

Towards the imaging of spin-wave mixing

Imaging and characterisation of
backward volume spin-waves
using boron vacancy centres

E. A. Aspeslagh Nielen



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by

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Emma Aspeslagh Nielen

Abstract

spin-waves are collective excitations of the magnetisation in a magnetic material and are a promising candidate for future information-processing technologies. To understand and exploit spin-wave devices, it is essential to be able to detect and image spin-waves with high spatial resolution. In this thesis, boron-vacancy (V_B^-) centres in hexagonal boron nitride were used to image spin-waves in a Permalloy film. The long-term goal is the spatial imaging of spin-wave mixing signals, where two spin-waves interact nonlinearly to generate a signal at the difference frequency.

To support these future experiments, magnet calibration procedures were developed to provide accurate control over both the strength and direction of the applied magnetic field. By introducing a corkscrew calibration trajectory and improved fitting procedures, the calibration time was reduced by approximately a factor of fifty, from around a weekend to about one hour. In addition, a simulation package with graphical user interfaces in both MATLAB and Python was developed, combining colour-centre dynamics and spin-wave dispersion calculations to rapidly determine suitable measurement conditions.

Backward volume spin-waves were systematically excited and imaged for different magnetic fields and excitation frequencies. Although clear changes in spin-wave activity were observed, the measured patterns appeared strongly scattered and did not resemble well-defined plane waves, making them unsuitable as reference material for future spin-wave mixing experiments. Furthermore, neither Fourier-transform nor power-spectral-density analysis yielded a reliable reconstruction of the backward volume dispersion. However, the inverse participation ratio proved to be a useful measure for quantifying changes in spin-wave activity and showed qualitative agreement with the expected dispersion.

Future work should therefore focus on Damon–Eshbach spin-waves, which exhibit more well-defined wavefronts and are expected to provide better reference material for spin-wave mixing measurements. Together, the magnet calibration procedures, simulation tools, and measurements presented in this thesis provide an important foundation for future spatial imaging of spin-wave mixing using colour-centre magnetometry.

*E. A. Aspeslagh Nielsen
Delft, June 2026*

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Introduction

Imagine your house is on fire. Luckily, you have four friends, five buckets, and a river nearby. What would you do? Would you ask each friend to run back and forth independently with a bucket of water, or would you form a chain and pass the buckets along? Of course, you would choose the second option, because it is much more efficient: the water is transported without everybody getting tired.

Yet our current electronics are based on the first option. Information is carried by moving electrons from one place to another, which leads to energy losses and heating [1]. Luckily, there is a potential alternative to some of the use cases of electronics, called magnonics [2, 3]. Here, the charge does carry the information; instead, the information is carried by a spin-wave: a propagating oscillation of the electron spins in the material [4]. Spin-waves are especially interesting because they can encode information not only in their amplitude, but also in their phase, wavelength, and frequency [3, 1].

To fully understand magnonics, the detection and imaging of spin-waves is essential. While electrical measurements can provide information about the presence of spin-wave excitations, they generally do not reveal their spatial structure. For example, it was shown that spin-waves propagating underneath a superconductor hybridise with Meissner currents, modifying their wavelength [5]. Such effects cannot be fully understood from electrical measurements alone, motivating the need for spin-wave imaging techniques.

Several techniques exist to detect spin-waves, including Brillouin light scattering [6], which uses inelastic light scattering to probe the wave-vector and frequency of spin-waves. Magneto-optical Kerr microscopy [7] exploits the magneto-optical Kerr effect to directly image the magnetisation, while X-ray microscopy [8] uses magnetic contrast in X-ray absorption to image spin-wave dynamics. In contrast to these techniques, in the van der Sar lab spin-waves are detected through their stray field: the oscillating magnetic field generated by the precessing spins in the magnetic material. This field is detected using quantum defects, in particular nitrogen-vacancy (NV^-) centres in diamond. [9, 10]

NV^- centres are atomic-scale defects in diamond that are highly sensitive to small oscillating magnetic fields. This enables nanoscale magnetic-field imaging of spin-waves [11]. However, NV^- -based measurements also have practical limitations. Scanning- NV^- measurements typically require integration with an AFM system, introducing the risk of tip crashes and sample damage during scanning; this type of measurement has a small field of view and is slow [9, 12]. In addition, NV^- centres in diamond membranes are limited in sensitivity and spatial resolution because the NV^- centres cannot always be positioned sufficiently close to the magnetic sample, which suppresses the detection of short-wavelength spin-wave stray fields. [13, 14, 9]

Recently, defects in hexagonal boron nitride (hBN) were discovered that could be used as an alternative. These defects are negatively charged boron-vacancy centres, V_B^- . Since hBN is a two-dimensional material, it can be placed directly on top of the magnetic material without any stand-off distance [15, 16]. The oscillating magnetic field produced by spin-waves can then be probed directly with a scanning

laser that excites the V_B^- centres, thereby significantly reducing measurement risk and time. Another advantage of V_B^- centres is that they operate at higher frequencies than the NV^- centre, allowing for the detection of spin-waves with higher frequencies. All this makes V_B^- centres an attractive platform for studying spin-waves in thin films [17].

One disadvantage of these colour centres is that they are only sensitive to oscillating magnetic fields near their electron spin resonance (ESR) frequency [16, 10]. As a result, only spin-waves with frequencies matching the ESR of the colour centre can be detected directly, which significantly limits the spin-wave band that can be imaged [9]. To overcome this limitation, one can use spin-wave mixing. By exciting two spin-waves with different frequencies, non-linear interactions can generate mixing products at the sum and difference frequencies [18]. By tuning the difference frequency to match the ESR frequency of the colour centre, the stray field generated by the mixing product can still be probed.

Spin-wave mixing has already been probed in thin YIG films [19]. However, an important next step is to image the generated mixing signal spatially. Before this can be done, it is essential to image and understand the stray field of the unmixed spin-waves. In this thesis, backward volume spin-waves in Permalloy are imaged in preparation for future spin-wave mixing experiments [20].

This thesis is structured as follows. First, the relevant theory of colour centres and spin-waves is introduced. The experimental setup is then described, followed by the determination of the spatial resolution of the setup, with a particular focus on V_B^- centres. Next the magnet calibrations performed for both the present measurements and future experiments are discussed. Thereafter, the coding package developed to combine spin-wave and colour-centre dynamics is presented. Finally, measurements of backward volume spin-waves are presented and characterised, after which the main conclusions of this thesis are discussed.

2

Colour centres for magnetometry

Colour centres, in particular the nitrogen vacancy (NV^-) and boron vacancy (V_{B}^-) centres, are used in this thesis because their photoluminescence changes when they are driven by resonant magnetic fields. I will first discuss the general properties of colour centres, focusing on their magnetic-field-dependent photoluminescence, after which I will go into detail on both the NV^- and V_{B}^- centres. Throughout this thesis, the V_{B}^- centre is used to detect spin-waves through their oscillating stray magnetic fields, while NV^- centres are used for the calibration of the applied magnetic fields.

2.1. Optical properties

A colour centre has spin-dependent optical properties that can be used to detect spin-waves. The colour centres considered here are spin-1 systems, with a ground-state spin triplet consisting of the states $|0\rangle$ and $|\pm 1\rangle$, as shown in Figure 2.1a. Their Hamiltonian, in frequency units, is given by:

$$H = DS_z^2 + \gamma B \cdot S \quad (2.1)$$

where D is the zero-field splitting, γ is the gyromagnetic ratio of the electron spin, B is the externally applied magnetic field, and S is the spin-1 operator of the colour centre. The first term describes the intrinsic splitting between the $|0\rangle$ and $|\pm 1\rangle$ states, while the second term describes the Zeeman interaction with the external magnetic field. This Hamiltonian shows that the resonance frequencies are set by the zero-field splitting and by the magnetic-field projection onto the colour-centre spin axis.

Optical excitation makes the spin state visible because different spin states have different probabilities to emit a photon. To excite the colour centre from the ground state to an excited state, a laser is applied. After which it can relax back to the ground state. If the spin state is preserved during this relaxation, a photon is emitted. However, for the $|\pm 1\rangle$ states there is also a non-radiative decay path through an intermediate state, which returns the system to the $|0\rangle$ ground state without emitting a photon. As a result, the emitted photoluminescence (PL) is spin dependent. [10]

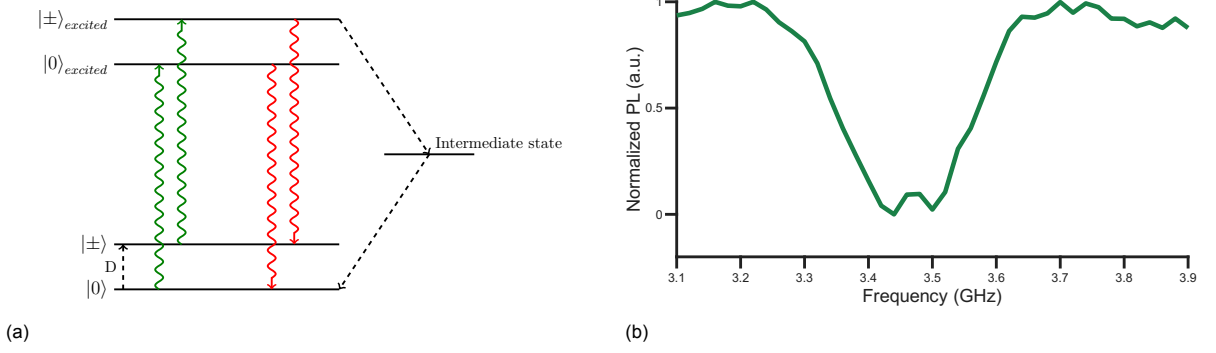


Figure 2.1: **Optical properties of colour centres.** (a) Energy levels of a spin-1 colour centre. The $|0\rangle$ and $|\pm 1\rangle$ states are split by the zero-field splitting D . Optical excitation is mostly spin preserving, but the $|\pm 1\rangle$ states can also decay through an intermediate state without emitting a photon. (b) ESR spectrum of V_B^- centres in hBN. The photoluminescence decreases when the microwave frequency is resonant with the transition near $D = 3.46$ GHz, giving an ESR dip.

An oscillating magnetic field at the resonance frequency between $|0\rangle$ and $|\pm 1\rangle$ drives transitions into the darker $|\pm 1\rangle$ states, resulting in a dip in photoluminescence when the colour centre is continuously pumped by a laser. Detecting these PL dips as a function of the applied microwave frequency is known as optically detected magnetic resonance (ODMR), a form of electron spin resonance (ESR), and is shown for V_B^- centres in Figure 2.1b. Since spin-waves produce oscillating magnetic stray fields, they can drive these spin transitions and can therefore be detected optically using colour centres [21, 17].

2.2. Magnetic field dependency of colour centres

An applied magnetic field lifts the degeneracy of the $|\pm 1\rangle$ states, resulting in two distinct ESR frequencies, as can be seen from equation 2.1. This interaction is also known as the Zeeman interaction. Figure 2.2 shows this splitting for both the V_B^- (green) and NV^- centre (blue), for a parallel, canted, and perpendicular applied magnetic field. To first order, the ESR splitting is determined by the magnetic-field component parallel to the defect axis, while perpendicular fields mainly cause state mixing and second-order shifts.

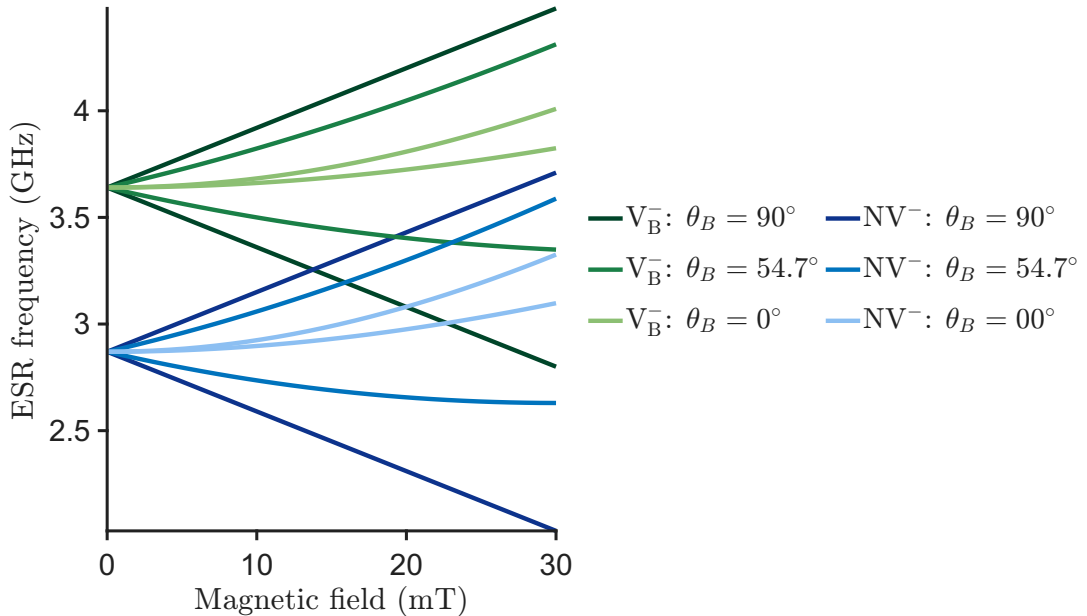


Figure 2.2: **ESR frequencies as a function of magnetic field for the V_B^- and NV^- centre.** The green curves correspond to the V_B^- centre, while the blue curves correspond to the NV^- centre. For each colour centre, three orientations of applied magnetic field with respect to the centre's axis are shown: parallel ($\theta_B = 0^\circ$), canted ($\theta_B = 54.7^\circ$), perpendicular ($\theta_B = 90^\circ$). The splitting between the two ESR branches increases with the magnetic-field component parallel to the defect axis while perpendicular fields mainly cause state mixing and second-order shifts.

2.3. Nitrogen vacancy centres

NV^- centres in diamond membranes are used in this thesis as vector magnetometers for calibrating the applied magnetic field. An NV^- centre consists of a substitutional nitrogen atom next to a vacancy in the diamond lattice. Because of the cubic diamond structure, the NV^- centre can have four possible orientations along the $\langle 111 \rangle$ crystal directions, as shown in Figure 2.3. Each orientation measures a different projection of the applied magnetic field, so each NV^- family can have a different ESR frequency. In total, up to eight ESR transitions can be observed, with two transitions for each NV^- orientation. In the $[100]$ diamond membranes used here, two NV^- families point partly towards positive z , and two point partly towards negative z , all at an angle of 54.7° with respect to the z -axis. Since this geometry is fixed by the diamond lattice, the NV^- membrane provides a reliable way to determine both the strength and orientation of the magnetic field.

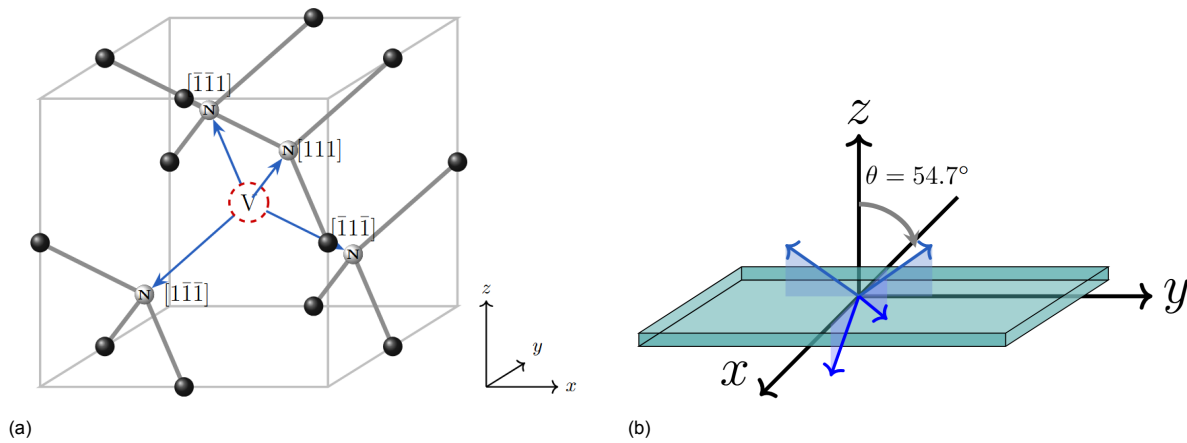


Figure 2.3: **Crystallographic structure of the NV^- centre and diamond membrane.** (a) Schematic diamond lattice with carbon atoms in black, a vacancy indicated by a red dashed circle, and the four possible nitrogen positions shown in white. An NV^- centre consists of one vacancy and one neighbouring nitrogen atom, giving four possible NV^- orientations along the $\langle 111 \rangle$ directions. (b) Schematic NV^- $[100]$ diamond membrane, with the surface in the xy -plane. The four NV^- axes make an angle of 54.7° with the z -axis: two point towards positive z and two towards negative z . The different blue shades and shaded regions indicate the direction of the NV^- axes relative to the membrane plane.

2.4. Boron vacancy centres

In this thesis, boron vacancy (V_B^-) centres in hexagonal boron nitride (hBN) are used to probe spin-waves. A key advantage of hBN is that thin flakes can be transferred directly onto the sample surface, resulting in a near-zero sensor-sample distance. As a consequence, the spatial resolution is primarily limited by the diffraction limit of the laser rather than by the sensor-sample separation. This is possible because hBN is a van der Waals material consisting of weakly bonded two-dimensional layers that can be exfoliated and transferred onto a sample, as shown in Figure 2.4a. The small sensor-sample distance makes hBN suitable for detecting short-wavelength spin-wave stray fields.

The larger zero-field splitting of the V_B^- centre, $D \sim 3.46$ GHz, allows it to detect higher-frequency spin-waves than the NV^- centre, $D \sim 2.87$ GHz, at zero field.

In the hBN the V_B^- centres are created; this can be achieved using either neutron irradiation [22] or helium ion implantation [23]. The V_B^- point defect is shown in Figure 2.4b. The quantisation axis of this defect, indicated by the green arrow in Figure 2.4b, is oriented out of plane. The advantage of this out-of-plane axis is that it simplifies the application of external magnetic fields, as we can use magnets with only an in-plane field component to tune the spin-wave band, while the V_B^- transition frequency is, to first order, relatively insensitive to in-plane magnetic fields [17].

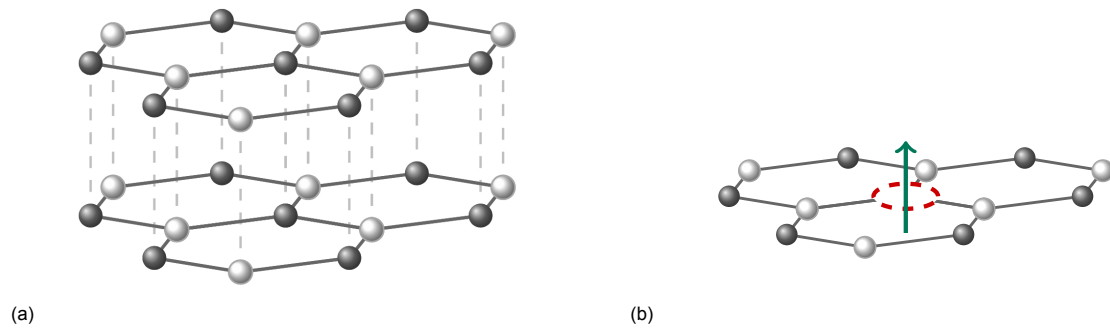


Figure 2.4: **Crystal structure of hBN, with boron atoms in grey and nitrogen atoms in white and the boron vacancy (red circle).** (a) Layered structure of hBN. It consists of clearly defined two-dimensional hexagonal layers held together by van der Waals bonds (dashed lines) (b) Schematic representation of a boron vacancy defect and its quantisation axis. The red dashed circle indicates the boron vacancy. The quantisation axis of the V_B^- centre is shown in green and is normal to the hBN plane.

3

Spin-wave excitation

This chapter describes spin-wave properties and how to characterise them using their dispersion. First, I will derive the spin-wave dispersion, after which I will show how we can excite them using microstrips. Then I will explain how colour centres detect the stray field produced by these spin-waves and how spin-wave mixing can enhance these processes.

3.1. Spin-wave dispersion

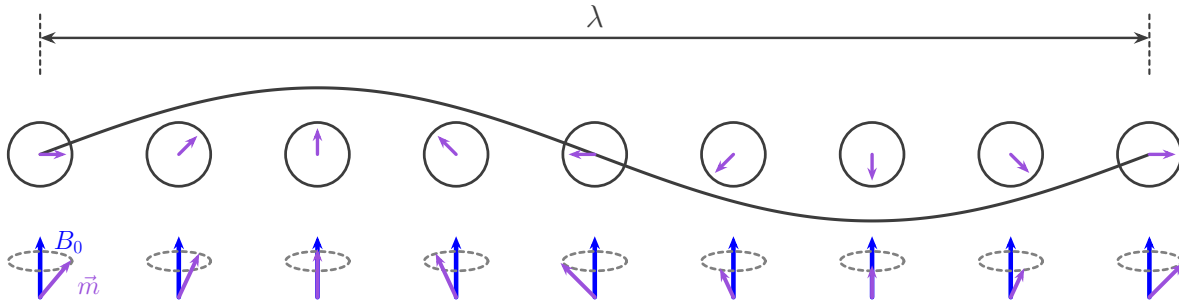


Figure 3.1: **Schematic representation of a propagating spin-wave with wavelength λ .** The top row shows the top view of a propagating spin-wave, while the bottom row illustrates the local magnetisation precession around the equilibrium magnetic field B_0 , shown in blue. The purple arrows indicate the magnetisation $\vec{m}$ and the dashed circles represent the precessional trajectories. The spatial phase difference between neighbouring spins gives rise to a spin-wave profile. A spin-wave is therefore a spatially periodic phase pattern of the local magnetisation precession.

Spin-waves are collective magnetic excitations in a magnetic material, shown in Figure 3.1. Their associated quasi-particle is called a magnon, hence the name of the field: magnonics. We describe these magnons using the Landau-Lifshitz-Gilbert (LLG) equation [24], which describes the dynamics of spins inside a magnetic material:

$$\frac{d\mathbf{m}}{dt} = -\gamma \mathbf{m} \times \mathbf{B}_{\text{eff}} - \alpha \frac{d\mathbf{m}}{dt} \times \mathbf{m}. \quad (3.1)$$

Here, $\mathbf{m}$ is the unit magnetisation vector. The first term describes the torque caused by the effective magnetic field present in the material, $\mathbf{B}_{\text{eff}}$, which causes the spins to precess around it. The second term describes damping, with α the Gilbert damping coefficient, which causes the spins to relax back to their equilibrium position. These dynamics are shown in Figure 3.2a.

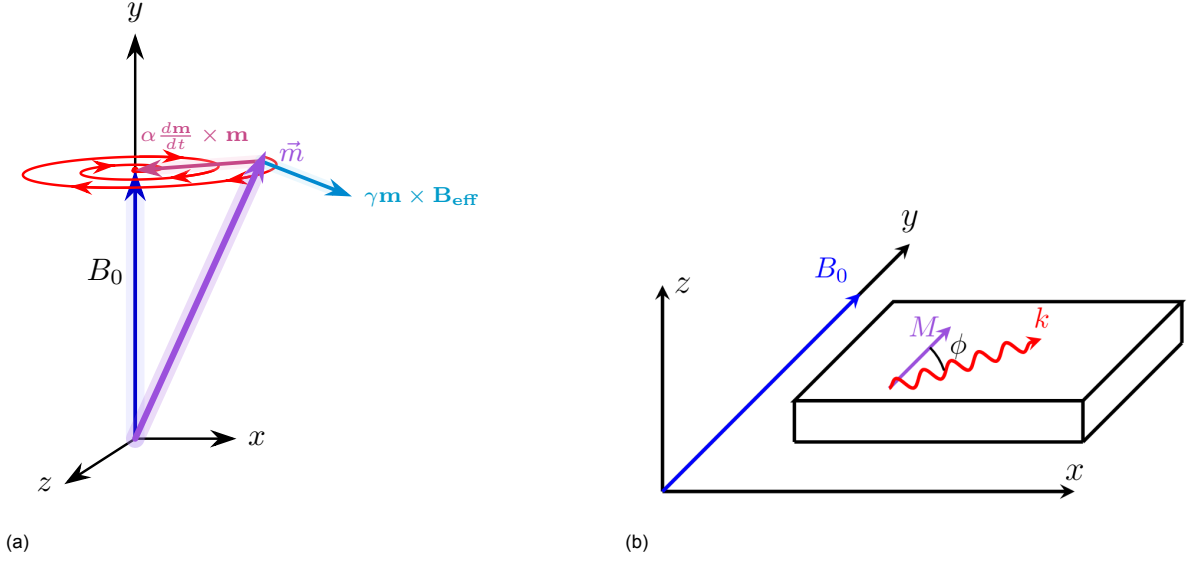


Figure 3.2: **Geometry of a single spin and magnetic thin film.** (a) A spin precessing around an applied magnetic field B_0 . The magnetic field causes a torque, shown in light blue, on the spin, which causes it to precess. The damping term is shown in pink as $\alpha \frac{d\mathbf{m}}{dt} \times \mathbf{m}$. (b) Geometry considered for the derivation. A thin film lies in the xy -plane, with an external magnetic field B_0 along the y -direction. M depicts the magnetisation of the thin film, which is also along y , k is the direction of the propagating spin-wave, and ϕ is the in-plane angle between M and k .

The effective magnetic field arises from multiple interactions inside the magnetic material:

$$\mathbf{B}_{\text{eff}} = \mathbf{B}_{\text{exchange}} + \mathbf{B}_{\text{demagnetising}} + \mathbf{B}_{\text{Zeeman}} + \mathbf{B}_{\text{AC}} \quad (3.2)$$

Here, $\mathbf{B}_{\text{exchange}}$ is the exchange field arising from the interaction between neighbouring spins, $\mathbf{B}_{\text{demagnetising}}$ is the demagnetising field due to the dipole-dipole interaction of spins, $\mathbf{B}_{\text{Zeeman}}$ represents the contribution from the externally applied magnetic field, and $\mathbf{B}_{\text{AC}}$ is the external oscillating field used to excite spin-waves. Together, the exchange, demagnetising and Zeeman interaction determine the spin-wave dispersion in the magnetic material, while $\mathbf{B}_{\text{AC}}$ excites the spin-waves. In the following sections, these individual contributions are discussed in order of increasing interaction range, for the geometry shown in Figure 3.2b.

3.1.1. Exchange interaction

The exchange interaction arises from the quantum-mechanical overlap of neighbouring electron wave-functions, combined with the Pauli exclusion principle, causing neighbouring spins to preferentially align parallel in a ferromagnet. The Heisenberg model describes the interaction between neighbouring spins as,

$$H = - \sum_{ij} J_{ij} \mathbf{s}_i \cdot \mathbf{s}_j, \quad (3.3)$$

where $\mathbf{s}_i$ is the i^{th} spin and J_{ij} is the exchange constant between neighbouring spins. For a ferromagnet, $J_{ij} > 0$, which is the case considered in this thesis. To minimise the energy of the system, the spins align to minimise the energy, illustrated in Figure 3.3.

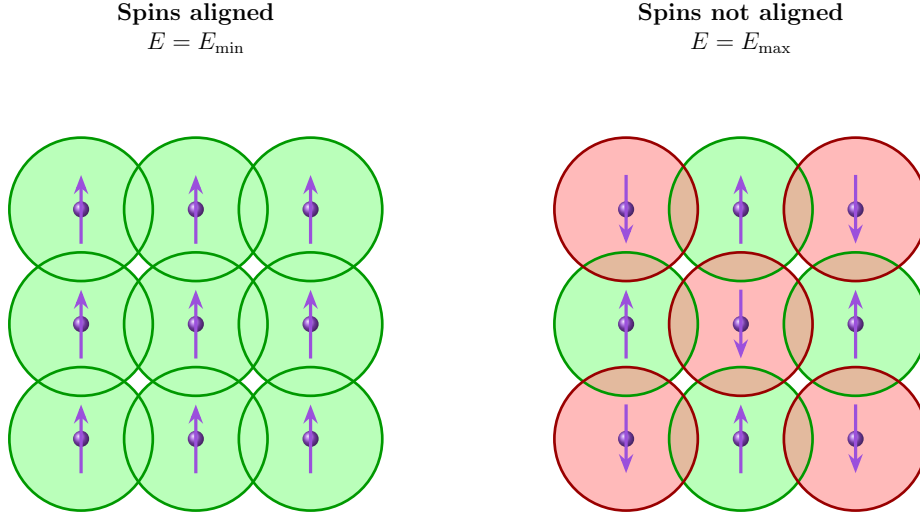


Figure 3.3: **Illustration of the exchange interaction for a ferromagnet.** The coloured circles schematically represent neighbouring electron wavefunctions. Green regions schematically indicate energetically favourable overlap configurations, where neighbouring spins are aligned parallel, lowering the total energy of the system. Red regions indicate energetically unfavourable configurations for antiparallel spin alignment, resulting in a higher total energy.

Using this fact and following the derivation detailed in Appendix A.0.1, we arrive at the isotropic magnetic field caused by this exchange interaction

$$\mathbf{B}_{\text{ex}}(\mathbf{k}, z) = -\frac{\omega_{\text{ex}}}{\gamma} k^2 \mathbf{m}(\mathbf{k}, z) \quad (3.4)$$

Here, $m_x(\mathbf{k}, z)$, $m_y(\mathbf{k}, z)$ and $m_z(\mathbf{k}, z)$ are the Fourier components of the magnetisation, ω_{ex} is the exchange-frequency coefficient, γ is the gyromagnetic ratio, and $k = |\mathbf{k}|$ is the magnitude of the spin-wave wave vector. The exchange interaction is isotropic, as it depends only on absolute k , and its k^2 dependence causes it to dominate the spin-wave dynamics at large wave vectors (small wavelengths).

3.1.2. Demagnetising interaction

The demagnetising field is the dipole field a spin experiences from all other spins in the material. This dipole field in a ferromagnet causes a torque on the magnetic moment of the spin that tries to pull the spin out of alignment and thus lowers the overall magnetisation, hence the name, demagnetising interaction. This interaction is illustrated in Figure 3.4a. Following the derivation detailed in Appendix A.0.2 for a thin film as seen in Figure 3.2b, we can define the following contribution of the dipolar interaction between spins,

$$\mathbf{B}_{\text{dem}}(\mathbf{k}) = \frac{\omega_{\text{demag}}}{\gamma} f(kt) \begin{pmatrix} \cos \phi (\cos \phi m_x(\mathbf{k}, z) + \sin \phi m_y(\mathbf{k}, z)) \\ \sin \phi (\cos \phi m_x(\mathbf{k}, z) + \sin \phi m_y(\mathbf{k}, z)) \\ m_z(\mathbf{k}, z) \end{pmatrix} - \frac{\omega_{\text{demag}}}{\gamma} \begin{pmatrix} 0 \\ 0 \\ m_z(\mathbf{k}, z) \end{pmatrix} \quad (3.5)$$

Here, ω_{demag} is the demagnetising frequency, γ is the gyromagnetic ratio, $f(kt)$ describes the film thickness-dependent dipolar form factor, ϕ is the angle between the spin-wave wave vector and the equilibrium magnetisation, and $m_{x,y,z}$ are the Fourier-transformed magnetisation components. The final term represents the static out-of-plane demagnetising field contribution. Noticeably, the demagnetising field is anisotropic, which is due to the fact that we have a thin film and not a bulk system. In contrast to the exchange interaction, the demagnetising interaction is anisotropic and depends on the propagation direction of the spin-wave. Since it originates from a long-range dipolar interaction, it is most important at small wave vectors (large wavelengths).

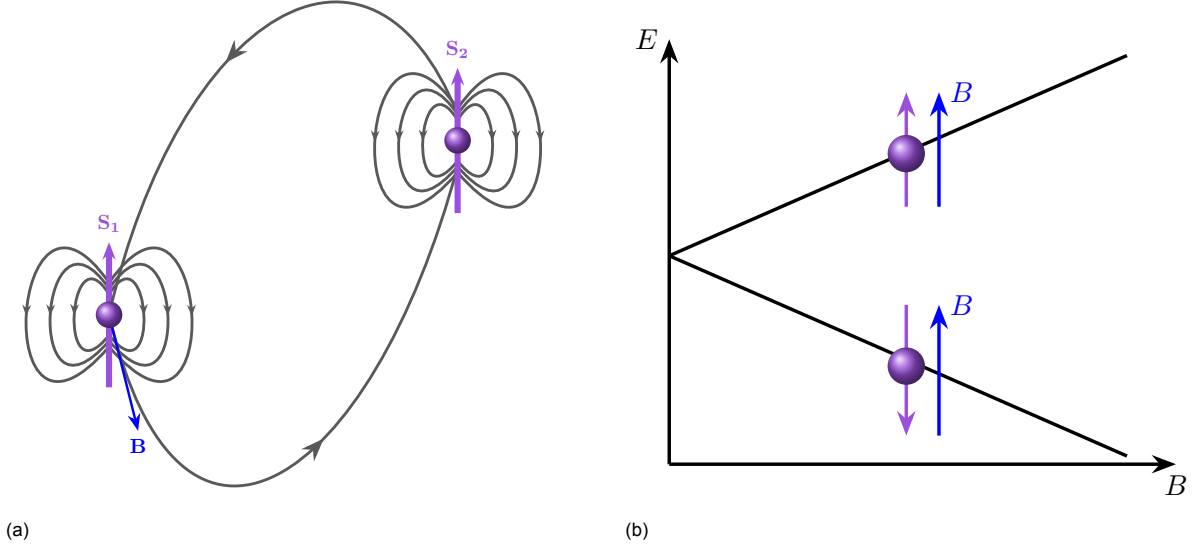


Figure 3.4: **Illustrations of the dipole-dipole and Zeeman interaction** (a) Schematic illustration of the dipole-dipole interaction between two spins in a material. The dipolar field generated by one spin, $\mathbf{S}_2$, produces a magnetic field at the position of another spin, $\mathbf{S}_1$. This field, B , produces a magnetic torque on the magnetic moment of the other spin, $\mathbf{S}_2$, causing it to misalign with the other spin. (b) An externally applied magnetic field couples differently to different spin states. States whose spins are aligned with the field have a higher energy than states that are anti-aligned, resulting in a splitting of the energy levels.

3.1.3. Zeeman interaction

The effect of an externally applied magnetic field is included via the Zeeman interaction. This interaction originates from the coupling between the magnetic moment of the electron and the external magnetic field. The Zeeman interaction introduces a uniform effective magnetic field given by

$$\mathbf{B}_{\text{Zeeman}} = \frac{\omega_B}{\gamma} \hat{y}, \quad \text{with} \quad \omega_B = \gamma B_0, \quad (3.6)$$

where B_0 is the externally applied magnetic field and γ is the gyromagnetic ratio of the electron. The spins then precess around this effective field with the Larmor frequency ω_B . As a result, spins aligned parallel and antiparallel to the field obtain different energies, as shown in Figure 3.4b, lifting the degeneracy of the spin states.

3.1.4. From effective fields to the spin-wave dispersion

To calculate the spin-wave dispersion, we can now add all contributing field terms together and insert them into the LLG equation 3.1. Before doing this, we linearise the LLG equation by assuming that the precessing motion of the spin is perpendicular to its equilibrium magnetisation, and that there is no oscillation along the direction of the equilibrium magnetisation. Full derivation detailed in Appendix A.1.

Spin-waves are created when the susceptibility tensor diverges. The susceptibility tensor describes how the material responds to an applied magnetic field, or equivalently, the "willingness" of the spins in the material to follow an external oscillating magnetic field. When the susceptibility diverges, this response becomes maximal, resulting in the excitation of spin-waves. The susceptibility tensor is given by,

$$\chi = \frac{\gamma}{\Lambda} \begin{pmatrix} i\omega & -\omega_2 + i\alpha\omega \\ -\omega_3 + i\alpha\omega & -i\omega \end{pmatrix} \quad (3.7)$$

with $\Lambda = \omega^2 - (\omega_2 - i\alpha\omega)(\omega_3 - i\alpha\omega)$. Here, ω_2 and ω_3 contain the Zeeman, exchange contributions, and the dipolar interaction. The anisotropic nature of the dipolar interaction causes the spin-wave dispersion to depend on the propagation direction ϕ . We can find the spin-wave dispersion relation by setting $\Lambda = 0$. The real part of this solution is:

$$\omega_{\text{spin-wave}} \approx \sqrt{\omega_2 \omega_3}. \quad (3.8)$$

Due to the anisotropy of the spin-wave dispersion, there are three important cases to consider:

- $k = 0$, also called the ferromagnetic resonance (FMR). All spins precess with the same phase, shown with a star in Figure 3.5.
- $\phi = 0, \pi$, these spin-waves propagate parallel to the magnetisation and are known as backward volume (BV) spin-waves, shown in blue in Figure 3.5.
- $\phi = \pm \frac{\pi}{2}$, these spin-waves propagate perpendicular to the magnetisation and are known as Damon-Eshbach (DE) spin-waves or surface spin-waves, shown in red in Figure 3.5.

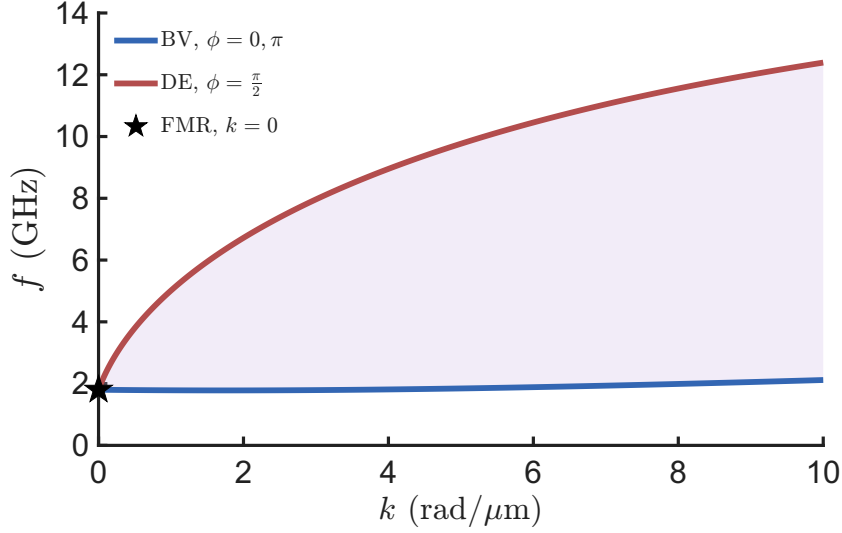


Figure 3.5: **1D spin-wave dispersion.** The Damon-Eshbach (DE) and backward volume (BV) spin-wave dispersion branches for an in-plane field of 4 mT in a 55 nm thick Permalloy film. The FMR at $k = 0$ is shown with a star. The shaded region shows the range of dispersion frequencies for propagation angles between the BV and DE branches.

3.2. Microstrip induced spin-wave excitations

In this thesis, spin-waves are excited using the microwave magnetic field generated by a microstrip. In Appendix A.2, the stray field generated by a microstrip is derived for the geometry shown in Figure 3.6.

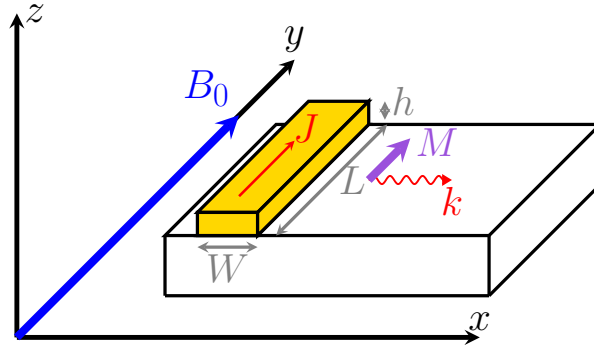


Figure 3.6: **Microstrip geometry.** A thin film in the xy plane with a microstrip of width W , height h and length L . The current distribution in the microstrip is along the $+y$ direction. It excites spin-waves perpendicular to its interface, along k .

The microstrip excites spin-waves with an efficiency set by the Fourier transform of its current distribution and shape. The result of the derivation of the microstrip stray field is, in Fourier space,

$$\widetilde{B}_x(\mathbf{k}, z, \omega) = 2\mu_0 \underbrace{J_y(\omega)}_{\text{current distribution}} \underbrace{e^{\kappa z} \frac{e^{-\kappa h} - 1}{\kappa} e^{-ik_y y}}_{\text{decay factor}} \underbrace{\frac{\sin(k_x w/2)}{k_x} \frac{\sin(k_y L/2)}{k_y}}_{\text{FT of rectangular microstrip}} \quad (3.9)$$

$$\widetilde{B}_z(\mathbf{k}, z, \omega) = -2i\mu_0 \underbrace{J_y(\omega)}_{\text{current distribution}} \underbrace{\frac{e^{\kappa z} e^{-\kappa h} - 1}{\kappa^2} e^{-ik_y y}}_{\text{decay factor}} \underbrace{\sin(k_x w/2) \frac{\sin(k_y L/2)}{k_y}}_{\text{FT of rectangular microstrip}} \quad (3.10)$$

or in short notation

$$\widetilde{B}_x(\mathbf{k}, z, \omega) = i \frac{\kappa}{k_x} \widetilde{B}_z(\mathbf{k}, z, \omega). \quad (3.11)$$

An important result that we will use later in this thesis is that the magnetic stray field generated by a microstrip is directly proportional to the Fourier transform of its shape and current distribution. From these expressions, several conclusions can be drawn. Firstly, not every wave vector is efficiently excited. The term $\sin(k_x \frac{w}{2})$ imposes a selection rule that suppresses excitations at

$$k_x = \frac{2n\pi}{w}, \quad (3.12)$$

where the sine term vanishes. Consequently, these wave vectors are less efficiently excited by the microstrip. This behaviour is illustrated in Figs. 3.7a and 3.7b.

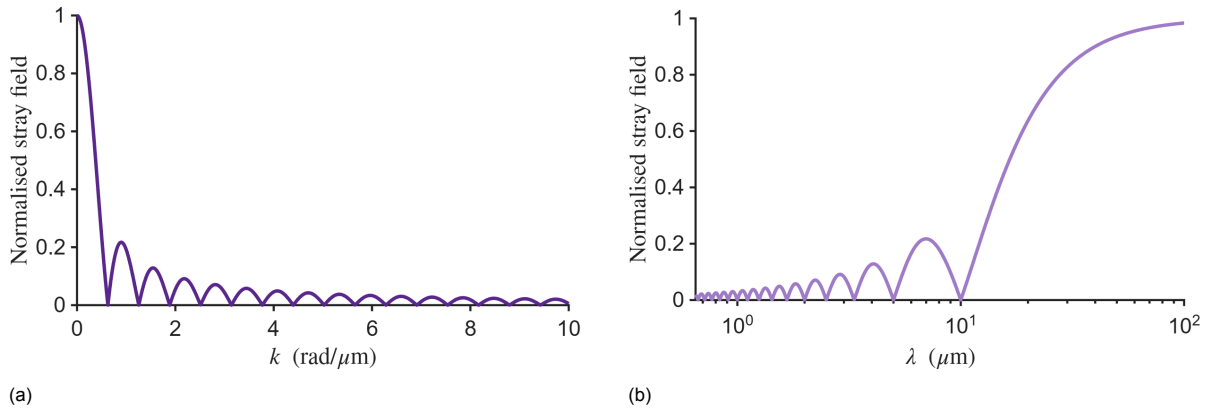


Figure 3.7: **Excitation efficiency of a microstrip.** (a) Wave vectors k that can be efficiently excited by a microstrip with a width of $10 \mu\text{m}$. (b) Corresponding spin-wavelengths λ that can be efficiently excited.

Additionally, the real parts of the microstrip field are shown in Figure 3.8. From this, we observe that the field component along x follows a profile similar to the shape of the microstrip itself. In contrast, the field component along z changes sign at the edges of the microstrip. This sign reversal indicates that the microstrip field is not purely linearly oscillating, but instead possesses a spatial curl or handedness. This behaviour can also be identified directly from equation 3.11, where the factor i shows that B_x and B_z are 90° out of phase.

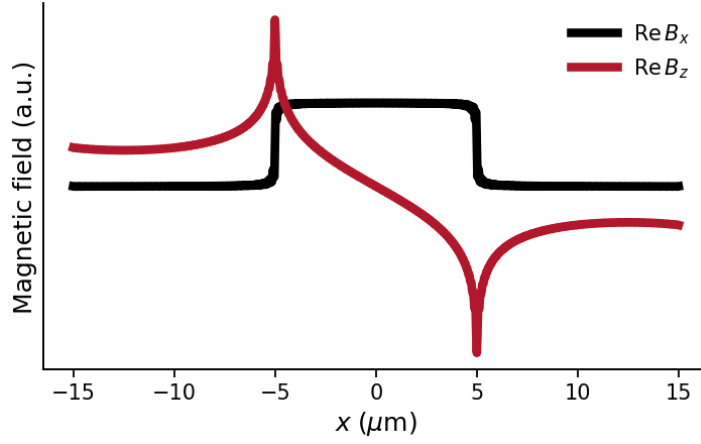


Figure 3.8: **Real parts of the magnetic stray field generated by the microstrip.** The in-plane component B_x (black) follows the shape of the microstrip, while the out-of-plane component B_z (red) changes sign at the strip edges. The phase difference between B_x and B_z gives the microstrip field a circularly polarised character.

Due to the spatial curl of the z -component, there is a difference in excitation efficiency of the DE and BV modes.

- When $M \perp k$, the magnetisation is parallel to the microstrip and Damon–Eshbach spin-waves are excited. In this geometry, the phase difference between B_x and B_z gives the $+k$ and $-k$ wave vectors opposite handedness. The spin-wave precession is driven by the magnetic field components perpendicular to the magnetisation direction; both B_x and B_z contribute to the excitation. This is illustrated in Figure 3.9, where the individual contributions of B_x (black) and B_z (red), as well as the resulting spin motion, are shown. Because spin-waves only couple efficiently to the microwave field with the correct handedness, one propagation direction is excited efficiently while the coupling for the opposite direction is suppressed. This results in directional spin-wave excitation.

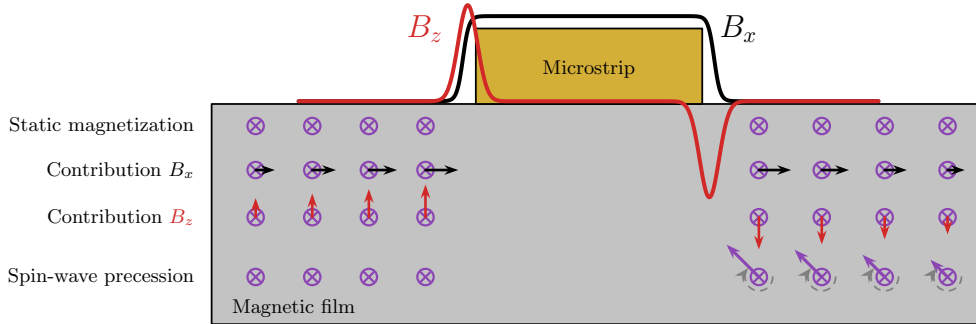


Figure 3.9: **Illustration of directional excitation of Damon–Eshbach spin-waves.** Cross-section of the microstrip in the x - z plane for the Damon–Eshbach geometry, where $M \perp k$. The microwave field components B_x (black) and B_z (red) are shown together with the resulting spin precession. Due to the sign reversal of B_z across the microstrip, the microwave field has opposite handedness on opposite sides of the microstrip. Only on one side does the field rotate with the same handedness as the spin-wave precession, resulting in efficient spin-wave excitation and directional propagation.

- When $M \parallel k$, the magnetisation is perpendicular to the microstrip and backward volume spin-waves are excited. In this geometry, the effective microwave field can be viewed as linearly polarised. The spin-wave precession is driven by the magnetic field components perpendicular to the magnetisation direction; in this case only B_z contributes, resulting in a linearly polarised field. A linearly polarised field can be decomposed into equal left- and right-circularly polarised components and therefore contains a component with the correct handedness for both propagation directions. This is illustrated in Figure 3.10, where the individual contributions of B_x (black)

and B_z (red), as well as the resulting spin motion, are shown. As a result, backward volume spin-waves are excited symmetrically on both sides of the microstrip.

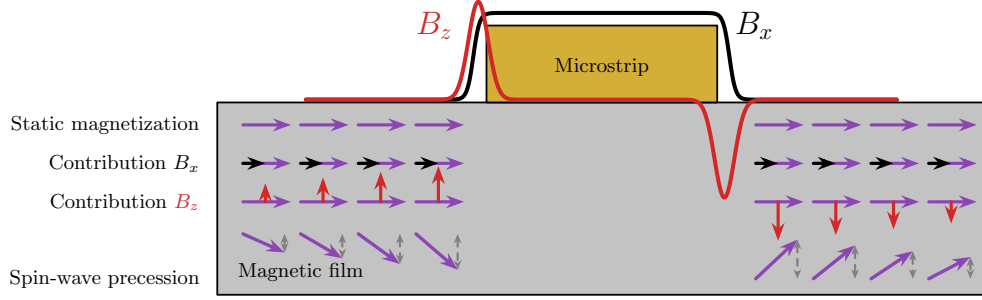


Figure 3.10: **Illustration of excitation of backward volume spin-waves.** Cross-section of the microstrip in the x - z plane for the backward volume geometry, where $M \parallel k$. The microwave field components B_x (black) and B_z (red) are shown together with the resulting spin precession. In this geometry, the effective microwave field is linearly polarised and can therefore be decomposed into left- and right-circularly polarised components. As a result, the microwave field can couple to spin-waves propagating in both directions, leading to symmetric spin-wave excitation on both sides of the microstrip.

3.3. Spin-wave detection with colour centres

In this thesis, V_B^- centres detect spin-waves through the circularly polarised components of the spin-wave and microstrip stray fields. In this section, I will elaborate on the photoluminescence (PL) response of the V_B^- centre to the magnetic stray fields generated by the microstrip and the spin-waves. After this, I will show how it can be used to obtain the spatial profiles of spin-waves and spin-wave mixing.

3.3.1. Spin-wave spatial profiles

A spin-wave generates a magnetic stray field that oscillates at the same frequency as the spin-wave itself. This stray field then drives the transition between the $|0\rangle$ and $|\pm 1\rangle$ of the colour centre and causes a PL dip when resonant. To be able to infer what the spatial profile of a spin-wave probed by a colour centre is, we need to know what the total magnetic field driving the colour centre is, which is the sum of the spin-waves and microstrip field.

The V_B^- (and NV^-) transitions $|0\rangle \rightarrow |\pm 1\rangle$ are driven by the right- and left-circularly polarised components of the AC magnetic field, respectively[25, 26]. These AC magnetic fields consist of both the microstrip and spin-wave stray fields projected onto the plane perpendicular to the quantisation axis. For the V_B^- centre, the quantisation axis is along $\hat{z}$, such that the relevant driving fields lie in the x - y plane. This results in the following circularly polarised magnetic field components (full derivation given in Appendix A.2.2).

$$B_{BV,\perp} = \frac{1}{\sqrt{2}} |\bar{m}_z e^{ikx} + B_{y,MS}| \quad (3.13)$$

$$B_{DE,\perp} = \frac{1}{\sqrt{2}} |(\bar{m}_x + \bar{m}_z) e^{ikx} + B_{x,MS}| \quad (3.14)$$

where $B_{BV,\perp}$ and $B_{DE,\perp}$ are the transverse magnetic-field components detected by the colour centre for the backward volume and Damon–Eshbach geometries, respectively. The quantities $\bar{m}_x$ and $\bar{m}_z$ are the amplitudes of the magnetisation components of the spin-wave. The terms $B_{y,MS}$ and $B_{x,MS}$ are the corresponding microstrip driving-field components, and the factor $1/\sqrt{2}$ converts the linearly expressed field components into the circularly polarised transverse component to which the colour centre is sensitive. These transverse magnetic fields are shown in Figure 3.11a. Here, the DE and BV branches exhibit the same spatial profile due to the normalisation of the magnetic field amplitudes. In addition, the microstrip is rotated between the DE and BV geometries, resulting in nearly identical driving-field profiles.

Using the following formula for the photoluminescence of the colour centre [27]

$$\frac{PL}{PL_0} = 1 - C = 1 - C_0 \frac{B_{\perp}^2}{B_{\perp}^2 + \Phi} \quad (3.15)$$

in which PL_0 is the photoluminescence in the absence of spin-wave excitation, C_0 is the maximum ODMR contrast, defined as the relative change in photoluminescence between the off- and on-resonance photoluminescence, and Φ is a parameter determined by the optical excitation and relaxation time. We normalise the magnetic stray field, making $\Phi = 1$. This gives the following spatial profiles for the BV and DE branches, shown in Figure 3.11b.

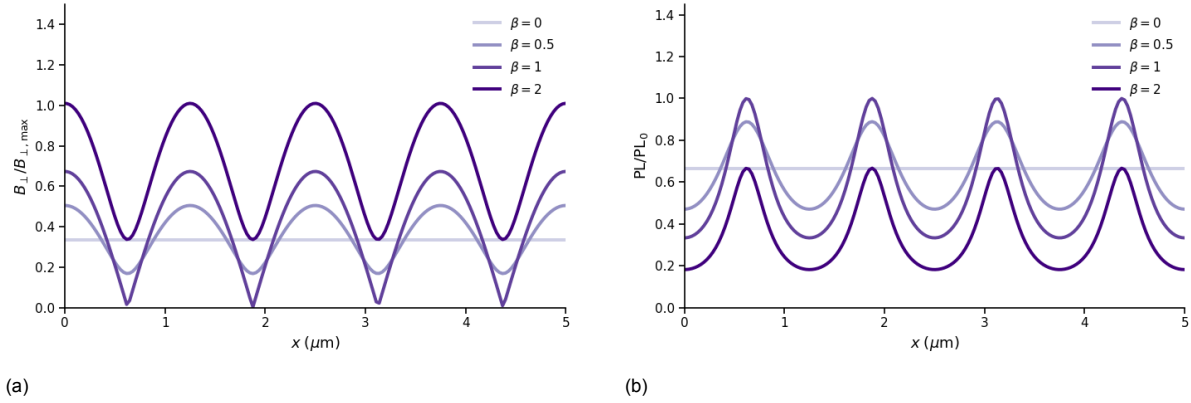


Figure 3.11: **Spatial magnetic stray field and PL response of spin-waves.** In both panels, the DE and BV branches exhibit the same spatial profile due to the normalisation of the magnetic field amplitudes. (a) Normalised magnetic field profile corresponding to the BV and DE configurations. The microstrip field is taken proportional to the spin-wave field, $B_{y(x),MS} = \beta B_{BV(DE),sw,y(x)}$, where β controls the relative strength of the external driving field. (b) Spatial dependence of the normalised photoluminescence for the DE and BV spin-wave branches.

3.3.2. Difference-frequency spin-wave mixing

When measuring spin-waves with either NV^- or V_B^- centres, one can only measure spin-waves at their respective electron spin resonance (ESR) frequency. As the spinwave and colour centre both get tuned with changing magnetic field, the bandwidth that one can measure is severely limited. This difficulty may be overcome using spin-wave mixing.

Spin mixing entails the excitation of multiple spin-waves at the same position and time, causing these spin-waves to interact with each other and form spin-textures of different frequencies. There are multiple ways for spin-waves to mix, but in this thesis, we will focus on so-called 'difference frequency generation' as described and measured before using NV^- centres [19]. Difference frequency generation is based on the fact that when two spin-waves mix, they generate a spin-texture with a frequency and k-vector equal to the difference between the original frequencies and k-vectors of the unmixed spin-waves.

To determine the spatial profile of the mixing product of two spin-waves, we again calculate the stray field generated by the mixed spin-waves for both DE and BV modes. This derivation is given in Appendix A.3. The important takeaway is that if we assume the magnetisation to oscillate in the x - z plane, while the length of the magnetisation remains constant, the component along the equilibrium magnetisation direction oscillates at the difference frequency, the sum frequency, and the double frequencies, as shown in equation 3.16,

$$\begin{aligned}
m_y(y, t) = & \left[1 - \frac{m_{x1}^2 + m_{z1}^2}{2} - \frac{m_{x2}^2 + m_{z2}^2}{2} \right. \\
& - \underbrace{\frac{m_{x1}^2 - m_{z1}^2}{2} \cos(2k_1 y - 2\omega_1 t)}_{\text{double frequency spin-wave 1}} \\
& - \underbrace{\frac{m_{x2}^2 - m_{z2}^2}{2} \cos(2k_2 y - 2\omega_2 t)}_{\text{double frequency spin-wave 2}} \\
& - \underbrace{\frac{(m_{x1}m_{x2} + m_{z1}m_{z2}) \cos(\Delta k y - \Delta \omega t)}{2}}_{\text{difference frequency}} \\
& \left. - \underbrace{\frac{(m_{x1}m_{x2} - m_{z1}m_{z2}) \cos((k_1 + k_2)y - (\omega_1 + \omega_2)t)}{2}}_{\text{sum frequency}} \right]^{1/2}.
\end{aligned} \tag{3.16}$$

Because this is a non-linear interaction, we must now go beyond the linearised form of the LLG equation and calculate the stray field for both the Damon–Eshbach branch and the backward volume branch, including the m_y term. From this, we find that in the ideal situation the stray field of the Damon–Eshbach branch does not depend on m_y , and therefore the detectable stray field does not oscillate at the difference frequency. However, if we assume a small misalignment between $\mathbf{M}$ and $\mathbf{k}$, so that the DE geometry is given by $\phi = \pi/2 + \delta\phi$, the stray field does contain a mixed component. This is a reasonable assumption for our measurements, as perfect alignment of the magnet has proven to be very difficult. The resulting stray field and PL signal detected by the colour centre are shown in Figure 3.12 for both DE and BV modes.

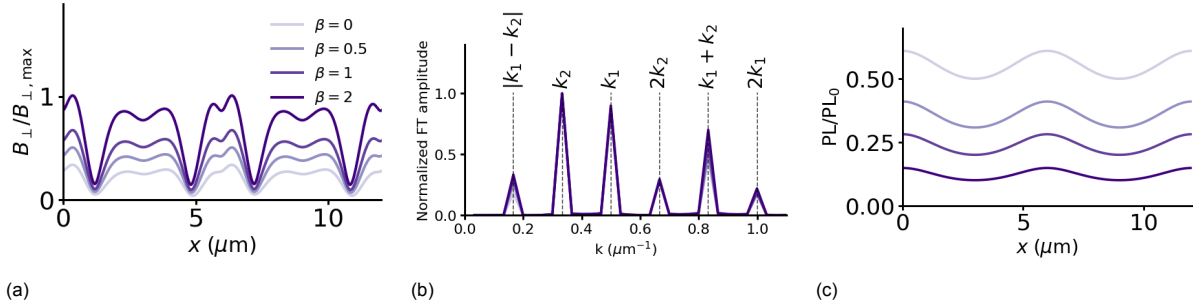


Figure 3.12: **Spatial magnetic stray field, Fourier transform and PL response of two mixed spin-waves.** Calculated spin-wave mixing signal for two spin-waves with wavelengths $\lambda_1 = 2 \mu\text{m}$ and $\lambda_2 = 3 \mu\text{m}$. This holds for both the backward volume branch and Damon–Eshbach branch, assuming a small angle mismatch. (a) Normalised stray field of two mixing spin-waves. (b) The spatial Fourier transform shows components at the wave vectors corresponding to the difference, sum, and double- k mixing terms. (c) Corresponding photoluminescence response, assuming the colour-centre resonance frequency matches the difference-frequency mixing product, $f_{\text{colour centre}} = \Delta f$.

The mixed spin-wave signal contains many different spatial frequency components, as can be seen from the stray field profile in Figure 3.12a. This is further confirmed by the Fourier transform shown in Figure 3.12b, where multiple mixing products can be identified. Since the colour centre can only detect a single resonance frequency at a time, only one specific mixing component can be measured. This is illustrated in Figure 3.12c, where the photoluminescence response corresponding to the difference-frequency mixing product is shown. Consequently, the colour centre acts as a frequency-selective probe that can isolate and image individual spin-wave mixing products.

Spin-wave detection

Spin-waves can be detected using the magnetic stray fields generated by their oscillating magnetisation to drive colour centres. In this chapter, we investigate how these stray fields are detected using V_B^- centres in hexagonal boron nitride. First, the experimental setup and sample geometry used throughout this thesis are introduced. Subsequently, the measurement scheme and spatial resolution of spin-wave imaging using V_B^- centres are analysed. Finally, the effect of the finite distance between the colour centres and the magnetic film is considered, showing how this leads to wave vector-dependent filtering of the detected spin-wave stray fields.

4.1. Experimental Setup

The measurements presented in this thesis were performed using a confocal microscope setup, referred to within our group as Room Temperature 1 (RT1). In this section, we first discuss the sample geometry used throughout the experiments in this thesis. Afterwards, we explain the working principle of the confocal microscope and how it is used to detect spin-waves.

4.1.1. Sample design

The sample used throughout this thesis consists of a GGG substrate on which a 55 nm thick Permalloy film is sputtered. Permalloy ($\text{Ni}_{80}\text{Fe}_{20}$) is a soft ferromagnetic material widely used in magnonics due to its low magnetic damping, low coercivity, and high magnetic permeability, making it highly suitable for the excitation and propagation of spin-waves [28, 29, 30]. On top of the Permalloy, a gold microstrip is fabricated with a width of $10\text{ }\mu\text{m}$ and a thickness of 80 nm. An hBN flake with V_B^- centres is subsequently transferred onto the microstrip and magnetic film. This flake consists of several hBN layers with a total thickness of approximately 55 nm. The sample geometry is shown in Figure 4.1.

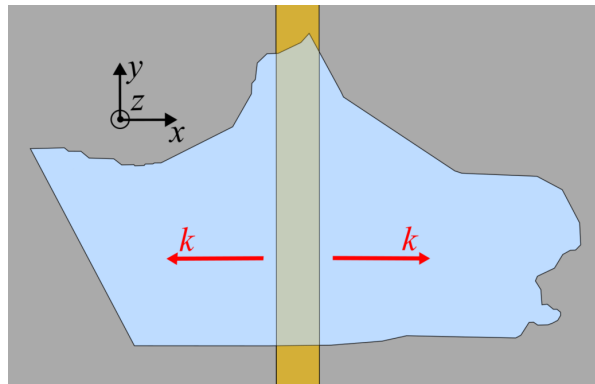


Figure 4.1: **Schematic view of the sample.** A 55 nm thick hBN flake (blue) is placed on top of a gold microstrip, 80 nm thick and $10\text{ }\mu\text{m}$ wide, and a 55 nm thick permalloy film (gray). The microstrip excites spin-waves in the permalloy, which propagate away from the microstrip with wave vector k .

4.1.2. Confocal microscope setup

The confocal microscope images spin-waves by scanning the laser spot and recording the V_B^- PL at each position. In this thesis, we used a confocal microscope because of its spatial resolution [31]. In Figure 4.2, a diagram is shown of the confocal microscope called RT1. In this setup, a green laser is first focused using multiple lenses (L_i) until it hits a dichroic mirror that reflects green light and lets red light (from the V_B^- or NV^- centre) through. After the dichroic mirror, the green light is reflected up to a scanning mirror, which then focuses it onto a downward mirror, which focuses the light through the objective and onto the sample. The scanning mirror is used to scan the laser across the sample, allowing for spatial images to be made. The red light from the colour centre is reflected up and let through by the dichroic mirror. After this, the red light is guided to the sensor, an avalanche photo detector (APD). The spin-waves and colour centres are tuned using the magnet placed below the sample. This setup therefore enables spatial imaging of spin-waves through the photoluminescence response of the colour centres.

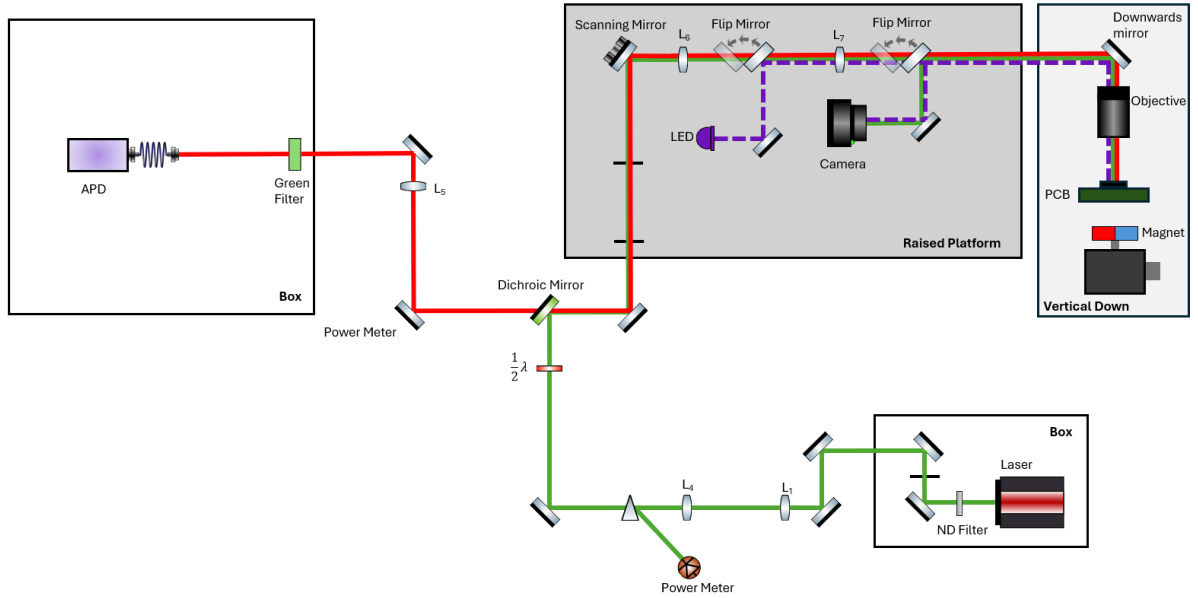


Figure 4.2: **Schematic overview of the Room Temperature 1 (RT1) confocal microscope setup used throughout this thesis.** A green laser is directed through a set of lenses, mirrors, and a dichroic mirror towards the objective, which focuses the light onto the sample containing the magnetic film and hBN flake with V_B^- centres. The emitted photoluminescence is collected by the same objective and separated from the excitation laser using the dichroic mirror and green filter before being detected by the avalanche photodiode (APD). A camera and LED illumination path are included for imaging and sample alignment. The setup additionally contains scanning mirrors for raster scanning and a movable permanent magnet below the sample to apply an external magnetic field. This setup enables spatial maps of the V_B^- PL response while independently tuning the applied magnetic field.

4.2. Spatial resolution of spin-wave detection using V_B^- centres in hBN

The spatial resolution of V_B^- -based spin-wave imaging is set by both the laser spot size and the decay of spin-wave stray fields away from the magnetic film. The detection of spin-waves using boron vacancies embedded in hexagonal boron nitride was first demonstrated in 2024 [17, 23]. The goal of this thesis is to extend this work to the spatial imaging of backward volume spin-wave modes in Permalloy. Backward volume spin-waves can possess very small wavelengths, while the V_B^- centre has a relatively large bandwidth of approximately 300 MHz. As a result, the spatial resolution achievable with V_B^- -based imaging may become limited. To determine the limits of this technique, it is important to understand the smallest spin-wave wavelength that can still be resolved using the V_B^- centre and our experimental setup.

4.2.1. Spin-wave measurement scheme using V_B^- centres

Each pixel in a V_B^- spin-wave image is an average over all colour centres inside the laser spot and the hBN thickness. The V_B^- centres are located inside the hBN flake, which is placed on top of the sample, as shown in Fig 4.3. During a measurement using the confocal microscope, a laser spot is scanned across the hBN flake. At each laser position, all V_B^- centres within the illuminated area are excited and emit PL based on the present spin-wave stray field.

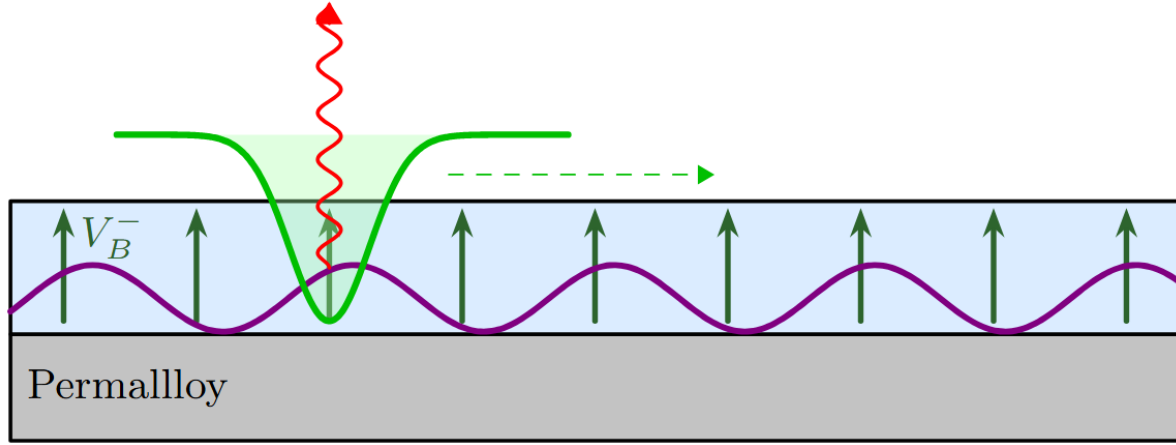


Figure 4.3: **Schematic overview of the spin-wave measurement using V_B^- centres in hBN.** A spin-wave propagates in the Permalloy layer, producing a spatially varying magnetic stray field (purple) at the position of the hBN flake. A green laser is scanned across the sample, causing the V_B^- centres to emit red photons based on the strength of the present spin-wave stray field. The green V_B^- arrows indicated here represent ensembles of V_B^- centres.

In Figure 4.4a, a spin-wave measurement using the hBN flake in the Damon–Eshbach geometry is shown. The sinusoidal spatial profile of the spin-wave can clearly be observed in the measurement. The microstrip modifies the local spin-wave amplitude through its driving field, making its position visible in the image as a slanted feature across the centre of the flake.

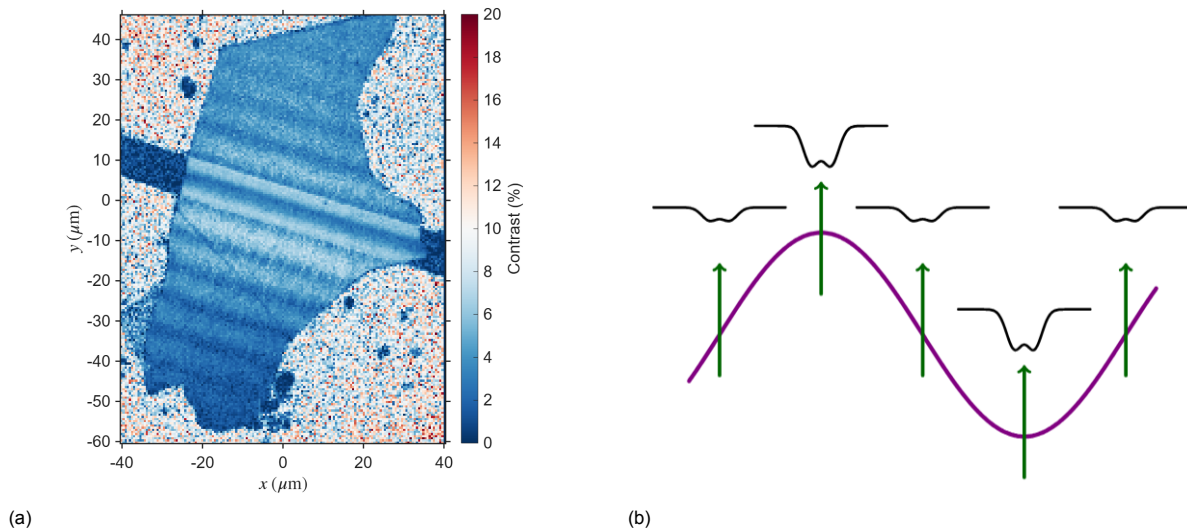


Figure 4.4: **Damon–Eshbach spin-wave measurement and contrast mechanism.** (a) Spatial scan over the hBN flake in the Damon–Eshbach geometry. The sinusoidal spatial profile of the spin-wave is clearly visible. The stripline, located across the middle of the flake, enhances the excitation of spin-waves directly beneath it, resulting in larger amplitudes and making the stripline position visible in the measurement. (b) Schematic illustration of the contrast mechanism. The local spin-wave stray field (purple) changes the depth of the V_B^- (green) photoluminescence dip, shown in black above the V_B^- . At spin-wave antinodes, the dip is deepest, while near nodes the dip is shallow. The depth of the dip defines the measured contrast.

The emitted photoluminescence (PL) depends on the local magnetic stray field generated by the spin-wave, as schematically shown in Figure 4.4b. At a spin-wave antinode, the driving field is largest,

and the PL dip of the V_B^- centre is deepest. Near a node, the driving field is small, and the dip becomes shallow. The difference in PL between these positions is defined as the contrast. The measured signal at each laser position is not the PL response of a single point, but rather the combined response of all V_B^- centres within the laser spot. In this sense, the measurement can be viewed as a convolution between the laser spot profile and the spin-wave-induced PL pattern of the V_B^- centres.

The laser spot profile is that of a Gaussian, and due to this, when the wavelength becomes comparable to it, the convolution will no longer be a coherent signal; thus, the limit for spin-wave measurement is comparable to the laser spot size.

4.2.2. Laser spot size

To determine the size of the laser spot, measurements were performed on a calibration sample containing gold dots with different diameters. When such a dot is smaller than the laser spot, the measured feature size is no longer determined by the actual dot size, but by the size of the laser spot itself. In that case, the dot effectively acts as a point-like object, and the measured image provides an estimate of the laser spot's spatial extent. Figure 4.5a shows a scan over several gold dots on the calibration sample.

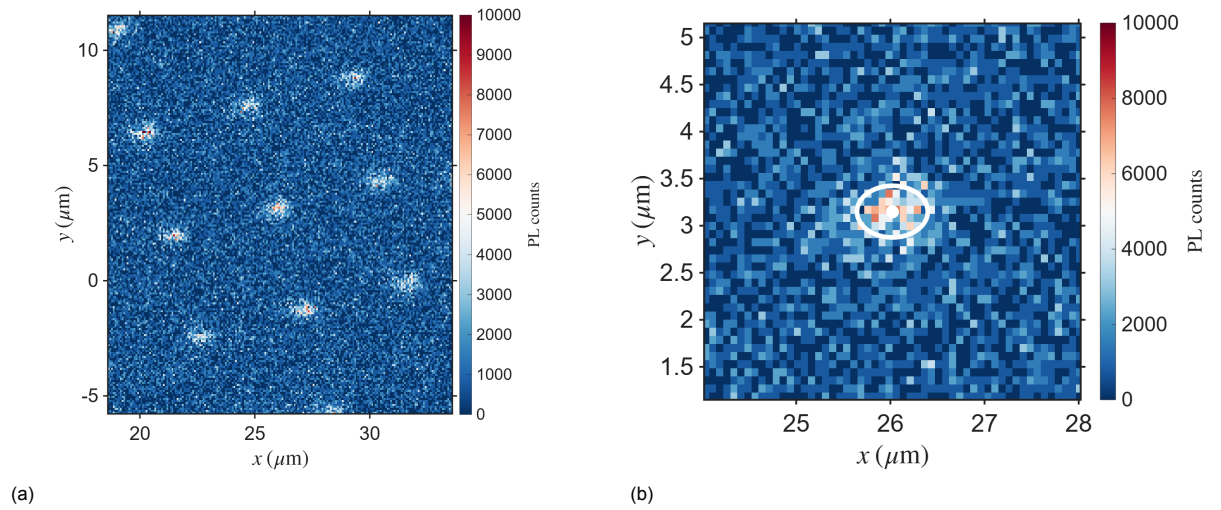


Figure 4.5: **2D scans of gold dots.** (a) Photoluminescence scan of the gold-dot calibration sample. The periodic array of dots of 100nm is used to estimate the effective optical spot size. (b) Zoom-in on a single dot with the fitted Gaussian FWHM contour overlaid. From the fit, the effective spot width is estimated as $w = 725$ nm.

To determine the size of the laser spot, Gaussian functions were fitted to the measured profiles of the gold dots. A close-up of this is shown in Figure 4.5b with the fitted gaussian shown in white. The laser spot size was then defined as the beam width w ,

$$w = \frac{\sqrt{2} \text{FWHM}}{\sqrt{\ln(2)}}, \quad (4.1)$$

where FWHM is the full width at half maximum of the fitted Gaussian. This definition corresponds to the Gaussian beam width parameter, where the intensity has decreased to $1/e^2$ of its maximum value. The widths obtained from all nine measured dots were averaged, yielding an effective laser spot size of approximately 725 nm. Thus the smallest possible spin-wave we can approximately measure, theoretically, is 725 nm.

4.2.3. Resolution estimate from radial Fourier profiles

To estimate the smallest spin-wave wavelength that we can still detect, we looked at the radial Fourier profiles of the measured spin-wave images. First, the two-dimensional Fourier transform of each image was calculated. Then, for every absolute wave vector, the Fourier power was averaged over a ring in Fourier space. This gives a one-dimensional profile that shows how much signal is present at each spatial frequency, independent of the direction of propagation.

At high k , the radial Fourier signal decreases because short-wavelength spin-waves become harder to detect. This can be caused by the finite laser spot size, the distance between the magnetic film and the V_B^- centres, and the noise in the measurement. To define a practical cut-off, the radial Fourier profiles were normalised, and the point where the signal dropped below $1/7 \approx 1/e^2$ was used as the detection limit.

This analysis was applied to the backward volume spin-wave measurements discussed in Chapter 7. Only measurements with visible spin-wave activity were included, to avoid extracting a cut-off from noise-dominated images. The resulting radial Fourier profiles are shown in Figure 4.6. The largest wave vector above the $1/7$ threshold is $k = 6.61 \text{ rad}/\mu\text{m}$. Using this radial Fourier method, the smallest spin-wave wavelength detected in the measurements is therefore approximately 950 nm.

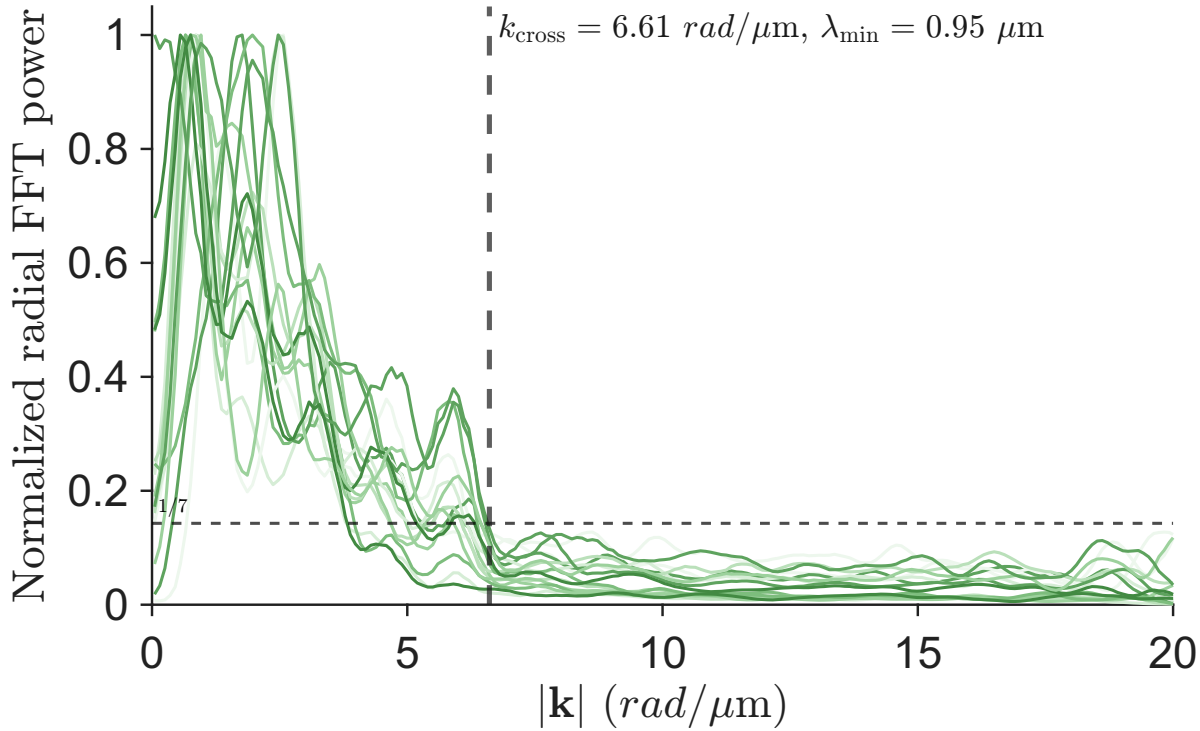


Figure 4.6: **Radial Fourier profiles used to estimate the smallest detected spin-wave wavelength.** For each backward volume spin-wave image, the two-dimensional Fourier power was averaged over rings with constant wave vector magnitude $k = \sqrt{k_x^2 + k_y^2}$. The profiles were normalised to their maximum value. The dashed line indicates the $1/7$ threshold used to define a practical detection limit. The largest wave vector above this threshold is $k = 6.61 \text{ rad}/\mu\text{m}$, corresponding to a smallest detected wavelength of $\lambda_{\min} \approx 950 \text{ nm}$.

4.3. Distance-dependent wave vector filtering

The finite distance between the magnetic film and the V_B^- centres acts as a wave vector-dependent filter on the detected spin-wave stray field. A spin-wave with wave vector k produces a stray field outside the magnetic film, so the field amplitude at a distance d scales approximately as

$$B_{\text{sw}}(k, d) \propto k e^{-kd}. \quad (4.2)$$

The factor k suppresses very long-wavelength spin-waves, because a nearly uniform magnetisation produces little stray field outside the film. The exponential factor suppresses very short-wavelength spin-waves, because their stray fields decay rapidly away from the surface. For a single sensor at distance d , the detected stray field is therefore largest around $k=1/d$.

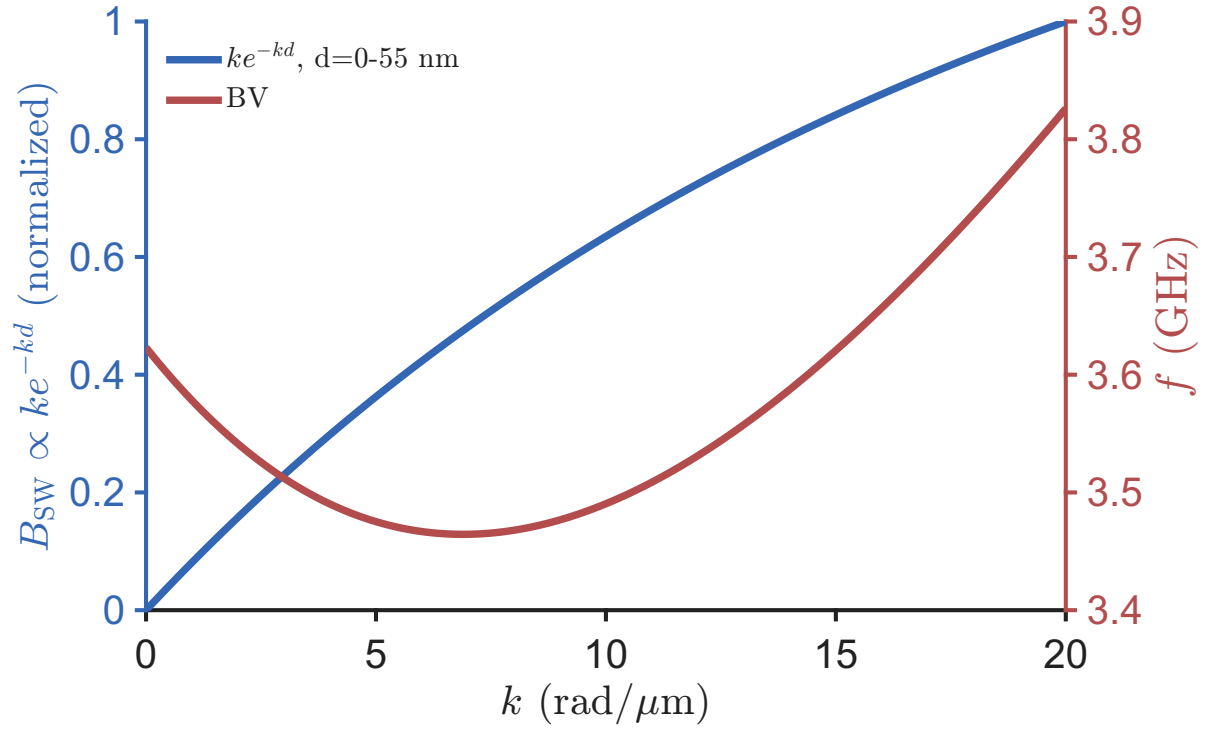


Figure 4.7: **The wave vector-dependent filter function and spin-wave dispersion.** The wave vector-dependent filter function, $f_k = ke^{-kd}$ for a separation distance of $d=0-55$ nm (blue), together with the backward volume spin-wave dispersion branch for an in-plane magnetic field of $B=156$ mT (red). The filter function peaks for $k = 1/d$, indicating that the V_B^- centres are most sensitive to spin-waves with wave vectors near this value.

In reality, the V_B^- centres are distributed throughout the hBN flake rather than located at a single height. Since the flake is 55 nm thick, the detected signal should be interpreted as an average over the sensor distances. A visual representation of this wave vector-dependent decay is shown in Figure 4.8 for an hBN flake containing V_B^- centres placed on top of a magnetic film.

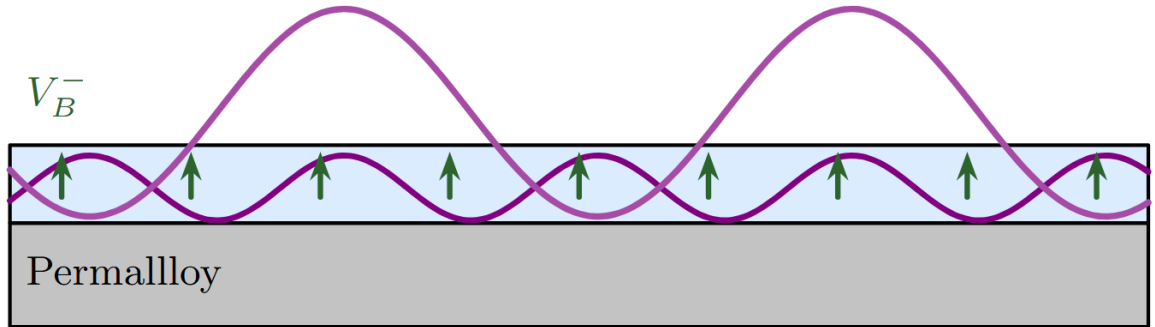


Figure 4.8: **Illustration of distance and k-vector dependent decay of spin-wave stray field.** Cross-section of a magnetic film with a microstrip used to excite spin-waves. An hBN flake containing V_B^- centres is placed on the magnetic film. The V_B^- centres are driven by the spin-wave stray fields, which decay as a function of both wave vector and distance from the film. Spin-waves with small wave vectors (large wavelengths, light purple) produce large stray fields and are therefore not efficiently detected, while spin-waves with larger wave vectors (small wavelengths, dark purple) generate stray fields that are less far from the surface and can be sensed by the V_B^- centres.

Static magnetic field control

To accurately tune the spin-wave band and ESR frequency of the colour centre, precise magnetic field control is required. Achieving such control is non-trivial, as the magnetic field depends on the position, orientation, and magnetisation of the permanent magnet, while small mechanical misalignments can lead to changes in both the field strength and direction at the sample position. Reliable spin-wave measurements therefore require a dedicated magnet calibration.

In this thesis, two dedicated magnet calibration procedures were developed and implemented. The first calibration was designed to provide accurate in-plane field control for backward volume spin-wave measurements using V_B^- centres. The second calibration was developed for future spin-wave mixing experiments, where both large magnetic fields and precise alignment with the colour-centre axis are required. Together, these calibration procedures provide the foundation for reliable magnetic-field control in the experiments presented in this thesis and for future mixing experiments.

5.1. Magnet calibration for backward volume spin-wave measurements

To be able to measure backward volume spin-waves in Permalloy, it is necessary to apply an in-plane magnetic field with full angular control. This angular control is required to achieve the backward volume (BV) geometry, where the magnetisation of the Permalloy is aligned parallel to the spin-wave propagation direction, which is perpendicular to the microstrip. Since the magnetisation is set by the externally applied magnetic field, the magnetic field must therefore also be oriented perpendicular to the microstrip. The calibration therefore needs to determine both the magnitude and in-plane angle of the applied field.

An in-plane magnetic field is preferred because the quantisation axis of the V_B^- centre is oriented out of plane. As a result, the spin transition frequencies are, to first order, relatively insensitive to in-plane magnetic fields [17]. This allows the application of magnetic fields required for spin-wave excitation while keeping the V_B^- transition frequency stationary.

To achieve this, a cylindrical bar magnet was mounted on a rotary stage, which itself was attached to a Thorlabs XYZ translation stage with a travel range of 5 cm along each axis. To accurately calibrate the magnetic field produced by the magnet, a PCB containing an ensemble NV^- diamond was placed at the sample position, schematically shown in Fig. 5.1a. This geometry allows for the NV^- centre to be used as a vector magnetometer.

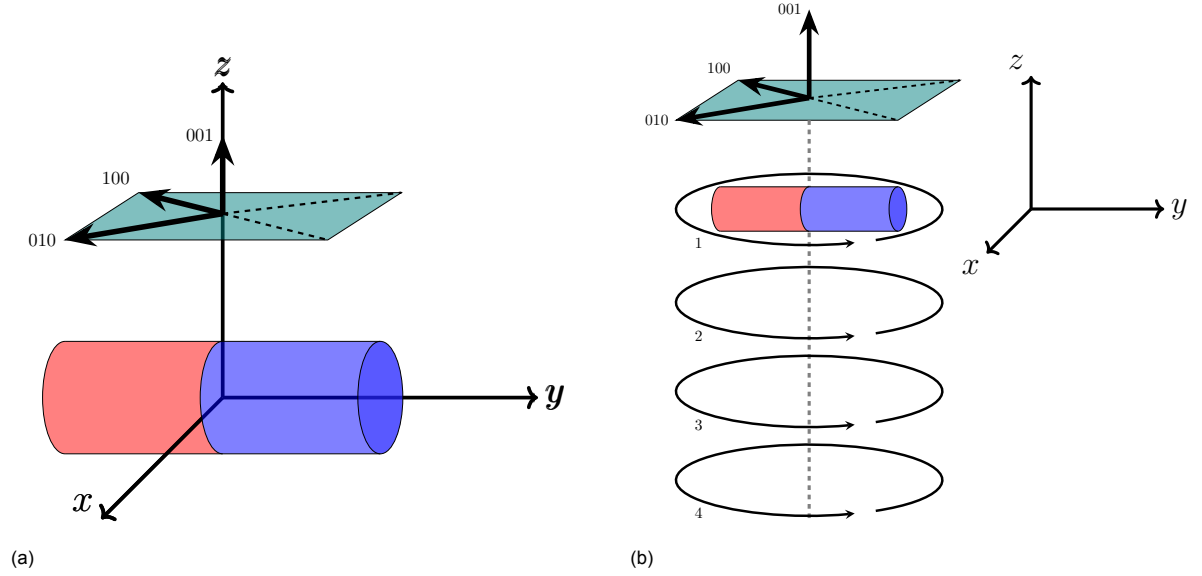


Figure 5.1: **Static and measurement geometry of the magnet.** (a) Schematic representation of the magnet calibration setup. A cylindrical neodymium N42 magnet with a diameter of 5 mm and a height of 14 mm is mounted on a rotary stage attached to an XYZ translation stage (not shown). The orientation of the NV⁻ ensemble axes, $[100]$, $[010]$, and $[001]$, defines the measurement frame used for the calibration. (b) Schematic drawing of the full magnet calibration rotations performed at four different magnet distances to the NV⁻ membrane. For each rotation angle, an ESR spectrum was recorded.

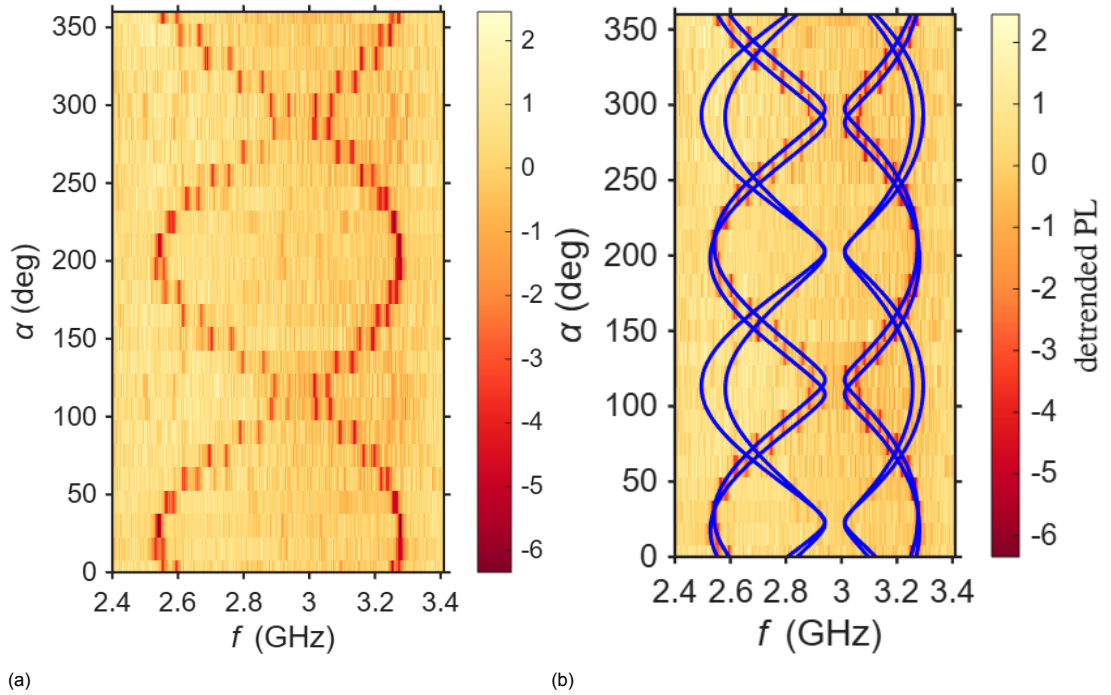


Figure 5.2: **In-plane magnet calibration measurement and fit.** (a) ESR measurement acquired during a full 360° rotation of the magnet at a fixed distance from the NV⁻ membrane. Each horizontal trace corresponds to one magnet angle. (b) The same measurement with the fitted ESR resonances overlaid in blue. The agreement between the measured and fitted resonance positions shows that the magnet model captures the field evolution during rotation.

A full rotation of the magnet maps how the NV⁻ ESR frequencies change with the in-plane field direction. For four different magnet distances to the NV⁻ membrane, ESR measurements were performed over a full 360° rotation of the magnet in the x - y plane. An example of such a measurement is shown in Fig. 5.2a), while the complete dataset is provided in Appendix B.1. The evolution of the ESR resonances with magnet angle can be used to calibrate the magnet position and geometry.

5.1.1. Magnet calibration procedure

To determine the magnetic field produced by the magnet, a fitting routine was implemented. This fitting procedure consisted of comparing the experimentally measured ESR signal to an ESR signal predicted by the magnetic field of an ideal cylindrical magnet. The magnet was modelled as an ideal solenoid as proposed in [32] using cylindrical integrals. For each trial magnet geometry, the magnetic field at the NV^- ensemble was converted into expected ESR dip positions, from which a synthetic ESR trace was constructed using Lorentzian dips; in Figure 5.3 a fitted synthetic ESR is shown together with the original trace. The fit then minimised the residual between the measured and synthetic ESR traces using non-linear least squares. This procedure extracts the magnet geometry from the full ESR spectrum rather than from selected dip positions. The resulting fit is shown in Fig. 5.2 b) in blue. As can be seen, this fitting procedure results in a fit that has reasonable correspondence with the measured data.

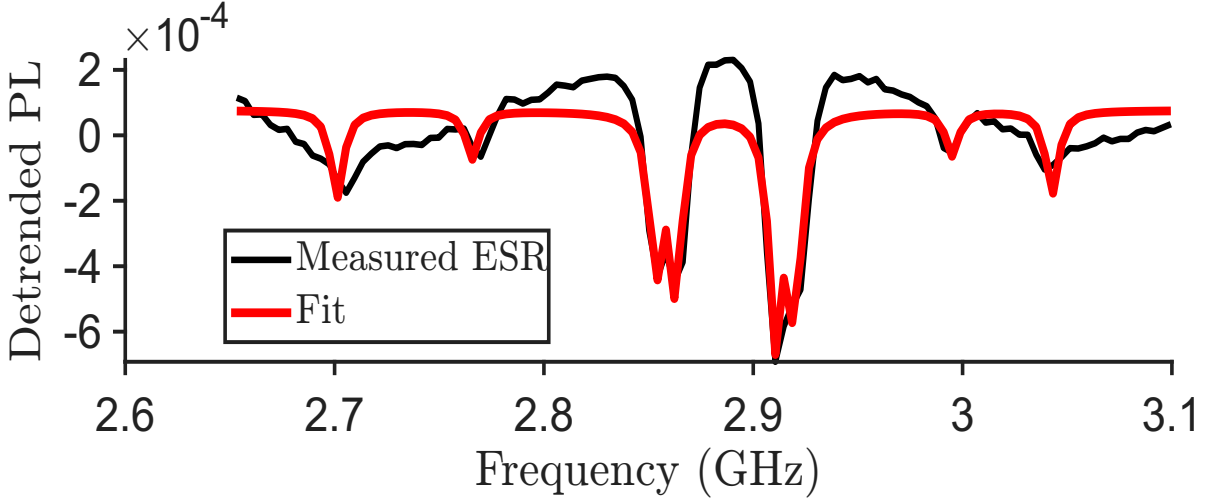


Figure 5.3: **Synthetic ESR trace fitted to a measured ESR trace.** A single detrended ESR trace is shown in black, while the synthetic ESR trace generated from the fitted resonance positions is shown in red. The synthetic trace contains one Lorentzian dip for each predicted NV^- transition and is used to compare the magnet model directly to the measured ESR signal.

5.2. Magnet calibration for spin-wave mixing experiments

For the spin-wave mixing experiments, the aim is to detect a difference-frequency signal that does not itself satisfy the linear spin-wave dispersion. Therefore, the ESR transition frequency of the colour centres has to be tuned down to the same frequency range of around 20-100 MHz. This corresponds to applying a magnetic field of approximately 102 mT along the colour centre axis. In addition, to ensure reproducibility of the measurements, it is necessary to magnetise the Permalloy film prior to each measurement using a large in-plane magnetic field of ~ 100 mT. Thus the magnet needs to be able to supply both.

5.2.1. Tilted-field design considerations

To correct for the resolution limitations of the V_B^- , we chose to continue our measurements using a NV^- membrane. Due to this, we had to account for the fact that the NV^- axis is oriented at an angle of 54.7° with respect to the z-axis. If the magnetic field is misaligned with respect to this axis, especially at higher field strengths, the NV^- can quench, and the PL dip will largely disappear[33].

To prevent quenching, given the limited freedom to move the magnet in-plane due to the piezo design, the magnet was tilted with respect to the z-axis at an angle of 65° . The tilt was chosen such that, when the magnet is moved only along z, the resulting magnetic field reaches an angle of 54.7° at a field strength of 100 mT. This choice was made because quenching effects become more severe at higher fields. At lower fields, small field misalignments affect the NV^- resonances less strongly. In addition, working at lower z-heights gives more freedom in the x-y plane, so small lateral movements can be used to compensate for the field angle. Since most measurements are performed around 100 mT, aligning the magnet well at this field gives the most reliable calibration.

5.2.2. Corkscrew calibration trajectory

To obtain a good fitting result, all eight ESR dips have to be present in the signal. To obtain measurements where most dips are visible, the magnetic field projection on all four NV^- families should be different. However, since the exact orientations of these families are not known, the most practical approach is to move the magnet in a circular path around the sensor, thereby ensuring sufficient misalignment. In addition, it is essential to perform measurements at multiple z -heights in order to accurately determine the strength of the magnet. To combine these two requirements, the magnet was moved along a corkscrew path while scanning downwards, shown in Fig. 5.4a.

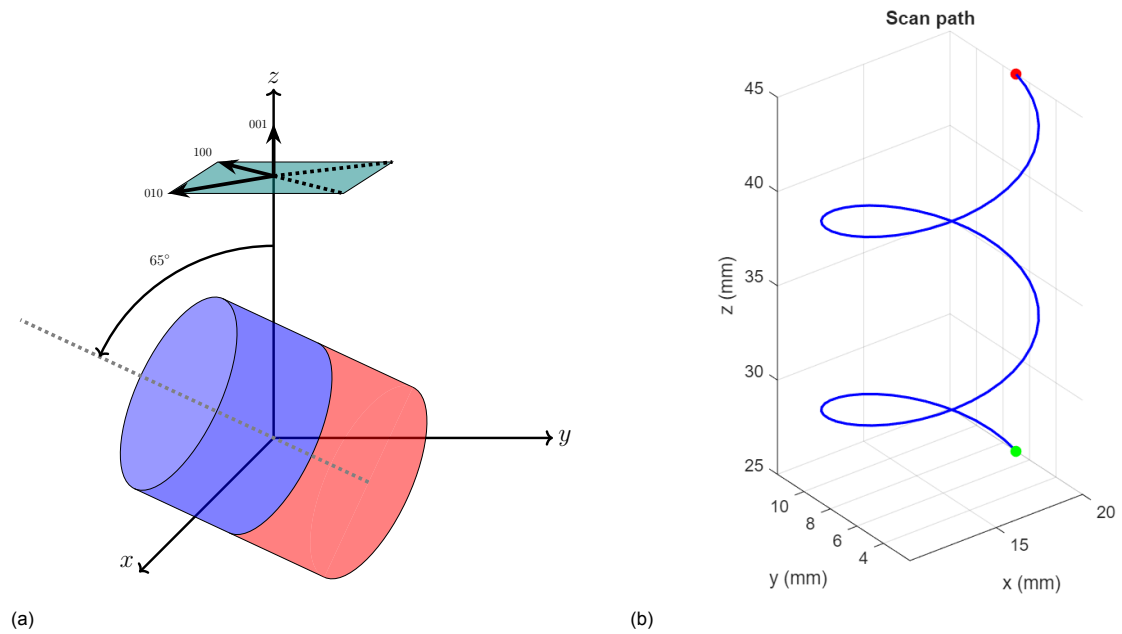


Figure 5.4: **Magnet geometry and calibration path of a canted magnet.** (a) Schematic of the tilted magnet used in the spin-wave mixing experiments. A cylindrical N52 magnet (diameter and length = 14.29 mm) is tilted by 65° with respect to the z -axis, such that the magnetic field is aligned with the NV^- axis at 100 mT. The orientation of the NV^- crystal axes with respect to the laboratory frame is shown above the magnet. (b) Corkscrew calibration path used to characterise the field of the tilted magnet. The magnet follows a circular trajectory while its distance to the sample is gradually varied, so that the field is measured for different strengths and orientations. This provides enough information to determine the magnet position, orientation, and field strength accurately.

5.2.3. Canted magnet fitting procedure

This corkscrew trajectory resulted in ESR traces as shown in Fig. 5.5a, where most spectra exhibit all eight ESR dips. An additional feature of this trajectory is the presence of symmetry points where multiple dips overlap. Although these overlap regions can complicate the fitting procedure, since multiple magnetic-field configurations may produce similar ESR spectra, they provide a useful visual indicator for assessing the quality of the fit. Before fitting, a fourth-order polynomial background was subtracted from each ESR trace to remove slow drift and a broad microwave-background feature in the measured frequency range. The corrected corkscrew data were fitted using the same procedure as for the previous magnet calibration.

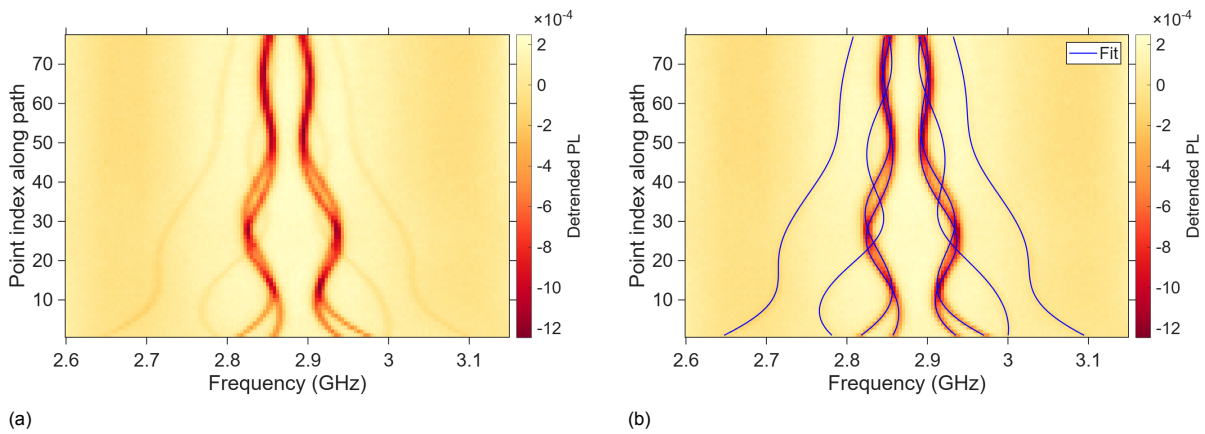


Figure 5.5: **Corrected ESR measurements obtained along the corkscrew calibration trajectory.** (a) Measured ESR spectra along the corkscrew trajectory after background correction. (b) The same ESR trajectory with the fitted ESR dip positions plotted on top (blue).

5.2.4. Minimum Number of ESR Spectra for Calibration

To determine how many ESR spectra are needed for an accurate calibration, a subsampling analysis was performed. Instead of fitting the full corkscrew trajectory, only a subset of the measured ESR spectra was used in the fit. The resulting magnetic field was then compared to the field obtained from the fit using the full dataset. This gives the mean deviation in magnetic field over the calibration path as a function of the number of sampled ESR spectra.

The result of this analysis is shown in Fig. 5.6. As expected, the error decreases when more ESR spectra are included in the fit. However, the improvement becomes much slower after roughly ten to twenty sampled spectra. Already with around ten ESR spectra, the mean field deviation is below 2% for both datasets. This shows that the corkscrew trajectory contains enough information to calibrate the magnet using only a small number of measurement points.

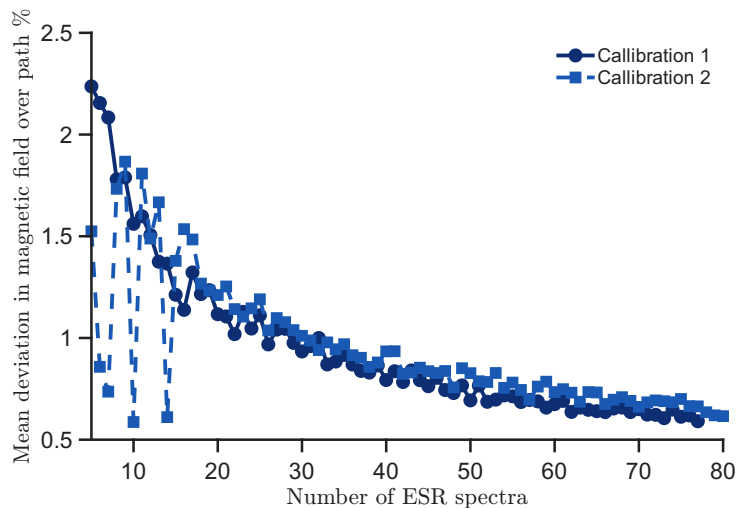


Figure 5.6: **Subsampling analysis of the corkscrew magnet calibration.** Mean magnetic-field deviation along the calibration path as a function of the number of ESR spectra used in the fit. Each point is obtained by fitting a subset of the corkscrew trajectory and comparing it to the field from the full dataset. The two curves show two independent calibration datasets. The deviation quickly falls below 2% after about ten ESR spectra, showing that the corkscrew path allows accurate calibration with only a small number of measurement points. The oscillations in dataset 2 for ($N < 16$) are likely caused by favourable or unfavourable sampling of high-symmetry points.

The corkscrew calibration path therefore significantly reduces the calibration time. Previously, a full magnet calibration could take an entire weekend, whereas the corkscrew method reduces this to approximately one hour. The corkscrew method therefore provides a practical workflow for future experiments requiring accurate magnetic-field control.

6

Visualising the spin-wave dispersion and colour-centre dynamics

To efficiently perform spin-wave measurements, it is important to have intuitive access to both the spin-wave dispersion and the colour-centre dynamics. This is because spin-waves can only be detected when they are resonant with the colour centre. For this reason, a simulation package was developed, in both MATLAB and Python, that simultaneously calculates the spin-wave dispersion and the colour-centre Hamiltonian as a function of the applied magnetic field. By finding the intersections between the spin-wave modes and the ESR transition frequencies, the package directly predicts the measurable spin-wave wave vectors and wavelengths. In addition, magnetic-field parameters can be swept to determine how the detectable spin-wave wavelengths evolve with field conditions, making the package a useful tool for selecting experimental parameters. Full documentation of the package is provided in Appendix C.

6.1. Simulation parameters

The goal of the simulation package is to provide a flexible framework for calculating both the spin-wave dispersion and the colour-centre response for a wide range of experimental conditions. The package supports different magnetic materials, optional uniaxial anisotropy, and the inclusion of a superconducting layer on top of the magnetic film[5, 34]. In addition, the dynamics of any spin-1 colour-centre can be included through their zero-field splitting, orientation, and Zeeman splitting, to determine which spin-wave modes are experimentally detectable. An overview of the main input and output parameters and outputs is given in Table 6.1.

To make it easy to map the parameter space, the package can also take multiple magnetic-field values at once. It then calculates the dispersion and colour-centre response for every possible field combination. For example, if the input is $B_x = 15$ mT, $B_y = [13, 19]$ mT, and $B_z = [0, 10, 20]$ mT, the package computes all six magnetic-field combinations. This makes it possible to quickly study how the measurable spin-wave wavelength changes over a range of experimental settings.

Category	Symbol	Unit	Description
Material parameters	t	m	film thickness
	M_s	A/m	Saturation magnetisation
	D_{ex}	m ² /s	Exchange stiffness constant
	D	GHz	Zero-field splitting of the spin-1 colour centre
	θ, ϕ	rad	Zenith and azimuthal angle of the colour centre
Experimental parameters	B_x, B_y, B_z	T	Applied magnetic field in Cartesian coordinates
	B, θ_B, ϕ_B	T, rad	Applied magnetic field in polar coordinates
	—	on/off	Magnetic-field dependence of the colour centre
Enabled physics	—	—	Standard spin-wave dispersion
	B_K	T	Include uniaxial anisotropy
	h_{SC}, λ_L	m	Include superconducting screening, Superconductor thickness, London penetration depth
Simulation parameters	N_k	—	Number of k points
	$k_{\text{min}}, k_{\text{max}}$	rad μm^{-1}	wave vector range
Output	$k_{\text{DE}}, k_{\text{BV}}$	rad μm^{-1}	Wave vectors of the spin-wave branches
	$f_{\text{DE}}, f_{\text{BV}}$	GHz	Branch frequencies
	v_g	m/s	Group velocity
	f^+	GHz	ESR transition frequency
	f^-	GHz	ESR transition frequency
	λ	μm	Measurable spin-wave wavelengths

Table 6.1: **Overview of the available settings in the spin-wave simulation GUI.** Numerical parameters can be entered directly, while the enabled-physics options determine which physical effects are included in the model. These parameters define the magnetic film, applied field, colour-centre orientation, and the wave vector range over which intersections with the ESR frequencies are searched.

6.2. The spin-wave dispersion and colour-centre Hamiltonian

The simulation package connects the spin-wave dispersion to the colour-centre ESR frequencies. For a magnetic film without anisotropy or a superconducting layer, the package uses the dispersion derived in Appendix A.1.1. When anisotropy or a superconducting layer is included, the corresponding models from Refs. [5, 34] are used. The coordinate system and angle definitions used in the package are shown in Figure 6.1. This gives one consistent geometry for the applied field, magnetisation, colour-centre axis, and spin-wave propagation direction.

Instead of saving the full two-dimensional dispersion, the package only evaluates the experimentally relevant branches. These are the backward volume and Damon–Eshbach branches, and, when anisotropy is included, the branches along the x and y directions. This is sufficient because the microstrip mainly excites spin-waves along specific directions, and it also reduces the computation time. The package then calculates the colour-centre ESR frequencies and searches for crossings with the spin-wave dispersion. These crossings correspond to spin-wave modes that can be detected by the colour centre. From the crossing wave vector, the wavelength is calculated using $\lambda = 2\pi/|k|$. In this way, the package directly predicts which spin-wave wavelengths are experimentally accessible.

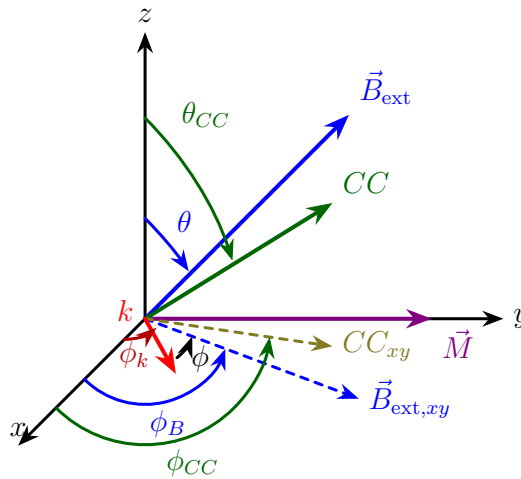


Figure 6.1: **Definition of the coordinate system and angles used in the simulation package.** The applied magnetic field $\vec{B}_{\text{ext}}$, colour-centre axis CC , magnetisation $\vec{M}$, and spin-wave wave vector $\vec{k}$ are shown with their polar and azimuthal angles.

6.3. Graphical User Interface

During experiments, it is inconvenient to manually run scripts and adjust parameters for every new magnetic-field setting. Therefore, I developed a graphical user interface (GUI) for the simulation package in both MATLAB and Python. The GUI provides direct access to all parameters and allows rapid calculation and visualisation of the spin-wave dispersion, colour-centre ESR frequencies, and their crossing points. An example is shown in Fig. 6.2.

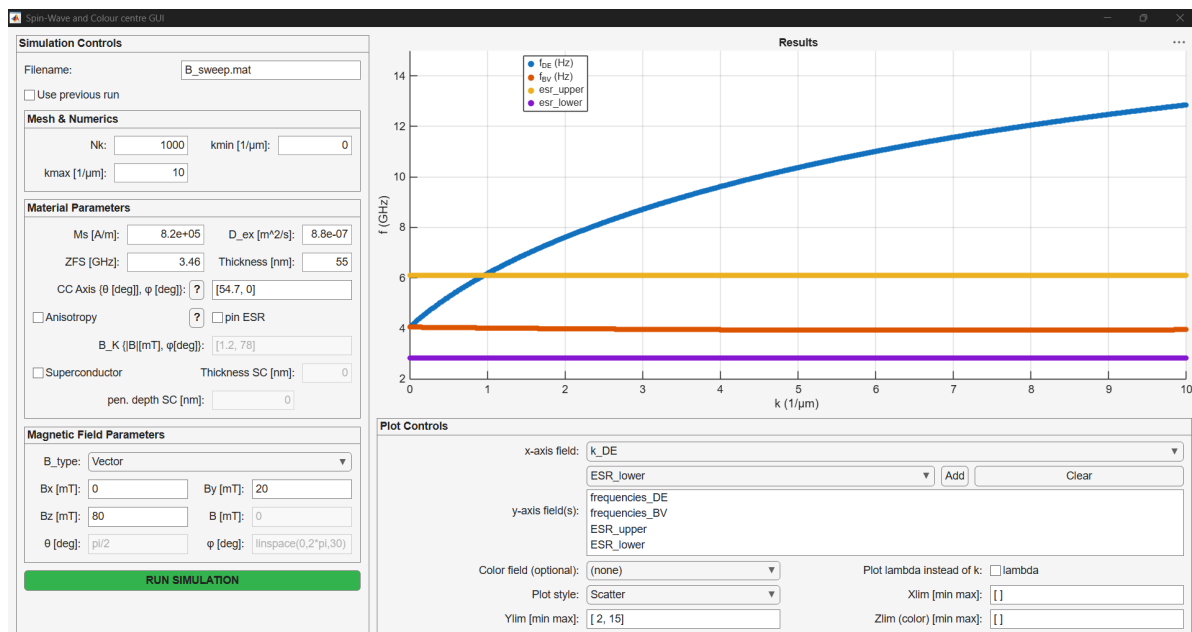


Figure 6.2: **Graphical user interface of the simulation package in MATLAB.** The GUI allows the user to quickly change the magnetic-field parameters, calculate the spin-wave dispersion and colour-centre ESR frequencies, and determine the crossing points from which the measurable spin-wave wavelengths are obtained.

Systematic measurement of backward volume spin-waves in Permalloy

The backward volume spin-wave measurements did not produce the clear plane waves needed as reference data for future mixing experiments. A systematic series was performed in which backward volume spin-waves were excited and imaged for different magnetic fields and excitation frequencies. These measurements were intended to test whether the unmixed spin-waves could provide a reference for later imaging of difference-frequency mixing products. Instead, the observed patterns were strongly scattered and could not be reliably used as clean reference plane waves. This chapter therefore focuses on characterising the observed backward volume spin-wave activity and understanding why the spatial patterns are not well-defined.

7.1. Frequency sweeps for different magnetic fields

To compare backward volume spin-waves with the spatial profiles of their mixing products, systematic measurements were performed for different static magnetic fields and excitation frequencies. This is possible due to the large frequency bandwidth of the V_B^- ESR, as shown in Figure 2.1b. In the backward volume geometry, the magnetisation is parallel to the spin-wave propagation direction. We define the geometry as shown in Fig. 7.1, with B_x along the spin-wave propagation direction, B_y along the microstrip and B_z out of plane.

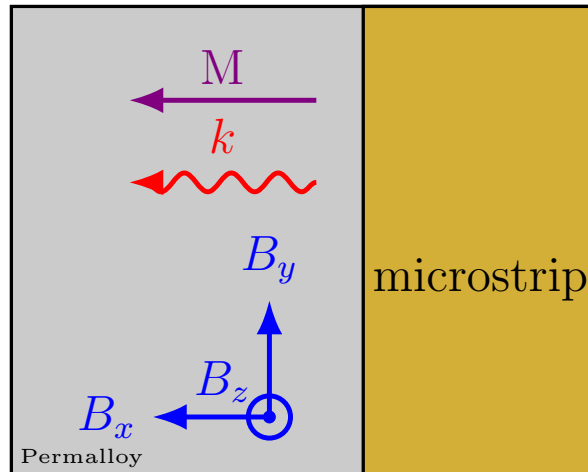


Figure 7.1: **Experimental geometry for backward volume spin-wave excitation.** A microstrip (gold) excites spin-waves in the Permalloy film (grey), propagating with wave vector k and magnetisation M (purple). The applied magnetic field components B_x , B_y , and B_z are indicated in blue, with B_z pointing out of the plane of the sample.

For each magnetic field value, the excitation frequency was varied to effectively sweep through the

spin-wave dispersion. A frequency sweep for one magnetic field value is shown in Fig. 7.2. The gold microstrip was used to excite the spin-waves. Strictly speaking, the measurements are not performed in a perfect backward volume geometry due to the small residual magnetic field component $B_y \approx 1.2$ mT. The small parallel field component originates from slight magnet misalignment during the measurements. Nevertheless, we refer to this configuration as backward volume, since the applied field is predominantly aligned with the spin-wave propagation direction.

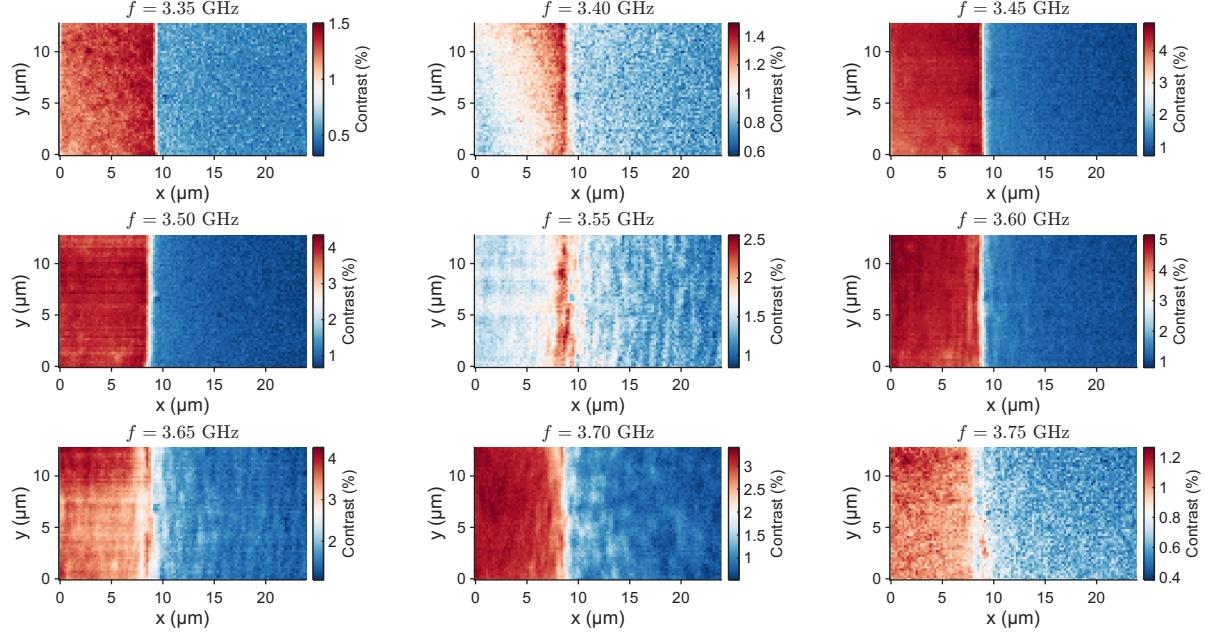


Figure 7.2: **Spatial maps of spin wave activity in the backward volume geometry** spatial spin-wave maps recorded near the microstrip ($x = 15\text{--}25\ \mu\text{m}$) for an applied magnetic field of $[B_x, B_y, B_z] = [16.2, 1.21, 0.4]$ mT, as a function of excitation frequency. Measurements at an external magnetic field of $[B_x, B_y] = [14.2, 1], [15.2, 1.1], [17.2, 1.2]$ mT, are included in Appendix D.1. The excitation frequency is varied across the expected backward volume band. The spatial pattern changes with frequency, but the images do not show clean plane-wave fronts.

As can be seen, different spin-wave activity is observed at different frequencies, as expected from Fig. 7.3, where the dispersion for this measurement is shown. The spatial profiles show some spin-wave activity, but no clear plane waves. In the other frequency sweeps at different fields, the same behaviour is observed, as can be seen in Appendix D.1. The grey lines indicate the measurement frequencies, and the red dots mark the crossings of these frequencies with the dispersion.

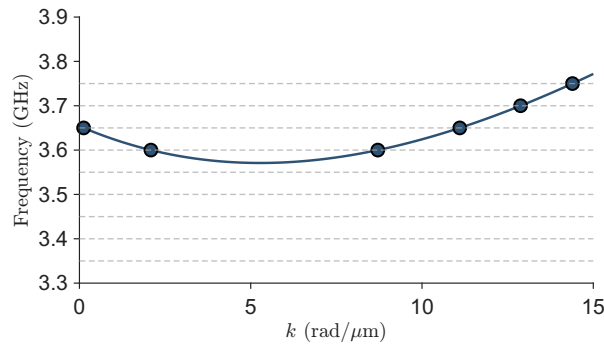


Figure 7.3: **Backward volume spin-wave dispersion.** Dispersion for the backward volume spin-waves for an applied magnetic field of $[B_x, B_y, B_z] = [16.2, 1.21, 0.4]$ mT. The frequencies used for the measurements in Fig. 7.2 are shown with grey lines, and with a blue dot where the dispersion is crossed.

The measured start of spin-wave activity is close to, but not exactly at, the calculated backward volume band edge. From the calculated dispersion in Fig. 7.3, the first BV spin-waves are expected near 3.6 GHz, while weak activity appears already around 3.55 GHz in Fig. 7.2. This difference is

comparable to the effect of the magnetic-field uncertainty, which is approximately 0.5 mT. The frequency sweep therefore shows clear spin-wave activity, but does not provide a precise reconstruction of the BV dispersion.

7.2. Quantitative characterisation of backward volume spin-wave activity

To determine whether the spin-wave activity observed in Figure 7.2 corresponds to backward volume spin-waves, the measured spatial patterns are compared to the expected dispersion. This is first attempted using the Fourier transform and power spectral density (PSD), which should in principle reveal the dominant wave vectors and therefore the corresponding wavelengths. However, because the measured patterns do not form clear plane waves, these methods do not always yield unambiguous results. Therefore, the FFT and PSD are used mainly as indicators of spin-wave activity rather than as a reliable way to fully reconstruct the dispersion.

7.2.1. Fourier transform

To compare this activity with the calculated dispersion, the corresponding wave vector, or equivalently the wavelength, must be extracted from the data. As a first approach, this is done using a Fourier transform. Before applying the transform, the microstrip region was removed, and a linear background was subtracted from the measured map to enhance the spin-wave signal. Figure 7.4a shows this processing procedure for the measurement at $f = 3.65$ GHz. Panel (a) shows the original measurement, and panel (b) shows the Fourier transform of the cropped and detrended image.

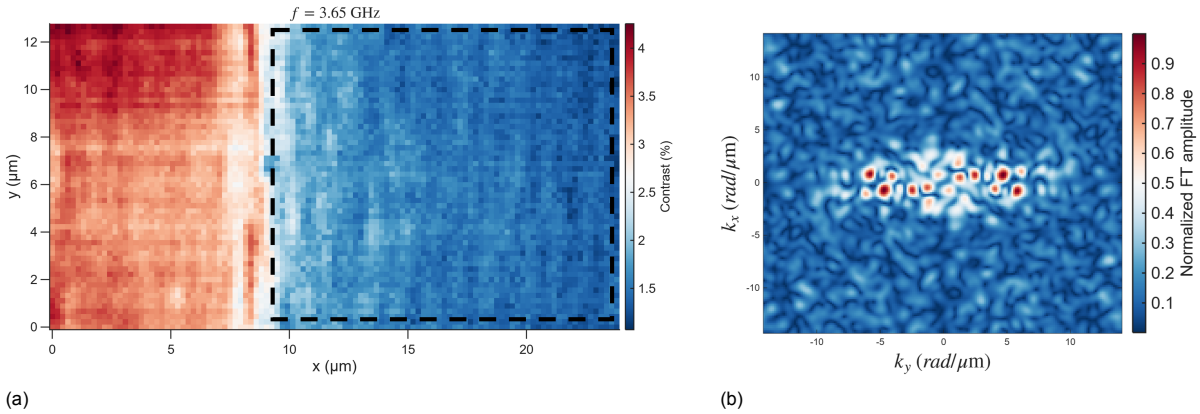


Figure 7.4: **Processing steps used to extract the spatial frequency content of the measured spin-wave map.** (a) Original measurement at $f = 3.65$ GHz with the cropping region indicated with a black dashed square. (b) Fourier transform of the cropped and background-subtracted data.

In Figure 7.4b, no clear isolated peak can be identified in the spectrum, meaning that no single dominant wave vector, and therefore no well-defined wavelength, can be extracted from this measurement. As can be seen in Appendix D.1, almost all spin-wave maps exhibit this behaviour, and no dispersion could be obtained from the Fourier transform.

7.2.2. Power Spectral Density

To reduce the noise in the measurements and to enhance the Fourier transform, power spectral density (PSD) was used. Compared to a direct Fourier transform of the raw data, the PSD has several advantages. First, by taking the autocorrelation, uncorrelated noise is suppressed while coherent, periodic features (such as spin-wave signals) are reinforced. As a result, the PSD provides a cleaner representation of the dominant spatial frequencies in the data [35].

Overall, the PSD can therefore be seen as a more robust measure of the underlying spin-wave activity, particularly in the presence of noise. Taking the PSD of the same measurement as shown in Fig. 7.4a yielded a less noisy frequency spectrum shown in Fig. 7.5a.

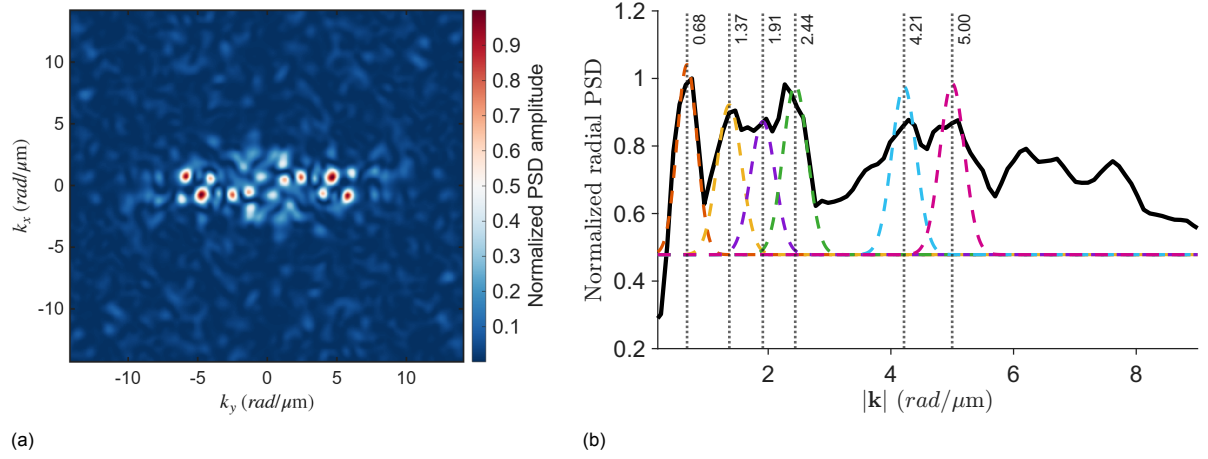


Figure 7.5: **Power spectral density analysis of the measurement shown in Figure 7.4.** (a) Two-dimensional PSD at $f = 3.65 \text{ GHz}$, showing peaks at specific wave vectors $\mathbf{k}$, corresponding to the dominant spatial frequency modes. (b) Radially averaged PSD, with six Gaussians fitted to peaks at $k = 0.68, 1.37, 1.91, 2.44, 4.21,$ and $5 \text{ rad}/\mu\text{m}$. The PSD reveals several spatial-frequency components, but no unique wave vector can be assigned.

To further investigate the spectral features, the radial PSD amplitude was calculated, as shown in Fig. 7.5b. In this spectrum, six Gaussians were fitted, which identified peaks at $k = 0.68, 1.37, 1.91, 2.44, 4.21,$ and $5 \text{ rad}/\mu\text{m}$. From the calculated dispersion in Fig. 7.5b, the expected wave vectors at 3.65 GHz are ($k = 0.1 \text{ rad}/\mu\text{m}$) and ($k = 11 \text{ rad}/\mu\text{m}$). This does not correspond to the observed k -values. One reason is that we are limited in resolution to $k_{\text{max}} = 6.6 \text{ rad}/\mu\text{m}$, as determined in section 4.2.3. Another possibility for the mismatch in k -values is that the applied magnetic field slightly deviates from the predicted values from the calibrations. The remaining peaks are likely caused by scattering of the backward volume spin-waves due to defects, edge roughness, or other inhomogeneities in the magnetic film. It should also be noted that the PSDs obtained from the other frequency sweeps were considerably less distinct. Thus preventing a reliable extraction of the full backward volume dispersion from the experimental data.

7.3. Origin of the scattered spin-waves

The measured spin-waves patterns do not appear as clear plane w. Several mechanisms could lead to this behaviour, including the large bandwidth of the V_B^- , scattering from defects, non-ideal excitation by the microstrip, or filtering due to the finite sensor distance. Before discussing these possible origins in the following sections, I first examine whether the measured patterns are indeed consistent with scattered spin-waves.

7.3.1. Isofrequency contour

To determine whether the spin-waves have scattered, the Fourier transform is compared to the isofrequency contour. The dispersion shown in Figure 7.3 only depends on the absolute value of the wave vector k . In reality, however, the dispersion differs for different k_x and k_y components. For a given frequency, this results in a so-called isofrequency contour, i.e. the set of all k -values that correspond to that frequency, as shown in Fig. 7.6a. An important thing to note is that the bandwidth of the V_B^- is approximately 300 MHz . As a result, during the measurements, all spin-wave modes with frequencies within this bandwidth contribute to the signal.

Consequently, the measured isofrequency contour is expected to appear broader than the contour corresponding to a single frequency. In Fig. 7.6, two isofrequency contours are therefore shown, corresponding to frequencies of $3.65 \pm 0.15 \text{ GHz}$. In the absence of scattering, the excited wave vector is restricted by the geometry of the microstrip, and one would therefore expect to observe spin-waves with a k -vector that lies on the dispersion. However, if scattering occurs, spin-waves can redistribute over different directions in k and frequency space. In this case, taking the Fourier transform of the spatial data would reveal an isofrequency contour, rather than a single localised peak.

The measured Fourier spectrum shows partial overlap with the calculated isofrequency contour,

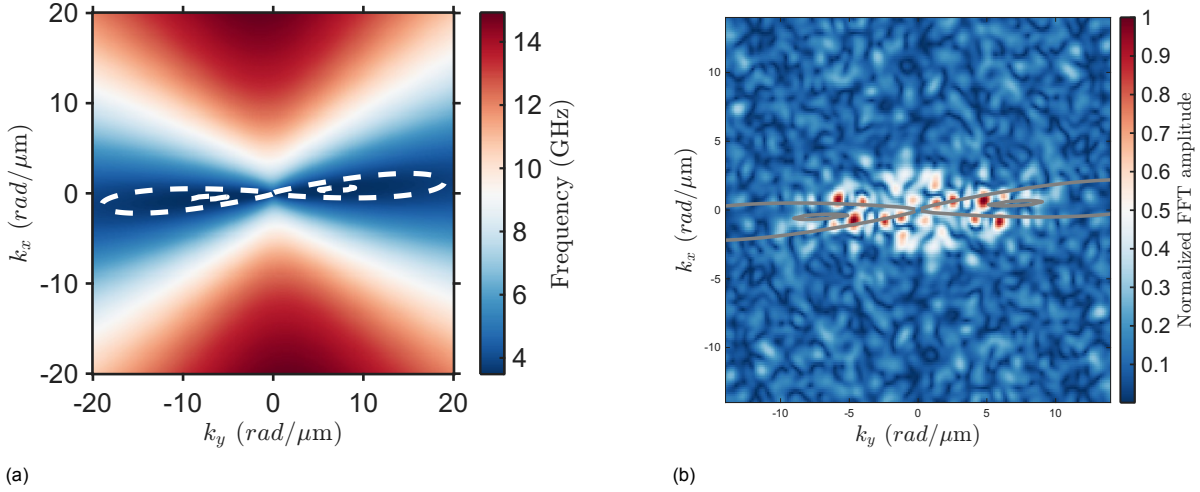


Figure 7.6: **Comparison between the calculated isofrequency contour and the measured Fourier spectrum.** (a) Two-dimensional spin-wave dispersion at $[B_x, B_y, B_z] = [16.2, 1.21, 0.4]$ mT, with the isofrequency contour at 3.65 ± 0.15 GHz shown as a white dashed line. (b) Fourier transform of the measured spin-wave map from Figure 7.4, with the calculated isofrequency contour overlaid in gray.

but the agreement is not exact. In Figure 7.6b, the Fourier transform of the measurement shown in Figure 7.4 is presented, with the corresponding isofrequency contour overlaid in gray. We can infer that the isofrequency contour is not fully in correspondence with the data; this is due to the uncertainty in the magnetic field. The relatively low signal for high k -values in the FFT is a consequence of the finite spatial resolution of the V_B^- centres as stated in section 4.2.3. Additionally, a central feature, appearing as a “blob” at the intersection of the cross-like structure, can be observed. A similar feature has been reported in [9]. The appearance of the isofrequency contour suggests that the backward volume spin-waves have undergone scattering into other wave vectors along the isofrequency contour. The next step is therefore to investigate the possible mechanisms responsible for this scattering.

7.3.2. Scattering due to non-ideal microstrip excitation

One possibility for the observed scattered spin-waves is that they did not scatter from defects in the permalloy but are instead excited directly by a defective microstrip. If we assume the microstrip to be an ideal rectangular conductor, it excites spin-waves with a well-defined wave vector perpendicular to the microstrip. However, in the presence of imperfections—such as edge roughness, defects, or inhomogeneities—additional wave vector components can be excited [36, 37, 38, 39, 40]. In this case, the microstrip no longer acts as a simple, single- k source, but instead generates a broader distribution of wave vectors along the isofrequency contour.

As discussed in Section 3.2, and repeated here for clarity, the stray field generated by a microstrip depends on both its current distribution and its spatial Fourier spectrum, as shown in equation 3.9, repeated here. Therefore, the range of k -values that can be excited is determined by these parameters.

$$\widetilde{B}_x(\mathbf{k}, z, \omega) = 2\mu_0 \underbrace{J_y(\omega)}_{\text{current distribution}} \underbrace{\frac{e^{\kappa z} e^{-\kappa h} - 1}{\kappa}}_{\text{decay factor}} e^{-ik_y y} \underbrace{\frac{\sin(k_x w/2)}{k_x} \frac{\sin(k_y L/2)}{k_y}}_{\text{FT of rectangular microstrip}} \quad (7.1)$$

This equation motivates a simple toy model in which either the microstrip shape is modified to mimic geometric imperfections, or the current distribution is changed to mimic defects or inhomogeneities inside the conductor. By calculating the Fourier spectrum of this non-ideal microstrip, selecting only the k -values that lie on the isofrequency contour, and transforming the result back to real space, we obtain a qualitative estimate of the spin-wave pattern that such imperfections could excite. In this simplified model, the out-of-plane stray field is disregarded, and the microstrip is assumed infinitely thin. Using this toy model, we investigate the effect of microstrip imperfections on the spatial profile of spin-waves.

Edge roughness

First, we consider the effect of edge roughness introduced during the microstrip fabrication process on the spin-wave pattern [41]. The microstrip is modelled as a rectangular bar with a random edge roughness of 400 nm, as shown in Fig. 7.7a. The edge roughness is relatively large, but is used here to clearly illustrate how edge imperfections can introduce scattering. Because the Fourier transform of the rough microstrip contains wave vector components that are not strictly perpendicular to the microstrip, additional spin-wave directions can be excited. This results in a scattered spin-wave pattern, approximately similar to that observed in the measurements above and as shown in Figure 7.7b.

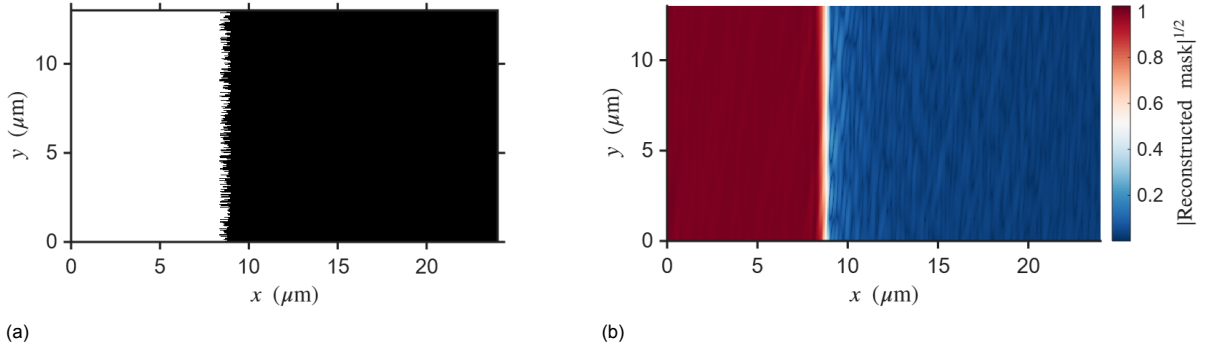


Figure 7.7: **Qualitative simulation of spin-wave excitation by a microstrip with edge roughness.** (a) Real-space simulation of the rough microstrip. (b) Reconstruction of the selected spin-wave pattern. The microstrip has a width of $10 \mu\text{m}$, a length of $50 \mu\text{m}$, an edge roughness amplitude of 400 nm, which is quite large but used for illustrative purposes, and a pixel size of 50 nm. The selected isofrequency band is centred around $f = 3.65 \text{ GHz}$ with a bandwidth of $\pm 0.15 \text{ GHz}$, comparable to the bandwidth of the V_B centre, and an external magnetic field of $[B_x, B_y, B_z] = [16.2, 1.21, 0.4] \text{ mT}$. The spin-wave pattern is similar to that of the measurements in Fig. 7.4.

Non-uniform current distribution

Next, we consider the effect of a non-uniform current distribution within the microstrip. As no defects are observed in the microstrip under visual inspection with the optical microscope, it is unlikely that edge roughness alone is responsible for the imperfect spin-wave excitation. Either these edge effects are too small to have a significant influence, or the origin of the imperfect excitation lies elsewhere. Local defects in the gold can modify the current flow. Examples include grain-boundary-related variations, local thickness changes, or regions with reduced conductivity [42, 43, 44, 45].

To model this, the microstrip is modelled as a perfect rectangular bar, with local conductivity defects introduced within it, see Fig. 7.8a. The current distribution is then calculated by solving the approximate conduction equation

$$\nabla \cdot (\sigma \nabla \phi_{el}) = 0,$$

where σ is the local conductivity and ϕ_{el} is the electric potential. The current density is then obtained from

$$\mathbf{J} = -\sigma \nabla \phi_{el}.$$

The simulated current-defect pattern resembles the measured scattered spin-wave maps, see Fig. 7.8b. This does not prove that current defects are present in the microstrip, but it shows that non-uniform current flow can scatter spin-waves. Current defects are therefore a plausible mechanism for the observed non-plane-wave spin-wave patterns.

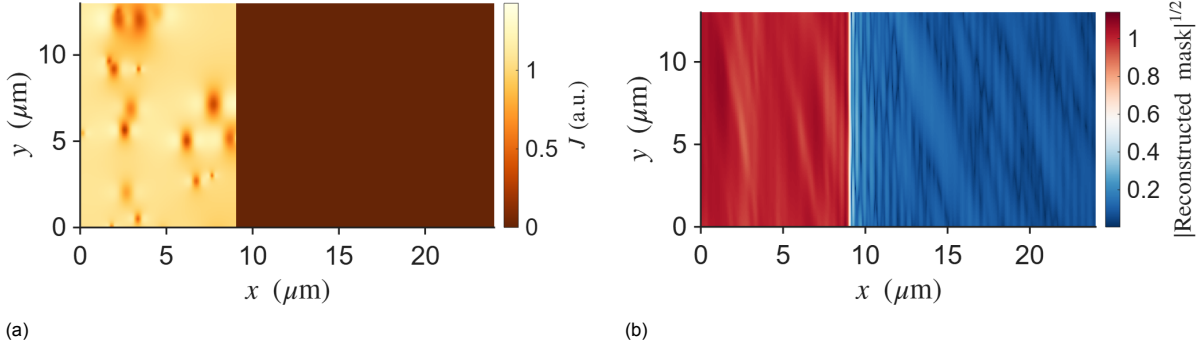


Figure 7.8: **Qualitative simulation of spin-wave excitation due to a non-uniform current distribution inside the microstrip.** (a) Calculated current-density distribution with local conductivity defects. (b) Reconstruction of the selected spin-wave pattern. The selected isofrequency band is centred around $f = 3.65$ GHz with a bandwidth of ± 0.15 GHz and an external magnetic field of $[B_x, B_y, B_z] = [16.2, 1.21, 0.4]$ mT. The observed spatial pattern resembles that of the measured spin-waves.

Overall, the above effects of edge roughness and electrical defects of a microstrip can contribute to spin-waves being scattered and therefore can be a cause of the spin-wave scattering effect we see in the measurements of Fig. 7.2

7.3.3. Defect induced scattering

Another mechanism that could contribute to the scattering of the backward volume spin-waves is scattering from imperfections within the Permalloy film itself. Due to the growth process of thin Permalloy films, the material is not perfectly homogeneous but consists of crystalline grains separated by grain boundaries, typically on length scales of approximately 10–20 nm [46, 47]. These structural inhomogeneities can lead to local variations in the effective magnetic properties of the film, such as the anisotropy and exchange interaction [46]. As a result, the effective magnetic field is not perfectly uniform throughout the film, providing a possible source of spin-wave scattering. This is especially relevant for short-wavelength spin-waves, as their dynamics is mediated by the exchange interaction. Because the backward volume modes studied here can have short wavelengths, local variations in exchange and anisotropy could contribute to the observed scattering.

7.3.4. Distance based filtering

In combination with the previous argument, if we assume that the microstrip defects are small, we would mainly expect scattering of spin-waves with relatively short wavelengths (large k). As shown in [9] for YIG, and as described in section 4.3, the stray field of spin-waves is spatially filtered depending on both the wave vector k and the distance to the sensor. As our flake is directly on top of the Permalloy, we are more sensitive to short spin-wavelengths, as discussed in section 4.3.

7.4. Qualitative characterisation of spin-wave activity

Here, I introduce the inverse participation ratio (IPR) and Watershed transform as measures to characterise the observed patterns and to assess whether they are consistent with the backward volume dispersion. In the previous sections, I showed that the measurements in Figure 7.2 and Appendix D.1 contain scattered spin-wave activity. However, the quantitative analysis did not yield a definitive match to the backward volume dispersion, as the dispersion could not be reliably extracted from the FFT or PSD. We therefore turn to the IPR and Watershed transform, which quantify the spatial and frequency structure of the measurements directly and do not rely on extracting a well-defined dispersion.

7.4.1. Inverse Participation Ratio

The first method used to quantify spin-wave activity is the inverse participation ratio (IPR). In quantum mechanics, the IPR is used to describe how localised a wavefunction is [48, 49]. It is defined as

$$\text{IPR} = \frac{\sum_i^N |\Psi_i|^4}{\left(\sum_i^N |\Psi_i|^2\right)^2}. \quad (7.2)$$

Although a spin-wave map is not a wavefunction, the same expression can still be used as a measure of how localised the signal is in an image by taking the IPR of the PSD. In Fig 7.9 an example is shown of the PSD for spatial images with one dominant wave, with multiple waves and with noise. The PSD of the single wave is strongly localised in two points, $(\pm k)$. Thus, when a measurement contains a single spin-wave, we would expect a localised PSD and thus a high IPR. For the case of the multiple waves, the PSD has a more spread-out spectrum and thus a lower IPR. In the case of noise, the PSD spectrum is itself noisy and very delocalized, yielding an even lower IPR. Hence, we can use the IPR to analyse the presence and spatial structure of spin-waves.

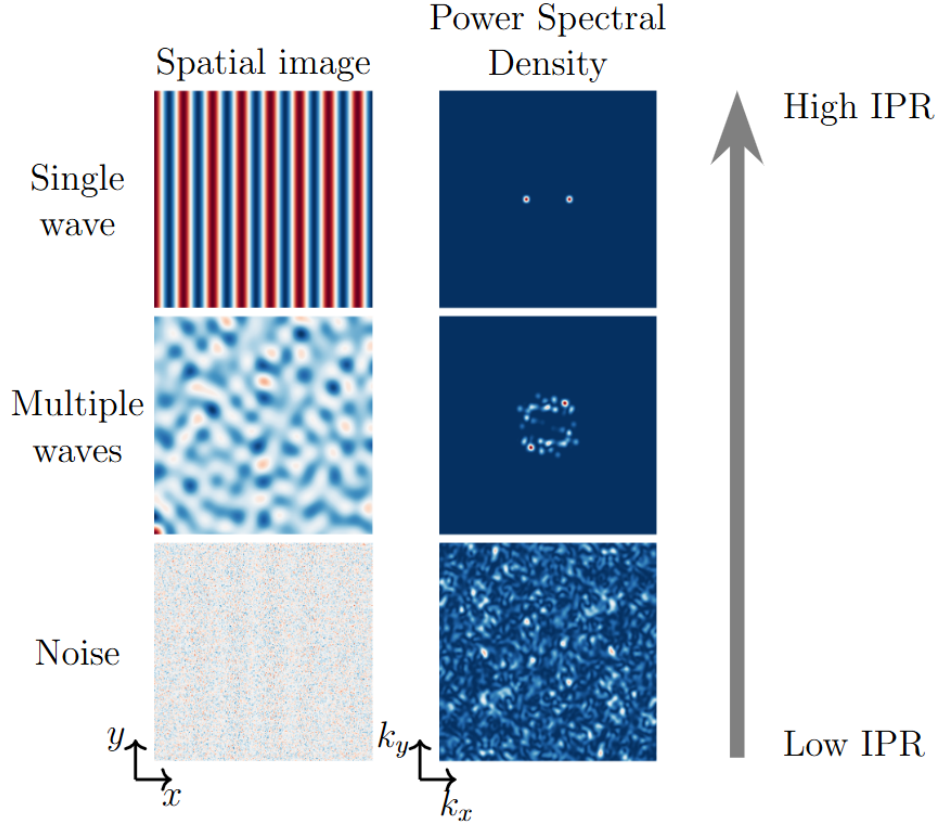


Figure 7.9: **Example of three spatial images with their corresponding PSD with varying IPR values.** The single wave has the most localised spectrum at $(\pm k)$ and thus the highest IPR. Multiple waves have a more spread-out PSD spectrum and a lower IPR. The IPR is even lower when the image and corresponding PSD are pure noise.

We expect an average IPR when spin-waves are excited at the bottom of the band, where two spin-waves are excited. A slightly higher IPR is expected when the dispersion is crossed only once until the excited k-vector becomes too large to be probed, and the lowest IPR is expected when the excitation frequency does not lie on the dispersion. In Fig. 7.10a, the backward volume dispersion and the IPR are shown for frequency sweeps at different magnetic fields. The IPR increases when the excitation frequency crosses the dispersion, peaks where the band is crossed once for the first time and decreases when the k-vectors become too large. This behaviour supports that the observed spatial structures in the measurements can originate from backward volume spin-waves.

A possible counterargument is that the apparent increase in spin-wave activity is caused by frequency-dependent $V_{\bar{B}}$ sensitivity rather than by spin-wave excitation. For example, frequencies near a deeper ESR dip could produce stronger PL contrast even if the spin-wave amplitude did not change. To check this, the ESR spectrum was compared with the frequency-dependent spin-wave activity, as shown in Appendix D.2. The strongest features in the spin-wave maps do not simply coincide with the largest ESR contrast. The frequency dependence of the measured activity therefore cannot be explained by the $V_{\bar{B}}$ ESR response alone.

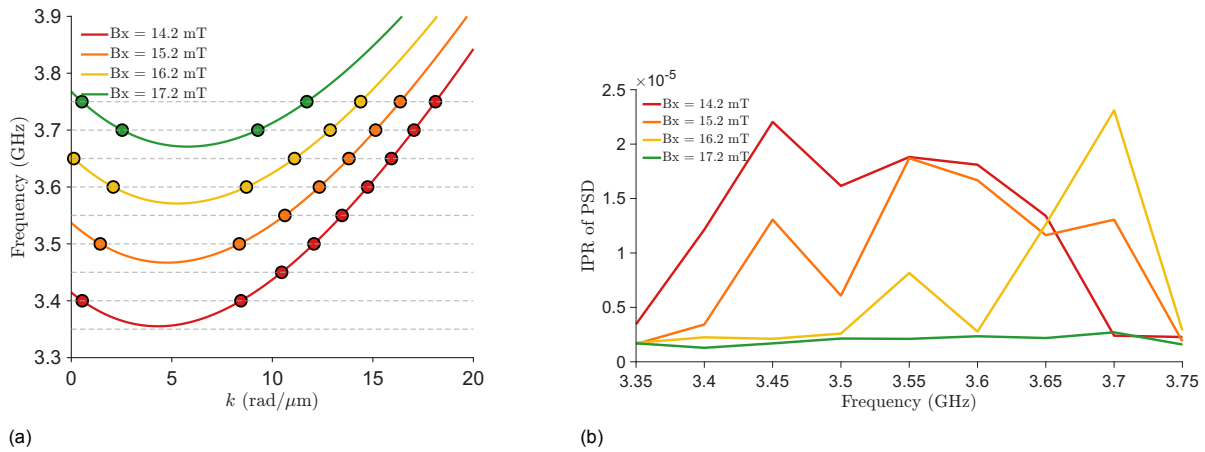


Figure 7.10: **Comparison between the backward volume spin-wave dispersion and the IPR of the PSD of the measured spin-wave maps** for applied fields of $[B_x, B_y] = [14.2, 1], [15.2, 1.1], [16.2, 1.1],$ and $[17.2, 1.2]$ mT. (a) Backward volume spin-wave dispersion with the excitation-frequency intersections marked. (b) IPR of the PSD-transformed spin-wave maps. A higher IPR corresponds to a more localised PSD.

7.4.2. Watershed transform

A second method to characterise spin-wave activity is to use a Watershed transform. Instead of analysing the localisation of the PSD this method looks directly at the spatial structure of the measured spin-wave maps. The Watershed transform is a way to divide an image into different regions by thinking of the image as a landscape. Bright and dark parts of the image are treated as hills and valleys. If we imagine filling this landscape with water from the lowest points, then water will start to collect in different valleys. When water from two valleys is about to meet, a boundary is drawn between them. In this way, the image is split into separate compartments [50]. This is graphically shown in Fig. 7.11 for a single wave, multiple waves and noise.

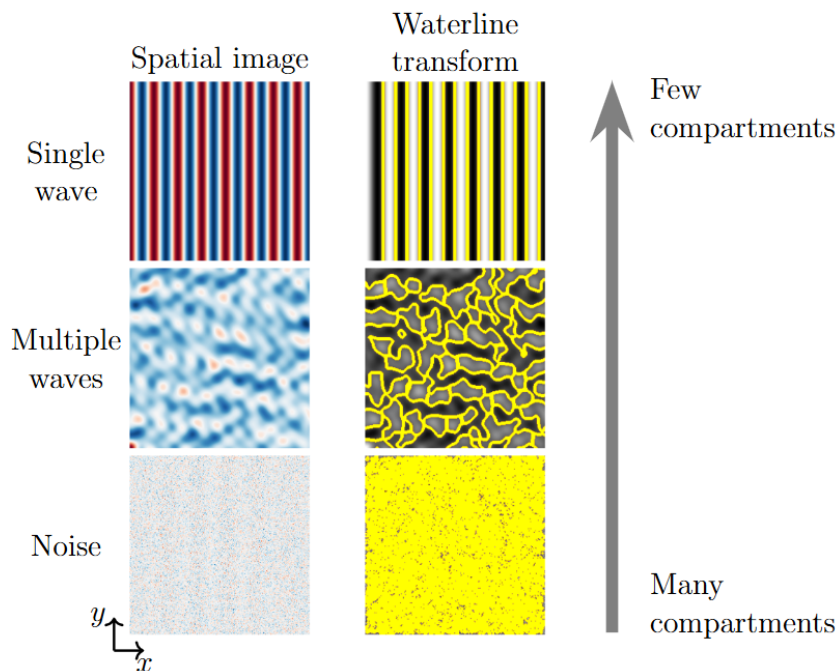


Figure 7.11: **Example of three spatial images with their corresponding Watershed transform compartments (yellow).** The single wave has the least spatial variation and thus the lowest amount of compartments. Multiple waves have more spatial variation and more compartments. The highest amount of compartments is when the image and Watershed transform are pure noise.

It can be seen that a single clear spin-wave signal has the fewest compartments, the signal with multiple waves has more compartments, and the noise has the most. Our spin-wave maps should yield the same result. Thus, fewer compartments mean a clearer spin-wave pattern, while many compartments mean the image is more fragmented and probably contains less spin-wave activity. In Fig. 7.12b the number of compartments is shown for different frequencies and magnetic fields. The dispersion in Fig. 7.10a is repeated here for easy comparison. Here we see the number of compartments drop when we expect the most spin-wave activity and rise again when the spin-waves become too small to probe. This also provides a possible measure for the backward volume spin-wave dispersion.

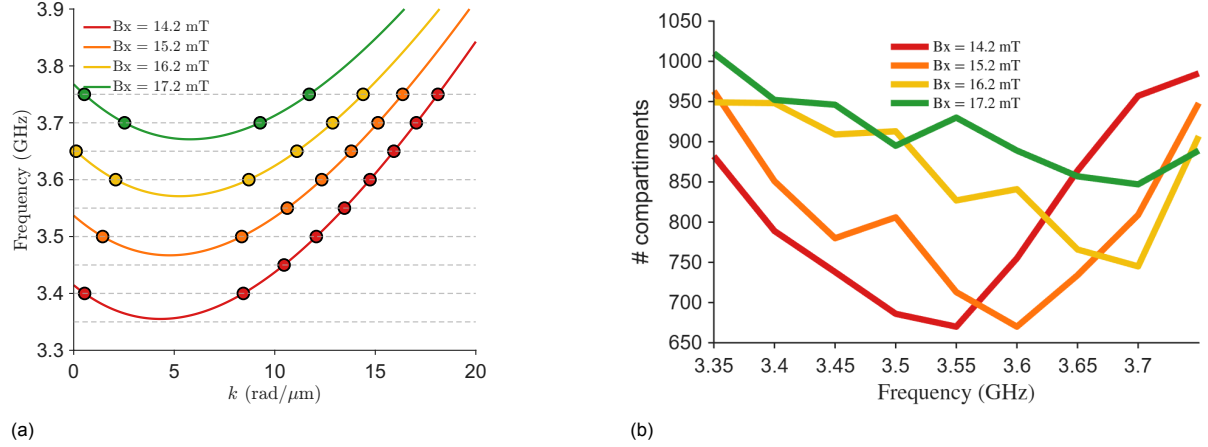
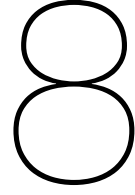


Figure 7.12: **Comparison between the calculated backward volume spin-wave dispersion and the experimentally extracted Watershed-compartment count** for applied fields of $[B_x, B_y] = [14.2, 1], [15.2, 1.1], [16.2, 1.1],$ and $[17.2, 1.2]$ mT. (a) Backward volume spin-wave dispersion for the different applied magnetic fields. The marked points indicate the intersections between the excitation frequencies and the dispersion. (b) Number of watershed compartments as a function of excitation frequency. A lower number of compartments corresponds to a more continuous spatial pattern, which is used here as a qualitative indicator of stronger spin-wave activity.

The watershed method provides a complementary measure to the IPR. While the IPR characterises the localisation of the signal in k -space, the watershed transform characterises the fragmentation of the signal in real space. Both methods are therefore sensitive to spin-wave activity, but from different perspectives. The IPR is expected to increase when the Fourier spectrum becomes more localised, while the number of watershed compartments is expected to decrease when the real-space spin-wave pattern becomes more pronounced.

The IPR and watershed analyses both indicate that the measured frequency-dependent activity is consistent with backward volume spin-wave excitation. The FFT and PSD do not provide a reliable wave-vector reconstruction, so the measurements cannot be used as clean plane-wave reference data for spin-wave mixing. However, the qualitative metrics show changes near the frequencies where BV modes are expected from the calculated dispersion. The next step is therefore to repeat the experiment in a geometry that produces cleaner wavefronts before attempting to image the difference-frequency mixing product.



Conclusion

The goal of this thesis was to move towards imaging the difference-frequency product generated by two mixing spin-waves. To prepare for this experiment, we took three main steps. First, we imaged backward volume spin-waves in Permalloy to test whether they could be used as a reference for the mixed signal. Second, I improved the magnetic-field control by developing and calibrating magnet systems for both the present measurements and future spin-wave mixing experiments. Third, I developed a simulation package with graphical interfaces in MATLAB and Python to combine the spin-wave dispersion with the colour-centre ESR frequencies. Together, these steps provide the experimental and computational groundwork for future imaging of spin-wave mixing.

8.1. Outlook

As discussed in this thesis, backward volume spin-waves are not ideal reference material for spin-wave mixing measurements. Although spin-wave activity was observed, the measured patterns do not resemble clear plane waves and instead appear strongly scattered. As a result, it is difficult to use them as a reference for the mixed signal.

As shown in Section 3.3.2, the Damon–Eshbach spin-wave branch can exhibit a stray field oscillating at the difference frequency by relaxing the requirement that the magnetisation is perpendicular to the propagation direction of the spin-wave. Experimentally, this can be achieved by applying the magnetic field at a small angle with respect to the microstrip. This would be my recommendation for future mixing experiments, as Damon–Eshbach spin-waves exhibit well-defined wavefronts and therefore provide better reference material.

Another possible route towards improved imaging of spin-wave mixing signals is the use of an NV^- scanning tip or NV^- membrane, as their resolution is better than the V_B^- centre.

An additional advantage of an NV^- scanning tip is that the sensor–sample distance can be chosen independently and is not fixed by the thickness of a transferred flake or membrane. This allows measurements to be performed at larger stand-off distances, which preferentially detect longer-wavelength backward volume spin-waves that are expected to scatter less.

Finally, when the backward volume dispersion is crossed twice, two spin-waves with the same frequency are excited. These spin-waves could still mix, but the resulting difference-frequency signal becomes a DC magnetic field rather than an oscillating field, since the frequency difference is zero. Such a signal could potentially be measured using an NV^- centre through DC magnetometry. This provides an additional route for studying spin-wave mixing that may be particularly useful for backward volume spin-waves.

Declaration of AI Use

Artificial intelligence tools were used during the preparation of this thesis for language support and programming assistance. ChatGPT (OpenAI) was used to improve grammar, spelling, style, and readability of the written text. In addition, ChatGPT was used to generate complete code files and scripts for data analysis, simulations, visualization, and figure generation. It was also used to assist in the conversion of code between MATLAB and Python.

All scientific content, research design, experimental work, data collection, analysis, interpretation of results, figures, and conclusions were developed independently by the author. Any AI-generated text or code was critically reviewed, tested, modified where necessary, and verified by the author before use. Responsibility for the correctness of all methods, analyses, software implementations, results, and conclusions remains entirely with the author.

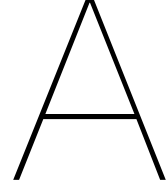
The author gratefully acknowledges ChatGPT (OpenAI) for assisting with language refinement, programming support, code translation, and the drafting of this declaration.

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Extended Theory

A.0.1. Exchange Interaction

The interaction between neighbouring spins is described in the Heisenberg model as

$$\hat{\mathcal{H}} = - \sum_{ij} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j \quad (\text{A.1})$$

where $\mathbf{S}_i$ is the i^{th} spin and J_{ij} is the exchange constant between neighbouring spins. Now we can write down the energy, using $\mathbf{S}_i \cdot \mathbf{S}_j = S^2 \cos \phi_{ij}$ and using the fact that the angle between the spins, ϕ_{ij} , is small, as alignment is energetically favourable, we get

$$E = -JS^2 \sum_{ij} \cos \phi_{ij} \approx -JS^2 \sum_{ij} \frac{\phi_{ij}^2}{2} \quad (\text{A.2})$$

now we can approximate ϕ_{ij} with $\phi_{ij} = |\mathbf{m}_i - \mathbf{m}_j| \approx |\nabla \mathbf{m}|$ taking the continuous limit into account. So now we can write our energy, also called the free energy as

$$F_{ex} = A_{ex} \sum_{i=x,y,z} \sum_{j=x,y,z} \left(\frac{\partial m_i}{\partial j} \right)^2. \quad (\text{A.3})$$

Here A_{ex} is the exchange constant for the continuous limit.

The exchange field is then related to the free energy as:

$$\mathbf{B}_{\text{exchange}} = - \frac{1}{M_s} \frac{\delta F_{ex}}{\delta \mathbf{m}}. \quad (\text{A.4})$$

To make the calculations easier, we take the Fourier transform for x and y of the free energy, using the fact that $\mathcal{F} \left(\frac{\partial m_i}{\partial j} \right) = ik_j m_i(\mathbf{k})$; we obtain

$$F_{ex} = A_{ex} \sum_{i=x,y,z} \left[\left(\frac{\partial m_i(\mathbf{k}, z)}{\partial z} \right)^2 + k^2 m_i^2(\mathbf{k}, z) \right] \quad (\text{A.5})$$

where we define $k = \sqrt{k_x^2 + k_y^2}$. Because the magnetisation across the film thickness is constant, the first term vanishes. Now we can calculate the exchange contribution to the effective magnetic field.

$$B_{\text{ex},i} = - \frac{2A_{ex}}{M_s} k^2 m_i(\mathbf{k}, z) \quad (\text{A.6})$$

defining $\omega_{\text{ex}} = \frac{2\gamma A_{ex}}{M_s}$

$$B_{\text{ex},i} = - \frac{\omega_{\text{ex}}}{\gamma} k^2 m_i(\mathbf{k}, z) \quad (\text{A.7})$$

A.0.2. Demagnetising interaction

The dipole-dipole field is the field a spin feels from another spin in the material at distance $\mathbf{r}'$. This interaction causes spins to misalign (in a ferromagnet). Due to this, the magnetic field gets "demagnetised" as spins flip and the overall magnetisation drops. Thus the demagnetising field, we need to consider the dipole field of a magnetic film, defined as

$$\mathbf{B}_{\text{dipole}} = \mu_0 M_s \int d\mathbf{r}' \Gamma(\mathbf{r} - \mathbf{r}') \mathbf{m}(\mathbf{r}') \quad (\text{A.8})$$

in which Γ is the dipole tensor, which describes the field $\mathbf{B}$ at $\mathbf{r}$ due to the magnetisation $\mathbf{m}$ at $\mathbf{r}'$ and it is defined as

$$\Gamma_{ij}(\mathbf{r}) = \partial_i \partial_j \frac{1}{4\pi|\mathbf{r}|}, \quad i, j = x, y, z. \quad (\text{A.9})$$

We first transform the dipole tensor into Fourier space for x and y as

$$\hat{\Gamma}_{ij}(\mathbf{k}, z) = \int dx dy e^{-i(k_x x + k_y y)} \Gamma_{ij}(\mathbf{r}) \quad (\text{A.10})$$

$$= \partial_i \partial_j \int dx dy e^{-i(k_x x + k_y y)} \frac{1}{4\pi|\mathbf{r}|} \quad (\text{A.11})$$

$$= \partial_i \partial_j \frac{e^{-k|z|}}{2k} \quad (\text{A.12})$$

which gives for the full dipole tensor in Fourier space.

$$\hat{\Gamma}_{ij}(\mathbf{k}, z) = \begin{pmatrix} -\frac{k_x^2}{2k} e^{-k|z|} & -\frac{k_x k_y}{2k} e^{-k|z|} & -\frac{i k_x}{2} e^{-k|z|} \\ -\frac{k_x k_y}{2k} e^{-k|z|} & -\frac{k_y^2}{2k} e^{-k|z|} & -\frac{i k_y}{2} e^{-k|z|} \\ -\frac{i k_x}{2} e^{-k|z|} & -\frac{i k_y}{2} e^{-k|z|} & \frac{k}{2} e^{-k|z|} - 1 \end{pmatrix}. \quad (\text{A.13})$$

Now we need to average over the thickness to get the demagnetisation contribution to the effective field, which gives us

$$\mathbf{B}_{\text{dem}}(\mathbf{k}) = \mathbf{B}_{\text{dipole}} = -\mu_0 M_s \bar{\Gamma}_{ij}(\mathbf{k}) \mathbf{m}(\mathbf{k}) \quad (\text{A.14})$$

To average over a film of thickness t :

$$\bar{\Gamma}_{ij}(\mathbf{k}) = \frac{1}{t} \int_0^t dz' \int_0^t dz \hat{\Gamma}_{ij}(\mathbf{k}, z - z'). \quad (\text{A.15})$$

Using the substitution $u = z - z'$, this becomes

$$\bar{\Gamma}_{ij}(\mathbf{k}) = \frac{1}{t} \int_{-t}^t (t - |u|) \hat{\Gamma}_{ij}(\mathbf{k}, u) du. \quad (\text{A.16})$$

Due to $\text{sgn}(u)e^{-k|u|}$ being odd, it vanishes during integration, so that

$$\bar{\Gamma}_{zx} = \bar{\Gamma}_{zy} = 0. \quad (\text{A.17})$$

and we now can compute the entire dipole tensor averaged over the thickness.

$$\bar{\Gamma}_{ij}(\mathbf{k}) = \begin{pmatrix} -f(kt) \frac{k_x^2}{k^2} & -f(kt) \frac{k_x k_y}{k^2} & 0 \\ -f(kt) \frac{k_x k_y}{k^2} & -f(kt) \frac{k_y^2}{k^2} & 0 \\ 0 & 0 & 1 - f(kt) \end{pmatrix}, \quad (\text{A.18})$$

defining the so-called finite-thickness dipolar form factor or dynamic demagnetisation factor,

$$f(kt) = 1 - \frac{1 - e^{-kt}}{kt}, \quad (\text{A.19})$$

so we have, defining $\omega_{\text{demag}} = \gamma\mu_0 M_s$ and ϕ being the inplane angle between $\mathbf{m}$ and $\mathbf{k}$.

$$\mathbf{B}_{\text{dem}}(\mathbf{k}) = \frac{\omega_{\text{demag}}}{\gamma} \begin{pmatrix} f(kt) \cos^2 \phi & f(kt) \sin \phi \cos \phi & 0 \\ f(kt) \sin \phi \cos \phi & f(kt) \sin^2 \phi & 0 \\ 0 & 0 & f(kt) - 1 \end{pmatrix} \begin{pmatrix} m_x \\ m_y \\ m_z \end{pmatrix}. \quad (\text{A.20})$$

A.1. Spin-wave Dispersion

We can now define $\mathbf{B}_{\text{eff}}$ in more specific terms, yielding

$$B_{\text{eff},x} = -f(kt) \sin \phi \frac{\omega_{\text{demag}}}{\gamma} (\sin \phi m_x - \cos \phi m_y) - \frac{\omega_{\text{ex}}}{\gamma} k^2 m_x + B_{\text{AC},x} \quad (\text{A.21})$$

$$B_{\text{eff},y} = \frac{\omega_{\text{demag}}}{\gamma} (-f(kt) \sin \phi \cos \phi m_x - f(kt) \cos^2 \phi m_y) - \frac{\omega_{\text{ex}}}{\gamma} k^2 m_y + \frac{\omega_B}{\gamma} \quad (\text{A.22})$$

$$B_{\text{eff},z} = \frac{\omega_{\text{demag}}}{\gamma} (f(kt) - 1) m_z - \frac{\omega_{\text{ex}}}{\gamma} k^2 m_z + B_{\text{AC},z} \quad (\text{A.23})$$

To obtain the spin-wave dispersion, we need to linearize the LLG equation (eq. 3.1). For this linearisation we use that $\mathbf{m} = (m_x, 1, m_z)$, $\dot{m}_y = 0$. Here we followed the derivation as done in [51] using the geometry as shown in Fig. 3.2b

$$-i\omega m_z = -\gamma (m_x B_{\text{eff},y} - B_{\text{eff},x}) + i\alpha\omega m_x \quad (\text{A.24})$$

$$-i\omega m_x = -\gamma (B_{\text{eff},z} - m_z B_{\text{eff},y}) - i\alpha\omega m_z \quad (\text{A.25})$$

The linearisation of the LLG equation states that for $B_{\text{eff},x}$ and $B_{\text{eff},z}$ we only need the dynamic parts and for $B_{\text{eff},y}$ we only need the static part with $k = 0$ on B_{eff} which gives us,

$$B_{\text{eff},x} = -f(kt) \sin^2 \phi \frac{\omega_{\text{demag}}}{\gamma} m_x - \frac{\omega_{\text{ex}}}{\gamma} k^2 m_x + B_{\text{AC},x} \quad (\text{A.26})$$

$$B_{\text{eff},y} = \frac{\omega_B}{\gamma} \quad (\text{A.27})$$

$$B_{\text{eff},z} = \frac{\omega_{\text{demag}}}{\gamma} (f(kt) - 1) m_z - \frac{\omega_{\text{ex}}}{\gamma} k^2 m_z + B_{\text{AC},z} \quad (\text{A.28})$$

From the linearisation we can combine equations A.25–A.28 to get the following matrix:

$$\begin{pmatrix} -i\omega & \omega_3 - i\alpha\omega \\ \omega_2 - i\alpha\omega & i\omega \end{pmatrix} \begin{pmatrix} m_x \\ m_z \end{pmatrix} = \gamma \begin{pmatrix} B_{\text{AC},x} \\ B_{\text{AC},z} \end{pmatrix} \quad (\text{A.29})$$

where we redefined

$$\omega_0 = \omega_B + \omega_{\text{ex}} k^2 \quad (\text{A.30})$$

$$\omega_2 = \omega_0 + \omega_{\text{demag}} (1 - f(kt)) \quad (\text{A.31})$$

$$\omega_3 = \omega_0 + \omega_{\text{demag}} f(kt) \sin^2 \phi \quad (\text{A.32})$$

We can now calculate the susceptibility tensor, which describes how the material responds to an applied magnetic field, from the above equations, using $\delta\mathbf{m} = \chi\mathbf{B}_{\text{AC}}$, which gives us:

$$\chi = \frac{\gamma}{\Lambda} \begin{pmatrix} i\omega & -\omega_2 + i\alpha\omega \\ -\omega_3 + i\alpha\omega & -i\omega \end{pmatrix} \quad (\text{A.33})$$

with $\Lambda = \omega^2 - (\omega_2 - i\alpha\omega)(\omega_3 - i\alpha\omega)$. We can find the spin-wave dispersion relation by setting $\Lambda = 0$, due to the fact that when the susceptibility diverges to infinity, the "willingness" of the spins to follow

the oscillating magnetic field ($\mathbf{B}_{AC}$) becomes maximal and thus spin-waves are created. The real part of this solution is:

$$\omega_{\text{spin-wave}} \approx \sqrt{\omega_2 \omega_3}. \quad (\text{A.34})$$

here α was neglected as it is small.

A.1.1. Arbitrary applied field angle

In the derivation above we assumed the magnetisation and applied magnetic field to be along y , which is generally not the case. To account for an out-of-plane tilt in the magnetisation we need to change the Zeeman and demagnetising contributions. The Zeeman contribution becomes, with θ the angle of the applied magnetic field with respect to the y -axis:

$$B_{\text{Zeeman}} = \frac{\omega_B}{\gamma} \cos(\theta) \quad (\text{A.35})$$

The transformation of the demagnetising field is quite a tedious derivation and is therefore not shown here, for reference see [52]. The resulting spin-wave dispersion gives,

$$\omega_{\text{spin-wave}} \approx \sqrt{\omega_2 \omega_3 - \omega_1^2}. \quad (\text{A.36})$$

in which

$$\omega_1 = \omega_{\text{demag}} f(kt) \sin \theta \sin \phi \cos \phi \quad (\text{A.37})$$

A.2. Exciting spin-waves

There are multiple ways of exciting spin-waves, but in thesis we use microstrips to excite them. Below I will derive the stray field of these microstrips and which spin-waves are excited by this.

The oscillating magnetic field generated by a microstrip generates an oscillation of the magnetisation in the material. This oscillation is described as

$$\mathbf{M}(\mathbf{r}) = \frac{1}{\mu_0} \int d\mathbf{r}' \Gamma(\mathbf{r}, \mathbf{r}') \mathbf{B}(\mathbf{r}'). \quad (\text{A.38})$$

We again apply a 2D Fourier transform for ease of calculation, which yields

$$\mathbf{M}(z, \mathbf{k}, \omega) = \frac{1}{\mu_0 t} \int_{-t}^0 dz' \chi(z, z', \mathbf{k}, \omega) \mathbf{B}(z', \mathbf{k}, \omega). \quad (\text{A.39})$$

To be able to calculate this, we first need the magnetic field generated by the microstrip, which can be retrieved from the current distribution $\mathbf{J}$. We use Ampère's law to obtain

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \nabla \times \int d\mathbf{r}' \frac{\mathbf{J}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} \quad (\text{A.40})$$

we can write $\mathbf{B}(\mathbf{r})$ as the curl of the vector potential $\mathbf{A}$ which gives us

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int d\mathbf{r}' \frac{\mathbf{J}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} \quad (\text{A.41})$$

Fourier transforming this gives

$$\mathbf{A}(\mathbf{r}, \omega) = \frac{\mu_0}{4\pi} \int d\mathbf{r}' \frac{\mathbf{J}(\mathbf{r}', \omega) e^{ik|\mathbf{r} - \mathbf{r}'|}}{|\mathbf{r} - \mathbf{r}'|}. \quad (\text{A.42})$$

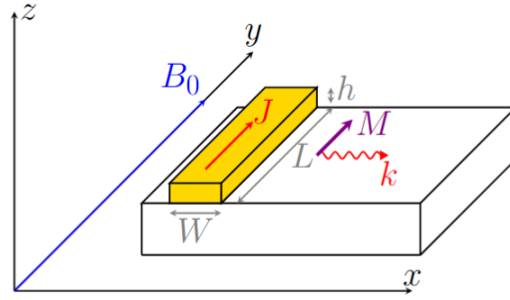


Figure A.1: A thin film in the xy plane with a microstrip with width W , height h and length L . The current density in the microstrip is along $+y$. It excites spin-waves along k .

We define our microstrip as shown in Fig. A.1 with the current density along y , thus we have a vector potential that only points in the y direction, which gives us the following magnetic field

$$\begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} = \begin{pmatrix} \frac{\partial A_y}{\partial z} \\ 0 \\ -\frac{\partial A_y}{\partial x} \end{pmatrix} \quad (\text{A.43})$$

now we can calculate $\mathbf{B}(\mathbf{r})$

$$B_z = -\frac{\partial}{\partial x} \left[\frac{\mu_0}{4\pi} \int d\mathbf{r}' \frac{J_y(\mathbf{r}', \omega) e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} \right] = 2i\mu_0 J_y(\omega) \frac{e^{-k_z z} e^{-k_z h} - 1}{k_z} \sin\left(k_x \frac{w}{2}\right) \sin\left(k_y \frac{L}{2}\right) \frac{e^{-ik_y y}}{k_y} \quad (\text{A.44})$$

$$B_x = 2i\mu_0 J_y(\omega) \frac{e^{-k_z z} e^{-k_z h} - 1}{k_z} \sin\left(k_x \frac{w}{2}\right) \sin\left(k_y \frac{L}{2}\right) \frac{e^{-ik_y y}}{k_y}. \quad (\text{A.45})$$

Now we want to know the microstrip fields close to the microstrip so we can use the near-field approximation $k_z = \sqrt{k^2 - k_x^2 - k_y^2} \approx i\kappa$, then

$$e^{ik_z z} \approx e^{-\kappa z}, \quad \frac{1 - e^{ik_z h}}{ik_z} \approx \frac{1 - e^{-\kappa h}}{\kappa} \quad (\text{A.46})$$

which gives us

$$B_x(\mathbf{k}, z, \omega) = 2\mu_0 J_y(\omega) e^{\kappa z} \frac{e^{-\kappa h} - 1}{\kappa} \frac{\sin\left(k_x \frac{w}{2}\right)}{k_x} \frac{\sin\left(k_y \frac{L}{2}\right)}{k_y} e^{-ik_y y} \quad (\text{A.47})$$

$$B_z(\mathbf{k}, z, \omega) = -2i\mu_0 J_y(\omega) e^{\kappa z} \frac{e^{-\kappa h} - 1}{\kappa^2} \sin\left(k_x \frac{w}{2}\right) \frac{\sin\left(k_y \frac{L}{2}\right)}{k_y} e^{-ik_y y} \quad (\text{A.48})$$

or in short notation

$$B_x(\mathbf{k}, z, \omega) = i \frac{\kappa}{k_x} B_z(\mathbf{k}, z, \omega) \quad (\text{A.49})$$

A.2.1. Stray field of a spin-wave

To calculate the stray field of a spin-wave, we first need to find what the magnetic stray field resulting from a magnetisation profile is. In Fourier space, this is described as:

$$\mathbf{B}(\mathbf{k}, z) = \mu_0 M_s \Gamma(\mathbf{k}, z) \mathbf{m}(\mathbf{k}), \quad (\text{A.50})$$

Using the dipole tensor from equation A.13 and averaging over the thickness, but this time for $r > r'$, i.e. above the film, we get

$$\hat{\Gamma}(\mathbf{k}, z) = \frac{1}{2}e^{-kz}(1 - e^{-kt}) \begin{pmatrix} \sin^2 \phi & \sin \phi \cos \phi & i \sin \phi \\ \sin \phi \cos \phi & \cos^2 \phi & i \cos \phi \\ i \sin \phi & i \cos \phi & -1 \end{pmatrix} \quad (\text{A.51})$$

and now we can calculate the stray field of the spin-waves using equations A.51 and $\mathbf{m} = [m_x(\mathbf{k}), 0, m_z(\mathbf{k})]^T$, which gives

$$\mathbf{B}_{\text{spin-wave}}(z, \mathbf{k}) = \frac{\mu_0 M_s}{2} e^{-kz}(1 - e^{-kt}) \begin{pmatrix} i \sin \phi m_z(\mathbf{k}) + \sin^2 \phi m_x(\mathbf{k}) \\ i \cos \phi m_z(\mathbf{k}) + \sin \phi \cos \phi m_x(\mathbf{k}) \\ -m_z(\mathbf{k}) + i \sin \phi m_x(\mathbf{k}) \end{pmatrix} \quad (\text{A.52})$$

A.2.2. Probing spin-waves

As stated above, in this thesis we use V_B^- 's to probe spin-waves, but the question arises, what is the total magnetic field that drives the defects as the spin-waves interfere with the field of the microstrip, or

$$\mathbf{B}_{\text{tot}} = \mathbf{B}_{\text{SL}} + \mathbf{B}_{\text{spin-wave}}. \quad (\text{A.53})$$

The V_B^- transitions $|0\rangle \rightarrow |-1\rangle$ and $|0\rangle \rightarrow |1\rangle$ are driven by the right- and left-circularly polarised components of the AC magnetic field, respectively [25, 26], in the plane perpendicular to the quantisation axis ($\hat{z}$), i.e. the x - y plane. This gives

$$B_{\perp, \pm} = \frac{1}{\sqrt{2}} |B_{\text{tot}, x} \pm i B_{\text{tot}, y}|. \quad (\text{A.54})$$

We take the BV branch (DE branch) such that $M \parallel k_y$ ($M \perp k_y$), by rotating the microstrip to be along x for the DE geometry, and we ignore the decay prefactor $\frac{\mu_0 M_s}{2} e^{-kz}(1 - e^{-kt})$, we have

$$\mathbf{B}_{\text{BV, tot}} = \begin{pmatrix} 0 \\ B_{y, SL} e^{-i\omega t} \\ B_{z, SL} e^{-i\omega t} \end{pmatrix} + \begin{pmatrix} 0 \\ i m_z(\mathbf{k}) \\ -m_z(\mathbf{k}) \end{pmatrix} \quad \mathbf{B}_{\text{DE, tot}} = \begin{pmatrix} B_{x, SL} e^{-i\omega t} \\ 0 \\ B_{z, SL} e^{-i\omega t} \end{pmatrix} + \begin{pmatrix} i m_z(\mathbf{k}) + m_x(\mathbf{k}) \\ 0 \\ -m_z(\mathbf{k}) + i m_x(\mathbf{k}) \end{pmatrix}. \quad (\text{A.55})$$

Since spin-waves are travelling elliptical waves, due to the elliptical excitation field of the microstrip, we define, taking into account the linearisation we used in the LLG equation,

$$\mathbf{m}(y, t) = M_s \hat{\mathbf{y}} + \begin{pmatrix} \bar{m}_x \\ 0 \\ -i \bar{m}_z \end{pmatrix} e^{i(ky - \omega t)} \quad (\text{A.56})$$

where $\bar{m}_x$ and $\bar{m}_z$ are the amplitudes of m_x and m_z . Filling this in and ignoring the static part of $\mathbf{m}$, we get:

$$\mathbf{B}_{\text{BV, tot}} = \begin{pmatrix} 0 \\ 1 \\ i \end{pmatrix} \bar{m}_z e^{i(ky - \omega t)} + \begin{pmatrix} 0 \\ B_{y, SL} \\ B_{z, SL} \end{pmatrix} e^{-i\omega t}, \quad \mathbf{B}_{\text{DE, tot}} = \begin{pmatrix} \bar{m}_z + \bar{m}_x \\ 0 \\ i \bar{m}_z + i \bar{m}_x \end{pmatrix} e^{i(ky - \omega t)} + \begin{pmatrix} B_{x, SL} \\ 0 \\ B_{z, SL} \end{pmatrix} e^{-i\omega t} \quad (\text{A.57})$$

$$B_{\text{BV}, \perp} = \frac{1}{\sqrt{2}} |\bar{m}_z e^{iky} + B_{y, SL}| \quad (\text{A.58})$$

$$B_{\text{DE}, \perp} = \frac{1}{\sqrt{2}} |(\bar{m}_x + \bar{m}_z) e^{iky} + B_{x, SL}| \quad (\text{A.59})$$

A.3. Difference frequency spin-wave mixing

For Damon–Eshbach (DE) waves, the spin-wave wavevector is perpendicular to the magnetisation direction, so that

$$\phi = \frac{\pi}{2}. \quad (\text{A.60})$$

In this case, the dipolar tensor (equation A.51) becomes, ignoring the common damping prefactor,

$$\Gamma_{\text{DE}}(\mathbf{k}, z) = \begin{pmatrix} 1 & 0 & i \\ 0 & 0 & 0 \\ i & 0 & -1 \end{pmatrix}. \quad (\text{A.61})$$

For backward-volume (BV) waves, the spin-wave wavevector is parallel to the magnetisation direction, giving

$$\phi = 0. \quad (\text{A.62})$$

The dipolar tensor then reduces to, again ignoring the common damping prefactor,

$$\Gamma_{\text{BV}}(\mathbf{k}, z) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & i \\ 0 & i & -1 \end{pmatrix}. \quad (\text{A.63})$$

Assuming a right-circularly polarised spin-wave, we write the magnetisation as

$$\mathbf{m}(\mathbf{k}) = \begin{pmatrix} m_x(\mathbf{k}) \\ m_y(\mathbf{k}) \\ -im_z(\mathbf{k}) \end{pmatrix}. \quad (\text{A.64})$$

Multiplying this magnetisation with the dipolar tensor gives the stray field for the BV and DE geometries. Omitting the common prefactor $\frac{1}{2}\mu_0 M_s e^{-kz}(1 - e^{-kt})$, which is absorbed into the field amplitudes, we obtain

$$\mathbf{B}_{\text{BV}}(\mathbf{k}, z) = \begin{pmatrix} 0 \\ m_y(\mathbf{k}) + m_z(\mathbf{k}) \\ im_y(\mathbf{k}) + im_z(\mathbf{k}) \end{pmatrix}, \quad \mathbf{B}_{\text{DE}}(\mathbf{k}, z) = \begin{pmatrix} m_z(\mathbf{k}) + m_x(\mathbf{k}) \\ 0 \\ im_z(\mathbf{k}) + im_x(\mathbf{k}) \end{pmatrix}. \quad (\text{A.65})$$

We now consider spin-wave mixing. In this case, the magnetisation is assumed to consist of two waves with different wavevectors and frequencies. We define

$$\Delta k = k_1 - k_2, \quad \Delta \omega = \omega_1 - \omega_2. \quad (\text{A.66})$$

The dynamic components of the magnetisation are written as

$$m_x(y, t) = m_{x1} e^{i(k_1 y - \omega_1 t)} + m_{x2} e^{i(k_2 y - \omega_2 t)}, \quad (\text{A.67})$$

and

$$m_z(y, t) = m_{z1} e^{i(k_1 y - \omega_1 t)} + m_{z2} e^{i(k_2 y - \omega_2 t)}. \quad (\text{A.68})$$

The longitudinal component is then determined from the normalisation condition

$$m_y(y, t) = \gamma(y, t) = \sqrt{1 - [\Re(\tilde{m}_x(y, t))]^2 - [\Re(\tilde{m}_z(y, t))]^2}. \quad (\text{A.69})$$

Substituting the two-wave expressions for $m_x(y, t)$ and $m_z(y, t)$ gives

$$\begin{aligned}
\gamma(y, t) = & \left[1 - \frac{m_{x1}^2 + m_{z1}^2}{2} - \frac{m_{x2}^2 + m_{z2}^2}{2} \right. \\
& - \frac{m_{x1}^2 - m_{z1}^2}{2} \cos(2k_1 y - 2\omega_1 t) \\
& - \frac{m_{x2}^2 - m_{z2}^2}{2} \cos(2k_2 y - 2\omega_2 t) \\
& - (m_{x1}m_{x2} + m_{z1}m_{z2}) \cos(\Delta k y - \Delta \omega t) \\
& \left. - (m_{x1}m_{x2} - m_{z1}m_{z2}) \cos((k_1 + k_2)y - (\omega_1 + \omega_2)t) \right]^{1/2}.
\end{aligned} \tag{A.70}$$

Strictly speaking, the real-space magnetisation should first be Fourier transformed, multiplied by the dipolar tensor in Fourier space, and then transformed back to real space. However, after evaluating the tensor multiplication for the BV and DE geometries, the remaining operation is linear in the magnetisation components. Therefore, apart from the common k -dependent prefactor, the real-space stray field can be written directly in terms of the real-space magnetisation components.

For the BV geometry this gives

$$\begin{aligned}
\mathbf{B}_{\text{BV}}(y, t) &= \mathcal{F}^{-1} [\mathbf{B}_{\text{BV}}(\mathbf{k}, z)] \\
&= \begin{pmatrix} 0 \\ \mathcal{F}^{-1}[m_y(\mathbf{k})] + \mathcal{F}^{-1}[m_z(\mathbf{k})] \\ i\mathcal{F}^{-1}[m_y(\mathbf{k})] + i\mathcal{F}^{-1}[m_z(\mathbf{k})] \end{pmatrix} \\
&= \begin{pmatrix} 0 \\ m_y(y, t) + m_z(y, t) \\ im_y(y, t) + im_z(y, t) \end{pmatrix}.
\end{aligned} \tag{A.71}$$

Substituting $m_y(y, t) = \gamma(y, t)$ and the two-wave expression for $m_z(y, t)$, the BV stray field becomes

$$\mathbf{B}_{\text{BV}}(y, t) = \begin{pmatrix} 0 \\ \gamma(y, t) + m_{z1}e^{i(k_1 y - \omega_1 t)} + m_{z2}e^{i(k_2 y - \omega_2 t)} \\ i\gamma(y, t) + im_{z1}e^{i(k_1 y - \omega_1 t)} + im_{z2}e^{i(k_2 y - \omega_2 t)} \end{pmatrix}. \tag{A.72}$$

For the DE geometry, the stray field is

$$\mathbf{B}_{\text{DE}}(y, t) = \begin{pmatrix} m_z(y, t) + m_x(y, t) \\ 0 \\ im_z(y, t) + im_x(y, t) \end{pmatrix}. \tag{A.73}$$

Substituting the two-wave expressions for $m_x(y, t)$ and $m_z(y, t)$ gives

$$\mathbf{B}_{\text{DE}}(y, t) = \begin{pmatrix} m_{z1}e^{i(k_1 y - \omega_1 t)} + m_{z2}e^{i(k_2 y - \omega_2 t)} + m_{x1}e^{i(k_1 y - \omega_1 t)} + m_{x2}e^{i(k_2 y - \omega_2 t)} \\ 0 \\ im_{z1}e^{i(k_1 y - \omega_1 t)} + im_{z2}e^{i(k_2 y - \omega_2 t)} + im_{x1}e^{i(k_1 y - \omega_1 t)} + im_{x2}e^{i(k_2 y - \omega_2 t)} \end{pmatrix}. \tag{A.74}$$

This can be written more compactly as

$$\mathbf{B}_{\text{DE}}(y, t) = \begin{pmatrix} (m_{z1} + m_{x1})e^{i(k_1 y - \omega_1 t)} + (m_{z2} + m_{x2})e^{i(k_2 y - \omega_2 t)} \\ 0 \\ i(m_{z1} + m_{x1})e^{i(k_1 y - \omega_1 t)} + i(m_{z2} + m_{x2})e^{i(k_2 y - \omega_2 t)} \end{pmatrix}. \tag{A.75}$$

From equations A.72 and A.75 we can see that the stray field of the Backward Volume wave contains the spin-wave mixing term γ and the Damon Eshbach stray field does not. However, as we will show later in this thesis, it is almost impossible to perfectly align your magnetic field perpendicular to the microstrip, so we must account for a small angle deviation, $\phi = \frac{\pi}{2} + \delta\phi$

This gives us for the Damon Eshbach branch, ignoring $\delta\phi^2$ as it is small,

$$\mathbf{B}_{\text{DE}}(y, t) = \begin{pmatrix} (m_{z1} + m_{x1})e^{i(k_1 y - \omega_1 t)} + (m_{z2} + m_{x2})e^{i(k_2 y - \omega_2 t)} - \delta\phi\gamma \\ \delta\phi((-m_{z1} + m_{x1})e^{i(k_1 y - \omega_1 t)} + (m_{z2} + m_{x2})e^{i(k_2 y - \omega_2 t)}) \\ i(m_{z1} + m_{x1})e^{i(k_1 y - \omega_1 t)} + i(m_{z2} + m_{x2})e^{i(k_2 y - \omega_2 t)} + i\delta\phi\gamma \end{pmatrix}. \quad (\text{A.76})$$

The total AC magnetic field driving the colour centre is the sum of the direct microstrip field and the spin-wave stray field,

$$\mathbf{B}_{\text{tot}} = \mathbf{B}_{\text{SL}} + \mathbf{B}_{\text{spin-wave}}. \quad (\text{A.77})$$

The V_{B}^- transitions $|0\rangle \rightarrow |-1\rangle$ and $|0\rangle \rightarrow |1\rangle$ are driven by the right- and left-circularly polarised components of the AC magnetic field, respectively. These circular components lie in the plane perpendicular to the quantisation axis $\hat{z}$, i.e. in the x - y plane. The relevant driving field is therefore

$$B_{\perp, \pm} = \frac{1}{\sqrt{2}} |B_{\text{tot}, x} \pm iB_{\text{tot}, y}|_{\omega=\omega_{\text{NV}}}. \quad (\text{A.78})$$

This expression allows us to evaluate how interference between the direct microstrip field and the two spin-wave modes contributes to the driving of the colour-centre transitions at the resonance frequency of the NV or V_{B}^- . We can see that both the Backward Volume and Damon Eshbach spin-waves now contain the spin-wave mixing term γ , thus for both we can detect these mixing product with our colourcentres.

A.3.1. Out of phase spin-wave mixing

If we assume that both the initial spin-waves are out of phase, we define,

$$\mathbf{m}(y, t) = \begin{pmatrix} m_{x1}(\mathbf{k}) \\ m_{y1}(\mathbf{k}) \\ -im_{z1}(\mathbf{k}) \end{pmatrix} e^{i(k_1 y - \omega_1 t + \phi_1)} + \begin{pmatrix} m_{x2}(\mathbf{k}) \\ m_{y2}(\mathbf{k}) \\ -im_{z2}(\mathbf{k}) \end{pmatrix} e^{i(k_2 y - \omega_2 t + \phi_2)}. \quad (\text{A.79})$$

where $m_{xj}(\mathbf{k})$, $m_{yj}(\mathbf{k})$, and $m_{zj}(\mathbf{k})$ are the complex amplitudes of the j -th spin-wave mode, k_j and ω_j are its wavevector and angular frequency, and ϕ_j is an arbitrary phase offset. The factor $-i$ in the z -component accounts for the 90° phase difference between the in-plane and out-of-plane components of the spin-wave precession. The total dynamic magnetisation is obtained as the superposition of the two modes.

This gives,

$$\begin{aligned} m_y(y, t) = & \left[1 - \frac{m_{x1}^2 + m_{z1}^2}{2} - \frac{m_{x2}^2 + m_{z2}^2}{2} \right. \\ & - \frac{m_{x1}^2 - m_{z1}^2}{2} \cos(2k_1 y - 2\omega_1 t + 2\phi_1) \\ & - \frac{m_{x2}^2 - m_{z2}^2}{2} \cos(2k_2 y - 2\omega_2 t + 2\phi_2) \\ & + (m_{x1}m_{x2} + m_{z1}m_{z2}) \cos(\Delta k y - \Delta\omega t + (\phi_1 - \phi_2)) \\ & \left. + (m_{x1}m_{x2} - m_{z1}m_{z2}) \cos((k_1 + k_2)y - (\omega_1 + \omega_2)t + (\phi_1 + \phi_2)) \right]^{1/2}. \end{aligned} \quad (\text{A.80})$$

The relative phases ϕ_1 and ϕ_2 do not alter the amplitudes of the generated mixing products, but only shift their spatial positions. The difference-frequency term acquires a phase offset of $\phi_1 - \phi_2$, while the sum-frequency term acquires a phase offset of $\phi_1 + \phi_2$.

B

Static magnetic field control measurements

B.1. In-Plane Magnet Calibration

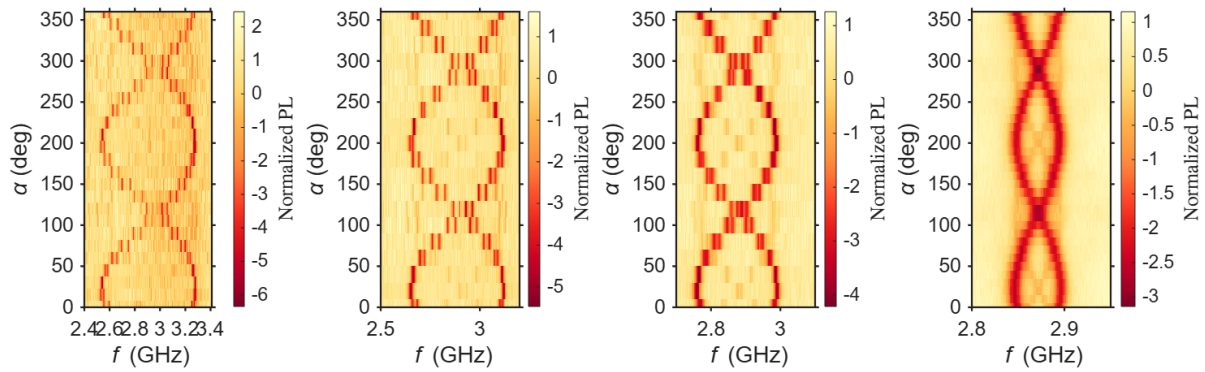


Figure B.1: ESR measurements obtained during rotation of the in-plane magnet for several magnet distances to the sensor. Each horizontal trace corresponds to one rotation angle of the magnet.

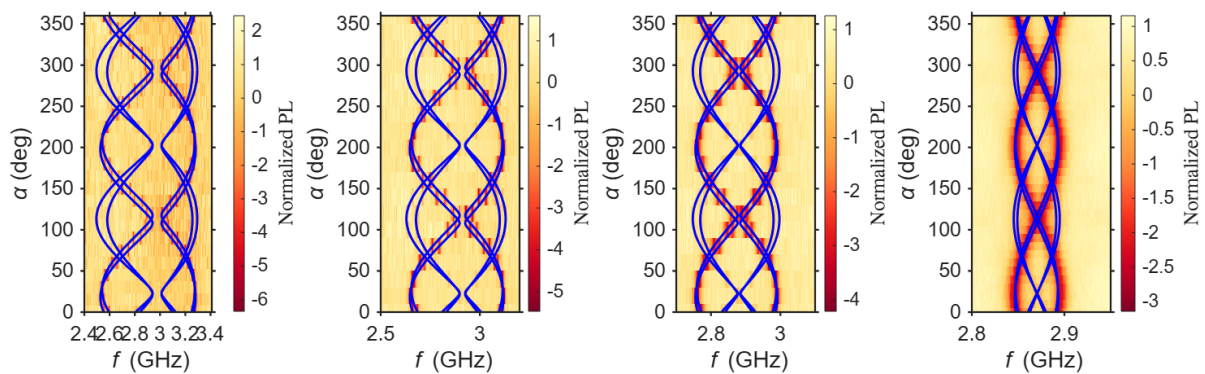
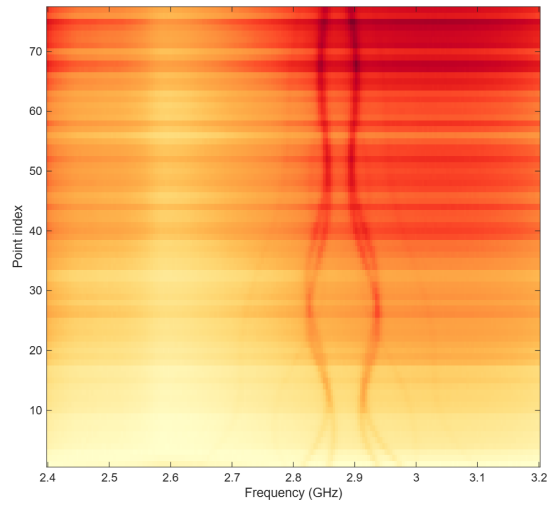
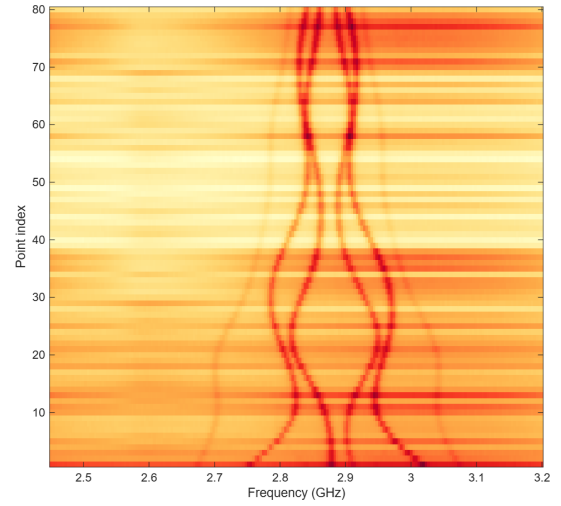


Figure B.2: ESR measurements obtained during rotation of the in-plane magnet for several magnet distances to the sensor. The blue lines on top are the corresponding fit.

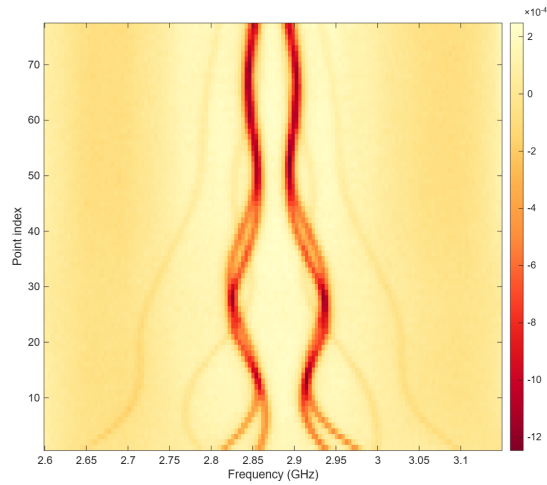
B.2. Canted Magnet Calibration



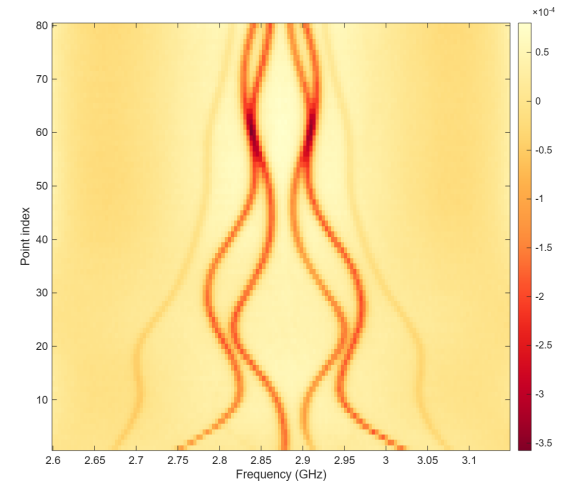
(a) Corkscrew centred below sample: ESR map



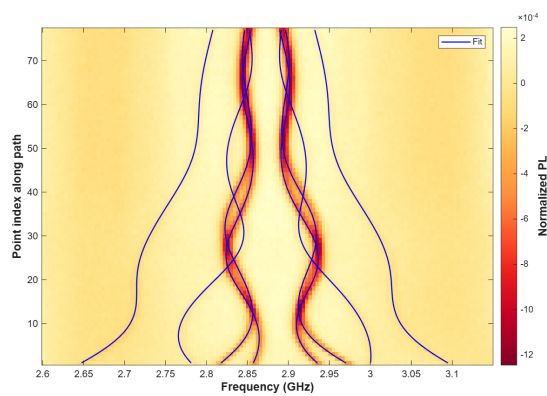
(b) Corkscrew off-center: ESR map



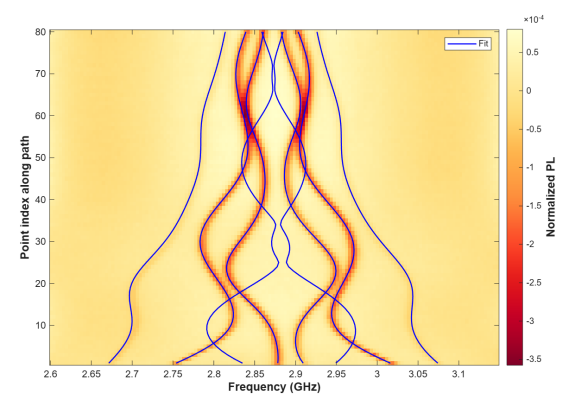
(c) Corkscrew centred below sample: detrended ESR map



(d) Corkscrew off-centre: detrended ESR map

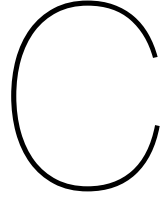


(e) Corkscrew centred below sample with fitted calibration path overlaid in blue



(f) Corkscrew off-centre with fitted calibration path overlaid in blue

Figure B.3: ESR maps measured along a corkscrew magnet calibration path for two different corkscrew positions. The left column shows the corkscrew centred below the sample, while the right column shows the corkscrew in an off-centre position. The top row shows the raw ESR maps, and the middle row shows the corresponding detrended ESR maps. The bottom row shows the same detrended ESR maps with the fitted calibration path overlaid in blue.



Documentation Dispersion Simulation

This appendix briefly documents the MATLAB and simulation framework. The Python framework works the same but is not yet documented.

C.1. Running the Simulation

The simulation can be executed either from a MATLAB script or through a graphical user interface (GUI).

C.2. Script Interface

The main simulation routine is called using

```
run_field_sweep(struct)
```

The input structure contains all material, magnetic-field, and plotting parameters.

C.3. Main Input Parameters

C.3.1. Mandatory

- .filename(string)** name for output .mat file
- .exchange_stiffness_D(float)** exchange stiffness [m^2/s]
- .Ms(float)** saturation magnetisation [$\frac{\text{A}}{\text{m}}$]
- .thickness(float)** film thickness [m]
- .zero_field_splitting(float)** zero field splitting of the colour center [Hz]
- .phi_colourcenter(float)** in-plane angle wrt the x-axis of the colour centre axis [deg], see Canting Colourcenter picture.
- .theta_colourcenter(float)** angle wrt the z-axis of the colour centre axis [deg], see Canting Colour-center picture.
- .Nk(int)** number of k points
- .kmin(int)** minimum k [$\frac{1}{\text{m}}$]
- .kmax(int)** maximum k [$\frac{1}{\text{m}}$]
- .B_type(string)** "Vector" or "Scalar", A flag to tell the simulation whether you are using $[x, y, z]$ or $[r, \phi, \theta]$
- .B(float or linspace)** magnitude of external magnetic field [T], if **.B_type="scalar"**

.phi(float or linspace) in-plane angle wrt the x-axis of the external magnetic field [*rad*], if **.B_type="scalar"**
.theta(float or linspace) angle of the external magnetic field wrt to the film normal [*rad*], if **.B_type="scalar"**
.Bx(float or linspace) magnitude of external magnetic field along x [*T*], if **.B_type="Vector"**
.By(float or linspace) magnitude of external magnetic field along y [*T*], if **.B_type="Vector"**
.Bz(float or linspace) magnitude of external magnetic field along z [*T*], if **.B_type="Vector"**

C.3.2. optional

.use_previous_run(boolean) True or False, reuse existing **.mat** file
.anisotropy(boolean) True or False, add anisotropy to simulation
.B_k(vector) uniaxial anisotropy field [*T*]
.height(float) Superconductor thickness [*m*]
.lambda_L(float) penetration depth [*m*]
.wavelength(boolean) converts all *k* values to wavelength λ using $\lambda = \frac{2\pi}{k}$
.plot(boolean) generate plot
.xaxis(string) x-axis variable in the plot, mandatory of **.plot=true**
.yaxis(string) y-axis variable in the plot, mandatory of **.plot=true**
.zaxis(string) z-axis variable in the plot, optional of **.plot=true**
.zcolor(string) color variable in the plot, optional of **.plot=true**

C.3.3. output

Below are all the variables, output variables, and how they can be used for plotting.

.frequencies_DE (array) Damon–Eshbach mode frequencies [Hz].
.frequencies_BV (array) Backward Volume mode frequencies [Hz].
.frequencies_X (array) Frequencies for X-direction cut [Hz].
.frequencies_Y (array) Frequencies for Y-direction cut [Hz].
.k_DE (array) Damon–Eshbach wavevector values [1/m].
.k_BV (array) Backward Volume wavevector values [1/m].
.k_X (array) Wavevector values for X-direction cut [1/m].
.k_Y (array) Wavevector values for Y-direction cut [1/m].
.Btheta_long (array) Polar field angle θ [rad], repeated for each frequency entry.
.Bphi_long (array) Azimuthal field angle ϕ [rad], repeated for each frequency entry.
.B_long (array) Field magnitude [mT], repeated for each frequency entry.
.ESR_lower (float) second electron spin resonance frequency [Hz].
.ESR_upper (float) first electron spin resonance frequency [Hz].
.Btheta (float) Polar field angle θ [rad].
.Bphi (float) Azimuthal field angle ϕ [rad].
.B (float) Magnetic field magnitude [mT].

- .Bx (float)** X-component of magnetic field [mT], if **.B_type="Vector"**.
- .By (float)** Y-component of magnetic field [mT], if **.B_type="Vector"**.
- .Bz (float)** Z-component of magnetic field [mT], if **.B_type="Vector"**.
- .Bx_long (array)** X-component of field [mT], repeated for each frequency entry, if **.B_type="Vector"**.
- .By_long (array)** Y-component of field [mT], repeated for each frequency entry, if **.B_type="Vector"**.
- .Bz_long (array)** Z-component of field [mT], repeated for each frequency entry, if **.B_type="Vector"**.
- .measurable_k_ESR_lower_DE (float)** Interpolated Damon–Eshbach wavevector at **ESR_lower**.
- .measurable_k_ESR_upper_DE (float)** Interpolated Damon–Eshbach wavevector at **ESR_upper**.
- .measurable_k_ESR_lower_BV (float)** Interpolated Backward Volume wavevector at **ESR_lower**.
- .measurable_k_ESR_upper_BV (float)** Interpolated Backward Volume wavevector at **ESR_upper**.
- .measurable_k_ESR_lower_X (float)** Interpolated X-cut wavevector at **ESR_lower**.
- .measurable_k_ESR_upper_X (float)** Interpolated X-cut wavevector at **ESR_upper**.
- .measurable_k_ESR_lower_Y (float)** Interpolated Y-cut wavevector at **ESR_lower**.
- .measurable_k_ESR_upper_Y (float)** Interpolated Y-cut wavevector at **ESR_upper**.

C.3.4. GUI Interface

The GUI provides the same functionality through interactive input fields and dropdown menus and can be started using

```
run_field_sweep_GUI
```

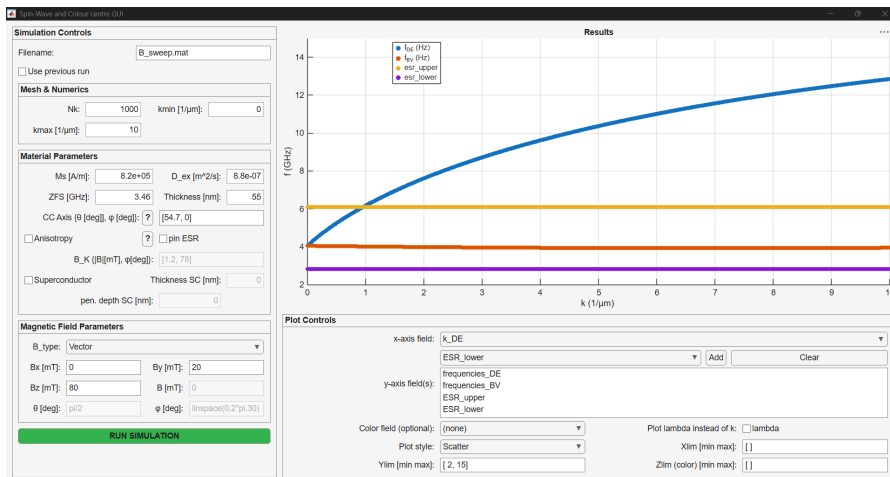


Figure C.1: Graphical user interface used for configuring and running spin-wave dispersion simulations.

C.4. Code Structure

The simulation framework consists of several MATLAB scripts and functions responsible for calculating spin-wave dispersions, ESR frequencies, and plotting routines.

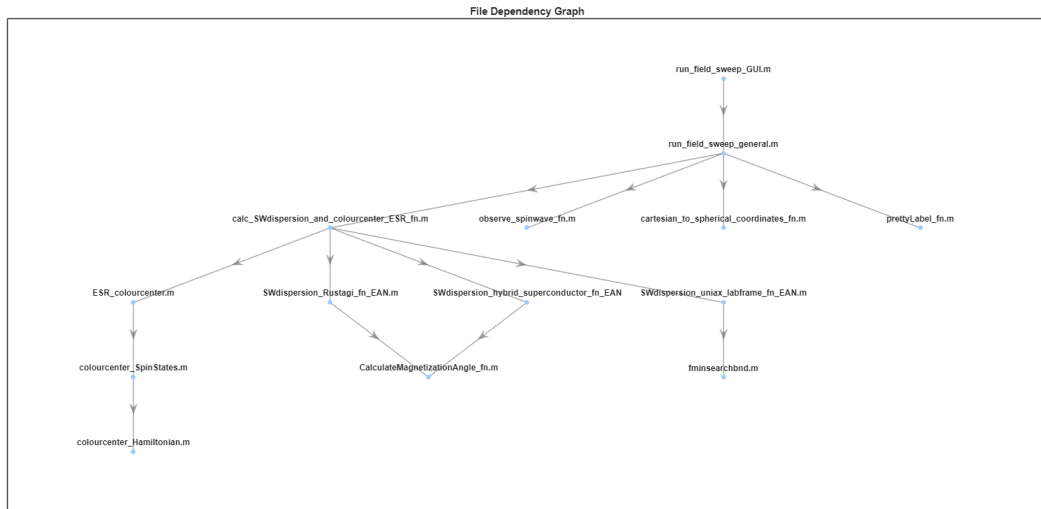


Figure C.2: Overview of the main MATLAB files and their dependencies used in the dispersion simulation framework.

The files are:

run_field_sweep_general.m Core function executing full parameter sweeps over field magnitude and orientation.

run_field_sweep_GUI_v4.m GUI version with additional plotting, export, and parameter controls.

calc_SWdispersion_and_colourcenter_ESR_fn.m Main simulation routine computing spin-wave dispersion (with /without anisotropy or superconductors) and colour-centre ESR. Also extracts the Demon-Eshbach, Backwards-Volume and X&Y cuts.

SWdispersion_Rustagi_fn_EAN.m Implements isotropic spin-wave dispersion following Rustagi *et al.*, Phys. Rev. B 102, 220403 (2020).

SWdispersion_uniax_labframe_fn_EAN.m Computes spin-wave dispersion with anisotropy in the laboratory coordinate frame.

SWdispersion_hybrid_superconductor_fn_EAN Computes spin-wave dispersion with a superconductor on top

CalculateMagnetizationAngle_fn.m Calculates equilibrium magnetization angles from applied field direction.

fminsearchbnd.m Bound-constrained variant of MATLAB's `fminsearch` for local optimisation, used in **SWdispersion_uniax_labframe_fn_EAN.m**.

observe_spinwave_fn.m Extracts wavevector from the given spin-wave dispersions matching the given ESR and sorts them into negative and positive group velocity.

cartesian_to_spherical_coordinates_fn.m Utility function converting Cartesian to spherical coordinate systems.

ESR_colourcenter.m Main entry point for ESR calculation of colour centres.

colourcenter_SpinStates.m Calculates spin eigenstates and projections for ESR analysis.

colourcenter_Hamiltonian.m Builds and diagonalises the spin Hamiltonian of the defect system.

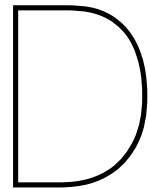
prettyLabel_fn.m Utility for consistent axis and legend labelling across figures.

C.5. Magnetic-Field Sweeps

The simulation iterates over all supplied magnetic-field configurations. For example, when using

```
Bz = linspace(0,10e-3,100)
```

while keeping B_x and B_y fixed, a one-dimensional field sweep is performed. If multiple field components are supplied as arrays, multidimensional sweeps are generated automatically.



Systematic measurement and analysis of Backward Volume Spin-waves

This appendix presents the full set of real-space and reciprocal-space measurements. For each field value and each excitation frequency, the measured 2D map is shown together with its two-dimensional Fourier spectrum and the corresponding normalised power spectral density. The reciprocal-space representations were obtained after detrending, windowing, zero-padding, and normalisation.

The calculated isofrequency contours are overlaid on the Fourier and PSD maps to compare the experimentally observed wave-vector distribution with the expected spin-wave dispersion. These supplementary measurements provide a more complete overview of the frequency and field dependence. (measurements are shown on the next pages).

Also, the IPR in comparison to the ESR frequencies is shown at the end.

D.1. Frequency and magnetic field sweeps

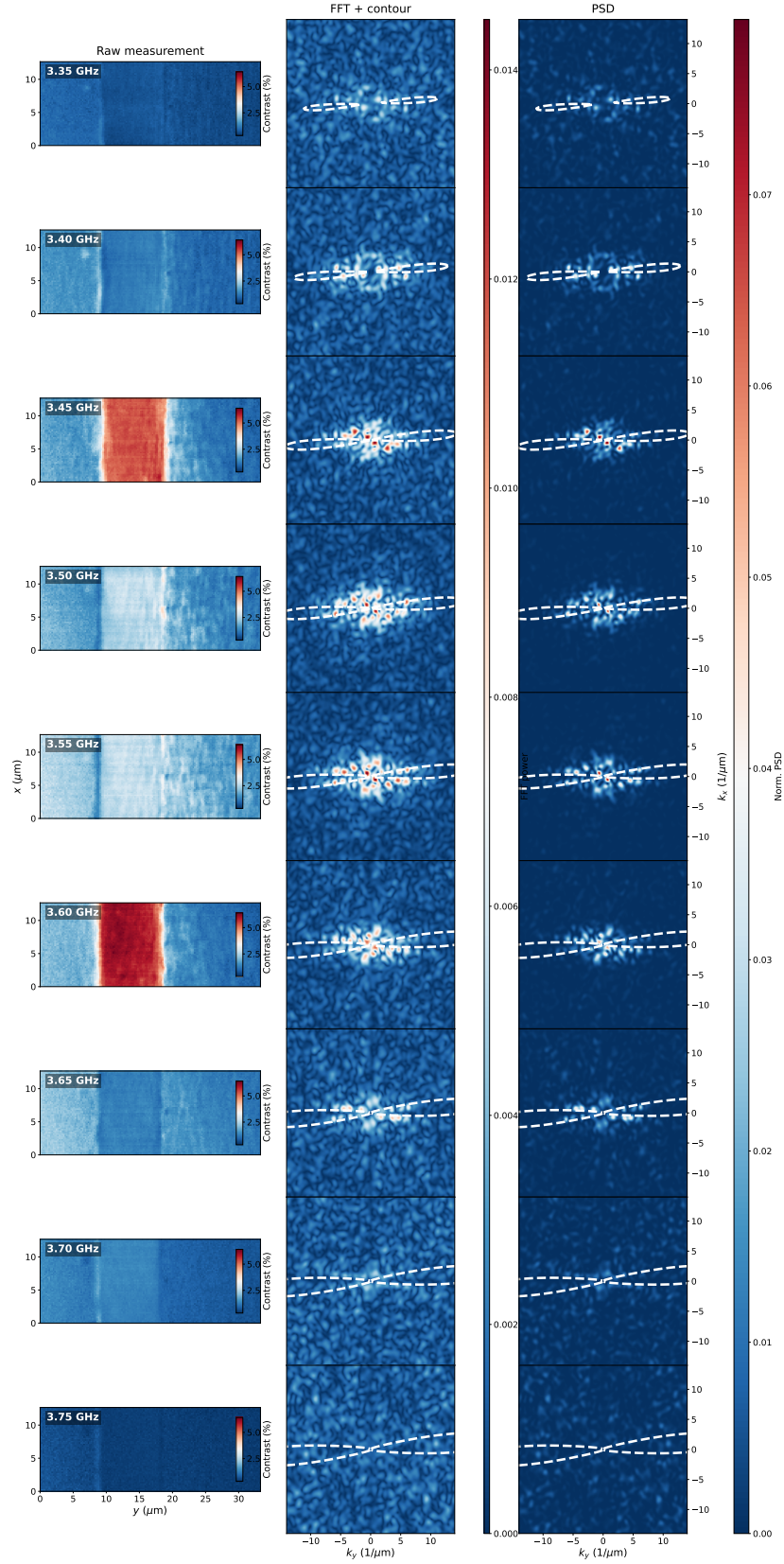


Figure D.1: **Frequency-dependent spin-wave response measured at an applied field of $B = (B_x, B_y, B_z) = (14.2, 1.1, 0.4)$ mT.** Each row corresponds to one excitation frequency. The left column shows the uncropped real-space contrast, $C = (1 - 1/\text{counter_norm}) \times 100\%$, with a common colour scale. The middle column shows the two-dimensional Fourier amplitude obtained from the cropped real-space data after linear detrending in both spatial directions, multiplication by a two-dimensional Hann window, zero padding by a factor of three, and normalisation such that the integrated FFT power equals unity. The right column shows the power spectral density obtained from the same cropped data by linear detrending, mean subtraction, Hann windowing, calculation of the two-dimensional autocorrelation, Fourier transformation, and normalisation by the integrated PSD. White dashed lines indicate calculated isofrequency contours from the Rustagi spin-wave dispersion using $M_s = 8.2 \times 10^5 \text{ A m}^{-1}$, film thickness $t = 55 \text{ nm}$, and exchange stiffness $D = 8.8 \times 10^{-7}$.

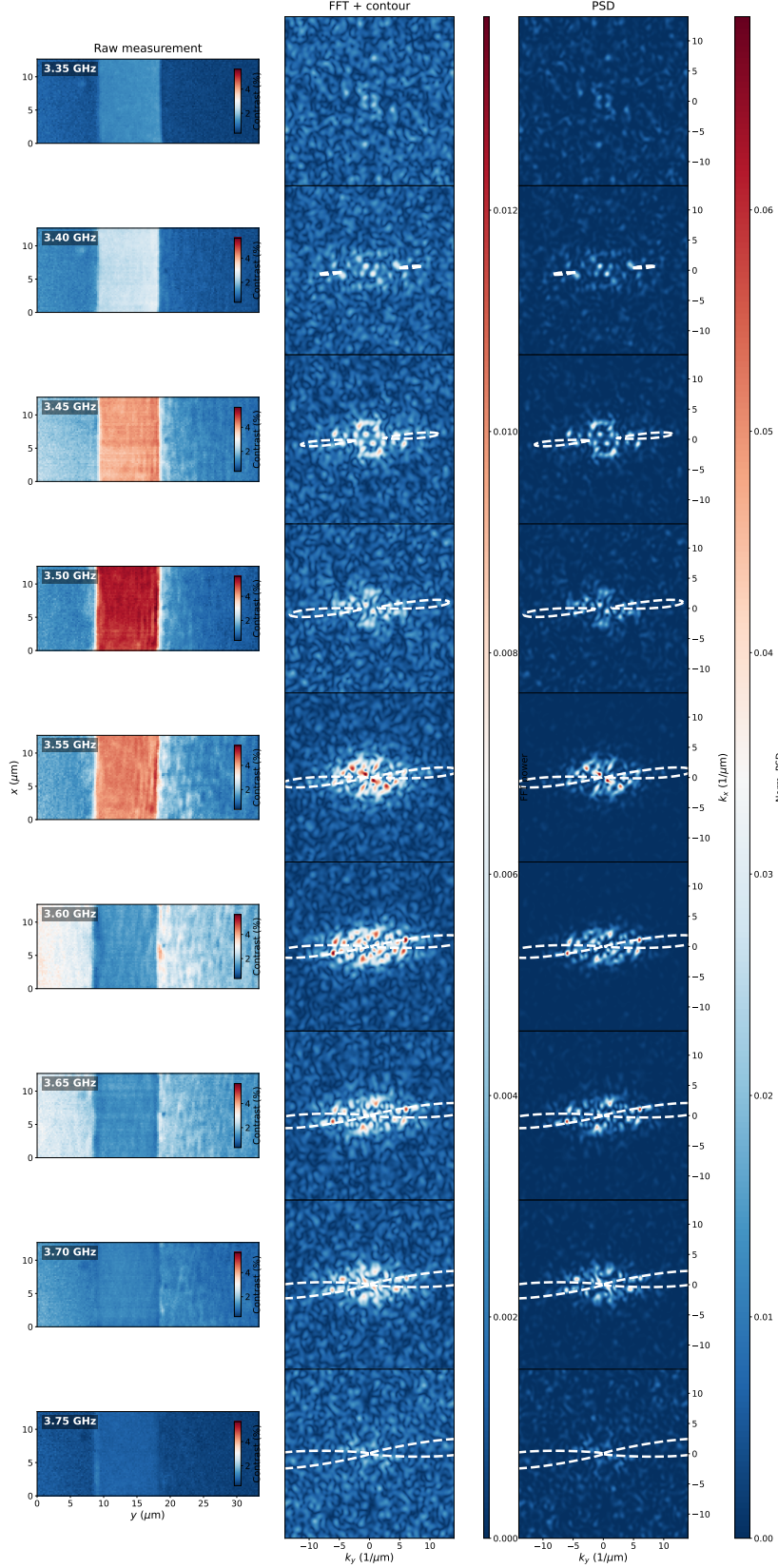


Figure D.2: **Frequency-dependent spin-wave response measured at an applied field of $B = (B_x, B_y, B_z) = (15.2, 1.2, 0.4)$ mT.** Each row corresponds to one excitation frequency. The left column shows the uncropped real-space contrast, $C = (1 - 1/\text{counter_norm}) \times 100\%$, with a common colour scale. The middle column shows the two-dimensional Fourier amplitude obtained from the cropped real-space data after linear detrending in both spatial directions, multiplication by a two-dimensional Hann window, zero padding by a factor of three, and normalisation such that the integrated FFT power equals unity. The right column shows the power spectral density obtained from the same cropped data by linear detrending, mean subtraction, Hann windowing, calculation of the two-dimensional autocorrelation, Fourier transformation, and normalisation by the integrated PSD. White dashed lines indicate calculated isofrequency contours from the Rustagi spin-wave dispersion using $M_s = 8.2 \times 10^5 \text{ A m}^{-1}$, film thickness $t = 55 \text{ nm}$, and exchange stiffness $D = 8.8 \times 10^{-7}$.

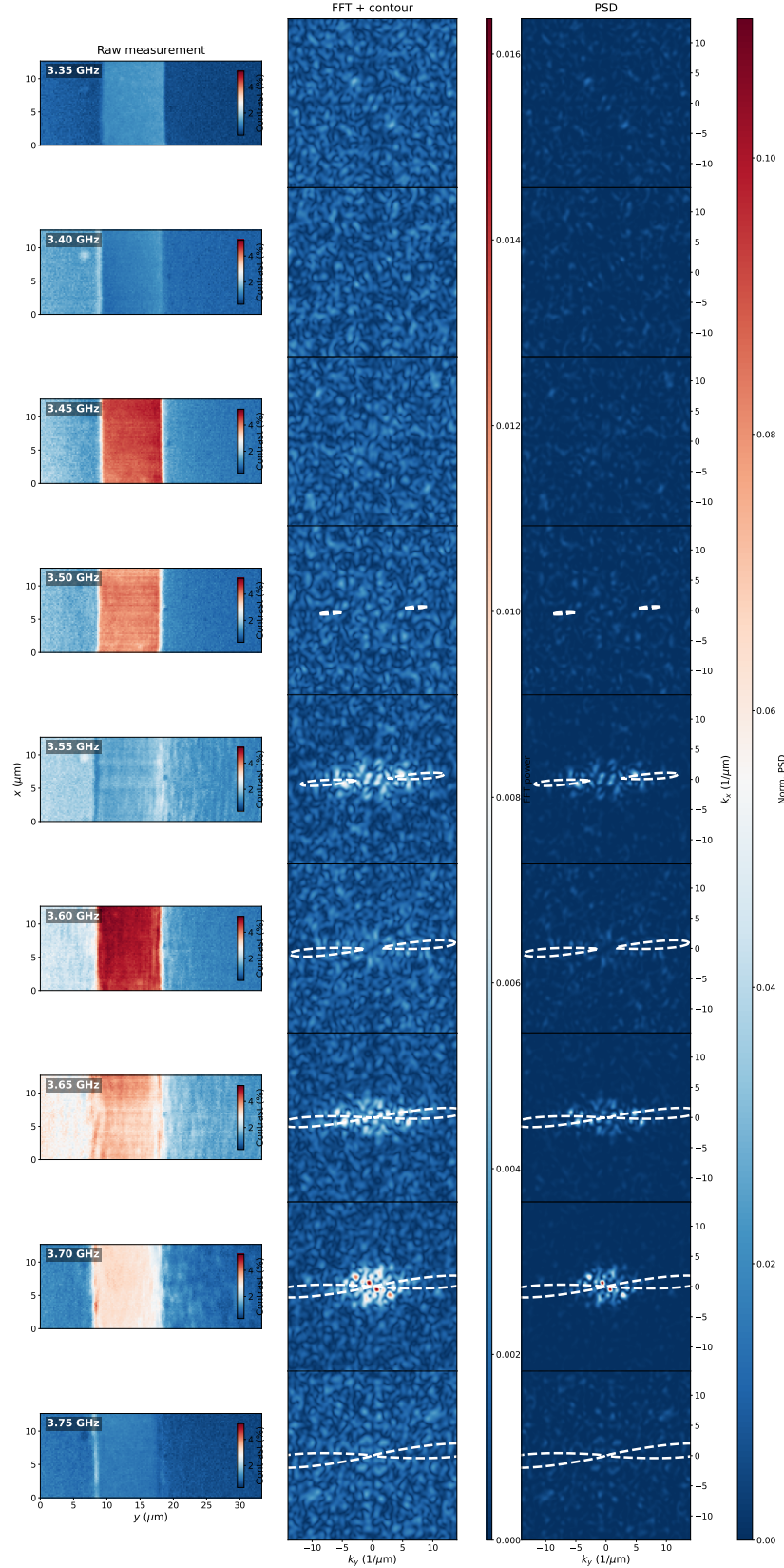


Figure D.3: **Frequency-dependent spin-wave response measured at an applied field of $B = (B_x, B_y, B_z) = (16.2, 1.1, 0.4)$ mT.** Each row corresponds to one excitation frequency. The left column shows the uncropped real-space contrast, $C = (1 - 1/\text{counter_norm}) \times 100\%$, with a common colour scale. The middle column shows the two-dimensional Fourier amplitude obtained from the cropped real-space data after linear detrending in both spatial directions, multiplication by a two-dimensional Hann window, zero padding by a factor of three, and normalisation such that the integrated FFT power equals unity. The right column shows the power spectral density obtained from the same cropped data by linear detrending, mean subtraction, Hann windowing, calculation of the two-dimensional autocorrelation, Fourier transformation, and normalisation by the integrated PSD. White dashed lines indicate calculated isofrequency contours from the Rustagi spin-wave dispersion using $M_s = 8.2 \times 10^5 \text{ A m}^{-1}$, film thickness $t = 55 \text{ nm}$, and exchange stiffness $D = 8.8 \times 10^{-7}$.

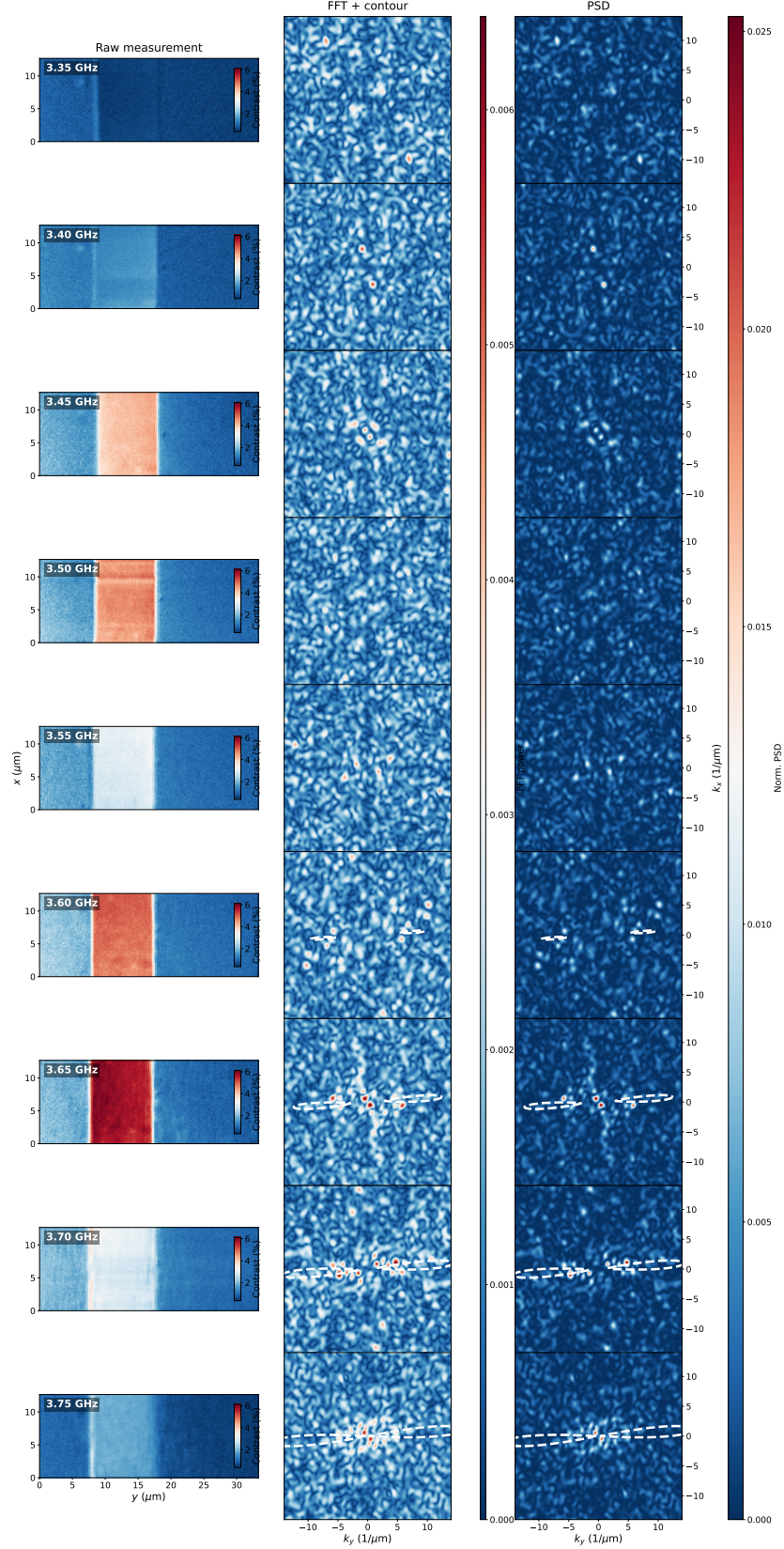


Figure D.4: **Frequency-dependent spin-wave response measured at an applied field of $B = (B_x, B_y, B_z) = (17.2, 1.2, 0.5)$ mT.** Each row corresponds to one excitation frequency. The left column shows the uncropped real-space contrast, $C = (1 - 1/\text{counter_norm}) \times 100\%$, with a common colour scale. The middle column shows the two-dimensional Fourier amplitude obtained from the cropped real-space data after linear detrending in both spatial directions, multiplication by a two-dimensional Hann window, zero padding by a factor of three, and normalisation such that the integrated FFT power equals unity. The right column shows the power spectral density obtained from the same cropped data by linear detrending, mean subtraction, Hann windowing, calculation of the two-dimensional autocorrelation, Fourier transformation, and normalisation by the integrated PSD. White dashed lines indicate calculated isofrequency contours from the Rustagi spin-wave dispersion using $M_s = 8.2 \times 10^5 \text{ A m}^{-1}$, film thickness $t = 55 \text{ nm}$, and exchange stiffness $D = 8.8 \times 10^{-7}$.

D.2. IPR

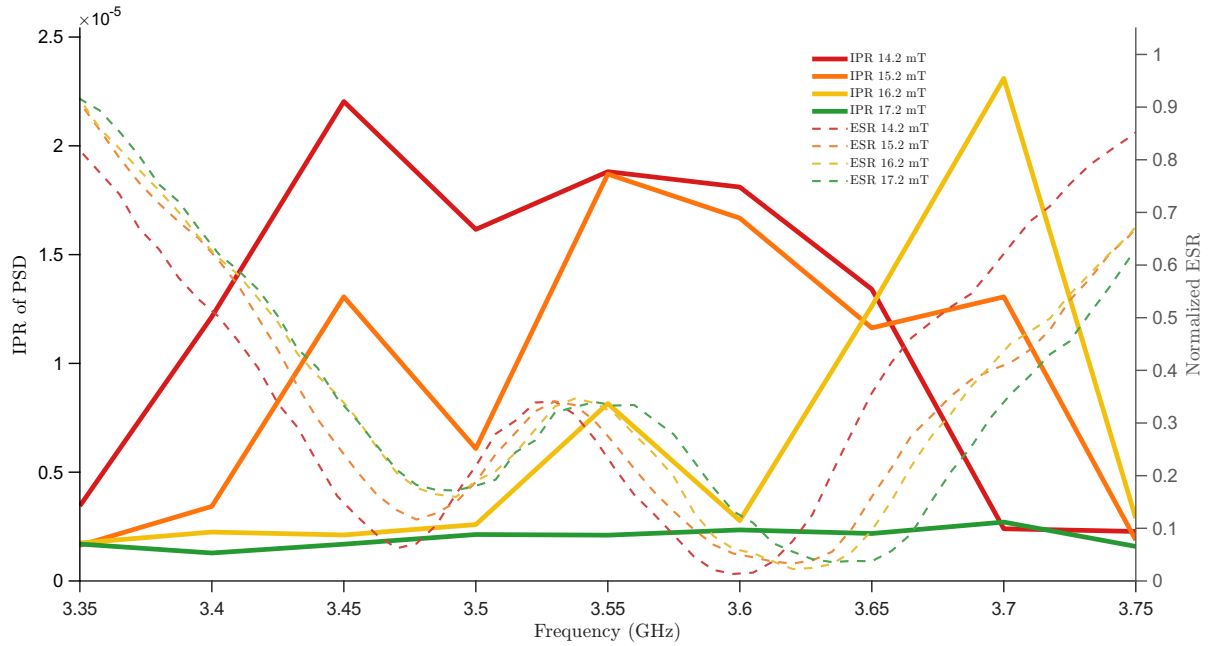


Figure D.5: **Comparison between the measured spin-wave activity and the colour-centre ESR response.** The normalised spin-wave activity is shown together with the corresponding smoothed ESR spectra (dashed lines) for different applied magnetic fields. The strongest spin-wave features do not consistently coincide with the deepest ESR dips, indicating that the observed frequency dependence is not solely determined by the colour-centre sensitivity but also reflects the excitation of backward volume spin-waves.