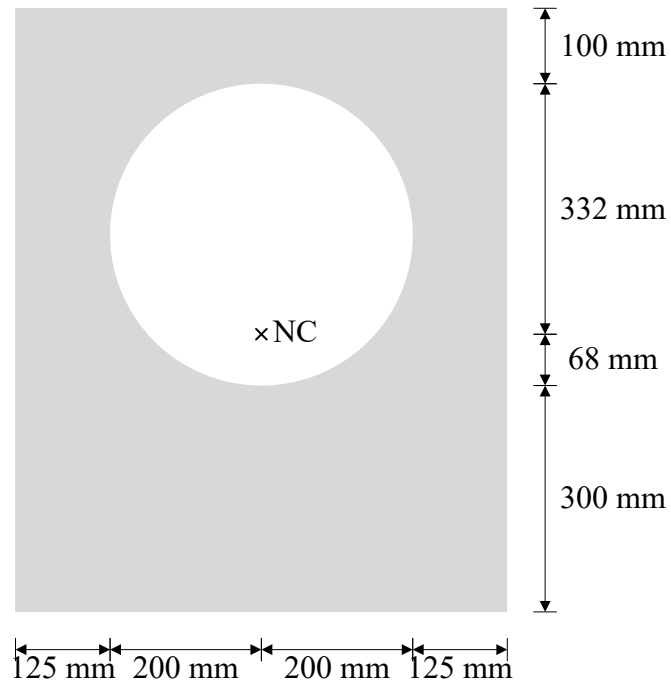
Given is the following cross-section:



5. Find the normal force centre

6. Find the second moment of area I_{zz}

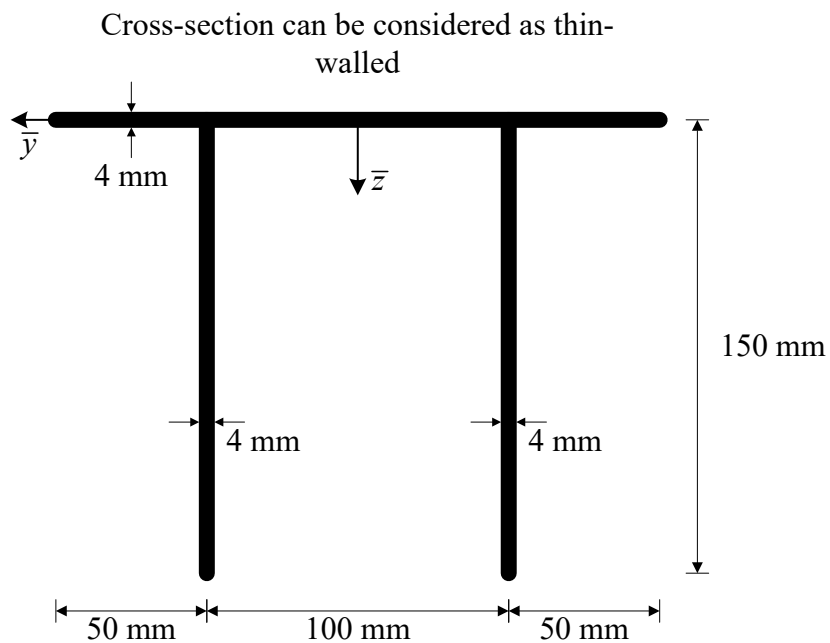
💡 Solution assignment 5



💡 Solution assignment 6

$$I_{zz} = 24.82 \cdot 10^9 \text{ mm}^4$$

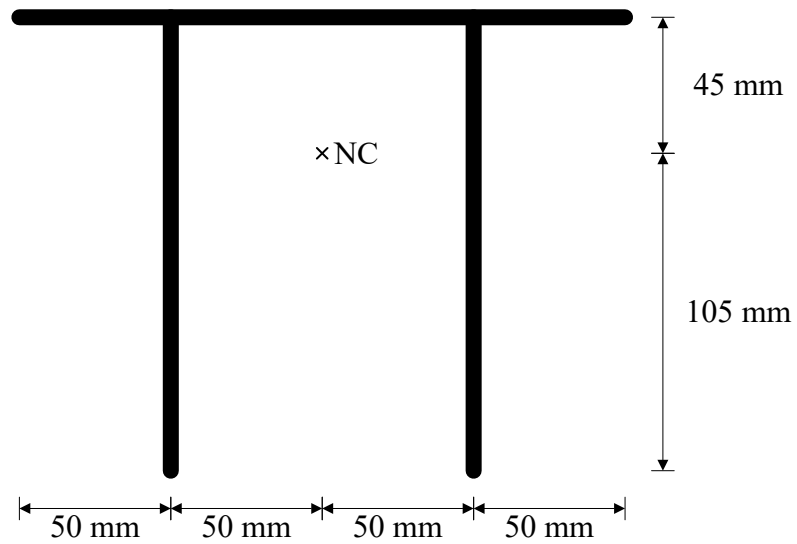
Given is the following cross-section:



7. Find the normal force centre

8. Find the second moment of area I_{zz}

💡 Solution assignment 7



💡 Solution assignment 8



$$I_{zz} = 4.95 \cdot 10^6 \text{ mm}^4$$

Week 12 (November 18th)

	Monday	Tuesday	Wednesday	Thursday	Friday	Up ho
Homework deadlines	Read theory and make homework exercise normal stresses Prerequisites	Read theory and make homework exercise shear stresses in bending elements Prerequisites			Read theory and make homework exercise core of cross- section Prerequisites	M Rea an ho ex tran te Ne
Class activities	Guided practice on normal stresses Prerequisites	Guided practice on shear stresses in bending elements Prerequisites			Guided practice on core of cross- section Prerequisites	

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▶

Homework deadline lesson

Monday November 18th

As a preparation for Monday's lesson you'll look into normal stresses due to normal forces and bending moments:

[Solve stresses in an extension element](#)

[Normal stresses](#)

[Solve stresses in a bending element](#)

[Normal stresses](#)

Please go through the topics which you can improve your skills on. Take at least a look at the examples and try a few exercises. During the lesson you'll make some advanced exercises.

Lesson November 18th

Contents

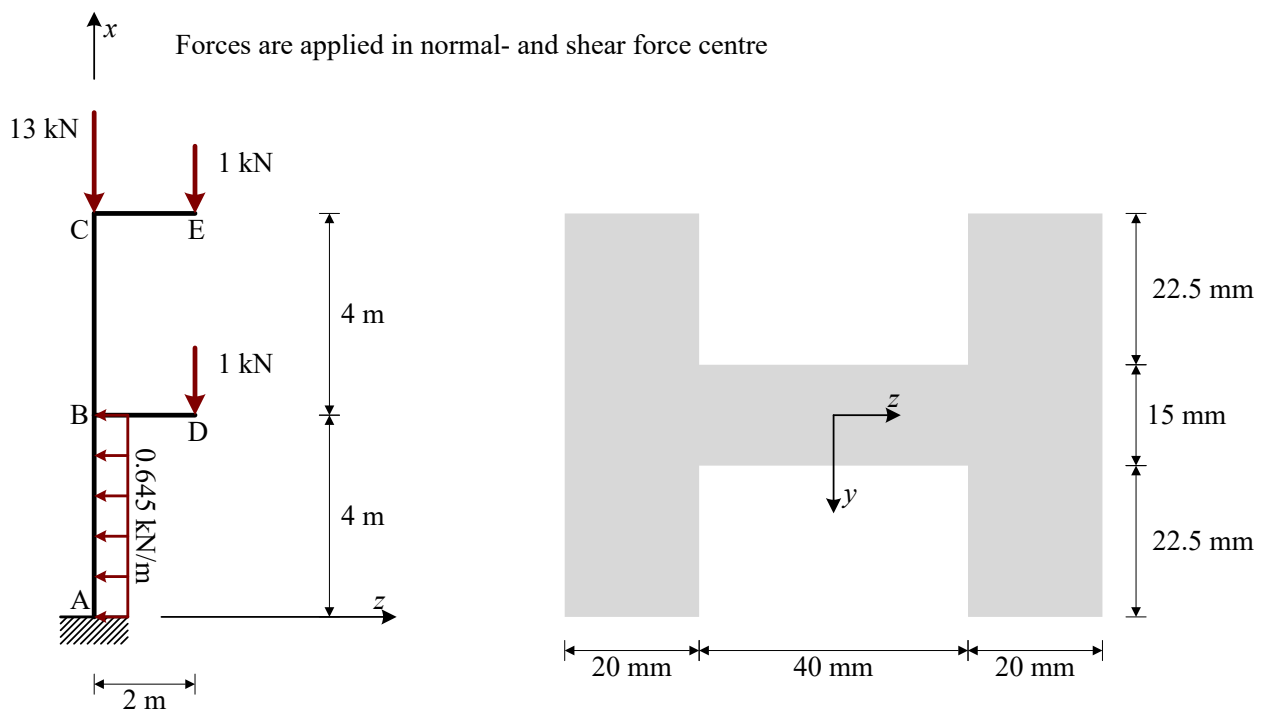
- Exercise normal stresses

During today's lesson you'll work on a complex exercise on the topic of normal stresses.

Please ask your questions regarding the [homework](#) as well!

Exercise normal stresses

Given is the following structure and cross-section:

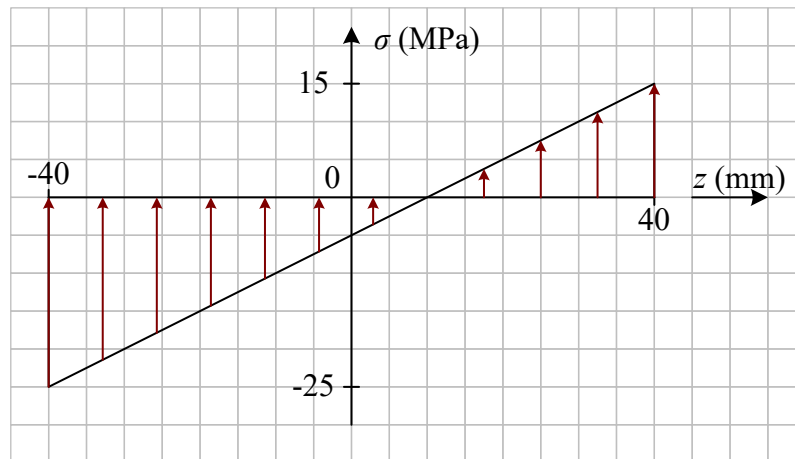


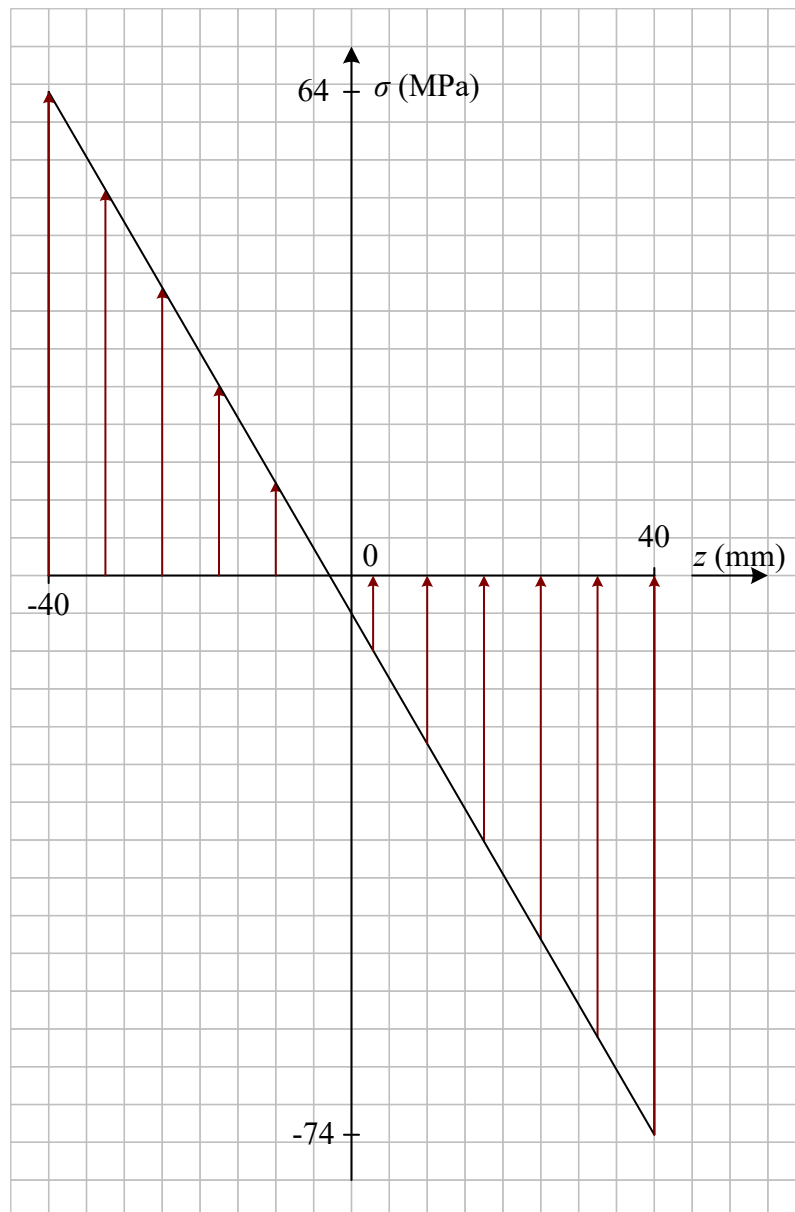
1. Find the relevant cross-sectional properties.
2. Find the normal stresses at cross-section A and draw the normal stress distribution in this cross-section.
3. Find the normal stresses in the cross-section just below B and draw the normal stress distribution in this cross-section.

💡 Solution assignment 1

- $A = 3000 \text{ mm}^2$
- $I_{zz} = 2.32 \cdot 10^6 \text{ mm}^4$

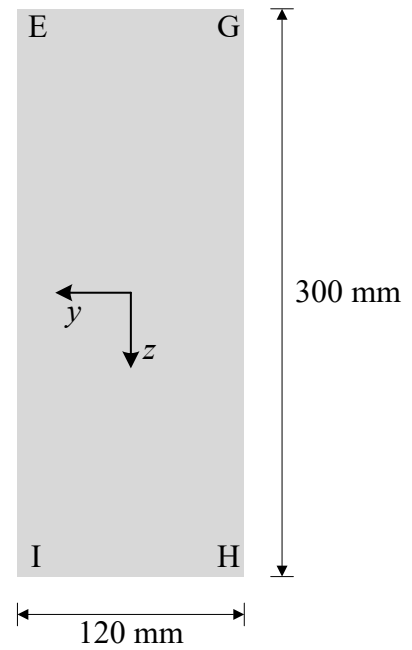
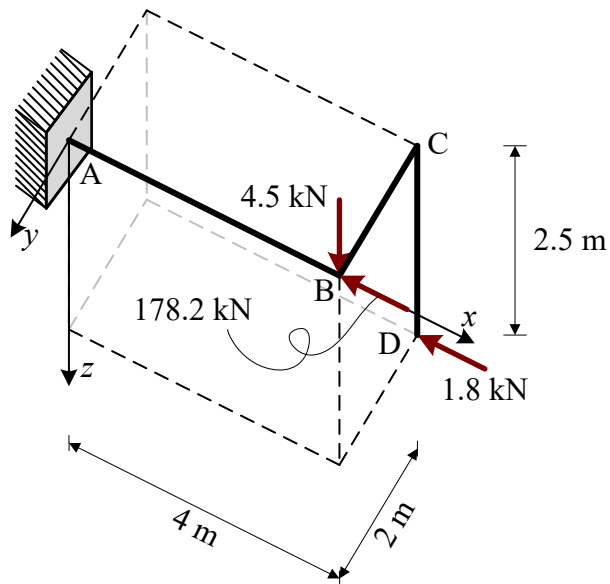
💡 Solution assignment 2





Given is the following structure and cross-section:

Forces are applied in normal- and shear force centre



4. Find the relevant cross-sectional properties.
5. Find the normal stresses at cross-section B in beam AB in points E, F, G, H.

💡 Solution assignment 1

- $A = 36000 \text{ mm}^2$
- $W_z = 1.8 \cdot 10^6 \text{ mm}^3$
- $W_y = 0.72 \cdot 10^6 \text{ mm}^3$

💡 Solution assignment 2

- $\sigma_E = 2.5 \text{ MPa}$
- $\sigma_G = -7.5 \text{ MPa}$
- $\sigma_H = -12.5 \text{ MPa}$
- $\sigma_I = -2.5 \text{ MPa}$

Homework deadline lesson

Tuesday November 19th

As a preparation for Tuesday's lesson you'll look into shear stresses due to bending moments:

[Solve stresses in a bending element](#)

[Shear stresses](#)

Please go through the topics which you can improve your skills on. Take at least a look at the examples and try a few exercises. During the lesson you'll make some advanced exercises.

Lesson November 19th

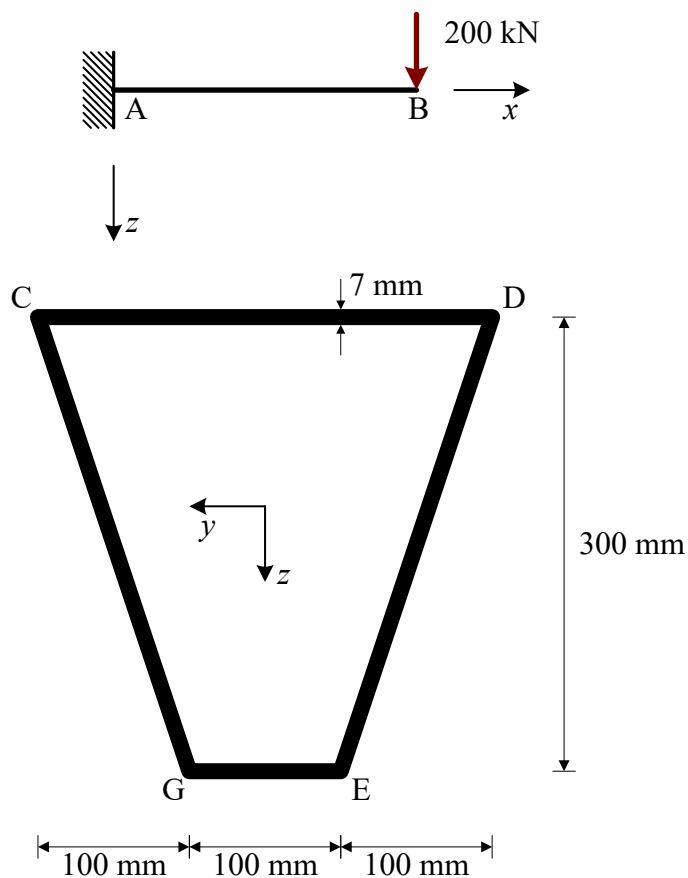
Contents

- Exercise shear stresses

During today's lesson you'll work on a complex exercise on the topic of shear stresses. Please ask your questions regarding the [homework](#) as well!

Exercise shear stresses

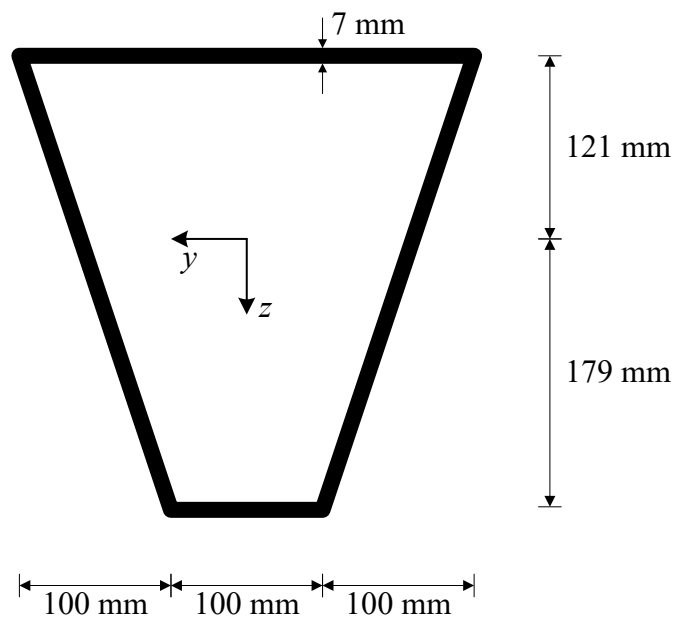
Given is the following structure and cross-section:



1. Find the relevant cross-sectional properties.
2. Find the shear stresses at cross-section A and draw the shear stress distribution in this cross-section.

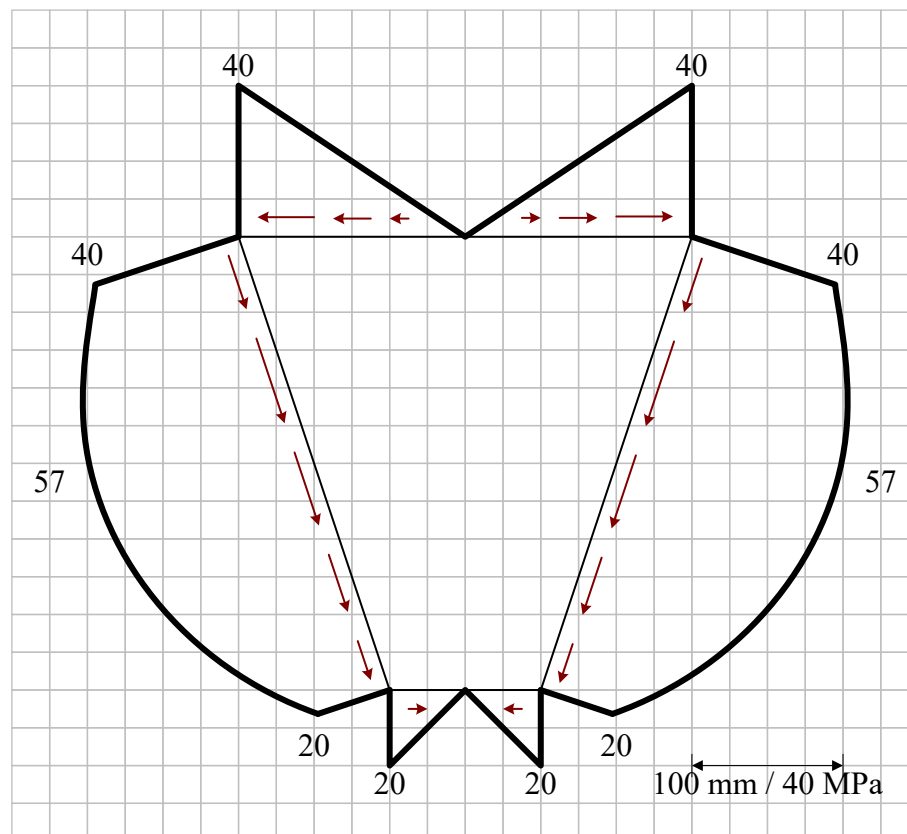
💡 Solution assignment 1

Normal force centre is given by:



$$I_{zz} = 90.1 \cdot 10^6$$

💡 Solution assignment 2



Homework deadline lesson

Friday November 22nd

As a preparation for Friday's lesson you'll look into the core of a cross-section:

[Core of a cross-section](#)

Please go through the topics which you can improve your skills on. Take at least a look at the examples and try a few exercises. During the lesson you'll make some advanced exercises.

Lesson November 22nd

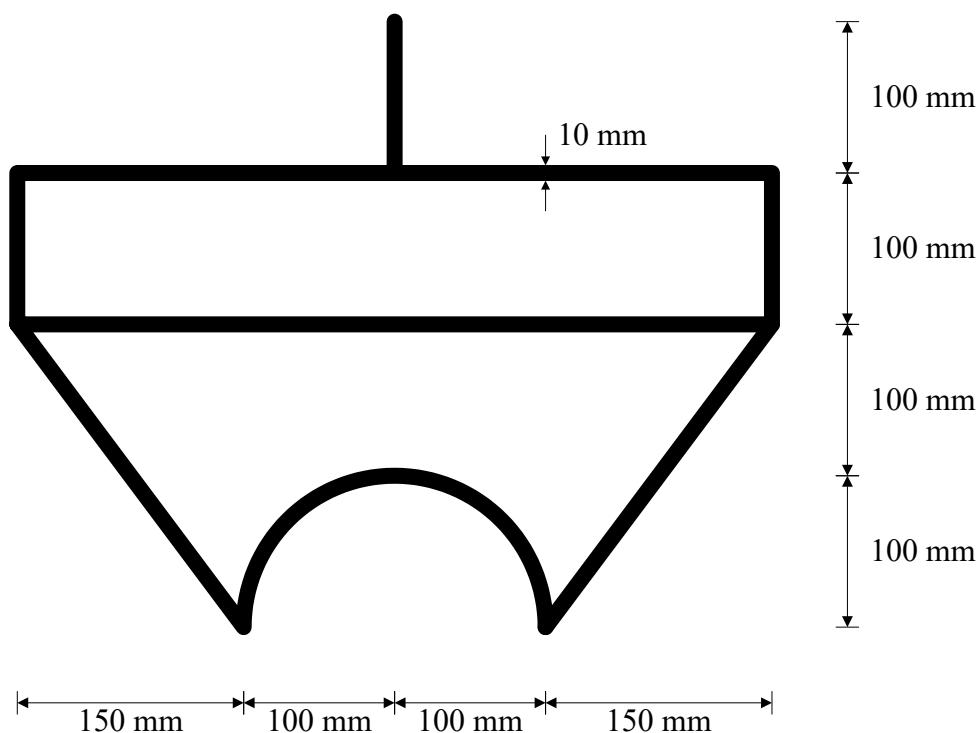
Contents

- Exercise Core

During today's lesson you'll work on a complex exercise on the topic of the core of a cross-section. Please ask your questions regarding the [homework](#) as well!

Exercise Core

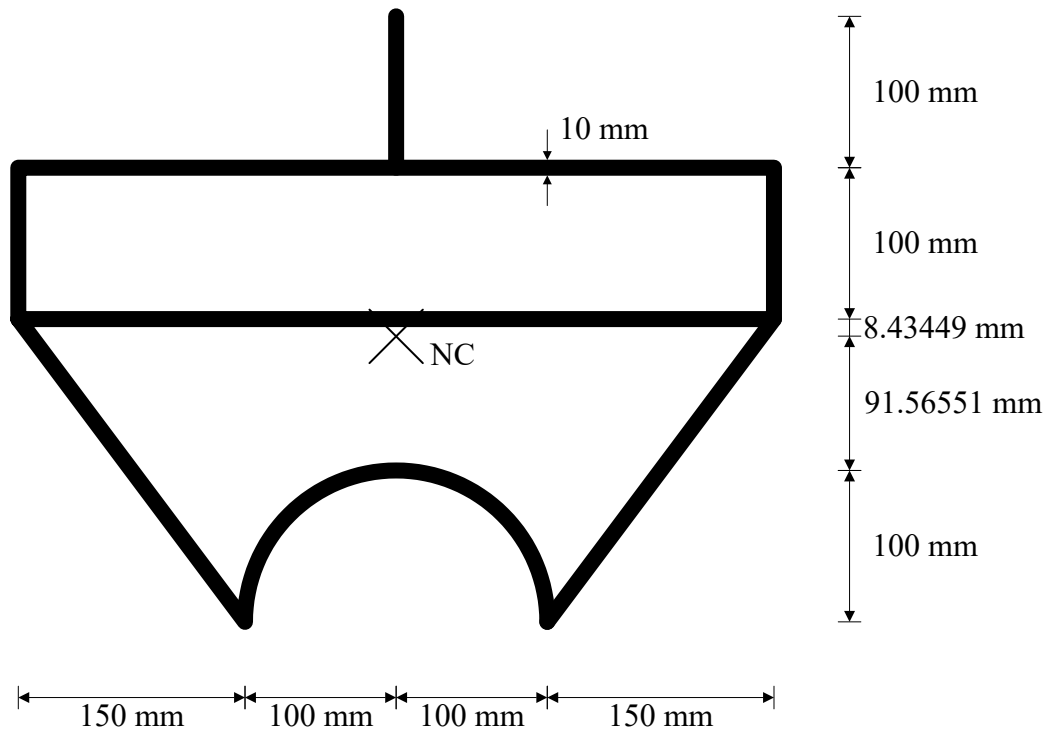
Given is the following structure and cross-section:



1. Find the relevant cross-sectional properties.
2. Find the shear stresses at cross-section A and draw the shear stress distribution in this cross-section.

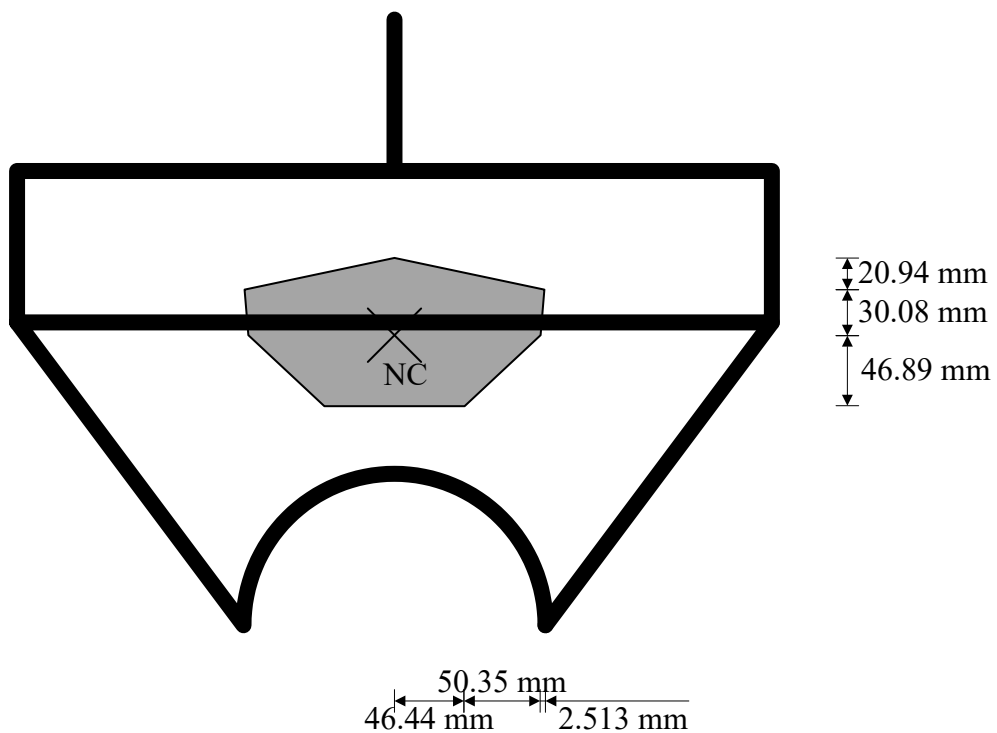
💡 Solution assignment 1

Normal force centre is given by:



- $A \approx 21141 \text{ mm}^2$
- $I_{zz} \approx 206.618 \cdot 10^6 \text{ mm}^4$
- $I_{yy} \approx 511.566 \cdot 10^6 \text{ mm}^4$

💡 Solution assignment 2



Week 13 (November 25th)

	Monday	Tuesday	Wednesday	Thursday	Friday	Upcoming
Homework deadlines	Read theory and make homework exercise transforming tensors New content	Read theory and make homework exercise Circle of Mohr New content			Read theory and make homework exercise torsional internal forces Prerequisites	Monday Read theory and make homework exercise torsional internal forces Prerequisites
Class activities	Guided practice on transforming tensors New content	Guided practice on Circle of Mohr New content			Guided practice on torsional internal forces Prerequisites	

Homework deadline lesson

Monday November 25th

As a preparation for Monday's lesson you'll look into Tensors and Transforming those:

[Model](#)

[Stresses in 3D](#)

[Strains in 2D](#)

[Tensors](#)

[Transformations](#)

[Analytical](#)

Please go through the topics which you can improve your skills on. Take at least a look at the examples and try a few exercises. During the lesson you'll make some advanced exercises.

Lesson November 25th

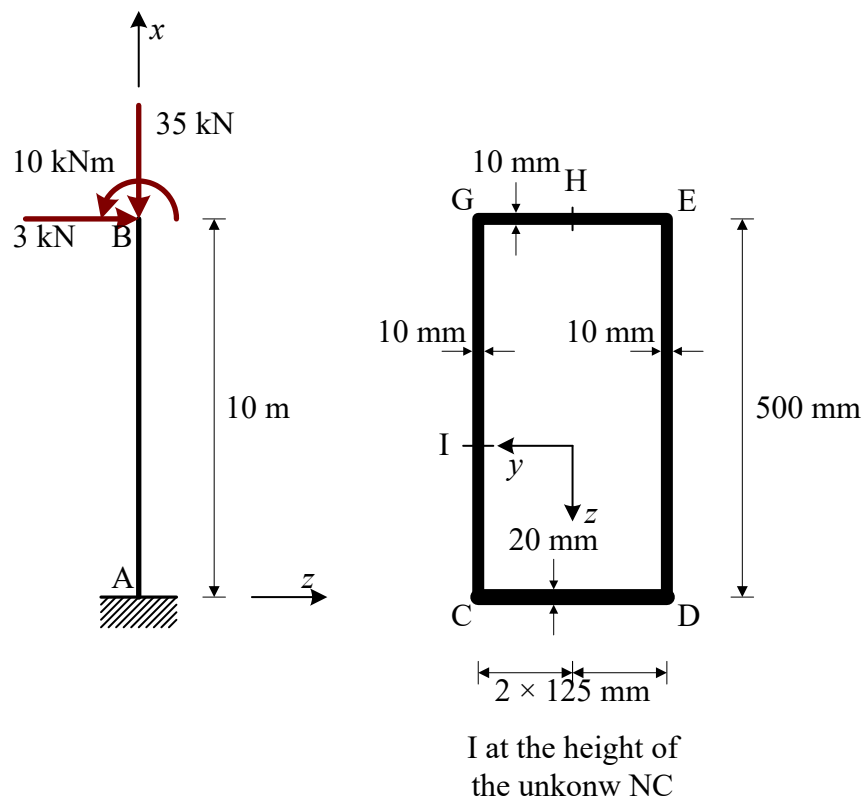
Contents

- Exercise Transforming tensors

During today's lesson you'll work on a complex exercise on the topic of the Transforming tensors. Please ask your questions regarding the [homework](#) as well!

Exercise Transforming tensors

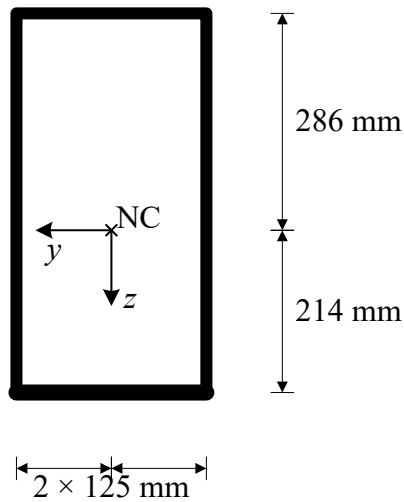
Given is the following structure and cross-section:



1. Find the relevant cross-sectional properties.
2. Find the normal and shear stresses just below G, in H, in I and just right of C in cross-section A.
3. Find the principal values of the stresses in the points just below G, in H, in I and just right of C in cross-section A.

💡 Solution assignment 1

Normal force centre is given by:



- $A \approx 17500 \text{ mm}^2$
- $I_{zz} \approx 655 \cdot 10^6 \text{ mm}^4$

💡 Solution assignment 2

- $\sigma_{\text{just below G}} = +6.73 \text{ MPa}$
- $\tau_{\text{just below G}} = +0.164 \text{ MPa}$
- $\sigma_H = +6.73 \text{ MPa}$
- $\tau_H = 0 \text{ MPa}$
- $\sigma_I = -2 \text{ MPa}$
- $\tau_I = +0.35 \text{ MPa}$
- $\sigma_{\text{just right of C}} = -8.53 \text{ MPa}$
- $\tau_{\text{just right of C}} = -0.12 \text{ MPa}$

💡 Solution assignment 3

- $\sigma_{1, \text{ just below G}} = +6.73 \text{ MPa}$
- $\sigma_{2, \text{ just below G}} = -0.0040 \text{ MPa}$
- $\sigma_{1, H} = +6.73 \text{ MPa}$
- $\sigma_{2, H} = 0 \text{ MPa}$
- $\sigma_{1, I} = 0.59 \text{ MPa}$
- $\sigma_{2, I} = -2.06 \text{ MPa}$
- $\sigma_{1, \text{ just right of C}} = 0.0018 \text{ MPa}$
- $\sigma_{2, \text{ just right of C}} = -8.5 \text{ MPa}$

Homework deadline lesson

Tuesday November 26th

As a preparation for Tuesday's lesson you'll look into Transforming tensors using Mohr's circle:

[Model](#)

[Stress-Strain relations](#)

[Transformations](#)

[Graphical](#)

Please go through the topics which you can improve your skills on. Take at least a look at the examples and try a few exercises. During the lesson you'll make some advanced exercises.

Lesson November 26th

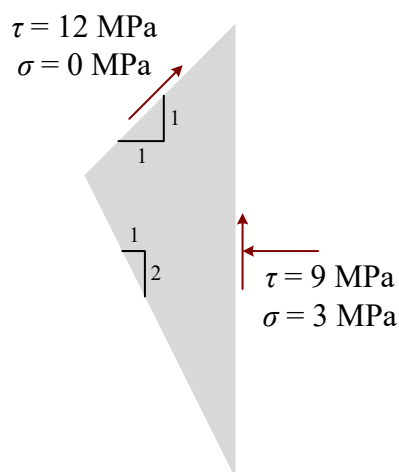
Contents

- Exercise Transforming tensors graphically

During today's lesson you'll work on a complex exercise on the topic of the Transforming tensors graphically. Please ask your questions regarding the [homework](#) as well!

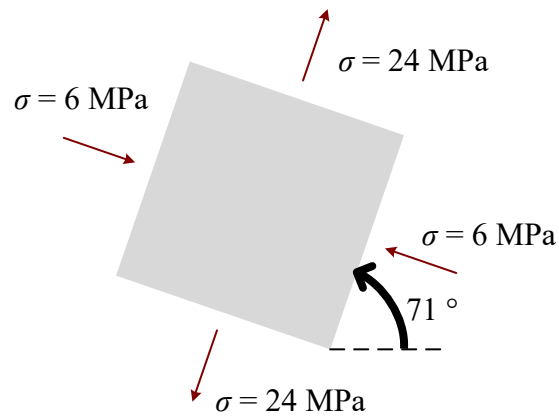
Exercise Transforming tensors graphically

Given is the following stress element:

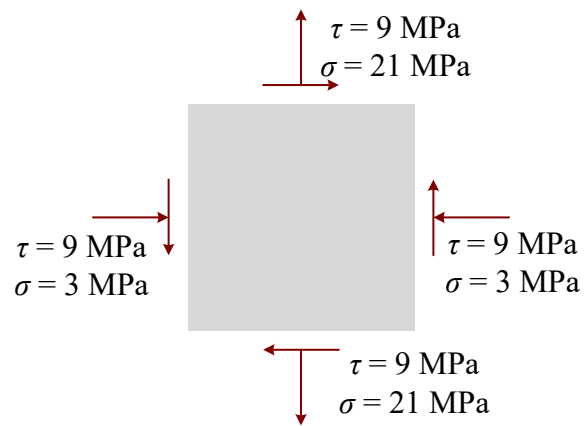


1. Find the principal stresses and their directions using Mohr's circle. Draw the stresses on a rotated square stress element.
2. Find the stresses on a square stress element with horizontal and vertical faces.

Solution assignment 1



Solution assignment 2



Homework deadline lesson

Friday November 29th

As a preparation for Tuesday's lesson you'll look into Internal forces and displacement due to torsion:

[Differential equations for equilibrium relations](#)

[... for torsion](#)

[Solve equilibrium](#)

[... of a body in torsion](#)

[Solve section forces torsion structures](#)

[... using equilibrium in a specific cross-section](#)

[Diagrams](#)

[... using differential equations](#)

[Model](#)

[Shear stress-strain diagram](#)

[Cross-sectional properties](#)

[Polar moment of inertia](#)

[Torsional moment of inertia thin-walled tubes](#)

[Strain diagram](#)

[... for torsion](#)

[Kinematic relations](#)

[... for torsion in circular cross-sections and thin-walled tubes](#)

[Constitutive relations](#)

[... for torsion](#)

[Solve displacements circular and thin-walled tubes for structures in torsion](#)

[... using constitutive equation](#)

[... using differential equations](#)

Please go through the topics which you can improve your skills on. Take at least a look at the examples and try a few exercises. During the lesson you'll make some advanced exercises.

Lesson November 29th

Contents

- Exercise Internal forces and displacement due to torsion

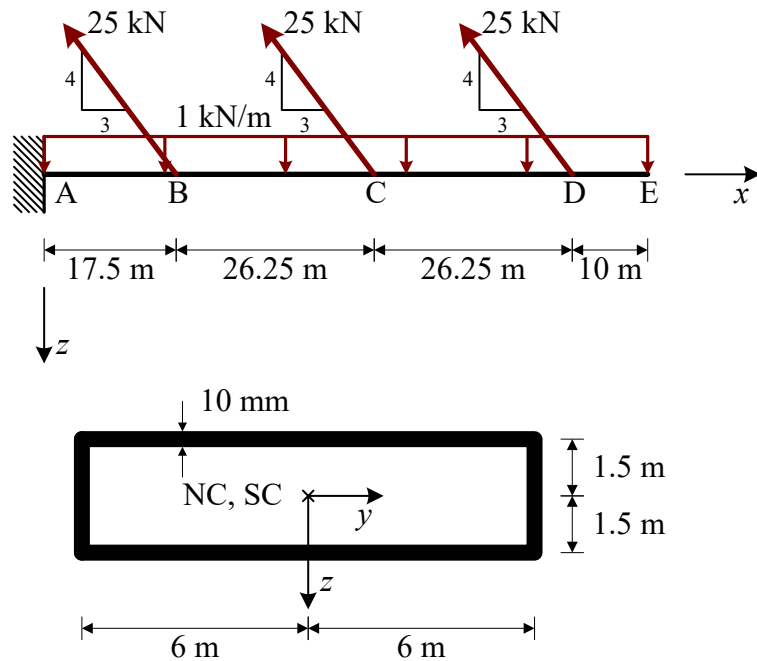
During today's lesson you'll work on a complex exercise on the topic of the Internal forces and displacement due to torsion. Please ask your questions regarding the [homework](#) as well!

Exercise Internal forces and displacement due to torsion

Note

In this exercises, you'll be given the location of the [shear force centre](#) ('SC'). This topic will be treated in the homework of [next week Wednesday](#). The only thing you need to know now is that the torsional moment can be calculated using the lever arm with respect to this shear force centre.

Given is the following structure:



$$E = 210 \text{ GPa}, G = 80 \text{ GPa}$$

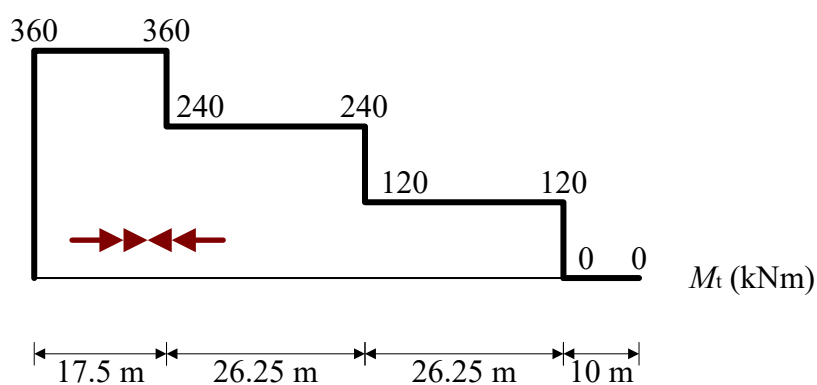
Distributed load is applied in shear force centre

Point loads are applied at $y = 6 \text{ m}$

It represents the Puente de la Mujer in Buenos Aires, Argentina. The cables are connected to the edge of the deck, leading to torsional forces.

1. Draw the torsional moment distribution in the structure.
2. Find the rotation around axis x at E

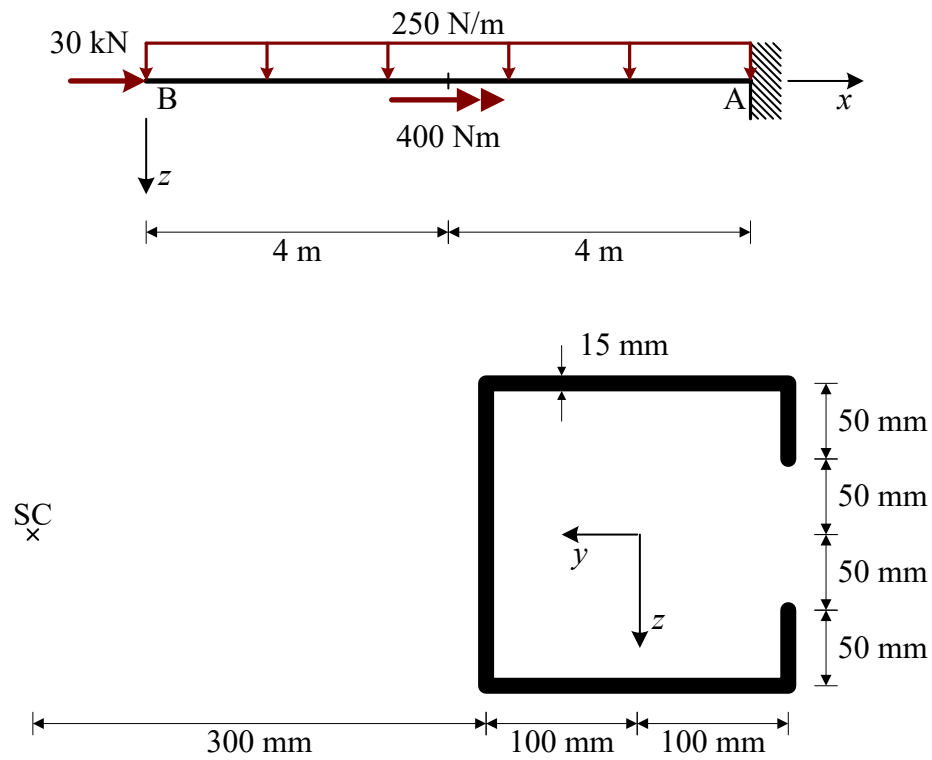
💡 Solution assignment 1



💡 Solution assignment 2

$$\varphi_{E, x} = -1.1 \cdot 10^{-4} \text{ rad}$$

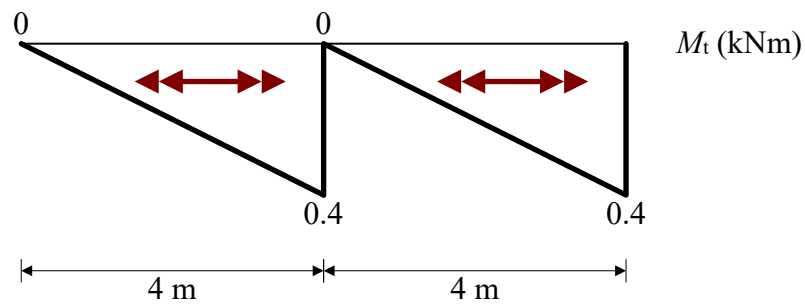
Given is the following structure:



$E = 210 \text{ GPa}$, $G = 80 \text{ GPa}$
Distributed load is applied at $y = 0$

3. Draw the torsional moment distribution in the structure.
4. Find the rotation around axis x at B

💡 Solution assignment 3



💡 Solution assignment 4

$$\varphi_{B, x} = -0.0254 \text{ rad}$$

Week 14 (December 2nd)

	Monday	Tuesday	Wednesday	Thursday	Friday	Up hoi
Homework deadlines	Read theory and make homework exercise shear stresses in torsional elements Prerequisites	Read theory and make homework exercise shear force centre Prerequisites			Read theory and make homework exercise failure criteria New content	Wee Pre ! assi st inde struc col me New Prei
Class activities	Guided practice on shear stresses in torsional elements Prerequisites	Guided practice on shear force centre Prerequisites			Guided practice on failure criteria New content	



Homework deadline lesson

Monday December 2nd

As a preparation for Monday's lesson you'll look into Shear stresses in torsional elements:

[Solve stresses in a torsional element](#)

[... in a thin-walled circular cross-section](#)

[... in solid circular bars and thick-walled circular tubes](#)

[... in a closed thin-walled non-circular cross-section](#)

[... in open thin-walled cross-sections](#)

[... for various cross-sections in combination with shear force](#)

Please go through the topics which you can improve your skills on. Take at least a look at the examples and try a few exercises. During the lesson you'll make some advanced exercises.

Lesson December 2nd

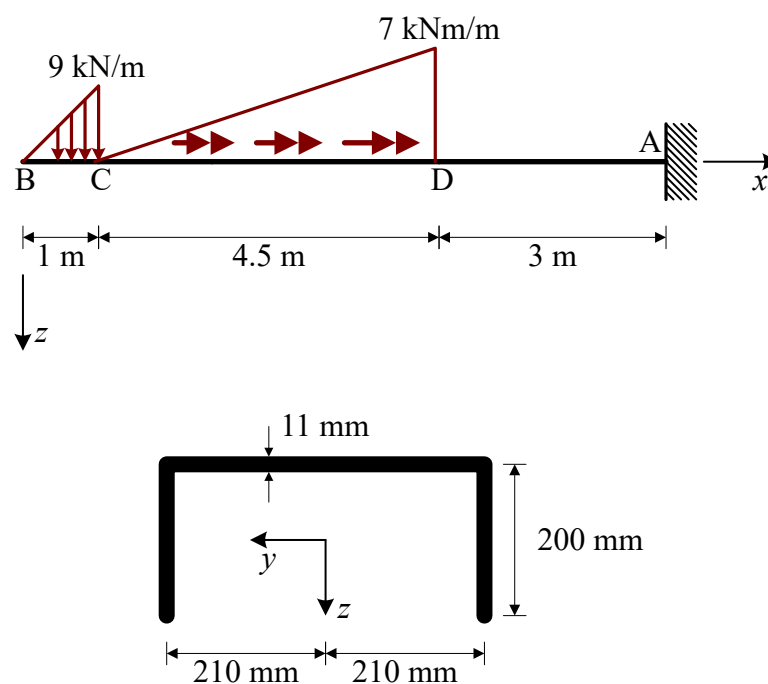
Contents

- Exercise Stresses due to torsion

During today's lesson you'll work on a complex exercise on the topic of the Stresses due to torsion. Please ask your questions regarding the [homework](#) as well!

Exercise Stresses due to torsion

Given is the following structure:



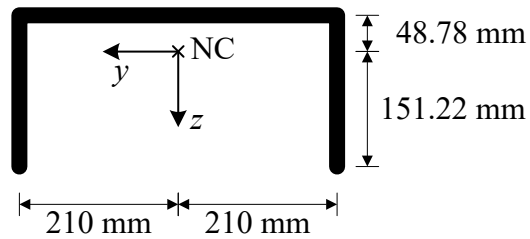
Shear force centre at $y = 0$.

Forces are applied in shear- and normal force centre

Cross-section can be modelled as thin-walled

1. Find the relevant cross-sectional properties.
2. Find the maximum shear stress and its location.
3. Find the principal stresses at that point and their directions. Draw a square stress element to represent those stresses.

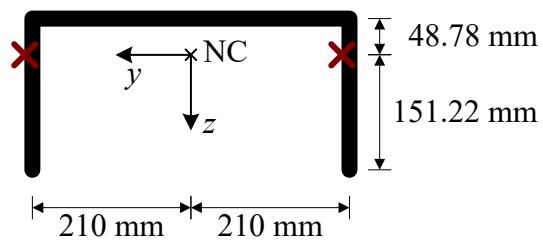
💡 Solution assignment 1



- $A = 9020 \text{ mm}^2$
- $I_{zz} = 37.2 \cdot 10^6 \text{ mm}^4$
- $I_t = 364 \cdot 10^3 \text{ mm}^4$

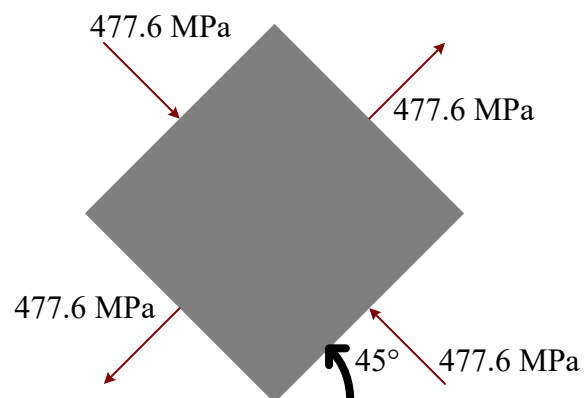
💡 Solution assignment 2

In segment AD in grey crosses:



$$|\tau_{\max}| = 477.6 \text{ MPa}$$

💡 Solution assignment 3



Homework deadline lesson

Tuesday December 3rd

As a preparation for Monday's lesson you'll look into the Shear Force centre:

[Cross-sectional properties](#)

[Shear force centre](#)

Please go through the topics which you can improve your skills on. Take at least a look at the examples and try a few exercises. During the lesson you'll make some advanced exercises.

Lesson December 3rd

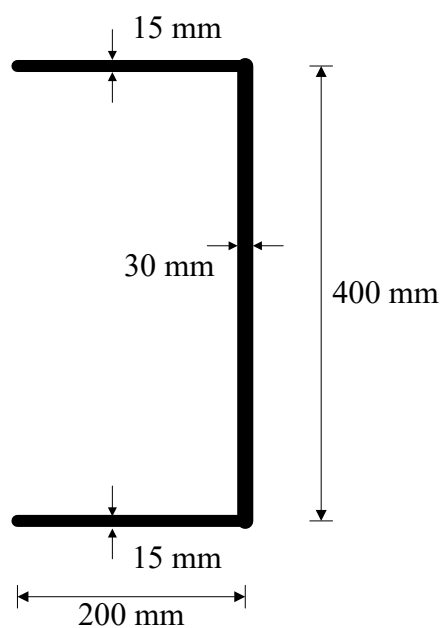
Contents

- Exercise Shear force centre

During today's lesson you'll work on a complex exercise on the topic of the Shear force centre. Please ask your questions regarding the [homework](#) as well!

Exercise Shear force centre

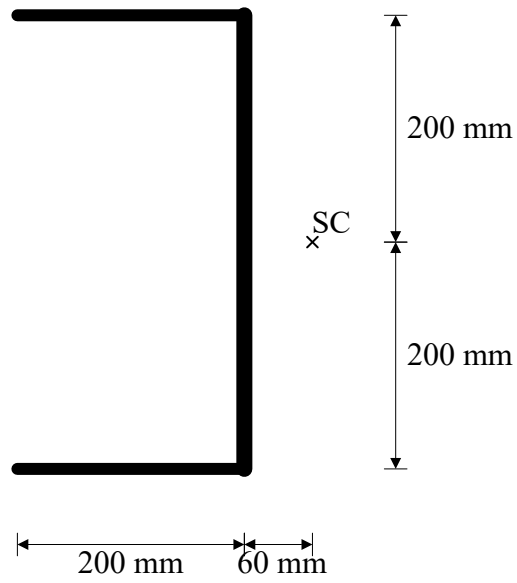
Given is the following cross-section



1. Find the relevant cross-sectional properties.
2. Find the location of the shear force centre

💡 Solution assignment 1

$$I_{zz} = 400 \cdot 10^6 \text{ mm}^4$$



Homework deadline lesson

Friday December 6th

As a preparation for Monday's lesson you'll look into the Failure criteria:

[Failure](#)

Please go through the topics which you can improve your skills on. Take at least a look at the examples and try a few exercises. During the lesson you'll make some advanced exercises.

Lesson December 6th

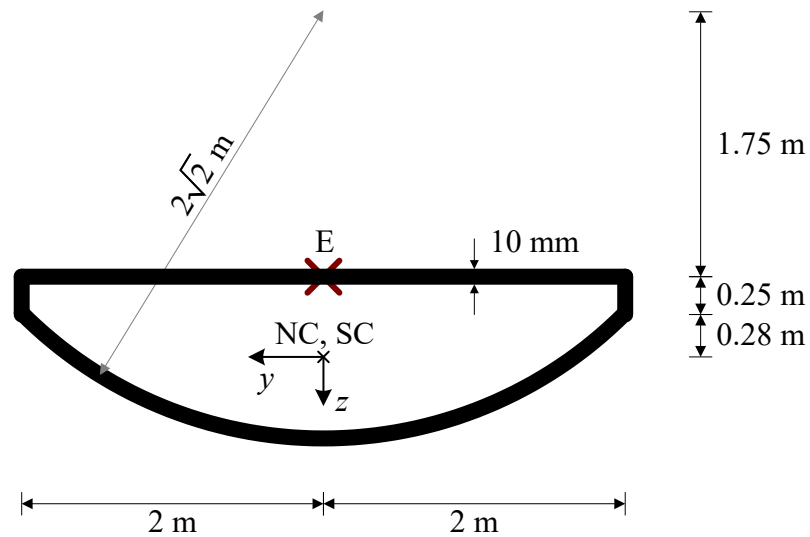
Contents

- Exercise Failure models

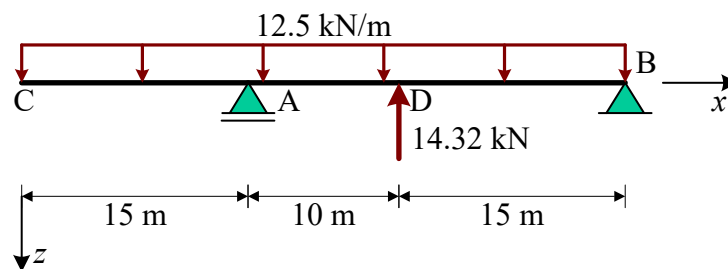
During today's lesson you'll work on a complex exercise on the topic of the Failure models. Please ask your questions regarding the [homework](#) as well!

Exercise Failure models

Given is the following structure. It represents the Prinses Amaliabrug, a bicycle bridge near Dordrecht.



$I_{zz} = 0.238 \text{ m}^4$
Cross-section can be modelled as thin-walled



Distributed load is applied at shear force center.

Support A is applied at $y = -2 \text{ m}$

Force of 14.32 kN is applied at $y = -2 \text{ m}$

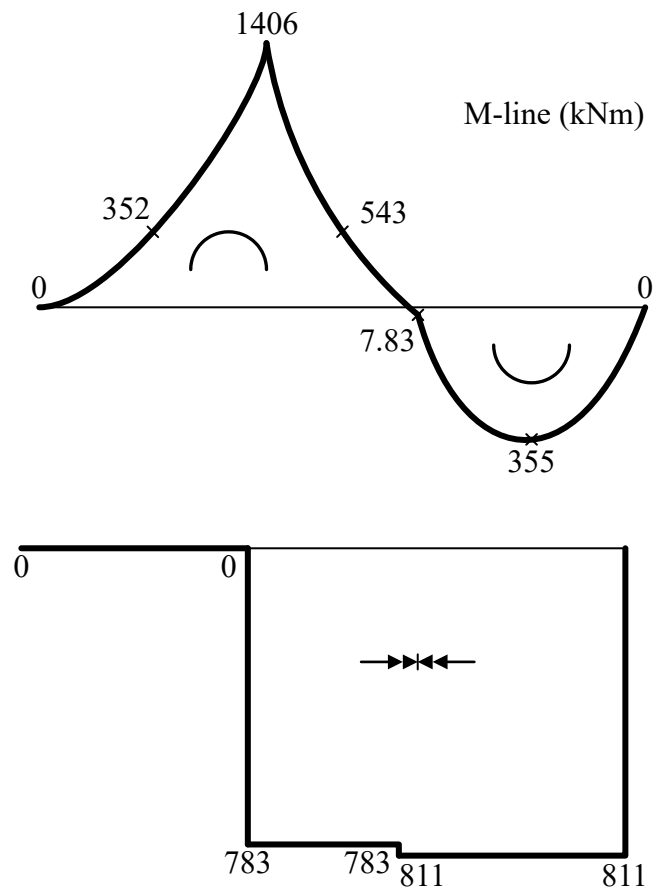
Support B is applied at both $y = 2 \text{ m}$ as $y = -2 \text{ m}$.

1. Find the relevant cross-sectional properties.
2. Find the bending- en torsional moment distribution.
3. Find the shear- and normal forces in cross-section just right of D at point E
4. Draw the stress state in cross-section just right of D at point E in the principal stress plane including Tresca's failure criterium for unknown yield stress.
5. What should be the minimum yield stress according to Tresca?

💡 Solution assignment 1

$$A_m = 3.23 \text{ m}^2$$

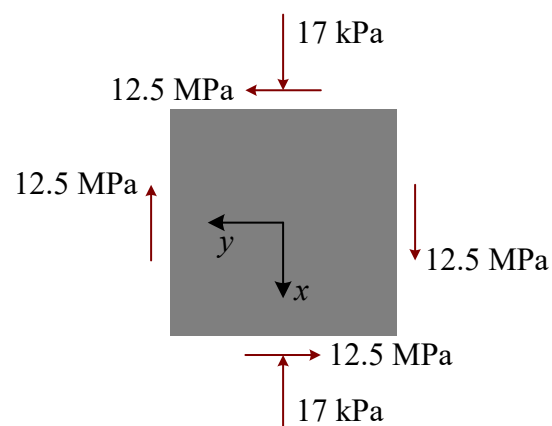
Solution assignment 2



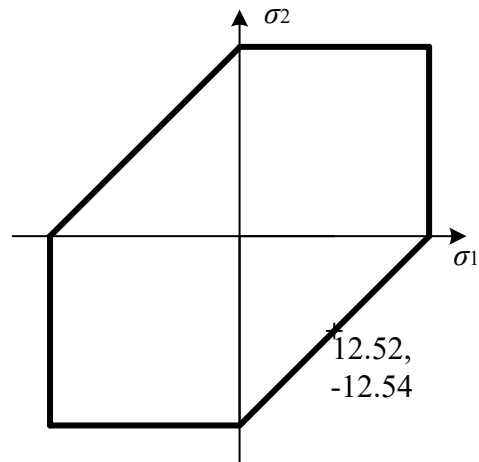
Solution assignment 3



- $\sigma_E = -0.0174 \text{ MPa}$
- $\tau_E = 12.53 \text{ MPa}$



💡 Solution assignment 4



💡 Solution assignment 5



- $f_y > 25.06 \text{ MPa}$

Week 15 (December 9th)

	Monday	Tuesday	Wednesday	Thursday	Friday	Upcoming homework
Homework deadlines		Prepare for exam assignments statically indeterminate structure and continuum mechanics New content Prerequisites				End-of-week homework Systems with 1 dof New content
Class activities		Exam assignment 2 statically indeterminate structure and 1 continuum mechanics New content Prerequisites		Review exam assignment 2 statically indeterminate structures and assignment 1 continuum mechanics New content Prerequisites		

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Preparation exam assignments Tuesday December 10th

Contents

- Assignment on statically indeterminate structures
- Assignment on continuum mechanics

Assignment on statically indeterminate structures

As a preparation for the exam assignment on Statically indeterminate structures you can repeat all the topics of Statically indeterminate structures, including the prerequisites. This content can be viewed at:

- All content from Week 3 Friday to Week 8 Friday in this book
- The stripped version of the [Mechanics-BSc book](#) (url includes `/2024-assignment1`).
- The following chapters of the full [Mechanics-BSc book](#):
 - Statics: all topics as a prerequisite except for `Differential equations for equilibrium relations` and `Solve section forces ... using differential equations`.
 - Stresses and displacements: all topics as a prerequisite, except for `Cross-sectional properties` and `Solve displacements ... using differential equations`.
 - Statically indeterminate structures: all topics

Assignment on continuum mechanics

As a preparation for the exam assignment on Continuum mechanics you can repeat all the topics of Continuum Mechanics, including the prerequisites. This content can be viewed at:

- All content from Week 11 Monday to Week 14 Friday in this book and:
 - Week 4 (September 23th)
 - Homework deadline lesson Monday September 23th except for `Solve displacements truss structures`
 - Week 5 (September 30th)
 - Homework deadline lesson Wednesday October 2nd
 - Lesson October 2nd
 - Week 6 (October 7th)
 - Homework deadline lesson Wednesday October 9th
 - Lesson October 9th
- The stripped version of the [Mechanics-BSc book](#) (url includes `/2024-assignment2`).
- The following chapters of the full [Mechanics-BSc book](#):
 - Statics: all topics as a prerequisite
 - Stresses and displacements: all topics as a prerequisite, except for `Solve displacements ...` and `kinematic relations`.
 - Continuum mechanics: all topics

Repeat those topics which you have difficulty with. If you've any questions along the way, please visit the TAs in 6.41 during lunch time.

Exam Tuesday December 10th

Contents

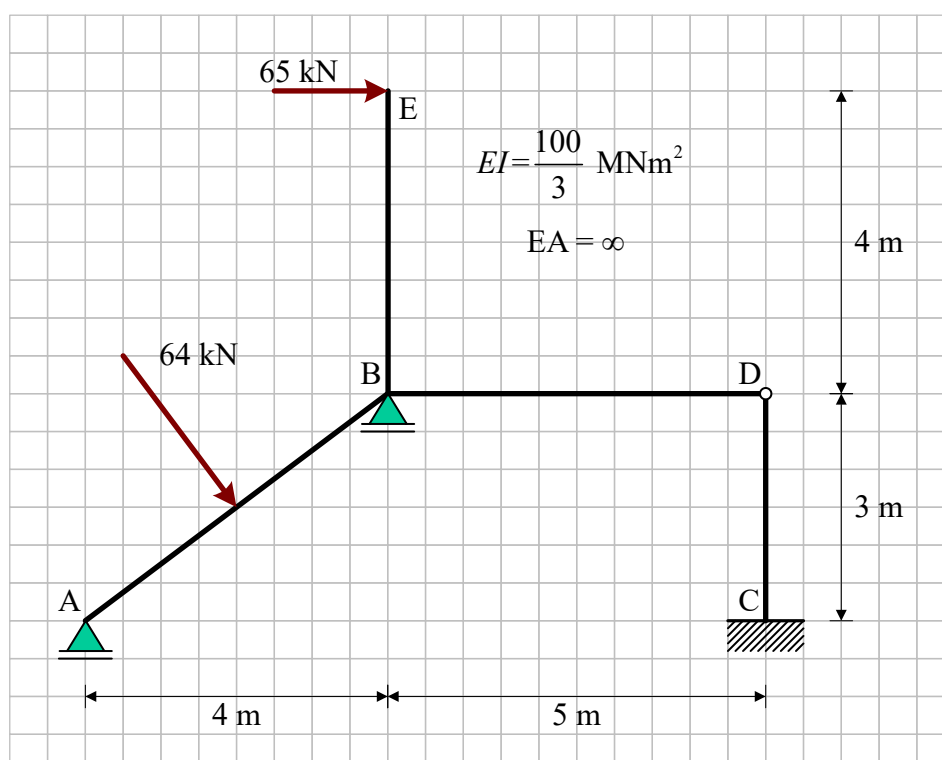
- Exam assignment 2 Statically indeterminate structures
- Exam assignment 1 Continuum mechanics

Today you'll make the second exam assignment covering Statically indeterminate structures including its prerequisites and/or the first exam assignment covering Continuum mechanics including its prerequisites. For more information about the exam see [the assessment information in course information](#)

Exam assignment 2 Statically indeterminate structures

Your own submission and its grading will be available on **ans*** [exam assignment Statically indeterminate structures 2](#) after the exam.

Given is the following structure:

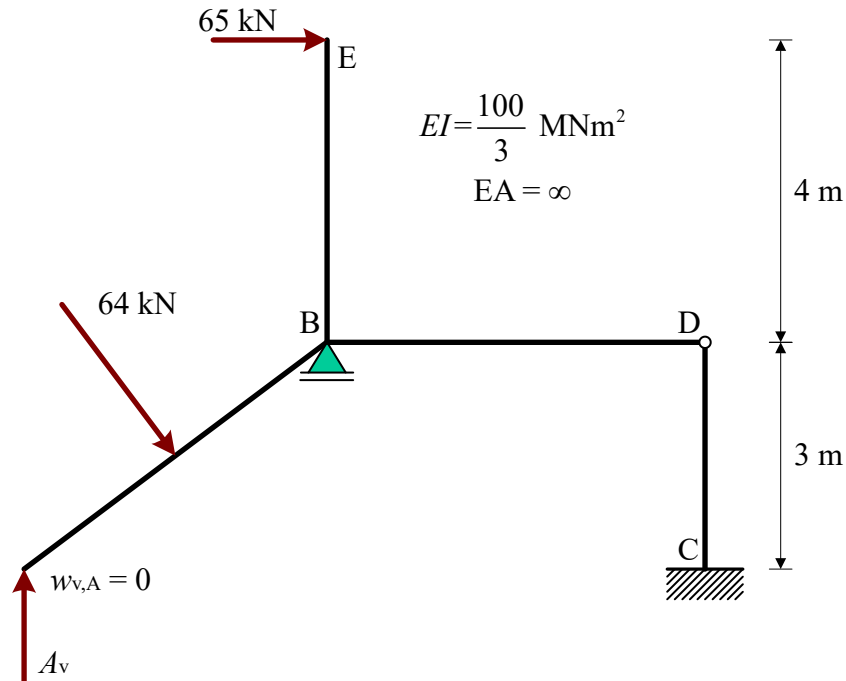


Calculate the displacement of E and the moments in all the members using a force-, displacement- or hybrid- ('hoekveranderingsvergelijkingen' with moveable nodes) method.

Solution

Convert the structure into a statically determinate structure with a displacement or force condition.

For example when using the force method:



Using forget-me-nots, this gives a rotation of B of:

$$\varphi_B = \frac{5(-4 \cdot A_v + 2.5 \cdot 64 - 4 \cdot 65)^3}{\frac{100}{3} \cdot 10^3} = -2.0 \cdot 10^{-4} A_v - 5.0 \cdot 10^{-3} (\circ)$$

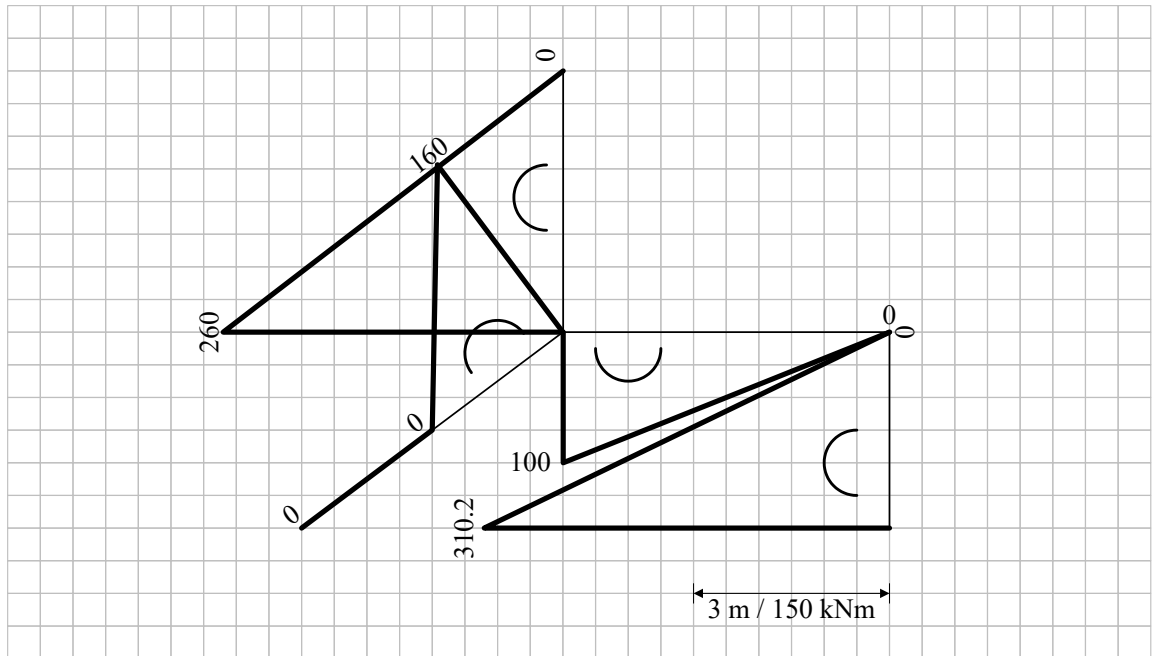
Again using forget-me-nots, this give a vertical displacement in A of:

$$w_{v,A} = \varphi_B \cdot 5 - \frac{A_v \cdot \frac{4}{5} \cdot 5^3}{3 \cdot \frac{100}{3} \cdot 10^3} + \frac{F \cdot 2.5^3}{3 \cdot \frac{100}{3} \cdot 10^3} + \frac{F \cdot 2.5^2}{2 \cdot \frac{100}{3} \cdot 10^3} \cdot 2.5 = -2.0$$

Using $w_{v,A} = 0$ gives $A_v = 0$ kN

Again using forget-me-nots for both rotation of B, bending of BE and bending of CD a displacement of $8.9518 \cdot 10^{-2}$ m is found.

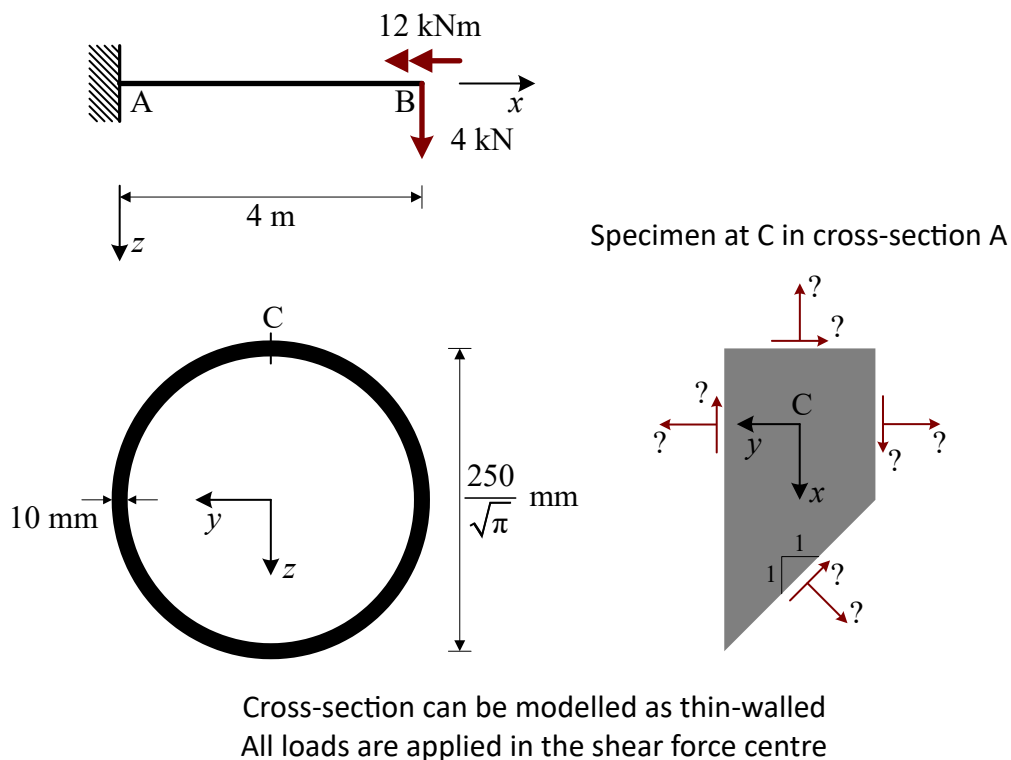
The bending moment diagram results in:



Exam assignment 1 Continuum mechanics

Your own submission and its grading will be available on [ans* exam assignment Continuum mechanics 1](#) after the exam.

Given is the following structure:



Find the stresses alongside all edges of the shown element from cross-section A point C, and determine the required yield stress according to the Von Mises failure criterium.

Solution



In C the shear stress due to bending is 0.

The normal stress due to bending is:

$$\sigma = \frac{4 \cdot 4000 \cdot \frac{1}{2} \cdot \frac{0.250}{\sqrt{\pi}}}{\pi \cdot \left(\frac{1}{2} \cdot \frac{0.250}{\sqrt{\pi}} \right)^3 \cdot 0.010} = 102.4 \cdot 10^6 \text{ Pa. Which is a tension stress.}$$

The shear stress due to torsion is:

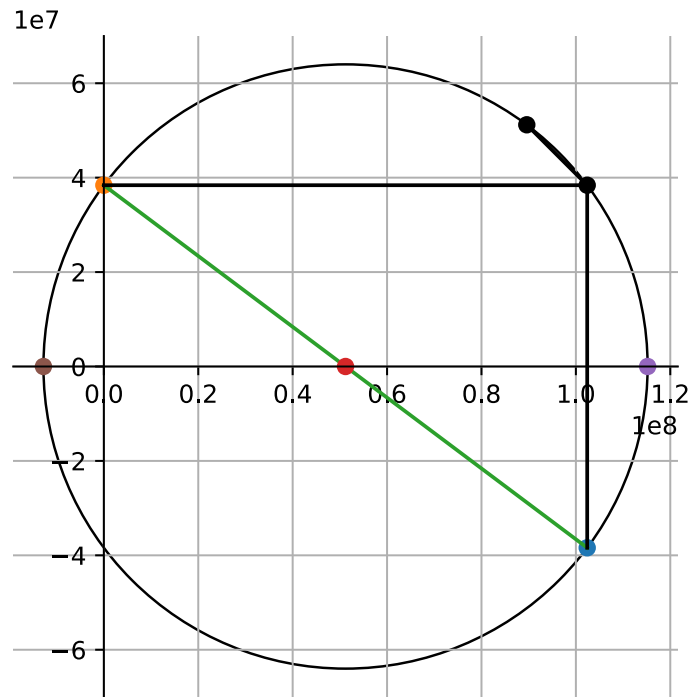
$$\tau = \frac{12000 \cdot \frac{1}{2} \cdot \frac{0.250}{\sqrt{\pi}}}{2 \cdot \pi \cdot \left(\frac{1}{2} \cdot \frac{0.250}{\sqrt{\pi}} \right)^3 \cdot 0.010} = 38.4 \cdot 10^6 \text{ Pa. This shear stress works}$$

clockwise on a positive x -plane and anticlockwise on a negative x -plane (corresponding of the top plane of the shown specimen).

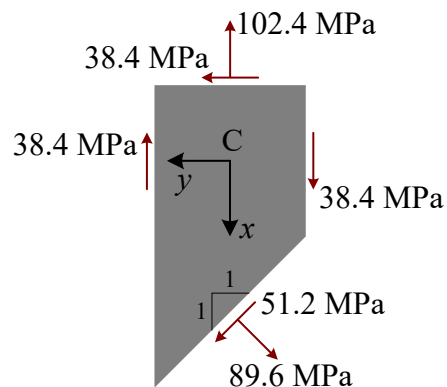
The stress at the bottom plane of the shown specimen can be found by rotating the right plane $-\frac{1}{4}\pi$ rad:

- $\sigma_{\bar{x}\bar{x}} = 102.4 \sin^2 \left(-\frac{1}{4}\pi \right) + 2 \cdot -38.4 \sin \left(-\frac{1}{4}\pi \right) \cos \left(-\frac{1}{4}\pi \right) = 89.6 \text{ N}$
- $\tau_{\bar{x}\bar{y}} = 102.4 \sin \left(-\frac{1}{4}\pi \right) \cos \left(-\frac{1}{4}\pi \right) - 38.4 \cos^2 \left(-\frac{1}{4}\pi \right) + 38.4 \sin \left(-\frac{1}{4}\pi \right)$

This can also be done using Mohr's circle:



This results in:



The principal stresses can be found using:

- $\sigma_1 = \frac{102.4}{2} + \sqrt{\left(\frac{102.4}{2}\right)^2 + 38.4^2} = 115.2 \text{ MPa}$
- $\sigma_2 = \frac{102.4}{2} - \sqrt{\left(\frac{102.4}{2}\right)^2 + 38.4^2} = -12.8 \text{ MPa}$

Now the Von Mises failure criterion can be evaluated:

$$f_y > \sqrt{\frac{1}{2} \left((115.2 + 12.8)^2 + (-12.8 - 0)^2 + (0 - 115.2)^2 \right)} = 12\sqrt{91} \text{ MPa}$$

or $f_y > \sqrt{102.4^2 + 3 \cdot (38.4)^2} = 12\sqrt{91} \text{ MPa} \approx 122 \text{ MPa}$

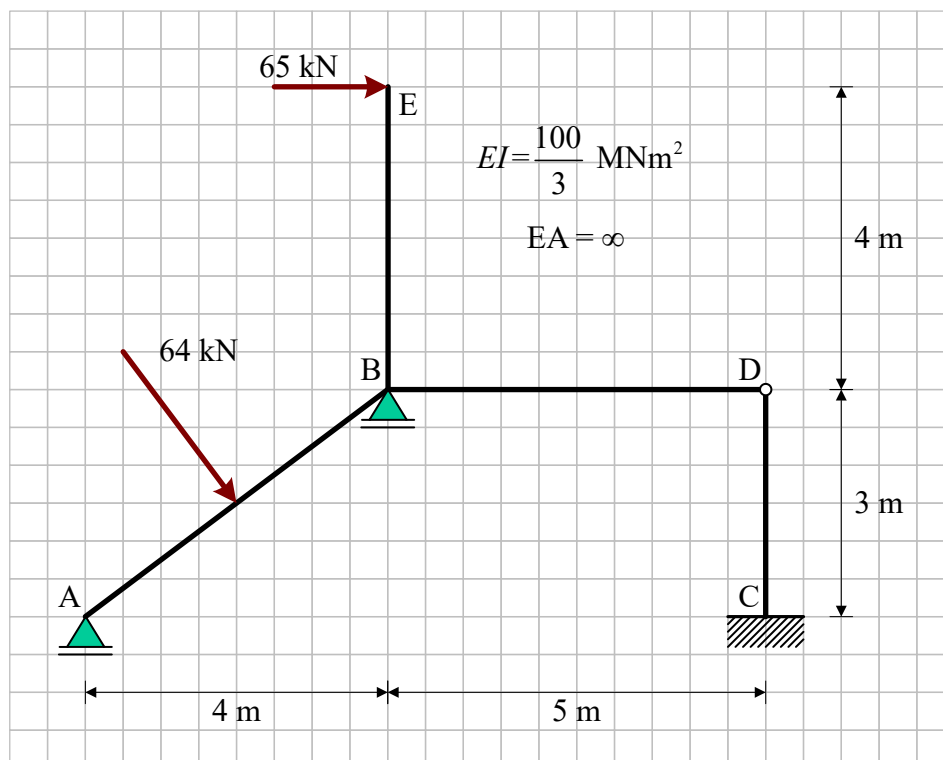
Lesson December 12th

Contents

- Results second exam assignment on Statically indeterminate structures
- Results first exam assignment on Continuum mechanics
- Create knowledge graph

Today we'll discuss the results of the first [exam assignment 2 on Statically indeterminate structures](#) and [exam assignment 1 on Continuum mechanics](#).

Results second exam assignment on Statically indeterminate structures

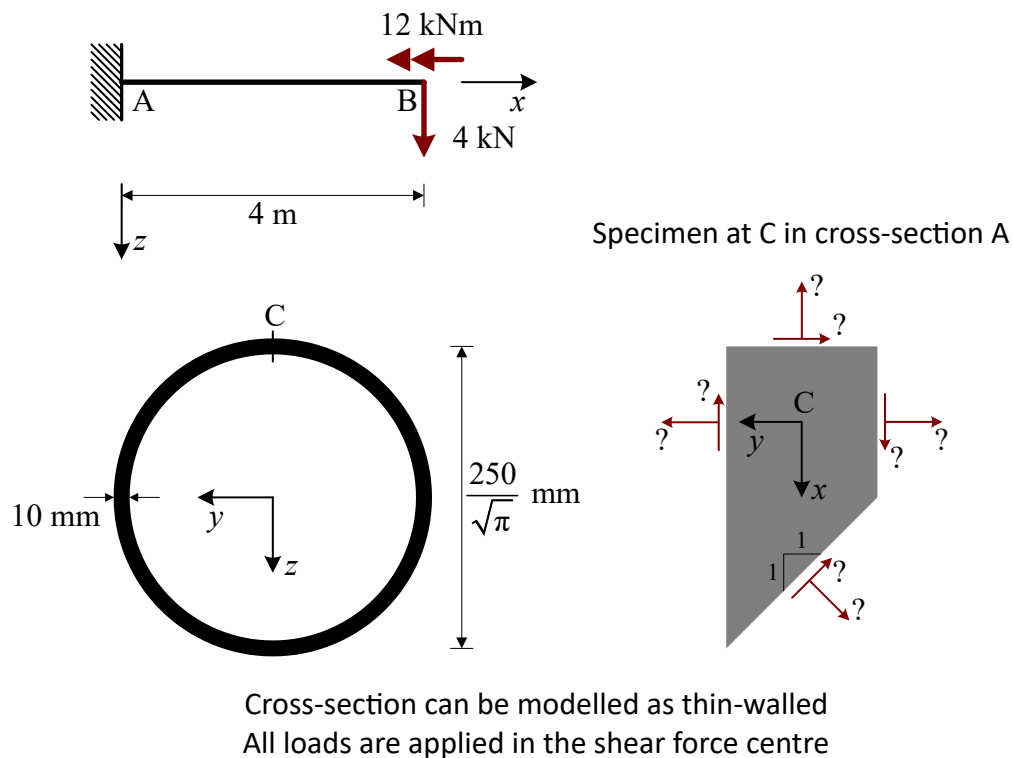


Approaches taken during the [exam assignment](#):

- Hinge at B causes local mechanism BE, could be dealt with by:
 - For 'hoekveranderingsvergelijkingen', first all cantilever parts should be replaced by moments and/or forces

- If it's the only mechanism (only one degree of freedom), use hybrid method ('hoekveranderingsvergelijkingen' with movable nodes)
- Mistakes in sign moments around B
- φ_B^{BE} set equal to rotation at tip of cantilever beam loaded by point load
- Displacement of E consists of:
 - Bending BE
 - Rotation B
 - Bending CD
- Mistakes in solving moment distribution

Results first exam assignment on Continuum mechanics



Approaches taken during the [exam assignment](#):

- Mistakes in calculating stresses
- Stresses in wrong direction acting on stress element
- Shear stresses on perpendicular faces not in equilibrium
- Normal stresses at both sides the x - and y -plane
- Incorrect direction of shear stresses after rotation
- Calculation of principal stresses inconsistent with stress element

- Rotation from rotation centre doesn't give correct new point
- Incorrect location of rotation centre
- A third nonzero principal stress

Create knowledge graph

In the [third lesson of the first week](#) you created a knowledge graph on the topics you've encountered in the prerequisite tests, and how they link to the learning objectives of this course. Let's try and extend today in a course graph. These course graphs will be incorporated in this book!

You'll start with a template containing the (abbreviated) learning objectives and prerequisite learning objectives:

This lecture

Overview

Domains



Your task is to add subjects to the learning objectives and the links between those.

1. Login at <https://prime-applets.ewi.tudelft.nl/login>
2. Wait for Tom van Woudenberg to add you to the correct Course Graph course
3. Open the correct Course Graph Course: <https://prime-applets.ewi.tudelft.nl/graph-editor/course-overview/CT1000>

4. Click on  (edit) for your own course graph
5. Click [Edit Layout](#) - [Domains](#) to check the layout and eventually change it a bit (remember to click [Save](#) when you're done)
6. Click [Edit Fields](#) - [Subjects](#) to add subjects. Try to keep the number of subjects per domain (which are the learning objectives) limited
7. Click [Subject relations](#) to show relations between subjects
8. Click [Edit Layout](#) - [Overview](#) to alter the layout (eventually use [Auto layout](#)) to a layout that makes sense to you
9. Repeat steps 6-9 until you're satisfied!

You'll be able to view your graph in this book (see subpages)!

Week 16 (December 16th)

	Monday	Tuesday	Wednesday	Thursday	Friday	Up hoi
Homework deadlines					End-of-week homework Systems with 1 dof New content	E : ho S wit i n Ne
Class activities	Lecture New content	Lecture New content		Lecture New content		

End-of-week homework week 16

As End-of-week homework you'll look into the systems with 1 dof: read chapter 1 and 2 and study the examples from the book *Mechanica: Stabiliteit van het evenwicht* ([Hartsuijker and Welleman, 2023](#)) carefully, start with the assignments in the book and on **ans*** [exercises](#) and finish these at the end of the week. Answers available in Dutch [here](#).

Week 17 (January 6th)

	Monday	Tuesday	Wednesday	Thursday	Friday	Up hours
Homework deadlines					End-of-week homework Systems with 1 and infinite number dofs New content	End-of-week homework Systems with 1 and infinite number dofs New content
Class activities	Lecture New content	Lecture New content		Lecture New content		

End-of-week homework week 17

As End-of-week homework you'll look into the Systems with 1 and infinite number dofs:
Read chapter 3 and 4 study the examples from the book *Mechanica: Stabiliteit van het evenwicht* ([Hartsuijker and Welleman, 2023](#)) carefully. Example 3 and 4 from chapter 4 is reading. Start with the assignments in the book and on **ans*** [exercises](#) and finish these at the end of the week.

Week 18 (January 13th)

	Monday	Tuesday	Wednesday	Thursday	Friday	Up ho
Homework deadlines					End-of-week homework Flexural buckling, Eulerian approach New content	Fric 20: fo ass co me anc Ne Pre
Class activities	Lecture New content	Lecture New content		Lecture New content		

End-of-week homework week 18

As End-of-week homework you'll look into the Flexural buckling, Eulerian approach: Read chapter 5 and 6 and study all the examples from the book *Mechanica: Stabiliteit van het evenwicht* ([Hartsuijker and Welleman, 2023](#)) carefully. Paragraph 6.1.2 and 6.1.3 are reading only. Start with the assignments in the book and on **ans*** [exercises](#) and finish these at the end of the week.

Week 20 (January 27th)

	Monday	Tuesday	Wednesday	Thursday	Friday	Upcoming homework
Homework deadlines					Prepare for exam assignments continuum mechanics and buckling New content Prerequisites	Monday week 30: Prepare for exam assignments statically indeterminate structure, continuum mechanics and buckling New content Prerequisites
Class activities					Exam assignment 2 continuum mechanics and 1 buckling New content Prerequisites	

Preparation exam assignments Tuesday December 10th

Contents

- Assignment on continuum mechanics
- Assignment on buckling

Assignment on continuum mechanics

As a preparation for the exam assignment on Continuum mechanics you can repeat all the topics of Continuum Mechanics, including the prerequisites. This content can be viewed at:

- All content from Week 11 Monday to Week 14 Friday in this book and:
 - Week 4 (September 23th)
 - Homework deadline lesson Monday September 23th except for `Solve displacements truss structures`
 - Week 5 (September 30th)
 - Homework deadline lesson Wednesday October 2nd
 - Lesson October 2nd
 - Week 6 (October 7th)
 - Homework deadline lesson Wednesday October 9th
 - Lesson October 9th
- The stripped version of the [Mechanics-BSc book](#) (url includes `/2024-assignment2`).
- The following chapters of the full [Mechanics-BSc book](#):
 - Statics: all topics as a prerequisite
 - Stresses and displacements: all topics as a prerequisite, except for `Solve displacements ...` and `kinematic relations`.
 - Continuum mechanics: all topics

Assignment on buckling

As a preparation for the exam assignment on Buckling you can repeat all the topics of Buckling, including the prerequisites. This content can be viewed at:

- All content from Week 16 Monday to Week 18 Friday in this book and:
 - Week 2 (September 9th)
 - Homework deadline lesson Wednesday September 11th except for
 - Solve equilibrium; of a 3D-particle; 3D-body; body in torsion
 - Solve support reactions for statically determinate structures
 - Solve section forces truss structure
 - Solve section forces frame structures
 - Homework deadline lesson Friday September 13th except for `Solve section forces and shape cable; ... using equilibrium in a specific cross-section`
 - Week 3 (September 16th)
 - Homework deadline lesson Monday September 16th
 - Week 4 (September 23th)
 - Homework deadline lesson Monday September 23th
 - Week 5 (September 30th)
 - Homework deadline lesson Friday October 4th
- The stripped version of the [Mechanics-BSc book](#) (url includes `/2024-assignment3`).
- The following chapters of the full [Mechanics-BSc book](#):
 - Statics: all topics as a prerequisite, except for:
 - Virtual work
 - Solve equilibrium; of a 3D-particle; 3D-body; body in torsion
 - Solve support reactions for statically determinate structures
 - Solve section forces truss structure
 - Solve section forces frame structures; ... using equilibrium in a specific cross-section; diagrams
 - Solve section forces and shape cable; ... using equilibrium in a specific cross-section.
 - Stresses and displacements: all topics as a prerequisite, except for:
 - Cross-sectional properties
 - Solve stresses in an extension element
 - Solve stresses in a bending element

- Solve stresses in a torsional element
 - Solve displacements truss structures; ... using Williot
 - Solve displacements circular and thin-walled tubes for structures in torsion
 - Solve displacements statically determinate structures subjected to temperature influences
- Buckling

Repeat those topics which you have difficulty with. If you've any questions along the way, please visit the TAs in 6.41 during lunch time.

Exam Friday January 31th

Contents

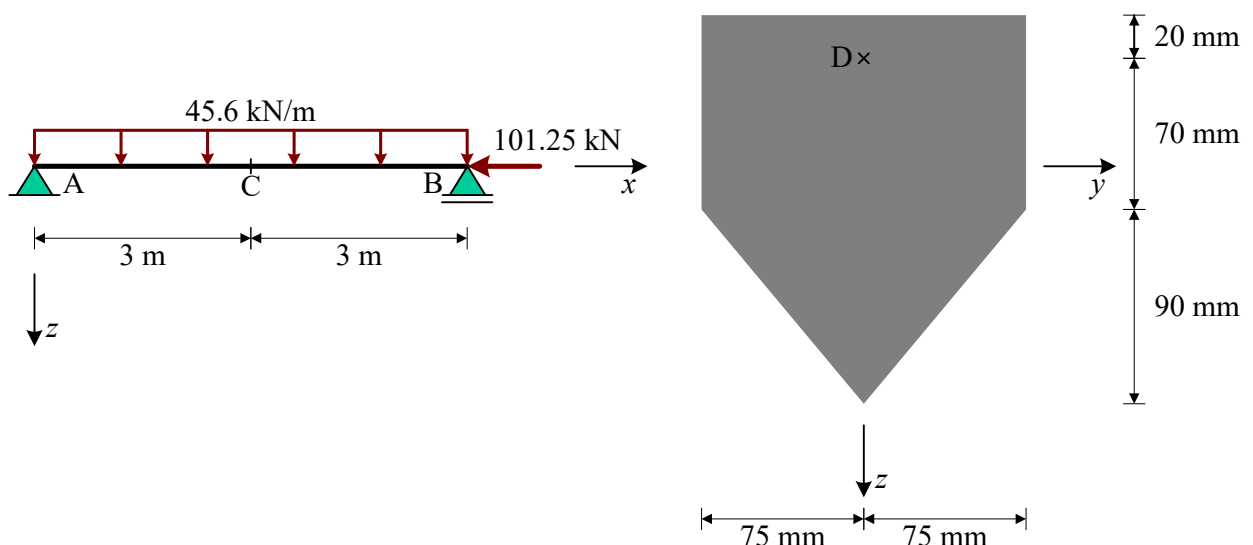
- Exam assignment 2 Continuum mechanics
- Exam assignment 1 Buckling

Today you'll make the second exam assignment covering Continuum mechanics including its prerequisites and/or the first exam assignment covering Buckling including its prerequisites. For more information about the exam see [the assessment information in course information](#)

Exam assignment 2 Continuum mechanics

Your own submission and its grading will be available on **ans*** [exam assignment Continuum mechanics 2](#) after the exam.

Given is the following structure:



All loads are applied at in the normal force centre

Alle krachten grijpen aan in het normaalkrachtencentrum

- Find the stress state with maximum shear stresses in point D and draw the (rotated) stress element for this stress state

- Find the principal stresses in point D and draw the (rotated) stress element for this stress state.
- Draw the required yield stress envelope for Tresca's yield criterion in the relevant principal stress plane including values.

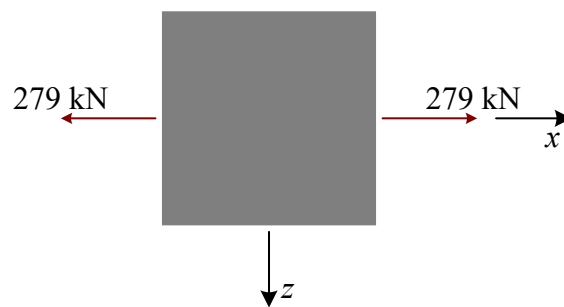
Solution

In D the shear stress 0.

Relevant cross-sectional properties are:

- $A = 20250 \text{ mm}^2$
- From top fibre downwards, the normal force centre is at 70 mm
- $I_{zz} = 37462500 \text{ mm}^4$

The normal stress is -25 MPa with no shear stresses.

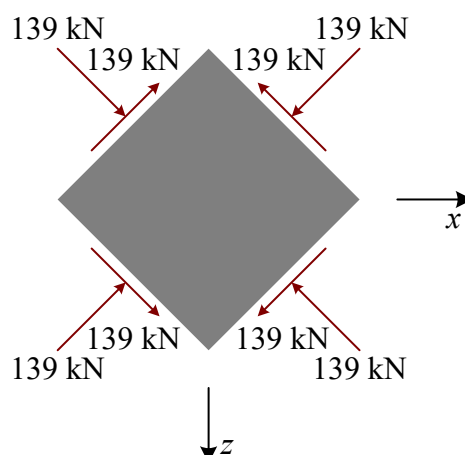


As this is the only stress, the principal stresses are: $\sigma_1 = -279 \text{ MPa}$, $\sigma_2 = \sigma_3 = 0 \text{ MPa}$.

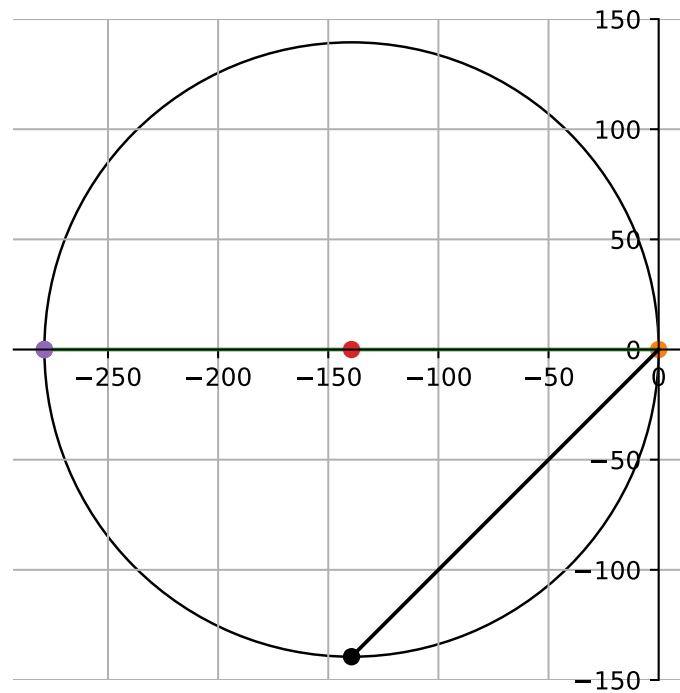
Because of $\sigma_2 = 0 \text{ MPa}$, the maximum shear stress is $\frac{279}{2} = 139 \text{ MPa}$ at 45° .

Or it can be found by rotating the right plane $\frac{1}{4}\pi$ rad:

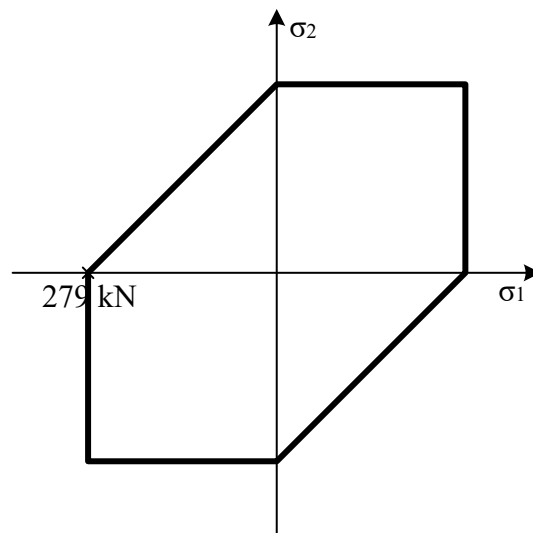
- $\sigma_{\bar{x}\bar{x}} = -279 \cos\left(\frac{1}{4}\pi\right)^2 = -139 \text{ MPa}$
- $\tau_{\bar{x}\bar{y}} = -279 \sin\left(\frac{1}{4}\pi\right) \cos\left(\frac{1}{4}\pi\right) = 139 \text{ MPa}$



This can also be found using Mohr's circle:



The Tresca yield criterion looks as follows:



Exam assignment 1 Buckling

The exam assignment was provided as shown [📄 here](#)

Solution



The solution is shown  [here](#)

Week 30 (April 14th)

	Monday	Tuesday	Wednesday	Thursday	Friday
Homework deadlines	Prepare for exam assignments statically indeterminate structure, continuum mechanics and buckling New content Prerequisites				
Class activities	Exam assignment 3 statically indeterminate structures, 3 continuum mechanics and 2 buckling New content Prerequisites				

Preparation exam assignments Monday April 14th

Contents

- Assignment on statically indeterminate structures
- Assignment on continuum mechanics
- Assignment on buckling

Assignment on statically indeterminate structures

As a preparation for the exam assignment on Statically indeterminate structures you can repeat all the topics of Statically indeterminate structures, including the prerequisites. This content can be viewed at:

- All content from Week 3 Friday to Week 8 Friday in this book
- The stripped version of the [Mechanics-BSc book](#) (url includes `/2024-assignment1`).
- The following chapters of the full [Mechanics-BSc book](#):
 - Statics: all topics as a prerequisite except for `Differential equations for equilibrium relations` and `Solve section forces ... using differential equations`.
 - Stresses and displacements: all topics as a prerequisite, except for `Cross-sectional properties` and `Solve displacements ... using differential equations`.
 - Statically indeterminate structures: all topics

Assignment on continuum mechanics

As a preparation for the exam assignment on Continuum mechanics you can repeat all the topics of Continuum Mechanics, including the prerequisites. This content can be viewed at:

- All content from Week 11 Monday to Week 14 Friday in this book and:
 - Week 4 (September 23th)
 - Homework deadline lesson Monday September 23th except for `Solve displacements truss structures`
 - Week 5 (September 30th)
 - Homework deadline lesson Wednesday October 2nd
 - Lesson October 2nd
 - Week 6 (October 7th)
 - Homework deadline lesson Wednesday October 9th
 - Lesson October 9th
- The stripped version of the [Mechanics-BSc book](#) (url includes `/2024-assignment2`).
- The following chapters of the full [Mechanics-BSc book](#):
 - Statics: all topics as a prerequisite
 - Stresses and displacements: all topics as a prerequisite, except for `Solve displacements ...` and `kinematic relations`.
 - Continuum mechanics: all topics

Assignment on buckling

As a preparation for the exam assignment on Buckling you can repeat all the topics of Buckling, including the prerequisites. This content can be viewed at:

- All content from Week 16 Monday to Week 18 Friday in this book and:
 - Week 2 (September 9th)
 - Homework deadline lesson Wednesday September 11th except for
 - `Solve equilibrium; of a 3D-particle; 3D-body; body in torsion`
 - `Solve support reactions for statically determinate structures`
 - `Solve section forces truss structure`
 - `Solve section forces frame structures`
 - Homework deadline lesson Friday September 13th except for `Solve section forces and shape cable; ... using equilibrium in a specific cross-section`
 - Week 3 (September 16th)
 - Homework deadline lesson Monday September 16th
 - Week 4 (September 23th)
 - Homework deadline lesson Monday September 23th

- Week 5 (September 30th)
 - Homework deadline lesson Friday October 4th
- The stripped version of the [Mechanics-BSc book](#) (url includes /2024-assignment3).
- The following chapters of the full [Mechanics-BSc book](#):
 - Statics: all topics as a prerequisite, except for:
 - Virtual work
 - Solve equilibrium; of a 3D-particle; 3D-body; body in torsion
 - Solve support reactions for statically determinate structures
 - Solve section forces truss structure
 - Solve section forces frame structures; ... using equilibrium in a specific cross-section; diagrams
 - Solve section forces and shape cable; ... using equilibrium in a specific cross-section.
 - Stresses and displacements: all topics as a prerequisite, except for:
 - Cross-sectional properties
 - Solve stresses in an extension element
 - Solve stresses in a bending element
 - Solve stresses in a torsional element
 - Solve displacements truss structures; ... using Williot
 - Solve displacements circular and thin-walled tubes for structures in torsion
 - Solve displacements statically determinate structures subjected to temperature influences
 - Buckling

Repeat those topics which you have difficulty with. If you've any questions along the way, please visit the TAs in 6.41 during lunch time.

Exam Monday April 14th

Contents

- Exam assignment 3 Statically indeterminate structures
- Exam assignment 3 Continuum mechanics
- Exam assignment 2 Buckling

Today you'll make the third exam assignment covering Statically indeterminate structures including its prerequisites and/or the third exam assignment covering Continuum mechanics including its prerequisites and/or the second exam assignment covering Buckling including its prerequisites. For more information about the exam see [the assessment information in course information](#)

Exam assignment 3 Statically indeterminate structures

Your own submission and its grading will be available here after the exam.

Exam assignment 3 Continuum mechanics

Your own submission and its grading will be available here after the exam.

Exam assignment 2 Buckling

Your own submission and its grading will be available here after the exam.

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How the book is made

This book is created using open source tools: it is a JupyterBook that is written using Markdown, Jupyter notebooks and Python files to generate some figures. Additional tooling is used from the [TeachBooks initiative](#) to enhance the editing and reading experience. The files are stored on a [public GitHub repository](#). The website can be viewed at https://teachbooks.io/bridging_mechanics/. Contact the authors for additional information.

About the Editors

Tom van Woudenberg



Tom van Woudenberg is a lecturer at Delft University of Technology. Equipped with a dedication to education in structural mechanics, I strive to cultivate a blended learning environment that engages students actively and rewards their efforts.

Tom graduated in 2020 at Delft University of Technology on structural optimisation. From August 2020 to August 2022, Tom worked at Amsterdam University of Applied Sciences as lecturer construction, specialised in structural mechanics. Since September 2022, Tom is working at Delft University of Technology.

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Bart Slingerland is a teaching assistant, who has partly written the content in this book.

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