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On the Velocity Potential in Michell's System and the Configuration of the Wave-ridges due to a Moving Ship.

(Non-uniform Theory of Wave Resistance-4)

By Tatsuro HANAOKA, *Member*

Introduction.

We have often experienced that the velocity potential, which the author gave in his other papers[1], is too much complicated for us to understand the wave-system due to the ship moving with constant velocity describing harmonic oscillations.

In this paper the author transformed the velocity potential in Michell's system. It will be very useful for a numerical calculation and an explanation of the wave-system, because it is written in convergent form of integral and classifies the wave-systems. And also we can obtain the configuration of the wave-ridges from the velocity potential by means of the saddle point method. We shall be newly interested in Michell's system from this matter.

1. The velocity potential in Michell's system due to the steady motion of a ship.

The author has already shown[2] that Havelock's velocity potential due to the steady motion of a ship can be transformed in Michell's one.

The velocity potential due to the pressure advancing with constant velocity V over the surface and the velocity potential[3] due to the ship advancing obliquely to her plane of symmetry with constant velocity can de transformed in Michell's system dy the same procedure. I will neglect the course of transformation and show the resulting expressions.

The Michell's systems can be written

$$
\begin{aligned}
\Phi = & \frac{1}{\pi^2 \rho V \kappa'} \iint p(x',y') \int_0^\infty \int_0^\infty \frac{m e^{-|y-y'|\sqrt{m^2+n^2}} \cos(nz+\varepsilon) \cos\varepsilon \sin m(x-x')}{\sqrt{m^2+n^2}} \, dm\, dn\, ds \\
& + \frac{1}{\pi \rho V \kappa'^2} \iint p(x',y') \int_0^{\kappa'} \frac{m^2 e^{zm^2/\kappa' - |y-y'| m \sqrt{1-m^2/\kappa'^2}} \sin m(x-x')}{\sqrt{1-m^2/\kappa'^2}} \, dm\, ds \\
& - \frac{1}{\pi \rho V \kappa'^2} \iint p(x',y') \int_{\kappa 1}^\infty \frac{m^2 e^{zm^2/\kappa'} \cos\{m(x-x') - |y-y'| m \sqrt{m^2/\kappa'^2-1}\}}{\sqrt{m^2/\kappa'^2-1}} \, dm\, ds
\end{aligned}
\tag{1·1}
$$

and

$$
\begin{aligned}
\Phi = & \frac{sgn\, y}{\pi^2 V} \iint (\phi_r' - \phi_l') \int_0^\infty \int_0^\infty \frac{1}{m} e^{-|y|\sqrt{m^2+n^2}} \cos(nz+\varepsilon) \cos(nz'+\varepsilon) \sin m(x-x')\, dm\, dn\, ds \\
& + \frac{sgn\, y}{\pi V \kappa'} \iint (\phi_r' - \phi_l') \int_0^{\kappa'} m\, e^{(z+z')m^2/\kappa' - |y| m \sqrt{1-m^2/\kappa'^2}} \sin m(x-x')\, dm\, ds \\
& + \frac{sgn\, y}{4 \pi V} \iint (\phi_r' - \phi_l') \left\{ \frac{|y|}{y^2+(z-z')^2} + \frac{|y|}{y^2+(z+z')^2} \right\} ds \\
& + \frac{sgn\, y}{\pi V \kappa'} \iint (\phi_r' - \phi_l') \int_{\kappa'}^\infty m\, e^{(z+z')m^2/\kappa'} \sin\left\{ m(x-x') - |y| m \sqrt{m^2/\kappa'^2-1} \right\} dm\, ds
\end{aligned}
\tag{1·2}
$$

where $\tan\varepsilon = -m^2/\kappa' n$, respectively.

2. The velocity potential in Michell's system due to the unsteady motion of a ship.

1) T.Hanaoka; Non-uniform Theory of Wave Resistance-2 and 3, in 1951 and 1952.
2), 3) T.Hanaoka; On the Fundamental Theory of the wave Resistance of the Ship Advancing with Constant Velocity, in 1951.

The velocity potential due to the unsteady motion of a ship can also de transformed in Michell's system by the similar procedure to the case of the steady motion.

I will take example by the symmetric velocity field of a deep draught ship and show the procedure.

The velocity potential is written

$$\begin{aligned}\Phi=&-\frac{e^{i\nu t}}{2\pi}\iint w(x',z')\left(\frac{1}{R_1}-\frac{1}{R_2}\right)ds\\&+\frac{\kappa' e^{i\nu t}}{2\pi}\iint w(x',z')\int_{-\pi/2}^{\pi/2}\oint_0^{\infty}ke^{k(z+z')}\left\{\frac{e^{-ik\varpi}}{(k\cos\alpha-\nu/V)^2-\kappa'k}+\frac{e^{ik\varpi}}{(k\cos\alpha+\nu/V)^2-\kappa'k}\right\}dk\,d\alpha\,ds\\&-\frac{i\kappa' e^{i\nu t}}{2\pi}\iint w(x',z')\int_{-\pi/2}^{\pi/2}\frac{\sec^2\alpha}{a_1'-a_2'}\left\{a_1'\,e^{a_1'(z+z')-ia_1'\varpi}-a_2'\,e^{a_2'(z+z')-ia_2'\varpi}\right\}d\alpha\,ds\\&+\frac{i\kappa' e^{i\nu t}}{2\pi}\iint w(x',z')\left\{\int_{-\pi/2}^{-\alpha_1}+\int_{\alpha_1}^{\pi/2}\right\}\frac{\sec^2\alpha}{b_1'-b_2'}\left\{b_1'\,e^{b_1'(z+z')+ib_1'\varpi}+b_2'\,e^{b_2'(z+z')+ib_2'\varpi}\right\}d\alpha\,ds\ \text{[4]}\end{aligned}\tag{2·1}$$

where $\Omega=\nu/V\kappa'$, $\varpi=(x-x')\cos\alpha+y\sin\alpha$,

$$\left.\begin{matrix}a_1'\\a_2'\end{matrix}\right)=\frac{\kappa'(1+2\Omega\cos\alpha\pm\sqrt{1+4\Omega\cos\alpha}}{2\cos^2\alpha},\quad\left.\begin{matrix}b_1'\\b_2'\end{matrix}\right)=\frac{\kappa'(1-2\Omega\cos\alpha\pm\sqrt{1-4\Omega\cos\alpha})}{2\cos^2\alpha}\tag{2·2}$$

$$\alpha_1=\begin{pmatrix}0, & \Omega<1/4,\\ \cos^{-1}1/4\Omega, & \Omega>1/4,\end{pmatrix}\qquad\left.\begin{matrix}R_1\\R_2\end{matrix}\right)=\sqrt{(x-x')^2+y^2+(z\mp z')^2}$$

If Φ_1, Φ_2 and Φ_3 denote the first, second and third terms of (2·1) resepectively, Φ_1 is trasformed in Michell's system

$$\Phi_1=-\frac{e^{i\nu t}}{\pi^2}\iint w(x',z')\int_0^{\infty}\int_0^{\infty}\frac{e^{-|y|\sqrt{m^2+n^2}}}{\sqrt{m^2+n^2}}\{e^{-im(x-x')}+e^{im(x-x')}\}\sin nz\sin nz'\,dm\,dn\,dS\tag{2·3}$$

by the very same procedure as in the steady motion.

On writing $m\sec\alpha$ for k and then n'/m for $\sec\alpha$ in Φ_2, we get,

$$\begin{aligned}\Phi_2=&\frac{\kappa' e^{i\nu t}}{\pi^2}\iint w(x',z')\Bigg[R_e\int_0^{\infty}\oint_0^{\infty}e^{(z+z')n'+i|y|\sqrt{n'^2-m^2}}\left\{\frac{1}{(m-\nu/V)^2-\kappa'n'}+\frac{1}{(m+\nu/V)^2-\kappa'n'}\right\}\\&\times\frac{n'\cos m(x-x')}{\sqrt{n'^2-m^2}}dn'dm-iR_e\int_0^{\infty}\oint_0^{\infty}e^{(z+z')n'+i|y|\sqrt{n'^2-m^2}}\left\{\frac{1}{(m-\nu/V)^2-\kappa'n'}\right.\\&\left.-\frac{1}{(m+\nu/V)^2-\kappa'n'}\right\}\frac{n'\sin m(x-x')}{\sqrt{n'^2-m^2}}dn'dm\Bigg]dS\end{aligned}\tag{2·4}$$

since the integrant of n' is pure imaginary in the range $0<n'<m$.

Now, we can get the formula

$$\oint_0^{\infty}\frac{n'e^{n'(z+z')+i|y|\sqrt{n'^2-m^2}}}{\{(m\mp\nu/V)^2-\kappa'n'\}\sqrt{n'^2-m^2}}dn'=i\int_0^{\infty}\frac{ne^{in(z+z')-|y|\sqrt{m^2+n^2}}}{(m\mp\nu/V)^2-i\kappa'n}dn+\frac{1}{2}\,\text{Res.}\tag{2·5}$$

where

$$\frac{1}{2}\,\text{Res.}=-\frac{i\pi(m\mp\nu/V)^2}{\kappa'}\cdot\frac{e^{(z+z')(m\mp\nu/V)^2/\kappa'+i|y|\sqrt{(m\mp\nu/V)^4/\kappa'^2-m^2}}}{\sqrt{(m\mp\nu/V)^4/\kappa'^2-m^2}},\qquad(m\mp\nu/V)^2>\kappa'n$$

$$\frac{1}{2}\,\text{Res.}=-\frac{\pi(m\mp\nu/V)^2}{\kappa'}\cdot\frac{e^{(z+z')(m\mp\nu/V)^2/\kappa'-|y|\sqrt{m^2-(m\mp\nu/V)^4/\kappa'^2}}}{\sqrt{m^2-(m\mp\nu/V)^4/\kappa'^2}},\qquad(m\mp\nu/V)^2<\kappa'n$$

by means of contour integration. And we can easily prove that

$$(m\mp\nu/V)^2>\kappa'm\ \text{ and }\ (m\mp\nu/V)^2<\kappa'm$$

coincide with

$$\begin{pmatrix}m<a_2 \ \text{ or }\ m>a_1\\ m<b_2\ \text{ or }\ m>b_1\end{pmatrix}\ \text{ and }\ \begin{pmatrix}a_2<m<a_1\\ b_2<m<b_1\end{pmatrix}$$

respectively, where

$$\left.\begin{matrix}a_1\\a_2\end{matrix}\right)=\frac{\kappa'(1+2\Omega\pm\sqrt{1+4\Omega})}{2},\quad\left.\begin{matrix}a_1\\b_2\end{matrix}\right)=\frac{\kappa'(1-2\Omega\pm\sqrt{1-4\Omega})}{2}\tag{2·6}$$

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4) refer 1).

Hence, when we substitute (2·5) in (2·4), we get

$$\begin{aligned}\Phi_2=&-\frac{\kappa' e^{i\nu t}}{\pi^2}\iint w(x',z')\int_0^\infty\int_0^\infty e^{-|y|\sqrt{m^2+n^2}}\left\{\frac{(m-\nu/V)^2\sin n(z+z')+\kappa' n\cos n(z+z')}{(m-\nu/V)^4+\kappa'^2n^2}e^{-im(x-x')}\right.\\&\left.+\frac{(m+\nu/V)^2\sin n(z+z')+\kappa' n\cos n(z+z')}{(m+\nu/V)^4+\kappa'^2n^2}e^{im(x-x')}\right\}\frac{n}{\sqrt{m^2+n^2}}\,dm\,dn\,dS\\&-\frac{e^{i\nu t}}{\pi\kappa'}\iint w(x',z')\left[-\left\{\int_0^{a_2}+\int_{a_1}^\infty\right\}\frac{(m-\nu/V)^2e^{(z+z')(m-\nu/V)^2/\kappa'-im(x-x')}}{\sqrt{(m-\nu/V)^4/\kappa'^2-m^2}}\sin|y|\sqrt{(m-\nu/V)^4/\kappa'^2-m^2}\,dm\right.\\&-\left\{\int_0^{b_2}+\int_{b_1}^\infty\right\}\frac{(m+\nu/V)^2e^{(z+z')(m+\nu/V)^2/\kappa'+im(x-x')}}{\sqrt{(m+\nu/V)^4/\kappa'^2-m^2}}\sin|y|\sqrt{(m+\nu/V)^4/\kappa'^2-m^2}\,dm\\&+\int_{a_2}^{a_1}\frac{(m-\nu/V)^2e^{(z+z')(m-\nu/V)^2/\kappa'-|y|\sqrt{m^2-(m-\nu/V)^4/\kappa'^2}-im(x-x')}}{\sqrt{m^2-(m-\nu/V)^4/\kappa'^2}}dm\\&\left.+\int_{b_2}^{b_1}\frac{(m+\nu/V)^2e^{(z+z')(m+\nu/V)^2/\kappa'-|y|\sqrt{m^2-(m+\nu/V)^4/\kappa'^2}+im(x-x')}}{\sqrt{m^2-(m+\nu/V)^4/\kappa'^2}}dm\right]dS\end{aligned}\tag{2·7}$$

as Michell's system of Φ_2.

Next, I will transform Φ_3 in Michell's system. If we put

$$\left.\begin{array}{ll}a_1'\cos\alpha=m_1\ , & b_1'\cos\alpha=m_3\\ a_2'\cos\alpha=m_2\ , & b_2'\cos\alpha=m_4\end{array}\right\}\tag{2·8}$$

we see that

$$\left.\begin{array}{l}m_1=a_1\ ,\quad m_2=a_2\ ,\quad m_3=b_1\ ,\quad m_4=b_2\ ,\quad \text{when } \alpha=0\\ m_1=\infty\ ,\quad m_2=0\ ,\quad m_3=\infty\ ,\quad m_4=0\ ,\quad \text{when } \alpha=\pi/2\end{array}\right\}\tag{2·9}$$

and $m_1,\ m_3>\nu/V,\ m_2,\ m_4<\nu/V$, when $0<\alpha<\pi/2$.

When we introduce (2·8) and (2·9) in (2.7) to interchange integral variable α with m, we have

$$\begin{aligned}\Phi_3=&-\frac{ie^{i\nu t}}{\pi\kappa'}\iint w(x_1',z')\left[\left\{-\int_0^{a_2}+\int_{a_1}^\infty\right\}\frac{(m-\nu/V)^2e^{(z+z')(m-\nu/V)^2/\kappa'-im(x-x')}}{\sqrt{(m-\nu/V)^4/\kappa'^2-m^2}}\cos|y|\sqrt{(m-\nu/V)^4/\kappa'^2-m^2}\,dm\right.\\&\left.-\left\{\int_0^{b_2}+\int_{b_1}^\infty\right\}\frac{(m+\nu/V)^2e^{(z+z')(m+\nu/V)^2/\kappa'+im(x-x')}}{\sqrt{(m+\nu/V)^4/\kappa'^2-m^2}}\cos|y|\sqrt{(m+\nu/V)^4/\kappa'^2-m^2}\,dm\right]dS\end{aligned}\tag{2·10}$$

If we sum up (2·3), (2·7) and (2·10), and keep in good shape, introducing new variables ε_1 and ε_2 which are defined by the formulas

$$\tan\varepsilon_1=-(m-\nu/V)^2/\kappa'n\ ,\quad \tan\varepsilon_2=-(m+\nu/V)^2/\kappa'n$$

we get

$$\begin{aligned}\Phi=&-\frac{e^{i\nu t}}{\pi^2}\iint w(x',z')\int_0^\infty\int_0^\infty\frac{e^{-|y|\sqrt{m^2+n^2}}}{\sqrt{m^2+n^2}}\left\{\cos(nz+\varepsilon_1)\cos(nz'+\varepsilon_1)e^{-im(x-x')}\right.\\&\left.+\cos(nz+\varepsilon_2)\cos(nz'+\varepsilon_2)e^{im(x-x')}\right\}dm\,dn\,dS\\&-\frac{e^{i\nu t}}{\pi\kappa'}\iint w(x',z')\left[\int_{a_2}^{a_1}\frac{(m-\nu/V)^2e^{(z+z')(m-\nu/V)^2/\kappa'-y\sqrt{m^2-(m-\nu/V)^4/\kappa'^2}-im(x-x')}}{\sqrt{m^2-(m-\nu/V)^4/\kappa'^2}}dm\right.\\&\left.+\int_{b_2}^{b_1}\frac{(m+\nu/V)^2e^{(z+z')(m+\nu/V)^2/\kappa'-|y|\sqrt{m^2-(m+\nu/V)^4/\kappa'^2}+im(x-x')}}{\sqrt{m^2-(m+\nu/V)^4/\kappa'^2}}dm\right]dS\\&-\frac{ie^{i\nu t}}{\pi\kappa'}\iint w(x',z')\left[-\int_0^{a_2}\frac{(m-\nu/V)^2e^{(z+z')(m-\nu/V)^2/\kappa'-im(x-x')-i|y|\sqrt{(m-\nu/V)^4/\kappa'^2-m^2}}}{\sqrt{(m-\nu/V)^4/\kappa'^2-m^2}}dm\right.\\&+\int_{a_1}^\infty\frac{(m-\nu/V)^2e^{(z+z')(m-\nu/V)^2/\kappa'-im(x-x')+i|y|\sqrt{(m-\nu/V)^4/\kappa'^2-m^2}}}{\sqrt{(m-\nu/V)^4/\kappa'^2-m^2}}dm\\&\left.-\left\{\int_0^{b_2}+\int_{b_1}^\infty\right\}\frac{(m+\nu/V)^2e^{(z+z')(m+\nu/V)^2/\kappa'+im(x-x')-i|y|\sqrt{(m+\nu/V)^4/\kappa'^2-m^2}}}{\sqrt{(m+\nu/V)^4/\kappa'^2-m^2}}\right]dS\end{aligned}\tag{2·11}$$

as the velocity potential in Michell's system concerning the symmetric velocity field of a deep draught ship.

In the other papers [5] the author has already given

5) refer 1)

$$\Phi=\frac{ie^{i\nu t}}{4\pi^2\rho V}\iint p(x',y')\int_{-\pi/2}^{\pi/2}\oint_0^\infty k e^{kz}\left\{-\frac{(k\cos\alpha-\nu/V)e^{-ik\varpi}}{(k\cos\alpha-\nu/V)^2-\kappa' k}+\frac{(k\cos\alpha+\nu/V)e^{ik\varpi}}{(k\cos\alpha+\nu/V)^2-\kappa' k}\right\}dk\,d\alpha\,dS$$
$$-\frac{e^{i\nu t}}{4\pi\rho V}\iint p(x',y')\int_{-\pi/2}^{\pi/2}\sec\alpha\left(M_1' e^{a_1' z-ia_1'\varpi}-M_2' e^{a_2' z-ia_2'\varpi}\right)d\alpha\,dS$$
$$-\frac{e^{i\nu t}}{4\pi\rho V}\iint p(x',y')\left\{\int_{-\pi/2}^{-\alpha_1}+\int_{\alpha_1}^{\pi/2}\right\}\sec\alpha\left(N_1' e^{b_1' z+ib_1'\varpi}+N_2' e^{b_2' z+ib_2'\varpi}\right)d\alpha\,dS \tag{2.12}$$

and

$$\Phi=\frac{e^{i\nu t}}{4\pi V}\iint \phi'(x',z')\int_{-\infty}^{x} e^{-i\nu(x-\chi)/V}\left(\frac{y}{R_1^3}+\frac{y}{R_2^3}\right)d\chi\,dS$$
$$+\frac{e^{i\nu t}}{4\pi^2 V}\iint \phi'(x',z')\int_{-\pi/2}^{\pi/2}\oint_0^\infty k e^{k(z+z')}\sin\alpha\left\{\frac{(k\cos\alpha-\nu/V)e^{-ik\varpi}}{(k\cos\alpha-\nu/V)^2-\kappa' k}+\frac{(k\cos\alpha+\nu/V)e^{-ik\varpi}}{(k\cos+\nu/V)^2-\kappa' k)}\right\}dk\,d\alpha\,dS$$
$$-\frac{ie^{i\nu t}}{4\pi V}\iint \phi'(x',z')\int_{-\pi/2}^{\pi/2}\tan\alpha\left(M_1' e^{a_1'(z+z')-ia_1'\varpi}-M_2' e^{a_2'(z+z')-ia_2'\varpi}\right)d\alpha\,dS$$
$$+\frac{ie^{i\nu t}}{4\pi V}\iint \phi'(x',z')\left\{\int_{-\pi/2}^{-\alpha_1}+\int_{\alpha_1}^{\pi/2}\right\}\tan\alpha\left(N_1' e^{b_1'(z+z')+ib_1'\varpi}+N_2' e^{b_2'(z+z')+ib_2'\varpi}\right)d\alpha\,dS \tag{2.13}$$

where

$$M_1'=\frac{a_1'^2-a_1'\dfrac{\nu}{V}\sec\alpha}{a_1'-a_2'}\qquad N_1'=\frac{b_1'^2+b_1'\dfrac{\nu}{V}\sec\alpha}{b_1'-b_2'}$$

$$M_2'=\frac{a_2'^2-a_2'\dfrac{\nu}{V}\sec\alpha}{a_1'-a_2'}\qquad N_2'=\frac{b_2'^2+b_2'\dfrac{\nu}{V}\sec\alpha}{b_1'-b_2'}$$

as a velocity potential due to the unsteady motion of a shallow draught ship and a velocity potential due to the unsymmetrical motion of a deep draught ship respectively. These formulas can also be transformed in Michell's system by the same process. I will show only the results.

Namely, (2·12) and (2·13) can be written

$$\Phi=\frac{ie^{i\nu t}}{2\pi^2\rho V\kappa'}\iint p(x',y')\int_0^\infty\int_0^\infty\frac{e^{-|y-y'|\sqrt{m^2+n^2}}}{\sqrt{m^2+n^2}}\{(m-\nu/V)\cos(nz+\varepsilon_1)\cos\varepsilon_1 e^{-im(x-x')}$$
$$-(m+\nu/V)\cos(nz+\varepsilon_2)\cos\varepsilon_2 e^{im(x-x')}\}dm\,dn\,dS$$
$$+\frac{ie^{i\nu t}}{2\pi\rho V\kappa'^2}\iint p(x',y')\Big[\int_{a_2}^{a_1}\frac{(m-\nu/V)^3}{\sqrt{m^2+(m-\nu/V)^4/\kappa'^2}}e^{z(m-\nu/V)^2/\kappa'-|y-y'|\sqrt{m^2-(m-\nu/V)^4/\kappa'^2}-im(x-x')}$$
$$-\int_{b_2}^{b_1}\frac{(m+\nu/V)^3}{\sqrt{m^2-(m+\nu/V)^4/\kappa'^2}}e^{z(m+\nu/V)^2/\kappa'-|y-y'|\sqrt{m^2-(m+\nu/V)^4/\kappa'^2}+im(x-x')}dm\Big]dS$$
$$+\frac{e^{i\nu t}}{2\pi\rho V\kappa'^2}\iint p(x',y')\Big[\int_0^{a_2}\frac{(m-\nu/V)^3}{\sqrt{(m-\nu/V)^4/\kappa'^2-m^2}}e^{z(m-\nu/V)^2/\kappa'-im(x-x')-i|y-y'|\sqrt{(m-\nu/V)^4/\kappa'^2-m^2}}dm$$
$$+\int_{a_1}^\infty\frac{(m-\nu/V)^2}{\sqrt{(m-\nu/V)^4/\kappa'^2-m^2}}e^{z(m-\nu/V)^2/\kappa'-im(x-x')+i|y-y'|\sqrt{(m-\nu/V)^4/\kappa'^2-m^2}}dm$$
$$+\left\{\int_0^{b_2}+\int_{b_1}^\infty\right\}\frac{(m+\nu/V)^3}{\sqrt{(m+\nu/V)^4/\kappa'^2-m^2}}e^{z(m+\nu/V)^2/\kappa'+im(x-x')-i|y-y'|\sqrt{(m+\nu/V)^4/\kappa'^2-m^2}}dm \tag{2.14}$$

and

$$\Phi=\frac{ie^{i\nu t}\,\mathrm{sgn}\,y}{2\pi^2 V}\iint\phi'(x',z')\int_0^\infty\oint_0^\infty e^{-|y|\sqrt{m^2+n^2}}\left\{\frac{e^{-im(x-x')}}{m-\nu/V}\cos(nz+\varepsilon_1)\cos(nz'+\varepsilon_1)\right.$$
$$\left.+\frac{e^{im(x-x')}}{m+\nu/V}\cos(nz+\varepsilon_2)\cos(nz'+\varepsilon_2)\right\}dm\,dn\,dS$$
$$+\frac{ie^{i\nu t}\,\mathrm{sgn}\,y}{2\pi V\kappa'}\iint\phi'(x',z')\left\{\int_{a_2}^{a_1}(m-\nu/V)e^{(z+z')(m-\nu/V)^2/\kappa'-|y|\sqrt{m^2-(m-\nu/V)^4/\kappa'^2}-im(x-x')}dm\right.$$
$$\left.-\int_{b_2}^{b_1}(m+\nu/V)e^{(z+z')(m+\nu/V)^2/\kappa'-|y|\sqrt{m^2-(m+\nu/V)^4/\kappa'^2}+im(x-x')}dm\right\}dS$$

$$+\frac{e^{i\nu t}\operatorname{sgn} y}{2\pi V}\iint\phi'(x',z')e^{-i\nu(x-x')/V}\int_0^\infty e^{-|y|\sqrt{(\nu/V)^2+n^2}}\cos nz\cos nz'\,dn\,dS$$

$$+\frac{ie^{i\nu t}\operatorname{sgn} y}{2\pi V\kappa'}\iint\phi'(x',z')\Big[\int_0^{a_2}(m-\nu/V)e^{(z+z')(m-\nu/V)^2/\kappa'-im(x-x')-i|y|\sqrt{(m-\nu/V)^4/\kappa'^2-m^2}}dm$$

$$+\int_{a_1}^\infty(m-\nu/V)e^{(z+z')(m-\nu/V)^2/\kappa'-im(x-x')+i|y|\sqrt{(m-\nu/V)^4/\kappa'^2-m^2}}dm$$

$$-\left\{\int_0^{b_2}+\int_{b_1}^\infty\right\}(m+\nu/V)e^{(z+z')(m+\nu/V)^2/\kappa'+im(x-x')+i|y|\sqrt{(m+\nu/V)^4/\kappa'^2-m^2}}dm\Big]dS \qquad (2\cdot14)$$

in Michell's system.

3. The configuration of the wave-ridges due to the steady motion of a ship.

The configuration of the wave-ridges due to the steady motion of a ship was theoretically investigated by Lord Kelvin. But we shall see that tbe same result is obtained from the velocity potential in Michell's system in this section.

Let us suppose that we have a pressure-point moving with constant velocity V along the axis of x in the negative direction. The first and second terms of (1·1) give the local disturbance, but the third term expresses the disturbance which extends to infinity. So we may analyse only the third term for this subject. Because surface elevation is given by the formula

$$\zeta=-\frac{V}{g}\left[\frac{\partial\Phi}{\partial x}\right]_{z=0},$$ the height of the wave is written

$$\zeta=-\frac{P_0}{\pi\rho g\kappa'^2}\int_{\kappa'}^\infty\frac{m^3\sin\{mx-|y|m\sqrt{m^2/\kappa'^2-1}\}}{\sqrt{m^2/\kappa'^2-1}}dm \qquad (3\cdot1)$$

where P_0 denotes the total disturbance-pressure, from (1·1).

To obtain an approximate formula of (3·1), we take the saddle point method.

Consequently,

$$\zeta=-\frac{P_0}{\pi\rho g\kappa'^2}\sum_{n=1}^{k}\frac{m_n^3}{\sqrt{m_n^2/\kappa'^2-1}}\left\{\frac{2\pi}{|F''(m_n)|}\right\}^{1/2}\sin\left\{F(m_n)+\operatorname{sgn}F''(m_n)\frac{\pi}{4}\right\} \qquad (3\cdot2)$$

where

$$F(m)=mx-|y|m\sqrt{m^2/\kappa'^2-1} \qquad (3\cdot3)$$

and m_n denote the roots of the equation $F'(m)=0$ in the range $\kappa'<m_n<\infty$, and k denote the number of the roots. If we put

$$m_n/\kappa'=\sec\theta_n,\quad 0<\theta_n<\pi/2 \qquad (3\cdot4)$$

$F''(m)$ is written

$$F''(m_n)=\frac{|y|}{\kappa'}\cdot\frac{1-3\sin^2\theta_n}{\sin^3\theta_n}$$

as a function of θ_n

Therefore, if we write $\sin^{-1}1/\sqrt{3}=\theta_0$, we get

$$\left.\begin{aligned}&F''(m_n)>0\quad\text{when}\quad\theta_n<\theta_0,\\&F''(m_n)<0\quad\text{when}\quad\theta_n>\theta_0.\end{aligned}\right\} \qquad (3\cdot5)$$

Next, we get

$$\frac{|y|}{x}=\frac{\tan\theta_n}{2\sec^2\theta_n-1} \qquad (3\cdot6)$$

from $F'(m_n)=0$. The relation between θ_n and $|y|/x$ is shown in fig. 1. We see that $k=2$ in (3·2), because one value of $|y|/x$ corresponds to two values of θ when $|y|/x<1/2\sqrt{2}$ and we can prove without difficulty that $\theta_1<\theta_0$ for one of θ_n and $\theta_2>\theta_0$ for the other.

And also substituting (3.6) in $F(m_n)$, we have

$$F(m_n)=\kappa'|y|/\sin\theta_n\cos^2\theta_n \qquad (3\cdot7)$$

Hence, the resulting expression of (3·2) is written

$$\zeta=-\frac{P_0\kappa'}{\pi\rho g}\frac{\sec^3\theta_1}{\tan\theta_1}\left\{\frac{2\pi\kappa'\sin^3\theta_1}{|y|(1-3\sin^2\theta_1)}\right\}^{1/2}\sin\{\kappa'|y|/\sin\theta_1\cos^2\theta_1+\pi/4\}$$
$$-\frac{P_0\kappa'}{\pi\rho g}\frac{\sec^3\theta_2}{\tan\theta_2}\left\{\frac{2\pi\kappa'\sin^3\theta_2}{|y|(3\sin^2\theta_2-1)}\right\}^{1/2}\sin\{\kappa'|y|/\sin\theta_2\cos^2\theta_2-\pi/4\} \tag{3·8}$$

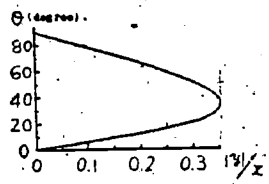


Fig. 1

under the consideration of (3·5) and (3·7). The two terms give the parts due to the transverse and diverging waves respectively. At $\theta_n=\theta_0$, where the two systems combine, there is a phase-difference of a quarter-period between them. The wave length in the direction of the axis of y is written

$$|y|=\frac{2\pi}{\kappa'}\sin\theta_n\cos^2\theta_n \tag{3·9}$$

from (3·8). Substituting (3·9) in (3·6), we get

$$x=\frac{2\pi}{\kappa'}\cos\theta_n(1+\sin^2\theta_n) \tag{3·10}$$

as a wave length in the direction of the axis of x.

(3·9) and (3·10) are also written

$$\left.\begin{aligned} x&=\frac{\pi}{2\kappa'}(5\cos\theta_n-\cos3\theta_n)\\ |y|&=\frac{\pi}{2\kappa'}(\sin\theta_n+\sin3\theta_n)\end{aligned}\right\} \tag{3·11}$$

This is the well-known formula for the configuration of the wave-ridges. In (3·8), interchanging the notations $|y|/\sin\theta_n$ with ϖ_n and $\sec\theta_n$ with $V\tau_n/2\varpi_n$, the expression of ζ coincides with the result already given[6].

4. The configuration of the wave-ridgas due to the unsteady motion of a ship

We can analyse the wave-system and get the configuration of the wave-ridges in non-uniform fluid field by the similar process to the case of the steady motion. In this case we may also consider only the third term of (2·14) which expresses the disturbance to extend to infinity. Since surface elevation is given by the formula

$$\zeta=-\frac{1}{g}\left[\frac{\partial\Phi}{\partial t}+V\frac{\partial\Phi}{\partial x}\right]_{z=0}$$

we get

$$\zeta=-\frac{i\bar{P}_0e^{i\nu t}}{2\pi\rho g\kappa'^3}\left\{-\int_0^{a_2}\frac{(m-\nu/V)^4}{\sqrt{(m-\nu/V)^4/\kappa'^2-m^2}}e^{-imx-i|y|\sqrt{(m-\nu/V)^4/\kappa'^2-m^2}}dm\right.$$
$$+\int_{a_1}^{\infty}\frac{(m-\nu/V)^4}{\sqrt{(m-\nu/V)^4/\kappa'^2-m^2}}e^{-imx+i|y|\sqrt{(m-\nu/V)^4/\kappa'^2-m^2}}dm$$
$$-\int_0^{b_2}\frac{(m+\nu/V)^4}{\sqrt{(m+\nu/V)^4/\kappa'^2-m^2}}e^{imx-i|y|\sqrt{(m+\nu/V)^4/\kappa'^2-m^2}}dm$$
$$\left.-\int_{b_1}^{\infty}\frac{(m+\nu/V)^4}{\sqrt{(m+\nu/V)^4/\kappa'^2-m^2}}e^{imx-i|y|\sqrt{(m+\nu/V)^4/\kappa'^2-m^2}}dm\right\} \tag{4·1}$$

from (2·14), where $\bar{P}_0e^{i\nu t}$ denotes the total disturbance-pressure. We see that the wave-system is constructed of four kinds of wave-groups, and we may call the wave-systems which are expressed by the first, second, third and fourth terms of (4·1) A_2-, A_1-, B_2- and B_1-waves respectively.

Applying the saddle point method to (4·1), it is written

$$\zeta=-\frac{i\bar{P}_0e^{i\nu t}}{2\pi\rho g\kappa'^2}\sum_{n=1}^{k}\left[-\frac{(m_{A_2n}-\nu/V)^4}{\sqrt{(m_{A_2n}-\nu/V)^4/\kappa'^2-m^2_{A_2n}}}\left\{\frac{2\pi}{|F''_{A_2}(m_{A_2n})|}\right\}^{1/2}e^{i\{F_{A_2}(m_{A_2n})+\mathrm{sgn}\,F''_{A_2}(m_{A_2n})\pi/4\}}\right.$$

6) H. Lamb, Hydrodynamics, 1932, p. 436.

$$+\frac{(m_{A_1n}-\nu/V)^4}{\sqrt{(m_{A_1n}-\nu/V)^4/\kappa'^2-m^2_{A_1n}}}\left\{\frac{2\pi}{|F''_{A_1}(m_{A_1n})|}\right\}^{1/2}e^{i\{F_{A_1}(m_{A_1n})+\text{sgn}\,F''_{A_1}(m_{A_1n})\pi/4\}}$$
$$-\frac{(m_{B_2n}+\nu/V)^4}{\sqrt{(m_{B_2n}+\nu/V)^4/\kappa'^2-m^2_{B_2n}}}\left\{\frac{2\pi}{|F''_{B_2}(m_{B_2n})|}\right\}^{1/2}e^{i\{F_{B_2}(m_{B_2n})+\text{sgn}\,F''_{B_2}(m_{B_2n})\pi/4\}}$$
$$-\frac{(m_{B_1n}+\nu/V)^4}{\sqrt{(m_{B_1n}+\nu/V)^4/\kappa'^2-m^2_{B_1n}}}\left\{\frac{2\pi}{|F''_{B_1}(m_{B_1n})|}\right\}^{1/2}e^{i\{F_{B_1}(m_{B_1n})+\text{sgn}\,F''_{B_1}(m_{B_1n})\pi/4\}} \quad (4\cdot2)$$

where

$$\left.\begin{aligned}F_{A_2}(m)&=-mx-|y|\sqrt{(m-\nu/V)^4/\kappa'^2-m^2}, && 0<m<a_2\\ F_{A_1}(m)&=-mx+|y|\sqrt{(m-\nu/V)^4/\kappa'^2-m^2}, && a_1<m<\infty\\ \begin{matrix}F_{B_2}(m)\\F_{B_1}(m)\end{matrix}&\Big\}=mx-|y|\sqrt{(m+\nu/V)^4/\kappa'^2-m^2}, && \begin{pmatrix}0<m<b^2\\ b_1<m<\infty\end{pmatrix}\end{aligned}\right\}\quad(4\cdot3)$$

and $m_{A_2n}, m_{A_1n}, m_{B_2n}, m_{B_1n}$ denote the roots of the equation $F'_{A_2}(m)=0$, $F'_{A_1}(m)=0\cdots$etc. in the ranges respectively.

If we put

$$\left.\begin{aligned}&(m_{A_1,A_2n}-\nu/V)^2/\kappa'_{A_1,A_2n}=\sec\theta_{A_1,A_2n}, \quad 0<\theta_{A_1,A_2n}<\pi/2\\ &\text{for } A_1\text{-}, A_2\text{-waves, and}\\ &(m_{B_1,B_2n}-\nu/V)^2/\kappa' m_{B_1,B_2n}=\sec\theta_{B_1,B_2n}\begin{pmatrix}0<\theta_{B_1,B_2n}<\pi/2, \text{ when } \Omega<1/4\\ \cos^{-1}1/4\Omega<\theta_{B_1,B_2n}<\pi/2, \text{ when } \Omega>1/4\end{pmatrix}\end{aligned}\right\}\quad(4\cdot4)$$

for B_1-, B_2-waves, we get

$$\left.\begin{aligned}&\frac{|y|}{x}=-\frac{\tan\theta_{A_2n}}{\tan^2\theta_{A_2n}-\sec^2\theta_{A_2n}\sqrt{1+4\Omega\cos\theta_{A_2n}}} && A_2\text{-wave}\\ &\frac{|y|}{x}=\frac{\tan\theta_{A_2n}}{\tan^2\theta_{A_1n}+\sec^2\theta_{A_1n}\sqrt{1+4\Omega\cos\theta_{A_1n}}} && A_1\text{-wave}\Big\}, 0<\theta_{A_2A_1n}<\pi/2\\ &\frac{|y|}{x}=\frac{\tan\theta_{B_2n}}{\tan^2\theta_{B_2n}-\sec^2\theta_{B_2n}\sqrt{1-4\Omega\cos\theta_{B_2n}}} && B^2\text{-wave}\quad 0<\theta_{B_2.B_1n}<\pi/2, \text{ when } \Omega<1/4\\ &\frac{|y|}{x}=\frac{\tan\theta_{B_1n}}{\tan^2\theta_{B_1n}+\sec^2\theta_{B_1n}\sqrt{1-4\Omega\cos\theta_{B_1n}}} && B_1\text{-wave}\quad \cos^{-1}1/4\Omega<\theta_{B_2.B_1n}<\pi/2, \text{ when } \Omega>1/4\end{aligned}\right\}\quad(4\cdot5)$$

from the conditions $F_{A_2}(m_{A_2})=0$, $F_{A_1}(m_{A_1})=0$, etc.

The relation between $\theta_{A_2,A_1,B_2,B_1n}$ and $|y|/x$ is shown in figs. 2 and 3. We see that $k=1$ or 2 in (4.2)

Namely,

$k=2$ for A_1-, B_1-waves, $k=1$ for A_2-, B_2-waves — when $\Omega<1/4$

$k=2$ for A_1-wave, $k=1$ for B_1-wave — when $\Omega>1/4$

$k=1$ for A_2-wave, $k=2$ for B_2-wave — when $1/4<\Omega<1/\sqrt{2}$

$k=2$ for A_2-wave, $k=1$ for B_2-wave — when $\Omega>1/\sqrt{2}$

We can get the boundary value $\Omega=1/\sqrt{2}$ from the condition

$$\left[\frac{\partial}{\partial\theta}\left(|y|/x\right)_{A_2,B_2}\right]_{\theta=\pi/2}=0$$

Next, substituting (4·4) in $F''(m)$, it is written

$$\left.\begin{aligned}F''_{B_2}(m_{A_2n})&=-\frac{2|y|}{\kappa'\sin\theta_{A_2n}}\left\{1-\frac{(1+4\Omega\cos\theta_{A_2n})(1+2\Omega\cos\theta_{A_2n}+\sqrt{1+4\Omega\text{ coe }\theta_{A_2n}})}{4\Omega^2\sin^2\theta_{A_2n}}\right\}\\ F''_{A_1}(m_{A_1n})&=\frac{2|y|}{\kappa'\sin\theta_{A_1n}}\left\{1-\frac{(1+4\Omega\cos\theta_{A_1n})(1+2\Omega\cos\theta_{A_1n}-\sqrt{1+4\Omega\cos\theta_{A_1n}})}{4\Omega^2\sin^2\theta_{A_1n}}\right\}\\ F''_{B_2}(m_{B_2n})&=-\frac{2|y|}{\kappa'\sin\theta_{B_2n}}\left\{1-\frac{(1-4\Omega\cos\theta_{B_2n})(1-2\Omega\cos\theta_{B_2n}+\sqrt{1-4\Omega\cos\theta_{B_2n}})}{4\Omega^2\sin^2\theta_{B_2n}}\right\}\\ F''_{B_1}(m_{B_1n})&=-\frac{2|y|}{\kappa'\sin\theta_{B_1n}}\left\{1-\frac{(1-4\Omega\cos\theta_{B_1n})(1-2\Omega\cos\theta_{B_1n}-\sqrt{1-4\Omega\cos\theta_{B_1n}})}{4\Omega^2\sin^2\theta_{B_1n}}\right\}\end{aligned}\right\}\quad(4\cdot6)$$

as the functions of θ. The relation between θ and $\kappa' F''/2|y|$ is shown in figs. 4 and 5. There is a phase-difference of a quarter-period in every wave-system, when $\theta \sim \kappa' F''/2|y|$ curves go across the axis of θ from one side to the other. We can prove without difficulty that $F''_{A_2}=0$, $F''_{A_1}=0$, …etc. are satisfied at the point θ_0, where $\frac{\partial}{\partial\theta}\left(|y|x\right)_{A_2}=0$. … etc.

When we subtitute (4·5) in $F(m)$ of (4·3) and express $F(m)$ as the functions of θ using (4·4), it is written

$$\left.\begin{aligned}
F_A(m_{A_2n}) &= -\frac{|y|\kappa'\sqrt{1+4\Omega\cos\theta_{A_2n}}\left(1+2\Omega\cos\theta_{A_2n}-\sqrt{1+4\Omega\cos\theta_{A_2n}}\right)}{2\tan\theta_{A_2n}\cos^3\theta_{A_2n}}\\
F_{A_1}(m_{A_1n}) &= -\frac{|y|\kappa'\sqrt{1+4\Omega\cos\theta_{A_1n}}\left(1+2\Omega\cos\theta_{A_1n}+\sqrt{1+4\Omega\cos\theta_{A_1n}}\right)}{2\tan\theta_{A_2n}\cos^3\theta_{A_2n}}\\
F_{B_2}(m_{B_2n}) &= -\frac{|y|\kappa'\sqrt{1+4\Omega\cos\theta_{B_2n}}\left(1-2\Omega\cos\theta_{B_2n}-\sqrt{1-4\Omega\cos\theta_{B_2n}}\right)}{2\tan\theta_{B_2n}\cos^3\theta_{B_2n}}\\
F_{B_1}(m_{B_1n}) &= -\frac{|y|\kappa'\sqrt{1-4\Omega\cos\theta_{B_1n}}\left(1-2\Omega\cos\theta_{B_1n}+\sqrt{1-4\Omega\cos\theta_{B_1n}}\right)}{2\tan\theta_{B_1n}\cos^3\theta_{B_1n}}
\end{aligned}\right\} \quad (4\cdot7)$$

Following the process of the steady case, we get the formulas to trace the configuration of the wave-ridges

$$\left.\begin{aligned}
x &= -\frac{\pi}{\kappa'}\cdot\frac{\left(\sin^2\theta-\sqrt{1+4\Omega\cos\theta}\right)\left(1+2\Omega\cos\theta+\sqrt{1+4\Omega\cos\theta}\right)}{\Omega^2\cos\theta\sqrt{1+4\Omega\cos\theta}}\\
|y| &= \frac{\pi}{\kappa'}\cdot\frac{\sin\theta\left(1+2\Omega\cos\theta+\sqrt{1+4\Omega\cos\theta}\right)}{\Omega^3\sqrt{1+4\Omega\cos\theta}}
\end{aligned}\right\} \quad A_2\text{-wave} \quad (4\cdot8)$$

$$\left.\begin{aligned}
x &= \frac{\pi}{\kappa'}\cdot\frac{\cos\theta-\cos 3\theta+4\cos\theta\sqrt{1+4\Omega\cos\theta}}{\sqrt{1+4\Omega\cos\theta}\left(1+2\Omega\cos\theta+\sqrt{1+4\Omega\cos\theta}\right)}\\
|y| &= \frac{\pi}{\kappa'}\cdot\frac{\sin\theta+\sin 3\theta}{\sqrt{1+4\Omega\cos\theta}\left(1+2\Omega\cos\theta+\sqrt{1+4\Omega\cos\theta}\right)}
\end{aligned}\right\} \quad A_1\text{-wave} \quad (4\cdot9)$$

$$\left.\begin{aligned}
x &= \frac{\pi}{\kappa'}\cdot\frac{\left(\sin^2\theta-\sqrt{1-4\Omega\cos\theta}\right)\left(1-2\Omega\cos\theta+\sqrt{1-4\Omega\cos\theta}\right)}{\Omega^2\cos\theta\sqrt{1-4\Omega\cos\theta}}\\
|y| &= \frac{\pi}{\kappa'}\cdot\frac{\sin\theta\left(1-2\Omega\cos\theta+\sqrt{1-4\Omega\cos\theta}\right)}{\Omega^2\sqrt{1-4\Omega\cos\theta}}
\end{aligned}\right\} \quad B_2\text{-wave} \quad (4\cdot10)$$

$$\left.\begin{aligned}
x &= \frac{\pi}{\kappa'}\cdot\frac{\cos\theta-\cos 3\theta+4\cos\theta\sqrt{1-4\Omega\cos\theta}}{\sqrt{1-4\Omega\cos\theta}\left(1-2\Omega\cos\theta+\sqrt{1-4\Omega\cos\theta}\right)}\\
|y| &= \frac{\pi}{\kappa'}\cdot\frac{\sin\theta+\sin 3\theta}{\sqrt{1-4\Omega\cos\theta}\left(1-2\Omega\cos\theta+\sqrt{1-4\Omega\cos\theta}\right)}
\end{aligned}\right\} \quad B_1\text{-wave} \quad (4\cdot11)$$

from (4·7). The forms of the curves defined by (4·8), (4·9), (4·10) and (4.11) are shown in figs. 6, 7 and 8. The curves become cusps at the points where $\frac{\partial}{\partial\theta}(|y|/x)=0$ and at the cusps there is a phase-difference of a quarter-period in the wave systems.

When $V\to0$, $\kappa'\to\infty$ and so $a_1, b_1\to\infty$, $a_2, b_2\to\nu^2/g$ and $\Omega\to0$. Therefore A_1-and B_1-waves vanish and the formulas to trace the form of the wave-ridges are written

$$x=\pm\frac{2\pi g}{\nu^2}\cos\theta, \qquad |y|=\frac{2\pi g}{\nu^2}\sin\theta$$

from (4·8) and (4·10). Hence, the forms of wave-ridges become concentric circles.

When $\nu\to0$, $a_1, b_1\to\kappa'$, $a_2, b_2\to0$ and $\Omega\to0$. Hence the form of the wave-ridges coincides with the configuration in an uniform fluid field, because A_2-and B_2-waves vanish and (4·9) and (4·11) coinside with (3·11) and the velocities of propagation of the two wave-systems become equal and opposite.

5. The relation between the ship's position and the configuration of wave-ridges.

We can get the tangents of the curves of the wave-ridges from (4·8), (4·9), (4·10) and (4·11). Namely,

$dy/dx=\tan\{\pi-(\pi/2-\theta)\}$ for A_2-waves,

$dy/dx=\tan\ -(\pi/2-\theta)$ for A_1-, B_2-, B_1-waves.

Hence, we find that θ denotes the angle which the normal to the carve makes with the positive direction of the axis of x, for A_2-wave and the angle vhich the normal makes with negative direction of the axis of x, for A_1-, B_2-, B_1-waves.

If x_0 denotes the point where the normal at a point (x_1, y_1) on the curve intersects the axis of x, and D_0 and D_1 denote the distances from x_0 to the origin and from x_0 to the point (x_1, y_1) respectively (refer fig. 9), we shall easily find that there are the relations

$$D_1=y_1/\sin\theta,\quad D_0=x_1\pm y_1\cot\theta \tag{5·1}$$

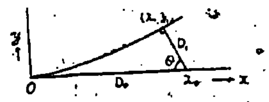


Fig. 9

between them. But we take negative sign for A_2-wave and positive sign for the other waves in the double sign of (5·1). Substituting (4·8), (4·9), (4·10) and (4·11) in (5·1), they are written

$$\left.\begin{array}{l}
\left.\begin{array}{l}
D_1=\dfrac{4\pi\cos^2\theta}{\kappa'\sqrt{1+4\Omega\cos\theta}\,(1+2\Omega\cos\theta-\sqrt{1+4\Omega\cos\theta})}\\[2ex]
D_0=\dfrac{2\pi V(1+\sqrt{1+4\Omega\cos\theta})}{\nu\sqrt{1+4\Omega\cos\theta}}
\end{array}\right\}\text{for } A_2\text{-wave,}\\[4ex]
\left.\begin{array}{l}
D_1=\dfrac{4\pi\cos^2\theta}{\kappa'\sqrt{1+4\Omega\cos\theta}\,(1+2\Omega\cos\theta+\sqrt{1+4\Omega\cos\theta})}\\[2ex]
D_0=\dfrac{2\pi V(\sqrt{1+4\Omega\cos\theta}-1)}{\nu\sqrt{1+4\Omega\cos\theta}}
\end{array}\right\}\text{for } A_1\text{-wave,}\\[4ex]
\left.\begin{array}{l}
D_1=\dfrac{4\pi\cos^2\theta}{\kappa'\sqrt{1-4\Omega\cos\theta}\,(1-2\Omega\cos\theta-\sqrt{1-4\Omega\cos\theta})}\\[2ex]
D_0=\dfrac{2\pi V(\sqrt{1-4\Omega\cos\theta}+1)}{\nu\sqrt{1-4\Omega\cos\theta}}
\end{array}\right\}\text{for } B_2\text{-wave,}\\[4ex]
\left.\begin{array}{l}
D_1=\dfrac{4\pi\cos^2\theta}{\kappa'\sqrt{1-4\Omega\cos\theta}\,(1-2\Omega\cos\theta+\sqrt{1-4\Omega\cos\theta})}\\[2ex]
D_0=\dfrac{2\pi V(1-\sqrt{1-4\Omega\cos\theta})}{\nu\sqrt{1-4\Omega\cos\theta}}
\end{array}\right\}\text{for } B_1\text{-wave.}
\end{array}\right\} \tag{5·2}$$

When we pick up only the terms of waves from (4·1) and put

$(m-\nu/V)^2/\kappa' m=\sec\alpha$ for A_2, $-A_1$-waves

$(m+\nu/V)^2/\kappa' m=\sec\alpha$ for B_2, $-B_1$-waves

we get $\nu/a_2'(\theta)$, $-\nu/a_1'(\theta)$, $\nu/b_2'(\theta)$ and $\nu/b_1'(\theta)$ as the velocities of the propagation of elementary waves in the direction of θ under the consideration of wave-vector.

These are the velocities concerning the coordinate which moves with constant velocity V along the axis of x, in the negative direction. In order to transform them in the velocities concerning the stationary coordinate, we must add them $-V\cos\theta$ for A_2-wave and $V\cos\theta$ for the other waves. If C_{A_2}, C_{A_1}, C_{B_2} and C_{B_1} denote the respective velocities of propagation concerning the stationary coordinate, these are written

$$\left.\begin{array}{l}
C_{A_2}=\dfrac{V\cos\theta(\sqrt{1+4\Omega\cos\theta}-1)}{1+2\Omega\cos\theta-\sqrt{1+4\Omega\cos\theta}}\\[2ex]
C_{A_1}=\dfrac{V\cos\theta(\sqrt{1+4\Omega\cos\theta}-1)}{1+2\Omega\cos\theta+\sqrt{1+4\Omega\cos\theta}}\\[2ex]
C_{B_2}=\dfrac{V\cos\theta(1-\sqrt{1-4\Omega\cos\theta})}{1-2\Omega\cos\theta-\sqrt{1-4\Omega\cos\theta})}\\[2ex]
C_{B_1}=\dfrac{V\cos\theta(1+\sqrt{1-4\Omega\cos\theta})}{1-2\Omega\cos\theta+\sqrt{1-4\Omega\cos\theta})}
\end{array}\right\} \tag{5·3}$$

Consequently, we get

$$D_0/V=2D_1/C_{A_2,A_1,B_2,B_1}$$

from (5·2) and (5·3) for every wave-system.

Hence, we get a theorem:—

The time that the elementary wave in the direction of θ which a ship makes at a past instant reaches to the wave-ridges, is equal to half the time till present from the instant.

The theorem is also satisfied in uniform fluid field, and it is well known that the analysis already given concerning configuration of wave-ridges in steady motion, started from this theoem.

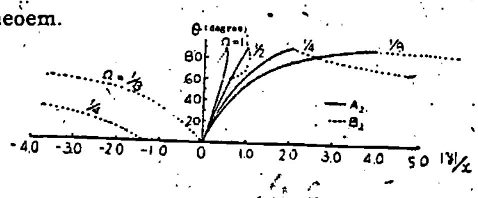


Fig. 2. $\theta\sim|y|/x$ curves of A_2-, B_2-waves.

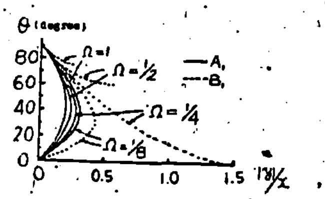


Fig. 3. $\theta\sim|y|/x$ curves of A_1-, B_1-waves

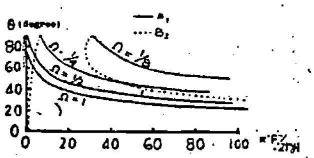


Fig. 4 $\theta\sim\kappa' F''/2|y|$ curves of A_2-, B_2-waves.

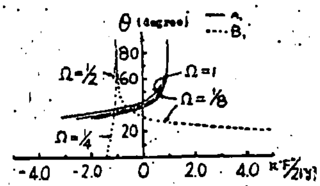
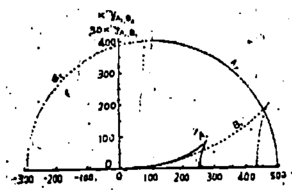
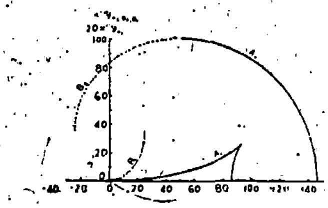


Fig. 5 $\theta\sim\kappa' F''/2|y|$ curves A_1-, B_1-waves.



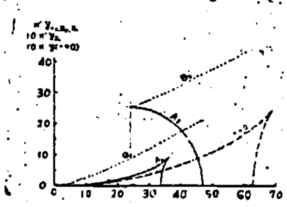
$\rightarrow\begin{cases}\kappa'\chi_{A_2,B_2}\\ 50\,\kappa'\chi_{A_1,B_1}\end{cases}$

Fig. 6. the configuration of wave-ridges at $\Omega=\dfrac{1}{8}$



$\rightarrow\begin{cases}\kappa' x_{A_2,B_2,B_1}\\ 20\,\kappa'\chi_{A_1}\end{cases}$

Fig. 7. the configuration of wave-ridges at $\Omega=\dfrac{1}{8}$



$\rightarrow\begin{cases}\kappa' x_{A_2,B_2,B_1}\\ 10\,\kappa' x_{A_1}\\ 10\,\kappa' x(\nu=0)\end{cases}$

Fig. 8. The configuration of wave-ridges at $\Omega=1/2$ (The configuration of wave-ridges at $\nu=0$ is shown to be compared with others)

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