

Experimental and Numerical Analyses of Piled Raft in Sand

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Experimental and Numerical Analyses of Piled Raft in Sand

by

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Cover: "View of an Arching Viaduct in Autumn" by Laura Meinhardt

This thesis is confidential and cannot be made public until August 25, 2026.

Preface

This document contains the M.Sc. thesis of Henry Gunawan (me), which studies the behaviour of piled raft foundations. It compares the behaviour of piled raft foundations with pile groups and rafts, using 3D finite element analysis (FEA) with SANISAND-MS as the constitutive model and centrifuge testing. Additionally, it includes a comparative analysis between the FEA and centrifuge testing. Beyond that, I also performed the calibration of a viscoelastic model to fit the cyclic behaviour of piled raft foundations. Throughout these few months, I have learned a lot during this process, and I am deeply grateful to Luca Flessati for offering me the opportunity to work on this project, which is part of the IDEA League project titled "INITIATION - Improving Lifespan of Existing Viaducts Foundations."

When I began this Master's program two years ago, I did not anticipate the path it would take. Who knew I would take B2 in the module of "Advanced Soil Mechanics" or that I would learn so much about coding, 3D FEA, scripting, and machine learning (even winning the crossover competition for plastic detection project) I even used and calibrated an advanced constitutive model. It has been a great pleasure to learn all of this while being at Seaway7 for a year during my Master's studies. Two values I always keep with me are living a life without regret and either go big or go home (though I've learned that sometimes we need to start small to go big, much like in centrifuge testing). I can confidently say that with what I've done and learned so far in this Master's program, I have no regrets at all, and I am proud of how far I've come.

In my honest opinion, I think the scope of this thesis is very large, with a huge emphasis on finite element analysis, in addition to centrifuge testing, and, who knew, at the end, I also needed to do and learn the calibration of an advanced constitutive model. There were moments when I thought this could be split into a few theses. However, I was fortunate to receive help from many people along the way. I extend my heartfelt thanks to Pietro Marveggio and Matteo Corigliano (the IDEA League is Pietro's project, and Matteo, a PhD candidate from Politecnico di Milano, is also helping on this project) for allowing me to participate and sharing data with me. I would also like to express my gratitude to Miguel and Vagelis for their feedback and willingness to supervise this thesis.

I am also grateful to my friends for their continuous 'support'. I would also like to thank my family for always supporting my decisions and having faith in me — papa, mama, and both of my sisters, Linda and Florence.

*Henry Gunawan
Delft, July 2024*

Abstract

The early 20th century marked a period of significant infrastructural development in Europe, resulting in the construction of numerous bridges and structures that have lasted to the present day. Many of these foundations, often designed as piled rafts but analysed as pile groups, were constructed using simplistic methodologies. These methods neglected the interactions between the raft, soil, and piles. Despite advancements in geotechnical engineering, current standards like Eurocode 7 still rely on this outdated approach, potentially leading to unnecessarily conservative and expensive retrofitting solutions.

Recent research highlights the significant contribution of rafts to the overall bearing capacity of foundations, suggesting that current design standards may underestimate the capacity provided by rafts. This thesis investigates the behaviour of piled raft foundations compared to pile groups and rafts, utilizing both 3D finite element analysis with the SANISAND-MS constitutive model and centrifuge testing. Furthermore, it explores the feasibility of using a viscoelastic model to capture the cyclic behaviour of piled raft foundations. The study focuses on dry sand conditions and quasi-static cyclic loading to address questions regarding load-displacement behaviour, load and moment distribution, internal forces on piles, and the modelling of cyclic loading conditions.

The findings demonstrate the superior performance of piled rafts in terms of stiffness and resistance compared to pile groups and rafts. Specifically, the vertical resistance ratio ($\beta_{PR,V}$) and combined resistance ratio ($\xi_{PR,V}$) indicate that accounting for the raft nearly doubles the vertical resistance of the piled raft foundation and aligns closely with the combined resistance of the pile group and raft as separate foundations. In lateral loading, the horizontal load ratio ($\beta_{PR,H}$) and moment resistance ratio ($\beta_{PR,M}$) suggest that the piled raft requires more horizontal load and moment to achieve the same displacement and rotation compared to the pile group alone. The raft contributes approximately 45% of the vertical load and 50% of the moment, reducing the axial load on individual piles by around 40%.

Analysis of internal forces reveals that shaft resistance is relatively small. Larger moments are observed in edge piles under axial loading, indicating the deformability of the raft and suggesting a likelihood of failure in the edge piles due to moments. Under lateral loading, the tension piles quickly reach maximum tensile resistance, with moments continuing to increase, highlighting a high probability of failure in the edge piles due to moments.

The study also demonstrates that modelling the elastoplastic cyclic behaviour of piled rafts using a viscoelastic model with frequency-independent viscosity is feasible. While not fully capturing all aspects of foundation behaviour, this approach adequately replicates stiffness and damping characteristics, making it useful for seismic design using a pseudostatic approach.

These findings offer potential for more efficient foundation designs, improved retrofitting strategies for existing structures, and increased understanding of the foundation behaviour of piled raft systems.

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Nomenclature

Abbreviations

Abbreviation	Definition
CPRF	Combined Piled Raft Foundation
FE	Finite Element
FEA	Finite Element Analysis
PG	Pile Group
PR	Piled Raft
R	Raft

Symbols

Symbol	Definition	Unit
A	Area	$[m^2]$
A_b	Base area	$[m^2]$
A_s	Shaft area	$[m^2]$
c'	Effective cohesion	$[kPa]$
c_u	Undrained cohesion	$[kPa]$
D	Diameter	$[m]$
D_{in}	Inner Diameter	$[m]$
D_{out}	Outer Diameter	$[m]$
D_r	Relative density	$[-]$
E	Young's modulus	$[kPa]$
EA	Axial stiffness	$[kN]$
EI	Flexural stiffness	$[kNm^2]$
$E_{oed,i}$	Interface Oedometer modulus	$[kPa]$
G	Shear modulus	$[kPa]$
G_0, G_{max}	Initial shear modulus	$[kPa]$
G_i	Interface shear modulus	$[kPa]$
G_s	Specific gravity	$[-]$
G_{soil}	Soil shear modulus	$[kPa]$
GF	Gauge Factor	$[-]$
H	Horizontal Load	$[kN]$
L	Length	$[m]$
M	Moment	$[kNm]$
N	Acceleration factor	$[-]$
R	Radius	$[m]$
R_{inter}	Interface reduction factor	$[-]$
U	Revolution per minute	$[RPM]$
V	Volume	$[m^3]$
V_{in}	Input Voltage	$[V]$
V_{out}	Output Voltage	$[V]$
W	Weight	$[N]$
e_0	Initial void ratio	$[-]$
e_{max}	Maximum void ratio	$[-]$
e_{min}	Minimum void ratio	$[-]$

Symbol	Definition	Unit
h	Height	$[m]$
u	Displacement	$[m]$
u_x, u_y, u_z	$x/y/z$ -axis displacement	$[m]$
α_{CPRF}	Piled raft coefficient	$[-]$
β_{PR}	Piled raft over pile group resistance	$[-]$
γ	Soil unit weight	$[-]$
ν	Poisson's ratio	$[-]$
ω	Angular velocity	$[rad/s]$
ϕ	Friction angle	$[^\circ]$
ρ	Density	$[kg/m^3]$
θ	Rotation	$[^\circ]$
ε	Soil strain	$[-]$
ε_q	Deviatoric strain	$[-]$
ε_v	Volumetric strain	$[-]$
ξ_{PR}	Piled raft over pile group raft and raft resistance	$[-]$

Introduction

1.1. Background

The early of 20th century marked a period of massive infrastructural development across Europe, with the construction of numerous bridges and structures that have lasted to this day. A report from the Joint Research Center of the EU [19] reveals that more than 1,234 km of road bridges, each over 100 meters long, are located along the major European transport corridors within the Trans-European Transport Network. However, many of these structures have now reached the end of their design life.

During that period, design methodologies were relatively simplistic due to the limited understanding of complex soil mechanics and the lack of advanced design techniques compared to current practices. Many foundations were constructed as piled rafts; however, they were designed and analysed as pile groups using simplified analytical methods. These methods neglected the complex interactions between the raft and the soil, as well as between the raft and the piles. Figure 1.1 illustrates the differences between a raft, a pile group, and a piled raft. Visually, a pile group is very similar to a piled raft, but Figure 1.2 demonstrates that the interactions between these foundations are entirely different.

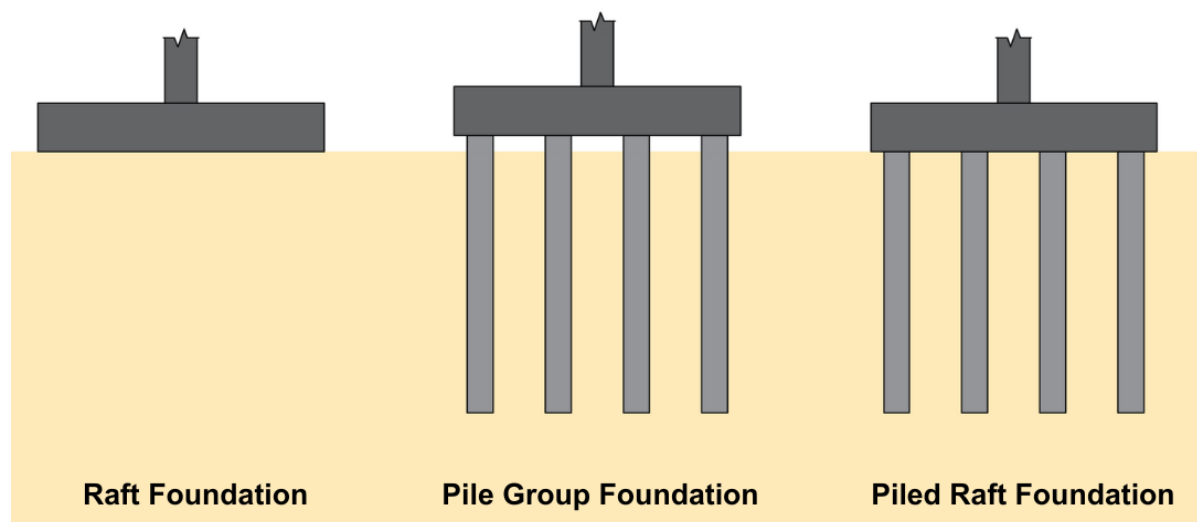


Figure 1.1: Types of foundations

Despite the advancements today, this simplified approach is still implemented in current standards such as Eurocode 7. In this standard, the contribution of the raft to bearing capacity is neglected and the moment capacity of the foundation is calculated based on the assumption that the most heavily loaded pile alone reaches its axial bearing capacity [1].

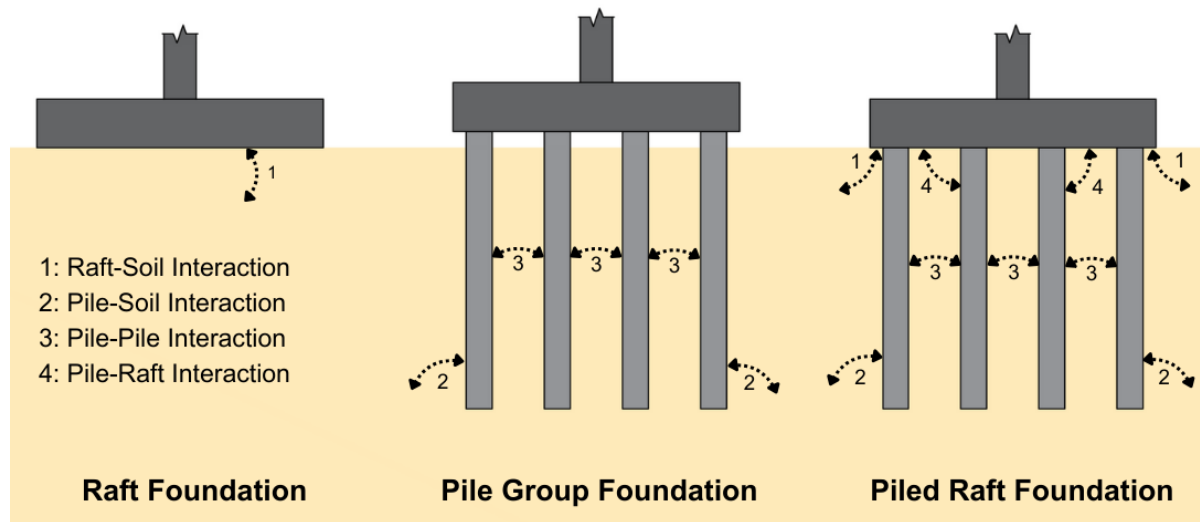


Figure 1.2: Foundation interactions

Recent research by Sakellariadis and Anastasopoulos in 2022, through numerical and experimental analyses, show the raft contribution is significant [49]. This finding is also supported by studies conducted by Brand *et al.* (1972), Cooke (1986), Conte *et al.* (2003), Randolph (1994), Katzenbach (2000), Poulos (2001), and others. These studies suggest that current design standards may be overly conservative by neglecting the raft's contribution and the system's ability to redistribute loads, leading to unnecessarily expensive retrofitting. Besides of axial loading, structures are also subjected to lateral and moment loading, while most of these research were only focused on the axial loading.

Another concern is the seismic design of the piled raft foundation. Using a pseudo-static approach, the applied acceleration a will be calculated based on a simplified response spectrum according to Eurocode 8 [2]. The value of the applied acceleration depends on the natural period (T_n) and a correction factor (η), which accounts for the equivalent dissipation of the system, as illustrated in Figure 1.3.

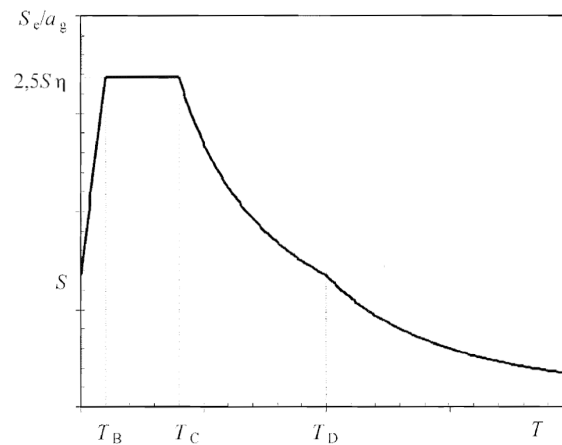


Figure 1.3: Simplified response spectrum according to Eurocode 8 [2]

In a rigid structure, the natural period is determined by the mass and stiffness of the structure. However, by considering soil deformability and dissipation, the system can achieve a lower maximum acceleration and a longer natural period, resulting in a more realistic and lower applied acceleration. Therefore, a simplified model that captures these responses is necessary.

A promising approach is to use a viscoelastic model, which incorporates both stiffness and viscosity parameters, to represent the soil deformability and dissipation, thereby representing a realistic behaviour of the system. If

successful, this model would provide a simplified and accurate prediction of the foundation's seismic response, validating its feasibility for use in seismic design of piled raft foundation.

1.2. Research Motivation and Objectives

Piled raft foundations offer significant advantages in supporting structures in challenging soil conditions, potentially outperforming traditional pile groups or raft foundations. This research aims to provide a comprehensive understanding of piled raft behaviour, focusing on their performance relative to other foundation types and their practical implications in geotechnical engineering.

The primary objective of this thesis is to:

1. Analyse and compare the behaviour of piled raft (PR) foundations to pile group (PG) and raft (R) foundations under various loading conditions.

Additionally, to broaden the scope and practical applicability of the research, a secondary objective is to:

2. Investigate the feasibility of modelling the cyclic behaviour of piled raft foundations using an equivalent linear viscoelastic model.

To maintain focus, this study concentrates on foundations in dry sand, using a single foundation type and quasi-static cyclic loading for cyclic behaviour assessment.

1.3. Research Questions

To address these objectives, this thesis explores the following research questions:

1. How does the load-displacement behaviour of piled rafts compare to pile groups and rafts under both vertical and lateral loading?
2. How are the loads and moments distributed between the raft and piles in piled raft foundations under various displacement and rotation conditions?
3. What are the critical failure modes and internal force distributions in piled raft foundations under different loading scenarios?
4. To what extent can an equivalent linear viscoelastic model capture the cyclic behaviour of piled raft foundations, and what are its limitations?
5. What are the practical implications of these findings for geotechnical engineering, particularly in terms of:
 - Assessment and potential retrofitting of existing pile foundations?
 - Design optimisation and cost-effectiveness for new constructions?
 - Overall performance enhancement of foundation systems?

By addressing these questions, this research aims to provide valuable insights for both the academic understanding of piled raft behaviour and the engineering practices in foundation design and assessment.

1.4. Research Outline

This thesis is structured as follows:

Chapter 2: Literature Review

Chapter 2 discusses the Eurocode 7 assessment on piled raft foundations, the concept of raft foundations, pile foundations, pile group foundations, piled raft foundations, and the cyclic behaviour of sand.

Chapter 3: Methodology

Chapter 3 discusses the constitutive model used in the finite element analysis and the fundamental of centrifuge testing.

Chapter 4: Reference Case

Chapter 4 presents the investigated reference case, providing an overview of the type of analysis conducted, the loading conditions applied, the soil characteristics, the foundation details, and the domain description.

Chapter 5: Finite Element Analysis

Chapter 5 discusses the finite element analysis conducted in this study. This includes the setup and results of the 3D finite element analysis using the SANISAND-MS constitutive model.

Chapter 6: Experimental Analysis

Chapter 6 focuses on the experimental analysis, detailing the centrifuge testing setup, results, and the calibration of parameters used to match the experimental findings.

Chapter 7: Conclusions and Recommendations

Chapter 7 presents the conclusions and limitations of the research and provides recommendations for future work.

2

Literature Review

This chapter provides an overview of relevant concepts related to the research motivation of this thesis. It begins with a discussion of the assessment by Eurocode 7 [1], followed by the concepts of raft foundation, pile foundation, pile group foundation, piled raft foundation, and the cyclic behaviour of sand.

2.1. Piled Raft Foundation Assessment by Eurocode 7

Eurocode 7 is the primary design standard for geotechnical engineering in Europe. However, it lacks specific guidelines for the assessment of piled raft foundations. This section is divided into two parts: assessment related to piled rafts and limiting values.

2.1.1. Piled Raft

There is no dedicated section in Eurocode 7 addressing the assessment of piled raft foundations; it is mentioned only briefly. In Section 7.1 (2), which covers general pile foundations, it states:

”The provisions of this Section should not be applied directly to the design of piles that are intended as settlement reducers, such as in some piled raft foundations.”

Additional mentions indirectly related to piled raft foundations can be found in other sections. For instance, Section 7.4.2 (1), which discusses design considerations for pile foundations, states:

”The behaviour of individual piles and pile groups and the stiffness and strength of the structure connecting the piles shall be considered.”

Furthermore, Section 7.6.2.1, which related to compressive ground resistance, includes the following points:

(5) The stiffness and strength of the structure connecting the piles in the group shall be considered when deriving the design resistance of the foundation.

(6) If the piles support a stiff structure, advantage may be taken of the ability of the structure to redistribute load between the piles. A limit state will occur only if a significant number of piles fail together; therefore, a failure mode involving only one pile need not be considered.

Beyond these references, Eurocode 7 does not provide further guidance on piled raft foundations.

2.1.2. Limiting Values

Settlement Limiting Value

Eurocode 7 Section 7.6.1.1(3) provides guidance on defining the ultimate limit state for piles in compression:

“For piles in compression, it is often difficult to define an ultimate limit state from a load-settlement plot showing a continuous curvature. In these cases, settlement of the pile top equal to 10% of the pile base diameter should be adopted as the ‘failure’ criterion.”

Rotation Limiting Value

According to Eurocode 7 Annex H (2):

“The maximum acceptable relative rotations for open framed structures, in filled frames and load bearing or continuous brick walls are unlikely to be the same but are likely to range from about 1/2000 to about 1/300, to prevent the occurrence of a serviceability limit state in the structure. A maximum relative rotation of 1/500 is acceptable for many structures. The relative rotation likely to cause an ultimate limit state is about 1/150.”

2.2. Raft Foundation

2.2.1. Ultimate Limit State

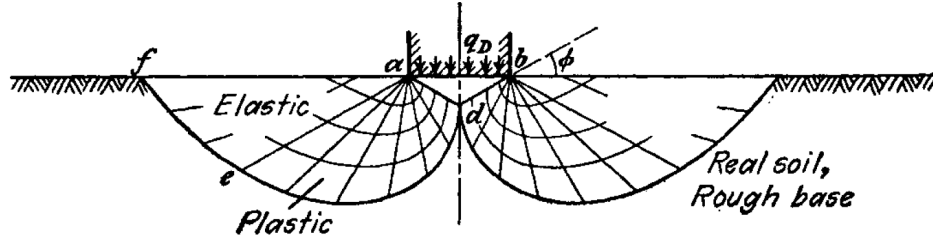


Figure 2.1: Boundaries of zone of plastic flow after failure of earth support (After Terzaghi, 1943 [51])

Raft foundation is a type of shallow foundation that has been used for centuries. However, the ultimate bearing capacity theory started when Terzaghi in 1943 [51] extended the plastic failure theory of Prandtl (1920) to evaluate the bearing capacity of soils for shallow strip footings, as shown in Figure 2.1. The way Terzaghi formulated it is by using the soil properties of cohesion, friction angle, and unit weight, then the force equilibrium and earth pressure coefficient then formulate the bearing capacity equation as:

$$q_u = q_c + q_q + q_\gamma = c'N_c + qN_q + \frac{1}{2}\gamma BN_\gamma \quad (2.1)$$

This formula, later known as Terzaghi's bearing-capacity equation. The coefficients N_c , N_q , and N_γ are called the bearing capacity factors which Terzaghi derived from passive earth pressure, publication by Prandtl (1920) and Reissner (1924) with designed values of:

- $N_q = \frac{(e^{(\frac{3}{4}\pi - \frac{\phi}{2}) \tan \phi})^2}{2 \cos^2(45^\circ + \frac{\phi}{2})}$
- $N_c = \cot \phi (N_q - 1)$
- $N_\gamma = \frac{1}{2} \tan \phi (\frac{K_P \gamma}{\cos^2 \phi} - 1)$

Later in Europe, Brinch Hansen in 1970 [18] extended the work of Terzaghi by including the inclination of the foundation base, shape of foundation, and inclination of the load parameters which are not considered in the Terzaghi method and then adapted and modified by Eurocode 7 becomes:

$$q_u = c'N_c b_c s_c i_c + q'N_q b_q s_q i_q + \frac{1}{2}\gamma' B' N_\gamma b_\gamma s_\gamma i_\gamma \quad (2.2)$$

with the design values of dimensionless factors shown in Table 2.1.

Category	Factor	Formula
Foundation Base Inclination	$b_q = b_\gamma$	$(1 - \alpha \tan \phi')^2$
	b_c	$b_q - \frac{1-b_q}{N_c \tan \phi'}$
Foundation Shape	Rectangular Shape	
	s_q	$1 + \frac{B'}{L'} \sin \phi'$
	s_γ	$1 - 0.3 \frac{B'}{L'}$
	s_c	$\frac{s_q N_q - 1}{N_q - 1}$
	Square/Circular Shape	
	s_q	$1 + \sin \phi'$
	s_γ	0.7
	s_c	$\frac{s_q N_q - 1}{N_q - 1}$
Load Inclination	i_c	$i_q - \frac{1-i_q}{N_c \tan \phi'}$
	i_q	$[1 - \frac{H}{V + A' c' \cot \phi'}]^m$
	i_γ	$[1 - \frac{H}{V + A' c' \cot \phi'}]^{m+1}$

Table 2.1: Eurocode 7 bearing capacity factor

2.3. Pile Foundation

In contrast to shallow foundations, deep foundations are designed to transfer load to the deeper soil layer. Among various type of deep foundation, the most common type is pile foundations.

2.3.1. Axial Capacity

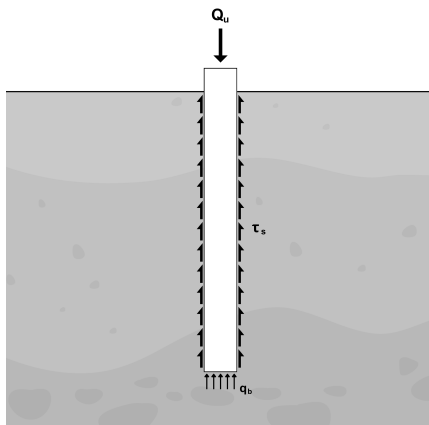


Figure 2.2: Axial loaded pile

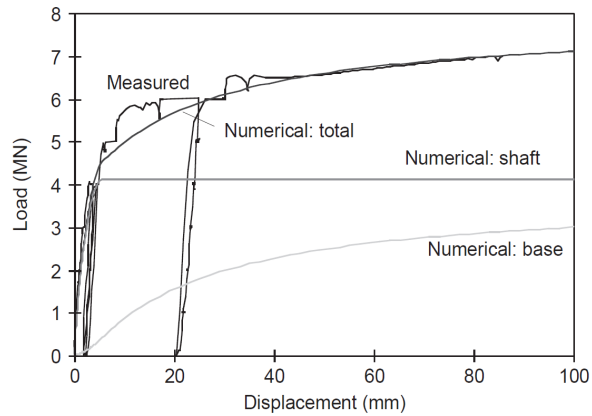


Figure 2.3: Single pile load displacement response (After Piling Engineering, 2009 [15])

A pile subjected to load parallel to its axis will carry the load by shear generated along the shaft and normal stress at the base of the pile. Consequently, the ultimate capacity, Q_u , under axial load is the sum of the base capacity, Q_b , with the shaft capacity, Q_s .

$$Q_u = Q_b + Q_s = A_b q_b + A_s \tau_s \quad (2.3)$$

The capacity of the shaft and base depends on the area of the pile base (A_b), the area of the pile shaft (A_s), and the soil characteristics, which affect the end-bearing pressure (q_b) and shear stress along the pile shaft (τ_s). Typically, the shaft capacity is mobilized at much smaller displacements (typically 0.5 to 2% of the pile diameter) compare to the base capacity, which may require displacements as large as 5 to 10% of the diameter in order to be fully mobilized [15]. To illustrate this, Figure 2.3 shows results from a load test on a bored pile, 0.8 m diameter with 20 m long. The measured response has been simulated numerically, showing the individual contributions of the pile's shaft and base alongside the total response. The shaft capacity of 4.1 MN is mobilized at a displacement of less than 6 mm (0.75% of the pile diameter), while the base response is still rising for displacements in excess of 80 mm (10% of the pile diameter).

At a displacement of 10% of its diameter, the pile's total capacity reaches 7 MN, with the shaft contributing 60% of this capacity. However, at a working load of 3.5 MN, it is shown that the shaft bears 95% of the load, remaining only 5% to the base. Once the capacity of the pile shaft is fully mobilized (at 4.1 MN), the overall "stiffness" of the pile-soil interaction decreases to that of the base alone, leading to a significant increase in displacement.

The above example illustrates a crucial aspect of pile design, with the needs to accurately estimate the value for shaft friction τ_s and end-bearing resistance q_b to calculate the shaft and base capacities.

Design Methods

The most popular design methods for estimating the axial capacity of sand, such as API RP2A approach, are based on conventional earth pressure theory. The first edition of the API RP2A design method for the axial capacity of siliceous sands was proposed in 1969 [3] and is still being used worldwide with updated editions. The 21st edition [4] for estimating pile axial capacity in siliceous sand uses the form:

$$\begin{aligned} q_b &= N_q \sigma'_{v0} \leq q_{b,lim} \\ \tau_s &= K_f \sigma'_{v0} \tan \delta = \beta_s \sigma'_{v0} \leq \tau_{s,lim} \end{aligned} \quad (2.4)$$

With the value of N_q and β_s suggested by API is shown in Table 2.2.

Soil Condition	Relative Density	End Bearing Factor, N_q	$q_{b,lim}$ (MPa)	$\beta_s = K_f \tan \delta$	$\tau_{s,lim}$ (kPa)
Med-Dense	$D_r = 35-65\%$	20	5	0.37	81
Dense	$D_r = 65-85\%$	40	10	0.46	96
Very Dense	$D_r = 85-100\%$	50	12	0.56	115

Table 2.2: Axial pile design value (After API, 2007 [4])

These approaches are fundamentally empirical, and recent research has demonstrated the limitation that API method has a tendency to overestimate the shaft capacity of long piles in loose sand, while underestimating the capacity of short piles in dense soils [29]. This concern leads to development of design methodologies that rely on cone penetration testing, which offers more reliable predictions due to the similarities between the cone and an axially loaded pile. However, some adjustments are necessary due to the scale effects and the limited displacements for pile design. To address these scale effects, an appropriate averaging method for the cone resistance is required.

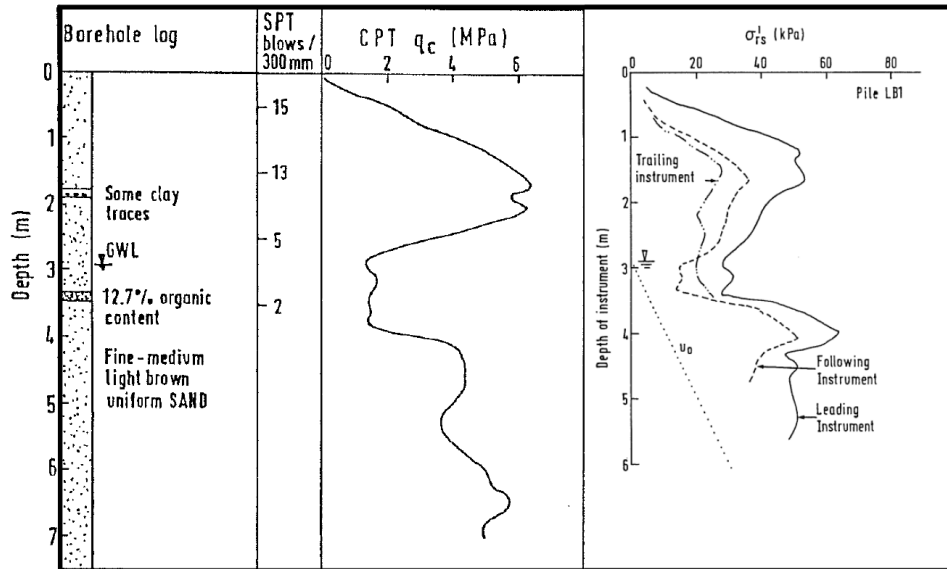


Figure 2.4: CPT q_c and stationary effective radial stresses during installation of ICP at Labenne (after Lehane, 1992 [28])

Nordlund (1963) [40] suggested calculating an average cone resistance (q_c) over a range from three pile diameters above to two diameters below the pile's base. Fleming and Thorburn (1983) [16] recommend a more detailed

method to this averaging, in order to put more weight on the minimum values and extending the averaging range from two diameters below to eight diameters above the base. On the other hand, the Dutch method proposed by Van Mierloo and Koppejan (1952) [52] advise for a conservative average to determine the representative q_c value through a conservative average between at least $0.7D$ and maximum $4D$ below the pile tip, and $8D$ above the pile tip.

To account for the limited displacements in pile design, it is recommended to reduce the value of cone resistance. Based on a serviceability displacement limit of 10% of the pile diameter, the ratio of q_b/q_c range from 0.2, as recommended by Jardine and Chow in 1996 [23], to 0.5 as recommended by Kraft in 1990 [31].

Method	Base Resistance Design Equation	Shaft Resistance Design Equation
Fugro-05	$\frac{q_{b0.1}}{q_{c,avg}} = 8.5 \left(\frac{p_a}{q_{c,avg}} \right)^{0.5} A_r^{0.25}$	Compression loading: $\tau_f = 0.08q_c \left(\frac{\sigma'_{v0}}{p_{ref}} \right)^{0.05} \left(\frac{h}{R^*} \right)^{0.9}$ for $\frac{h}{R^*} \geq 4$ $\tau_f = 0.08q_c \left(\frac{\sigma'_{v0}}{p_{ref}} \right)^{0.05} 4^{-0.9} \left(\frac{h}{4R^*} \right)$ for $\frac{h}{R^*} \leq 4$ Tension loading: $\tau_f = 0.08q_c \left(\frac{\sigma'_{v0}}{p_{ref}} \right)^{0.15} [\max(\frac{h}{R^*}, 4)]^{-0.85}$
ICP-05	if $D_i \geq 2.0(D_r - 0.3)$ or $D_i \geq 0.083 \frac{q_{c,avg}}{p_a} D_{CPT}$, pile is “unplugged”: $\frac{q_{b0.1}}{q_{c,avg}} = A_r$. If not, pile is “plugged”: $\frac{q_{b0.1}}{q_{c,avg}} = \max[0.5 - 0.25 \log(\frac{D}{D_{CPT}}), 0.15, A_r]$	$\tau_f = a[0.029bq_c \left(\frac{\sigma'_{v0}}{p_{ref}} \right)^{0.13}$ $[\max(\frac{h}{R^*}, 8)]^{-0.38} + \Delta\sigma'_{rd}] \tan \delta_f$ open-ended pile in tension: $a = 0.9$, other: $a = 1$ tension pile: $b = 0.8$, compression pile: $b = 1$
NGI-05	$q_{b0.1} = \min[\text{plugged } q_{b0.1}, \text{unplugged } q_{b0.1}]$ Plugged: $q_{b0.1}/q_{c,tip} = F_{Dr} = 0.7/(1 + 3Dr^2)$ Unplugged: $q_{b0.1} = q_{b,ann} A_r + q_{b,plug}(1 - A_r)$; $q_{b,ann} = q_{c,tip}$ and $q_{b,plug} = 12\tau_{f,avg} L/D_i$	$\tau_f = \frac{z}{L} p_{ref} F_{Dr} F_{sig} F_{tip} F_{load} F_{mat} \geq \tau_{min}$ $F_{Dr} = 2.1(D_r - 0.1)^{1.7}$ $D_r = 0.4 \ln(\frac{q_{c1N}}{22})$ $F_{sig} = \left(\frac{\sigma'_{v0}}{p_{ref}} \right)^{-0.25}$ $F_{tip} = \text{driven open-ended: } 1, \text{ close-ended: } 1.6$ $F_{load} = 1$ for tension, 1.3 for compression $F_{mat} = 1$ for steel, 1.2 for concrete $\tau_{min} = 0.1\sigma'_{v0}$
UWA-05	$\frac{q_{b0.1}}{q_{c,avg}} = 0.15 + 0.45A_{r,eff}$	$\tau_f = \frac{f_t}{f_c} [0.03q_c A_{r,eff}^{0.3}$ $[\max(\frac{h}{D}, 2)]^{-0.5} + \Delta\sigma'_{rd}] \tan \delta_f$ $A_{r,eff} = 1 - IFR(\frac{D_i}{D})^2$ $IFR = \frac{\Delta L_p}{\Delta z}$ $\frac{f_t}{f_c} = 1$ for compression, 0.75 for tension
Note	$p_a = 100 \text{ kPa}$; $D_{CPT} = 36\text{mm}$; $A_r = 1 - (\frac{D_i}{D})^2$ $A_{r,eff} = 1 - \text{FFR}(\frac{D_i}{D})^2$ $\text{FFR} = IFR(\frac{\Delta L_p}{\Delta z})$ over final 3D penetration $\text{FFR} = \min[1, (\frac{D_i(m)}{1.5})^{0.2}]$ if not measured $D_r = 0.4 \ln[\frac{q_{c,tip}}{22(p_a \sigma'_{v0})^{0.5}}]$ $q_{c,avg} = q_c$ averaged $\pm 1.5D$ over pile tip for Fugro-05 & ICP-05 $q_{c,avg} = q_c$ averaged with Dutch technique for UWA-05	$\tau_f = \text{local ultimate shaft friction}$ $\delta_f = \text{interface friction angle}$ $p_{ref} = 100\text{kPa}$; $R^* = (R^2 - R_i^2)^{0.5}$ $\Delta\sigma'_{rd} \approx \frac{4G}{D}$ $G = 185q_c q_{c1N}^{-0.7}$ (operational level shear modulus) $q_{c1N} = \frac{q_c/p_a}{(\sigma'_{v0}/p_a)^{0.5}}$

Table 2.3: Axial pile design method (Modified After Xu, 2007 [56])

The focus on averaging methods and adjustment due to displacements concern primarily to determine the base resistance. Regarding the shaft resistance, the mechanism governing shaft friction has been studied by Lehane in 1992 [28], using the Imperial College Pile, which was designed to measure the radial effective stresses while simultaneously measuring the shear stresses at different locations along the pile shaft, as shown in Figure 2.4. These experiments demonstrated for the first time that the local radial effective stresses increased and decreased synchronously with cone resistance (q_c), indicating the critical role of sand state in mobilizing shaft capacity. Another important finding from these experiments was the friction fatigue effect, showing the equalized radial effective stress σ'_{rc} is dependent to the pile tip relative distance (h/D). These findings led Lehane and Jardine in 1994 [30] to formulate an equation for estimating the equalized radial effective stress for a closed-ended displacement pile as follows:

$$\sigma' = a q_c (h/R)^c = 0.024 q_c (h/R)^{-0.33} \quad (2.5)$$

The great confidence level of the CPT approach and these findings, led various research institution to conduct parallel studies. these led to the development of other CPT-based design methodologies, such as Fugro-05 [27], ICP-05 [24], NGI-05 [8], and UWA-05[8]. These methods have all been included in the commentary of the 21st edition of the API design guidelines [4] and shown in Table 2.3.

2.3.2. Lateral Capacity

Piled foundations are commonly subjected to horizontal loading, such as wind, waves, and seismic activities. When a pile is subjected to lateral loading, normal stresses increase on the side facing the load and decrease on the opposite side. This loading causes soil displacements that are radially outward in front of the pile and radial inward behind it. Eventually, near the ground surface, a gap may open up between the soil and the back of the pile, with soil failure in front of the pile in a wedge-shaped mechanism, as shown in Figure 2.5.

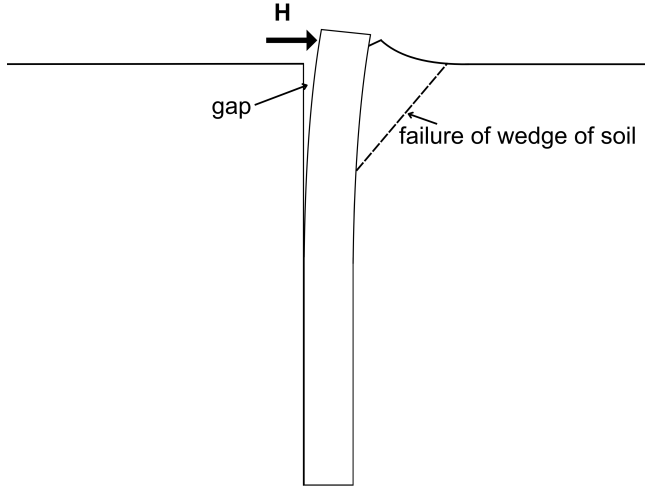


Figure 2.5: Displacement of pile under lateral load

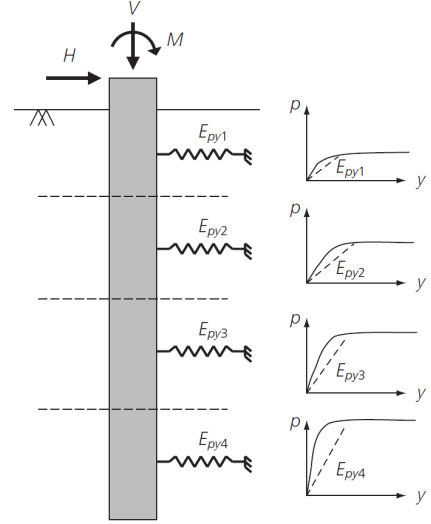


Figure 2.6: Winkler p - y model for lateral loading
(After Doherty and Gavin, 2011 [14])

The common method for analysing laterally loaded piles are based on the Winkler foundation model, commonly known as the p - y method. This method assumes the pile as a beam on an elastic foundation, supported by a series of discrete springs representing the soil reaction. These springs, as illustrated in Figure 2.6, can be characterized by a linear or non-linear response, which correlates the soil reaction force p at various depths to the lateral displacement y . The spring stiffness E_{py} is defined by the secant modulus of the p - y curve at any given point.

The concept of Winkler foundation was first introduced in 1867 [54], while the p - y model itself was proposed by Reese and Matlock in 1956 [48]. Matlock's experimental research in 1970 [39] further validated the model by demonstrating that the soil resistance at any depth along the pile is independent of the deflections at other depths, supporting the assumption of uncoupled springs. The first empirical p - y curves, specifically for cohesionless soils, were formulated by Reese *et al.* in 1974 [47], based on lateral load testing of two identically instrumented piles at Mustang Island, Texas.

Analytical solutions are feasible when assuming a constant or linearly increasing spring stiffness k along the pile's length. For a pile with a specified bending rigidity, EI_p , embedded in soil with a uniform subgrade reaction coefficient k , there exists a critical length beyond which the pile's response similar to an infinitely long pile, given by:

$$L_c = 4 \left[\frac{EI_p}{k} \right]^{1/4} \quad (2.6)$$

Therefore, in the case of long and slender piles, the effect of loads applied at their tops reduced with depth. For such piles, the ground-level deflection u and rotation θ , resulting from these loads, can be described by the

equations:

$$\begin{aligned} u &= \sqrt{2} \frac{H}{k} \left(\frac{L_c}{4} \right)^{-1} + \frac{M}{k} \left(\frac{L_c}{4} \right)^{-2} \\ \theta &= \frac{H}{k} \left(\frac{L_c}{4} \right)^{-2} + \sqrt{2} \frac{M}{k} \left(\frac{L_c}{4} \right)^{-3} \end{aligned} \quad (2.7)$$

For scenarios where the subgrade reaction coefficient increases linearly with depth, expressed as $k = nz$, similar expressions for the critical length and the deformation responses due to load can be derived. The critical length, L_c , for a linearly increasing subgrade reaction is calculated as:

$$L_c = 4 \left[\frac{EI_p}{n} \right]^{1/5} \quad (2.8)$$

And the associated ground-level displacements are:

$$\begin{aligned} u &= 2.43 \frac{H}{n} \left(\frac{L_c}{4} \right)^{-2} + 1.62 \frac{M}{n} \left(\frac{L_c}{4} \right)^{-3} \\ \theta &= 1.62 \frac{H}{n} \left(\frac{L_c}{4} \right)^{-3} + 1.73 \frac{M}{2} \left(\frac{L_c}{4} \right)^{-4} \end{aligned} \quad (2.9)$$

Advancements in computing now enable the adoption of non-linear spring models to represent soil behaviour more accurately, moving beyond a singular coefficient of subgrade reaction toward a complete load transfer curve. The application of finite difference models to simulate pile bending further allows a complete load deflection curve to be obtained, especially under circumstances where cyclic loading is a concern, the analysis of load transfer curves becomes critically important for design.

2.4. Pile Group Foundation

Pile foundations usually consist not of a single pile but a group of piles where the structural load is transferred to the raft, which then distributes the load across the piles. The efficiency of a single pile is generally affected by the presence of other piles.

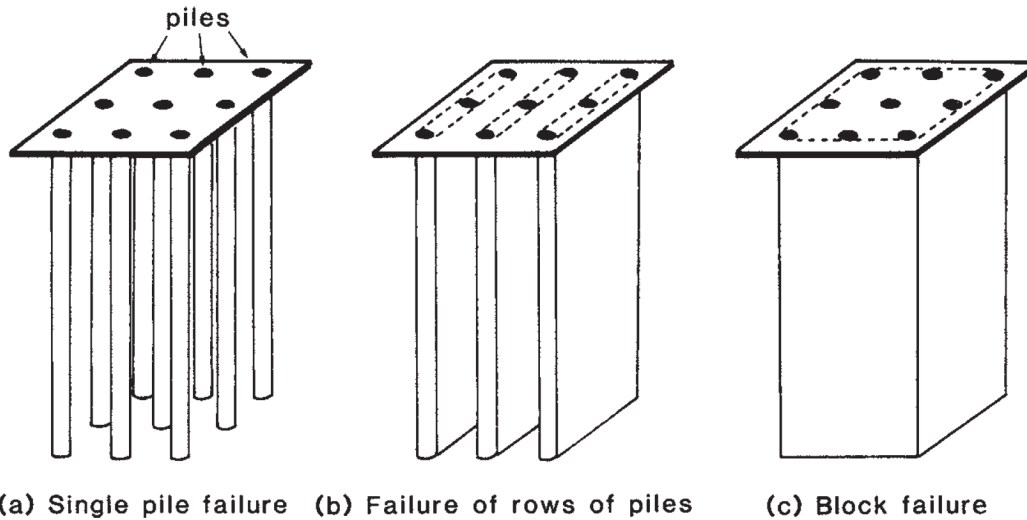


Figure 2.7: Block failure of pile groups (After Piling Engineering, 2009 [15])

In this foundation, failure can occur either from the failure of individual piles or the block. Block failure may arise when piles are positioned too closely, significantly decreased their capacity compared to an isolated pile. Whether under axial or lateral loads, block failure involves soil movement between the piles, leading to failure planes, as illustrated in Figure 2.7.

2.4.1. Axial Capacity

The axial capacity of a pile group ($Q_{u,G}$) is typically calculated as the product of the single pile's capacity (as discussed in Chapter 2.3), the number of piles (n) in the group and an efficiency coefficient η :

$$Q_{u,G} = \eta n Q_u \quad (2.10)$$

The proper value for efficiency coefficient is determined based on empirical evidence, primarily from small-scale natural gravity or centrifuge tests. The Converse-Labarre formula offers a simplified method to estimate efficiency:

$$\eta = 1 - \frac{\arctan(\frac{s}{D})[(f-1)g + (g-1)f]}{\frac{\pi}{2}fg} \quad (2.11)$$

where s represents the spacing distance between piles, while f and g represent the number of rows and columns of piles in the group.

In sandy soils, the block capacity of most pile groups, particularly for displacement piles due to the compaction effect during installation, will exceed the sum of individual pile capacities ($\eta \geq 1$). However, accurately quantifying this increased capacity is challenging, leading to the conservative design practice of assuming a pile group efficiency of 1 in sand.

2.4.2. Lateral Capacity

The lateral capacity of a pile group rarely becomes a critical design consideration, except in the case involving block failure (as shown in Figure 2.7(b)), where the load applies parallel to the individual blocks. This failure mode will govern when the shearing resistance of the soil between the piles is less than the limiting resistance of an isolated pile.

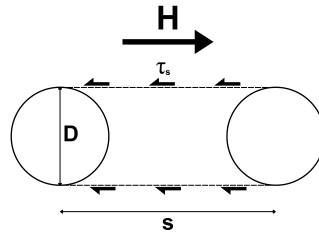


Figure 2.8: Block failure under lateral load

For non-cohesive soil, the limiting force per unit length on a single pile can be represented as $P_u = K_p^2 \sigma'_v D$ [15]. The limiting shear stress on the sides of the soil block presented in Figure 2.8 can be expressed as:

$$\tau_s = K \sigma'_v \tan \phi' \quad (2.12)$$

Here, the earth pressure coefficient K ranges between K_0 and K_p . Thus, block failure occurs at pile spacing when:

$$\frac{s}{D} < \frac{K_p^2}{2K \tan \phi'} \quad (2.13)$$

For most practical pile spacing, this condition is satisfied; therefore, block failure is not expected to occur, allowing the limiting force per unit length on piles behind another pile to be estimated by:

$$P_u = 2K \sigma'_v \tan \phi' s \quad (2.14)$$

The exact value for the coefficient K remains a subject of discussion, although an average value may be used as an approximation to account for the varying stress field between the piles.

2.4.3. Closed Form Analytical Solution

Recent studies by Di Laora (2019) [13] and Di Laora *et al.* (2022) [12] introduced a closed form analytical solutions to assess the load-bearing capacity of pile groups under combined loads, including vertical, lateral, and moment forces. These analytical solutions enable the construction of interaction diagram to predict the failure envelopes of pile groups. Iovino *et al.* (2021) [21] expanding the closed-form solutions through a numerical approach to define failure envelopes for pile groups under inclined and eccentric loads. However, it's crucial to note that the primary focus on these studies remains on pile groups, despite most foundation was constructed as piled rafts rather than as pile groups.

2.5. Piled Raft Foundation

The piled raft foundation represents a composite foundation system where the raft is designed to distribute the load both directly to the soil beneath and the piles. This type of foundation offers a higher load-bearing capacity and economic value, particularly when strong soil exists near to the surface.

Despite these advantages, the analysis of piled raft foundation is challenging due to the soil-structure interactions involved, including raft-soil, pile-soil, pile-pile, and pile-raft interactions. Moreover, Eurocode 7 does not provide specific guidance for the design of piled raft foundations, except for a general mention in Section 7.1 point (2) stating “The provision of this Section should not be applied directly to the design of piles that are intended as settlement reducers, such as in some piled raft foundations”.

The theoretical analyses of piled raft foundations was first discussed in the late 20th century by Poulos in 1968 [43], Butterfield and Banerjee in 1971 [6], Davis and Poulos in 1972 [11], and further advanced by Randolph in 1983 [45] and 1994 in “Design methods for pile groups and piled rafts” [46], the “Piled raft foundation projects in Germany” case study by Katzenbach *et al.* in 2000 [25], and “Piled raft foundations: design and applications” by Poulos in 2001 [44].

Randolph proposed a simple approach to assess the combined stiffness of a piled raft system by using an average interaction factors between pile and raft, α_{rp} . The α_{rp} is derived from the equation:

$$\begin{Bmatrix} w_p \\ w_r \end{Bmatrix} = \begin{bmatrix} \frac{1}{k_p} & \frac{\alpha_{pr}}{k_r} \\ \frac{\alpha_{rp}}{k_p} & \frac{1}{k_r} \end{bmatrix} \begin{Bmatrix} P_p \\ P_r \end{Bmatrix} \quad (2.15)$$

where the subscripts p represent the pile groups and r represents the raft, w denotes the settlement, P represents the loads, and k is the stiffness.

By applying the reciprocal theorem, which set the off-diagonal terms in the flexibility matrix to be equal, the relationship between the interaction factors is set as:

$$\frac{\alpha_{pr}}{\alpha_{rp}} = \frac{k_r}{k_p} \quad (2.16)$$

Given that piles and raft share the same average settlement, this expression allows the overall stiffness k_{pr} and the proportion of load carried by the raft to be calculated from:

$$\begin{aligned} k_{pr} &= \frac{k_p + (1 - 2\alpha_{rp})k_r}{1 - \alpha_{rp}^2 \frac{k_r}{k_p}} \\ \frac{P_r}{P_r + P_p} &= \frac{(1 - \alpha_{rp})k_r}{k_p + (1 - 2\alpha_{rp})k_r} \end{aligned} \quad (2.17)$$

The stiffness of the raft and pile group can be evaluated conventionally based on elastic theory. Further analyses by Clancy and Randolph in 1993 [7] show that as the group size increases, the value of α_{rp} tends toward a constant value of 0.8, independent of the pile spacing, slenderness ratio, or stiffness ratio. This leads to a simplification of Equation 2.17:

$$\begin{aligned} k_{pr} &= \left(\frac{1 - 0.6 \frac{k_r}{k_p}}{1 - 0.64 \frac{k_r}{k_p}} \right) k_p \\ \frac{P_r}{P_p} &= \frac{0.2}{1 - 0.8 \frac{k_r}{k_p}} \frac{k_r}{k_p} \end{aligned} \quad (2.18)$$

In contrast to Randolph, Katzenbach used a 3D finite element approach and defined a piled raft coefficient, α_{CPRF} , as the ratio between the sum of the resistance provided by the piles to the total resistance of the foundation:

$$\alpha_{CPRF} = \frac{\Sigma R_{pile}}{R_{tot}} \quad (2.19)$$

This coefficient enables a distinction between a pure pile group $\alpha_{CPRF} = 1$ and a spread foundation $\alpha_{CPRF} = 0$, illustrating a spectrum of foundation behaviour based on the load contribution between the piles and the raft. Furthermore, Katzenbach also illustrates a qualitative example on the dependency between the piled raft coefficient α_{CPRF} and the settlement, as shown in Figure 2.9. According to research by Katzenbach in 2002 [17], piled raft foundations has been built with a coefficient of α_{CPRF} between 0.3 and 0.9, with an optimal range for technical and economical considerations suggested being around 0.5 and 0.7.

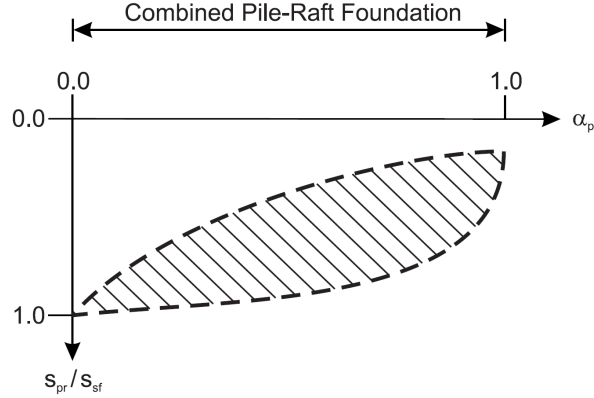


Figure 2.9: Qualitative example of a possible settlement reduction of a piled raft in function of the piled raft coefficient α_{pr} (After ISSMGE Combined Pile-Raft Foundation Guideline, 2013 [26])

To assess the vertical bearing capacity $Q_{PR,ult}$ of piled raft ($0 < \alpha_{CPRF} < 1$), Poulos in 2000 [42] suggested considering it as the smaller of two calculations: the ultimate capacity of the block Q_{BF} containing the piles plus the raft's portion outside the pile group, and the sum of the ultimate loads of the raft $Q_{R,ult}$ and piles $Q_{G,ult}$:

$$Q_{PR,ult} = Q_{UR,ult} + Q_{G,ult} \quad (2.20)$$

The expression above was also proposed by Liu *et al.* in 1985 [35]. However, by considering the interaction between the raft and piles, alongside the soil properties changes during installation, Liu *et al.* (1994) [36] and Borel (2001) [5] further refined Equation 2.20 to:

$$Q_{PR,ult} = \alpha_{UR}Q_{UR,ult} + \alpha_GQ_{G,ult} \quad (2.21)$$

Here, α_{UR} and α_G are coefficients affecting the failure load of the raft and the pile group, respectively, when combined as a piled raft. The initial proposal by Poulos corresponds to $\alpha_{UR} = \alpha_G = 1$, represents a scenario where the performance of the raft and piles are not changed by their interaction.

In 2001, Poulos [44] takes a step further in the paper of “Piled Raft Foundations: Design and Applications” by proposing a three-stage design process for piled raft foundations, starting with a preliminary assessment of the impact of pile numbers on load capacity and settlement based on the method proposed by Randolph, followed by a more detailed assessment of piling requirements (such as piles location and general characteristics), and concluding with a detailed design phase to obtain the optimum design of piles, detailed distribution of settlement and force distribution using several approaches, such as “strip on springs” approach, “plate on springs” approach, boundary element methods, method combining boundary element analysis for the pile and finite element analysis for the raft, simplified finite element analysis, or three-dimensional finite element analysis. Poulos also reviews the necessary geotechnical parameter assessment for the design process and presents several case studies, including the Westend 1 Tower in Frankfurt, a five-story building in Urawa, Japan, the Messe Turm Tower in Frankfurt, and other more.

Highlighting even more about the significance of the raft in piled raft foundations, Mandolini's papers in 2005 and 2006 documented the load distribution through both case histories and experimental tests in clay. The findings, detailed in Table 2.4 and 2.5 respectively, reveal that the raft's contribution to the overall load-bearing capacity varies significantly, ranging from 8% to 70%.

Case	Reference	Structure	s/d [-]	Ag/A [-]	raft load [%]	L/B [-]
1	Van Impe & De Clerq (1994)	Multispan bridge	3.8	0.70	27	1.00
2	Yamashita et al. (1993)	Building Urawa	8.0	0.90	51	0.64
3	Cooke et al. (1981)	Stonebridge park	3.6	0.90	23	0.65
4	Sommer et al. (1991)	Messe Turm	6.4	0.83	45	0.52
5	Joustra et al. (1977)	Apartment block	5.2	0.90	22	0.70
6	Hight & Green (1976)	Dashwood house	3.0	0.90	19	0.50
7	Jendeby (1986)	House 1	6.5	0.90	8	2.10
8	Jendeby (1986)	House 2	10.5	0.90	66	2.20
9	Jendeby (1986)	Uppsala house	11.2	0.90	64	2.20
10	Russo (1996)	Garigliano bridge	3.0	0.88	20	4.50
11	Katzenbach et al. (2000)	Messe Torhaus	3.5	0.80	20	1.14
12	Katzenbach et al. (2000)	Westend 1 – DG Bank	6.0	0.52	50	0.63
13	Katzenbach et al. (2000)	Japan Centre	5.5	0.45	60	0.60
14	Katzenbach et al. (2000)	Forum	6.0	0.55	62	0.70
15	Katzenbach et al. (2000)	Congress Centre	5.8	0.62	60	1.00
16	Katzenbach et al. (2000)	Main Tower	3.3	0.70	15	0.50
17	Katzenbach et al. (2000)	Eurotheum	5.2	0.55	70	0.80
18	Katzenbach et al. (2000)	Treptowers	6.5	0.86	52	0.38
19	Hooper (1979)	National Westminster Bank	3.8	0.91	29	0.50
20	Hooper (1979)	Hide Park Cavalry Barracks	4.3	0.72	39	0.90
21	Present report	Tank 12 Harbour Napoli	5.8	0.82	50	0.92
22	Present report	Tank 14 Harbour Napoli	5.0	0.82	46	1.10

Table 2.4: Case histories with observations of the load sharing (After Mandolini *et al.*, 2005 [38])

To quantitatively assess this contribution, Mandolini introduced two coefficients, β_{PR} and ξ_{PR} , defined as:

$$\beta_{pr} = \frac{Q_{PR,ult}}{Q_{G,ult}} \quad (2.22)$$

$$\xi_{pr} = \frac{Q_{PR,ult}}{Q_{UR,ult} + Q_{G,ult}} \quad (2.23)$$

These coefficients indicate that the raft's involvement increased the ultimate bearing capacity beyond that of pile groups alone, with β_{PR} consistently exceeding unity. Particularly in tests where larger settlements were observed, β_{PR} proved to be higher. Conversely, the coefficient ξ_{PR} ranging between 0.8 and 1, implies that the combined ultimate capacity of the piled raft is at least 80% of the total capacities of its components separately. Therefore, assuming the piled raft's capacity solely with the pile group appears overly conservative [50].

Test	Reference	L/d	s/d	n	B_R/d	β_{PR}	ξ_{PR}
1 g-model	Cooke (1986)	48	3	49	22	1.29	0.83
	Cooke (1986)	48	3	25	16	1.44	0.88
	Cooke (1986)	48	3	9	10	1.31	0.81
	Cooke (1986)	24	3	49	22	1.74	0.86
	Cooke (1986)	24	3	25	16	1.55	0.8
	Cooke (1986)	24	3	9	10	1.94	0.97
	Cooke (1986)	48	4	49	28	1.62	0.85
	Cooke (1986)	48	4	25	20	1.89	1
	Cooke (1986)	48	4	9	12	1.71	0.94
	Cooke (1986)	24	4	25	20	2.1	0.81
	Cooke (1986)	24	4	9	12	2.5	1.03
Centrifuge	Conte et al. (2003)	28.6	4	9	14.3	2.26	1
	Conte et al. (2003)	30.2	4	49	28.6	2.43	1
	Conte et al. (2003)	28.6	4	9	28.6	9.57	0.89
	Conte et al. (2003)	14.3	4	49	28.6	3.74	1.04
	Conte et al. (2003)	28.6	4	49	28.6	2.45	0.83
Full scale	Brand et al. (1972)	40	5	4	7	1.18	-
	Brand et al. (1972)	40	4	4	6	1.06	-
	Brand et al. (1972)	40	3	4	5	1.13	-
	Brand et al. (1972)	40	2.5	4	4.5	1.08	-
	Liu et al. (1994)	45	4	16	15	1.07	-
	Sales (2000)	33.3	5	4	6.7	1.33	0.89
	Borel (2001)	27.1	-	1	4.4	1.9	1.02

Table 2.5: Experimental findings obtained from tests on piled rafts resting in clay (After L. de Sanctis and A. Mandolini, 2006 [50])

Recent analysis in 2022 by Ioannis *et al.* through 3D FEA and centrifuge testing on foundations with two piles also verify the contribution of the raft is not negligible [49]. In 2023, Corigliano *et al.* [9] analyse piled raft in clay by comparing FEA results with conventional methodologies also revealed significant disparities in the failure envelopes. The analysis showed the conventional approach tend to underestimate the capacity of piled raft.

2.6. Cyclic Behaviour of Drained Sand

Cyclic loading and its impact are practical relevance for many problems in geotechnical engineering. It may be caused by traffic, machinery, wind, waves, and earthquakes. The cyclic loading on soil can lead to an accumulation of permanent deformations or to a possible liquefaction due to a build-up of excess pore water pressure. In the case of this thesis, excess pore water pressure is not considered. Therefore, the main consideration is on the accumulation of permanent deformation.

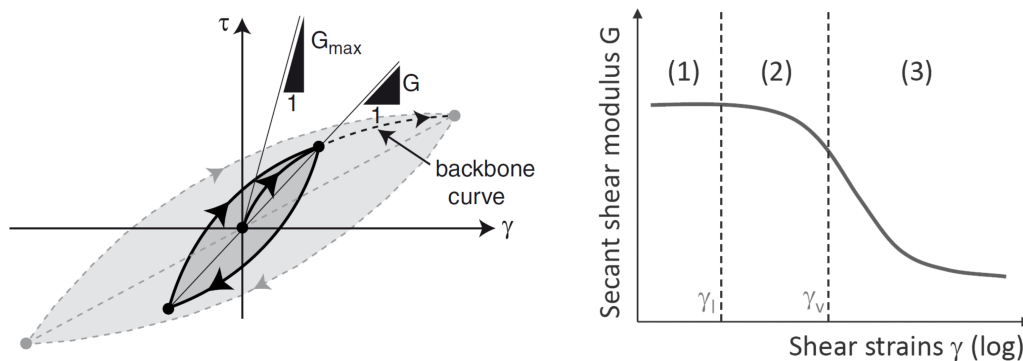


Figure 2.10: a) Definition of the secant shear modulus G (After Wichtmann, 2012) b) Decrease of G with increasing shear strain (After Soil Dynamics Lecture by Luca Flessati)

Dynamic properties are used to define the soil behaviour in this case, including shear modulus (G), Poisson's ratio (ν), and damping (D). The secant shear modulus $G = \tau^{amp} / \gamma^{amp}$ corresponds with the inclination of a straight line passing through two extreme points of the hysteresis.

Depending on the load magnitude and current state of the soil, it will lead to different behaviour in the soil. As for sand, cyclic behaviour can be categorized depends on the strain level, such as small strains, medium strains, and large strains. The conventional threshold between three of these categories are the inception of non-linear behaviour, γ_l , and inception of shear-volumetric coupling, γ_v , as shown in Figure 2.10 (b).

2.6.1. Small Strains

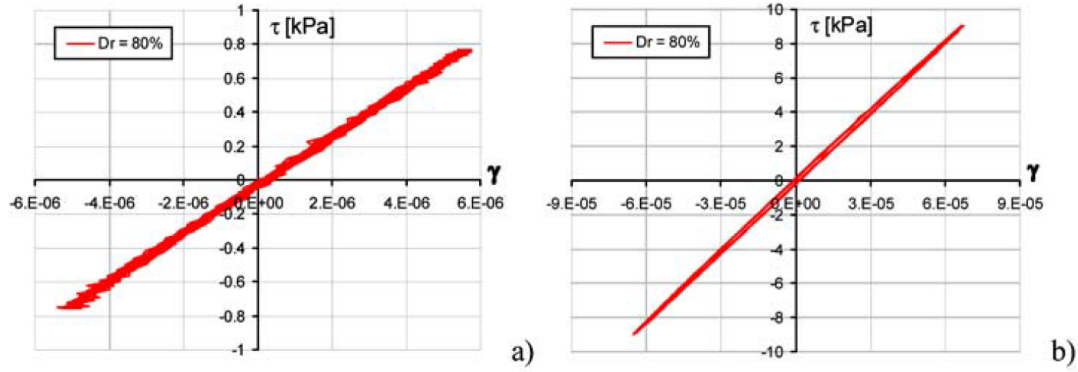


Figure 2.11: Cyclic torsional shear tests on a dense dry Toyoura specimen: stress-strain cycles at a) small strains, b) medium strains for a loading frequency 0.5 Hz. (After Zambelli *et al.*, 2007 [57])

Sand behaviour is categorized as small strain when there is no accumulation of shear strain over the cycle, therefore there is no degradation of the shear modulus G observed. Depending on the soil condition, small strain usually happened when the shear strains $\gamma < 10^{-5}$. In this case, the behaviour of the sand is expected to be linear and elastic, as shown in Figure 2.11.

The secant shear modulus G in small strains takes its maximum G_{max} and is independent of the shear strain amplitude. It sometimes also addressed as “dynamic shear modulus” and termed as G_{dyn} or G_0 . It can be calculated using wave velocity with $G_{max} = \rho v_s^2$. Although this value can be tested from the laboratory using bender element or resonant column, it is preferable to determine from in-situ measurement to avoid the sample being disturbed.

2.6.2. Medium Strains

Medium strain is a transitional zone between small strain to large strain. In this case, there is a hysteresis loop shown from the shear modulus, where the shear modulus is not linear, but the behaviour over cyclic is stable.

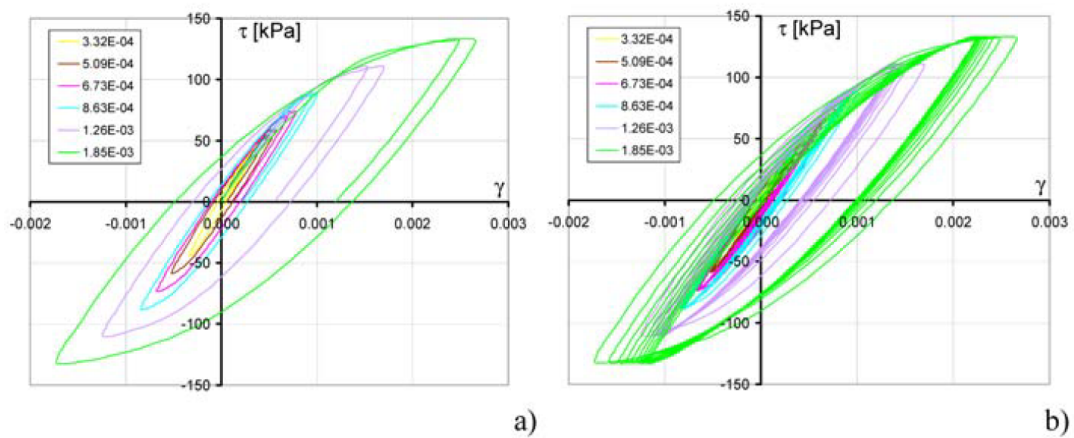


Figure 2.12: Cyclic torsional shear test at medium strains on a dense dry Toyoura specimen: stress-strain cycles a) 1^o cycle, b) all nine cycles for a loading frequency 0.1Hz. (After Zambelli *et al.*, 2007 [57])

Based on the cyclic torsional shear test performed in Toyoura sand by Zambelli *et al.* with symmetric load, the hysteresis loop of the sand specimens changes over cycle as shown in Figure 2.12. As illustrated in Figure 2.13, the shear modulus G decrease while the damping D increase over strain. Over cycle, the shear modulus G increase while the damping ratio D decrease. This behaviour shows the sand specimen compacted and getting stiffer during the cyclic process.

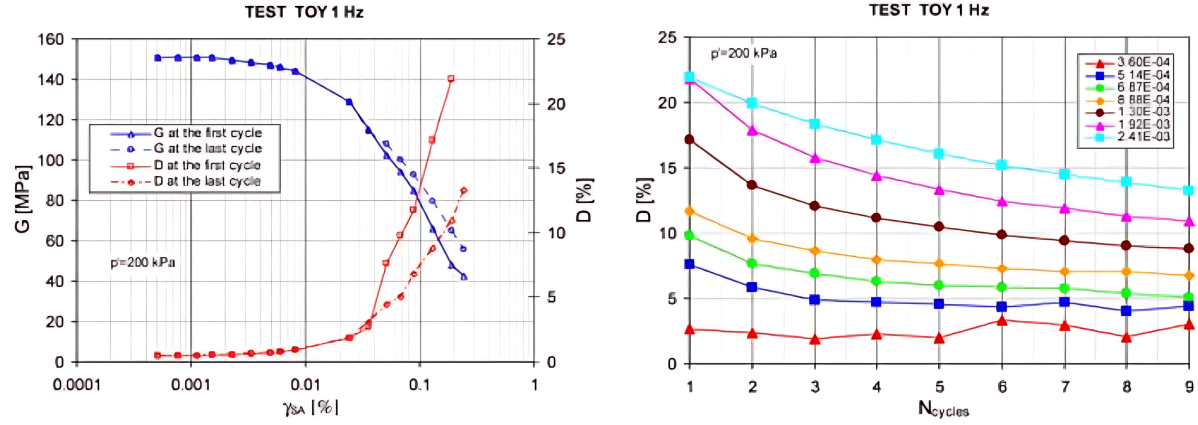


Figure 2.13: Shear moduli and damping ratio versus single amplitude shear strain, measured during cyclic torsional shear tests on dense dry Toyoura specimens (After Zambelli *et al.*, 2007 [57])

2.6.3. Large Strains

In the case of large strains, there is an observable hysteresis curve, with the response becoming unstable as there is a noticeable degradation of shear modulus G over strain over cycle. This behaviour shown by a cyclic triaxial tests with asymmetric loads done by Wichtmann in 2005 [53] as shown in Figure 2.15, when the load amplitude is small, the soil sample tends to ratcheting. But when the load become too large, the soil shows an unstable cyclic response, where until a certain limit will go toward incremental collapse.

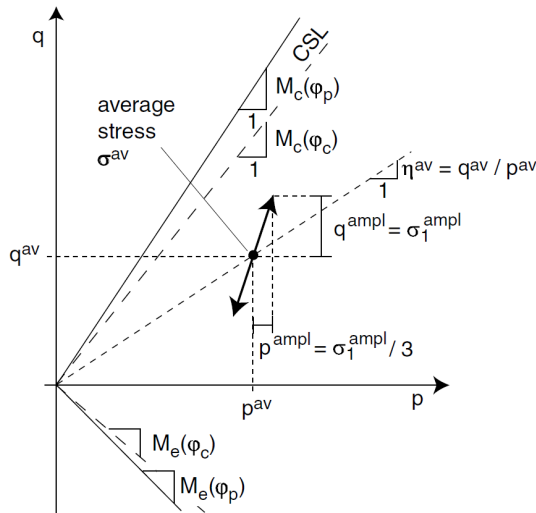


Figure 2.14: State of stress in cyclic triaxial test (After Wichtman *et al.*, 2005 [53])

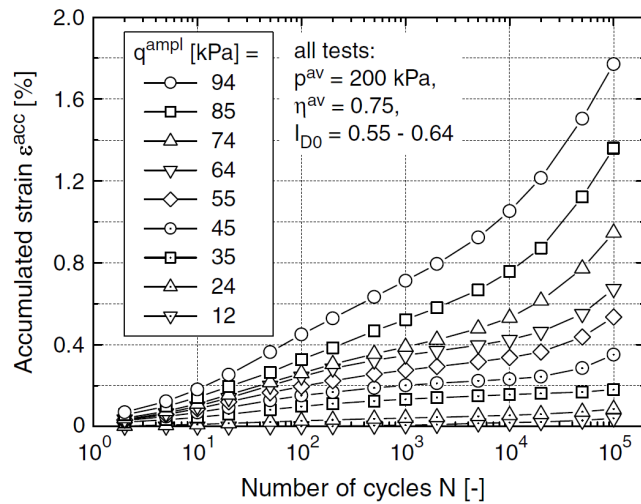


Figure 2.15: Accumulation curves $\varepsilon^{acc}(N)$ in tests with different stress amplitudes q^{amp} (After Wichtman *et al.*, 2005 [53])

2.6.4. Capturing Cyclic Behaviour of Sand with Viscoelastic Model

One of the laboratory testing methods to evaluate the behaviour of soils under cyclic loading conditions is the cyclic torsional shear test. Figure 2.16 shows the device used for the torsional shear test.

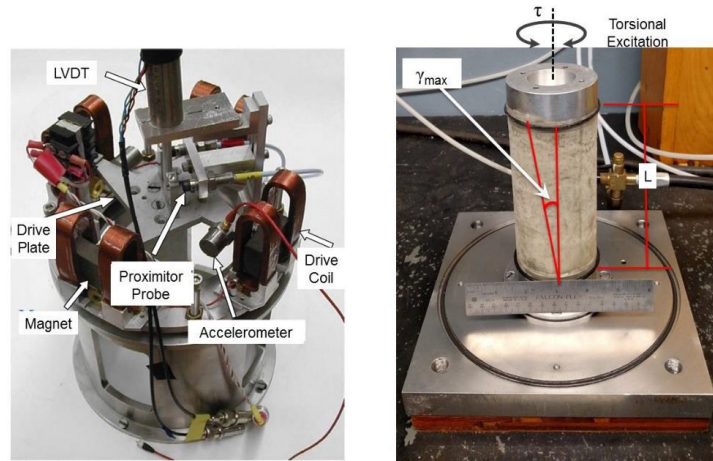


Figure 2.16: Torsional shear test device (After Parra *et al.*, 2016 [41])

Figures 2.17 shows the calibration of the viscoelastic model to fit the results from cyclic torsional shear tests. This figure demonstrates that the viscoelastic model is able to capture the cyclic behaviour of sand as observed in cyclic torsional shear tests. However, while this is the case on a microscopic scale, the ability of the viscoelastic model to capture the behaviour of piled raft foundations under cyclic loading remains uncertain.

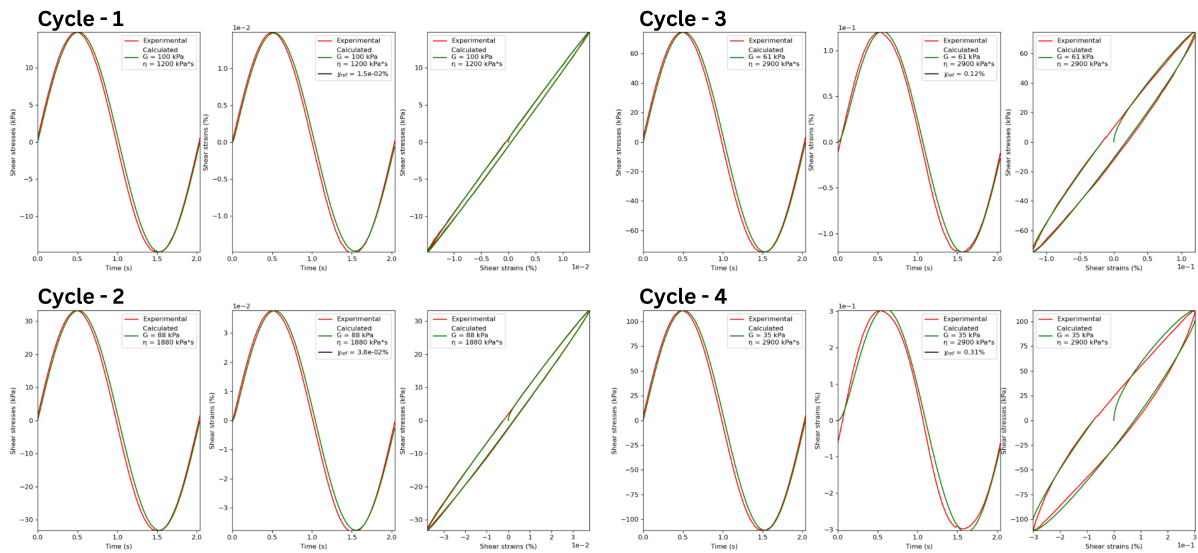


Figure 2.17: Viscoelastic calibration on cyclic torsional shear test result (Personal work)

2.7. Conclusion

The literature review studied for this thesis has highlighted various discussions, particularly focusing on the assessment of Eurocode 7, the analysis of raft, pile, pile groups, and piled raft foundations, and lastly, the cyclic behaviour of sand.

Eurocode 7, commonly used as a design code, provides a robust framework for general applications. However, it lacks detailed information on piled raft foundations, which have been commonly used in the industry.

The assessment of rafts, piles, and pile groups has significantly developed over the century. It starts with Terzaghi's bearing capacity equation in 1943, progressing to the Brinch Hansen formulation in 1970, which is commonly used in Europe and also defined in Eurocode 7. The axial capacity of single piles has been extensively researched, leading to the development of design concepts based on the CPT method due to its similarity. Furthermore, the concept of 'friction fatigue' in the shaft capacity has also been observed. As a result, several design

methods, such as Fugro-05, ICP-05, NGI-05, and UWA-05, have been proposed. In addition to axial capacity, the concept of lateral capacity is addressed through the Winkler foundation model, also known as the p - y model.

The axial capacity of pile groups has been analysed through analytical solutions derived from single pile equations with an efficiency factor. However, the determination of this efficiency factor is recommended through empirical evidence, such as small-scale testing. While the lateral capacity of pile groups generally is not a critical concern, potential of block failure need to be analysed. To assess the overall pile groups design, Di Laora in 2019, followed by Iovino in 2021 with a numerical approach, proposed a closed-form analytical solutions to create the failure envelopes of pile groups.

In the context of piled raft foundations, the complex interaction between soil, piles, and raft in piled rafts, which complicates analysis, was not thoroughly addressed until the late 20th century. Poulos first discussed it in 1968, followed by Butterfield and Banerjee in 1971, and then Davis and Poulos in 1972. Studies have shown that the raft significantly contributes to the overall bearing capacity in piled raft, with Mandolini and Russo in 2005 presenting case histories that shows the raft's contributions ranging from 8 to 70%. Sanctis and Mandolini in 2006 further validated this range with experimental findings in clay from various testing methods. Following these studies, several design methodologies were proposed. In 1994, Randolph introduced an analytical design approach. In 2000, Katzenbach published a paper showcasing the piled raft foundation in Germany through FEA and field-testing. Poulos further refined this approach in 2001 in the paper "Piled Raft Foundations: Design and Applications," proposing a three-stage design process starting from preliminary evaluation, detailed assessment, and optimization of foundation design.

Recent studies also showed the importance of considering the raft's contribution in piled raft foundations, as highlighted by Ioannis et al. and Corigliano et al., who used 3D Finite Element Analysis to demonstrate the significant bearing capacity provided by rafts. These findings also pointed out that conventional methodologies have been underestimating the capacity of piled raft foundations.

Cyclic loading, particularly in dry sand, presents additional challenges. The behaviour under cyclic loading varies with strain levels, and existing studies categorize the cyclic behaviour of sand into small, medium, and large strains, each showing distinct characteristics and stability issues. While the viscoelastic model has proven capable of capturing the cyclic behaviour of sand, there remains a research gap in understanding how these cyclic behaviours can be effectively modelled using equivalent linear viscoelastic models, particularly for piled raft foundations.

In summary, the literature highlights the need for further research into the behaviour of piled raft foundations. Additionally, investigating the feasibility of using viscoelastic models to capture cyclic behaviour in such systems is recommended as a simpler approach.

3

Methodology

3D finite element analysis using PLAXIS 3D and experimental testing with TU Delft centrifuge will be performed in this thesis. This chapter provides the concept related to constitutive model as a basis of finite element analysis and the concept of centrifuge testing.

3.1. Constitutive Model

3.1.1. General Formulation

A constitutive model provides a mathematical description of a soil's mechanical behaviour, aiming to replicate outcomes observed in experimental tests. The formulation of a basic constitutive model for soil mechanics includes several key ingredients, such as rheological schemes, elastic laws, yield criteria, plastic potential, plastic flow rules, and loading and unloading criteria.

Rheological Scheme

The rheological scheme represents the deformation and flow behaviour of materials. This scheme is used to define how the soil deforms under load, capturing both elastic (ε^{el}) and plastic (ε^{pl}) deformations, formulated as:

$$\varepsilon = \varepsilon^{el} + \varepsilon^{pl} \quad (3.1)$$

In terms of triaxial formulation, this becomes:

$$\begin{bmatrix} \varepsilon_v \\ \varepsilon_q \end{bmatrix} = \begin{bmatrix} \varepsilon_v^{el} \\ \varepsilon_q^{el} \end{bmatrix} + \begin{bmatrix} \varepsilon_v^{pl} \\ \varepsilon_q^{pl} \end{bmatrix} \quad (3.2)$$

Elastic Law

The elastic law describes the relationship between stress and strain within the elastic region. Formulated based on Hooke's law, the elastic law is defined as:

$$\begin{bmatrix} \varepsilon_v^{el} \\ \varepsilon_q^{el} \end{bmatrix} = \begin{bmatrix} 1/K & 0 \\ 0 & 1/3G \end{bmatrix} \begin{bmatrix} p' \\ q \end{bmatrix} \quad (3.3)$$

Yield Criterion & Plastic Potential

The yield criterion defines the conditions under which the soil transitions from elastic to plastic behaviour. while the plastic potential, often related but not identical to the yield surface, guiding how the material strain in plastic behaviour.

Yield criterion is evaluated through a yield function, f , as a function of the stress state. Various yield functions, such as Mohr-Coulomb, Tresca, Drucker-Prager, Von Mises, and Matsuoka-Nakai, are used in soil mechanics. For instance, as illustrated in Figure 3.1, when the yield function $f < 0$, the stress state is within the yield contour indicating elastic behaviour. At $f = 0$, plastic behaviour initiates, and a stress state with $f > 0$ is impossible. Therefore, the yield criterion is defined as:

$$f(\sigma) = 0 \quad (3.4)$$

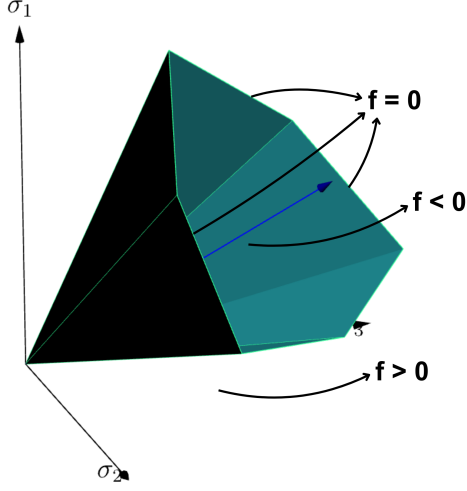


Figure 3.1: Mohr-Coulomb yield function (Modified after Gertraud Medicus work on SoilModels.com)

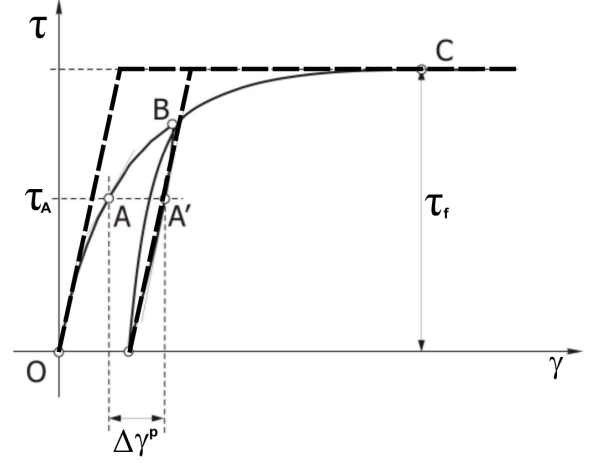


Figure 3.2: 1D Linear Elastic-Perfectly Plastic stress-strain curve

In a 1D Linear Elastic-Perfectly Plastic (LEPP) model (see Figure 3.2), the yield criterion and plastic potential is defined as:

$$f = g = \tau - \tau_f = 0 \quad (3.5)$$

In triaxial LEPP formulations, it evolves to:

$$\begin{aligned} f(q, p') &= 0 \\ f &= q - \frac{6 \sin \phi'}{3 - \sin \phi'} p' \end{aligned} \quad (3.6)$$

The plastic potential, g , is a function used to derive the plastic flow rules. In some cases, it is defined as equal to the yield function, $g = f$, or known as *associated flow rule*, as shown in Equation 3.5. However, formulation with associated flow rule leads to excessive plastic volumetric strains. Therefore, a non-associated flow rule, where g is distinct from f is recommended for a more accurate interpretation of plastic behaviour. In a triaxial LEPP scenario, the plastic potential is given by:

$$\begin{aligned} g(q, p') &= C \\ g &= q - \frac{6 \sin \psi'}{3 - \sin \psi'} p' = C \end{aligned} \quad (3.7)$$

Plastic Flow Rule

The plastic flow rule, derived from the plastic potential, defines the evolution of soil strain under plastic deformation. It is generally defined as:

$$d\gamma^{pl} = d\Lambda \frac{\partial g}{\partial \tau} \quad (3.8)$$

For triaxial LEPP formulations, it is expressed as:

$$\begin{bmatrix} d\varepsilon_v^{pl} \\ d\varepsilon_q^{pl} \end{bmatrix} = d\Lambda \begin{bmatrix} \frac{\partial g}{\partial p'} \\ \frac{\partial g}{\partial q} \end{bmatrix} \quad (3.9)$$

Loading/Unloading Criterion

The loading/unloading criterion specifies the material behaviour across cycles of loading and unloading, distinguishing between elastic and plastic responses. This criterion is important for identifying whether the applied loading produces plastic deformations or if unloading results in an elastic recovery. It is generally defined by the equation:

$$d\Lambda f = d\Lambda df = 0 \quad (3.10)$$

This equation serves as a fundamental principle in determining the state of deformation based on the loading path and history.

Extensions to Basic Constitutive Models

In addition to the main ingredients of constitutive models, advanced formulations include several additional features to improve the model's capabilities and accuracy. These features include hardening/softening rules, rate-dependent behaviour, fabric and structure, anisotropy, and temperature effect, among others.

3.1.2. SANISAND-04

The SANISAND-04 (Simple ANIsotropic SAND) model, developed by Dafalias and Manzari in 2004 [10], is a constitutive model commonly used to represent sand behaviour under various loading conditions. It is a critical state-based constitutive model incorporates anisotropy, dilatancy, bounding surface plasticity, and fabric evolution. These features make SANISAND-04 proficient at predicting the complex behaviour of sands.

In the SANISAND-04 framework, the principal effective stresses are denoted by σ_1 , with $\sigma_2 = \sigma_3$, and the corresponding principal strains by ε_1 , with $\varepsilon_2 = \varepsilon_3$. The model defines triaxial variables such as the deviatoric stress $q = \sigma_1 - \sigma_3$, the isotropic stress $p = \frac{1}{3}(\sigma_1 + 2\sigma_3)$, the deviatoric strain $\varepsilon_q = \frac{2}{3}(\varepsilon_1 - \varepsilon_3)$, and the volumetric strain $\varepsilon_v = \varepsilon_1 + 2\varepsilon_3$. A stress ratio variable $\eta = q/p$ is introduced in the formulation.

In the formulation of SANISAND-04, fifteen parameters must be defined, summarizing the aspects of elasticity, critical state, yield surface, plastic modulus, dilatancy, and the fabric-dilatancy tensor. These parameters are summarized in Table 3.1.

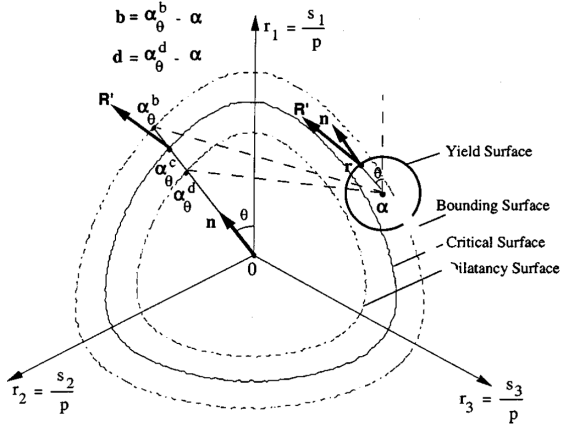
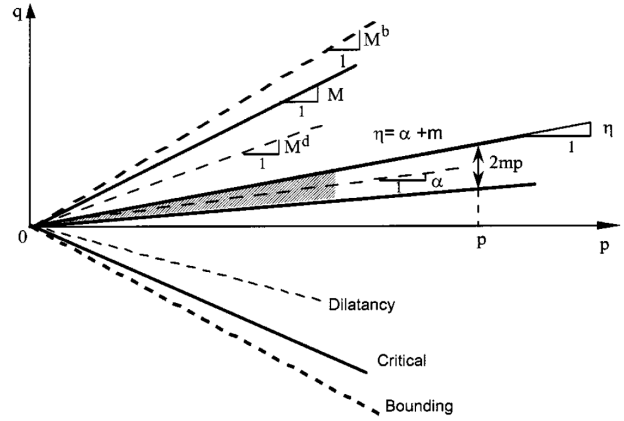
Category	Constant	Variable
Elasticity	Shear modulus at small strains	G_0
	Poisson's ratio	ν
Critical state	Slope of the critical state line	M
	Compression-to-extension strength ratio	c
	CSL slope parameter	λ_c
	Initial void ratio	e_0
	CSL intercept parameter	ξ
Yield surface	Yield locus opening parameter	m
Plastic modulus	Initial plastic modulus	h_0
	Hardening modulus	c_h
	Bounding surface parameter	n^b
Dilatancy	'Intrinsic' dilatancy parameter	A_0
	Dilatancy rate parameter	n^d
Fabric-dilatancy tensor	Maximum fabric-dilatancy tensor	z_{max}
	Fabric-dilatancy tensor scaling parameter	c_z

Table 3.1: SANISAND-04 constitutive model parameter

Following this, Table 3.2 shows all related mathematical equations used in the SANISAND-04 model for both triaxial and multiaxial conditions, while Figure 3.3 and 3.4 illustrates the yield, critical, dilatancy, and bounding plane incorporates with SANISAND-04, both in π and q, p planes.

Triaxial equations	Multiaxial equations	Constants
Critical state line	$e_c = e_0 - \lambda_c(p_c/p_{at})^\xi$	e_0, λ_c, ξ
Elastic deviatoric strain increment $d\varepsilon_q^e = dq/3G$	$d\mathbf{e}^e = ds/2G$ $G = G_0 p_{at} [(2.97 - e)^2 / (1 + e)] (p/p_{at})^{1/2}$	G_0
Elastic volumetric strain increment	$d\varepsilon_v^e = dp/K$ $K = 2(1 + \nu)G/3(1 - 2\nu)$	ν
Yield surface $f = \eta - \alpha - m = 0$	$f = [(\mathbf{s} - p\boldsymbol{\alpha}) : (\mathbf{s} - p\boldsymbol{\alpha})]^{1/2} - \sqrt{2/3}pm = 0$	m
Plastic deviatoric strain increment $d\varepsilon_q^p = d\eta/H$ $H = q/(M^b - \eta)$ $M^b = M \exp(-n^b\Psi)$ $h = h_0/ \eta - \eta_n $	$d\mathbf{e}^p = \langle L \rangle \mathbf{R}'$ $K_p = (2/3)ph(\alpha_\theta^b - \alpha) : \mathbf{n}$ $\alpha_\theta^b = \sqrt{2/3}[g(\theta, c)M \exp(-n^b\Psi) - m]\mathbf{n}$ $h = b_0/(\alpha - \alpha_{in}) : \mathbf{n}$ $b_0 = G_0 h_0 (1 - c_h e)(p/p_{at})^{-1/2}$	M, c, n^b h_0, c_h
Plastic volumetric strain increment $d\varepsilon_v^p = d \varepsilon_q^p $ $d = A_d(M^d - \eta)$ $M^d = M \exp(n^d\Psi)$ $A_d = A_0(1 + \langle sz \rangle)$	$d\mathbf{e}_v^p = \langle L \rangle D$ $D = A_d(\alpha_\theta^d - \alpha) : \mathbf{n}$ $\alpha_\theta^d = \sqrt{2/3}[g(\theta, c)M \exp(n^d\Psi) - m]\mathbf{n}$ $A_d = A_0(1 + \langle \mathbf{z} : \mathbf{n} \rangle)$	n^d A_0
Fabric-dilatancy tensor update $dz = -c_z \langle -d\varepsilon_v^p \rangle (sz_{max} + z)$	$d\mathbf{z} = -c_z \langle -d\varepsilon_v^p \rangle (z_{max}\mathbf{n} + \mathbf{z})$	c_z, z_{max}
Back-stress ratio tensor update $d\alpha = d\eta$	$d\alpha = \langle L \rangle (2/3)h(\alpha_\theta^b - \alpha)$	

Table 3.2: SANISAND-04 constitutive equations

Figure 3.3: Schematic of yield, critical, dilatancy, and bounding surfaces on the stress ratio π plane (After Dafalias and Manzari, 2004 [10])Figure 3.4: Schematic of the yield, critical, dilatancy, and bounding lines in q, p space (After Dafalias and Manzari, 2004 [10])

Based on the equations presented, Figure 3.5 shows the drained soil response simulated with the SANISAND-04 model. This figure demonstrates the soil's behaviour both before and after interacting with the yield surface over a loading cycle. Initially, the soil shows linear stiffness; however, upon reaching the yield surface, nonlinear behaviour emerges. Part (a) of the figure captures how the soil densifies over cycles under symmetric loading conditions, while part (b) illustrates a ratcheting response of the soil under drained conditions with asymmetric loads.

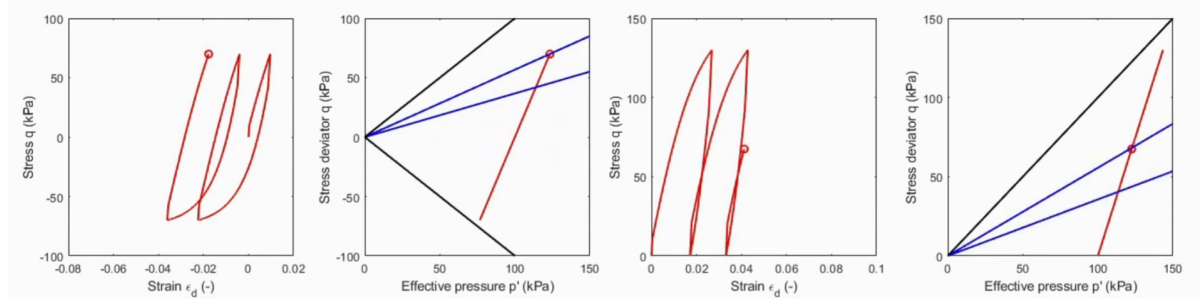


Figure 3.5: a) Symmetric loads, b) asymmetric loads drained soil response with SANISAND-04 (After Luca Flessati's soil dynamics lecture)

To validate the SANISAND-04 model, Dafalias and Manzari compared its predictions against experimental findings from drained compression tests on Toyoura Sand conducted by Verdugo and Ishihara in 1996. Figure 3.6 showcases this comparison, illustrating the model's capable to replicate the observed experimental responses.

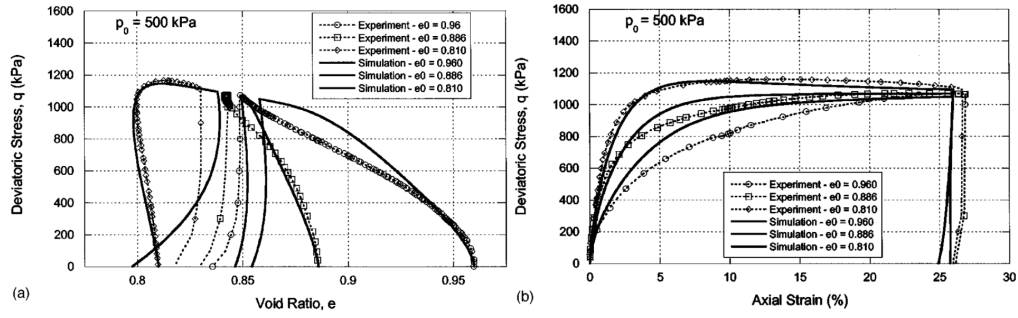


Figure 3.6: Simulations of SANISAND-04 versus experiments in drained compression tests on Toyoura sand (Verdugo and Ishihara, 1996); $(\sigma'_3)_{in} = 500$ kPa (After Dafalias and Manzari, 2004 [10])

3.1.3. SANISAND-MS

The SANISAND-MS model formulated by Liu *et al.* in 2019 [33] is developed to address the modelling of sand behaviour under cyclic loading conditions. Although the primary focus is for offshore foundations, this concept is applicable onshore to such as cyclic conditions due to high-speed traffic loads, vibrating machines, and earthquakes, and others. It is an evolution of the SANISAND model, incorporating additional features to accurately model high-cyclic loading conditions and the associated sand behaviour. Building on the same bounding surface plasticity theory, it replaced the fabric dilatancy tensor in SANISAND-04 with a new memory surface (MS) concept to better represent the sand's response to cyclic loads. The memory surface tracks the evolution of soil fabric and its effects on the soil's mechanical response, allowing for a better representation of cyclic ratcheting and strain accumulation in a drained condition.

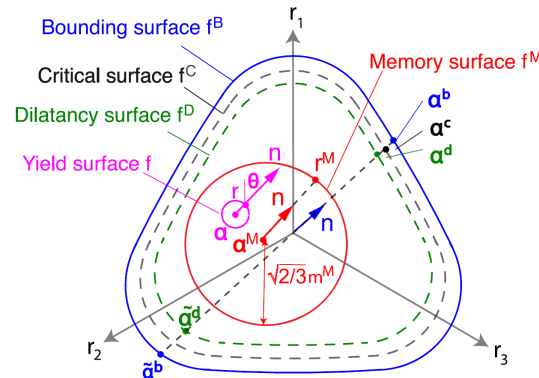


Figure 3.7: SANISAND-MS loci in the deviatoric stress ratio plane (After Liu *et al.*, 2020 [34])

As shown in Figure 3.7, the model features five conical loci: the narrow conical yield surface centred along its axis, the critical state surface, bounding surface, and dilatancy surface centred along the p -axis and the newly defined memory surface. The memory surface, representing the stress ratios previously experienced by the soil, undergoes kinematic hardening, similar to the yield surface. It also can expand and shrink to simulate the formation and damage of soil fabric. Figure 3.8 gives an example of the stress state of soil inside SANISAND-MS loci, especially the development memory surface when dealing with load.

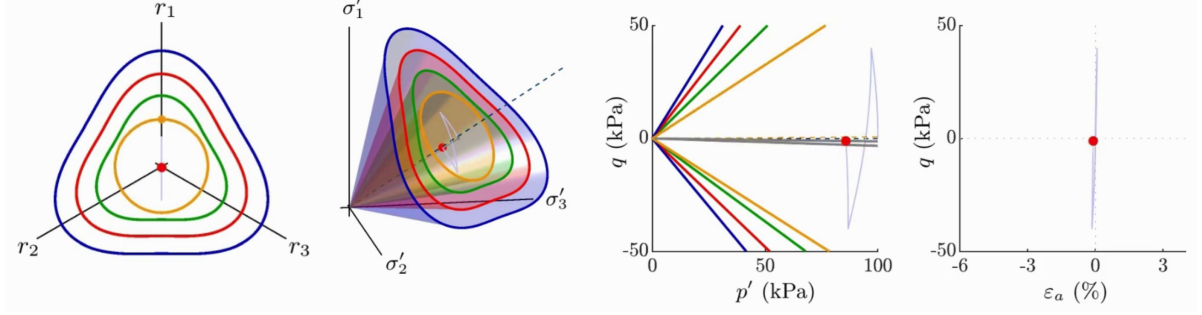


Figure 3.8: Growth of MS over cycle in undrained soil with SANISAND-MS (Modified after Gertraud Medicus work on SoilModels.com)

Table 3.3 shows all the constitutive equation and parameters needed to formulate SANISAND-MS.

Aspect	Constitutive Equation	Parameter Description
Elastic moduli	$G = G_0 p_{atm} \left[\frac{(2.97 - e)^2}{1 + e} \right] \left(\frac{p}{p_{atm}} \right)^{1/2}$ $K = \frac{2(1 + \nu)G}{3(1 - 2\nu)}$	G_0 : dimensionless shear modulus ν : Poisson's ratio
Critical state line	$e_c = e_0 - \lambda_c \left(\frac{p_c}{p_{atm}} \right)^\xi$	e_0 : reference critical void ratio λ_c, ξ : CSL shape parameters
Yield function	$f = \sqrt{(\mathbf{s} - p\boldsymbol{\alpha}) : (\mathbf{s} - p\boldsymbol{\alpha})} - \sqrt{2/3}pm$	m : yield locus opening parameter
Memory function	$f^M = \sqrt{(\mathbf{s} - p\boldsymbol{\alpha}^M) : (\mathbf{s} - p\boldsymbol{\alpha}^M)} - \sqrt{2/3}pm^M$	
Deviatoric plastic flow	$d\mathbf{e}^p = \langle L \rangle \mathbf{R}'$ $\mathbf{R}' = B\mathbf{n} - C \left[n^2 - \left(\frac{1}{3} \right) \mathbf{I} \right]$ $\mathbf{n} = \frac{(\mathbf{r} - \boldsymbol{\alpha})}{\sqrt{\frac{2}{3}m}}$ $B = 1 + \frac{3(1-c)}{(2c)g(\theta)\cos 3\theta}$ $C = \frac{3\sqrt{3/2(1-c)g(\theta)}}{c}$ $g(\theta) = \frac{c}{[(1+c) - (1-c)\cos 3\theta]}$	
Volumetric plastic flow	$d\varepsilon_{vol}^p = \langle L \rangle D$ $D = \left[A_0 \exp \left(\beta \frac{\langle \tilde{b}_d^M \rangle}{b_{ref}} \right) \right] (\mathbf{r}_d^\theta - \mathbf{r}) : \mathbf{n}$ $r_\theta^d = \sqrt{\frac{2}{3}}g(\theta)M \exp(n^d\Psi)\mathbf{n}$ $\tilde{b}_d^M = (\tilde{r}_\theta^d - \tilde{r}^M) : \mathbf{n}$ $b_{ref} = (r_\theta^b - r_{\theta+\pi}^b) : \mathbf{n}$ $r_{\theta+\pi}^b = \sqrt{\frac{2}{3}}g(\theta + \pi)M \exp(-n^b\Psi)(-\mathbf{n})$	A_0 : 'intrinsic' dilatancy parameter β : dilatancy memory parameter n^d : void ratio dependence parameter
Yield surface evolution	$d\boldsymbol{\alpha} = \left(\frac{2}{3} \right) \langle L \rangle h(\mathbf{r}_\theta^b - \mathbf{r})$ $\mathbf{r}_\theta^b = \sqrt{\frac{2}{3}}g(\theta)M \exp(-n^b\Psi)\mathbf{n}$	M : critical compression stress ratio n^b : void ratio dependence parameter

Aspect	Constitutive Equation	Parameter Description
	$h = \frac{b_0}{(\mathbf{r} - \mathbf{r}_{in}) : \mathbf{n}} \exp \left[\mu_0 \left(\frac{p}{p_{atm}} \right)^{0.5} \left(\frac{b^M}{b_{ref}} \right)^2 \right]$ $b_0 = G_0 h_0 (1 - c_h e) / \sqrt{\frac{p}{p_{atm}}}$ $b^M = (\mathbf{r}^M - \mathbf{r}) : \mathbf{n}$	c : compression-extension ratio μ_0 : ratcheting parameter h_0, c_h : hardening parameters
Memory surface evolution	$dm^M = \sqrt{\frac{3}{2}} d\alpha^M : \mathbf{n} - \frac{m^M}{\zeta} f_{shr} \langle -d\varepsilon_{vol}^p \rangle$ $d\alpha^M = \frac{2}{3} \langle L^M \rangle h^M (\mathbf{r}_\theta^b - \mathbf{r}^M)$ $h^M = \frac{1}{2} \left[\frac{b_0}{(\mathbf{r}^M - \mathbf{r}_m) : \mathbf{n}} + \sqrt{\frac{3}{2}} \frac{m^M f_{shr} \langle -D \rangle}{\zeta (\mathbf{r}_\theta^b - \mathbf{r}^M) : \mathbf{n}} \right]$	ζ : memory surface shrinkage parameter

Table 3.3: SANISAND-MS constitutive equations and material parameters

As shown in Table 3.4, SANISAND-MS change the fabric-dilatancy tensor with a proposed memory parameter, therefore substituting z_{max} and c_z with μ_0 , β and ζ . Liu *et al.* add the ratcheting parameter μ_0 to keep track of the hardening coefficient h , β in the tensor of deviatoric plastic flow direction D , and ζ in the evolution of yield locus opening parameter m . These new parameters linked to the proposed memory surface can be identified by best-fitting cyclic test results, possibly from both drained and undrained triaxial cyclic tests. In total, there are 16 parameters needed in SANISAND-MS, in comparison with SANISAND-04 which needs 15 parameters.

No.	Variable	Description	SANISAND-MS	SANISAND04
Elasticity				
1	G_0	Dimensionless shear modulus	✓	✓
2	ν	Poisson's ratio	✓	✓
Critical State Line				
3	e_0	Reference critical void ratio	✓	✓
4	λ_c	CSL slope parameter	✓	✓
5	ξ	CSL intercept parameter	✓	✓
Yield Function				
6	m	Yield locus opening parameter	✓	✓
Volumetric Plastic Flow				
7	A_0	'Intrinsic' dilatancy parameter	✓	✓
8	n^d	Void ratio dependence parameter for dilatancy	✓	✓
Yield Surface Evolution				
9	M	Critical stress ratio (CSL slope)	✓	✓
10	n^b	Void ratio dependence parameter for yield surface	✓	✓
11	c	Compression-to-extension strength ratio	✓	✓
12	h_0	Initial plastic modulus	✓	✓
13	c_h	Hardening modulus	✓	✓
Memory				
14	μ_0	Ratcheting parameter	✓	-
15	β	Dilatancy memory parameter	✓	-
16	ζ	Memory surface shrinkage parameter	✓	-
Fabric-Dilatancy Tensor				
17	z_{max}	Maximum fabric-dilatancy tensor	-	✓
18	c_z	Fabric-dilatancy tensor scaling parameter	-	✓
Total Variables:			16	15

Table 3.4: Comparison of variables in SANISAND-MS and SANISAND04 models

From the calibration of SANISAND-MS against the monotonic drained triaxial test by Wichtmann (2005), Liu *et al.* shows the model is capable to replicate the experimental results, therefore proving the model is realistic to model drained behaviour of sand.

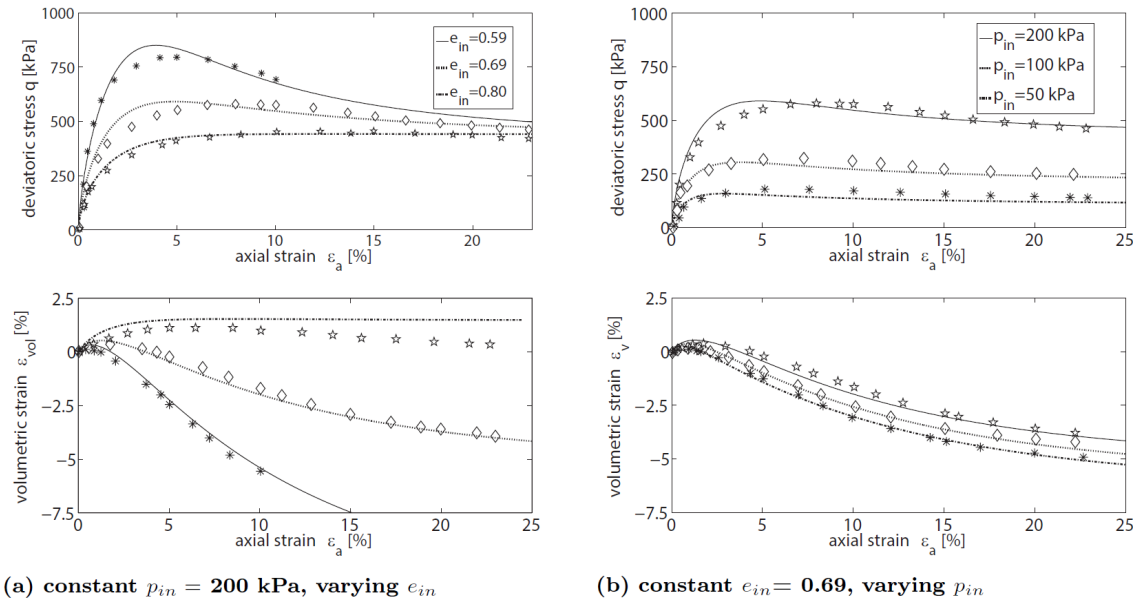


Figure 3.9: Calibration of model parameters against the monotonic drained triaxial test results by Wichtmann (2005) – experimental data denoted by markers (After Liu *et al.*, 2019 [33])

3.2. Centrifuge Testing

Centrifuge testing represents a form of small-scale physical modelling. As highlighted by Wood (2005) [55], physical modelling is performed to examine specific characteristics of prototype behaviours. While full-scale testing is considered a form of physical modelling—replicating all aspects of the prototype at full scale—most physical models are constructed on a much smaller scale. This approach is chosen to better at gathering information on expected response patterns, often necessitating parametric studies where key model parameters are varied to understand their effects. This process typically requires multiple model tests, including repetitions for increased confidence in the results. Physical modelling thus serves to simplify and clarify the understanding of mechanisms, such as soil-structure interactions, under specific boundary conditions, providing insights into their responses to various loads and environmental factors.

3.2.1. History

The concept of centrifuge modelling was first proposed by E. Phillips in 1869, suggesting that gravity-driven processes could be simulated on a smaller scale through centrifugal acceleration. In 1931, P. Bucky initiated the first applications of centrifuge testing to study slope stability in open-pit mining. A significant advancement came in 1960 when A. Schofield established the foundational principles of soil mechanics centrifuge modelling, along with the development of centrifugal machines and instrumentation. In the same year, M. Mikasa was pioneering in applying geotechnical centrifuge for studying soil consolidation and its interaction with urban infrastructure, such as tunnels. The Cambridge 10m beam centrifuge in the United Kingdom, introduced in 1971, has been a central tool for geotechnical modelling, capable of reaching a centrifugal acceleration of approximately 130g at a 4.125m radius. By 1990, TU Delft in the Netherlands had developed a 1.2m radius beam centrifuge, facilitating quick and economical tests and ongoing updates of sensors, actuators, and model containers¹. Until 2011, there are approximately 110 geotechnical centrifuge worldwide [22].

3.2.2. Fundamentals of Centrifuge Modeling

Centrifuge modelling is a technique that scales down the physical dimensions of a prototype by a factor of N , while simultaneously scaling up the stresses by the same factor. This approach ensures that the stress-strain response of the soil in the model matches that of the prototype under similar stress conditions. The process relies on the centrifuge's ability to generate a radial outward acceleration $a_{cf} = Ng = \omega^2 R$ which increases gravity's effects by a factor of N , where g is the acceleration due to gravity, ω is the angular velocity, and R is the radius at which the model is positioned from the rotation axis.

¹History of centrifuge modelling in geotechnical engineering, After Miguel Cabrera work

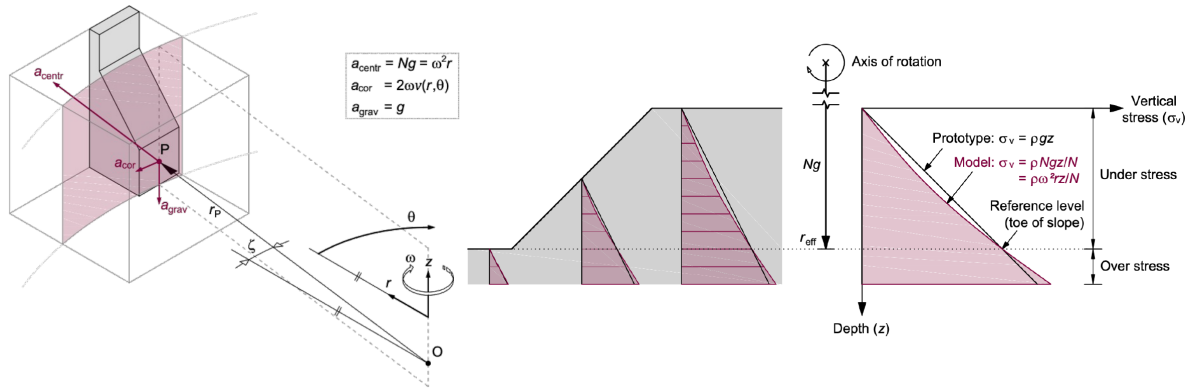


Figure 3.10: Centrifuge plane and profile of vertical stress in slopes (After Idinger, 2016 [20])

The angular velocity (ω) is related to the rotation speed of the centrifuge, measured in revolutions per minute (U), by the equation $\omega = \frac{2\pi U}{60}$. This relationship allows for the calculation of U required to achieve a desired scaling factor (N) for a given radius (R), using the equation:

$$U = \frac{60}{2\pi} \sqrt{\frac{Ng}{R}} \quad (3.11)$$

Due to the circular acceleration field around the rotation axis, as illustrated in Figure 3.10, it is important to account for the variation in centrifugal acceleration across the model. Further from the rotation axis, will be influenced with higher acceleration. This effect must be accurately corrected to ensure the precision of the testing.

3.2.3. Scaling Principles

The principles of scaling are crucial in centrifuge modelling, allowing for the appropriate translation of real-world phenomena into manageable models. Table 3.5 outlines the scaling principles for various quantities.

Parameter	Symbol	Model/Prototype	Units
General scaling principles (slow events)			
Length	L	$1/N$	m
Area	A	$1/N^2$	m^2
Volume	V	$1/N^3$	m^3
Mass	m	$1/N^3$	$Nm^{-1}s^2$
Stress	σ	1	Nm^{-2}
Strain	ε	1	-
Force	F	$1/N^2$	N
Bending moment	M	$1/N^3$	Nm
Work	W	$1/N^3$	Nm
Energy	E	$1/N^3$	J
Time (consolidation)	t	$1/N^2$	s
Dynamic events			
Time (dynamic)	t_{dyn}	$1/N$	s
Frequency	f	N	s^{-1}
Displacement	u	$1/N$	m
Velocity	v	1	ms^{-1}
Acceleration/Acceleration due to gravity (g)	N	N	ms^{-2}

Table 3.5: Scaling principles (Modified after Madabushi, 2014 [37])

4

Reference Case

This chapter provides an overview related to the type of analysis, loading conditions, material geometry, and domain description of the reference case.

4.1. Type of Analysis and Loading

To address the research questions mentioned in Chapter 1.3, three types of foundations will be analysed under different loading conditions:

- **Piled Raft Foundation**
 - Axial loading analysis
 - Lateral loading analysis
 - Cyclic loading analysis
- **Pile Group Foundation**
 - Axial loading analysis
 - Lateral loading analysis
- **Raft Foundation**
 - Axial loading analysis
 - Lateral loading analysis

The methodologies for each type of analysis are as follows:

- **Axial Loading Analysis:** A vertical load is applied at the top of the pier until the vertical displacement (u_z) reaches 80 cm ($\frac{u_z}{D} = 1$) or failure occurs.
- **Lateral Loading Analysis:** An initial vertical load is applied, followed by lateral loading at the top of the pier until the rotation (θ) reaches 4° .
- **Cyclic Loading Analysis:** An initial vertical load is applied, followed by cyclic loading at the top of the pier until the horizontal load (H) reaches 80% of the maximum horizontal load (H_{max}), with an 10% increase in lateral load per cycle.

4.2. Soil

The soil used in this thesis is dry Geba sand, characterized by a relative density (D_r) of 70% to 80%. As an initial study, for the finite element analysis part of this thesis, the parameters of Geba sand for the SANISAND-MS model will be adopted from a previous study.

4.3. Foundation

Figure 4.1 shows the prototype scale geometry of the foundation that will be the focus of analysis throughout this thesis. For the centrifuge testing, a scale factor $N = 100\times$ is applied, meaning the model geometry will be $100\times$ smaller than the prototype.

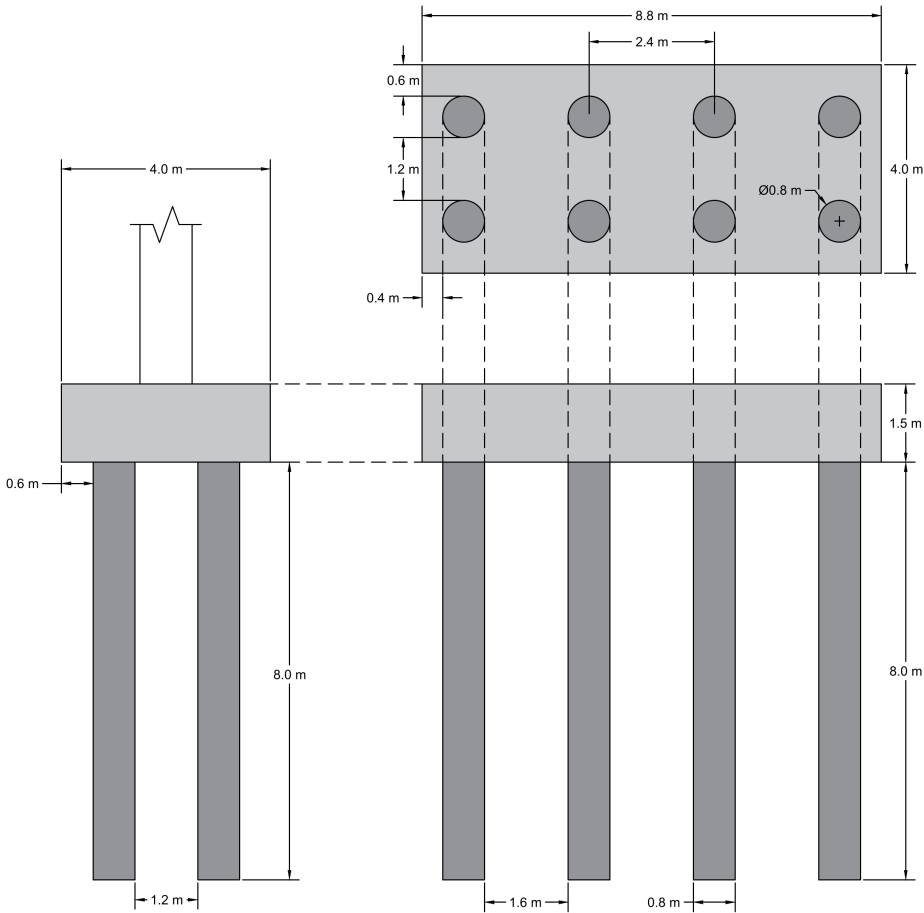


Figure 4.1: Prototype scale foundation geometry

In both the FEA and experimental testing, steel is selected for modelling the raft and the pier components, whereas concrete is used for the pile in FEA, and aluminium in the experimental setup. The foundation properties utilized in both FEA and experimental testing are summarized in Table 4.1.

Properties	FEA				Experimental Testing			
	Unit	Pier	Raft	Pile	Unit	Pier	Raft	Pile
Material	[-]	Steel	Steel	Concrete	[-]	Steel	Steel	Aluminium
Length	m	1	8.8	8	mm	1	8.8	8
Width	m	1	2	-	mm	1	2	-
Height	m	10	1.5	-	mm	10	1.5	-
Outer Diameter	m	-	-	0.8	mm	-	-	0.8
Inner Diameter	m	-	-	-	mm	-	-	0.6
Volume	m ³	10	26.4	4.02	mm ³	10	26.4	1.76
Unit Weight	kN/m ³	78.5	78.5	25	kN/m ³	78.5	78.5	27
Quantity	[-]	1	1	4	[-]	1	1	4
Weight	kN	785	2072.4	402.12	kN	7.85E-07	2.07E-06	1.90E-07
Total Weight	kN	3259.524			3.0474E-06			

Table 4.1: Material properties

As indicated in Table 4.2 comparing the experimental setup to the FEA results, there is a similarity in the pile axial stiffness (EA) between the two, with a minor difference of approximately 2.08%. This minor difference is chosen due to the focus on axial loading behaviour of the pile. However, the difference in the flexural stiffness (EI) is more pronounced, at about 59.51%. This significant differences in EI values will be a point of discussion when analysing and comparing the bending moment results derived from both methodologies.

Property	Unit	Experimental	FEA	Difference	Difference [%]
Material	[-]	Aluminium	Concrete	-	-
D_{out}	m	0.8	0.8	-	-
D_{in}	m	0.6	-	-	-
A	m ²	0.22	0.50	-	-
E	kPa	30×10^6	70×10^6	-	-
I	m ⁴	1.37×10^{-2}	2.01×10^{-2}	6.36×10^{-3}	31.64%
EA	kN	15.39×10^6	15.08×10^6	3.14×10^5	2.08%
EI	kNm ²	96.2×10^4	60.3×10^4	3.59×10^5	59.51%

Table 4.2: Aluminium and concrete properties differences

4.4. Domain Description

The domain of the experiment is determined by the size of the basket that fits inside the centrifuge. Due to space constraints, the basket designated for the experimental tests measures $L \times W \times H = 41 \text{ cm} \times 15 \text{ cm} \times 16.5 \text{ cm}$, and the foundation will be modelled as half. Consequently, in the finite element analysis, the boundary dimensions are scaled up by a factor of $N = 100 \times$ to $L \times W \times H = 41 \text{ m} \times 15 \text{ m} \times 16.5 \text{ m}$, as illustrated in Figure 4.2.

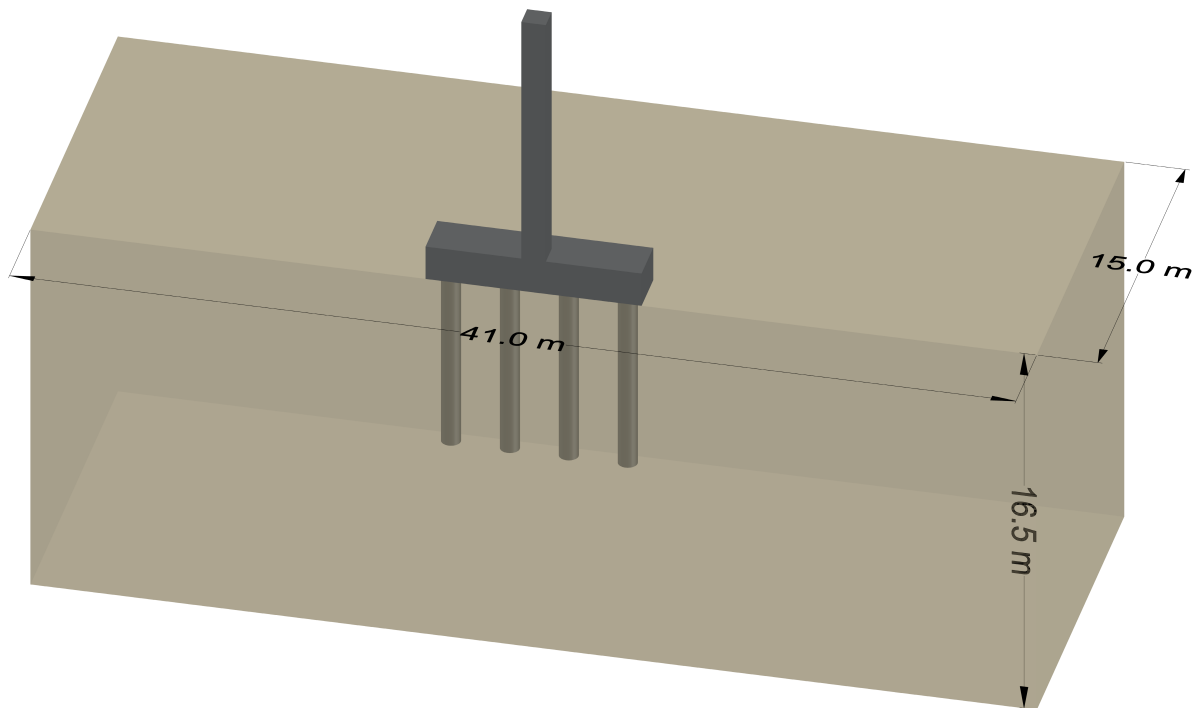


Figure 4.2: Domain

Finite Element Analysis

5.1. Element and Mesh

The finite element analysis (FEA) is carried out using PLAXIS 3D. The model geometry is subdivided into elements, each consisting of 10 nodes. In this project, the finite element (FE) elements are categorized as soil, foundation (including piles, raft, and pier), and foundation-soil interface. Table 5.1 shows the number of elements and nodes used in the FEA.

Foundation Type	Elements	Nodes
Pile Group	34,338	53,795
Piled Raft	34,474	54,183
Raft	32,108	49,720

Table 5.1: Element and nodes

As mentioned in Chapter 4, the foundation is modelled as half. Figure 5.1 illustrates the general appearance of the mesh.

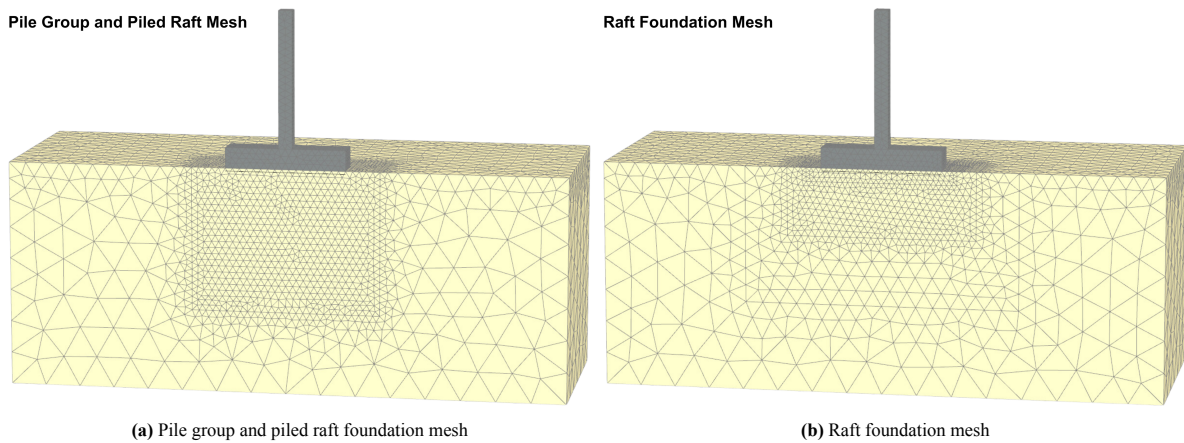


Figure 5.1: Finite Element mesh

5.2. Materials

5.2.1. Soil

The SANISAND-MS constitutive model, requiring 16 parameters, is chosen for this analysis, as discussed in Chapter 3.1.3. Table 5.2 highlights these parameters, including $M_c^{b,max}$, a parameter introduced in PLAXIS to increase the stability of SANISAND-MS. As mentioned in Chapter 4.2, the calibration of these parameters was performed in a previous study.

Aspect	Parameters	Description	Value
Elasticity	G_0	Shear modulus coefficient	400
	ν	Poisson's ratio	0.05
Critical State Line	$\bar{e}_0$	Reference critical void ratio	0.935
	λ_c	CSL slope in compressibility plane	0.0015
	ξ	CSL constant	0.7
Yield Function	m	Size of the yield surface	0.1
Volumetric Plastic Flow	A_0	Dilatancy parameter	1.104
	n^d	Dilatancy surface parameter	0.75
Yield Surface Evolution	M_c	Critical stress ratio in compression	1.25
	n^b	Bounding parameter	0.96
	c	Extension-to-compression ratio	0.702
	h^0	Hardening parameter	8.64
	c_h	Hardening parameter	1.09
Memory	μ_0	MS ratcheting parameter	500
	β	MS dilatancy parameter	0
	ζ	MS shrinkage parameter	0.0001
Void Ratio	e_{max}	Maximum void ratio	1.07
	e_{min}	Minimum void ratio	0.64
	e_0	Initial void ratio	0.769
Stability	$M_c^{b,max}$	Maximum bounding stress ratio in compression	1.92

Table 5.2: Parameters for Geba sand using the SANISAND-MS constitutive model

5.2.2. Foundation

The foundation is represented as volume elements, and their behaviour is assumed to be linear and isotropic. The foundation size is referred to in Section 4.3. The raft and pier are assigned the properties of steel, while the piles are assigned the properties of concrete, as indicated in Table 5.3.

Description	Parameters	Pier and Raft	Piles	Dimension
Material	[-]	Steel	Concrete	[-]
Unit weight	γ	78.5	25	kN/m^3
Young Modulus	E	200×10^6	30×10^6	kN/m^2
Poisson Ratio	ν	0.3	0.2	[-]

Table 5.3: Foundation material parameter

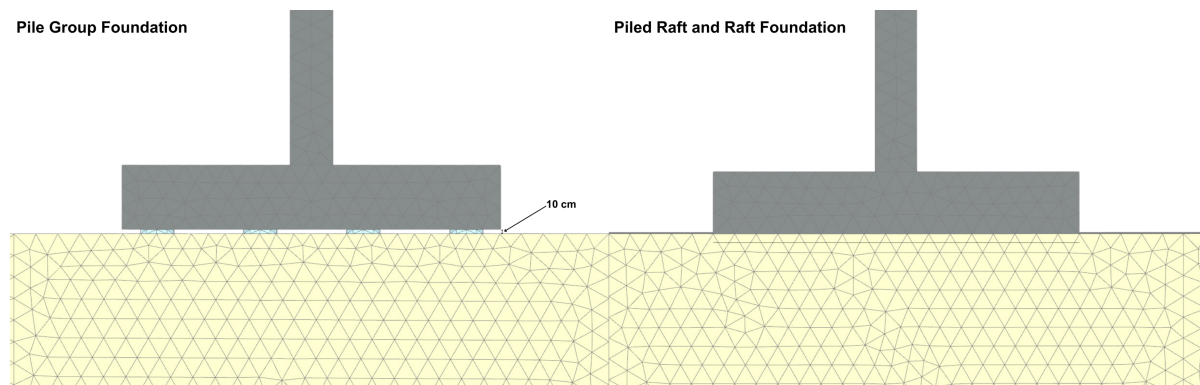


Figure 5.2: Modelling approaches for pile group, piled raft, and raft in FEA

Figure 5.2 shows the difference in how the pile group, piled raft, and raft are modelled in the FEA. In the pile group, a 10 cm gap is assigned between the raft and soil to avoid any contact. Figure 5.3a shows the general model of the foundation mesh and its material.

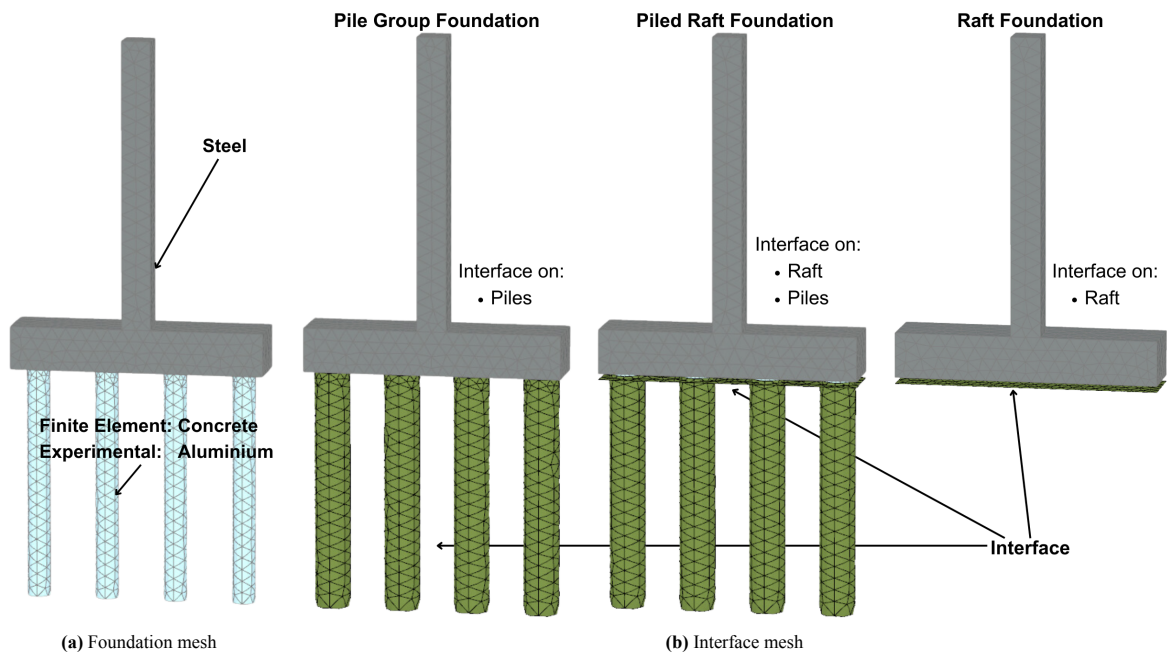


Figure 5.3: Foundation and interface mesh

5.2.3. Foundation-Soil Interface

The interface is modelled with the Mohr-Coulomb constitutive model, with stress-dependency defined using a power law formulation. Table 5.4 shows the parameters assigned to the interface, while Figure 5.3b shows the area where the interface is assigned in each foundation type.

Parameters	Description	Value	Dimension
E_{oed}	Oedometer Stiffness	315×10^3	kPa
c_{inter}	Interface Cohesion	0.01	kPa
ϕ_{inter}	Interface Friction Angle	20	$^\circ$
ψ_{inter}	Interface Dilatancy	0	$^\circ$

Table 5.4: Interface parameter

5.3. Boundary Conditions

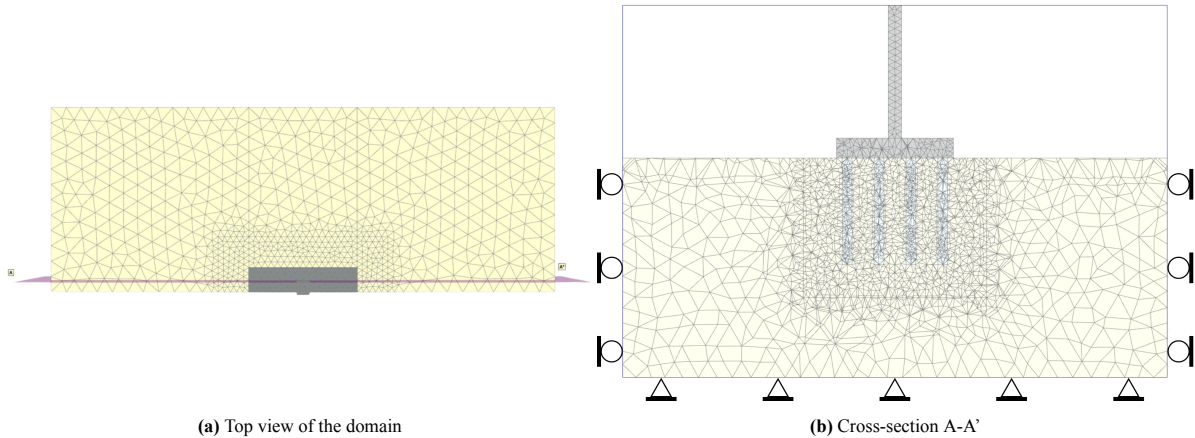


Figure 5.4: Boundary condition

Figure 5.4 shows the model boundary conditions for the FEA. All vertical boundaries (sides of the domain) are roller-supported, allowing for vertical displacement but restricting horizontal movement. The bottom boundary is fixed, preventing both vertical and horizontal displacements.

5.4. Load Phase

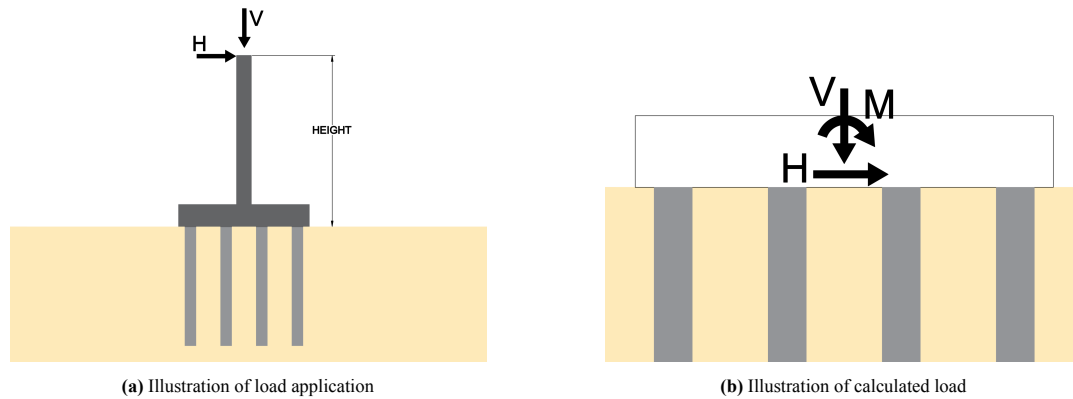


Figure 5.5: Illustration of applied and calculated load

Figure 5.5 shows how the load is applied and calculated. V , H , and M represent the vertical, horizontal, and moment loads, respectively. The moment (M) is calculated as the product of the horizontal load (H) and the height from the top of the pier to the ground surface ($M = H \times \text{height}$). In general, the stages set up in PLAXIS 3D can be categorized into three different analyses: axial loading, lateral loading, and cyclic loading.

In axial loading analysis, the phases are:

- **Initial Phase:** During this phase, soil stress is computed following the K_0 procedure. Deformation is not considered.
- **Equilibrium Phase:** The foundation is assumed to be wished in place, and the effects of its installation are not taken into account. Plastic calculations are employed.
- **Axial Loading Phase:** Axial loads are incrementally increased using prescribed loading at the top of the pier until the target displacement is reached.

In lateral loading analysis, the phases are:

- **Initial Phase**
- **Equilibrium Phase**
- **Initial Axial Loading Phase:** Axial loads are incrementally increased until 5 or 10 MN.
- **Lateral Loading Phase:** Horizontal loads are incrementally increased using prescribed loading at the top of the pier until the target rotation is reached.

In cyclic loading analysis, the phases are:

- **Initial Phase**
- **Equilibrium Phase**
- **Initial Axial Loading Phase:** Axial loads are incrementally increased until 5 or 10 MN.
- **Cyclic Loading Phase:** Horizontal loads are gradually increased by 10% of the capacity every cycle until 70% of the maximum resistance is reached. Figure 5.6 shows an illustration of the applied horizontal load, while the maximum resistance is determined from monotonic loading.

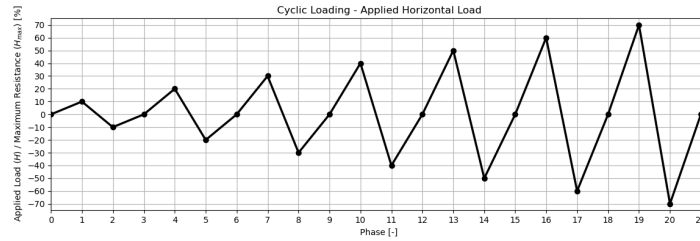


Figure 5.6: Applied horizontal load in cyclic loading analysis

5.5. Axial Loading Result

5.5.1. Load-Displacement Analysis

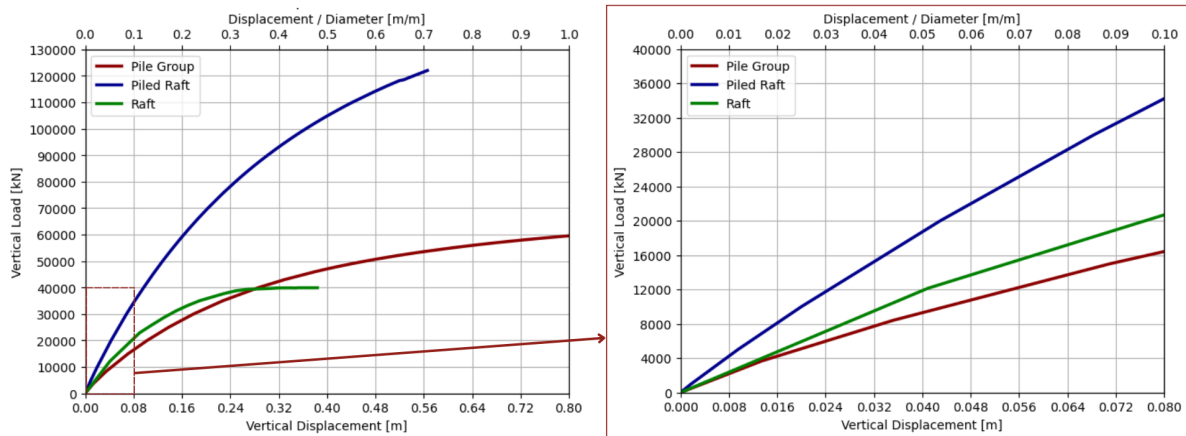


Figure 5.7: Axial load-displacement curve

Figure 5.7 shows the load-displacement behaviour of the three foundation types. The pile group analysis reached a vertical displacement (u_z) of 80 cm with a vertical load (V) of 60 MN. The piled raft foundation analysis was stopped at a vertical displacement u_z of 56 cm, with V reaching a very large value of 120 MN, while the raft foundation reached its ultimate capacity at approximately u_z of 28 cm with V_u of 40 MN.

This analysis demonstrates that for both the pile group and the piled raft, limiting the pile foundation capacity based on serviceability limits is reasonable. The load-displacement curves do not plateau, even at very large displacements—above 70% of D for the piled raft and 100% of D for the pile group. According to Eurocode 7, Section 7.6.1.1(3), the “failure” criterion is defined as the settlement of the pile top equal to 10% of the pile base diameter. In this case, this criterion corresponds to a settlement of 8 cm. By examining the figure, the initial stiffness of the foundation up to u_z of 8 cm (10% D) is almost linear, indicating that the response is mostly in the elastic zone with some plasticity.

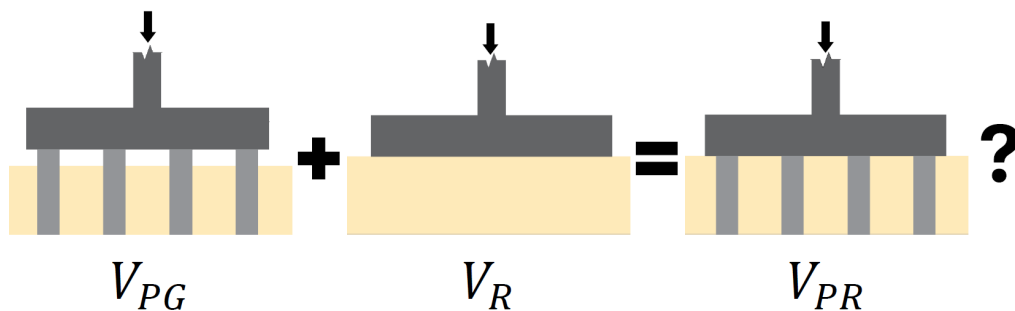


Figure 5.8: Illustration of axial load-displacement relationship between piled raft, pile group, and raft

Figure 5.8 illustrates the concept of load-displacement behaviour for a piled raft that will be examined in this analysis. The vertical resistance of the piled raft (V_{PR}) is compared to the vertical resistance of the pile group (V_{PG}) and the raft (V_R) at the same amount of vertical displacement (u_z). The goal is to determine how the behaviour of the piled raft differs from that of the pile group and the combined behaviour of the pile group and the raft. Using the parameters β_{PR} and ξ_{PR} as defined by Mandolini (2005) [38]:

$$\beta_{PR,V} = \frac{V_{PR}}{V_{PG}}, \quad \xi_{PR,V} = \frac{V_{PR}}{V_{PG} + V_R}$$

Figures 5.9 and 5.10 show β_{PR} and ξ_{PR} up to a displacement of 40 cm. It can be seen that both β_{PR} and ξ_{PR} increase with increasing displacement. β_{PR} starts at 1.89 at $u_z = 0$ cm and increases to 2.23 at $u_z = 40$ cm. Similarly, ξ_{PR} increases from 0.82 at zero displacement, reaches 1 at $u_z = 18$ cm, and further increases to 1.21 at $u_z = 40$ cm. These results are generally consistent with the findings of Sanctis and Mandolini (2006) [50], which state that β_{PR} is always above unity. However, while Sanctis and Mandolini reported ξ_{PR} values ranging from 0.82 to 1.00, these analyses show ξ_{PR} extending up to 1.21. This discrepancy may be attributed to the different soil types used in the studies; Sanctis and Mandolini's research was conducted in clay, whereas this analysis involves sand.

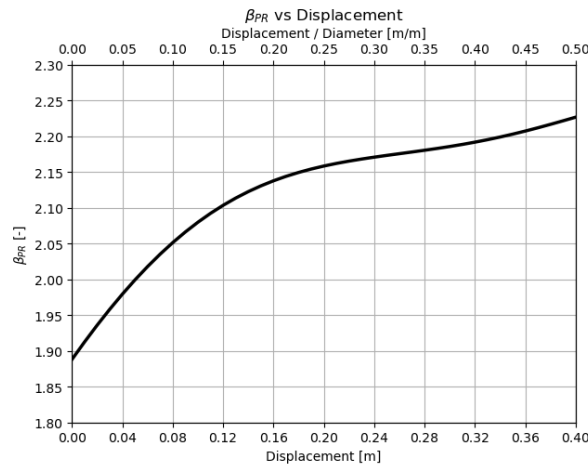


Figure 5.9: β_{PR} vs. displacement curve

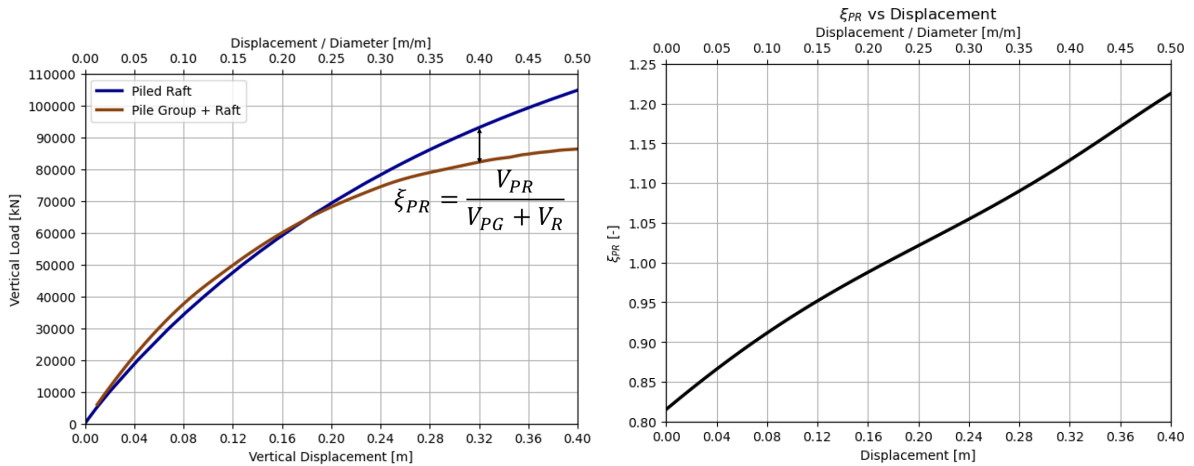


Figure 5.10: ξ_{PR} vs. displacement curve

To show the development of the deviatoric strain and vertical displacement, a cross-section will be plotted at the location indicated in Figure 5.11. Figure 5.12 presents the deviatoric strain contour with a legend ranging from 0 to 15%, while Figure 5.13 displays the vertical displacement with a normalised legend.

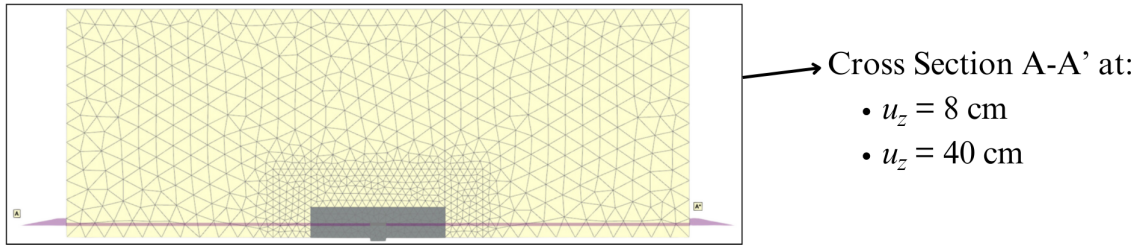


Figure 5.11: Cross-Section A-A'

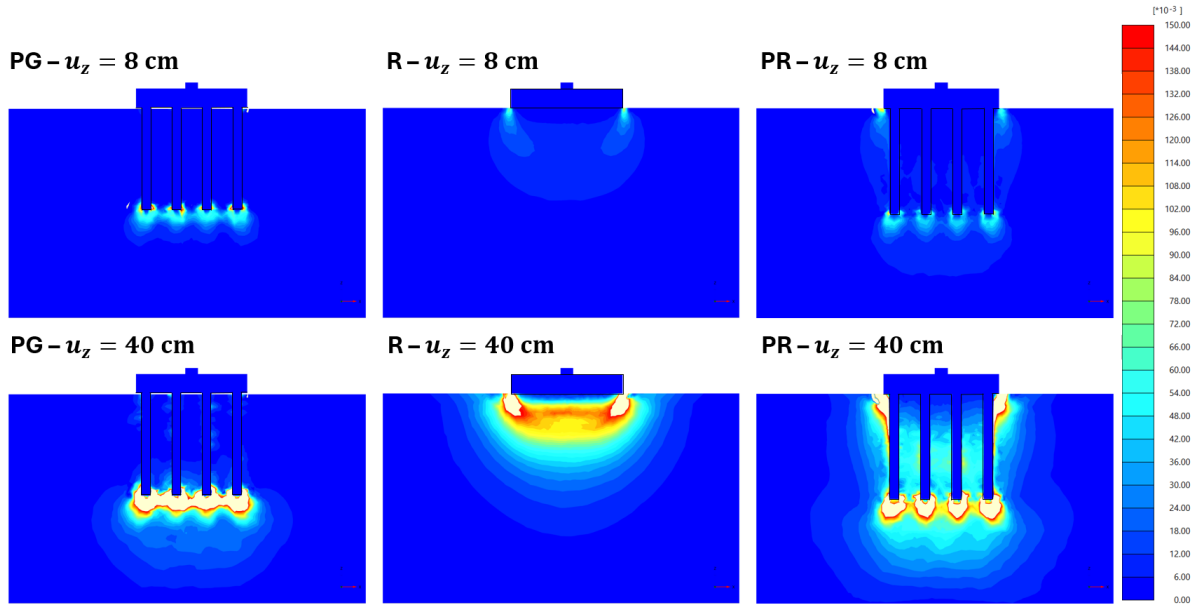


Figure 5.12: Deviatoric strain contour in axial loading analysis

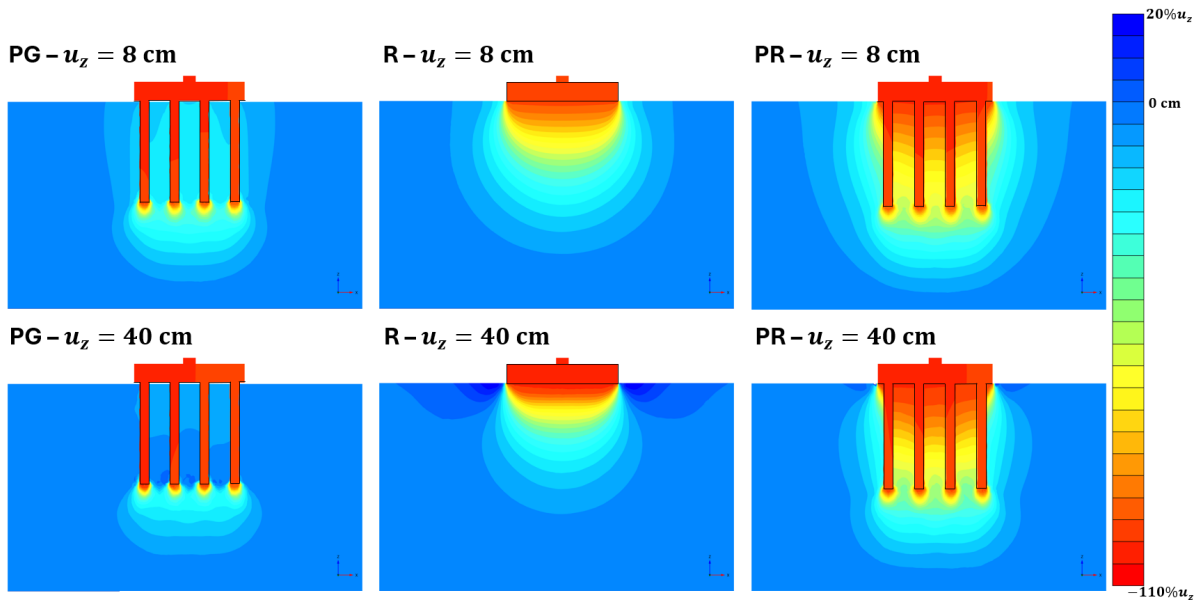


Figure 5.13: Vertical displacement contour in axial loading analysis

In Figure 5.12, at a displacement of 8 cm, the deviatoric strain distribution for the piled raft is not exactly similar

to the combined deviatoric strain distribution of the pile group and raft. This is due to the presence of the piles, which prevent the development of deviatoric strains beneath the piled raft foundation. At a displacement of 40 cm, the deviatoric strain distribution for the piled raft differs even more compared to that of the raft foundation. The piles in the piled raft prevent the soil inside the foundation from developing deviatoric strains, whereas the soil beneath the raft foundation shows very high deviatoric strains (greater than 10%). This is also evidenced by the vertical displacement contour in Figure 5.13, where at 40 cm of displacement, the raft has developed its failure mechanism around the foundation.

5.5.2. Load Contribution Analysis

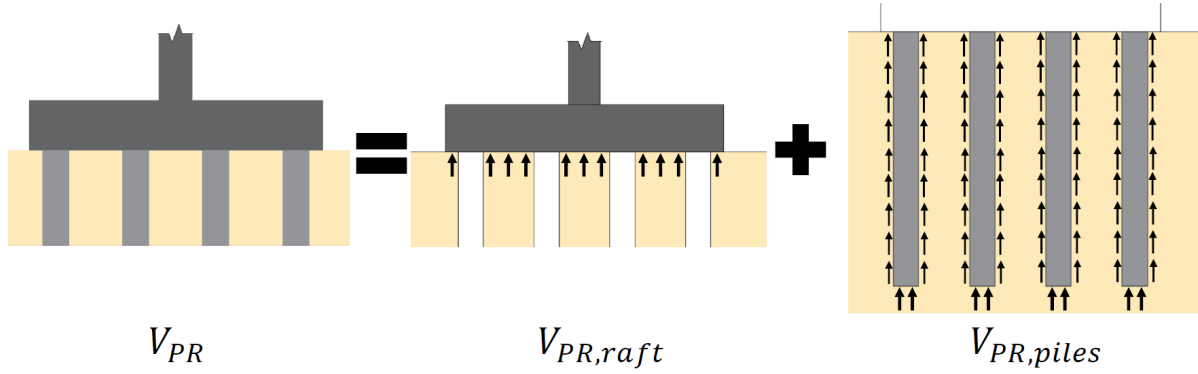


Figure 5.14: Illustration of axial load contribution in piled raft foundation

The vertical resistance of a piled raft (V_{PR}) is a combination of the resistance from the raft ($V_{PR,raft}$) and the resistance from the piles ($V_{PR,piles}$). Figure 5.14 illustrates the concept of separating the vertical resistance of a piled raft into these two contributions. The axial resistance of the piles ($V_{PR,piles}$) is obtained from the axial load at the top of the pile, as extracted from the FEA. The raft resistance ($V_{PR,raft}$) is then calculated using the following equation:

$$V_{PR,raft} = V_{PR} - V_{PR,piles}$$

Piles and Raft Contribution in Piled Raft

Figure 5.15 shows the load contributions between the piles and the raft in a piled raft foundation. Up to a displacement of around 5 cm, the load contribution is approximately 50% for both the piles and the raft. Beyond this point, the contribution of the piles increases, reaching about 60% at a displacement of 40 cm (50% of D), while the raft's contribution decreases to 40%.

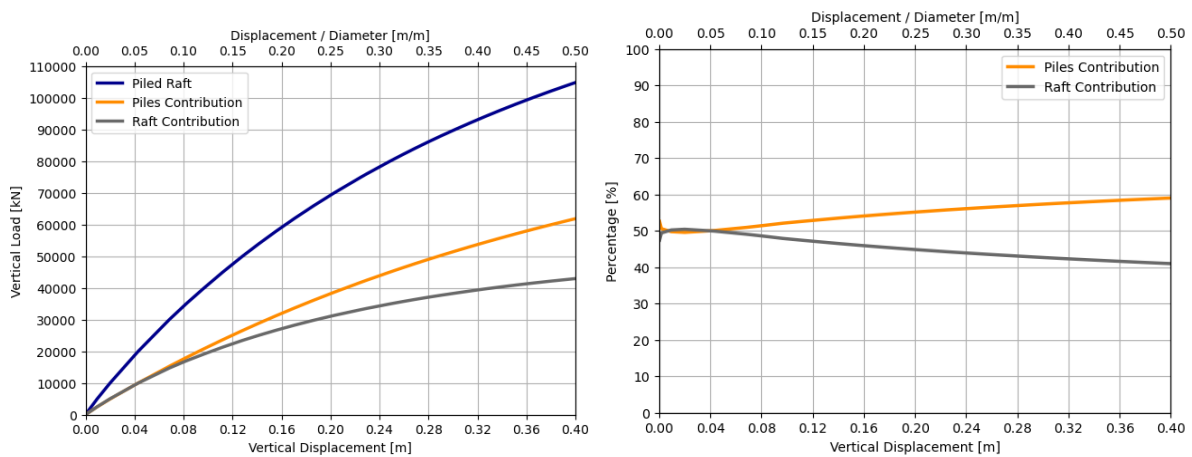


Figure 5.15: Load contribution in piled raft foundation

This behaviour is likely due to the soil beneath the raft experiencing very large strains. As the soil below the raft

reaches high strain levels, the system seeks equilibrium by redistributing the load to other areas capable of bearing the load, such as the soil around the piles. Additionally, the load distribution from the raft to the underlying soil increases the normal stress on the soil around the piles, enabling it to support more load. This demonstrates the interaction between the raft and the piles.

The pile contribution values range between 0.5 and 0.6. Research by Katzenbach (2002) [17] suggests that the optimal range for technical and economic considerations is between 0.5 and 0.7. Therefore, this piled raft configuration can be considered optimal.

Piles and Raft Contribution in Piled Raft Compared to Pile Group and Raft

In addition to comparing the contributions of the piles and raft within a piled raft foundation, the piles' contribution can be compared to that of a pile group, while the raft's contribution can be compared to that of a raft foundation. Figures 5.16 and 5.17 show these comparisons, respectively.

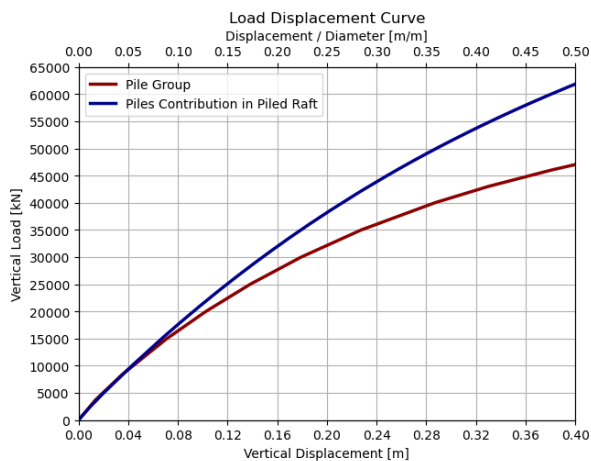


Figure 5.16: Pile contribution in piled raft vs. pile group

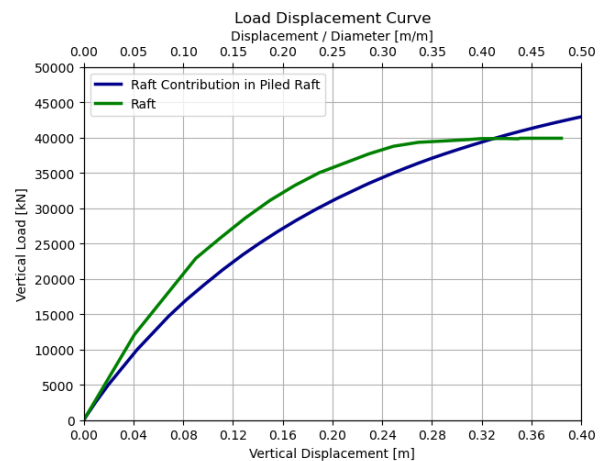


Figure 5.17: Raft contribution in piled raft vs. raft foundation

As shown in Figure 5.16, which compares the pile contributions in a piled raft and a pile group, the stiffness of the piles in both the pile group and the piled raft is similar up to a displacement of 8 cm. Beyond this point, the stiffness in the pile group foundation decreases significantly compared to the piles in the piled raft. This behaviour is expected due to the effect of the raft in the piled raft. The load transferred by the raft to the soil increases the normal stress in the soil, thereby increasing the resistance of the piles.

For the raft, as shown in Figure 5.17, the raft foundation initially displays higher stiffness and reaches its ultimate capacity at a displacement of 28 cm with an ultimate resistance of approximately 40 MN. In contrast, the raft contribution in the piled raft has a lower initial stiffness but continues to increase its resistance without reaching an ultimate capacity. This is due to the ability of the piled raft to redistribute loads between the piles and the raft, thereby preventing failure. At a displacement of 32 cm (40% of D), the load carried by the raft in the piled raft exceeds that carried by the standalone raft. The same redistribution mechanism allows the piled raft to continually increase its overall resistance.

Pile Base and Shaft Contribution

Figure 5.18 shows the contributions of the shaft and base in the piles for both the pile group and the piled raft. For the same amount of displacement, the contributions of both the shaft and the base in the piled raft are higher compared to the pile group. Additionally, the base contribution is significantly higher than that of the shaft, with the base contributing approximately 85-90%. This is rather different from what typically occurs in piles, where most of the load is usually carried by the shaft.

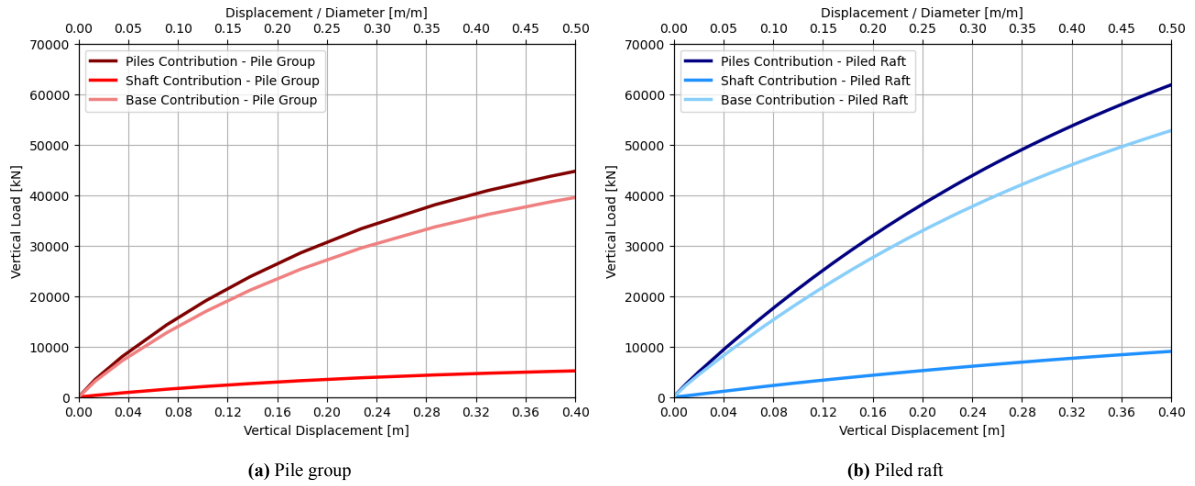


Figure 5.18: Pile shaft and base contribution

5.5.3. Pile Internal Force Analysis

In this section, the pile axial forces and moments over depth will be analysed. Six different vertical displacements (u_z) will be taken for comparison: 4 cm (5% D), 8 cm (10% D), 16 cm (20% D), 24 cm (30% D), 32 cm (40% D), and 40 cm (50% D).

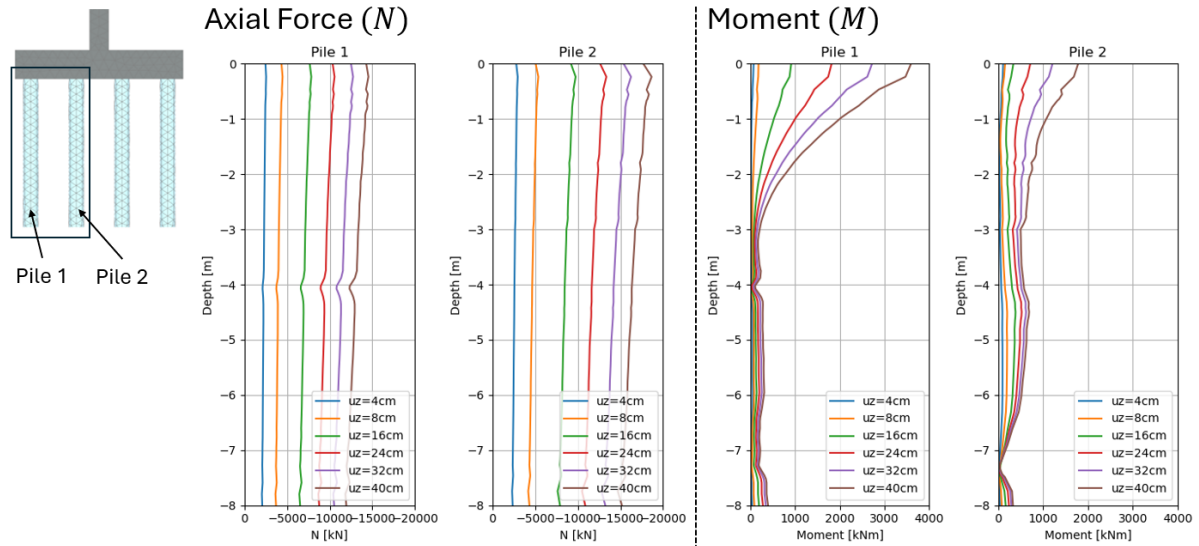


Figure 5.19: Internal force of piles in axial loaded piled raft

As shown in Figure 5.19, the axial load distribution indicates that the middle pile (Pile 2) is subjected to a higher axial load compared to the edge pile (Pile 1). This is due to the deformability of the raft and the concentration of load at the centre of the raft. Additionally, the edge pile (Pile 1) experiences a higher moment at the top compared to the middle pile (Pile 2), which is attributed to the bending of the raft. This suggests a high probability of structural failure at the top of the edge piles due to the internal moments. Such a failure is typically characterized as bending or flexural failure, where the pile may experience cracking, yielding, or even buckling under excessive moment.

5.6. Lateral Loading Result

As shown in Figure 5.5, the vertical and lateral loads are applied at the top of the pier. In this analysis, two heights are considered: 10 m and 20 m, with initial vertical loads of 5 MN and 10 MN, respectively. Following

the application of the initial vertical load, the foundation is subjected to lateral loading until the rotation at the top of the raft reaches $\theta = 4^\circ$. This section will provide the results and analysis for the 20 m height with a 10 MN initial vertical load, while the results for the other scenarios are provided in Appendix A.

5.6.1. Load-Displacement Analysis

Figure 5.20 shows the moment-rotation and horizontal load-displacement curves for the foundation with a height of 20 m and an initial vertical loading of 10 MN. In general, two key observations can be made. First, the raft foundation reaches failure, whereas the pile group and piled raft foundations do not. Additionally, it can be seen that initially, the piled raft is stiffer compared to the pile group. However, beyond a certain displacement and rotation, the piled raft and pile group exhibit almost the same gradient or stiffness. The point at which the pile group and piled raft start to have the same gradient or stiffness occurs at nearly the same rotation or horizontal displacement, where the raft foundation fails.

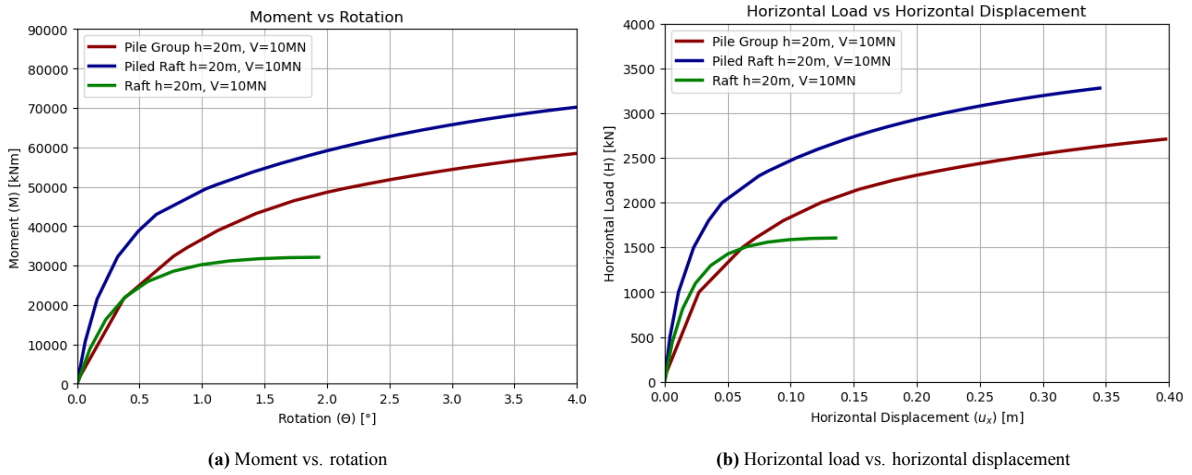


Figure 5.20: Load displacement curve in lateral loading analysis

Moment-Rotation

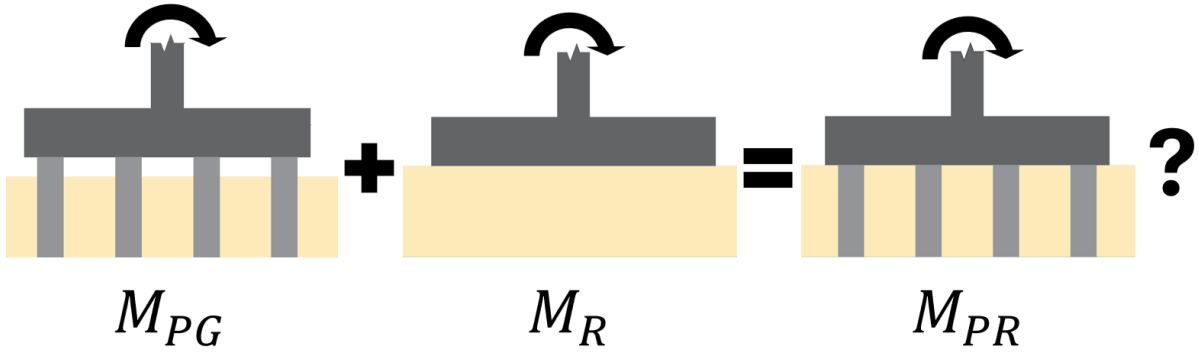


Figure 5.21: Illustration of moment relationship between piled raft, pile group, and raft

Similar to the concept mentioned in Chapter 5.5.1, Figure 5.21 illustrates the concept of moment-rotation behaviour for a piled raft that will be examined in this analysis. The moment resistance of the piled raft (M_{PR}) is compared to the moment resistance of the pile group (M_{PG}) and the raft (M_R) at the same amount of rotation (θ). The goal is to determine how the behaviour of the piled raft differs from that of the pile group and the combined behaviour of the pile group and the raft. Using the parameters $\beta_{PR,M}$ and $\xi_{PR,M}$:

$$\beta_{PR,M} = \frac{M_{PR}}{M_{PG}}, \quad \xi_{PR,M} = \frac{M_{PR}}{M_{PG} + M_R}$$

Figure 5.22 shows the $\beta_{PR,M}$ values, while Figure 5.23 shows the $\xi_{PR,M}$ values. Both $\beta_{PR,M}$ and $\xi_{PR,M}$ tend to decrease as rotation increases, eventually becoming constant. At $\theta < 2^\circ$, $\beta_{PR,M}$ starts from 2.3, decreases to 1.2, and then stabilises at 1.2 as rotation increases. This indicates that at $\theta < 2^\circ$, it takes 130% to 20% more moment to achieve the same rotation in the piled raft foundation compared to the pile group. At $\theta > 2^\circ$, 20% more moment is required to achieve the same rotation in the piled raft foundation compared to a pile group. Interestingly, $\theta \approx 2^\circ$ corresponds to the rotation at which the raft foundation fails. Beyond this rotation, the moment-rotation slopes for the piled raft and pile group become identical, causing $\beta_{PR,M}$ to remain constant.

Initially, $\xi_{PR,M} \approx 1$, suggesting that the stiffness of the piled raft is nearly the same as the combined stiffness of the pile group and raft considered as separate foundations. As rotation increases, $\xi_{PR,M}$ decreases and stabilises at around 0.75 after $\theta > 0.5^\circ$. This indicates that it takes 25% less moment to achieve the same rotation in the piled raft foundation compared to the combined moment of the pile group and raft as separate foundations.

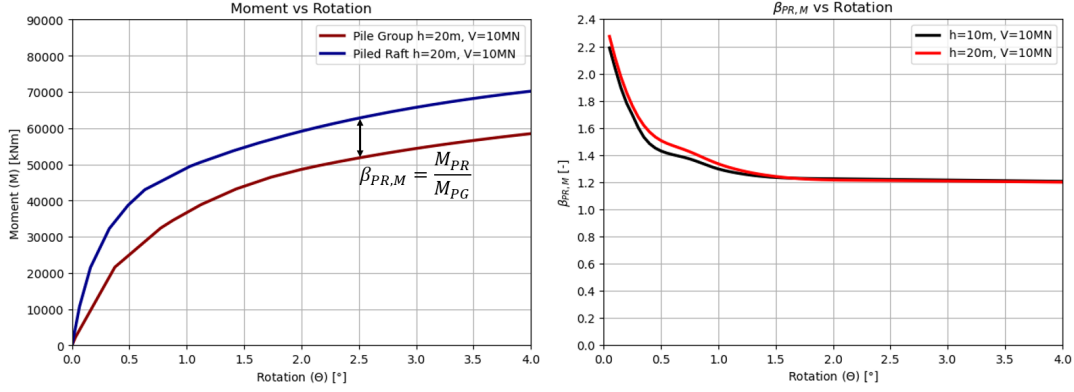


Figure 5.22: $\beta_{PR,M}$ vs. rotation

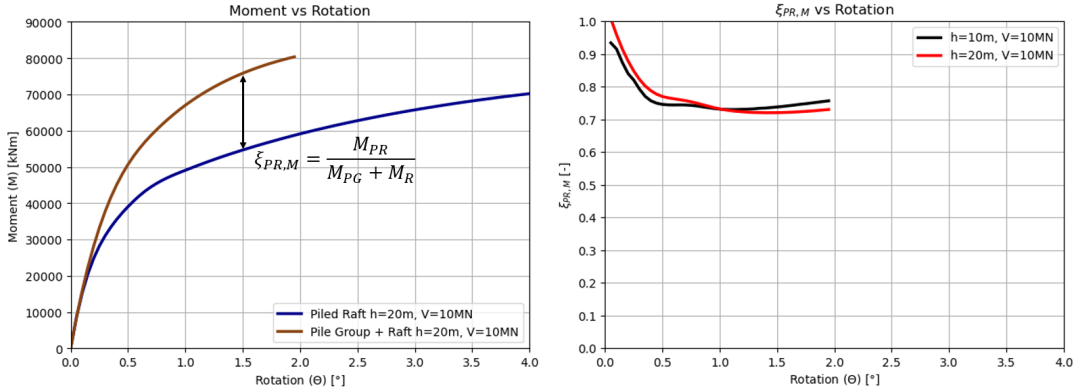


Figure 5.23: $\xi_{PR,M}$ vs. rotation

Horizontal Load-Displacement

Similar to the concept analysed for moment-rotation, the parameters $\beta_{PR,H}$ and $\xi_{PR,H}$ are used:

$$\beta_{PR,H} = \frac{H_{PR}}{H_{PG}}, \quad \xi_{PR,H} = \frac{H_{PR}}{H_{PG} + H_R}$$

Figure 5.24 shows the $\beta_{PR,H}$ values, while Figure 5.25 shows the $\xi_{PR,H}$ values. Similar to $\beta_{PR,M}$ and $\xi_{PR,M}$, both $\beta_{PR,H}$ and $\xi_{PR,H}$ tend to decrease as horizontal displacement increases, eventually becoming constant. At $u_x < 0.14$ m, $\beta_{PR,H}$ starts from 2.1, decreases to 1.3, and then stabilises at 1.3 as horizontal displacement increases. This indicates that at $u_x < 0.14$ m, it takes 110% to 30% more horizontal load (H) to achieve the same horizontal displacement in the piled raft foundation compared to the pile group. At $u_x > 0.14$ m, 30% more horizontal load is required to achieve the same horizontal displacement in the piled raft foundation compared to a pile group. Interestingly, $u_x \approx 0.14$ m corresponds to the horizontal displacement at which the raft foundation

fails. Beyond this horizontal displacement, the horizontal load-displacement slopes for the piled raft and pile group become identical, causing $\beta_{PR,H}$ to remain constant.

For $\xi_{PR,H}$, the values range from 0.7 to 0.8, indicating that it takes 20% to 30% less horizontal load to achieve the same horizontal displacement in the piled raft compared to the combined horizontal load of the pile group and raft as separate foundations.

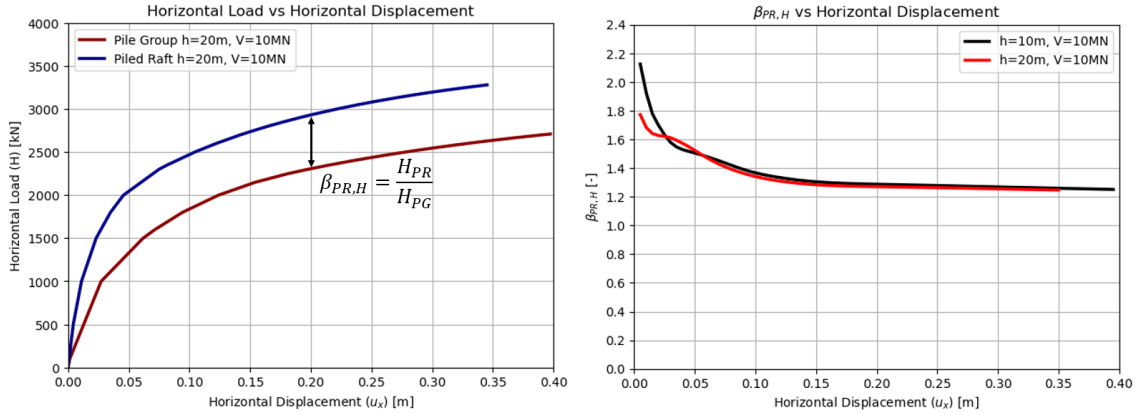


Figure 5.24: $\beta_{PR,H}$ vs. horizontal displacement

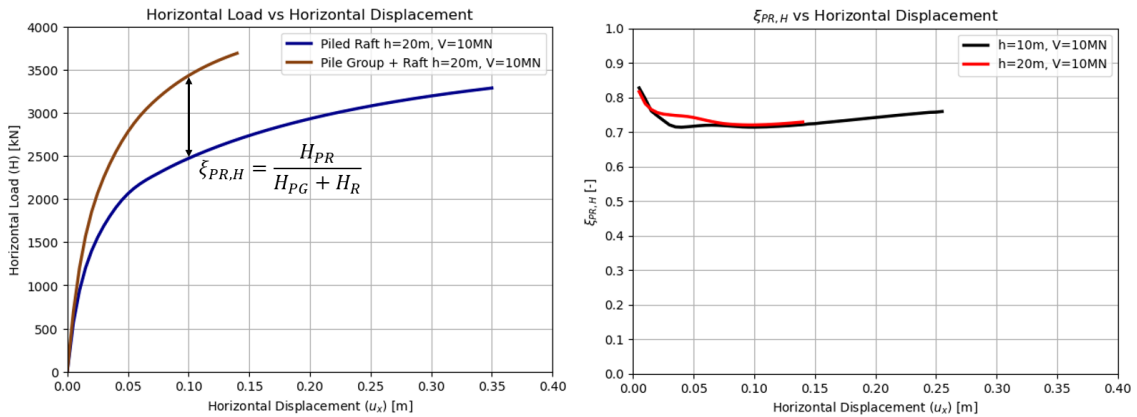


Figure 5.25: $\xi_{PR,H}$ vs. horizontal displacement

Cross-Sections

Examining the same cross-section indicated in Figure 5.11 at $\theta = 0.5^\circ$ and $\theta = 2.0^\circ$, Figure 5.26 presents the deviatoric strain contour with a legend ranging from 0 to 15%, while Figure 5.27 displays the total displacement. Figure 5.26 shows that in the raft foundation, due to rotation, the raft is loading the soil beneath the right side of the foundation. At a larger rotation of $\theta = 2.0^\circ$, Figure 5.27 shows that the left side of the raft foundation essentially loses contact with the soil.

For the pile group and piled raft, although the piled raft is a combination of a raft foundation and a pile group foundation, the deviatoric strain mainly develops at the bottom of the pile in compression and on the right side of the foundation due to the foundation pushing the soil. The deviatoric strain development on the left side is due to extension. This deviatoric strain development indicates a high possibility of a gap forming on the left side of the foundation and the failure of a wedge of soil on the right side, as shown in Figure 2.5.

Furthermore, the total displacement shown in Figure 5.27 indicates a centre of rotation for all scenarios. For the pile group and piled raft, the centre of rotation shifts towards the centre of the foundation as the rotation increases.

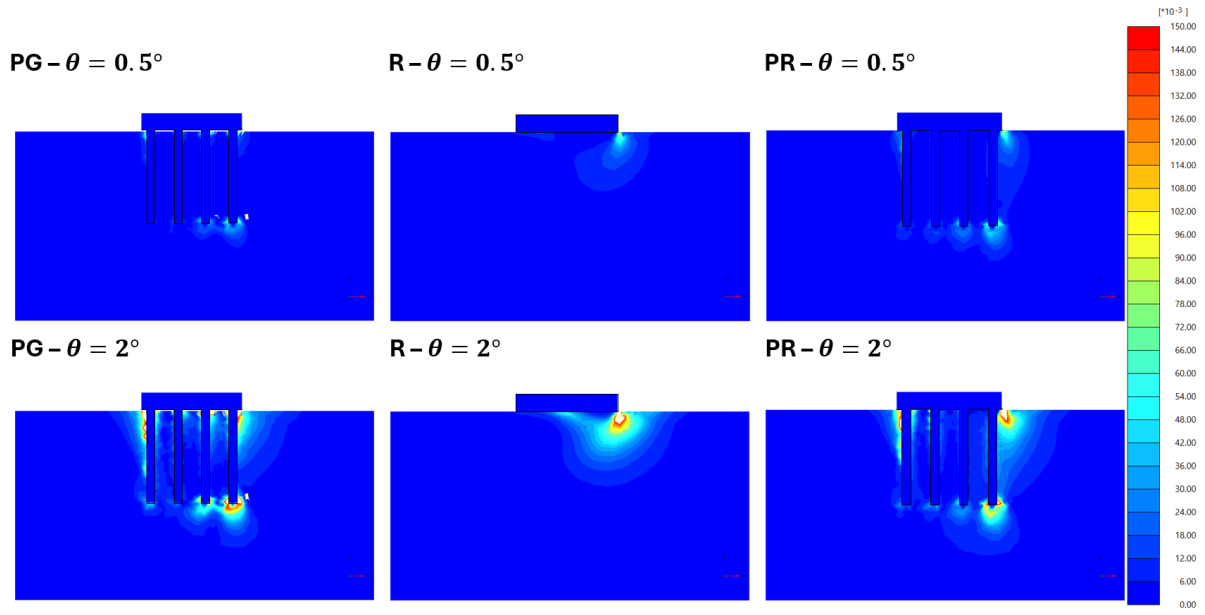


Figure 5.26: Deviatoric strain contour in lateral loading analysis

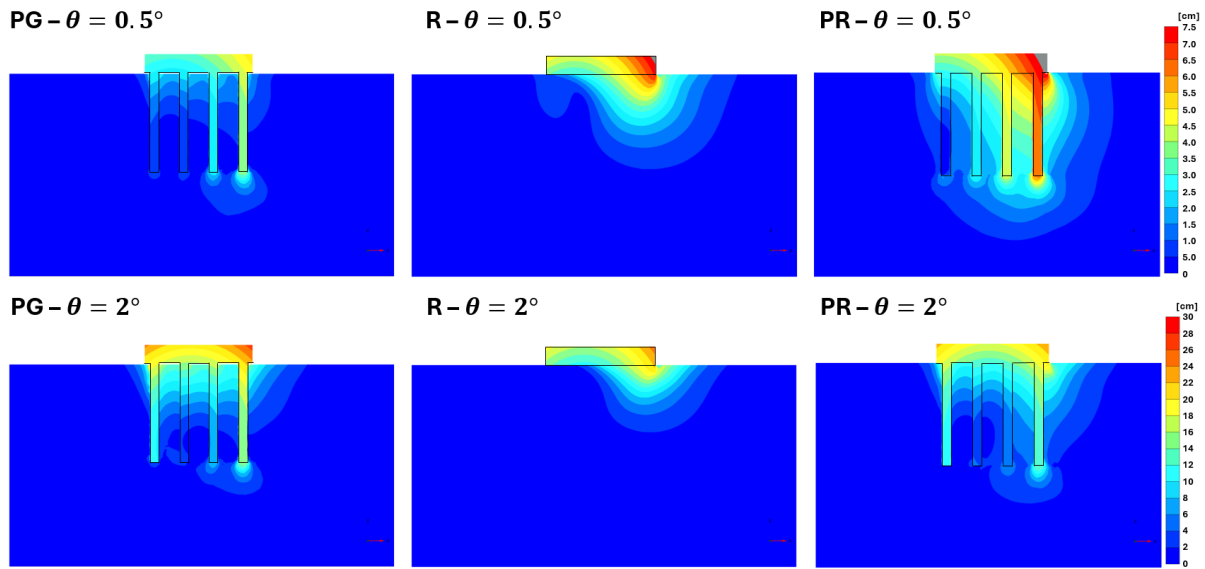


Figure 5.27: Total displacement contour in lateral loading analysis

5.6.2. Moment Contribution Analysis

This section discusses the moment contributions of the raft and piles in a piled raft foundation. The moment resistance of a piled raft foundation (M_{PR}) is a combination of the resistance from the raft (M_{raft}) and the piles (M_{piles}). In the piles, some of the moment will be transferred to the moment at the top of the pile ($M_{pile,M}$) and the axial load ($M_{pile,N}$), causing some piles to be in compression and others in tension. Typically, the moment contributions from the raft (M_{raft}) and the moment at the top of the pile ($M_{pile,M}$) are disregarded, which can lead to an expensive design in the piles. This section focuses on quantifying these moment contributions to determine whether it is appropriate to disregard them.

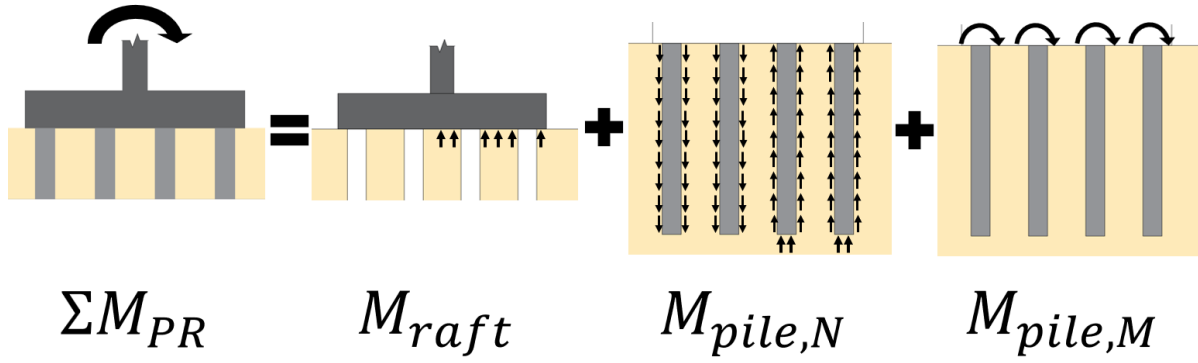


Figure 5.28: Illustration of moment contribution in piled raft foundation

As illustrated in Figure 5.28, the acting moment in the entire foundation can be calculated as:

$$M = M_{pile,N} + M_{pile,M} + M_{raft} \quad (5.1)$$

where M is the applied moment, $M_{pile,N}$ is the moment contribution of the piles due to axial load, $M_{pile,M}$ is the moment contribution at the heads of the piles, and M_{raft} is the moment contribution of the raft. The moment contribution of the piles due to axial load, $M_{pile,N}$, can be calculated as the product of the axial load at the top of the pile (N_{pile}) and the distance from the pile to the centre of the foundation. The moment contribution at the heads of the piles ($M_{pile,M}$) can be extracted from PLAXIS 3D. Therefore, M_{raft} can be calculated using:

$$M_{raft} = M - M_{pile,N} - M_{pile,M} \quad (5.2)$$

Figure 5.29 shows the moment distribution in the piled raft over rotation for a height of 20 m. It can be seen that the contributions of $M_{pile,N}$ and M_{raft} are significant. Figure 5.29b quantifies the M_{raft} contribution relative to the rotation. It is evident that the raft has a large contribution, providing approximately 40%-50% of the total applied moment.

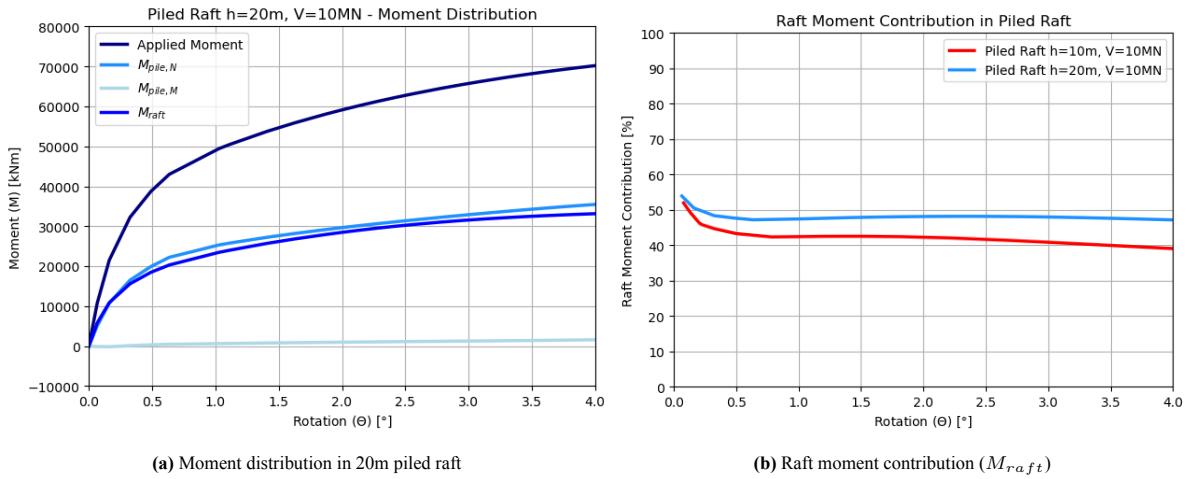


Figure 5.29: Moment distribution and raft contribution in piled raft foundation

Pile $M_{pile,N}$ Contribution

While the previous discussion focused on the piled raft, this section analyses the pile contribution. Figure 5.30 shows the pile contributions $M_{pile,N}$ in both the pile group and the piled raft.

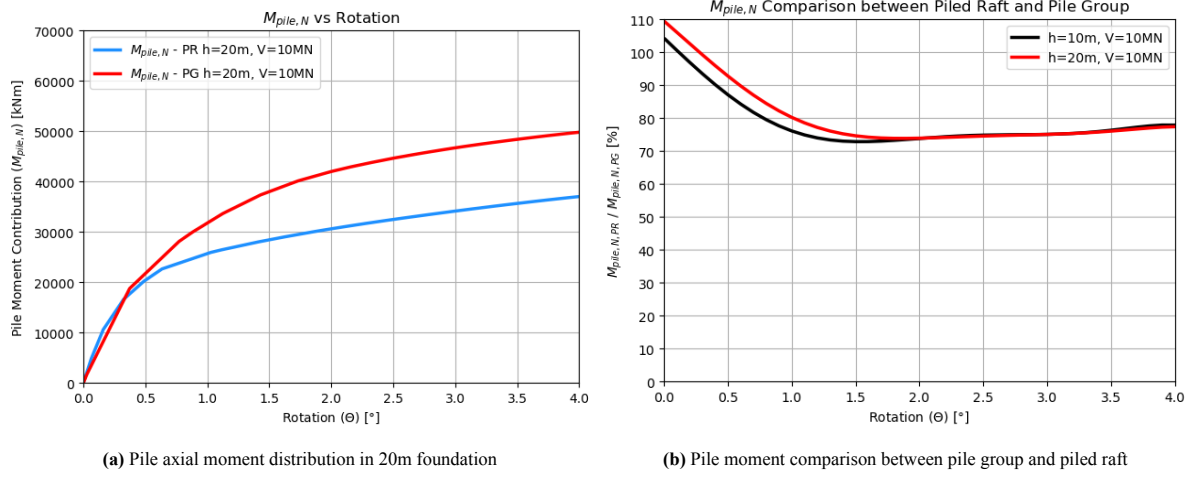


Figure 5.30: Pile axial moment ($M_{pile,N}$) distribution and comparison between pile group and piled raft

It can be seen that $M_{pile,N}$ in the pile group is almost equal to the applied moment with some contribution from $M_{pile,M}$, whereas $M_{pile,N}$ in the piled raft is smaller. Figure 5.30b compares $M_{pile,N}$ in the piled raft to $M_{pile,N}$ in the pile group at the same amount of rotation. The comparison shows that initially, the moment contribution of the piles in the piled raft is almost the same as in the pile group. As the rotation increases, the moment contribution of the piles in the piled raft decreases to around 75% of that in the pile group, indicating that the piles in the piled raft carry 25% less moment compared to the piles in the pile group. This suggests that by accounting for the contribution of the raft, the piles can be designed smaller due to the reduced load they need to carry.

5.6.3. Pile Internal Force Analysis

To provide more detail about the axial forces of each pile, Figure 5.31 shows the axial force versus rotation for each pile. It can be seen that pile 4, which is in compression, carries the most compressive load, while piles 1 and 2 reach their maximum tensile resistance at approximately 700 kN. Similar to previous results, each pile in the piled raft carries less axial load compared to the piles in the pile group.

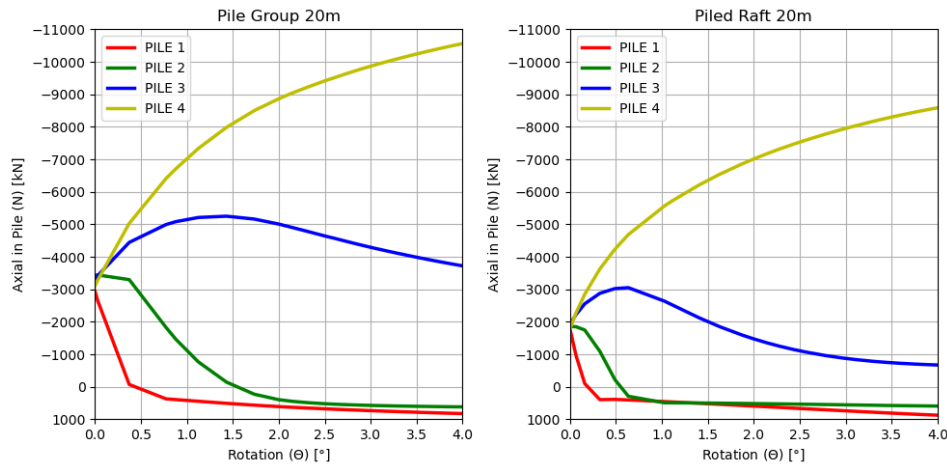


Figure 5.31: Axial force in piles vs. rotation in pile group and piled raft

Moreover, the pile axial forces and moments over depth will be analysed. Six different rotations will be shown for comparison at (θ): 0.5° , 1.0° , 1.5° , 2.0° , 3.0° , and 4.0° .

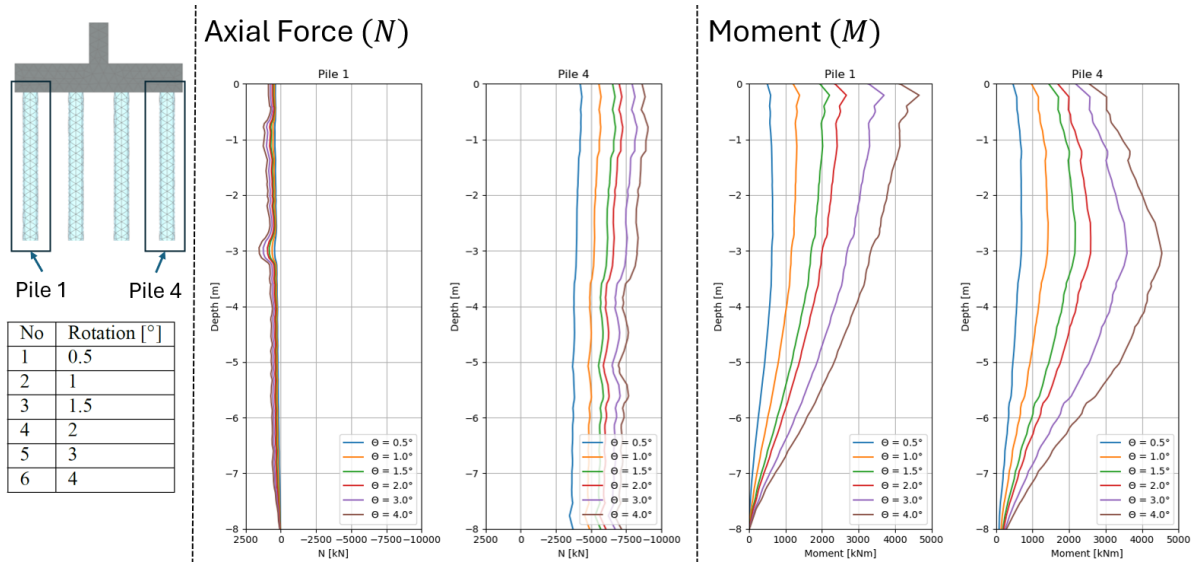


Figure 5.32: Internal force of piles in lateral loaded piled raft

As shown in Figure 5.32, Pile 1 has reached its tensile resistance at the first plot of $\theta = 0.5^{\circ}$, while Pile 4 (in compression) shows that the compression keeps increasing with rotation. Despite this, the moment in Pile 1 continues to increase, particularly at the top. Pile 1 and Pile 4 show different moment behaviours: Pile 1 behaves similarly to a cantilever, with the maximum moment at the top, whereas Pile 4 behaves with the maximum moment near the middle, although the moment at the top also continues to increase.

5.7. Cyclic Loading

The goal of the cyclic analysis is to determine if a viscoelastic model can accurately capture the behaviour of a piled raft foundation subjected to cyclic loading, which will be useful for seismic design as discussed in Chapter 1. A viscoelastic model will be used to fit the load-displacement curve and produce the stiffness, viscosity, and damping curves. This analysis is conducted with three different heights, i.e., 10 m, 15 m, and 20 m, each with an initial vertical load of 5 MN. The horizontal load is gradually increased by 10% of the maximum horizontal load until reaching 70% of the maximum load. The maximum horizontal load for the piled raft is set to 4000 kN for a height of 10 m, 3000 kN for a height of 15 m, and 2500 kN for a height of 20 m. Table 5.5 tabulates the maximum horizontal load applied at each cycle, and Figure 5.33 shows the cyclic loading in each phase.

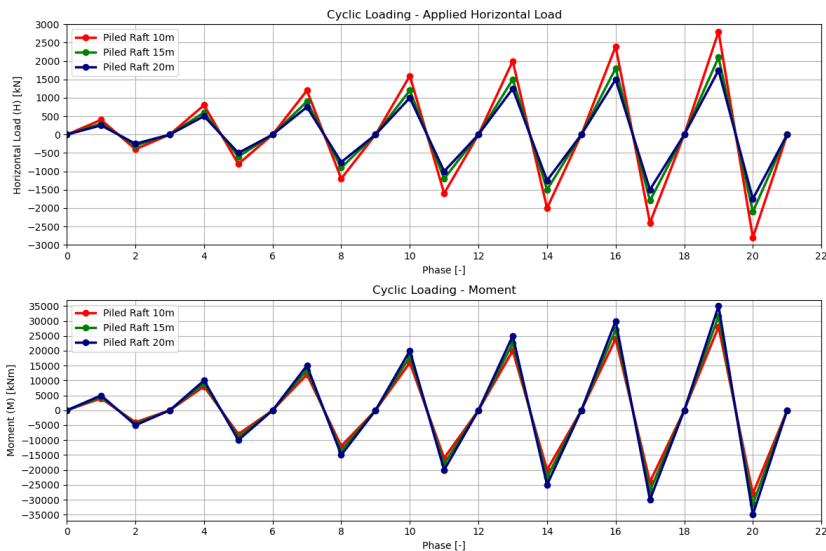


Figure 5.33: Applied horizontal load for every phase in cyclic loading

	Maximum Horizontal Load [kN]						
Cycle	1	2	3	4	5	6	7
Height: 10m	400	800	1200	1600	2000	2400	2800
Height: 15m	300	600	900	1200	1500	1800	2100
Height: 20m	250	500	750	1000	1250	1500	1750

Table 5.5: Maximum horizontal load in every cycle

5.7.1. Load Displacement Analysis

Figure 5.34 shows the moment-rotation and horizontal load-displacement curves for a 20 m high foundation. The elastoplastic behaviour and stiffness degradation are visible from the backbone of the curve.

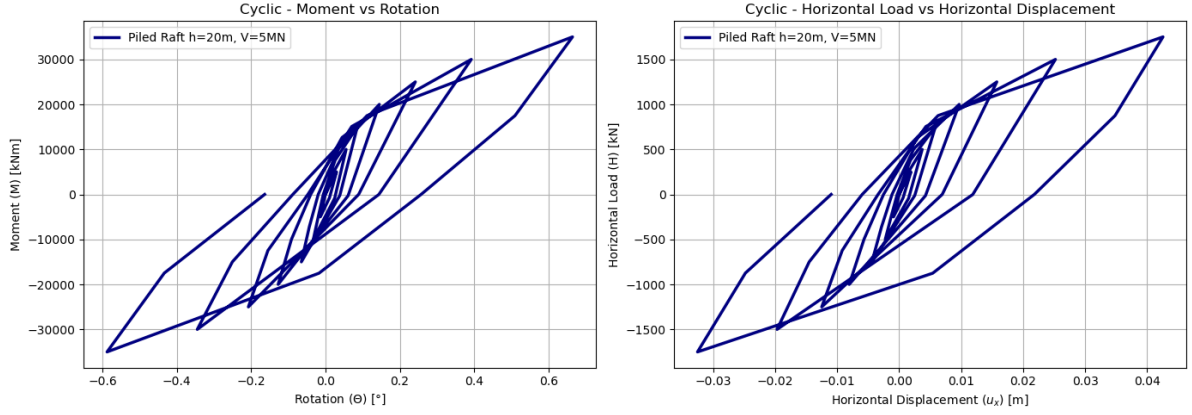


Figure 5.34: Cyclic moment vs. rotation and horizontal load vs. displacement curve

5.7.2. Viscoelastic Model

A viscoelastic model will be used to capture the elastoplastic behaviour of the foundation by fitting the load-displacement curve with optimal stiffness and viscosity values for each cycle. Due to the nature of this analysis, which is conducted using an elastic-plastic model, there is no time dependency. Therefore, the viscosity in the viscoelastic model needs to be modified to be frequency independent. The equilibrium equation is defined as:

$$H = G_H u_x + \frac{\eta_H}{\omega} \dot{u}_x \quad (5.3)$$

At an instant between time t and t_{i-1} , the equation above can be written as:

$$\frac{H_i + H_{i-1}}{2} = G_H \left(\frac{u_{x,i} + u_{x,i-1}}{2} \right) + \frac{\eta_H}{\omega} \left(\frac{u_{x,i} - u_{x,i-1}}{dt} \right) \quad (5.4)$$

Rearranging the equation above leads to:

$$u_{x,i} = \frac{dt(H_i + H_{i-1}) - (dtG_H - 2\frac{\eta_H}{\omega})u_{x,i-1}}{dtG_H + 2\frac{\eta_H}{\omega}} \quad (5.5)$$

Similarly, for moment and rotation:

$$M = G_M \theta + \frac{\eta_M}{\omega} \dot{\theta} \quad (5.6)$$

The equation above can be rearranged to:

$$\theta_i = \frac{dt(M_i + M_{i-1}) - (dtG_M - 2\frac{\eta_M}{\omega})\theta_{i-1}}{dtG_M + 2\frac{\eta_M}{\omega}} \quad (5.7)$$

Given that viscosity is defined as frequency independent, the damping ratio is defined as:

$$D = \frac{\eta}{2G} \quad (5.8)$$

5.7.3. Viscoelastic Model Result

To optimize the viscoelastic model and fit it to the finite element results, H and M were arranged as harmonic loads. An optimization function was defined to match the calculated u_x and θ with the finite element results by minimizing the loss function:

$$L = \sum (u_{x,calculated} - u_{x,FE})^2 \quad (5.9)$$

The code used for this optimization process is provided in Appendix C.

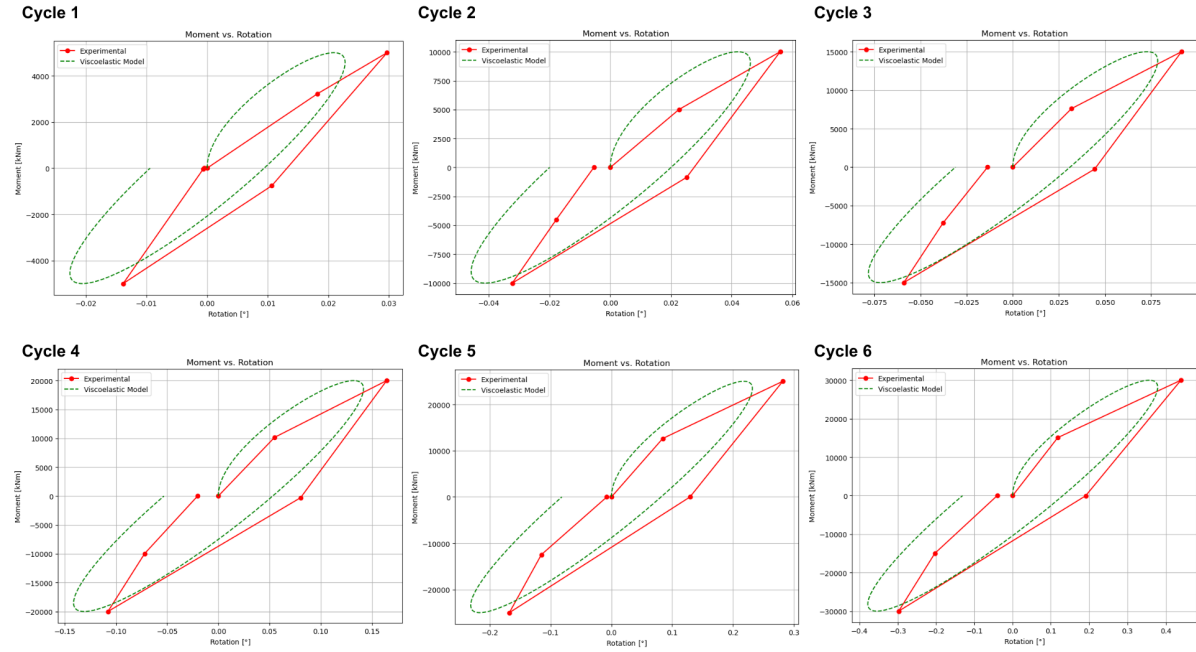


Figure 5.35: Viscoelastic calibration in the moment-rotation results of a piled raft with a height of 20 meters

Figure 5.35 shows the calibration for the moment-rotation curves of the piled raft foundation with a height of 20 m. Although the viscoelastic model does not fully capture the cyclic behaviour of the piled raft foundation, it does adequately represent the stiffness and area of the response, fitted using the viscosity. Therefore, the stiffness degradation, viscosity, and damping ratio over cycles can be plotted.

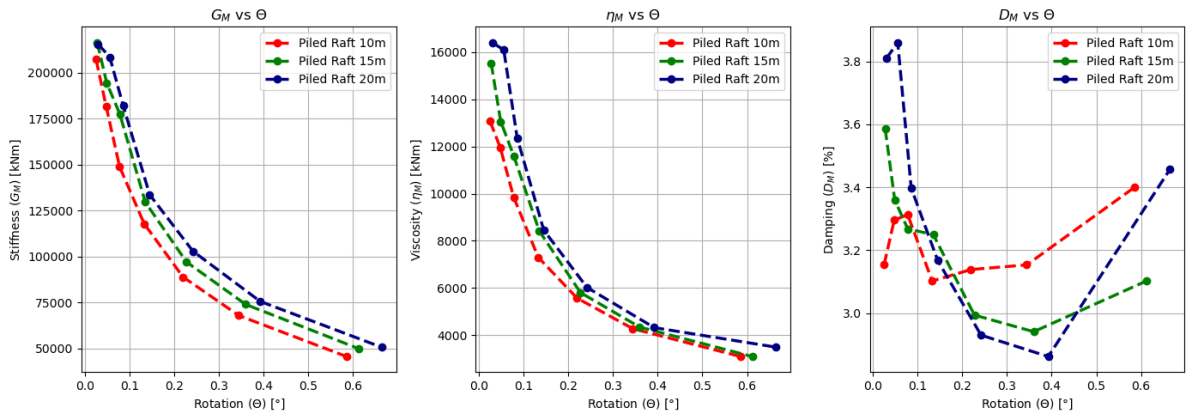


Figure 5.36: Stiffness degradation, viscosity, and damping results - Moment vs. rotation

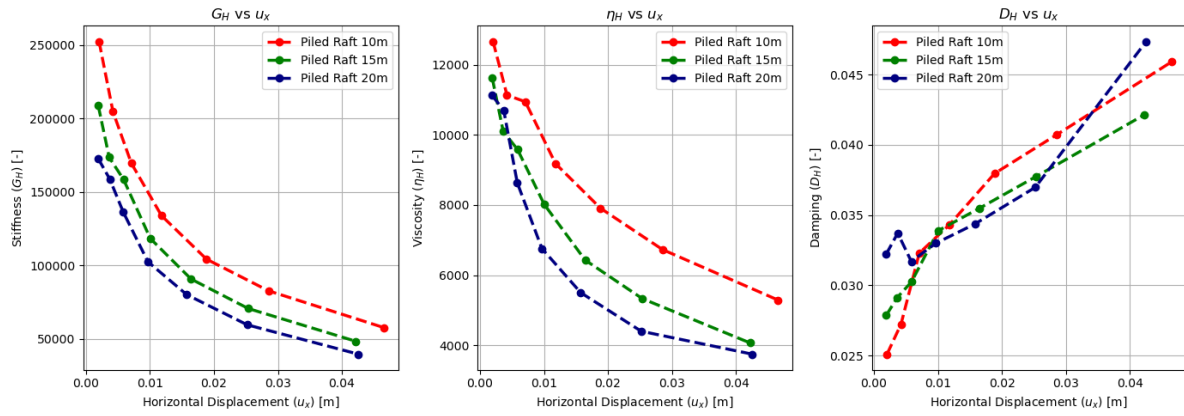


Figure 5.37: Stiffness degradation, viscosity, and damping results - Horizontal load vs. horizontal displacement

Figures 5.36 and 5.37 show the calculated stiffness, viscosity, and damping based on the viscoelastic model for the moment and horizontal load, respectively. It can be observed that the stiffness and viscosity gradually decrease with increasing displacement and rotation. The damping ratio for horizontal loading gradually increases with displacement, while for the moment, it shows a "U" trend, initially decreasing and then increasing.

6

Experimental Analysis

The experimental testing described in this thesis is part of the IDEA League project titled "INITIATION," led by Pietro Marveggio. The experiments were primarily conducted by Pietro and Matteo Corigliano. As a master student, I had the opportunity to assist them in this research. While this thesis focuses on one aspect of the experimental analysis, the remaining data will be further analysed as part of the INITIATION project.

6.1. Centrifuge Specification

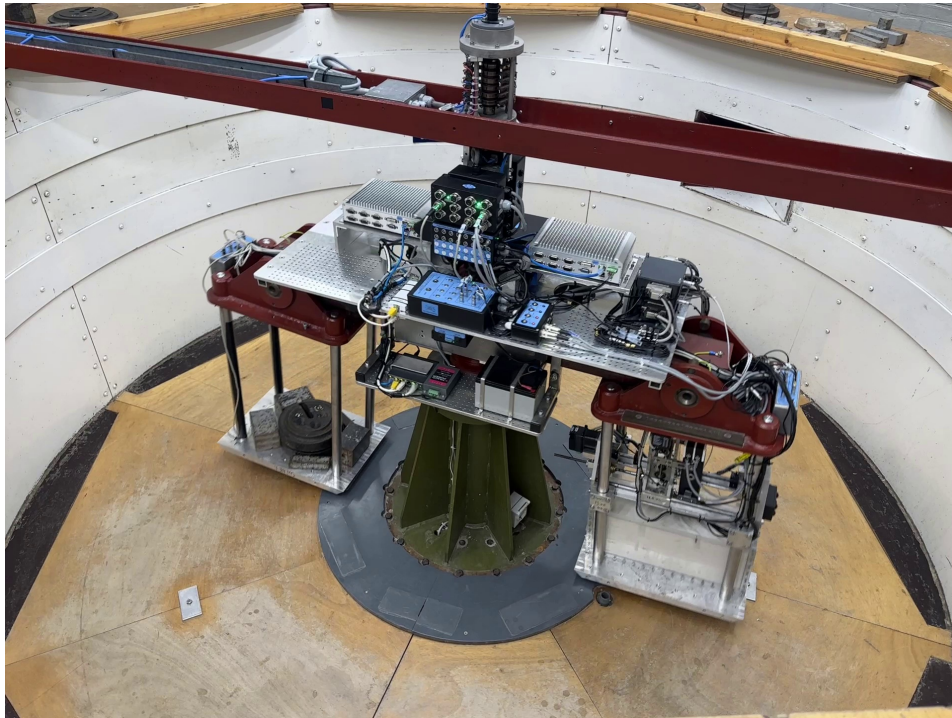


Figure 6.1: TU Delft geotechnical centrifuge

The centrifuge used in this thesis is the beam centrifuge at TU Delft, as shown in Figure 6.1, with a target scale factor of $N = 100g$. Tests with $N = 50g$ have also been performed, and the results will be presented in Appendix B. Table 6.1 shows its main specifications.

Radius	1.215 m
Basket Size	400 x 400 x 500 mm
Maximum Payload	30 kg
Maximum Acceleration	300 g
Maximum Rotation Velocity	450 RPM
Maximum Data Logging Frequency	200 kHz
Number of Data Logging Channels	46
Data Transfer System	Wireless router and fiber-optic slip rings

Table 6.1: Specifications of the TU Delft geotechnical centrifuge

6.2. Model Strongbox

The model is designed as a half-model, with the foundation positioned at the middle front of the model strongbox. In total of 3 strongboxes were used in this experiment, with Figure 6.2 shows the strongbox B as example. The rest are tabulated in Table 6.3 with the size of the inside of the strongbox.

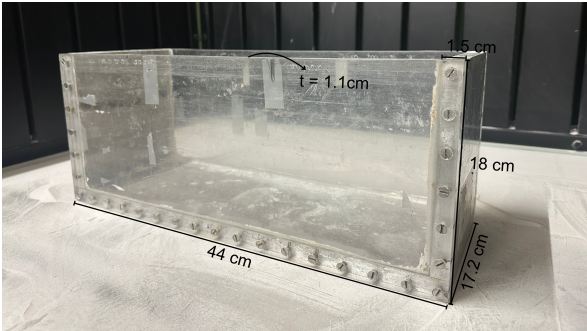


Figure 6.2: Strongbox B

ID	Weight [kg]	Inner Length [cm]	Inner Width [cm]
A	7.569	41	14.5
B	6.931	41	15
C	7.460	41	14.7

Figure 6.3: Strongbox details

6.3. Material

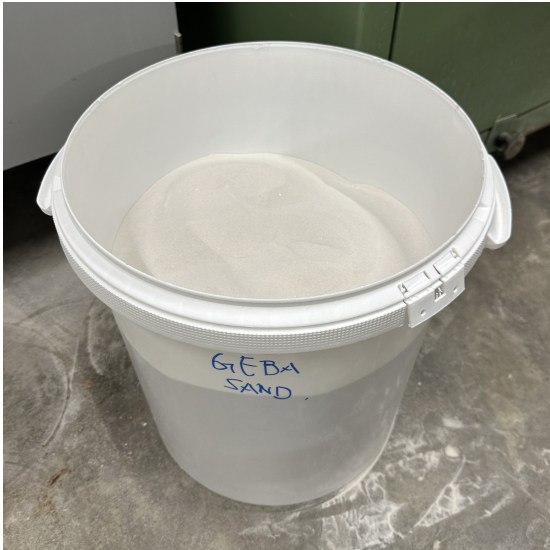


Figure 6.4: Geba Sand

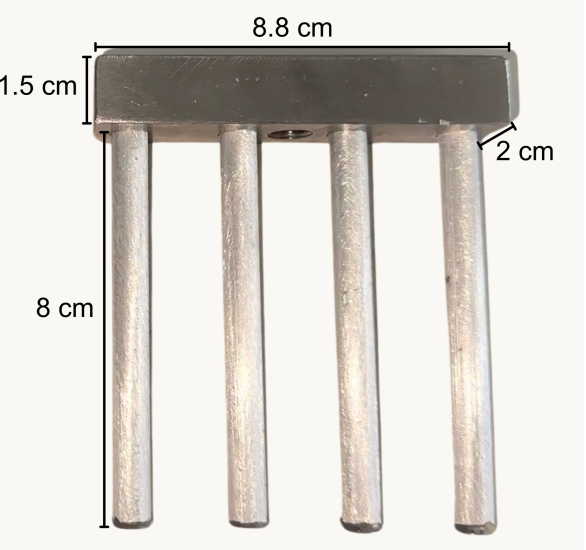


Figure 6.5: Small-scale foundation model

6.3.1. Soil

The soil used is Geba sand, as mentioned in Chapter 4.2, with a target relative density of around 70 to 80%. Figure 6.4 shows the Geba sand used in the experiments.

6.3.2. Foundation Model

With $N = 100g$, the foundation model size will be 100 times smaller. Figure 6.5 shows the small scale foundation model and its dimensions.

6.3.3. Strain Gauge

Strain gauges are used in this experiment as load cells to measure the forces acting on the foundation. The strain gauge used has a Gauge Factor (GF) of $2.35 \pm 2\%$. Figure 6.6 shows the detailed strain gauge specifications, and Figure 6.7 provides additional details about the strain gauge dimensions.



Figure 6.6: Strain gauge specifications

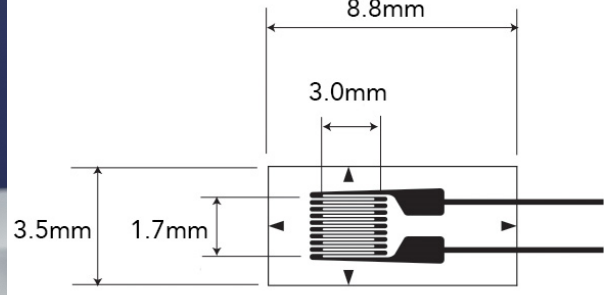


Figure 6.7: Strain gauge dimensions

A strain gauge typically consists of a conductive wire arranged in a grid pattern on a flexible carrier. When subjected to strain, the wire deforms, causing a change in its electrical resistance. This change in resistance can be measured to determine the amount of strain. The concept used to calculate the resistance in the strain gauge is the Wheatstone bridge, shown in Figure 6.8. In this thesis, a quarter Wheatstone bridge is used. Figure 6.9 shows the configuration connecting the strain gauges to the terminal, which will be mounted on the foundation. The quarter Wheatstone bridge connection works such that when an input voltage is applied, the output voltage can be calculated using Equation 6.1.

$$V_{out} = V_{in} \left(\frac{R_{G1}}{R_{G1} + R_{R1}} - \frac{R_{R2}}{R_{R2} + R_{R3}} \right) \quad (6.1)$$

According to this equation, when $\frac{R_{G1}}{R_{G1} + R_{R1}} = \frac{R_{R2}}{R_{R2} + R_{R3}}$, there will be no potential difference, and the output, V_{out} , will be zero. In this case, the Wheatstone bridge is balanced. When a change in resistance occurs in any of the resistors, the system becomes imbalanced, producing a potential difference that can be read as an output voltage. The calibration and analysis of load cells were not performed as part of this thesis.

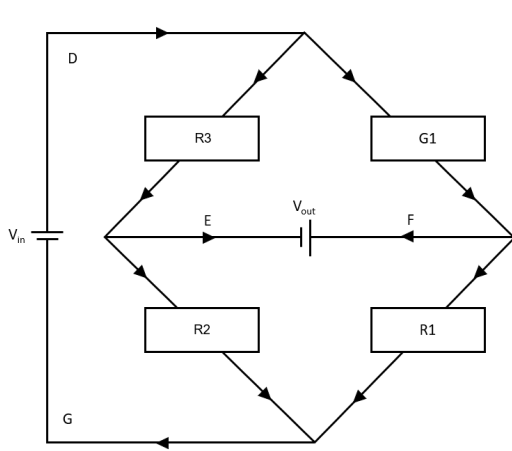


Figure 6.8: Quarter Wheatstone bridge load cell diagram

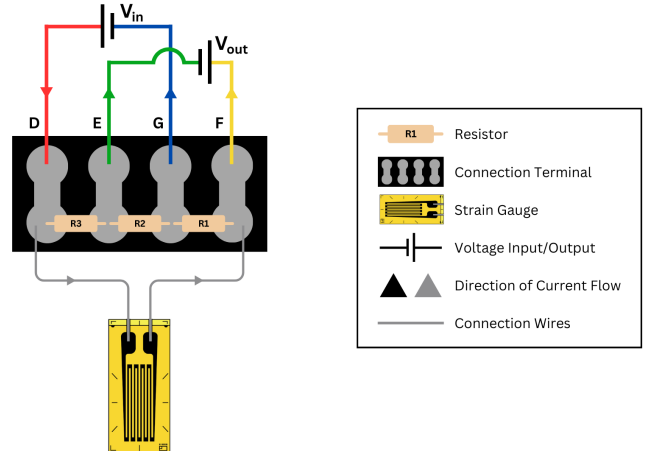


Figure 6.9: Load cell configuration

6.4. Experiment Preparation

6.4.1. Soil Preparation

The soil is prepared using the dry pluviation method. Figure 6.10 shows the pluviation setup, where the process is performed with an adjustable falling height to maintain consistency. Figure 6.11 illustrates the pluviation process, visually demonstrating the effectiveness of the net in distributing the soil uniformly.

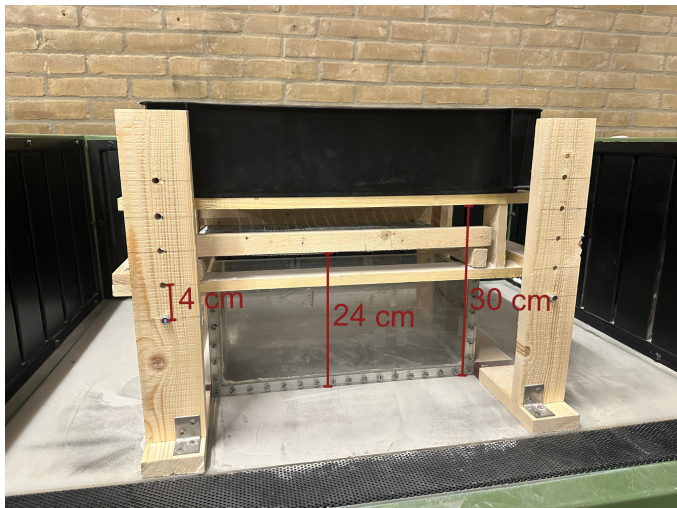


Figure 6.10: Pluviation setup



Figure 6.11: Pluviation process

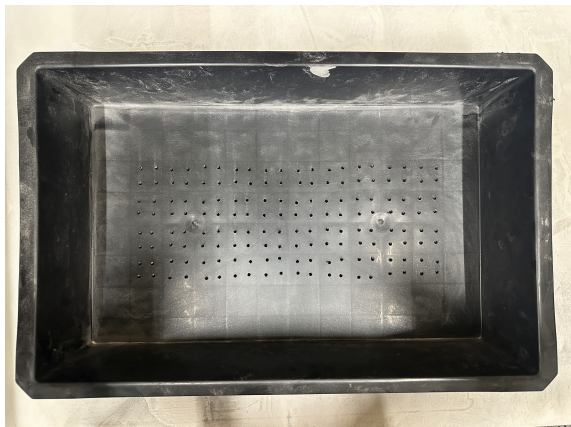


Figure 6.12: Pluviation hole

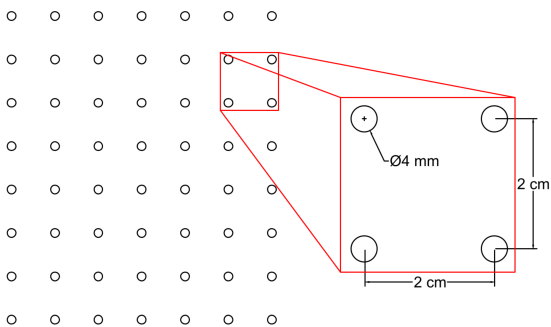


Figure 6.13: Pluviation hole dimensions

Figures 6.12 and 6.14 show the actual net and hole used in the experiment, while Figures 6.13 and 6.15 illustrate the detailed dimensions of the hole and the net.

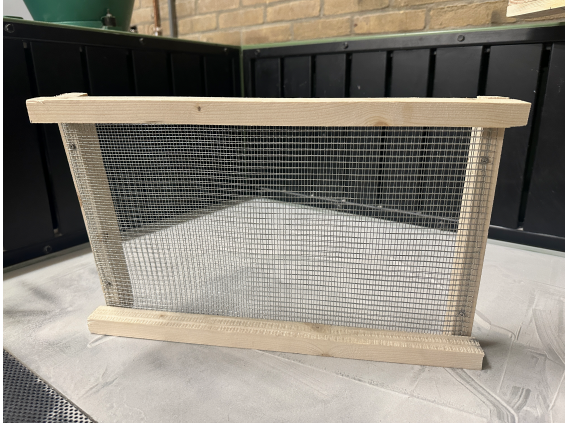


Figure 6.14: Pluviation net

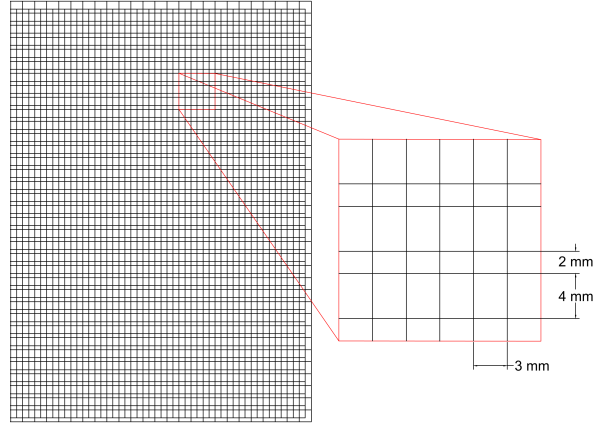


Figure 6.15: Pluviation net dimensions

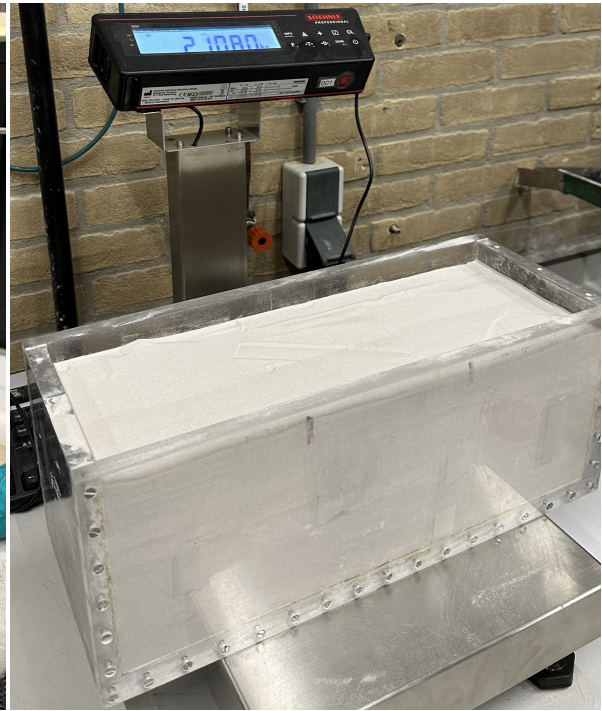
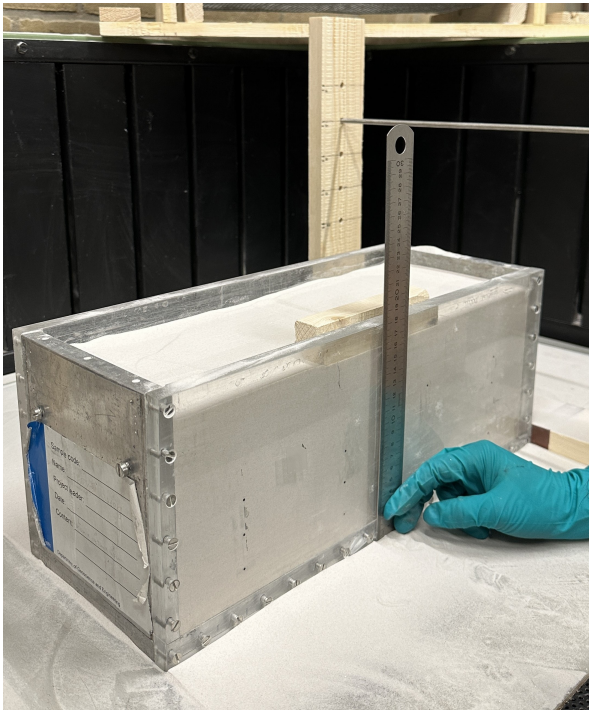


Figure 6.16: Soil sample measurement

After the pluviation process is completed, the soil sample height is measured and weighed, as shown in Figures 6.16. To achieve the target relative density, the soil's relative density can be calculated using the mass of the soil, total volume, and the void ratio as follows:

$$D_r = \frac{e_{max} - e}{e_{max} - e_{min}} = \frac{1.07 - e}{1.07 - 0.64} \quad (6.2)$$

$$e = \frac{V_T}{V_s} - 1 = V_T \cdot \frac{G_s \rho_w}{m_s} - 1 = V_T \cdot \frac{2.67 \cdot 1000 \text{ kg/m}^3}{m_s} - 1 \quad (6.3)$$

As documented by Li *et al.* (2022) [32], Geba sand has a specific gravity (G_s) of 2.67, a maximum void ratio (e_{max}) of 1.07, and a minimum void ratio (e_{min}) of 0.64.

6.4.2. Foundation Preparation
Foundation Model

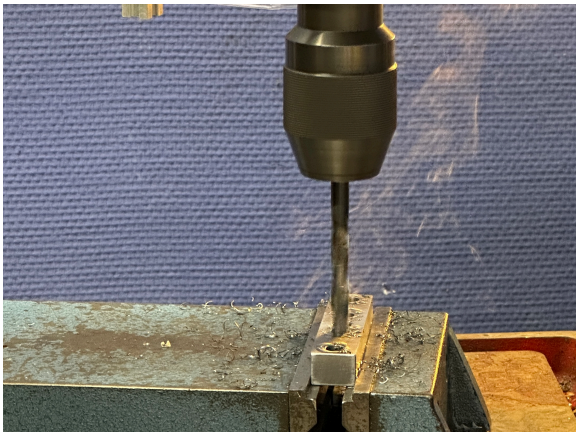


Figure 6.17: Foundation drilling process



Figure 6.18: Gluing

The foundation model preparation includes drilling the raft to fit the piles and then gluing them, as shown in Figures 6.17 and 6.18, respectively.

Strain Gauge

Figure 6.19 illustrates the positions of the strain gauges on the foundation. Figure 6.20 shows the foundation with the attached strain gauge load cells, with the terminal and cables mounted at the top.

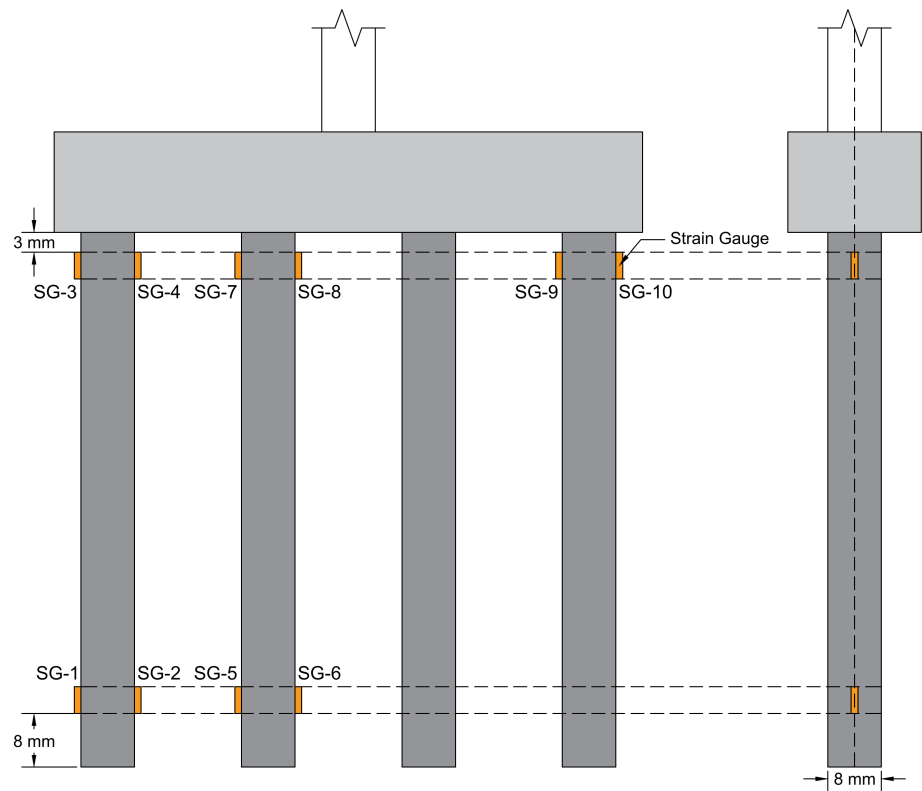


Figure 6.19: Illustration of foundation with strain gauges

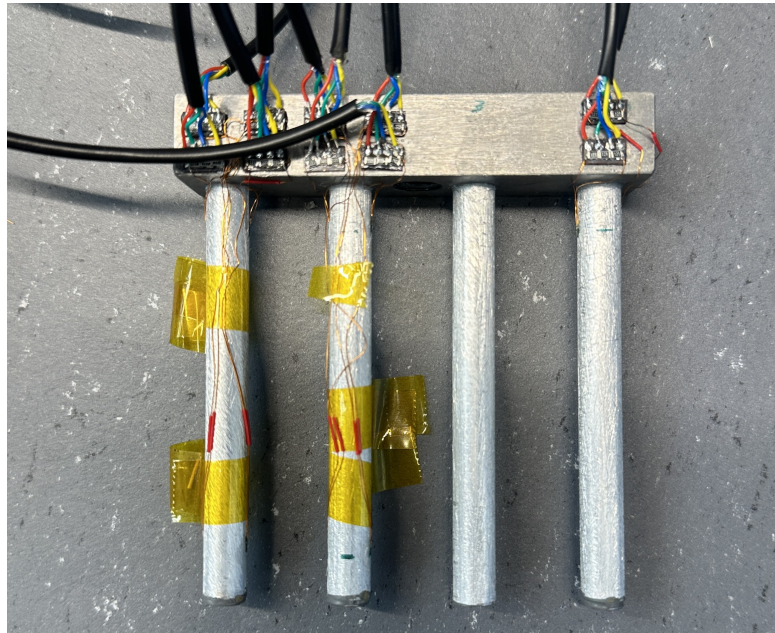


Figure 6.20: Foundation with strain gauge load cells

6.5. Centrifuge Test

6.5.1. Test Set Up

Centrifuge Preparation

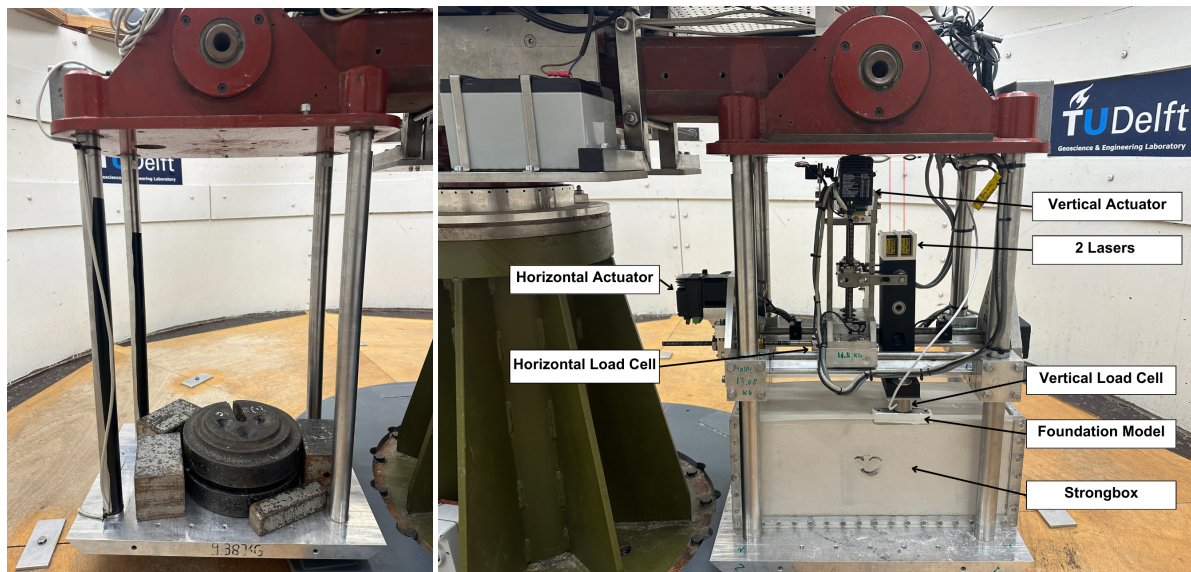


Figure 6.21: Centrifuge configuration

The centrifuge preparation begins by moving the strongbox to one arm of the centrifuge. After that, the foundation is connected to the load cell and installed into the strongbox. Then, the foundation is connected to the connection and actuator. Since the centrifuge used is a beam centrifuge, it is necessary to balance the other arm. In this experiment, a total of 38 kg was placed on the other arm to ensure the centrifuge arm is balanced. Figure 6.21 shows the setup of the foundation inside the centrifuge and the weight on the other arm.

Centrifuge Rotation Speed

Due to the centrifugal force from rotation, with a target of $N = 100\times$, the centrifuge rotation speed needs to be calculated. Figure 6.22 illustrates the schematic distances for the centrifuge.

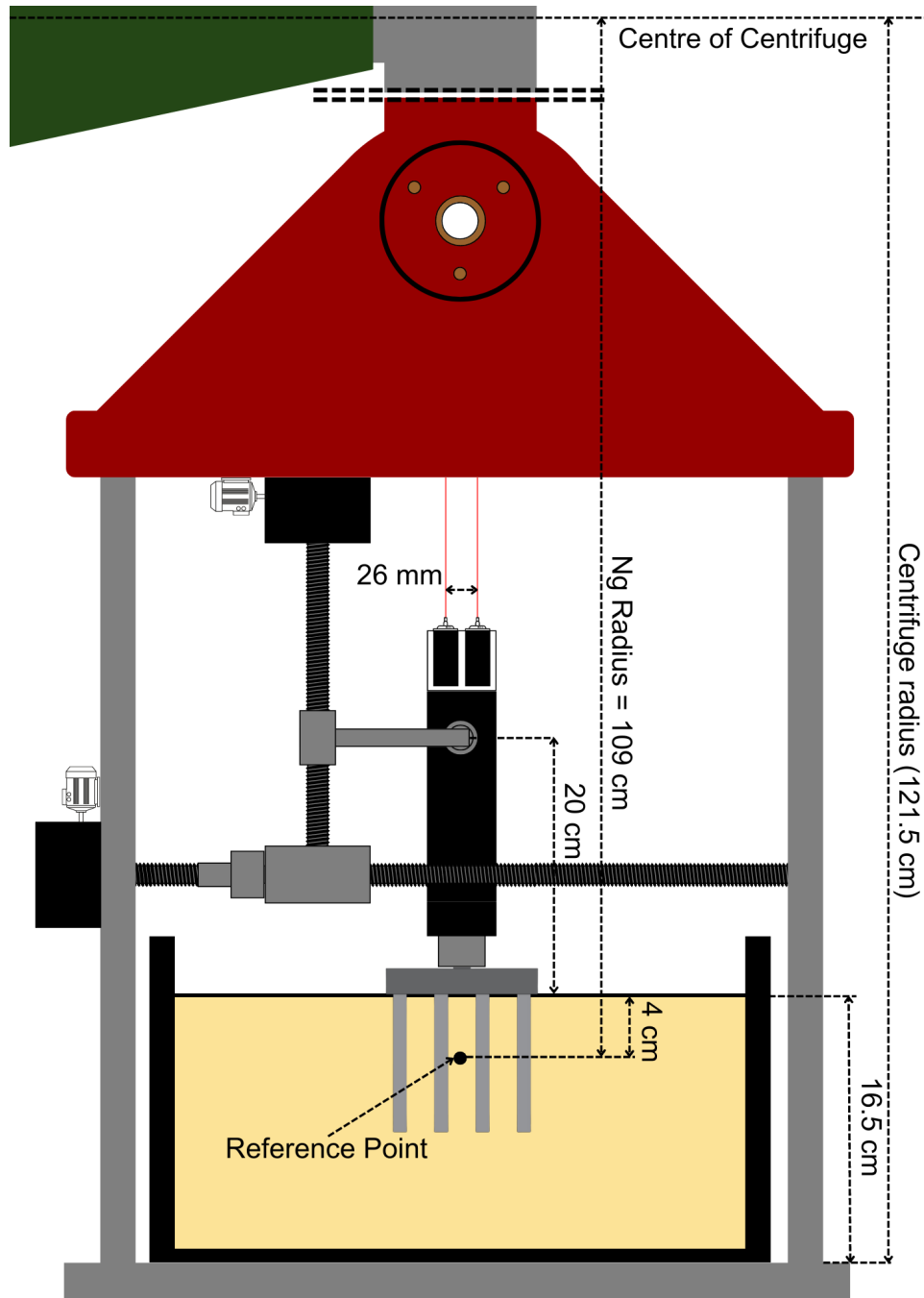


Figure 6.22: Test setup illustration with dimensions

Given the centrifuge rotation speed calculation provided in Equation 3.11, the rotation speed can be calculated as:

$$U = \frac{60}{2\pi} \sqrt{\frac{100 \times 9.81 \text{ m/s}^2}{1.09 \text{ m}}} \approx 286.5 \text{ RPM} \quad (6.4)$$

6.5.2. Test Execution

Test execution was conducted by remotely controlling and observing from the control room, as shown in Figure 6.23.



Figure 6.23: Remote desktop view of the centrifuge

The axial loading analysis was performed by moving the vertical actuator downward, then measuring the load from the load cell and the distance from the laser. The horizontal loading analysis was conducted by initially moving the vertical actuator downward until the initial vertical load was reached, followed by moving the horizontal actuator. The rotation of the foundation was measured from the difference between the two lasers applied at the top, with the distance between the two lasers being 26 mm.

6.6. Axial Loading Result

For the axial loading analysis, three tests were chosen as representatives for each foundation type: V_PG_100g_1 for the pile group, V_PR_100g_2 for the piled raft, and V_R_100g_1 for the raft foundation. Table 6.2 highlights the details. Detailed information can be found in Appendix B.

Test ID	Foundation Type	Relative Density	N Design
V_PG_100g_1	Pile Group	81%	100g
V_PR_100g_2	Piled Raft	77%	100g
V_R_100g_1	Raft	78%	100g

Table 6.2: Axial loading test details

Figures 6.24 shows the test results before upscaling, with the background noise visible in the vertical displacement measurements.

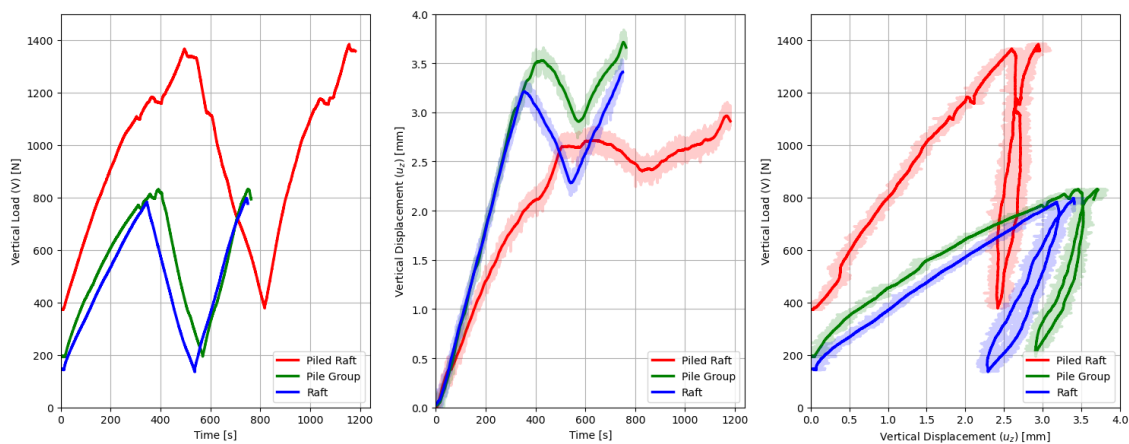


Figure 6.24: Axial loading result from centrifuge test at 100g

Significant noise was observed in the vertical displacement measurements due to the laser sensor. To address this issue, filtering was applied. Figure 6.25 presents the noise from the laser measurement in the frequency domain, obtained through Fourier transformation. The measurements were sampled at a frequency of 10 Hz. According to the Nyquist theorem, the maximum frequency that can be accurately represented without aliasing is half the sampling rate, which is 5 Hz. As a result, the frequency spectrum is plotted up to 5 Hz. From this figure, it is clear that no characteristic frequencies are present. Therefore, a moving average filter with a window size of 200 data points was applied to smooth the data.

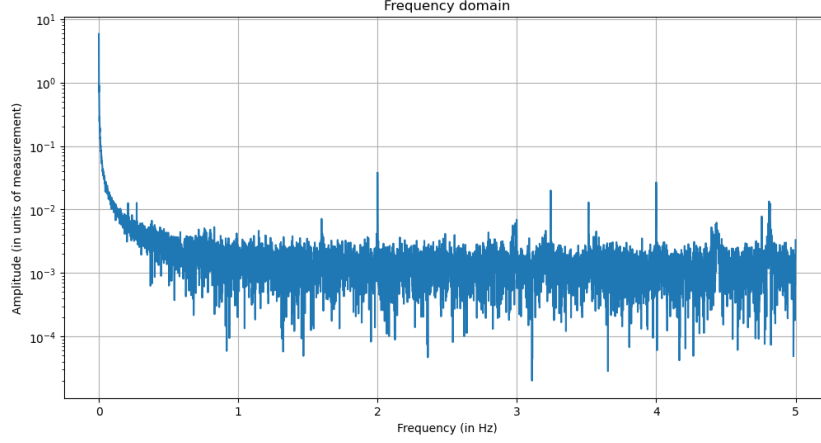


Figure 6.25: Displacement in the frequency domain

After filtering, upscaling was performed using the following equations:

$$F_{prototype} = N^2 \times F_{model}$$

$$u_{prototype} = N \times u_{model}$$

6.6.1. Load-Displacement Analysis

Figure 6.26 shows the load-displacement curves for all foundation types. With a pile diameter of 80 cm at $N = 100g$, the pile group and piled raft were loaded axially until u_z reached 40-50% of the pile diameter (D). The load-displacement curves do not show any plateau, suggesting that the Eurocode approach of capping the ultimate limit state (ULS) at 10% of D for serviceability may be practical, despite the actual resistance being significantly higher. Using the equation $\beta_{PR,V} = \frac{V_{PR}}{V_{PG}}$, it can be seen from the centrifuge testing results that $\beta_{PR,V}$ increase from 1.60 to 1.90 as the displacement increases. This indicates that the piled raft is significantly stronger compared to the pile group alone. A similar trend was observed in the finite element analysis, with $\beta_{PR,V}$ ranging from 1.89 to 2.23.

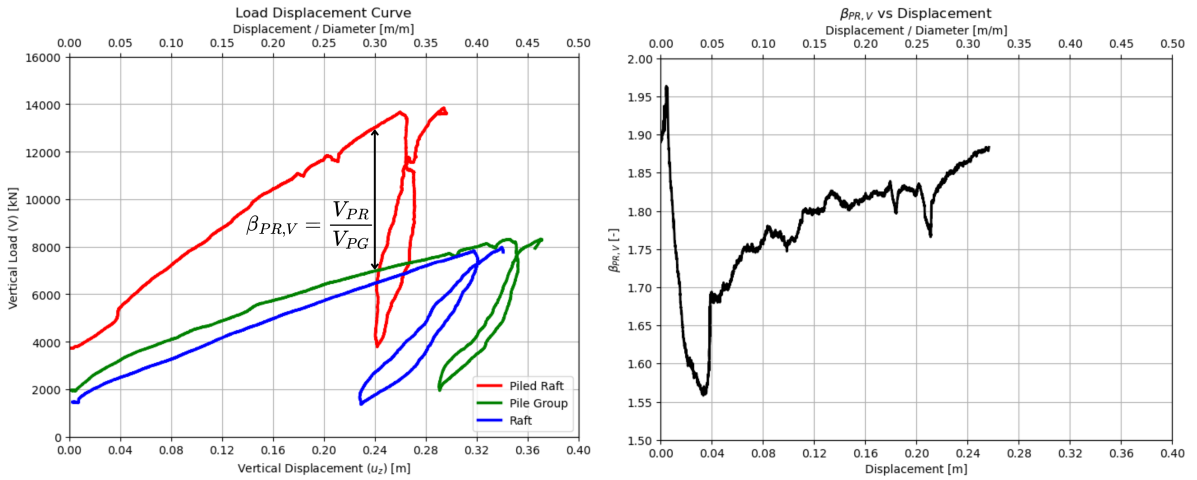


Figure 6.26: Axial load-displacement curve and $\beta_{PR,V}$ from centrifuge test

Figure 6.27 shows the comparison between the piled raft and the combination of the pile group and raft. From the load-displacement curves, it can be seen that the stiffness of the piled raft is almost identical to that of the pile group and raft combined. Using the equation $\xi_{PR,V} = \frac{V_{PR}}{V_{PG} + V_R}$, $\xi_{PR,V}$ remains constant at around 0.98, while the result from finite element analysis shows $\xi_{PR,V}$ gradually increasing with displacement from 0.8 to 1.2.

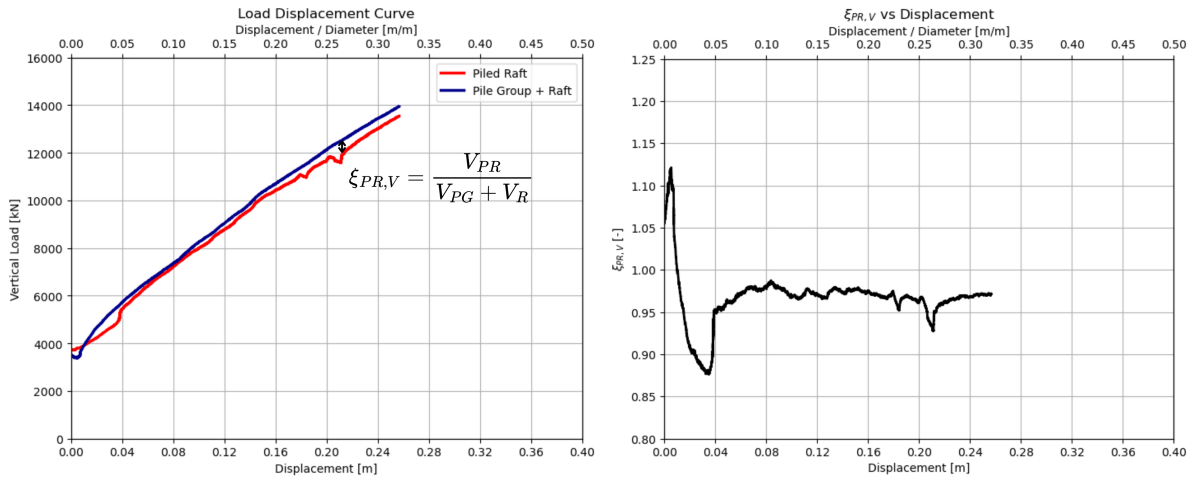


Figure 6.27: Axial load-displacement curve and $\xi_{PR,V}$ from centrifuge test

The results from the centrifuge testing highlight that the piled raft foundation, while significantly stronger compared to the pile group alone, behaves almost similarly to the combined system of the pile group and raft. The $\beta_{PR,V}$ value exceeding 1 indicates the piled raft carries more load at the same displacement compared to the pile group, demonstrating its higher resistance. On the other hand, the $\xi_{PR,V}$ value closes to 1 suggests that the resistance of the piled raft is nearly equivalent to the sum of the pile group and raft acting as separate foundations. This indicates an effective distribution of loads within the piled raft system to achieve optimal performance.

Moreover, when comparing the results presented by L. de Sanctis and Mandolini in Table 2.5, where $\beta_{PR,V}$ ranges from 1.06 to 3.74 and $\xi_{PR,V}$ ranges from 0.8 to 1.03, the results from this centrifuge test fall within this range. Although the tests highlighted by L. de Sanctis and Mandolini were performed in clay with comparison at one ultimate value, while this one is in sand with $\beta_{PR,V}$ and $\xi_{PR,V}$ ranging with displacement, the findings are still consistent.

Conclusions and Recommendations

7.1. Conclusions

This research has provided various insights into the behaviour of piled raft foundations, addressing the primary objective of comparing their performance to pile groups and rafts, as well as the secondary objective of investigating cyclic behaviour modelling. The key findings, aligned with the research questions, are as follows:

1. The load-displacement behaviour of piled rafts (PR) demonstrates superior stiffness compared to pile groups (PG) and rafts (R) in all scenarios. Finite element analysis and centrifuge testing reveal that:
 - Vertical stiffness of piled rafts is 1.60 to 2.23 times higher than that of pile groups.
 - The vertical resistance ratio, $\beta_{PR,V} = \frac{V_{PR}}{V_{PG}}$, ranges from 1.89 to 2.23, indicating that the inclusion of the raft nearly doubles the foundation's vertical resistance.
 - The combined resistance ratio, $\xi_{PR,V} = \frac{V_{PR}}{V_{PG}+V_R}$, varies from 0.8 to 1.2 in finite element analysis and remains constant at around 0.98 in centrifuge testing.
 - For lateral loading, piled rafts exhibit about 1.25 times greater horizontal load and moment resistance compared to pile groups.
2. The research reveals the load distribution within piled raft foundations:
 - The raft contributes approximately 45% of the vertical load and about 50% of the moment.
 - In lateral loading, the presence of the raft reduces the axial load carried by the piles by around 40%.
3. Analysis of internal forces highlights:
 - Shaft resistance is found to be minimal.
 - Under axial loading, edge piles experience larger moments, indicating raft deformability and potential failure of edge piles due to moment.
 - In lateral loading, piles in tension reach maximum tensile resistance rapidly (at approximately 0.3° of rotation), with continued increase in moment at the pile top, suggesting high probability of failure due to excessive moments.
4. The study demonstrates that modelling the elastoplastic cyclic behaviour of piled rafts using a viscoelastic model with frequency-independent viscosity is feasible. While not fully capturing all aspects of foundation behaviour, this approach adequately replicates stiffness and damping characteristics for seismic design using an equivalent linear approach.
5. The findings have significant practical implications for geotechnical engineering:
 - Retrofitting: Many foundations designed as pile groups may actually function as piled rafts, potentially reducing or eliminating the need for retrofitting in existing structures due to higher stiffness and bearing capacity than expected in the early design.

- **Design Optimization:** The superior performance of piled rafts in both vertical and lateral loading allows for more efficient and cost-effective foundation designs, particularly for high-rise buildings and structures in challenging soil conditions.
- **Performance Enhancement:** The integrated action of piles and rafts leads to significant increases in both axial and lateral stiffnesses, improving overall foundation performance.

In conclusion, this research focuses on our understanding of piled raft foundation behaviour and provides practical insights that can significantly impact geotechnical engineering practice. The findings demonstrate the superior performance of piled rafts in both vertical and lateral loading conditions, offering opportunities for more efficient and economical foundation designs. By leveraging these insights, engineers can develop more resilient foundation systems for new constructions and adopt a more informed approach to assessing and retrofitting existing structures.

7.2. Limitations

Assessment of the strain gauge data from the centrifuge tests and the analysis of horizontal load were not performed in this thesis. However, as this thesis supports the IDEA League project titled “INITIATION - Improving Lifespan of Existing Viaducts Foundations,” led by Pietro, the project will continue with further analysis.

7.3. Recommendations

Appendix A presents load-displacement results under varying initial axial loads and heights, comparing horizontal versus vertical displacements, vertical displacement versus rotation, and shifts in the centre of rotation for both pile groups and piled rafts. These results can be further analysed in future research to develop interaction domains, create analytical solutions, and explore other areas of interest.

The constraints of the centrifuge’s limited space led to a very small model, making it vulnerable to small-scale effects. Additionally, the domain size can create a boundary value problem, as the distance between the pile tip and the domain boundary was only one pile length. While this scenario might represent very dense soil conditions, using larger models with increased foundation and domain sizes would reduce the likelihood of small-scale effects and avoid boundary value problems.

Regarding the centrifuge testing, this thesis focuses on macro-scale load-displacement comparisons. A more detailed analysis of the centrifuge testing data, such as strain gauge results, would allow for the calculation of internal forces within each pile and the contributions of the raft and piles. This detailed analysis would enable a more comprehensive comparison between finite element results and centrifuge tests.

The viscoelastic model used in this thesis was selected for its simplicity. While it successfully captured stiffness and viscosity, it did not fully replicate the cyclic loading behaviour of the piled raft foundation. Therefore, developing a more advanced viscoelastic model that can accurately capture this behaviour is recommended. Additionally, the Python script created for optimizing the viscoelastic model is provided in Appendix C for further use.

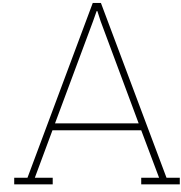
Although the calibration of Geba Sand using SANISAND-MS is not performed in this thesis due to a lack of data, a Python script with an automated calibration built with a machine learning function that calibrates the soil lab test results against PLAXIS Soil Test is provided in Appendix D.

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Finite Element Result

A.1. Horizontal Loading Result

A.1.1. Load Displacement

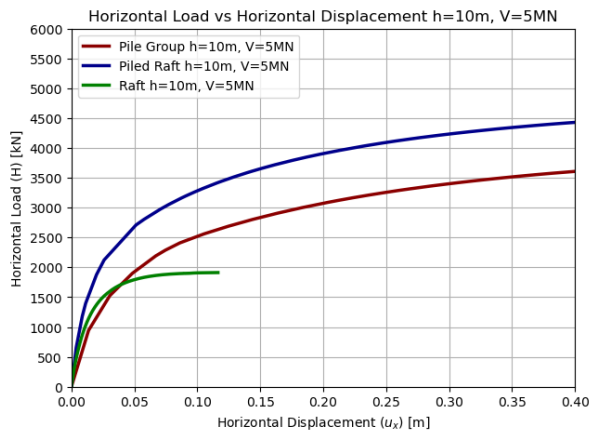


Figure A.1: Horizontal load vs. horizontal displacement, $h = 10m$, $V_{in} = 5MN$

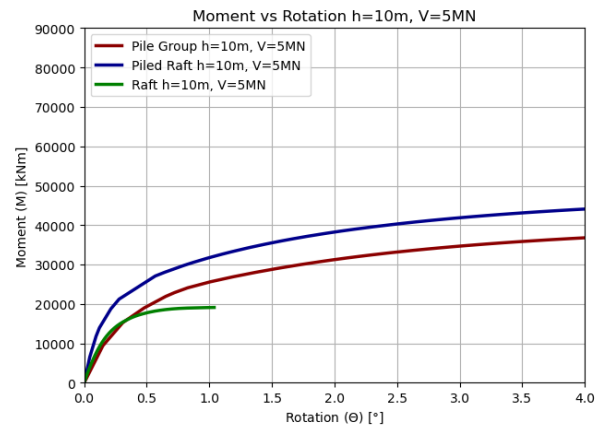


Figure A.2: Moment vs. rotation, $h = 10m$, $V_{in} = 5MN$

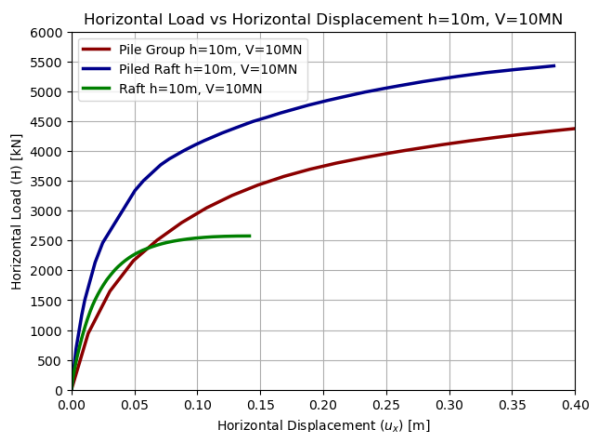


Figure A.3: Horizontal load vs. horizontal displacement, $h = 10m$, $V_{in} = 10MN$

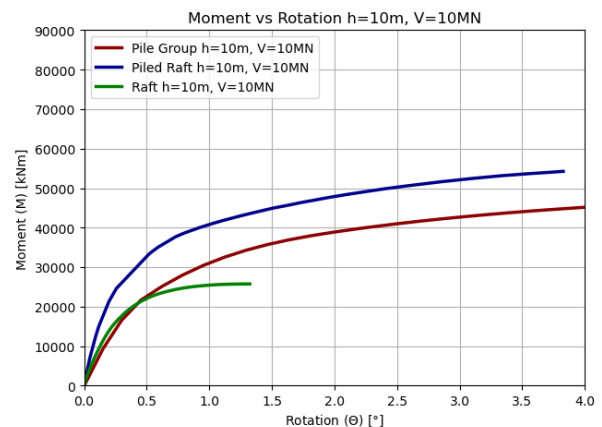


Figure A.4: Moment vs. rotation, $h = 10m$, $V_{in} = 10MN$

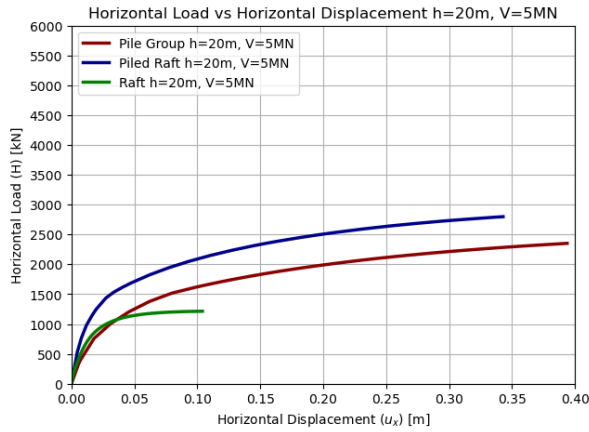


Figure A.5: Horizontal load vs. horizontal displacement, $h = 20m$, $V_{in} = 5MN$

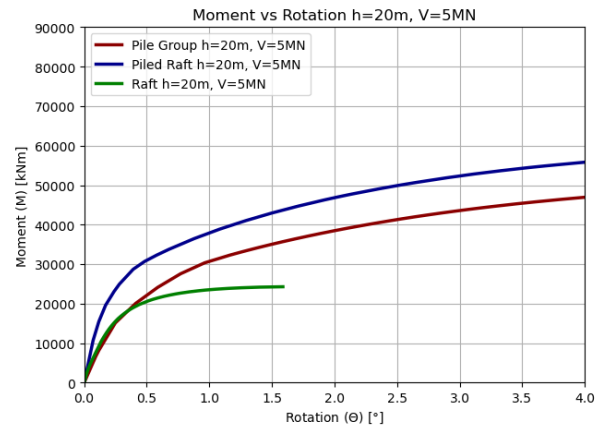


Figure A.6: Moment vs. rotation, $h = 20m$, $V_{in} = 5MN$

A.1.2. Horizontal Displacement vs. Vertical Displacement and Vertical Displacement vs. Rotation

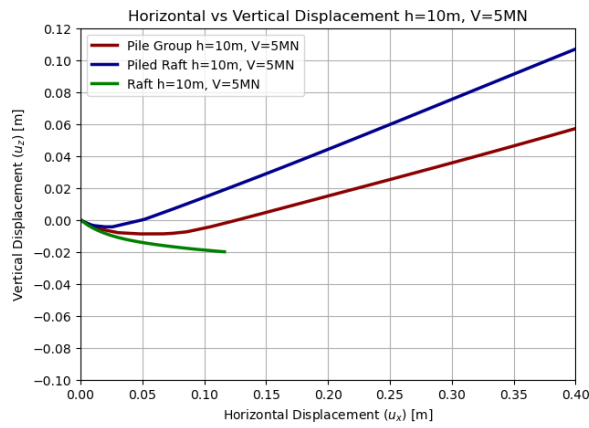


Figure A.7: Horizontal displacement vs. vertical displacement, $h = 10m$, $V_{in} = 5MN$

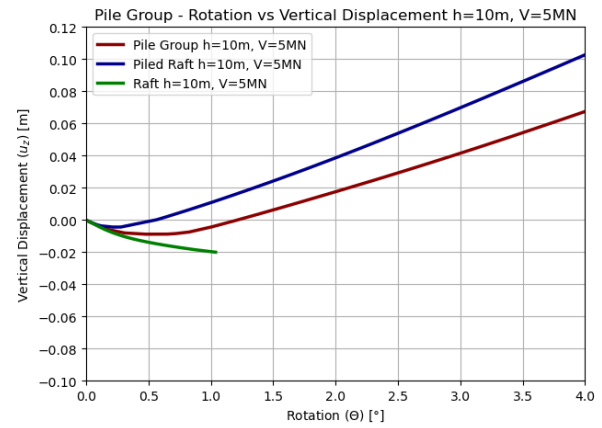


Figure A.8: Vertical displacement vs. rotation, $h = 10m$, $V_{in} = 5MN$

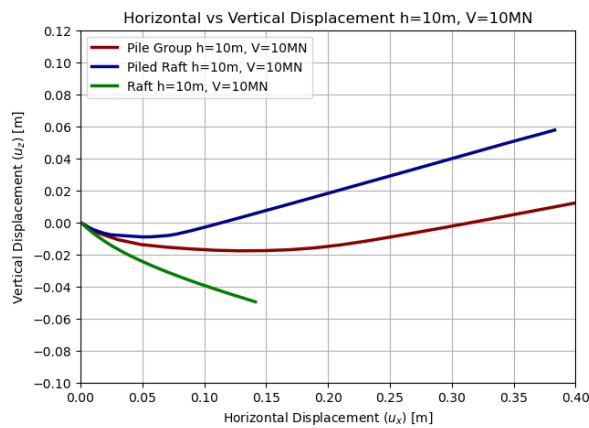


Figure A.9: Horizontal displacement vs. vertical displacement, $h = 10m$, $V_{in} = 10MN$

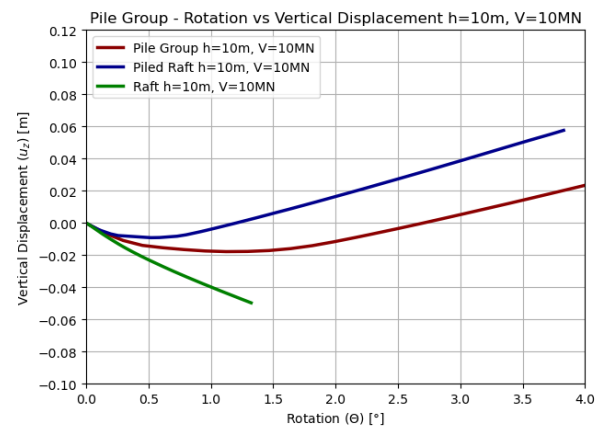


Figure A.10: Vertical displacement vs. rotation, $h = 10m$, $V_{in} = 10MN$

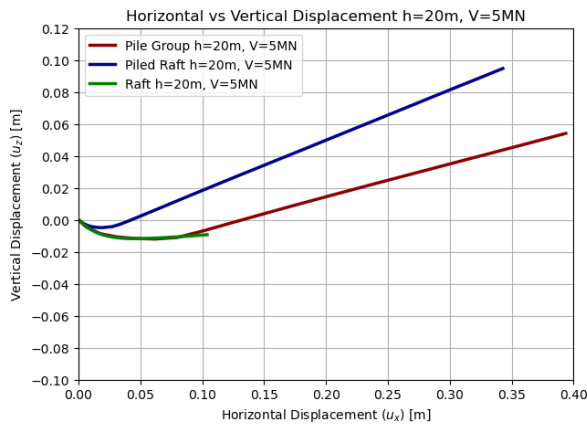


Figure A.11: Horizontal displacement vs. vertical displacement, $h = 20m$, $V_{in} = 5MN$

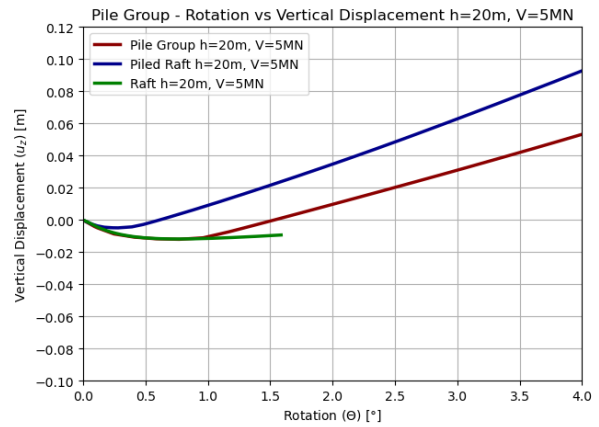


Figure A.12: Vertical displacement vs. rotation, $h = 20m$, $V_{in} = 5MN$

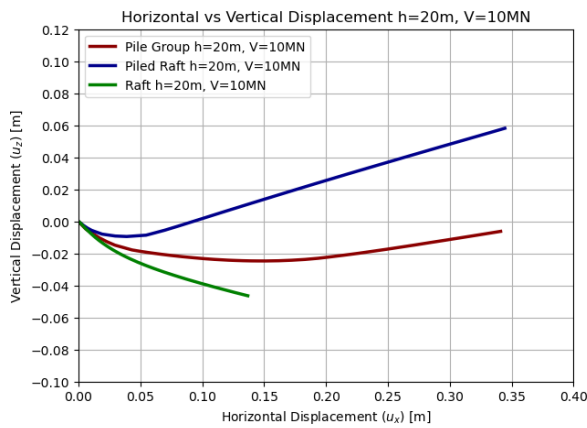


Figure A.13: Horizontal displacement vs. vertical displacement, $h = 20m$, $V_{in} = 10MN$

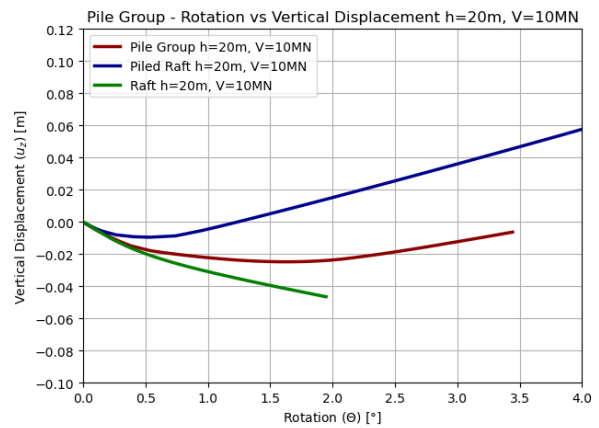


Figure A.14: Vertical displacement vs. rotation, $h = 20m$, $V_{in} = 10MN$

A.1.3. Centre of Rotation

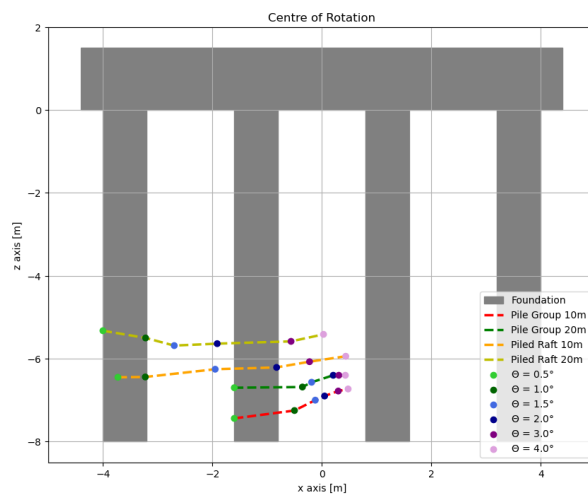


Figure A.15: Centre of rotation

A.2. Cyclic Loading Result

A.2.1. Load Displacement

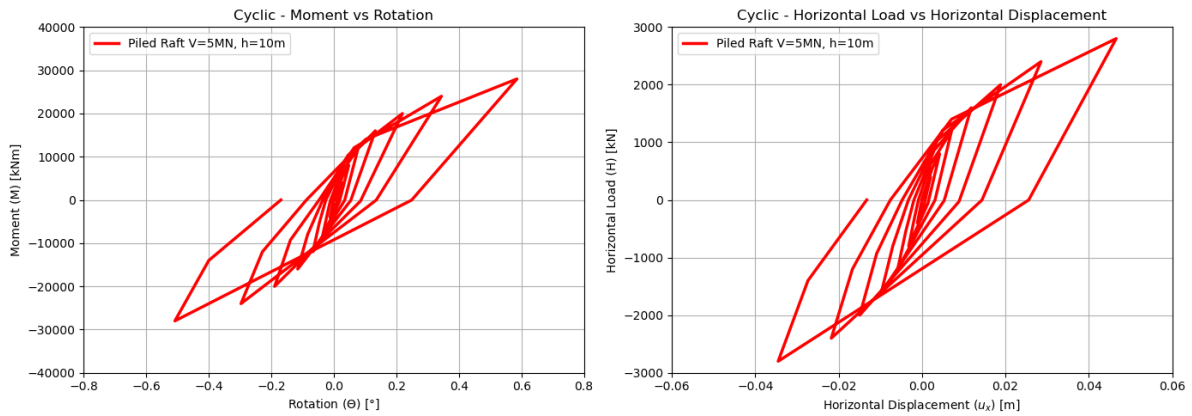


Figure A.16: 10m foundation cyclic load displacement curve

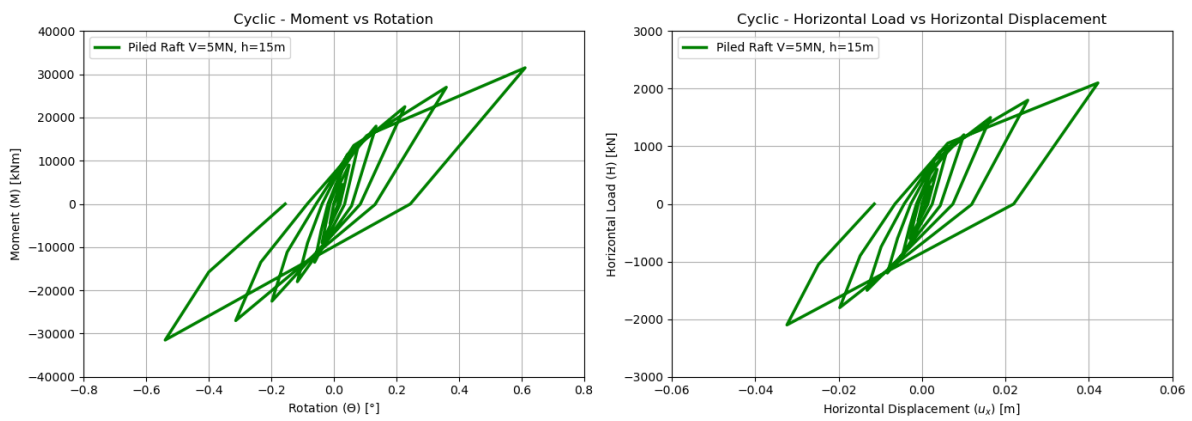


Figure A.17: 15m foundation cyclic load displacement curve

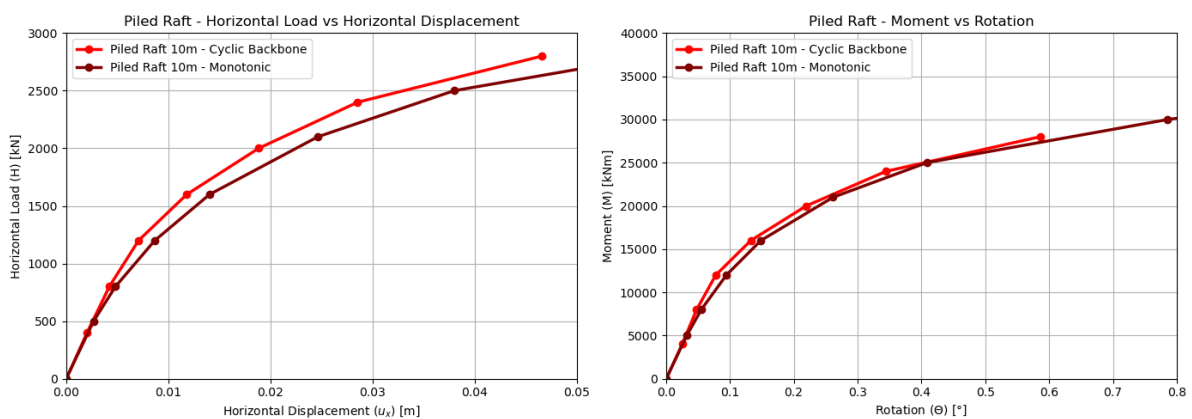


Figure A.18: 10m foundation cyclic backbone vs. monotonic load displacement curve

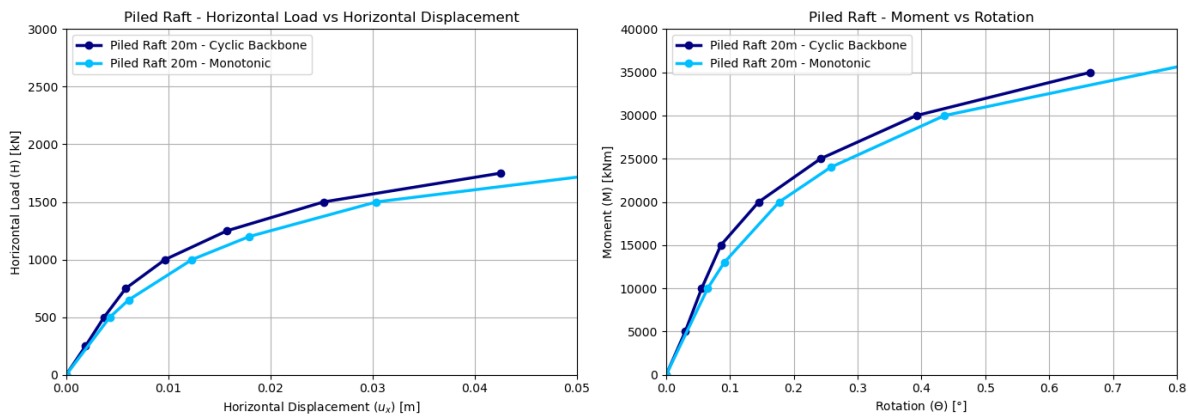


Figure A.19: 20m foundation cyclic backbone vs. monotonic load displacement curve

A.2.2. Viscoelastic Calibration
Moment vs. Rotation - Height = 10m

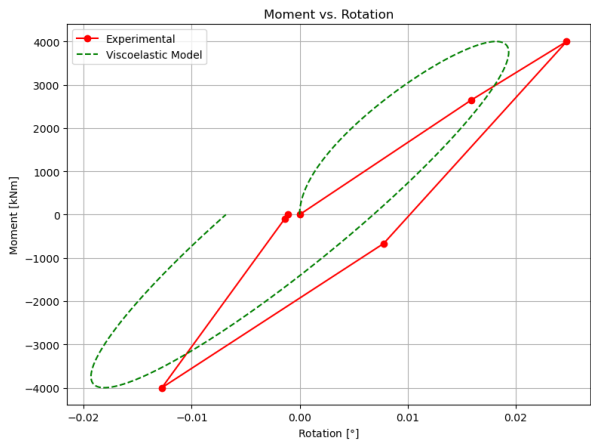


Figure A.20: Moment vs. rotation 10m 1st cycle

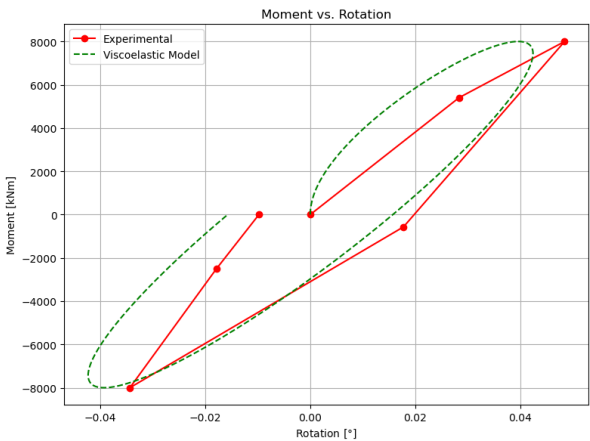


Figure A.21: Moment vs. rotation 10m 2nd cycle

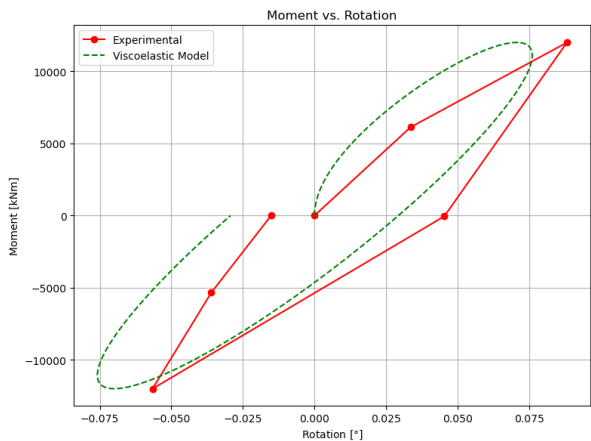


Figure A.22: Moment vs. rotation 10m 3rd cycle

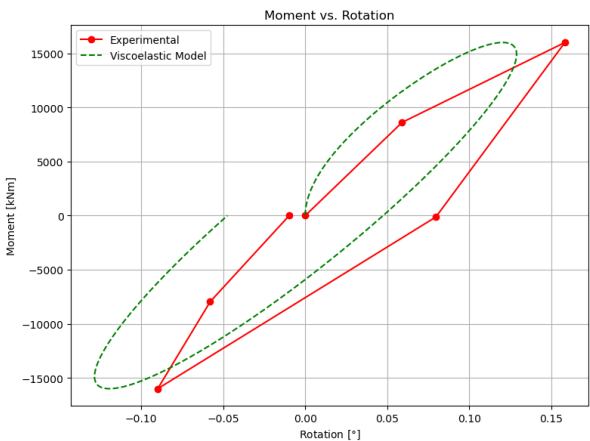


Figure A.23: Moment vs. rotation 10m 4th cycle

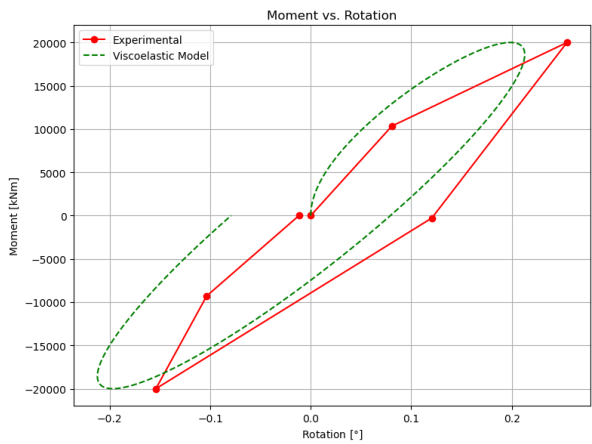


Figure A.24: Moment vs. rotation 10m 5th cycle

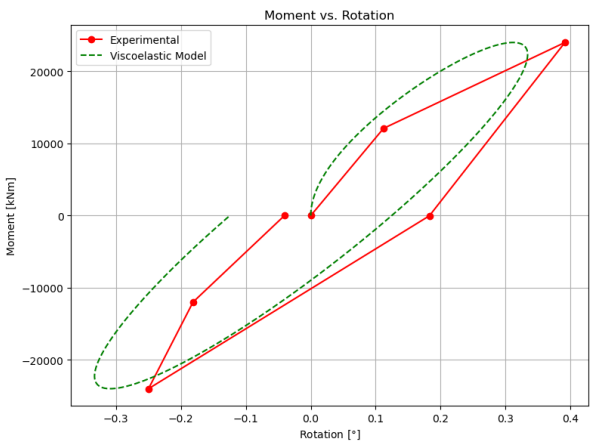


Figure A.25: Moment vs. rotation 10m 6th cycle

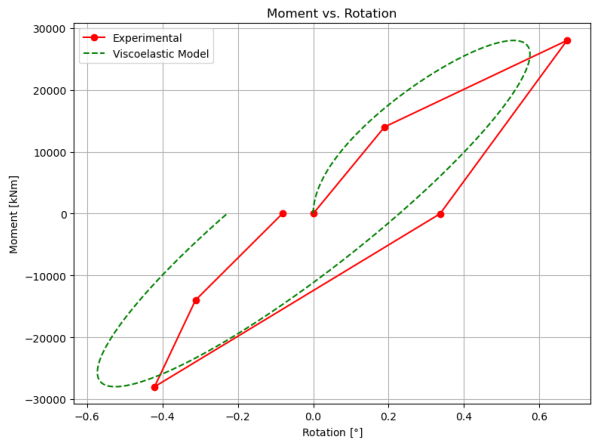


Figure A.26: Moment vs. rotation 10m 7th cycle

Moment vs. Rotation - Height = 15m

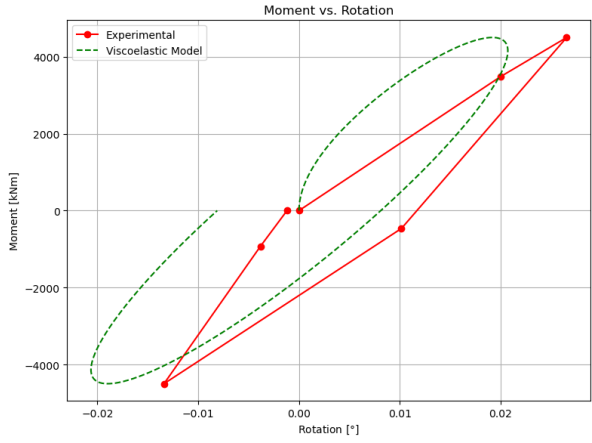


Figure A.27: Moment vs. rotation 15m 1st cycle

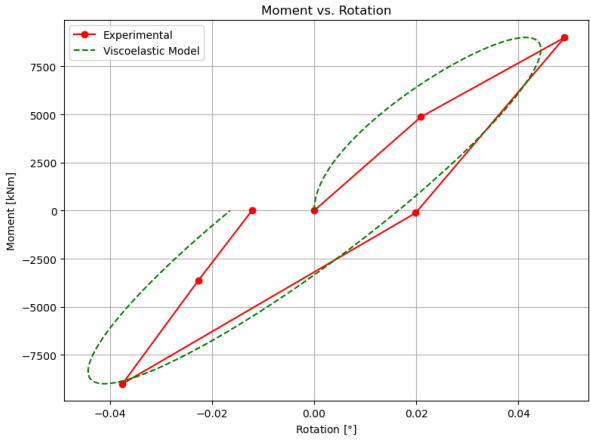


Figure A.28: Moment vs. rotation 15m 2nd cycle

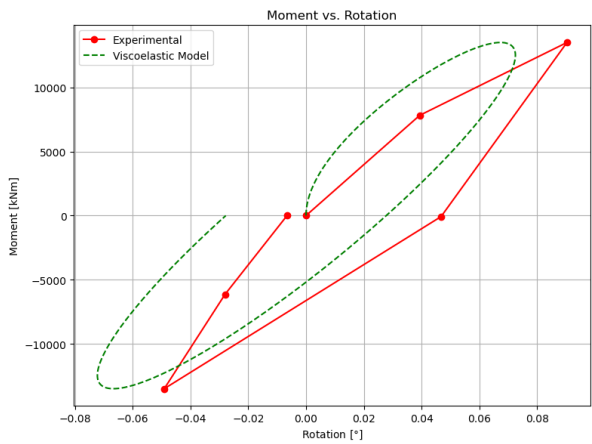


Figure A.29: Moment vs. rotation 15m 3rd cycle

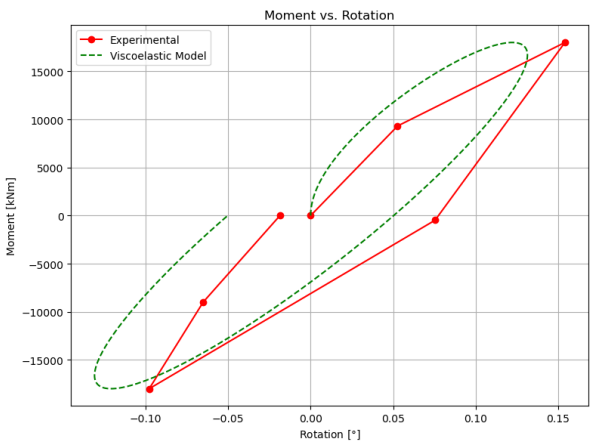


Figure A.30: Moment vs. rotation 15m 4th cycle

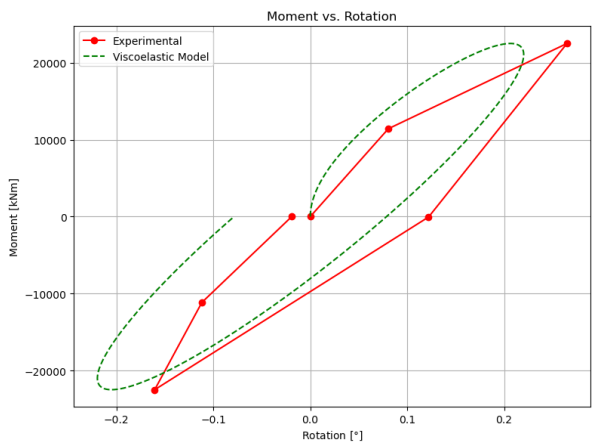


Figure A.31: Moment vs. rotation 15m 5th cycle

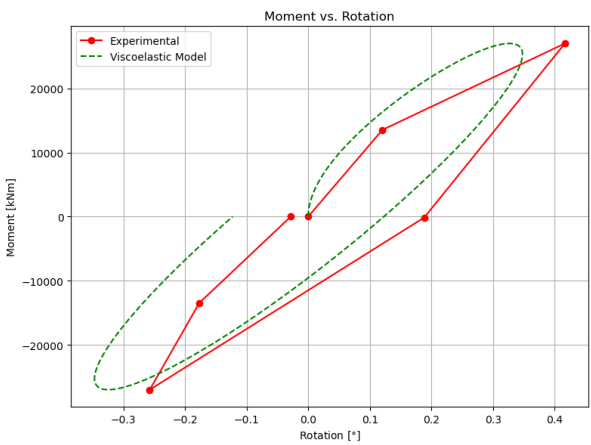


Figure A.32: Moment vs. rotation 15m 6th cycle

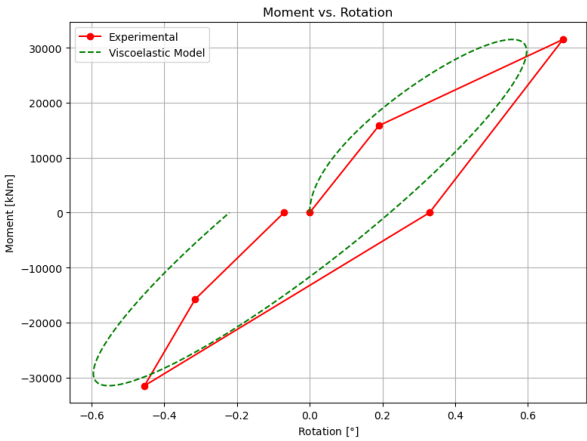


Figure A.33: Moment vs. rotation 15m 7th cycle

Moment vs. Rotation - Height = 20m

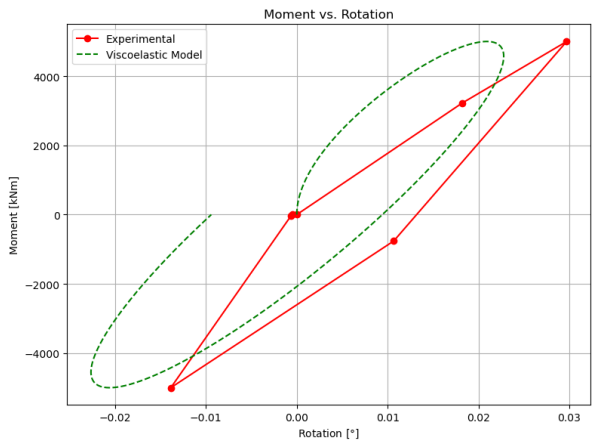


Figure A.34: Moment vs. rotation 20m 1st cycle

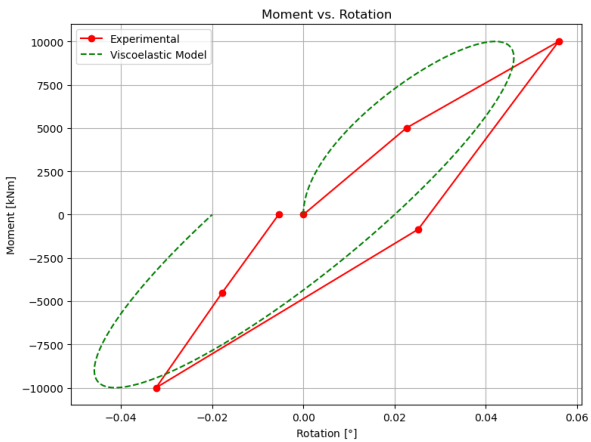


Figure A.35: Moment vs. rotation 20m 2nd cycle

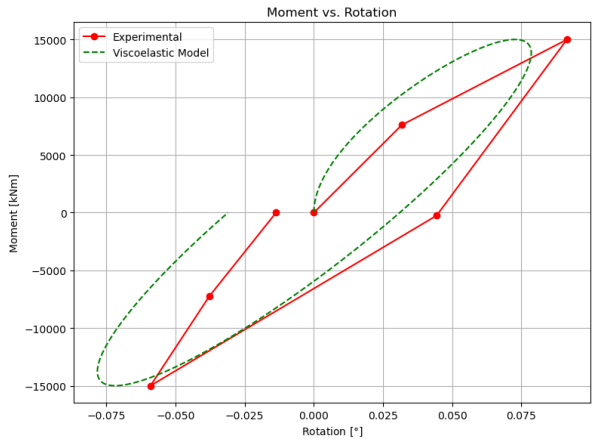


Figure A.36: Moment vs. rotation 20m 3rd cycle

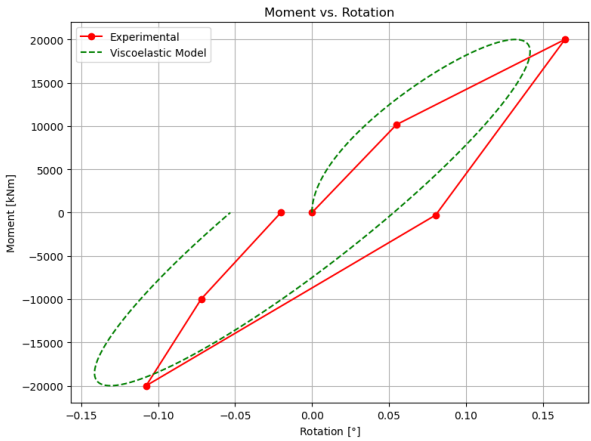


Figure A.37: Moment vs. rotation 20m 4th cycle

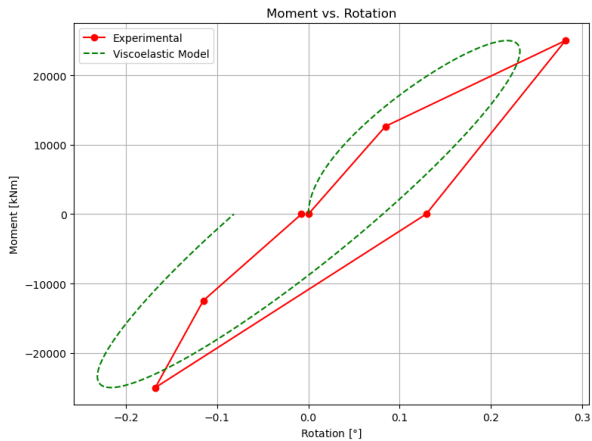


Figure A.38: Moment vs. rotation 20m 5th cycle

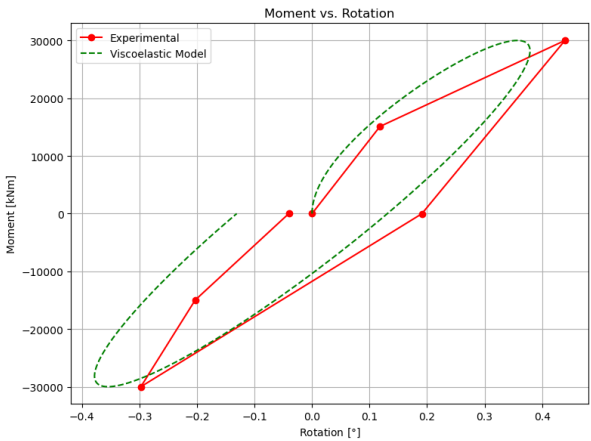


Figure A.39: Moment vs. rotation 20m 6th cycle

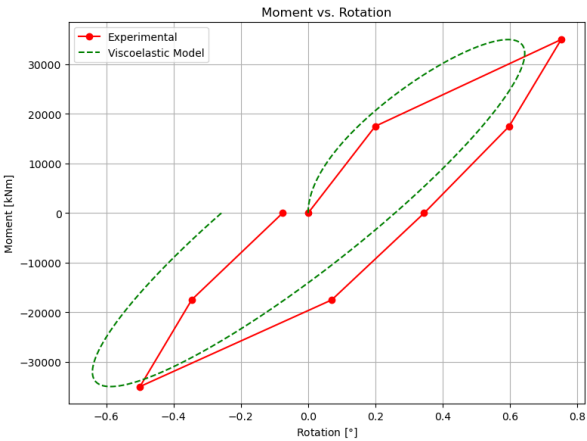


Figure A.40: Moment vs. rotation 20m 7th cycle

Horizontal Load vs Horizontal Displacement - Height = 10m

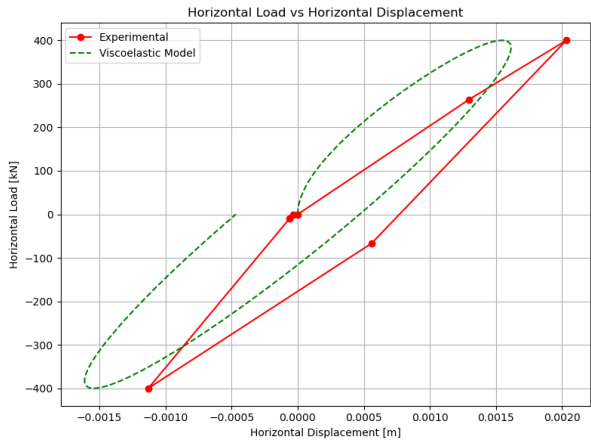


Figure A.41: Horizontal load vs displacement 10m 1st cycle

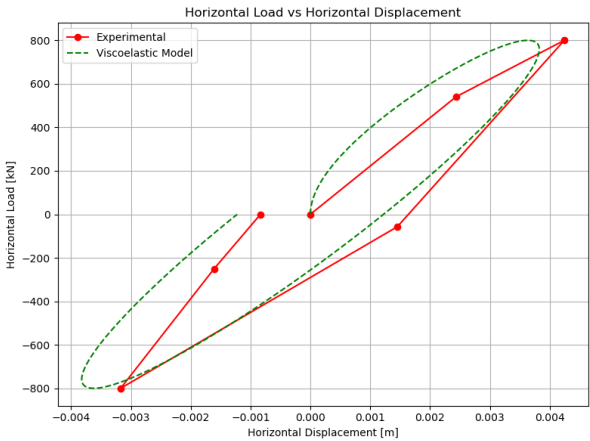


Figure A.42: Horizontal load vs displacement 10m 2nd cycle

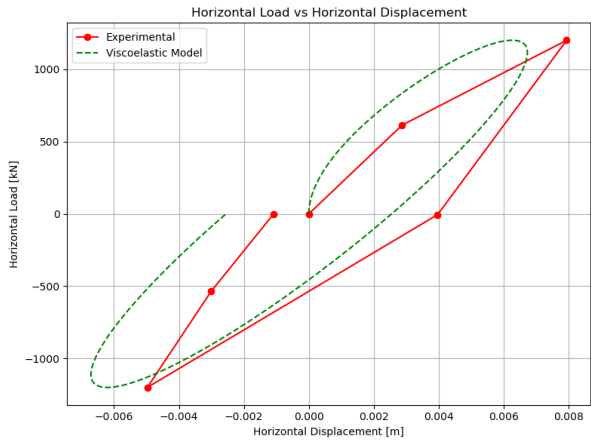


Figure A.43: Horizontal load vs displacement 10m 3rd cycle

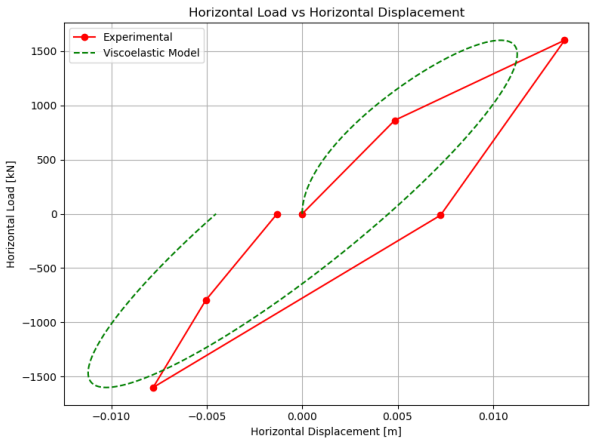


Figure A.44: Horizontal load vs displacement 10m 4th cycle

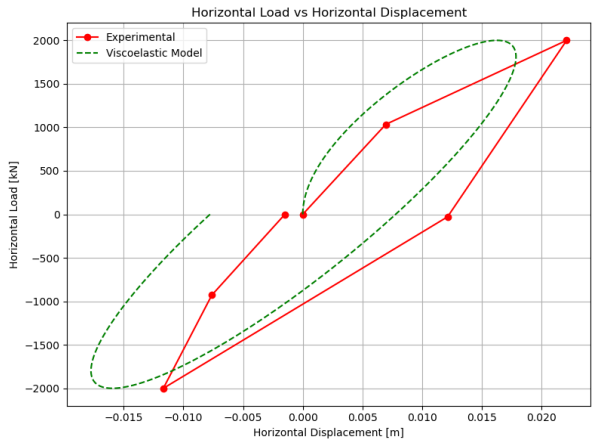


Figure A.45: Horizontal load vs displacement 10m 5th cycle

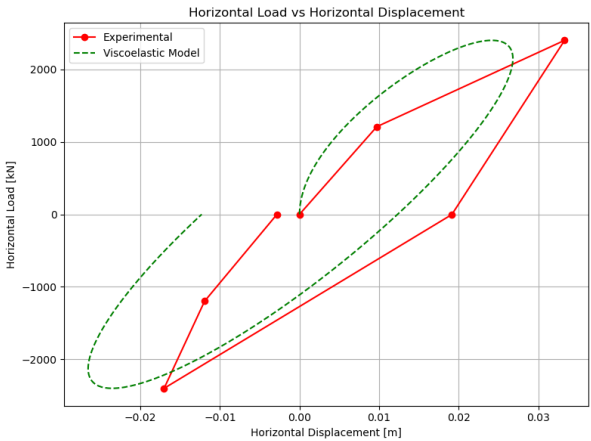


Figure A.46: Horizontal load vs displacement 10m 6th cycle

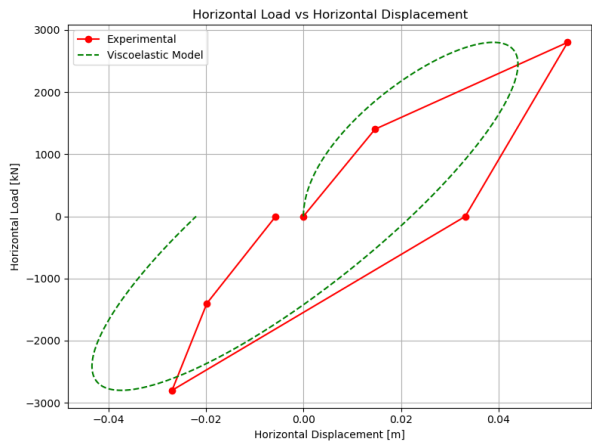


Figure A.47: Horizontal load vs displacement 10m 7th cycle

Horizontal Load vs Horizontal Displacement - Height = 15m

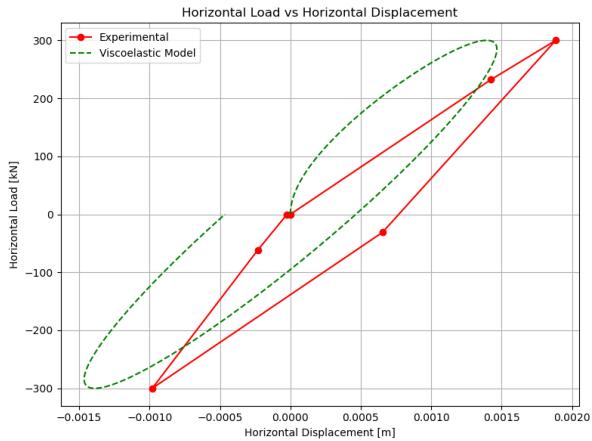


Figure A.48: Horizontal load vs displacement 15m 1st cycle

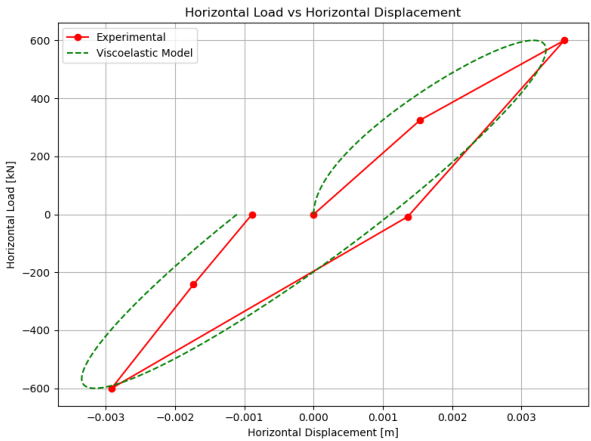


Figure A.49: Horizontal load vs displacement 15m 2nd cycle

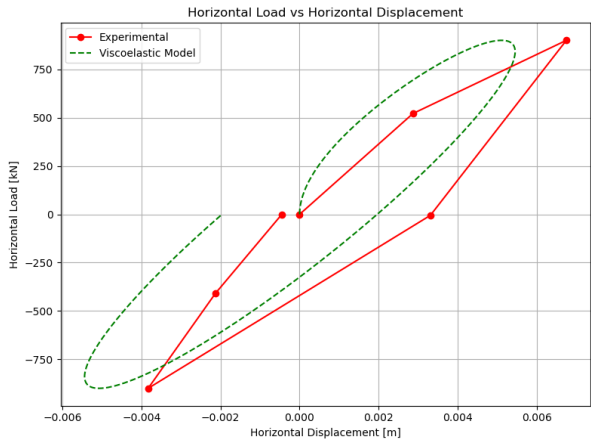


Figure A.50: Horizontal load vs displacement 15m 3rd cycle

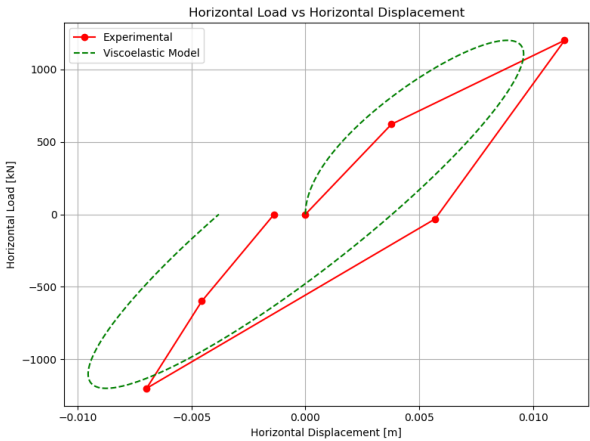


Figure A.51: Horizontal load vs displacement 15m 4th cycle

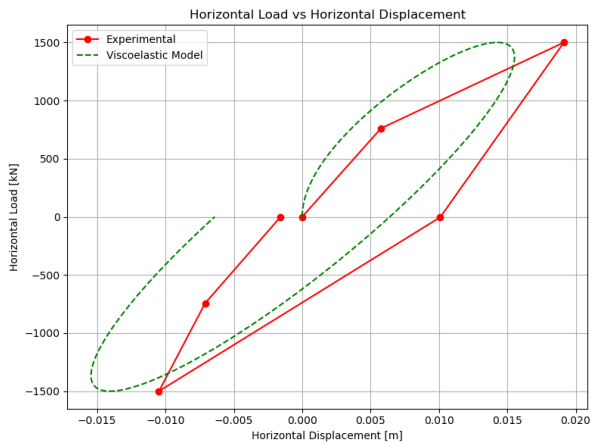


Figure A.52: Horizontal load vs displacement 15m 5th cycle

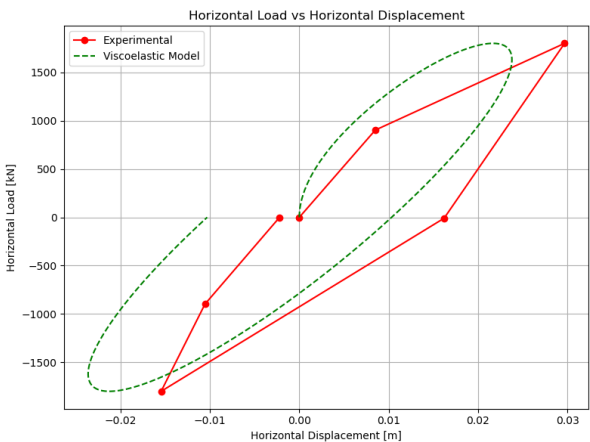


Figure A.53: Horizontal load vs displacement 15m 6th cycle

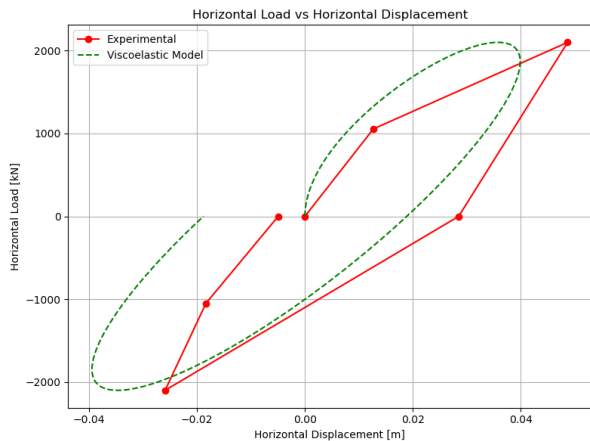


Figure A.54: Horizontal load vs displacement 15m 7th cycle

Horizontal Load vs Horizontal Displacement - Height = 20m

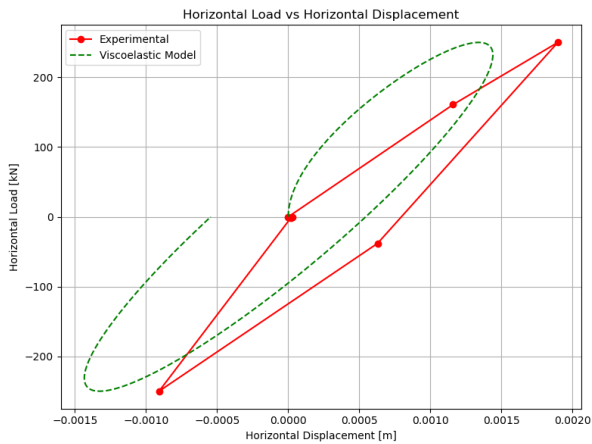


Figure A.55: Horizontal load vs displacement 20m 1st cycle

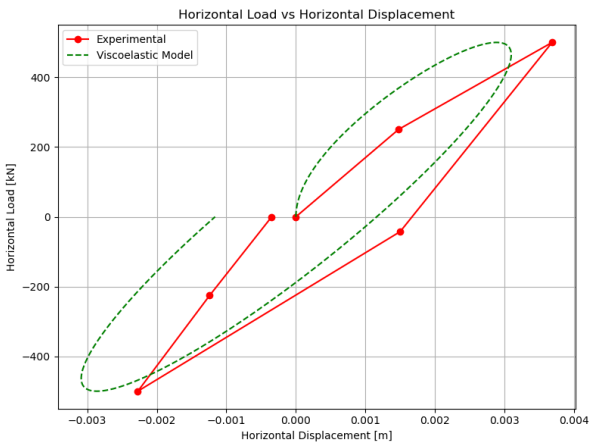


Figure A.56: Horizontal load vs displacement 20m 2nd cycle

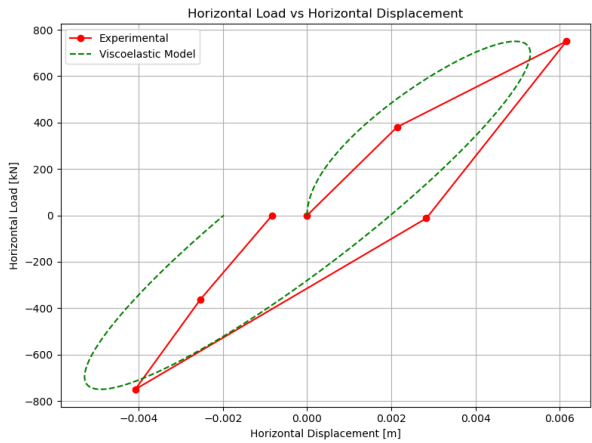


Figure A.57: Horizontal load vs displacement 20m 3rd cycle

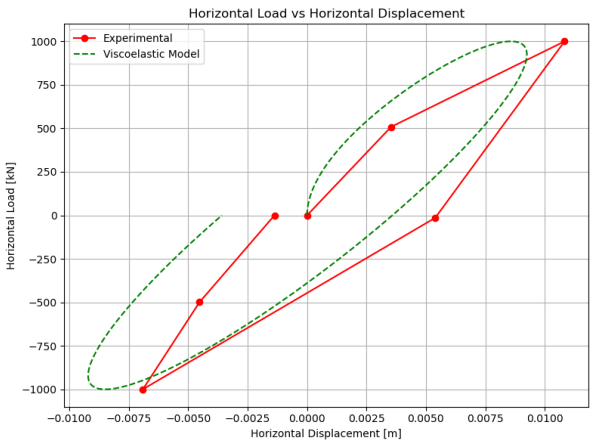


Figure A.58: Horizontal load vs displacement 20m 4th cycle

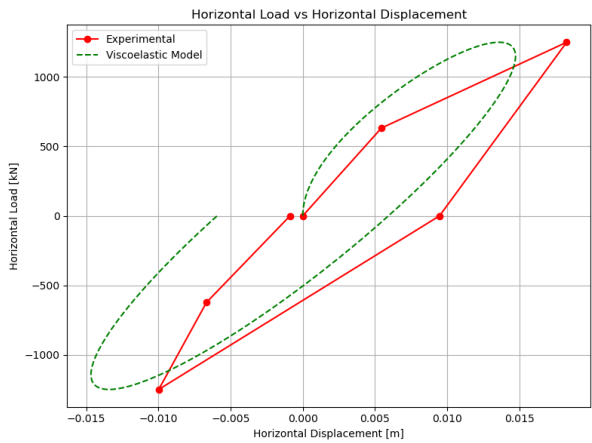


Figure A.59: Horizontal load vs displacement 20m 5th cycle

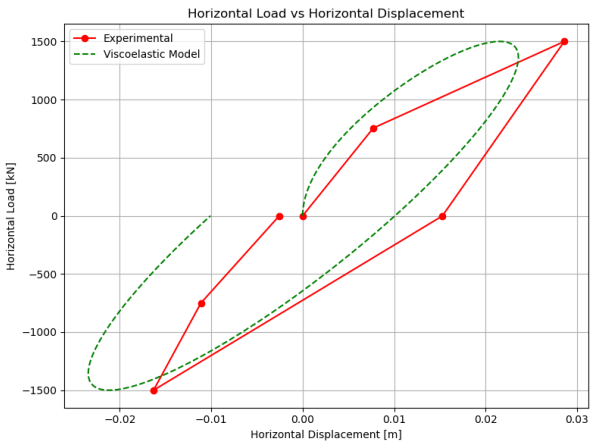


Figure A.60: Horizontal load vs displacement 20m 6th cycle

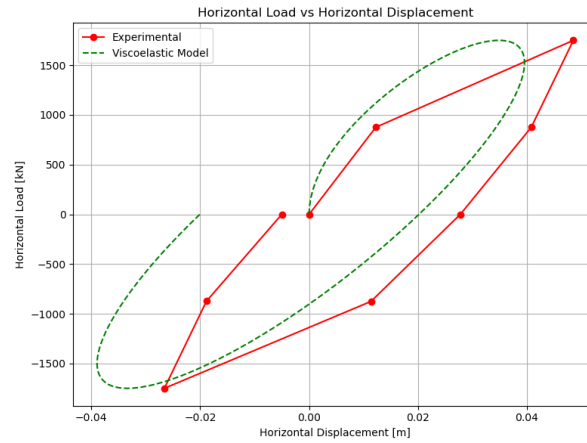


Figure A.61: Horizontal load vs displacement 20m 7th cycle

A.2.3. Viscoelastic Result

Moment vs. Rotation

Cycle	Height=10 m			Height=15 m			Height=20 m		
	G [kNm]	η [kNm]	D [%]	G [kNm]	η [kNm]	D [%]	G [kNm]	η [kNm]	D [%]
1	207,095.81	13,063.49	3.15	216,357.83	15,513.09	3.59	215,381.53	16,409.13	3.81
2	181,450.96	11,959.03	3.29	194,330.66	13,058.40	3.36	208,447.45	16,093.94	3.86
3	148,709.15	9,854.58	3.31	177,541.66	11,603.70	3.27	181,928.13	12,362.40	3.40
4	117,742.39	7,305.02	3.10	129,483.09	8,414.94	3.25	133,510.77	8,457.89	3.17
5	88,819.73	5,575.12	3.14	96,975.44	5,806.20	2.99	102,790.92	6,023.51	2.93
6	67,842.40	4,278.62	3.15	73,864.55	4,344.48	2.94	75,499.81	4,321.29	2.86
7	45,512.60	3,095.04	3.40	49,896.94	3,095.57	3.10	50,516.98	3,493.00	3.46

Table A.1: Moment vs. rotation viscoelastic result

Horizontal Load vs. Horizontal Displacement

Cycle	Height=10 m			Height=15 m			Height=20 m		
	G [kN/m]	η [kN/m]	D [%]	G [kN/m]	η [kN/m]	D [%]	G [kN/m]	η [kN/m]	D [%]
1	252,457.27	12,669.30	2.51	208,693.07	11,625.26	2.79	172,781.95	11,126.28	3.22
2	204,554.12	11,136.51	2.72	173,508.71	10,105.13	2.91	158,787.45	10,704.66	3.37
3	169,550.90	10,947.84	3.23	158,479.45	9,585.00	3.02	136,180.10	8,632.94	3.17
4	133,791.05	9,181.21	3.43	118,281.61	8,015.85	3.39	102,418.95	6,757.61	3.30
5	104,165.59	7,907.05	3.80	90,636.80	6,429.22	3.55	80,022.45	5,498.20	3.44
6	82,630.31	6,727.86	4.07	70,571.24	5,328.07	3.77	59,485.80	4,397.86	3.70
7	57,586.30	5,288.86	4.59	48,264.42	4,065.80	4.21	39,530.35	3,744.15	4.74

Table A.2: Horizontal load vs. horizontal displacement viscoelastic result

B

Experimental Result

B.1. Test Detail

B.1.1. Test Performed at N=100g

Test ID	Box ID	m [kg]	L [m]	W [m]	Gross Weight [kg]	Soil h [m]	ρ [kN/m ³]	e [-]	D_r [-]
V_PG_100g_1	A	7.569	0.41	0.145	21.21	0.148	15209	0.72	81%
V_PG_100g_2	B	6.931	0.41	0.15	20.41	0.142	15141.3	0.73	79%
V_PR_100g_1	C	7.46	0.41	0.147	21.72	0.154	15071.9	0.74	77%
V_PR_100g_2	A	7.569	0.41	0.145	21.73	0.155	15075.8	0.74	77%
V_R_100g_1	C	7.46	0.41	0.147	21.37	0.15	15094	0.74	78%

Table B.1: Experimental test detail for test performed with N=100g

B.1.2. Test Performed at N=50g

Test ID	Box ID	m [kg]	L [m]	W [m]	Gross Weight [kg]	Soil h [m]	ρ [kN/m ³]	e [-]	D_r [-]
V_PR_50g_1	C	7.460	0.410	0.147	21.54	0.152	15077.4	0.74	77%
V_PR_50g_2	C	7.460	0.410	0.147	21.90	0.156	15066.4	0.74	77%
V_PG_50g_1	C	7.460	0.410	0.147	21.67	0.153	15117.2	0.73	78%
V_PG_50g_2	A	7.569	0.410	0.145	20.89	0.143	15371.6	0.70	85%
V_PG_50g_3	B	6.931	0.410	0.150	21.85	0.158	15061.8	0.74	77%
V_PG_50g_4	C	7.460	0.410	0.147	21.90	0.156	15066.4	0.74	77%
H_PR_50g_1	B	6.931	0.410	0.150	21.15	0.150	15120.7	0.73	79%
H_PR_50g_2	A	7.569	0.410	0.145	22.12	0.159	15101.2	0.73	78%
H_PR_50g_3	C	7.460	0.410	0.147	21.90	0.156	15066.4	0.74	77%

Table B.2: Experimental test detail for test performed with N=50g

B.2. Axial Loading
B.2.1. Test Performed at N=100g
Measurement

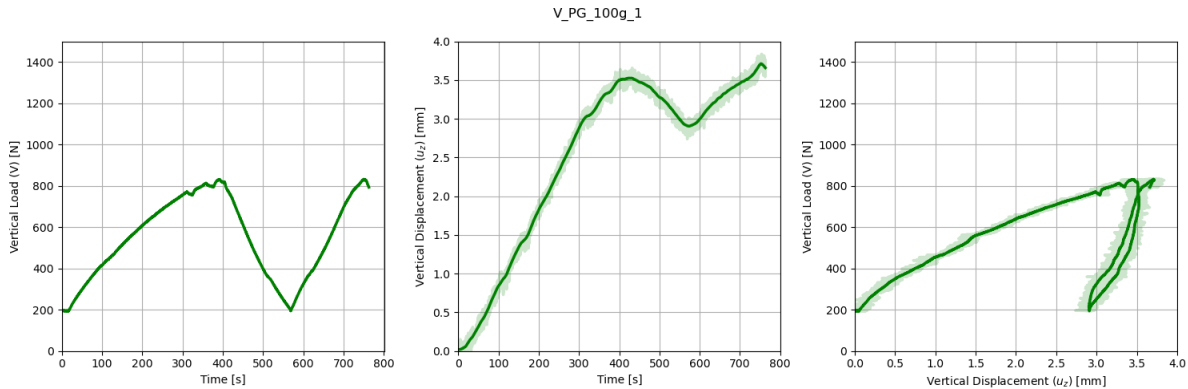


Figure B.1: V_PG_100g_1 measurement

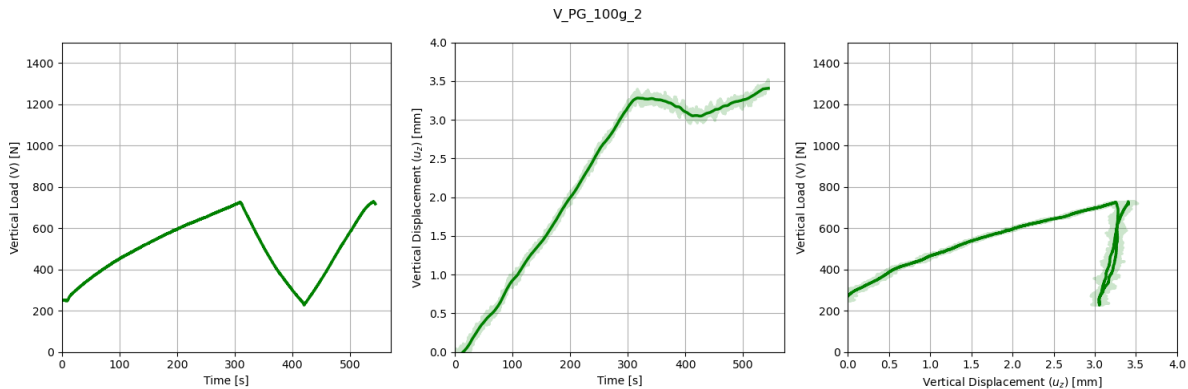


Figure B.2: V_PG_100g_2 measurement

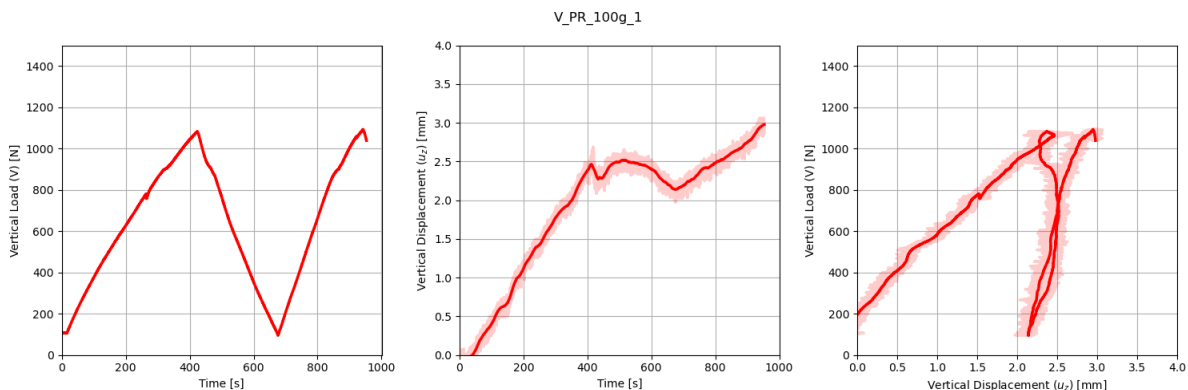


Figure B.3: V_PR_100g_1 measurement

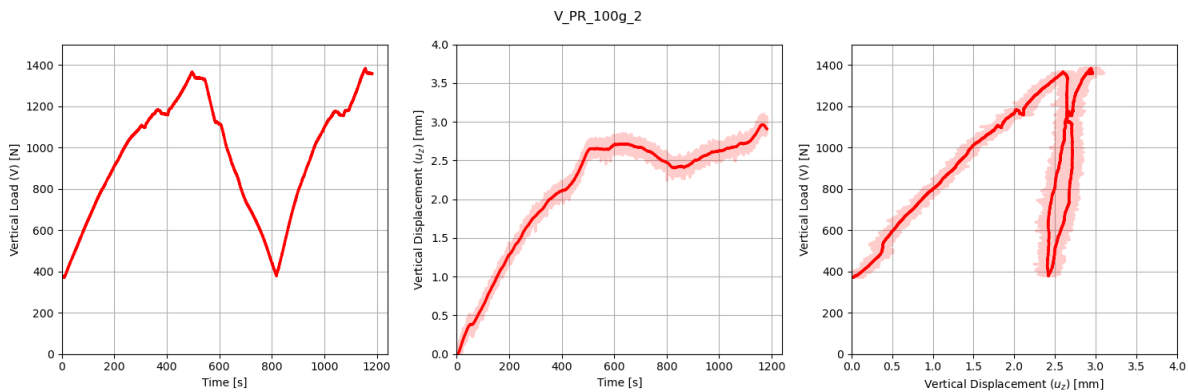


Figure B.4: V_PR_100g_2 measurement

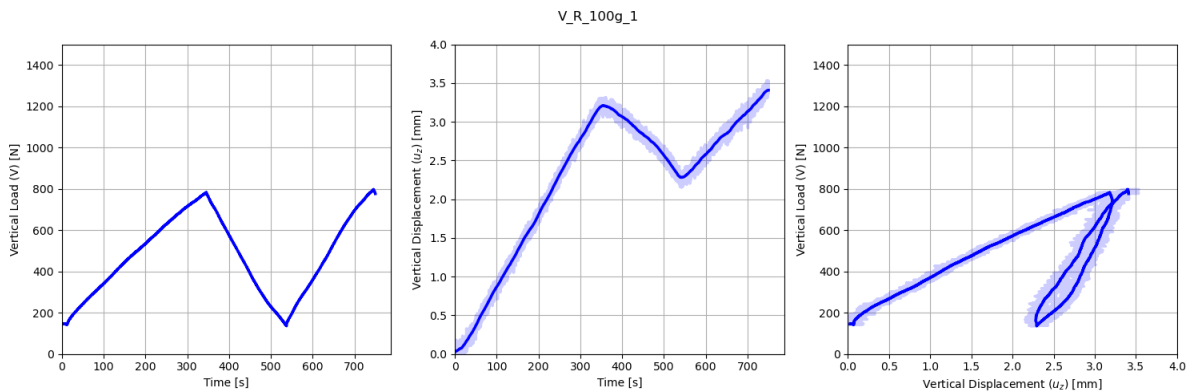


Figure B.5: V_R_100g_1 measurement

Upscale Result

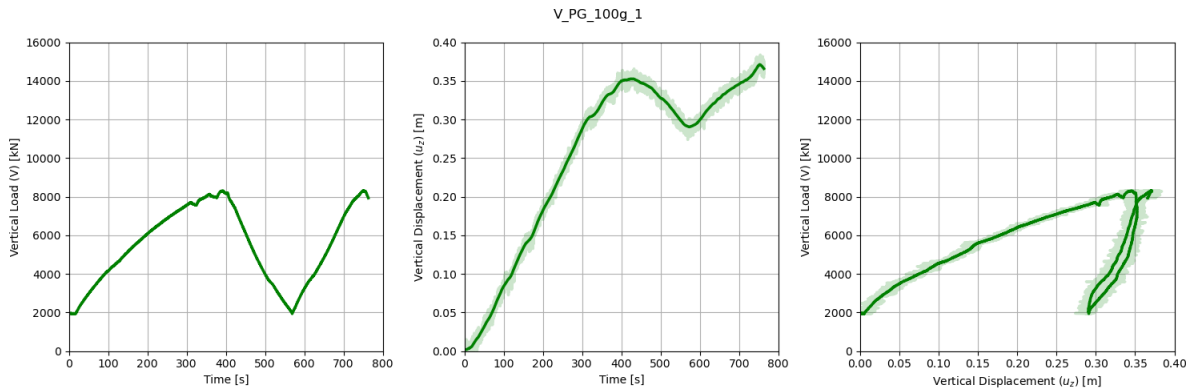


Figure B.6: V_PG_100g_1 upscaled

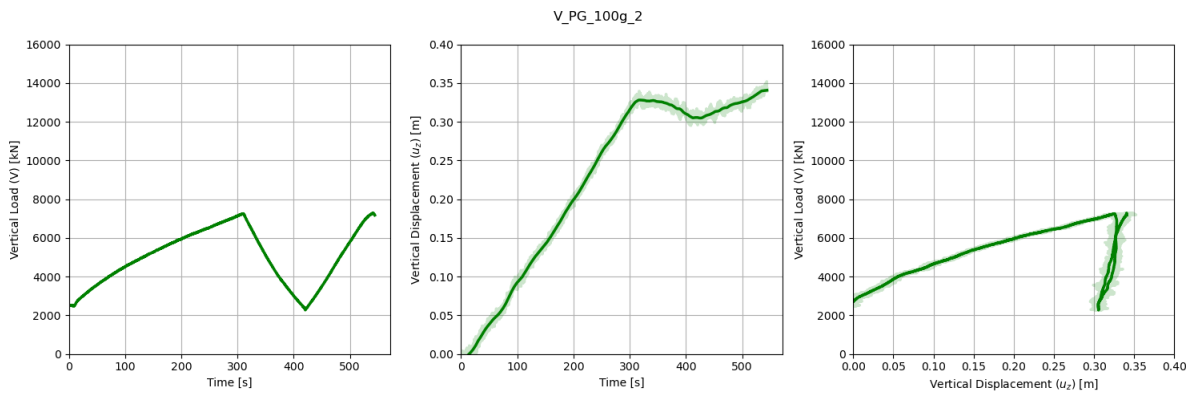


Figure B.7: V_PG_100g_2 upscaled

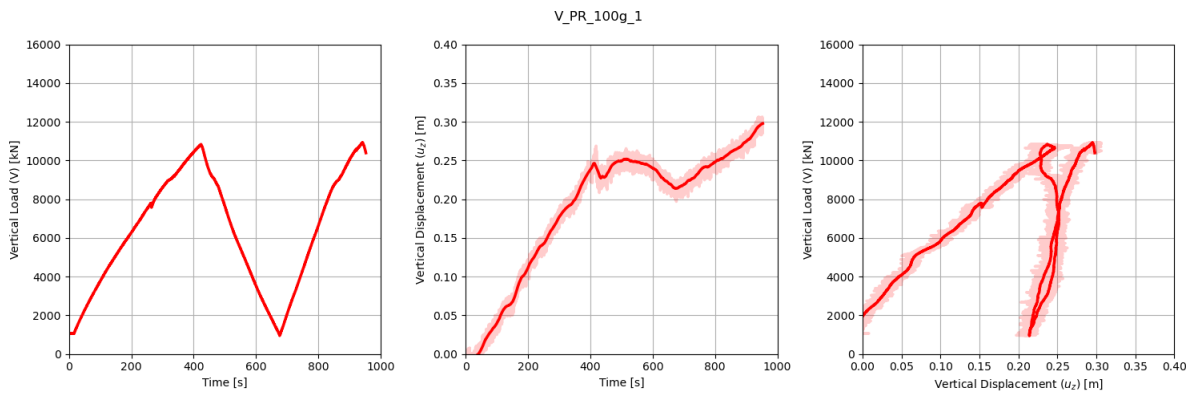


Figure B.8: V_PR_100g_1 upscaled

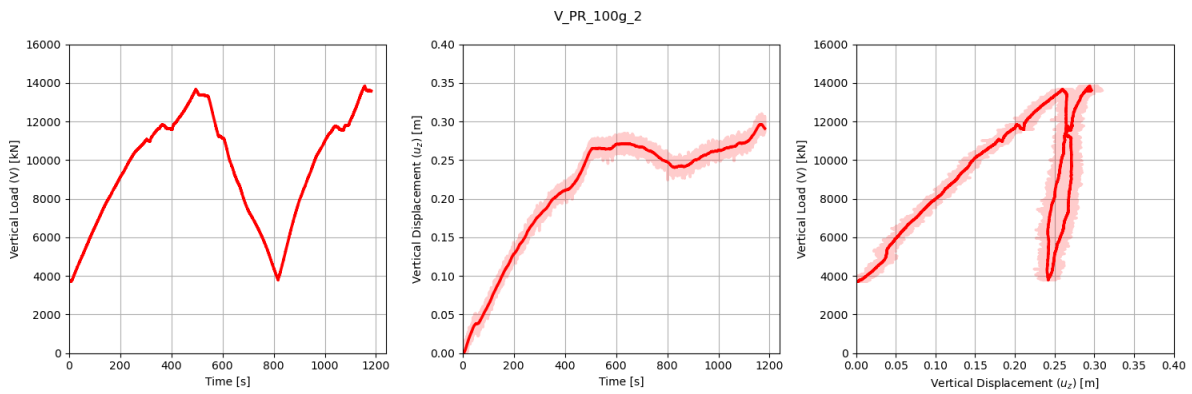


Figure B.9: V_PR_100g_2 upscaled

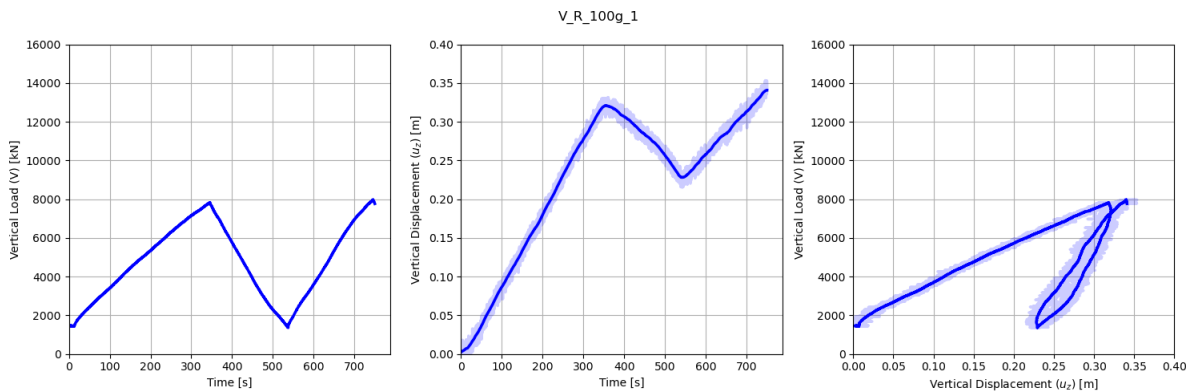


Figure B.10: V_R_100g_1 upscaled

B.2.2. Test Performed at N=50g
Measurement

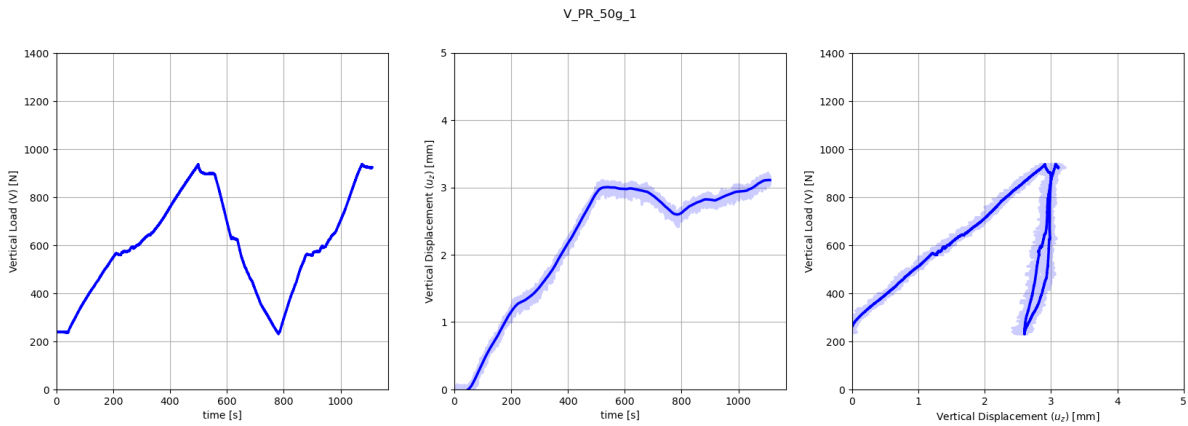


Figure B.11: V_PR_50g_1 measurement

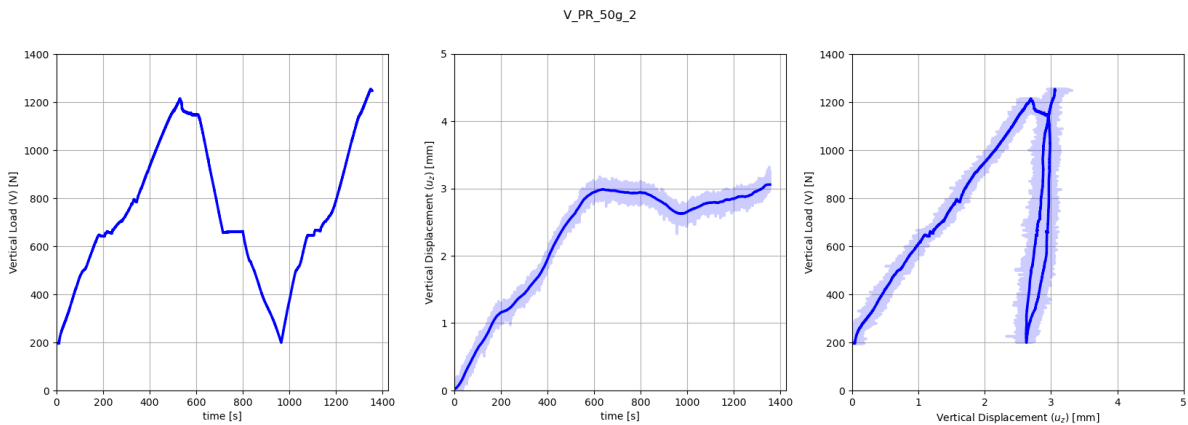


Figure B.12: V_PR_50g_2 measurement

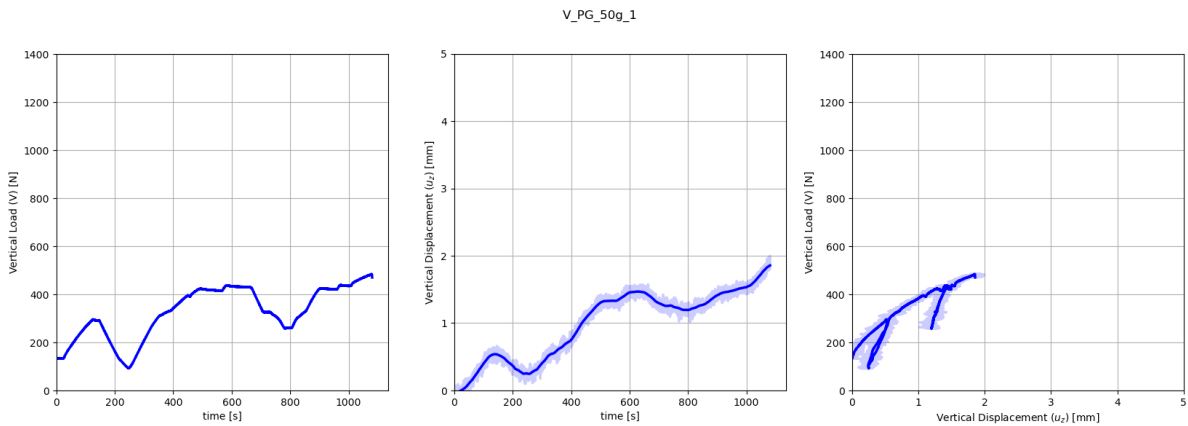


Figure B.13: V_PG_50g_1 measurement

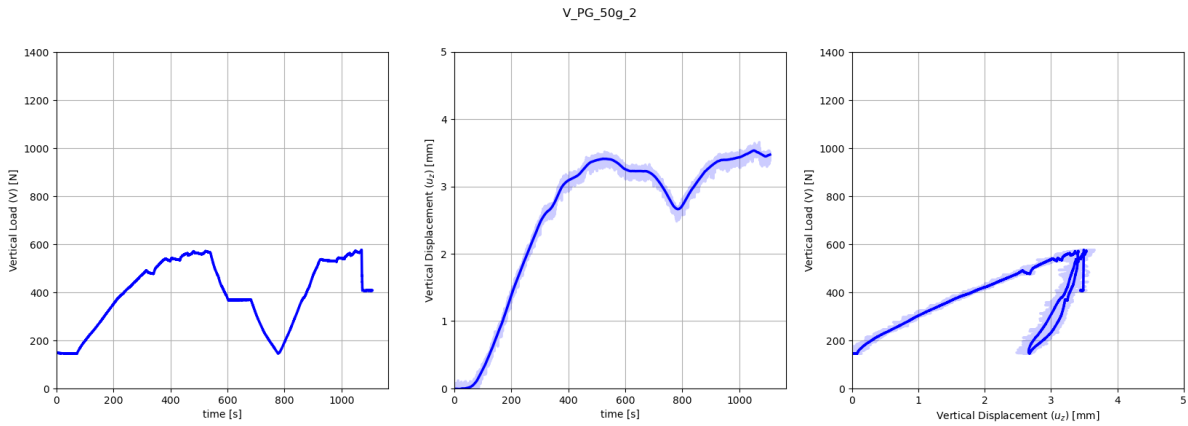


Figure B.14: V_PG_50g_2 measurement

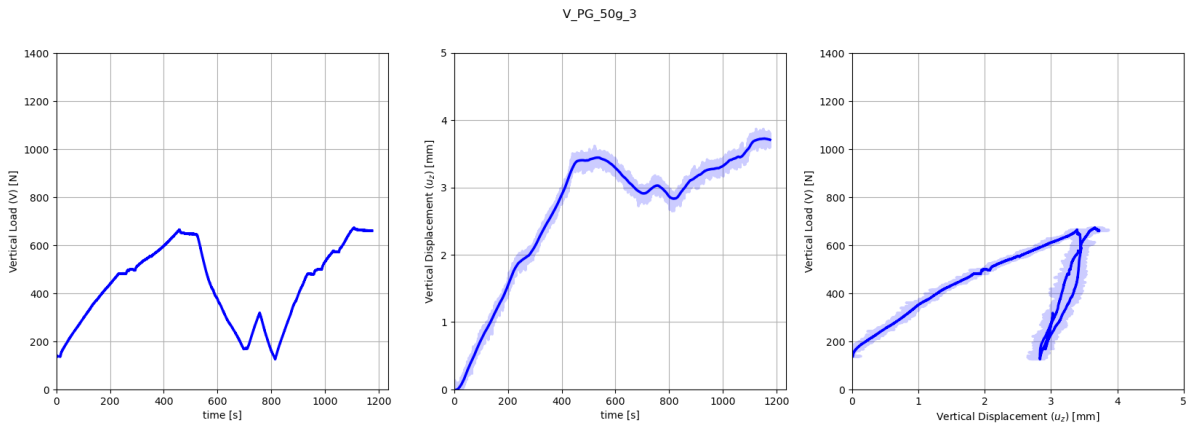


Figure B.15: V_PG_50g_3 measurement

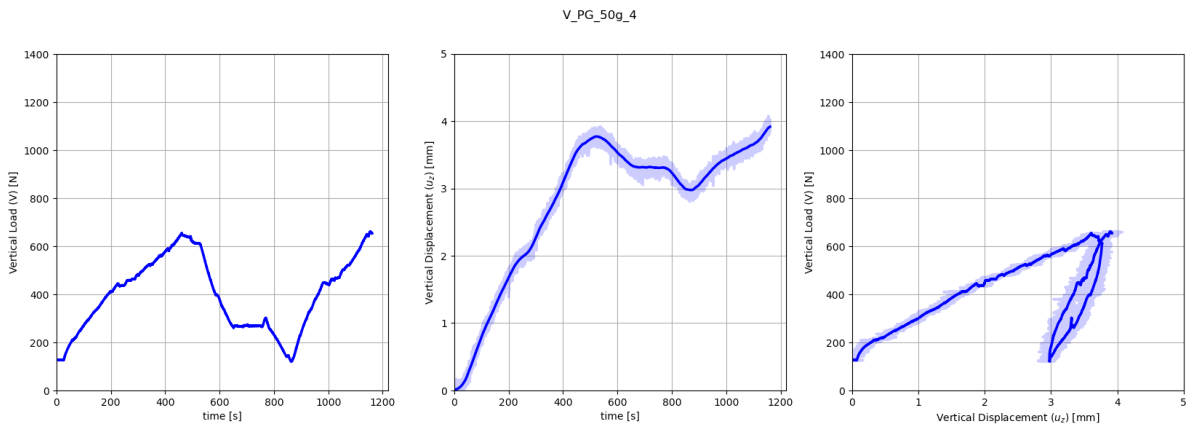


Figure B.16: V_PG_50g_4 measurement

Upscale Result

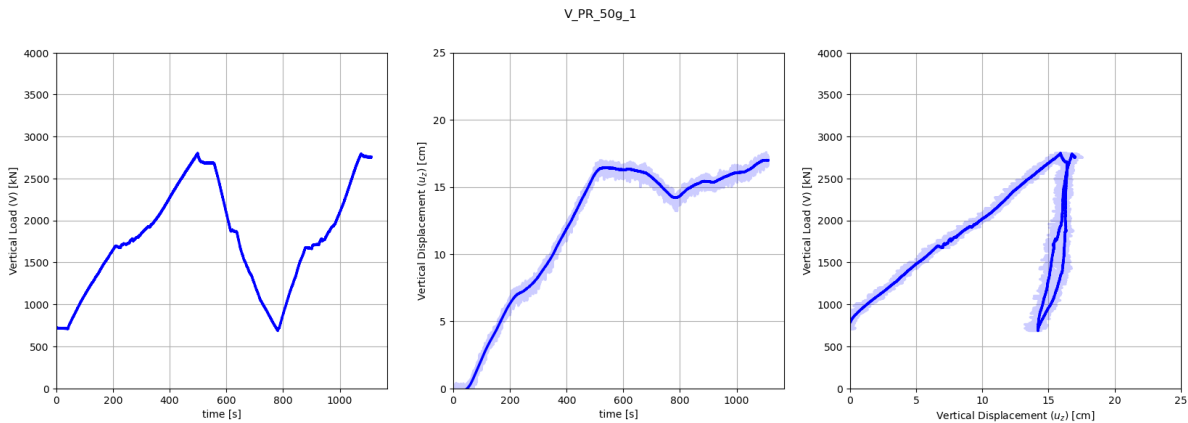


Figure B.17: V_PR_50g_1 upscaled

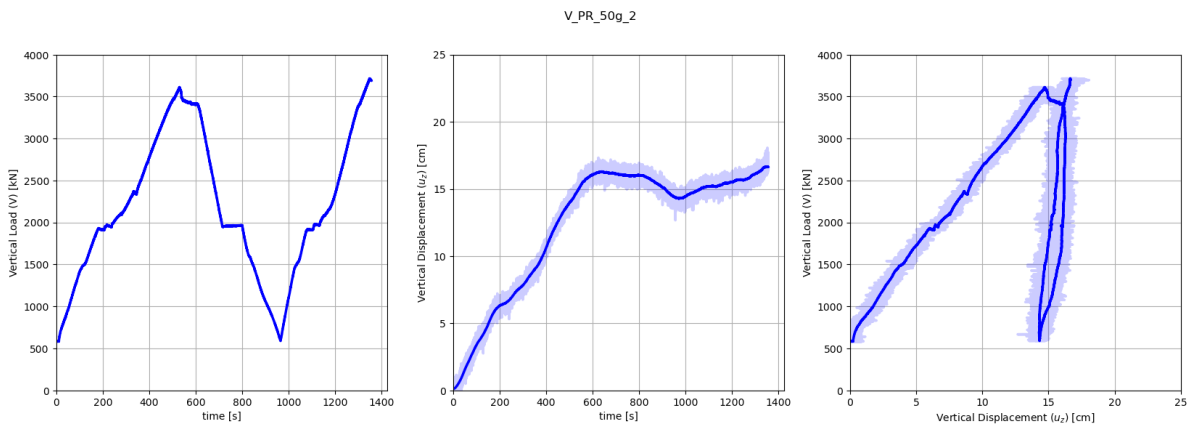


Figure B.18: V_PR_50g_2 upscaled

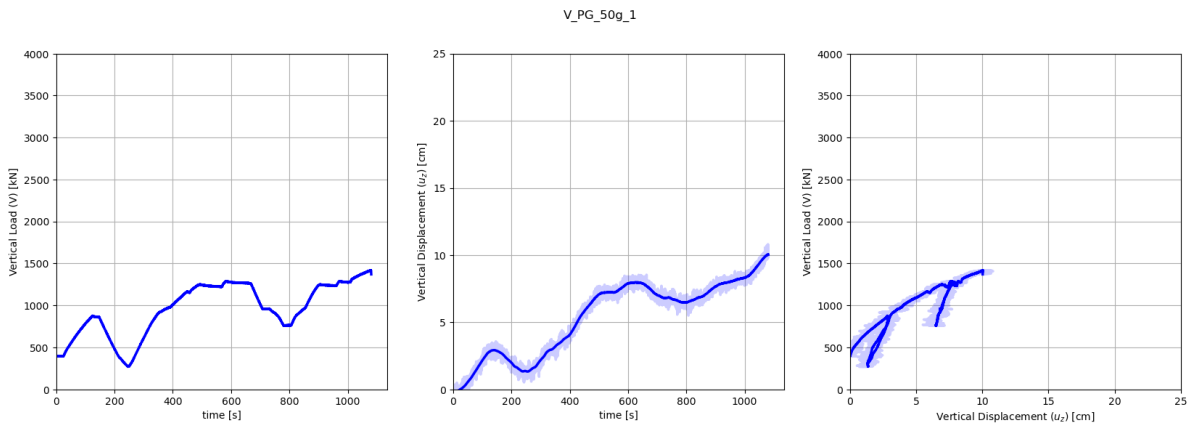


Figure B.19: V_PG_50g_1 upscaled

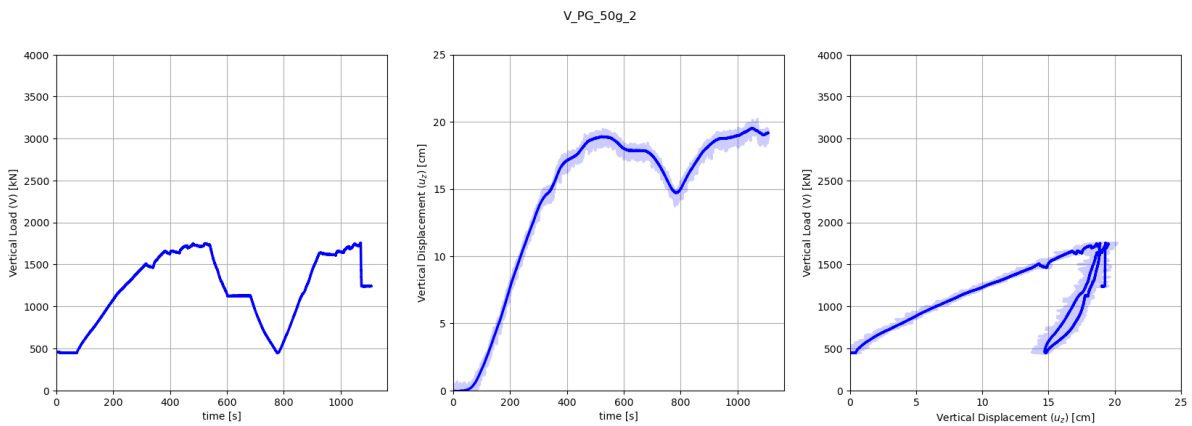


Figure B.20: V_PG_50g_2 upscaled

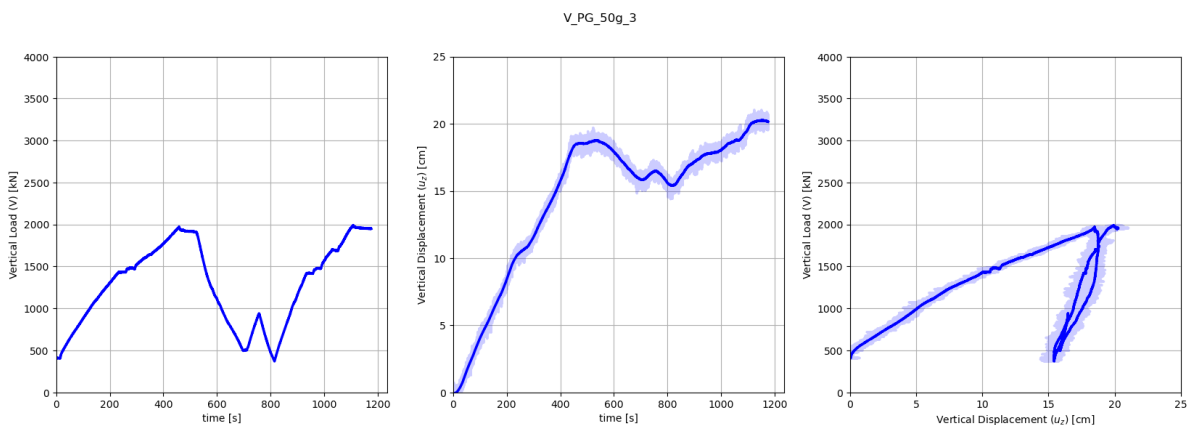


Figure B.21: V_PG_50g_3 upscaled

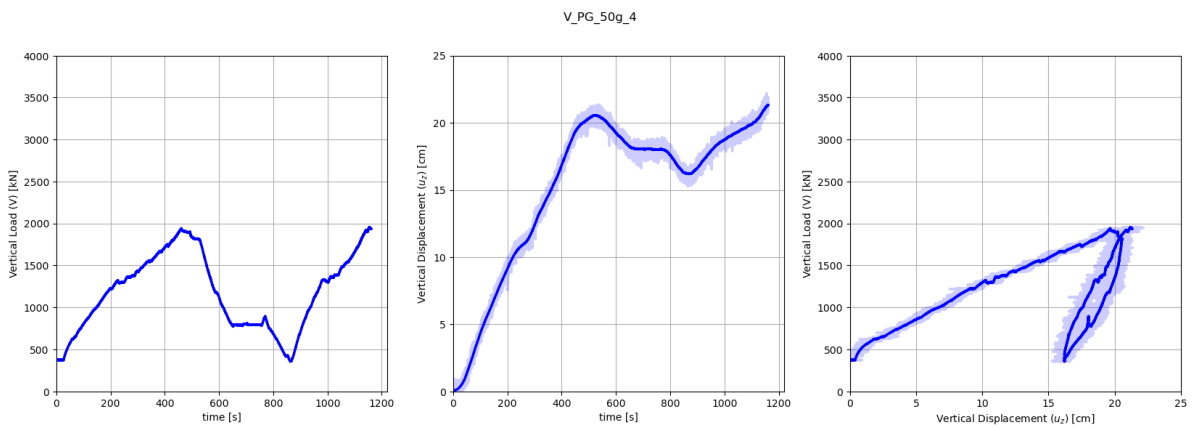
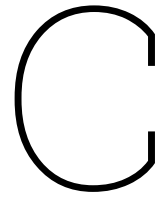


Figure B.22: V_PG_50g_4 upscaled



Viscoelastic Calibration Code

```
1 """
2 Viscoelastic Model: Fitting cyclic data with viscoelastic, using optimizing function.
3 """
4
5 #library imports
6 import math
7 import numpy as np
8 import pandas as pd
9 import matplotlib.pyplot as plt
10 from matplotlib.ticker import MultipleLocator
11 from scipy.interpolate import make_interp_spline
12 from scipy.optimize import minimize
13
14 # Path
15 file_path = r'..\FINAL\REPORT\CYCLIC.xlsx'
16
17
18 # Read
19 df = pd.read_excel(file_path, sheet_name="1")
20 PR_10m = df.loc[df['Height[m]'] == 10]
21
22
23 # Function
24 def preprocess_data(data, t_min, t_max, gam_col):
25     """Preprocesses the data by normalizing time."""
26     data_filtered = data[(data['time[s]'] >= t_min) & (data['time[s]'] <= t_max)].copy()
27     data_filtered['time[s]'] -= t_min
28     return data_filtered
29
30 def setup_simulation(t_exp, tau_exp, N):
31     """Sets up the time calibration and amplitude for simulation based on FE data."""
32     Tmax = t_exp[-1]
33     dt = Tmax / N
34     t_cal = np.linspace(0, Tmax, N)
35     Amp = np.max(tau_exp)
36     om = 2 * np.pi / Tmax
37     tau_cal = Amp * np.sin(om * t_cal)
38     gam_cal = np.zeros(N)
39     return t_cal, tau_cal, gam_cal, dt
40
41 def viscoelastic_model(dt, N, G, eta, tau_cal, gam_cal):
42     """Simulates the viscoelastic model over time."""
43     for i in range(1, N):
44         gam_cal[i] = (dt * (tau_cal[i] + tau_cal[i-1]) + (2 * eta - G * dt) * gam_cal[i-1]) /
45             (dt * G + 2 * eta)
46     return gam_cal
47
48 def loss_function(params, gam_smooth, tau_cal, t_cal):
49     """Calculates the weighted RMS loss for optimization."""
```

```

49     G, eta = params
50     gam_cal = viscoelastic_model(t_cal[1] - t_cal[0], len(t_cal), G, eta, tau_cal, np.
        zeros_like(tau_cal))
51     weights = np.linspace(1, 2, len(gam_smooth)) # Increasing weight towards the end
52     rms = np.sum(weights * (gam_smooth - gam_cal)**2)
53     return rms
54
55 def optimize_parameters(gam_smooth, tau_cal, t_cal):
56     """Finds optimal G and eta using Nelder-Mead optimization."""
57     initial_guess = [10, 10] # Initial guesses for G and eta
58     result = minimize(lambda params: loss_function(params, gam_smooth, tau_cal, t_cal),
59                       initial_guess, method='Nelder-Mead')
60     return result.x, result.success
61
62 def plot_results(t_exp, tau_exp, gam_exp, t_cal, tau_cal, gam_cal, gam_smooth):
63     """Creates plots of the stress-time, strain-time, and stress-strain curves."""
64     fig, axes = plt.subplots(1, 3, figsize=(15, 5))
65     ax1, ax2, ax3 = axes
66
67     ax1.plot(t_exp, tau_exp, 'ro-', label="FE")
68     ax1.plot(t_cal, tau_cal, 'g--', label="Calculated")
69     ax1.set(title='Stress-Time', xlabel='Time(s)', ylabel='Force or Moment')
70     ax1.legend()
71     ax1.grid()
72
73     ax2.plot(t_exp, gam_exp, 'ro-', label="FE")
74     ax2.plot(t_cal, gam_cal, 'g--', label="Calculated")
75     ax2.set(title='Strain-Time', xlabel='Time(s)', ylabel='Rotation or Displacement')
76     ax2.legend()
77     ax2.grid()
78
79     ax3.plot(gam_exp, tau_exp, 'ro-', label="FE")
80     ax3.plot(gam_cal, tau_cal, 'g--', label="Calculated")
81     ax3.set(title='Stress-Strain', xlabel='Rotation or Displacement', ylabel='Force or Moment')
82     ax3.legend()
83     ax3.grid()
84
85     plt.tight_layout()
86     plt.show()
87
88 def main(data, t_min, t_max, tau_col, gam_col):
89     data_processed = preprocess_data(data, t_min, t_max, gam_col)
90     t_exp = data_processed['time[s]'].to_numpy()
91     gam_exp = data_processed[gam_col].to_numpy()
92     tau_exp = data_processed[tau_col].to_numpy() # Adjust unit if necessary
93
94     N=1000
95     t_cal, tau_cal, gam_cal, dt = setup_simulation(t_exp, tau_exp, N)
96     spl = make_interp_spline(t_exp, gam_exp, k=3)
97     gam_smooth = spl(t_cal)
98
99     [G, eta], success = optimize_parameters(gam_smooth, tau_cal, t_cal)
100    print(f'Optimized G: {G}, eta: {eta}, Success: {success}')
101
102    gam_cal = viscoelastic_model(dt, N, G, eta, tau_cal, gam_cal)
103    plot_results(t_exp, tau_exp, gam_exp, t_cal, tau_cal, gam_cal, gam_smooth)
104
105    max_tau = np.max(tau_exp)
106    max_gam = np.max(gam_exp)
107
108    return G, eta, success, max_tau, max_gam
109
110 # Result
111 results_PR10_moment = pd.DataFrame(columns=['Cycle', 'G', 'eta', 'Success', 'MaxTau', 'MaxGam', 'Damping'])
112 omega = 2*np.pi
113
114 for i in range(7):
115     print(f"Cycle: {i+1}")
116     G, eta, success, max_tau, max_gam = main(PR_10m, i, i + 1, 'M[kNm]', 'Rotation[degree]')

```

```

    )
117     new_row = pd.DataFrame({
118         'Cycle': [i + 1],
119         'G': [G],
120         'eta': [eta],
121         'Success': [success],
122         'Max_Tau': [max_tau],
123         'Max_Gam': [max_gam],
124         'Damping': [eta*omega/(2*G)]
125     })
126     results_PR10_moment = pd.concat([results_PR10_moment, new_row], ignore_index=True)
127
128
129 # Plotting
130 # Create a figure with two subplots side by side
131 fig, (ax1, ax2, ax3) = plt.subplots(1, 3, figsize=(14, 5)) # Adjusted figsize for better
    display
132
133 # Plot for Horizontal Load vs. ux
134 ax1.plot(results_PR10_moment["Max_Gam"], results_PR10_moment["G"], label="Piled_Raft_10m",
    linewidth=2.5, color="r")
135 ax1.set_title("$G_M$ vs $\\Theta$")
136 ax1.set_xlabel("Rotation_($\\Theta$) [°]")
137 ax1.set_ylabel("Stiffness_($G_M$) [-]")
138 ax1.legend(loc="best")
139 ax1.grid(True)
140
141 # Plot for Moment vs. Rotation
142 ax2.plot(results_PR10_moment["Max_Gam"], results_PR10_moment["eta"], label="Piled_Raft_10m",
    linewidth=2.5, color="r")
143 ax2.set_title("$\\eta_M$ vs $\\Theta$")
144 ax2.set_xlabel("Rotation_($\\Theta$) [°]")
145 ax2.set_ylabel("Viscosity_($\\eta_M$) [-]")
146 ax2.legend(loc="best")
147 ax2.grid(True)
148
149 ax3.plot(results_PR10_moment["Max_Gam"], results_PR10_moment["Damping"], label="Piled_Raft_10
    m", linewidth=2.5, color="r")
150 ax3.set_title("$D$ vs $\\Theta$")
151 ax3.set_xlabel("Rotation_($\\Theta$) [°]")
152 ax3.set_ylabel("Damping_($D$) [-]")
153 ax3.legend(loc="best")
154 ax3.grid(True)
155
156 plt.tight_layout()
157 plt.show()

```

D

SANISAND-MS Calibration Code

In general, the calibration is divided to 5 sections:

- Calibration of G_0 Based on Oedometer tests
- Calibration of M_c based on critical state from triaxial consolidated drained (CD) test
- Calibration of e_0 , λ_c , ξ Based on e_c
- Calibration of h_0 , c_h , n_b based on triaxial CD $q - \varepsilon_{ax}$
- Calibration of A_0 , n_d based on triaxial CD $\varepsilon_{vol} - \varepsilon_{ax}$

```
1 """
2 Code created to calibrate SANISAND-MS Model based on Oedometer test, Triaxial test, and
   PLAXIS Soil Test.
3 """
4
5 ### Initial Setup
6
7 # General Libraries
8 import os
9 import sys
10 import shutil
11 import subprocess
12 import time
13 import math
14 import numpy as np
15 import pandas as pd
16 from math import pi
17
18 # Scientific Libraries
19 import scipy
20 from scipy.optimize import minimize
21 from scipy.interpolate import interp1d
22
23 # Machine Learning Libraries
24 from skopt import gp_minimize
25 from skopt.space import Real, Integer
26 from skopt.utils import use_named_args
27
28 # Visualization Libraries
29 import matplotlib.pyplot as plt
30 from matplotlib.ticker import MultipleLocator
31
32 # Email Handling Libraries
33 import smtplib
34 from email.mime.multipart import MIMEMultipart
35 from email.mime.text import MIMEText
36
37
38 #Library for Plaxis scripting
```



```

39 from plxscripting.easy import*
40
41 #Path
42 plaxis_script_lib_path=r'C:\ProgramData\Seequent\PLAXIS_Python_Distribution_V2'
43 working_folder = r'\.plaxis3d'
44 simulation_name = r'\script_test_model'
45 results_path = working_folder + simulation_name + '.p3d'
46 plaxis_python_path=r'C:\ProgramData\Seequent\PLAXIS_Python_Distribution_V2\python\Lib\site-
    packages'
47 sys.path.insert(0,os.path.join(plaxis_script_lib_path,plaxis_python_path))
48
49 #Server
50 in_port = 10000
51 out_port = 10001
52 address = 'localhost'
53 pw = '...'
54
55 s_i, g_i = new_server('localhost', port=in_port, password=pw )
56 s_o, g_o = new_server('localhost', port=out_port, password=pw )
57
58
59 ## General Function
60 ### Function to run PLAXIS Soil Test
61 def run_oed_each(e_ini):
62     """Run Oedometer Test in PLAXIS Soil Test"""
63     g_i.set(g_i.Material.User17, e_ini)
64     g_i.calculate(g_i.Oedometer)
65     df = pd.DataFrame({
66         'Vertical_stress[kPa]': g_i.Oedometer.Results.SigyyE,
67         'Vertical_strain[%]': g_i.Oedometer.Results.Epsyy
68     })
69     df['Vertical_stress[kPa]'] = -df['Vertical_stress[kPa]']
70     df['Vertical_strain[%]'] = df['Vertical_strain[%]'] *100
71     return df
72
73 def run_oed():
74     plaxis_oed_1= run_oed_each(0.8817) #Initial void ratio
75     plaxis_oed_2= run_oed_each(0.8127) #Initial void ratio
76     plaxis_oed_3= run_oed_each(0.9701) #Initial void ratio
77     plaxis_oed_4= run_oed_each(0.7262) #Initial void ratio
78     return plaxis_oed_1, plaxis_oed_2, plaxis_oed_3, plaxis_oed_4
79
80 def run_txcd_each(e_ini, cell_pressure):
81     """Run Triaxial CD Test in PLAXIS Soil Test"""
82     g_i.set(g_i.Triaxial.Behaviour, "Drained")
83     g_i.set(g_i.Material.User17, e_ini)
84     g_i.set(g_i.Triaxial.CellPressure, cell_pressure)
85     g_i.calculate(g_i.Triaxial)
86     df = pd.DataFrame({
87         'Axial_strain[%]': g_i.Triaxial.Results.Epsyy,
88         'Volumetric_strain[%]': g_i.Triaxial.Results.TotalVolumetricStrain,
89         'Axial_stress[kPa]': g_i.Triaxial.Results.SigmaEffective1,
90         'Deviatoric_stress[kPa]': g_i.Triaxial.Results.DeviatoricStress
91     })
92     df['Axial_strain[%]'] = -df['Axial_strain[%]'] *100
93     df['Volumetric_strain[%]'] = -df['Volumetric_strain[%]'] *100
94     df['Axial_stress[kPa]'] = df['Axial_stress[kPa]']
95     df['Deviatoric_stress[kPa]'] = df['Deviatoric_stress[kPa]']
96     return df
97
98 def run_txcd():
99     plaxis_txcd_1 = run_txcd_each(0.8144, 55) #Initial void ratio and consolidation pressure
100     plaxis_txcd_2 = run_txcd_each(0.8144, 205) #Initial void ratio and consolidation pressure
101     plaxis_txcd_3 = run_txcd_each(0.8144, 405) #Initial void ratio and consolidation pressure
102     return plaxis_txcd_1, plaxis_txcd_2, plaxis_txcd_3
103
104 def run_txcu_each(e_ini, cell_pressure):
105     """Run Triaxial CU Test in PLAXIS Soil Test"""
106     g_i.set(g_i.Triaxial.Behaviour, "Undrained")
107     g_i.set(g_i.Material.User17, e_ini)
108     g_i.set(g_i.Triaxial.CellPressure, cell_pressure)

```

```

109     g_i.calculate(g_i.Triaxial)
110     df = pd.DataFrame({
111         'Axial_strain[%]': g_i.Triaxial.Results.Epsyy,
112         'Mean_effective_stress[kPa]': g_i.Triaxial.Results.MeanEffStress,
113         'Axial_stress[kPa]': g_i.Triaxial.Results.SigmaEffective1,
114         'Deviatoric_stress[kPa]': g_i.Triaxial.Results.DeviatoricStress
115     })
116     df['Axial_strain[%]'] = -df['Axial_strain[%]'] * 100
117     df['Mean_effective_stress[kPa]'] = -df['Mean_effective_stress[kPa]']
118     df['Axial_stress[kPa]'] = df['Axial_stress[kPa]']
119     df['Deviatoric_stress[kPa]'] = df['Deviatoric_stress[kPa]']
120     return df
121 def run_txcu():
122     plaxis_txcu_1 = run_txcu_each(0.8998,55) #Initial void ratio and consolidation pressure
123     plaxis_txcu_2 = run_txcu_each(0.8998,205) #Initial void ratio and consolidation pressure
124     plaxis_txcu_3 = run_txcu_each(0.8998,405) #Initial void ratio and consolidation pressure
125     return plaxis_txcu_1, plaxis_txcu_2, plaxis_txcu_3
126
127
128 ### Function for Plotting
129 def plot_oed(lab_oed, plaxis_oed, e_values):
130     """Plot Oedometer Test between Laboratory Testing and PLAXIS Soil Test"""
131     fig, ax = plt.subplots(1, oed_count, figsize=(18, 5))
132     fig.suptitle("OEDOMETER TEST", fontsize=16)
133     for i in range(oed_count):
134         ax[i].plot(lab_oed[i]["Vertical_stress[kPa]"].values, lab_oed[i]["Vertical_strain[%]"].values, "o", label=f"Lab_e0={e_values[i]}", linewidth=2.5, color="r")
135         ax[i].plot(plaxis_oed[i]["Vertical_stress[kPa]"].values, plaxis_oed[i]["Vertical_strain[%]"].values, label=f"PLAXIS_e0={e_values[i]}", linewidth=2.5, color="b")
136         ax[i].set_xlabel("Vertical Stress[kPa]")
137         ax[i].legend(loc="best")
138         ax[i].set_xlim(0, 1000)
139         ax[i].set_ylim(-4, 0)
140         ax[i].grid()
141     ax[0].set_ylabel("Vertical Strain[%]")
142     plt.show()
143
144 def plot_txcd(lab_txcd, plaxis_txcd, e_txcd, sigmaxx_txcd):
145     """Plot Triaxial CD Test between Laboratory Testing and PLAXIS Soil Test"""
146     fig, ax = plt.subplots(2, txcd_count, figsize=(20, 10))
147     fig.suptitle(r"TRIAXIAL CD", fontsize=16)
148     def plot_txcd_each(i, lab_txcd, plaxis_txcd, e, s):
149         ax[0, i].plot(lab_txcd["Axial_strain[%]"].values, lab_txcd["Volumetric_strain[%]"].values, "o", label=f"Lab_e0={e}, s={s} kPa", linewidth=2.5, color="r")
150         ax[1, i].plot(lab_txcd["Axial_strain[%]"].values, lab_txcd["Deviatoric_stress[kPa]"].values, "o", label=f"Lab_e0={e}, s={s} kPa", linewidth=2.5, color="r")
151         ax[0, i].plot(plaxis_txcd["Axial_strain[%]"].values, plaxis_txcd["Volumetric_strain[%]"].values, label=f"PLAXIS_e0={e}, s={s} kPa", linewidth=2.5, color="b")
152         ax[1, i].plot(plaxis_txcd["Axial_strain[%]"].values, plaxis_txcd["Deviatoric_stress[kPa]"].values, label=f"PLAXIS_e0={e}, s={s} kPa", linewidth=2.5, color="b")
153     for i in range(txcd_count):
154         plot_txcd_each(i, lab_txcd[i], plaxis_txcd[i], e_txcd[i], sigmaxx_txcd[i])
155     ax[0, 0].set_ylabel("Volumetric strain[%]")
156     ax[1, 0].set_ylabel("Deviatoric stress[kPa]")
157     for i in range(2):
158         for j in range(txcd_count):
159             ax[i, j].set_xlabel("Axial strain[%]")
160             ax[i, j].legend(loc="best")
161             ax[i, j].set_xlim(-1, 27)
162             ax[0, j].set_ylim(-10, 1)
163             ax[1, j].set_ylim(0, 1800)
164             ax[i, j].grid()
165     plt.show()
166
167
168 def plot_txcu(lab_txcu, plaxis_txcu, e_txcu, sigmaxx_txcu):
169     """Plot Triaxial CU Test between Laboratory Testing and PLAXIS Soil Test"""
170     fig, ax = plt.subplots(2, txcu_count, figsize=(20, 10))
171     fig.suptitle(r"TRIAXIAL CU", fontsize=16)
172     def plot_txcu_each(i, lab_txcu, plaxis_txcu, e, s):
173         ax[0, i].plot(lab_txcu["Mean_effective_stress[kPa]"].values, lab_txcu["Deviatoric_

```

```

stress[kPa]").values, "o", label=f"Lab_{e0}_{e}_{s}_kPa", linewidth=2.5, color="r")
174 ax[1, i].plot(lab_txcu["Axial_strain_%"].values, lab_txcu["Deviatoric_stress_kPa"].values, "o", label=f"Lab_{e0}_{e}_{s}_kPa", linewidth=2.5, color="r")
175 ax[0, i].plot(plaxis_txcu["Mean_effective_stress_kPa"].values, plaxis_txcu["Deviatoric_stress_kPa"].values, label=f"PLAXIS_{e0}_{e}_{s}_kPa", linewidth=2.5, color="b")
176 ax[1, i].plot(plaxis_txcu["Axial_strain_%"].values, plaxis_txcu["Deviatoric_stress_kPa"].values, label=f"PLAXIS_{e0}_{e}_{s}_kPa", linewidth=2.5, color="b")
177 for i in range(txcu_count):
178     plot_txcu_each(i, lab_txcu[i], plaxis_txcu[i], e_txcu[i], sigmaxx_txcu[i])
179 ax[0, 0].set_ylabel("Deviatoric_stress_kPa")
180 ax[1, 0].set_ylabel("Deviatoric_stress_kPa")
181 for i in range(2):
182     for j in range(txcu_count):
183         ax[0, j].set_xlabel("Mean_effective_stress_kPa")
184         ax[1, j].set_xlabel("Axial_strain_%")
185         ax[i, j].legend(loc="best")
186         ax[0, j].set_xlim(0, 2000)
187         ax[1, j].set_xlim(-1, 27)
188         ax[0, j].set_ylim(0, 2500)
189         ax[1, j].set_ylim(0, 2500)
190         ax[i, j].grid()
191 plt.show()
192
193
194 ### Preprocessing Laboratory Data
195 #Path
196 lab_path = r'..\\LAB_TEST.xlsx'
197 lab_oed = pd.read_excel(lab_path, sheet_name="OED")
198 lab_txcd = pd.read_excel(lab_path, sheet_name="TRIAX-CD")
199 lab_txcu = pd.read_excel(lab_path, sheet_name="TRIAX-CU")
200
201 #Number of Test to be used
202 oed_count = 4
203 txcd_count = 3
204 txcu_count = 3
205
206 # Extract test data for each type
207 lab_oed = [lab_oed.loc[lab_oed['Test'] == i] for i in range(1, oed_count+1)]
208 lab_txcd = [lab_txcd.loc[lab_txcd['Test'] == i] for i in range(1, txcd_count+1)]
209 lab_txcu = [lab_txcu.loc[lab_txcu['Test'] == i] for i in range(1, txcu_count+1)]
210
211 # Value for Oedometer Test
212 e_oed = [0.8817, 0.8128, 0.9701, 0.7262] #initial void ratio
213
214 # Value for TXCD Test
215 e_txcd = [0.8144, 0.8144, 0.8144] #initial void ratio
216 sigmaxx_txcd = [55, 205, 405] #Consolidation Pressure
217
218 # Value for TXCU Test
219 e_txcu = [0.8998, 0.8998, 0.8998] #initial void ratio
220 sigmaxx_txcu = [55, 205, 405] #Consolidation Pressure
221
222 #Calculate mean effective stress
223 for i in range(len(lab_txcd)):
224     lab_txcd[i]["Mean_effective_stress_kPa"] = (lab_txcd[i]["Axial_stress_kPa"] + 2*
225         lab_txcd[i]["Radial_stress_kPa"])/3
226     lab_txcd[i]["e"] = ((1-(lab_txcd[i]["Volumetric_strain_%"])/100))*(1+e_txcd[i]))-1
227 for i in range(len(lab_txcu)):
228     lab_txcu[i]["e"] = e_txcu[i]
229
230
231 ### Initial parameters
232 initial_parameters = {
233     "sanisandms_G0": 95, # G0 Shear modulus constant
234     "sanisandms_nu": 0.05, # nu Poisson's ratio
235     "sanisandms_M_c": 1.275, # M_c Slope of CSL in q:p pl
236     "sanisandms_c": 0.7, # c M_e/M_c
237     "sanisandms_lambda_c": 0.018, # lambda_c CSL parameter (e:p pl

```

```

238     "sanisandms_e_0": 0.935,      # e_0      Void ratio on CSL at
239     "sanisandms_xi": 0.3,        # xi      CSL parameter (e:p pla
240     "sanisandms_m": 0.01,       # m      m_0, opening of yield s
241     "sanisandms_h_0": 30,       # h_0     Plastic modulus const
242     "sanisandms_c_h": 3,        # c_h     Plastic modulus constan
243     "sanisandms_n_b": 1.2,      # n_b     Plastic modulus const
244     "sanisandms_A_0": 0.9,      # A_0     Dilatancy constant
245     "sanisandms_n_d": 0.9,      # n_d     Dilatancy constant
246     "sanisandms_mu_0": 500,     # mu_0    Memory-surface const
247     "sanisandms_zeta": 0.0001,  # zeta    Memory surface shrink
248     "sanisandms_beta": 0,       # beta    Dilatancy constant
249     "sanisandms_e_ini": 1,      # e_in
250     "sanisandms_e_max": 1.07,   # e_max
251     "sanisandms_e_min": 0.64,   # e_min
252     "sanisandms_M_c_bmax": 1.92,
253     "sanisandms_El_Scheme": 1
254 }
255
256 # Set material parameters in PLAXIS
257 for i, (param, value) in enumerate(initial_parameters.items(), start=1):
258     g_i.set(getattr(g_i.Material, f"User{i}"), value)
259
260
261 ### Initial simulation in PLAXIS Soil Test
262 #Run the simulation
263 plaxis_oed = run_oed()
264 plaxis_txcd = run_txcd()
265 plaxis_txcu = run_txcu()
266
267 #Plot the result
268 plot_oed(lab_oed, plaxis_oed, e_oed)
269 plot_txcd(lab_txcd, plaxis_txcd, e_txcd, sigmaxx_txcd)
270 plot_txcu(lab_txcu, plaxis_txcu, e_txcu, sigmaxx_txcu)
271
272
273 ### OPTIMIZATION
274 def calculate_normalization_factors(oed_count, triaxcd_count, triaxcu_count, lab_oed,
275     lab_triaxcd, lab_triaxcu):
276     """Normalize scale for error calculation"""
277     strain_scale_oed = [abs(lab_oed[i]["Vertical_strain_%"]).max() for i in range(oed_count
278     )]
279     strain_scale_triax = [abs(lab_triaxcd[i]["Volumetric_strain_%"]).max() for i in range(
280     triaxcd_count)]
281     stress_scale = [abs(lab_triaxcd[i]["Deviatoric_stress_kPa"]).max() for i in range(
282     triaxcd_count)]
283     dev_stress_scale_triaxcu = [abs(lab_triaxcu[i]["Deviatoric_stress_kPa"]).max() for i in
284     range(triaxcu_count)]
285     mean_stress_scale_triaxcu = [abs(lab_triaxcu[i]["Mean_effective_stress_kPa"]).max() for
286     i in range(triaxcu_count)]
287     return strain_scale_oed, strain_scale_triax, stress_scale, dev_stress_scale_triaxcu,
288     mean_stress_scale_triaxcu
289
290 def interpolate_simulation(sim_data, lab_data, x_col, y_col):
291     """ Interpolates the simulation data to match the lab data points """
292     interp_func = interpfd(sim_data[x_col], sim_data[y_col], fill_value="extrapolate")
293     interpolated_y = interp_func(lab_data[x_col])
294     return interpolated_y
295
296 def set_material(G0, nu, M_c, c, lambda_c, e_0, xi, m, h_0, c_h, n_b, A_0, n_d, mu_0, zeta,
297     beta):
298     g_i.set(g_i.Material.User1, G0)
299     g_i.set(g_i.Material.User2, nu)
300     g_i.set(g_i.Material.User3, M_c)
301     g_i.set(g_i.Material.User4, c)
302     g_i.set(g_i.Material.User5, lambda_c)
303     g_i.set(g_i.Material.User6, e_0)
304     g_i.set(g_i.Material.User7, xi)
305     g_i.set(g_i.Material.User8, m)
306     g_i.set(g_i.Material.User9, h_0)
307     g_i.set(g_i.Material.User10, c_h)
308     g_i.set(g_i.Material.User11, n_b)

```

```

301     g_i.set(g_i.Material.User12, A0)
302     g_i.set(g_i.Material.User13, n_d)
303     g_i.set(g_i.Material.User14, mu_0)
304     g_i.set(g_i.Material.User15, zeta)
305     g_i.set(g_i.Material.User16, beta)
306
307 #Normalize factor for error
308 strain_scale_oed, strain_scale_txcd, stress_scale_txcd, dev_stress_scale_txcu,
    mean_stress_scale_txcu = calculate_normalization_factors(
309     oed_count, txcd_count, txcu_count, lab_oed, lab_txcd, lab_txcu
310 )
311
312
313 ### Calibrate GO Based on Oedometer Test
314 #Define initial parameter again based on the new function
315 GO, nu, M_c, c, lambda_c, e_0, xi, m, h0, c_h, n_b, A0, n_d, mu_0, zeta, beta = [100, 0.05,
    1.24, 0.6, 0.01, 0.946, 0.54, 0.01, 10.0, 1.006, 1.33, 0.414, 4.02, 500, 0.0001, 0]
316
317
318 # Define the search space
319 space = [
320     Real(60, 150, name='GO'),
321 ]
322
323 @use_named_args(space)
324 def objective_function_GO(GO):
325     """Function to change the material, run PLAXIS oedometer test, and calculate error"""
326     # Round parameters to desired precision
327     GO = round(GO, 2)
328
329     # Material parameters
330     param_list = [GO, nu, M_c, c, lambda_c, e_0, xi, m, h0, c_h, n_b, A0, n_d, mu_0, zeta,
        beta]
331     set_material(GO, nu, M_c, c, lambda_c, e_0, xi, m, h0, c_h, n_b, A0, n_d, mu_0, zeta,
        beta)
332
333     # Run the PLAXIS simulation with the given parameters
334     simulated_data_oed = run_oed()
335
336     # Interpolate simulated data to match lab data for oedometer tests
337     interpolated_strain_oed = [
338         interpolate_simulation(simulated_data_oed[i], lab_oed[i], "Vertical_stress_kPa", "
            Vertical_strain_%")
339         for i in range(oed_count)
340     ]
341
342     # Calculate the error for oedometer tests
343     error_oed = [
344         np.sum(((lab_oed[i]["Vertical_strain_%"] - interpolated_strain_oed[i]) /
            strain_scale_oed[i]) ** 2) / len(lab_oed[i])
345         for i in range(oed_count)
346     ]
347
348     total_error = sum(error_oed)
349     print(f"Parameters: {param_list}, Error: {total_error}")
350     return total_error
351
352 # Perform Bayesian Optimization
353 res = gp_minimize(objective_function_GO, space, n_calls=200, random_state=0, n_initial_points
    =20)
354
355 # Best parameters and corresponding error
356 best_params = res.x
357 best_error = res.fun
358
359 print("Best parameters:", best_params)
360 print("Best error:", best_error)
361
362 GO = best_params[0]
363
364

```

```

365 ### Calibrate  $M_c$  based on TXCD
366 def calculate_M_c(lab_txcd, txcd_count, lab_txcu=None, txcu_count=0):
367     """Calculate the ratio between  $M_c=q/p$ """
368     ratios_txcd = [
369         lab_txcd[i]["Deviatoric_stress_kPa"].iloc[-1] / lab_txcd[i]["Mean_effective_stress_kPa"].iloc[-1]
370         for i in range(txcd_count)
371     ]
372
373     # If lab_txcu and txcu_count are provided, calculate the ratios for triaxial CU tests
374     ratios_txcu = []
375     if lab_txcu is not None and txcu_count > 0:
376         ratios_txcu = [
377             lab_txcu[i]["Deviatoric_stress_kPa"].iloc[-1] / lab_txcu[i]["Mean_effective_stress_kPa"].iloc[-1]
378             for i in range(txcu_count)
379         ]
380
381     # Combine the ratios and calculate the average
382     all_ratios = ratios_txcd + ratios_txcu
383     M_c = np.mean(all_ratios)
384
385     return M_c
386
387 def plot_pq(lab_txcd, lab_txcu, M_c):
388     # Create figure
389     plt.figure(figsize=(10, 6))
390
391     # Plot the triaxial CD test data
392     for i in range(len(lab_txcd)):
393         plt.plot(lab_txcd[i]["Mean_effective_stress_kPa"].values, lab_txcd[i]["Deviatoric_stress_kPa"].values, label=f'TXCD_Test_{i+1}')
394
395     # Plot the triaxial CU test data
396     for i in range(len(lab_txcu)):
397         plt.plot(lab_txcu[i]["Mean_effective_stress_kPa"].values, lab_txcu[i]["Deviatoric_stress_kPa"].values, label=f'TXCU_Test_{i+1}')
398
399     # Determine the maximum mean effective stress from all data
400     max_mean_stress = max([data["Mean_effective_stress_kPa"].max() for data in lab_txcd + lab_txcu])
401
402     # Generate the line for  $M_c$ 
403     mean_stress_range = np.linspace(0, max_mean_stress, 500)
404     M_c_line = M_c * mean_stress_range
405     plt.plot(mean_stress_range, M_c_line, label=f' $M_c = {M_c:.4f}$ ', linestyle='--', color='black')
406
407     # Set labels and grid
408     plt.xlabel("p [kPa]")
409     plt.ylabel("q [kPa]")
410     plt.xlim(0, max_mean_stress)
411     plt.ylim(0, M_c_line[-1])
412     plt.grid()
413     plt.legend()
414     plt.title(r" $p$  vs  $q$ ,  $M_c$ ")
415     plt.show()
416
417 #Run the function
418 M_c = calculate_M_c(lab_txcd, txcd_count, lab_txcu, txcu_count)
419 plot_pq(lab_txcd, lab_txcu, M_c)
420
421 ### Calibrate  $e_0$ ,  $\lambda_c$ ,  $\xi$  Based on  $e_c$ 
422 def error_function(params, lab_txcd, lab_txcu, txcd_indices=None):
423     xi, e_0, lambda_c = params
424     errors = []
425
426     if txcd_indices is None:
427         txcd_indices = range(len(lab_txcd))
428
429

```

```

430 # Calculate errors for triaxial CD tests
431 for i in txcd_indices:
432     predicted_e = e_0 - lambda_c * (lab_txcd[i]["Mean_effective_stress_kPa"].iloc[-1] /
433         100) ** xi
434     actual_e = lab_txcd[i]["e"].iloc[-1]
435     errors.append(actual_e - predicted_e)
436
437 # Calculate error for the triaxial CU test
438 predicted_e = e_0 - lambda_c * (lab_txcu[0]["Mean_effective_stress_kPa"].iloc[lab_txcu
439     [0]["Mean_effective_stress_kPa"].idxmin()] / 100) ** xi
440 actual_e = lab_txcu[0]["e"].iloc[lab_txcu[0]["Mean_effective_stress_kPa"].idxmin()]
441 errors.append(actual_e - predicted_e)
442
443 return np.sum(np.square(errors))
444
445 def fit_parameters(lab_txcd, lab_txcu, txcd_indices=None):
446     initial_guess = [0.5, 0.908, 0.0142]
447     bounds = [(0, 1), (0, 2), (0, 0.1)]
448
449     result = minimize(error_function, initial_guess, args=(lab_txcd, lab_txcu, txcd_indices),
450         bounds=bounds, method='L-BFGS-B')
451
452     if result.success:
453         fitted_params = result.x
454         print(f"Fitted parameters: xi={fitted_params[0]}, e_0={fitted_params[1]}, lambda_c={
455             fitted_params[2]}")
456         return fitted_params
457     else:
458         raise ValueError("Optimization failed")
459
460 def plot_fitted_curve(lab_txcd, lab_txcu, xi, e_0, lambda_c, txcd_indices=None):
461     plt.figure(figsize=(10, 6))
462
463     if txcd_indices is None:
464         txcd_indices = range(len(lab_txcd))
465
466     # Plot triaxial CD tests
467     for i in txcd_indices:
468         plt.scatter((lab_txcd[i]["Mean_effective_stress_kPa"].iloc[-1] / 100) ** xi,
469             lab_txcd[i]["e"].iloc[-1], label=f"TXCD Test {i+1}")
470
471     # Plot triaxial CU test
472     plt.scatter((lab_txcu[0]["Mean_effective_stress_kPa"].iloc[lab_txcu[0]["Mean_effective_
473         stress_kPa"].idxmin()] / 100) ** xi, lab_txcu[0]["e"].iloc[lab_txcu[0]["Mean_
474         effective_stress_kPa"].idxmin()], label="TXCU Test 1")
475
476     # Plot the fitted line
477     x = np.linspace(0, 5, 500)
478     y = e_0 - lambda_c * x
479     plt.plot(x, y, label=f'r'$e_0$={e_0}, xi={xi}, lambda_c={lambda_c}', linestyle='--
480         ', color='black')
481
482     plt.xlabel(r"$(p/p_{at})^{\xi}$")
483     plt.ylabel(r"void_ratio, $e$")
484     plt.xlim(0, )
485     plt.ylim(0.8, )
486     plt.grid()
487     plt.legend()
488     plt.show()
489
490 # Fit parameters using TXCD (and TXCU)
491 fitted_params = fit_parameters(lab_txcd, lab_txcu, txcd_indices=[0, 2])
492 plot_fitted_curve(lab_txcd, lab_txcu, *fitted_params, txcd_indices=[0, 2])
493 xi, e_0, lambda_c=fitted_params
494
495 ### Calibrate $h_{0}$, $c_{h}$, $n_{b}$ Based on TXCD $q-\varepsilon_{ax}$
496 # Define the search space
497 space = [
498     Real(4, 8, name='h0'),
499     Real(1, 3, name='n_b'),

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```

493     Real(0.95, 1.2, name='c_h'),
494 ]
495
496 @use_named_args(space)
497 def objective_function_h0nbch(h0,n_b,c_h):
498     # Round parameters to desired precision
499     h0 = round(h0, 2)
500     n_b = round(n_b, 2)
501     c_h = round(c_h, 3)
502
503     # Material parameters
504     param_list = [G0, nu, M_c, c, lambda_c, e_0, xi, m, h0, c_h, n_b, A0, n_d, mu_0, zeta,
505                   beta]
506     set_material(G0, nu, M_c, c, lambda_c, e_0, xi, m, h0, c_h, n_b, A0, n_d, mu_0, zeta,
507                 beta)
508
509     # Run the PLAXIS simulation with the given parameters
510     simulated_data_txcd = run_txcd()
511
512     # Interpolate simulated data to match lab data for TXCD tests
513     interpolated_dev_stress_txcd = [
514         interpolate_simulation(simulated_data_txcd[i], lab_txcd[i], "Axial_strain[%]", "
515                               Deviatoric_stress[kPa]")
516         for i in range(txcd_count)
517     ]
518
519     # Calculate the error for TXCD tests
520     error_txcd_dev_stress = [
521         np.sum(((lab_txcd[i]["Deviatoric_stress[kPa]"] - interpolated_dev_stress_txcd[i]) /
522                stress_scale_txcd[i]) ** 2) / len(lab_txcd[i])
523         for i in range(txcd_count)
524     ]
525
526     total_error = sum(error_txcd_dev_stress)
527     print(f"Parameters: {param_list}, Error: {total_error}")
528     return total_error
529
530 # Perform Bayesian Optimization
531 res = gp_minimize(objective_function_h0nbch, space, n_calls=200, random_state=0,
532                  n_initial_points=20)
533
534 # Best parameters and corresponding error
535 best_params = res.x
536 best_error = res.fun
537
538 print("Best parameters:", best_params)
539 print("Best error:", best_error)
540 h0, n_b, c_h = best_params
541
542 ### Calibrate $A_{0}$, $n_{d}$ Based on TXCD $\epsilon_{vol}-\epsilon_{ax}$
543 # Define the search space
544 space = [
545     Real(0.4, 1.2, name='A0'),
546     Real(1, 4, name='n_d'),
547 ]
548
549 @use_named_args(space)
550 def objective_function_A0nd(A0,n_d):
551     # Round parameters to desired precision
552     A0 = round(A0, 3)
553     n_d = round(n_d, 2)
554
555     # Material parameters
556     param_list = [G0, nu, M_c, c, lambda_c, e_0, xi, m, h0, c_h, n_b, A0, n_d, mu_0, zeta,
557                   beta]
558     set_material(G0, nu, M_c, c, lambda_c, e_0, xi, m, h0, c_h, n_b, A0, n_d, mu_0, zeta,
559                 beta)
560
561     # Run the PLAXIS simulation with the given parameters
562     simulated_data_txcd = run_txcd()

```



```

557
558 # Interpolate simulated data to match lab data for TXCD tests
559 interpolated_vol_strain_txcd = [
560     interpolate_simulation(simulated_data_txcd[i], lab_txcd[i], "Axial_strain[%]", "
561                           Volumetric_strain[%]")
562     for i in range(txcd_count)
563 ]
564
565 # Calculate the error for TXCD tests
566 error_txcd_vol_train = [
567     np.sum(((lab_txcd[i]["Volumetric_strain[%]"] - interpolated_vol_strain_txcd[i]) /
568            strain_scale_txcd[i]) ** 2) / len(lab_txcd[i])
569     for i in range(txcd_count)
570 ]
571
572 total_error = sum(error_txcd_vol_train)
573 print(f"Parameters: {param_list}, Error: {total_error}")
574 return total_error
575
576 # Perform Bayesian Optimization
577 res = gp_minimize(objective_function_A0nd, space, n_calls=200, random_state=0,
578                  n_initial_points=20)
579
580 # Best parameters and corresponding error
581 best_params = res.x
582 best_error = res.fun
583
584 print("Best parameters:", best_params)
585 print("Best error:", best_error)
586 A0, n_d = best_params
587
588 ### Simulation Based on New Calibrated Parameter
589 print(G0, nu, M_c, c, lambda_c, e_0, xi, m, h0, c_h, n_b, A0, n_d, mu_0, zeta, beta)
590 set_material(G0, nu, M_c, c, lambda_c, e_0, xi, m, h0, c_h, n_b, A0, n_d, mu_0, zeta, beta)
591
592 #Run the test
593 plaxis_oed = run_oed()
594 plaxis_txcd = run_txcd()
595 plaxis_txcu = run_txcu()
596
597 #Plot the result
598 plot_oed(lab_oed, plaxis_oed, e_oed)
599 plot_txcd(lab_txcd, plaxis_txcd, e_txcd, sigmaxx_txcd)
600 plot_txcu(lab_txcu, plaxis_txcu, e_txcu, sigmaxx_txcu)

```