Generative Solar-Climatic Configuration

A model for feed-forward optimization of building envelopes as to solar energy potential

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ABSTRACT

The typical solar energy potential simulation workflows used in the AEC industry require an abundance of information regarding the detailed geometry and materialization of the design. These requisites render them incompatible with early-stage design decision-making pertaining to form-finding. By performing solar energy potential simulations at the pre-conceptual design phase, the necessary time for later-stage environmental assessments and design improvements is reduced considerably, while building designs with high performance but low environmental footprint are attained.

This research proposes a computational framework to navigate voxel-based morphologies of building envelopes in a performative design space. It investigates a generative workflow through an embedded multi-criteria optimization of solar-related indicators, which are mapped in a solar energy potential field. The formulation of this field consists of an a priori assessment of the solar energy potential in every discrete volumetric unit (voxel) and a vectorized description of the interdependency of them. The astronomical size of this solution space renders the use of metaheuristic methods more appropriate. More specifically, a subtractive strategy that incorporates an MCDA approach is being applied in order to reach user-defined optimization targets. The novelty and potential of the proposed methodology lies in streamlining the early decision making process for designers and architects and expanding the morphological possibilities. Through this framework, the performative design space is effectively navigated and nearly optimal solutions are generated, to act as suggestive mechanisms for informed architectural decisions.

ACKNOWLEDGMENTS

I started this master's thesis project with limited background in the fields involved but a great enthusiasm as a springboard. Almost seven months later I am really grateful for my mentors and friends that persuaded me to move forward with this decision and step out of my comfort zone. This project began and finished during the times of the pandemic, posing additional challenges in the process. Nevertheless, this whole experience has proven to be an incredible learning opportunity for me and a fruitful experience overall. It goes without saying that it would never be the same without the presence of the people involved, either physically or digitally.

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Anastasia Florou
Delft, June 2021

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Acronyms

AEC Architecture, Engineering and Construction
MCDA Multi Criteria Decision Analysis
BIM Building Information Modeling
RSE Reverse Solar Envelope
SE Solar Envelope
RC Relative Compactness
PoR Program of Requirements
HOY Hour Of the Year
SF Skyview Factor
DN Direct Normal
EPW Energy Plus Weather (file)
TP Toy Problem
WP Weighted Product
MOORA Multi Objective Optimization on the basis of Ration Analysis
TOPSIS Technique for Order of Preference by Similarity to Ideal Solution
DNIrr Direct Normal Irradiation
DNIll Direct Normal Illumination
DS Direct Skylight

Glossary of terms

Lattice	a numerical field within a discrete 3D space
Voxel	represents a value within a lattice
Mass	a group of voxels representing the volume of the building
Massing	the process of obtaining the mass from a given design space
Envelope	boundary of the mass
Zone	a group of voxels within a mass representing a specific attribute
Zoning	the process of obtaining the zone from a given mass
Stencil	the neighborhood definition in a discrete 3D space
Decision variable	a quantity/ value that the decision-maker controls
Decision space	the range of values that the decision variables can take
Design criteria	attributes of the design space that define the success of a solution
Performance indicator	an aggregated value that quantifies the performance towards a goal
Spatial function	space usage or occupation type

1 Research Framework

1.1 Context and motivation

The building industry is one of the highest direct energy-consuming sectors worldwide. Apart from the energy consumption for the buildings' construction, their operational energy expenditures are also notable [1]. Therefore, energy efficient building design becomes a field of high priority in the European and global energy strategies and a subject of several intergovernmental agreements [2].

In order to achieve higher energy efficiency in buildings, the integration of energy simulation tools in the design process is indispensable. The conceptual design is proven to be the phase with the most significant potential of reducing the final energy demand of the building [3]. Early design decisions have a great influence on the environmental impact and construction costs. Hence, enhancement of the building's energy performance should initiate from the conceptual design phase, increasing the project's optimisation prospects.

Nonetheless, the typical environmental simulation workflows used in the Architecture, Engineering and Construction (AEC) industry are geared towards feedback evaluation of an already existing, elaborated and materialized design [4]. Arguably, it is not straightforward to utilize such tools in early-stage design decision making pertaining to form-finding. Hence, such tools are widely used in later design stages to evaluate already taken decisions, leaving a narrow space for design adaptations and further morphological explorations [5].

There are many factors, quantifiable and not, affecting early design decisions and resulting in a multitude of design criteria. However, only limited criteria are being considered in this stage, addressing mainly the operational, social and cost requirements [6]. New criteria, such as the compactness, the solar potential and shading impact of the building envelope, as well as the allocation of functions according to their demands, can be integrated in the conceptual design phase, creating a consistent base to enhance the building's final performance. By performing solar energy potential simulations at the pre-conceptual design phase, the necessary time for later-stage environmental assessments and design improvements is reduced considerably, while building designs with high performance but low environmental footprint are attained.

The multitude of the criteria involved in this stage implies the possibility of the emergence of conflicts among them. Therefore, a deeper comprehension of the interaction of the design parameters is necessary, to understand the combination of factors that lead to the fulfillment of certain criteria. The plethora of factors that need to be considered and assessed simultaneously, renders the need of design decision assisting tools as a requisite. The aim of this thesis project is to propose a computational framework, which leads to more optimal building designs by facilitating decision making in early design stages. This is achieved by simplifying the design scenario and focusing on the most influential design variables in order to produce an approximation of a building's three dimensional configuration, of nearly-optimal solar potential for a given program of requirements.

1.2 Research objective

Considering the aforementioned problems and limitations in the field of energy-efficient building design, the research objective of the thesis becomes to develop and implement a computational framework, which would assist designers in early-stage design decision making, through the production of nearly optimal three dimensional building configurations.

When the design process is shifted towards the shape of the building, it is reasonable to wonder if there exists a conceivable envelope, most probably invisible to bare eyes, which can achieve a few conflicting optimization targets. Hence, the objective of this thesis focuses on how to reveal such solar-climatic envelopes and more specifically, on how can a field of potentials be mapped, how a solution space can be generated and navigated so as to generate a nearly optimal alternative. However, prior to getting into the realm of problem-solving, a key contribution of this research will be on a generalizable mathematical formulation of such problems. Given the focus of this thesis on form-finding, a standardized voxelization-based approach is being adopted, in order to minimize the bias of the framework towards particular geometries.

The proposed method is tested through a case study within an urban context, for generating a building envelope with maximized solar potential for an urban mixed-use complex, in a temperate climate in the northern hemisphere. However, the purpose of the proposed workflow is to accommodate for different optimization goals e.g. minimizing solar gain throughout summer and maximizing solar gain throughout winter. Ideally, this should result in an algorithm / methodology, without the use of commercial software, that is easily accessible and usable for designers and architects. Another significant evaluation factor will be the calculation time. However, obtaining proper results through the application of the model is more significant than the optimization of the calculation time.

1.3 Research question

The posed problem and research objective lead to the formulation of the following main research question:

> *"How to develop a computational framework, for early-stage design approximation of a building's envelope shape (massing), in order to maximize its solar energy potential?"*

To be able to systematically reach the answer of the main research question, the following subordinate questions have been formulated:

- *How to translate the solar potential of a building to performance criteria?*
- *How to define the most important criteria for the early stage design?*
- *How to turn the performance criteria into performance indicators in order to evaluate them?*
- *How to validate such a computational framework/ model?*

1.4 Research scope

From the formulation of the research objectives and questions, it becomes evident that this research project derives from multiple disciplines. The principal scientific fields involved are: Architectural and Climate design, Computer science and Mathematics. The combination and intersection of these fields and their branches is illustrated in Figure 1.

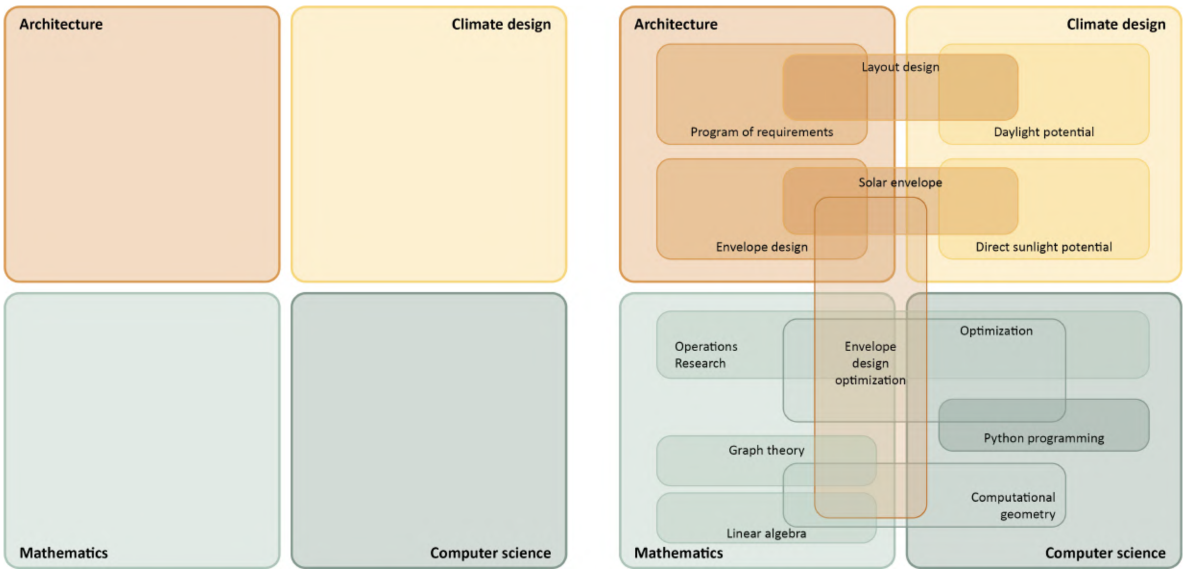

Figure 1: *Euler diagram illustrating the scope of the research*

Primary goal of this thesis is to contribute to the field of generative design for sustainable buildings, particularly by handling the challenge of facilitating decision making during the conceptual design phase, through the development of a usable model. The usability of the tool lies in its transparency, its ability to adapt and easily be handled and interpreted by the focus user, who in this situation is the architect/designer.

The tool aims to assist decision making rather than govern this process by proposing a determinative design. To be more precise, while this thesis tackles the approximation of a building's massing, the proposal of a finalized building shape and detailed facade design fall out of the scope. Other subjects that relate to the thesis but fall out of the scope of the project are the materiality of the building envelope, as well as its structural stability.

As far as the climate design part is concerned, this thesis focuses on early stage simulations and, in particular, on the solar irradiation and illumination analysis. It also accounts for the overshadowing of the surrounding buildings. Although it is recognized that additional factors also affect the final building energy performance, they are considered to either have a minimal effect compared to the one of the direct and diffused sunlight or to request more data and input parameters than the ones available in this design stage.

In contrast to the majority of the architectural research projects, this research will not lead to one dominant design outcome. The development, testing and validation of the methodology is going to be performed through the utilization of toy problems and then a case study situation is going to be used in the final phase, to demonstrate the results. The case study area is in Rotterdam, the Netherlands, and it has been chosen because of its interesting and diverse context, as well as because of the available information, documented due to its exploitation for other courses taught in TU Delft. A more detailed introduction of the case study area and specifications will be presented in Section 5.

1.5 Problem statement

As mentioned in the previous sections, the aim of the proposed computational framework is to maximize the solar-climatic potential of a building, while meeting functional requirements, through an early massing and zoning approximation. Since the research focus is on the pre-conceptual design phase, it becomes clear that only a few, if any, design parameters are already established. As a consequence, performing detailed environmental simulations becomes futile. Some assumptions need to be made, rendering the pursuit of one absolute best solution unrealistic. Since the optimal solution becomes impossible to reach, (meta)heuristic methods need to be established in order to reach nearly optimal results in a reasonable time-span.

In the context of this research, a voxel-based generative design approach is going to be adopted. More specifically, a subtractive method is going to be applied both for the production of the solar envelope and the final building shape. In these terms, the problem can be expressed as:

> *"Given an initial set of voxels, which of them should be removed in order to maximize the solar potential of the envelope and meet the given functional requirements, while maintaining its compactness and avoiding the shading of its context?"*

1.6 Research methodology

The methodology of this thesis finds its fundamentals in researching through development and design. It becomes evident that the research approach that is more relevant to this project is situated closer to the rational-atomistic side, as depicted in Figure 2. The research has been structured in four main sections, as illustrated in Figure 3. The initial part is the research framework of the thesis, starting from the motivation that induces the formulation of the research objectives and questions, based on which the problem statement is developed. The second part is covered by the literature review process for which a separate integrated methodology was followed, which will be elaborated in Section 2.1. If deemed that, after the knowledge gained from the literature review part, the research objectives and questions need refinement, they are amended accordingly.

Having set the research framework of the project, the mathematical formulation and the establishment of the tool workflow follow. In this part, the nature of the inputs, the outputs and the mathematical formulas involved are analyzed thoroughly. This step also indicates the beginning of the third part of the research, which is the tool development. The mathematical formulation set the foundation for the algorithmic design part, where Python programming

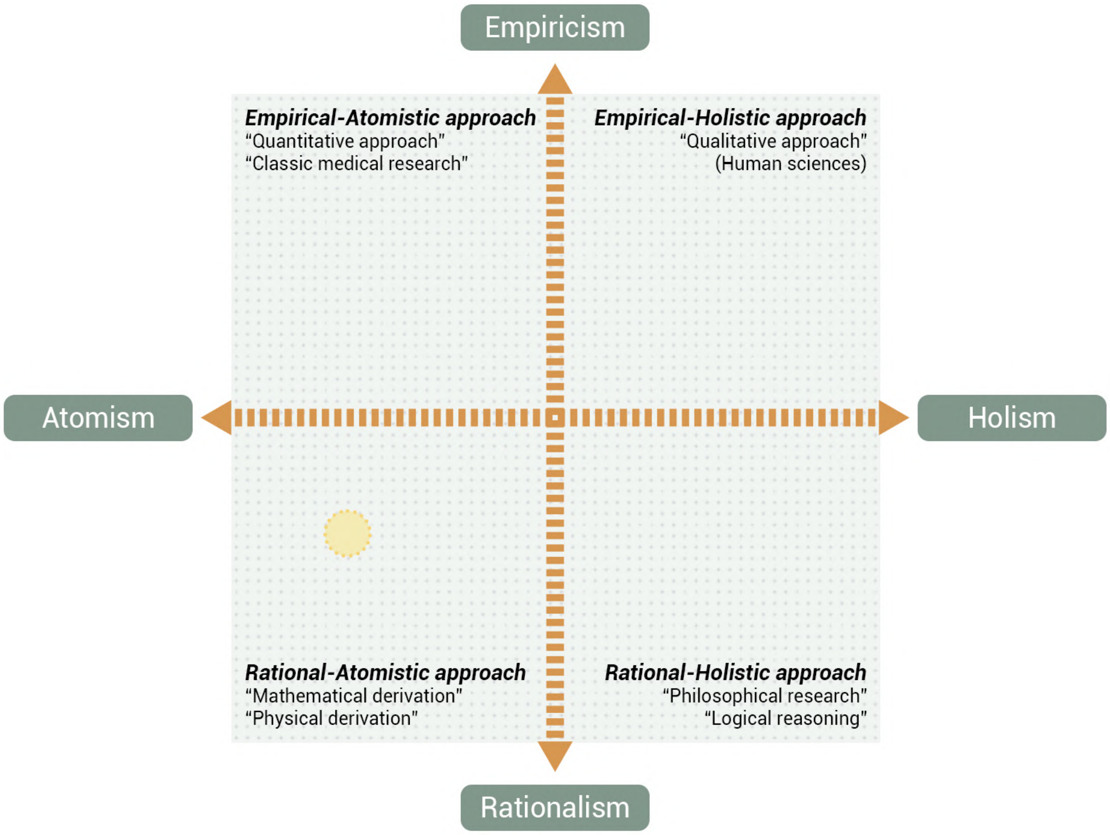

Figure 2: *The 4 continents on the world map of philosophy of science (adjusted from: Philosophy of Science)*

language is used. For the development and the verification of the tool several toy problems are utilized, leading to an iterative process of constant assessment of results and adaptation of the tool framework.

In parallel, the case study is being defined, by collecting all the essential data. As far as the location and context information are concerned, the original data is modelled and utilized while for the program of requirements, a fictional scenario is developed. When the iterative verification process yields proper results, the model is applied to the aforementioned case study to conduct a final validation and evaluation.

This process is also part of the fourth segment of the presented research methodology, which is the evaluation and conclusion phase. In this part, the results of the research are being assessed and conclusions are drawn. Fundamental evaluation criteria are the usability of the tool, the computation time it requires and its ability to produce diverse, yet equally functional and efficient, results. The reflection section covers the limitations of the research, as well as its strengths and weaknesses, leading to the suggestion of further developments.

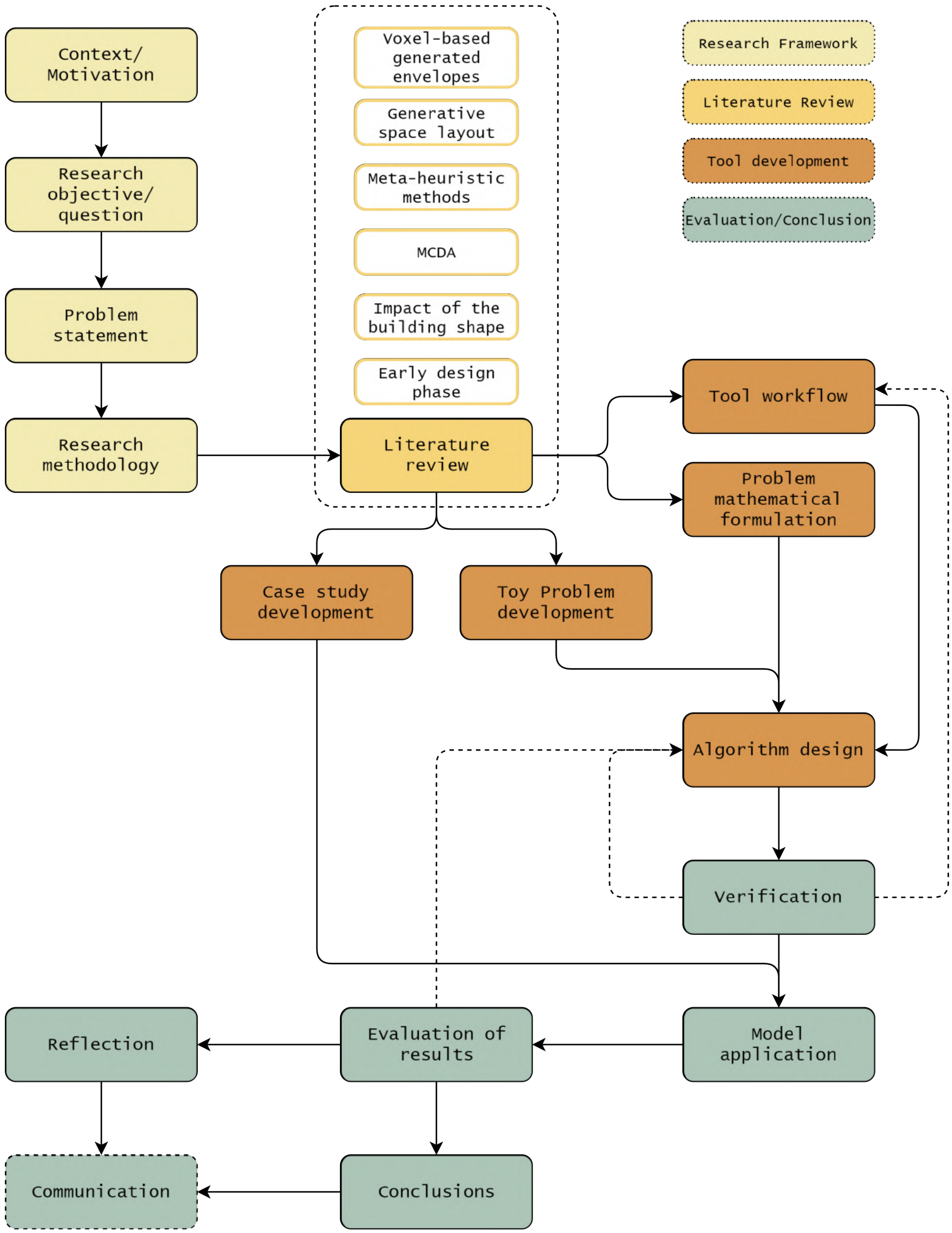

Figure 3: *Research Methodology flowchart*

1.7 Planning and organization

Following the establishment of the research methodology, the different phases were broken down into work packages and then mapped in the form of a Gantt chart. This chart (Figure 4) depicts the goal and the duration of each task, expressed in a number of weeks, and assists in keeping track of the progress and the sequence of the tasks. In summary, it can be seen that P1 and P2 phases were primarily dedicated to the process of literature study, that would lead in the establishment of the appropriate theoretical background, in order to proceed to the problem formulation. During the P3 phase, the problem was formulated and elaborated through mathematical terms and an initial application of it in code, through toy problems, was initiated. During the P4 phase the main algorithm was developed, improved through several assessments and then applied to a case study. By the end of the P4 period the core of the thesis was completed and P5 served as a period of final improvements in the verbal and graphical representation of the project.

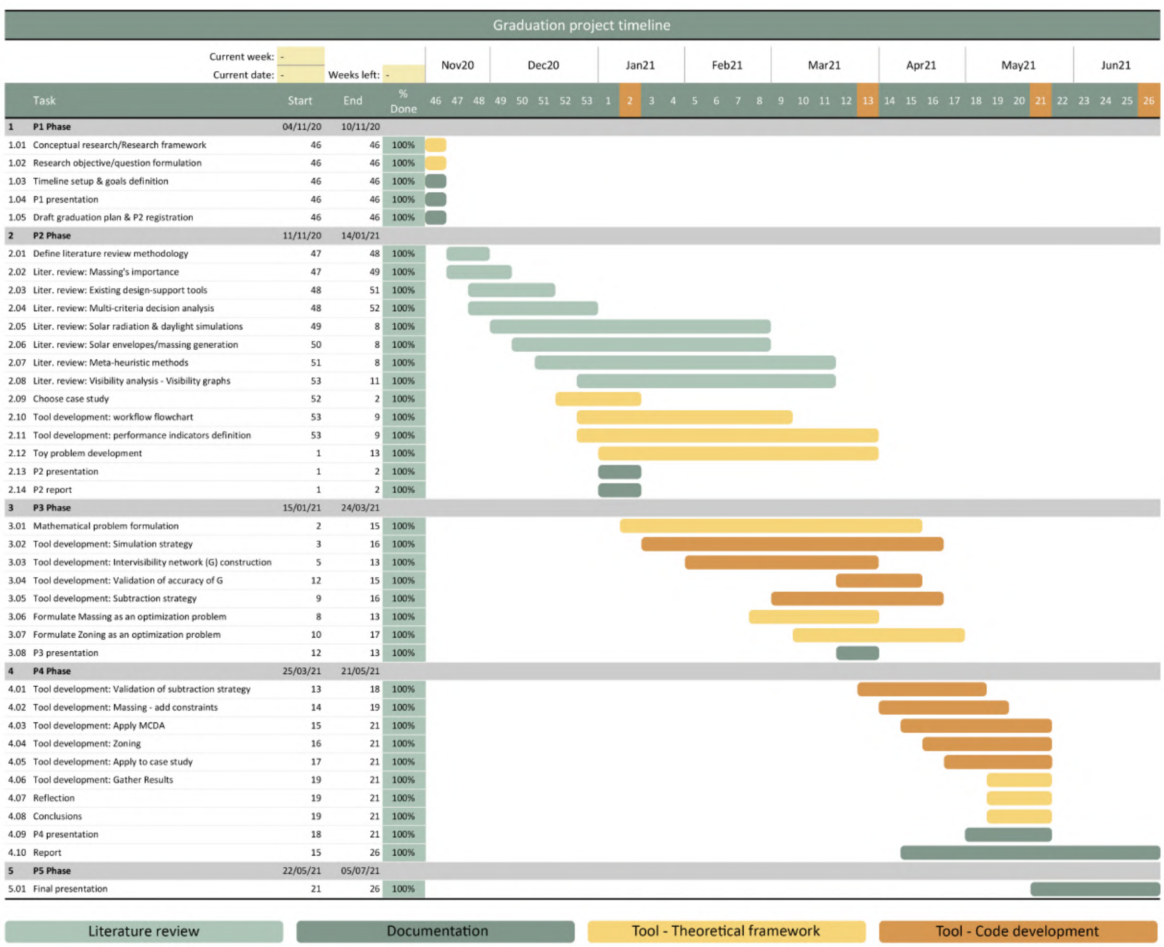

Figure 4: *Timeline depicted in a Gantt chart*

2 Literature Review

2.1 Methodology

In order to conduct an effective literature review, a systematic methodology was followed, as illustrated in Figure 5. The starting point of this methodology is the analysis of the research questions to identify the keywords which are essential for the formulation of the search terms. Based on these search terms, the search queries were defined and then fed to the following search engines: Google Scholar, Semantic Scholar, Scopus, Dimensions and CORE. The first stage of the relevance and reliability check is performed by reading the abstract and the conclusion of each paper. If the piece of literature is found to be irrelevant but still reliable it is being stored in a back-up database for possible future reassessment. For the relevant papers, a full text reading follows with some parallel note keeping. If after this process the paper is evaluated as irrelevant is being stored in the backup database. Otherwise, the piece of literature is added to Mendeley, which constitutes the main database, accompanied with a short summary, the main keywords and the relevant points with the ongoing research. The paper at hand is also analyzed with regards to its most significant references which are evaluated for future full-text reading. After reading and evaluating all the gathered literature, the analysis of the available information determines if the process is completed or if there is the need to reexamine the back-up database or even reiterate through the search engines. In the following section the literature review analysis and results are going to be presented per research category, as each one is characterized by different research criteria and goals.

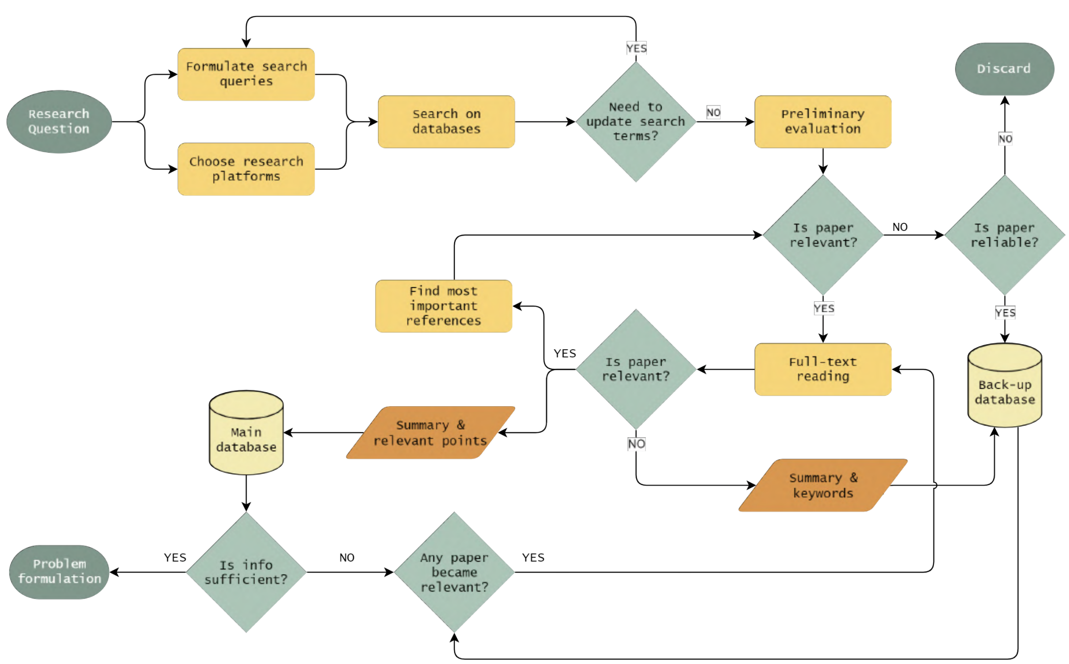

Figure 5: *Literature review methodology flowchart*

2.2 Early-design phase

This section of the literature review focuses on the exploration of the effect of early-design decisions on the final performance of buildings and its goal is to validate the purpose of the proposed model and define its potential according to data presented by relevant studies. Subsequently, tools developed to support decision making in this design phase will be briefly presented, to examine the several approaches that have already been applied, detect their benefits and drawbacks and identify the possible research gap in the field.

2.2.1 Importance and potential

A key factor in the enchantment of the building's energy performance is the implementation of new strategies and decision-supporting processes, tackling the early stage design [7]. A feature, highlighting the importance of early design phase, suggests that 20% of the design decisions taken, subsequently influence 80% of all design decisions [8]. Thus, early design considerations can have a significantly positive impact in the final energy performance and cost of the building, as conceptually illustrated in Figure 6.

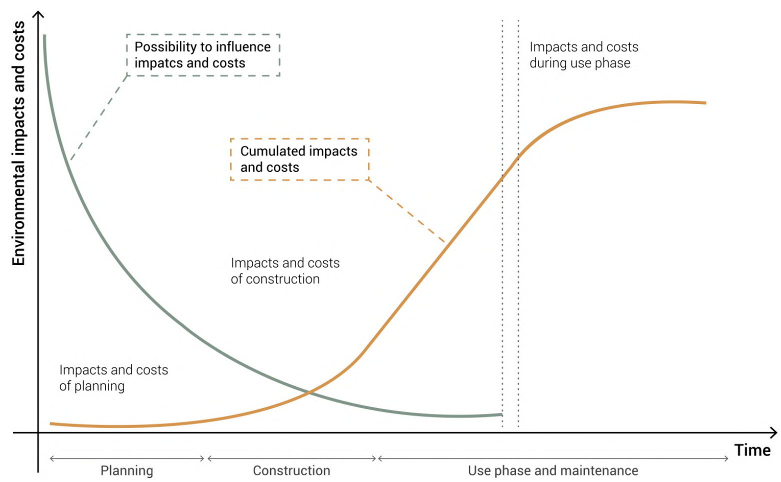

Figure 6: *Influence of design decisions on life cycle impacts and costs (source: [9])*

The greatest percentage of greenhouse gas emissions, that emanate from the building sector, are principally caused by the end use of electricity for heating, cooling, artificial lighting and mechanical ventilation [10]. The envelope shape has critical influence on all these factors [11]. Thus, the priority should shift towards high-performing building envelopes that are designed for the maximum exploitation of their location potential and local climate conditions. Such designs should tackle the decrease of interior peak loads and overall electricity demands, resulting in estimated energy savings between 10% and 40% [12].

From the analysis of the aforementioned literature it can be concluded that early design stages provide the highest potential for generating high-performing buildings, creating the opportunity to reduce energy demands by up to 40%. The absence of design specifications in this stage impose some environmental analysis limitations but increases the potential of discovering nearly-optimal design solutions. The significance of the building envelope shape is highlighted, as the principal building component that can contribute to this reduction.

2.2.2 Design decisions support tools

As mentioned in section 1.1, although various whole-building environmental simulations software have been developed, these are not amenable to the early stages of the design process. This is mainly due to the high time demand of the process and the specification degree of the required input [13]. As a result, such tools are providing foreordained results that evaluate the already determined design rather than proactively assist the design process by providing guidelines [3]. With the purpose to stimulate the utilization of building performance simulation through the early design phases, various research groups have developed novel tools.

Østergård et al. [7] suggest an iterative approach that implements Morris sensitivity analysis to help the designer identify the importance of each design parameter in the final performance, by integrating Monte Carlo simulations. A Monte Carlo framework is also applied by J. S. Hygh et al. [13], who present a modeling approach to appraise the building's energy performance, with the use of EnergyPlus. In the framework of this study, standardized regression coefficients are applied to evaluate the sensitivity of the total energy demand to several design criteria. The implementation of sensitivity analysis modelling is also present in the research conducted by S. Attia et al. [3]. This paper presents a tool that is used to promptly evaluate the energy efficiency of design alternatives, allowing for comparative assessments.

A design decision assisting tool that supports the iterative exploration of the design space is the MITDesign Advisor [14]. This tool allows the users to assess the energy performance of preliminary design, while including a design optimizer that implements a branching fuzzy-logic classifier to interpolate within the multi-parameter design space. A. Schlueter and F. Thesseling [15] are addressing the same issue by presenting a prototypical tool, integrated into a building information modelling (BIM) software, that allows for immediate energy and exergy calculations based on a statistical mathematical calculation model.

S. Petersen and S. Svendsen [16] propose a model which utilizes differential sensitivity analysis to depict how design variables will influence the energy performance of the building. This is achieved by generating design recommendations, through parameter variations, but it can be applied only to rectangular single-sided rooms. Another research that is clearly focusing on the influence of the envelope design on the final energy performance, is the one presented by V. Granadeiro et al. [17]. This study suggests a methodology consisting of a flexible shape grammar-based design system which generates alternative envelope shapes, while incorporating energy simulations in the process. Another novel form-finding method is the one proposed by F. De Luca et al. [18]. This paper introduces a computational workflow, called Reverse Solar Envelope (RSE), that generates solar envelopes according to their contextual shading. The aim of the tool is the generation of early massing options that can then be used in early stage performance analysis.

Although it is acknowledged that there are more such tools, it is considered that an adequate, diverse portion of them has been examined, in order to draw reliable conclusions. To facilitate this process, all the aforementioned information is gathered in Table 1. Studying these research works, it can be observed that, while such tools are addressing the issue of early design decision support, they are still of evaluative nature, providing insufficient or no form-finding guidance. The first six cited methods require the pre-existence of a geometric model, rendering them constrained by their own definitions and design boundaries. These studies are also neglecting parameters linked to the immediate surroundings and local topography.

Only the models presented by V. Granadeiro et al. [17] and F. De Luca et al. [18], include a form generation feature. In the first example the shape is generated according to the set grammars but not with the consideration of its energy performance. In the later research the form generation is based on the shading impact of the envelope on the surrounding urban environment. However, also in this case the environmental performance analysis is used only as an evaluation method of the generated massing options.

As far as the performance indicators are concerned, it is noticeable that the first seven tools set the total energy demand as the primary factor, while some of them are also considering thermal comfort and daylight. The simulation methods used for the assessment of these factors are diverse. While some make use of already established tools, others are based directly on mathematical models and energy equations. The common point of all of them is the simplicity of the utilized models, the lack of detailed data and the need for simulation time reduction.

Table 1: *Early-design support tools comparative matrix*

Research team	Year	Inputs		Performance indicators	Evaluation method	
		Geometric model	Var. design parameters		Existing tool	Custom methodology
T. Ostergarda et al.	2017	+	+	energy demand thermal comfort daylight	−	+
J.S. Hygh et al.	2012	+	+	energy demand	+	−
S. Attia et al. (ZEBO)	2012	+	+	thermal comfort energy demand	+	−
B. Urban et al. (MIT Design Advisor)	2007	+	+	heating & cooling load thermal comfort / daylight	−	+
A. Schlueter et al. (DPV)	2008	+	+	energy demand	−	+
S. Petersen et al. (iDbuild)	2010	+	+	energy demand indoor environment	+	−
V. Granadeiro et al.	2012	−	+	energy demand	+	−
F. De Luca et al. (RSE)	2021	−	−	contextual shading	−	+

2.3 Building shape

The form of the building dictates its visual expression as well as various performance factors. The consideration of the building morphology is essential, not only for its architectural expression, but also for ensuring adequate solar potential and for balancing the final energy demand. Moreover, every building is positioned within an environment, and as a result, its form should respect the surrounding elements. In this section, the concept of the solar envelope is introduced, as the proposed solution to the contextual shading considerations. Furthermore, different factors expressing the geometric properties of the building are presented and discussed, with respect to their relation to energy evaluations.

2.3.1 Solar envelope

The concept of solar envelope was introduced by Ralph L. Knowles in the 1970s and is referring to the maximum buildable volume which allows its context to receive the required direct solar access [19]. That is, imaginary boundaries within which buildings can be designed and built while respecting surroundings premises. More and more countries enact laws to guarantee minimum solar access to new and existing buildings. These laws are establishing minimum solar rights in defined time frames [20].

One approach of forming solar envelopes is this of using discrete points. This has been applied in tools like SolVelope [21], SolCAD [22] and SustArc [23]. According to this method, the plot under study is populated by a grid of points which are then moved vertically to their maximum possible height, that allow enough daylight for the surrounding facades. The second approach is using solid geometry and is based on four operations: cutting planes, intersecting planes, intersecting volumes and volume subtraction. These operations are widely applied in extensions for commercially available design software like SketchUP, AutoCAD and Revit [18].

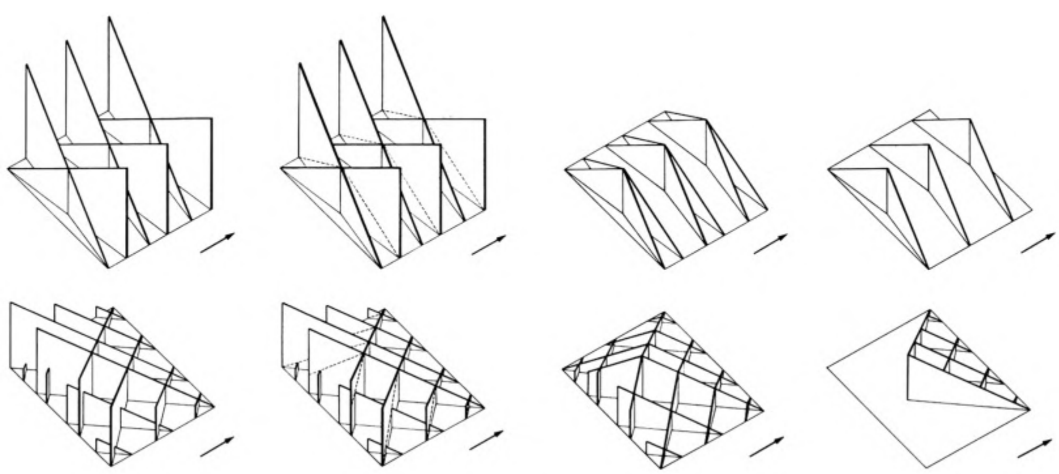

Figure 7: *Solar envelope method using vertical planes (source: [24])*

The majority of the existing methods are based on additive approaches. However, these methods are proven to be more adequate for low- and mid- rise buildings in uniform urban settings [25]. Subtractive methods that aim to produce larger and more accurate solar envelopes have been developed by I. de Araujo [26] and F. De Luca [18]. These methods are both based on a voxelized geometry and the utilization of a ray-tracing algorithm and are contributing to the partial relaxation of the constraints of the conventional approaches, while offering more design freedom.

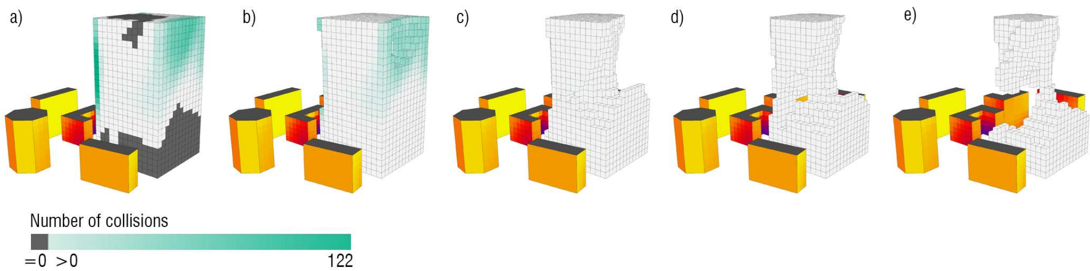

Figure 8: *Solar envelope subtractive voxel-based method (source: [26])*

2.3.2 Shape characteristics and energy performance

The shape of the building is one of its most characteristic properties. Except for defining its visual identity it also plays a significant role in its energy performance [27]. For this reason, various mathematical formulas have been proposed in an attempt to numerically define the building geometry. The intention of these numerical factors is usually to express the compactness of the envelope as its most influential geometric characteristic.

For instance, Aksoy and Inalli [28] express the building compactness through the shape factor (SF) and examine its effect on the heating demand. The shape factor, in this case, is calculated as the ratio of the length to the depth of a conventional building storey. In a later study, Bostancioglu [29] makes use of the ratio of the external surface area to the initial floor area, to estimate its impact on construction, operational and energy costs. Nevertheless, the most commonly used formula for evaluating the compactness of a building is the one dividing the total facade area of the building to the volume that these surfaces enclose. This ratio, when linked to the ratio of a reference building of the most compact shape, results in the relative compactness (RC) indicator (equation 1). Relevant studies have demonstrated that this factor better approximates the subjective classification of shape compactness by designers [30]. When using the sphere as a reference for the most compact shape, RC is expressed through equation 2. However, since buildings are usually of orthogonal shapes, in such studies RC is expressed in relation with a cube shape, resulting in equation 3. It becomes clear that relative compactness as an indicator is pure shape-dependent and not size-dependent.

$$C_{rel\_general} = \frac{(A/V)_{building}}{(A/V)_{reference}} = \frac{A_{ref}}{A_{building}} \tag{1}$$

$$C_{rel\_sphere} = \frac{4.84V^{2/3}}{A} \tag{2}$$

$$C_{rel\_cube} = \frac{6V^{2/3}}{A} \tag{3}$$

The cube-reference equation (3) has been used for several studies that evaluate its relation with various building-performance factors. Ourghi, AlAnzi, and Krarti [31], present a simplified analysis method to predict the link of the RC factor to the annual final energy use and cooling demand of office buildings. The context of this research is focused on commercial buildings in cooling dominated countries and within these boundaries it is justified that the higher the building relative compactness the lower the cooling load and energy use.

The same formula has also been adopted by Catalina, Virgone and Iordache [32] in the context of their research on identifying to which degree relative compactness does not negatively influence other performance indicators, such as daylighting. Their argumentation line is based on the fact that, while a compact shape minimizes the costs and thermal energy consumption, it may affect the indoor comfort and increase the electrical consumption for artificial lighting. Their results highlight the fact that the building morphology is a significant design parameter when it comes to energy performance, with a critical impact on the indoor illuminance levels.

Pessenlehner and Mahdavi [33] initiate their research by illustrating the limitations of using the RC factor, some of which are linked to its inability to express a specific morphology of a building, its orientation, as well as the amount and characteristics of transparent elements. They evaluate the reliability of this indicator for energy-related assessments and they conclude that while it is a reliable factor in the prediction of heating loads and transmission losses, it can be disadvantageous in overheating prediction, which is mostly linked to the amount and orientation of glazing.

It can be concluded that, from the various factors that have been proposed in order to express the geometric efficiency of a standard building design, the relative compactness, as proposed by Mahdavi et al. [30], is one of the most widely used for design and energy assessment purposes. It has also been a subject of several evaluations regarding its applicability spectrum and effectiveness, revealing its advantages and limitations. It becomes clear, that while it appears to be suitable for final energy use predictions it is more efficient to be combined with additional indicators regarding daylighting, overheating etc.

2.4 Multi Criteria Decision Analysis

The sustainability performance of buildings depends on various criteria. At the same time the performance of these criteria is affected by many, usually conflicting, factors that are determined during the design process. Multiple-criteria decision analysis (MCDA) is a sub-discipline of operations research which facilitates decision making and is particularly useful for complex problems featuring conflicting objectives.

This section is dedicated in a review of the most wide-applied MCDA methods, focusing on their application fields and on the principal features that guide the selection of the most appropriate method. The goal of this part is the identification of the most important factors when choosing a method for this project and the creation of a comparative matrix. Thus, although a great amount of literature review on several MCDA methods has been conducted for the production of Table 2, it is considered that their detailed presentation falls out of the scope of this chapter.

The main stages of a MCDA process can be defined as: the criteria selection, criteria weighting, evaluation process and final aggregation [34]. During the criteria selection, it is important to identify the features of the information that each method can handle. These features are divided according to their determinism, to certain, uncertain and fuzzy and according to their literal meaning to ordinal, cardinal or mixed. Moreover, the weighting approaches are crucial for the final outcome of the process as they determine if they act as importance coefficients or trade-offs. This is also affected by the ability to include thresholds or veto values. Generally, for the weight assignment it is essential to consider the deviation degree among the criteria and their level of dependency [34]. Regarding the aggregation methods, three main processes can be identified: the aggregation to one single criterion, the establishment of an outranking relation among the criteria and some mixed methods.

As far as the input characteristics are concerned, for energy-efficient building designs where multiple factors, qualitative and quantitative, are affecting the final decision it is more practical to use a method that supports mixed type of information. However, in the framework of this project, that the production of energy efficient envelopes is based on mathematical formulations and computational processes, it is possible to produce exclusively cardinal data. Generally, in decision occasions where environmental criteria are involved, non compensatory or partial compensatory methods are usually adopted [35]. For the same reason, the possibility of threshold values is also favorable in such decision making situations.

While the single criterion methods are the most widely applied ones, they impose some risks and, in some occasions, a high degree of subjectivity. On the contrary, outranking approaches seem more complicated in their application but they can handle the inaccuracy of criteria values through the establishment of preference thresholds. As a consequence they are believed to be more suitable for energy-related problems [35].

Using these conclusions as a guideline, two or three methods are going to be selected during the tool development phase in order to evaluate their suitability with the proposed design methodology. The general process that is going to be followed in order to select a suitable MCDA method is depicted in the flowchart in Figure 9.

Table 2: *MCDA methods comparative matrix*

Method	Aggregation		Compensation		Thresholds	Data features				
	Single criterion	Outranking	Total	Partial		Ordinal	Cardinal	Certain	Uncertain	Fuzzy
AHP	+	–	–	+	–	–	+	+	+	–
ANP	+	–	–	+	–	–	+	+	+	–
ELECTRE	–	+	–	+	+	+	+	+	–	–
EVAMIX	+	–	–	+	–	+	+	+	–	–
Fuzzy AHP	+	–	–	+	–	–	+	+	+	+
Fuzzy TOPSIS	+	–	+	–	–	–	+	+	+	+
IDRA	–	+	–	+	–	–	+	+	+	–
MAUT	+	–	–	+	–	–	+	–	+	–
MAVT	+	–	–	+	–	–	+	+	–	–
NAIADE	–	+	–	+	+	+	+	+	+	–
ORESTE	–	+	–	+	+	+	–	+	–	–
PROMETHEE	–	+	–	+	+	+	+	+	–	–
REGIME	–	+	–	+	+	+	–	+	–	–
SAW	+	–	+	–	–	–	+	+	–	–
TOPSIS	+	–	+	–	–	–	+	+	–	–

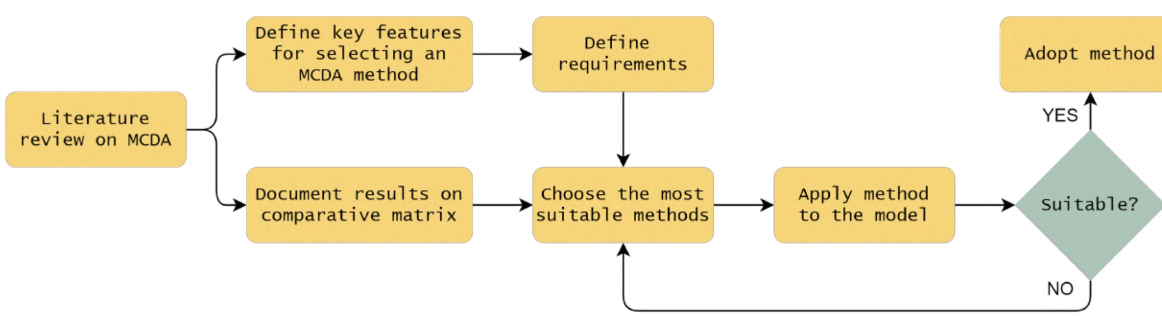

Figure 9: *Process of choosing a suitable MCDA method*

2.5 Performance-based voxel-generated envelopes

As computational evaluative workflows become increasingly utilized in the architectural design process, some studies on generative design initiate the discussion on voxel-based generated envelopes. Such studies will be presented in this section in chronological order, with the aim to identify their scope and methodology as well as their advantages and limitations, in an effort to investigate the common practices and advances in this field.

Leidi and Schlüter [36] propose a computational design methodology for early stage evaluation of urban design alternatives, called Volumetric Site Analysis (VSA). The performance indicators that are taken into account include passive, active and expressive criteria, such as solar heat gains, daylight, airflow, urban visibility etc. These are handled as physical properties of points in space, which result from the volumetric discretization of the urban site. The urban environment is translated in a multidimensional multivariate vectorial dataset and then visualized through a flexible visualization framework that facilitates the interpretation of the computed information. The combination of these features lead to a better understanding of the simulation output and increases the effectiveness of the conceptual design phase.

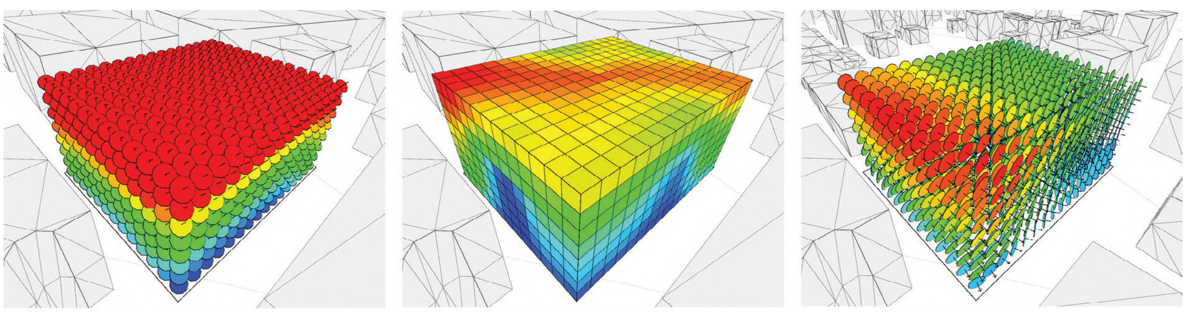

Figure 10: *Example of simulations of different indicators, through different visualization elements (source: [36])*

Jyioti [37] suggests a performance driven generative framework for high rises, through the exploration of the performance-driven design space. This research is also focusing on a dense urban context, while aiming to integrate the environmental simulations part earlier in the design process, to support informed morphological decisions. The performance criteria that are taken into account are the daylight, direct sunlight and solar irradiation, for which different seasonal optimization goals are established. The optimization process combines formal and performative perspectives and applied various methods (e.g. Recursive Accumulation, Recursive Erosion etc) to produce optimized voxel colonies that are then combined to produce the final outcome.

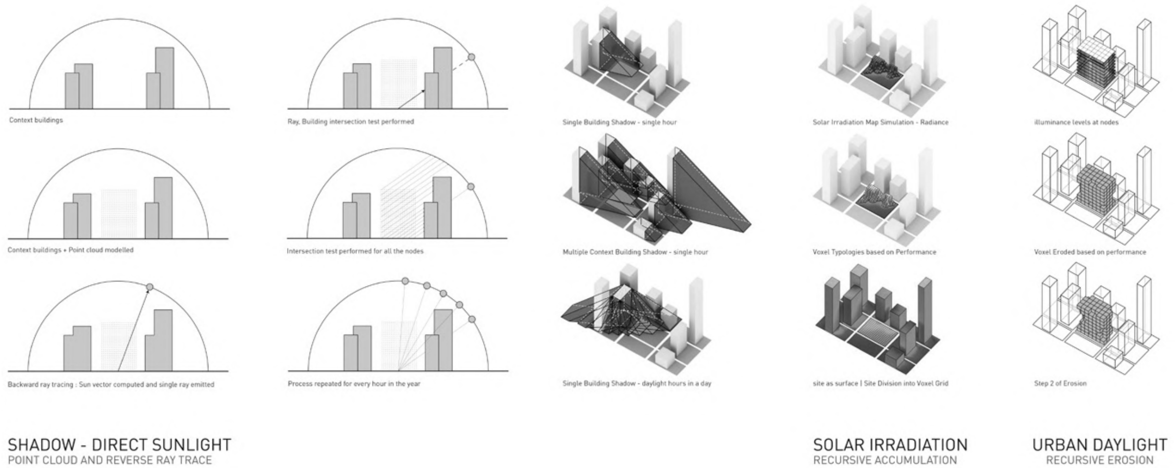

Figure 11: *Parameters: solar irradiation, urban daylight, direct sunlight; Process: recursive accumulation, recursive erosion, shadow wedge, and point cloud intersection. (source: [37])*

Another study focusing on urban morphogenesis is the one presented by Darmon [26]. The methodology of this research is based on rules-based algorithms pertaining to solar access. Supplementary parameters, regarding comfort criteria and construction feasibility aspects, are also taken into account. The performance indicators are calculated based on ray-tracing methods relative to the sun positions. In the context of this research, five shape generation methods, both subtractive and additive, are being tested and compared. The results demonstrate the design flexibility that voxels-based generative methods provide and their additional benefits on constructibility, such as the assistance of prefabrication and the subsequent reduction on its carbon impact.

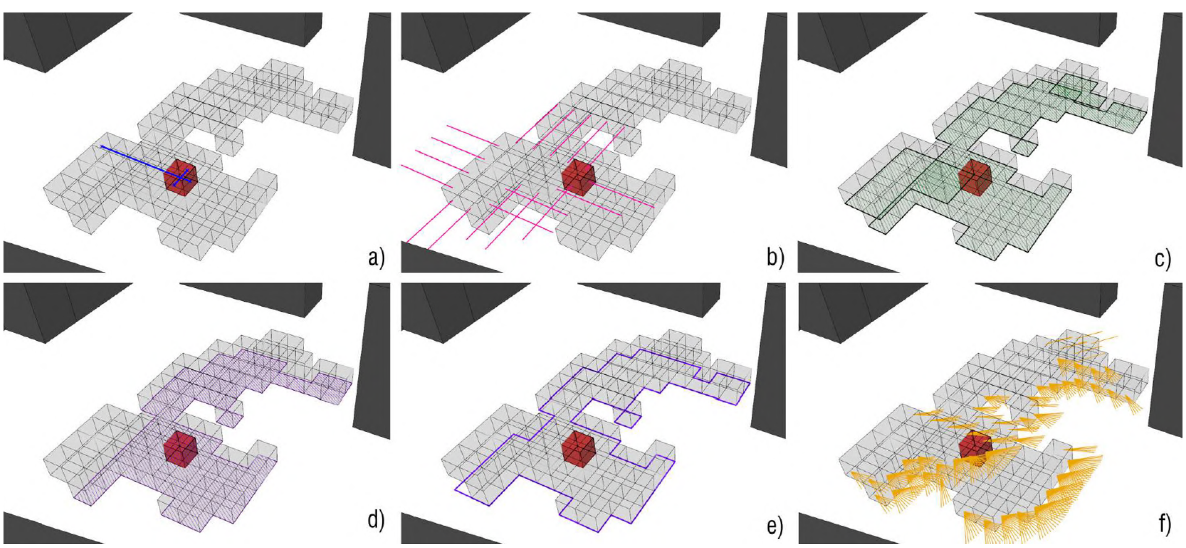

Figure 12: *Algorithm Evaluation criteria: a) Building Thickness b) Neighborhood proximity c) Building Gross Area d) Building Ground Floor Area e) Building Footprint f) Direct Solar Access (source: [26])*

3 Proposed Methodology

3.1 Framework

The objective of this research is the development of a feed-forward optimization model for building envelopes and, as a side contribution, the generalized formulation of such problems. This section provides an introduction to the posed problem and its analysis into sub-problems, for which the mathematical formulation, the methodology and the algorithmic design is presented. The framework of the presented research can be presumed as a process consisting of three main stages as presented in Figure 13. The first stage concerns the compliance with building regulations and its purpose is to define the maximum volume that encapsulates all the environmental laws of a specific case. This is achieved by setting the bounds of the design space within which the optimization of the envelope's topology takes place (Stage 2). In this stage, the solution space is explored to find a nearly optimal building shape which achieves a combination of optimization targets. The outcome of Stage 2 is the massing of the building, which in Stage 3 is divided into several zones, representing spatial functions, according to the given PoR. While the whole framework is presented, the focus of this research is mainly on Stage 2 and, thus, this part will be presented more thoroughly. The first stage is briefly discussed while stage 3 serves to demonstrate the entire workflow, but will not be further explored in this thesis.

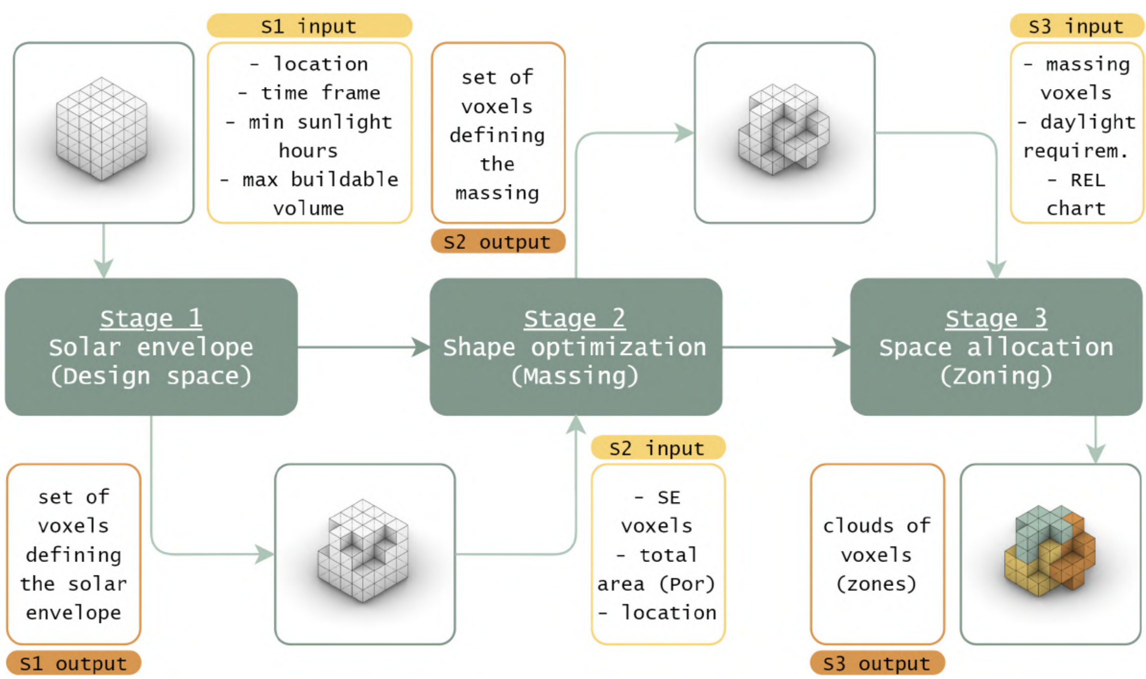

Figure 13: *Proposed methodology framework*

3.2 Stage 1 - Environmental envelope

3.2.1 Problem formulation

This stage is necessary in order to ensure that the model's output complies with the building regulations and, unlike the two following ones, concerns a problem with a narrow universe of possible solutions, whose size depends mainly on the methodology that is followed.

Taking into account the location of our case study, the Netherlands, the existing relevant building regulations concern the maximum building height and the minimum sunlight hours that each facade should receive. Hence, for this specific case the environmental envelope concept coincides with the solar envelope one. In section 2.3.1 this concept was introduced and the main methodologies for its production were briefly presented. Conventional methods usually result in smaller and more restrictive shapes than the more recent voxel-based approaches. As a result, a subtractive voxel-based method was adopted for this stage, in order to ensure a wider solution space for the next stage.

To explain the problem in a more streamlined manner, we will make use of a hypothetical simplified situation or a "toy" problem, method that have served as an expository tool in many scientific fields [38]. We assume a simple urban context and a plot in it. The plot is extruded to the maximum buildable height, as stated by the relevant regulations and is populated with a three dimensional array of $\mathbf{n}$ voxels (Figure 14c).

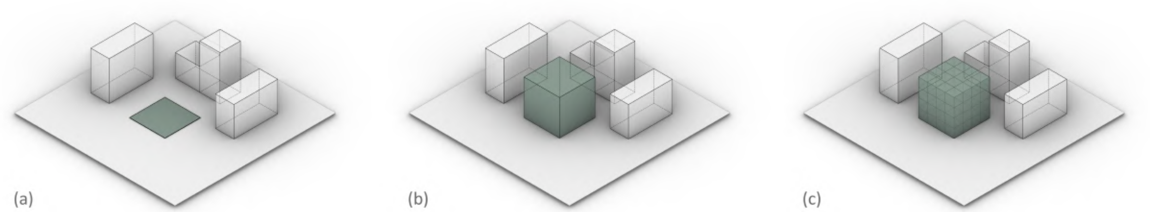

Figure 14: *Set of stage 1 of the toy problem*

This problem can be formulated as follows:

> *"Given a specific location (plot and context) and a building code, it is sought to define the minimum voxels that need to be removed, in order for the surrounding facades to receive at least the minimum permitted direct sunlight hours."*

and it is subject to the following **constraints**:

- the total remaining area must be at least equal to the sum of the zone areas
- all voxels should have at least one neighboring voxel

These constraints are established in order to ensure that the resulting solar envelope is big enough to accommodate the architectural program, and that the remaining space is usable. This second parameter refers to the prevention of floating voxels (i.e. voxels with no neighbor) which can be interpreted as isolated segments of space.

3.2.2 Methodology

The methodology which is used in this stage is based on a subtractive approach. The solar envelope is generated through the elimination of three dimensional cells from a voxelized domain, according to their compliance with urban regulation regarding minimum direct sunlight hours. As illustrated in the flowchart in Figure 16 the process can be arranged in four main steps: (I) Envelope voxelization, (I) Sun vectors computation, (III) Sun rays intersections, (IV) Removal process - Envelope generation.

Input
The necessary input for this stage can be grouped in two categories
Location data

- longitude and latitude (float)
- plot outline (curve / mesh / surface)
- context geometry (mesh)

Building code (regulations) data

- predefined analysis period (start month-day and end month-day)
- minimum sunlight hours requirement (float)
- maximum buildable height (float)

(I) Envelope voxelization
After setting up the required input the process begins by extruding the plot outline to the maximum allowed height. The maximum buildable volume is then discretized in a three dimensional array of $\mathbf{n}$ number of cubic cells (Figure 15a). The algorithm allows for the selection of the size of the voxels. A smaller number will lead to more accurate results and a more flexible design space for the next massing process but will be more computationally intensive.

(II) Sun vectors computation
Given the longitude and latitude from the input stage, the sun path for this exact location is being generated, using the sun path module of the ladybug library for Python [39]. Subsequently, the hours of the year (HOYS) corresponding to the predefined analysis period are being calculated. The sunpath and the computed HOYS are then fed to the sun rays generation algorithm that provides the sun vector for each HOY.

(III) Sun rays intersections

This process is performed in two stages. The first group of rays that is generated are the ones originating from the voxels centroids shooting towards the computed sun objects (Figure 15b). The intersection of these rays with the context mesh is performed in order to discard the ones that are being obstructed by it (Figure 15c). For the remaining rays a backwards shooting, in the opposite direction of the sun object and towards the context mesh, is taking place (Figure 15d). Every ray that intersects with the context mesh is stored as one hour of direct sun obstruction.

Table 3: *Framework of Algorithm 1: Sun Rays Intersection Algorithm*

Input	Data Type	Notes
B Array of voxels	$\mathbf{b} := [b_i]_{n\times 1}$	Array of all the discrete volumetric elements of the maximum buildable envelope
T Array of sun vectors	$\mathbf{t} := [t_{k,h}]_{m\times 3}$	Array of sun vectors corresponding to the hours of the year of the pre-defined period
Context mesh	$\mathbf{M}$	The surface mesh representing the context of the building
Output	**Data Type**	**Notes**
C Contextual Shading index	$\mathbf{c} := [c_{i,k}]_{n\times m}$	Array of the contextual shading of each voxel for each ray/hoy
Problem: Find which hours of the predefined period, each voxels obstructs the direct sun from its surroundings.		

Algorithm 1: Sun Rays Intersections Algorithm

```
SunRaysIntersections (B, T, M):
    C ← [0]_{n×s}
    foreach voxel b in B do
        ct ← extract centroid of b
        foreach sun vector t in T do
            ray ← a ray with the source of ct and direction of −t
            I ← check intersection of M and ray
            if not I then
                ray′ ← a ray with the source of ct and direction of t
                I′ ← check intersection of M and ray′
                if I then
                    C[b, t] ← 1
    return C
```

(IV) Removal process - Envelope generation

After the two-stage ray intersection process, the amount of sun blocking hours for each voxel has been calculated. The sun blocking hours are aggregated per day in order to define the daily contextual shading indicator for each voxel. When subtracting these values from the available sunlight hours of each day, the hours that each voxel allows the surrounding to receive solar rays is computed and stored in a vector. If any of the values in the sun access vector is less than the minimum allowed according to the relevant input, the corresponding voxel is removed. This process is also explained through the algorithm 2. At the end of this process the solar envelope has been generated (Figure 15f).

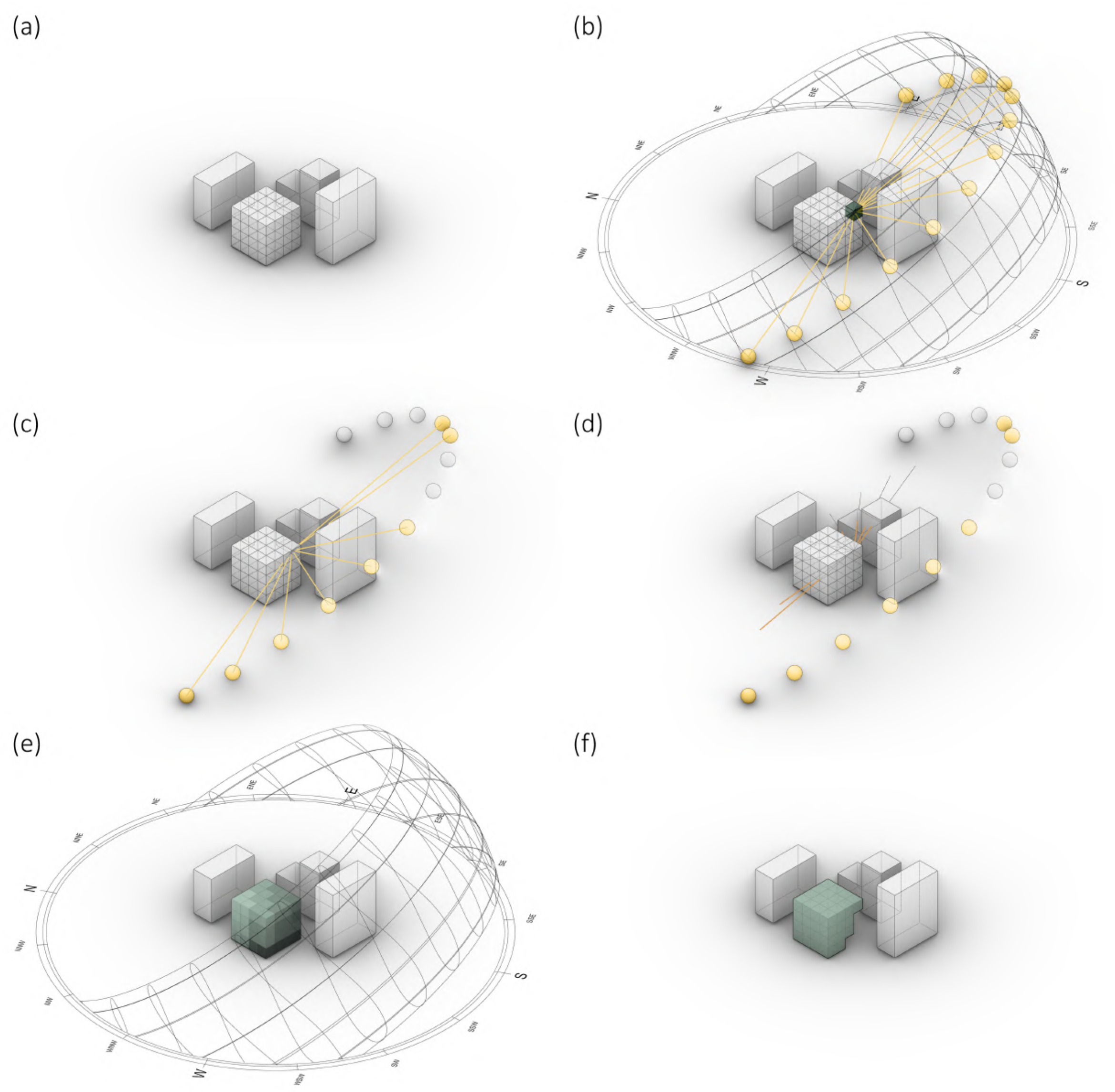

Figure 15: *Diagrammatic representation of Stage 1 steps through a toy problem*

Table 4: *Framework of Algorithm 2: Envelope Generation Algorithm*

Input	Data Type	Notes
C Contextual Shading index	$\mathbf{c} := [c_{i,k}]_{n\times m}$	Array of the contextual shading of each voxel for each ray/hoy
H Array	$\mathbf{h} := [h_k]_{m\times 1}$	Array of hoys of direct sunlight within the predefined period
B Array of voxels	$\mathbf{b} := [b_i]_{n\times 1}$	Array of all the discrete volumetric elements of the maximum buildable envelope
Min sun hours	**h_min**	Minimum hours of direct sunlight according to the building code
First and last day	**doy_start**, **doy_end**	First and last day of the year of the predefined period
Output	**Data Type**	**Notes**
V Array of voxels	$\mathbf{v} := [v_i]_{n\times 1}$	Array of voxels that define the solar envelope
Problem: Find which voxels of the envelope obstruct its context more than permitted and remove them.		

Algorithm 2: Envelope Generation Algorithm

EnvelopeGeneration *(***C**, **H**, **V**, **h_min**, **doy_start**, **doy_end***)***:**
- $d \leftarrow$ number of days between doy_start and doy_end
- $D \leftarrow [0]_{d\times 1}$ // initialize vector for number of sunlight hours per day
- $SA \leftarrow [0]_{n\times d}$ // sunlight hours access that each voxel allows to the context per day
- **foreach** *hoy* h *in* **H** **do**
  - $day \leftarrow \lfloor h/24 \rfloor$ // calculate doy from hoy
  - $D[day] \leftarrow +1$
- **foreach** *voxel* b *in* **B** **do**
  - **foreach** day *in* **D** **do**
    - $d\_hrs \leftarrow D[day]$ // total hours of sunlight for this day
    - $c\_d \leftarrow$ sum $C[b, (d\_hours)]$ // sum hours of contextual shading of this day
    - $SA[b, day] \leftarrow d\_hrs - c\_d$
  - **if** *any* $SA[b] < h\_min$ **then**
    - $B[b] \leftarrow 0$ // remove this voxel from the initial array
- $V \leftarrow$ reshape B according to initial envelope lattice
- return V

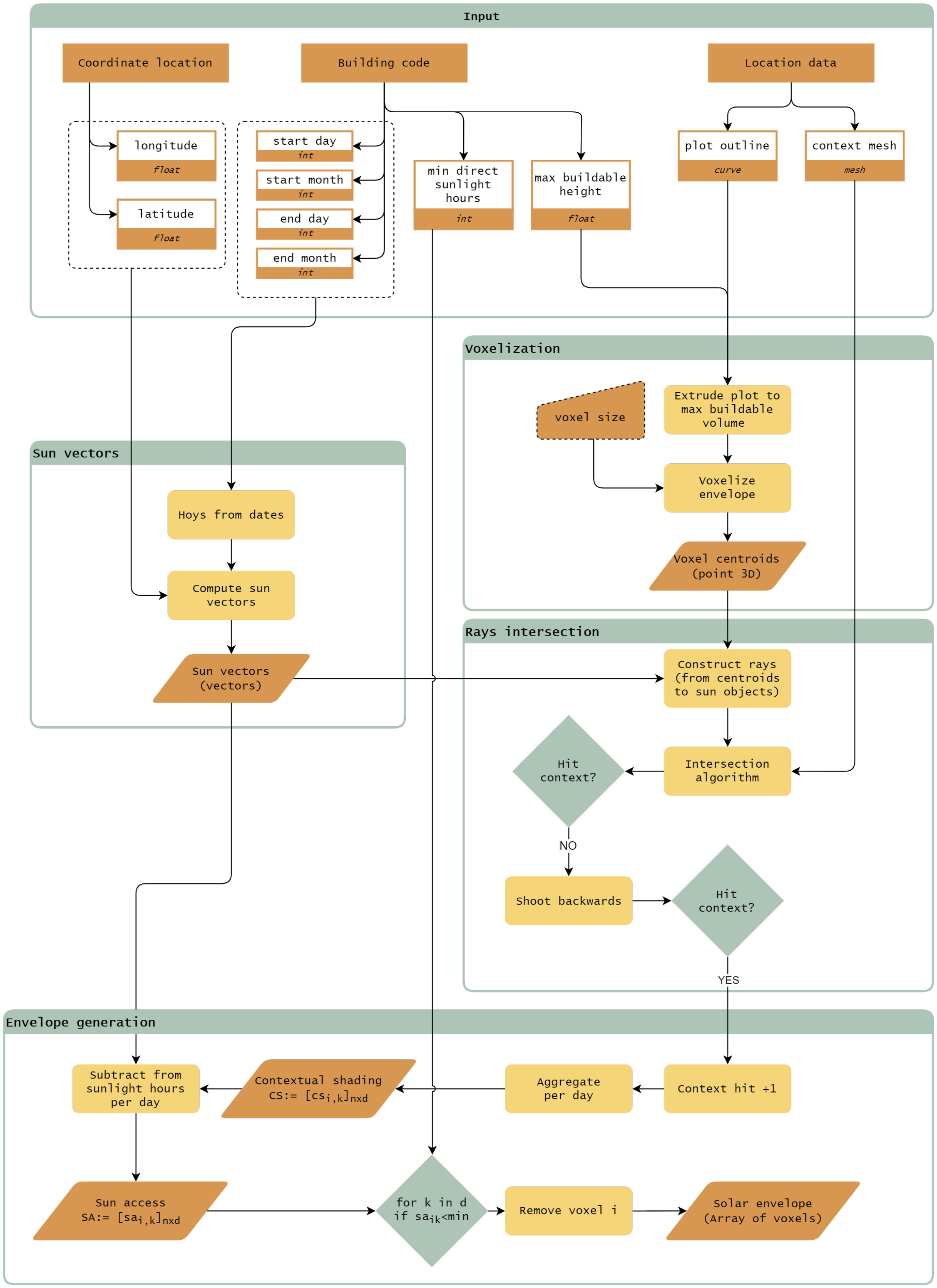

Figure 16: *Flowchart of the developed computational methodology*

3.3 Stage 2 - Massing

3.3.1 Problem formulation

The outcome of the first stage is the array of voxels that define the solar envelope. This array constitutes the boundaries of the design space within which the optimization of the topology of the final envelope is performed. The tessellation of the design space into three dimensional cells allows for the application of a Topology Optimization method, analogous to the ones found in structural design and mechanics operations. The duality of the topology and the geometric nature of the cells lead to a process that results in an optimal shape through the optimization of the topological model.

A subtractive approach is going to be adopted also in this stage. The goal is to calculate reliable, intuitive and comparable numeric indicators referring to performance criteria, which will denote a virtual design space. This results in a decision space mapped through a field of potentials, which can be navigated in order to assess the effect of each design decision and compare the potential of several building shape designs.

In order to explain the problem in simplified terms, the same toy problem is used. The outcome of the toy problem from stage 1 is a set of voxels, representing the solar envelope of the building under study. The problem of this stage is expressed as which voxels should be removed, in order for the resulting shape to have a nearly optimal solar potential.

The solar potential in this stage is defined by the combination of three factors:

- the Direct Normal (Sunlight) Irradiation
- the Direct Normal (Sunlight) Illumination
- the Direct Skylight Hours

It becomes clear that more focus is given to the importance of the direct sunlight part, linked to the first two factors. The third factor serves as an initial estimation of the direct skylight part. The problem of this stage is formulated as follows:

> *"Given a specific location (plot and context) and a building's programme of requirements (PoR), indicating the amount of surface area per spatial function (zone), it is sought to find a nearly optimal envelope topology (and thus shape), according to the following objectives and constraints:"*

Objectives:

- maximize the aggregated Direct Normal Irradiation of the envelope
- maximize the aggregated Direct Normal Illumination of the envelope
- maximize the aggregated Direct Skylight Hours of the envelope
- maximize the compactness of the envelope

Constraints:

- the total remaining area must be at least equal to the sum of the surface areas per spatial function (zone) as indicated in the PoR.
- the total remaining area must not exceed the maximum allowed floor space (according to building regulations).

3.3.2 Mathematical representation

This stage of the proposed methodology aims in finding an optimal solution to the posed problem, concerning the shape of the envelope. As a consequence, it can be expressed as an optimization problem. In this section the mathematical formulation of this problem is presented. In the following table the main mathematical notations are presented.

Table 5: *The nomenclature of mathematical notations*

Description	Notation	Notes
Array of Indices of Voxels	$\mathbf{v} := [v_i]_{n\times 1}$	an array of spatial indices (Morton Code) of all discrete volumetric elements of the rasterized/discretized envelope that define the Solar Envelope
Vector of Opacities	$\mathbf{x} := [x_i]_{n\times 1}$	Vector representing the transparency state of all the discrete volumetric elements of the rasterized envelope
Array of Rays	$\mathbf{r} := [r_k]_{m\times 1}$	Array of all the rays that are supposed to be shot from voxels toward the visibility objective
Ray weights vector	$\mathbf{w} := [w_k]_{m\times 1}$	Vector with the weights corresponding to the rays of $\mathbf{r}$
Matrix of visible rays per voxel	$\mathbf{U} := [U_{k,i}]_{m\times n}$	a matrix representing the visibility of unobstructed r_k for v_i, given a partially obstructing context, whose entries indicate if v_i receives a ray r_k.
Intervisibilities Graph	$\mathbf{G} := [G_{i,j,k}]_{n\times n\times m}$	Directed multigraph of visibility dependency of voxels regarding a particular visibility target represented as a tensor (a stack of matrices), whose dimensions respectively correspond to [obscuring voxels, obscured voxel, vision rays], i.e. $G_{i,j,k}$ will be 1 if a voxel v_i obscures a voxel v_j for receiving a ray r_k, and 0 otherwise.
Visibility Evaluation function	$f(\mathbf{x})$	Function that aggregates the scores in space and in time to result in one performance indicator value (based on $\mathbf{G}, \mathbf{U}, \mathbf{x}$ and $\mathbf{w}$

Some elementary definitions will help make the formulation more comprehensible:

- the design space consists of all the discrete elements i ($i \in [0,n) \subset \mathbb{N}$) of the solar envelope
- the transparency vector $\mathbf{x} = [x_i]_{n\times 1} \in [0,1]^n$ contains all the design variables (occupation status/ opacity of each voxel), and represents one possible envelope scenario
- if $k (k \in \mathbb{N})$ is the minimum number of voxels that need to be kept at the end of the optimization process, then vector $\mathbf{x}$ should contain k number of 1s and $(n-k)$ 0s
- in order to calculate one score for each solution/configuration, resulting from one possible transparency vector $\mathbf{x}_i$, we need to aggregate in time and in space, through an evaluation scheme $f(\mathbf{x})$.

Visibility Evaluation Function

The evaluation method that is developed is based on a vectorized process of data retrieval from a multigraph that represents the intervisibilities of the envelope discrete elements with regards to m rays corresponding to one visibility objective ($\mathbf{G} := [G_{i,j,k}]_{n\times n\times m}$).

The function that is being constructed will work as an estimator of the total visibility potential of a particular configuration, that corresponds to a decision variable $\mathbf{x}_i$, which is a vector of opacities by the size of the number of voxels in the configuration. The total obstruction caused by a certain configuration can be found as follows:

$$\mathbf{G}^{\mathbf{T}}\mathbf{x}$$

This results in a matrix of size $m \times n$ which is filled with integers that indicate the number of times that each voxel is obstructed from each ray because of the other voxels. In this particular situation what matters is just the obstruction of one voxel irrespective of it is blocked by one or more voxels. To account for that the minimum of 1 and every entry of the previously computed matrix is calculated:

$$\min(\mathbf{J}, \mathbf{G}^T\mathbf{x})$$

where $\mathbf{J}$ is a matrix of 1s of the size $m \times n$. By subtracting this array from J the unobscured rays per voxel are revealed:

$$\mathbf{J} - \min(\mathbf{J}, \mathbf{G}^T\mathbf{x})$$

To account for the fact that the context also prevents the visibility of some rays from certain voxels, the Hadamard product of the matrix of unobscured rays $\mathbf{U}$ by the previously defined term:

$$\mathbf{U} \odot \left(\mathbf{J} - \min(\mathbf{J}, \Omega^T\mathbf{x})\right)$$

To also account for the removed/non-existent voxels, the dot product with the transparency vector is performed:

$$\left(\mathbf{U} \odot \left(\mathbf{J} - \min(\mathbf{J}, \Omega^T \mathbf{x})\right)\right) \mathbf{x}$$

Finally, to account for the possible various importances of the rays, the inner product of the above computed vector by the vector of the ray weights is calculated and define the integral visibility function, as a function of the decision variable $\mathbf{x}$:

$$\mathbf{w}^T \left(\mathbf{U} \odot \left(\mathbf{J} - \min(\mathbf{J}, \Omega^T \mathbf{x})\right)\right) \mathbf{x}$$

where $\mathbf{x}$ is the opacity of the voxels and $\mathbf{w}$ the weight of each ray. The visibility evaluation function is formulated as follows:

$$f(\mathbf{x}) = \mathbf{w}^T \left(\mathbf{U} \odot \left(\mathbf{J} - \min(\mathbf{J}, \Omega^T \mathbf{x})\right)\right) \mathbf{x} \tag{4}$$

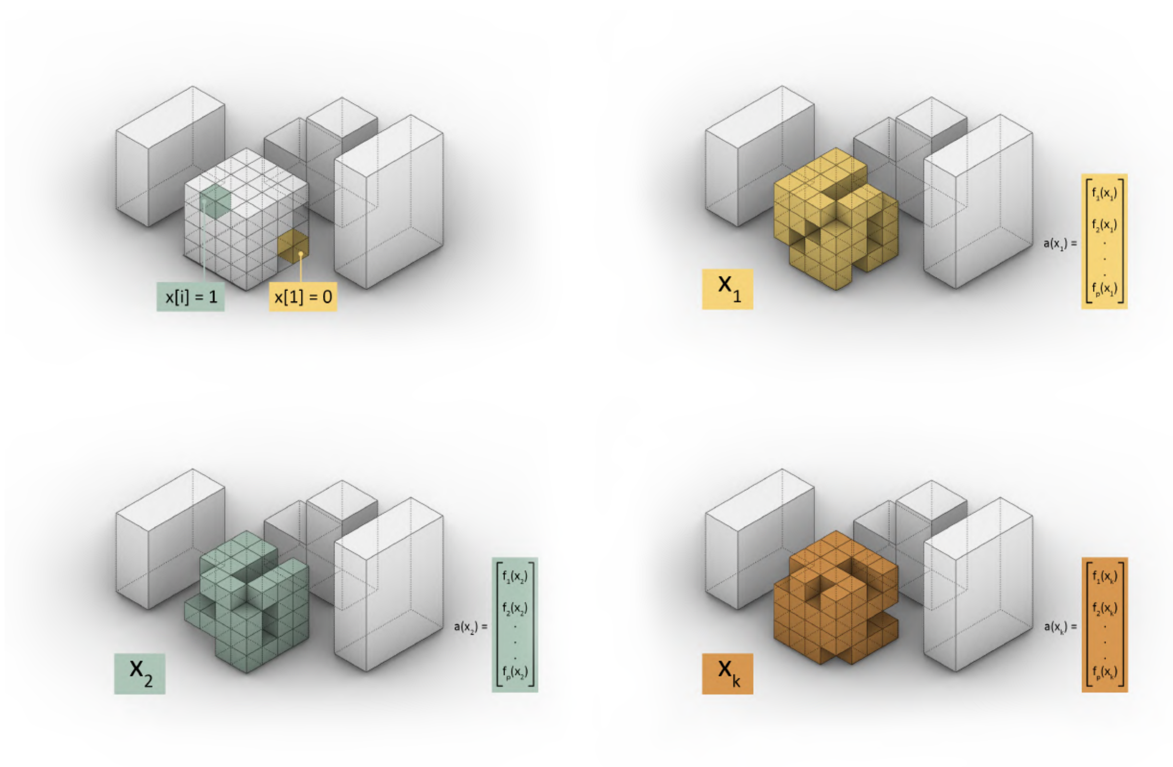

Figure 17: *Starting and three alternative situations - scenarios of the toy problem*

This evaluation scheme can be applied for the calculation of the score of a configuration $\mathbf{x}$ with regards to a performance indicator defined by a ray tracing process towards a visibility target. Hence, it can be applied for the 4 out of 5 criteria included in the objectives of this optimization problem. If only one performance indicator defines the final solar potential, then the problem can be simply expressed as:

"Find the decision vector $\mathbf{x}$ *for which* $f(\mathbf{x})$ *is maximized."*

Nevertheless, if more performance indicators $p(p \in \mathbb{N})$ are considered, then there is the need to aggregate the scores in space per performance indicator, by applying the evaluation scheme for each one of them, resulting in a vector of scores for each solution:

$$a(\mathbf{x}) = [f_1(\mathbf{x}), f_2(\mathbf{x}), \dots f_p(\mathbf{x})]$$

such that, all the possible solutions can be expressed as an array: $[\mathbf{A}]_{k \times p}$. Given that the problem under study refers to the shape optimization of a building, some additional objectives should be established in order to make the solutions feasible. In the context of this research, the relative compactness of the envelope is considered as the most significant one and can be expressed as an additional function $f_c(\mathbf{x})$. In this case, the solution becomes a Pareto optimal (Figure 18).

"A scenario vector $\mathbf{a_x}$ *is considered Pareto optimal if there is not another scenario vector* $\mathbf{a_y}$*, such that* $f_p(\mathbf{a_y}) \geq f_p(\mathbf{a_x})$ *for all performance indicators* p *and* $f_g(\mathbf{a_y}) > f_g(\mathbf{a_x})$ *for at least one index* g*."*

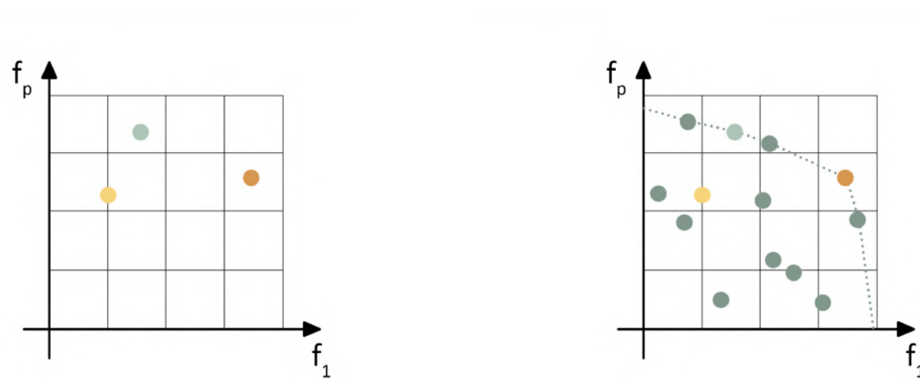

Figure 18: *Hypothetical Pareto front (dashed line) of the toy problem, expanded with imaginary alternatives*

As previously mentioned, all the possible choices of k out of n define all the possible k-combinations as solutions to the problem, and can be expressed through the binomial coefficient (Equation 5).

$$\binom{n}{k} = \frac{n(n-1)\dots(n-k+1)}{k(k-1)\dots 1} \tag{5}$$

The number of feasible solutions for such a problem can be huge, and for this reason a nearly-optimal solution is sought.

3.3.3 Methodology

One of the major drawbacks of generative design applications in architecture is the computational time and power needed for iterative evaluation processes. In the context of this research, a new methodology for evaluating light-related indicators (e.g. skylight, direct sunlight, solar energy potential) is proposed in order to tackle this impediment. This methodology is based on the concept of pre-computing the interdependencies of volumetric elements of a 3D spatial matrix, with regards to visibility objectives (e.g. the sun) and storing the information in form of a weighted directed multi-graph. Based on this strategy, the whole process of shape optimization can be arranged in four main steps: (I) Definition of visibility objectives & Computation of reference vectors, (II) Computation of visibilities & intervisibilities, (III) Definition of performance indicators & their evaluation strategy, (IV) Compute a nearly-optimal envelope.

(I) Definition of visibility objectives & Computation of reference vectors

In the problem formulation of this stage the indicators that define the solar potential, in the context of this research, were defined. From these indicators it becomes evident that the visibility objectives are the sun and the sky. After establishing an analysis period, for which the simulations are going to be performed, and a daily and/or hourly step, the hours of the year (HOYS) of this period are computed and the sun vectors $\mathbf{t_{sun}}$ ($t_{sun} \in m$) are extracted according to them. For the skydome, a simple hemisphere is created and homogeneously subdivided according to a resolution input. The vectors $\mathbf{t_{sky}}$ ($t_{sky} \in o$) for this simple skydome are then computed based on these subdivisions. The sun vectors correspond to a temporal integration while the sky vectors to a spatial integration. These two sets of reference vectors are used in the next step for the creation of the rays array ($[\mathbf{R}_k]_{m\times 1}$).

(II) Computation of visibilities & intervisibilities

The intervisibility networks are constructed one by one with regards to each emitting direction vector, and aggregated per emitter. Rays are constructed with the source of the voxels' centroids and the direction of the vector towards the visibility target and their intersections with each element of the voxelated envelope are stored in the graph. The resulting graph indicates which other voxels, each voxel blocks with regards to each vector and thus it also contains directional information. Algorithm 3 will present the general logic that is followed for the construction of the intervisibilities graph for one set of vectors.

Table 6: *Framework of Algorithm 3: Intervisibility Graph Construction Algorithm*

Input	Data Type	Notes
Array of Indices of Voxels	$\mathbf{v} := [v_i]_{n\times 1}$	an array of all discrete volumetric elements of the rasterized/discretized envelope that define the Solar Envelope
Array of Rays	$\mathbf{r} := [r_k]_{m\times 1}$	Array of all the rays that are supposed to be shot from voxels toward the visibility target
Context mesh	$\mathbf{M}$	The surface mesh representing the context of the building
Output	**Data Type**	**Notes**
Intervisibilities Graph	$\mathbf{G} := [G_{i,j,k}]_{n\times n\times m}$	Directed multigraph of visibility dependency of voxels regarding a particular visibility target represented as a tensor, whose dimensions respectively correspond to [obscuring voxels, obscured voxel, vision rays].
Matrix of visible rays	$\mathbf{U} := [U_{k,i}]_{m\times n}$	a matrix representing the visibility of unobstructed r_k for v_i, given a partially obstructing context.
Problem: Find the graph that describes how visibility of voxels depend on each other in case of each ray.		

Algorithm 3: Visibility Graph Construction Algorithm

```
VisibilityGraphConstruction (V, T):
    foreach voxel v in V do
        ct ← extract centroid of v
        m ← create cuboid mesh representing v
    M′ ← aggregate m over V
    G ← [0]_{n×n×m}
    U ← [0]_{m×n}
    foreach voxel v in V do
        foreach ray r in R do
            V′ ← find voxels in M′ that intersect with (r, v)
            foreach voxel v′ in V′ do
                G[v′, v, r] ← 1
            I ← check intersection of M and ray (r, v) // source of v and direction of r
            if not I then
                U[v, r] ← 1
    return G, U
```

(III) Definition of performance indicators & their evaluation strategy

The evaluation method that is developed is based on the pre-computed reference rays and the intervisibilities graphs that are created according to them. The functions that are developed for the evaluation of each performance indicator require the transparency state of the envelope (expressed through vector $[\mathbf{x}]_{\mathbf{n}\times\mathbf{1}}$) as input.

Direct Normal Irradiation & Direct Normal Illumination

The evaluation scheme (Equation4) already encompasses the vectorized process of computing which rays reach an envelope described by a transparency vector $[\mathbf{x}]_{\mathbf{n}\times\mathbf{1}}$. Based on this information the Direct Normal Irradiation and Illumination can be computed by assigning the corresponding weights to these rays. This results in the following equations:

$$f_{DNIrr}(\mathbf{x}) = \mathbf{w}_{irr}^{T}\left(\mathbf{U}\odot\left(\mathbf{J}-\min(\mathbf{J},\Omega^{T}\mathbf{x})\right)\right)\mathbf{x} \tag{6}$$

$$f_{DNIll}(\mathbf{x}) = \mathbf{w}_{ill}^{T}\left(\mathbf{U}\odot\left(\mathbf{J}-\min(\mathbf{J},\Omega^{T}\mathbf{x})\right)\right)\mathbf{x} \tag{7}$$

As discussed above, vector $\mathbf{w}$ consists of the weights of each computed ray. These weights can refer to literal values, e.g. direct normal radiation and illuminance, that can be extracted through the corresponding EPW file. Nonetheless, the adjustability of this array allows for the assignment of temporal weights accentuating the desired sunlight hours and thus defining the optimization target for each situation.

Direct Skylight

The Direct Skylight factor refer to the amount of rays, corresponding to sky patches, that reach the envelope. This measure serves as an indicator for the amount of the direct skylight that the building receives. In this stage a simple, homogeneously divided skydome is considered. However, different custom skydome models, which are compatible with ray tracing simulations, can be integrated as well. The process for calculating this factor is again based on the evaluation scheme (Equation 4). From its application, the total number of sky rays that reach the envelope is calculated (Equation **??**). The angle of incidence is neglected. While this unit-less measure still serves as an indicator for comparative studies, another factor that can be derived from it is the ratio of the total rays received to the total rays shot initially.

$$f_{DS}(\mathbf{x}) = \mathbf{w}_{sky}^{T}\left(\mathbf{U}\odot\left(\mathbf{J}-\min(\mathbf{J},\Omega^{T}\mathbf{x})\right)\right)\mathbf{x} \tag{8}$$

Relative compactness

While this indicator is not directly linked to the solar potential of the envelope, it affects its energy performance regarding transmission losses and its feasibility as a design solution. This factor is calculated with reference to an idealized cubical configuration, by applying equation 3 discussed in Section 2.3.2. Based on the transparency state of the envelope, the total outer surface area is calculated as well as the total volume of the voxels that constitute the envelope resulting in the following formula:

$$f_{rc}(\mathbf{x}) = \frac{6(V(\mathbf{x}))^{2/3}}{A(\mathbf{x})} \tag{9}$$

(IV) Compute a nearly-optimal envelope

As mentioned above, the most computationally heavy part of the rays-mesh intersections has been performed already in the second step. Furthermore, a vectorized evaluation scheme has been developed, based on which the functions to evaluate the ray-related performance indicators are formulated (Equations 6, 7, 8). In this step, this information is utilized in order to find the decision variable / transparency vector $\mathbf{x}$ which describes an envelope of a nearly-optimal solar potential.

As presented in Section 3.3.2, as the size of the envelope and the level of its discretization increases the number of possible solutions rises exponentially. For this reason, the proposed methods are based on (meta)heuristics. Three such methods are presented as part of this research, namely:

- Iterative Evaluation
- Minimization of objective function
- Cost Index evaluation

Iterative Evaluation

This method is based on the concept of an iterative assessment of the effect of removing each one of the voxels from the transparency vector $[\mathbf{x_i}]_{\mathbf{n}\times\mathbf{1}}$, in the final solar potential of the resulting building shape. One iteration of this process can be described through the following steps:

Algorithm 4: Evaluation process - Single iteration outline Algorithm

```
EvaluationProcess-SingleIteration (F(x), x):
    S ← [0]_{n,p} // initialize scenarios matrix (voxels x criteria)
    foreach voxel v in x do
        x' ← copy x
        x'[v] ← 0 // remove from transparency vector
        foreach criterion p do
            S[v, p] ← F(x) // apply the evaluation scheme for every ray-related criterion
        z ← worst performing voxel according to S // apply MCDA
        x[z] ← 0 // remove from transparency vector
    return x
```

This process is repeated until all the needed voxels are removed. Although all the iterations are based on the update of the intervisibilities graphs, which have already been computed, it can still be computationally heavy for really big design domains-envelopes. In this case, some additional filters can be integrated with the aim of reducing the amount of iterations. The first filter concerns the detection of only the outer voxels of the envelope and, hence, the evaluation of the effect of removing only voxels from this subset. This filter reduces the computation time and may result in more compact envelopes but it does not allow for the creation of more complex spaces like atria. The comparison of these two alternatives is discussed in more detail in Section 4.2.2. The second strategy concerns the number of voxels that are removed per iteration. This study investigates if and how many more voxels can be removed per iteration without compromising the accuracy of the evaluation process (Section 4.2.3).

The result of the evaluation process is the transparency vector of the envelope with a nearly optimal solar potential. By simply reshaping this transparency vector according to the shape of the initially fed solar envelope lattice, the final massing can be derived. Undoubtedly, there is not only one optimal solution for this problem, and hence the specification of a "nearly" optimal result. The application of different MCDA methods and the assignment of different weights on the criteria, will yield different results (examined in Section 4.2.4). A more generalized version of this process is presented through Algorithm 5.

Table 7: *Framework of Algorithm 5: Envelope Generation Algorithm*

Input	Data Type	Notes
$\mathbf{V}$ Array of voxels	$\mathbf{v} := [v_i]_{n\times 1}$	Array of voxels that define the solar envelope
$\mathbf{G}$	$[g_{i,j,k}]_{n\times n\times m}$	Directed multigraph of visibility dependency of voxels
$\mathbf{U}$	$[u_{k,i}]_{m\times n}$	Matrix representing the unobscured rays per voxel
$\mathbf{z}$	$[z_h]_{o\times 1}$	Vector of total area per zone (spatial function) according to PoR
$\mathbf{w}$	$[w_k]_{m\times 1}$	Vector of weights corresponding to each ray
$\mathbf{p}$	p [int]	Number of performance criteria
Output	**Data Type**	**Notes**
$\mathbf{E}$	$[e_i]_{n\times 1}$	Array of voxels of the final chosen envelope (Massing)
Problem: Remove the worst performing voxels according to the defined performance criteria.		

Algorithm 5: Envelope Generation - Iterative Evaluation Algorithm

```
MassingIterativeEvaluation (V, G, U, z, w, p):

    A_tot ← sum values of z // calculate total area needed
    u ← unit of Solar Envelope lattice // voxel's edge length
    v_vol ← u^3 // voxel's volume
    // calculate number of voxels that need to be removed
    to_remove ← V.size − ⌈A_tot / v_vol⌉

    x ← create transparency vector [x]_{n×1} from envelope lattice V

    removed ← 0 // number of removed voxels

    // remove voxels until all needed voxels are removed
    while removed < to_remove do

        // initialize scenarios matrix
        S ← [0]_{n×p} // new scenario when a new voxel is temporarily removed
        in_ids ← indices of in-envelope voxels (where x = 1)

        // iterate through the indices of the voxels which are still in the envelope
        foreach index i in in_ids do

            x_i ← copy transparency vector x
            x_i[i] ← 0 // temporarily remove i from envelope

            // what is the effect of temporarily removing i in the aggregated indicators
            foreach indicator p in P do
                S[i, p] ← f(x)

            w_id ← apply MCDA to scenarios matrix S // find worst performing voxel
            x[w_id] ← 0 // remove worst performing voxel

            removed ← +1
    E ← reshape x according to envelope lattice shape

    return E
```

Minimization of objective function

This method is based on the utilization of already established optimization algorithms that aim in the minimization of an objective function. The problem under study is subject to some constraints and the values that the decision variable can take are binary. For these reasons, a Bound-Constrained minimization method is more suitable. For the implementation of this stage the SciPy library for Python is used [40].

The exact method that was chosen is the "trust-constr", a trust-region algorithm which, according to the corresponding documentation: *"is the most versatile constrained minimization algorithm implemented in SciPy and the most appropriate for large-scale problems"*. Since the posed problem is an equality constrained problem, the implementation of the algorithm that is used is based on the Byrd-Omojokun Trust-Region SQP method. [41].

The developed Visibility Evaluation Function (Equation 4) is used for the formulation of the objective function. Since the goal is to minimize this function, the opposite of the scalar product is used ($-1f(\mathbf{x})$). The constraints concern the architectural program and the minimum area needed and, as a consequence, the minimum amount of voxels that need to remain in the configuration after the optimization process. A maximum number of iterations can also be defined. The effect of this limit on the execution time and the final score of the envelope is presented in Section XX. The data that has been given as input for the algorithm can be seen in table 8.

Table 8: *Optimization - minimization algorithm input*

Input	**Formula**
Objective function	$f(\mathbf{x}) = \mathbf{w}^T\left(\mathbf{U} \odot \left(\mathbf{J} - \min(\mathbf{J}, \Omega^T\mathbf{x})\right)\right)\mathbf{x}$
Constraints	$c(\mathbf{x}) = sum(\mathbf{x}) \leqslant min\_voxels$
Bounds	[0,1]

The outcome of the optimization process is the decision variable $\mathbf{x}$ that is considered nearly-optimal, according to the algorithm used. Subsequently, this vector $\mathbf{x}$ can be used to generate the corresponding envelope.

Cost Index evaluation

This method is based on the removal of voxels from a configuration according to their cost index. The cost index is a numerical value that shows how "costly" or "annoying" is every voxel for the configuration and is calculated based on the information provided by the pre-computed arrays of visible rays ($[\mathbf{U}]_{m\times n}$) and intervisibilities ($[\mathbf{G}]_{n\times n\times m}$).

The development of a formula that would calculate this cost is based on the computation of two factors. First one is the Obscuring index, which expresses the visibility potential that one voxel prevents from the others and is calculated as follows:

$$Obscuring\_index = [\mathbf{G}^T_{k,i,j}]_{m\times n\times n\times n}[\mathbf{x}_i]_{n\times 1}$$

This results in a matrix of size $m \times n$ whose entries represent the amount of times a voxel prevent as ray from reaching the other voxels. The second factor is the Obscured index, which expresses the visibility potential that is denied from a voxel because of the other voxels and is calculated as follows:

$$Obscured\_index = [\mathbf{G}^T_{k,j,i}]_{m\times n\times n\times n}[\mathbf{x}_i]_{n\times 1}$$

This results in a matrix of size $m \times n$ whose entries represent the amount of times each ray is obscured from a voxel because of the other voxels.

A voxel is considered costly for a configuration when it obstructs a great amount of rays from the rest of the voxels (high obscuring index) while not being much obscured itself. This is expressed through the subtraction:

$$Obscuring\_index - Obscured\_index = [\mathbf{G}^T_{k,i,j}]\mathbf{x} - [\mathbf{G}^T_{k,j,i}]\mathbf{x}$$

To account for the partially obscuring context, the Hadamard product with the visible rays matrix is performed:

$$\left([\mathbf{G}^T_{k,i,j}]\mathbf{x} - [\mathbf{G}^T_{k,j,i}]\mathbf{x}\right) \odot \mathbf{U}$$

Finally the dot product of this matrix with the weights of the rays (aggregation over columns) result in a vector of size $1 \times n$ that consists of the cost index value of every voxel:

$$C(\mathbf{x}) = \mathbf{w}^T \left([\mathbf{G}^T_{k,i,j}]\mathbf{x} - [\mathbf{G}^T_{k,j,i}]\mathbf{x}\right) \odot \mathbf{U} \tag{10}$$

After having calculated the cost index of every voxel and the number of voxels that need to be removed (v), the first v voxels with the highest cost index value are removed from the configuration. If the performance indicators are defined by more than one visibility targets then a different cost index value is calculated for each indicator and an MCDA method is integrated in the removal process.

4 Verification, Validation & Benchmarking

In a traditional sense, the verification and validation processes refer to an application of the model under study in real-world examples. In the context of this research, the more technical verification of the proposed methodology is going to be applied in the already presented toy problem, to ensure that the model provides accurate results. After securing this part, the validation is going to be performed by making use of the model on a case study (Section 5), to make sure that it meets the needs of the focused user and provides reasonable and usable outputs.

4.1 Visibility Evaluation Function

The backbone of the proposed methodology is the introduction of directed multigraphs, that capture the intervisibilities of elements in 3D space with regards to a visibility objective, as a faster and more effective way to simulate light-related indicators. The methodology of creating these graphs was presented in detail in Section 3.3.3. It is argued that these graphs can be used to retrieve data and calculate performance indicators with simple matrix computations, without the need to re-perform exhaustive ray-mesh intersections.

In order to verify the results of this process these steps were followed:

- three envelope scenarios of different discretization degrees (64, 150 and 343 voxels) were generated according to three randomly produced transparency vectors (x_1, x_2, x_3) and placed in different location and urban contexts.
- for each envelope the visible rays matrix ($\mathbf{U}$) and the intervisibilities graph ($\mathbf{G}$) was computed
- for each of the envelopes two process were executed:
  Process 1
  - compute ray-mesh intersections
  - assign weights (w) to rays
  - aggregate scores in time and in space to reach a final score $(s1_i, i \in \{1,2,3\})$

  Process 2
  - the evaluation scheme (Equation 4) was applied with the input of $\mathbf{G}$ and the same weights $\mathbf{w}$ for the rays, to compute the final score $(s2_i, i \in \{1,2,3\})$

At the end of this process two things were compared: the deviation among the two scores $s1$ and $s2$ per envelope and the simulation time of the two methods ($t1$ and $t2$). The results are illustrated in Table 9.

Table 9: *Visibility Evaluation Function verification results*

Envelope	Voxels number	s1i - s2i	t1 [ms]	t2 [ms]
1	64	0	1320	19
2	150	0	8240	125
3	343	0	40400	202

The outcome of this verification process indicate that the application of the evaluation scheme gives the exact same results as the intersection simulations for all the cases ($s1_i - s2_i = 0, for i \in \{1,2,3\}$). Moreover, from the comparison of the average execution time of the two processes it arises that the proposed methodology is notably faster than the conventional one and the difference becomes even more significant as the number of voxels that form the envelope increases.

4.2 Method 1: Iterative Evaluation

In the fourth step of Section 3.3.3 the iterative evaluation process was presented. In order to verify this method the process is going to be initially broken down into smaller sub-process and their results are going to be discussed. Additionally, the two strategies for reducing the code run time are going to be studied and finally the effect of changing the MCDA method that is involved in the process is going to be presented.

4.2.1 Method break down

This iterative evaluation method is based on the creation of a Scenarios matrix that captures the scores of each alternative (removal of a different voxel) with regards to some performance indicators. This matrix of alternatives is processed through an MCDA method, from which the best solution is decided.

In order to verify this process and ensure that it gives nearly-optimal results, a simplified scenario has been developed. A toy problem of 3x3x3 voxelated envelope is modelled within the same context as the previous toy problems (Figure 19). The proposed methodology is followed to construct the intervisibilities graph. To simplify the case even more, only the sun is considered as a visibility objective and the performance criteria that will define the configuration score are the total direct normal irradiation that reaches the envelope (Equation 4), as well as, its relative compactness (Equation 9). The algorithm will be applied for the removal of five voxels. At the end of each iteration the score of each alternative will be plotted in order to spot and assess the one selected by the MCDA process. The Weighted Product method was chosen for this study.

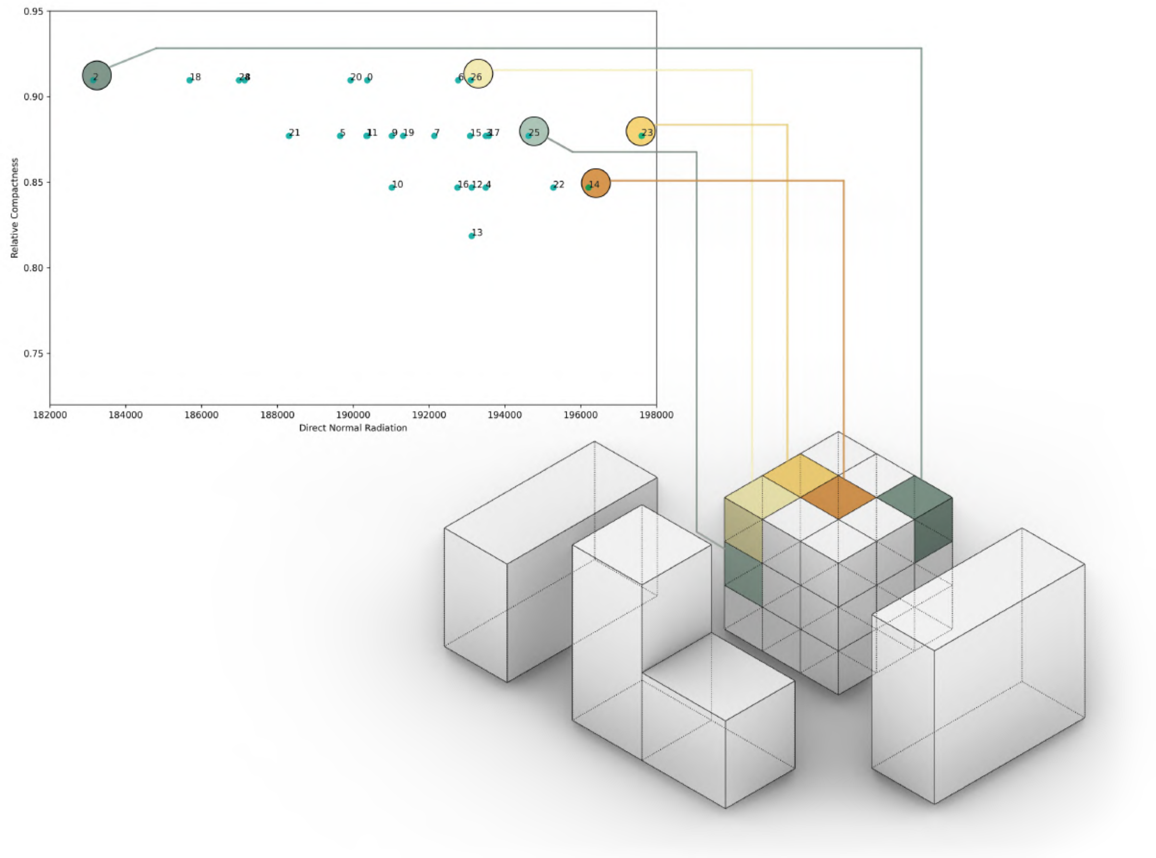

Figure 19: *Plot of alternatives after the first iteration.*

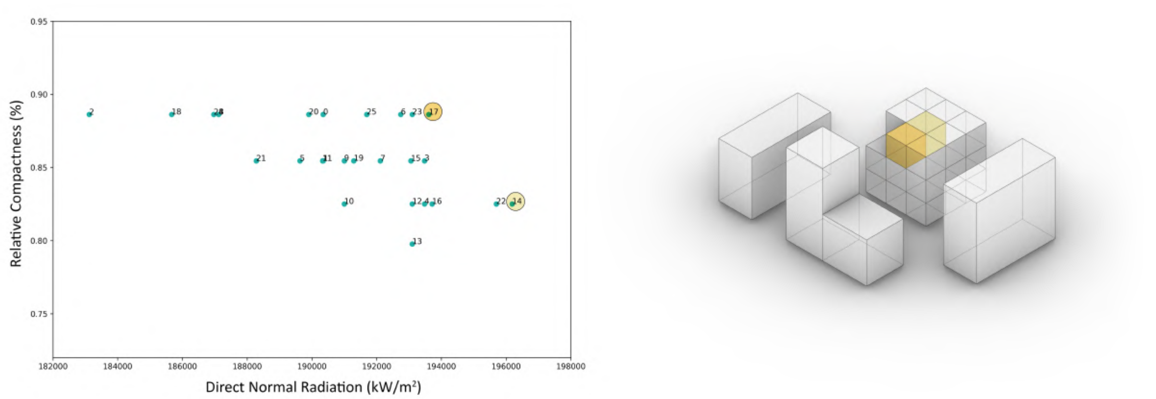

Figure 20: *Plot of alternatives after the second iteration. The dark yellow voxel is the one that was chosen by the MCDA process to be removed.*

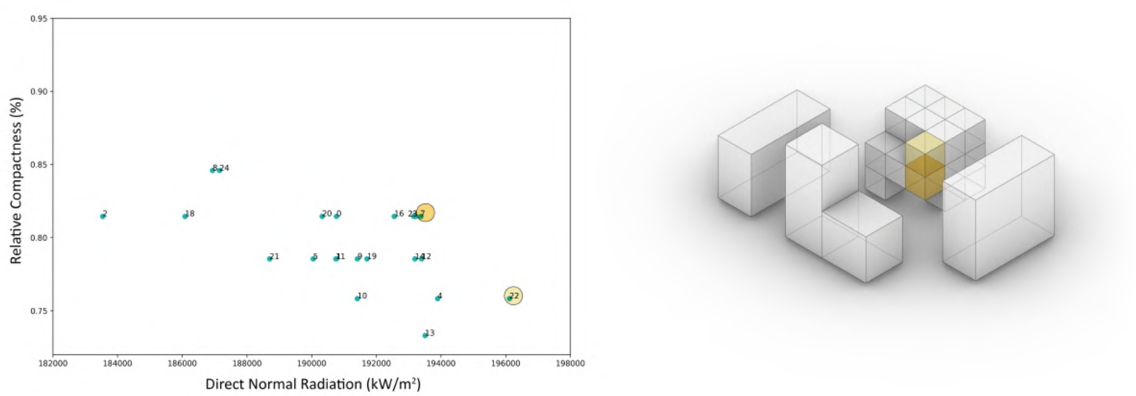

Figure 21: *Plot of alternatives after the last iteration. The dark yellow voxel is the one that was chosen by the MCDA process to be removed*

When plotting and comparing all the alternatives after each of the five iterations it can be observed, that the voxel that is chosen to be removed by the model is always part of the Pareto front of the alternatives. The same procedure was repeated for different MCDA methods as well (MOORA and TOPSIS). In some cases, an alternative voxel was chosen but it was always a Pareto optimal.This fact verifies the effectiveness and the ability of the iterative removal process, to yield nearly optimal results. For brevity purposes only the first method is presented in this section. The diagrammatic representation of all the iterations can be be found in Appendix A.2.

The same process was followed for an envelope of a higher discretization level and for three performance indicators. The iterative evaluation is broken down again into sub-processes, each of which will tackle the application of the iterative process only for one performance indicator, in order to study how each one affects the final result.

The toy problem / configuration has the following setup (Figure 22):

- Discretization: 64 voxels
- Location: Amsterdam
- Voxels to be removed: 10
- Performance indicators:
  - Direct Normal Irradiation
  - Direct Skylight
  - Relative Compactness

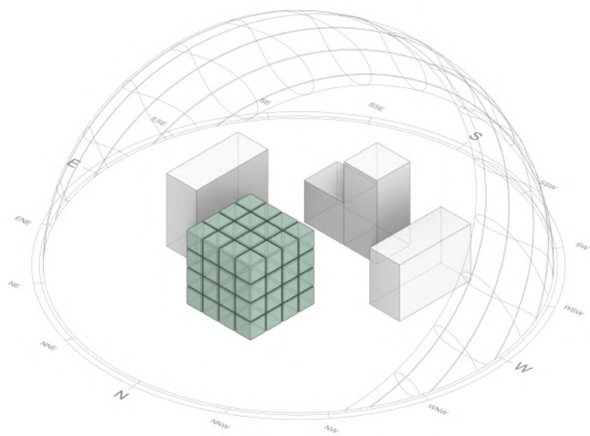

Figure 22: *Toy Problem set up*

The following figures illustrate the effect that the removal of each one of the voxels has in each one of the indicators in three stages: the first iteration, the fifth and the last (tenth) one. The ranking of the voxels after the application of MCDA for all the indicators is visualized in the fourth column.

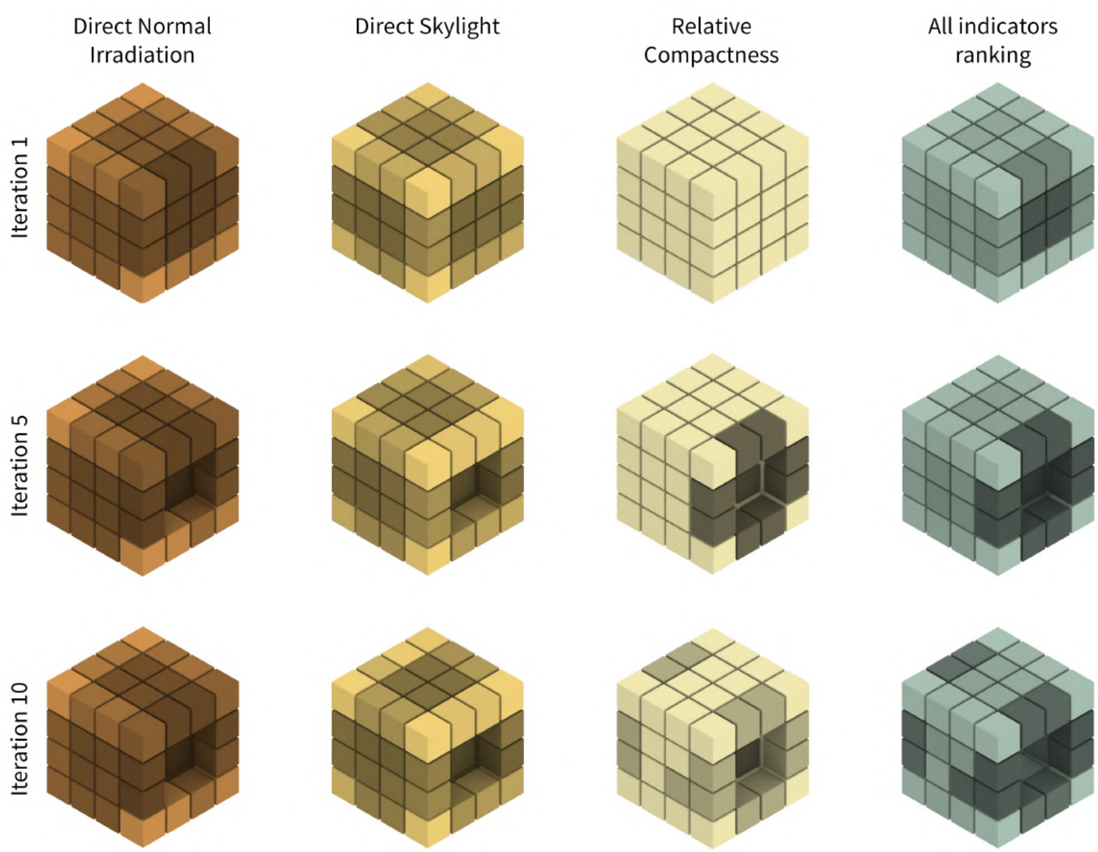

Figure 23: *Iterative evaluation stages: Visualization of voxel ranking per performance indicator*

The following charts show the score per indicator of the envelope that is resulting if each voxel is removed individually. The removed one is highlighted. For the first four iterations the position of the removed voxel is also illustrated. The charts for the rest of the iterations can be found in Appendix A.2.

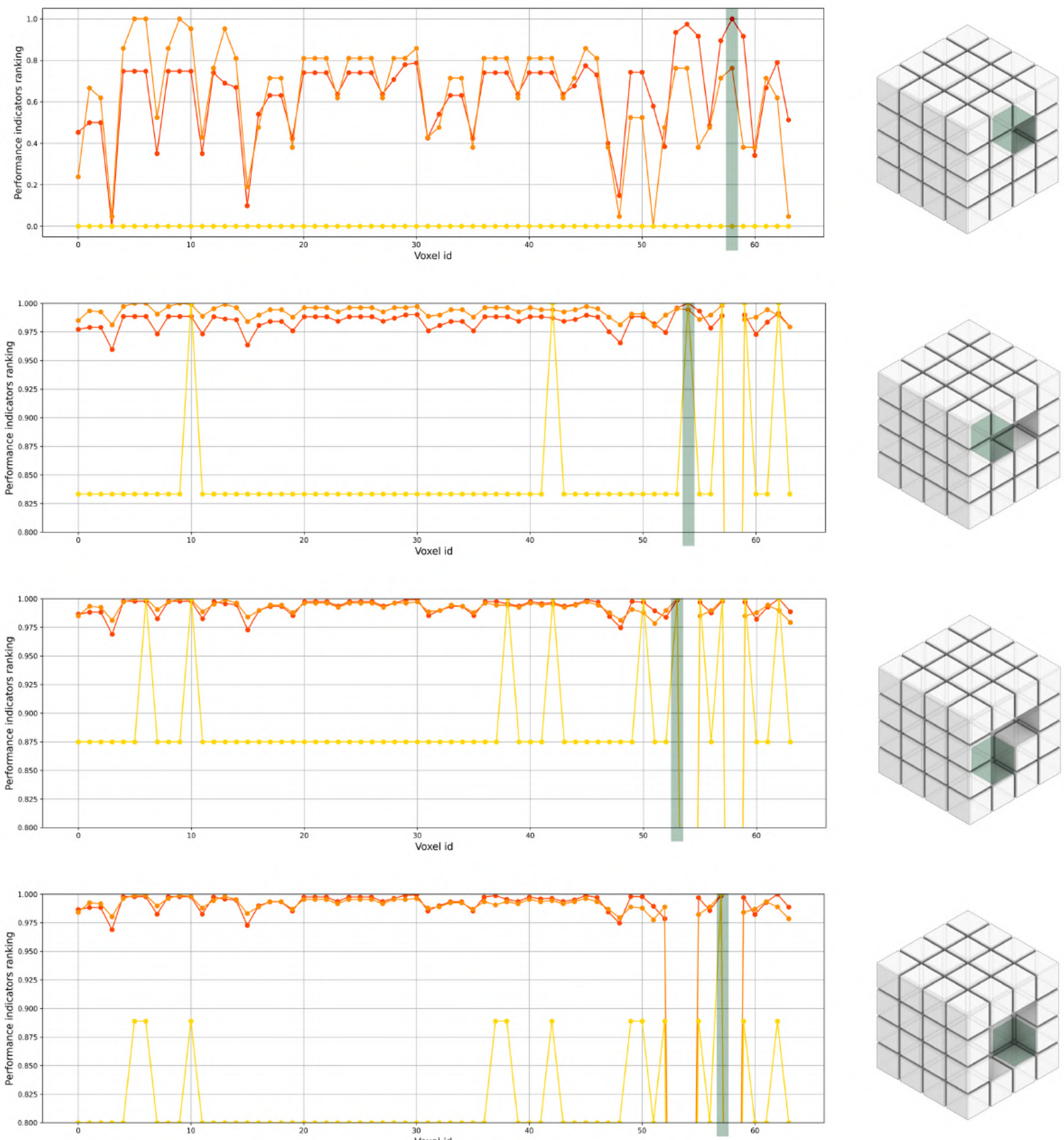

Figure 24: *Plotted scores for the first 4 iterations (red: DNI, orange: DS, yellow: RC)*

An additional verification process includes the application of the iterative evaluation process for each one of the three performance indicators and the comparison of the resulting envelope with the one that combined process produces with the use of MCDA.

For this comparison the above presented Toy Problem was used and the MCDA method that was chosen is the Weighted Product. The results are illustrated in the following figures and the comparison of the envelopes per performance indicator are depicted in Table 10.

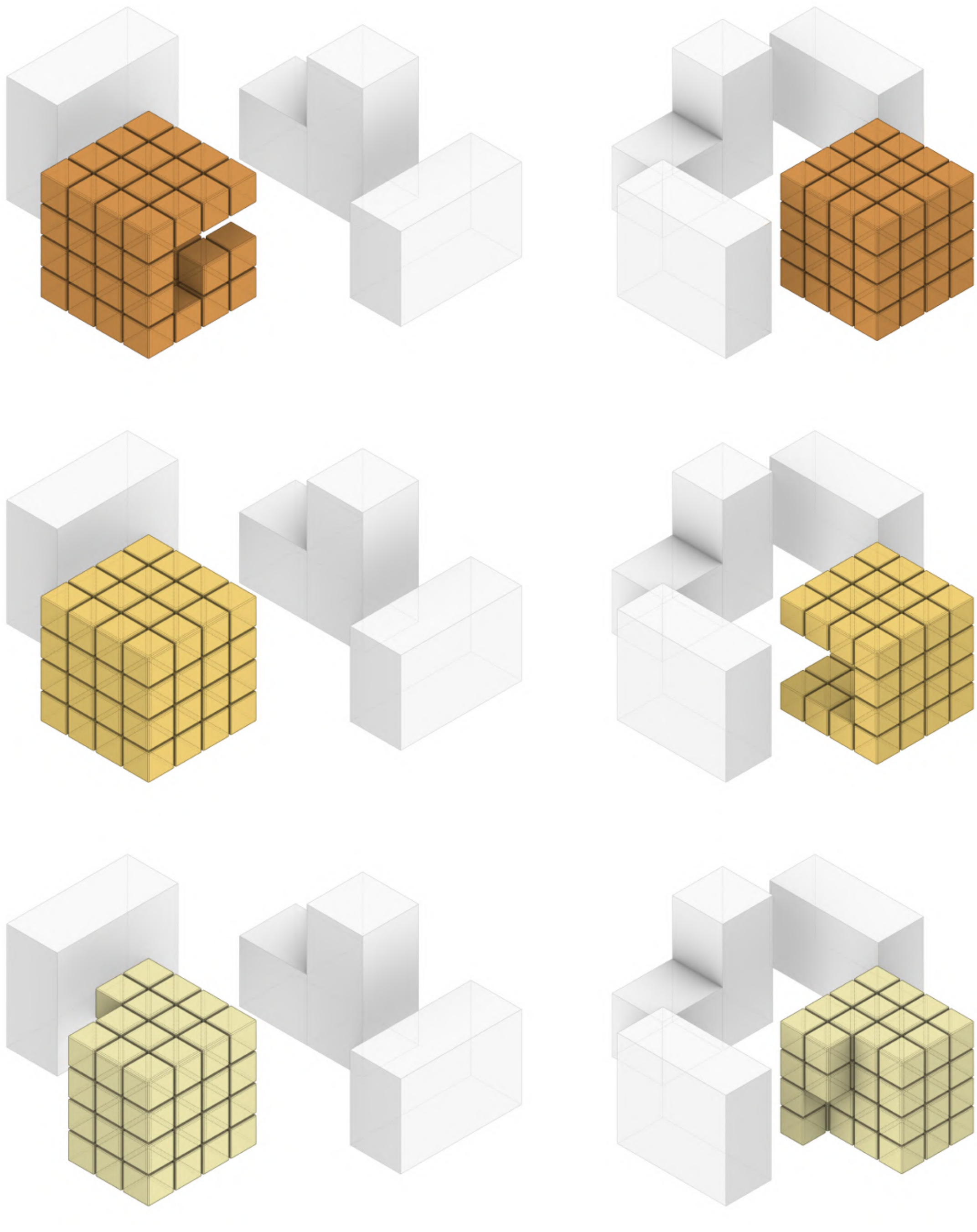

Figure 25: *Resulting envelopes if only one performance indicator is taken into account. From top to bottom: DNI, DS, RC.*

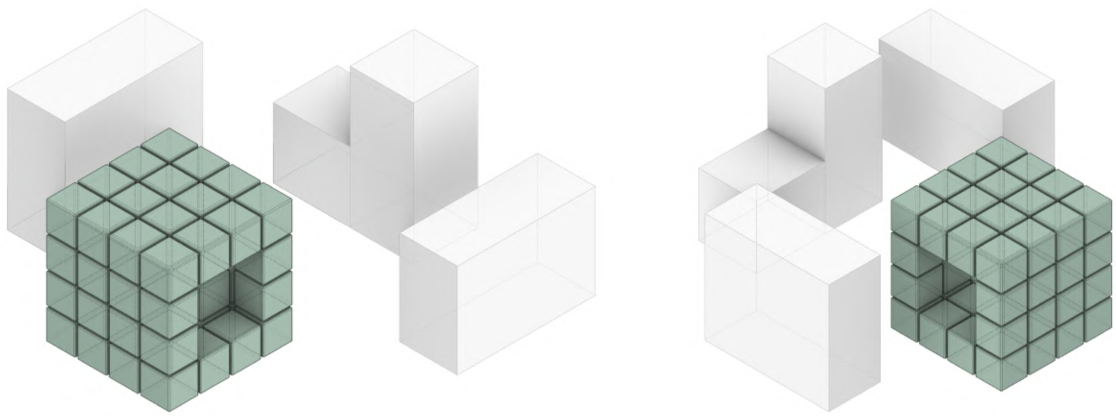

Figure 26: *Resulting envelope when all the performance indicators are taken into account.*

Table 10: *Envelopes scores per indicator*

-	DNI (kWh/m2)	DS (sky rays)	RC (%)
Envelope 1 (according to DNI)	390	1050	52
Envelope 2 (according to DS)	380	1090	65
Envelope 3 (according to RC)	340	990	75
Envelope final (all indicators)	390	1080	67

From Table 10 it can be observed that the envelopes that is generated when all the performance indicators are taken into account has the most balanced performance. It achieves a high solar potential, while maintaining its compactness. After breaking down the process in all its components and observing each one of it separately and their combination, this second additional study verifies the results of the first simplified situation. It also facilitates the comprehension of the stages of the iterative evaluation process.

4.2.2 Removal strategy - Outer voxels detection

In the fourth step of Section 3.3.3 the evaluation process is presented. In that part, the integration of a filter in the iterations is discussed as a possible solution to the high computational demand of the process. This filtering refers to the identification of the outer layer of voxels, in each iteration, in order to reduce the required execution time. The concern that is being raised is the possibility of this filtering to reduce the morphological freedom of the process outcome, as it will not allow for the creation of atria-like spaces. These two approaches will be compared on the basis of their computation time demand, the resulting shape of the envelope and their scores per performance indicator. The comparative study is going to be conducted for three different MCDA methods, namely MOORA, TOPSIS and Weighted Product. The above-presented toy problem of the 5x5x5 full voxelated envelope, with 38 voxels to be removed, is going to be used.

Table 11: *Comparison of the two removal strategies (Situation 1 - MOORA method)*

Situation 1- MOORA method						
	Direct Normal Irradiation (kWh/m2)	Direct Normal Illumination (klx h)	Skyview factor (%)	Relative Compactness (%)	Runtime (Sec)	Common indices
All voxels	529,0	49824	16,0	73,4	159,3	37/38
Outer voxels	531,9	50100	16,0	73,4	141,0	

Table 12: *Comparison of the two removal strategies (Situation 1 - TOPSIS method)*

Situation 1 - TOPSIS method						
	Direct Normal Irradiation (kWh/m2)	Direct Normal Illumination (klx h)	Skyview factor (%)	Relative Compactness (%)	Runtime (Sec)	Common indices
All voxels	515,3	48711	15,4	78,7	201,8	38/38
Outer voxels	515,3	48711	15,4	78,7	158,2	

Table 13: *Comparison of the two removal strategies (Situation 1 - Weighted Product method)*

Situation 1 - Weighted Product method						
	Direct Normal Irradiation (kWh/m2)	Direct Normal Illumination (klx h)	Skyview factor (%)	Relative Compactness (%)	Runtime (Sec)	Common indices
All voxels	584,7	55018	16,3	59,2	208,1	38/38
Outer voxels	584,7	55018	16,3	59,2	171,3	

As it can be seen from the the results through Tables 11 - 13, the two strategies lead to almost identical results. For the MOORA method, only one different voxel is removed, while for the other two the exact same indices are selected by the model. As expected, the computation time is reduced in all three examples.

The same comparative study is repeated with an envelope of 7x7x7 in the same geographic location but with a different urban context (Figure 27). Only the MOORA method is applied in this study, since it was the only method that showed deviation in the results. The resulting scores from this process can be seen in Table 14.

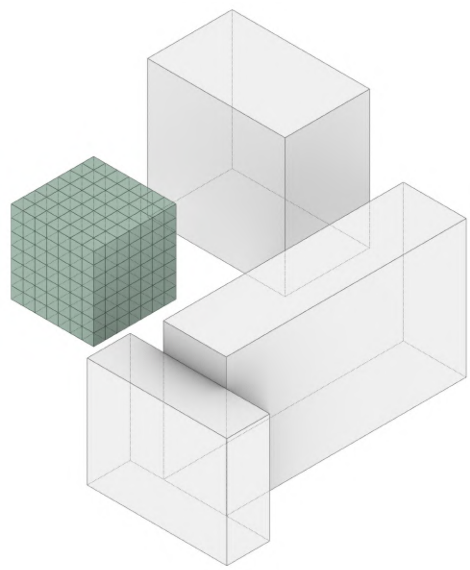

Figure 27: *Toy Problem set up - Situation 2*

Table 14: *Comparison of the two removal strategies (Situation 2 - MOORA method)*

Situation 2 - MOORA method						
	Direct Normal Irradiation (kWh/m2)	Direct Normal Illumination (klx h)	Skyview factor (%)	Relative Compactness (%)	Runtime (Sec)	Common indices
All voxels	547	51765	9,8	80,3	13287	103/103
Outer voxels	547	51765	9,8	80,3	7838	

In this case the exact same voxels are removed. The reduction in runtime is even more significant and it is predicted to be even greater for bigger envelopes. Undeniably, more exhaustive analysis through different examples, is needed to confirm these outcomes. However, these preliminary results reveal stimulating possibilities for the effectiveness of the integration of this alternative in the removal process.

4.2.3 Removal strategy - Step Integration

Another strategy that is studied as a possible solution to the high computational demand of the removal process, is the integration of a step per iteration. According to this method, after finishing one evaluation of the effect of the removal of each voxel and the application of MCDA, more than one voxels are removed. In order to verify this strategy, different steps were tested.

The setup of the toy problem for this verification is as follows:

- urban context: same as in the previously presented toy problems
- cubic voxelated envelope of 7x7x7
- performance criteria: total direct normal irradiation, relative compactness
- all criteria are equally weighted (0,5 each)
- TOPSIS method was used for the decision making process
- the steps that were tested are: 2, 3, 5 and 10 voxels per iteration

The voxelated envelope that was chosen is of higher resolution than the previous verification processes, as it was considered necessary to reach more accurate conclusions.

The comparison criteria are:

- Direct Normal Irradiation reaching the voxel
- Relative Compactness of the resulting envelope
- Runtime

The results of this verification process are presented in Table 15.

Table 15: *Comparative matrix of different number of steps*

Step	Direct Normal Irradiation (kWh/m2)	Relative Compactness (%)	Runtime (Sec)	Common removed voxels
1	1060,0	75,8	5316	-
2	1060,0	75,8	2096	103/103
3	1060,2	75,8	1728	100/103
5	1046,9	76,3	791	91/103
10	1015,8	78,5	419	65/103

As it can be observed by the tabular data and the plots of Figure 28, the integration of the step strategy reduces the runtime considerably. In the situations where one, two or three voxels are removed per iteration, the resulting envelopes have almost identical scores. The code runtime when the step is equal to 3 is approximately three times less than the one of the initial model. A noticeable fact is that as the step number increases, envelopes with higher relative compactness but lower solar potential are achieved. The same verification process was repeated for three different resolutions of the same envelope. Through this study a noticeable fact was that as the number of voxels increased, the step number that led to almost identical results also increased. Although, further simulations with different configurations are necessary to reach more concrete results, this initial study allows for the assumption that the integration of a step that is less than 1% of the total number of voxels, would still lead to nearly optimal envelopes, while significantly reducing the run time.

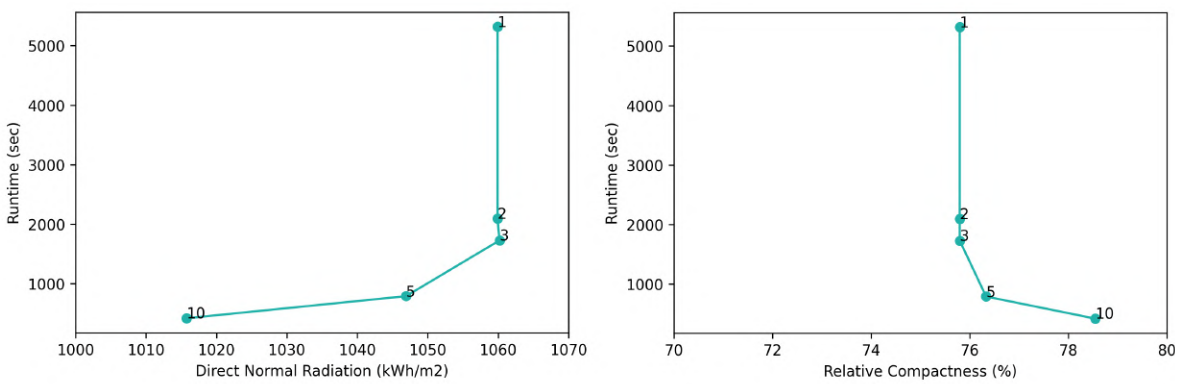

Figure 28: *Plots of the scores to the runtime per step integration*

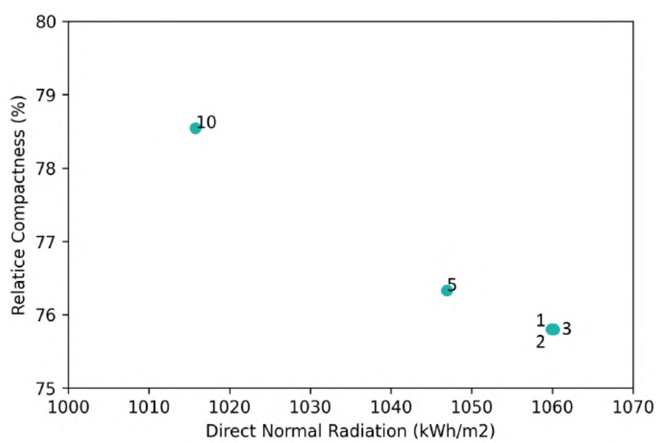

Figure 29: *Plots of the scores resulting from different step numbers*

4.2.4 MCDA methods comparison

A significant part of the iterative removal process, which leads to the final nearly-optimal shape, is the application of MCDA to decide which voxels to be removed from the initial envelope. Undoubtedly, the utilization of different methods and different criteria weights will lead to different results. Although the scope of this research does not include the investigation and suggestion of a single method, a preliminary comparative study has been done and will be briefly presented in this section. This study is focusing on the effect of the application of different methods on the final scores for each criterion. For each selected method, a small introduction is given and the resulting envelope is presented. In the last part, the results of the different methods are compared through some graphs. For the application of all the methods the library “scikit criteria” [42] for Python was used.

The methods that are going to be compared are:
- TOPSIS
- Weighted Product
- MOORA

Naturally, the toy problem set up is the same for each piece of code, that is:
- urban context: same as in the previously presented toy problems
- cubic voxelated envelope of 5x5x5
- performance criteria:
  - total direct normal irradiation reaching the envelope
  - total direct normal illumination reaching the envelope
  - skyview factor
  - relative compactness
- all criteria are equally weighted (0.25 each)

(I) TOPSIS

TOPSIS refers to the "Technique for Order of Preference by Similarity to Ideal Solution" and is based on the concept that the chosen alternative should have the shortest geometric distance from the positive ideal solution and the longest euclidean distance from the worst solution [42]. TOPSIS is a method of compensatory aggregation, which means that it allows for trade-offs between criteria.

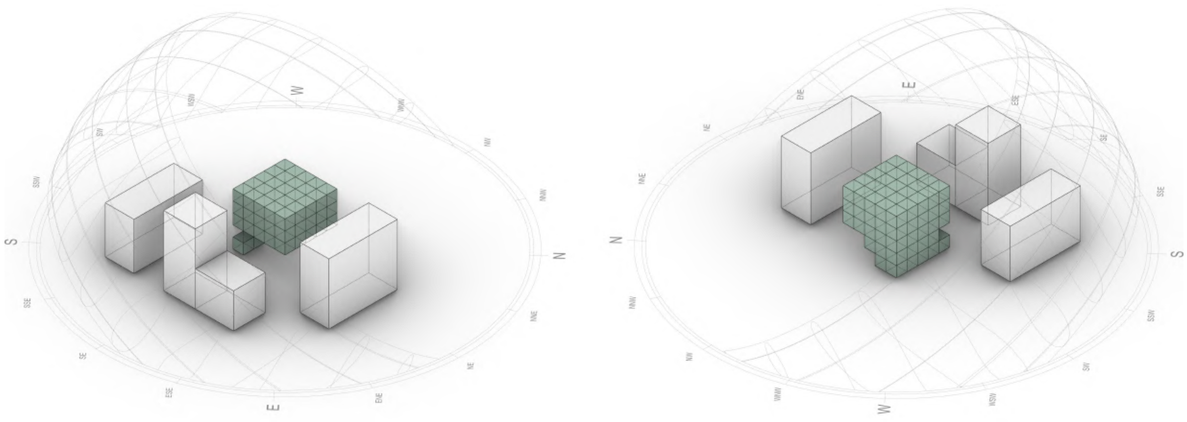

Figure 30: *TOPSIS envelope result*

(II) Weighted Product

The weighted product model (WPM) is one of the most popular multi-criteria decision analysis methods. The process is based on a comparison of decision alternatives by multiplying a number of ratios, one for each criterion. It is similar to the weighted sum model but it can additionally be used for multi-dimensional problems. That is, problems were the criteria that defined the alternatives are defined through different units. This is why it is considered more appropriate for this case [43].

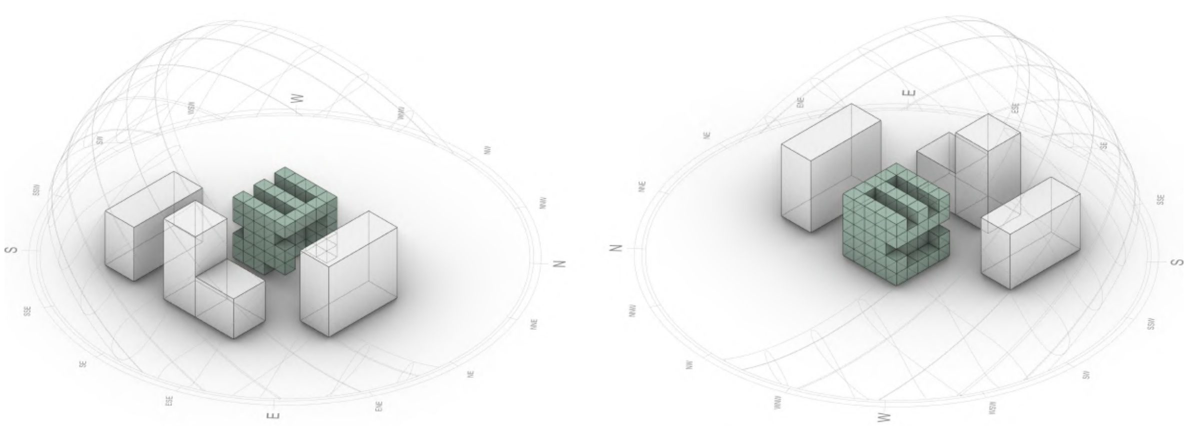

Figure 31: *Weighted Product envelope result*

(III) MOORA

MOORA refers to a family of multi-objective optimization methods on the basis of ratio analysis. In this particular case, the RefPointMOORA method was chosen, which ranks the alternatives from a reference point selected with the Min-Max Metric of Tchebycheff [42].

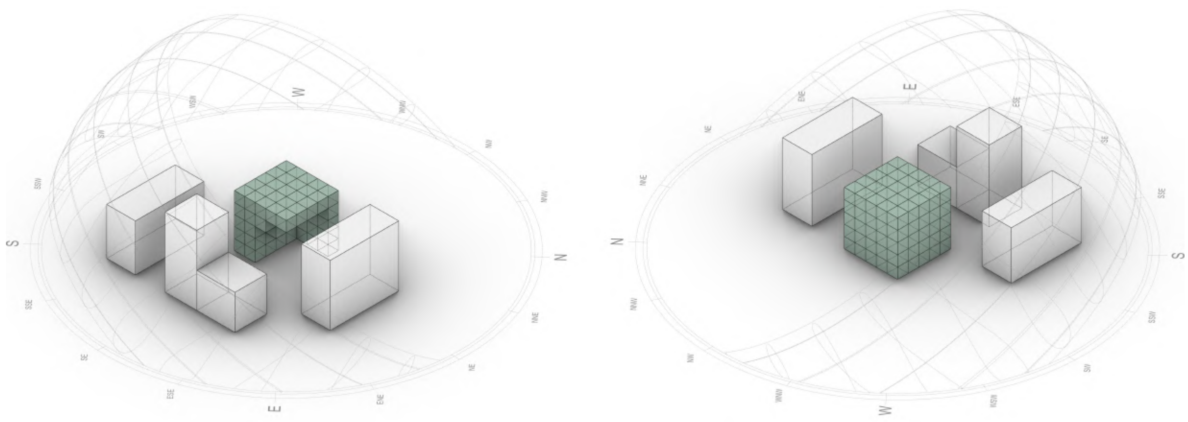

Figure 32: *MOORA envelope result*

Results

The computed scores of each envelope per performance indicator are provided in Table 16. When comparing the tabular data, it can be seen that, while the envelope resulting from the Weighted Product model has the highest potential of solar irradiation and illumination, it also has the lowest relative compactness. This fact is also evident for the 3D visualization of this envelope (Figure 31). On the other hand, the TOPSIS envelope reaches a notably high relative compactness score but has the least potential. The MOORA method appears to be balancing better these factors. As far as the computation time and the skyview factor are concerned, no big deviations are observed.

Table 16: *Computed scores per indicator for each applied MCDA method*

Method	Direct Normal Irradiation (kWh/m2)	Direct Normal Illumination (klx h)	Skyview factor (%)	Relative Compactness (%)	Runtime (sec)
TOPSIS	515,3	48711	15,4	78,7	229,46
WP	584,7	55018	16,3	59,2	226,14
MOORA	529,0	49824	16,1	73,4	234,52

Although this preliminary comparative study allows for some initial deductions, it is still quite limited to be used for definite conclusions. A research on the effect of the criteria weights on the output would also be beneficial. Based on the aforementioned observations, MOORA is going to be initially applied to the case study. If the timeline allows for it or if proven necessary, also the weighted product method will be used after adjusting the criteria weights to ensure a compact envelope.

4.3 Method 2: Objective function minimization

4.3.1 Maximum iterations and optimality

This Section studies the effect of the maximum iterations number in the optimization process. To investigate the effect of this parameter in the product of the process only the DNI indicator will be taken into account and will serve as the objective function for optimization (Equation 11). The toy problem that is going to be used is the one that consists of 150 voxels. The optimization method that is being selected from the ones available by the SciPy library is the 'trust-constr' minimization one.

$$f_(\mathbf{x}) = \mathbf{w}_{irr}^{T}\left(\mathbf{U} \odot \left(\mathbf{J} - \min(\mathbf{J}, \Omega^{T}\mathbf{x})\right)\right)\mathbf{x} \tag{11}$$

The following table shows the run time for each occasion and the resulting score of the objective function, as well as the percentage of how close this score is to the maximum achieved. The maximum achieved score is calculated by applying the iterative process for the exact same parameters.

Table 17: *Optimization process - Maximum iterations effect*

Max iterations	DNI (kWh/m2)	time (sec)	Closeness to optimal (%)
500	590	2.65	82
1000	615	3.41	86
1500	590	3.72	82
2000	625	2.95	87
3000	625	2.92	87
5000	610	3.39	85
10000	600	3.48	84

By observing the tabular data it becomes clear that there is no obvious connection between the number of maximum iterations and the achieved scores, at least for this problem size. From the application of the algorithm it also becomes evident that the optimization process terminates before reaching the number of maximum iterations. A notable fact is that an envelope of at least 82% optimality is always achieved. Further studies with bigger problem sizes are needed to reach more concrete conclusions.

4.4 Diversity of results

A really important factor, which affects the usability of the proposed tool, is its ability to generate diverse results when some crucial parameters change (e.g. the urban context). This verifies the fact that the model investigates a great amount of the design space before proposing nearly-optimal solutions and can act as a useful and practical suggestive mechanism in real-world projects. In this section some comparative studies are going to be presented. Four parameters are taken into account:

- envelope resolution (number of voxels)
- urban context (surrounding buildings)
- location (longitude & latitude)
- optimization target (season to optimize solar potential for)

A more detailed Toy Problem of around 400 voxels was chosen for these studies in order to increase the validity of the results. For each study, three of the above mentioned parameters remain fixed while one is used as a variable. The three proposed methods are applied separately to compare the results for each one of them and also reach possible generalized conclusions, such as their average run time.

4.4.1 Variable 1: Resolution

This first study investigates the effect of the descretization degree of the envelope on the final shape. The two resolution levels that are tested concern an envelope of approximately 100 voxels and one of 600. The set up of the toy problem can be seen in Figure 33. The fixed parameters are:

- Location: Amsterdam
- Context: Sparse mid-high buildings
- Optimization target: Maximize for the whole year

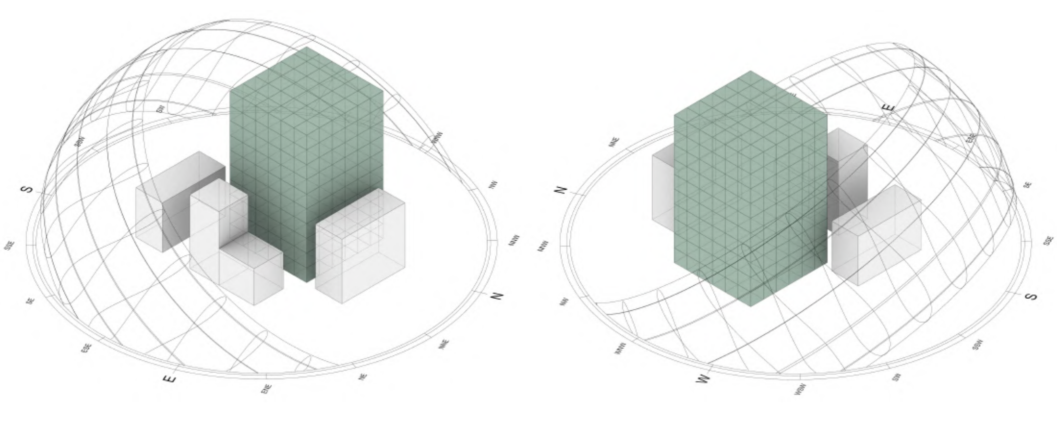

Figure 33: *Results diversity - Study 1: Toy problem set up*

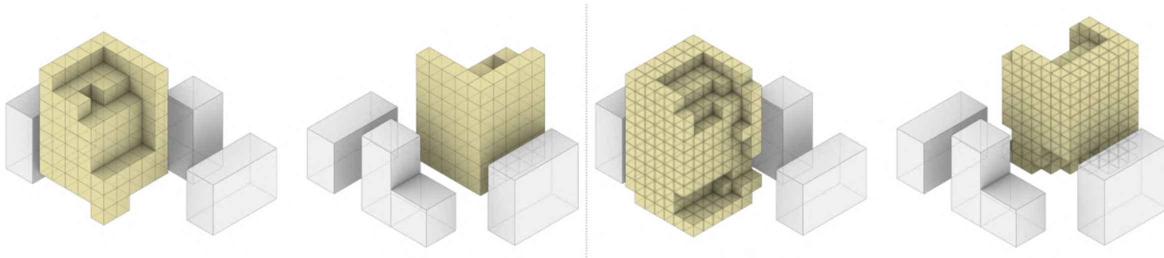

Figure 34: *Study 1 - Resolution - Method 1: left-low resolution, right-high resolution*

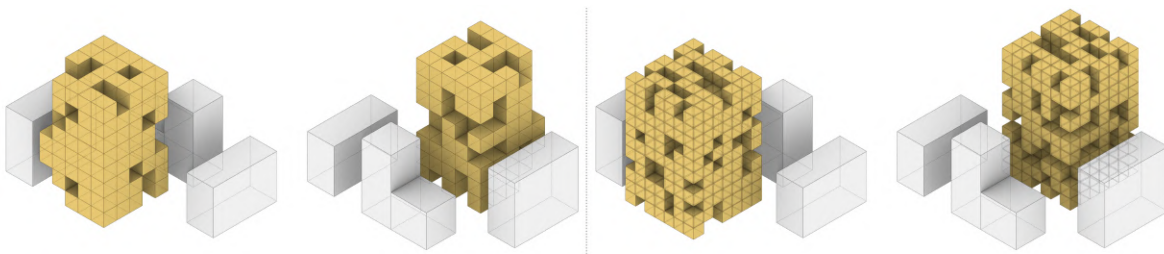

Figure 35: *Study 1 - Resolution - Method 2: left-low resolution, right-high resolution*

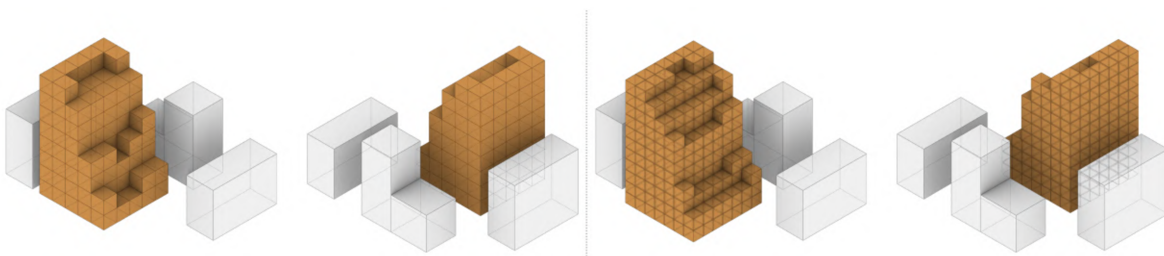

Figure 36: *Study 1 - Resolution - Method 3: left-low resolution, right-high resolution*

From the above presented figures it can be seen that resolution of the envelope affects the final shape but not in a crucial degree. The patterns are almost the same with dense and void areas being formed in the same places, especially for the first and the third method. This is a positive fact since it implies that using even low resolution envelopes for really fast simulations, still yields reliable results.

4.4.2 Variable 2: Context

The second study concerns the effect of the urban context in the shape of the building. The shape and position of the surrounding facades affects the contextual shading of the envelope and thus its solar potential. The three context alternatives that are being studied can be seen in the following Figure.

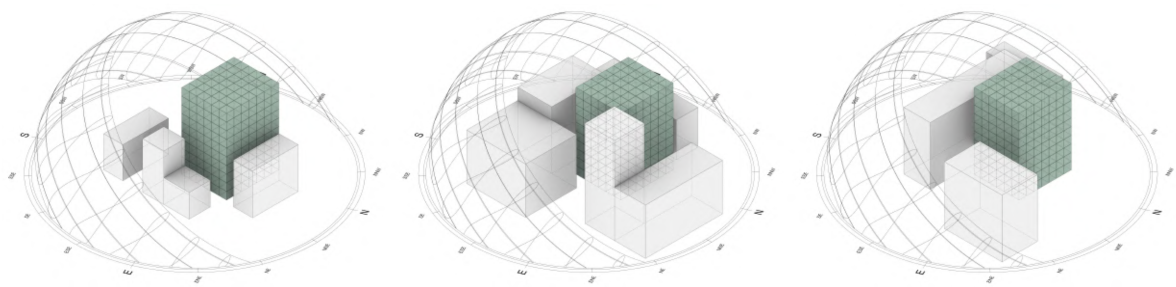

Figure 37: *Results diversity - Study 2: Toy problem set up*

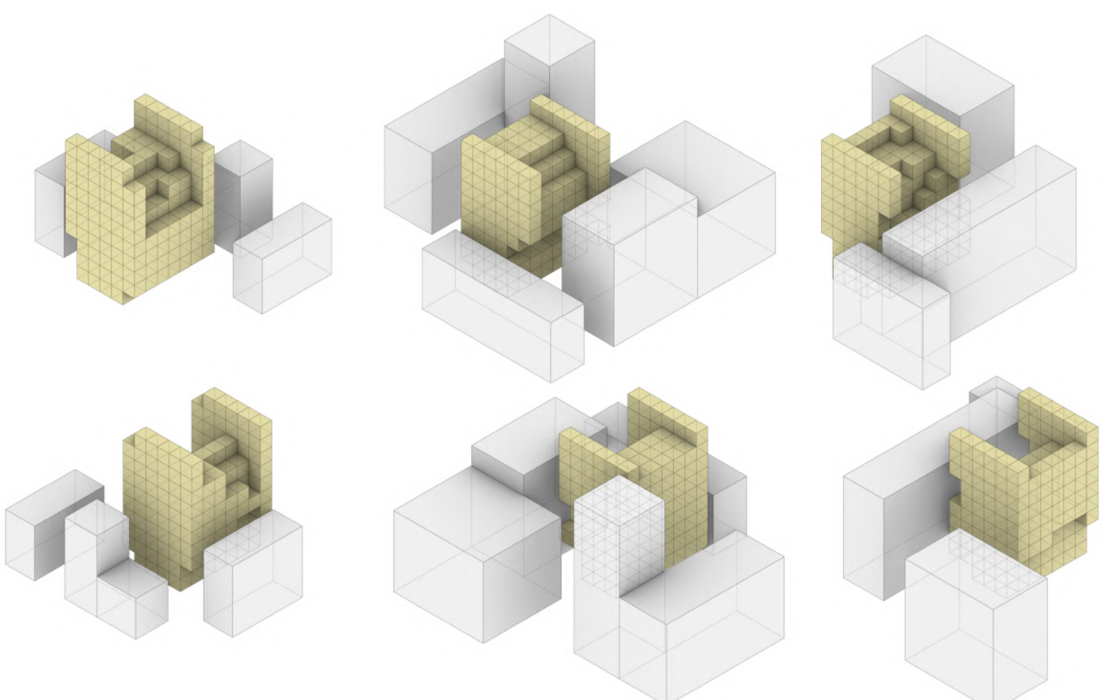

Figure 38: *Study 1 - Context - Method 1: left - context1, middle - context2, right - context3*

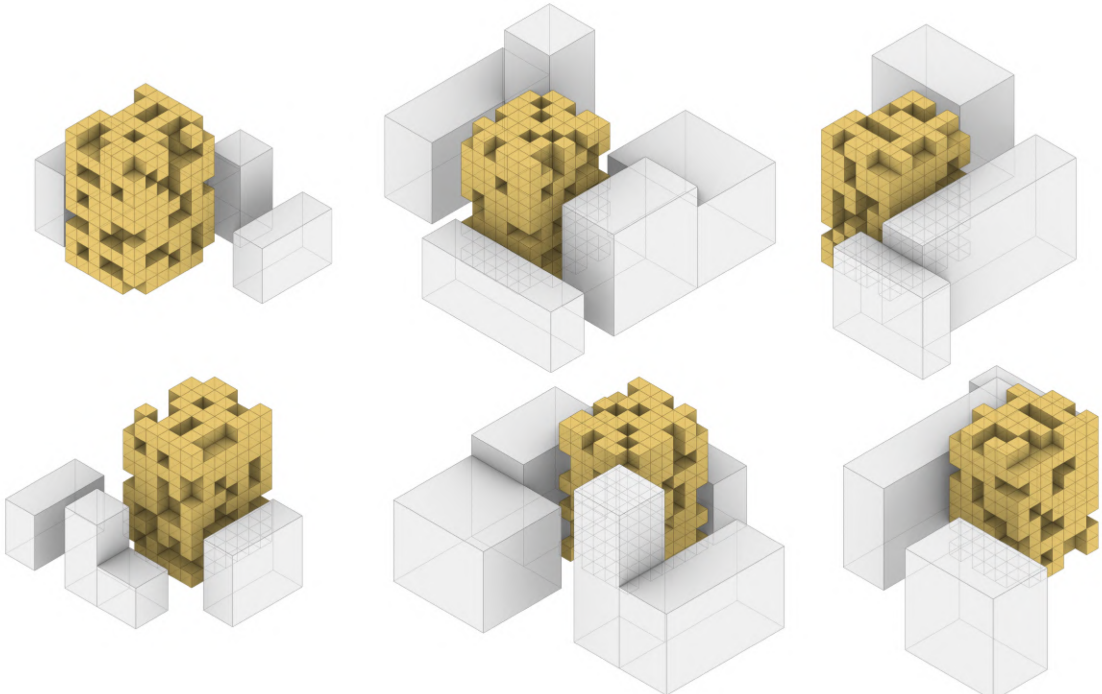

Figure 39: *Study 1 - Context - Method 2: left - context1, middle - context2, right - context3*

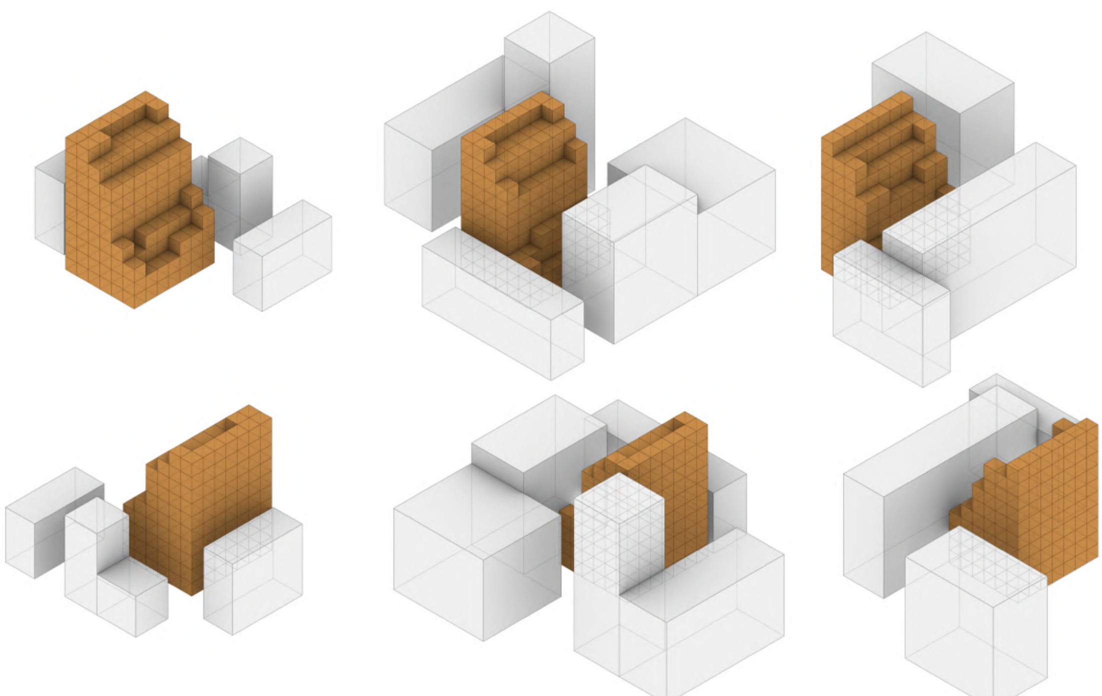

Figure 40: *Study 1 - Context - Method 3: left - context1, middle - context2, right - context3*

Figures 38-40 present the results of this comparative study. It can be observed that the most diverse shapes are the ones that result from the second method. However, these results also indicate a hint of randomness. The cost index evaluation method (3) seems to produce really similar results while the iterative method (1) gives diverse results but with a reasonable deviation, given the fixed parameters (location & optimization target).

4.4.3 Variable 3: Location

This study investigates the effect of the location of the plot under study on the final envelope shape. In this case, the fixed parameters are the urban context and the optimization period and the location alternatives are: Amsterdam(Netherlands), Athens(Greece) and Oslo(Norway). The Sunpath of these three locations and the set up of the Toy Problem can be seen in the Figure below. The scores of the resulting envelopes for each method are illustrated in Table 18.

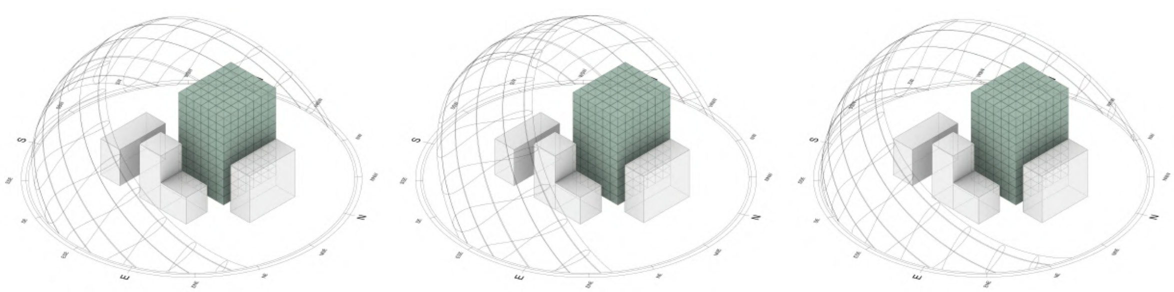

Figure 41: *Results diversity - Study 3: Toy problem set up*

Table 18: *Diversity of results - Study 2 - Scores per method*

	Amsterdam (NL)			**Athens (GR)**			**Oslo (NO)**		
	DNI (kWh/m2)	**RC (%)**	**time (sec)**	**DNI (kWh/m2)**	**RC (%)**	**time (sec)**	**DNI (kWh/m2)**	**RC (%)**	**time (sec)**
Method 1	1350	64	1015	2440	73	960	1095	69	985
Method 2	1350	45	180	2620	40	170	1055	40	180
Method 3	1280	74	1	2450	72	1	990	71	1

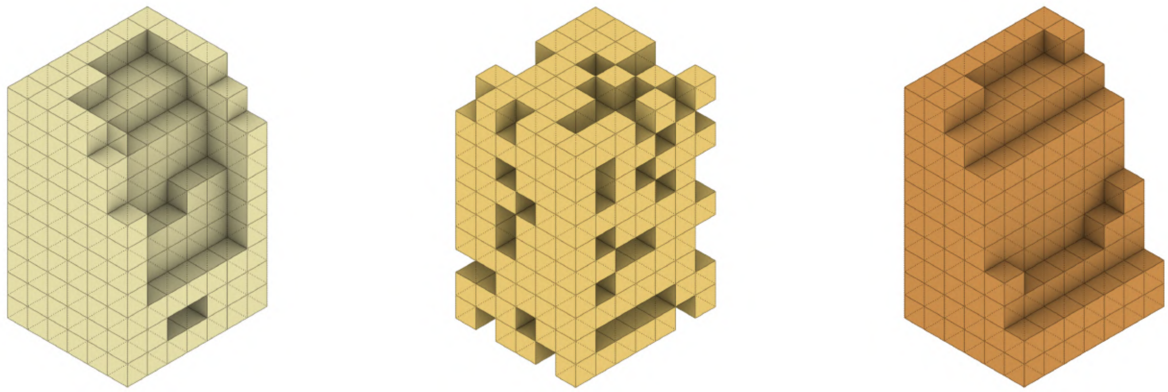

Figure 42: *Study 3 - Location: Amsterdam*

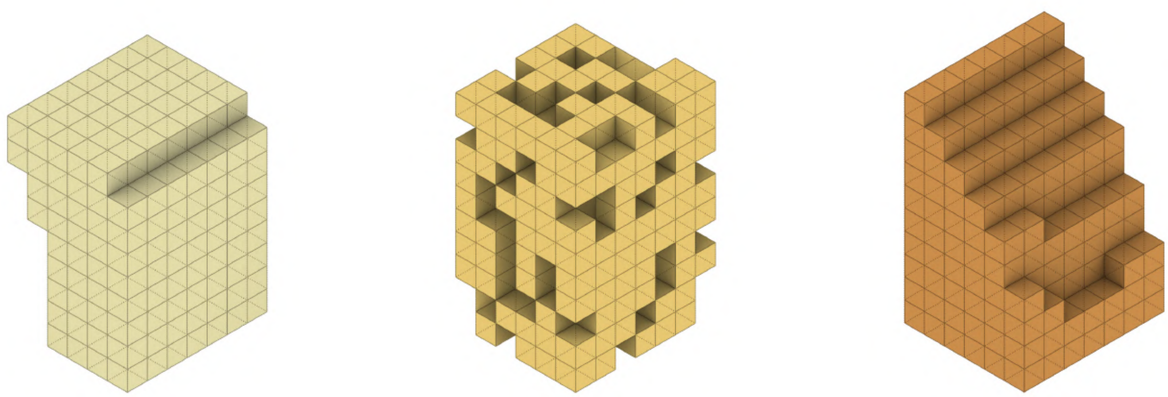

Figure 43: *Results diversity - Study 3: Athens*

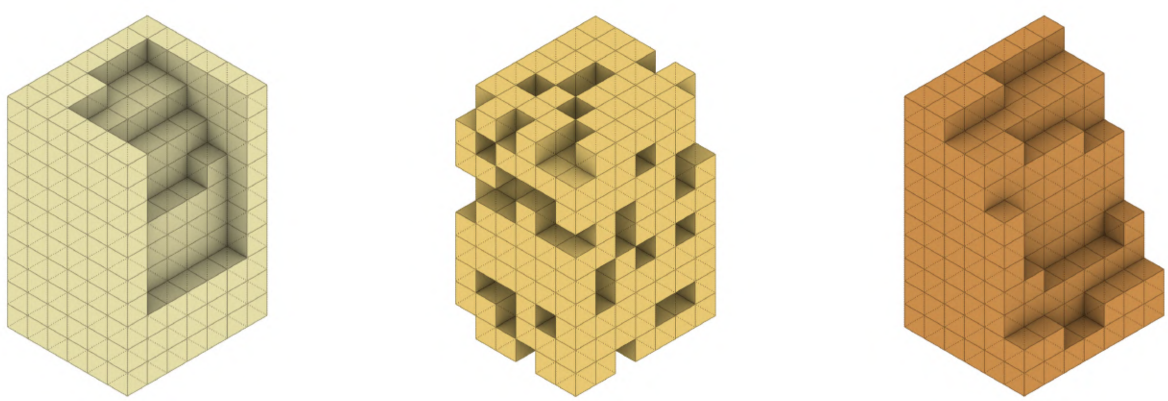

Figure 44: *Results diversity - Study 3: Oslo*

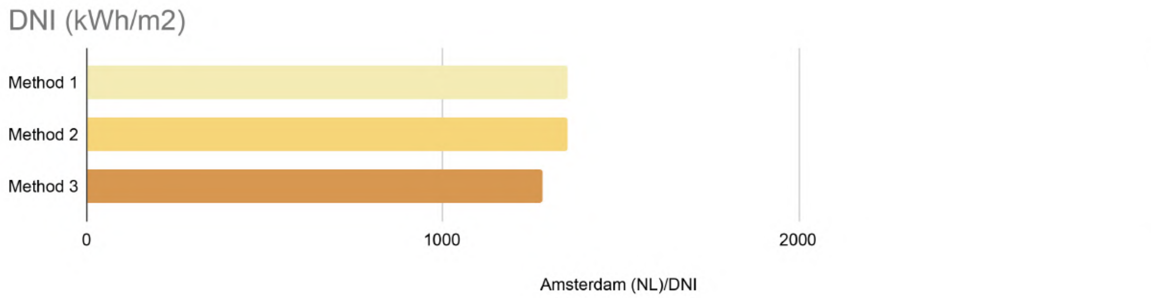

Figure 45: *Study 3 - Location: Amsterdam - Methods' scores*

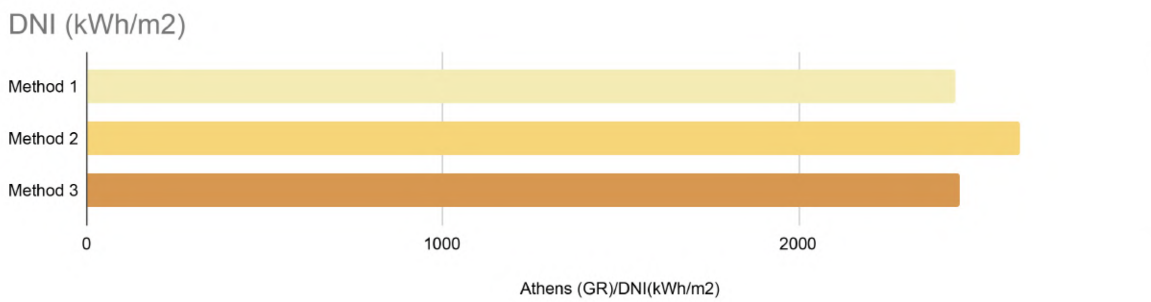

Figure 46: *Study 3 - Location: Athens - Methods' scores*

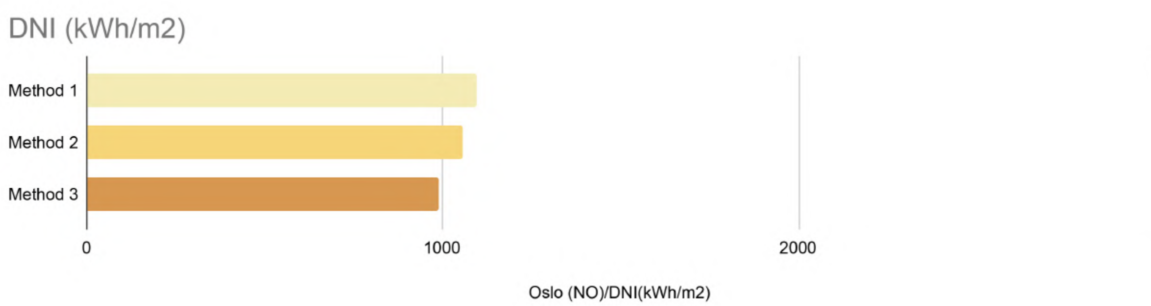

Figure 47: *Study 3 - Location: Oslo - Methods' scores*

From the above presented envelope results and their corresponding scores we can see that, like in the previous study (Section 4.4.2), the third method produces the most similar envelopes. Method 2 results are characterized again from some degree of randomness but some patterns can also be recognized. Method 1 seems to give more similar results depending on the climate. For the heating dominating countries (Amsterdam and Oslo) the envelope shapes are way more alike than the one for the cooling-dominated climate (Athens).

4.4.4 Variable 4: Optimization target

This final study concerns the effect of the period for which the optimization is performed on the envelope shape. The computed scores per season are also plotted in order to see if, except of the shape, there is an actual effect in the solar potential per season. The three methods are used for three different optimization goals:

- target 1: maximize for winter
- target 2: maximize for winter and autumn
- target 3: maximize for winter and minimize for summer

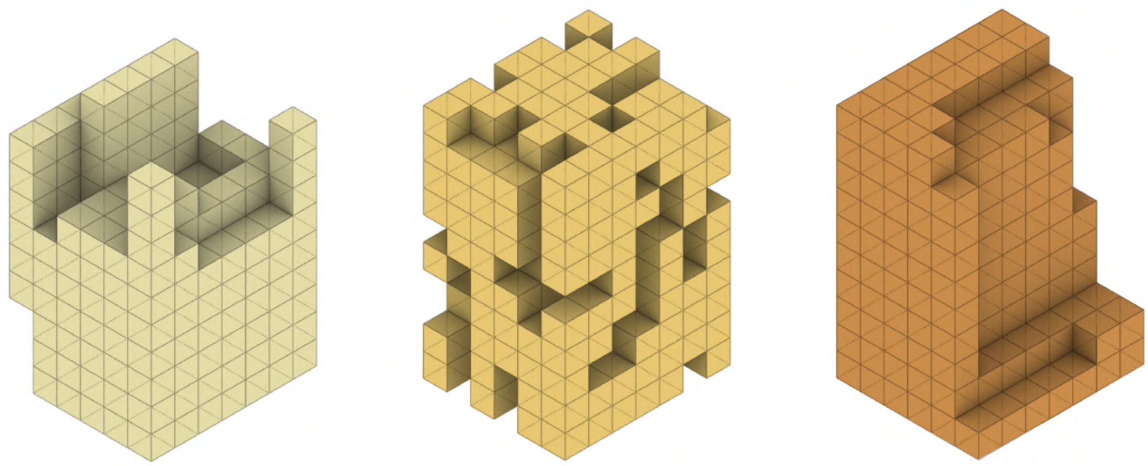

Figure 48: *Study 4 - Optimization Target 1*

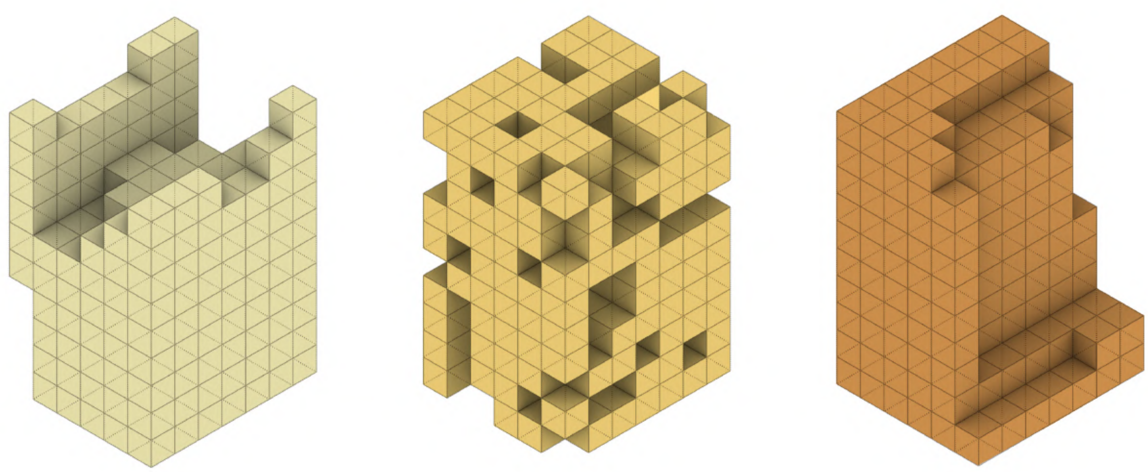

Figure 49: *Study 4 - Optimization Target 2*

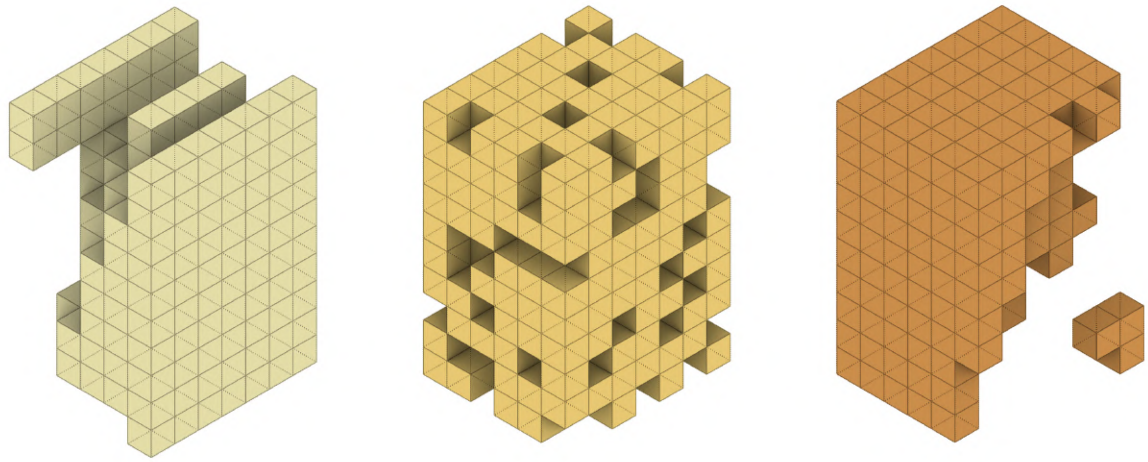

Figure 50: *Study 4 - Optimization Target 3*

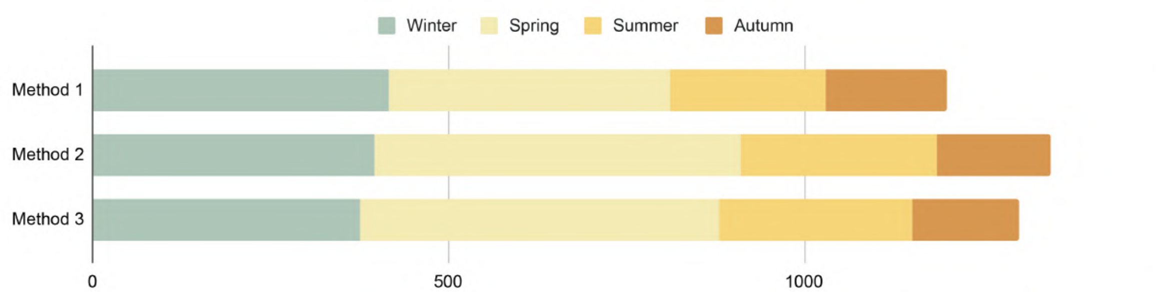

Figure 51: *Study 4 - Optimization Target 1: Methods' scores*

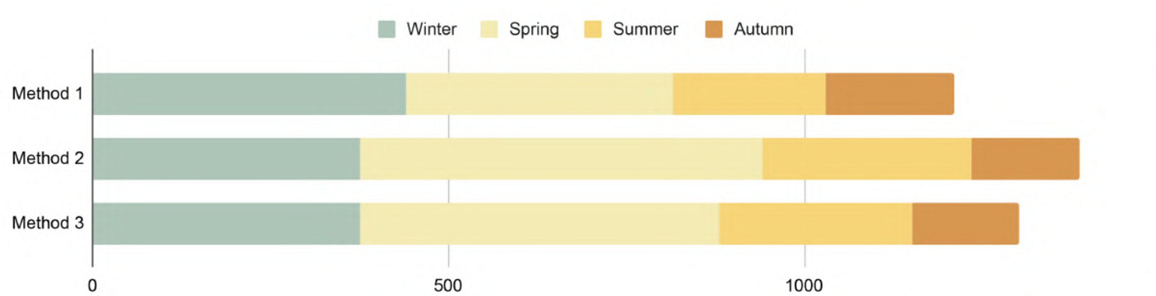

Figure 52: *Study 4 - Optimization Target 2: Methods' scores*

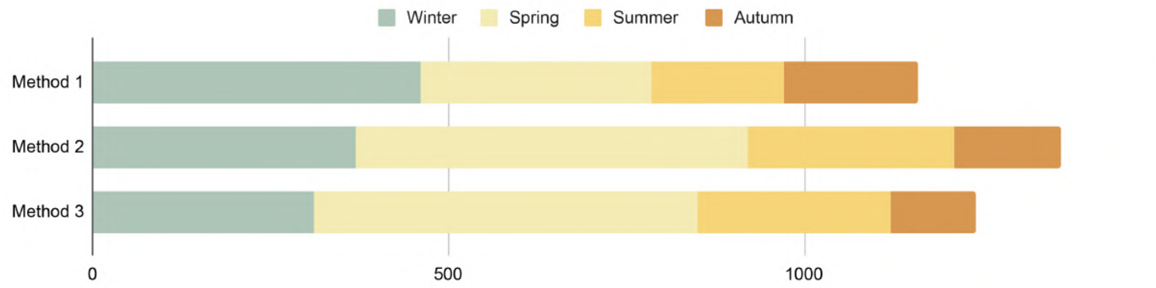

Figure 53: *Study 4 - Optimization Target 3: Methods' scores*

From the above presented Figures we can observe that the change in the optimization period target causes indeed an alteration of the shape of the envelope in all three methods. It is notable that, while in the previous studies the third Method had the least of deviation among the results, in this case it shows a great difference among the envelopes from target 1 ad 3 (Figures 48 and 50 third column). Method 2 results are governed also in this case from a degree of morphological randomness, although they yield the best results with regards to solar potential. Method 1 produces similar results for the first 2 target which alter greatly for the third one.

4.4.5 Methods comparison

These comparative studies, except of investigating the potential of the proposed methods to produce diverse, yet optimal, results, also reveled several additional properties for each method. To begin with, the iterative evaluation (Method 1) is by far the most time consuming one, as for these studies of a 400 voxel envelope it took an average of 800 second per run. The second method (minimization of objective function) is significantly faster with an average run time of 200 seconds, while the fastest is the third method (cost index evaluation) that it takes less than a second to run.

As far as the morphological aspect is concerned, the second method produces envelopes of high solar potential but of shape that is hard to be interpreted and used as a design guideline. These shapes are also characterized by a relatively low relative compactness. The above presented studies included two performance indicators, the Direct Normal Irradiation (for this part will be mentioned as solar potential) and the Relative Compactness. Method 1 and 2 produce envelopes with the highest energy potential, while 1 and 3 with the highest relative compactness (RC). Interestingly, only the first method had the RC factor included in the evaluation process. Methods 2 and 3 are design in such a way to optimize only for the solar potential. However, the third method works in such a way as it has the RC factor embedded in the process.

5 Case Study

The proposed methodology was tested through the different verification processes, as presented in Section 4, which resulted in the justification of the model's framework. In this Section the model is going to be implemented in a case study to evaluate its applicability in real-life situations. Through this process is sought to ensure that the product of this research meets the needs of the focus user, which in this case is the designer/architect. The results are going to be assessed through the prism of the practicality of the tool and its ability to produce diverse and usable results.

5.1 Site & Program

The selected site is located in Rotterdam and more particularly is the urban block between the streets Vijverhofstraat, Zomerhofstraat, Schoterbosstraat, and Teilingerstraat. This site has been chosen as it features a dense, yet diverse, urban context, with taller buildings on the Southwest side and smaller-scale building blocks on the North and East. This urban morphology was deemed to be adequate and interesting for the evaluation of the proposed model. The architectural challenge concerns the design of a mixed use living and working building of approximately 190.000 m^2. The spatial functions included in the architectural program and their corresponding areas are presented in Table 19.

Table 19: *Case Study - Architectural Program*

Spatial functions	Unit area (m^2)	Unit height (m)	Units number	Total area (m^2)	Total volume (m^3)
Housing_Large	120	3,5	100	12000	42000
Housing_Medium	90	3,5	60	5400	18900
Housing_Studios	40	3,5	80	3200	11200
Office_Large	1000	3,5	10	10000	35000
Office_Medium	500	3,5	20	10000	35000
Co-working_space	1000	3,5	6	6000	21000
Retail	150	3,5	30	4500	15750
Parking	700	3,5	3	2100	7350
Cafe	100	3,5	4	400	1400
Restaurant	200	3,5	2	400	1400

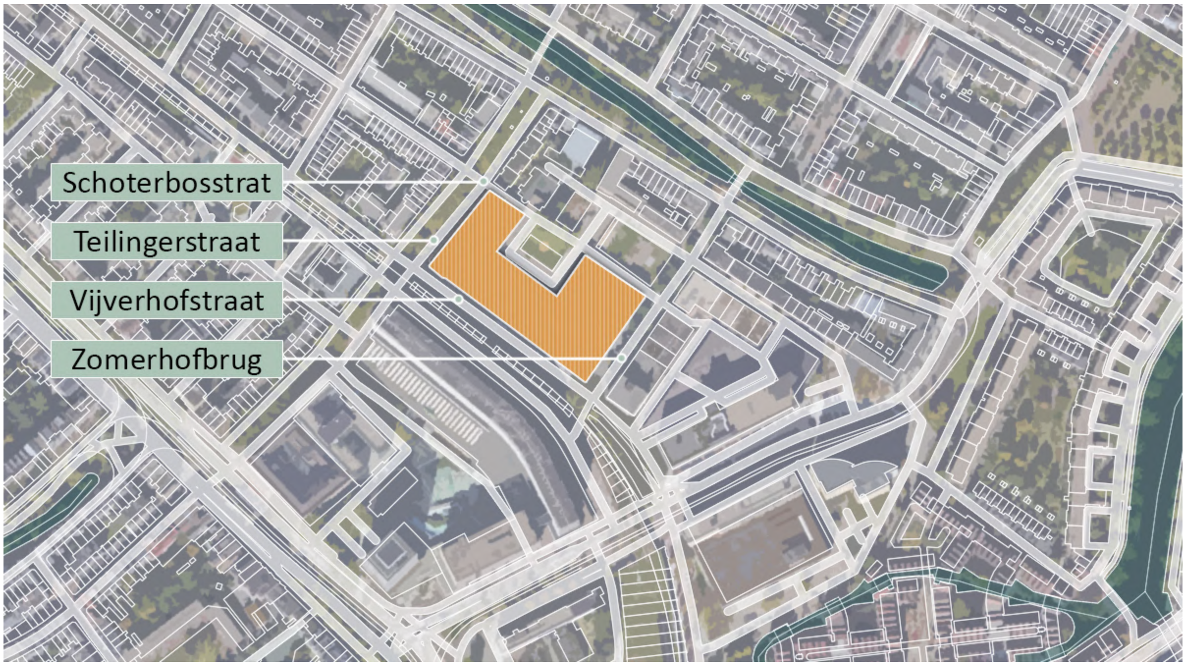

Figure 54: *Case Study site top view*

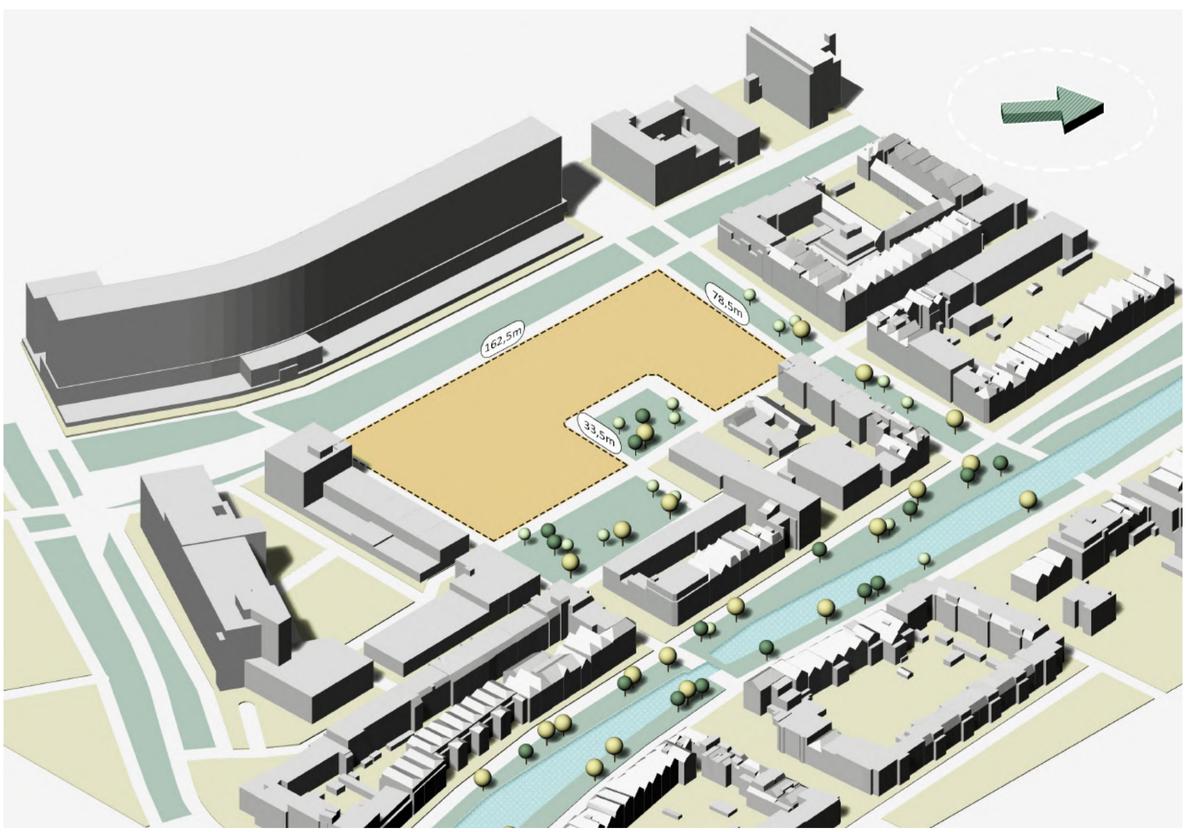

Figure 55: *Diagrammatic 3D representation of the Case Study site*

5.2 Stage 1 - Solar Envelope

The first stage concerns the Solar envelope formation. The building regulations that are taken into account for this specific case, derive from the Dutch building code. According to this, every facade should receive a minimum of 2 hours of direct sunlight in a time frame between February and October [44]. The input that was given in the model for this stage can be seen in Table 20 and the initial set up in Figure 56.

Table 20: *Case study Stage 1 - Input*

Voxel_size (m)	Time frame				Building regulations		Location data	
	start_month	start_day	end_month	end_day	min_hours	max_height	longitude	latitude
6	2	19	10	21	2	30	4.3571	520.116

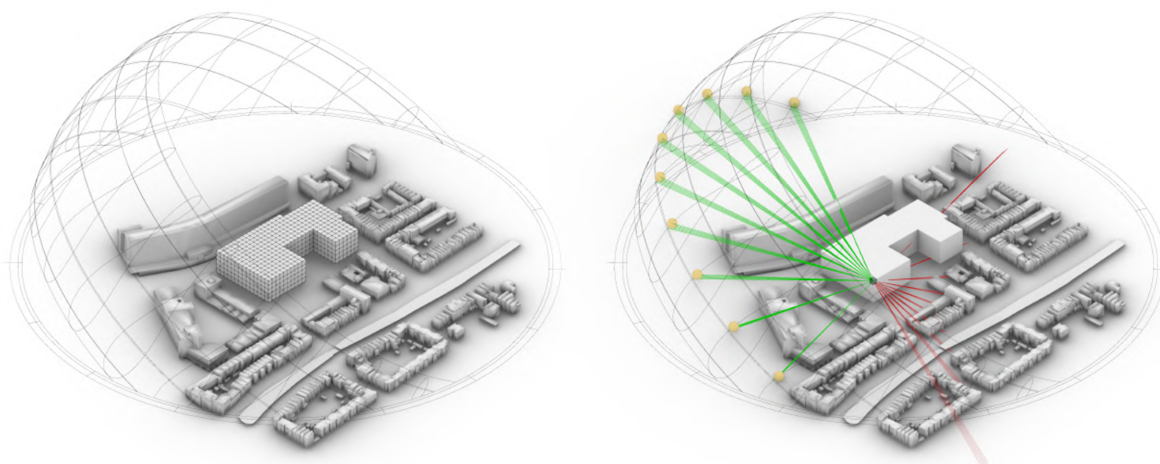

Figure 56: *Case Study Stage 1 set up and example of sun rays extension (red) towards the context for one voxel*

After the application of the first stage of the model, the contextual shading index, representing the total hours that each voxel shades its surroundings during the predefined period, is visualized in Figure 57-left. When aggregating this information per day, all of the voxels appear to comply with the aforementioned regulations. However, given the limitations of this methodology (discussed in Section 6.2.1), a safety factor is introduced in order to ensure the validity of the results. Based on the scores after the integration of the safety factor, eight voxels from the North side of the envelope are removed (Figure 57-right).

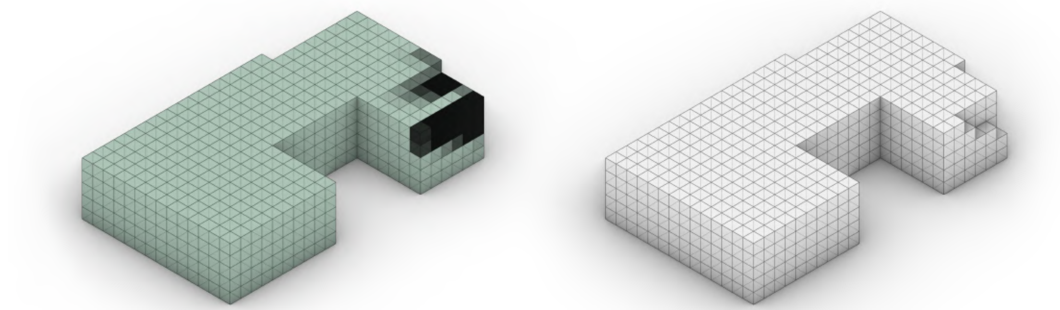

Figure 57: *Case Study Stage 1 contextual shading results and solar envelope after voxels removal*

5.3 Stage 2 - Massing

The result of Stage 1 is used as input for Stage 2 of the model which will define the shape of the envelope. An envelope of the same shape but a higher degree of descretization is used. Given the timeline of this thesis project and the characteristics of the envelope-generation proposed methods as discussed in Section 4.4.5, the cost index evaluation method is argued to be the most suitable method to demonstrate this Stage of the Case Study. In a previous version of this report the iterative evaluation process, before its final modifications, had also been applied in the same case study and the results can be found in Appendix A.3. The result of Stage2 according to the chosen method is visualized in Figure 58.

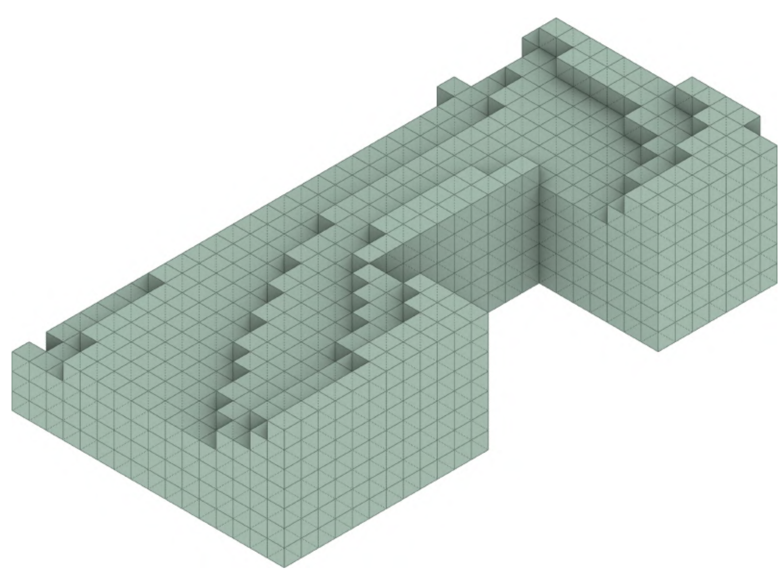

Figure 58: *Case Study Stage 2 result isometric*

6 Discussion & Conclusion

6.1 Discussion of results

After presenting the proposed methodology in detail, it becomes clear that the verified vectorized visibility evaluation process, which is based on the pre-computation of intervisibilities of spatial elements, carries numerous advantages and a great potential for performative massing studies. This process was broken down into its stages and it was verified accordingly.

A significant discussion point emerges from the utilization of this process in order to produce envelopes of optimal solar potential. For this purpose, three different methods have been proposed, all based on the pre-computed intervisibilities. Advantages and drawbacks of these methods have been briefly presented in Section 4.4.5. These conclusions are also summarized in the following comparative Table 21. The criteria for which the methods are compared are: computational power, diversity of results, ease of (design) interpretation, solar potential, relative compactness, control over the process. The assigned values (+, ++, +++) show the classification among the three options (+ worst, +++ best).

Table 21: *Comparative table of proposed methods*

	Computational power	Diversity of results	Solar potential	Relative compactness	Ease of interpretation	Control over the process
Method 1 (Iterative evaluation)	+	+++	+++	++	++	+++
Method 2 (Objective function)	++	+	++	+	+	+
Method 3 (Cost index evaluation)	+++	++	+	+++	+++	++

As it can be observed from the comparative tabular data, the cost index evaluation method is significantly faster and computationally feasible than the other two. However, it appears to produce envelopes according to a repetitive pattern that leads to less diverse shapes but more compact ones. The iterative evaluation process balances in a notable level the solar potential and visual diversity aspect but is remarkably slower as a method. The method that is based on the objective function minimization yield results of high solar potential but low relative compactness and usually of shapes that are hard to be interpreted in a design manner.

In Section 5 the application of the model in a selected Case study was presented. After the completion of the first stage no voxels were found to exceed the building regulation limits. However, after the application of a safety factor some of them in the Northern side of the envelope were removed (Figure 59). The removed voxels constituted roughly 0.5% of the whole envelope and thus a great design space is left for the application of the second stage.

Figure 59: *Case study - Solar envelope results*

For the completion of the second stage the resolution of the envelope was increased even more, resulting in a model of approximately 3000 voxels. According to the general proposed methodology, three different methods have been proposed for this stage (Section 3.3.3). Given the time limitations of the processes and the size of the problem, the third method of the cost index evaluation was applied. After analyzing all the different factors that are included in the cost index calculation, the resulting shape is considered to be logical.

6.2 Limitations

6.2.1 Stage1

The first stage consists of the solar envelope definition. Although the presented methodology accounts for many factors, it also carries some limitations. To begin with, the accuracy of the results are highly dependent on the urban morphology. For example, the outcome of this process tends to be more accurate in cases that the urban context consists of relatively big monolithic buildings and less accurate for a more segmented, complex neighborhood. The reason for this is that the shading impact of each voxel is aggregated per day and per volumetric unit for the whole analysis period. In this way, what is calculated is the voxel's daily shading impact on all the context buildings as a total. As a result, a voxel may seem to obstruct excess hours of sunlight when in reality it could just obstruct just a few hours from different buildings during different times of the day. A segmentation of the surroundings in separate building blocks and the aggregation of the shading results per each building could solve this restraint.

Another limitation lies in the fact that the context is being modeled and treated as a set of uniform solid blocks. No discretization of the facade surfaces is taken into consideration, nor a separation of facade elements in opaque and transparent parts. This, combined with the fact that the rays have the voxels as sources and not the facade elements, highlights the focus of this methodology on the calculation of a more generalized indicator of the obstruction of one voxel to its surroundings as a total. This indicator is also considered independently of the shading that the surrounding buildings cast to each other. Hence, after the simulation, the building may seem to not overshadow the context as an entity but may be contributing to an overshadowing combined with its surroundings.

One more factor to consider is the intervisibilities of the voxels themselves. The spatial interrelations of the volumetric elements with regards to the visibility objective, in this case the sun, are also affecting the calculation of this indicator but they are not taken into account until the next proposed stage.

Although these restraints are acknowledged, the development of an exact, error-proof solar envelope generation method falls out of the scope of this research. As already mentioned, this stage serves mostly to present the proposed framework more comprehensively and to ensure the feasibility of the next stage, which is also the main focus of this research. Despite those limitations, the proposed methodology seems to be quite effective for a nearly-accurate estimation of the solar envelope shape and it could be even proven to be enough for projects that take place in countries without specific regulations on direct sunlight of facades. There is room for modifications and improvements that would render this methodology even more precise and in perfect accordance with building regulations.

6.2.2 Stage2

While the proposed methodology for the Massing stage provides interesting possibilities, it also carries some limitations. To begin with, for simplification reasons of this stage, the volumetric units are perceived as solid elements and not as a combination of surfaces with different orientations. Hence, although directional information about the rays is being stored, it can not be used in its full potential at this stage. This also affects the evaluation and score aggregation process, as the weights that are assigned to the rays are not reduced according to the angle of incidence with the hit surface. Despite the fact that the calculated factors are not perfectly accurate, they still serve their purpose as indicators. Comparative studies among alternatives are still feasible, but the calculated numbers should not be treated as literal values for further calculations. This additional level of information is taken into account in the next stage, where the computed values require a higher degree of accuracy.

At this development state, the proposed methodology regards three solar-related properties. However, more importance is given to the indicators linked to the direct sunlight, while only the skyview factor is related to the direct skylight part. For the required level of detail of the pre-conceptual design phase, this was deemed to be enough. Nonetheless, future developments could include the integration of a more intricate factor regarding diffused sunlight. As far as the performance indicators are concerned, another limitation lies in the fact that only measurable properties can be included. This does not affect the estimation of the solar potential directly but it may affect the ability of the proposed framework to account for additional non-quantifiable aspects that affect the conceptual design stage.

One limitation of a more technical nature, is the necessary computation time and power. This factor is highly dependent on the level of the discretization of the envelope. Although a higher resolution provides more accurate results and shapes that are easier to be interpreted morphologically, it exponentially increases the simulation time. This can be overcome by the optimization of the algorithm itself or the replacement of some of the sub-methods with more efficient ones. Future developments on the performance of computers and laptops will also assist in overcoming this limitation.

6.3 Conclusions & Future development

The current research investigates a mathematical derivation of building envelopes to support a novel computational framework for voxel-based designs of a nearly-optimal solar potential. The proposed workflow introduces a generative subtractive method for feed forward building form optimization. Eventually, this integrated design approach may assist designers and architects in taking informed decisions during the pre-conceptual design phase, based on solar access information.

The proposed method divides the full buildable volume, corresponding to a plot, into a three dimensional domain, consisting of 3D cells-voxels. Visibility objectives are defined and reference vectors are constructed according to them. The intervisibilities of the voxels are computed with regards to each visibility target and are then used in an embedded vectorized optimization process. The MCDA method, the criteria weights and the optimization target can be defined and then modified by the user, resulting in a variety of nearly-optimal results. This feature can facilitate comparative studies among design alternatives, with regards to performance and form qualities, leading to designs of high sustainability performance.

Verification and validation studies show that the proposed framework supports the generation of nearly-optimal envelopes while avoiding the repetition of exhaustive ray-tracing simulations. The application through the selected case study reveals some morphological flaws that can be overcome through minor adaptations of the model. Some other considerable limitations include the computational power demand that remains high, especially for bigger design domains, and the more precise computation of the solar-related performance indicators. This may demand the face-wise construction of the inter-visibilities graphs. Therefore, future work is foreseen to primarily take into consideration these points of concern. Further research is also demanded in order to develop the third part of the proposed methodology, concerning the zoning of the envelope, in order to make the framework more complete. Interesting, yet not urgent, future developments may include the creation of a simple interface to be usable from users who are not familiar with coding environments and the integration of additional form-related criteria.

A Appendices

A.1 Appendix 1 - Reflection

Introduction

This part of the report contains the reflection of the thesis that was presented in the main body of the report. The ultimate goal of this research project is to develop a computational framework for a feed forward optimization of building envelopes based on a generalized mathematical formulation of the massing design problem. This design problem refers to the discovery of a building shape which will have nearly-optimal solar potential. The reflection will focus on the process that was followed and define its wider scope in the research field, the master's program and its broader socio-technical relevance.

Process and Planning

To begin with, an extensive literature review was necessary to cover the required background in mathematics regarding graph theory, linear algebra, topology and other relevant topics. The literature study was also extended to projects covering building shape optimization aspects. Understanding principles of light-related simulations were also significant at this point. The initially proposed methodology included the separate and sequential formulation of the mathematical problem under study and then its application through a computational workflow, using Python. However, these two aspects were developed less linear than expected. Additional support from the mentors, especially in the mathematical formulation aspect, was highly valuable. The initial planning developed for the P2 presentation was followed with slight adjustments, leading to a smooth transition between the several thesis deadlines. The only larger-scale adjustment concerned the elimination of the final stage of the proposed design methodology (i.e. the Zoning), as after the P3 presentation it was deemed to be more important to focus on the further development of the first two stages and just define the framework of this last one.

Research and Design

Within the Building Technology Graduation Studio, this thesis fits in the junction of the chairs of Design Informatics and Climate Design. This also becomes clear from the supervisors selection. This thesis could be characterized as a design by research and development project. The research and experimentation part plays a significant role in the final outcome, which is the model that produces the design product. However, as the research refers to building envelopes, initial speculations of the design product can be made, guiding the evaluation of the usability of the tool.

Results and Applicability

Despite the emergent challenges of this thesis, primarily linked to the initially limited mathematical and computational background of the author, the presented results are highly promising and stimulating. The main research question, as well as the sub-questions, are considered to have been answered in an adequate level. Of course there are still some limitations, as discussed in the relative section, mainly concerning the computational power that is required for the application of the model. This parameter led the whole verification and validation process to be executed through simplified toy problems and not the case study. Further improvements in the algorithm itself and in the process will allow for the calibration of the model according to several case studies, rendering it even more applicable to real-life projects.

Scientific and Social relevance

The context and motivation of this thesis already reveals the importance of sustainability in the build environment. It is evident that, although important steps have been made towards better performing buildings, these are still proven to not be enough. In a design world that architecture is driven by performance, integrated computational workflows become an undeniable requirement. However, in the efforts to not compromise in architectural quality, a harmonious synergy between such tools and the designer should be achieved. For this reason, the goal of the proposed model is to facilitate early-design decisions without detecting them. It reveals the portion of the design space which serves particular optimization targets and serves as a guideline for the design development. In this way a more smooth integration into the design process is achieved. At the same time, the algorithmic and open source nature of such models allow for their reproducibility and their constant improvement, contributing to the advancement of the relative scientific field.

Dilemmas and Issues

As already discussed, the main dilemmas derive from the lack of extensive verification and validation of the model. Its application in multiple and diverse case studies is considered necessary, although it could not fit in the proposed timeline. Furthermore, a comparative study among the two design workflows, the traditional one that incorporates environmental simulations later in the design stage and the proposed one, would reveal the real extent of the contribution of this thesis in reducing the design time, while reaching better results.

Another dilemma is how this model affects the real role and contribution of the architect in the whole design process. In a world where everything is slowly but steadily replaced by automated technological processes, it is important to not forget the real essence of architecture and its unquantifiable properties and qualities as well as its human-orientation. For this reason, as aforementioned, the proposed model is envisioned to assist the designer, without determining the design outcome.

Additional issues and obstacles in the research process were primarily linked to my lack of Python programming language knowledge and my limited mathematical background. The combination of these factors slowed down the progress several times but thanks to the constant support from my mentors, advancement was always ensured. Despite these challenges, this thesis was also the perfect opportunity for me to expand my horizons and acquire new valuable skills.

A.2 Appendix 2 - Verification & Validation details

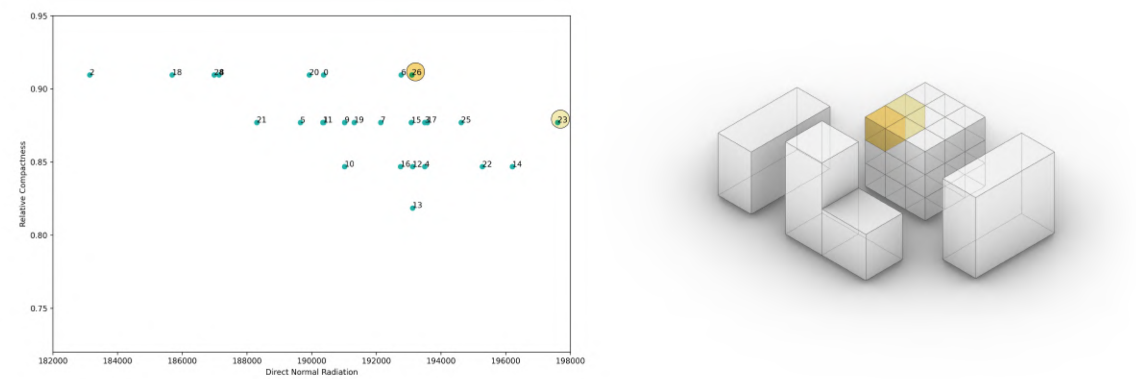

Figure 60: *Iterative process verification: Example1 - Iteration 1*

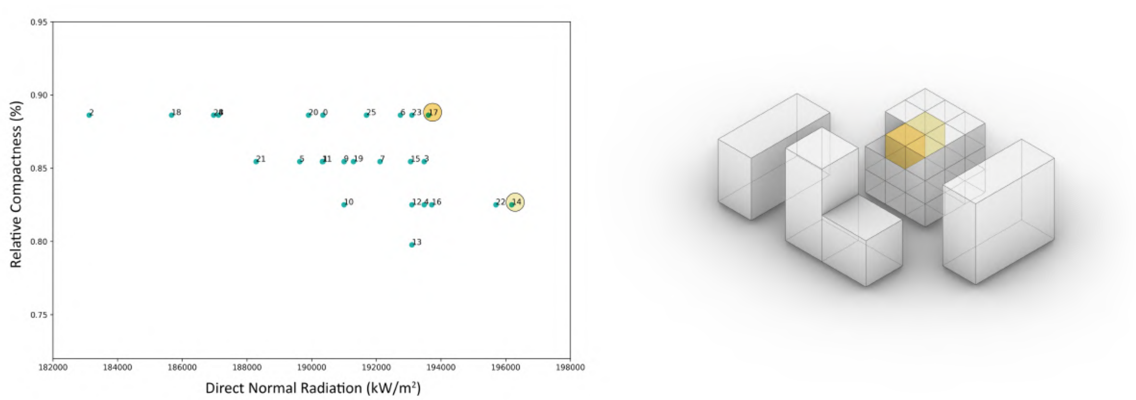

Figure 61: *Iterative process verification: Example1 - Iteration 2*

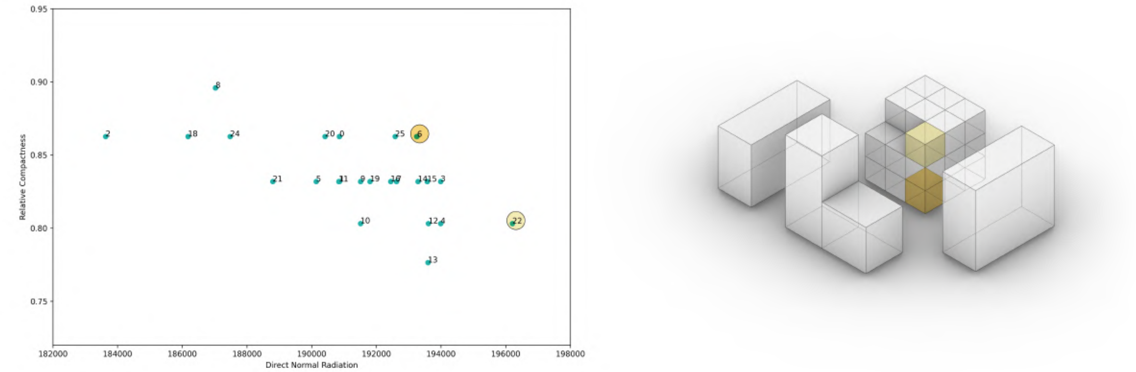

Figure 62: *Iterative process verification: Example1 - Iteration 3*

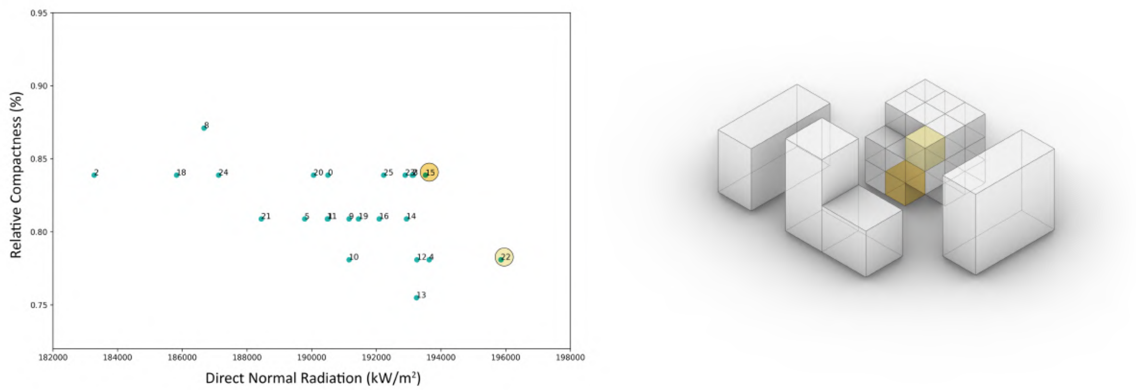

Figure 63: *Iterative process verification: Example1 - Iteration 4*

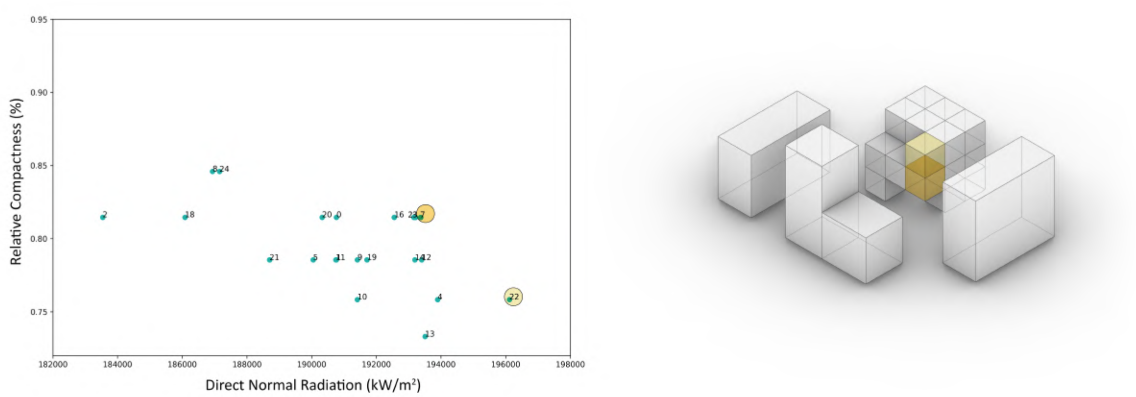

Figure 64: *Iterative process verification: Example1 - Iteration 5*

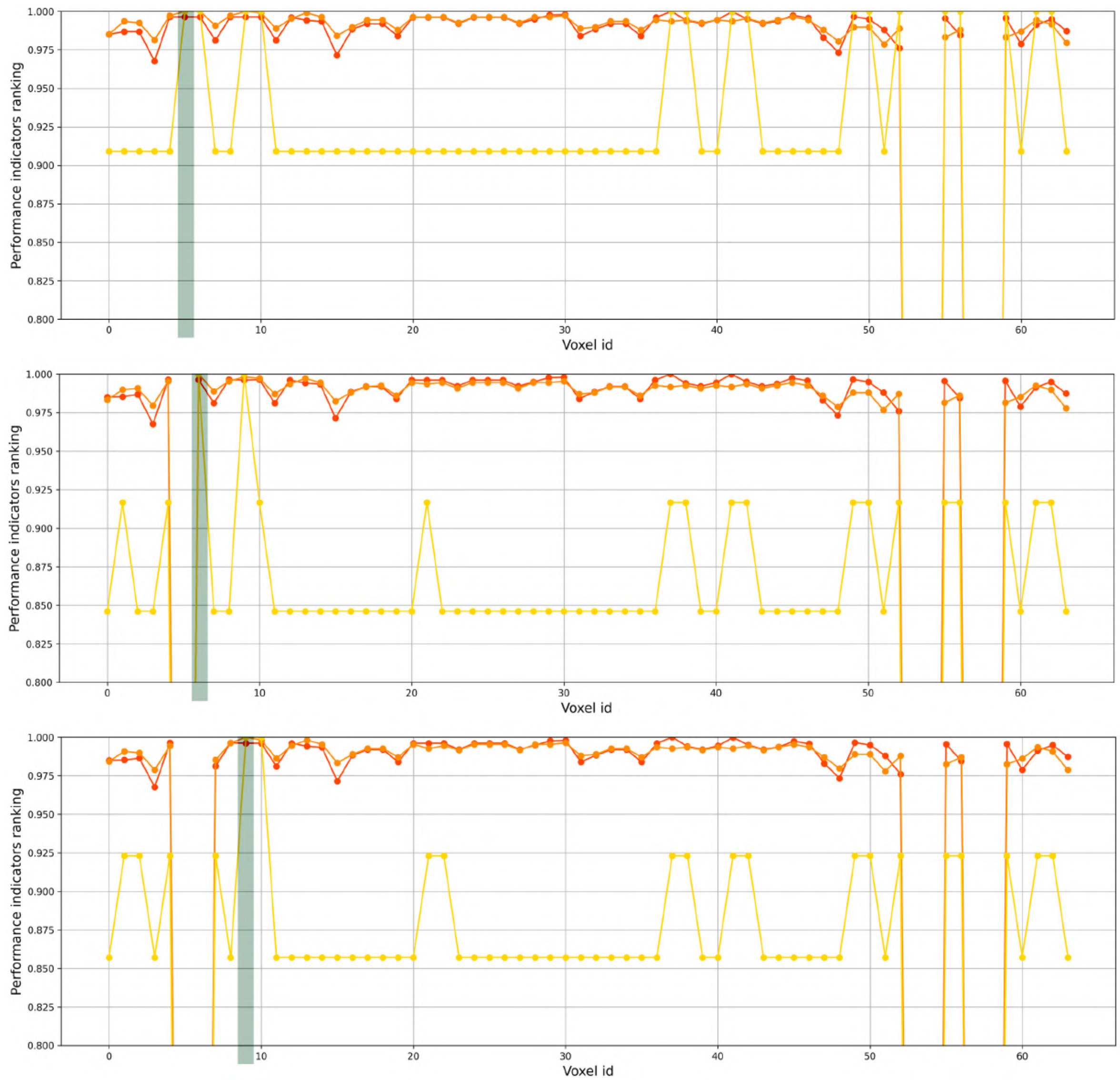

Figure 65: *Iterative process verification: Example2 - Iterations 5-7*

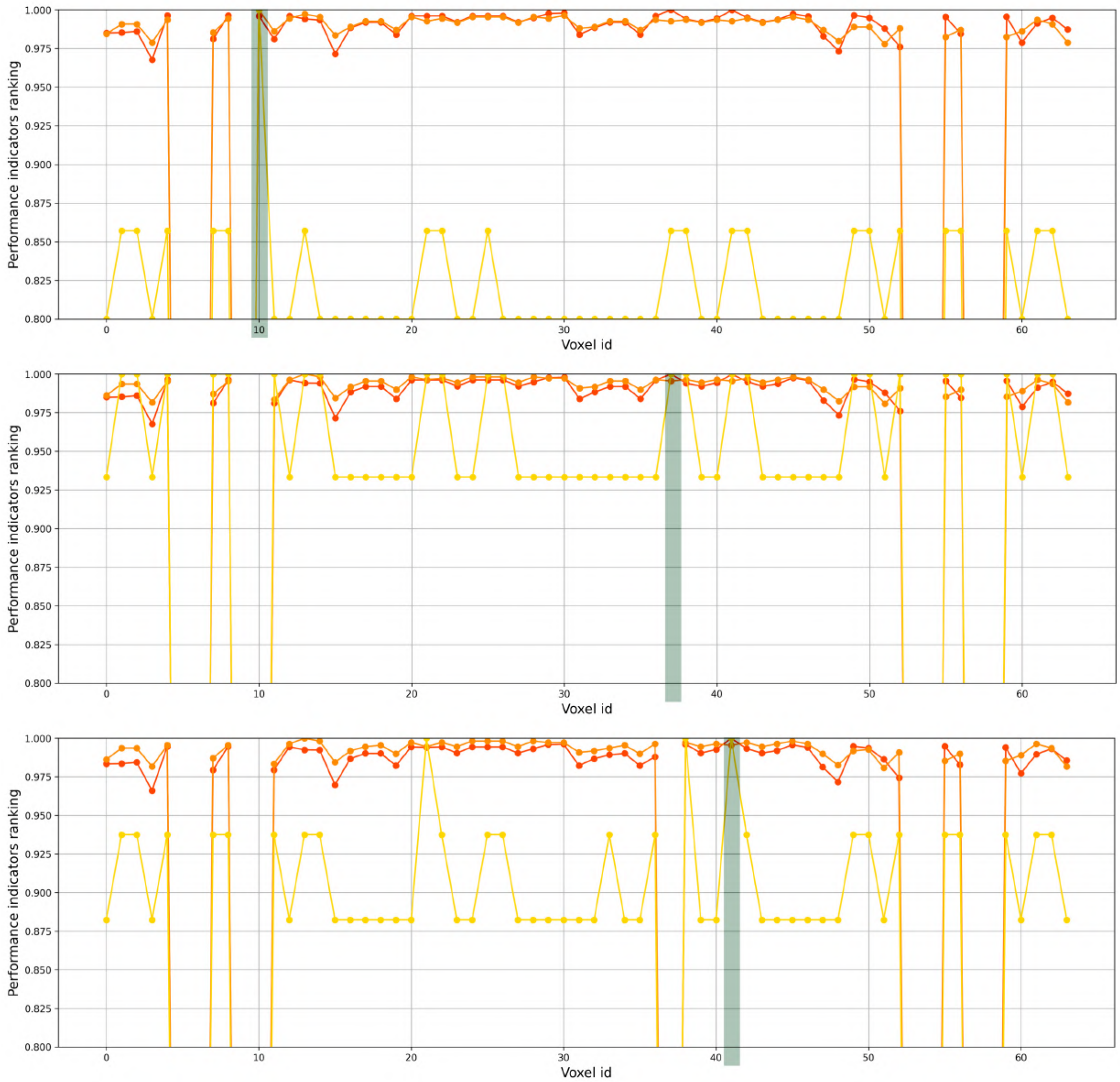

Figure 66: *Iterative process verification: Example2 - Iterations 8-10*

A.3 Appendix 3 - Case Study - Iterative evaluation process

The input regarding the architectural program for this stage is depicted in Table 22. In order to reduce the computation demand of the process the strategy of evaluating only the outer voxels has been adopted, as well as a step of 20 voxels removal per iteration. Additional information about the input to the model is depicted in Table 23.

Table 22: *Case study Stage 2 - Architectural program input*

Spatial functions	Total volume (m3)		
Housing_Large	42000		
Housing_Medium	18900		
Housing_Studios	11200		
Office_Large	35000	**Volume needed (m3)**	189000
Office_Medium	35000	**Voxel's edge (m)**	6
Co-working_space	21000	**Voxel's volume (m3)**	216
Retail	15750	**Voxels needed**	875
Parking	7350	**Step (voxels)**	20
Cafe	1400	**Voxels to remove**	570
Restaurant	1400	**Iterations**	29

Table 23: *Case study Stage 2 - General model input*

MCDA criteria weights				MCDA method	Removal strategy	Optimization (maximization) target
Direct Normal Irradiation	Direct Normal Illumination	Skyview factor	Relative compactness			
0.25	0.25	0.25	0.25	MOORA	outer voxels	year

As it can be seen by the data in Table 23 the optimization goal is the maximization of the solar potential of the envelope throughout the year. The selected MCDA method is MOORA and all the criteria are equally weighted. In Figure 68 the result of the process is illustrated through two isometrics and in Figure 68 as part of the urban context.

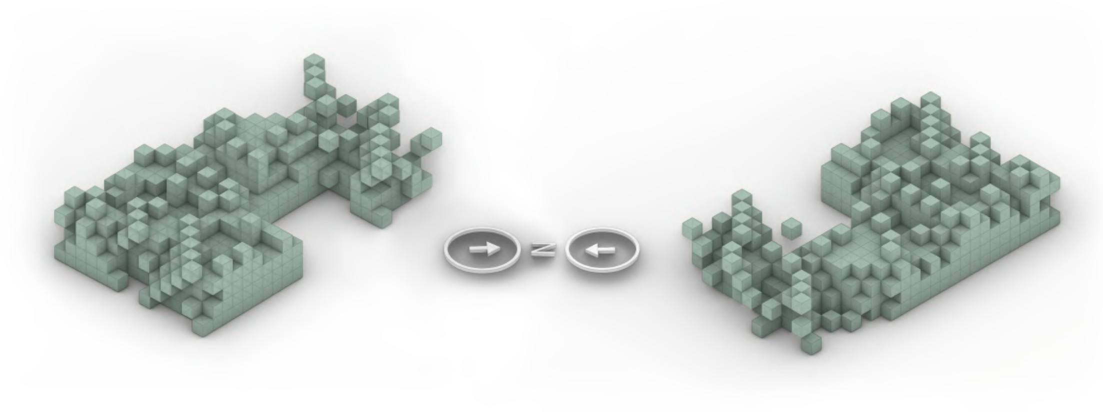

Figure 67: *Case Study Stage 2 result isometrics*

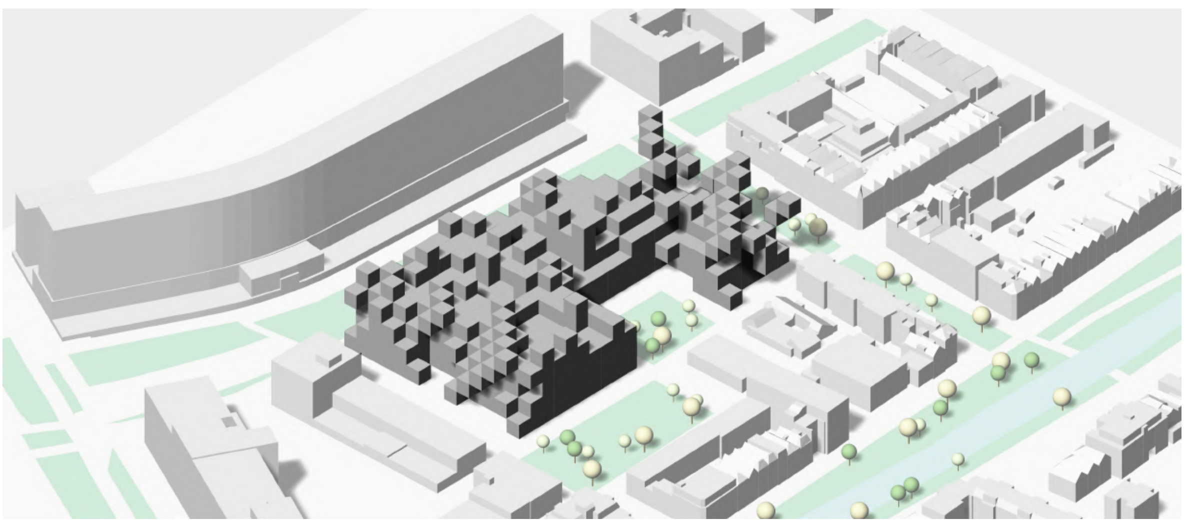

Figure 68: *Case Study Stage 2 result in urban context*

After the first four iterations a tendency of removing voxels from the South-east, South-west and the top of the envelope can be observed (Figure 69 left). When almost half of the needed voxels are removed, the envelope starts losing its contiguity on the West side (Figure 69 center). During the rest of the iterations voxels from the top and the North-west side continue being removed, affecting the initial symmetry of the envelope. The most intact part of the envelope is located in the East side (Figure 69 right). The end result illustrates a massing mostly concentrated towards the South-east side of the plot with an average height of 20 meters. The gradual removal of voxels per iteration is illustrated in Appendix A.3 through Figure 71.

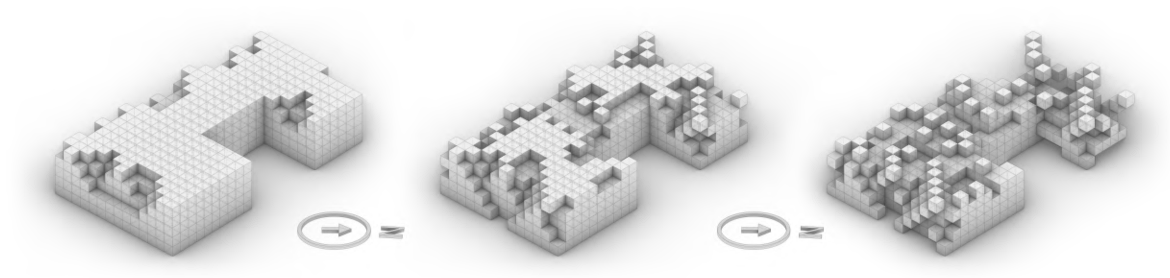

Figure 69: *Case study - Stage 2: three intermediate steps*

On the North-west side some scattered voxels can be observed (Figure 70). While the Relative Compactness was introduced as an objective in the whole process this was not enough to prevent the phenomenon of “floating” voxels (i.e. voxels with not even one neighbor). Such voxels would create non-usable spaces.

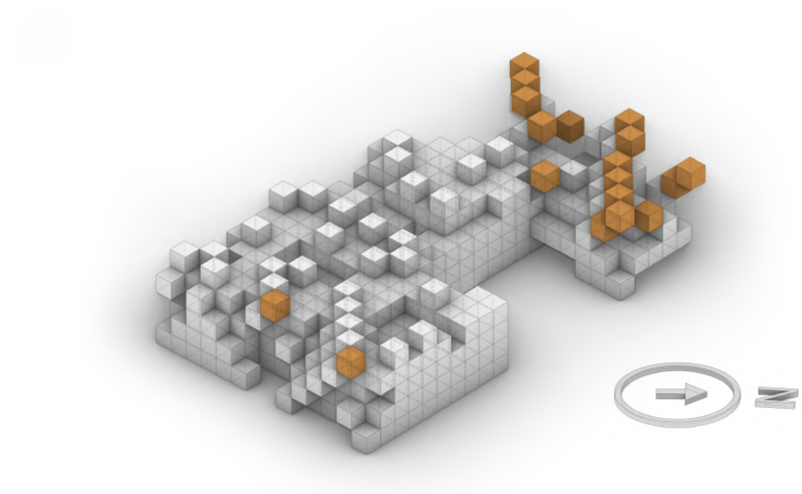

Figure 70: *Case study - Stage 2: floating voxels*

Some manual adjustments could make the resulting configuration more usable for design purposes (Figure **??**). Nonetheless, this can also be tackled within the algorithm with some additional considerations. Three possible solutions to the problem are:

- the increase of the weight that corresponds to the relative compactness objective during the MCDA evaluation
- the integration of a “correction” algorithm after the massing process that would reposition such voxels to the closest possible position
- the introduction of a “penalty” point in case the removal of a voxel would cause the loss of contiguity of the envelope

Undeniably, the utilization of smaller voxels would also increase the resolution of the produced envelope and as a consequence would facilitate the interpretation of the results.

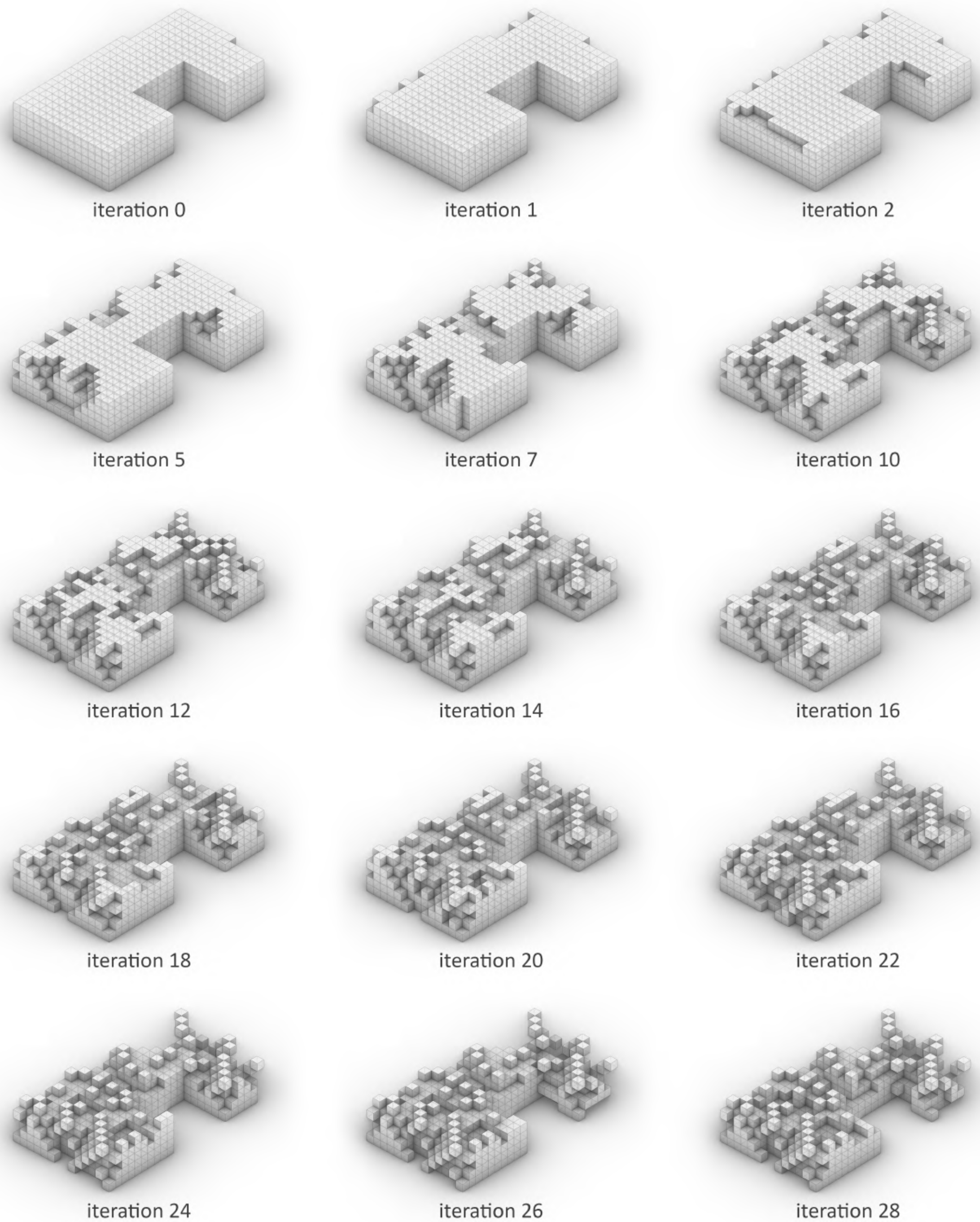

Figure 71: *Case Study - Intermediate steps/iterations of the Massing process*

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