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# Dynamics of rotating rings using a higher order theory



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### Introduction

Dynamics of rotating rings on elastic foundation (REF) is a classical problem in solid mechanics. Various models are developed for rotating rings, especially in the field of tire research. It is well accepted nowadays that the effect of the rotation-induced hoop stress needs to be incorporated to accurately predict the dynamic behaviors of rings rotating at high speeds. To capture this effect, a nonlinear strain-displacement relation needs to be employed. However, the choice of the nonlinear strain-displacement relation is ambiguous in the literature; different adoption does influence the predictions for the ring. Another point which needs to be pointed out is that when the inner surface of a ring is connected to a relatively stiff foundation, both the through-thickness variations of transverse normal and shear stresses play an important role.

### **Higher order theory**

In this paper, a new higher order theory is developed for the in-plane vibrations of a high speed rotating ring in which the inner surface is connected to an immovable hub by distributed springs while the outer surface is stress-free. The transverse normal stress and transverse shear stress at the inner surface are non-zero but related to the displacements of the inner surface to which the foundation is directly connected. Higher order terms are introduced o satisfy the radial and shear stress boundary conditions at both inner and outer surfaces. To include the pre-stress caused by rotation, the nonlinear engineering strain is applied, which is the second order truncation of the geometrically exact strain-displacement relation. Because of the inclusion of higher order terms, the coupling of in-plane flexural, extensional and shear motions is considered by the new nonlinear governing equations.

### **Results and discussion**

The nonlinear governing equations are linearized around the static axisymmetric equilibrium caused by steady rotation. Since higher order terms are included, the linearized equations can thus deal with high frequency vibrations of rotating rings and rings of moderate thickness. The free vibrations are first examined using the linear governing equations. Instability, which is usually overlooked in the literature, has been predicted for high-speed rotating thin rings [1]. Resonant speeds are predicted, at which modes appear as a stationary displacement pattern to a space-fixed observer. It is shown that the shear deformation and rotatory inertia are important even for thin rings which are elastically supported. Bending dominant motions always experience resonances at lower speeds than those of extensional or shear motions. Forced vibration of a rotating ring subjected to a stationary point load is investigated. Wave-like deformations are found when the ring rotates at speeds higher than the minimum resonant speed, which confirms the existence of the resonant speeds predicted from the free vibration analysis.

## References

[1] Lu, T., A. Tsouvalas, and A. V. Metrikine. "The in-plane free vibration of an elastically supported thin ring rotating at high speeds revisited." *Journal of Sound and Vibration* 402 (2017): 203-218.