Reliability based Ultimate Limit State Design in Finite Elements and compliance with Eurocode7

by

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Summary

Geotechnical design problems may be characterized by a certain degree of uncertainty, due to insufficient soil data and transformation of test results in soil parameters. In common practice, engineers perform deterministic analyses according to design standards as Eurocode 7, where the uncertainties are taken into account through partial factors for loads and soil properties to attain certain specified target reliabilities. For complex soil structure interaction problems, partial factor method is difficult to adopt, as the design standards consider geotechnical standards with single failure mechanism. This is especially problematic for Ultimate Limit state designs where both stiffness and strength properties are dominant.

With the advent of limit state design philosophy in Eurocodes, the use of reliability methods in Finite Element Analysis for complex situations has become more and more of interest. Reliability analyses allow to explicitly define the single uncertainties in the model by using an appropriate probabilistic distribution for each source of uncertainty. The reliability index and the probability of failure with respect to a predefined condition are calculated. The problem with using reliability based probabilistic design is the absence of simple computational approaches that can be easily implemented. Monte Carlo simulations are commonly used to solve soil structure interaction problems. For a large and complex soil-structure interaction problem, it is computationally intensive to complete even a single run. This practical disadvantage can be solved only by a computationally efficient method.

A special purpose application to perform probabilistic analysis in PLAXIS 2D, called PROBANA has been recently developed at Plaxis B.V. PROBANA performs direct probabilistic calculations in the finite element framework, using First Order Reliability Method or Monte Carlo Method. In this thesis, PROBANA (FORM) is used to perform reliability analysis for three benchmarks, and the results from PROBANA – FORM are compared with Point Estimate Method (PEM) and other stochastic Methods. The results from FORM are found to be comparable with that of PEM. It is concluded that PEM is less accurate due to assumptions made by PEM in the underlying output distribution and FORM is more accurate and practical as it is computationally less intensive compared to other stochastic methods such as the Monte Carlo analysis.

An extensive comparison of the reliability based method with Eurocode design method shows possibilities to implement reliability methods with EC7. One such approach is proposed, and demonstrated with the benchmarks.

List of Symbols

Symbol	**Description**	**Units**
c'	Cohesion	kPa
ϕ'	Friction angle	$^\circ$
X_k	Characteristic Value	$-$
X_d	Design Value	$-$
γ	Partial factor	$-$
σ	Standard deviation	$-$
β	Reliability Index	$-$
μ_{msf}	Mean Safety factor	$-$
X	Any Soil Property	$-$
p_f	Failure Probability	$-$
$g(x)$	Limit State function	$-$
E	Young's Modulus	kN/m^2
γ_{sat}	Saturated Unit weight	kN/m^3
γ_{unsat}	Unsaturated Unit weight	kN/m^3
ψ	Dilatancy angle	$^\circ$
ν	Poisson's ratio	$-$
ρ	Correlation coefficient	$-$
α	Importance factor	$-$
q	Bearing Capacity	kN/m^2

List of Abbreviations

ULS	Ultimate Limit State
SLS	Serviceability Limit State
RBD	Reliability Based Design
FORM	First Order Reliability Method
PEM	Point Estimate Method
DA	Design Approach
EC7	Eurocode 7
PROBANA	PROBAbilisitc ANAlysis
RC	Reliability Class
MC	Monte Carlo
COBYLA	COnstrained Optimisation BY Linear Approximation
PROBANA	PROBAbilisitc ANAlysis
COV	Coefficient Of Variation
pdf	Probability density function
HS	Hardening Soil Model
MC	Mohr Coloumb Soil Model
FEM	Finite Element Method
msf	Mean Safety Factor
FoS	Factor of Safety

Contents

1

Introduction

1.1. General

A structure is only as good as its foundation. Hence foundation design plays a crucial role in the overall stability of a structure. Geotechnical structures are designed according to design codes i.e. Eurocode 7. Like the other Eurocodes, EC7 uses the approach of limit state design. EC7 considers uncertainty in the design by the use of total, partial or load and resistance factors of safety. Their use is mainly based on past experiences and often leads to over conservative designs.

Numerical analysis, mainly Finite Element method has been widely used in practical geotechnical design, primarily to assess ground deformations (serviceability limit states – SLS). However, its use in assessing the safety (ultimate limit states – ULS) of designs is increasing and there are several advantages to be gained from the use of numerical analysis in ULS design such as checking of multiple failure mechanisms which are not pre-determined. Partial factoring is relatively straightforward when using traditional design methods where failure modes are predefined, but less straightforward when using numerical analysis. Additionally, no guidance is offered by Eurocode to implement partial factors in finite element methods.

The traditional method used in geotechnical design is deterministic. The deterministic design approach considers a single value for soil property and ignores the inherent variability of the soil. Probabilistic analyses are increasingly being employed as it is a more realistic way of describing uncertainty in variables. It considers the variability of input parameters and provides the probability of failure based on a given probability distribution. But the additional computational effort involved in a stochastic analysis has not been found acceptable. This calls for the need to have computationally efficient methods.

Reliability-based design (RBD) is an approach currently gaining popularity for geotechnical engineering. But its implementation poses several challenges, one being the additional time in order to perform it. Usually Monte Carlo is performed when a finite element model is involved. However, it is deemed impractical to be used in industries owing to its computational complexity and time constrains. This thesis focuses on investigating a Reliability-based ULS Design approach that can be employed in industries with better computational ease.

1.2. Problem Definition

In a geotechnical engineering design, uncertainties are unavoidable. Quantifying the uncertainties and associated risks are crucial in the overall design. The current version of Eurocode 7 accommodates three design approaches that use partial factors to account for uncertainties. For complex soil structure interaction problems, applying a fixed partial factor can result in unrealistic failure mechanisms. Also, human intuition is not suited for reasoning with uncertainties. Hence, reliability based design is widely being applied to complex real-world problems using stochastic techniques. RBD gives the probability of failure and reliability index by explicitly considering uncertainty in the design.

The problem with using Reliability-based design is the absence of simple computational approaches that can be easily implemented. Monte Carlo simulations are commonly used to solve soil structure interaction problems. For a large and complex soil-structure interaction problem, it is computationally intensive to complete even a single run. This practical disadvantage can be solved only by a computationally efficient method.

Point Estimate method is a relatively simple technique and is being used in geotechnical practice for reliability calculations. Despite its simplicity it has been proven to be accurate in many practical situations. It is computationally less intensive and user friendly. A recent research (Kamp, 2016) shows possibilities in combining PEM with a finite element program to obtain a Reliability-based design, without the use of partial factors. Although this method considers only uncertainty of model parameter and does not address uncertainties in geometric parameters and water table and the like, it has been shown to produce satisfactory results. This thesis aims at making a comparison between PEM based FEM and Monte Carlo based Finite Element Approach. It is also aimed at automating this method for practical purposes.

The increasing use of the finite element method (FEM) in geotechnical design has raised the question of the compliance of this design approach with Eurocode 7 for the ultimate limit state (ULS). Additionally, Eurocode 7 does not provide any guidance on the direct use of fully probabilistic reliability methods for geotechnical design. This thesis aims to compare extensively the proposed PEM approach and EC7 and investigates its possibilities to replace the partial factor method of Eurocode 7. Achieving a user friendly environment for introducing probabilistic concepts into finite element modelling that complies with the Eurocode is the motive of this thesis.

1.3. Research Objectives

A framework for introducing probabilistic concepts employing point estimate method into finite element calculations using the Parameter variation feature of Plaxis has been verified in a recent research (Kamp, 2016).This approach produced satisfactory results when compared with existing stochastic models. It was also shown to provide comparable results as the deterministic calculations according to Eurocode 7 (Design approach 3). Although this has been verified, some aspects require further investigation

This research aims at a further investigation of this method to understand and analyse the possibilities to apply this method in engineering practice. An extensive comparison of this method with Eurocode 7 is to be made to analyse if this approach is a suitable alternative to the partial factor approach of Eurocode 7.

A recently developed tool in PLAXIS allows reliability based probabilistic analysis using First order reliability method. This allows direct probabilistic calculations in the finite element framework. Probabilistic calculations are performed using this tool and is compared with Point Estimate Method. Since modern geotechnical design codes also have an underlying reliability basis, it is extensively studied if such a reliability based design can be implemented in the Eurocode7. Hence the main research goals of this thesis are understated:

1. Extensive comparison between Point Estimate Method, First Order Reliability Method in the framework of PLAXIS and other stochastic methods.

2. Investigate the compliance of reliability based design with Eurocode 7 to understand potential possibilities of adopting reliability based design in engineering practice

3. Automation of the Point Estimate Method in PLAXIS by developing post processing routines using Python Programming interface.

1.4. Research Outline

- **Validation of PROBANA**
  The accuracy of the optimisation algorithm used in PROBANA is validated using 2 benchmarks – A vertical cut problem and a Prandtl wedge problem. This is done in chapter 3.

- **Comparison of FORM / PEM / Monte Carlo results**
  Probabilistic analysis is performed using FORM (PROBANA), and the results are

compared with previous research (Kamp, 2016) for three benchmarks. This is discussed in Chapter 4.

- **Comparison of Reliability based design with Eurocode 7**
  Reliability based design is compared with Eurocode based design. Their differences and reasons for the same are studied. Ways to incorporate reliability methods in Eurocode 7 are investigated, and demonstrated. This is also discussed in Chapter 3
- **Postprocessing PEM results in Plaxis**
  A Python code is written to automate Parameter variation results from PLAXIS to perform Point Estimate Method. This is discussed in chapter 4.

2
Literature Review

This chapter discusses the relevant literature used in this research. Since this research mainly concerns implementing reliability based designs within geotechnical design standards, structural reliability theories and Eurocode 7 are described and related literature is discussed. First order reliability method (FORM) and the newly developed probabilistic tool used to perform FORM are dealt in detail. Point Estimate Method is discussed briefly and results from (Kamp, 2016), which is basis of this research are summarized.

2.1. Geotechnical Design Standard: Eurocode 7

Eurocode 7 (Part 1: General Rules, Part 2: Ground Investigation and testing) is the European design standard for geotechnical structures. Eurocode 7 (EC7) is based on limit state design philosophy where the performance of a structure is defined based on a set of limit states beyond which the structure fails. Eurocode adopts two design philosophies – Ultimate limit state and serviceability limit state to check the performance of a structure.

1. Ultimate limit state (ULS) is associated with collapse or other similar forms of structural failure.

2. Serviceability limit states (SLS) correspond to conditions beyond which the specified requirements of the structure are not met.

For well-designed structures, ultimate limit states have low probabilities of occurrence whereas serviceability limit states have high probabilities of occurrence. Limit state design considers all possible failure modes and requires that for all design situations, no relevant limit state is exceeded. In practice, a geotechnical engineer knows from experience which limit states governs a design situation. EN1990 specifies two design methods.

- The partial factor method
- An alternative method based on probabilistic methods

Partial factor design method is the most commonly adopted method, and is the Eurocode design method. It involves applying partial factors to characteristic values, and the resulting design value ensures that the limit state isn't exceeded. Eurocode 7 provides no guidance in using direct probabilistic methods for geotechnical designs. Serviceability limit states (SLS) checks if the design action effect does not exceed the limiting value of the deformation of the structure. Eurocode 7 specifies a partial factor of unity for SLS.

2.1.1. Partial factor Method

Partial factor method is used in limit state designs to account for uncertainties in the parameter values. This method ensures that no relevant limit state is exceeded by factoring the actions and material properties or resistances with a partial factor. According to Eurocode 7, design value of a soil property, X_d is computed as

$$X_d = \frac{X_k}{\gamma_m} \tag{2.1}$$

In Equation 2.1, X_k is the characteristic value, and γ_m is the partial factor. The partial factor values for different soil properties are defined in the code. Characteristic value, X_k is discussed in the following section.
Serviceability limit states (SLS) checks if the design action effect, E_d does not exceed the limiting value of the deformation, C_d of the structure (Equation 2.2). Eurocode 7 specifies a partial factor of unity for SLS.

$$E_d \leq C_d \tag{2.2}$$

Ultimate Limit State Design (ULS) involves checking if design action effect, E_d does not exceed the design resistance, R_d (Equation 2.3).

$$E_d \leq R_d \tag{2.3}$$

The factored values are called the design values, and they are obtained as shown in Equation 2.4 and Equation 2.5. Characteristic values are multiplied with the partial factor to obtain design action effect while design resistance characteristic resistance values are divided by partial factors. The resistance thus diminished is required to be greater or equal to amplified actions.

$$E_d = \gamma_m X_k \tag{2.4}$$

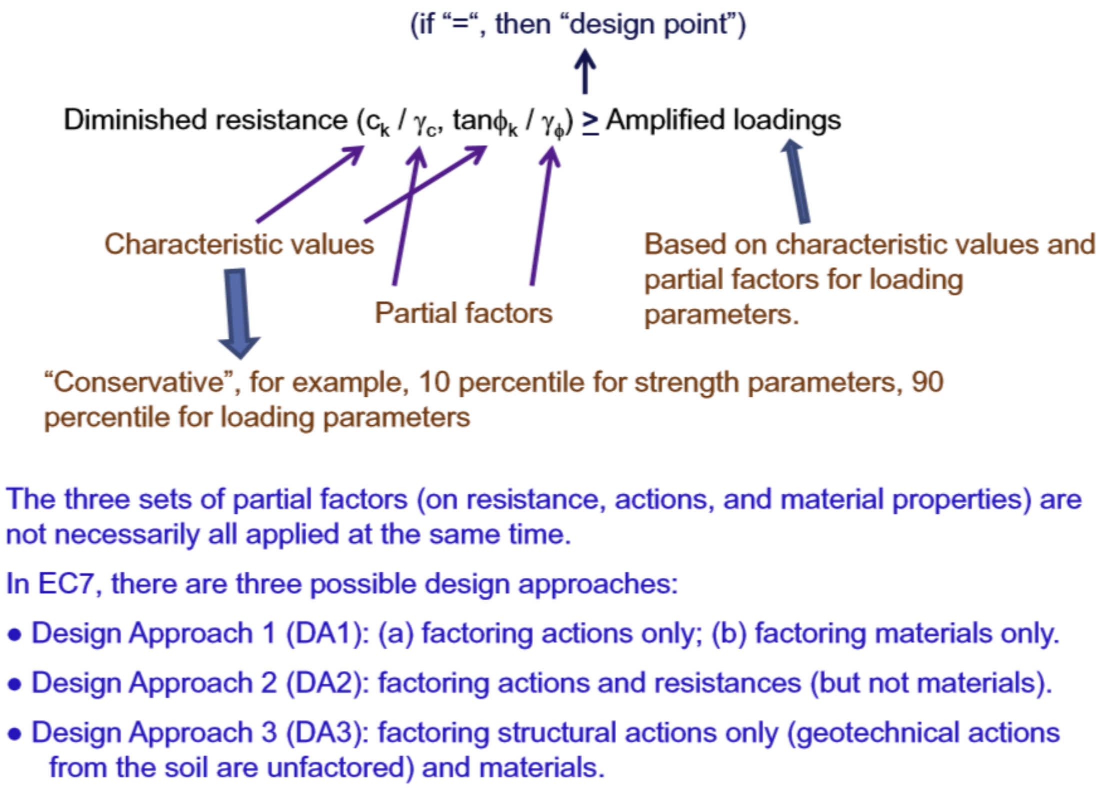

Figure 2.1: Eurocode 7 design approaches (Low and Phoon (2015))

$$R_d = \frac{R_k}{\gamma_m} \tag{2.5}$$

EC7 provides three design approaches and recommended values for partial factors specific to each design approach, for ULS. These design approaches are depicted in Figure 2.1.

Partial factors calibration:

Figure 2.2 shows an overview of various methods available for calibrating partial factors. EN 1990 mentions two ways to determine the numerical values of the partial factors.

- **Based on traditional building design:**
  This method referred as method a, involves deterministic calculations based on historical and empirical methods. In this method, calibration of partial factors is based on long tradition of building design to have similar safety levels as already existing structures. This method aims at achieving a target reliability index by applying partial factors. Calibration of Eurocode is primarily based on this method.

- **Based on probabilistic methods:**
  EN 1990 specifies two probabilistic calibration methods – FORM / Level II and

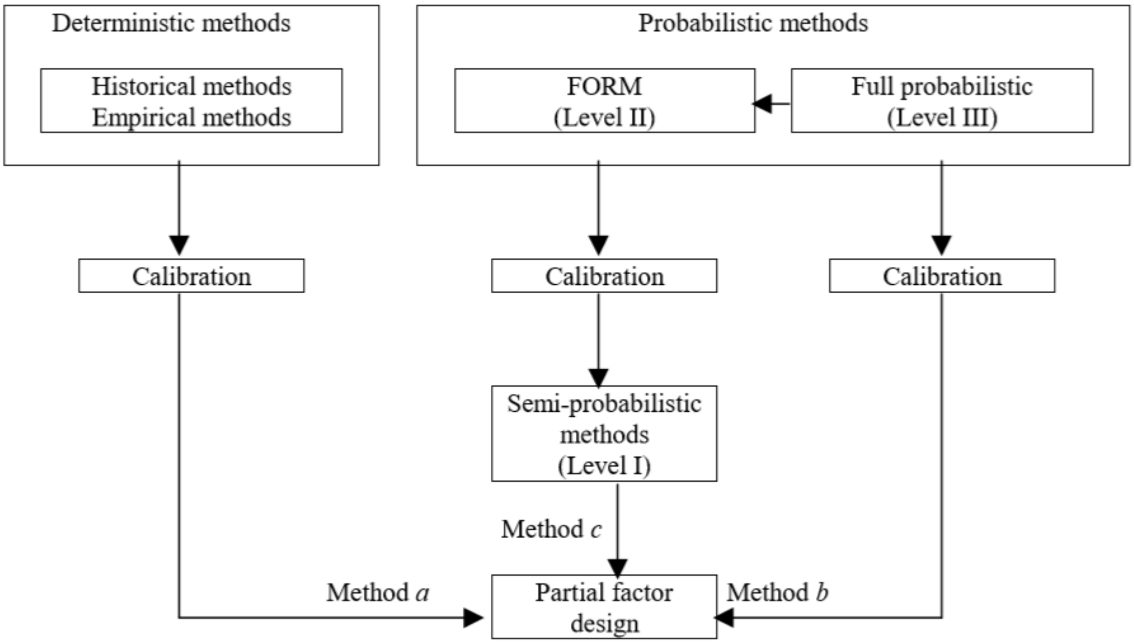

Figure 2.2: Overview Reliability Methods (Gulvanessian et al. (2002))

> Fully probabilistic / Level III. FORM / Level II methods are approximate reliability methods which are based on linear or second order approximations of the limit state function. Partial factors are calibrated based on reliability values obtain from Method b uses fully probabilistic methods to ca: Calibration based on fully probabilistic methods

Level III / Fully probabilistic methods give the best estimate of the reliability level of a structure, but they are seldom used for design because of the high computationally cost and insufficient statistical data available. Probabilistic methods give an estimate of the reliability level of the structure, which are used to calibrate partial factors whereas traditional building design calibrate partial factors based on target reliability values. According to EN1990, while using probabilistic methods to calibrate partial factors, the reliability levels of the structure should be as close as possible to the target reliability index. The Eurocodes have been primarily based on method a. Method c or equivalent methods have been used for further development of the Eurocodes.

Definition of characteristic value:
Eurocode 7 defines the characteristic value as a

> *cautious estimate of the value affecting the occurrence of the limit state.*

Characteristic values basically takes into account the inherent variability of the ground and the uncertainty involved in determining the parameters. These values are chosen based on the limit state so that the parameters would have different characteristic values for different failure mechanisms. However, Eurocode 7 does not provide information regarding how cautious this estimate should be. Eurocode7 further defines characteristic value using statistics, as understated.

> *If statistical methods are used, the characteristic value should be derived such that the calculated probability of a worse value governing the occurrence of the limit state under consideration is not greater than 5 %.*

Assuming normal distribution, this is written as:

$$X_k = \mu_x - 1.645\sigma_x \tag{2.6}$$

X_k is the characteristic value, μ_X is the mean soil property, σ_X is the standard deviation assuming a normal distribution. Though the characteristic value is a conservative estimate of the mean value, it has some shortcomings:

- It is often difficult to accurately quantify the 5% fractile due to limited data that a site investigation provides. Limited soil data provides a much accurate mean property value compared to the 5% fractile value.
- The 5% fractile value is based on the assumption that there are no local weak zones in the soil. Orr (2017) explains that the volume of the soil in the failure zone influences the characteristic value. If a slope and a foundation are on the same ground, failure zone in the case of a stability of a slope involves a much larger volume of soil than that of a spread foundation. Hence the mean values that governs the stability of the slope and the foundation are different. If the spread foundation were on a weaker zone, the characteristic value must be a more cautious estimate than 5%.

Hicks (2012) explains that characteristic values are problem dependent with the two governing factors being averaging of soil properties along the failure path, and the tendency of the failure path to follow the path of least resistance. Characteristic values are commonly considered as a conservative estimate, but argues that the ignoring the tendency of the failure path to follow the path of least resistance could lead to unconservative estimates. Therefore the reduction of the mean values along the failure path should be taken into account.

2.2. Structural Reliability Analysis Concepts

The primary goal of a structural design is to ensure sufficient safety throughout its lifetime. This is ensured by the following design criteria

$$Resistance > Load \tag{2.7}$$

Though this might sound straight forward at first sight, the uncertainties associated with the load and resistance parameters make this slightly complex. If the uncertainties are to be considered, it is difficult to ensure the validity of this criteria in absolute terms. Rather the probability that the criteria fails to be satisfied or in other words, the probability of failure is evaluated. Reliability of the structure with respect to the criterion can thus be computed. This is the basis of a reliability based design.
Reliability of a structure is the probability of successful performance of the structure with respect to the design criteria. The design criteria can be written as

$$Z = R - S \tag{2.8}$$

R is the resistance and S is the Load. Z is performance criteria which depends on the load and resistance design parameters. This function defines the failure surface in the design parameter space, which is the boundary between the safe and unsafe regions. Equation 2.8 can be generalised as:

$$Z = g(x_1, x_2, ...x_n) \tag{2.9}$$

where $g(x)$ constitutes the n basic variables x_1, x_2,.. x_n of the performance function. The performance function owes it's name to the fact that it is a measure of the performance of any structure. Like any mathematical equation, the performance function could have three outcomes as follows:

- $g(x) > 0$: Safe region
- $g(x) = 0$: Limit state
- $g(x) < 0$: Failure region

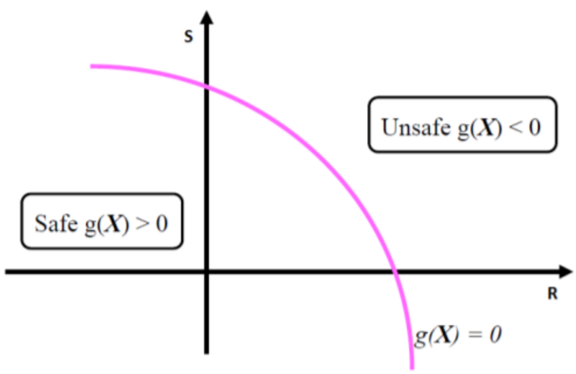

Figure 2.3: Limit State Concept

In Figure 2.3, the curve is the performance function. The region to the right of the curve is unsafe where $g(x) < 0$ while the region to the left of the curve is the safe region ($g(X) > 0$). The boundary or the curve represents the combination of the variables that are on the verge of failure i.e. at limit state. The basic goal of RBD is to adjust a set of design parameters such that a prescribed target probability of failure is not exceeded. The probability of failure, p_f is the integration of the joint probability density function over the failure region. This is the fundamental equation in reliability analysis.

$$p_f = \iint_{g(x)<0} f_X(x_1, x_2, ...x_n) dx_1 dx_2 ... dx_n \tag{2.10}$$

$f_X(x_1, x_2, ...x_n)$ is the joint probability density function of the random variables x_1, x_2,.. x_n. For statistically independent variables, the joint probability density function is the product of individual probability density functions. Evaluating this multiple integral is highly complex, which is the fundamental problem in reliability analysis. Therefore, analytical approximations of this integral is made. One such method is the First order reliability Method (FORM).

As Point Estimate Method (PEM) and First Order Reliability Method (FORM) are extensively used in this thesis, an overview of these methods is given in the following section.

2.2.1. Point Estimate Method

Point Estimate Method (PEM) is a level II semi probabilistic method proposed by Rosenbluth (195, 1981) that has been adopted in many geotechnical reliability analyses. It approximates the continuous pdf of each stochastic input with 2 discrete evaluation points which are at a distance of 1 standard deviation from the mean, on either sides of the mean of the pdf. Deterministic computations are performed with different combinations of all the evaluation points. This results in 2^n computations, where n is the number of stochastic parameters. A weighted average of all the deterministic computations is made and a response distribution is assumed for the for the output, from which the reliability index is evaluated.

Christian and Baecher (2002) suggested that when the number of stochastic inputs is greater than five or six, the number of evaluations becomes too large for practical applications. The computational efficiency of methods proposed over the years is considerably less than Rosenbluth (1975, 1981). Kamp (2016) investigates the applicability of Rosenbluth's PEM to evaluate the reliability of geotechnical structures. Kamp (2016) demonstrates PEM for a case where the soil structure interactions play an important role. It was shown that PEM provides comparable results with Crude Monte Carlo Simulation and Eurocode 7. In this research, the results of Kamp (2016) are compared with an approximate method, called First Order Reliability Method, and the differences in efficiency and accuracy are investigated.

Kamp (2016) provides a detailed description of the PEM concept. The same is briefed here.

Figure 2.4 depicts Rosenbluth's PEM for a bivariate case. The two stochastic input variables, X_1 and X_2 are represented by the corresponding distributions. Each stochastic input X has two evaluation points - X_- and X_+ on both sides of mean. Since the number of stochastic variables considered here is 2, PEM results in 4 combinations, represented by the 4 dots in the parameter space. In the context of this thesis, each of these dots refers to a deterministic analysis with the corresponding combination of X_1 and X_2. For n stochastic inputs, PEM results in 2^n combinations. Each dot or deterministic result is associated with a weight which is computed with the expression provided in Equation 2.12.

$$P_{(s1,s2,...sn)} = \frac{1}{2^n} \cdot [1 + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} (s_i)(s_j)\rho_{ij}] \tag{2.11}$$

s_i is +1 for evaluation point at X_+ and -1 for evaluation point at X_-. The m^{th} moment is calculated by:

$$E[Y^m] = \sum P_i \cdot y_i^m \tag{2.12}$$

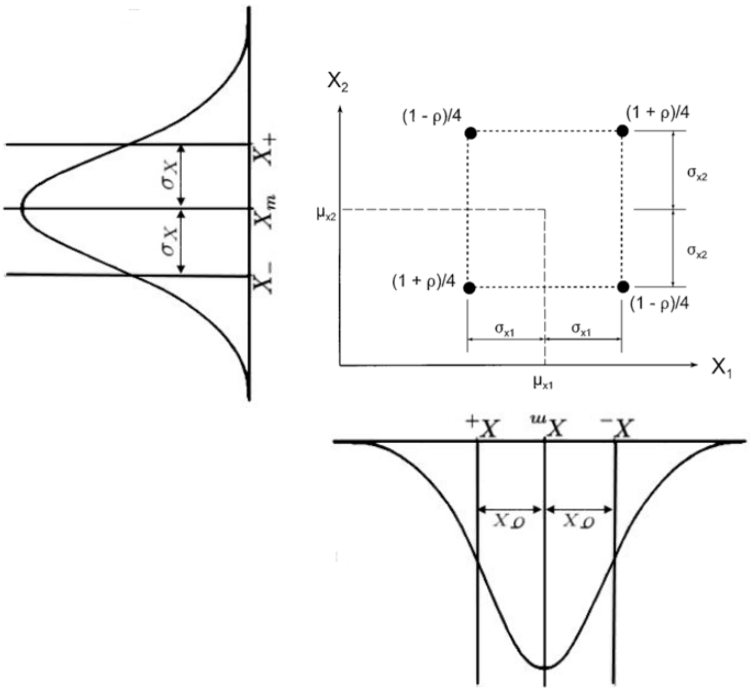

Figure 2.4: Rosenbluth's PEM for bivariate correlated or uncorrelated case

Other PEM related Methods

Since the formulation of PEM by Rosenbluth (1981), many researchers have tried to improvise the method to reduce the estimating points without hampering the accuracy. Two relatively simple methods for reducing the evaluation points to 2n or

2n+1 has been put forth by Harr (1989) and Hong (1996, 1998). Harr (1989) deals with the case in which the variables may be correlated but their skewness coefficient are zero. Hong(1996; 1998) deals with the other problem of uncorrelated variables with significant skewness. Li (1992) presented an explicit expression to assess the expected value of the function. This expression considers the kurtosis coefficients of the input distribution while it only requires (N^2+3N+2)/2 function evaluations.

2.2.2. First Order Reliability Method

FORM stands for First Order Reliability Method. The term first order indicates that the it is a first order approximation of the performance function. FORM constitutes two approaches – First Order Second Moment method (FOSM) and Advanced First Order Reliability Method (AFOSM) or Hasofer – Lind Method. When the term 'FORM' is used, one usually refers to the Hasofer Lind method.

Hasofer - Lind method involves transforming the random variables to a reduced space of co-ordinates. This transformation of the coordinate space is performed to aid in the computation of reliability Index. The reliability index is defined in this new reduced space. The Hasofer-Lind reliability index β_{HL} is defined as the minimum distance between the origin and the limit state surface. Thus, the determination of this point has two important aspects – Optimization of the distance to find the right minimum distance point, with the Constraint that the point lies on the limit state surface. This minimum distance point on the limit state surface is called the 'design point'. The design point represents the most probable point of failure – MPP. The physical meaning of reliability index in this definition is the minimum distance between the origin to the limit state surface in the reduced space of random variables. This point on the limit state surface is the most probable point of failure or the design point. The actual problem here is to determine the design point that leads to the least distance between the origin and the limit state surface. This becomes a constrained optimization problem where the distance between the origin and the limit state surface is optimized / minimized by constraining the design point to lie on the limit state.

In figure 2.5, $g = E - R$ is the limit state function. R is the structural resistance and E is the action effect. A structure is considered safe when $E_d < R_d$. CEN (2012) mentions that design values should be based on FORM. The main steps of FORM are summarized as follows:

- Transformation of the original coordinate space, X to the standard / reduced co-ordinate space, U and limit state function from $g(X) = 0$ to $g'(U) = 0$.
- Approximate or linearize the limit state function at a certain point (design point).
- Performed constrained optimization to determine the design point.
- Reliability index, β is the distance between design point and the origin in the

standard space.

- Failure Probability is $p_f = \phi(-\beta)$

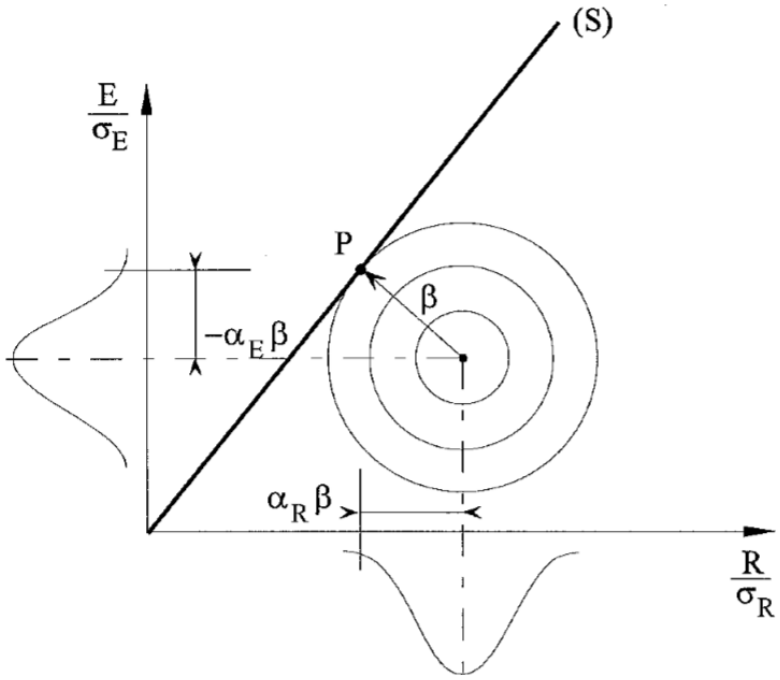

Figure 2.5: FORM design point and Reliability index (CEN (2012))

A second order approximation can reduce the linearization errors. Such a modification is referred to as Second Order Reliability Method, SORM.

2.3. Probabilistic tool in PLAXIS - PROBANA

PROBANA (stands for probabilistic analysis) is a recently implemented tool in Plaxis which couples reliability analysis with Finite elements. This tool uses the facilities offered by OpenTURNS, an open source library for treating uncertainties. PROBANA offers the option of performing reliability analysis using FORM or Monte Carlo. It requires the statistical distribution of the stochastic input parameters and outputs a reliability index, probability of failure, importance factors and design values for the corresponding stochastic inputs variables. FORM uses an optimisation approach to compute the reliability index. PROBANA performs the optimisation using COBYLA (COnstrained optimisation BY Linear Approximation, Powell, 1994). This algorithm finds the point on the failure surface (limit state function) which has the minimal distance from the mean point in the standard space. This distance is the Reliability index. It starts from the mean values of the parameters and iteratively converges to the design point. COBYLA uses four error tolerances for convergence. The errors considered are:

1. Absolute error: The distance in absolute value between points in successive iteration.
2. Relative error: The relative distance between points in successive iteration.
3. Residual error: The orthogonality error indicating the lack of orthogonality between the limit state surface and the minimum distance vector that links the origin in the standard space and the point from the iteration.
4. Constraint error: The distance between the design point and the constraint function defined by the threshold.

The errors ensure optimisation of the distance between limit state function and the mean values. This error ensures that the design point lies on the limit state function. The probabilistic tool defines default tolerance of all the errors as 0.001. It should be noted that constraint error has units of the threshold function, and therefore should be carefully chosen. For example, if the criterion of interest is safety factor, the default value of 0.001 is not practical. A constraint error of 0.01 is valid in this case. The default number of iterations is 100. The algorithm converges if one of the following criteria is met:

- The maximum number of iterations is reached.
- The absolute error and relative errors are above the tolerances
- The residual and constraint error are above the tolerances.

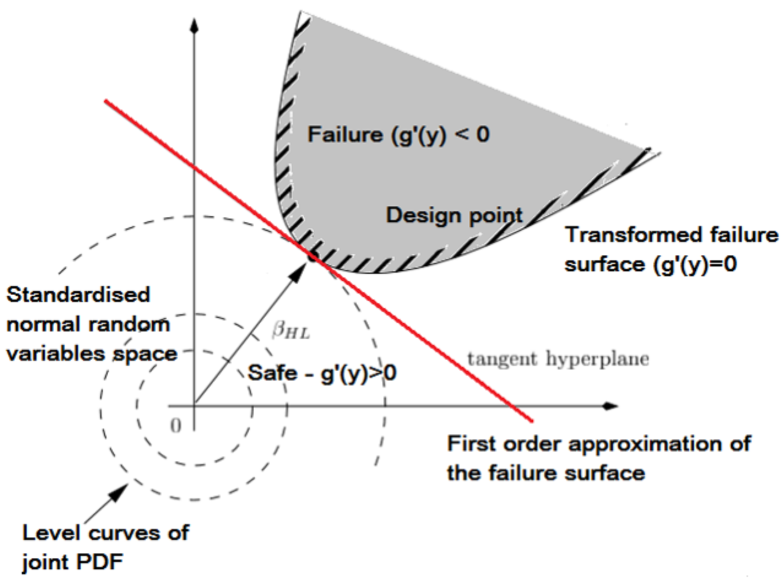

Figure 2.6: FORM design point and Reliability index

OpenTurns FORM analysis:

The main steps of FORM analysis by OpenTURNS method is summarised as follows:

1. Isoprobabilisitic transformation: Transformation of the random variables from original space (X) to the standard normal space (U), and limit state function from $g(X) = 0$ to $g'(U) = 0$. Isoprobabilisitic transformation ensures rotation invariance.

2. Find the design point which is the point verifying the event of maximum likelihood. The design point is the point on the limit state boundary the nearest to the origin of the standard space. Thus, u is the result of a constrained optimization problem.

3. Approximate the limit state surface in the standard space with a linear surface at the design point In the standard space (U), iterations are performed to find the point on the failure surface $g'(U) = 0$ closest to the origin. This is the design point or the most probable point (MPP).

In Chapter 4, the optimization algorithm is validated with two problems having known analytical solutions.

3

Validation of the tool - PROBANA

Probana is a recently developed tool implemented in Plaxis that uses FORM (or Monte Carlo) to perform reliability based probabilistic analysis in a Finite element framework. The accuracy of PROBANA is tested by performing analyses of problems with known analytical solutions. These problems are analysed with a single stochastic parameter, as manual computation of failure probability is only possible with one random variable. Therefore, this validation is intended to perform an accuracy check in the underlying optimization method adopted by Probana – FORM.

3.1. Stability of a Vertical Cut

The stability of a vertical excavation is the problem considered. The hypothetical vertical cut is 12 m deep and has a unit weight of $18kN/m^2$. The basic problem geometry is shown in Figure 3.1. The soil parameters used in the finite element model are given in the Table 3.1.

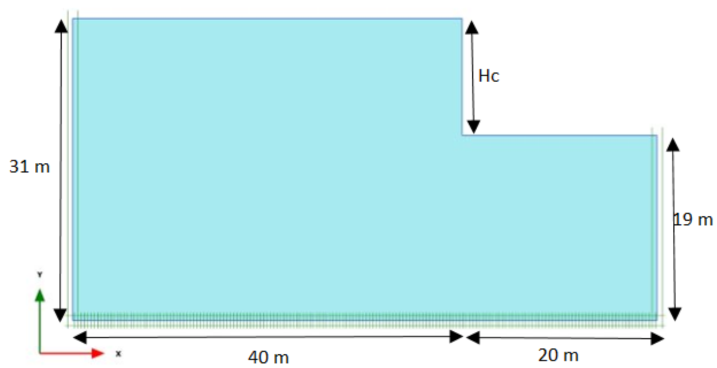

Figure 3.1: Vertical Cut Problem geometry

Table 3.1: Soil Parameters Vertical cut

Parameter	Name	Value	Unit
Material Model	Model	Mohr Coulomb	-
Drainage type	Type	Undrained (B)	-
Young's Modulus	E	5000	$[kN/m^2]$
Poisson's Ratio	ν	0.3	-
Unit weight	$\gamma_{unsat}/\gamma_{sat}$	18/24	$[kN/m^3]$
Undrained shear strength	$S_{u,ref}$	133.34	$[kN/m^2]$
Friction angle	Φ	0	°
Dilatancy angle	Ψ	0	°
Tensile strength	σ_t	10.00E6	$[kN/m^2]$
Tension cut off	-	Off	-

The factor of safety of the slope can be expressed as in Equation 3.1

$$F = \frac{N_0 c}{\gamma H} \tag{3.1}$$

N_0 is the stability number that depends on the slope angle. For vertical slopes, the value of N_0 is 3.83 according to Swedish slip circle method, and plane slip surface gives a N_0 value of 4. c is the cohesion, γ is the unit weight and H is the slope height. From a deterministic analysis in Plaxis with the parameters specified in Table 3.1, a stability number, N_0 of 3.85 is obtained. Hence the model can be described with Equation 3.2

$$F = \frac{3.85 S_u}{\gamma H} \tag{3.2}$$

First a deterministic analysis is performed, which is further extended to a probabilistic model by coupling with PROBANA.

3.1.1. Deterministic Model PLAXIS

A deterministic analysis of the vertical cut is performed in Plaxis with the parameters specified in Table 3.1 and a deterministic factor of safety of 2.39 is obtained. Figure 3.2 depicts this. S_u is the undrained shear strength, γ is the unsaturated unit weight, H_c is the critical height, F is the factor of safety.

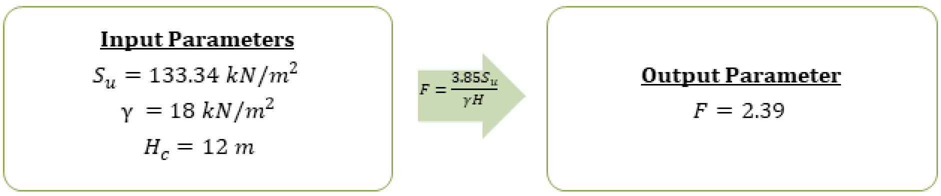

Figure 3.2: Deterministic Model

3.1.2. Probabilistic Model – Plaxis Probana

A probabilistic analysis is performed by coupling the deterministic plaxis model with PROBANA. Undrained shear strength, S_u is considered as the stochastic input. The statistical properties of S_u are shown in Figure 3.3. The probability of failure of the slope is to be computed using PROBANA. This requires defining a performance function for the criterion considered. In this case, the criterion being Factor of safety, the performance function is defined as Equation 3.3.

$$Z = \mu_F - 1 \tag{3.3}$$

μ_F is the mean safety factor and 1 is the threshold value of the safety factor as $F < 1$ corresponds to slope failure. PROBANA essentially gives the probability of failure, i.e. in this case, the probability that Factor of safety is less than 1. The aim of this problem is to validate this probability of failure with a manually calculated failure probability. Due to the linearity of the problem, the failure probability can be conveniently calculated.

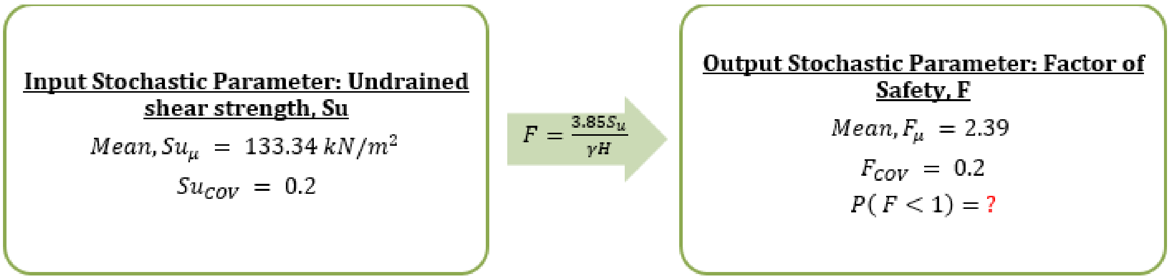

Figure 3.3: Probabilistic Model

Figure 3.3 depicts the probabilistic model with the statistical distribution of the input and output stochastic parameters. The mean values of S_u and F correspond to the deterministic model, which is the starting point for the probabilistic calculations. A coefficient of variation of 0.2 is assumed for the undrained shear strength (NEN9997-1). The coefficient of variation of factor of safety is the same as that of S_u, as S_u and F have a linear relationship. The failure probability of the slope is computed in Probana, for a threshold F value of 1. Probana gives a failure probability of 0.0017. This probability of failure from PROBANA is compared with the manually calculated probability of failure.

The mean undrained shear strength value, Su_μ (133.34 kPa) is chosen such that $F_{\mu-2.9\sigma}$ corresponds to safety factor of 1 i.e. Probana threshold safety factor.

$$F_{\mu-2.9\sigma} = \frac{3.85 * Su_{\mu-2.9\sigma}}{\gamma H} = 1 \tag{3.4}$$

With this condition, Su_μ is back calculated as 133.34 KPa (Equation 3.5).

$$Su_{\mu-2.9\sigma} = 56kPa \implies Su_\mu = 133.34kPa \tag{3.5}$$

The reason for choosing threshold F corresponding to $Su_{\mu-2.9\sigma}$ is elaborated here. This problem aims at validating occurrences of very low probabilities. In a normal distribution 99.7 % values lies in the range of $\mu \pm 3\sigma$. Probability of occurrence is determined by computing the corresponding area under the normal distribution. In this validation problem, PROBANA failure probability is validated with manually computed probability. For accurately computing the failure probability manually, it should lie in the range $\mu \pm 3\sigma$. Therefore, probability less than $\mu - 2.9\sigma$ are considered.

3.1.3. Manually computing probability of failure

The coefficient of variation of factor of safety is the same as that of S_u, as S_u and F have a linear relationship. Thus, with COV of 0.2, and mean Factor of safety of 2.39 from the deterministic model, the output distribution of factor of safety, F is drawn as shown in Figure 3.4. The failure probability is manually computed as the area of region below F < 1 i.e. 0.00186. It is verified if this failure probability is the same as the probability that Probana gives.

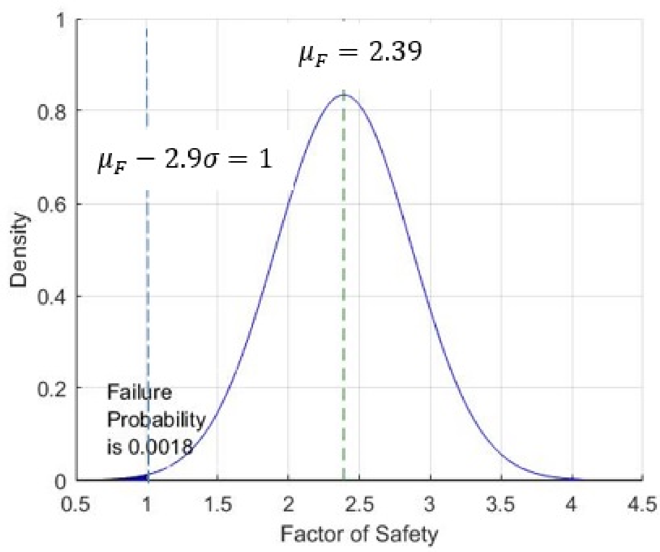

Figure 3.4: Output distribution (factor of safety) and manually computed failure probability

The expected failure probability is the area of the shaded portion in Figure 3.4 i.e. 0.00186. Probana gives a failure probability very close to the expected probability (Table 3.2). This verifies the optimisation algorithm (COBYLA) used in Probana.

Table 3.2: Probability Validation

Expected Probability	0.00186
Probana FORM Probability	0.0017
Error	5.91%

3.2. Bearing Capacity of a shallow foundation

The determination of bearing capacity of a strip footing on a cohesive frictionless material is considered here. A footing of width 2 m and surcharge load of $10kN/m^2$ is modelled as shown in Figure 3.5. A displacement controlled test is performed in PLAXIS 2D to analyse the bearing capacity of the strip footing. The probability of failure of the footing is computed using PROBANA for a specific threshold bearing capacity, and the results are compared with an analytical solution.

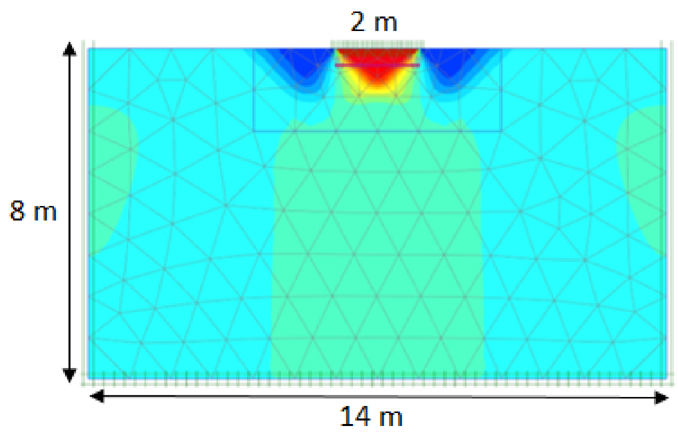

Figure 3.5: Strip footing PLAXIS 2D Model

Table 3.3: Soil Parameters

Parameter	Name	Magnitude	Unit
Model	Mohr Coulomb		
Type	Undrained (B)		
Young's modulus	E	250,000	$[kN/m^2]$
Poisson's ratio	ν	0.2	[-]
Cohesion	C_u	100	$[kN/m^2]$
Friction angle	ϕ	0	[deg]
Dilatancy angle	Ψ	0	[deg]
Tension cut off	Off		

Closed form solution:

Prandtl (1920) published an analytical solution for the bearing capacity of a maximum strip load on a weightless infinite half-space. The collapse load from Prandtl's Wedge solution can be found as:

$$q = (2 + \pi) * C_u = 5.14C_u \tag{3.6}$$

In Equation 3.6, C_u is the soil cohesion and q is the bearing capacity.

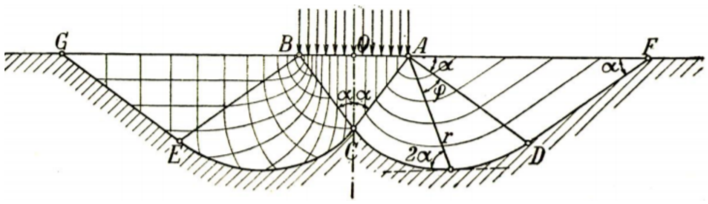

Figure 3.6: Prandtl wedge - Collapse load

From Table 3.3, Undrained shear strength C_u of the footing in the problem considered here is 100 kN/m^2. The bearing capacity of the strip footing is analytically calculated as shown in Equation 3.7.

$$q = (2 + \pi)C_u = 5.14C_u = 5.14 * 100 = 514kN/m^2 \tag{3.7}$$

3.2.1. Deterministic analysis PLAXIS

A displacement controlled test in performed in PLAXIS and the bearing capacity is analysed. The displacement vs bearing capacity graph is shown in Figure 3.7. (The bearing capacity of the footing in PLAXIS is calculated as the Reached Force, F_y divided by the width of the footing, B which is 2m) Figure 3.7 shows a comparison between PLAXIS finite element solution and closed form solution. PLAXIS gives a bearing capacity of 521.5 kN/m^2 compared to the closed form solution of 514 kN/m^2.

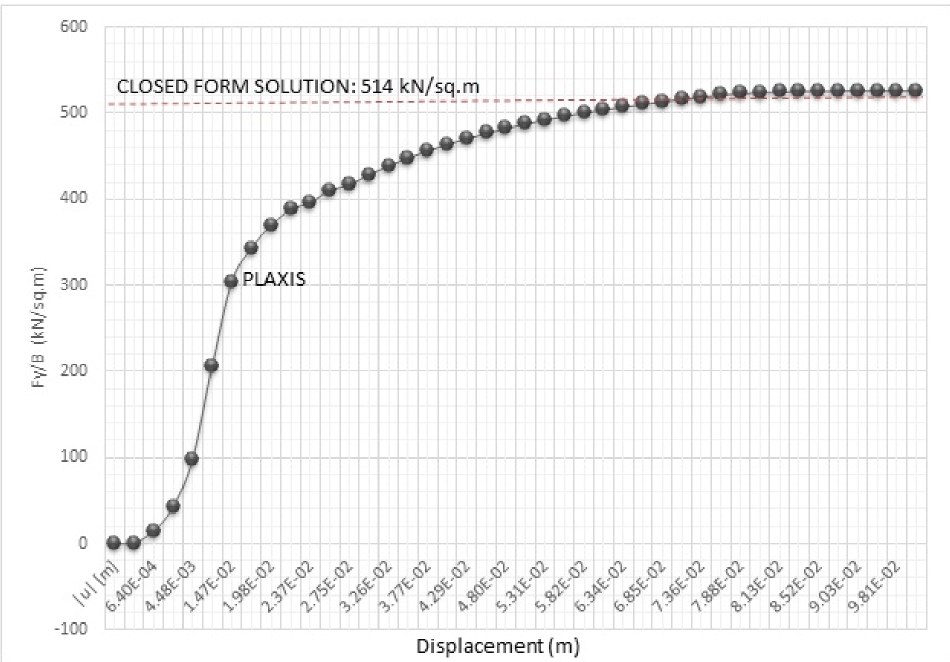

Figure 3.7: Bearing Capacity vs Displacement from PLAXIS and comparison with analytical solution

3.2.2. Probabilistic analysis in PROBANA - FORM

A probabilistic analysis is performed by considering the undrained shear strength as stochastic. A coefficient of variation of 0.2 is assumed, and the mean value of 100 kN/m^2 from the deterministic model is used to define the statistical distribution of S_u. The deterministic PLAXIS model is coupled to PROBANA and the probability of failure is determined. Like the previous validation benchmark, a threshold value equal to $\mu + 2.9\sigma$ is used, so that the probability of failure that is validated is sufficiently low and also manually computable.

Calculation of the threshold bearing capacity:

Mean $C_u = 100kN/m^2$

From deterministic analysis, Mean q = 521.5 kN/m^2

For a linear single stochastic model, COV of input and output distribution are the same.

Therefore, COV of $C_u = 0.2 = COV of q$

Threshold q = Mean $q + 2.9\sigma = 521.5 + (2.9) * (0.2) * (521.5) = 823.97kN/m^2$

Table 3.4 gives the input values used to perform the probabilistic analysis in PROBANA.

Table 3.4: PROBANA Input

Mean S_u	100 kN/m^2
COV S_u	0.2
Threshold Bearing capacity	823.97 kN/m^2
Errors	Default

Note: In this benchmark, the threshold bearing capacity value is the upper limit, μ + 2.9σ in the normal distribution, while in the previous benchmark, the threshold factor of safety was set to the lower limit, μ - 2.9σ. This is because the threshold value for bearing capacity is the maximum allowable bearing capacity of the footing whereas threshold value for the factor of safety is the minimum safety factor for which the vertical cut would be stable. Therefore the choice of upper limit or lower limit for the threshold depends on the problem.

The probability of failure corresponding to the threshold bearing capacity is manually calculated as the area of the normal distribution above the threshold value. This is almost the same as the failure probability from PROBANA as shown in Table 3.5.

Table 3.5: Expected Probability and PROBANA Probability

Expected Probability	0.00186
Probana FORM Probability	0.00187

3.3. Conclusion

The optimization algorithm of PROBANA has been validated for two benchmarks with known analytical solution. The manually computed probability of failure and the probability of failure from PROBANA are almost the same. This verifies that COBYLA - the algorithm used in FORM gives accurate results. Though, the optimization algorithm is validated, the efficiency of this for complex geotechnical designs requires more investigation.

4

Reliability based design & comparison with Eurocode 7

This chapter summarizes results of reliability analysis on three benchmarks and a case study. A comparison is made between Point Estimate Method (PEM), and First order reliability Method (FORM) with other stochastic Methods and Eurocode 7. Two approaches are suggested to incorporate Reliability based designs in Eurocode 7. These approaches are briefly explained here, and demonstrated in detail with the first benchmark. The benchmarks and the case study are same as the previous research (Kamp, 2016) for comparison purposes.

4.1. Reliability Analysis Methodology

Two reliability based methods – Point Estimate Method (PEM) and First Order Reliability Method (FORM) are compared and how Reliability based design compliments the Eurocode 7 design approach is investigated using three benchmarks and a case study. Point Estimate Method is a rather simple and straightforward approach to perform a semi probabilistic analysis in the framework of finite elements. FORM, also performed in a finite element framework, is a more complex method in terms of its underlying mathematical complexity, but is principally supposed to give more accurate results compared to PEM. This is investigated by comparing PEM and FORM with other stochastic methods.

4.1.1. Point Estimate Method (PEM)

PEM is performed using the Parameter Variation Feature in Plaxis. Parameter Variation feature in Plaxis directly gives the different deterministic PEM combinations. The methodology used is summarized below.

1. **Deterministic Analysis**: Model the problem in Plaxis and perform deterministic analysis.

2. **Sensitivity Analysis**: Analyze the sensitivity of each parameter and identify the dominant parameters of the problem. The sensitivity analysis feature available in Plaxis is used for this.

3. **Parameter Variation Analysis**: Using the dominant parameters identified in the sensitivity analysis, and by judgment, the stochastic inputs are decided. Stochastic inputs are the input parameters with considerable uncertainty. Select these parameters and perform Parameter variation analysis with the Parameter Variation feature in Plaxis. Here, it is required to define the maximum and minimum values of each parameter. This is computed with mean, μ and the COV defined in NEN9997-1 . The deviation is calculated as shown in Equation 4.1. The minimum and maximum values are one deviation to the left and right of the mean.

$$\sigma = COV * \mu \tag{4.1}$$

4. **Post-processing results**: The PEM combinations are obtained in the previous step. From this, the reliability index is to be calculated. A script is written for automatic post-processing of the Plaxis results. The script, on activating retrieves the output from step 3 and computes the reliability index. This process is explained below:

   - For the PEM combinations obtained in step 3, Mean and Standard deviation (the first two statistical moments) are calculated based on which a distribution is assumed for the output e.g. normal or log-normal.
   - In order to compute the reliability index, the user is asked to define a limit state function. The limit state function represents the failure limit, or the threshold.
   - The reliability index is computed as the distance between the mean and the limit state line in units of the standard deviation. This is expressed in the following equation:

$$\beta_{normal} = \frac{Mean - Threshold}{\sigma} \tag{4.2}$$

4.1.2. First Order Reliability Method (FORM)

FORM is performed using a recently developed tool in Plaxis, called Probana which is coupled with Plaxis. Probana stands for Probabilistic analysis. It performs iterations to search for the design point using a constrained optimisation approach called COBYLA and further calculates the Reliability Index and Probability of Failure.

1. **Deterministic Analysis**: Model the problem in Plaxis and perform deterministic analysis. The result of the deterministic analysis is the mean response, and also the starting point in the probabilistic analysis

2. **Probabilistic analysis**:

   - Connect PROBANA to Plaxis by activating the Remote Scripting server in PLAXIS. This connects the deterministic model to PROBANA.
   - PROBANA displays all the soil parameters of the deterministic model. Select the stochastic parameters, and define their statistical properties (Mean and Standard deviation).
   - Assign correlation values for correlated parameters.
   - Define a response threshold based on the limit state function and choose appropriate tool calculation features.
   - PROBANA outputs the reliability index, Probability of failure, design points and the sensitivity coefficients.

The strategy used to compare the results of PEM, FORM and Eurocode 7 design approach are discussed in the following section.

4.2. Comparison Reliability based design and Eurocode 7

4.2.1. PEM and FORM

To compare the extent to which PEM and FORM agree, the reliability index obtained from both methods are compared. The reliability index from PEM is obtained by assuming an appropriate output distribution, whereas FORM calculates reliability index based on a corresponding probability of failure.

4.2.2. FORM and Eurocode 7

To compare FORM results with Eurocode7, the design point from FORM is compared with the design values from Eurocode7. In FORM, the design point is a point on the limit state surface that separates safe combinations of parametric values from the unsafe set of parametric values. The design point in FORM is the most probable combination of parametric values at failure (Kong and Phoon, 2015).

Geotechnical Ultimate Limit State analyses are performed according to Eurocode 7 with design parameters. Basis for the design parameters are cautious estimates of the mean values, which are generally replaced by characteristic values. The characteristic value for every parameter is taken as the 5 percent fractile value as defined in the Eurocode7. Considering a normal distribution of a parameter X with mean value, X_m and standard deviation, σ_x, the characteristic value is defined as:

$$X_k = X_m - 1.64 \cdot \sigma_x \tag{4.3}$$

Design parameters are computed by factoring the characteristic value with a partial safety factor. The partial safety factors are nationally determined for every soil property irrespective of how the characteristic value is determined. The partial safety factor ensures a certain level of safety or reliability to the structure.

$$X_d = \frac{X_k}{\gamma_m} \tag{4.4}$$

To compare FORM and Eurocode 7 design values, both should correspond to the same reliability level. FORM outputs a reliability index based on the design point that it converges to, whereas Eurocode 7 computes design values for a corresponding target reliability index. For comparison purposes, it must be ensured that FORM design point and Eurocode design values correspond to the same reliability index. This is done by calibrating the limit state function used in FORM according to EC7 design values.

This is illustrated by the flow chart shown in Figure 4.1. Here, both Eurocode 7 and

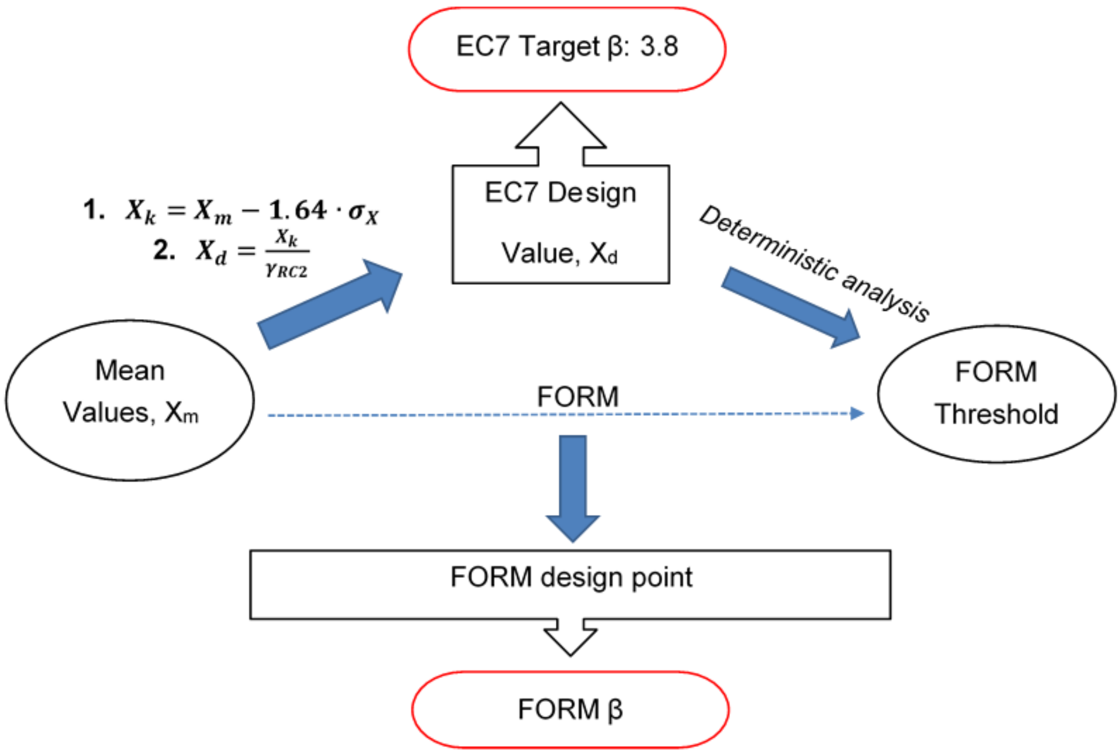

Figure 4.1: Flowchart - Calibrating FORM according to Eurocode 7

FORM starts from the same point i.e. the Mean value point, X_m. Using partial factors defined in EC7, the design values are computed. These design values correspond to a reliability index of 3.8. It is to be noted that there are three classes of partial factors which correspond to different reliability levels. In the flowchart shown in Figure 4.1, Reliability class 2, which corresponds to a β value of 3.8 is used. A deterministic plaxis calculation is done using EC7 design parameter values, and the result of this deterministic analysis is considered as the FORM threshold. This ensures that both FORM and EC7 correspond to same reliability level.

4.3. Coupling Reliability Analysis with Eurocode 7

In the previous section, it is discussed how to calibrate reliability analysis according to EC7 reliability level. If reliability analysis can complement the EC7 design, both can be coupled to obtain more practical designs. Two such approaches to couple reliability analysis with EC7 are explained here, and later demonstrated with the benchmark 1. The Eurocode for basis of structural design, EN1990 (CEN, 2002) describes basis for partial factor design and reliability analysis. EN1990 allows for possibilities to include reliability based designs provided target reliabilities are achieved. According to EN1990, partial factors can be determined in one of the following ways:

1. Based on calibration to an extensive experience of building tradition.
2. Based on statistical evaluation of experimental data and field observations, carried out within the framework of probabilistic reliability theory.

EN1990 mentions that while using probabilistic reliability methods, the ULS partial factors should be calibrated such that the reliability levels are as close as possible to the target reliability index. The design values are calculated by using the expressions provided in EN 1990:2002. According to Table C3, EN1990, for a normal distribution, the design value is computed as Equation 4.5.

$$X_d = \mu - \alpha\beta\sigma \tag{4.5}$$

X_d is the design value, μ is the mean value, α is the sensitivity coefficient, σ is the standard deviation. The partial factor is obtained by dividing the characteristic value by the design value. Two approaches to determine partial factors / design values using reliability based approach (FORM) are proposed here.

Approach 1:

1. Identify the stochastic parameters of a problem. Obtain the mean values and coefficient of variation of all the parameters:
   Mean and COV are the two statistical parameters required for a Level 1 reliability evaluation. The mean value is the average value from experimental data or field observation. COV of parameters are obtained from NEN9997-1.
2. Obtain EC7 design values of all parameters:
   This is done by dividing the characteristic values, X_k by the corresponding partial factors, γ_m from Eurocode7.
3. Determine the deterministic ULS threshold value and set up the limit state equation:

ULS threshold is obtained by performing a deterministic analysis of the problem with the design values of the parameters from Step 2 (In this thesis this value is referred as EC7 ULS). Setting up the limit state equation is explained in the benchmark 1.

4. Perform FORM analysis with the mean value points as the starting point and the EC7 ULS as the threshold, and determine the corresponding reliability index, β and FORM sensitivity factors, α_i.

5. Calculate design value X_d, with FORM β and FORM α_i values.

$$X_d = \mu(1 - \alpha \cdot \beta.COV) \tag{4.6}$$

6. Calculate partial factor

$$\gamma_m = (1 - 1.64 \cdot COV)/(1 - \alpha \cdot \beta.COV) \tag{4.7}$$

Approach 2:

1. Obtain EC7 design values of all parameters, and determine the deterministic ULS threshold value and set up the limit state equation, as in the previous approach.

2. Perform FORM analysis with the mean value points as the starting point and the EC7 ULS as the threshold, and determine the corresponding reliability index, β and FORM design points. Since these design points correspond to the EC7 Ultimate limit state function, these design points from FORM analysis can be directly used instead of EC7 partial factor method.

Limitations of the approaches: The reliability method used in these approaches, FORM works by assigning a sensitivity coefficient, α to each stochastic parameter. However, this is a relative sensitivity coefficient, which implies that, α values of all stochastic parameters from FORM adds up to a total of 1. When parameters have very low sensitivity, the partial factor from approach 1 is less than 1, or in approach 2, the FORM design values are higher than the mean values. This is unacceptable as Eurocode 7 aims to determine a conservative estimate for the parameters. Therefore, for parameters that have very low sensitivity coefficients in FORM, this method should not be used to compute design values or partial factors. Rather, partial factors from Eurocode 7 should be used. These approaches are illustrated in the next section with a benchmark.

4.4. Benchmark 1: Slope Stability Problem

Slope stability is one of the most common geotechnical problems. A simple slope is considered here and its stability is analysed using Finite element reliability analysis in Plaxis. In general, Plaxis performs a finite element calculation based on deterministic values of input parameters which gives a deterministic value of output quantities. This chapter explores the feasibility of a semi probabilistic method using Point Estimate Method and a probabilistic approximation method using FORM on a slope stability problem. The flowchart in Figure 4.2 shows the different methods used in

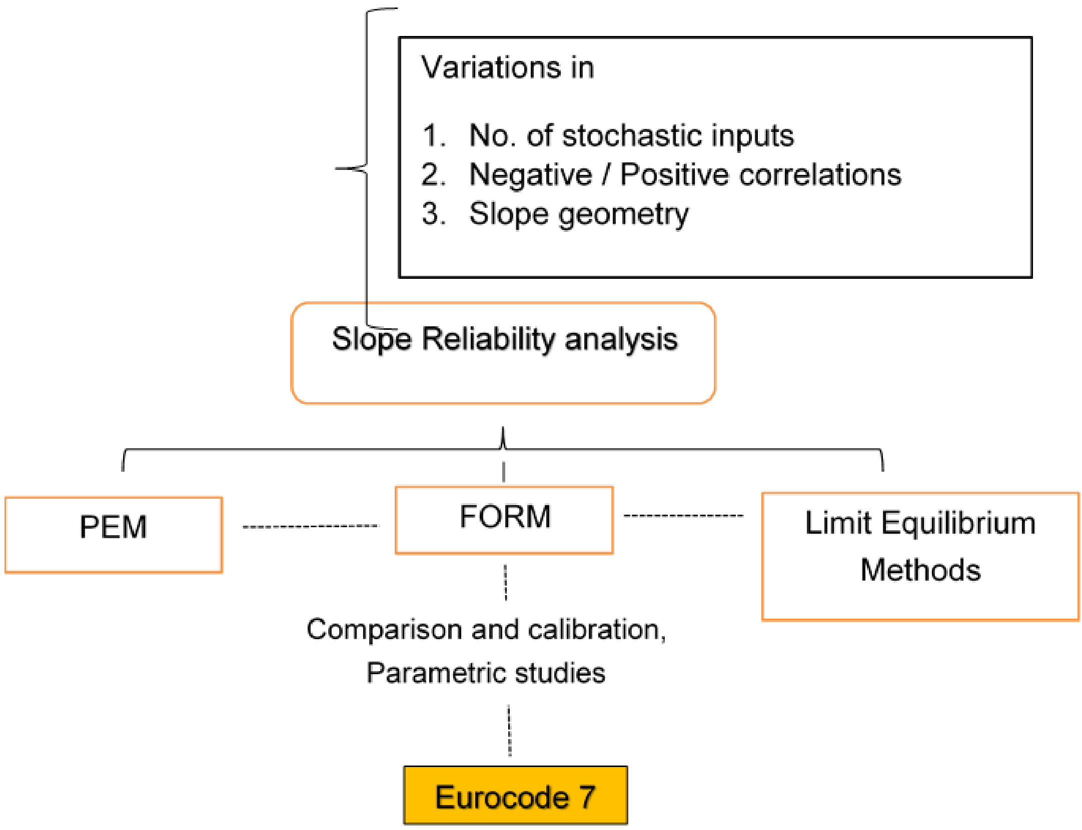

Figure 4.2: Flowchart showing different analysis and variations of the problem

the reliability analysis, and the variations that were performed on the slope to obtain a clearer perspective of RBD. The main reliability methods used here are PEM and FORM. These are compared with each other and with a limit equilibrium method. PEM was performed using the parameter variation feature in Plaxis. FORM is performed using PROBANA, a probabilistic tool that is coupled with Plaxis. Extensive comparison and calibration of FORM with Eurocode 7 is done, and some parametric studies are carried out. Different variations on the slope is done by varying the number of stochastic parameters, and slope geometry. The influence of positive and negative correlation on reliability has also been studied. For each variation, a comparison between all the reliability methods and Eurocode 7 is done.

Problem Geometry and Soil Parameters

The geometry of the hypothetical slope considered in this benchmark is shown in Figure 4.5. The slope considered here is homogeneous and has a height of 10 m and slope inclination of 1:2. The slope is fully drained as the ground water level is assumed to be located at a great depth. The soil properties of the slope are given in Table 4.1. The soil behaviour is modelled using both Mohr Coulomb model and Hardening model in PLAXIS 2D. (Kamp, 2016) provides convincing results using

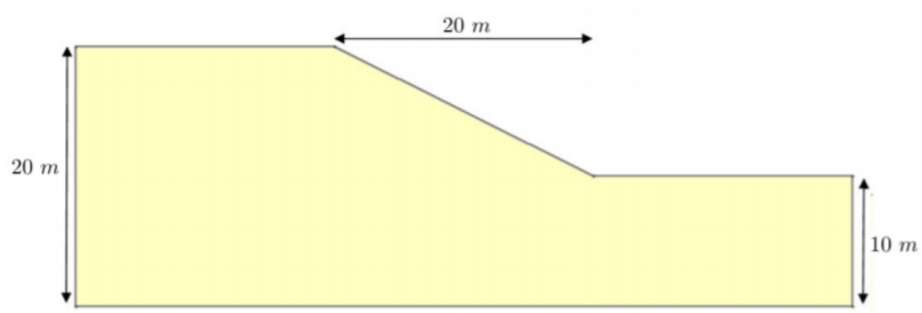

Figure 4.3: Slope Problem Geometry

Table 4.1: Mean Soil Properties (Normal Distribution)

Soil Type: Sand				
Property	**Symbol**	**Mean μ_x**	**Units**	**COVx**
Cohesion	c'	5	$[kN/m^2]$	0.20
Friction angle	ϕ'	35	[deg]	0.10
Unit weight	γ	21	$[kN/m^3]$	-

Point Estimate Method for the slope considered here. Hence the same slope is investigated again using an advanced model (Hardening Soil) in Plaxis coupled with Point Estimate Method.

Mohr – Coulomb Model Parameters

The factor of safety of this slope is evaluated, considering its strength properties i.e. cohesion c' and friction angle φ'. The factor of safety is determined using strength reduction in Plaxis. For the preliminary analysis, the soil behaviour is simulated using a Linearly Elastic Perfectly Plastic model (Mohr Coulomb). The basic Mohr Coulomb model parameters of the soil slope are given in Table 4.2. With these parameters, a deterministic factor of safety of 1.824 is obtained using strength reduction in Plaxis.

Table 4.2: Mohr Coulomb Basic Parameters

Parameter	Name	Magnitude	Unit
Young's modulus	E	20,000	[kN/m^2]
Poisson's ratio	ν	0.3	[-]
Cohesion	c'_{ref}	5	[kN/m^2]
Friction angle	ϕ'	35	[deg]
Unit weight	$\gamma_{unsat}/\gamma_{sat}$	21/24	[kN/m^2]
Dilatancy angle	ψ	0	[deg]

Hardening Soil Model Parameters:

The slope is analysed using Hardening model to simulate more realistic soil behaviour. The model parameters for the soil slope are shown in Table 4.3. The hardening model parameters are chosen such that they are equivalent to the Mohr Coulomb parameters previously used. E_{50}^{ref} for the Hardening model is considered as the basic stiffness modulus used in the Mohr – Coulomb model. Default value is used for E_{ur}^{ref}(In plaxis by default, $E_{ur}^{ref} = 3E_{50}^{ref}$). For sand, usually E_{oed}^{ref} is considered almost same as E_{50}^{ref}. These parameters gave a deterministic factor of safety of 1.81 using $\varphi - c$ reduction in Plaxis.

Table 4.3: Hardening Model Parameters for the soil slope

Parameter	Magnitude	Unit
Unit weight, $\gamma_{unsat}/\gamma_{sat}$	21/24	$[kN/m^3]$
Triaxial Stiffness, E_{50}^{ref}	20000	$[kN/m^2]$
Oedometer Stiffness, E_{oed}^{ref}	20000	$[kN/m^2]$
Unloading stiffness, E_{ur}^{ref}	60000	$[kN/m^2]$
Reference stress, p_{ref}	100	$[kN/m^2]$
Power, m	0.5	[-]
Poisson's ratio, ν_{ur}	0.2	[-]
Cohesion, c	5	$[kN/m^2]$
Friction angle, ϕ	35	[deg]
Dilatancy angle, ψ	0	[deg]
NC stress ratio, K_0^{nc}	0.4624	[-]
Failure Ratio, R_f	0.9	[deg]
Tensile Strength, σ_t	0.0	$[kN/m^2]$
Initial Stress ratio, K_0	0.4624	[-]

4.4.1. Semi Probabilistic Reliability Analysis using PEM

A semi probabilistic analysis is performed using Point Estimate method. PEM results in 2^n combinations. This is performed using the Parameter variation feature in Plaxis supplemented by a Python Script for automatic Postprocessing of results. This has been already done in (Kamp, 2016) for the same benchmark, using Mohr – Coulomb soil Model. Here, hardening model is used to model more realistic soil behaviour.

Firstly, a performance function is defined for the criterion considered. In this case, the criterion being Factor of safety, the performance function is defined as

$$Z = \mu_{msf} - 1 \tag{4.8}$$

μ_{msf} is the mean of the output distribution (factor of safety) and 1 is the threshold value of the safety factor as $fos < 1$ corresponds to slope failure.
PEM performs 2^n evaluations, where n corresponds to the number of stochastic variables. The evaluation points for these combinations are obtained from the input distribution of the stochastic variables, located one standard deviation from the mean on either side of the distribution. Deterministic analysis is performed for these combinations which can be automatically done using Parameter variation feature in Plaxis. Each deterministic analysis results in a factor of safety. The mean factor of safety, μ_{msf} and standard deviation σ_{msf} of the combination is calculated.

Table 4.4: PEM combinations with corresponding Factors of safety

Combination	**Cohesion**	**Friction angle**	**Factor of Safety**	
			MC	**HS**
Deterministic	5	35	1.824	1.81
1	4	31.5	1.574	1.562
2	6	31.5	1.956	1.680
3	4	38.5	1.682	1.942
4	6	38.5	2.087	2.077
		μ_{msf}	*1.825*	*1.815*
		σ_{msf}	*0.206*	*0.204*
		β_{normal}	*4.009*	*3.99*

The reliability index is evaluated as the distance between mean and threshold in units of standard deviation. This can be mathematically expressed as:

$$\beta_{normal} = \frac{(\mu_{msf} - threshold)}{\sigma_{msf}} \tag{4.9}$$

The threshold factor of safety considered here is 1. This assumes that the factor of safety follows a normal distribution. For a log-normal distribution, the reliability index is:

$$\beta_{lognormal} = \frac{ln \frac{\mu_{msf}}{\sqrt{(1+COV_{msf}^2)}}}{\sqrt{(ln\,(1+COV_{msf}^2))}} \tag{4.10}$$

With respect to the safety criterion, there isn't much difference in the results as the failure criterion in both models is the same. This was performed as a check to verify the results of Mohr Coulomb with a more advanced model.

4.4.2. Fully Probabilistic Reliability Analysis using Plaxis - PROBANA

The probabilistic tool is based on FORM – First Order Reliability Method, which uses an iterative constrained optimisation method to determine the design point and the Reliability Index. The concepts of FORM and the working of PROBANA are discussed in Chapter 2. Table 4.5 shows the statistical distribution of the stochastic parameters. The mean values are the deterministic values of the input parameters. Standard deviation is mean multiplied by the COV of the parameter, where COV values are from NEN9997-1. The limit state function with respect to factor of safety is defined like PEM.

Table 4.5: Statistical parameter of the input variables

Parameter	Distribution	Mean, μ_x	COVx	Standard Deviation, σ_x
c'	Normal	5	0.2	1.0
φ'	Normal	35	0.1	3.5

$$Z = \mu_{msf} - 1 \tag{4.11}$$

This limit state function indicates that the starting point in FORM analysis is the mean values and the threshold is 1. FORM starts from mean and iteratively converges to the threshold.

The results from the probabilistic analysis in PLAXIS using FORM are summarised in Table 4.6 and Table 4.7. The algorithm converges for 67 iterations. Table 4.6 shows the probability of failure and the corresponding reliability index obtained for

Table 4.6: FORM Failure probability and Reliability Index

Probability of failure	$3.26 * 10^{-6}$
Reliability Index	4.51

Table 4.7: FORM Design values

Parameters	Mean Values	Design Point	Importance Factors
Cohesion, c'	5.0	3.40	12.6%
Friction angle, φ'	35.0	20.24	87.4%

the slope. The design points (Table 4.7) give the combination of c' and φ' that results in failure. This point lies or almost lies on the failure surface (i.e. the limit state function). One of the attractive features of FORM is that it also provides an importance factor of each parameter i.e. it gives an estimate of the contribution each parameter has on the probability of failure.
The same was repeated using Hardening model to model the soil behaviour and quite expectedly, the results were the same.

4.4.3. Comparison – PEM, FORM and Monte Carlo

The reliability index from PEM, FORM and Monte Carlo analysis (limit equilibrium method) are compared in Table 4.8. Significant difference in reliability values from the different methods could be explained by the different underlying approach of each method.
In all the three methods, Reliability Index is calculated as the distance between the mean Factor of safety and the threshold in units of standard deviations. The difference lies in the approach used by each method. PEM and Monte Carlo performs iterations with different combinations of the inputs, and assumes or fits the outcomes of the iterations with a statistical distribution, based on which the Reliability index is computed. FORM works in a physical space of input stochastic variables, where it iteratively searches for the design point. There also exists other differences between PEM and FORM namely PEM has to make assumptions regarding the output distribution, whereas FORM neither makes assumptions nor reveals the nature of the output distribution. These differences explain the difference in results. In theory, Monte Carlo gives the best estimate of the reliability index, and FORM is supposed to give a good approximation of the Reliability Index compared to PEM. But the efficiency and accuracy of FORM also depends on the optimization algorithm.
Considering that Monte Carlo values are the most accurate, PEM β values and FORM β values are compared to Monte Carlo β values. Table 4.8 shows that PEM β_{normal} values are closer to Monte Carlo whereas FORM β values are considerably different from Monte Carlo. The similarity in PEM and Monte Carlo results can be explained by the similar underlying approach of both methods. PEM can be perceived as a subset of Monte Carlo Analysis. The β value of FORM is an approximate average of β_{normal} and $\beta_{lognormal}$ values of PEM. It is to be investigated if this trend is the same for other benchmarks.

4.4.4. Comparison – FORM and Eurocode7

The design points from FORM are compared with the design values from Eurocode 7. To compare FORM and Eurocode 7 design values, both should correspond to the same reliability level. FORM outputs a reliability index based on the design point that it converges to, whereas Eurocode 7 computes design values for a correspond-

Table 4.8: Reliability Index: PEM, FORM and Monte Carlo

Method	β_{msf}	
FORM	4.51	
MC	4.181	
	β_{normal}	$\beta_{lognormal}$
PEM	4.009	5.29

ing target reliability index.
In this section, it is investigated how to calibrate FORM analysis according to Eurocode 7 design standards, so that Reliability based design can be incorporated in Eurocode 7. NEN9997-1 recommends target reliability index for two reference periods (1 and 50 years), and associates partial factors to the Reliability index. Table 4.9 shows partial factors from NEN9997-1 Annex A for slope stability problems. These target reliabilities are intended to be primarily used in design of new structures. For a structure of Reliability Class 2, the reliability index of 3.8 should be used provided that probabilistic models of basic variables are related to the reference period of 50 years.

Table 4.9: Partial factors for different Reliability classes (Slope stability set M2)

	Reliability Class		
Soil parameter	**RC1(β = 3.3)**	**RC2 (β = 3.8)**	**RC2 (β = 4.3)**
Friction angle, $\gamma_{\varphi'}$	1.2	1.25	1.3
Cohesion, $\gamma_{c'}$	1.3	1.45	1.6
Undrained shear strength, γ_{cu}	1.75	1.75	2.0
Volume unit weight, $\gamma_{\gamma'}$	1.0	1.0	1.0

In this investigation, the limit state function in FORM probabilistic analysis is defined based on the design values according to Eurocode7 as shown in Equation 4.12.

$$Z = FOS_{mean} - FOS_{(EC7design)} = FOS(c'_m, \varphi'_m) - FOS(c'_d, \varphi'_d) \tag{4.12}$$

The design values of the soil parameters are computed by factoring the characteristic values with the partial factors specified in Eurocode (Table 4.9). A deterministic calculation is done in Plaxis with the design values of the parameters, c_d and φ_d, and the design safety factor is obtained (This safety factor will be referred here as the design safety factor). The deterministic factor of safety with the design values is 1.106 (Table 4.10). This value is defined as the threshold in the FORM limit state function. Hence the limit state function is.

$$Z = FOS_{mean} - 1.106 \tag{4.13}$$

Table 4.10: Deterministic analysis with mean and design values

	Cohesion, c'	**Friction angle, Φ'**	**Deterministic FoS**
Mean values, X_m	5	35	1.824
Design values, X_d	2.32	24.14	1.106

By Eurocode's definition, with the above limit state function where the design safety factor is the threshold value, a reliability index of 3.8 (EC7 target β) is targeted. However, the actual reliability index is often different from EC7's target reliability index. The actual reliability index is determined from FORM. It is investigated how the EC7 target reliability index, compares with that of the actual reliability index of the slope. This concept is depicted by the flowchart shown in Figure 4.1.

Table 4.11: Comparison of FORM design points and EC7 design values

Parameters	**Mean**	**FORM design point**	**EC7 Design Value**
c'	5	3.68	2.32
Φ'	35	22.29	24.14
Reliability Index, β		3.85	3.80

Table 4.11 shows the FORM design point and resulting reliability index with the limit state function defined as in Equation 4.13. The reliability index from FORM is 3.85, which is very close to EC7's target reliability index of 3.8. To understand if this similarity in the β values is a coincidence, the same was repeated for a slope with a different slope angle and soil properties (this is discussed in the following section). It should be noted that although β values match, the design points that FORM back calculates for the threshold corresponding to EC7 design values is different. FORM design values for Φ' is lower than EC7 design value. This raises the question of whether EC7 gives an optimistic estimate. The reason for this is the differences in importance factor that FORM and EC7 assigns to these parameters. Both FORM design points (3.68, 22.29) and EC7 design values (2.32, 24.41) lie on the same limit state function in the parameter space, but despite this, FORM chooses a different combination of design points than that of EC7. To interpret the reasons for this difference, the underlying reliability theory used to compute the partial factors in Eurocode 7 is understood. The Eurocode for basis of structural design, EN 1990:2002 + A1 (EC0) describes the basis for partial factor design and reliability analysis. According to EC0, partial factor can be determined in one of the following ways:

1. Based on calibration to a long experience of building tradition
2. Based on statistical evaluation of experimental data and field observations, carried out within the framework of probabilistic reliability theory

EN 1990 mentions that while using probabilistic reliability methods, the ULS partial factors should be calibrated such that the reliability levels are as close as possible to the reliability index. The design values are calculated by using the expressions provided (Table C3, EN 1990:2002). The partial factor is obtained by dividing the design value by the characteristic value. Based on this, Vrouwenvelder et al. (2012) presented the following formula for γ_m using theory of reliability. This formula assumes that the variable parameters are normally distributed.

$$\gamma_m = \frac{(1 - 1.64 \cdot COV)}{(1 - \alpha \cdot \beta.COV)} \tag{4.14}$$

(The numerator is the characteristic value when multiplied with the mean, and the denominator is the design value when multiplied with the mean) α is the sensitivity coefficient. This coefficient in principle should follow from a probabilistic FORM calculation. The partial factor should hold good for all design situations which has different sensitivity factors. Vrouwenvelder et al. (2012) conceives the value of α as an average of many cases. β, the target reliability index, is related to the reliability level required. Coefficient of variation, COV considers the uncertainty of the parameter based on observations in the laboratory or field. EC7 uses an average value of sensitivity coefficient α, and a target reliability, β to compute the partial factors, whereas FORM gives the actual sensitivity coefficient for each parameter, and the actual EC7 ULS Reliability index. If Reliability analysis were to be used, the partial factors could be improved and more accurately calculated. This brings us to the following question.

How to incorporate Probabilistic Reliability analysis in EC7?

These approaches describe how to calibrate partial factors from Eurocode 7 according to Reliability based design.

Approach 1:

1. Obtain EC7 design values of all parameters, and determine the deterministic ULS threshold value and set up the limit state equation.

2. Perform FORM analysis with the mean value points as the starting point and the EC7 ULS as the threshold, and determine the corresponding reliability index, β and FORM sensitivity factors, α_i.

3. Calculate design values of all the parameters with FORM β and FORM α_i values, $X_d = \mu(1 - \alpha \cdot \beta \cdot COV)$.

4. Calculate partial factor, $\gamma_m = \frac{(1-1.64 \cdot COV)}{(1-\alpha \cdot \beta.COV)}$

Table 4.12 shows the partial factors for the above benchmark using this approach.

Table 4.12: FORM sensitivity coefficient and calculated partial factors

	Cohesion, c'	Friction angle, φ'
Sensitivity coefficient, α_i	11.6%	88.4%
Design Point, X_d	4.594	23.088
Partial Factor, γ_i	0.73	1.26

Approach 2:

1. Obtain EC7 design values of all parameters, and determine the deterministic ULS threshold value and set up the limit state equation.

2. Perform FORM analysis with the mean value points as the starting point and the EC7 ULS as the threshold, and determine the corresponding reliability index, β and FORM design points. Since these design points correspond to the EC7 Ultimate limit state function, these design points from FORM analysis can be directly used instead of EC7 partial factors.

Table 4.13 shows a comparison between the design values using the different approaches and Eurocode7. FORM approach 1 is a subtle way of incorporating reliability analysis in Eurocode7. This is proposed to make a smooth transition from the traditional partial factor approach used in Eurocode7. In terms of implementation in engineering practice, FORM approach 2 is still a farfetched goal, as more research is necessary to back the reliability of FORM results, for complex design situations.

Table 4.13: Design values from approach 1 and 2 comparison

	Cohesion, c_d	Friction angle, φ_d
FORM Approach 1	4.594	23.088
FORM Approach 2	3.68	22.29
Eurocode 7	2.32	24.14

Limitations of the approaches: The reliability method used in these approaches, FORM works by assigning a sensitivity coefficient, α to each stochastic parameter. However, this is a relative sensitivity coefficient, which implies that, α values of all stochastic parameters from FORM adds up to a total of 1. When parameters have very low sensitivity, the partial factor from approach 1 is less than 1, or in approach 2, the FORM design values are higher than the mean values. This is unacceptable as Eurocode 7 aims to determine a conservative estimate for the parameters. Therefore, for parameters that have very low sensitivity coefficients in FORM, this method should not be used to compute design values or partial factors. Rather, partial factors from Eurocode 7 should be used.

4.4.5. Slope – Different geometry and Soil Properties

Here, a slope with a different geometry and mean soil properties is analysed to verify if the similarity in β values between FORM and EC7 (Table 4.14) holds true for a slope with slightly different geometry and soil properties. The slope has a height of 10 m and an inclination of 1:1.62 as shown in Figure 4.4. The soil properties are given in Table 4.14. The slope has a deterministic factor of safety of 1.93 with mean parameter values.

Figure 4.4: Slope stability problem with different slope geometry

Table 4.14: Soil Properties

Soil Type: Sand				
Property	**Symbol**	**Mean μ_x**	**Units**	**COVx**
Cohesion	c’	8	$[kN/m^2]$	0.20
Friction angle	ϕ’	40	[deg]	0.10
Unit weight	γ	21	$[kN/m^3]$	-

The reliability of the slope is analysed using FORM with the limit state function defined according to EC7 ULS, and the reliability index from FORM is compared with EC7’s β value of 3.8. The limit state function is defined as follows, according to EC7 Ultimate Limit states.

$$Z = \mu_{msf} - 1.134 \tag{4.15}$$

Table 4.15: FORM design point and EC7 design values

Parameters	**Mean**	**FORM design point**	**EC7 Design Value**
c’	8	5.52	3.71
Φ’	40	25.06	27.85
Reliability Index, β		4.04 ($p_f = 2.65 * 10^{-5}$)	3.80

Table 4.15 compares FORM design points with EC7 design values. FORM reliability

index is higher than EC7's β of 3.8. This implies that EC7 design values do not always exactly correspond to the β value it targets. Probabilistic reliability analysis not only considers uncertainties in a rational manner, but also reveals the actual reliability level of the structure.

4.4.6. Negatively correlated cohesion and friction angle

In this section, the influence of including correlations between soil properties on reliability results are investigated. Comparisons are made between different Reliability methods, and with Eurocode7.

Often, a low cohesion corresponds to high friction angle and vice versa. In other words, cohesion, c and friction angle, φ are negatively correlated. A correlation coefficient of -0.5 is assumed between c' and φ' as shown in Equation 4.16.

$$\rho_{(c',\varphi')} = -0.5 \tag{4.16}$$

Comparison – PEM, FORM and Monte Carlo

This investigation makes two comparisons for the slope in Figure 4.5 with soil properties mentioned in Table 3.1. The results are shown in Table 4.16.

1. *Comparison in β values between PEM, FORM and MC for correlated c' and φ'.* The reliability index, β from FORM is approximately an average of PEM β_{normal} and PEM $\beta_{lognormal}$. MC β values and PEM β_{normal} are similar, for reasons explained before. This trend in β values is same as observed in the previous investigations.

2. *Comparison of β values between correlated c' and φ' and uncorrelated c' and φ'.* On comparing β values between correlated parameters and uncorrelated parameters, correlated parameters result in higher β values than uncorrelated parameters. The reason for this is explained here. A hypothetical deterministic safety analysis with correlated soil properties would result in a higher safety factor than with uncorrelated properties. This is because when parameters are negatively correlated, they work in coordination with each other to achieve safety. So, as one parameter is factored up, the other parameter is factored down. Conversely, when parameters are independent, each should independently make sure that the slope is safe. Hence negatively correlated parameters leads to a significant rise in higher reliability.

Comparison – FORM and EUROCODE7

As discussed earlier, to compare the design values from FORM and EC7, both should correspond to the same reliability level. This is ensured by choosing a threshold value in FORM equivalent to EC7 ULS.

Table 4.16: Reliability Indices for correlated input parameters

	PEM		MC	FORM
	β_{normal}	$\beta_{lognormal}$		
Correlated, $\rho_{c,\varphi} = -0.5$	4.702	6.215	4.88	5.41
Uncorrelated, $\rho_{c,\varphi} = 0$	4.009	5.29	4.18	4.51

Hence, the limit state function is defined as Equation 4.17, where 1.106 is the deterministic factor of safety with the design values of c' and φ'.

$$Z = \mu_{msf} - 1.106 \quad (4.17)$$

FORM is performed for the slope in Figure 4.5, assuming a correlation coefficient, $\rho_{c,\varphi}$ of -0.5, for the limit state function defined in Equation 4.17.

Table 4.17: FORM results - design point and sensitivity coefficients for negatively correlated c' and phi'

Parameters	FORM design point	FORM sensitivity coefficient, α_i
c'	5.80	3.3%
Φ'	19.71	96.7%

Table 4.18: FORM design point and Eurocode design values for correlated input parameters

Parameters	Mean	FORM design point	EC7 Design Value
c'	5	5.80	2.32
Φ'	35	19.71	24.14
Reliability Index, β		4.65	3.80

Table 4.17 shows the FORM results - the design points and the sensitivity coefficients of c' and φ'. The sensitivity coefficient of c' is much lower while the sensitivity coefficient of φ' is much higher when compared to the case where correlation was not considered. Table 4.18 compares FORM design points and reliability index with EC7 design values and target reliability index. For c', the design point is much higher than the mean value point whereas for φ', the design point is much lesser than the mean value point. These discrepancies are explained by the simple fact that, because FORM considers the factor of safety to be more sensitive to φ' than to c', negative correlation factors down φ' while factoring up c'.

Negative correlation has no influence on EC7 design values. EC7 does not consider correlations, and it is difficult to judge how to factor up c while factoring down φ' by intuition. It is also to be noted that reliability index varies considerably with inclusion of correlations. Not accounting for negative correlations between parameters may lead to underestimation of reliability values.

4.4.7. Positively correlated cohesion and friction angle (Hypothetical case)

To investigate the effect of positive correlation on reliability index, c' and φ' are assumed to have a positive correlation. This is unrealistic and is performed only to understand how a positive correlation affects reliability results. This investigation is done using FORM, and the reliability level and design values are compared with that of EC7.

$$\rho_{(c',\varphi')} = 0.5 \tag{4.18}$$

Table 4.19: FORM results - design values and sensitivity coefficients for positively correlated input parameters

Parameters	FORM design point	FORM sensitivity coefficient, α_i
Cohesion, c'	2.40	40.7%
Friction angle, ϕ'	24.04	59.3%

Table 4.19 shows FORM results – the design values and the sensitivity coefficients of each parameter. The sensitivity coefficient of c' is higher compared to the case where c' and φ' were uncorrelated. This is because positive correlation between two parameters leads to factoring up both parameters simultaneously.

Table 4.20: FORM and EC7 design values

Parameters	Mean	FORM design point	EC7 Design Value
c'	5	2.40	2.32
Φ'	35	24.04	24.14
Reliability Index, β		3.35	3.80*

Table 4.20 compares FORM design points with EC7 design values. FORM design points are almost the same as the design values from EC7, but FORM reliability index is lower than EC7's target β value of 3.8. The partial factors adopted in EC7 appears to be equivalent to assuming a positive correlation between c' and φ', which is unrealistic. This could also lead to a reliability index lower than the EC7's target reliability index, like in this case. This explains that ignoring correlations between variables may also lead to unconservative designs.

As FORM β values showed significant difference compared to PEM and Monte Carlo, the slope stability problem is investigated further with only one stochastic parameter in the next section. The parameter with least sensitivity is ignored.

4.4.8. Single stochastic Parameter Model

The uncertainty in cohesion was ignored as the importance factor of c' was very low (12.6%). Keeping the distribution of the friction angle same (COV of 0.1), probabilistic analysis was performed using FORM in Plaxis. The results of the analysis are discussed in this section. The results from FORM are compared with PEM and Monte Carlo (performed in Slide 7.0).

Table 4.21: Comparison - FORM, PEM and Monte Carlo

	β_{msf}			
FORM	4.786 ($p_f = 8.49 * 10^{-7}$)			
PEM	4.162			
	Fellenius	*Bishop*	*Janbu*	*Spencer*
Monte Carlo	4.14	4.32	4.10	4.32

FORM reliability indices are significantly different from PEM and Monte Carlo. The possible explanation for this is the different approach that FORM uses, compared to PEM and Monte Carlo. Though PEM is based on finite element method and Monte Carlo is based on limit equilibrium methods, they yield comparable results.
PEM and Monte Carlo are more similar than different. Though they are based on completely different methods, they have similar approaches in determining the reliability index. PEM can be visualized as a subset of Monte Carlo. This could explain the similarity in the results between PEM and MC. To compare FORM and Eurocode7, the limit state function is defined according to EC7 ULS. The deterministic factor of safety with the design values of soil parameters is 1.2698. Hence the limit state equation is defined as Equation 4.19.

$$Z = \mu_{msf} - 1.2698 \tag{4.19}$$

Table 4.22: Comparison - FORM and EC7 design values

Parameters	**Mean**	**FORM design point**	**EC7 Design Value**
Φ'	35	24.19	24.14
Reliability Index, β		3.08	3.80

Table 4.22 shows that the design point from FORM and the EC7 are the same. This is because when only one parameter is stochastic, there is only one value that lies on the limit state function. When there are more than one stochastic parameters, there is scope for more than one combination of parameters to lie on the same limit state function.
The reliability index from FORM is 3.08 compared to EC7's target β of 3.8. According to EC0, when only one parameter is stochastic, the sensitivity coefficient of the parameter is 1. When only one parameter is stochastic, it has a sensitivity coefficient of 1, be it FORM or EC7. Hence both FORM and EC7 design value is the same. This

explains that the difference in partial factors / design values between FORM and EC7 are mainly due to the different sensitivity coefficients that EC7 assumes and FORM computes.
With a β value of 3.08, the design value is calculated with Equation 4.5 as follows:

$$\mu - \beta \cdot \alpha \cdot \sigma = 35 - 3.08 * 1 * 3.5 = 24.22$$

With a β value of 3.8, the design value is calculated Equation 4.5 as follows:

$$\mu - \beta \cdot \alpha \cdot \sigma = 35 - 3.8 * 1 * 3.5 = 21.70$$

This shows that EC7 design value of 24.14 does not achieve the target reliability index of 3.8. A design value much lower, i.e. 21.7 is required to satisfy Eurocode's target reliability index of 3.8.

4.4.9. Parametric Study

In this section, it is studied how the statistical distribution of a stochastic parameter influences the design point values and Reliability Index. This is done for different cases where

1. Only φ' is stochastic
2. c' and φ' are stochastic

Case 1: Only φ' is stochastic

The limit state equation used for this study is based on EC7 ULS, as FORM results are compared with EC7 for different statistical distributions.

$$Z = \mu_{msf} - 1.2698 \tag{4.20}$$

Table 4.23: FORM and EC7 design values for different phi' distribution; COVc' = 0

Φ' Distribution		**Eurocode 7**		**FORM**	
Mean	COV	φ_k	φ_d	φ_d	β
32	0.052	29.26	24.143	24.19	4.69
34	0.085	29.26	24.143	24.19	3.39
35	0.100	29.26	24.142	24.19	3.08
37	0.128	29.26	24.143	24.19	2.70
39	0.152	29.26	24.143	24.15	2.50

Table 4.23 shows the statistical distribution of friction angle, the design values from Eurocode 7 and the results from FORM analysis. Mean and coefficient of variation

of the distribution is chosen such that each distribution results in the same characteristic value. The characteristic value and design value are calculated according to Eurocode 7. Characteristic value is the 5 % fractile and design value is calculated by factoring the characteristic value with the partial factor.
FORM analysis gives the reliability index and design point for each case. The reliability index reduces with higher mean and coefficient of variation. This is because greater coefficient of variations leads to higher variability or greater spread in the values, giving a lower reliability index. The FORM design point however is the same for all distributions. This is because irrespective of the distribution, FORM always finds that value of the friction angle that corresponds to the threshold safety factor, since all other values are fixed and φ' is the only stochastic variable. For a problem with a single stochastic parameter, the design point is the same irrespective of the statistical distribution of the parameter, while the Reliability index and probability of failure depends on the coefficient of variation of the distribution.
It is also observed that although the design points are the same irrespective of the distribution of the input parameter, FORM gives significantly different β values. Table 4.23 shows that, for COV values higher than 0.08, EC7 target reliabilities are not satisfied. EC7 assumes a certain degree of uncertainty and assigns a single value to account for uncertainties, which have possibilities of leading to unconservative designs.

Case 2: c' and φ' stochastic

Considering both c' and φ' as stochastic, Reliability index and design values are determined for a range of COVφ' having the same EC7 characteristic value with constant COVc' of 0.2.
The limit state function is defined based on EC7 ULS to ensure that FORM corresponds to the same reliability level assumed in EC7.

$$Z = \mu_{msf} - 1.106 \tag{4.21}$$

Table 4.24: FORM analysis for φ' distributions having same characteristic value. COV c' is 0.2

Φ' Distribution		Eurocode 7 Design Values		FORM Design Values / Reliability In.		
Mean	COV	φ_d	c_d	φ_d	c_d	β
34	0.085	24.14	2.32	22.31	3.67	4.26
35	0.100	24.14	2.32	22.29	3.68	3.85
37	0.128	24.14	2.32	21.80	4.05	3.35
39	0.152	24.14	2.32	21.44	4.31	3.04

Table 4.24 shows the distributions of φ', and design values from FORM and Eurocode7. The distributions (Mean and COV's) of φ' are chosen such that they result in the same characteristic value. This is done in order to analyze how FORM reliability and design points vary for distributions with the same characteristic value. Different distributions imply different levels of parametric uncertainty. Hence higher COVφ, quite expectedly gives lower β values.
For different mean values, and standard deviations, FORM gives a unique combination of design values whereas EC7 design values are the same irrespective of the statistical distribution. Although FORM gives a unique combination of design values, they are not significantly different for each distribution, despite having different β values. This could be due to several reasons, some of which are listed below:

1. Here the number of stochastic variables is as less as 2. Thus, there is not much scope for FORM to have significantly different combinations of design points. With greater number of stochastic variables, the differences are expected to be greater.

2. The typical COV range for friction angle is 0.05 – 0.15. This range is large enough to have different β values, but not large enough to have different combinations of design points. This can be conceived as a broader case of Case 1 where FORM always picks the same friction angle.

This also reveals an apparent disadvantage of the partial factor approach adopted in EC7. EC7 targets a reliability index of 3.8, but this is not achieved when the COV of friction angle is higher than 0.1. This is understandable because NEN9997-1 assumes a COV value of 0.1 for friction angle. It is also important to note that this is probably not the case for a different slope. Applying the same partial safety factor for different parametric uncertainty does not lead to a fixed Reliability index or probability of failure, as assumed in Eurocode7.

4.4.10. Conclusions

In this chapter, stability of a simple slope is analyzed using Finite element reliability analysis. PEM, FORM and Monte Carlo methods are used to perform reliability analysis. These methods are compared with each other and with Eurocode7. Based on the results, the merits and demerits of the methods are studied, and two approaches are proposed and to incorporate reliability based design in EC7. This is also demonstrated using this benchmark. The conclusions of reliability based design and it's compliance with Eurocode 7 based on this benchmark are summarized below:

1. Comparison between PEM and FORM was one of the main objectives of this investigation. A trend in reliability index values is observed wherein the reliability index from FORM is an approximate average of PEM β_{normal} and $\beta_{log\ normal}$

values. On comparing PEM and FORM results against Monte Carlo, PEM and Monte Carlo results show minor differences whereas considerable differences are observed between FORM and Monte Carlo β values.

2. Having to assume an output distribution in PEM hampers the accuracy of the result. FORM neither makes an assumption nor reveals the nature of the output distribution. PEM is far more straightforward compared to FORM which is mathematically complex. FORM associates each parameter in the parametric space with an importance factor and determines the design point based on this. Whereas in PEM, it is first required to identify the sensitivity value of the parameters beforehand. FORM is a more superior method to PEM owing to lesser assumptions.

3. In PEM, the number of iterations required increases exponentially with the number of stochastic parameters. FORM also requires more iterations as the number of stochastic parameters increases but it is possible to optimize this speed this by defining the right calculation features in FORM algorithm.

4. Comparison is made between FORM and Eurocode7. FORM and EC7 fundamentally differ in the sensitivity coefficients it assigns to the different parameters. FORM works with relative sensitivity factors, whereas EC7 partial factors are independent. A disadvantage with FORM's relative sensitivity coefficients sometimes result in higher design values, which is unacceptable according to Eurocode design standards.

5. Using the proposed methodology, actual reliability level of a structure for EC7 Ultimate limit states can be determined, and compared to the target reliability specified in Eurocode 7.

4.5. Benchmark 2: Shallow Foundation

In this benchmark, the reliability index of the structure is assessed based on the bearing capacity of a footing. The bearing capacity of a shallow foundation is analyzed using Plaxis. A footing of width 2 m and surcharge load of $10kN/m^2$ is modelled as shown in Figure 4.5. The footing is assumed to be rigid. The ground water is assumed to be located at great depth and hence it's influence is not considered in this benchmark. A prescribed displacement of 0.4 m is applied to simulate settlements. The Mohr Coulomb and Hardening Model Parameters used are given in Table 4.25 and Table 4.26 with which a deterministic bearing capacity of 359.35 kN/m^2 and 356 kN/m^2 respectively were obtained.

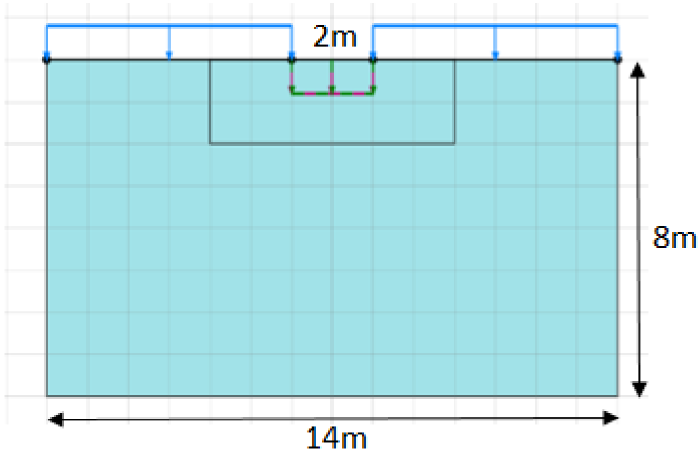

Figure 4.5: Geometry Shallow Foundation

Table 4.25: Mohr Coulomb Model Parameters

Parameter	Name	Magnitude	Unit
Young's modulus	E	20,000	$[kN/m^2]$
Poisson's ratio	ν	0.3	[-]
Cohesion	c_{ref}	5	$[kN/m^2]$
Friction angle	ϕ	25	[deg]
Dilatancy angle	ψ	0	[deg]

4.5.1. Semi Probabilistic analysis using PEM

The reliability index of the bearing capacity was obtained using Point Estimate Method. The limit state equation with respect to the bearing capacity criterion considered is

$$Z = p - p_d \tag{4.22}$$

p is the mean bearing capacity and p_d is the design bearing capacity obtained with the design parameters factored with partial factors specified in Eurocode 7(Table4.27).

Table 4.26: Hardening soil model parameters

Parameter	**Magnitude**	**Unit**
Unit weight $\gamma_{unsat}/\gamma_{sat}$	15/20	$[kN/m^3]$
Triaxial Stiffness, E_{50}^{ref}	20000	$[kN/m^2]$
Oedometer Stiffness, E_{oed}^{ref}	20000	$[kN/m^2]$
Unloading stiffness, E_{ur}^{ref}	60000	$[kN/m^2]$
Reference stress, p_ref	100	$[kN/m^2]$
Power, m	0.5	[-]
Poisson's ratio, ν_{ur}	0.2	[-]
Cohesion, c	5	$[kN/m^2]$
Friction angle, ϕ	25	[deg]
Dilatancy angle, ψ	0	[deg]
NC stress ratio, K_0^{nc}	0.5774	[-]
Failure Ratio, R_f	0.9	[-]
Tensile Strength, σ_t	0.0	$[kN/m^2]$
Initial Stress ratio, K_0	0.5774	[-]

The normal and log normal reliability indices are evaluated as shown below.

$$\beta_{normal} = \frac{\mu_Z}{\sigma_Z} \tag{4.23}$$

$$\beta_{lognormal} = \frac{ln\frac{\mu_{msf}}{\sqrt{(1+COV_{msf}^2)}}}{\sqrt{(ln\,(1+(COV)_{msf}^2))}} \tag{4.24}$$

A detailed description of steps involved in performing PEM for this benchmark is given in the previous research (Kamp, 2016). The bearing capacities obtained using MC model and Hardening Soil Model for all PEM combinations is shown in Table 4.28.

Table 4.27: Deterministic bearing capacity with mean and design values of the parameters

	c' (kPa)	γ (kN/m^3)	ϕ' (deg)	$Bearing\ Capacity$ (kN/m^2)	
				MC	HS
Mean Deterministic	5	15	25	359.35	356.60
Design Deterministic	2.10	12.54	18.35	135.70	131.80

Table 4.28: PEM combinations and results for Mohr Coulomb and Hardening soil model

	c' (kPa)	γ (kN/m^3)	ϕ' (deg)	$Bearing\ Capacity$ (kN/m^2)	
				MC	HS
1	4	13.5	22.5	254.30	251.40
2	4	13.5	27.5	433.40	402.95
3	6	13.5	22.5	292.90	285.70
4	6	13.5	27.5	461.35	428.75
5	4	16.5	22.5	273.90	270.90
6	4	16.5	27.5	458.35	428.35
7	6	16.5	22.5	309.70	306.35
8	6	16.5	27.5	502.00	463.10
			μ_p	373.24	354.69
			σ_p	93.35	78.88
			β_{normal}	2.54	2.82

4.5.2. Probabilistic analysis using FORM in Plaxis

The footing is analyzed using FORM in PROBANA. Table 4.29 shows the statistical distribution of the stochastic input parameter considered in the reliability analysis (NEN9997-1). The standard deviation is derived from the typical COV values of the soil properties.

Table 4.29: Statistical distribution of Input Parameters

Parameter	Distribution	Mean Value	Standard Deviation
c'	Normal	5	1
φ'	Normal	25	2.5
γ'	Normal	15	1.5

The constraint function is defined, for a threshold of the design bearing capacity. The function can be written as:

$$Z = p - p_{design} \tag{4.25}$$

p is the mean bearing capacity and p_d is the design bearing capacity. By definition, the bearing capacity is the average maximum vertical force between the soil and the footing. This assumes that the stress is constant along the footing. This is inconsistent with the bearing capacity obtained using Plaxis, where the maximum bearing capacity is at the mid-point of the footing. Thus, the limit state function is modified to:

$$Z = \sigma'_{yy} - \sigma'_{(yy,d)} \tag{4.26}$$

σ'_{yy} is the vertical stress just beneath mid-point of the plate (footing) with the mean parameters and $\sigma'_{(yy,d)}$ is the vertical stress beneath the mid-point of the plate with the design parameters (1 % fractile).
A first order reliability analysis of the footing is done with the stochastic parameters given in Table 4.29 and a threshold bearing capacity of p_{design} in PROBANA. Table 4.30. shows the probability of failure and reliability index from FORM.

Table 4.30: FORM Results

Probability of failure	0.988
Reliability Index	-3.706

Table 4.31: FORM design point and importance factors

Parameters	Mean Values	Design Point	Importance Factors
Cohesion, c'	5.0	3.83	0.098
Friction angle, φ'	35.0	16.21	0.899
Unit weight, γ'	15.0	14.70	0.0028

The probability of failure is almost 1 and the reliability index is negative. This is because the region that PROBANA considers as the probability of failure is the safe region which is complementary to the actual region. The actual probability of failure of the footing with respect to the bearing capacity is the area of the normal distribution curve for bearing capacity values below the threshold bearing capacity, p_{design}.

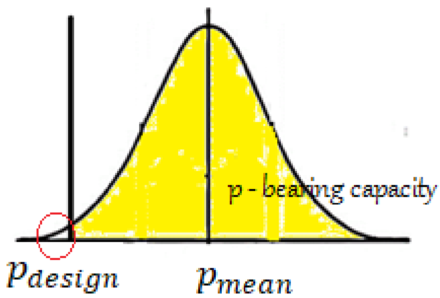

Figure 4.6: Output distribution

This is depicted in Figure 4.6, wherein the unshaded portion is the actual probability of failure but what is computed is the complementary area. This is a mathematical issue, and hence the absolute value of the reliability index can be considered as the actual value. The actual probability of failure is obtained by subtracting the probability of failure from 1. This issue can be avoided by considering a different formulation of the limit state function. The limit state function can be rewritten in terms of factor of safety as:

$$Z = \frac{p_d}{p_{mean}} - 1 \tag{4.27}$$

However, this is not yet possible with PROBANA's graphical user interface.

4.5.3. Comparison – RBD and Eurocode 7

Table 4.32 compares reliability indices from FORM, PEM and Monte Carlo. In this case, β values from FORM are comparable to $\beta_{lognormal}$ values from PEM and Monte Carlo. This augments the previously observed trend where FORM gives reliability index values which lies midway between PEM β values.

Table 4.32: PEM vs FORM Reliability Index

Method	β_{msf}	
FORM	3.706	
	β_{normal}	$\beta_{log\ normal}$
PEM	2.52	3.95
MC	2.14	3.48

To compare FORM and EC7 design values, both should correspond to the same reliability level. The limit state function considered for this problem already ensures this by considering the deterministic bearing capacity with the design parameter values as the threshold. Table 4.33 shows that the reliability index from FORM is very close to the expected beta value of 3.8. Similar to the previous benchmark, the design points from FORM are different from that of EC7. This is because of the differences in importance factors that FORM and EC7 assigns to the parameters.

Table 4.33: Comparing FORM design points and EC7 design values

Parameters	Mean	FORM design point	EC7 Design Value
c'	5	3.83	2.1
Φ'	25	16.21	18.35
γ'	15	14.70	12.54
Reliability Index, β		3.706	3.80

4.5.4. Conclusions

Some of the conclusions from this benchmark are summarized below.

- Based on the problems faced while performing reliability analysis of this shallow foundation problem, one of the main finding from this benchmark is that FORM requires careful formulation of the limit state function so that right area is calculated as the probability of failure.

- In this benchmark, the previously observed trend in reliability results from different reliability methods is augmented. The results from this benchmark also augment the fact that defining the deterministic value with EC7 design values as the threshold value in FORM is a good approach to determine the actual reliability of the problem, which may then be the EC7's target reliability index.

4.6. Benchmark 3: Cantilever Retaining Wall

A retaining wall of height 5 m considered in the previous research (Kamp, 2016) has been analyzed for its stability. Finite element reliability analysis of the retaining wall is performed using Plaxis coupled with FORM and PEM. The geometry of the retaining wall considered is shown in Figure 4.7. The problem has two soil layers, a foundation layer 10 m thick and a backfill layer that is 5 m. The backfill layer is divided in 5 layers, 1m each and each 1 m layer is added as a separate phase in Plaxis.

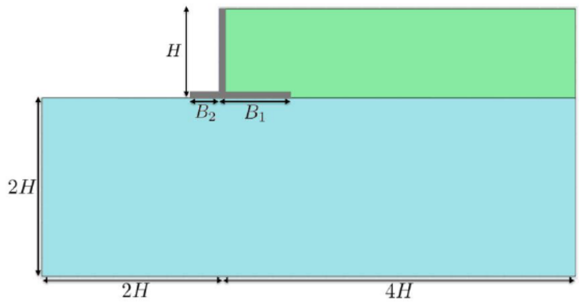

Figure 4.7: Geometry Retaining wall

Mean soil properties and the standard deviation used for the probabilistic analysis are shown in Table 4.34. The problem was previously modelled as a Mohr coulomb model. Here an advanced model – the hardening soil model is used for simulating more realistic soil behaviour. The parameters of Mohr Coulomb soil model and the hardening soil model are given in Table 4.35 and Table 4.36.

Table 4.34: Mean Soil Properties - Backfill and Foundation soil and COV

	Foundation Soil		**Backfill Soil**		**Units**	**COV**
Property	**Symbol**	**Magnitude**	**Symbol**	**Magnitude**		
Young's modulus	E_f	40,000	E_b	20,000	kPa	0.10
Unit weight	γ_f	22	γ_b	20	kN/m^3	0.05
Cohesion	c_f	5	c_b	5	kN/m^2	0.20
Friction angle	Φ_f	25	Φ_b	25	deg	0.10

4.6.1. Semi Probabilistic analysis using PEM

Reliability index is computed with respect to safety criterion using Point Estimate Method. A sensitivity analysis in Plaxis gives the influence of different input parameters based on a specified criterion. This has been done in previous research (Kamp, 2016) and the dominant properties are identified and these variables are considered stochastic in this analysis.

Table 4.35: Mohr Coulomb Model Parameters

	Foundation Soil		Backfill Soil		Units
Parameter	**Symbol**	**Magnitude**	**Symbol**	**Magnitude**	
Young's modulus	E_f	40,000	E_b	20,000	[kPa]
Poisson's ratio	ν_f	0.3	ν_b	0.3	[-]
Unit weight	γ_f	22	γ_b	20	
Cohesion	c_f	5	c_b	0	$[kN/m^2]$
Friction angle	Φ_f	35	Φ_b	30	[deg]
Dilatancy angle	ψ_f	0	ψ_b	0	[deg]

Table 4.36: Hardening Soil Model Parameters

Parameter	Foundation Soil	Backfill Soil	Units
Unit weight $\gamma_{unsat}/\gamma_{sat}$	20/25	22/25	$[kN/m^3]$
Triaxial Stiffness, E_{50}^{ref}	20000	40000	$[kN/m^2]$
Oedometer Stiffness, E_{oed}^{ref}	20000	40000	$[kN/m^2]$
Unloading stiffness, E_{ur}^{ref}	60000	120000	$[kN/m^2]$
Reference stress, p_{ref}	100	100	$[kN/m^2]$
Power, m	0.5	0.5	[-]
Poisson's ratio, ν_{ur}	0.2	0.2	[-]
Cohesion, c	0	5	$[kN/m^2]$
Friction angle, φ	30	35	[deg]
Dilatancy angle, ψ	0	0	[deg]
NC stress ratio, K_0^{nc}	0.5	0.4264	[-]
Failure Ratio, R_f	0.9	0.9	[deg]
Tensile Strength, σ_t	0.0	0.0	$[kN/m^2]$
Initial Stress ratio, K_0	0.5	0.4264	[-]

Performance function:

$$Z = \mu_{msf} - 1 \tag{4.28}$$

The reliability index is calculated as follows.

$$\beta_{normal} = \frac{\mu_Z}{\sigma_Z} \tag{4.29}$$

$$\beta_{lognormal} = \frac{ln \frac{\mu_{msf}}{\sqrt{(1+COV_{msf}^2)}}}{\sqrt{(ln\ (1 + COV_{msf}^2))}} \tag{4.30}$$

Table 4.37 shows PEM results for HS and MC model. The deterministic factor of safety with MC and HS soil model were almost the same. Hence it was expected to have similar PEM results with MC an HS soil model.

Table 4.37: PEM results

	Factor of Safety	
	Mohr Coulomb	Hardening Soil
μ_{msf}	2.02	2.08
σ_{msf}	0.185	0.186
β_{normal}	5.53	5.52

It is also important to investigate how PEM captures the influence of stiffness properties. This will involve considering the stiffness properties as stochastic variables. In hardening model, the stiffness property is represented by E_{50}, E_{oed}, and E_{ur}. It was however not possible to use the Parameter variation feature to perform PEM combinations with the Hardening model stiffness parameters, because parameter variation feature gives unrealistic combinations of the Hardening Stiffness parameters, E_{oed}, E_{ur} and E_{50}.

4.6.2. Probabilistic analysis using FORM in Plaxis

FORM based probabilistic analysis was done using PROBANA. The results are compared with PEM and EC7. Table 4.38 shows the mean properties and standard deviation of the stochastic input parameters.

Table 4.38: Statistical properties of Input parameters

Parameter	Distribution	Mean Value	Standard Deviation
c_f'	Normal	5	1.0
φ_f'	Normal	35	3.5
φ_b'	Normal	30	3.0

4.6.3. PEM and FORM comparison

A comparison is done between reliability indices from PEM, FORM and Monte Carlo. PEM is performed with Parameter variation feature in Plaxis, FORM is done using PROBANA and Monte Carlo in Phase 2. (Phase 2 results are taken from Kamp, 2016) PEM and FORM are performed using the following limit state function. The limit state function defined with respect to safety factor is:

$$Z = \mu_{msf} - 1 \tag{4.31}$$

Table 4.39 shows β values from PEM, FORM and MC. PEM and MC have comparable results, whereas FORM results is comparable to the average of β_{normal} and $\beta_{log\ normal}$ values from PEM and MC. As the results follow the same trend that was observed

Table 4.39: PEM, FORM and MC comparison β values

Method	β_{msf}	
FORM	6.22 $p_f = 2.489 * 10^{-10}$	
	β_{normal}	$\beta_{lognormal}$
PEM	5.51	7.63
MC	5.23	7.35

in the previous benchmarks, the reasons are the same as explained in the previous benchmarks.

4.6.4. FORM and EC7 Comparison

As explained in the previous benchmarks, to compare FORM and EC7, they should both correspond to the same reliability level. This is done by defining the threshold in the limit state function as the design safety factor. The deterministic safety factor corresponding to EC7 design values is 1.26. Thus, the limit state function is defined as:

$$Z = \mu_{msf} - 1.26 \tag{4.32}$$

Table 4.40: FORM design points and sensitivity coefficients

Parameters	Mean Values	FORM design point	FORM sensitivity coefficient, α_i
c_f'	5	3.83	0.068
Φ_f'	35	20.52	0.851
Φ_b'	30	26.18	0.080

Table 4.40 gives the design points and sensitivity coefficients of the stochastic parameters. From FORM sensitivity coefficients, it is seen that the foundation soil friction angle is the most sensitive, and hence the design point of the Φ_f' is the much far away from its corresponding mean value, whereas for the other parameters that have lower sensitivity coefficient, the design value is closer to the mean value.
Table 4.41 compares FORM design points with EC7 design values. FORM design points are lower than EC7 design values for sensitive parameters (Φ_f') and higher than EC7 design values for parameters with low sensitivity (c_f', Φ_b').

Table 4.41: FORM and EC7 Comparison

Parameters	Mean	FORM design point	EC7 Design Value
c_f'	5	3.83	2.32
Φ_f'	35	20.52	24.14
Φ_b'	30	26.18	20.53
Reliability Index, β		4.48 $p_f = 3.68 * 10^{-6}$	3.80

4.6.5. Conclusions

Finite element reliability analysis of a cantilever retaining wall is performed with FORM and PEM. Reliability based design is compared with EC7 design, and the compliance of RBD with EC7is studied. The trend in reliability results from different methods were similar to the previous benchmarks, and thus not repeated here. FORM determines the importance of each parameter in the FORM parameter space, which influences the design point of the parameter that it determines. FORM works in a physical space of parameters where parameters with high importance are varied from the mean point to a greater extent than parameters with low importance. Design points of parameters with high importance are farther away from the corresponding mean values and vice-versa. This means that the design point of parameters indirectly influence each other. This has both merits and demerits. The merit: FORM identifies the importance of each parameter with respect to the problem scenario. The demerit is that this can sometime result in very low design values for parameters with less importance. Though from a mathematical point of view, this may seem correct, it is not practical to have design values higher than mean values.

4.7. Case study: Cantilever Retaining Wall on Piles

This case study is an attempt to investigate reliability based design for a more realistic geotechnical structure. A cantilever retaining wall on foundation piles is investigated here. Kamp (2016) performs Reliability analysis with Point Estimate Method for this case study.

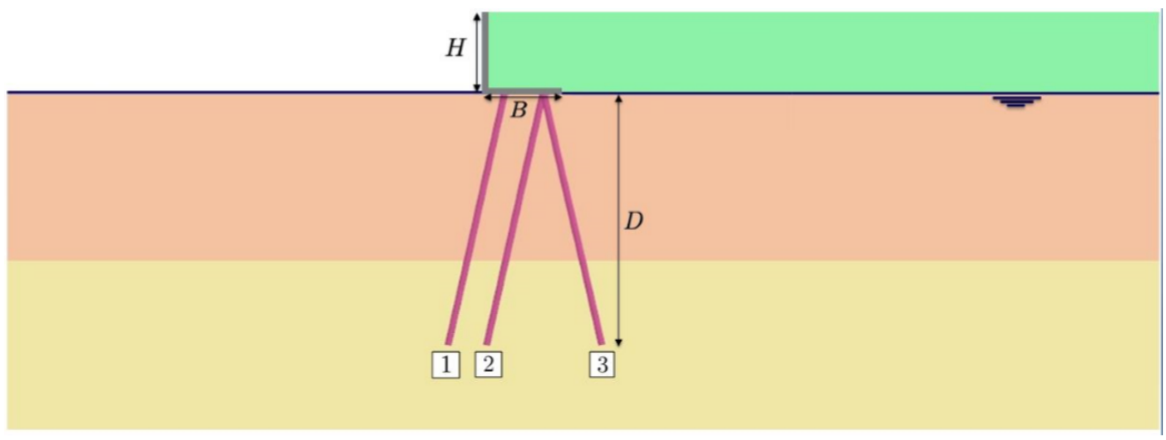

Figure 4.8: Geometry: Cantilever Retaining wall on piles

This structure has a complex failure mechanism as both strength and stiffness parameters influence the failure. The backfill soil induces a horizontal pressure on the retaining wall, due to which the wall rotates. This rotation of the retaining wall generates bending moments in the piles. Also, soil deformations induce lateral pressure in the piles. Therefore, a slight deviation in the soil stiffness on which the piles are located would affect the structural forces on the piles. This makes it slightly complicated to use the partial factor method to design such a structure.

The geometry of the problem and is shown in Figure 4.8. The soil properties can be found in Table 4.42. Since the retaining wall is founded on a clay layer, foundation piles are required. The length of the foundation slab is B = 4 m, the wall height H = 5 m. Both the wall and the foundation slab have a thickness of d = 0.5 m. The piles are connected to the retaining wall at ground level and the tip is in the sand layer at a depth of D = 15 m. The backfill layer has a thickness of 5 m, the clay layer has a thickness of 10 m and the foundation sand layer is 10 m thick. The groundwater level is located at ground level.

The mean soil properties of the foundation layers and the backfill are given in Table 4.42.

A deterministic analysis is performed, which is then extended to probabilistic using Parameter Variation feature for PEM and by coupling to PROBANA for FORM. In the following section, PEM and FORM results are compared, and they are compared to

Table 4.42: Mean Soil Properties and COV

Parameter	Foundation Sand	Foundation Clay	Backfill Sand	Unit	COV
Young's modulus, E	80000	2000	20000	[kPa]	0.25
Unit weight, $\gamma_{unsat}/\gamma_{sat}$	18 / 20	10 / 17	18 / 19	$[kN/m^3]$	0.05
Cohesion c'	0	5	0	$[kN/m^2]$	0.20
Friction angle φ'	35	20	30	[deg]	0.10

Table 4.43: Mohr Coulomb Model Parameters

	Foundation Sand	Foundation Clay	Backfill Sand	Unit
Young's modulus, E	80000	2000	20,000	[kPa]
Poisson's ratio, v	0.3	0.3	0.3	[-]
Cohesion, c	0	5	0	$[kN/m^2]$
Friction angle, φ	35	20	30	[deg]
Dilatancy angle, ψ	0	0	0	[deg]
Reduction factor, R_{inter}	0.8	0.7	0.8	[-]

Eurocode design standard.

4.7.1. PEM and FORM comparison

Stochastic variables are decided based on sensitivity analysis (Kamp, 2016). In the analysis here, the variables that are considered stochastic are φ_s, φ_c, c_c, $\gamma_{c,sat}$. The statistical properties of these parameters are given in Table 4.44.

Table 4.44: Stochastic input and the statistical properties

Parameter	Distribution	Mean Value	Standard Deviation
c_c'	Normal	5	1.0
Φ_c'	Normal	20	2.0
Φ_s'	Normal	35	3.5
$\gamma_{c,sat}$	Normal	17	0.85

PEM and FORM are performed with the statistical distribution for a limit state equation defined in , and results obtained are given in Table 4.45.

$$Z = \mu_{msf} - 1.10 \tag{4.33}$$

FORM gives a higher reliability index in comparison to PEM, and the reasons for this are the same as explained in the previous benchmarks.

Table 4.45: PEM and FORM comparison

Method	β_{msf}
FORM	8.22 ($p_f = 9.8 * 10^{-17}$))
PEM	6.65

4.7.2. FORM and EC7

As explained in the previous benchmarks, to compare FORM and EC7, they should both correspond to the same reliability level. This is done by defining the threshold in the limit state function as the design safety factor. The deterministic safety factor corresponding to EC7 design values is 1.66. Thus, the limit state function is defined as:

$$Z = \mu_{msf} - 1.66 \tag{4.34}$$

Table 4.46: FORM and EC7 comparison

Parameters	Mean Values	FORM Design Point	EC7 design value
Cohesion, c_c'	5.0	3.73	2.24
Friction angle, φ_s'	35.0	26.09	25.03
Friction angle, φ_c'	20.0	15.05	14.05
Saturated unit weight, γ_{sat}	17.0	14.51	15.61
	Reliability Index, β	4.79	3.80

Table 4.46 compares FORM design points with EC7 design values. FORM design points are lower than EC7 design values for sensitive parameters and higher than EC7 design values for parameters with low sensitivity. The reliability index from FORM is higher than EC7's value of 3.8 which suggests that selecting the right combination of design points is crucial for an optimised design. Further investigation is required to assess the feasibility of reliability based design for complex geotechnical problems.

5

Python Scripting

A python script is written to post process results from parameter variation for Point Estimate Method. Kamp (2016) uses Plaxis' Parameter Variation feature to generate the determinisitic combinations required to perform Point Estimate Method. For a problem with n number of stochastic inputs, parameter variation results in 2^n deterministic results. PEM requires computing a weighted average of the results from the deterministic files, from which a reliability index is to be determined. For large n, it is impossible to post process the results from parameter variation manually. Hence a code is written to aid in post processing the results to obtain PEM results.

Parameter Variation

Step 1: Deterministic Analysis
Perform deterministic analysis of the project and obtain deterministic results.
Step 2: Sensitivity Analysis
Select soil Parameters, and relevant criterion. Perform Sensitivity Analysis to identify dominant parameters
Output: A Plaxis folder is generated with sensitivity analysis combinations
Step 3: Parameter Variation
Select Stochastic Parameters using sensitivity analysis results and engineering judgement, and define upper and lower bound value for each soil parameter.
For PEM, upper and lower bounds for soil parameters are defined as follows:
Upper bound is reference value - 1 Std. deviation
Lower bound is reference value + 1 Std. deviation
Output: A Plaxis folder is generated with all deterministic combinations of parameters selected
Step 3: Post-processing using Script

The working of the script is explained in the following section.
Output: Point Estimate Method results

Working of the Script:

The user is required to give some information regarding the project in the script. Entries to be given by the user in the script are:

- path of ParVar folder (The folder where the Parameter Variation results Plaxis files are located).
- Type of result. The already defined Result Types are factor of safety and reached force.
  Note: The user can include more criterions and result types in the script.
- Phase name of the result phase.
  Note: Phase name should be same as the phase name defined in the phases explorer (case sensitive)

With this information, the script retrieves results from each deterministic plaxis file from the ParVar folder. The output phase of each file is opened and the result is retrieved and printed on screen. A snippet of the output safety factors for benchmark 1 (Slope stability Problem) discussed in Section 4.4 is shown below.

```
SAFETY FACTOR: 1.56173226237179
SAFETY FACTOR: 1.94531236451223
SAFETY FACTOR: 1.68280026634672
SAFETY FACTOR: 2.07856400290755
```

Each safety factor is a result of a deterministic computation, as explained previously. PEM requires computing a weighted average of all the results to calculate the reliability index. This is shown in the snippet below.

```
correlations: <y/n>n
PEM Mean: 1.8171022240345724
PEM sigma: 0.2049700801892578
What is the threshold limit?1
Reliability index_Normal: 3.9864463305088544
```

PEM mean refers to the weighted average of the deterministic results shown above. PEM sigma is the standard deviation of the output results. The user is asked for the threshold limit of the safety factor. In this case, the criterion being safety factor, 1 is entered as the threshold. The reliability index is therefore computed.

Technical Issues: Parameter variation feature is not directly accessible in the Plaxis - Python interface due to access limitations. Correlations between input parameters are not incorporated in the script due to similar reasons.

6

Conclusions and Recommendations

This chapter summarizes the conclusions of this research, and some recommendations based on the conclusions.

6.1. Answers to the Research questions

In this section, the research objectives are recalled and matched to the key outcomes of this thesis.

RO1 : Extensive comparison between Point Estimate Method, First Order Reliability Method in the framework of PLAXIS and other stochastic methods.

1. FORM, and PEM are computationally efficient reliability methods, when compared to other stochastic methods such as Monte Carlo. The differences in reliability indices obtained from PEM, FORM and Monte Carlo could be explained by the different underlying approach of each method. For the cases considered in this research, Point Estimate Methods and Monte Carlo gave comparable results. This could be attributed to the fact that Point Estimate Method is after all a statistical approximation or rather a subset of Crude Monte Carlo. Conversely, FORM results are less comparable with PEM. However, for the cases presented in this research, assuming a lognormal distribution for the output resulted in comparable results between PEM and FORM. The reason for this discrepancy could be because FORM gives no information regarding the output distribution, whereas PEM makes assumptions regarding the nature of the output distribution.

2. Point Estimate Method is a straightforward method, and the underlying concept is easier to comprehend which is an advantage from a practical standpoint. But PEM requires assuming an output distribution and the result heavily relies on

this assumption which hampers the accuracy of PEM. FORM is mathematically complex, as searching for the most probable point of failure in FORM is a numerically challenging task and sometimes the errors degrade the accuracy of the results. FORM is based on a constrained optimisation approach, and the accuracy of FORM depends on the optimisation algorithm adopted. However, a relative advantage of such analytical methods is that they provide physical interpretation and do not require much computation time.

3. FORM directly associates each parameter with a sensitivity coefficient, which influences the design points. The most sensitive parameters have lower design points and vice versa. FORM works in a n-dimensional space where the sensitivity of parameters are considered to compute the design point. Whereas, in PEM, it is required to first identify the sensitive parameters by performing a sensitivity analysis, and the dominant parameters are assumed stochastic. Sensitivity coefficients do not play a role in determining the reliability index in PEM.

RO2: Investigate the compliance of Reliability based design with Eurocode 7 to understand potential possibilities of adopting reliability based design in engineering practice

1. Reliability based design is an efficient and practical approach to design complex geotechnical designs where partial factor method is not easily adoptable. Using the proposed methodology, actual reliability level of a structure for EC7 Ultimate limit states can be determined, and compared to the target reliability specified in Eurocode7.

2. Eurocode 7 is not flexible with regard to defining different target reliability levels for different problems. With reliability based design, it is possible to determine design values for different target reliability levels.

3. Reliability analysis using FORM also gives an estimate of the importance factors of each parameter in a problem which can be used to optimize the partial factors.

4. FORM and EC7 fundamentally differ in the sensitivity coefficients it assigns to the different parameters. FORM works with relative sensitivity factors, whereas EC7 partial factors are independent. Relative sensitivity coefficients sometimes result in higher design values, which is unacceptable according to Eurocode design standards.

5. EC7 does not consider correlations between soil properties. There are possibilities of this leading to unconservative designs. It was observed that EC7 design values coincided with FORM design values when unrealistic correlations were assumed in FORM computations.

RO3: Automation of the Point Estimate Method in PLAXIS by developing post processing routines using Python Programming interface.
A Python Script was developed to postprocess Plaxis results from Parameter variation feature, which is used to perform Point Estimate Method.

6.2. Main Conclusions

1. This research shows few cases where Eurocode7 design values do not satisfy the target reliability values. The proposed methodology using FORM can be used to determine the actual reliability index of a problem, which can be compared with the target value based on partial factor approach from Eurocode7.
2. Point Estimate Method is a computationally efficient method that can be used to perform preliminary reliability analysis in a finite element framework to determine the approximate reliability level of a structure. However due to various assumptions made in PEM, the results may not be very accurate.
3. In comparison with PEM, FORM is a more accurate method to perform reliability analysis. Also FORM provides information regarding the importance of different parameters which can be used to understand the problem better.

6.3. Recommendations

Some recommendations to improve the tools used for reliability analysis, and further research is summarized here.

1. Parameter Variation feature in Plaxis with Hardening Soil Model gives unrealistic combinations of E_{oed}, E_{ur} and E_{50}. This limits the use of this feature only to Mohr Coulomb Model. It is recommended to improve this by creating constraints to consider the Hardening stiffness parameters as a single unit while performing Parameter Variation. Another recommendation for Parameter Variation feature would be to include a file with the results from all Plaxis files of the PaRVar folder, so that users can spend less time to extract information from individual Plaxis files.
2. PROBANA (FORM) uses an optimisation algorithm from OpenTurns called COBYLA to perform reliability analysis. The efficiency and accuracy of FORM depends on this optimisation algorithm. The significance of the errors used in COBYLA is still unclear and more investigation of this algorithm is required to obtain better insight of it's limitations.
3. Eurocode 7 aims at determining design values of parameters corresponding to a target reliability index. Further investigation is recommended to formulate a method where PROBANA back calculates the design parameters based on a

target reliability index. This could serve as a practical tool to be used along with Eurocode7.

4. The accuracy of failure probability in PROBANA can be improved by using second order approximations instead of first order approximation. Second Order Reliability Method (SORM) can be used to reduce linearization errors.

5. The inherent spatial variability of soil is not taken into account. Random Finite Element Methods can be used to include the spatial variability of the soil.

6. The speed of performing FORM in PROBANA depends on the calculation features of the COBYLA. The values of the calculation features are problem dependent. To make the tool more computationally efficient, more investigation is required to determine optimum value of these calculation features.

7. In this thesis, a methodology is proposed to determine the actual reliability of a problem. To test if the proposed methodology does provide actual reliabilities, it is recommended to conduct more research with different geotechnical scenarios.

8. This thesis does not fully explore the possibilities of reliability analysis on the case study (Cantilever retaining wall on Piles) in Section 4.6. However it was shown that a reliability analysis can be efficiently performed with PROBANA - FORM for the case study, despite having complex failure mechanisms. The results from Kamp (2016) provides reasons to believe that the effects of soil structure interaction are captured by Point Estimate Method. Therefore it is recommended to investigate the same in PROBANA with different limit states.

9. A main limitation of the reliability method used here is that Level II methods like PEM or FORM cannot be used for problems with many stochastic parameters. To perform stochastic analysis for problems with more stochastic problems, it is recommended to investigate more stable methods like Directional Sampling.

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Appendix A

Li's PEM (1992)

Li's Method derives $E[Y_m]$ from the Taylor Series Expansion of the function $Y = f(x_i)$, as the sum of three terms: A central term evaluated at the mean values of the random variables, a term that takes into account the values of Y evaluated at discrete points that approximates the pdf of each X_i, and A term that accounts for pair wise correlations among the variables X_i as:

$$E[Y^m] = \varphi Y^m + \sum_{(i=1)}^{N} (\gamma_{(i+)} y_{(i+)}^m + \gamma_{(i-)} y_{(i-)}^m) + \sum_{(i=1)}^{(N-1)} \sum_{(p=i+1)}^{N} \eta_i p y_i p^m \tag{1}$$

Li's method gives almost the same mean and standard deviation as that of Rosenbluth's PEM. It is computationally much efficient than Rosenbluth's PEM for higher number of input parameters. For 10 variables, Rosenbluth's PEM requires 1024 computations whereas Li's Method only requires 66. Li's Method gives four statistical output parameters, which can be used to fit a better output distribution. Both methods consider correlations between variables and asymmetry of the input distribution. Rosenbluth's PEM is based on Gaussian quadrature whereas Li's method is based on Taylor Series Expansion. Both methods are unsuitable for non-linear models. In case of Li's methods, advanced methods have been proposed to consider non linearity but by disregarding correlations.
Figure 1 shows an excel template with Li's PEM (1992).

Table 1: Comparison between Rosenbluth (1981) and Li (1992)

Rosenbluth, 1981	**Li, 1992**
Gaussian Quadrature procedure	Taylor series expansion of the function
Two statistical input and output	4 Statistical inputs
2n computations	$(n^2 + 3n + 2)/2$ computations
Not suitable for non-linear functions	Not suitable for non-linear functions

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
1	**No. of variables, n**	2															
2	**Computational amount**		6														
3																	
4				Statistical Properties						Evaluation Points		Probability at each point					
5		**Mean μ**	**COV**	**SD σ**	**Skew γ**	**Kurtosis κ**	x'_{i+}	x'_{i-}	x_{i+}	x_{i-}	x_0	p_{i+}	p_{i-}	p_0			
6	**Cohesion, c**	5	0.2	1	0	3	1.73205	-1.73205	6.73205	3.26795	5	0.166666667	0.166667	0.666667			
7	**Friction angle,φ**	35	0.1	3.5	0	3	1.73205	-1.73205	41.06218	28.93782	35	0.166666667	0.166667	0.666667			
8																	
9							Corelation Matrix				**Function**	**Value**	**Weights**				
10	**Function**	**c**	**φ**	**FoS**		Parameter	**c**	**φ**			Y^m	1.805	0.166667	0.300833			
11	$f(\mu_c,\mu_\varphi)$	5	35	1.805		**c**	1	-0.5			Y_{c+}	1.915	0.333333	0.638333			
12	$f(x_{c+},\mu_\varphi)$	6.73	35	1.915		**φ**	-0.5	1			Y_{c-}	1.683	0.166667	0.2805			
13	$f(x_{c-},\mu_\varphi)$	3.27	35	1.683							$Y_{\varphi+}$	2.166	0.333333	0.722			
14	$f(\mu_c,x_{\varphi+})$	5	41.06	2.166			**c**	**φ**	η_i		$Y_{\varphi-}$	1.505	0.166667	0.250833			
15	$f(\mu_c,x_{\varphi-})$	5	28.94	1.505		**c**	1	-0.16667	0.8333333		$Y_{c\varphi}$	2.289	-0.16667	-0.3815			
16	$f(x_{c+},x_{\varphi+})$	6.73	41.06	2.289		**φ**	-0.16667	1	0.8333333				$E[Y^m]$	1.811			
17								η=	1.6666667				Moments		**Output Statistics**		
18													E[Y]	1.811	Mean	1.811	
19													$E[Y^2]$	3.305597	SD	0.160859	
20													$E[Y^3]$	6.072118	Skewness	-1.93152	
21															Reliability	5.041678	

Figure 1: Benchmark 1 (Li's PEM (1992) on Slope stability problem Excel calculation

Appendix B

PROBANA Output (Convergence)

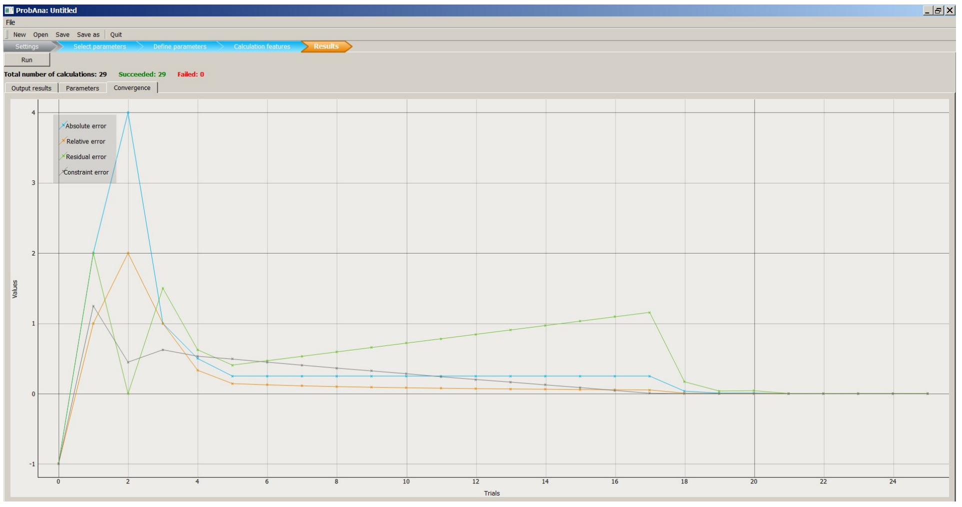

Figure 2: PROBANA errors convergence (Slope stability Problem)

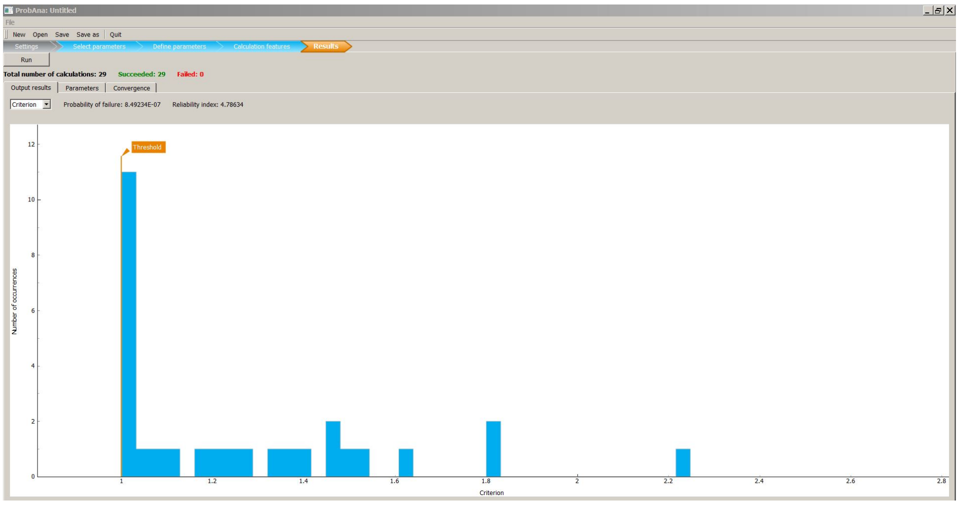

Figure 3: Benchmark 1 (PROBANA Threshold Convergence Slope stability Problem