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Solving the station-based one-way carsharing network planning problem with relocations and non-linear demand

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ABSTRACT

One-way station-based carsharing systems allow users to return a rented car to any designated station, which could be different from the origin station. Existing research has been mainly focused on the vehicle relocation problem to deal with the travel demand fluctuation over time and demand imbalance in space. However, the strategic planning of the stations' location and their capacity for one-way carsharing systems has not been well studied yet, especially when considering vehicle relocations simultaneously. This paper presents a Mixed-integer Non-linear Programming (MINLP) model to solve the carsharing station location and capacity problem with vehicle relocations. This entails considering several important components which are for the first time integrated in the same model. Firstly, relocation operations and corresponding relocation costs are taken into consideration to address the imbalance between trip requests and vehicle availability. Secondly, the flexible travel demand at various time steps is taken as the input to the model avoiding deterministic requests. Thirdly, a logit model is constructed to represent the non-linear demand rate by using the ratio of carsharing utility and private car utility. To solve the MINLP model, a customized gradient algorithm is proposed. The application to the SIP network in Suzhou, China, demonstrates that the algorithm can solve a real world large scale problem in reasonable time. The results identify the pricing and parking space rental costs as the key factors influencing the profitability of carsharing operators. Also, the carsharing station location and fleet size impact the vehicle relocation and carsharing patronage.

1. Introduction

Traffic congestion and its main consequences on productive time loss and air pollution are regarded as the main issues resulting from the urbanization process due to the quick growth of private cars usage (Beckmann, 2013). Moreover the increasing car ownership imposes great pressure on parking in urban areas. Therefore, reducing the use of private cars is essential in order to tackle traffic congestion, thus reducing air pollution, time lost, and save land resources. Carsharing is a transport demand management measure that was first adopted informally in the 1940s, when groups of citizens needed to save travel costs due to a rise in gasoline prices. A representative example was the system called "Sefage" in Zurich, Switzerland, in 1948 (Correia and Antunes, 2012; Shaheen and Cohen, 2012), a non-governmental club consisting of citizens who were willing to share vehicles in their neighborhoods. This earliest vehicle sharing was only used by a small number of people but the concept had been created. Nevertheless it was only in the 1980s that carsharing started to become more popular in Europe and in the USA (Shaheen et al., 1999). By October 2014, there were 33 countries operating carsharing systems encompassing around 4,800,000 members and over 104,000 vehicles in more than 1531

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cities (Shaheen and Cohen, 2014). In recent years, carsharing is becoming even more attractive and pervasive around the world due to its low price and flexible car-return policies. Carsharing is expected to significantly increase its market penetration in the next ten years (Zhou et al., 2017). Herbawi et al. (2016) pointed out that carsharing, if used for a greater part of the mobility needs of travelers, can be a bridging mode between private cars and public transportation.

Carsharing systems can be divided into two categories: round-trip and one-way, based on the operation mode (Jorge et al., 2015a, Shaheen et al., 2015; Boyacı et al., 2017). The older more traditional companies provide round-trip carsharing services, in which the vehicle should be returned to the original rental station. One-way carsharing allows users to return the vehicle to any designated station and the price is usually determined based on a combination of trip duration and trip distance (such as GoGet) or simply based on the trip duration (such as EVCARD). Therefore, one-way carsharing can attract more trip motives other than just the occasional shopping or leisure trip as is usually the case for round-trip systems (such as ZipCar). Due to the daily fluctuations of urban travelling demand, one-way carsharing systems cannot serve all potential demand increase (resulting from its added flexibility) because many vehicles may be needed in one station whilst another may be packed with vehicles that are not needed at that moment. However given the very significant improvement in service convenience, one-way carsharing is witnessing a soaring popularity in Europe and the USA with many companies expanding their services (e.g., Car2go, DriveNow, GoGet and Hertz 24/7). At this stage, carsharing operators seem to be focusing mainly on growing, either expanding geographic coverage in cities where they are operating or by offering the system in more cities around the world. Yet companies are still facing great challenges in the planning and operation of their systems.

The asymmetric demand between pairs of stations, leading to an obvious solution of vehicle relocation along a day, is common practice in carsharing systems (Jorge et al., 2014) but also bike sharing (Li et al., 2016b). The vehicle relocation problem would actually be easier and cheaper to do if it would be possible to move the cars easily in a truck. Vehicle routing has been extensively studied to rebalance bicycles among stations in public bicycle sharing systems, therefore the same algorithms could be potentially used for cars as well (Ho and Szeto, 2017). However, relocating a bundle of vehicles together by heavy trucks may not even be possible to do in a city center. The main reason is that loading and unloading vehicles needs large operational space and time. Instead, staff-based relocations are more frequently utilized in carsharing operations. To maximize the benefit of such relocations operators are faced with the need to weigh the relocation costs and the potential trips that clients may be able to do with those vehicles (Nair and Miller-Hooks, 2011).

There are three relocation mechanisms based on the available information, namely, a static method to maintain a minimum threshold of available cars at each station; a predictive method based on historical demand; and an exact based on perfect knowledge on future demand (Barth and Todd, 1999). Optimization methods were usually adopted for predictive relocation (Bruglieri et al., 2014). Kek et al. (2009) studied the relocation personal assignment problem by establishing a Mixed Integer Linear Programming (MILP). Furthermore, with the same Singapore case-study from Kek et al. (2009), Nair and Miller-Hooks (2011) proposed a Divide-and-conquer algorithm to manage fleet redistribution considering asymmetric and variable demand. Weikl and Bogenberger (2015) divided a study region into macroscopic zones, and developed a MILP model for inter zone relocation and a rule-based method for microscopic intra zone relocation. On the other hand, a simulation-based method was also used for testing a rolling-horizon relocation mechanism with perfect demand information by Barth and Todd (1999). They developed a simulation model for a community car club in Southern California to analyze the performance of the system. Boyacı et al. (2017) developed an integrated framework using Mixed-integer Non-linear Programming (MINLP) optimization and simulation to make operational decisions related to vehicle relocations and staff allocation. In addition to the above mentioned operator-based strategies, user-based strategies, where various incentives are presented to encourage users to carry out vehicle relocation (Jorge et al., 2015b), have been proposed. Febbraro et al. (2012) used discrete event simulation to test a user-based carsharing relocation model by offering a fare discount. From the research mentioned above, it is possible to conclude that vehicle relocation is a complex problem to solve. To combine it with other operational decisions presents an even greater challenge.

One of those challenges in carsharing systems modeling lies in the demand sensitivity to the level of service that is being offered to the clients which is a function of the number of stations, walking distance, price, car availability, etc. For example, demand loss is inevitable when there are not enough cars at a station. To circumvent such problem, some studies have assumed a potential abundant demand for this type of service which would be significantly larger than vehicle supply. It is considered that clients would choose carsharing for sure if this was available to them. Nourinejad and Roorda (2014) introduced a dynamic model to maximize the total profit by servicing part of the total carsharing demand rather than all demand. Jorge et al. (2015b) considered elastic demand where carsharing requests were assumed to be linearly decreasing with regard to price. They developed a MINLP based on elastic pricing to obtain a higher balance between demand and supply at each station. The optimal dynamic pricing was obtained when no relocations were needed. However, other factors that affect the utility of carsharing services are ignored the current literature, which presents an important research gap that we want to tackle in the present study.

The last challenge arises from the strategic decision of where to locate and how many parking places to provide in the network of carsharing stations (Jian et al., 2016). A few studies have investigated the joint optimization of station capacity and fleet size for a given network of stations (Cepolina and Farina, 2012; Fassi et al., 2012; Hu and Liu, 2016). Cepolina and Farina (2012) drew upon the convenience of microscopic simulation to design the fleet dimension of personal intelligent vehicles in pedestrian commercial streets. Hu and Liu (2016) developed a mixed queuing network model to address the reservation policy and road congestion effect in one-way carsharing systems. Fassi et al. (2012) presented a discrete event simulation method to evaluate the carsharing network's growth strategies to meet the future demand. Some studies looked into the carsharing depot location problem. For example, Correia and Antunes (2012) proposed three MILP models to optimize the depot location, station capacity as well as the vehicle fleet size under what they called a controlled, full and conditional service scheme of selecting customers. A classical Lisbon case with 75 candidate stations and 1776 trips was used to demonstrate the model. Li et al. (2016a) proposed a continuum approximation method for electric vehicle carsharing station location and fleet size problem with recharging constraints. Deng and Cardin (2018) used discrete event simulator to jointly optimize the allocation of carsharing parking spots and vehicles. However, those studies typically assumed fixed demand where vehicle relocation was either ignored or only one-time rebalancing was allowed during the night which constitutes another research gap.

In this paper, we are adopting the vantage point of a one-way carsharing operator who intends to enter the market by deploying carsharing stations in a region using regular internal combustion vehicles. The operator aims to maximize the total profit by defining the long term resource allocation (carsharing stations location, station capacity, and fleet size) and the short term operation strategies (vehicle relocation) simultaneously. In addition, we consider dynamic and asymmetric travel demand, where the requests to travel between traffic zones fluctuates along the day and is not symmetrically distributed in space. As such, vehicle relocations are allowed both during the operation hours, to satisfy customers' requests, and at the end of the operation period in order to rebalance the vehicle inventory back to its initial level. Travel and relocation times are affected by traffic congestion during peak hours. The competition with private cars is accounted for by using a logit model to determine the potential demand for carsharing which is further subject to vehicle availability. The utility of using carsharing for each zone OD pair depends on the walking costs to access a station in the origin and destination traffic zones, the in-vehicle travel time and the travel costs (it is assumed that the traveler has a perfect knowledge of the travel time and travel costs). Therefore the demand for carsharing and the strategic location decisions of the operator are related to each other, making the problem potentially difficult to solve. In this paper we aim to propose a mathematical model to jointly optimize the station location, station capacity and fleet size considering vehicle relocations and non-linear demand for a station-based one-way carsharing system. Due to the difficulty in solving such type of problems a customized gradient algorithm is also developed to get near-optimal solutions in reasonable time. To the best of our knowledge, this paper is the first attempt to address the three issues simultaneously through an exact optimization approach and corresponding solving algorithm.

The remainder of this paper is organized as follows. In Section 2, the dynamic one-way carsharing model and its solution algorithm are introduced. Section 3 presents the application to the Suzhou Industrial Park (SIP), Suzhou, China. Conclusions are drawn in Section 4.

2. Model formulation

2.1. Assumptions

The assumptions for applying our model are the following:

- The demand of carsharing is known in advance through an on-line booking system or by prediction through historical data. The total travel demand of taking carsharing and driving private cars is known and fixed.
- Only two modes: carsharing and private car are considered. Other modes such as public transport, bicycle, and walking are not being considered as competitors. In other words, the potential travel demand of carsharing comes from original users of private cars. The proportion of carsharing demand is calculated based on the utility functions of both modes of transport.
- Not all carsharing demand is satisfied since this is subject to vehicle availability.
- The installed stations in a zone is uniformly distributed. The catchment regions of the carsharing stations are simplified and considered to be circles.
- At least one parking spot is provided in a station.
- A carsharing station only serves demand in its own traffic zone.
- One staff member from the carsharing organization can relocate only one vehicle at a time.
- Vehicles are relocated at the beginning of the time step.
- The distribution of this travel demand in different zones over time is similar in all days and is representing a typical working day.

2.2. Notation

Primary notation used throughout this paper is listed in alphabetical order as follows:

Parameters

a_i	Area of traffic zone i
b	Value of travel time
c_o	Opening costs of a carsharing station per day (e.g. marking and sign installation)
c_f	Fixed costs per vehicle per day including depreciation costs and maintenance costs for shared cars or private cars
c_g	Gasoline consumption costs per time step
c_p	Rental costs of a parking spot per trip for private cars
c_r	Vehicle relocation costs per time step
c_s	Rental costs of a parking spot per day for shared cars
g_{ij}^t	Travel time in time steps from zone i to zone j where $i \neq j$ departing at time instant t

J: $\{i\}$	Set of traffic zones
q_{ij}^t	Total travel demand from traffic zone $i \in J$ to traffic zone $j \in J$ where $i \neq j$ in time step $t \in T$
r	Rental price of a shared car per time step
s	Speed of walking
T: $\{t\}$	Set of time steps
U_{ij}^{pc}	Utility of private cars for travelers from zone i to zone j where $i \neq j$
Decision variables	
N_{ij}^t	Number of repositioned (relocated) cars from traffic zone $i \in J$ to traffic zone $j \in J$ where $i \neq j$ at the beginning of time step $t \in T$
P_{ij}^t	Proportion of trips serviced by carsharing from traffic zone $i \in J$ to traffic zone $j \in J$ where $i \neq j$ in time step $t \in T$
S_i	Number of parking spots in zone i
V_i^t	Number of shared cars in zone i at the beginning of time step t
X_i	Number of carsharing stations in zone i
Auxiliary variables	
c_{wi}	Costs of walking to access carsharing stations in zone i
P_{ij}^{cs}	Potential proportion of travelers taking carsharing from zone i to zone j where $i \neq j$
Q_{ij}^t	Satisfied carsharing travel demand from traffic zone $i \in J$ to traffic zone $j \in J$ where $i \neq j$ in time step $t \in T$
U_{ij}^{cs}	Utility of carsharing for travelers from zone i to zone j where $i \neq j$

2.3. Problem setting

The problem that we address in this paper is how to design a one-way station-based carsharing system, including station location, stations' capacity, and fleet size in order to maximize the profit of the system. This is not entirely new however we are adding, in relation to previous literature, a series of components that have not been considered at the same time in any model known to the authors. As defined before, we consider that carsharing is in competition with private cars and that travelers' mode choice behavior is captured by a logit model $P_{ij}^{cs} = \exp(U_{ij}^{cs})/[\exp(U_{ij}^{cs}) + \exp(U_{ij}^{pc})]$ where P_{ij}^{cs} denotes the probability of travelers choosing carsharing, U_{ij}^{cs} and U_{ij}^{pc} are the utilities of carsharing and private car respectively. Travelers will weigh the utilities of carsharing and private cars in conducting the mode choice. In this paper, we assume travelers decide if they would like to use carsharing based on the utility values before travelling. The probability P_{ij}^{cs} can be interpreted as the maximum potential demand proportion for carsharing if available, assuming that the demand for private cars can always be attended. However, since some stations could have zero cars to rent in real-time operation, the actual number of travelers using carsharing is reduced. Let P_{ij}^t denote the proportion of trips serviced by carsharing from traffic zone i to traffic zone j in time step t where $P_{ij}^t \leqslant P_{ij}^{cs}$ and the set is defined as $\mathbf{P} := \{P_{ij}^t\}$. In this way, we allow for demand losses subject to carsharing vehicle availability.

Carsharing operators aim to maximize the total profit by weighting the income from servicing travelers and the station opening costs, parking fees and personnel costs. The decision variables are described as follows. The operator decides the number of carsharing stations $\mathbf{X} := \{X_i\}$ in each traffic zone with varying number of parking spots $\mathbf{S} := \{S_i\}$. Every station in zone i has an equal number of S_i/X_i parking spots. This setting is expected to maximize the probability of getting a vehicle for the carsharing users arriving at any station in the zone i. During operation, variables $\mathbf{V} := \{V_i^t\}$ determine the number of available cars in traffic zone $i \in J$ at the beginning of time step t. In particular, V_i^1 is the initial number of vehicles allocated to traffic zone i at the beginning of the day and $\sum_{i \in J} V_i^1$ is the fleet size. To meet the predicted future demand, the operator relocates N_{ij}^t vehicles from zone i to traffic zone j starting at the beginning of time step t (time instant t). In particular, variable $N_{ij}^{|T|+1}$ determines the last number of relocations at the end of an operation day (time instant $|T| + 1$), which is carried out to reset the vehicle inventory back to the initial number V_i^1 for the next day. Note that time instant t stands for the beginning of time step t. There are $|T|$ time steps and $|T| + 1$ time instants. Let the set of relocation decision variables be $\mathbf{N} := \{N_{ij}^t\}$.

2.4. Mathematical model

The optimization model for solving the problem defined above is established as the following MINLP:

$$\mathbf{P1} \quad \max_{\mathbf{S},\mathbf{X},\mathbf{P},\mathbf{N},\mathbf{V}} \theta = (r - c_g) \sum_{t \in T} \sum_{i \in J} \sum_{j \in J} g_{ij}^t q_{ij}^t P_{ij}^t - c_o \sum_{i \in J} X_i - c_s \sum_{i \in J} S_i - c_f \sum_{i \in J} V_i^1 - c_r \sum_{t \in T} \sum_{i \in J} \sum_{j \in J} g_{ij}^t N_{ij}^t \tag{1}$$

Subject to:

$$V_i^{t+1} = V_i^t - \sum_{j \in J} q_{ij}^t P_{ij}^t - \sum_{j \in J} N_{ij}^t + \sum_{j \in J} q_{ji}^{m_{jit}} P_{ji}^{m_{jit}} + \sum_{j \in J} N_{ji}^{m_{jit}}, \quad \forall\ i \in J, t = 1,2,...|T|-1, m_{jit} = max\{0, t + 1 - \lceil g_{ij}^t \rceil\} \tag{2}$$

$$V_i^{|T|+1} - \sum_{j\in J} q_{ij}^{|T|+1}P_{ij}^{|T|+1} - \sum_{j\in J} N_{ij}^{|T|+1} + \sum_{t=m_{ji|T|}}^{|T|+1}\sum_{j\in J} q_{ji}^tP_{ji}^t + \sum_{t=m_{ji|T|}}^{|T|+1}\sum_{j\in J} N_{ji}^t = V_i^1, \forall\, i\in J, m_{ji|T|} = max\{0,|T|+1-\lceil g_{ij}^{|T|}\rceil\} \tag{3}$$

$$\sum_{j\in J}(q_{ij}^tP_{ij}^t + N_{ij}^t) \leqslant V_i^t,\quad \forall\, i\in J, t\in T \tag{4}$$

$$V_i^t \leqslant \lambda S_i,\quad \forall\, t\in T \tag{5}$$

$$S_i \leqslant \xi X_i,\quad \forall\, i\in J \tag{6}$$

$$c_{wi} = \frac{b}{s}\sqrt{\frac{a_i}{\pi X_i}},\quad \forall\, i\in J \tag{7}$$

$$P_{ij}^{cs} = \frac{\exp(U_{ij}^{cs})}{\exp(U_{ij}^{cs}) + \exp(U_{ij}^{pc})},\quad \forall\, i\in J, j\in J \tag{8}$$

$$U_{ij}^{cs} = -\bar{g}_{ij}(r+b) - c_{wi} - c_{wj},\quad \forall\, i\in J \tag{9}$$

$$U_{ij}^{pc} = -\bar{g}_{ij}(c_g+b) - c_p - \frac{c_f}{\alpha},\quad \forall\, i\in J \tag{10}$$

$$P_{ij}^t \leqslant P_{ij}^{cs}, \forall\, i\in J, j\in J, t\in T \tag{11}$$

$$0 \leqslant P_{ij}^t \leqslant 1, \forall\, i\in J, j\in J, t\in T \tag{12}$$

$$X_i \leqslant x_{i,\max}\quad \forall\, i\in J \tag{13}$$

$$Q_{ij}^t = P_{ij}^tq_{ij}^t,\quad \forall\, i\in J, j\in J, t\in T \tag{14}$$

$$N_{ij}^t, Q_{ij}^t, S_i, V_i^t, X_i \in integer^+,\quad \forall\, i\in J, j\in J, t\in T \tag{15}$$

The objective function (1) is to maximize the daily profit of the carsharing operator. The revenue comes from the carsharing income collected from the users. The total cost consists of 5 components, namely, gasoline consumption costs, carsharing station capital investment, parking spot rental costs, vehicle fixed costs, and vehicle relocation costs as represented by the five terms in (1) respectively.

The model comprises 14 sets of constraints. Eq. (2) define the conservation constrains for the number of available sharing cars in each zone over time. The number of cars in zone i at time instant $t+1$ is equal to the number of existing cars at time instant t minus the number of cars rented from zone i in time step t and the number of cars relocated from zone i at time instant t, plus the number of rented cars returning back to zone i in time step t (or between time instants t and $t+1$) and the number of relocated cars entering zone i in time step t (between time instants t and $t+1$). Subscript m_{jit} finds the time instant at which vehicles that have departed from zone j can return to zone i exactly during time instants t and $t+1$. Constraints (3) require the vehicle relocations $N_{ij}^{|T|+1}$ at the end of the day to reset the inventory back to the initial level so as to satisfy the demand in the next morning. Superscript $m_{ji|T|}$ identifies the time instant at which the vehicles departing from zone j could return to zone i exactly during time instants $|T|$ and $|T|+1$. Terms $\sum_{t=m_{ji|T|}}^{|T|}\sum_{j\in J} q_{ji}^tP_{ji}^t + \sum_{t=m_{ji|T|}}^{|T|}\sum_{j\in J} N_{ji}^t$ in (3) calculate the number of vehicles that will return to zone i at the end of the operation day (which could be out of the operation hours meaning after time instant $|T|+1$) either by travelers dropping off vehicles or by means of staff relocation. Constraints (4) assure that the number of cars V_i^t is larger than the sum of the client trips ($q_{i,j}^tP_{i,j}^t$) and the vehicles being relocated from the station (N_{ij}^t). Constraints (5) capture the relationship between parking spaces and available vehicles. The number of parking spots S_i should be larger than the number of cars V_i^t present at each zone at the time instant t, where λ is an elasticity coefficient. It guarantees that there will be at least $(1-\lambda)S_i$ parking spots available for parking. Constraints (6) impose that parking spots are rented only when a zone has at least one station present there, where ξ is a sufficiently large number. In a carsharing station, at least one parking spot is rented. The non-linear constraints (7) calculate the walking costs c_{wi} based on the radius of walking in each zone, where s is the average walking speed and b is the value of time. Stations partition their zone (which has an area of a_i) into X_i catchment regions. Based on the calculation function of the circle radius, each region will therefore have an average walking radius of$\sqrt{a_i/\pi X_i}$. The non-linear constraints (8) represent the logit model to calculate the demand of carsharing. The utility function U_{ij}^{cs} of carsharing in constraints (9) is defined as the weighted sum of the rental costs and walking costs. The utility U_{ij}^{pc} of private vehicles in (10) is defined as the weighted sum of vehicle fixed costs, parking and average travel time costs. In Constraints (10), α is the average number of private vehicle trips per day, and c_f/α calculates the fixed cost of using private vehicles per trip. According to constraints (7)–(10), the walking costs c_{wi} non-linearly decrease with the number of stations X_i, whereas the demand proportion of carsharing (P_{ij}^{cs}) is non-linearly related to the walking costs c_{wi}. Constraints (11) ensure that the actual carsharing usage rate P_{ij}^t is less than the proportion of potential carsharing users P_{ij}^{cs}. In this way, the maximum potential demand for carsharing $P_{ij}^{cs}q_{ij}^t$ also fluctuate during the day. The actual satisfied demand $P_{ij}^tq_{ij}^t$, where $P_{ij}^tq_{ij}^t \leqslant P_{ij}^{cs}q_{ij}^t$, depends on the availability of vehicles in operation. Constraints (12)–(15) specify the domain of decision variables and auxiliary variables.

2.5. Solution algorithm

In this model, constraints (7)–(10) relate station locations and the demand for using the system. Also, the large set of integer decision variables including the number of carsharing stations, the number of parking spots, fleet size and the number of relocated cars, further add to the computation complexity of the problem. There are several non-linear solvers in the programming platform AMPL which could deal with a small scale problem. However, to solve a large-scale MINLP problem such as the one described above, an efficient customized solution algorithm is needed. As such, we develop a gradient procedure by using the notion of service attractiveness denoted by set $\boldsymbol{\rho} := \{\rho_i, \forall i \in J\}$ following a similar concept proposed by (An and Lo, 2014a, 2014b, 2015, 2016). We cannot guarantee the global optimum of the solutions obtained by this algorithm. However, the gradient method can ensure the solution quality improves (the total profit increases) step by step until a local optimum is reached. This algorithm helps search for a solution to this complex MINLP that cannot be solved by the mature commercial solvers such as Gurobi, CPLEX or Xpress. In the future study, we will try to improve the solution efficiency by employing some Heuristic methods. The notion of $\boldsymbol{\rho}$ decouples the relation between the carsharing station location and the resultant carsharing demand. Given a ρ_i, we first calculate the number of carsharing stations X_i, which will be explained below. Then the actual walking costs c_{wi} can be obtained by plugging the value of X_i into Eq. (7). After walking costs c_{wi} in original zone i and c_{wj} in destination zone j are determined, the proportion of potential carsharing demand P_{ij}^{cs} can be obtained from Eqs. (8)–(10), which also serves as the upper bounds of constraints (11). As such, the non-linear constraints (7)–(10) are removed for a given set of ρ_i. The remaining problem is simplified to a mixed integer linear programming problem **P2** and can be readily solved by commercial solvers such as Gurobi, CPLEX or Xpress. The formulation is:

$$\textbf{P2} \quad \max_{\mathbf{S,P,N,V}} \theta(\boldsymbol{\rho}) = (r-c_g)\sum_{t\in T}\sum_{i\in J}\sum_{j\in J} g_{ij}^t q_{ij}^t P_{ij}^t - c_o\sum_{i\in J} X_i - c_s\sum_{i\in J} S_i - c_f\sum_{i\in J} V_i^1 - c_r\sum_{t\in T}\sum_{i\in J}\sum_{j\in J} g_{ij}^t N_{ij}^t \tag{16}$$

Subject to: (2)–(6) and (11)–(15).

The objective value with respect to $\boldsymbol{\rho}$ can thus be obtained. The next step is to search for a promising direction for $\boldsymbol{\rho}$ such that the objective value is increased sequentially until the stopping criteria are satisfied. In the following, we first delineate the procedure to calculate the number of carsharing stations X_i for a given ρ_i as in the first step of the solution algorithm.

In Constraints (8)–(10), the proportion P_{ij}^{cs} is decided by the utility that travelers give to each mode. For potential carsharing users, as the travel time and charge rate of carsharing are considred to be known before travelling, the walking costs will be the only variable influencing their mode choice. Let ρ_i ($\rho_i \neq 0$) be the attractiveness of zone i based on the weighted average travel time $\overline{g}_i = \sum_t\sum_j g_{ij}^t q_{ij}^t / \sum_t\sum_j q_{ij}^t$ over all trips starting from zone i. $2\overline{c}_{wi}$ indicates the average walking costs leaving the origin traffic zone i and arriving in any of the $|J|-1$ destination zones. If ρ_i equals 0, the walking costs $\overline{c}_{wi}$ are positive infinite and X_i is 0. No demand leaving from or arriving at zone i can be met by carsharing.

$$\begin{aligned}\rho_i &= \frac{\exp(-\overline{g}_i(r+b)-2\overline{c}_{wi})}{\exp(-\overline{g}_i(r+b)-2\overline{c}_{wi})+\exp(-\overline{g}_i(c_g+b)-c_p-\frac{c_f}{\alpha})}\\ &= \frac{1}{1+\exp(\overline{g}_i r+2\overline{c}_{wi}-\overline{g}_i c_g-c_p-\frac{c_f}{\alpha})}\end{aligned} \tag{17}$$

Based on Eq. (17), we can obtain the equation of the average walking costs:

$$\overline{c}_{wi} = \frac{1}{2}\ln\left(\frac{1}{\rho_i}-1\right)-\frac{\overline{g}_i r}{2}+\frac{\overline{g}_i c_g}{2}+\frac{c_p}{2}+\frac{c_f}{2\alpha} \tag{18}$$

Given ρ_i and $\overline{g}_i$, we can obtain $\overline{c}_{wi}$ as the average walking costs in zone i. In other words, if the average walking costs for trips starting from zone i are $\overline{c}_{wi}$, the average proportion of carsharing demand for those trips can be achieved at ρ_i.

In a zone there is a negative relation between the walking costs c_{wi} and the number of carsharing stations X_i. Hence, the maximum walking costs $c_{wi,\max}$ to access a carsharing station in zone i occur when only one carsharing station is installed, i.e. $c_{wi,\max} = b/s\sqrt{a_i/\pi}$ when $X_i = 1$.

$$X_i = \begin{cases} 1 & \overline{c}_{wi} > c_{wi,\max} \\ \min\left\{x_{i,max}, \left\lceil \frac{a_i}{\pi(\overline{c}_{wi}s/b)^2}\right\rceil\right\} & 0 < \overline{c}_{wi} \leqslant c_{wi,\max} \\ x_{i,max} & \overline{c}_{wi} \leqslant 0 \end{cases} \tag{19}$$

The case $\overline{c}_{wi} > c_{wi,\max}$ indicates that travelers will choose carsharing as long as there is a carsharing station. Hence, we take $X_i = 1$. The case $\overline{c}_{wi} \leqslant 0$ indicates that the target mode share ρ_i of carsharing cannot be achieved unless walking can bring additional benefits (other than costs). Hence, the maximum number of stations $x_{i,max}$ should be installed to get P_{ij}^{cs} as close to ρ_i as possible. Otherwise, by substituting $\overline{c}_{wi}$ into Eq. (7), we obtain the number of installed stations X_i by applying Eq. (19).

Proposition 1. *The feasible region of ρ_i in (0,1) can map to the feasible region of X_i in $[0,+\infty)$.*

Proof. See Appendix A. □

Proposition 2. *Using the average travel time $\overline{g}_i$ and average walking distance $\overline{c}_{wi}$ from zone i can better smooth the mapping from ρ_i to X_i through the entire feasible region.*

Proof. See Appendix B. □

The following pseudo-code shows the customized gradient algorithm. Let k be the iteration number and θ be the objective value.

Initialization

Step 1: Set $k = 1$, initialize $\boldsymbol{\rho} = \boldsymbol{\rho}^k$.

Step 2: Given $\boldsymbol{\rho}^k$, calculate the average walking costs $\bar{c}_{wi}$ through Eq. (18), and then calculate X_i through Eq. (19). Use X_i in original zone i and X_j in destination zone j to get the actual walking costs c_{wi} and c_{wj} in Eq. (7) and then substitute c_{wi} and c_{wj} into Eqs. (8)–(10) to obtain the carsharing demand proportion $P_{ij}^{cs,k}$. The total profit $\theta^k(\boldsymbol{\rho}^k)$ can be obtained by solving **P2**.

Step 3: Update the maximum objective value, and save it as θ^* .

Step 4: If $\frac{|\theta^k - \theta^{k-1}|}{\theta^k} < \varepsilon$, stop. Otherwise, proceed to Step 5.

Step 5: Determine the optimal $\boldsymbol{\rho}$.

Step 5.1: Calculate the partial derivative of θ^k with respect to ρ_i as below.

Step 5.1.0: Given $\boldsymbol{\rho}^k$ as the following $|J|$ dimensional vector and its corresponding objective value θ^k,

$\boldsymbol{\rho}^k = [\rho_1^k \;\; \rho_2^k \;\; \dots \;\; \rho_{|J|}^k]$

do the following for each element in $\boldsymbol{\rho}^k$:

Step 5.1.1:

a. Set

$\boldsymbol{\rho}_i^k = [\rho_1^k,\rho_2^k \;\; \dots \;\; \rho_i^k + \delta_i^k \;\; \dots \;\; \rho_{|J|-1}^k,\rho_{|J|}^k]$,

where δ_i^k is a small positive number.

b. If $\rho_i^k + \delta_i^k > 1$, go to Step e. Otherwise, proceed to Step c.

c. Solve **P2** given $\boldsymbol{\rho}_i^k$. If $\theta_i^k \neq \theta^k$, go to Step 5.1.2; otherwise set $\delta_i^k = \delta_i^k + \delta^0$, where the small positive number δ^0 is the step size.

d. Go back to Step a.

e. Set $\boldsymbol{\rho}_i^k = [\rho_1^k,\rho_2^k \;\; \dots \;\; \rho_i^k - \delta_i^k \;\; \dots \;\; \rho_{|J|-1}^k,\rho_{|J|}^k]$;

solve **P2** with given $\boldsymbol{\rho}_i^k$. If $\theta_i^k \neq \theta^k$, go to Step 5.1.2; otherwise set $\delta_i^k = \delta_i^k + \delta^0$.

f. If $\rho_i^k - \delta_i^k > 0$, go to Step e. Otherwise, the sensitively of ρ_i^k is set to zero.

Step 5.1.2: The sensitively of the element ρ_i^k can be obtained:

$$\frac{\Delta\theta^k}{\Delta\rho_i^k} = \frac{\theta_i^k - \theta^k}{\rho_i^{k'} - \rho_i^k}$$

where $\rho_i^{k'} = \rho_i^k + \delta_i^k$ or $\rho_i^{k'} = \rho_i^k - \delta_i^k$.

Step 5.1.3: If $i \neq |J|$, set $i = i + 1$, and go to Step 5.1.1. Otherwise, the whole set of sensitivities of θ^k with respective to $\boldsymbol{\rho}^k$ has been obtained, denoted by the vector $\nabla\theta^k$.

Step 5.2: Take the negative sensitivity vector $-\nabla\theta^k$ as the descent direction. The step size π_k is chosen in the same way as in Wang and Lo (2008):

$$\pi_k = \lambda_k \frac{\theta^k - \gamma_1\theta^*}{\|\Delta\theta^k\|^2}$$

If θ^* is not improved in the fifth iteration, set $\lambda_{k+1} = \frac{2}{3}\lambda_k$, which will reduce the step size gradually, and the convergence is guaranteed consequently. $\gamma_1\theta^*$ is an estimate of the minimum of θ^k. The value of parameters γ_1, λ_k should be specified for different problems.

Step 5.3: Calculate the reliability $\bar{\boldsymbol{\rho}}^k = \boldsymbol{\rho}^k + \pi_k\nabla\theta^k$. Project the new value $\bar{\boldsymbol{\rho}}^k$ onto the feasible space [0,1]; this result is set to be the service attractiveness $\boldsymbol{\rho}^{k+1}$ for the next step. Set $k = k + 1$, and go to Step 2.

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3. Application to the case-study of Suzhou Industrial Park

3.1. Setting up the case study

The case study region used in this paper for a demonstration of the application of the model is the Suzhou Industrial Park (SIP) in Suzhou, China, comprising 104 zones with a total area of 278 km^2 as shown in Fig. 1. The city has a population of 1.03 million and 620 thousands trips per day done in private cars. According to the land use features, the zones are divided into three types: Residential zones from Zone 1 to Zone 40, industrial zones from Zone 41 to Zone 80, commercial zones from Zone 81 to Zone 100, and

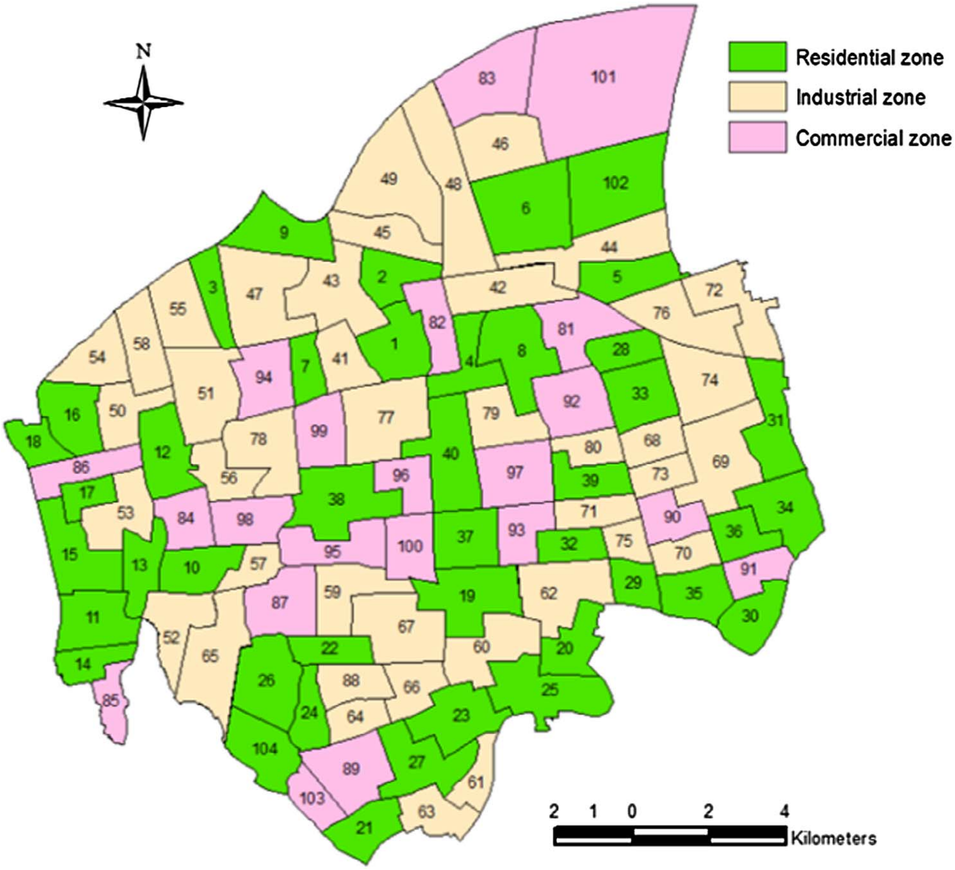

Fig. 1. Zoning of the case-study area.

zones from 101 to 104 are the undeveloped land with zero travel demand. In this case study, we consider a daytime operation of 13 h from 7:00 to 20:00 partitioned into 26 time steps with a 0.5-h duration.

The travel demand in SIP is randomly generated. The distribution of average departures in a traffic zone is presented in Fig. 2.

The free flow travel time on the shortest path is taken as the base travel time between any two zones, which is calculated in ArcGIS network analyst software using the actual road network of SIP in 2012. The actual travel time between two zones changes with the real-time traffic situation (see Table 1).

The values of the parameters needed for the model proposed in this paper are from a one-way carsharing operation report of EVCARD company (http://www.evcard-sh.com) as shown in Table 2. The Chinese Yuan (¥) to US dollar ($) exchange rate is 0.15 as of November 4, 2017.

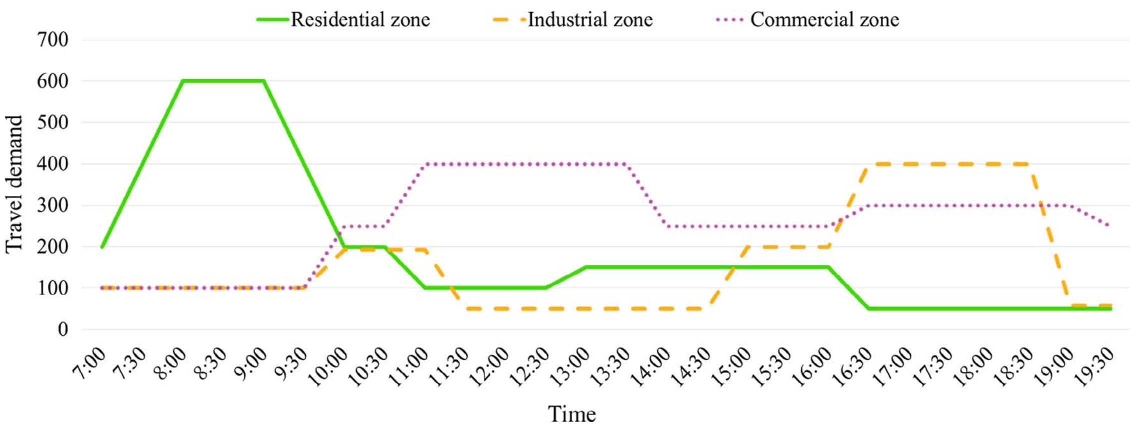

Fig. 2. Distribution of departures.

Table 1
Growth factors of travel time in relation to the shortest travel time.

Time	7:00–7:59	8:00–9:59	10:00–10:59	11:00–11:59	12:00–13:59
Growth factor	1.0	1.5	1.3	1.1	1.2
Time	14:00–15:59	16:00–17:59	18:00–18:59	19:00–19:59	20:00
Growth factor	1.1	1.5	1.3	1.0	1.0

Table 2
Parameters for the case-study application.

Parameter	c_f	c_p	c_s	c_o	r	c_g	c_r	b	s	α	λ	$x_{i,max}$
Value	56	10	12	1	60	20	85	56	6000	2	1	10
Unit	¥/veh*day	¥/veh*trip	¥/veh*day	¥/day	¥/h	¥/h	¥/h	¥/h	meter/h	–	–	–

Vehicle fixed costs for a day c_f are calculated based on the depreciation costs of a vehicle valued at ¥100,000 for 10-year service life (in Suzhou vehicles are still being used for higher periods compared to western countries) and the annual maintenance costs of ¥10,000 per year. c_p, c_s, r, c_g and c_r come from the average market pricings. c_o represents the opening costs of a parking station estimated around ¥360 per year including lane painting and indicator panel costs. The value of time b is calculated based on an average monthly income of ¥10,000 for car owners in Suzhou. Each zone can open at most 10 stations, i.e. $x_{i,\max} = 10\ ,\quad \forall\ i \in J$. α indicates that the average number of private vehicle trips is twice per day. The parameter λ is taken as 1 in the case study indicating no additional parking spaces are reserved for returned vehicles.

3.2. Optimization results

This model was run in an i7 processor @ 2.4 GHz, 8.00 Gb RAM computer with a Windows 7 64 bit operation system. The MINLP model was first solved by commercial solver MINOS in AMPL Education Version 12.2. It was then solved by applying the gradient procedure proposed in this paper. The algorithm was programmed in Python where the Gurobi 7.0.2 solver was installed. We set the step size $\delta^0 = 0.1$, the constant term $\gamma_1 = 1$, $\lambda_k = 1$, the starting point $\boldsymbol{\rho}^1$ as 0.5 and the stopping criteria ε as 1.0%. In the base scenario of the case study, the computation time for one iteration of the customized gradient algorithm is around 167 min by taking advantage of the parallel computing technology, and it takes 7 iterations until the stopping criteria was met. The daily profit is ¥7,659,543 ($1,148,932). However, MINOS could not get a feasible solution after 24 h' calculation.

To make a further comparison regarding computation efficiency for the two methods, we increase the time step duration to 1 h (with 13 time steps) and solve the model by MINOS and the proposed algorithm separately. MINOS stopped after 272 min' calculation and obtained its best solution with the total profit of ¥3,675,390 ($551,309) whereas the proposed algorithm obtained a profit of ¥6,009,654 ($901,448) in 505 min. The satisfied carsharing demand given by AMPL is 34.81% whilst our algorithm results in a share of 72.88%. Although MINOS found a solution much faster than the gradient method, its solution quality is significantly worse. Results demonstrate that the proposed gradient solution algorithm outperforms the commercial solver MINOS in terms of solution quality. Besides, we compare the optimization results and computation time when setting the time step size as 1 h, 0.5 h and 0.25 h (see Table 3). Results show that the total profit (the objective value) is increased by 46.56% while the computation time is increased by 645.07%. A more efficient solution algorithm is thus necessary to investigate a large scale problem considering short time step durations. In the next sections we use the optimization results with the time step of 0.5 h obtained by the gradient algorithm to draw insights for the planning and operations of carsharing systems.

3.2.1. Carsharing patronage

The carsharing market share, namely the satisfied carsharing demand or carsharing patronage is on average 82% of the total travelers, which is calculated by Eq. (20). The market share of carsharing in this paper refers to the percentage of travel demand that would be serviced by carsharing instead of private vehicles according to the mode choice model. We note that as referred in the assumptions only private cars is considered as competitors. The high patronage rate demonstrates the outstanding attractiveness of the one-way carsharing services comparing to private cars when only instrumental attributes such as costs and time are being considered.

Table 3
Optimization results with different time step duration.

Time step duration (min)	Market share of carsharing	Profit (1000¥)	Revenue (1000¥)	Computational time per iteration (min)	No. of iterations
60	72.88%	6010	11,759	72	7
30	83.58%	7660	15,275	167	7
15	90.24%	8808	16,053	529	7
Changes (%)	+23.82	+46.56	+36.52	+645.07	0

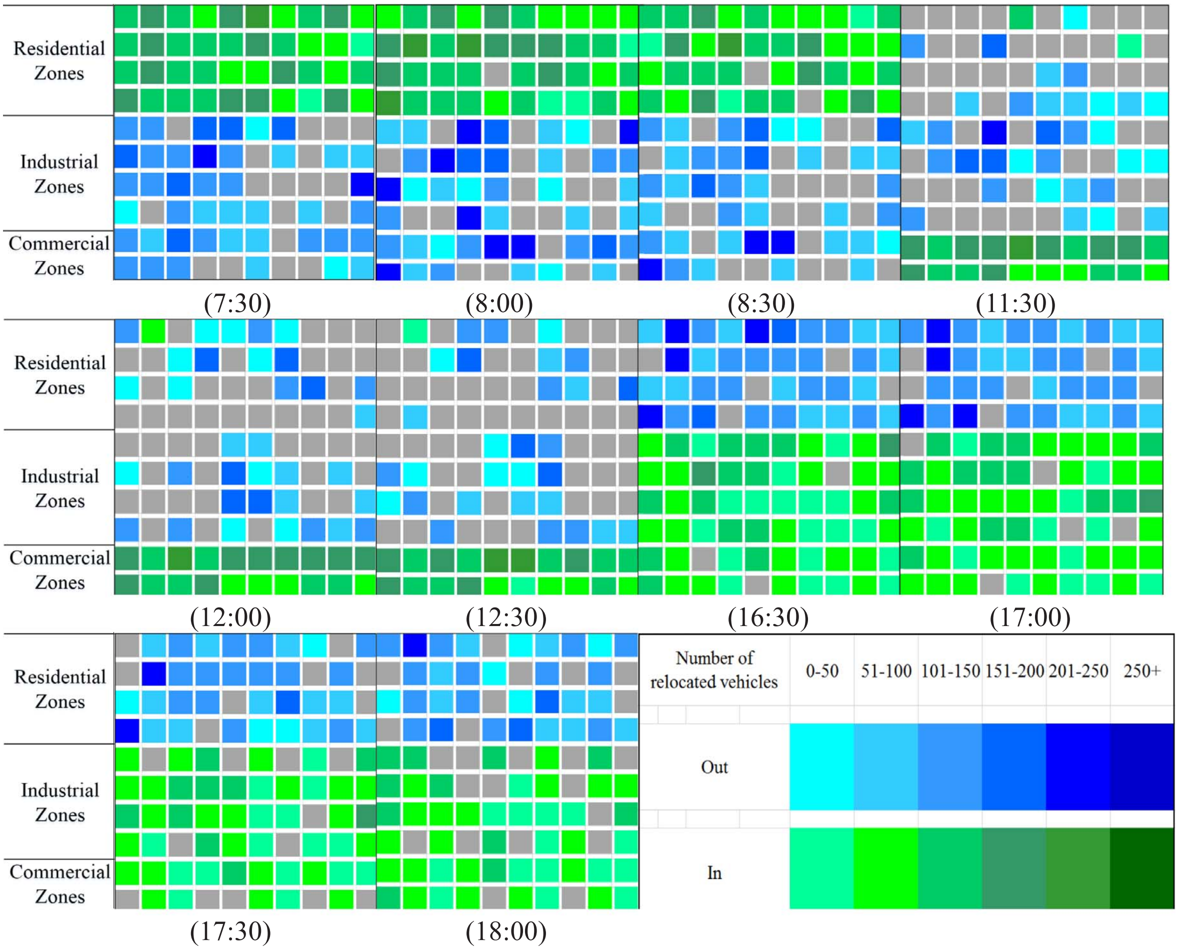

Fig. 3. Relocation operations along the day.

$$P = \frac{\sum_{t\in T}\sum_{i\in J}\sum_{j\in J} P_{ij}^{t} q_{ij}^{t}}{\sum_{t\in T}\sum_{i\in J}\sum_{j\in J} q_{ij}^{t}} \tag{20}$$

$$P_i^t = \frac{\sum_{j\in J} P_{ij}^{t} q_{ij}^{t}}{\sum_{j\in J} q_{ij}^{t}} \quad \forall\ i \in J, t \in T \tag{21}$$

Eq. (21) is used to calculate the average market share in each zone i and time step t. The market shares in all traffic zones are larger than 0.90 except at 8:00–9:00 in residential zones, at 16:30–18:30 in industrial zones and at 11:00–15:00, 16:30–19:00 in commercial zones. As shown in Fig. 2, the travel demand is larger than the average value in the aforementioned three periods when the low market shares occur. At the morning peak hours, the market share in residential zones ranges from 0.60 to 0.61. In the noon peak hours, the market share in commercial zones is about 0.67–0.89. Similarly, at the afternoon peak hours, the market shares are significantly lower than the other time slots in industrial zones and commercial zones, which are about 0.70–0.72 and 0.85–0.88 respectively. The substantial unsatisfied travel demand happening at the morning peak hours is more than at the afternoon peak hours although the total travel demand is similar. This phenomenon is probably due to the highly concentrated demand in a relative short duration in the morning peak. The carsharing fleet has 31,207 vehicles which service 509,125 trips. The average number of trips per vehicle per day is 16.31. By the end of 2016, there were over 400,000 private cars in SIP Suzhou (SIP, 2017). This demonstrate the high usage of carsharing comparing to private vehicles.

3.2.2. Vehicle relocation

Fig. 3 shows the number of relocated vehicles in the 100 traffic zones located in the constructed area indexed from 1 to 100. There are 58,510 relocations happening in the study area for the whole day, with an average of 22.5 relocations per zone per time step. We select 10 representative time steps in which the number of relocations is larger than 3000 to illustrate the relocation distribution. The cell number increases from left to right first and then increases from top to bottom. The top-left and bottom-right cells represent zone 1 and zone 100, respectively. The cells in the top four rows are residential zones while the fifth to the eighth row from the top are

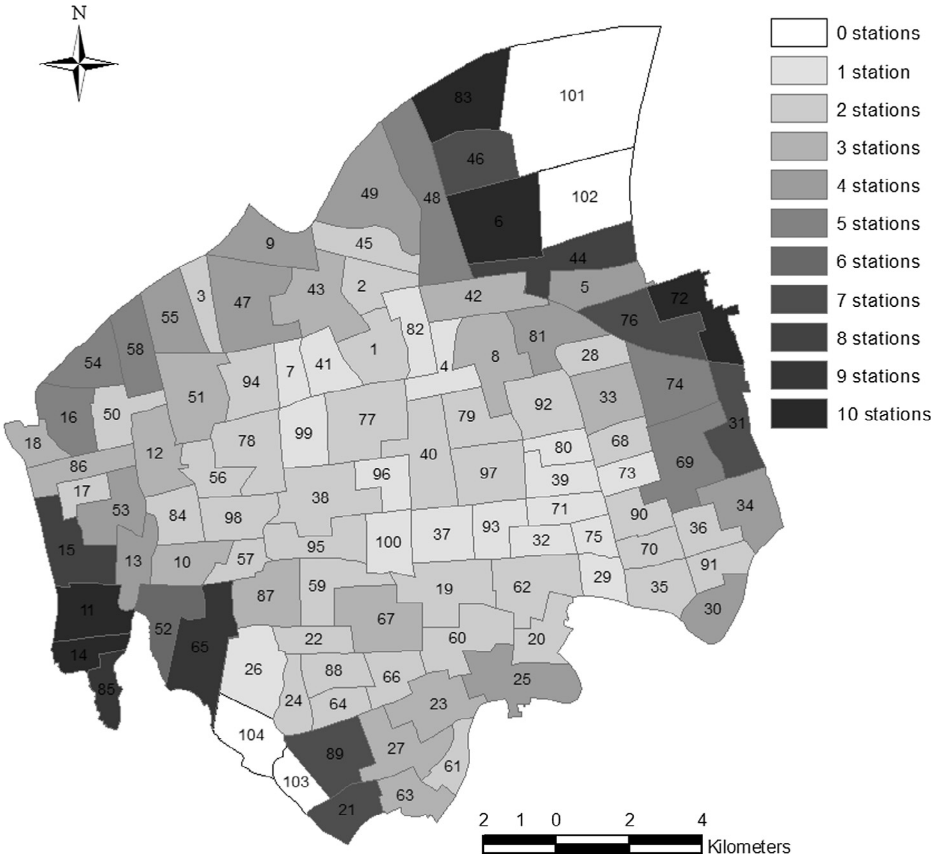

Fig. 4. Distribution of carsharing stations.

industry zones, and the bottom two rows are commercial zones. We use different colors to show the number of relocated vehicles. The blue/green scales represent the number of vehicles relocated from/to the zone. A darker color indicates more vehicle relocations. The gray scales indicate that no relocation operation occurs from/to the zone.

The relocation operations mainly happen in 3 periods of time (see Fig. 3), at 7:30–8:30, 11:30–12:30 and 16:30–18:00, which is highly consistent with the three demand peaks (morning, noon and afternoon). At the morning peak, a great amount of vehicles are relocated from industrial zones and commercial zones to residential zones to handle the increasing departures from the residential zones. From 10:00 to 13:30, the number of departures declines in residential zones whereas it rises gradually in commercial ones. In anticipation of such demand growth, many vehicles are relocated into commercial zones at 11:30–12:30. Though the demand in industrial zones starts to rise at 10:00 simultaneously, the existing vehicles are enough due to the accumulation resulting from the morning commute trips. In the afternoon, the number of relocation operations begins to increase at 16:30 to prepare for the large travel demand generating from industry and commercial zones in the afternoon peak.

3.2.3. Optimal number of stations, capacity and walking costs

On average, the company installs 3.4 carsharing stations in a zone and rents 312.1 parking spots in each zone. The average costs of walking are ¥5.16 for a carsharing user. The number of stations and the number of parking spots per station deviates considerably in each zone, and they are not linearly dependent on the zone size. We find that the average number of parking spots per zone in the residential zones (360) is significantly larger than that of the industrial zones (284) and the commercial zones (273). Besides, the average station capacity in the residential zones (147) is also larger than that in the industrial zones (118) and the commercial zones (141). The average number of stations in the residential, industrial and commercial zones are 3.60, 3.45 and 2.95, respectively. For the whole SIP, the number of car sharing stations in a zone ranges from 1 to 10, with an average of 3.26.

Fig. 4 shows the distribution of carsharing stations across zones. The number of stations created in the zones located at the edge of SIP (except for the zones with no stations) is significantly larger than the one in the central zones of SIP. The main reason is that the walking costs should be small enough to attract carsharing users in remote zones where users' average travel distance is relatively large. The results are consistent with Eq. (21), in which the carsharing demand proportion decreases with the increase of walking costs and travel distance. In other words, carsharing is more attractive for shorter distance travelers whose origin and destination zone have large station densities. In the central zones, the short travel distance makes the carsharing attractive enough for potential users. Hence the operator tends to build fewer stations with more parking spots in each station for management convenience. A

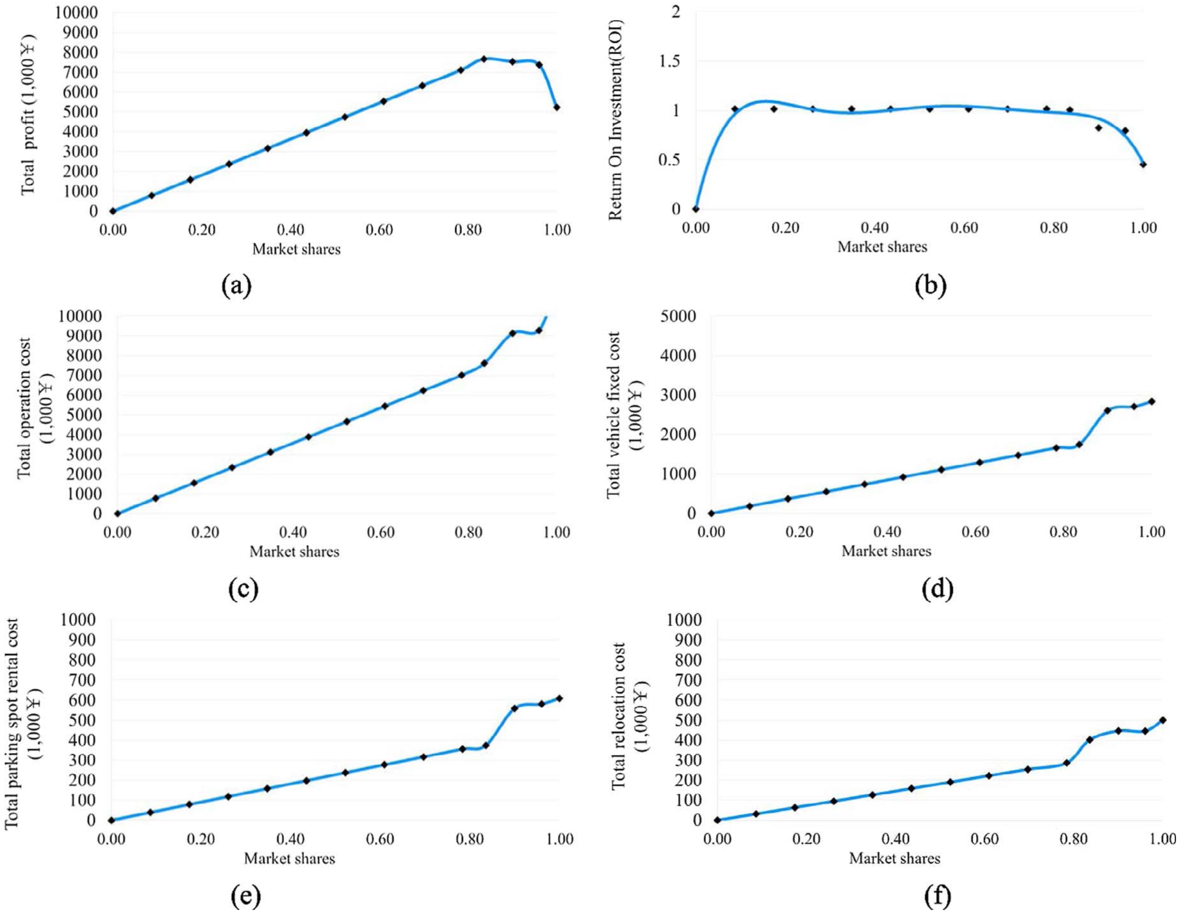

Fig. 5. Sensitivity analysis of the market shares.

regression analysis establishes an exponential function with a goodness of fit $R^2 = 0.69$ to quantify the relationship between the travel distance and the station density.

$$X_i = 0.07\exp(6.3\bar{g}_i) \tag{22}$$

3.3. Sensitivity analysis on cost component

To conduct a sensitivity analysis with respect to the cost component, we increase ρ from 0.1 to 0.7 with a step size of 0.1 first, and from 0.8 to 1 with a step size of 0.05, and then obtain the optimization results by solving **P2**. Please note that the descent direction finding process in Steps 3–5 is not needed here. The resulting market share is calculated by Eq. (20). We further built a return on investment (ROI) model (Eq. (23)), which equals the daily carsharing profit divided by the total operation costs per day (the fuel consumption costs, the carsharing station capital investment or depreciation costs, the parking spot rental costs, the vehicle fixed costs, and the vehicle relocation costs). We plot the optimization results in Fig. 5, setting the market share as the horizontal coordinate and the cost component as the vertical coordinate.

$$ROI = \frac{\theta}{c_g \sum_{t\in T}\sum_{i\in J}\sum_{j\in J} g_{ij}^t q_{ij}^t P_{ij}^t + c_o\sum_{i\in J}X_i + c_f\sum_{i\in J}V_i^1 + c_s\sum_{i\in J}S_i + c_r\sum_{t\in T\cup\{|T|+1\}}\sum_{i\in J}\sum_{j\in J} g_{ij}^t N_{ij}^t} \tag{23}$$

As shown in Fig. 5a, when the market share increases from 0 to 0.8, the total profit quickly rises at a steep rate. After reaching the peak, the total profit drops significantly when pushing the market share to 100%. This indicates that satisfying all the travel demand is not a good strategy for operators to get the maximum profit. The ROI profile fluctuates around 1, but it decrease rapidly when the market share increases from 0.8 to 1.0. The operator needs to install more carsharing stations, rent more parking spots and keep a larger fleet size to maintain a large market share (see Fig. 5d and e). Fig. 5f shows that the total relocation costs increase continuously, which indicates that relocation operation can help to balance the demand and the supply.

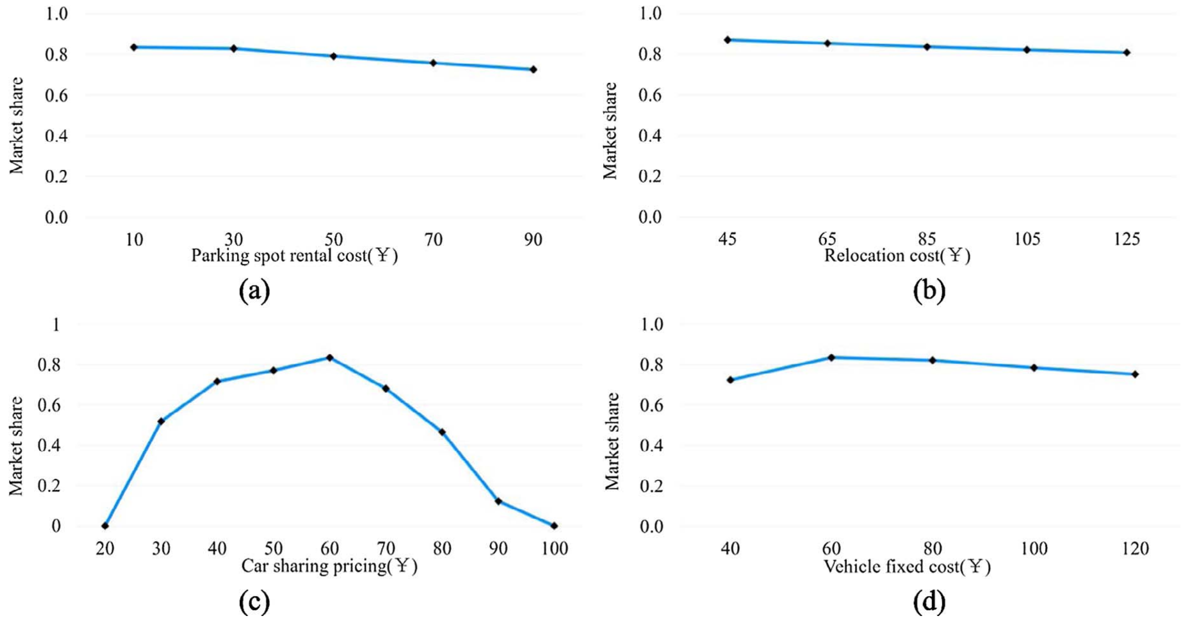

Fig. 6. Sensitivity analysis on market share.

3.4. Sensitivity analysis on market share

In this section, we conduct the sensitivity analysis on the market share when changing the costs of renting a parking spot c_s, vehicle relocation costs c_r, carsharing pricing r and vehicle fixed costs c_f.

The parking station rental cost and average carsharing market share are negatively related (see Fig. 6a). When the station rental cost increases from ¥10 to ¥90 per hour, the satisfied carsharing demand patronage drops from 84% to 72%. To promote the carsharing usage, offering subsidies on its parking can encourage operators to provide more services. With the increase of the relocation cost, the market share decreases but at a slow speed (see Fig. 6b). In Fig. 6c, when the carsharing pricing is as low as 20 which equals the gasoline consumption cost, the operator would not provide carsharing services for there is 0 profit. When the carsharing pricing is ¥60 per hour, the market share reaches the peak indicating the most significant benefits to the society as most travel demand can be satisfied. When the price reaches 100, the utility of carsharing declines and hence the carsharing demand is around 0 according to the logit model. In Fig. 6d, the market share increases by 11% from 72% to 83% when the vehicle fixed cost increases from ¥40 to ¥60. This is due to the fact that carsharing operators and private car owners use the same type of cars. When a more expensive car mode is adopted, the utility of private cars to travelers decreases whereas the utility of carsharing is maintained the same (assuming the car rental price unchanged). This leads to potential carsharing demand increase. However, the profit declines marginally by 8% from 83% to 75% when the vehicle fixed cost increases from ¥60 to¥120. Though the potential carsharing demand increases, the higher vehicle fixed cost forces operators to buy less vehicles to ensure its profitability. As a result, the average number of serviced demand drops when the vehicle fixed cost is larger than ¥60 despite the fact that more people are willing to take carsharing.

3.5. Sensitivity analysis on total profit

In this section, we conduct the sensitively analysis on the total profit when changing the costs of renting a parking spot c_s, vehicle relocation costs c_r, carsharing pricing r and vehicle fixed costs c_f.

When the parking spot rental cost increases from ¥10 to ¥50, the total profit reduces from ¥7,700,000 to ¥5,798,894 by 24.69% and the total parking spot rental cost increase from ¥312,070 to ¥1,908,900 by 511.69%. The total profit declines ¥23,764 on average if the unit rental cost increases by ¥10. Fig. 7b shows that increasing the relocation cost mildly cuts down the operator' profits. More analysis on vehicle relocation will be provided in the following section. In Fig. 7c, the total profit first increases with the rise of carsharing pricing, and then drops when the pricing is larger than ¥70. However, in Fig. 7c, the market share reaches the peak when setting the carsharing pricing as ¥60. We can see that the total profit increases continuously when the pricing increases from ¥60 to¥70. The main reason is that the high unit revenue is larger than the loss of unserved travel demand. Once the pricing exceeds ¥70, the travel demand decreases quickly, and the total profit starts to decline. In this case, when setting the carsharing pricing at ¥70 can bring the maximum profit for the operator. In Fig. 7d, the total profit increases first and then drop with the vehicle fixed cost, which is consistent with the results in Fig. 6d.

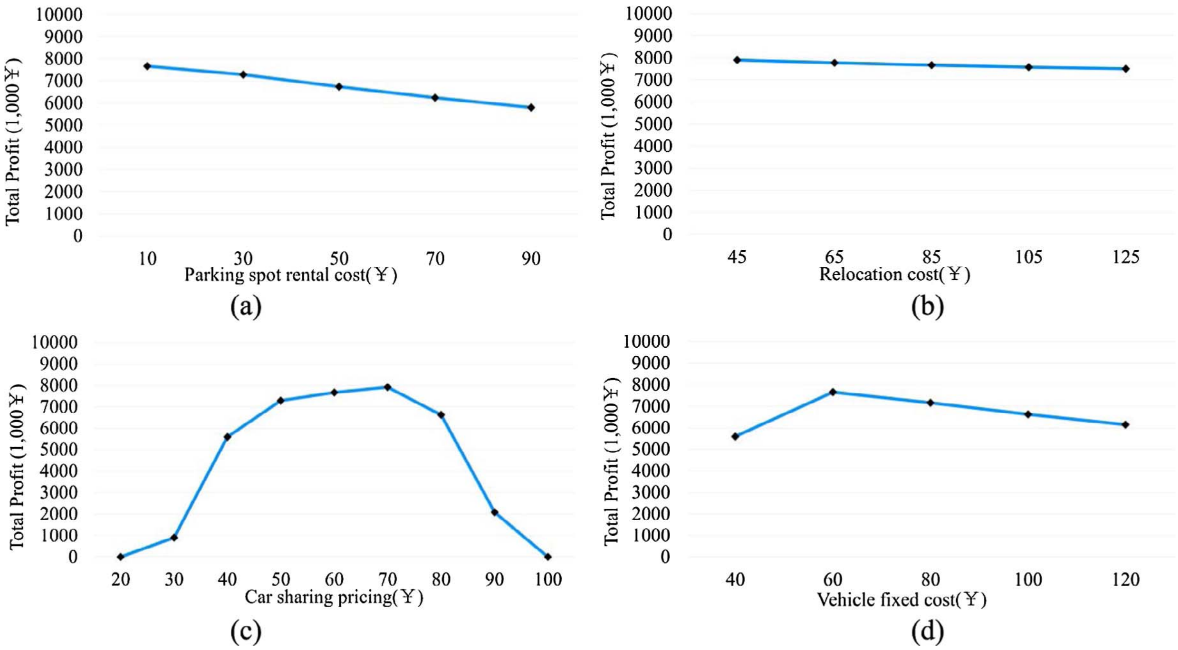

Fig. 7. Sensitivity analysis of the total profit.

3.6. Sensitivity analysis on relocation operations

To further analyze the impacts of vehicle relocations, we first increase the unit relocation cost in the aforementioned case with 104 zones, and then optimize it using the benchmark parameter settings. The infinite unit relocation cost (see Table 4) indicates that vehicle relocations are not implemented. The number of relocation trips drops from 58,510 to 0 when the unit relocation cost increases from 45 to infinity. The total profit decreases by 10.30%, and the market share drops by 14.39%. Vehicle relocation is of some help to the operator in increasing profit and to the society for satisfying travel demand. When setting the unit relocation cost as ¥45, ¥85 and ¥125, we find that the average travel time of a relocation is 5.61 min, 5.00 min and 4.49 min, and the relocation cost per trip is ¥4.20, ¥7.08 and ¥7.86, respectively. It indicates that the operators give up the long-distance relocations with the increase of unit relocation cost. It also explains why the profit does not decrease a lot when the unit relocation cost is tripled from ¥45 to ¥125.

3.7. Remarks

In this case study, the profit of carsharing operators may have been overestimated. When predicting the potential carsharing travel demand, as explained before, we have built a logit model that assumed users' intention to choose carsharing depends on the predictable travel time and walking costs. It ignores the psychological characteristics of users and all other aspects that influence mode choice (Kim et al., 2017). For example, a luxury private vehicle is usually preferred by high-end users, which reduces people's enthusiasm to switching to carsharing. Besides, using one-way independent trips can also generate excessive carsharing demand: for example, if a person travels in the morning by private car he/she won't be able to use the carsharing in the afternoon because he/she has to bring the car home (a tour based model would be needed to do this evaluation). In addition, underestimating the operation costs may cause a larger profit. In SIP, Suzhou, China, to promote carsharing, the local government provides a large discount when the operators rent parking spots. Moreover we did not take existing private vehicle ownership into consideration. A family owning a private car will not consider the fixed costs (purchase, maintenance) but probably only the usage costs (oil, parking) in their mode choice for a particular trip, since the fixed costs are regarded as sunk costs, which have to be paid no matter how intensively the

Table 4
Results of carsharing relocation sensitively analysis.

Unit relocation cost (¥/h)	No. of relocation (1000)	Relocation cost (1000¥)	Market share of carsharing	Profit (1000¥)	Revenue (1000¥)	Vehicle fixed cost (1000¥)	Parking Spot rental cost (1000¥)
45	76	306	87.04%	7897	15,664	1845	395
85	59	401	83.58%	7660	15,275	1748	374
125	45	410	80.84%	7497	14,910	1674	359
+ ∝	0	0	71.87%	7084	13,410	1539	330
Changes (%)	−100	−100	−17.49	−10.30	−14.39	−16.59	−16.46

private cars are being used. We also ignored the personnel allocation problem in the vehicle relocation operations by only specifying the relocation costs per hour. Given the significant amount of relocations, hiring and organizing enough people to carry out the relocation could be challenging and may lead to side effects such as congestion which are not being considered.

In this study, we have looked at an ideal environment for carsharing operators assuming people have not purchased private cars and have full flexibility to switch to carsharing. Such assumption may not reflect the reality of car ownership at this moment, but could provide a benchmark to measure the maximum potential of carsharing systems. Carsharing systems induce an enormous mode shift from private cars. The daily profit (as high as ¥7,659,543 or $1,148,932) of the carsharing operators estimated in this paper are reasonable given the significant market share of 80%. How to redistribute this profit by introducing competition among multiple carsharing operators could be an interesting topic to do further research. In addition, the proposed model and solution algorithm can be easily adapted to improve the realism of the results by adding other components to the utility functions. However, the composition of such utility functions should be carefully estimated.

4. Conclusion

This paper addressed the one-way station-based carsharing network design problem. Considering relocation operations and non-linear travel demand, a MINLP model was established to maximize the total profit for carsharing operators. A customized gradient algorithm was developed to solve the model. A large-scale case study was conducted to address the carsharing system design in the SIP, China, with 104 traffic zones and 13 h' operation duration. Although it needed a longer computation time, the customized gradient algorithm obtained a significantly higher profit than the commercial solver MINOS. According to the optimization results, the profit is maximized when the carsharing market share reaches 83%. The satisfied carsharing travel demand is significantly reduced in the peak hours due to insufficient vehicles. According to the sensitively analysis, the total profit and ROI decrease when market share is larger than 80%. It implies that chasing a large market share could harm the earned profit and investment return. Secondly, the total profit and carsharing market share start by gradually increasing and then drastically decreasing with the rise of carsharing prices. It indicates that pricing is the key issue that affects the carsharing system performance. Thirdly, relocation costs have a negative, yet marginal, impact on the total profit. When carsharing penetrates into the market, relocation operations can bring benefits to operators and users.

To make the problem trackable, we made several simplifications in this study, including the fact that the carsharing company and the private car users adopt the same type of vehicles; travelers are homogeneous in value of time; staff allocation for vehicle relocation is not considered; a set of parameters based on the city of Suzhou in China are used. These assumptions can be relaxed for further studies. In addition, with the rapid development of electric vehicle (EV), the one-way carsharing system using EVs has great potential to serve as an environmental friendly substitute mode for travelers. The battery capacity, recharging time, recharging station location problem should be taken into consideration when establishing the optimization model. In addition, adopting flexible pricing strategies, which will affect the carsharing utility function and the travel demand, is another interesting extension for future studies.

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Appendix A. Proof of Proposition 1

Proposition 1. *The feasible region of ρ_i in $(0,1)$ can map to the feasible region of X_i in $[0,+\infty)$.*

Proof. According to Eq. (18), $\bar{c}_{wi}$ approaches $-\infty$ when ρ_i approaches 1 while $\bar{c}_{wi}$ approaches $+\infty$ when ρ_i approaches 0 given all the other parameters c_f, c_p, $\bar{g}_{ij}$, r, c_g, α, λ fixed. Hence the feasible region for $\bar{c}_{wi}$ is $(-\infty,+\infty)$. According to Eq. (19), we can always find a corresponding $\bar{c}_{wi}$ for any feasible value of integer variable X_i. In this way, no feasible solution in X_i will be missed out by varying the continuous variable ρ_i from 0 to 1. □

Appendix B. Proof of Proposition 2

Proposition 2. *Using the average travel time $\bar{g}_i$ and average walking distance $\bar{c}_{wi}$ from zone i can better smooth the mapping from ρ_i to X_i through the entire feasible region.*

Proof. Let us consider trips starting from zone i. According to Eqs. (8)–(10), the demand proportion of carsharing P_{ij}^{cs} decreases with the increase of travel time $\bar{g}_{ij}$ to zone j and walking distance c_{wj} in zone j. In terms of the fixed vehicle fixed costs as well as the parking station rental costs, this part will occupy more proportion in the total travel costs when the travel distance is short. Hence, users will choose the private car more than carsharing when they have a long trip. Hence, the potential proportion of carsharing for users whose travel distance is short is larger than those users who have long travel distance. □

As such, the minimum demand proportion $\underline{P_{ij}^{cs}}$ among destinations $j \in J$ is achieved at the maximum travel time $\max_{\forall j \in J}\{\bar{g}_{ij}\}$ and

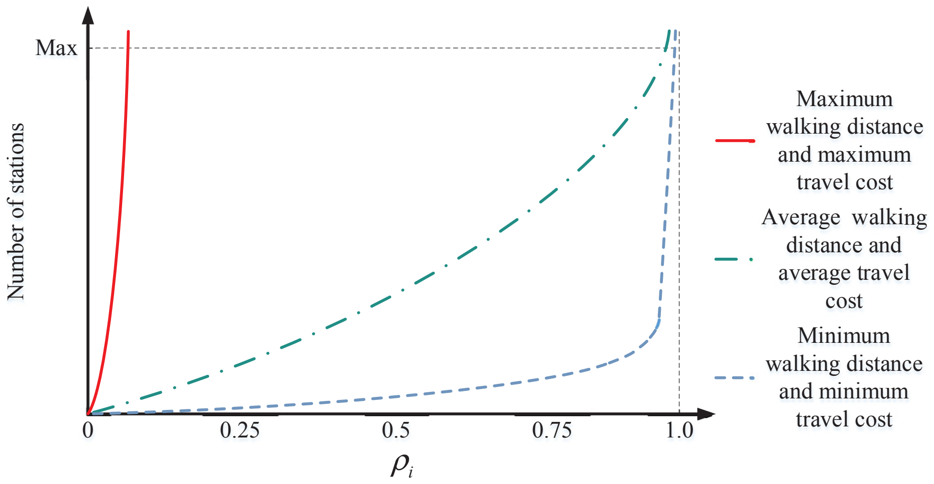

Fig. B1. Relationship between the proportion and number of stations.

with the maximum walking costs $c_{wj,\max}$ when only one station is installed. On the other hand, the maximum demand proportion $\overline{P_{ij}^{cs}}$ is achieved at the minimum travel time $\min_{\forall j\in J}\{\overline{g}_{ij}\}$ and zero walking costs $c_{wj} = 0$. If we set $\rho_i = \underline{P_{ij}^{cs}}$ by its lower bound, the resultant P_{ij}^{cs} shall be larger than ρ_i. When ρ_i increases from 0 to 1, P_{ij}^{cs} would increase from $\varkappa_j$ to 1, where $\varkappa_j > \rho_i$ and $\varkappa_j$ increases significantly with the decrease of $\overline{g}_{ij}$. As such, the effective searching region in ρ_i would cluster on the left around 0. Similarly, the searching region over ρ_i would cluster around 1 on the right if $\rho_i = \underline{P_{ij}^{cs}}$. To verity this finding, we excreted one scenario in the benchmark case study and plot out the relationship between ρ_i and X_i in Fig. B.1. The horizontal coordinate represents the given proportion ρ_i while the vertical coordinate represents the number of stations X_i calculated by Eq. (19). The red line uses $\rho_i = \underline{P_{ij}^{cs}}$ with the maximum walking costs and travel time while the blue line takes $\rho_i = \overline{P_{ij}^{cs}}$. The green line to shows the result based on the average value.

We can observe the number of stations X_i always increases with the rise of ρ_i as proved by Eqs. (18) and (19). However, the obtained number of stations increases at a high speed when the maximum walking costs and $\max_{\forall j\in J}\{\overline{g}_{ij}\}$ are used. On the contrary, when using the minimum walking costs and $\min_{\forall j\in J}\{\overline{g}_{ij}\}$, the marked increase occurs when ρ_i is close to 1. The green line shows that using the average value can smooth out the changing rate in the number of stations.

To further capture the impact of ρ_i selection, we calculate average demand proportion as $\sum_{j\in J} P_{ij}^{cs} q_{ij} / \sum_{j\in J} q_{ij}$ and plot out its relationship in Fig. B.2. As shown by the red line in Fig. B.2, when using $\max_{\forall j\in J}\{\overline{g}_{ij}\}$, the rise in carsharing demand for short trips is larger than the rate of rise of average demand proportion for long trips. The average demand proportion increases to 1 at a high speed. Therefore, using the average travel time and average walking costs with the green line can find the system optimum with a reasonable step size.

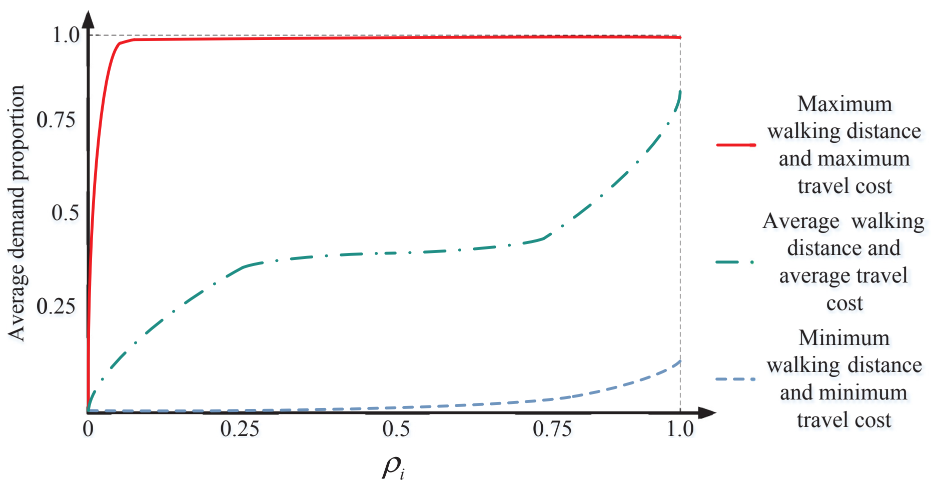

Fig. B2. Relationship between the proportion and average demand proportion.

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