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AN INVESTIGATION OF SURFACE DEFORMATIONS OCCURRING AT THE CONTACT EDGES OF STATICALLY LOADED DISCS

by

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'An investigation of surface deformations occurring at the contact edges of statically loaded discs'

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SUMMARY

An investigation has been carried out, using statically loaded steel and 'Perspex' discs, into elastic displacements occurring at the axial extremities of the Hertzian flat.

By employing the technique of 'Photostress', together with optical magnification, it has been found possible to inspect in detail the disposition of strain around these points.

Using a 'step grinding' technique a qualitative three-dimensional envelope of surface displacements has been constructed. The effect of these displacements is considered in relation to 'edge pitting' of discs.

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1. Introduction

Early theoretical studies on contact conditions between gear teeth were undertaken by Martin (1) and later by Gatcombe (2), Cameron (3,4) and Poritsky (5).

Experimental investigations, however, presented certain practical difficulties. Apart from the necessity of producing geometrically similar tooth profiles to a high order of accuracy, there remained the almost insurmountable problem of obtaining physical measurements of the conditions within the contact zone proper; since these are both transitory in nature and confined within relatively small boundaries.

In an attempt to reduce the magnitude of the problem while simulating, kinematically, some of the conditions of tooth contact, testing machines consisting of circular discs rotating under load in line contact were evolved and used to investigate such factors as type of lubricant, material, heat treatment and surface finish in addition to the various conditions giving rise to material and lubrication failure (6,7,8,9).

With the advent of the disc-testing machine theoretical analysis was further extended to embrace the conditions arising specifically in this type of contact. The early analysis by Martin (1) was based on the assumptions that the disc peripheries did not deflect elastically under the applied load and that the viscosity of the lubricant in the contact zone was unaffected by pressure or local temperature (see fig. 1). However, the resulting pressure curves, derived for highly loaded conditions, were found to have maxima in excess of the corresponding Hertzian static curves (see figures 1 and 2). Under these magnitudes of pressure the viscosity of the lubricant would be considerably increased. Moreover, the Hertzian pressures had been shown both experimentally and analytically to give significant elastic deflections. Thus both of Martin's original assumptions carried no validity under these conditions.

Later attempts on the problem, culminating in the extensive work of Grubin (10), Crook (11) and Dowson and Higginson (12), included the disc's elasticity together with pressure/temperature effects in the fluid equations for the lubricant. For these reasons the expression 'Elasto-Hydrodynamic Lubrication' is used to describe this type of contact. Mutually consistent shapes for film thickness and pressure curves have now been obtained with the aid of digital computers (12, 13) (See figure 3).

2. Investigation

A basic assumption, common to all theoretical analyses of discs, is that the cylinders are infinitely wide axially, or more explicitly, that side leakage of the lubricant is neglected. The axial pressure gradient is therefore assumed to be zero.

While this assumption is undoubtedly valid over almost the entire contact width, it clearly cannot apply at the extremities since this would demand a discontinuity of pressure. If the contact flats of the discs are assumed parallel (10, 11, 13) and separated by a distance 'h' ins. in the dynamic state, the axial side leakage/unit length of flat may be expressed by the equation

$$Qz = -\frac{h^3}{12\eta}\left(\frac{\partial P}{\partial z}\right)_{ext.}$$

where $-\left(\frac{\partial p}{\partial z}\right)_{ext.}$ is the axial pressure gradient at the extremities

and η is the working viscosity.

Now film thicknesses for contacts of this nature vary between 10^{-5} ins. and 10^{-4} ins. for normal working conditions, although thicknesses outside this range have been reported (12,13).

Thus, even assuming the normal working viscosity to be as low as 10^{-5} Reyns (SAE 10) and with small side leakage (say 10^{-3} cu.in/sec per unit contact length) the magnitude of the $\frac{\partial P}{\partial z}$ term is still considerable (between 10^5 and 10^7 lb/in^2/in). In addition, experimental evidence indicates that, in the axial direction, the contact surfaces tend to approach each other at the extremities of contact (17). Should this be so the magnitude of the $\partial P/\partial z$ term may be several orders greater than those quoted, since this magnitude varies as $\frac{1}{h^3}$.

It was with these considerations in mind that the experimental work described herein was initiated, with a view to establishing what effect, if any, these large pressure gradients might have on the plane sides of the discs themselves.

3. Method

A technique known as 'Photostress' is available commercially for the examination of surface stresses in engineering materials. It combines some of the advantages of Photoelasticity Brittle Lacquers and Electrical Strain Gauges. The method, developed by Zandman (14), consists essentially in coating the surface of the loaded member with a special plastic skin. When viewed under polarised light the surface shear strain distribution appears as a series of contrasting colour bands or fringes. These may be interpreted both qualitatively and quantitatively in terms of maximum shear strain and its gradient. The chief advantage of the technique, however, is that strain distributions may be observed directly.

As strain distributions more than actual strain magnitudes were initially of interest, it was decided to employ the 'Photostress' technique together with optical magnification on the plane faces of steel test discs and to compare the observed strain pattern with a pattern in 'depth', obtained by usual photoelastic methods, on geometrically similar discs of 'Perspex'.

4. Apparatus

The steel discs were EN34 case hardened to a depth of approximately 0.040". Peripheral diameter was 3.25" and track 0.187" wide. (see figure 4). Initial tests were carried out statically, since this simplified the task of observing and recording strain patterns without the risk of extraneous interference from dynamic sources.

Since it was intended to carry out dynamic observations at a later date, the static tests were performed initially in an I.A.E. gear testing machine suitably modified. These tests provided, therefore, a true basis of comparison for the later dynamic ones projected.

Two 'Photostress' instruments were employed for observations of the strain patterns. These are marketed by the 'Budd Co.' of America under the trade names of 'Large Field Meter' and 'Small Field Meter' and are commercially available in this country. The 'Large Field Meter' is intended primarily for the examination of relatively large areas of stress; the principle of operation of this instrument is shown in figure 5. Circular polarization may be achieved by the incorporation of two quarter-wave plates.

The 'Small Field Meter' is basically similar to a metal microscope and employs an analogous optical arrangement. Its use is confined to localised point observations or examination of areas of steep strain gradient. Circular polarization is not possible with the standard meter. The instrument was, however, modified to give this.

5. Procedure and results

Initial trials were carried out using the 'Perspex' discs. These were mounted in the machine and loaded to give a Hertzian Flat of approximately 0.060 ins. Previous experiment had shown that a flat of this size could be obtained using the steel disc without risk of permanent deformation. To facilitate observations the nearside face of each test disc (relative to the incident light ray) was painted with reflective paint.

Observations were made with the 'Large Field Meter'; a typical field pattern delineated from test photographs is reproduced in figure 7. The fringe pattern or isochromatics are seen to follow the normal quartic distribution associated with the Herzian stresses in this type of loading (16). The effect of the retaining bolts is localised which broadly agrees with general photoelastic observations.

The results obtained from observations on the steel discs, however, revealed an unexpected and totally different pattern from the previous test (see figure 6). The disposition of the isochromatics depart radically from the normal 'pattern in depth' while the configuration around the zone of intimate contact appears to indicate a small closed loop of isochromatics immediately above the Hertzian flat and parallel to its length. It is to be borne in mind that the original photographs were in colour, enabling small details and localised areas to be easily discriminated.

Since the surface strain pattern for the steel specimen appeared to show a radical difference both from conventional experimental and analytical 'patterns in depth' (see Figs. 8 and 9)(15.16), the validity of the result was at once suspected. Consideration was given to the possibility of the test machine imposing internal restraints on the specimens since unequal edge loading, for example, might seriously modify the usual isochromatic pattern.

A further series of tests was therefore carried out external to the machine; a simple straining frame being designed and built for this purpose. This design allowed freedom of movement to one disc, the other being fixed, and eliminated the possibility of asymmetrical loading. Results from this test were,however, identical to those initially found.

The validity of the findings having been established, attention was focussed on the area immediately surrounding the Hertzian flat since pattern divergences at this point appeared most marked.

For the purpose of these tests the 'Small Field Meter' was used in conjunction with the discs mounted in the straining frame (see fig. 10), since this arrangement greatly facilitated both ease of observation and small adjustments to the discs themselves.

Results from these trials for both the 'Perspex' and steel specimens are shown pictorially in Figs. 11 and 12. It is at this higher magnification that the divergences of the isochromatic patterns become most marked, overall distributions across the full width of the Hertzian flat being displayed.

As might be suspected from the illustration at the lower magnification (see Fig. 7) the higher magnification for the 'Perspex' specimen yielded no further information apart from rendering the conventional distribution across the flat more clearly (see Fig. 12). Results from the steel specimen, however, indicated what had previously been suspected; that in fact a closed isochromatic loop existed adjacent to and parallel with the contact plane proper (see Fig. 11).

Consideration of this configuration appeared to indicate the existence of stress concentrations acting in a direction parallel to the disc axes and of a sufficiently high order to cause an outward displacement or bulging of the disc material at its plane surface around the area of the contact flat. If this postulate was in fact correct, then evidence of local damage at this point might be available from various sources concerned with disc experiments. With this in mind evidence was sought for in relevant work on the subject and was immediately forthcoming.

Dawson (17), in work connected with the pitting of lubricated rollers, reported a concentration of pits 'around the two extreme edges of the track', - though the calculated thickness 'was about seven times greater than the total surface roughness', (see Fig. 13) and attributes this failure to metallic contact caused by side leakage of the lubricant.

While experimental evidence indicates that metallic contact can occur in this region (18) the direct effect of high stress accompanying the axial bulging of the plane surfaces of the discs around the contact point cannot be ignored. Indeed from a close study of the disc photograph reproduced from this paper it appears that plastic deformation in this direction has taken place at some of the pits (see Fig. 13 at A and B).

Further evidence was provided in a paper by Hamilton (19), who, in investigating the stress distribution in rolling contacts by photoelastic methods, employed a glass disc in contact with a soft copper flat. He reported, regarding the loading of the disc against the flat: 'since the stresses are compressive it should have been possible to apply heavy loads without cracking the glass but in fact some difficulty was encountered due to small chips breaking away from the edge of the disc' (our underlining).

El-Sisi and Sawki (20), in investigations concerning the lubrication of discs operating at nearly pure sliding reported on a series of scuffing trials' - 'The discs were also damaged at several points near the edges' and reproduce a photograph of one of the test discs showing extensive damage at both peripheral extremities.

From the evidence contained in these papers, it was obvious that edge damage could be sustained under a diverse variety of conditions. These include discs operating on relatively thick and thin oil films, discs without lubricant and discs under differing dynamic conditions of rolling and sliding. In the case of the glass disc, factors such as surface heat treatment, surface fatigue, fluid pressure effects and, probably, surface finish stresses could hardly be said to apply. The disc, it will be noted, was unlubricated.

It appeared logical therefore, in the light of the diversity of the tests, to associate this particular type of failure, common to all discs, with those factors which were themselves common to all tests. The conditions uniquely satisfying this criterion were found to reduce to two, namely; the peripheral geometry of the discs and the method of loading them. These, when considered in combination, implied some limiting stress distribution in the disc material proper as a likely cause of failure. Further, the 'Photostress' evidence directly supported this rationale, since the indicated stress concentrations, acting in the manner described, would have their greatest effect at the contact extremities.

As noted earlier, a stress concentration of this nature would be accompanied by an outward axial displacement or 'bulging' of the plane surfaces immediately at right angles to the contact flat, roughly akin to the 'bulge' of a motor-car tyre at the road surface. The 'bulge shape' of the test specimens would, naturally, be different for obvious reasons but the analogy is sufficient to illustrate the point. It is to be noted that the test specimens, being geometrically similar, would suffer no resistive forces axially, when under load. This would not apply to the case of a disc loaded against a plane.

An axial displacement being postulated, the next step, logically, was to ascertain whether it did in fact exist, and, if so, to determine a means whereby its general configuration might be found to an acceptable degree of accuracy. The task was rendered difficult both by the fact that specialised measuring equipment was not available and by the knowledge, inferred from a study of the contact photographs, that its physical size was likely to be small. (It was originally thought that the overall length approximated to that of the Hertzian flat; i.e. some 0.06 ins.) The problem of obtaining a reasonably accurate three-dimensional envelope appeared, therefore, somewhat formidable.

Initial attempts, using a 'Sigma' comparator to take traverses across the plane sides of the loaded discs, yielded inconclusive results; the technique being too crude for the degree of accuracy required. A second attempt involved surface grinding the loaded discs on their plane sides until a general level was indicated, a chordal traverse by 'Talysurf' being made through the appropriate area upon relaxing the load. The result was encouraging as a distinct hollow was recorded indicating a local axial bulging in the loaded condition (see Fig. 14).

After several fruitless attempts, a simple technique was devised for obtaining the required envelope, the method being as follows:

The straining frame, carrying the loaded discs, was accurately aligned on the table of a horizontal grinder, the plane faces of the discs being sprayed with marking blue around the contact point. 'Kiss grinding' was then commenced, terminating, after sparking off, at vertical steps of 10^{-4} ins. A photograph was then taken of the exposed surface at the end of each step. Upon suitably enlarging the complete photographic record the bulge contours could be plotted in plan at each level; a grid, scribed in the 'blue', acted as reference. A facsimile of the complete contour plot is reproduced in Fig. 15. It is seen from this that the extent of the deformation is greater than was initially anticipated.

To facilitate further study of the bulk deformation a three-dimensional model was constructed, to suitable scales, from the record of the contours. A photograph of the complete model is shown in Fig. 16. Vertical displacements are indicated by numbers on a three-dimensional grid. Units in the plane of the disc sides are of 0.05 ins. side. Vertical elevations, corresponding to local axial displacements, are in units of 10^{-4} ins. An enlargement of the configuration in the vicinity of the Hertzian flat is shown in Fig. 17.

6. Discussion and Comments

From the work already carried out the following general observations appear relevant:

1. That the phenomenon of edge failure, or pitting of loaded rollers running in line contact, can occur under a wide variety of test conditions ranging from virtually full hydrodynamic lubrication with relatively thick oil films to conditions of mixed friction and boundary lubrication with applied loading approaching that of scuffing. In addition, failure by edge splintering has been recorded using a loaded glass disc rolled in dry contact with a copper flat.

2. On the evidence presented it would appear that, in disc contacts of this nature, there is, in addition to deflections in the plane of the discs, an outward displacement or bulging of the disc material at and around the axial extremities of the contact zone and that this displacement implies the existence of axial stresses in these regions.

3. The magnitude of the measured displacements, if accepted quantitatively, appear to indicate associated stresses approaching the acceptable upper limit of yield stress for the Martensitic structure at the test disc periphery (appr. $150^{T}/in^{2}$) if an axial fibre of the contact surface is considered under simple one-dimensional strain. The situation, however, is far more complex than this since plane deformation does not occur in these regions. It is emphasised, further, that the profiles of the 'bulge' so far obtained are to be interpreted only as indicative of the general contours and not their absolute magnitudes; the technique used to obtain them being by no means faultless.

Possibilities of distortions of the straining frame under load, the physical removal of layers of metal from the stressed area; albeit a small volume, the chance of movement of the discs themselves under the grinding process are all factors which could influence the final result to an unknown degree.

While bearing the foregoing points in mind a pragmatical approach would indicate that the localised conditions at the extremities of heavily loaded and geometrically similar contacts cannot be lightly ignored, particularly when it is remembered that, in the dynamic case, the condition is cyclic. The fact that the bulge itself was readily detectable by the method described indicates both the order of its elevation above the surface and that of the stress producing it. In addition, the tightly closed loop in Fig. 11 infers both a considerable absolute strain and a high strain gradient.

With regard to the edge damage or pitting effect occurring in the manner described, the existence of axial stresses in these regions would, almost certainly, be contributory to this type of failure, if not its direct cause. It is worthy of emphasis when considering the bulk stresses in the material surrounding the contact zone that the plane sides of the disc constitute a free surface or non-reactive boundary. If appears reasonable, therefore, to expect displacements to be manifested normally to these planes.

The fact that failure can occur at the contact extremities, even with discs adequately lubricated, is obviously of some significance. It was noted when studying Ref. 17 that the modification of the discs section described therein (see Fig. 18) eliminated the edge pitting effect previously mentioned; the author of the paper attributing this to the modifications inhibiting any edge contact which may have been present during the initial trials.

It is felt, however, that any modification achieving this end would require considerable changes in the axial pressure profile for the fluid within the contact zone proper, particularly at the extremities. Since the actual modification to the cross section appears to leave the contact geometry (and therefore the pressure distribution within it) relatively unaffected, it is difficult to appreciate how, if edge failure arose in the manner suggested, the modification succeeded in preventing it. Furthermore, if the absolute magnitude of the fluid pressure profile had fallen sufficiently low at these points to allow some metallic contact to take place, then the reactive direct compressive stress in the disc material would be of the same order as this fluid pressure and significantly less than that pertaining to the centre of the contact area. Since the normal stresses at both points are cyclic with the disc's rotation it would be reasonable to expect failure in the more highly stressed region.

In considering the part played by asperity interference at the contact extremities (should this occur) it is to be borne in mind that the condition of lubrication at these points is almost certainly one of 'mixed friction' since the lubricant is flowing axially outward, across the zone, under the influence of the steep pressure gradient. As the supply of this lubricant is both relatively copious and continuous the possibility of approaching a condition of 'boundary lubrication' seems remote.

Bearing these several factors in mind it would appear that the effect of any partial breakdown of hydrodynamic conditions at the points in question may not be so important as might, initially, be supposed (18).

Viewing the modification of the disc section in relation to an axial stress system acting at the contact extremities, it is seen that the effect of the modification is to considerably reinforce the previously plane surface in resisting outward axial displacement under the action of the internal bulk stresses (see Fig. 18).

If localised stresses, arising from this displacement, played a major part in edge failure, then the suppression of this by the modification becomes self-explanatory.

7. Conclusions

It is to be emphasised that the work here described is of a preliminary nature and on that basis should not be interpreted quantitatively.

As an initial practical step towards some form of analysis of the stresses existing in and around the 'bulge' it is considered important to obtain a clear three-dimensional plot of its configuration. This would naturally require an order of accuracy significant to any calculation attempted, since the method might have to be inductive in nature. With this in mind work has been directed towards the design and construction of a suitable loading rig and measuring system. Details of this arrangement are given in the appendix to this paper (see Fig. 20 and Fig. 21).

It is of interest in passing to refer once again to the work done by Hamilton (19). As mentioned earlier, some difficulty was encountered by him in attempting to initiate yield in a copper test strip by rolling a loaded glass disc over it; the disc, in point of fact, splintering at the edges. However, when the experiment was repeated using a test flat of annealed aluminium a track was obtained without damage to the disc. While he does not state in this paper that the applied load was the same in both cases, such a situation might indeed be possible since the disc, after initial sinking into the surface of the softer specimen, would find some support at its plane peripheral edges from the walls of the track it had impressed. (See Fig. 19). It would be interesting to investigate the possibility using a disc of some brittle material on hard and soft surfaces under constant load since this might yield useful information on both the initiation of 'edge-pitting' and its possible prevention.

The inference to be drawn from this preliminary investigation would be that attention paid to the geometry at the working extremities of produced profiles on roller and gear contact surfaces might yield worthwhile returns, both in working life and in increased duty parameter.

The application of the 'Photostress' technique, together with optical magnification, is thought to offer distinct possibilities for the direct study of surface stress effects in the material surrounding contacts of the type described. In addition, the effect of the lubricant on these surface stress distributions may also be investigated.

By modifying the basic technique slightly and employing a 'chopping' type synchronised shutter, it has been found possible to record, dynamically, surface stress effects in loaded lubricated discs running at 3000 r.p.m. It is believed that this is the first time that measurements of this nature have been recorded. Results, so far, appear to be encouraging. Further work along this line is now proceeding.

8. Acknowledgments

The work herein described was carried out in the Propulsion Department, The College of Aeronautics, Cranfield. A grant from the 'Worshipful Company of Coachmakers and Coachharness Makers Inc.' enabled one of us* to continue with experimental development of the original research.

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Appendix

Apparatus for Obtaining the Envelope of Axial Displacements at the Contact Extremities

Outline. The method of obtaining the displacement envelope outlined in this paper suffers from the disadvantage that one side only of any pair of discs may be investigated at a given time.

The technique here described enables this difficulty to be circumvented since surface profiles of both faces of the discs may be obtained under any one loading.

Loading Arrangement. Fig. 20 shows a photograph of the loading arrangement. It is seen that two pairs of discs are used, each pair being bolted to a circular distance piece to form a 'bobbin'; final grinding of all disc faces being done after assembly.

Load is applied through two loading blocks; one being self-aligning in the diametral plane; after placing the complete assembly between the jaws of a large machine vice. The applied load is obtained by measuring the movement of the 'bobbin' centres; previous calibration having been carried out in a 'Denison' testing machine.

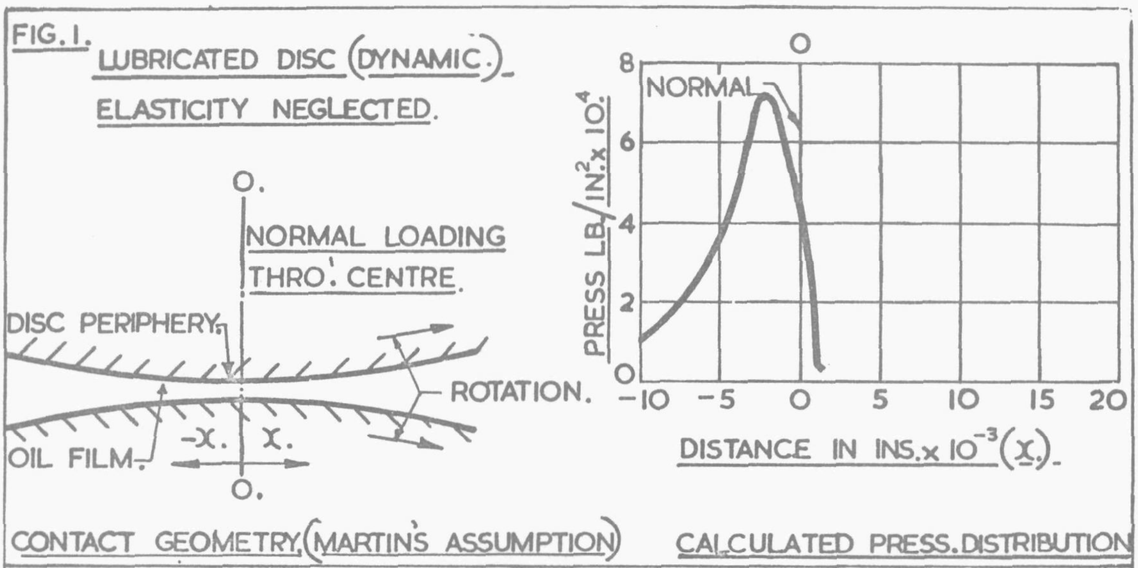
Measuring Instrument. The measuring instrument (see Fig. 21) consists, essentially, of a preloaded arm carrying a displacement transducer, the arm rotating about a spindle fixed to one of the 'bobbins' and ground in situ with it. The transducer is a 'Ferranti Micro-Comparator' capable of discriminating movements of the order of 10^{-6} ins.

Method. In operation the comparator is swung into position (see Fig. 21 Plan) allowing a small diamond stylus at the end of the comparator arm to make contact with the plane surface of the disc. The arm is then slowly traversed through the relevant area, its angular position being sensed by a fine-wire-wound potentiometer. Outputs from this and the comparator are fed to an amplifier and thence to an 'X-Y' recorder. Thus, for any constant-radius track, the locus of the displacements may be found by subtracting the loaded and unloaded curves, and the complete envelope of the displacements constructed.

After obtaining the required profiles on one side of the disc the comparator is lowered and the operation repeated for the other side. By the means outlined it is hoped to obtain three-dimensional envelopes for a range of loads.

List of figures

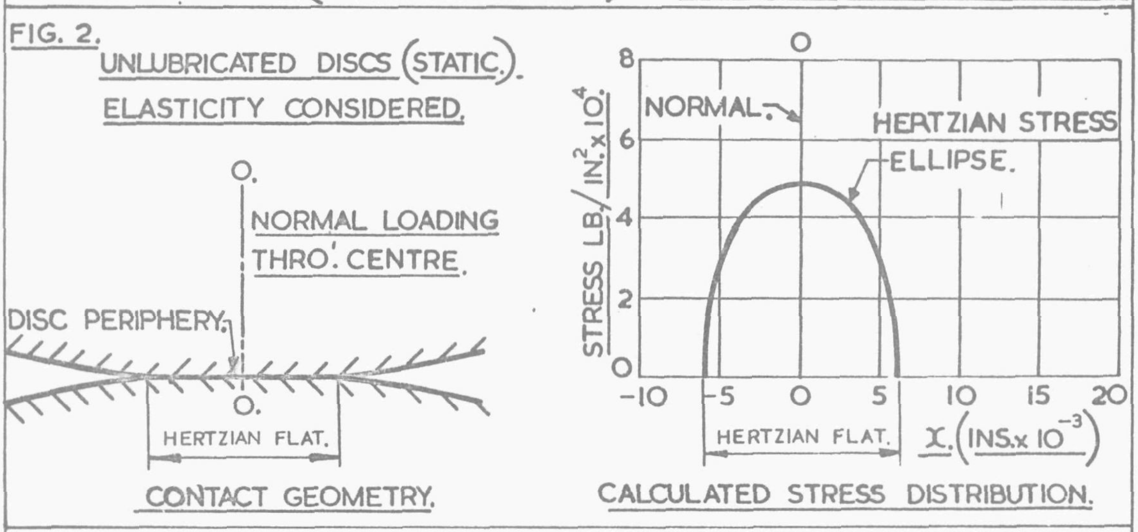
FIG. 1.


LUBRICATED DISC (DYNAMIC.) ELASTICITY NEGLECTED.

CONTACT GEOMETRY. (MARTIN'S ASSUMPTION)

CALCULATED PRESS. DISTRIBUTION

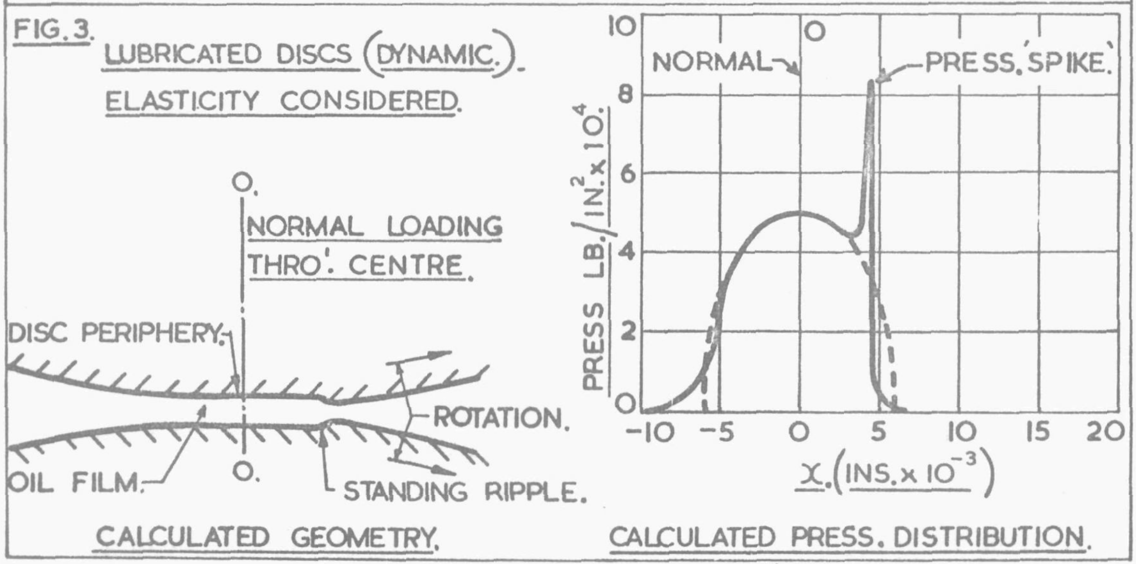
FIG. 2.


UNLUBRICATED DISCS (STATIC.) ELASTICITY CONSIDERED.

CONTACT GEOMETRY.

CALCULATED STRESS DISTRIBUTION.

FIG. 3.


LUBRICATED DISCS (DYNAMIC.) ELASTICITY CONSIDERED.

CALCULATED GEOMETRY.

CALCULATED PRESS. DISTRIBUTION.

CURVES SHOWN FOR TYPICAL VALUES: DISC RADIUS 1". SPEED 50 R.P.S.
(STEEL DISCS) OIL VISCOSITY 3μ REYNS. LOAD 500 LB/IN. WIDTH.

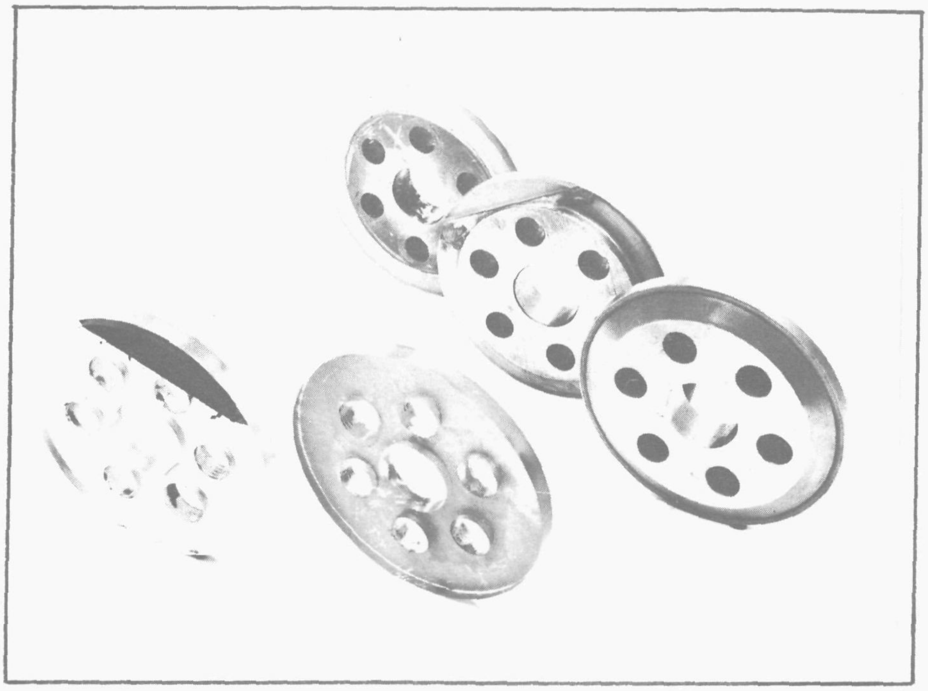


FIG. 4 DISCS USED IN EXPERIMENT.

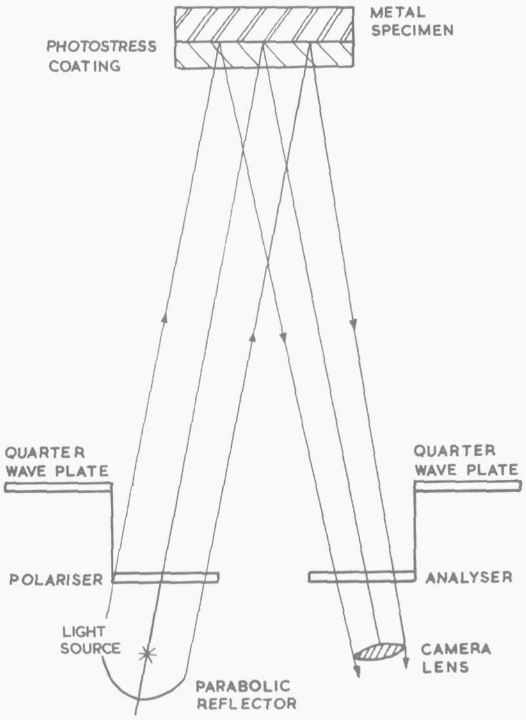


FIGURE. 5. DISC MACHINE — SCHEMATIC DIAGRAM FOR THE BUDD LARGE FIELD METER.

EXPERIMENTAL AND ANALYTICAL STRESS PATTERNS.

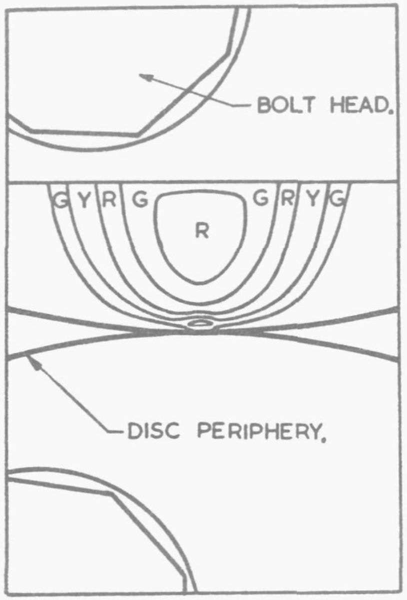


FIG. 6.

DIAGRAM OF ISOCHROMATIC PATTERN USING "PHOTOSTRESS" ON A STEEL DISC.

MAGNIFICATION 4 X.

(LARGE FIELD METER)

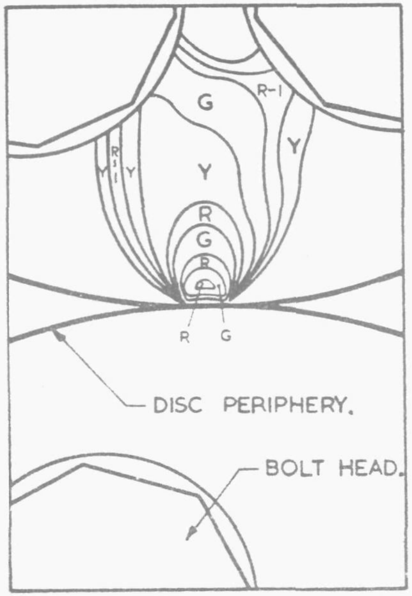


FIG. 7.

CONVENTIONAL PHOTOELASTIC PATTERN 'IN DEPTH' USING A DISC OF "PERSPEX".

MAGNIFICATION 4 X.

(LARGE FIELD METER)

COLOUR KEY

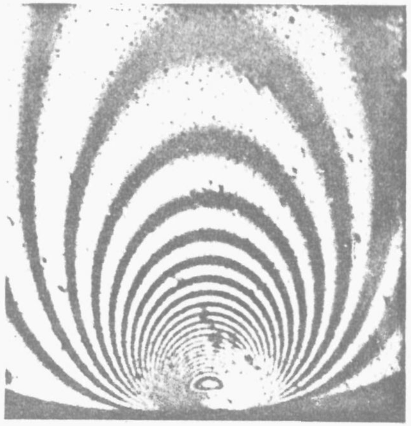
R = RED

R~I = RED~INDIGO

G = GREEN

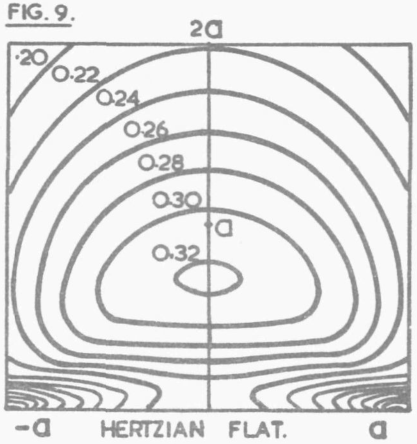
Y = YELLOW

FIG. 8.



PHOTOGRAPH OF EXPERIMENTAL STRESS PATTERN PLANE DISC LOADED AGAINST A FLAT.

FIG. 9.



THEORETICAL TWO-DIMENSIONAL CONTOURS FOR SUBSURFACE SHEAR STRESS IN TERMS OF HALF HERTZ FLAT LENGTH 'a' AND MAX HERTZ STRESS Po (MAX 0.322 Po @ DEPTH 0.705 a.)

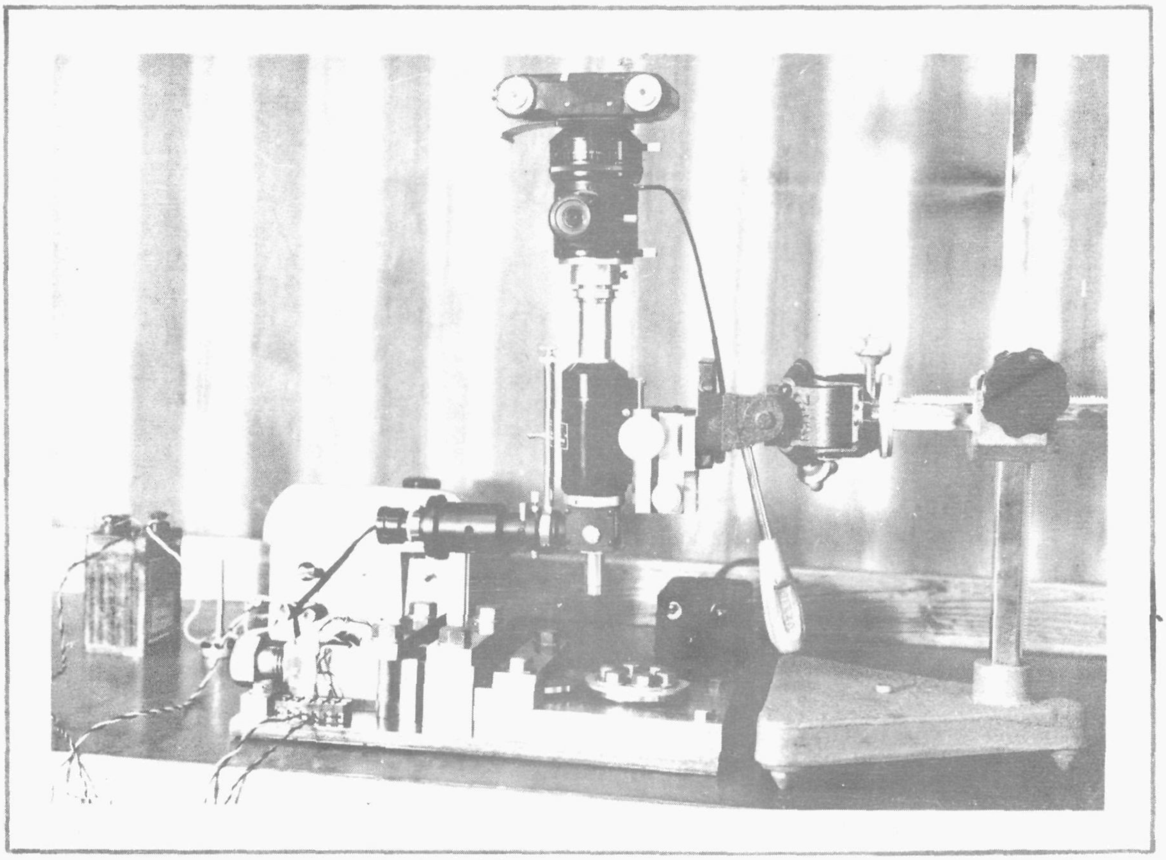


FIG. 10 STRAINING FRAME WITH SMALL FIELD METER

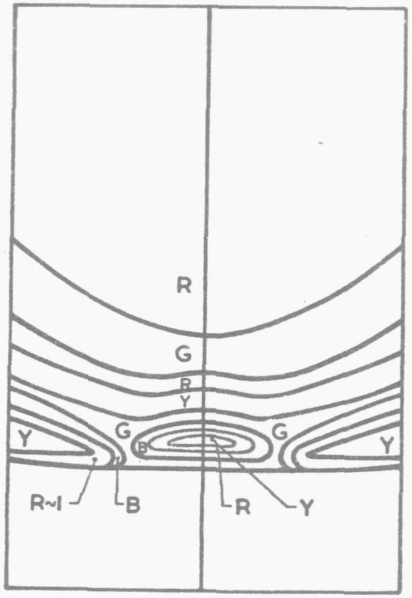


FIG. 11.

ENLARGEMENT OF FIG. 6 IN CONTACT AREA.

MAGNIFICATION 25 X

(SMALL FIELD METER.)

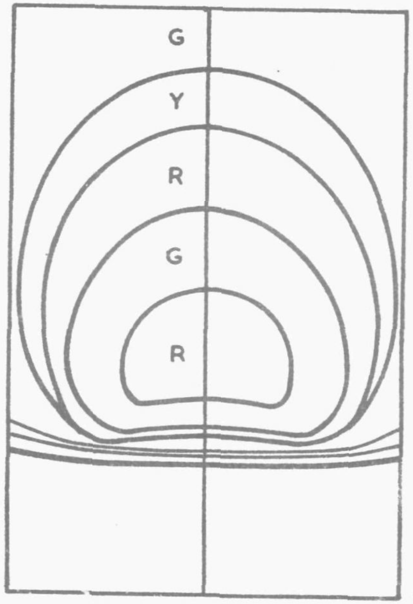


FIG. 12.

ENLARGEMENT OF FIG. 7 IN CONTACT AREA.

MAGNIFICATION 25 X.

(SMALL FIELD METER.)

COLOUR KEY

R = RED.
R~I = RED~INDIGO
G = GREEN
Y = YELLOW
B = BLUE

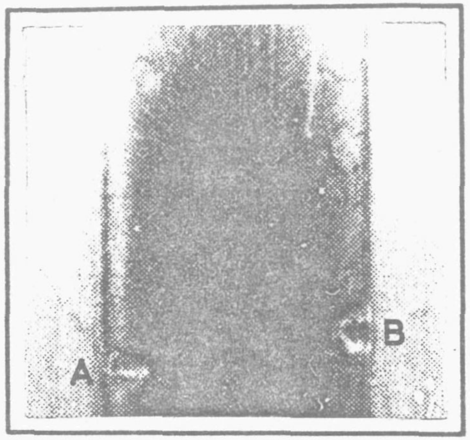


FIG. 13 PHOTOGRAPH OF EDGE PITTING (AFTER DAWSON)

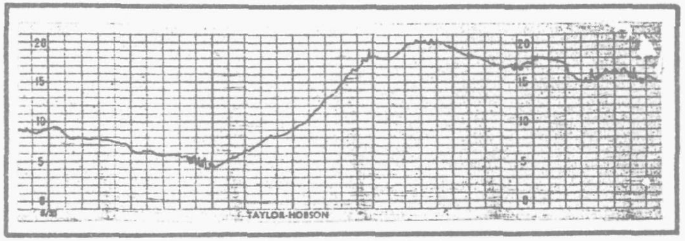
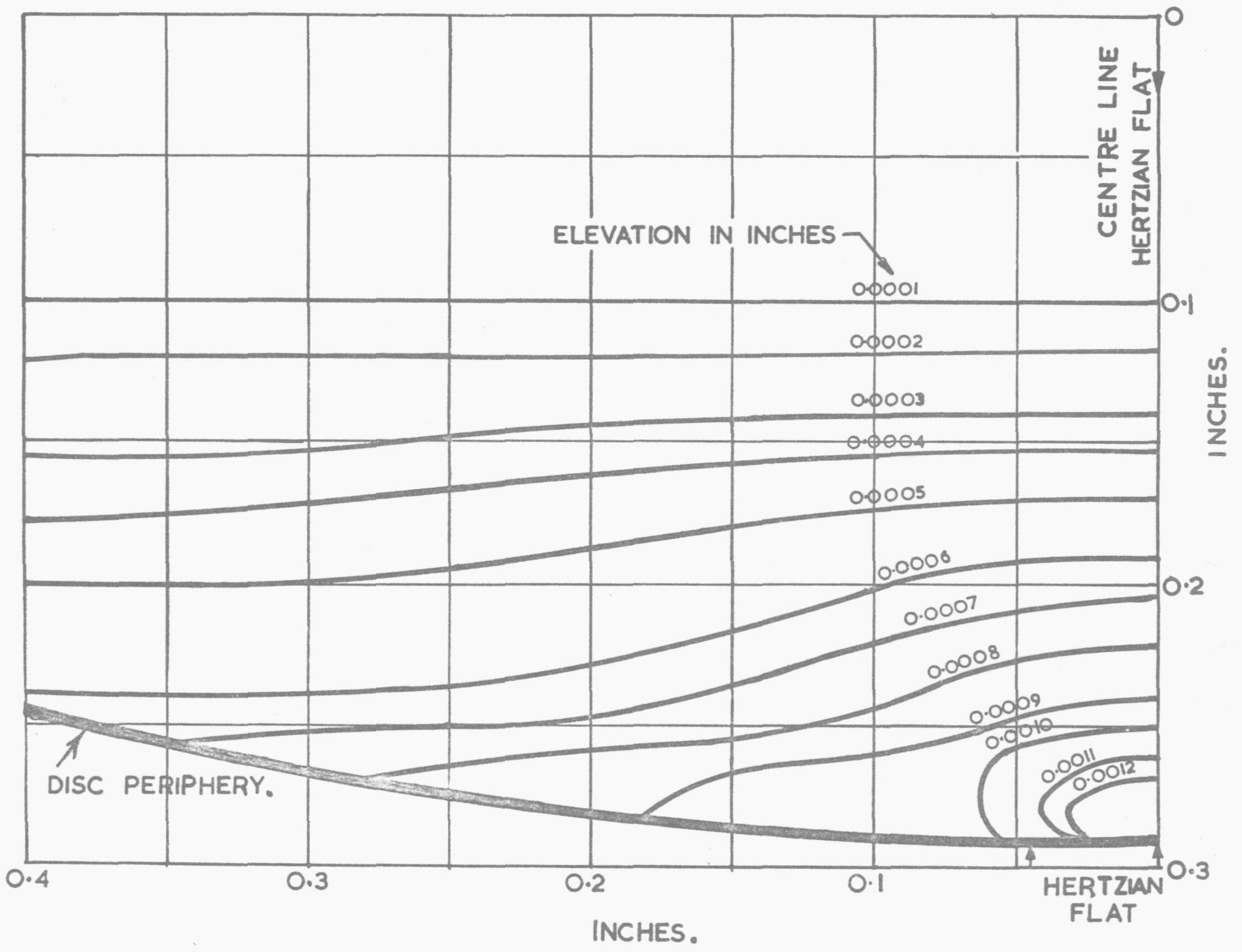


FIG. 14 "TALYSURF" TRACE OF DISC SURFACE
Scale: Vertical 2000:1 Horizontal 100:1

FIG. 15. CONTOURS OF AXIAL DISPLACEMENT FOR PLANE SURFACE OF DISC.



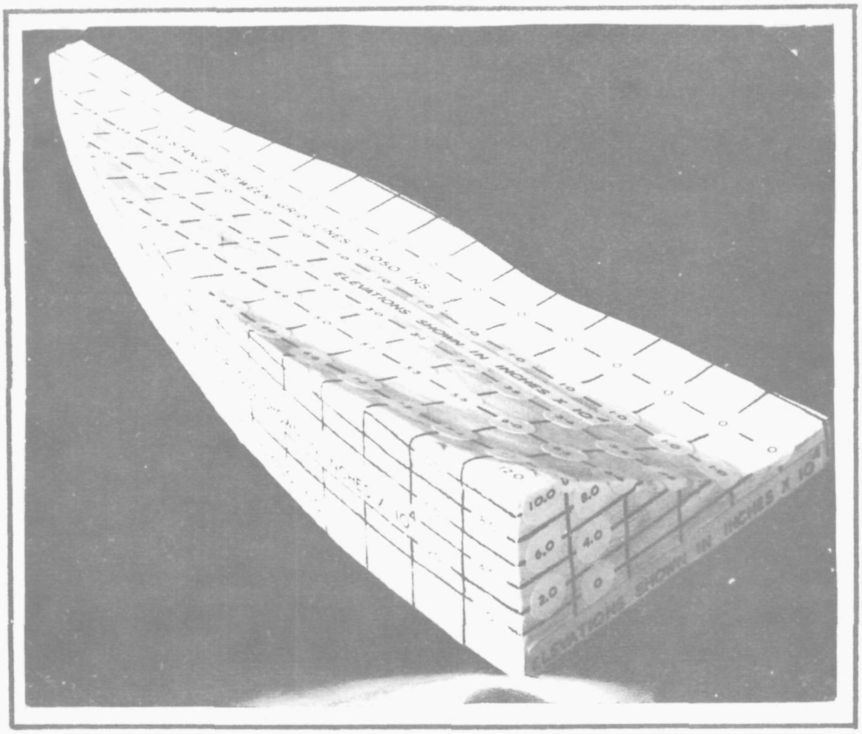


FIG. 16 THREE-DIMENSIONAL MODEL OF SURFACE DISPLACEMENTS

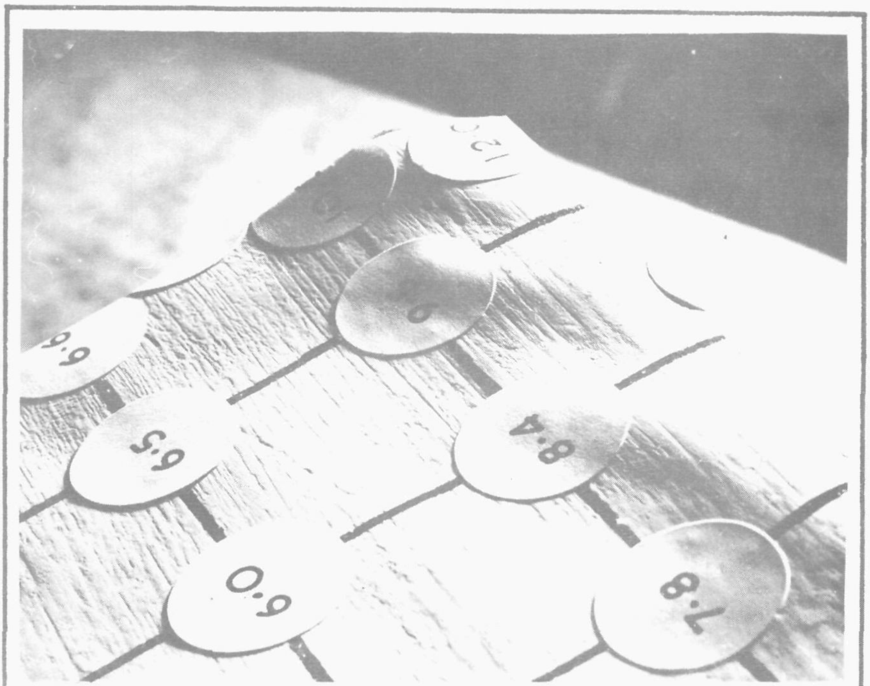
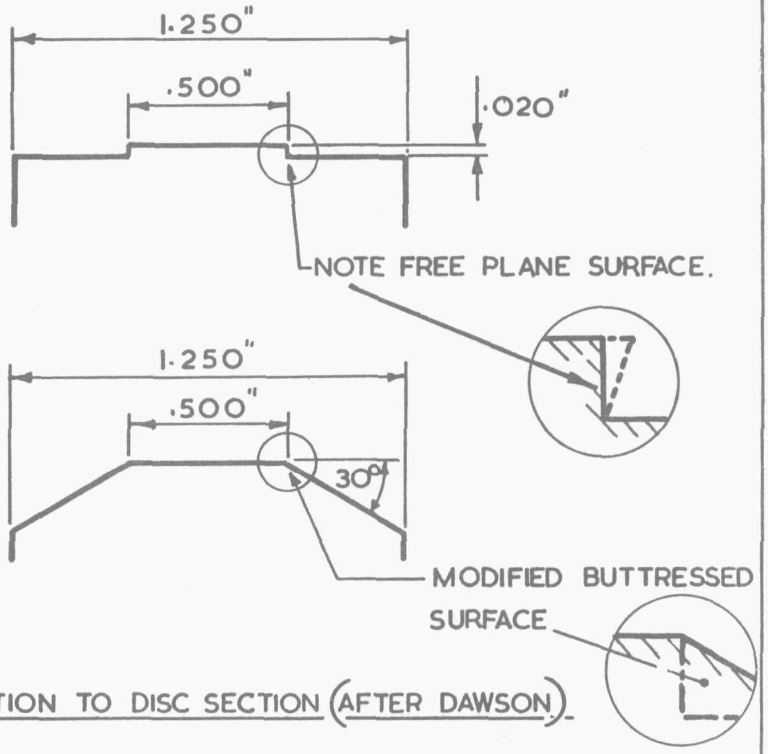


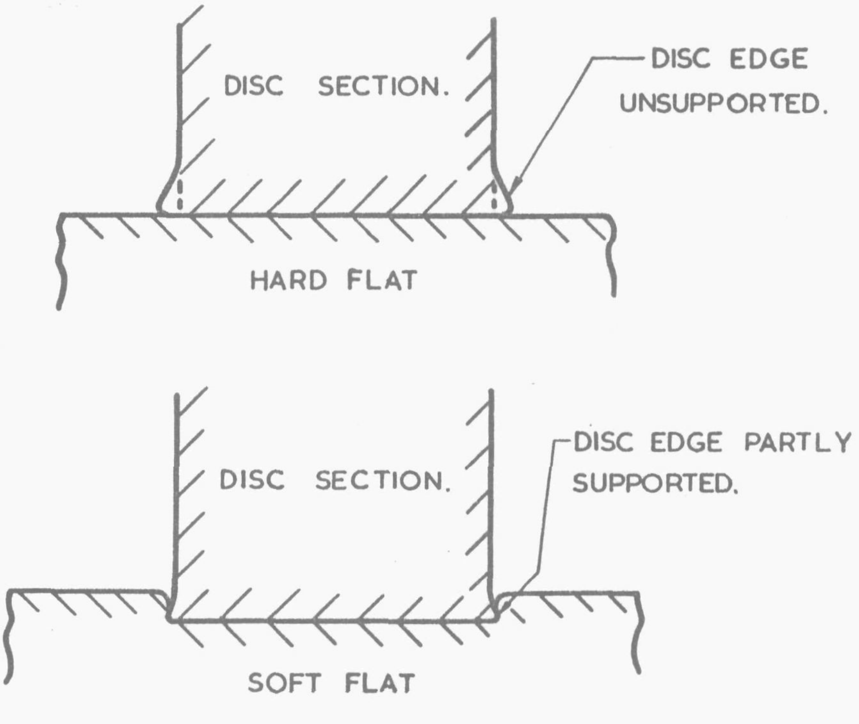
FIG. 17 DISPLACEMENTS IN THE VICINITY OF THE HERTZIAN CONTACT

FIG. 18



MODIFICATION TO DISC SECTION (AFTER DAWSON)

FIG. 19



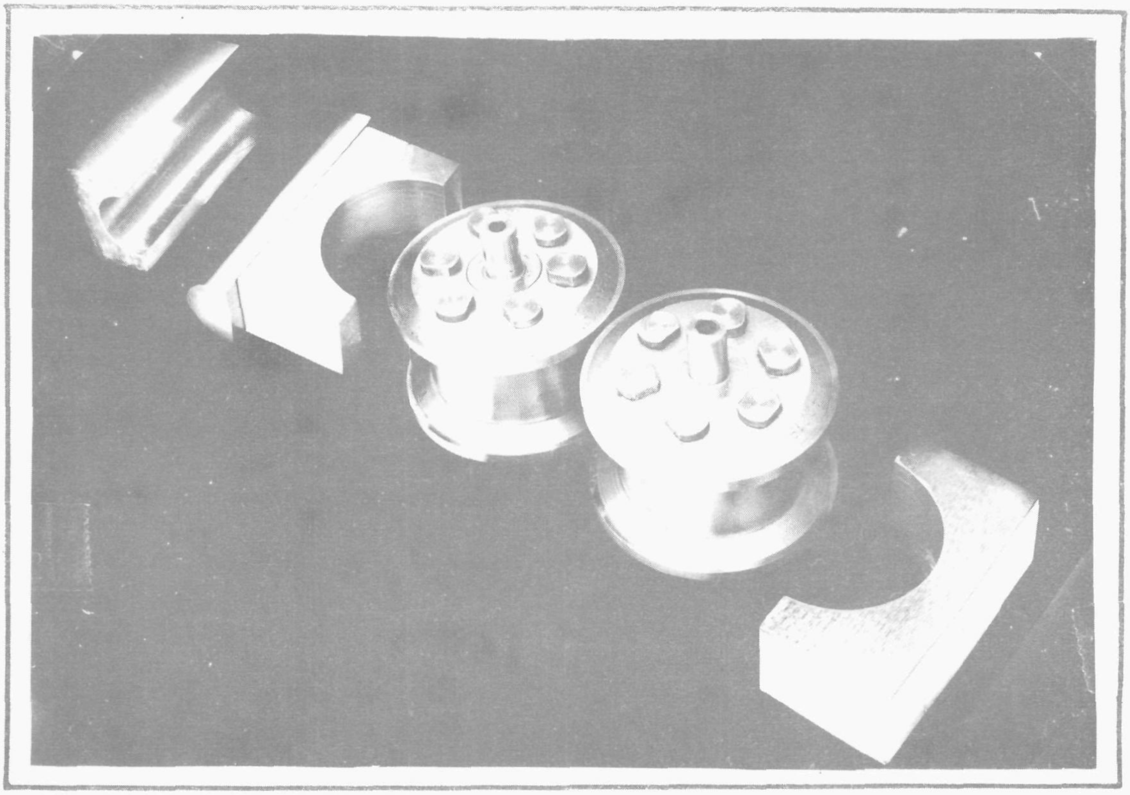
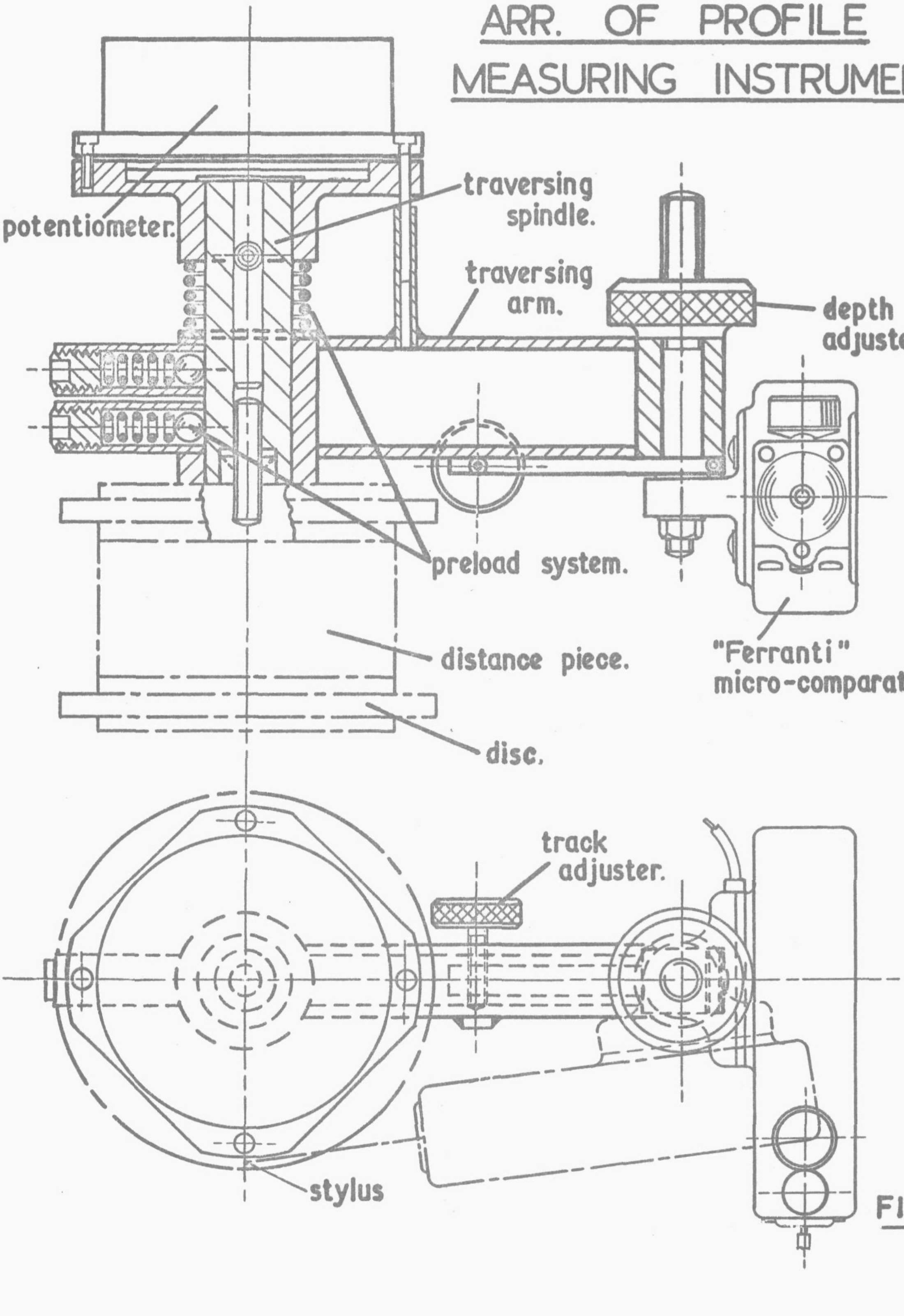


FIG. 20 DISC LOADING ARRANGEMENT (AS DESCRIBED IN APPENDIX)



ARR. OF PROFILE MEASURING INSTRUMENT.

FIG. 21.