1-D morphological river models:
Schematisation and interpretation

by

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SUMMARY

This study contains two topics which are important for the use of a one-dimensional computer model, calculating morphological changes in time.

Firstly, for a good prediction of the morphological changes by the model a good reproduction of the water movement and the sediment movement is necessary. One of the conditions for a good reproduction of the process is a good schematisation of the river cross-section in the model. In the river cross-section, a good reproduction of all the parameters playing a role in the morphological process is required. Several schematisation-methods are treated, considering a fluctuation in the discharge and irregularly shaped cross-sections.

The second topic of this study considers the interpretation of the calculated morphological change to the real cross-section. The calculated morphological change is distributed over the width of the cross-section in several ways, each distribution with his own physical background. The results show an influence of the distribution-option on some practical parameters.

For both topics data are used of the Da River in Vietnam and of the River Waal in the Netherlands.

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CONTENTS

1 INTRODUCTION

1.1 General

The complex process of the movement of water and the movement of sediment over the movable river bottom is called river hydraulics. These movements may vary considerably in magnitude, both in time and space. The insight in the behaviour of a river under certain circumstances is necessary when we want to predict its morphological evolution. These predictions are of basic importance for the assessments of flooding risks. They are also important in estimating the impact of natural changes, e.g. in climate changes, and of human interferences in river systems. The results give insight in the protection measures needed and management policies, called river engineering. Predictions can be made by means of scale models or mathematical models. Nowadays, due to the availability of cheap computer power, and to their more general applicability, mathematical models are often the most suitable tools for studies of river morphology.

The use of a model is a three-step approach:

- The river is schematised into a model
- The new situation is calculated in the model
- This solution is interpreted for the real river

This study regards mathematical models for calculating large-scale, long-term morphological changes due to human interference or natural changes. Such a program overlaps the transition period to reach a new equilibrium for longitudinal profiles of a large river basin.
The phenomena of erosion and material deposition are of a three-dimensional nature, but the simulation of these complex phenomena and the accuracy achieved are not necessary for the large-scale and long-term predictions. Apart from the huge demand of computer power, and the accumulation of errors deriving from the repeated computations, a serious problem is represented by the definition of appropriated boundary conditions which should remain valid during the simulation (Di Silvio, 1993).
One-dimensional models are generally considered most suitable for these predictions. They are based on strong simplifications of the problems and inevitably give less accurate information than models based on more space dimensions, but, on the other hand, they require less data (data are often difficult to obtain) and usually can be run on a microcomputer.

1.2 Short overview

Regarding a mathematical model for the calculation of morphological changes, large numerical time-steps and space-steps are recommended, which allows some simplification of the whole process. Two important assumptions are the validity of the sediment transport as a function of the flow velocity, and a quasi-steady state of the water movement. The last assumption makes it possible to decouple the water movement and the sediment movement for each time step in the model (Chapter 2). This is only valid when no storage outside the cross-section is assumed.

The derivation of a one-dimensional model from a fully three-dimensional one is called "down scaling from 3D to 1D" (Crosato 1995). This procedure must be performed accurately, because the relevant information on the main parameters playing a role in the processes to be modelled, should not be lost. One of the conditions for a good schematisation of reality in the model is a good schematisation of the cross-section of the river, regarding these parameters. The first aim of this study is to schematise the river cross-section for a one-dimensional mathematical model which calculates morphological changes, considering an irregular shape of the cross-section and a variation in the discharge.

A characteristic approach for a one-dimensional model is that some morphological parameters (flow velocity, bed level, water depth) are averaged over the cross-section. With this approach, the morphological change will be expressed as a change in time of the bed level $\Delta z_b(x,t)$ over a sediment carrying width $B_{sed}(x)$, which has to be constant in time.

In Chapter 3 this approach is worked out. Herefore some measured bed elevations of the River Da, in Vietnam are used, together with some relations between the discharge Q and the water level h, the Q-h rating curves. In section 3.2, some important parameters are described, together with their schematisation-notation. In the following sections some schematisation-options are studied, each giving a good reproduction of two parameters in the initial state, which is assumed to be in equilibrium. However, for a good reproduction of the water movement and the sediment movement, more than two parameters have to be schematised well. When the cross-section of the river has a not rectangular form, and a variation in the discharge is considered, a good schematisation of whole the process, including flow profiles, will not be possible with this approach. This is also caused by the fact that in this approach the sediment carrying width is not a function of time, what is not realistic with a varying discharge (c.q. water level).

In Chapter 4 a different approach is followed. The water depth and the bed level are not averaged over the cross-section anymore, and the morphological change in time will be expressed as a change in time of the cross-sectional area $\Delta A(x,t)$. The cross-section is schematised in the model with a notation of the width as function of the water level. This schematisation gives a good reproduction of the water movement, as well in the initial state as in a flow profile, and needs less input data compared to the notation of the real cross-section with the bed level as function of the width.

This means that a good schematisation of the sediment movement is left, and because of a good reproduction of the flow velocity, more in particular a good schematisation of the sediment carrying width is left. In Chapter 4 a method is developed to determine the sediment carrying width. With this method, the sediment carrying width will become an artificial parameter which is calculated by the model at each space step for each time step. This way a fluctuation in time of this parameter can be followed. The calculated $B_{sed}(x,t)$ is a function of the geometry of the cross-section, of the water level and of the used sediment transport formula, information which is all known in the quasi-steady approach in the model. The calculated $B_{sed}(x,t)$ is not a function of the local flow situation, what makes this method also valid in a flow profile. This approach is tested with some measurements of sediment transport at the River Waal, in the Netherlands, with a satisfying result.

The second aim of this study is to find a realistic interpretation of the morphological changes for the real river, the so-called "up scaling", and to study the influence of this interpretation. With the result of Chapter 4, this means that the calculated change of the cross-sectional area $\Delta A(x,t)$ has to be divided over the width of the cross-section, so a two-dimensional result is achieved.
In Chapter 5 some suggestions are given how to divide the morphological change over the width, each one with an own physical background.
For the situation of withdrawing water upstream, the new equilibrium situation at the river mouth is calculated, regarding the influence of these divisions. It shows that the use of different division-options results in different values for some practical parameters in the new equilibrium situation.

2 BASIC THEORY

2.1 General

Two phenomena play an important role where rivers are concerned, the movement of water and that of sediment. These two elements may vary considerably in magnitude, both in time and space.

2.2 Water movement

The Saint Venant equations give a good description of the one-dimensional water movement, with a variation in time (t) and longitudinal direction (x). The two partial differential equations, containing mean values over the cross-section, are:

The equation for water motion:

$$\frac{\partial Q}{\partial t} - \alpha' \frac{2Q}{A_s}\frac{\partial A}{\partial t} + gA_s\left(1 - \alpha' \frac{Q^2 B_s}{gA_s^3}\right)\frac{\partial a}{\partial x} - \alpha' \frac{Q^2}{A_s^2}\frac{\partial A_s}{\partial x}\Big|_a + gA_s\frac{\partial z_b}{\partial x} + \frac{Q|Q|}{C^2 R A_s^2} = 0 \qquad (2.1)$$

with: Q = discharge
A_s = conveying cross-section
B_s = stream width, surface width of conveying part
g = acceleration due to gravity
a = water depth
z_b = bottom level
C = Chézy coefficient
R = hydraulic radius
α' = coefficient

The equation of continuity of water:

$$B\,\frac{\partial z_w}{\partial t} + \frac{\partial Q}{\partial x} = 0 \qquad (2.2)$$

with: B = storage width (width at water surface)
z_w = water level

The coefficient α' is a correction term considering a non-uniform velocity distribution over width and depth in the conveying cross-section A_s, which carries the total discharge. In case of a rectangular form, α' can be assumed approximately unity. However, for a cross-section with a different shape this will not be the case. Assuming a constant surface slope over the width and a vertical mean velocity u at each instant according to $u = C\sqrt{a i_b}$, this results in:

$$\alpha' = A \int_{y_1}^{y_2} a\ 2\ C^2\ dy \left(\int_{y_2}^{y_1} C\ a^{3/2}\ dy \right)^{-2} \tag{2.3}$$

The resulting α' is usually quite close to unity. Moreover, in practical calculations the factor α' is often of little significance. When there are parts in the cross-section which do not carry any (net) flow at all, the terms α' and R concern A_s, in which the non-solid parts of the wetted perimeter will also show a shear stress because there is a significant gradient of the mean velocity. There are indications that this shear stress may be comparable to the bottom shear stress.

There are some simplifications of the equation for water movement. Considering a river with steady uniform flow, this becomes:

$$gA_s \frac{\partial z_b}{\partial x} + \frac{Q|Q|}{C^2 R A_s^2} = 0 \tag{2.4}$$

In another form:

$$Q = A\ C\ \sqrt{Ri} \tag{2.5}$$

This last formula is the Chézy-equation for steady uniform flow.

2.3 Sediment movement

Regarding the one-dimensional situation, the following partial differential equation is available for alluvial rivers:

The equation for continuity of sediment:

$$B \frac{\partial z}{\partial t} + \frac{\partial S}{\partial x} = 0 \tag{2.6}$$

with: S = sediment transport as bulk volume

The equation for sediment motion:

$$S = B\ f\ (U,\ parameters) \tag{2.7}$$

For the last equation an important assumption is made. The local transport is governed by the local hydraulic parameters of which the local mean velocity U, especially, varies in time and space. In other words, the sediment transport is a function of the water velocity, $s = f(u)$, assuming an equilibrium profile of the sediment concentration in vertical direction, $\phi_e(z)$. Because the adaptation time T_a and the adaptation length L_a are much smaller than the time-step and space-step respectively, this assumption is justified. There are several sediment transport formulas, each for a different flow or topographical situation. In this study, the general form is used:

$$S = B\ m_0\ U^n \tag{2.8}$$

The coefficient m_0 and the exponent n contain important characteristic parameters which govern the transport process, such as the grain size D, the Chézy coefficient C, the relative density $\Delta=(\rho_s-\rho)/\rho$ and the ripple factor μ. Most of these parameters will be assumed to be constant in time and space, while in reality there will be variations. For example, there will be variation in the grain diameter D in longitudinal direction. Also, the Chézy coefficient C, determined by the form of the cross-section and the water level, will vary in time and space, as well in longitudinal as in transverse direction.
Regarding the sediment movement, two other assumptions are made:

* Uniform bed material is supposed to be present
* Fixed banks are postulated. Or, in other words, the erodibility of the banks is much smaller than that of the bed

2.4 Quasi-steady flow

To prove the validity of assuming a quasi-steady situation, it is hypothesized for the time being that $B(x,t) = B =$ constant. Consequently, the four partial differential equations can be written for the unit of width.

Equation for water motion:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + g\frac{\partial a}{\partial x} + g\frac{\partial z}{\partial x} = -g\frac{u|u|}{C^2 a} \tag{2.9}$$

Equation for continuity of water:

$$\frac{\partial a}{\partial t} + u\frac{\partial a}{\partial x} + a\frac{\partial u}{\partial x} = 0 \tag{2.10}$$

or

$$\frac{\partial a}{\partial t} + \frac{\partial q}{\partial x} = 0 \qquad (2.11)$$

Equation for continuity of sediment:

$$\frac{\partial z}{\partial t} + \frac{\partial s}{\partial x} = 0 \qquad (2.12)$$

Equation for sediment motion:

$$s = f(u, parameters) \qquad (2.13)$$

with: $u(x,t)$ = flow velocity
$s(x,t)$ = sediment transport
$a(x,t)$ = water depth
$z(x,t)$ = bed level

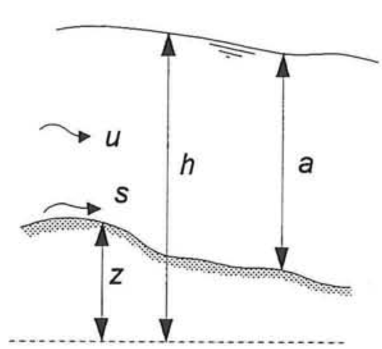


Fig. 2.1

The two sediment equations can be combined into

$$\frac{\partial z}{\partial t} + \frac{\mathrm{d}\, f(u)}{\mathrm{d}\ u}\frac{\partial u}{\partial x} = 0 \qquad (2.14)$$

This equation and the two water equations form a system of three partial differential equations with the three dependent variables u, a and z.

According to the cubic equation (de Vries, 1959) three celerities $c = \mathrm{d}x/\mathrm{d}t$ of this hyperbolic system are found:

$$-c^3 + 2uc^2 + (ga - u^2 + gf_u)c - ugf_u = 0 \qquad (2.15)$$

and in dimensionless form:

$$\Phi^3 - 2\Phi^2 + (1 - Fr^{-2} - \Psi Fr^{-2})\Phi + \Psi Fr^{-2} = 0 \qquad (2.16)$$

with: $\Phi = c/u$ = relative celerity
$Fr = u/\sqrt{ga}$ = Froude number
$\psi = f_u/a$ = dimensionless transport parameter

In this equation there are three celerities. Two of them, c_1 and c_2, express the propagation of a disturbance at the water surface. The third, c_3, expresses the propagation of a disturbance of the bed.
It was noted by de Vries (1959) that for low Froude numbers ($Fr < 0,6$) the water and sediment equations can be decoupled, because $|c_{1,2}| >> c_3$. This gives the following advantages:

- with respect to the water movement, the time interval for the computations (a few days) is too small to record some important bed changes, thus, it can be assumed that the propagation of the bed disturbance c_3 is zero. The propagation of the water surface disturbances, $c_{1,2} = u \pm \sqrt{ga}$ is not influenced by the mobility of the bed. A fixed bed can be assumed.

- regarding river-bed movement, the disturbance at the bed has the celerity:

$$c_3 = \frac{\Psi}{1-Fr^2} \times u \qquad (2.17)$$

the two celerities $c_{1,2}$ form the characteristic $dt = 0$ in the x,t-plane. In other words, the water flow can be considered quasi-steady.

This is illustrated in figure 2.2.

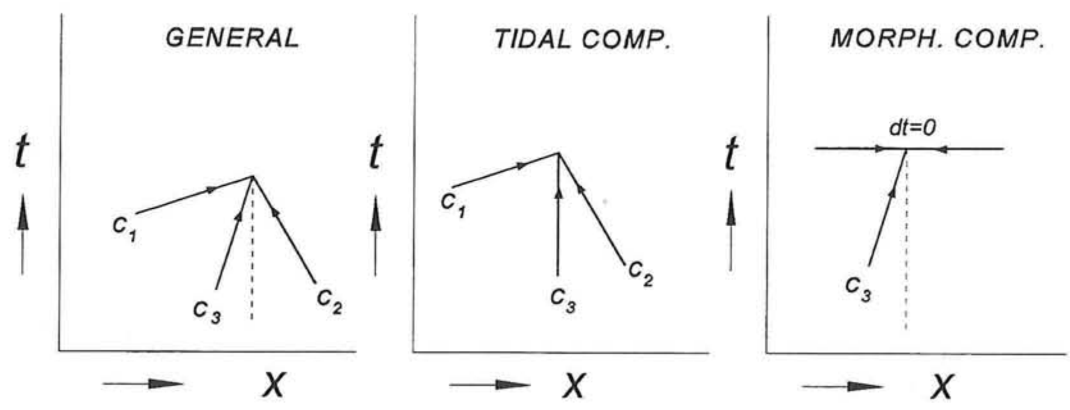


Fig. 2.2

Inserting $\partial u/\partial t = 0$ and $\partial a/\partial t = 0$ in the water flow equations it can easily be shown that the partial differential equations are reduced to ordinary differential equations. Because of the large time steps in morphological calculations the time derivatives become negligible compared to the others terms in the equations (convective, pressure and friction).

In the equation for water motion it is no problem to neglect the time derivatives. The equation is reduced to the one for steady non-uniform flow.

The equation for continuity of water should be considered more carefully. If $\partial a/\partial t = 0$, this implies $\partial q/\partial x = 0$. To understand the implications it has to be remarked that from $\partial q/\partial x = 0$ it follows that $q(x,t)$ becomes $q(t)$. Hence, at every time t there is steady flow. For each time step, however, a different discharge can be used, so that the fluctuation over a year is well-reproducible.

The fact that $\partial q/\partial x = 0$ implies the important restriction that there is no storage outside the cross-section, for the whole river reach. Only then a quasi-steady calculation is allowed.

Concerning a large river reach and considering a variation in the discharge over a year, in some cases it is not realistic to assume $B(x,t) = B =$ constant. This approximation only serves to indicate the quasi-steady approximation (with the cubic equation). In reality, the width B can become a function of time and space. Then the general equations have to be used again, in quasi-steady form:

$$\frac{\partial Q}{\partial x} = 0 \tag{2.18}$$

$$gA_s \left(1 - \alpha' \frac{Q^2 B_s}{g A_s^3} \right) \frac{\partial a}{\partial x} - \alpha' \frac{Q^2}{A_s^2} \frac{\partial A_s}{\partial x} \Big|_a + gA_s \frac{\partial z_b}{\partial x} + g \frac{Q|Q|}{C^2 R A_s^2} = 0 \tag{2.19}$$

2.5 Varying discharge

Each river has a seasonal fluctuation in the discharge. The morphological changes due to the entire river regime have to be known. In the past, when computational capacity was restricted, there was need to simplify the river regime by using a single discharge, correlated with some kind of average river characteristics.
All parameters in the river theory are discharge-dependent and change over time in a different fashion.
It has been proved sufficiently that a single discharge will not give a good reproduction of the morphological changes caused by the real sequence of discharges (De Vries, 1993a).
For calculations of long term morphological changes, where time dependent fluctuations are not primarily of interest, the quasi steady approach worked out in section 2.4 makes it possible to use a probability distribution of the discharge Q, giving a good representation of the river regime. It has to be mentioned that the chosen time step in the computer program has to make it possible to simulate the river regime.

2.6 Morphological changes

Morphological changes occur when a discontinuity disturbs an equilibrium and causes a flow profile in the river. This implies a gradient in the water velocity, $\partial U/\partial x$, followed by a gradient in the sediment transport $\partial S/\partial x$, which causes the morphological change $\partial z/\partial t$, according to the equation of continuity of sediment. So a good calculation of morphological changes requires a good schematisation of the sediment transport, and its change in time. The general form of the equation for sediment movement:

$$S = B_{sed}\, m_0\, U^n \tag{2.20}$$

with B_{sed} = sediment carrying width

shows that this demands a good schematisation of the mean water velocity U and its change in time. Regarding the simplified one-dimensional equation of a flow profile:

$$\frac{\partial h}{\partial x} = i_b \frac{1 - \dfrac{Q|Q|}{C^2 R A^2 i_b}}{1 - \dfrac{Q^2 B}{g A^3}} \tag{2.21}$$

it follows that the important parameters for the water movement are the wetted area A in the cross-section, the hydraulic radius R and the discharge Q. These have to be approximated well in the model, if a good reproduction of the sediment transport and its change in time is wanted. For changes in time, the surface width B_{sur} influences a good reproduction of a flow profile, because of its impact on the interaction between the changes in the wetted area ΔA, the changing water level and the changing hydraulic radius in equation 2.21.

The parameters A, R and B_{sur} will vary along the river and vary with the varying discharge. Finally, considering m_0 and n constant in time and place (and known), the sediment carrying width B_{sed} completes a good reproduction of the sediment transport. In reality, this parameter will also vary in space and with the discharge.

2.7 Assumptions for this study

This schematisation-study is for mathematical models calculating morphological changes. These models are one-dimensional, representing only longitudinal bed profiles and longitudinal free surface profiles. The sediment transport is a function of time and hydraulic flow conditions. For this study, the following hydraulic assumptions are made:

* No storage in the cross-section. $A_s = A$
* Froude number $Fr < 0.6$. This means that the water movement and the sediment movement can be decoupled.
* Quasi-steady state of the water movement.
* The Chézy value C is constant over the width
* The water-surface slope is equal over the width of the river.
* The pressure is hydrostatic, which implies that streamline curvature is small and the vertical acceleration is negligible
* Uniform bed material
* Fixed banks

3 SCHEMATISATION

3.1 General

Step one of the use of a model is the schematisation of reality into a model. One of the boundary conditions is a good reproduction of the initial state.
In a characteristic one-dimensional approach the morphological parameters are averaged over the cross-section. These are:

* The flow velocity U
* The water depth a
* The bed level z

Because most river cross-sections show a large width compared to the depth, a possible schematisation, often used in movable bed models, is a rectangular cross-section in the model (subscript m) as representative of the real cross-section (subscript r). This makes it also possible to work with the equations for unit of width. The calculated morphological change will be expressed by a change in bed level $\Delta z(x,t)$ over a sediment carrying width. Using this method, two problems arise which have not been solved yet:

* *Schematisation* of the initial condition $z_m(x,o)$ from $z_r(x,y,o)$.

* *Interpretation* of the computed values $z_m(x,t)$ to the desired prediction of $z_r(x,y,t)$

The schematisation problem will be studied in this Chapter and Chapter 4. The interpretation problem will be worked out in Chapter 5.

Keeping the water level in the model the same as in reality, this leaves two unknown parameters for the rectangular schematisation, B_m and z_m, which can be found by proper schematising of some important parameters (see figure 3.1).

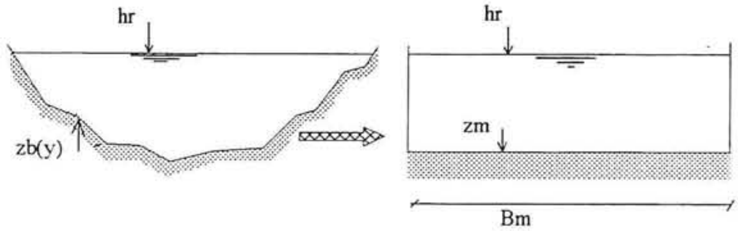


Fig. 3.1

There are three difficulties regarding the schematisation.

* Firstly, a good reproduction of morphological changes requires a good reproduction of more parameters playing a role in the morphological process, as well from the water movement as from the sediment movement.

* Secondly, when a discontinuity occurs, the change of the water movement has to be followed as good as possible, while it causes the morphological change.

* Thirdly, the schematisation has to be valid for various discharges, with respect to the seasonal fluctuation in the discharge.

In section 3.2 some parameters playing an important role in the process are mentioned, together with their possible schematisation in a rectangular cross-section.
In section 3.3, the first two difficulties are studied for one discharge, considering a calculation of the morphological change expressed by a change in the bed level $\Delta z(x,t)$.
In section 3.4, the variation in the discharge is considered.

3.2 Schematisation of parameters

Now the parameters, mentioned in section 2.6, will be calculated, which results in some options for schematisation.

For the water movement, it is necessary to select $B_m = B_r$, when a good reproduction of the storage is wanted. For this study a quasi-steady state of the water movement is assumed, so this reproduction is not necessary. Two other important parameters for a good reproduction of the water movement are the wetted area A_m and the discharge Q_m. During calculation time, the important water flow velocity U (for the sediment movement) will be calculated according to $U = Q/A$, with the discharge Q as a hard boundary condition. When the initial state is assumed to be in equilibrium, the following two equations apply. The wetted area in the model, A_m will be the same as in reality (A_r), acccording to the *A-integral* (over the width of the cross-section):

$$B_r\ (h_r - z_m) = \int_o^{B_r} (h_r - z(y))\mathrm{d}y \tag{3.1}$$

the wetted area in the model, A_m will be the same as in reality (A_r).

Using the Chézy equation for steady uniform flow, the discharge Q can be expressed and schematised accurately with the *Q-integral*:

$$C\ \sqrt{i_b}\ B_r\ (h_r - z_m)^{3/2} = C\ \sqrt{i_b} \int_0^{B_r} (h - z(y))^{3/2} \mathrm{d}y \tag{3.2}$$

considering a constant Chézy value C over the width.

For the hydraulic radius R two options are discussed here. First the general form of $R = A/P$, in which P is the wetted perimeter. However, it should be considered that to reduce the amount of input data of the initial real cross-section, the input can be given by the $B_r = B_r(h_r)$-notation, and not by the regular $z_r = z_r(B_r)$-notation. It can be proved that for a constant value of α (see eq. 3.3), the integral over the width

$$\int_0^{B_r} (h_r - z(y))^{\alpha} \mathrm{d}y \tag{3.3}$$

has the same value for both methods of notation (De Vries and Wang, 1995). Both methods are illustrated in figure 3.2.

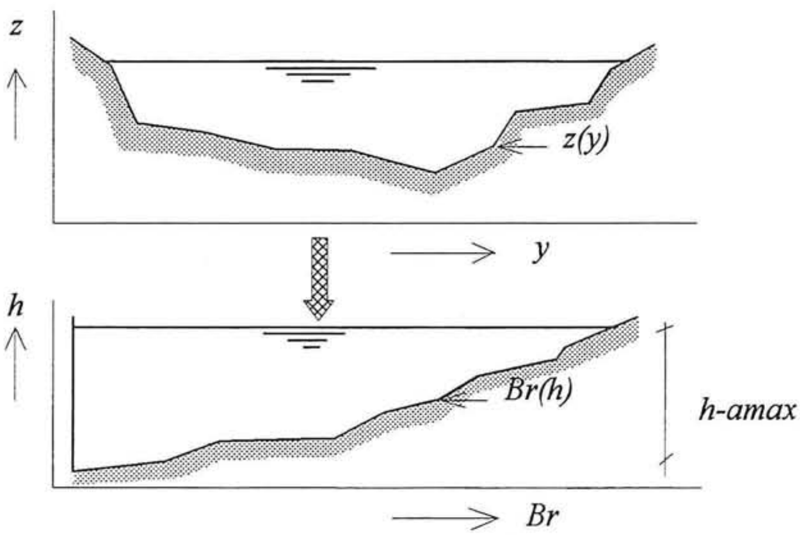


Fig. 3.2

For the wetted perimeter P, however, the values of both options will be different for irregularly shaped cross-sections. When there is a large width to depth ratio, the difference will mainly be caused by the y- axis value $(h_r - z_{max})$ in the $B_r = B_r(h_r)$ notation. So leaving this value out of the calculation of the wetted perimeter, this gives a good approximation of the real wetted perimeter.

For nearly rectangular profiles, however, it is not necessary to leave this value out of the calculation of P. In that case both methods of notation results in the same P.

Another notation of the hydraulic radius, especially useful for irregularly shaped cross-sections, is proposed by Engelund (Jansen et.al. 1979, p. 50). The dynamics of the river flow are influenced basically by the bottom shear stress. For irregularly shaped cross-sections the bottom shear stress will vary within the cross-section. Theoretical treatment of this value can only be applied to the very special case of steady, uniform, two-dimensional flow. The transverse derivatives vanish.
This way, the following simplified form of the formula for water movement is obtained.

$$g \frac{\partial z_w}{\partial x} - \frac{1}{\rho}\frac{\partial \tau_{xz}}{\partial z} = 0 \tag{3.4}$$

Integrating this formula over the depth, and using the boundary condition:

$$\tau_{xz} = \tau_b \left(1 - \frac{z}{a}\right) \tag{3.5}$$

the following mean bottom stress, τ_b, is obtained:

$$\tau_b = -\rho\, g\, a \frac{\partial z_w}{\partial x} \tag{3.6}$$

Using $P = A/R$ this gives:

$$\frac{A_s}{R}\tau_b = \int_0^{B_r} \tau_{xb}\, dy = \rho\, g\, A_s \frac{\partial z_w}{\partial x} \tag{3.7}$$

Another possible resistance law for the bottom friction is a relation between the depth mean velocity u (for unit of width), the shear velocity u_*, and τ_b. This gives:

$$\tau_b = \frac{\rho\, g}{C_1^2}\left(\frac{Q}{A_s}\right)^2 \tag{3.8}$$

where

$$Q = \int_0^{B_r} a\, u \,\mathrm{d}y = \sqrt{\left(\frac{\partial z_w}{\partial x}\right)} \int_0^{B_r} C\, a^{3/2} \mathrm{d}y \qquad (3.9)$$

Here C_1 is a representative Chézy value for the main river-channel. The combination of these equations gives the following hydraulic radius:

$$R = \left(\frac{1}{A} \int_0^{B_r} \frac{C}{C_1} a^{3/2}\, \mathrm{d}y \right)^2 \qquad (3.10)$$

valid only in steady uniform flow. However, it can also be used for slowly varying or slightly non-uniform flow. For this study, it is allowed to use this method, because the following two assumptions are made:

* a quasi-steady state
* low Froude numbers (< 0.6)

Using this method, it will be shown that the circle is closed, with respect to the Chézy equation for steady uniform flow and the three schematised parameters. So a good schematisation of A and Q results in a good schematisation of R.

To show its importance, in Appendix A both expressions of R are compared for some cross-sections, where the Chézy value C is presumed constant over the width, which may not be realistic. It is shown that both values are quite the same for cross-sections with an approximating rectangular form (difference about 10%). However, for cross-sections approximating a triangular form, the difference increases to up to 30%.

Regarding a good schematisation of the water movement in the initial state, one parameter is left, i.e. the surface width B_{sur}. As we consider only cross-sections without storage (an assumption for a quasi steady flow) an accurate representation of the storage width is not necessary.

However, all these schematisations are irrelevant with respect to the sediment transport, except that the important factor, the water velocity U, will be simulated quite well. For the sediment transport, the same Chézy equation for steady uniform water motion provides an option for schematisation according to the *S-integral*

$$m_0\ C^n\ \sqrt{i_b}^n \int_0^{B_r} (h-z(y))^{n/2}\mathrm{d}y = m_0\ C^n\ \sqrt{i_b}^n\ B_m\ (h-z_m)^{n/2} \tag{3.11}$$

where m_0, n, i_b and C are the same on both sides.

For these schematisations sufficient data are necessary. In reality, these are not always available. The most common data are discharge rating-curves and bed elevations at only a limited number of places. However, with combinations of these and the assumption of an initial state in equilibrium, the schematisations according to the *integrals* are possible.

As well the sediment transport schematisation as the discharge schematisation are on both sides multiplied by a certain power of $C\ \sqrt{i_b}$. For this study a constant Chézy value C over the width is assumed. This value can be calculated next way. When it is required that:

$$C\ \sqrt{i_b} \int_0^{B_r} (h-z(y))^{3/2}\mathrm{d}y = Q_r \tag{3.12}$$

in which Q_r is the measured discharge in steady uniform state, $C\ \sqrt{i_b}$ is found.
Another possibility concerns the hydraulic radius R, according to the equation for water motion:

$$C\ \sqrt{i_b} = \frac{Q_r}{A_r\sqrt{R}} \tag{3.13}$$

As mentioned before, there are two ways to find R, giving quite different values for some cross-sections. Regarding the resulting $C\ \sqrt{i_b}$ for both calculations, the *Q-integral* is only representative, when the hydraulic radius R according to Engelund is used. With this $C\ \sqrt{i_b}$ value, the *S-integral* will also be more realistic.

The value (which is) found can be checked when the averaged bottom slope over the river is calculated with some discharge rating-curves along the river. Assuming a steady uniform flow, the water surface slope is equal to the bottom slope. This gives a Chézy value which should be realistic. For a varying water level, the Chézy value will fluctuate as well, what is realistic.
In Appendix B, where the *integral*-theory is checked with data of the River Waal, the Chézy value is illustrated as well, with his fluctuation.

3.3 One discharge

3.3.1 General

First, the problem of schematising the cross-section in a rectangular profile will be studied for one discharge. Regarding the initial equilibrium state, we can calculate some important parameters and schematise them well in a rectangular cross-section for the model. This will be worked out in subsection 3.3.2.
In subsection 3.3.3 it will be studied wether the possible schematisation options give a good reproduction of the flow profile. This is important for a good reproduction of the morphological changes.

3.3.2 Rectangular profile for initial state

There are three parameters left which have to be schematised well. To determine the rectangular cross-section with the two unknowns, B_m and z_m, two parameter-schematisations are neccessary. Combining two parameters, there are three options:

* A and Q
* A and S
* Q and S

according to: $A_m = A_r$:

$$B_m\,(h_r - z_m) = \int_o^{B_r} (h_r - z(y))\,\mathrm{d}y \tag{3.14}$$

$Q_m = Q_r$:

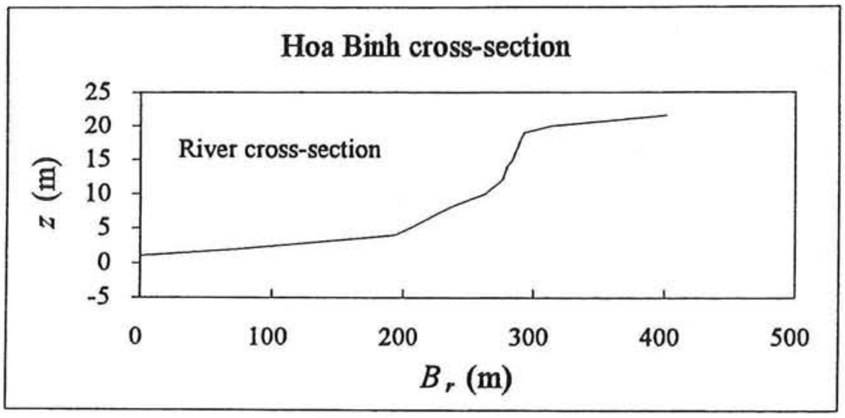
$$B_m\,(h_r - z_m)^{3/2} = \int_0^{B_r} (h_r - z(y))^{3/2}\,\mathrm{d}y \tag{3.15}$$

$S_m = S_r$:

$$B_m\,(h_r - z_m)^{n/2} = \int_0^{B_r} (h_r - z(y))^{n/2}\,\mathrm{d}y \tag{3.16}$$

During this study, some theories and calculations are illustrated with two example cross-sections with a nearly rectangular shape (Hoa Binh) and a nearly triangular shape (Trung Ha).

These are real cross-sections from the Da River, a tributary of the Red River in Vietnam. For both cross-sections a discharge rating-curve and a bed level elevation are available. The following figure shows both shapes in the $B_r(h_r)$ notation. The names of the cross-sections are connected to places nearby.



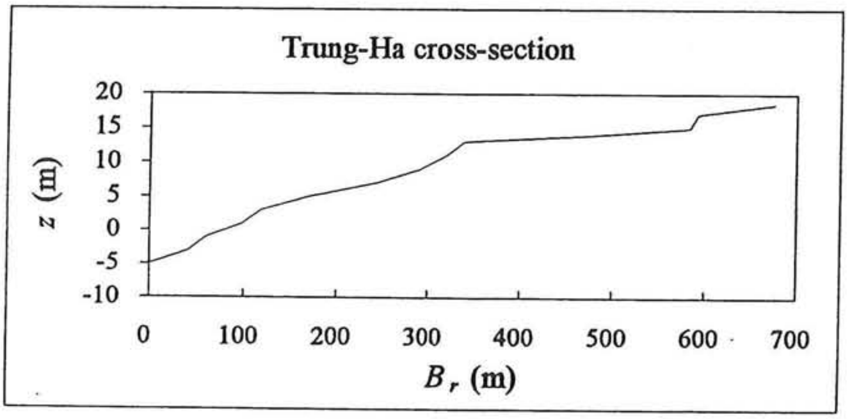


Fig 3.3

When the real cross-section has a nearly rectangular form, the resulting values of B_m and z_m of the three options will be aproximately the same. However, when the shape of the cross-section is different, the options will give different values. In Appendix A the results for several cross-sections will be shown. An overall outcome is that the Q,A-method results in the largest width and smallest depth, and the Q,S-method the opposite.
This means that each option does not gives a good reproduction of the total process in the initial state.
During calculation time, the flow velocity U will be calculated according to $U = Q/A$ with the discharge Q_r as a hard boundary condition.

The Q,A-option shows a good reproduction of the one-dimensional water movement, with the parameters A, Q, U and R. Only the real sediment transport will be larger than the obtained $B_m\ m_0\ U^n$. The effect will be an incorrect time scale of morphological changes.

The A,S-option has a different equilibrium depth, belonging to the discharge Q_r. This implies a lower water level, followed by a smaller wetted area A and thus a larger water velocity U in the model. This problem can be solved by calibrating the Chézy value until the same water level is reached with respect to a steady uniform flow.

Option Q,S also gives a smaller wetted area A, followed by an error in R and a larger water velocity U compared to reality. In this case, the equilibrium depth is correct.
The sediment transport S is correct in the initial state.

3.3.3 Reproduction of the flow profile

When a discontinuity disturbs the initial equilibrium state, morphological changes occur. These are caused by a gradient in the water velocity dU/dx. Consequently, for a good reproduction of the morphological changes, a good reproduction of the flow profile, and thus the flow velocity gradient, is needed.
In fig 3.4 an example is given of two flow profiles. The induced discontinuity is a withdrawal of water.

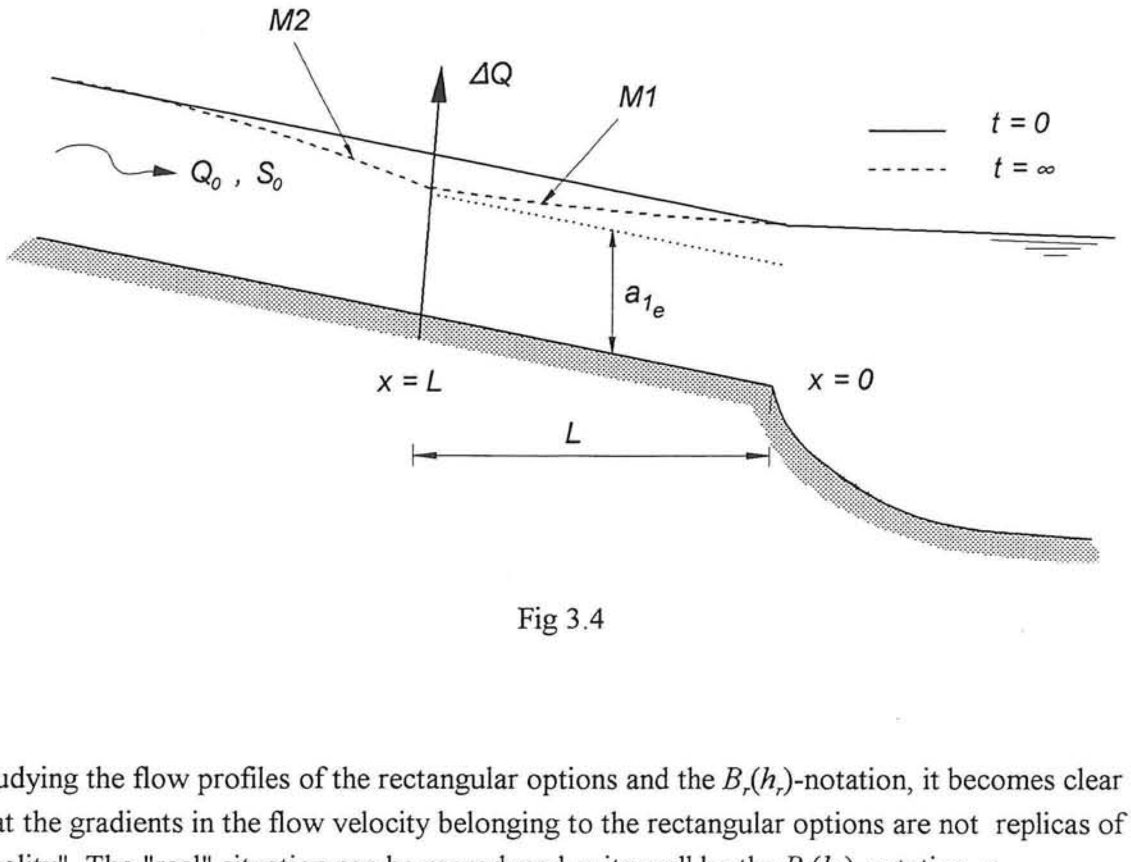


Fig 3.4

Studying the flow profiles of the rectangular options and the $B_r(h_r)$-notation, it becomes clear that the gradients in the flow velocity belonging to the rectangular options are not replicas of "reality". The "real" situation can be reproduced quite well by the $B_r(h_r)$-notation, a schematisation where the width is given as function of the water level. This is no problem when only a reproduction of the water movement is required. Regarding the one-dimensional equation of a flow profile

$$\frac{\partial h}{\partial x} = i_b \frac{1 - \dfrac{Q|Q|}{C^2RA^2i_b}}{1 - \dfrac{Q^2B}{gA^3}} \tag{3.17}$$

the deviation of the flow-velocity gradients is caused by two errors.
Firstly, by a wrong reproduction of one of the parameters of equation 3.17 in a rectangular schematisation.
Secondly, a wrong reproduction of the flow profile is caused by the deviation of the water surface profile of the rectangular cross-sections, which causes another interaction between the changing parameters of equation 3.17. In the flow profile, the three options of schematisation have a different interaction between the changes in the water level, the changes in the wetted area A and the changes in the hydraulic radius R.
The Q,A-option approximates the water movement best, because of the best reproduction of the surface width. This implies the same relative change of the flow velocity in the flow profile compared to "reality". The Q,A-option also approximates the parameters with influence in the flow profile equation quite well. This implies the same values in the flow profile.
For example, the Q,S-option shows a much larger fluctuation of the flow-velocity compared to "reality", and different values as well.

A good reproduction of this gradient (and interaction between the parameters) demands a good reconstruction of the real surface width in the area of the water levels and also a good reproduction of some parameters. A solution is "calibrating" the schematisation- profile.
The Q,A-schematisation starts following the $B_r(h_r)$-notation (going up) at the height where the calculated B_m of this option reaches the width of the $B_r(h_r)$-notation.
Now the wetted area A increases compared to the old Q,A-schematisation. To compensate this, the z_m is lifted up until A_m equals A_r for the water level h_r belonging to $B_r(h_r) = B_m$, according to

$$B_m\,(h_r - z_m) = \int_o^{B_m} (h_r - z(y))\mathrm{d}y \tag{3.18}$$

The calibrated schematisation has a hydraulic radius which differs from reality. This can be solved by calibrating the Chézy-value.
For the Q,A-option, the "calibrated" Q,A-option and the $B_r(h_r)$-schematisation the water velocity gradients are plotted in figure 3.5. Regarding the resulting gradients, the different values for the coefficients C and R also cause a deviation from "reality". However, much smaller than the results of the rectangular Q,A-option. One has to consider that the plotted gradient of the Q,A-option already was the best reproduction of the three rectangular options.

The figure illustrates a situation where at $N = 200$ water is withdrawn. At $N = 0$, the river flows in a lake. Figure 3.5 shows that the "calibrated" cross-section gives a quite good reproduction of "reality".

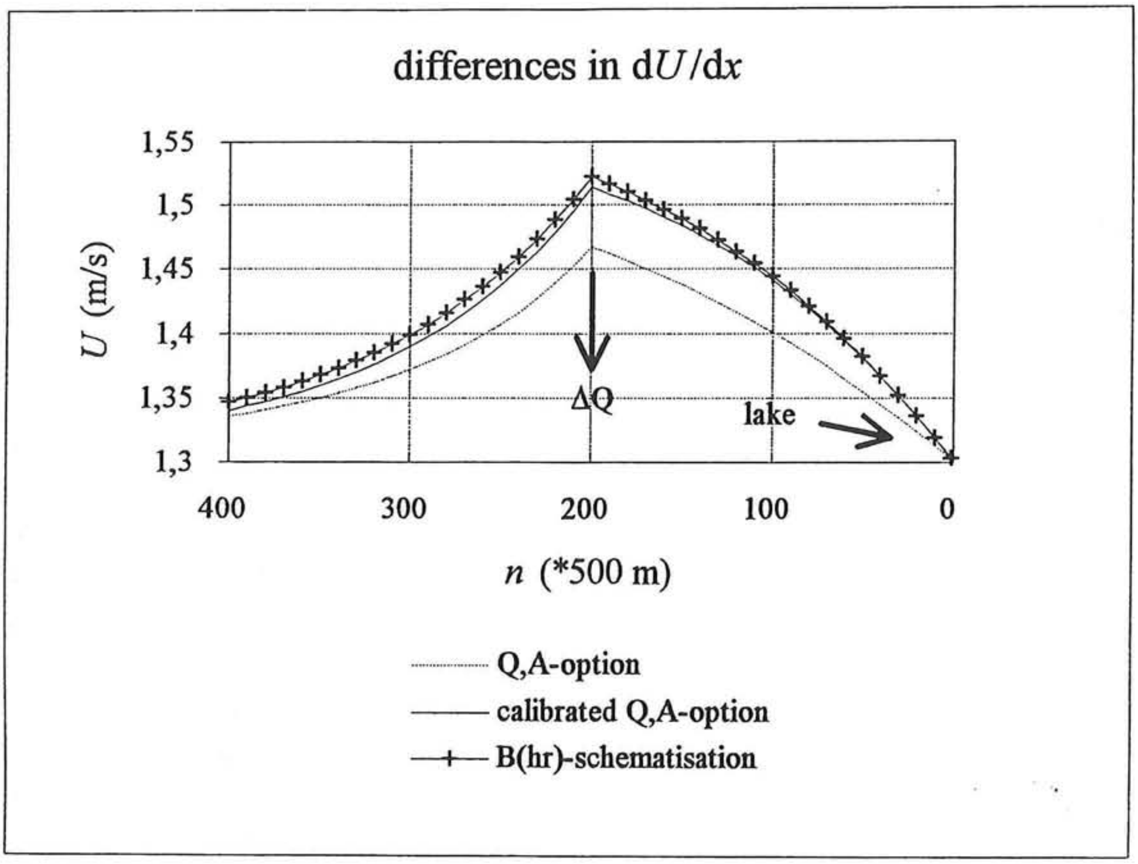


Fig 3.5

Now, the schematisation of the cross-section is not rectangular anymore, which means that the general equations for water movement have to be used instead of the equations for unit of width.

The sediment transport will be calculated according to $S = B_m U^n$. With the "calibrated" schematisation, the gradient in the sediment transport will be schematised quite accurately, but the quantity will be wrong, because the B_m has nothing to do with the sediment transport or morphology. The final morphological changes at $t = \infty$ will be calculated correctly. However, the time scale of the morphological change will be wrong.

To achieve a schematisation of the real quantity of sediment transport in the cross-section, The B_m must be calculated according to $S_m = S_r$:

$$\int_0^{B_r} (h_r - z(y))^{n/2} \mathrm{d}y = B_m R^{n/2} \tag{3.19}$$

with

$$R = \left(\frac{1}{A} \int_0^{B_r} \frac{C}{C_1} a^{3/2} \, dy \right)^2 \tag{3.20}$$

with $C = C_1$ = constant over the width.

For this B_m, the profile is "calibrated" again. In practice, the calculated B_m will be close to the surface width, so a nearly rectangular cross-section will be obtained, with (smaller) errors in the water velocity gradient when a discontinuity occurs. The hydraulic radius will also differ from the real value, which means that the Chézy value has to be adapted, which gives the small known error.
When in equation 3.19 the hydraulic radius R according to $R = A/P$ would be used, which is smaller for irregularly shaped cross-sections, the calculated B_m would be larger than the surface width.

The final result is a definition of the initial condition $z_m(x,0)$ from $z_r(x,y,0)$. Regarding the parameters of the water movement, the wetted area and the surface width (for only a small range of height) are well-reproducible. The hydraulic radius will be smaller, so the Chézy value has to be adapted. The schematisation gives a good reproduction of the sediment transport in the initial state. The morphological changes will be calculated according to $\Delta z(x,t)$ over B_m. However, this cross-section is only valid for one discharge.

3.4 Varying discharge

3.4.1 General

In reality there will be a fluctuating discharge. As mentioned before, a single discharge cannot provide the same morphological changes as the real sequence of discharges (De Vries, 1993a). So the schematisation profile has to be valid for a sequence of discharges.

For each discharge, the sediment transport will be different, caused by a different water velocity U, but also caused by a variation in the sediment carrying width B_{sed}, calculated according to equation 3.19. Again, this will especially apply to irregularly-shaped cross-sections. For almost rectangular cross-sections, the fluctuation in B_{sed} will be small.
The limitation of expressing the morphological changes as $\Delta z(x,t)$ is that also one $B_{sed}(x)$ is required. When at each time step during the calculation, the morphological change till that moment has to be given well by $\Delta z(x,t)$, a fluctuation of $B_{sed}(x)$ is not allowed, because it would imply a fluctuation in the total morphological change ($\Delta A_{sed}(x,t) = \Delta z(x,t) * B_{sed}(x)$).

In view of these findings it becomes difficult to find one fixed schematisation, valid for all the discharges, regarding the conditions mentioned in section 2.6. Using a statistical method, with a probability density $p(Q)$, it is possible to find one $z_m\,(x,0)$ and one $B_{sed}(x)$.

3.4.2 Schematisation

The general idea of calculating with one fixed schematisation profile, is to express the calculated morphological change as $\Delta z(x,t)$ over $B_{sed}(x)$, according to

$$B_{sed}\,\frac{\partial z}{\partial t} + \frac{\partial S}{\partial x} = 0 \tag{3.21}$$

A possibility to determine one $B_{sed}(x)$ is to find a *mean* $B_{sed}(x)$ by schematising the total sediment transport in a year (V). Having a probability density $p\{Q\}$ and a discharge rating-curve, the *mean* $B_{sed}(x)$ will be found with

$$B_{sed}(mean) = \frac{\sum\left\{\,p(h_r)\;S(h_r)\,\right\}}{\sum\left\{\,p(h_r)\;U(h_r)^n\,\right\}} \tag{3.22}$$

in which

$$\sum\,\left\{\,p(h_r)\;S(h_r)\,\right\} = \sum\left\{p(h_r)\{\,C\,\sqrt{i_b}(h_r)\}^n\int_0^{B_r}(h_r-z(y))^{n/2}\mathrm{d}y\right\} \tag{3.23}$$

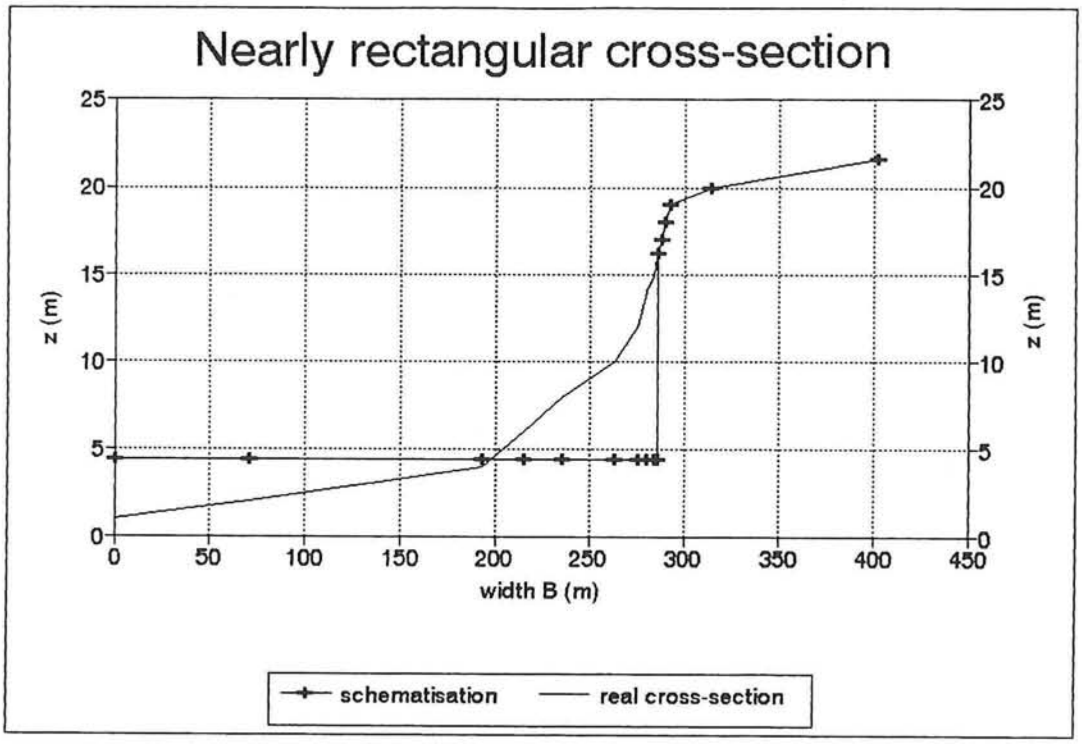
and

$$\sum\,\left\{\,p(h_r)\;U(h_r)^n\,\right\} = \sum\left\{p(h_r)\left\{\frac{C\,\sqrt{i_b}\,\int_o^{B_r}(h_r-z(y))^{3/2}\mathrm{d}y}{\int_o^{B_r}(h_r-z(y))\mathrm{d}y}\right\}^n\right\} \tag{3.24}$$

Subsequently, the profile will be "calibrated" (as in subsection 3.3.3). This method has been worked out for two cross-sections, a nearly rectangular and a nearly triangular cross-section. The results are shown in figure 3.6 and figure 3.7. Two types of errors occur using this method.

The nearly rectangular cross-section shows a rather good schematisation of the parameters of the water movement for all discharges. However, the *mean* $B_{sed}(x)$ is too large for the small discharges and too small for the large discharges in the sequence over a year. These two errors compensate each other, so sediment transport that in reality occurs in time of high discharges, now occurs in time of low discharges. In other words, the fluctuation in the sediment carrying width during the year is not followed by the fixed schematisation profile.
The second type of error is illustrated by the nearly triangular cross-section. The fixed schematisation-profile causes errors in the parameters of the water movement for the lower discharges, because the *mean* $B_{sed}(x)$ will be larger than their surface width.

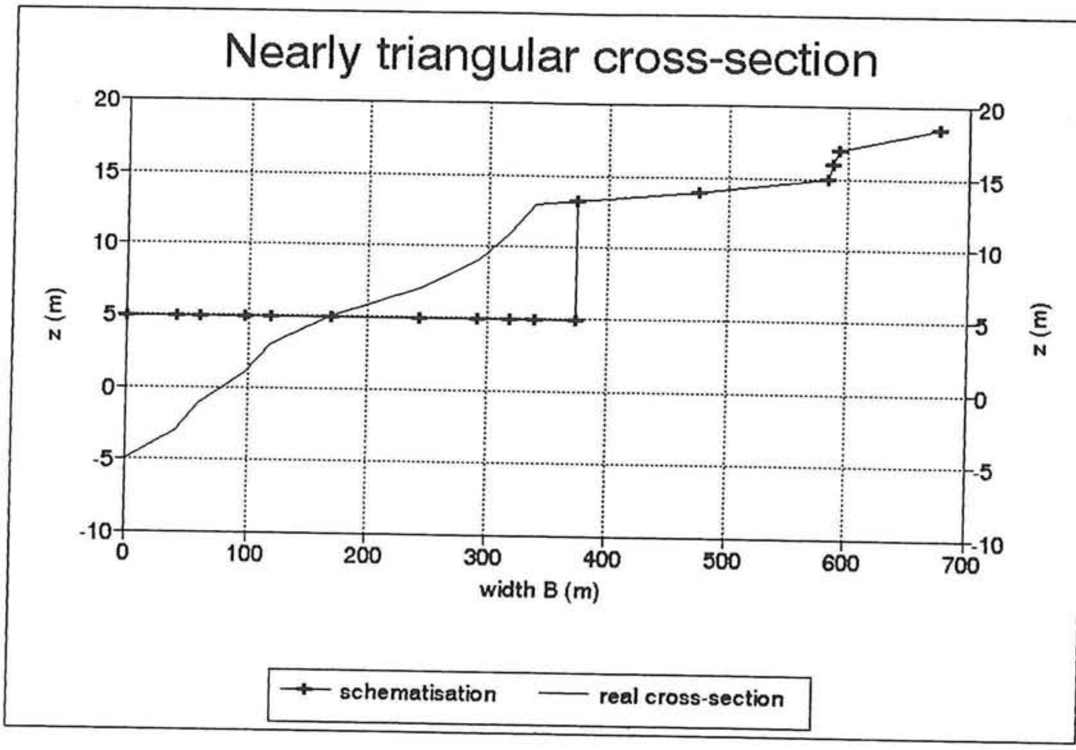
These two errors are especially caused by the general one-dimensional approach of the morphological parameters a and z, which are averaged over the cross-section. Working with one z_m and the expression of the morphological change by $\Delta z(x,t)$ makes a *mean* $B_{sed}(x)$ necessary. On the last page of this chapter, an overview is given of all errors that occur when an averaged water depth and an averaged bed level are required, together with the solution which will be worked out in chapter 5.



for each occuring discharge, the obtained parameter for the schematisation
is compared with the real parameter

Qr	hr	Bopm/Bop	Rm/Rr	Qm/Qr	Am/Ar	Sm/Sr	Csqrti
462	12.78	1.03	0.88	0.93	0.99	1.11	0.06
500	12.88	1.03	0.88	0.94	0.99	1.11	0.07
522	12.9	1.03	0.89	0.94	0.99	1.11	0.07
533	12.92	1.03	0.89	0.94	0.99	1.11	0.07
516	13.02	1.03	0.89	0.94	0.99	1.1	0.07
526	13.03	1.03	0.89	0.94	0.99	1.1	0.07
557	13.06	1.03	0.89	0.94	1	1.1	0.07
553	13.11	1.03	0.89	0.94	1	1.1	0.07
636	13.29	1.03	0.89	0.94	1	1.09	0.08
807	13.33	1.03	0.89	0.94	1	1.09	0.1
652	13.36	1.03	0.89	0.94	1	1.09	0.08
725	13.45	1.03	0.9	0.94	1	1.09	0.09
958	13.74	1.02	0.9	0.95	1	1.08	0.11
1210	14.32	1.02	0.91	0.95	1	1.06	0.13
1290	14.43	1.02	0.91	0.95	1	1.06	0.14
1580	14.88	1.01	0.92	0.96	1	1.05	0.16
1820	15.28	1.01	0.93	0.96	1	1.04	0.17
2600	15.94	1	0.93	0.97	1	1.04	0.22
2720	16.04	1	0.94	0.97	1	1.03	0.23
4520	18.14	1	0.95	0.98	1	1.02	0.3
4480	18.2	1	0.95	0.98	1	1.02	0.3
6500	20.28	1	0.97	0.98	1	0.98	0.35

Fig. 3.6



For each occuring discharge, the obtained parameter for the schematisation is compared with the real parameter

Qr	hr	Bopm/Bop	Rm/Rr	Qm/Qr	Am/Ar	Sm/Sr	Csqrti
516	9.58	1.25	0.54	0.67	0.91	2.34	0.09
522	9.63	1.24	0.55	0.67	0.91	2.3	0.09
462	9.71	1.24	0.55	0.68	0.91	2.24	0.08
526	9.78	1.23	0.56	0.68	0.92	2.18	0.09
572	9.88	1.23	0.57	0.69	0.92	2.11	0.1
557	9.9	1.23	0.57	0.69	0.92	2.1	0.09
533	9.91	1.23	0.57	0.69	0.92	2.09	0.09
553	9.94	1.22	0.57	0.7	0.92	2.07	0.09
636	10.04	1.22	0.58	0.71	0.93	2.01	0.1
500	10.05	1.22	0.58	0.71	0.93	2	0.08
725	10.08	1.22	0.58	0.71	0.93	1.98	0.12
807	10.29	1.21	0.6	0.72	0.94	1.87	0.13
652	10.74	1.18	0.63	0.75	0.95	1.67	0.09
958	10.88	1.17	0.63	0.76	0.96	1.62	0.13
1290	11	1.17	0.64	0.77	0.96	1.58	0.18
1210	11.51	1.15	0.67	0.8	0.97	1.44	0.15
1580	11.73	1.14	0.68	0.81	0.98	1.39	0.19
1820	11.77	1.14	0.69	0.81	0.98	1.39	0.22
2600	12.04	1.13	0.7	0.82	0.98	1.33	0.3
2720	12.18	1.13	0.71	0.83	0.99	1.31	0.31
4480	14.04	1	0.79	0.89	1	1.07	0.39
4520	14.07	1	0.79	0.89	1	1.06	0.39
6500	14.8	1	0.82	0.91	1	0.94	0.5

Fig. 3.7

<u>1-dimensional approach: parameters *S, U, z* and *a* averaged over cross-section</u>

One discharge

<u>rectangular profile with two unknown: z_m and B_m</u>

Q,A-option	initial state:	A, U, a_e, R good, S too small
	flow profile:	wrong interaction between ΔA, ΔR and Δh. Best reproduction of three options because of most realistic B_{sur} result is a wrong time scale of the morphological changes
Q,S-option	initial state:	a_e and S good, A too small thus U too large and R too small
	flow profile:	wrong interaction, smaller B_{sur}
A,S-option:	initial state:	a_e too large, Q as boundary condition, A smaller thus U larger, thus S larger. R smaller
	flow profile:	worst interaction, smallest B_{sur}

<u>solution: not rectangular profile with:</u>

B_{sed} with *S-integral*

z_m with *A-integral* until $h_r(B_{sed})$

$h_r > h_r(B_{sed})$: real cross-section

initial state:	A, U, and S good, R not good, so C has to be adapted
flow profile:	quite good, good B_{sur}

Varying discharge

required: one z_m and one B_{sed}.

solution: statistical method with probability density $p\{Q\}$

errors because: no variation in B_{sed} possible for varying discharge errors in parameters for smaller discharges, because $B_{sed} > B_{sur}$, especially for irregular profiles.

<u>Solution: 1-dimensional approach: no averaged *a* and *z*, *A*, *S* and *R* with *integrals*</u>

cross-section: $B = B(h_r)$. water movement: initial state and flow profile good good for all discharges and all shapes.
sediment movement: calculation B_{sed} with S- and R-*integrals*: valid for initial state and flow profile good

variation of parameters over the width is integrated in the 1-dimensional calculation, so a 2-dimensional accuracy is achieved.

4 SCHEMATISATION: WIDTH AS FUNCTION OF WATER LEVEL

4.1 General

Leaving the general idea of schematising the morphological change as a change of the bed level $\Delta z(x,t)$ over a sediment carrying width $B_{sed}(x)$, a one-dimensional approach arises, in which the morphological parameters a (water depth) and z (bed level) are not averaged over the cross-section anymore.

Instead of averaging a and z_b over the cross-section, the cross-section is given as $B_m(h_r) = B_R(h_r)$. Regarding the water movement, for all occurring discharges the conditions of section 2.6 are no problem anymore. The parameter A (wetted area) is calculated with the *integral*-theory, while Q is given as boundary condition. An averaged water flow velocity U over the cross-section can be calculated according to $U = Q/A$. For all water levels, the parameters in the one-dimensional equation for the flow profile are reproduced correctly.
This is also important for a good reproduction of the sediment movement. However, for the sediment movement a sediment carrying width is still missing. In a computer model, it is possible to give it as a parameter, which will be worked out in this Chapter.

The result is a one-dimensional model (only the longitudinal direction), while at each space-step x the morphological parameters are calculated with a two-dimensional accuracy (transverse direction) for a varying discharge. The morphological change will be expressed as $\Delta A_{sed}(x,t)$, a change in the cross-sectional area under the cross-section till a certain reference level. This area is illustrated in figure 4.1

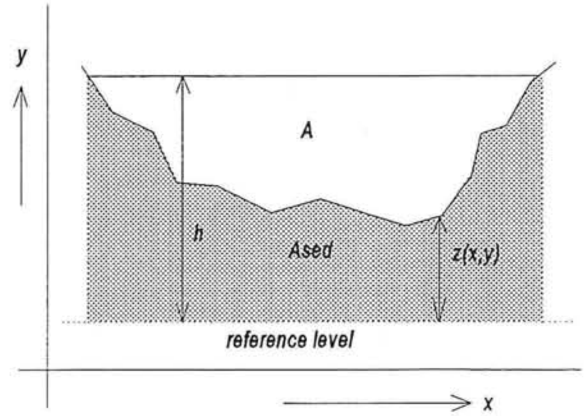


Fig 4.1

The parameters U, A_{sed} and S will be given as one value as function of x. The equation for continuity of sediment becomes:

$$\frac{\partial A_{sed}}{\partial t} + \frac{\partial S}{\partial x} = 0 \tag{4.1}$$

It can be assumed that most morphological changes occur in the deeper part of the river, so that the calculated $\Delta A_{sed}(x,t)$ will be representative for all water levels.

4.2 Sediment carrying width

4.2.1 Definition

In the steady uniform flow situation, a good reproduction of the sediment transport, with a variation over the width, is given by:

$$S = C^n \, i_b^{n/2} \int_0^{B_r} a^{n/2} \mathrm{d}y \tag{4.2}$$

Using the Chézy law for the one-dimensional reproduction of the sediment transport, this gives:

$$S = B_{sed} \, C^n \, i_b^{n/2} \, R^{n/2} \tag{4.3}$$

Using the Engelund equation for R, the sediment carrying width B_{sed} becomes

$$B_{sed} = \frac{\int_0^{B_r} (h-z(y))^{n/2} \mathrm{d}y}{\left(\dfrac{\int_0^{B_r} (h-z(y))^{3/2} \mathrm{d}y}{\int_0^{B_r} (h-z(y)) \mathrm{d}y} \right)^n} \tag{4.4}$$

Thus, for a steady uniform situation, the sediment carrying width can be found this way, as function of the water level and the geometry of the cross-section, $B_{sed}(h_r)$.

4.2.2 Sediment carrying width as function of the water level

However, morphological changes occur in a non-uniform situation, when there is a flow profile over the river. Regarding a one-dimensional computer program, the water movement, and thus the gradient in the water velocity, which is important with respect to morphological changes, is schematised accurately when the real cross-section is used. The sediment transport will be calculated with:

$$S = B_{sed}\ (Q/A)^n \tag{4.5}$$

The question is whether the equation obtained for the sediment carrying width B_{sed}, as function of the water level, can be used in a flow profile. It can be proved that the calculation of the sediment carrying width is not dependent on the flow conditions, but is dependent on the geometry of the cross-section and the water level.

Considering a flow profile, which gives the same water level, only with a discharge which is β times the discharge which belongs to that water level when a steady uniform flow is considered, the sediment transport is calculated in the following way.
First the discharge Q_r is distributed over the width of the real cross-section proportionally to $a^{3/2}(y)$. The proportions of this distribution are plotted in the figure 4.2, for the nearly rectangular profile

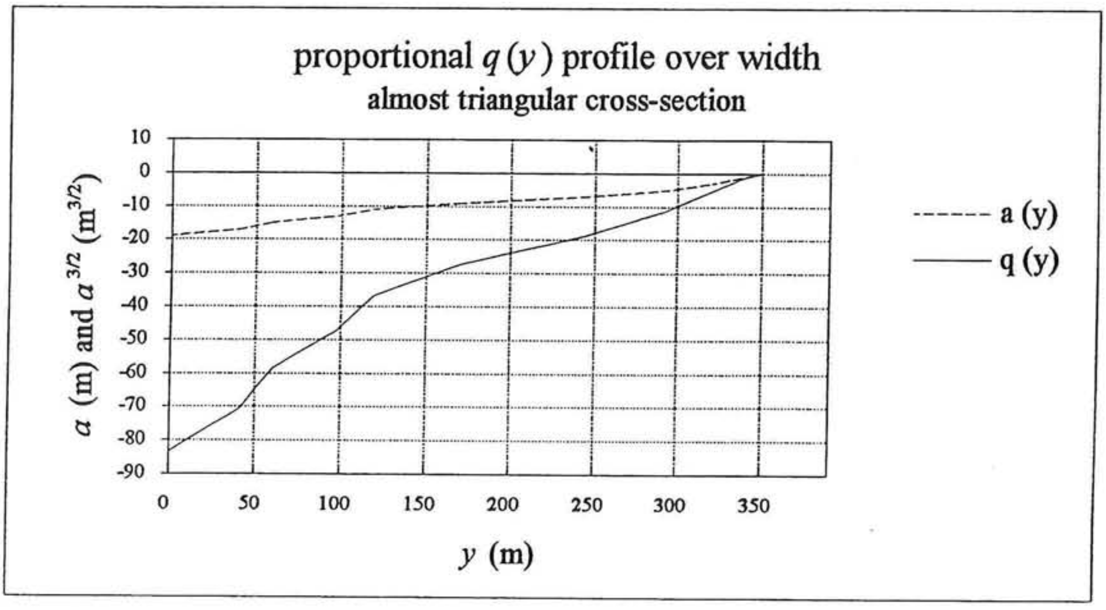


Fig. 4.2

resulting in $q_r(y)$ according to

$$q_r(y) = \frac{Q_r}{\int_0^{B_r} a^{3/2} dy} a^{3/2}(y) \tag{4.6}$$

Then, the depth mean velocity u is calculated as function of y, delivering a $u(y) = q(y)/a(y)$ profile over the cross-section width.
Now the total sediment transport can be calculated according to:

$$S = \int_0^{B_r} (\bar{u}(y))^n dy \tag{4.7}$$

In the computer program the one-dimensional mean water velocity $U = Q/A$ is used. The sediment carrying width B_{sed} is:

$$B_{sed} = \frac{\int_0^{B_r} (\bar{u}(y))^n dy}{(Q/A)^n} \tag{4.8}$$

The result is a sediment carrying width which is the same as the one belonging to that water level for a steady uniform flow.

$$B_{sed}\,(stat.\ uniform) = B_{sed}\,(flow\ profile)$$

This proves that the sediment carrying width is not dependent on the flow conditions, but is dependent on the geometry of the cross-section and on the water level.

When the water level h is calculated with the one-dimensional flow profile equation, $B_{sed}(h)$ and $A(h)$ can be calculated, resulting in a good representation of the total sediment transport through the total cross-section.

To show the variation of $B_{sed}\,(x,t)$ as function of $h_r(x,t)$, and the dependence of the geometry on the variation of $B_{sed}(x,t)$, for both the nearly rectangular and the nearly triangular cross-section, this variation is given in figure 4.3 (for the formula of Engelund and Hansen, 1967, with $n = 5$)

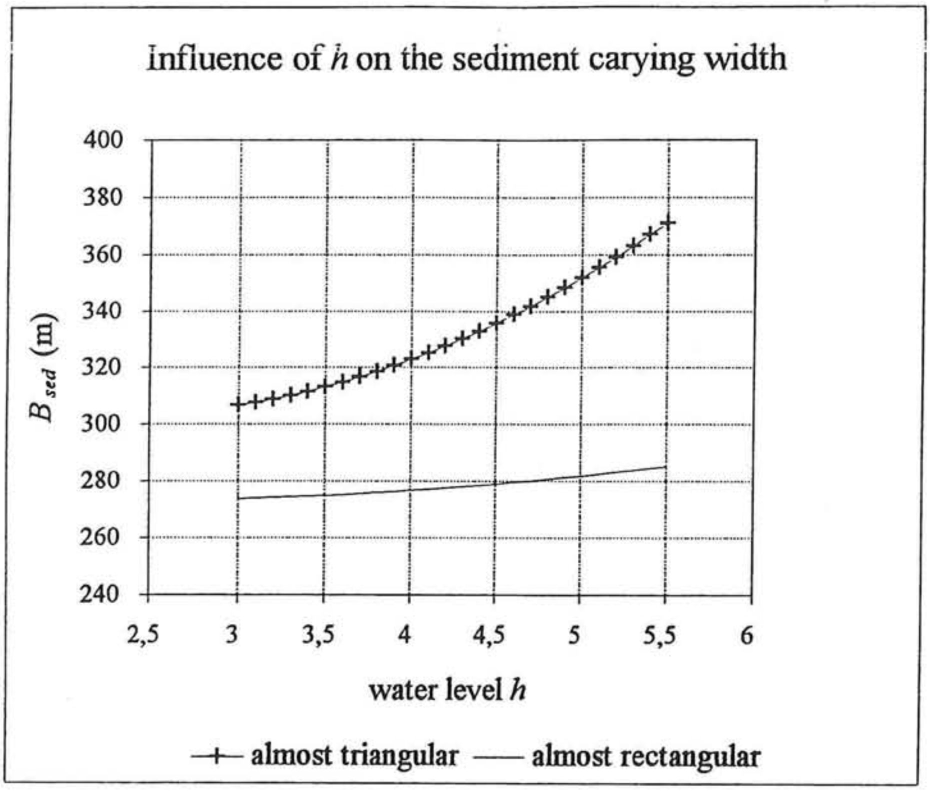


Fig 4.3

This figure shows a small variation of $B_{sed}(x,t)$ for the nearly rectangular cross-section, compared to the nearly triangular cross-section.
In Appendix A the variation of $B_{sed}(x,t)$ as a function of h_r is given for some more cross-sections.

4.2.3 Sensitivety of the exponent *n*

The general formula for sediment transport, $S=B_{sed}*m*U^n$, shows a dependence on the exponent *n,* which contains, together with *m,* all parameters which govern the transport process.
Using equation 4.4 for the sediment carrying width, there is also an influence of *n* on $B_{sed}(x,t)$.
For a non-rectangular profile, an increase in *n* causes an increase in $B_{sed}(x,t)$.

This will be demonstrated by an example of a composite cross-section, shown in figure 4.4, with $B_1 = B_2 = B$.

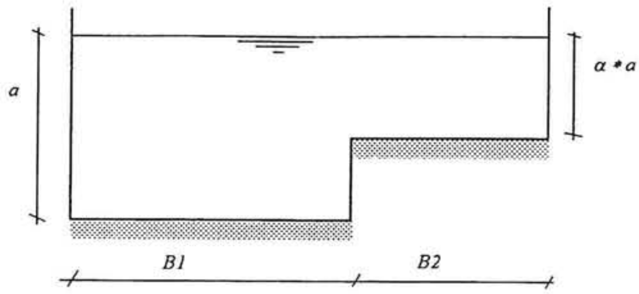


Fig. 4.4

The function for B_{sed} becomes

$$B_{sed} = B \frac{1 + \alpha^{n/2}}{\left(\dfrac{1 + \alpha^{3/2}}{1 + \alpha} \right)^n} \tag{4.9}$$

With $\alpha < 1$ and $n > 3$, this function gives an increasing B_{sed} with an exponential character for an increasing n.

Depending on the geometry of the cross-section, this influence can be either small or large. For the nearly rectangular and the nearly triangular cross-section, the results are plotted in figure 4.5.

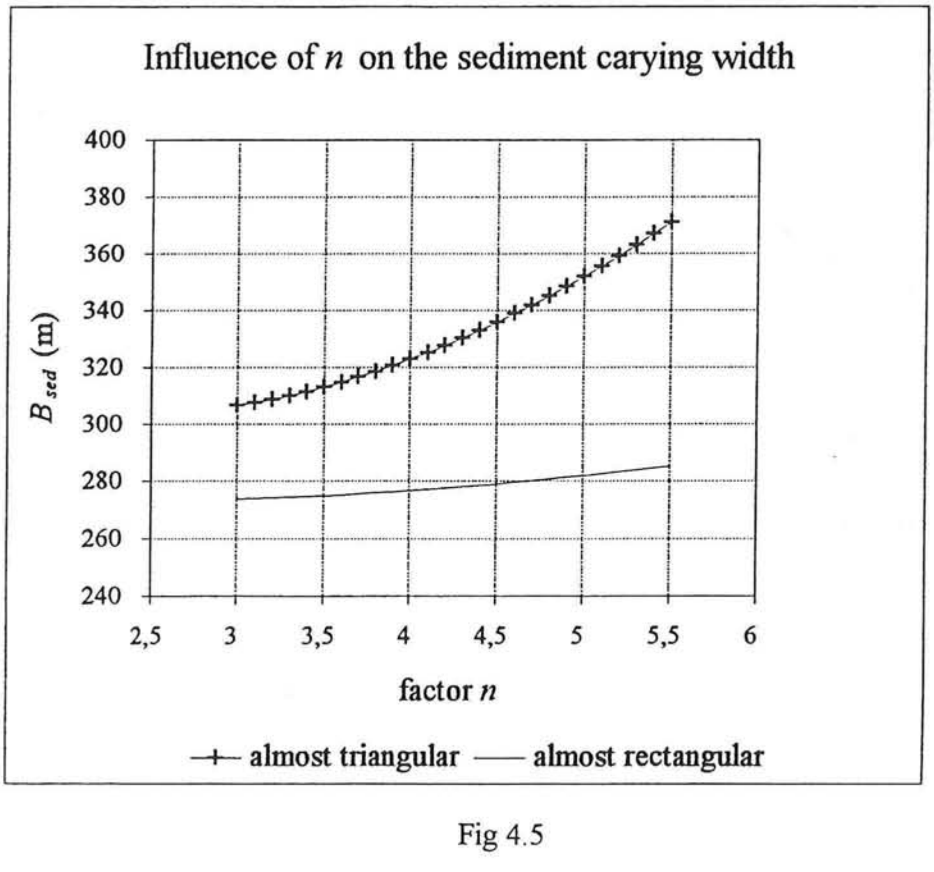


Fig 4.5

The general formula for sediment transport thus becomes:

$$S = B_{sed}(n)\ m\ U^n \tag{4.10}$$

Consequently, it is concluded that the sediment carrying width $B_{sed}(x,t)$ is a function of:

* The geometry of the cross-section
* The chosen transport formula (n)
* The occurring water level $h_r(x,t)$

For calculations with one discharge, it is advisable to use this method also, because it is to be expected that the sediment carrying width changes in a flow profile with varying water levels.

4.3 Chézy value as a function of the water level

In the initial state it is possible to determine the Chézy value C according to

$$C\ \sqrt{i_b} = \frac{Q_r}{\sqrt{R}} \tag{4.11}$$

When an averaged bottom slope is assumed this gives a Chézy value. Because the water movement is reproduced correctly, it is not necessary to calibrate this C, which means it can be used in the model, and is constant over the width.
When there is a fluctuation in the water level, there will be a fluctuation in the Chézy value as well, which is realistic, regarding the Colebrook-White formula (for values of R/k below 1000)

$$C = 18\ \log\ \frac{12\ R}{k} \tag{4.12}$$

In which k is the bottom roughness. An increasing R induces an increasing C.

Again, this fluctuation will be smaller for nearly rectangular cross-sections compared to irregularly shaped cross-sections.

4.4 Validation of *integral*-theory

To use the approach with the $B_r(h_r)$-schematisation in the model, and the calculation of the sediment carrying width for each space-step at each time-step (the *integral*-theory), the data needed are Q-h rating curves and bed elevations of cross-sections. Moreover approximating values of the coefficient m_0 and the exponent n have to be known.

It is possible to validate the *integral*-theory when measurements of the sediment transport are done and Chézy values are known. These data are not very often available .

However, for the River Waal these data are available at R.W.S. Arnhem, and some tests have been done. The available data are Chézy-values belonging to some discharges and measurements of the sediment transport in combination with discharges for a characteristic reach of the River Waal, between Tiel and Dodewaard. Also approximating values of n and m_0 are known: $n = 4$ and $m_0 = 0.000042$.

The methods of measurement used for the sediment transport of bed material are divided into bed load (transport) and suspended load (transport).

For the bed load, two methods are used, both plotted in figure 4.6.. First the dune-tracking method. With an echo-sounder the dune-velocity is estimated. These are the input data for a software package, which gives the bed load as output data. This method is plotted as +. The second method used here is the *BTMA*-method (bodem transport meter Arnhem), which is plotted as ○.

For the suspended load and the bed load together, the used method is called "akoestische zand transport meter" the *AZTM*, indicated in figure 4.6 by Δ.

Two mathematical calculation methods are also plotted. One for the bed load, according to the Meyer-Peter and Müller formula, and one for suspended load of bed material, according to the Engelund and Hansen formula.

It has to be mentioned that the tests with the *integral*-theory are only valid for the conveying cross-section between the groynes. These tests assume only sediment transport in the summer bed. For water levels higher than the top of the groyne, errors will arise in the results of the calculation according to the *integral*-theory, because of the discharge over the groynes. The calculation contains the influence of the wetted area in the cross-section above the groynes as good as possible. Here a reasonable height of the top of the groynes is used, because the height is not exactly known for each cross-section.

The result is a quite good approximation of the total sediment transport of the bed material (△) by the *integral*-theory, as can be seen in figure 4.6. Here, for one cross-section only the calculation is plotted which takes the influence of the wetted area above the groynes into account. In Appendix B, the results for some more cross-sections are given, together with the data used.

In the following tables, the measured and calculated results of some parameters are given for one cross-section. The results of the calculated parameters will be compared with the measured parameters for some discharges (in Table 1, 2 and 3). Here also the results of the calculation without the influence of the wetted area above the groynes are given. Note the errors in this calculation for the higher discharges:

Calculation one: without influence of wetted area above the groynes
Calculation two: with influence of wetted area above the groynes

$Q = 1345\ m^3/s$:

Table 1

	velocity U (m/s)	depth a (R) (m)	Chézy C ($m^{1/2}/s$)
measured	1.05	4.94	43.73
calculation 1	1.047	4.90	43.73
calculation 2	1.047	4.90	43.2

$Q = 2145\ m^3/s$:

Table 2

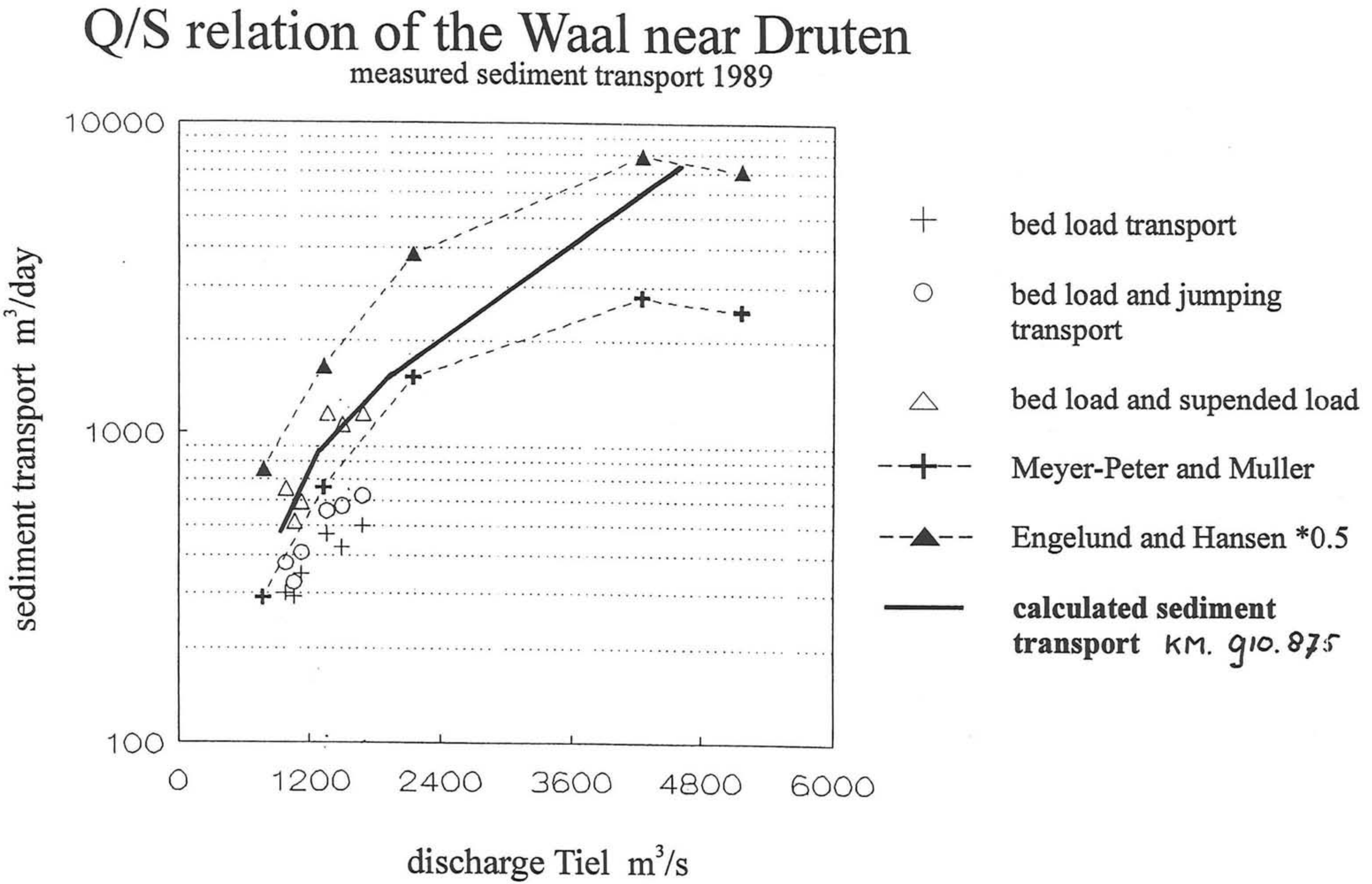
	velocity U (m/s)	depth a (R) (m)	Chézy C ($m^{1/2}/s$)
measured	1.24	6.64	44.28
calculation 1	1.22	6.51	43.6
calculation 2	1.19	6.51	43.3

$Q = 4247\ m^3/s$:

Table 3

	velocity U (m/s)	depth a (R) (m)	Chézy C ($m^{1/2}/s$)
measured	1.43	8.98	44.93
calculation 1	1.73	8.76	53
calculation 2	1.51	8.76	48.9

The results show that the *integral*-theory approximates the measured data quite good, certainly when the calculation contains the influence of the discharge over the groynes.



Q/S relation of the Waal near Druten
measured sediment transport 1989
10000
1000
100
0
1200
2400
3600
4800
6000
sediment transport m³/day
discharge Tiel m³/s
bed load transport
bed load and jumping transport
bed load and supended load
Meyer-Peter and Muller
Engelund and Hansen *0.5
calculated sediment transport KM. 910.875

Fig. 4.6

5 INTERPRETATION OF MORPHOLOGICAL CHANGES

5.1 General

As mentioned before, the use of a model is a three-step approach:

* The river is schematised into a model
* The new situation is calculated for the model-river
* The solution is interpreted for the real river

Regarding a one-dimensional mathematical model, in Chapter 3 the first and the third step where translated into two problems which have not been solved yet:

* *Schematisation* of the initial condition $z_m(x,0)$ from $z_r(x,y,0)$.

* *Interpretation* of the computed values $z_m(x,t)$ for the required prediction of $z_r(x,y,t)$

Chapter 3 shows that it is not possible to calculate an average $z_m(x,0)$ from $z_r(x,y,0)$, which gives together with a sediment carrying width a good reproduction of the total morphological process for a sequence of discharges and an irregular cross-section. A solution is to leave the one-dimensional approach, in which the water depth and the bed level are averaged over the cross-section. With the results of Chapter 4, step three is no longer a problem in finding a good interpretation of the computed values $z_m(x,t)$ for the required $z_r(x,y,t)$, but finding a good interpretation of the calculated morphological change, expressed as a change in the cross-sectional area $\Delta A_{sed}(x,t)$, for the real cross-section.

There are two problems regarding the changes:

* distribution over the real cross-section (see also Cunge 1980)

* Change in time of some significant parameters during calculation time has to be followed as well as possible

5.2 Distribution over real cross-section

According to the one-dimensional equation for continuity of mass, the morphological change is given as a change in the cross-sectional area $\Delta A_{sed}(x,t)$, the eroded or deposited sediment in a cross-section (see figure 6.1.)

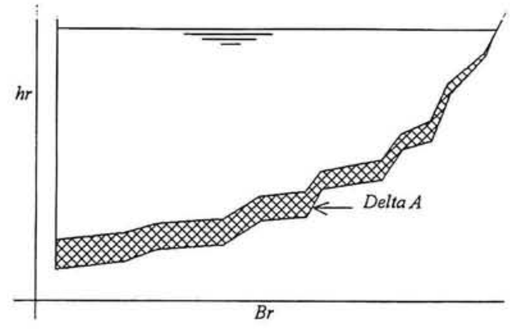


Fig. 5.1

Physically, it is not right to distribute the $\Delta A_{sed}(x,t)$ equal over the whole cross-section. More realistic options are distributions proportional to

* the water depth, a^1
* the water velocity, $a^{1/2}$
* the sediment transport capacity, $a^{5/2}$

This way a two-dimensional result in space is calculated with a one-dimensional calculation.

5.3 Influences on new equilibrium state at river mouth

5.3.1 General

A study will be performed on a cross-section at the mouth of a river, flowing in a lake with a constant water level. Upstream, there will be a discontinuity, i.e. the withdrawal of water, $\Delta Q = \gamma * Q_0$.
To get a first impression of the morphological change regarding the three options of distribution, only the two equilibrium situations are considered.

The new equilibrium state, at $t=\infty$, has two conditions: $Q_1 = (1-\gamma) * Q_0$
$S_1 = S_0$

With:

$$Q = A\ C\ \sqrt{R}\ \sqrt{i_b}$$

$$S = B_{sed}\ C^n\ \sqrt{R^n}\ \sqrt{i_b}^n$$

Using the integrals discussed before, S becomes:

$$S = C^n \, i_b^{n/2} \frac{\int_0^{B_r} (h-z(y))^{n/2} \mathrm{d}y}{\left(\frac{\int_0^{B_r} (h-z(y))^{3/2} \mathrm{d}y}{\int_0^{B_r} (h-z(y)) \mathrm{d}y} \right)^n} \left(\frac{\int_0^{B_r} (h-z(y))^{3/2} \mathrm{d}y}{\int_0^{B_r} (h-z(y)) \mathrm{d}y} \right)^n \tag{5.1}$$

which gives:

$$S = C^n \sqrt{i_b}^{\,n} \int_0^{B_r} (h-z(y))^{n/2} \mathrm{d}y \tag{5.2}$$

and Q becomes:

$$Q = C \cdot \sqrt{i_b} \cdot \int_0^{B_r} (h-z(y))^{3/2} \mathrm{d}y \tag{5.3}$$

Regarding the two conditions for $t=\infty$, the new bottom slope can be defined in two ways:

$$\sqrt{i_{b1}} = \frac{\alpha \sqrt{i_{b0}} \int_0^{B_r} (h-z_0(y))^{3/2} \mathrm{d}y}{\int_0^{B_r} (h-z_1(y))^{3/2} \mathrm{d}y} \tag{5.4}$$

and:

$$i_{b1}^{\ n/2} = \frac{\int_0^{B_r} (h-z_0(y))^{n/2} \ \mathrm{d}y \cdot i_{b0}^{\ n/2}}{\int_0^{B_r} (h-z_1(y))^{n/2} \ \mathrm{d}y} \tag{5.5}$$

By way of iteration, the new equilibrium state can be calculated, considering the correlation between the three changing parameters. This is done by enlarging the deposited material $\Delta A_{sed}(x,t)$ and distributing it proportional to a^{ξ}, until the two bottom slopes are equal.

There are two coefficients which have an influence on the final result, i.e. the distribution coefficient ξ and the sediment transport exponent n.
There are some important practical differences caused by these two parameters. For navigation aspects these are the maximum water depth a_{max}, or in other words the width with the minimum water depth needed for navigation. For navigation, the mean water velocity U is also important. For inundation danger upstream an important parameter is the final bottom slope i_b, which influences the water level h upstream.

5.3.2 Influence of distribution coefficient ξ

First, the influence of the distribution coefficient ξ on these parameters is shown. Again, the influence will be greater when a cross-section is less rectangular. In the complex calculation for the new equilibrium state an increasing ξ has the following influences:

* a decreasing ΔA, and thus an increasing $U = Q/A$
* a decreasing a_{max}
* an increasing i_b

which will be shown in the figures 5.2, 5.3 and 5.4, where it is calculated for the nearly triangular and the nearly triangular cross-section with $n = 5$, for the formula of Engelund and Hansen (1967). The discontinuity which causes the morphological changes is a withdrawal of water.

For the *A-integral*, and especially the *S-* and *Q-integrals*, the deeper parts of the river determine the magnitude of the value. To obtain a lower value for the *Q-integral*, less deposit material is needed when more is distributed over the deeper parts, which happens with an increase in the ξ-value, as can be seen in figure 5.2.

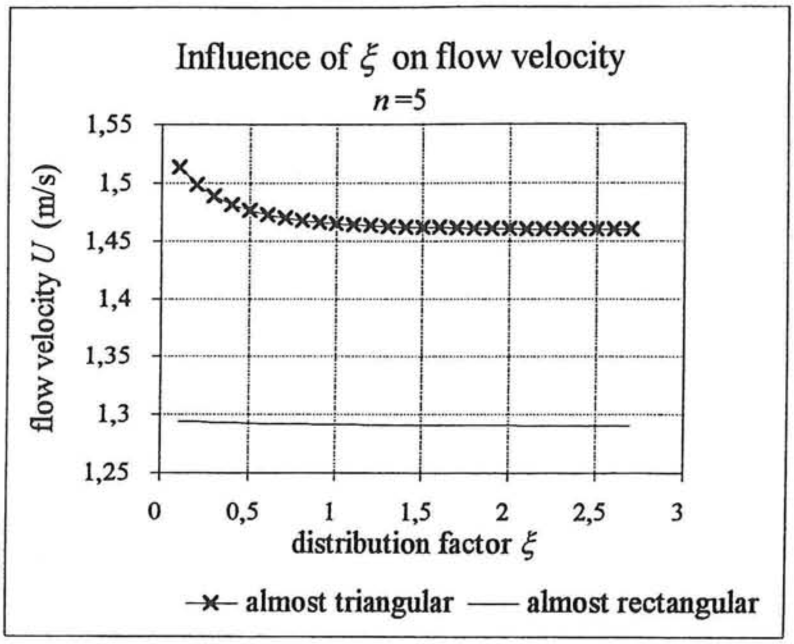


Fig. 5.2

With an increase in ξ it stands to reason that the deeper parts will get proportionally more deposited material, which implies a decrease in a_{max} (fig 5.3). However, when the minimum navigation depth is much smaller than the maximum water depth, the distribution coefficient is still of some importance. For this situation, an increase in ξ implies an increase in the possible navigation width.

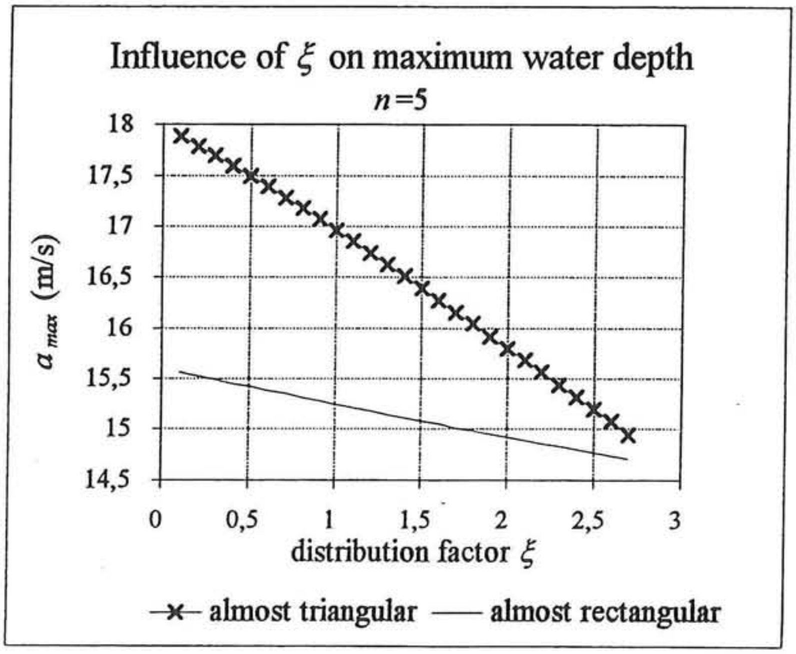


Fig. 5.3

An increase in ξ causes an increase in i_b in the final new equilibrium state, and thus in the water level upstream. This requires higher river dikes upstream. This influence is plotted in figure 5.4.

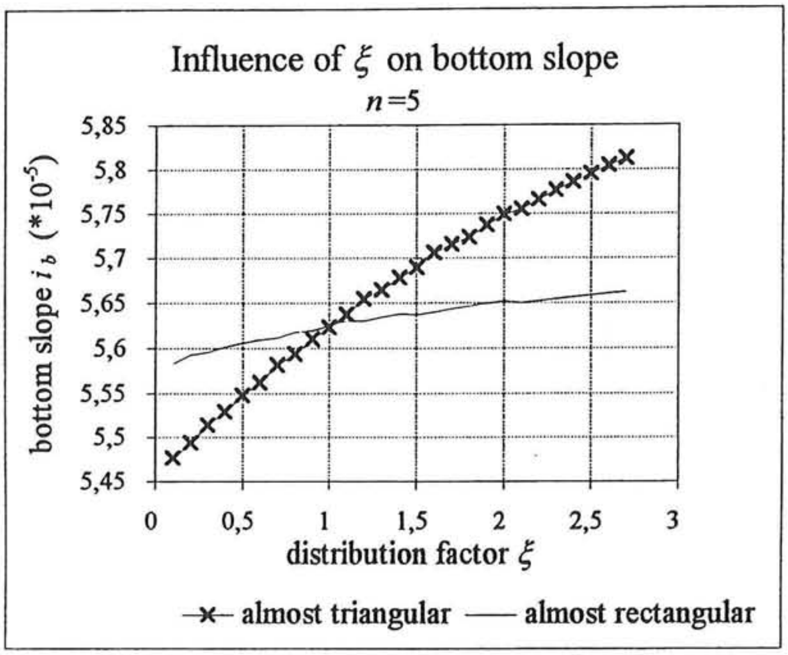


Fig 5.4

All figures show less influence of a varying distribution parameter for a nearly rectangular cross-section, which is physically correct.

5.3.3 Influence of the exponent *n*

The sediment transport exponent *n* influences the range of fluctuation caused by the distribution parameter ξ.
For the i_b this fluctuation decreases for a decreasing *n*, and becomes zero for $n = 3$. For the nearly triangular cross-section, with a larger fluctuation, this is shown in figure 5.5, by plotting the variation of i_b under influence of the exponent *n* for the three realistic distribution options.

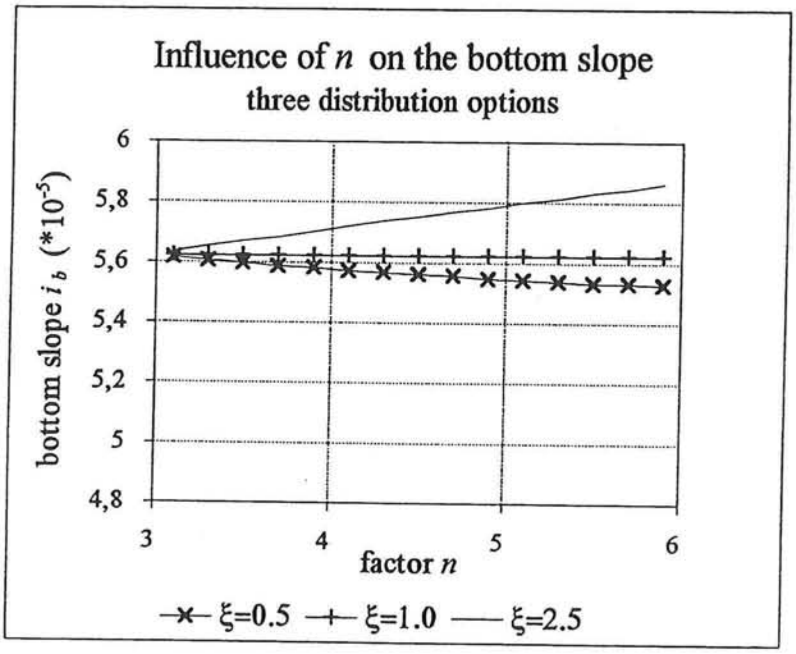


Fig. 5.5

For the two important parameters for navigation, the ranges change also for a varying exponent *n*. (Figure 5.6 and figure 5.7).

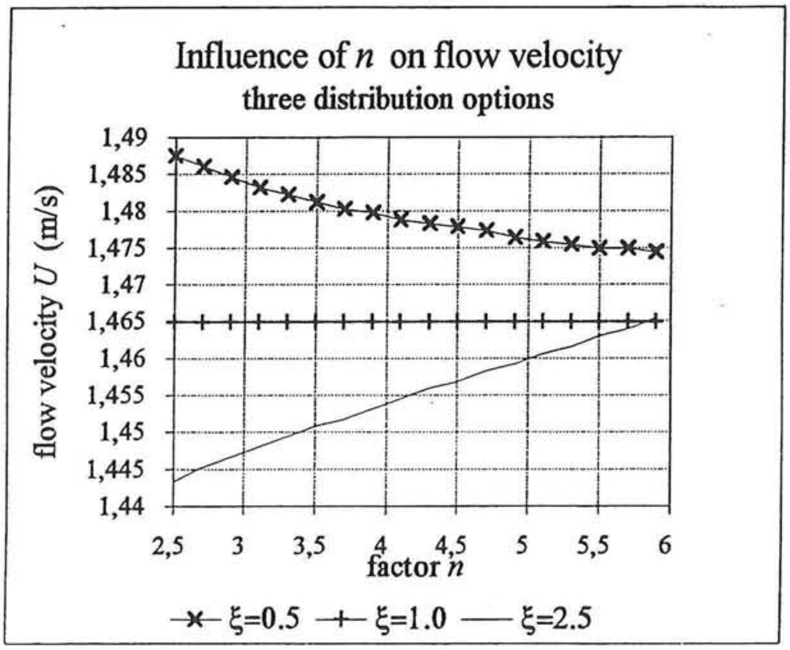


Fig. 5.6

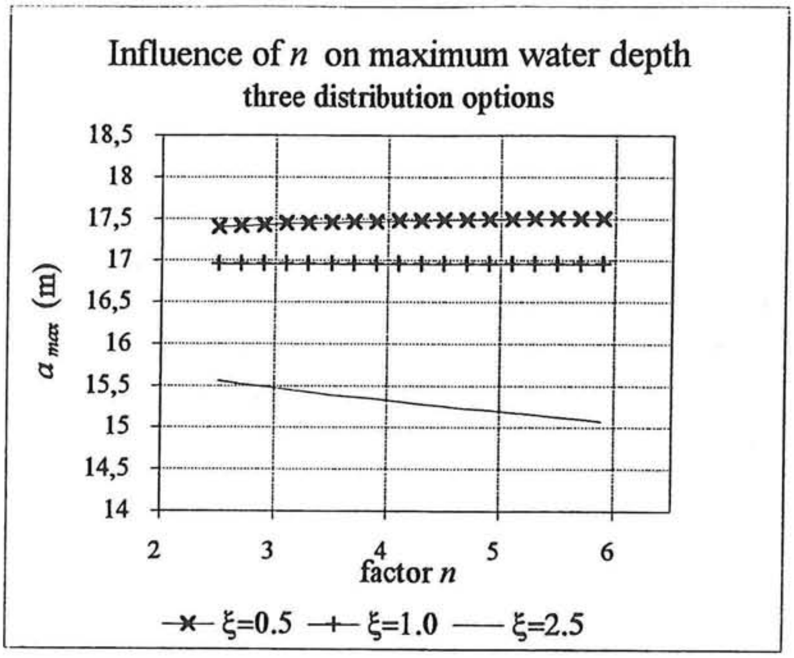


Fig. 5.7

The river engineer has to decide which choice to make concerning the two coefficients, n and ξ. For the distribution coefficient ξ it is not possible to say which one will be correct, because no experiments have been performed studying this topic. Working with a safety factor, the ξ with the most negative result for the particular problem can be taken. The exponent n depends on the physical situation of each problem, and can be *calculated.*

5.4 Change of parameters during calculation time

Considering the quasi-steady situation and the input of the real cross-section, a one dimensional computer program works with the following set of simplified equations:

$$gA\left(1-\frac{Q^2B_s}{gA^3}\right)\frac{\partial a}{\partial x}+gA\frac{\partial z_b}{\partial x}+\frac{Q|Q|}{C^2RA^2}=0 \tag{5.6}$$

$$\frac{\partial Q}{\partial x}=0 \tag{5.7}$$

$$\frac{\partial A_{sed}}{\partial t}+\frac{\partial S}{\partial x}=0 \tag{5.8}$$

$$S = B_{sed}(n)\ m\ U^n \tag{5.9}$$

With respect to these equations, there are some important parameters with a large influence on the total process of water movement and sediment movement in time. During the process of morphological changes, these parameters change as well. These changes have to be followed as good as possible.

In the equation for water motion the discharge Q is given as boundary condition. The Chézy-coefficient C is assumed to be constant over the whole cross-section width. It is possible to give C as a function of the water level. The change of the parameter for the wetted area A will be followed well, with $\Delta A_{sed}(x,t)$ as the morphological change. In case of a cross-section with no storage, the parameter for the stream width B_s will be the same as the surface width. When the morphological change $\Delta A_{sed}(x,t)$ will be distributed proportionally to a^{ξ}, the change in this parameter is negligibly small.

This leaves three parameters which have to be considered. The hydraulic radius R, the bottom slope i_b and the sediment carrying width B_{sed}. Especially B_{sed} and R are two parameters which change according to the real change of the cross-section. This means that $\Delta A_{sed}(x,t)$ has to be distributed several times during the calculation time.

Here the distribution coefficient ξ also influences the change in time. Taking the example used in section 5.3, at the river mouth, and the rectangular cross-section with a withdrawn of water of 11%, the following changes are found for the B_{sed}:

$\xi = 1/2$: $B_{sed1} = 0.97\ B_{sed0}$
$\xi = 1$: $B_{sed1} = B_{sed0}$
$\xi = 5/2$: $B_{sed1} = 1.02\ B_{sed0}$

Physically, it is not known wether the B_{sed} will increase or decrease in case of sedimentation, and the differences are small, so it is not possible to conclude which distribution method will be the right one regarding this parameter. However, these results, for one water level, show that the sediment carrying width is not changing very much when morphological changes occur.

Another important process which has to be considered with respect to a good schematisation of reality in time, is the variation of the water level, which will influence the distribution of $\Delta A_{sed}(x,t)$. Now, there will also be influence on the sediment carrying width which belongs to another time-step (with another water level). This requires a distribution of the morphological change after each time-step.

In section 5.3 a situation at the mouth of the river was studied, with no influence of a varying discharge on the water level. Here it does not matter if a morphological change $\Delta A_{sed}(x,t)$ is distributed only at $t = \infty$, or several times during the calculation (except when the changes of B_{sed} and R in time are required), because the proportions of the variation in depth is kept the same for the latter option.

However, upstream there will be a fluctuation in the water level in time for two reasons. Firstly, because of a change in time of the bottom slope i_b. Secondly, because of a varying discharge, each with his own morphological change. In this point of view, it also should be mentioned that the larger discharges have more influence on the morphological changes than the smaller discharges.

When at each time step the resulting $\Delta A_{sed}(x,t)$ is distributed according to the accompanying water level, it influences proportions of the variation of depth for other water levels. However, it would be a good simulation of reality, but requires much calculation time. It depends on the seasonal fluctuation of the water level, wether this is necessary for each time step. Another disadvantage is that each numerical space step gives different (y,z) coordinates representing the cross-section, which will give a large amount of data, regarding the amount of y,z points of the real cross-section.

A possible solution is not to distribute the calculated $\Delta A_{sed}(x,t)$ and to follow the change of the parameters as closely as possible. As mentioned above, in case of one water level this will be no problem for the B_{sed}, (x,t) because there will be almost no change. However, due to the fluctuation in the water level, and the correlation mentioned, this solution will cause small inaccuracies. So $B_{sed}(x,t)$ will be calculated at each time- and space-step with the B_{sed}-equation, considering the initial bed elevations. This calculation will not take much extra calculation time.

The change in the bottom slope can be followed quite well with the following calculation in the program:

$$\Delta i_b = (\Delta A_{k+1} - \Delta A_k)/(B_{sed}\ \Delta x)$$

where Δx is the numerical space step, and k the number of the space steps.

This leaves the hydraulic radius. Using the Q,A-schematisation option of a rectangular cross-section mentioned in Chapter 3, a z_m and B_m are found, giving a good approximation of R as well. The B_m is used to approximate the changing R in the following way

$$R = R_{initial} - \Delta A/B_m$$

This calculation of $R_{initial}$ and B_m for each time- and place-step (because of the varying water level in time and place) will not take much extra calculation time either.

6 CONCLUSIONS

6.1 General

This study contains two topics. The schematisation of a river cross-section for a one-dimensional morphological computer model and the interpretation back to reality of the calculated morphological change. The next two sections will mention the conclusions of this study for these two topics.

6.2 Schematisation of the cross-section

In the morphological process, more parameters play an important role. In section 3.2, some important parameters are described, together with their schematisation-notation.
All these notations are an integral (over the width of the cross-section) of a certain power α of the water depth α:

$$\int_0^{B_r} (h-z(y))^{\alpha} \mathrm{d}y \qquad (6.1)$$

For the following parameters the power α is:

wetted area A:	$\alpha = 1$
discharge Q:	$\alpha = 3/2$
Sediment transport S:	$\alpha = n/2$

It is important that all these parameters are schematised well in the model.

In Chapter 3 a one-dimensional approach is worked out in which the water depth and the bed level are averaged over the cross-section, together with the flow velocity. Here the morphological change is expressed by a change in the bed level, $\Delta z(x,t)$ over a sediment carrying with $B_{sed}(x)$.
Some schematisation-options are studied, each giving a good reproduction of two parameters mentioned above, assuming an initial state which is in equilibrium. However, for a good reproduction of the water movement and the sediment movement, more than two parameters have to schematised well. When the cross-section of the river has a not rectangular form, and a variation in the discharge is considered, a good schematisation of whole the process, including the flow profiles, will not be possible with this approach. This is also caused by the fact that in this approach the sediment carrying width is not a function of time, what is not realistic with a varying discharge (c.q. water level) and irregularly shaped cross-sections.

In Chapter 4 another approach is followed, in which the water depth a and the bed level z are not averaged over the cross-section in the model anymore. The cross-section is given as $B_m(h_r) = B_R(h_r)$. The parameters A and S are calculated with the *integrals* and Q is given as a hard boundary condition. The morphological changes will be expressed as a change in the cross-sectional area $\Delta A_{sed}(x,t)$, So one $z_m(x)$ and one $B_{sed}(x)$ are not needed anymore. The parameters U, A and S will be given as one value as function of x.
The result is a good schematisation of the water movement in the computer model for a varying discharge and all shapes of a cross-section. The sediment transport in the initial state as well as in a flow profile will be reproduced with

$$S = B_{sed} \, (Q/A)^n \tag{6.2}$$

With a good reproduction of the water movement, the only missing parameter is the sediment carrying width. This parameter is calculated according to

$$S = B_{sed} \, R^{n/2} \tag{6.3}$$

which gives with the use of the hydraulic radius according to Engelund

$$B_{sed} = \frac{\displaystyle\int_0^{B_r} (h-z(y))^{n/2} \mathrm{d}y}{\left(\dfrac{\displaystyle\int_0^{B_r} (h-z(y))^{3/2} \mathrm{d}y}{\displaystyle\int_0^{B_r} (h-z(y)) \mathrm{d}y} \right)^n} \tag{6.4}$$

It will supply, together with $U = Q/A$, a satisfying reproduction of the sediment transport in a steady uniform flow as well as in a flow profile. It has been proved that the flow condition which causes the water level has no influence. Thus the sediment carrying width can be found this way, as function of the water level, $B_{sed}(h_r)$. Each time step, the water level is known because the assumed quasi-steady state of the water movement allows it to decouple the water movement and the sediment movement. So first the water movement will be calculated over the whole length of the river. Afterwards, the sediment movement will be calculated, containing the calculation of the sediment carrying width.
Overall, the sediment carrying width, becomes a function of the transport formula (n) used, the geometry of the cross-section and the water level. This way, a one-dimensional mathematical model (x-direction) reaches a two dimensional accuracy (horizontal) at each longitudinal space-step.

The notation of the hydraulic radius R according to Engelund, used in the formula for the sediment carrying width, gives a better result than the general formula $R = A/P$, especially for irregular cross-sections.
For this approach the necessary data are a Q-h rating curve, some characteristic bed elevations of the river reach and the type of sediment transport formula valid for the situation studied. With this approach a first indication of the sediment transport, the fluctuation over a year, and possible morphological changes can be given. For the river Waal the results of this approach are compared with measurements of the sediment transport for various discharges. The results were satisfying. It has to be recommended to compare this approach also with measurements in an irregularly shaped cross-section, when data are available.

6.3 Interpretation of calculated morphological changes

Step three in the three-step approach of the use of a model is the interpretation of the morphological change for the real river, which is worked out in Chapter 5. With the results of Chapter 4, this results in a good interpretation of the calculated change in the cross-sectional area $\Delta A_{sed}(x,t)$ for the real cross-section. There are two problems regarding the changes:

* distribution over the real cross-section

* Change in time of some parameters has to be followed in time as good as possible

For the distribution, three realistic options are available. These are distributions proportional to

* the water depth, a^1
* the water velocity, $a^{1/2}$
* the sediment transport capacity, $a^{5/2}$

This way a two-dimensional result is calculated with a one-dimensional calculation in time.
In Chapter 5, a study is performed on the influence of the distribution coefficient ξ, from a^{ξ}. It concerns a cross-section at the mouth of a river flowing in a lake with a constant water level. Because of water withdrawal upstream, aggradation occurs in the river mouth. The final equilibrium state is calculated for all three distribution options, showing the influence of ξ. There are some practical important differences caused by a variation in the distribution coefficient. An increase in ξ causes:

* a decrease in the final ΔA_{sed}, and thus an increase in the flow velocity, according to $U = Q/A$. This can be important for navigation on the river.

* a decreasing a_{max}. Proportionally more sediment is deposited in the deeper parts of the river. This too can be important for navigation, considering the navigation width needed with a minimum water depth.

* an increasing i_b. This has influence on the water level upstream, and thus on the inundation danger upstream. Higher river dikes are necessary.

The magnitude of the fluctuation is strongly influenced by the shape of the cross-section. For the nearly rectangular cross-section, the influence of ξ is almost zero, which physically stands to reason.

It is not possible to say which distribution will be the right one, because no experiments, studying this topic, have been performed. However, with these results a river engineer can use the distribution option with the most negative result for the particular problem he is dealing with. This way a safety-factor is integrated in his predictions.

In Chapter 5, the fluctuation of the above mentioned practical parameters are plotted as function of the distribution coefficient ξ.

There is also an influence of the exponent n (from $s = m\ U^n$) on the practical parameters. This exponent influences the range of fluctuation caused by the distribution parameter ξ.

Finally, the change in some parameters is studied when morphological changes occur. These are parameters playing a role in the process to be modelled, so the changes have to be followed accurately. For one water level (at the river mouth near a lake), there will be no difference between a distribution of the morphological change at each time step, or only a distribution of the morphological change $\Delta A_{sed}(x,t)$ at the end of the calculation, because the proportional variation of the depth stays the same. It is also possible to accurately follow the change in all the parameters, when only $\Delta A_{sed}(x,t)$ is known at each time-step.

However, for a variation of the water level upstream, this proportional variation will be different for each water level, followed by different distributions. These are followed again by changes in the proportional variation of the depth for other water levels. Because of the correlation between the water levels and their distribution, it is only physically right when the morphological change is distributed each time-step. Otherwise it is not possible to assess the change of some parameters sufficiently. This requires more calculation time of the model, and more memory.

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MAIN SYMBOLS

a	depth	[L]
A	cross-sectional area	[L^2]
A_s	conveying cross-sectional area	[L^2]
B	width	[L]
B_s	stream width	[L]
b	subscript for bottom	[-]
c	celerity	[LT^{-1}]
C	Chézy coefficient	[$L^{0.5}T^{-1}$]
D	particle diameter	[L]
Fr	Froude number	[-]
$f(...)$	function of	[-]
g	acceleration of gravity	[LT^{-2}]
h	water level	[L]
i_b	mean slope of bottom	[-]
k	bed roughness	[L]
L	length	[L]
m	subscript for model	[-]
m_0	coefficient in $s = m_0 u^n$	[-]
n	exponent in $s = m_0 u^n$	[-]
P	wetted perimeter	[L]
$p\{...\}$	probability density	[-]
Q	discharge	[L^3T^{-1}]
q	discharge per unit width	[L^2T^{-1}]
R	hydraulic radius	[L]
r	subscript for real	[-]
S	sediment transport (bulk volume)	[L^3T^{-1}]
s	sediment transport per unit width	[L^2T^{-1}]
sed	subscript for sediment carrying	[-]
Re	Reynolds number	[-]
t	time	[T]
U	mean velocity in x-direction	[LT^{-1}]
u	mean velocity in x-direction	[LT^{-1}]
x	coordinate in flow direction	[L]
y	transverse coordinate	[L]
z	bed level	[L]
α'	correction coeff. uniform flow distribution	[-]
Δ	relative density	[-]
ξ	distribution coefficient	[-]
ρ	density of water	[ML^{-3}]
ρ_s	density of sediment	[ML^{-3}]
τ	shear stress	[$ML^{-1}T^{-2}$]

τ_b	bottom shear stress	$[ML^{-1}T^{-2}]$
Φ	relative celerity	[-]
ψ	dimensionless transport parameter	[-]

APPENDIX A

This appendix regards the behaviour of some significant parameters for several differently shaped cross-sections from the River Da in Vietnam. When the cross-section has a more irregular shape, the variation in these parameters will become greater.

Firstly, the two options for the hydraulic radius, mentioned in section (4.2), are compared. These are the general formula:

$$R = A/P \tag{A.1}$$

and that according to the Engelund method:

$$R = \left(\frac{1}{A} \int_0^{B_r} \frac{C}{C_1} a^{3/2} \, \mathrm{d}y \right)^2 \tag{A.2}$$

with $C = C_1$ = constant. This parameter is important for the calculation of the sediment carrying width. The Engelund method shows more realistic values for irregularly shaped cross-sections.

Secondly, the differences between B_m and z_m, the two schematisation-parameters for a rectangular cross-section, are shown for the three options of two equations, which are necessary to calculate the two unknown parameters. This is mentioned in sub-section (4.3.1). The three combinations are:

* A and Q
* A and S
* Q and S

according to:

$A_m = A_r$:

$$B_m \, (h_r - z_m) = \int_o^{B_r} (h_r - z(y)) \mathrm{d}y \tag{A.3}$$

$Q_m = Q_r$:

$$B_m\,(h_r - z_m)^{3/2} = \int_0^{B_r} (h_r - z(y))^{3/2}\,\mathrm{d}y \tag{A.4}$$

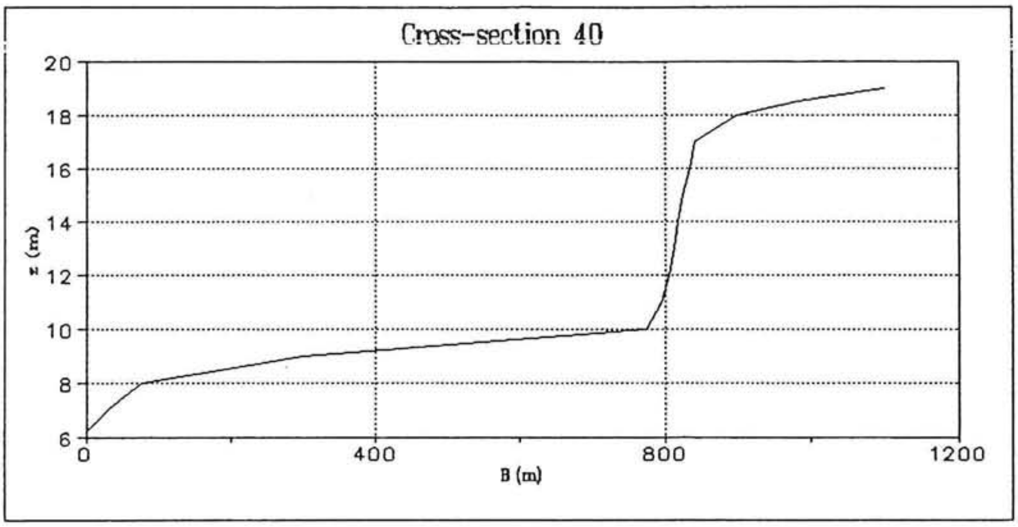
$S_m = S_r$:

$$B_m\,(h_r - z_m)^{n/2} = \int_0^{B_r} (h_r - z(y))^{n/2}\,\mathrm{d}y \tag{A.5}$$

Thirdly, the behaviour of the sediment carrying width as function of the water level is demonstrated, as mentioned in subsection (5.4.2) with

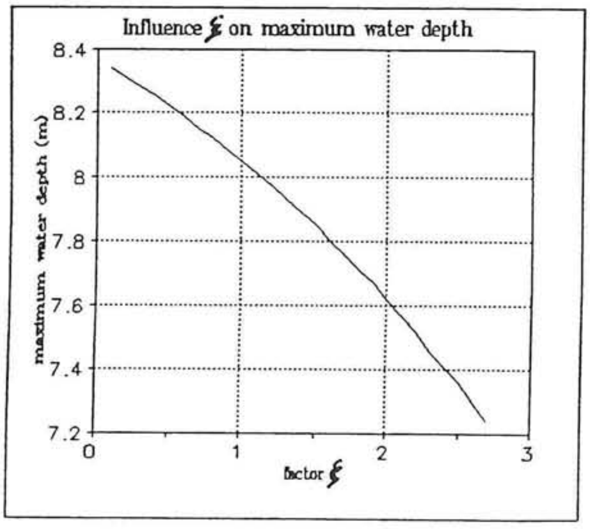
$$B_{sed} = \frac{\int_0^{B_r} (h - z(y))^{n/2}\,\mathrm{d}y}{\left(\dfrac{\int_0^{B_r} (h - z(y))^{3/2}\,\mathrm{d}y}{\int_0^{B_r} (h - z(y))\,\mathrm{d}y}\right)^{n}} \tag{A.6}$$

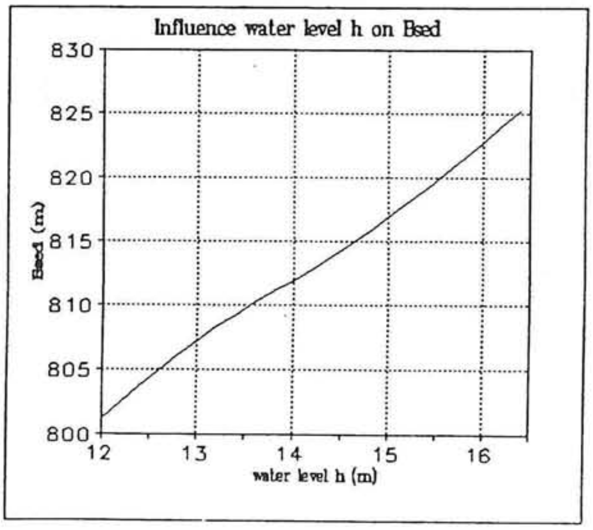
Finally, the influence of the distribution factor ξ on the maximum water depth in the new equilibrium state is demonstrated. This parameter is calculated at the river mouth, as mentioned in section (6.3). A realistic distribution will be proportional to a^{ξ}, with a being the water depth, and ξ is varying between 1/2 and 5/2.

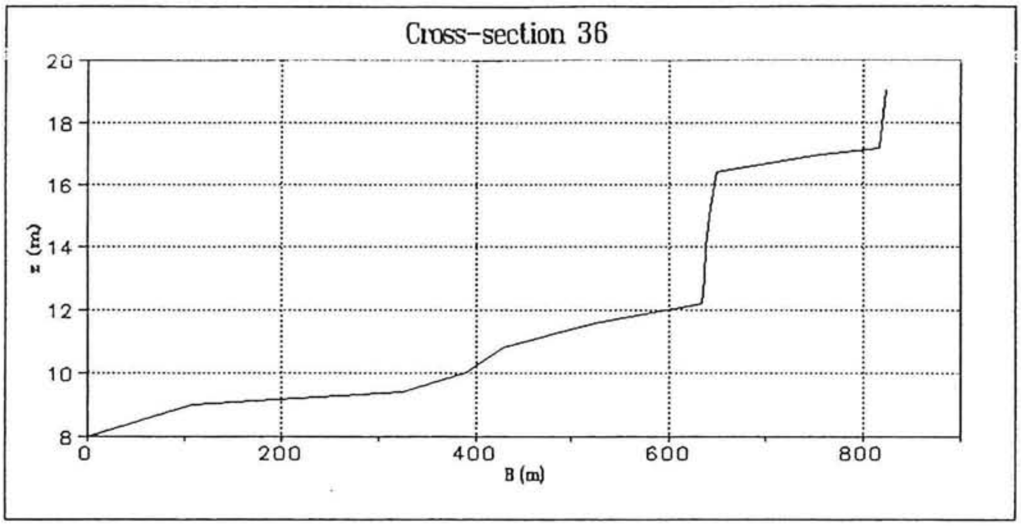


	h = 12.64 m	h = 15.25 m
R (A/P)	3.58	6.07
R (Engel.)	3.75	6.26
ratio	1.05	1.03

	h = 14.75 m		h = 17.98 m	
	B_m	z_m	B_m	z_m
Q,A-option	771.9	8.89	805.3	9.00
A,S-option	751.0	8.78	795.3	8.92
Q,S-option	722.6	8.72	783.4	8.88

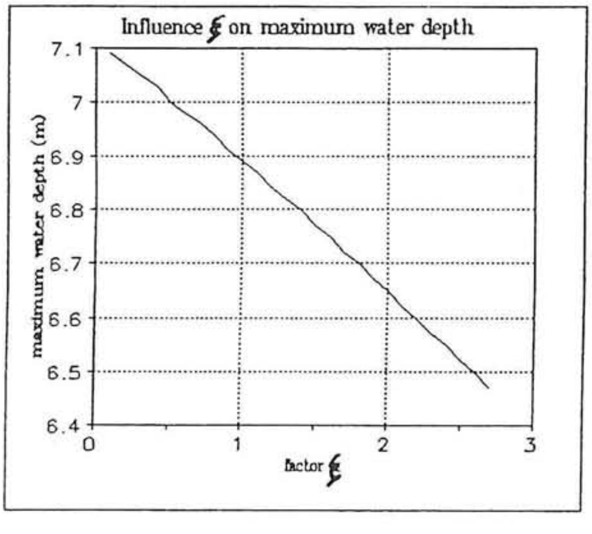


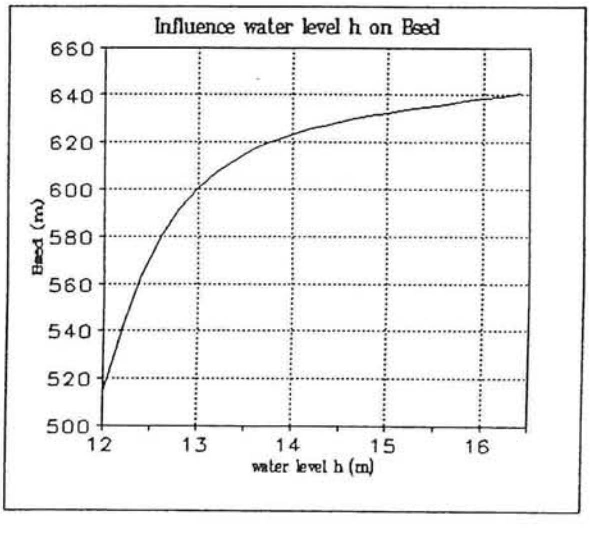


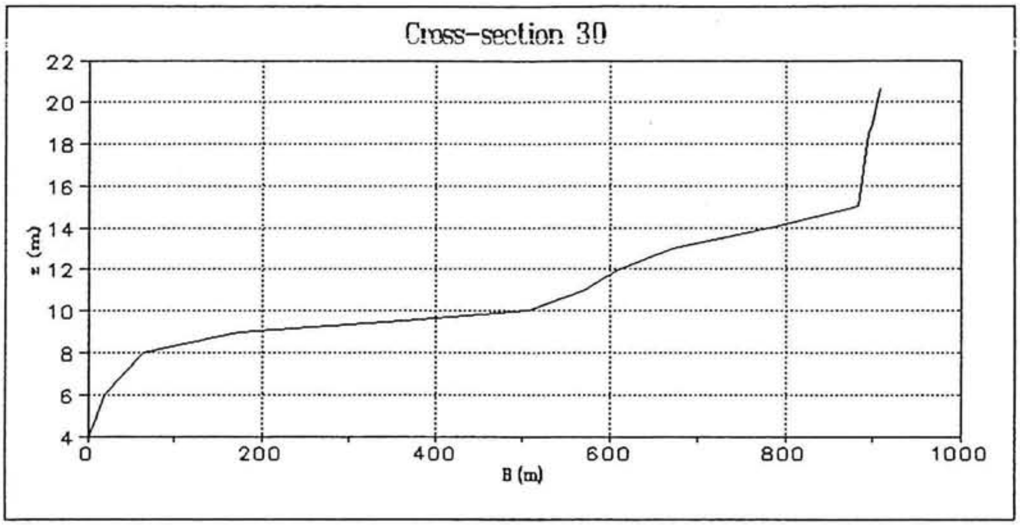


	h = 13.03 m	h = 15.75 m
R (A/P)	3.04	5.69
R (Engel.)	3.45	5.95
ratio	1.14	1.05

	h = 13.03 m		h = 15.75 m	
	B_m	z_m	B_m	z_m
Q,A-option	560.4	9.58	617.8	9.80
A,S-option	534.6	9.41	605.3	9.68
Q,S-option	502.8	9.32	589.9	9.61

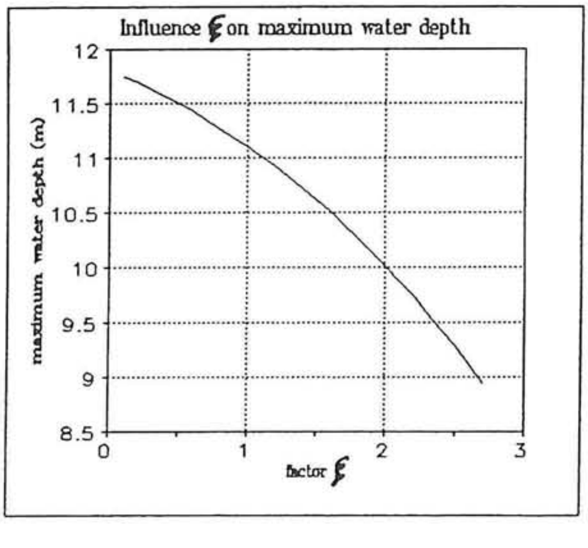


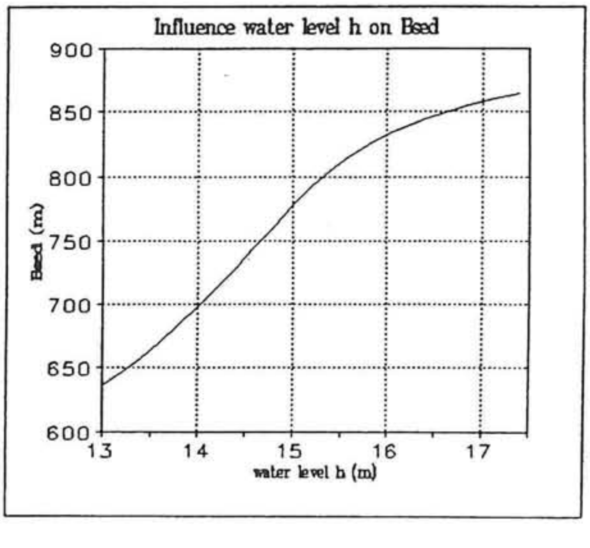


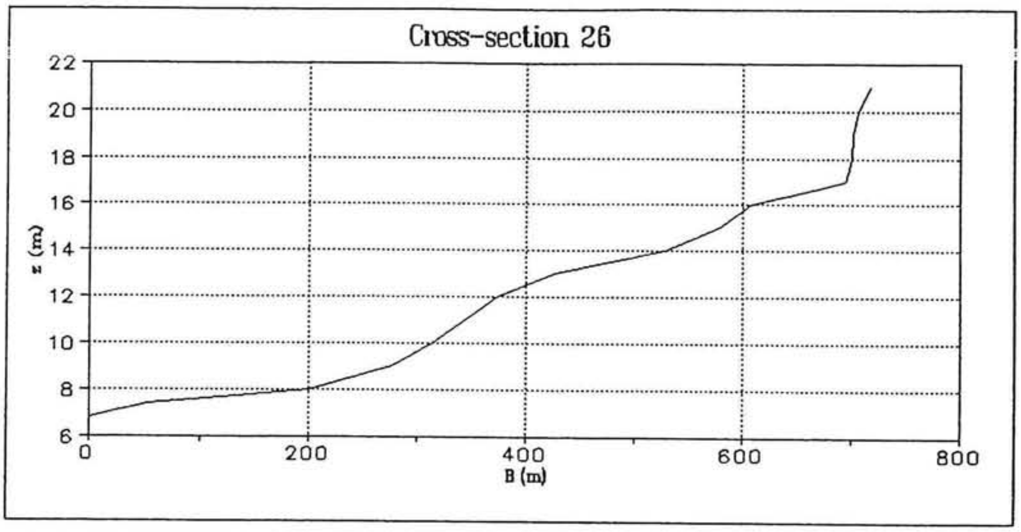


	h = 13.62 m	h = 16.51 m
R (A/P)	3.77	5.90
R (Engel.)	4.53	6.68
ratio	1.20	1.13

	h = 13.62 m		h = 16.51 m	
	B_m	z_m	B_m	z_m
Q,A-option	614.7	9.09	785.1	9.83
A,S-option	571.5	9.79	748.4	9.49
Q,S-option	537.5	8.67	702.5	9.31

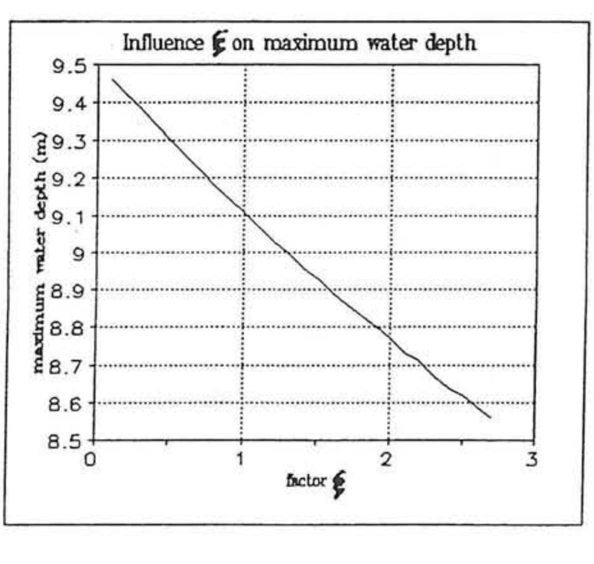


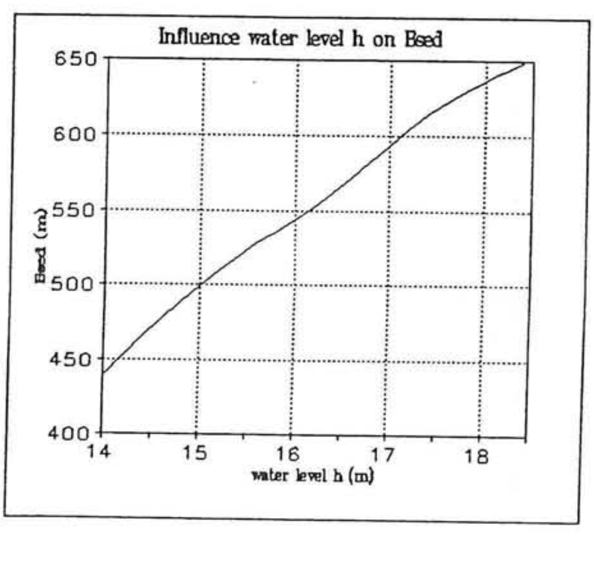




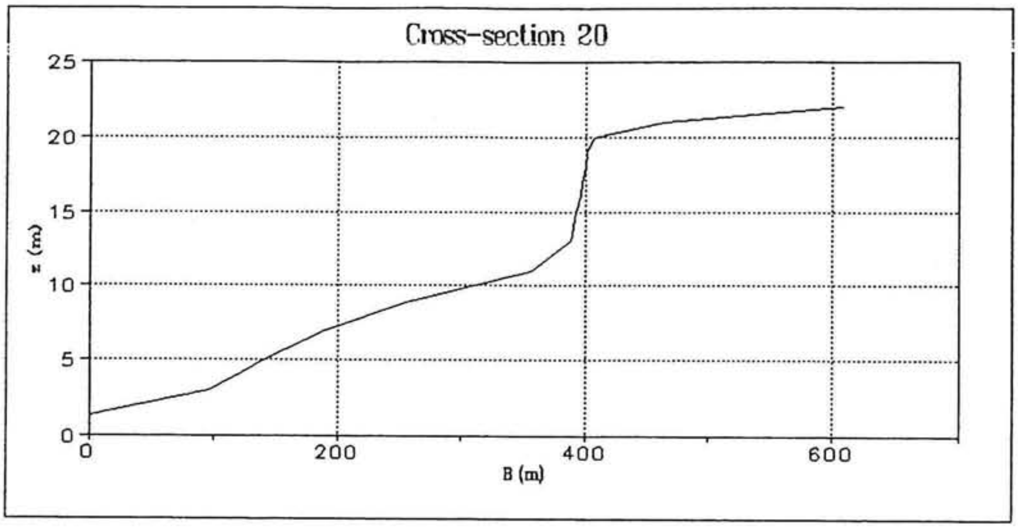
	h = 14.03 m	h = 17.04 m
R (A/P)	4.17	5.79
R (Engel.)	5.41	7.40
ratio	1.30	1.28

	h = 14.03 m		h = 17.04 m	
	B_m	z_m	B_m	z_m
Q,A-option	408.4	8.62	543.3	9.64
A,S-option	388.3	8.34	510.2	9.16
Q,S-option	364.7	8.19	471.9	8.91



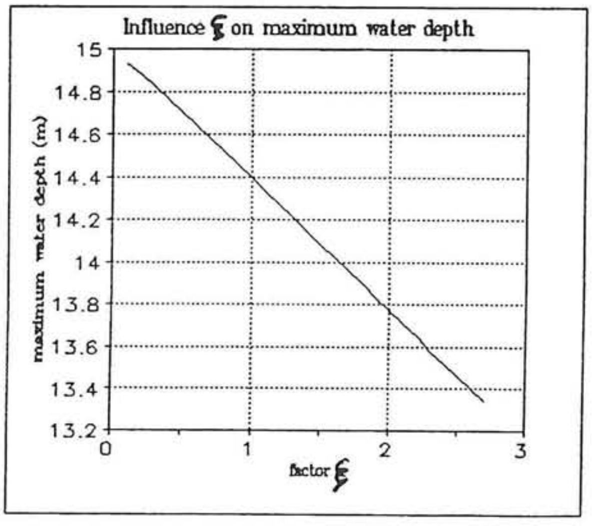


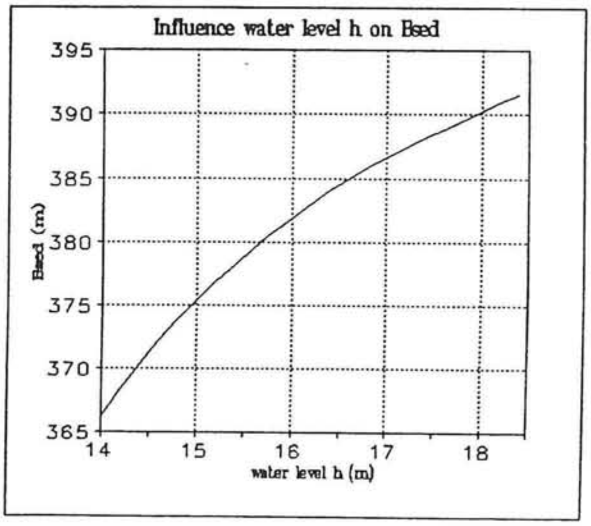
A.6

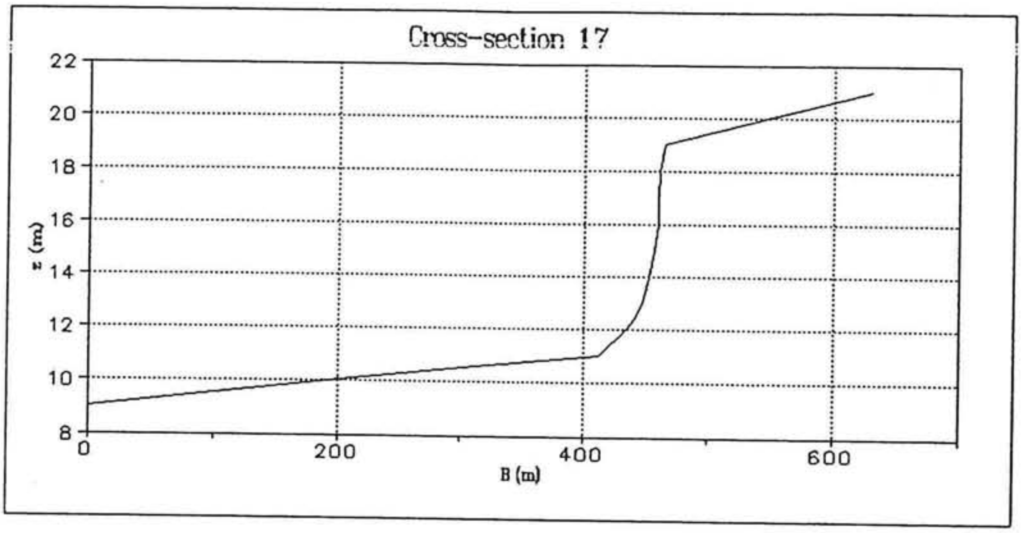


	h = 14.38 m	h = 17.49 m
R (A/P)	7.65	10.56
R (Engel.)	8.91	11.59
ratio	1.16	1.10

	h = 14.75 m		h = 17.98 m	
	B_m	z_m	B_m	z_m
Q,A-option	336.2	5.47	363.4	5.90
A,S-option	314.7	4.87	349.0	5.39
Q,S-option	290.5	4.56	331.0	5.13

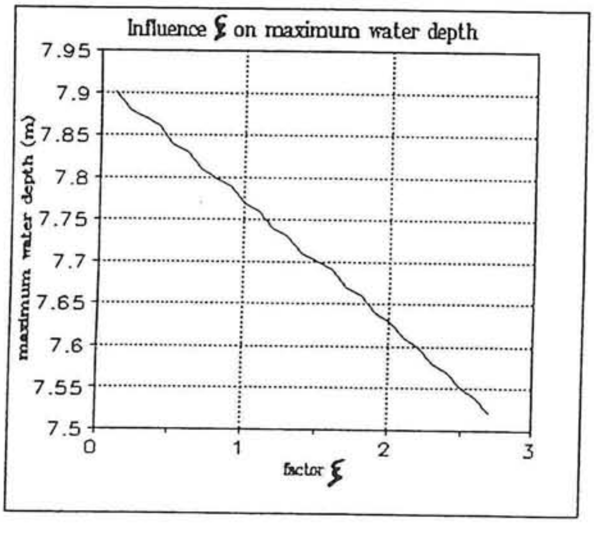


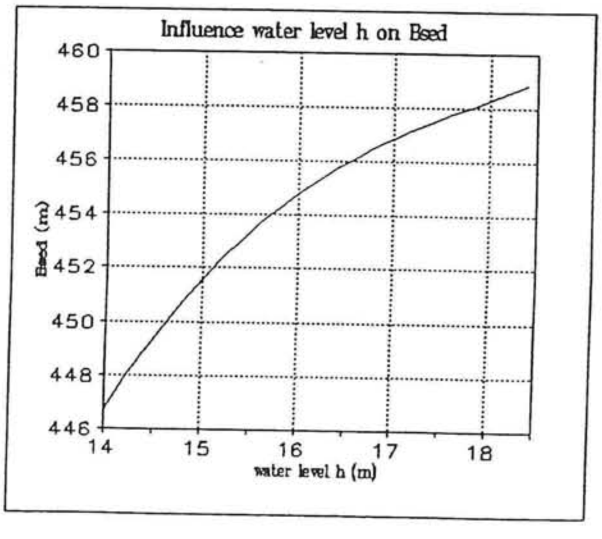


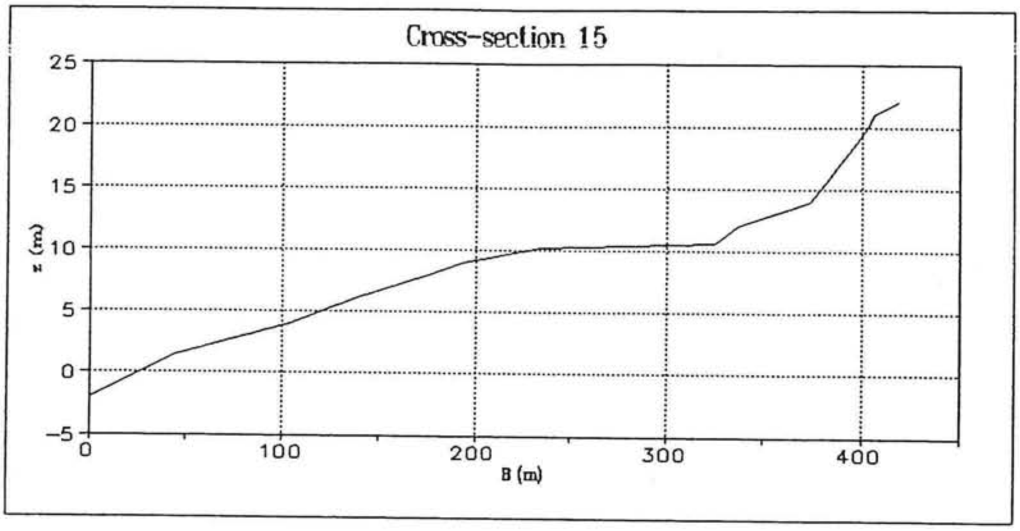


	h = 14.57 m	h = 17.74 m
R (A/P)	4.32	7.40
R (Engel.)	4.47	7.55
ratio	1.03	1.02

	h = 14.57 m		h = 17.74 m	
	B_m	z_m	B_m	z_m
Q,A-option	602.6	9.73	676.9	10.18
A,S-option	585.5	9.59	655.7	9.93
Q,S-option	565.4	9.52	628.2	9.80

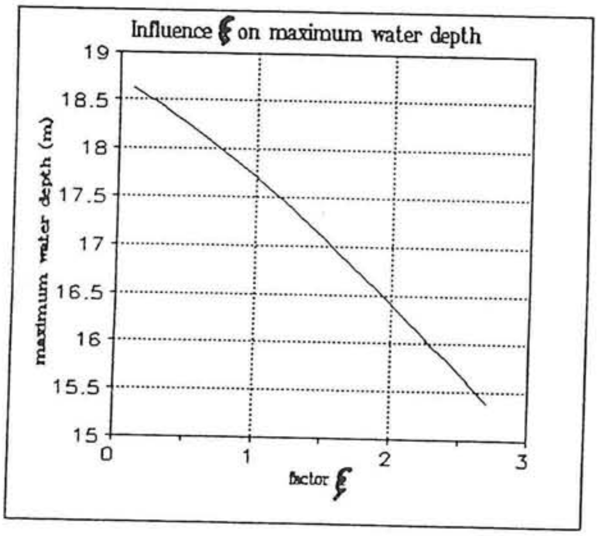


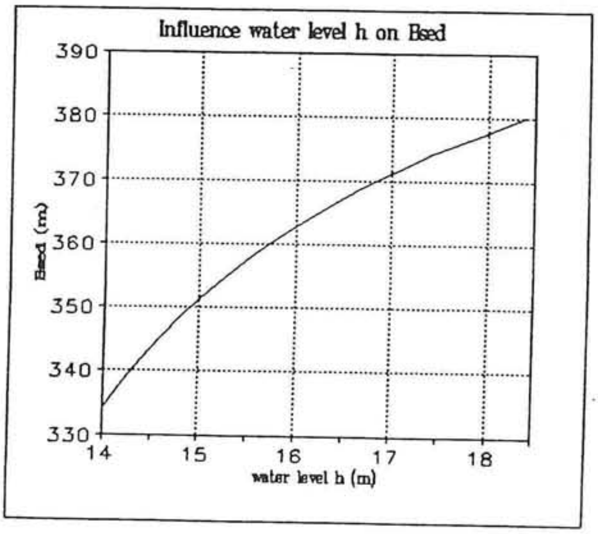


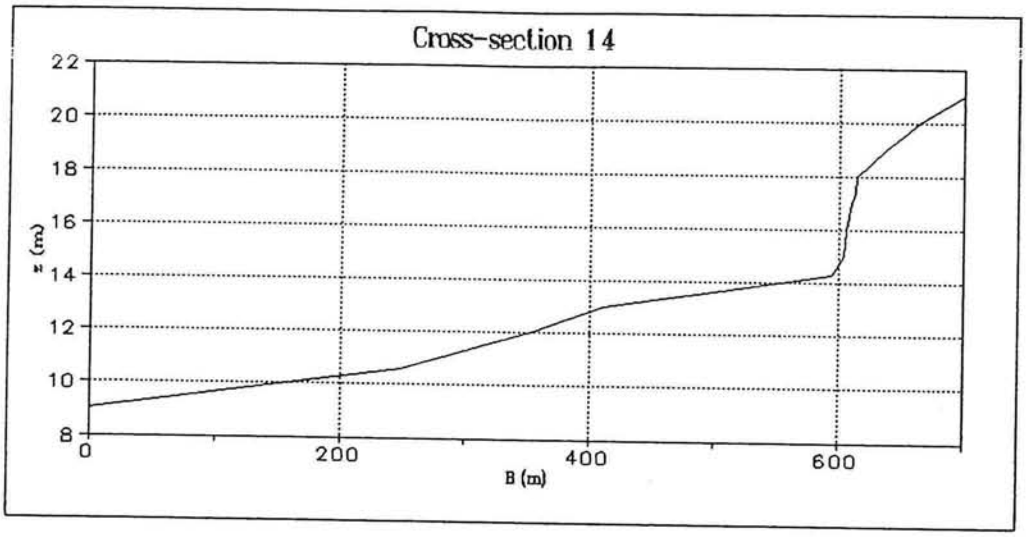


	h = 14.71 m	h = 17.92 m
R (A/P)	7.46	10.31
R (Engel.)	9.39	11.88
ratio	1.26	1.15

	h = 14.71 m		h = 17.92 m	
	B_m	z_m	B_m	z_m
Q,A-option	300.6	5.33	341.4	6.04
A,S-option	273.0	4.38	319.4	5.22
Q,S-option	242.0	3.87	293.4	4.78

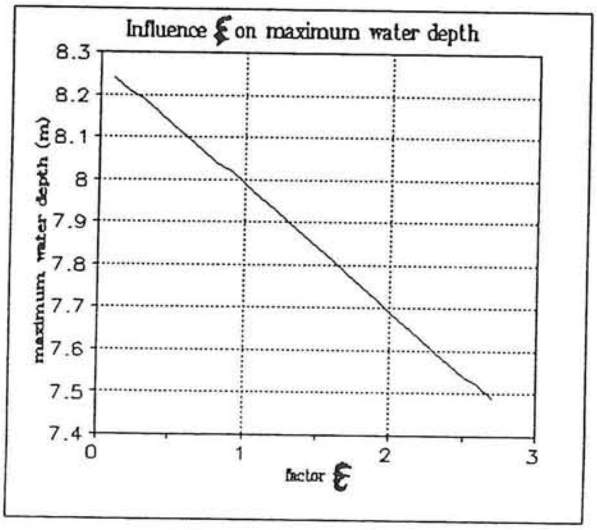


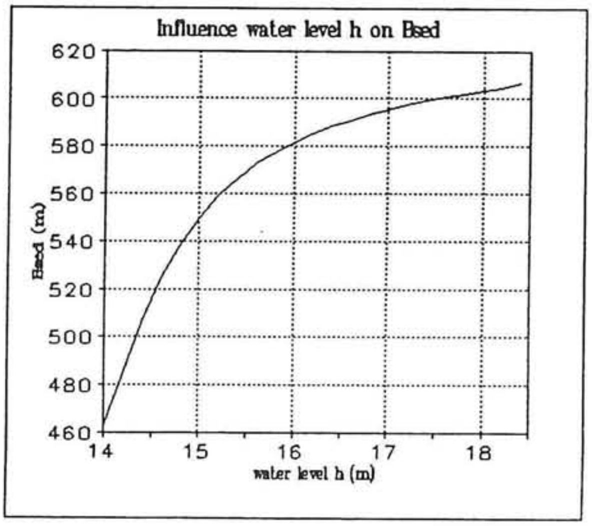




	h = 14.75 m	h = 17.98 m
R (A/P)	3.20	6.32
R (Engel.)	3.97	6.75
ratio	1.24	1.07

	h = 14.75 m		h = 17.98 m	
	B_m	z_m	B_m	z_m
Q,A-option	483.9	10.78	575.0	11.23
A,S-option	451.9	10.51	558.0	11.02
Q,S-option	415.5	10.36	536.6	10.91





APPENDIX B

This Appendix regards the vadility of the *integral*-theory for some more cross-sections of the River Waal. As mentioned in section 4.4, sufficient data of the Chézy values and of the sediment transport for various discharges have to be available.
Firstly, the available data will be mentioned. These are:

* a Q-h rating-curve at Tiel, called "Relatie waterstand-afvoer". This is a rating-curve belonging to a characteristic cross-section of the reach studied between Tiel and Dodenwaard. The water levels can be interpolated to other cross-sections (with help of the information on "dwarspeiling Waal Km 910-915"). The rating-curve is measured in 1992. (see pa. B.3)

* The bed elevations of a river reach between Tiel and Dodenwaard. This map is called "dwarspeiling Waal Km 910-915'. These bed elevations are given with respect to O.L.R., a reference level used on the Dutch rivers. With the information given on this map, it is possible to interpolate the Q-h rating-curve over the given reach. The measurements took place in 1994. The top of the groynes has a height of ca. + 2.75 m to O.L.R.

* Measurements of important parameters for the River Waal on the reach " Dodenwaard - Tiel. This table gives Chézy-values belonging to some discharges which occur. The measurements are taken in 1989. (see pa. B.4)

* Finally, a Q,S-relation is given, measured for the River Waal at Druten, which is also situated on the reach studied between Dodenwaard and Tiel. The measurements took place in 1989. The methods of measurement are explained in section 4.2. (see pa B.5).

It has to be mentioned that the data have been measured in a range of five years. However, the River Waal is a river of which the morphological changes are considered slow, which makes it possible to integrate these data.

The two types of data mentioned first are necessary as input for the *integral*-theory. The last two types mentioned make it possible to validate the *integral*-theory in two ways. This have been performed for some more cross-sections in the reach studied.

Firstly, the measured Chézy values are compared with the Chézy values calculated according to the *integral*-theory. It has to be stated that two calculations were performed, one with and one without the influence of the discharge over the groynes for higher discharges. The influence starts at $Q > 1800$. The results given in Table (B.1) confirm the results from section 4.2, i.e. a good approximation of the Chézy values by the *integral*-theory.

Secondly, The measured sediment transport is compared with the two calculations, confirming a realistic reproduction of the sediment transport by the *integral*-theory, especially by the second calculation, with the influence of discharge over the groynes. These results are plotted in the figures on pages B.6 till B.9.

MEASURED AND CALCULATED CHÉZY-VALUES

Table (B.1)

cross-section	discharge Q (m^3/s)	measured C ($\sqrt{m/s}$)	Calculation 1 of C ($\sqrt{m/s}$)	Calculation 2 of C ($\sqrt{m/s}$)
Waal Km 912.625	1350	43.7	43.8	43.8
	2150	44.3	43	42.8
	4250	44.9	51.2	46.7
Waal Km 910.875	1350	43.7	40.5	40.5
	2150	44.3	41.3	40.4
	4250	44.9	51.3	47.1
Waal Km 911.813	1350	43.7	41.9	41.9
	2150	44.3	40.7	39.9
	4250	44.9	47.9	45.1
Waal Km 911.938	1350	43.7	41.3	41.3
	2150	44.3	41.7	41.2
	4250	44.9	50.4	47.6

Relatie waterstand-afvoer
m + NAP - m3/s

Waterstand te: TIEL-WAAL (SLUIS)
Rivier : WAAL

WS	Q	WS	Q	WS	Q	WS	Q
1.10		4.10	1314	7.10	2957	10.10	7058
20		20	1353	20	3040	20	7228
30		30	1392	30	3127	30	7400
40		40	1431	40	3218	40	7574
50	360	50	1470	50	3310	50	7750
60	494	60	1511	60	3409	60	7926
70	428	70	1553	70	3512	70	8103
80	462	80	1597	80	3621	80	8281
90	496	90	1642	90	3736	90	8460
2.00	530	5.00	1690	8.00	3860	11.00	8640
10	566	10	1736	10	3981	10	8824
20	602	20	1785	20	4110	20	9008
30	638	30	1834	30	4244	30	9192
40	674	40	1884	40	4382	40	9376
50	710	50	1935	50	4520	50	9560
60	746	60	1987	60	4668	60	9744
70	782	70	2041	70	4816	70	9928
80	818	80	2096	80	4966	80	10112
90	854	90	2152	90	5117	90	10296
3.00	890	6.00	2210	9.00	5270	12.00	10480
10	928	10	2269	10	5430	10	
20	966	20	2329	20	5590	20	
30	1004	30	2391	30	5750	30	
40	1042	40	2455	40	5910	40	
50	1080	50	2520	50	6070	50	
60	1119	60	2588	60	6232	60	
70	1158	70	2657	70	6395	70	
80	1197	80	2729	80	6559	80	
90	1236	90	2803	90	6724	90	
4.00	1275	7.00	2880	10.00	6890	13.00	

25-08-1992
directie Gelderland
Afdeling Informatie, ANII

B.3

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RIVIER : BOVENRIJN EN WAAL
TRAJECT: DODEWAARD - TIEL

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AFVOER	DEBIET LOBITH (M3/S)	DEBIET TPL. (M3/S)	DIEPTE (M)	VERHANG *10-4 (-)	STROOM-SNELHEID (M/S)	CHEZY-WAARDE (M1/2/S)	K-WAARDE (M)	DARCY-WEISBACH (-)	N-FACT. MANNING (S/M1/3)	FROUDE (-)	REYNOLDS (-)	KRITISCHE SCHUIFSP. (N/M2)	SCHUIFSP. (N/M2)
Q 90%	1100	865	4.13	1.161	.81	36.79	.448	.0580	.0344	.127	2915796.	.6097	4.7038
Q 50%	1950	1345	4.94	1.161	1.05	43.73	.221	.0410	.0298	.150	4533810.	.6202	5.6264
Q 10%	3200	2145	6.64	1.186	1.24	44.28	.276	.0400	.0310	.154	7230499.	.6411	7.7254
Q 1%	6400	4247	8.98	1.127	1.43	44.93	.344	.0389	.0321	.152	11249345.	.6581	9.9282
Q .3%	7800	5163	9.47	1.061	1.43	45.07	.356	.0386	.0323	.148	11857266.	.6576	9.8568
Q MHW	16500	10370	12.39	.902	1.53	45.88	.420	.0373	.0332	.139	16655129.	.6649	10.9634

AFVOER	SCHUIFSP. SNELHEID (M/S)	STROOM-VERM. (N/MS)	DIM.LOZE SCHUIFSP. (-)	TAU-STER ACCENT (-)	KR.SCH. SNELH. (M/S)	REYNOLDS-SCHUIFSN. (-)	REYNOLDS-KORREL (-)	REYNOLDS U*D50/VISK (-)	H-ACCENT ENGELUND (M)	H-ACCENT MANNING (M)	RIB.FAC. ME-PE+MU	PARAM. "Y" (-)
Q 90%	.0686	3.7892	.3059	.0965	.0247	248250.	100.	57.	1.30	.97	.347	9.421
Q 50%	.0750	5.8918	.3659	.1478	.0249	324755.	100.	62.	1.99	1.43	.437	6.250
Q 10%	.0879	9.5986	.5024	.1961	.0253	511497.	100.	73.	2.59	1.82	.426	4.673
Q 1%	.0996	14.1908	.6456	.2447	.0257	784197.	100.	83.	3.40	2.33	.416	3.719
Q .3%	.0993	14.0817	.6410	.2419	.0256	824008.	100.	83.	3.57	2.44	.415	3.758
Q MHW	.1047	16.8155	.7130	.2644	.0258	1136996.	100.	87.	4.60	3.06	.410	3.418

B.4

AFVOER	G*DM3/ /VISK2 (-)	U* DM/ /VISK (-)	U/W50 (-)	U*/W50 (-)	DIEPTE /D50 (-)	SNELH./ V(G*D50) (-)	SCH.SP./ ROL*G*DM (-)	C-90 (M1/2/S)	***ZANDTRANSPORT*** FRIJLINK (M3/M/S)	ME-PE+MU (M3/M/S)	ENG+HANS (M3/M/S)	ACK+WHIT (M3/M/S)	KALINSKE (M3/M/S)
Q 90%	28060.	93.	6.7	.57	4347.	8.34	.1875	74.50	.000008	.000008	.000070	.000041	.000120
Q 50%	28060.	102.	8.7	.63	5200.	10.85	.2243	75.90	.000041	.000038	.000155	.000120	.000150
Q 10%	28060.	119.	10.4	.73	6989.	12.87	.3079	78.21	.000095	.000080	.000351	.000233	.000213
Q 1%	28060.	135.	11.9	.83	9453.	14.81	.3957	80.57	.000161	.000132	.000677	.000384	.000294
Q .3%	28060.	135.	11.9	.83	9968.	14.80	.3929	80.99	.000158	.000130	.000669	.000379	.000292
Q MHW	28060.	142.	12.8	.87	13042.	15.89	.4370	83.09	.000192	.000158	.000904	.000472	.000330

GEMIDDELD SEDIMENTTRANSPORT PER JAAR :

FRIJLINK :	371000.	M3/JAAR
ME-PE+MU :	322000.	M3/JAAR
ENG+HANS :	1495000.	M3/JAAR
ACK+WHIT :	998000.	M3/JAAR
KALINSKE :	1143000.	M3/JAAR

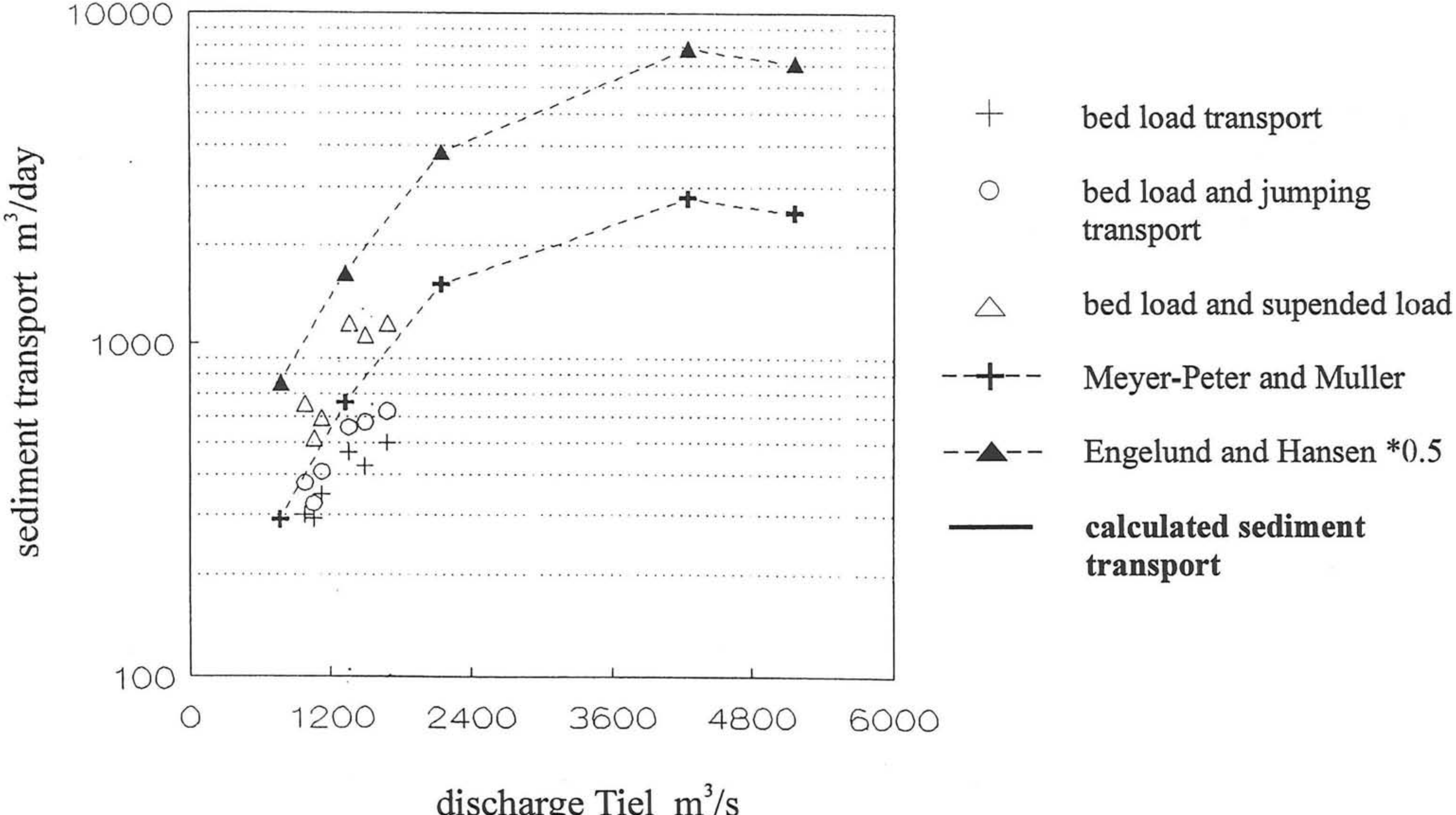
TRAJECTGEGEVENS:

TRAJECTLENGTE = 11975 M
NORMAALBREEDTE = 260 M
BODEMVERHANG = 1.294 *10-4
VALSNELHEID D-50 = .12 M/S
DICHTHEID WATER = 1000. KG/M3

D-10 = .00047 M   D-65 = .00140 M   D* = 36.
D-35 = .00077 M   D-90 = .00360 M
D-50 = .00095 M   D-M = .00155 M
KINEMATISCHE VISCOSITEIT = 1.141 *10-6 M2/S
DICHTHEID SEDIMENT = 2650. KG/M3

Q/S relation of the Waal near Druten

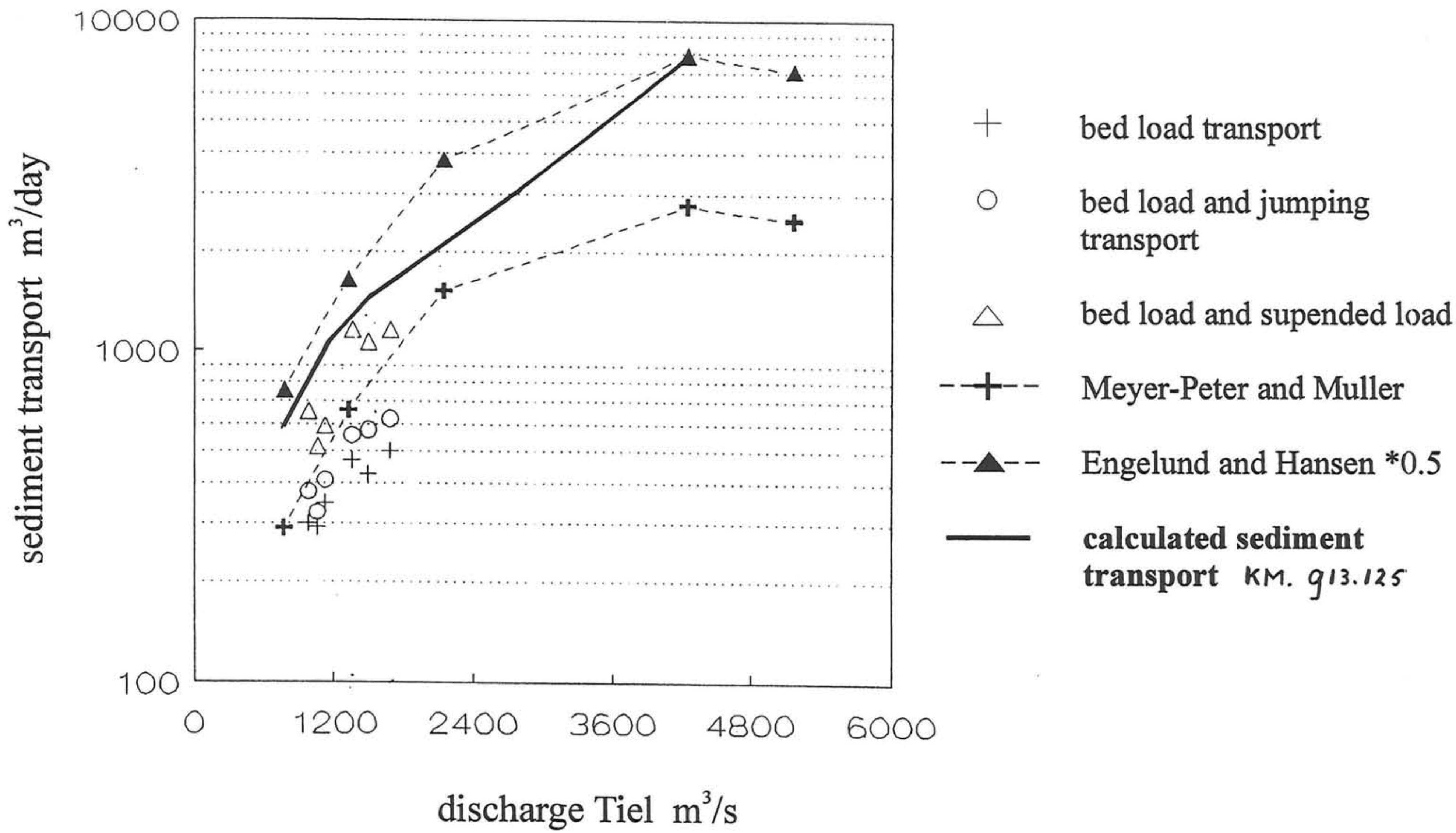
measured sediment transport 1989



B. 6

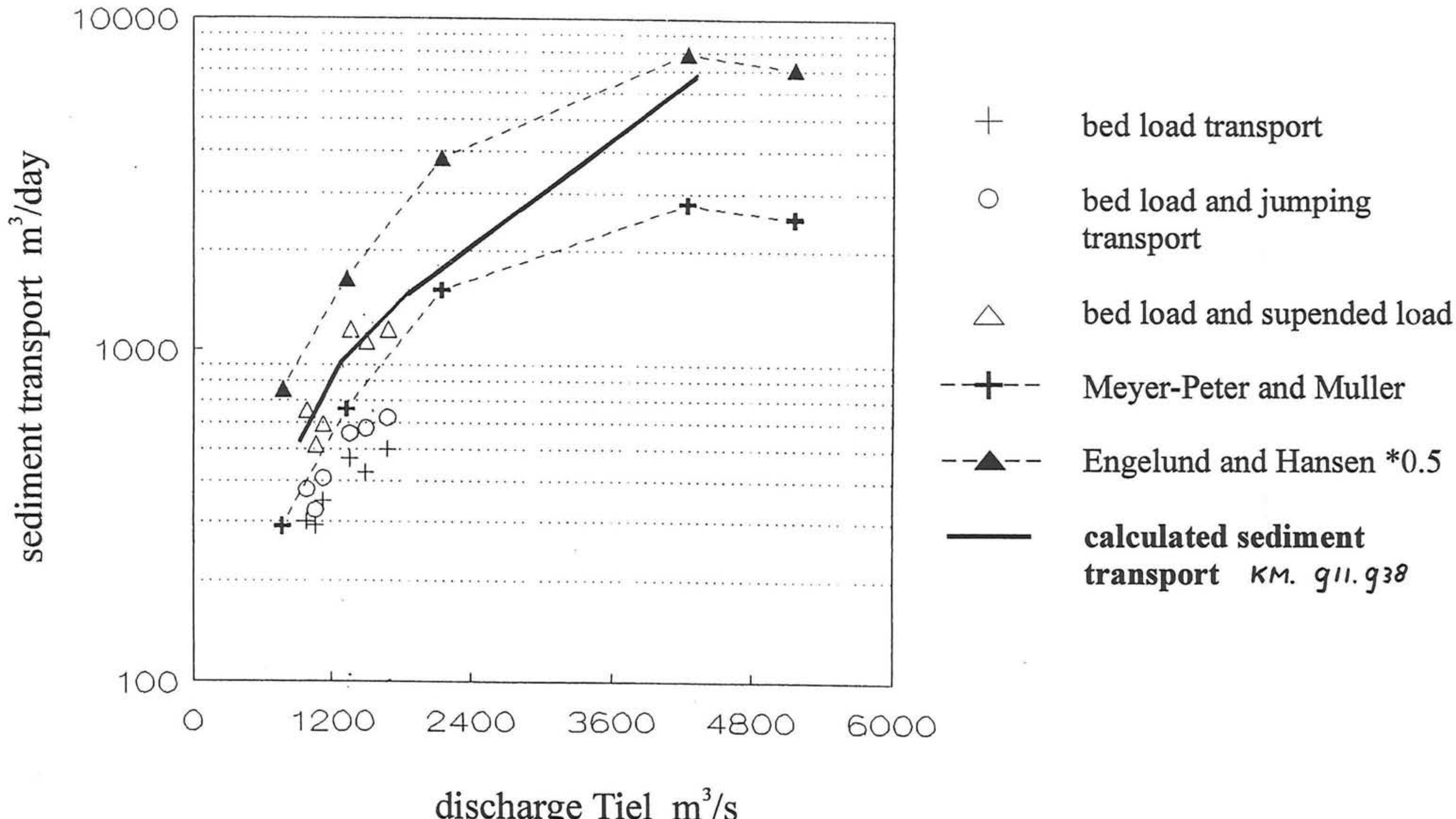
Q/S relation of the Waal near Druten

measured sediment transport 1989



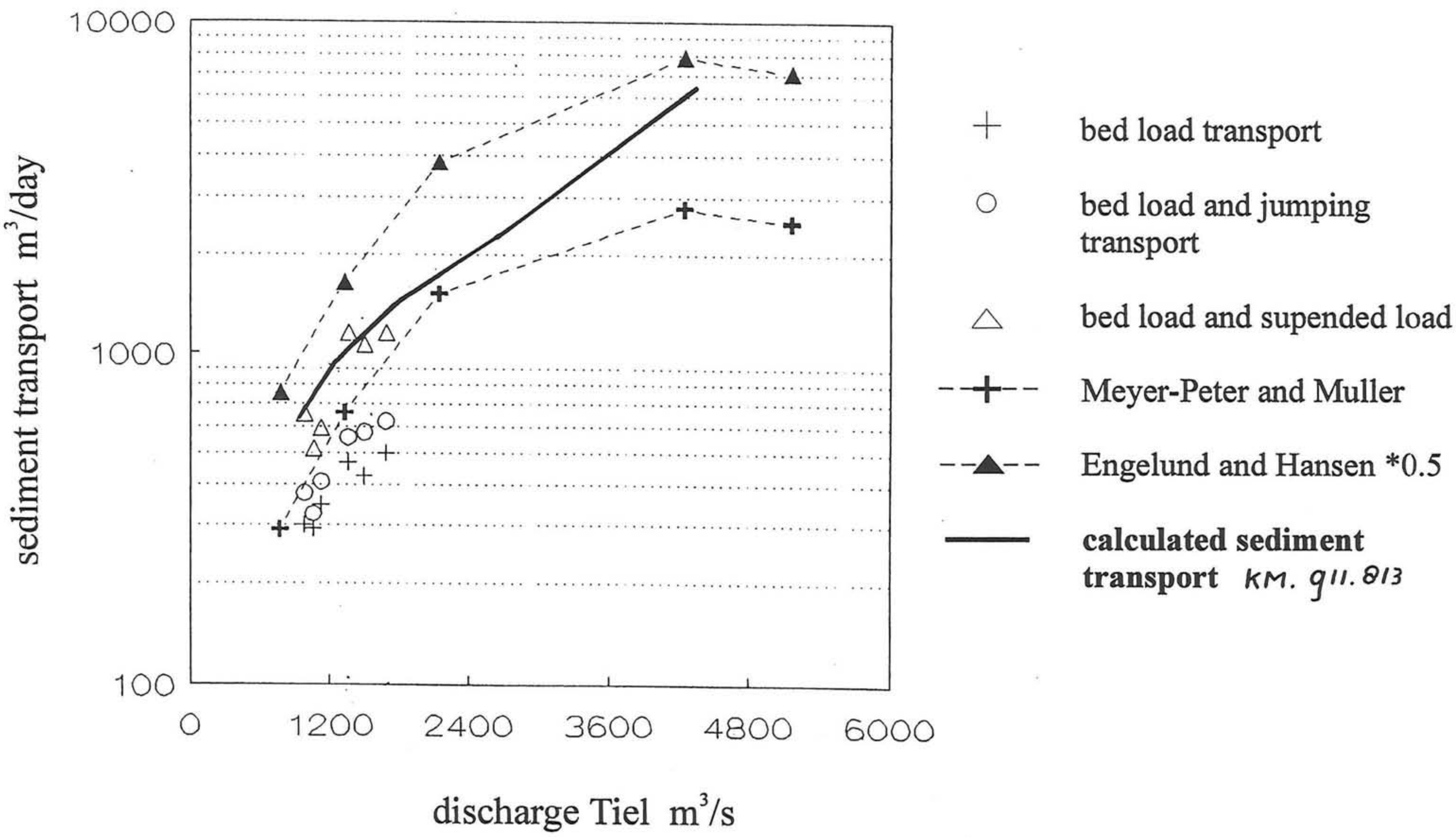
Q/S relation of the Waal near Druten

measured sediment transport 1989



Q/S relation of the Waal near Druten

measured sediment transport 1989



Q/S relation of the Waal near Druten

measured sediment transport 1989