Vessel route choice model by optimal control and calibration

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***Abstract*—To predict vessel traffic operations and improve safety and capacity in ports and inland waterways, a new maritime traffic model is developed. In this model, vessel behavior is categorized into a tactical level (route choice) and an operational level (the dynamics of the vessel behavior). This new maritime traffic model comprises two parts: the route choice model resulting in the vessel's preferred route, and the operational model describing the maneuvering behavior including interactions between vessels.**

This paper presents the vessel route choice model, which is based on disutility or cost minimization. The cost is determined by characteristics of the infrastructure, such as sailing time and distance to the bank. It is assumed that the bridge team will try to follow a preferred route that minimizes the cost to the destination. To calculate this preferred route to a certain destination, the so-called value function is defined as the minimum disutility function in continuous time and space. Subsequently, the value function is solved using dynamic programming and a numerical solution approach.

In this paper, Automatic Identification System (AIS) data of unhindered vessel behavior in the Port of Rotterdam, the Netherlands, are used for the calibration of the route choice model in four directions, as well as validation. These results could be used to improve vessel traffic management and provide a basis for predicting vessel behavior at operational level.

I. INTRODUCTION

With the development of international trade, the usage of vessels for transportation increases all over the world. It is getting more and more important to find the balance between safety and capacity in busy ports and inland waterways: when measures are taken to increase capacity, usually the safety decreases. To optimize ports and waterway design and improve maritime traffic management, modeling tools can be used.

Vessel behavior including its speed and path is difficult to predict, especially in ports and inland waterways. A lot of factors influence vessel behavior, such as waterway's geometry, human factors and external conditions including wind and visibility. Currently, some maritime models focus on calculating the risk probability of collisions or groundings (Degre et al., 2003, Fowler et al., 2000, Pedersen, 1995), while other models mainly consider the hydrodynamics of vessels (Sariöz et al., 2003, Sutulo et al., 2002, Yoon et al., 2003) or simulate the routing in a shipping network (Hsu et al., 2007, Kosmas et al., 2012, Norstad et al., 2011). In addition, most maritime simulation models focus on vessel dynamics and traffic for open seas. These models cannot be applied in constrained ports and waterways due to the fact that different factors affect sailing behavior in ports and waterways than in open seas (e.g. influence of banks and influence of water depth). Little research has been performed regarding the vessel route choice, interaction between vessels and human factors influencing maritime traffic in ports and inland waterways. In order to improve maritime traffic management and optimize ports and waterway design, a new maritime traffic model needs to be developed to predict and describe vessel traffic in ports and inland waterways.

In our research, vessel behavior is categorized into a tactical level and an operational level, which is similar to the classification of pedestrian behavior (Hoogendoorn, 2001). The tactical level includes vessel route choice in inland waterways without external influences. The vessel route choice at the tactical level serves as the basis for vessel behavior at the operational level. The operational level includes the external influences and dynamics of the vessel behavior, e.g. all decisions related to the sailing taken for the coming short time period. In other words, at the operational level, it is hypothesized that vessels follow the preferred route generated at the tactical level as much as possible, while taking into account external influences and human factors. Therefore, this new maritime traffic model will comprise two parts: the route choice model resulting in preferred routes, and the operational model describing the sailing behavior including interactions between vessels, which was proposed in our previous research (Hoogendoorn et al., 2013).

This paper presents the vessel route choice model. For vessel route choice, it is assumed that disutility or cost of each route for the vessel is determined by characteristics of the infrastructure, such as sailing time and distance to the bank. The bridge team will try to follow a route that minimizes the disutility to reach their destination, being the preferred route. In this research, the so-called value function is defined as the minimum disutility function in continuous time and space. From this value function the preferred route can be derived from the present position to the destination, which leads to the least disutility to the vessel. In other words, the bridge team will navigate their vessel in the direction in which the cost decreases most rapidly. The value function is obtained using dynamic programming and a numerical solution approach.

To calibrate the route choice model, Automatic Identification System (AIS) data are used. In recent research, AIS data have been proven to be a powerful tool to investigate maritime traffic (Aarsæther et al., 2009, Mou et al., 2010, Xiao et al., 2012). Automatic Identification System (AIS) is an onboard system transmitting vessel information (position, velocity, destination, etc.) between nearby vessels and shore stations. In this paper, AIS data in the Port of Rotterdam are provided by the Maritime Research Institute Netherlands (MARIN).

The remainder of this paper is structured as follows. Firstly, the vessel behavior theory at the tactical level is proposed, followed by an optimal route choice model for vessels in ports and restricted waterways. Then, the calibration process and results for the vessel route choice model are described. After that, validation by cross-comparison is presented. Finally, conclusions and recommendations for future research are presented.

II. Vessel Behavior at the Tactical Level

Our research focuses on the vessel behavior in the two-dimensional space, including vessel velocity and path. Previous research showed that a lot of factors influence vessel behavior, such as vessel characteristics (e.g. vessel type and size), waterway geometry and external conditions including wind, visibility and current (Ince et al., 2004).

In this paper, we present a vessel route choice model, which is at the tactical level. In the approach, the bridge team is considered as the “brain” of the vessel. In the vessel route choice theory, it is assumed that disutility or cost of each route is determined by characteristics of the infrastructure, which will be discussed in the next section. To identify the preferred route, the bridge team will predict and minimize this expected disutility or cost C.

In our research, we investigate vessel behavior in a waterway stretch, which is defined by two cross sections. These two cross sections can be considered as the entrance and the destination for vessels sailing in this direction. The vessel route x(·) is a continuous function, uniquely determined by the velocity trajectory v(·) through the waterway. Since the position is the derivative of the velocity, so optimizing the velocity also optimizes the route. Then, the utility optimization for the vessel route will yield the optimized velocity choice at the tactical level.

It should be noted that both vessel course and speed are included in this optimized velocity. As we discussed before, vessel speed is influenced by external influences (e.g. wind and visibility) and is determined by the bridge team according to the traffic situation and the infrastructure at the operational level. Therefore, the vessel route choice model will mainly consider vessel course, rather than vessel speed.

As implied in (1), the optimal course (over a time period) is the velocity that minimizes the cost, given the current time and position of the vessel:

$$\psi^*(\cdot) = \arg\min C\,(v(\cdot)|t_0, x_0) \tag{1}$$

where t_0 and x_0 are the current time and position of the vessel. This way, the vessel route choice problem becomes the optimization for vessel course in the research area.

In the next section, we will discuss the expected disutility and the solution for the vessel route choice optimization.

III. Route Choice Model by Optimal Control

For vessel behavior at the tactical level, it is assumed that the bridge team chooses a route by predicting and minimizing the expected disutility, which is determined by characteristics of the infrastructure. The contribution of these characteristics to the cost C will be proposed in this section.

The decision making process of the bridge team is feedback-oriented. That means for each time step, the bridge team will reconsider the expected disutility and make the choice for the preferred route in the next time steps to minimize the expected cost. This is a continuous feedback control system including input (velocity) and the controlled output (location).

As we know, vessels sometimes deviate from their planned path when they encounter other vessels. To flexibly adapt vessels to other routes, the expected minimum perceived disutility for all locations x and instants t is proposed. The so-called value function $W(t,x)$ is defined as expected minimum perceived disutility function in continuous time and space (Fleming et al., 2006). Based on $W(t,x)$, the optimal route for vessels can be determined.

A. *Vessel Kinematics under Uncertainty*

As mentioned above, velocity and location are considered as control input and output, respectively. To apply the control, consider the location *x* (the state) and the velocity *v* (the control) for a vessel. The vessel position at instance t $x(t)$ is known to the bridge team and expressed by $\hat{x}$. Then, the bridge team will predict the route costs and determine the future position $x(\tau)$ for $\tau > t$ using the vessel kinematics:

$$\mathrm{d}x = v\mathrm{d}t + \mathrm{d}\varepsilon \quad \text{subject to } x(t) = \hat{x} \tag{2}$$

where $v = v(\tau)$ denotes velocity of the vessel for $\tau > t$. The term $\mathrm{d}\varepsilon$ represents the small disturbance, which is $N(0,\sigma^2)$-distributed. The white noise reflects the uncertainty in the expected traffic conditions and is caused by human factors or randomness of future conditions.

Here, we investigate vessels sailing in ports and waterways, where they sail at relatively low speed. This speed is around 10 knots, which is normally far below the physical limitation of the vessel. So this physical limitation is not considered in our research.

B. *Generalized Expected Utility*

Consider a part of the waterway between two cross sections, which are set as the entrance and the destination respectively.

Let $[t, t_t)$ denote the planning period of the bridge team, where t and t_t are respectively the current time and the terminal time (planning horizon). The vessel is expected to reach its destination during this time period.

Let t_a denote the time of arrival at the destination, and let $T = \min(t_t, t_a)$. Consider an arbitrary control $v_{[t,T)}$ resulting in the trajectory $x_{[t,T)}$, the expected disutility or cost C is defined as

$$C\big(T, v_{[t,T)}\big) = \int_t^T L(\tau, x(\tau), v(\tau))\mathrm{d}\tau + \phi(T, x(T)) \tag{3}$$

where L and ϕ respectively denote the so-called running cost and the terminal cost. The running cost $L(\tau, x(\tau), v(\tau))$ reflects the cost incurred in a small time period $[\tau, \tau + \mathrm{d}\tau)$, given the location $x(\tau)$ at time τ and control velocity $v(\tau)$. The terminal cost $\phi(T, x(T))$ reflects the penalty incurred due to the vessel ending up at position $x(T)$ at the terminal time T, but not at the destination.

C. *Specification of Terminal Cost*

As defined in the last section, the terminal time T either equals the final time t_t of the planning period or the time t_a at which the vessel arrives at the destination. The terminal cost is defined as

$$\phi(T, x(T)) = \begin{cases} 0, & T < t_t \\ \phi, & T = t_t \end{cases} \tag{4}$$

The terminal cost ϕ thus reflects the penalty for not having arrived at the destination at the end of the prediction horizon. When the vessel arrives at the destination in time, the penalty is zero, so the vessel will aim to reach the destination within the prediction horizon.

D. *Specification of Running Cost*

By definition, the running cost L reflects the influence of different characteristics of the infrastructure considered by the bridge team. For simplicity, it is assumed that these attributes are independent and the running cost is linear-in-parameters as follows:

$$L(t, x, v) = \sum_{k=1,2,\ldots} c_k L_k(t, x, v) \tag{5}$$

where L_k denotes the contributions on vessel route choice of k different characteristics of the infrastructure, and c_k are relative weights for these factors. It should be noted that all weights cannot be uniquely determined from AIS data, since only the relative importance of the weights can be determined. Furthermore, weight factors c_k are different for different vessel groups according to AIS data analysis. For example, small vessels follow a path closer to their starboard bank compared to large vessels.

The data analysis showed that both banks and the vessel characteristics have influence on vessel route choice (Shu et al., 2013b). In our approach, we consider the following characteristics of the infrastructure in the running costs for a specific vessel category: expected sailing time, the waterway bend effect, discomfort due to proximity to banks and sailing at a certain speed. These running costs are described below.

Sailing time. For this sailing time, we define as follows:

$$L_1(t, x, v) = 1 \tag{6}$$

The definition above results in the route cost

$$\int_t^T c_1 \cdot L_1(\tau, x(\tau), v(\tau)) d\tau = c_1(T - t) \tag{7}$$

It means that the contribution of expected sailing time on running cost equals the expected sailing time, multiplied by the weight c_1. The weight factor c_1 reflects the time-pressure for the bridge team to arrive at the destination in time.

Sailing speed. To arrive at the destination in time, it is necessary to have an appropriate speed. However, high speed means high energy consumption, which will result in high cost. Speed choice is thus a trade-off between the time remaining to sail to the destination and the energy consumption due to sailing at a certain speed. For simplicity we assume the energy consumption to be a quadratic function of the vessel speed as follows:

$$L_2(t, x, v) = v^2 \tag{8}$$

The waterway bend effect. By including the sailing time and sailing speed, as stated in the previous paragraph, we assume that vessels prefer to sail in a straight line towards their destination. In bended waterways, this implies that vessels will cut corners. Applying sailing time and sailing speed in the cost function, example tracks (red lines) in two sailing directions are shown in Fig. 1, where the cut corners behavior is obvious.

However, previous AIS data analysis (Shu et al., 2013b) showed that vessels normally sail along the centerline of the waterway in the bend area of the waterway. To introduce the waterway bend effect on vessel routes, a term is therefore added to the running cost to make sure vessels are sailing along the waterway in the bend area.

An example of a bend waterway is shown in Fig. 2, where the bend area is shadowed. In the figure, two sides of the bend area are considered as a part of concentric circles. $R_0(x)$ denotes the distance to the center of the circle for the position x. $d_c(x)$ is the distance to the convex bank.

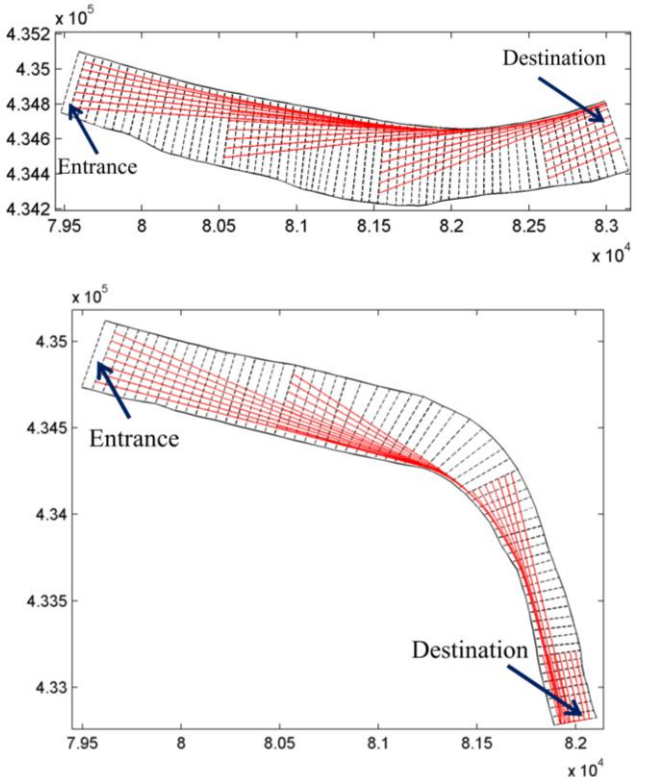

Figure 1. Example routes applying sailing time and sailing speed in the cost function.

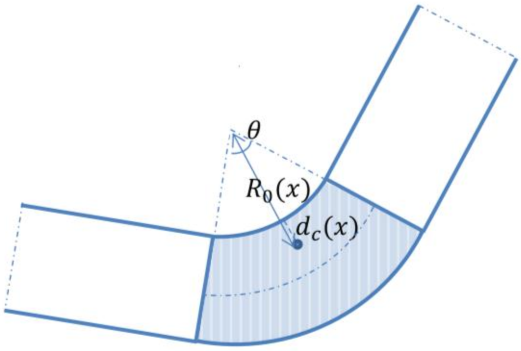

Figure 2. Example of bend waterway and parameters.

To reflect the influence caused by a bend in waterways, a linear decreasing utility from the convex bank is defined as follows:

$$L_3 = -d_c(x)/R_0(x) \tag{9}$$

It should be noted that this cost is only added in the bend area of the waterway. Then, in the bend area, L_3 provides repulsion from the convex bank.

Discomfort due to proximity to banks. As we know from AIS data analysis, vessels normally keep a certain distance to the bank, which in our research has been defined as the five meter water depth line (Shu et al., 2013b). The bridge team will adjust its course to make sure that their vessel is not too close to either portside bank or starboard bank. In our approach, it is assumed that a vessel is influenced by the bank when it is closer to the bank than a certain threshold distance.

The parameters for bank influence are shown in Fig. 3, where the vessel sails from the left to the right and its present location is *x*. Let $d_1(x)$ and $d_2(x)$ denote the distance to the portside bank and the starboard bank respectively. $D(x)$ is the width of the waterway at the position *x*. $D(x) \cdot r_1$ and $D(x) \cdot r_2$ describe how far both banks influence the vessel. In other words, r_1 and r_2 describe the percentage of the waterway width.

This way, the waterway is divided into three areas as shown in the figure. The vessel is influenced by the portside bank only when it sails in Area 1, thus when $d_1(x)$ is smaller than $D(x) \cdot r_1$. The starboard bank influences the vessel in a similar way. In Area 3, the vessel is not influenced by either bank.

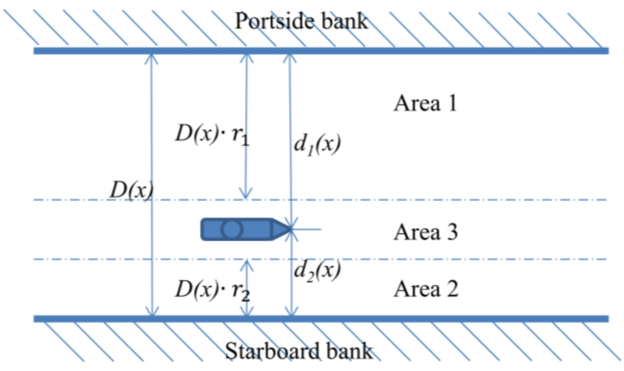

Figure 3. Waterway area division according to portside and starboard bank.

We add the influence of the two banks in the expected route cost as a monotonously decreasing (linear) function of the distance to the bank in the corresponding area. Running cost components L_3 and L_4 denote the contributions from the portside and starboard bank, respectively. They are defined as

$$L_4(t,x,v) = \begin{cases} 0, d_1(x) > D(x)\cdot r_1 \\ \dfrac{D(x)\cdot r_1 - d_1(x)}{D(x)\cdot r_1}, d_1(x) \le D(x)\cdot r_1 \end{cases} \tag{10}$$

$$L_5(t,x,v) = \begin{cases} 0, d_2(x) > D(x)\cdot r_2 \\ \dfrac{D(x)\cdot r_2 - d_2(x)}{D(x)\cdot r_2}, d_2(x) \le D(x)\cdot r_2 \end{cases} \tag{11}$$

E. Dynamic Programming and Numerical Solution

To solve the route choice problem in continuous time and space, the so-called value function $W(t,x)$ is defined as the expected minimum perceived disutility function. To solve the value function, a dynamic programming approach and a numerical solution approach are used in the model. The solution of $W(t,x)$ describes the minimum cost to the destination for a vessel located at position *x* at instant *t*. Based on this solution, the optimal vessel course can be determined. For details, we refer to previous work (Hoogendoorn et al., 2004).

IV. ROUTE CHOICE MODEL CALIBRATION

Using AIS data, the calibration of the route choice model is introduced in this section. We firstly introduce the AIS data and unhindered vessel behavior, being the vessel behavior without influence of other vessels and external influences. As we discussed before, these influences are considered at the operational level, but not at the tactical level. Then, the objective function for calibration is formulated. Finally, the estimation for bank effect is preformed to get the parameter c_3 in four sailing directions.

A. AIS Data and Unhindered Behavior

In our research, the class of small General Dry Cargo (GDC) vessels less than 3600 gross tonnage is used since this vessel category contains the largest amount of AIS data in the four sailing directions. AIS data of these vessels in the Botlek area in the Port of Rotterdam from January 2009 to April 2011 are selected.

As shown in Fig. 4, we selected four AIS data sets in these four directions for the route choice model calibration.

Figure 4. Research area

- Data set 1: vessels sail from Sea to Nieuwe Maas.
- Data set 2: vessels sail from Nieuwe Maas to Sea.
- Data set 3: vessels sail from Sea to Oude Maas.
- Data set 4: vessels sail from Oude Maas to Sea.

To compare lateral positions of these tracks and easily calculate the average path, 69 cross sections (for Data set 1 and Data set 2) and 68 cross sections (for Data set 3 and Data set 4) with intervals around 50 meters are defined in the research area. These cross sections are approximately perpendicular to the waterway axis and used to select AIS data. Endpoints of these cross sections are located at the five meters water depth line. For the areas without five meters water depth line, such as entrances to basins or the Oude Maas junction, endpoints are created such that the boundary remains smooth. In the model, these endpoints of cross sections will form the effective waterway bank for vessel sailing.

In our previous research (Shu et al., 2013a), AIS data analysis provided insight into vessel behavior. It was found that vessels deviate from their planned path when they encounter other vessels, especially during overtaking.

As mentioned before, vessel route choice is at the tactical level, where the influence of vessel encounters is not considered. To eliminate the influence of vessel encounters, empirical vessel paths are classified into hindered paths and unhindered paths according to the influence of other vessels.

Here, a path is defined as unhindered if the distance to other vessels is at least 2 km, during the whole trip of the vessel. AIS data of unhindered paths are then used for the calibration.

However, these unhindered paths concentrate in the right part of the waterway. To be able to estimate the influence of the banks, more data are needed to describe the vessel route choice in the areas close to banks. To provide more data in these areas, a part of hindered vessel paths is used. For hindered vessel paths, vessels normally deviate from their planned path and sail into the area closer to the banks. It is assumed that the influence of other vessels ends after the encounter. At that moment both vessels have the largest relative deviation when they are closest to each other. Hindered vessel paths after the encounter can then be considered as unhindered and used for calibration as well. Including these, the tracks of the AIS data set used for calibration cover most of the waterway.

The unhindered paths of AIS data used for calibration are selected in four sailing directions. The unhindered paths from data set 1 are shown as an example in Fig. 5.

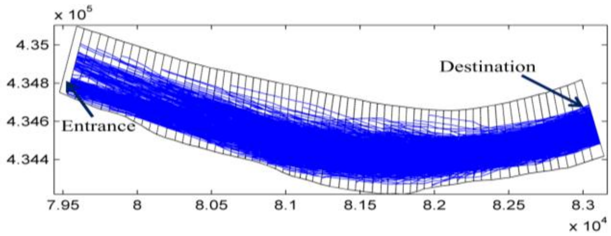

Figure 5. unhindered paths used for calibration in data set 1.

B. *Objective Function*

As we know, vessels have a two-dimensional motion including vessel speed and course. As we discussed before, only vessel course is considered in the objective function in the calibration of the route choice model. The calibration process aims at minimizing the difference between vessel course measured from AIS data and vessel course predicted by the vessel route choice model.

As we can see in Fig. 5, vessel paths concentrate in the right part of the waterway and they are not uniformly distributed. Overlapping paths provide similar inputs to the calibration. To combine a lot of repetitive inputs, a “meshgrid” of 10 m×10 m is used to generate a course field, which is an average course of the AIS data in each cell. This course field will be used to compare the difference with the course field generated in the route choice model.

To use the numerical solution approach, we discretize the waterway into small 5m×5m-cells and define the time step as 0.5 seconds. Then, the value function can be solved for the whole research area, resulting in the optimal course field. As we discussed before, only vessel course is considered in the route choice model calibration. This optimal course field will be used to compare with the course field from the AIS data.

Let α_{data} denote vessel course in the course field from Fig. 5. Correspondingly, α_{sim} is the optimized course for the same point in the “meshgrid” calculated by the route choice model based on a given β. For these m “meshgrid” points where we have AIS data, the average square error is defined as

$$E(\beta) = \frac{1}{m}\sum_{i=1}^{m}(\alpha_{data} - \alpha_{sim})^2 \qquad (12)$$

This way, the calibration problem becomes a multi-variable nonlinear optimization problem, which could be solved by the function “fminsearch” in MATLAB.

$$\beta^* = \arg\min E(\beta) \qquad (13)$$

As mentioned before, only the ratio between the weights can be determined by AIS data. Without loss of generality, we can set $c_1 = 1$. Since the speed is not relevant in route choice, we fix the parameter c_2 to give vessel a speed around the average speed 5 m/s in this area. ($c_2 = 0.035$)

C. *Estimation of Bank Effect*

Firstly, we define the bend area in the four directions. For these two waterway stretch, we define the bend area as shown in Fig. 6.

In the first waterway stretch, we define two bend areas, since the bend strength is different in these two parts of the waterway. In the route choice model, this bend area definition will be used to determine the bend effect.

For the bend effect, it has influence on the lateral position of vessels, similar as the influence of banks. If we put them together in the calibration, they will influence each other and result in different parameters of bank influence for different waterway geometry. So, we estimate the bend effect before the calibration.

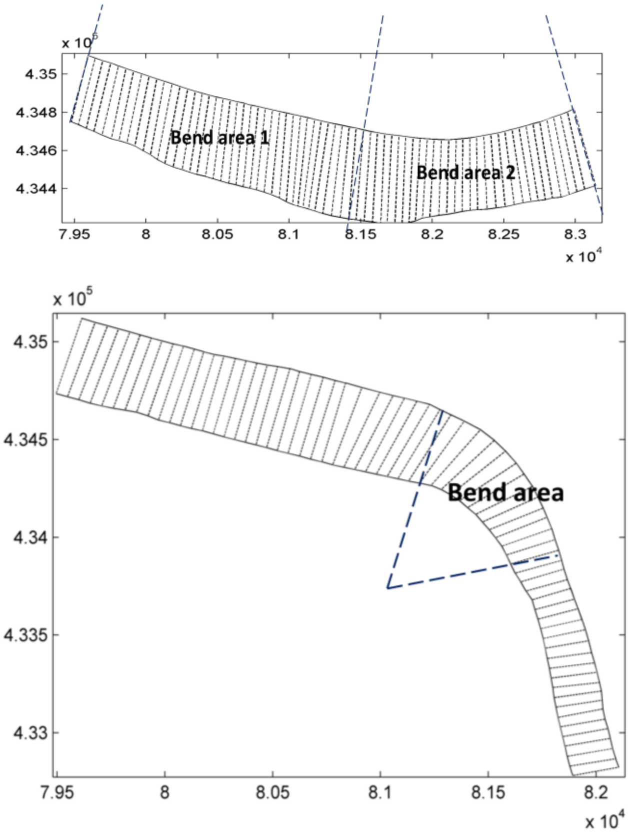

Figure 6. Bend area definition in two waterway stretches.

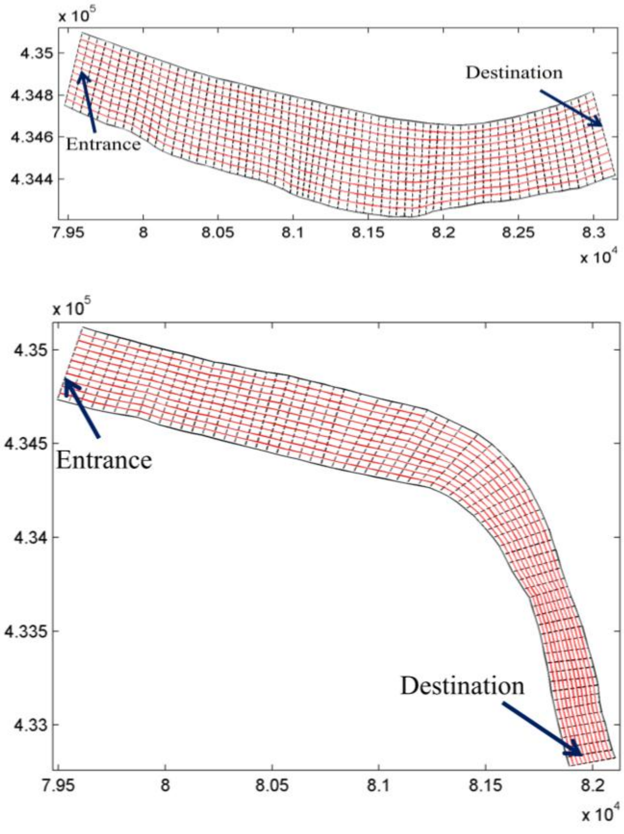

Figure 7. Routes on contour lines used to optimize bend effect.

TABLE I. OPTIMIZED PARAMETERS FOR BEND EFFECT

	Data set 1	Data set 2		Data set 3	Data set 4
$c_{3,area1}$	2.75	3.29	c_3	1.54	1.51
$c_{3,area2}$	1.63	1.38			

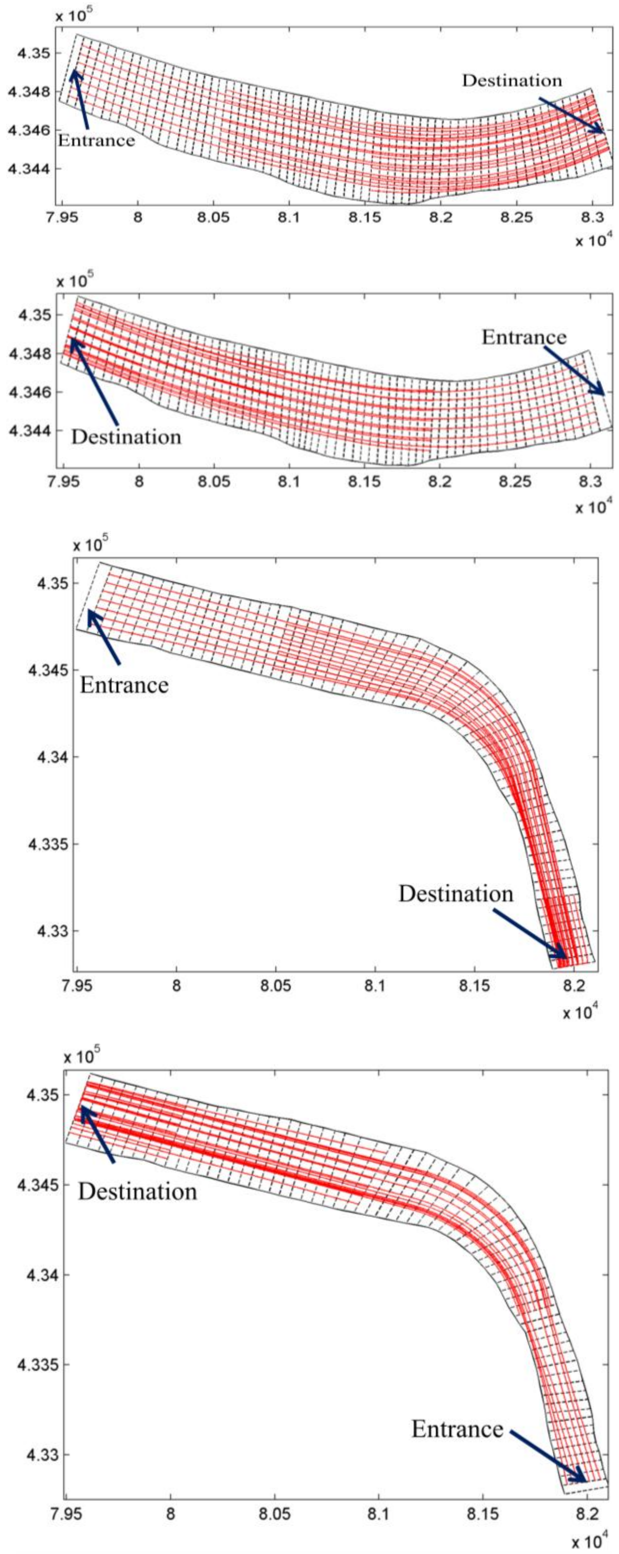

Figure 8. Vessel routes under optimized parameter for bend effect.

To get the optimized parameter for the bend effect, we use the routes on contour lines of relative lateral position. Sailing directions from Sea to Nieuwe Maas and from Sea to Oude Maas are shown as an example in Fig. 7. Actually, the routes for the other two directions overlap with these routes in the figure, but in opposite directions.

In this optimization for bend effect, we only add items of sailing time, sailing speed and the influence of the bend effect in the route choice model. For the parameters c_1 and c_2, we use the value given in the last section.

For the bend effect, the intersection of two boundary lines of the bend area is assumed as the center of the arc. Then the distance between the center and the position x is taken as the $R_0(x)$.

In this way, the optimized results for the four directions are shown in TABLE I. These values are determined by the waterway geometry and bend area division. It can be seen that the optimized results of c_3 vary from 1.38 to 3.29. This can be explained by the deviation of the center, which might result in the large difference for $R_0(x)$.

Based on these results, we can generate the vessel routes without cutting corners in four directions as shown in Fig. 8. It can be seen that all routes are approximately parallel to each other without bend effect.

Based on these optimized results for the bend effect in four directions, we can calibrate the remaining parameters for bank influence, as shown in the vector $\beta^T = (c_4, c_5, r_1, r_2)$.

V. Calibration Results

By applying the described optimization method and objective function, the best fit of the route choice model to the AIS data is found. The calibration results are summarized in TABLE II.

Based on the optimized results, the value function $W(t, x)$ can be generated in four directions. In Fig. 9, contour lines of the value function $W(t, x)$ are used to shown the changes of value function in all four directions.

According to the value function in Fig. 9, the optimized vessel routes could be determined in four directions, as shown in Fig. 10.

TABLE II. CALIBRATION RESULTS FOR DIFFERENT DATA SETS

	Data set 1	Data set 2	Data set 3	Data set 4
c_4	0.0199	0.0241	0.0354	0.0219
c_5	0.0123	0.0047	0.0067	0.0113
r_1	0.45	0.674	0.465	0.415
r_2	0.254	0.38	0.287	0.256

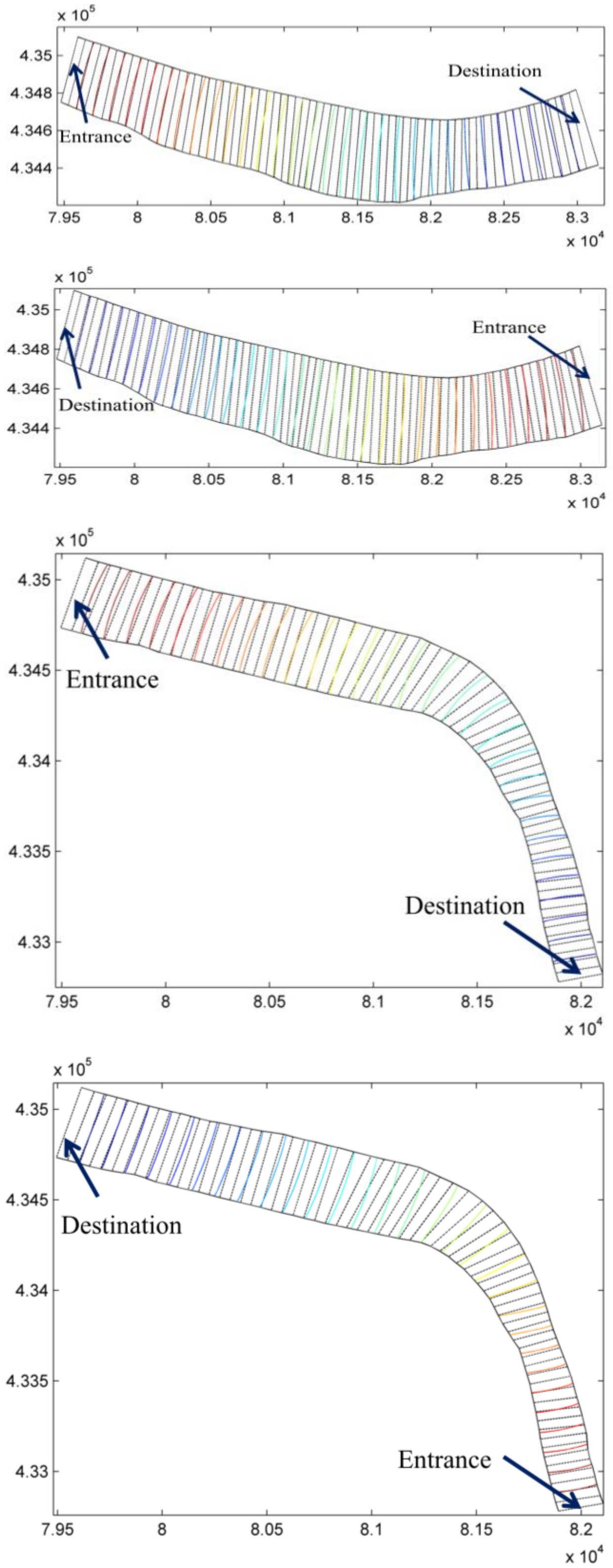

Figure 9. Contour lines for optimized value function in four directions.

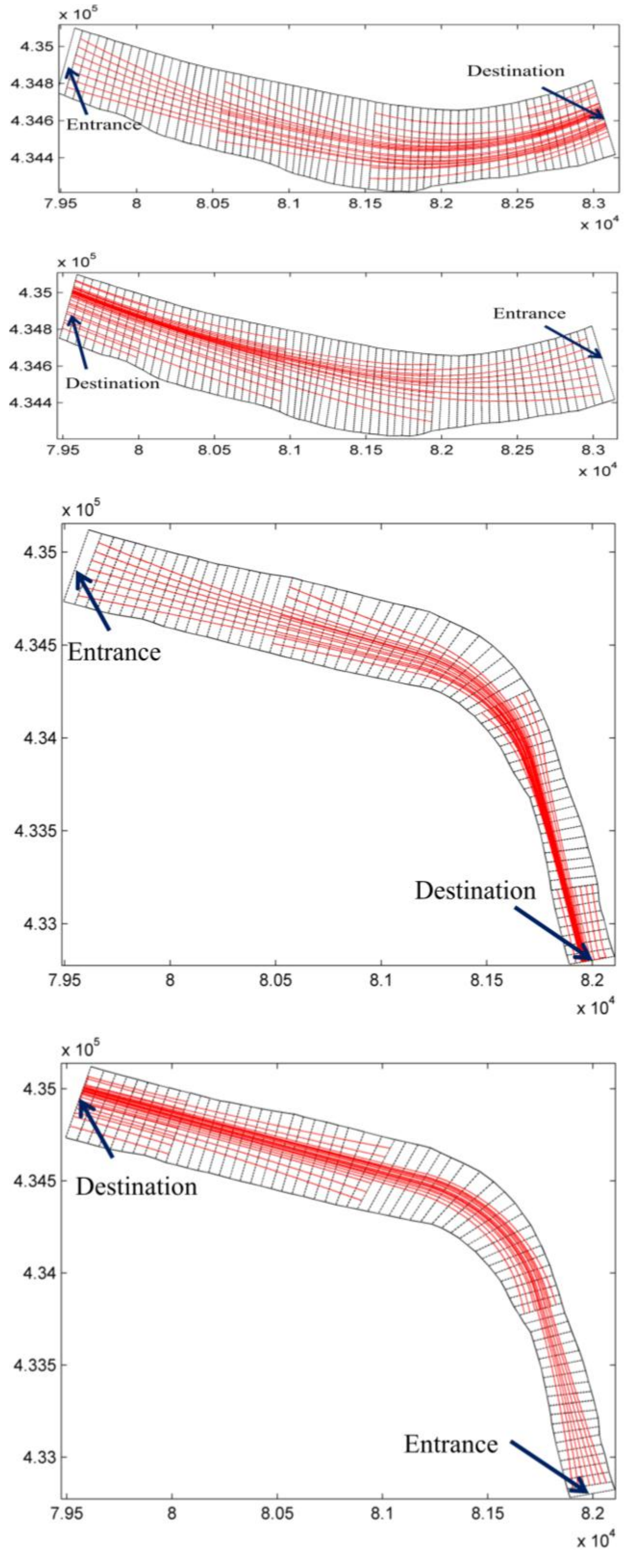

Figure 10. Optimized routes in four directions.

Comparing the contour lines and the optimized routes, it can be seen that the optimized course is the direction perpendicular to the contour lines, which is the direction the value function decreases most rapidly. The shape of the contour lines indicates that vessels will be pushed away from the bank when they are too close to the bank.

For these optimized routes, the part of routes close the destination is different from the first part. When vessels are close to the destination, they go straight to the destination, not being influenced by banks.

But in general, most part of these optimized routes in these four directions is plausible. They correspond with the pattern we found in the AIS data.

- Vessels concentrate in the right part of the waterway.
- When vessels sail too close to the bank, they will be pushed away from the bank.
- In the bend area, the vessels do not cut corners.

We compare the optimized parameter sets for different directions in TABLE II. For the calibrated results for data set 2, the values show most difference with other data sets. This might be explained by the cross current in the junction, which will influence vessel behavior, especially vessel's position. In this area, vessels sometimes sail very close to the starboard bank under the influence of current. That is why the portside bank influence range r_1 is larger than in the other cases, while the starboard bank influence is very weak.

The results for data set 1 and data set 4 are very similar to each other. Compared to the results of data set 1 and data set 4, optimized parameters for data set 3 show stronger bank influence of portside bank, but weaker influence of the starboard bank. This might be explained by the fact that, when vessels sail into the current, they prefer to stay more close to the starboard bank.

Combining the results for data set 1 and data set 4, it can be seen that r_1 is around 0.43, and r_2 approximately 0.25. The parameters r_1 and r_2 describe the percentage of the waterway width, in which both banks contribute to the cost function. That means the portside bank has influence on vessel route in 43% of the waterway width, but the starboard bank has a smaller influence range, which is 25% of the width. The remaining area of around 32% of the waterway is the area where banks do not have influence on vessel route. This area could be considered as the Area 3 shown in Fig. 4, where the vessels will concentrate. This is also corresponding to the phenomenon observed in Fig. 5.

In addition, the parameters c_4 and c_5 for portside bank and starboard bank are around 0.021 and 0.0118. The bank contribution in the cost function is a linear decreasing function (from 1 to 0) as shown in (10) and (11). Although the value of c_4 is around twice that of c_5, the total bank influence provides similar repulsion because the portside bank has about twice the distance (r_1) compared to the starboard bank (r_2).

Compared to the contribution of sailing time (equal to 1), these two values seem very small, but they cannot be neglected as they provide the repulsion of both banks.

TABLE III. CROSS-COMPARISON OF THE CALIBRATED PARAMETERS

	Set 1	Set 2	Set 3	Set 4
Parameters of set 1	14.32	20.55	13.7	15.5
Parameters of set 2	17.85	16.34	14.06	19.25
Parameters of set 3	15.53	18.68	12.95	16.71
Parameters of set 4	14.39	20.6	13.77	15.46

VI. VALIDATION BY CROSS-COMPARISON

Finally, we validate the obtained calibrated parameters by applying these values to the other datasets. This cross-comparison can be used to check the reliability of the obtained parameters and automatically takes into account the variance of the calibrated parameter values.

We use the calibrated parameters in TABLE II of each data set for the other three data sets. The obtained errors can be found in TABLE III. It can be seen in the table that the errors are from 13.8 to 20.6. The error in the data set 2 is larger than the errors in other data sets, except when the calibrated parameters by themselves are used. That means that the variance in this data set is large and as we discussed before, this might be caused by the current in the junction.

VII. CONCLUSIONS AND RECOMMENDATIONS

In this paper, an approach is proposed to generate vessel route choice in continuous time and space for ports and inland waterways. Dynamic programming and a numerical solution method are used to solve the value function, which can be used to generate optimal courses.

After the estimation of the bend effect, this vessel route choice model is calibrated based on AIS data of unhindered paths and (the unhindered part of) hindered paths. The calibrated results of the vessel route choice model show plausible preferred routes in the research area, which help us to understand the desired vessel behavior (route). According to the calibrated results, both banks have similar influence on vessel route. Both banks exert repulsion when the vessel sails too close to the bank.

The vessel route choice model will serve as input to the maneuvering model at the operational level. Both route choice model and maneuvering model form the new maritime traffic model, which describes maritime traffic by predicting single vessel behavior. Furthermore, the vessel route choice model provides the preferred routes for vessels. This can be used to improve management of waterway traffic, e.g. Vessel Traffic Services (VTS), as well as design and extension of ports and inland waterways.

In future research, it is planned to include the other infrastructural elements in the route choice model, such as buoys. This way, the actual sailing environment will be reflected in the model and make the model more generic. Furthermore, AIS data sets from other areas will be used to further validate the model.

ACKNOWLEDGMENT

The research in this paper is part of the research program "Nautical traffic model based design and assessment of safe and efficient ports and waterways", sponsored by the Netherlands Organization for Scientific Research (NWO). The authors like to thank MARIN and the Port of Rotterdam for providing AIS data. The fellowship of Yaqing Shu at Delft University of Technology is supported by the Chinese Scholarship Council (CSC).

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