A paradox in dynamic traffic assignment

Dynamic extension of the Braess paradox

Victor L. Knoop
Transport & Planning
Delft University of Technology, Delft, The Netherlands
Email: v.l.knoop@tudelft.nl

Serge P. Hoogendoorn
Transport & Planning
Delft University of Technology

Bart van Arem
Transport & Planning
Delft University of Technology

1 Introduction

The Braess paradox [1] is well known by traffic engineers. It states that adding a link to a network can, in special conditions, lead to an *increase* in total travel travel. Braess' work is based on a static network with link travel times. There are several conditions, like the maximum increase of travel time [2], which are derived for this case. However, with dynamic queuing models, the paradox changes. This paper will show that even for a very small network the addition of a link can increase the travel time (section 2). It will be also argued that this is in fact a common situation for real-world networks. The paper also presents a possible solution for the road layout avoiding the extra delay in section 3.

In this extended abstract we will not explain the queuing model in detail. We use a conceptual dynamic queuing model. The only important features are (1) the flow on a link is restricted to capacity and (2) if demand exceeds capacity, a queue will grow upstream of the bottleneck. For the extended abstract we assume that the vehicle speed up to capacity is the free flow speed. This assumption simplifies the calculations in the following section, but is not essential for the concept.

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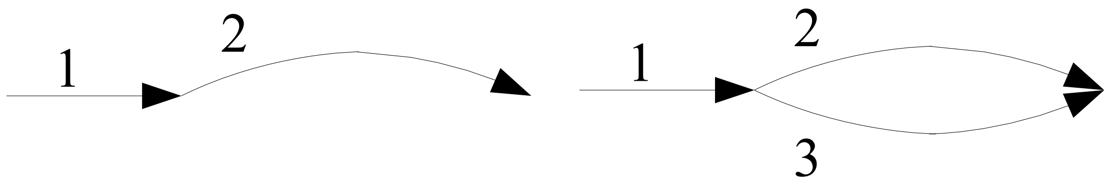


(a) Without extra link (b) With extra link

Figure 1: Network

2 Network and demand

Figure 1a shows the very simple network which we will be considering in this paper. The effect occurs for more networks, but this simple network will be used to show the relevant process. The link capacity is indicated with C and the traffic demand is Q. For this network, $C_1 > Q$ and $C_2 > Q$ (the properties of these links and all links which will be introduced are also shown in table 1). Since the capacity is sufficient, there is no congestion in the network and, because in this extended abstract we assume a free flow speed up to capacity, the average travel time ($T^{\text{avg}}_{\text{without}}$) is the sum free flow travel time on link 1 and link 2:

$$T^{\text{avg}}_{\text{without}} = T_1^{\text{free flow}} + T_2^{\text{free flow}} \tag{1}$$

Now consider adding link 3 (figure 1b), which has a capacity $C_3 < Q$ and a free flow travel time travel time $T_3^{\text{free flow}} = (T_2^{\text{free flow}} + \tau)$ with $\tau > 0$. Since there is no bottleneck on link 2 or link 3, traffic will be in free flow conditions. From the diversion point onwards, it will therefore be faster to take link 3. In a Wardrop equilibrium [3], users will only take the path with the lowest cost (in this case being travel time), meaning the traffic demand to link 3 is the full demand Q. However, since $C_3 < Q$ this will create congestion upstream of the diversion point, i.e. on link 1. All travellers, also travellers which might turn to link 2 will envisage this congestion. Therefore the average travel time is

$$T^{\text{avg}}_{\text{with}} = T_1^{\text{cong}} + T_3^{\text{free flow}} \tag{2}$$

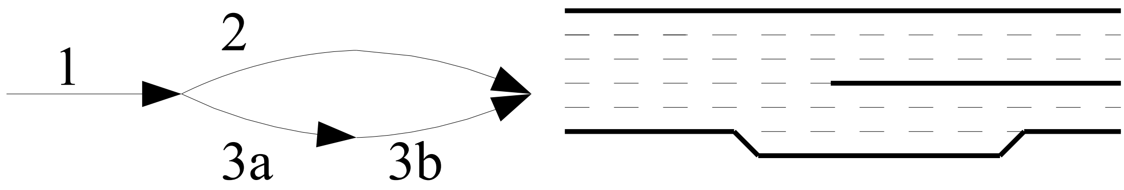
The difference in travel times can be calculated from the equation 1 and equation 2. In the limit that $\tau \to 0$, the extra delay D is:

$$D = T^{\text{avg}}_{\text{with}} - T^{\text{avg}}_{\text{without}} = T_1^{\text{free flow}} + T_2^{\text{free flow}} - T_1^{\text{cong}} - T_3^{\text{free flow}} = T_1^{\text{free flow}} - T_1^{\text{cong}} > 0 \tag{3}$$

This increase in travel time is only due to the addition of a link. As long as $Q > C_3$ the queue will grow and the delay will increase, theoretically to infinity. For the static network with link travel times, the possible travel time is bounded to twice the original travel time [2]; this no longer holds for the dynamic case.

Table 1: Properties of the links

Link	1	2	3	3a	3b
Capacity C	$> Q$	$> Q$	$< Q$	$> C_{3b}$	$= C_3$
Free flow travel time	$T_1^{\text{free flow}}$	$T_2^{\text{free flow}}$	$T_2^{\text{free flow}} + \tau$	$T_{3a}^{\text{free flow}}$	$= T_3^{\text{free flow}} - T_{3a}^{\text{free flow}}$



(a) Graph

(b) Implementation in lanes

Figure 2: The solution avoiding extra travel time

Although the network might seem artificial, it is actually a situation which can often occur in practice. Imagine a road approaching a town (link 1). To get to the other side, there is a motorway around the town, or a highway through the town. Often the motorway link will be take more time in case traffic in town is undisturbed.

3 Solution

To avoid this problem, the network designer has to make sure that the travel time on link 2 is larger than on link 3, or that the queue because of the restricted capacity of link 3 will not delay travellers turning onto link 2. This is possible by redesigning the network as shown in figure 2a. If one designs the network such that $C_{3b} < C_{3a}$, a queue will arise if on link 3a the demand exceeds the capacity of link 3b. If link 3a is long enough, traffic to link 2 is not delayed by this queue. Furthermore, once there is a queue on link 3a, the travel time over link 3a increases, which will make more travellers taking link 2 instead. In terms of road layout, this solution is relatively easy to implement. An example is shown in figure 2b.

Another solution would be to artificially increase travel time on link by means of traffic management (for instance, by introducing traffic lights). However, also with the these solutions, the total travel time will not be lower than the original travel time. If the link is not constructed for travel time reduction but for other reasons (e.g., access to a part of the town), then these solutions are useful.

4 Conclusions

The paper presented a paradox based on the Braess paradox. It shows that even in a very simple network layout the addition of an extra link can cause an increase of total travel time. It is furthermore shown that this delay is not bounded. The paper also provides solutions to avoid the extra delay. The network element which causes this delay is very common in real-world networks. Future research should show how large this problem in fact is.

References

[1] D. Braess. Über ein Paradoxon aus der Verkehrsplanung. *Unternehmensforschung*, 12:258–268, 1968.

[2] Tim Roughgarden and Éva Tardos. How bad is selfish routing? *J. ACM*, 49(2):236–259, 2002.

[3] J. Wardrop. Some theoretical aspects of road traffic research. *Proceedings of the Institute of Civil Engineers, Part II*, I(2):325–378, 1952.