Using Golay Sequences to Improve the Range Performance of Hybrid Codes for MIMO Radar

Schöpe, Max Ian; Driessen, Hans; Yarovoy, Alexander

DOI
10.23919/IRS.2018.8448132

Publication date
2018

Document Version
Final published version

Published in
2018 19th International Radar Symposium (IRS)

Citation (APA)
Schöpe, M. I., Driessen, H., & Yarovoy, A. (2018). Using Golay Sequences to Improve the Range Performance of Hybrid Codes for MIMO Radar. In H. Rohling (Ed.), *2018 19th International Radar Symposium (IRS) : Proceedings* (pp. 1-10). Cuvillier Verlag. https://doi.org/10.23919/IRS.2018.8448132

Using Golay Sequences to Improve the Range Performance of Hybrid Codes for MIMO Radar

Max Ian Schöpe*, **Hans Driessen***, **Alexander Yarovoy***

*Microwave Sensing, Signals and Systems (Faculty EEMCS)
Delft University of Technology
Mekelweg 4, 2628 CD Delft, The Netherlands
email: m.i.schope@tudelft.nl

Abstract: *In this paper, waveforms for MIMO phased array radar to enhance cross-range resolution are investigated. The problem of high sidelobes in range created by the use of Hybrid Codes with a single waveform and spatial coding is considered and a method to reduce these sidelobes by the use of Golay sequences as spatial codes is proposed. It is shown that the proposed method achieves the same range performance as a phased array radar with one waveform, despite creating additional sidelobes in Doppler.*

1. Introduction

A common way to use phased array radars is to transmit the same waveform simultaneously in all transmit channels. By adjusting the phase shifts in between the different channels, one can steer the transmit beam into different directions. In the following, this concept is called mono-color transmission. Multiple Input Multiple Output (MIMO) radar can offer some advantages over such a traditional radar system. For instance, due to the fact that it can transmit independent signals in every channel [1], it offers additional degrees of freedom compared to traditional phased array radar. If the transmit signals would be mutually orthogonal, several improvements could be achieved, amongst others a virtual array that is bigger than the physical one [2], which would lead to a better angular resolution. In addition to that, so-called colored transmission could be accomplished [3, 4]. Applying this technique could help to deal with clutter coming from specific directions. In [3] it is also argued that colored transmission can improve the clutter rejection in an airborne application, since the clutter is spreading in Doppler due to a wide transmit beam. If true orthogonality was achieved, the signals of the different channels would not form a distinct beam, but rather radiate in all directions uniformly. Such a radar would thus have the full angular coverage.

As each transmitted signal can be clearly associated with the according transmit direction in colored transmission, digital beamforming (DBF) can be applied on transmit through signal processing on receive. Theoretically, one can observe an unlimited number of beams while keeping the amount of measurements the same as for a single beam [4]. However, the computational effort on the receiver side increases. Since full orthogonality cannot be achieved in

978-3-7369-9545-1

reality, two alternative techniques have been proposed by Babur et al. in [4] and [5] to realize a similar performance. These techniques are called Circulating Codes and Hybrid Codes (also called Delft Codes).

This paper is based on the results of the Master's thesis results in [6] and introduces a technique to improve the range resolution for Hybrid Codes for static and slow targets by the use of Golay sequences and multiple pulses. After this introduction, section 2 gives a short overview of the Circulating and Hybrid Codes. The new method is introduced in section 3, while simulation results are shown in section 4. Finally, section 5 gives conclusions and recommendations for future research, as well as a possible usage scenario.

2. Available Techniques

2.1. Circulating Codes

As described in [4], the concept of Circulating Codes is fairly simple: there is only one waveform used, which is shifted in time between the N_{tx} different transmit channels of the radar system. The signal in the nth channel can be described as

$$s_T^n(t) = s(t - (n-1) \cdot \Delta t) \ , \tag{1}$$

where $s(t)$ is the basic signal and Δt is the relative time shift between two channels. When the time shift is set to $\Delta t = 1/B$, the Circulating Codes achieve the full angular coverage of 180° [4]. However, the Circulating Codes do not constantly illuminate the whole 180° as is expected from a real MIMO system with orthogonal signals, but rather cover the whole angular area with a narrow beam sweep over all angles during one pulse [6]. The total transmit signal $s_T(t)$ is the sum of the signals coming from each channel and depends on the transmit angle, since it determines the relative phase differences.

Compared to other waveforms for MIMO radar, the Circulating Codes offer much lower range sidelobes [7]. This makes them very attractive for practical applications. However, the extremely low sidelobes come with the price of an N_{tx} times wider peak width than for mono-color transmission, which leads to a decreased range resolution [4].

2.2. Hybrid Codes

To improve the range resolution of the Circulating Codes, a variation of the Circulating Codes has been proposed, the so-called Hybrid Codes [5]. The idea is to add a spatial code over the transmit antenna elements to influence the autocorrelation function. All transmit antenna elements are also used on receive, as in the equations of [5], thus there are N transmit and N receive antennas whose positions along the x-axis are denoted as $\vec{x}$.

The basic signal is very similar to the one for the Circulating Codes and in [5] it is written as

$$s_T^n(t) = c_n \cdot s(t - (n-1) \cdot \Delta t) \ , \tag{2}$$

where c_n is the spatial code along the antenna elements, usually consisting of combinations of -1 and +1. Just like the Circulating Codes, the Hybrid Codes do not illuminate the full 180 degrees constantly. But in this case, not only one beam is sweeping over all the angles during the pulse, but several ones [6]. Naturally, it is also possible to use code elements different to -1 and +1. Using this idea, it was proposed to apply a so-called mismatched filtering instead of matched filtering, to be able to decrease the sidelobes around the peak. Since it has been covered extensively in literature [8, 9], this concept is not further addressed here.

Babur et al. show in [5] that the ambiguity function of the Hybrid Codes around the peak is defined by the autocorrelation function of the spatial code:

$$|\chi_{\theta'=\theta_0}(i)|^2 \cong \begin{cases} \left|\sum_{n=1}^{N} c_n \cdot c_{i-n}\right|^2 & for\ |i| < N \\ 0 & else \end{cases}, \tag{3}$$

where θ' is the hypothesis about the target direction, θ_0 is the direction of the digital beam-forming on receive, i is the index of the ambiguity function and c_n is the spatial code along the antennas. This means that the main "corridor" of width $2N - 1$ around the target peak in the ambiguity function is defined by the spatial code. So, if a code with a good autocorrelation function is chosen, the resulting peak is as narrow as for mono-color transmission [5].

2.3. Research Problem Formulation

It has been shown in [5] that the Hybrid Codes recover the range resolution of mono-color transmission. Still, there are a lot of high sidelobes in range, which might cover weak targets or lead to false detections. The method proposed in section 3 tries to decrease these sidelobes further. This comes at the price of additional sidelobes in Doppler.

3. Proposed Technique

Here, the use of so-called Golay sequences as spatial codes for the Hybrid Codes is proposed. The Golay sequences were first introduced by Marcel Golay in 1961 in [10] and have the property that the sum of the autocorrelation functions of a pair add up constructively and cancel out the sidelobes. Taking a Golay pair a_i and b_i which both have length n, their autocorrelation functions are given as

$$R_{a,j} = \sum_{i=1}^{n-j} a_i a_{i+j} \quad \text{and} \quad R_{b,j} = \sum_{i=1}^{n-j} b_i b_{i+j}\ , \tag{4}$$

as stated in [10], where i denotes the index of the codes and j is the lag or shift applied in the autocorrelation. If these two autocorrelation functions are added up, the result is

$$R_{a,j} + R_{b,j} = \begin{cases} 0 & for\ j \neq 0 \\ 2n & for\ j = 0 \end{cases}. \tag{5}$$

Thus, the result of this calculation is 0 for all indexes that are unequal 0. Obviously, this code pair has excellent properties for a radar application, because in theory the code produces no sidelobes at all. In this section it is examined, how Golay pairs can be used in combination with Hybrid Codes to improve the resolution of the resulting peak in range.

3.1. Usage of Hybrid Codes with Golay Pair

The easiest way to make use of Golay pairs in Hybrid Codes, is to transmit two consecutive pulses, each containing one of the two codes of a Golay pair. A matched filter is applied to the received reflected signal.

At first, the simple case of static targets is examined. Since the target has no speed, no Doppler effect can be observed. In addition, there is no phase difference between the two pulses due to a displacement. When it is assumed that there are no target fluctuations and the received signal is just a delayed version of the complete transmit signal $s_T(t)$, the processing operation can be described as

$$y(\tau_1, \tau_2) = \int_{-\infty}^{\infty} (s_{T,1}(t) + s_{T,2}(t - t_d)) \cdot (s_{T,1}(t - \tau_1) + s_{T,2}(t - t_d - \tau_2))^* \, dt \; , \qquad (6)$$

where y is the processed matched filter output of the Golay pair, τ_1 and τ_2 are the times that the signals need to travel to the target and back, $s_{T,1}(t)$ and $s_{T,2}(t)$ are the originally transmitted signals with Golay code 1 and 2 respectively and t_d is the separation of the pulses in time, here equivalent to the pulse repetition time (PRT).

Since we assume static targets, the traveling times for the two signals are the same and thus $\tau_1 = \tau_2 = \tau$. Furthermore, for a large t_d, we can neglect the cross-terms of the processing operation (see section 3.2). Therefore, the following simplified equation can be obtained:

$$y(\tau) = \int_{-\infty}^{\infty} s_{T,1}(t) s_{T,1}^*(t - \tau) dt + \int_{-\infty}^{\infty} s_{T,2}(t - t_d) s_{T,2}^*(t - t_d - \tau) dt \; , \qquad (7)$$

where τ is the delay of the received signals in time. Since the object is not moving, the resulting main peak is the same, no matter how much time has passed between the two pulses. Since many applications involve the observation of moving targets, the influence of their movement on the measurements needs to be taken into account. Thus, the phase shift due to the displacement of the target and the according Doppler shift are now added to (7):

$$\begin{aligned} y(\tau_1, \tau_2) = & \int_{-\infty}^{\infty} s_{T,1}(t) s_{T,1}^*(t - \tau_1) e^{4\pi(v \cdot t)/\lambda_c} dt \\ & + \; e^{-j4\pi(v \cdot t_d)/\lambda_c} \cdot \int_{-\infty}^{\infty} s_{T,2}(t - t_d) s_{T,2}^*(t - t_d - \tau_2) e^{4\pi(v \cdot (t - t_d))/\lambda_c} dt \; , \end{aligned} \qquad (8)$$

where v is the velocity of the target and λ_c is the carrier wavelength. The phase shift due to the displacement is represented by the factor $exp(-j4\pi(v \cdot t_d)/\lambda_c)$. If the velocity of the target is assumed to be constant during its illumination with the two pulses, the Doppler frequency shift can be neglected. The use of the proposed technique is possible with a long or a short separation between the two pulses, which is explained in the next two subsections.

3.2. Transmitting with Long Separation in Time

There are two different cases of long separation. Firstly, a matched filter can be applied to the received reflected signal from each pulse individually. This means that the first pulse has to be received before the second one will be sent and the PRT has to be adjusted accordingly. Afterwards, the results from both matched filter operations are coherently added up to remove the sidelobes. Since the pulses are handled individually, there will be no cross-terms that produce extra sidelobes. Secondly, a matched filter can be applied to both signals jointly, with a long delay time in between. The cross-term influences in (6) produce peaks at a distance of $(c \cdot t_d)/2$ from the main peak. Therefore, when it is assumed that the separation in time t_d is very big with respect to the length of the transmit pulses, we can neglect these terms because the extra peaks will be cut off by the matched filter. In both cases, (7) and (8) can be applied. The use of this technique with moving targets is difficult since the phase shift due to their displacement influences the signals if the pulses are transmitted at different times.

In the case of moving targets, the effect of the phase shift can be removed, if the velocity of the target is known. In order to do this, the second pulse must be multiplied by the negated phase shift according to the assumed velocity. If this is done for an interval of assumed velocities, a peak indicates the Doppler value, where the effect of the phase shift is reduced to a minimum [6] and the result is similar to static targets. By summing the matched filter results of several Hybrid Code pulses with Golay pairs consecutively, the peak in the Doppler domain becomes more narrow [6]. Although most of the sidelobes in Doppler can be reduced in this way, there are still ambiguities in Doppler according to the PRT. In addition to that, several targets with very different velocities will produce sidelobes within each others range-angle ambiguity function.

3.3. Transmitting with Short Separation in Time

Alternatively, the two Golay pulses can also be transmitted right after each other, with no or only a very short pause in between. In this case, there is still a phase shift, but it can be neglected if the separation in time is very short with respect to the velocity of the target. Since the time separation t_d is very small, the cross-terms have to be taken into account again. From (6), assuming that $\tau_1 \approx \tau_2 \approx \tau$, this leads to

$$y(\tau) = \int_{-\infty}^{\infty} (s_{T,1}(t) + s_{T,2}(t - t_d)) \cdot (s_{T,1}(t - \tau) + s_{T,2}(t - t_d - \tau))^* \, dt \; , \tag{9}$$

where t_d is the delay between the two pulses. Here, the matched filter is applied to a signal consisting of the sum of both consecutively transmitted pulses.

4. Simulation Results

In both [4] and [5], linear frequency modulated (LFM) signals showed very good performance with Circulating and Hybrid Codes. Therefore, they are used as basic signals for the simulations.

Furthermore, the Barker 7 code is used as spatial code in these simulations, because of its very good autocorrelation properties. Since Golay pairs do not exist for every length, the simulations focus on a pair of length 8. Thus, the assumed number of transmit antenna elements is either 7 or 8 with a separation of a half wavelength. Following the example of [4] and [5], only one receive antenna element is assumed in the simulations, in order to focus on the effect of the DBF on transmit. The usage of several elements and DBF on receive would lead to an improvement of the results shown below. The LFM sweep has a bandwidth of 2.55MHz at a center frequency of 10GHz. Its length is 100µs at a sampling frequency of 1.25GHz. A single point target is assumed at 0m and 0° in azimuth.

4.1. Transmitting Hybrid Codes with Barker 7 Code

The Matlab simulation results for Hybrid Codes with Barker 7 code are shown in Fig. 1. Since a Barker 7 spatial code is used, only 7 antenna elements are assumed on transmit. The simulation matches the results from [5]. A narrow peak is visible at 0° and 0m, surrounded by a 1km wide sidelobe plateau with sidelobes between -16.5db and -10.4dB. The highest sidelobes in angle are at about -12.7dB. By adding weightings over the reference signal and the usage of all transmit antenna elements on receive, the performance can be improved.

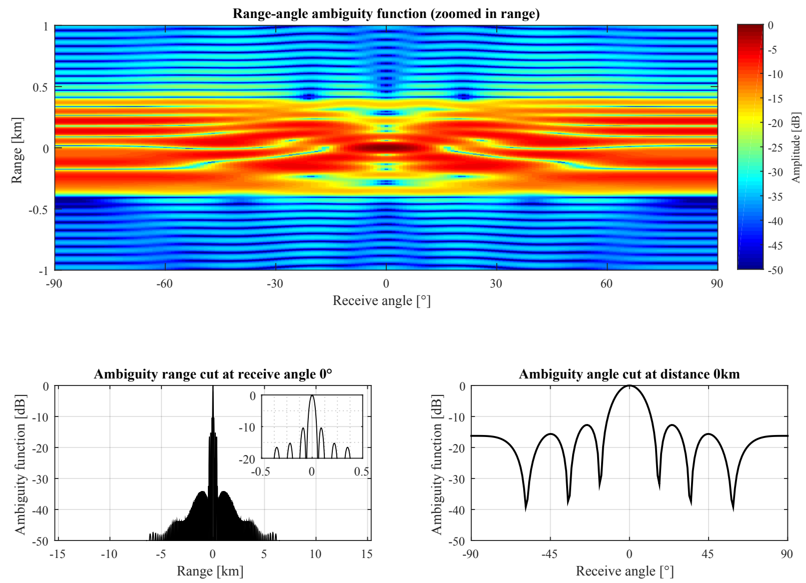

Figure 1: Simulation results for one Hybrid Codes pulse with Barker 7 code in Matlab. Top: two-dimensional ambiguity function in range and angle. Bottom left: range cut at angle 0°. Bottom right: angle cut at range 0km.

4.2. Transmitting Hybrid Codes with Golay Pair with Long Separation in Time

In this simulation, two Hybrid Code pulses are assumed to be transmitted, each with one Golay code of a pair as spatial code. The antenna is therefore assumed to have 8 elements. As can be observed in Fig. 2, a very narrow peak in range is formed at 0°. The sidelobes in range are at about -13.3dB, which is lower than in the case of Hybrid Codes with Barker 7 code. In addition to that, there is a smaller number of sidelobes in range. It is obvious that the sidelobe corridor in azimuth has a width of about 0.5 kilometers. This is half as wide as for the Hybrid Codes with Barker 7 code. The angular pattern is similar, due to the usage of a very similar antenna configuration. The highest angular sidelobes are at -11.8dB. By adding weightings and the usage of all elements on receive, the performance can be improved.

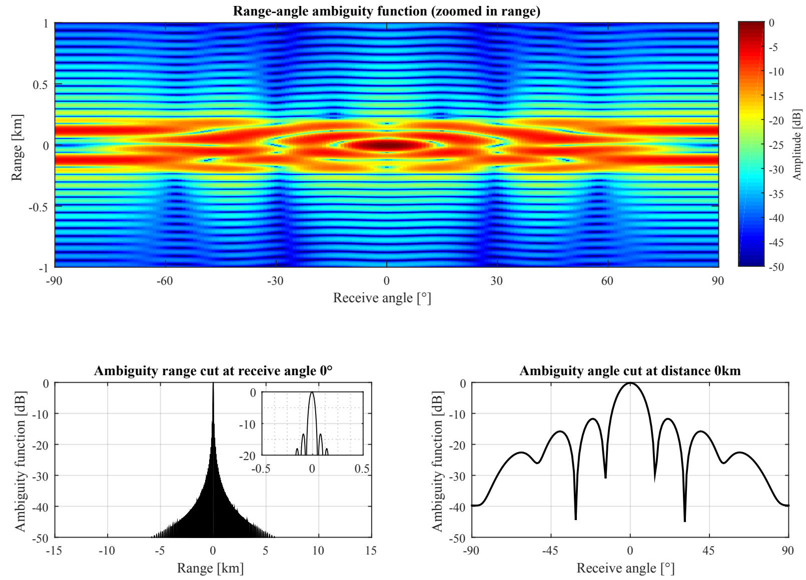

Figure 2: Simulation results for Hybrid Codes pulses with Golay sequences in Matlab using a long separation and assuming a static target. Top: two-dimensional ambiguity function in range and angle. Bottom left: range cut at angle $0°$. Bottom right: angle cut at range 0km.

When the object is not static as assumed in Fig. 2, then the phase shift due to the displacement of the target during the PRT needs to be corrected for (see section 3.2). In this example, we use the same parameters as above, and the target now has a velocity of 15m/s. For the phase shift correction it is assumed that the target has a velocity somewhere in between -60m/s and +60m/s. For a finite number of velocity values, the correction is applied. Fig. 3 shows that two peaks become visible. One at 15m/s, where the velocity got properly corrected and where the range cut is the same as for static targets. The other peak at -60m/s is an ambiguity. Obviously, the presence of multiple targets with different velocities will lead to sidelobes at other velocities.

4.3. Transmitting Hybrid Codes with Golay Pair with Short Separation in Time

When the Golay pair is transmitted with a short time separation, the phase shift due to the displacement of a moving target has a much smaller impact and is assumed to be negligible. Thus, this simulation does not involve a moving target. Both an up- and a down-chirp LFM signal are used in the simulation to avoid high sidelobes due to the cross-terms. The result is shown in Fig. 4. The highest sidelobes in range are at -13.3dB, which corresponds to the result in Fig. 2. In addition to that, there are many sidelobes at distances bigger than 2km at slightly

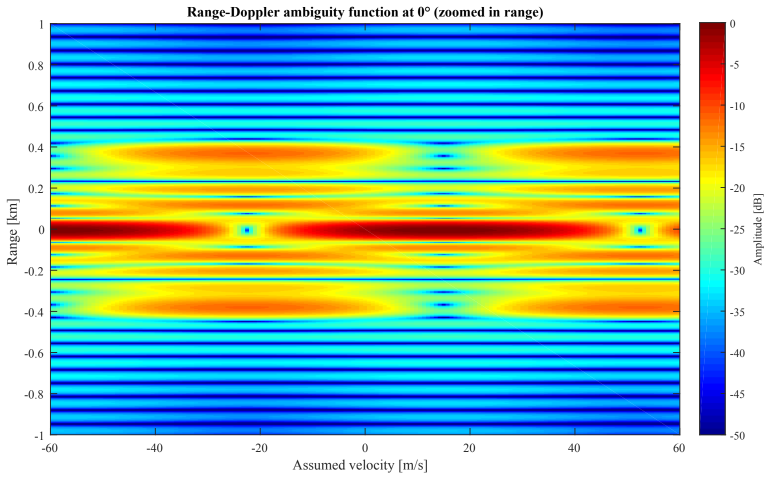

Figure 3: Simulation results for Hybrid Codes pulses with Golay sequences in Matlab using a long separation and assuming a moving target. It shows the range ambiguity cut for the correction of 161 different assumed velocities between -60m/s and +60m/s.

below -30dB. The angular sidelobes can be found at -12.7dB. By adding weightings and the usage of all elements on receive, the performance can be slightly improved.

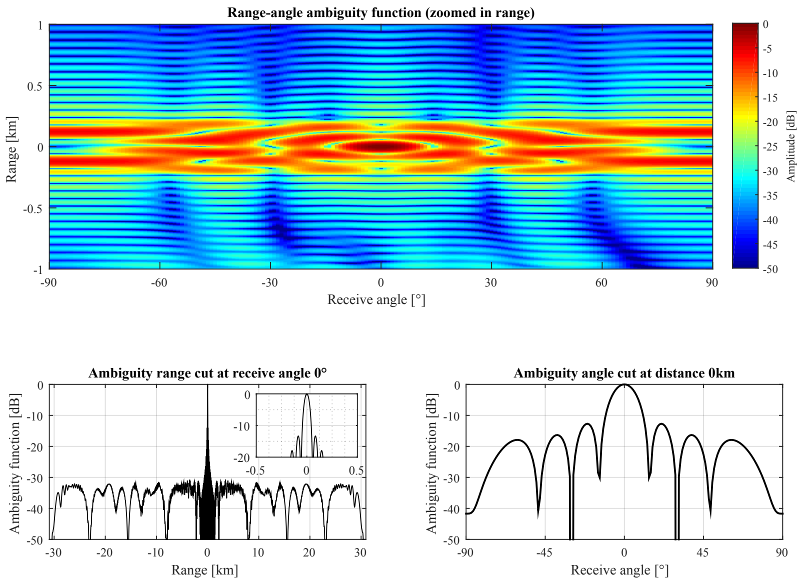

Figure 4: Simulation results for Hybrid Codes pulse with Golay sequences in Matlab using a short separation with one LFM up- and one LFM down-chirp. Top: two-dimensional ambiguity function in range and angle. Bottom left: range cut at angle $0°$. Bottom right: angle cut at range 0km.

4.4. Comparison with Mono-Color Transmission

The use of Hybrid Codes in combination with a Golay pair delivers the same sidelobe structure in the range cut of the ambiguity function as the mono-color transmission. This can be clearly seen in Fig. 5 where different range cuts at 0° are being compared. Thus, all additional sidelobes that were present in the range cut of Hybrid Codes with Barker 7 code have been removed.

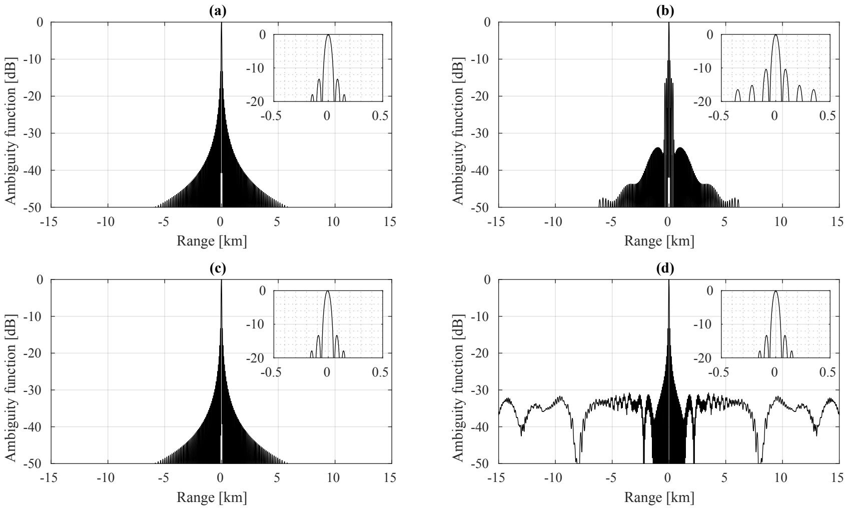

Figure 5: Range-cuts of the ambiguity function at angle 0° for different waveforms. (a): Mono-color transmission with single LFM signal. (b): Hybrid Codes with Barker 7 spatial code. (c): Hybrid Codes with Golay pair and long separation. (d): Hybrid Code with Golay pair and short separation.

5. Conclusions

In this paper, we analyzed the Hybrid Codes for MIMO radar and proposed a new method which combines the Hybrid Codes with Golay pairs to recover the range performance of mono-color transmission, while still providing similar properties to a MIMO radar with orthogonal signals.

The simulations of the Hybrid Codes in combination with Golay pairs showed a very narrow peak in range for static targets, as expected from the theory. Two ways of using Golay sequences have been explained. When a long separation between the Golay pulses is used, small phase shifts due to the displacement of a moving target have a big influence on the final result. This effect can be reduced, but only for one velocity at a time. If there are multiple targets with very different velocities, this method is suboptimal. Apart from that, a very short separation in time can be used. In that case, the phase shift will be small enough to omit it. The average level of sidelobes in range is higher than for the individual processing.

A possible use-case is the observation of ship traffic near the coast. Since ships are comparatively slow targets, the influence of the phase shift due to the displacement of the targets will be comparatively small. It is assumed that the ships do not exceed a speed of 50km/h, which is equivalent to about 16.67m/s. If the same radar configuration as in the simulations is used, a PRT of about 450μs leads to an observable velocity range of ±16.67m/s and a maximum observable distance of 67.5km. If a long separation is used, the sidelobes in range due to the displacement can be decreased by the use of a sequence of several Golay pairs. Alternatively, a very short or no separation can be applied, which removes the sidelobes in Doppler, but increases the general range sidelobe level.

To conclude, the usage of Hybrid Codes with Golay pairs for MIMO radar manages to improve the range resolution of the Circulating Codes while still providing very low sidelobes compared to other MIMO radar waveforms. These advantages can be used for slow or static targets. Future research needs to be conducted on the effects of the additional sidelobes due to multiple targets with different velocities

Acknowledgment: The authors would like to thank François Le Chevalier for his support during various discussions in person and via email. His additional explanations of the Circulating and Hybrid Codes, as well as his opinion about the application of Golay sequences in this context were greatly appreciated.

References

[1] J. Li and P. Stoica, "MIMO Radar Signal Processing", 1st ed., Hoboken, NJ, USA: John Wiley & Sons, Inc., 2009. ISBN 9780470391488.

[2] M. S. Davis, G. A. Showman, and A. D. Lanterman, "Coherent MIMO Radar: The Phased Array and Orthogonal Waveforms", *IEEE Aerospace and Electronic Systems Magazine*, vol. 29, no. 8, pp. 76-91, August 2014.

[3] F. Le Chevalier, "Space-time transmission and coding for airborne radars", *Radar Science and Technology (CIE - Chinese Institute of Electronics)*, vol. 6, no. 6, 2008.

[4] G. Babur, P. Aubry, and F. Le Chevalier, "Space-Time Radar Waveforms: Circulating Codes", *Journal of Electrical and Computer Engineering (Hindawi)*, vol. 2013, Article ID 809691, 8 pages, 2013. Available: https://www.hindawi.com/journals/jece/2013/809691/.

[5] G. Babur, P. Aubry, and F. Le Chevalier, "Simple transmit diversity technique for phased array radar", *IET Radar, Sonar & Navigation*, vol. 10, no. 6, pp. 1046-1056, 2016. ISSN: 1751-8784.

[6] M. I. Schöpe, "Improvement of Hybrid Codes for MIMO radar using multi-pulse waveforms and signal processing", M.Sc. thesis, TU Delft, Delft, the Netherlands, October 2017.

[7] H. Sun, F. Brigui and M. Lesturgie, "Analysis and comparison of MIMO radar waveforms", *2014 International Radar Conference*, Lille, 2014, pp. 1-6.

[8] T. Faucon, G. Pinaud, and F. Le Chevalier, "Mismatched filtering for circulating space-time codes", *IET International Radar Conference 2015*, Hangzhou, China, 2015, pp. 1-7.

[9] O. Rabaste and L. Savy, "Mismatched Filter Optimization via Quadratic Convex Programming for Radar Applications", *2014 International Radar Conference (RADAR)*, Lille, France, 2014.

[10] M. J. E. Golay, "Complementary Series", *IRE Transactions on Information Theory*, vol. 7, no. 2, pp. 82-87, 1961. ISSN 2168-2712.