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WATER GRAVITY WAVES GENERATED BY A MOVING LOW PRESSURE AREA

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ABSTRACT

Hurricanes (typhoons) have an intense low pressure area which often moves rapidly over the surface of the ocean. The coupled water gravity waves generated by such a moving low pressure area have been studied in a towing tank. It was found that the waves generated in this manner had a phase velocity identical to the velocity of the moving low pressure area. When the low pressure area was moving at velocities less than the shallow water wave velocity ($\sqrt{gh}$), the waves were of nearly constant period--the period being that which when considered with the water depth would provide a wave phase velocity identical to velocity of the moving low pressure area. As the group velocity of waver gravity waves is less than the phase velocity the wave groups were formed behind the moving disturbance. If the low pressure area is considered to be a gust it provides a mechanism for generating "wind waves" of the period observed in the ocean if the forward speed of the gust is taken as the mean speed of the wind. When the velocity of the moving low pressure area approached $\sqrt{gh}$ the characteristics of the waves changed (at $\sqrt{gh}$ phase and group velocities become identical in the linear theory). The wave disturbance consisted of a large wave followed by a few smaller waves. After the pressure area movement was stopped, the first wave would move ahead of the tail of the disturbance, the distance between the two phenomena increasing with increasing distance travelled by the waves. For certain combinations of initial conditions several waves were formed with this characteristic. These waves had the appearance of the records on the

tide gages at certain locations of the East Coast of the U. S. during Hurricane Carol (and other hurricanes). A study of Hurricane Carol with respect to its speed of forward movement and the water depth over which it moved showed that it should have generated waves of the type obtained in the laboratory and which were actually recorded.

INTRODUCTION

A hurricane (typhoon) is a small, roughly circular tropical disturbance containing poor weather, heavy rains, a very low pressure area, and strong counterclockwise winds. The low pressure area in the center is often 2 to 3 inches (of mercury) lower than the surrounding air. Maximum wind often exceed 100 knots. The speed of advance of a hurricane and its path vary considerably; however, the speed may exceed thirty knots in the more northern latitudes, as can be seen in Table I (U.S. Dept. of Commerce, Weather Bureau, 1957).

TABLE I. Average forward speed (knots) 2 hours before and after entering the coast, by regions. (United States Hurricanes with central pressures below 29.00 inches)

Texas 1900-1949	Mid-Gulf 1887-1955	Florida Peninsula	Florida Keys	Atlantic (N. of Fla. - S. of Hatteras) 1893-1955	Atlantic (Cape Hat. and northward) 1893 - 1955
4	6	6	8	7	16
8	7	6	8	9	18
8	7	7	9	9	22
8	9	8	10	12	23
10	10	8	10	14	24
10	10	8	11	15	29
11	11	10	13	16	30
11	11	10	13	16	33
11	14	10	16	17	40
11	16	11		18	47
12	16	11		21	
13	16	11		22	
14	21	11		26	
15	23	11			
15	25	12			
	28	13			
		13			
		14			
		14			
		17			
Mean					
11	14	11	11	16	28

There is evidence (Dunn, 1951; Unoki and Nakano, 1955; Harris, 1956) that long water gravity waves are associated with some hurricanes. It was suggested by D. Lee Harris of the U. S. Weather Bureau that it might be possible to model the moving pressure area of a hurricane using a suction fan and a towing carriage. A preliminary test showed results which were interesting enough to warrant a more detailed study.

No attempt was made to model exactly the shape of the pressure pattern within a hurricane. No attempt was made to include the high winds and the associated wind waves; it was believed that these waves would mask the phenomenon being studied--the coupled wave.

A study of a similar type phenomenon had been made by Press and Oliver (1955); they studied the flexural waves induced in a thin plate of aluminum by pressure pulse generated by a spark source. The experimental set-up was such that three phenomena could be studied; (1) the equivalent of a "ground shot"; i.e., only flexural waves in the aluminum; (2) a combination ground wave in the aluminum and the wave induced by the air pressure wave travelling along the plate; and, (3) only the air pulse induced flexural wave in the aluminum plate. The last phenomena was analagous to the study described herein. Press and Oliver found that the induced wave had a period such that the phase velocity associated with this period was the same as the velocity of the air pulse. It was found that the group of waves travelled with the group velocity associated with the measured wave period and that the group consisted primarily of waves of constant period rather than a dispersive group of the type associated with the ground blast waves.

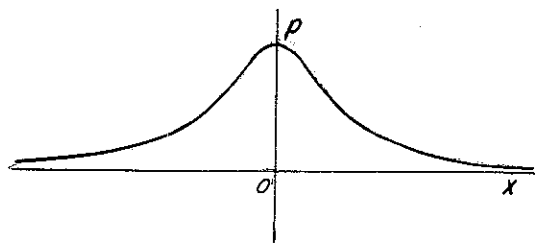
THEORY

Lord Kelvin (1906) developed a theory for a diffused line pressure source moving over a body of infinitely deep water. Havelock (1909, 1922,

1925-26), working along similar lines developed the theory for water of any depth, with the same assumptions being made as in the development of the linear theory.

Kelvin and Havelock found that the moving pressure source would generate transverse waves with phase velocities equal to the velocity of the moving pressure source. The wave periods would be the period associated with the particular wave velocity and water depth. The group of waves would form to the rear of the disturbance as the group velocity of water gravity waves is less than the phase velocity.

Havelock (1909) studied a diffused line pressure of the following sorts:



$$p = f(x) = \frac{P}{\pi} \frac{\alpha}{\alpha^2 + x^2}$$

where

$$P = \int_{-\infty}^{\infty} f(x)\, dx = \text{constant}$$

with x being measured in the direction of motion, p being the pressure and α expressing the degree of diffusion (it has the dimension of length, the "width" of the pressure line) and should be small compared with the normal wave dimensions. The amplitude of the waves formed by such a system is given by

$$a = \frac{C \cdot \frac{2\pi}{\lambda} P e^{-\frac{2\pi\alpha}{\lambda}}}{\left(1 - \frac{4\pi h/\lambda}{\sinh 4\pi h/\lambda}\right)}$$

where λ = wave length, h = water depth and C is a constant (probably $\frac{2}{\gamma}$ where γ is the specific weight of water). The term $(1 - \frac{4\pi h/\lambda}{\sinh 4\pi h/\lambda})$ expresses the effect of the difference between group velocity and phase velocity upon the accumulation of energy in the waves.

The relationship between the wave height (twice the wave amplitude, a) and the wave velocity has been plotted in Figure 1 for a box two feet long travelling over water 1.06 ft. in depth, taking α equal to one-half the length of the box which represents the "hurricane" in our series of tests. On the same graph the experimental points of wave height versus velocity of the pressure area have been plotted. It is apparent that the solution, which depends upon linear wave theory, becomes invalid as the wave celerity approaches $\sqrt{gh}$ because the wave amplitude becomes very large, which violates the assumption of the linear theory.

Havelock (1922) developed a more useful theory for a moving diffused pressure point which relates the wave resistance to the ratio of the wave celerity to $\sqrt{gl}$, where l is the radius of the pressure disturbance. Havelock's curves have been reproduced in Figure 2, and although they are not directly applicable to the diffused line source they do indicate the general relationship observed in our tests.

EXPERIMENTAL EQUIPMENT

The experimental equipment consisted primarily of a suction fan mounted at the top center of a plywood box (bottom side open) mounted beneath a towing carriage on the wave-towing tank (Figure 3). The box extended to within about an inch of the tank walls and to within about one inch of the water surface. Three box lengths were used; 2 ft., 4 ft., and 6 ft. Some data on the pressures within the boxes are shown in Figure 4.

A number of parallel wire resistance wave meters (Wiegel, 1956) were

mounted on the structure. These meters were mounted as shown in Figure 5 for the tests in water depths of 1.06, 2.26, 3.08 and 4.07 feet. The meters were mounted in slightly different positions for the tests in water depth of 0.50 and 0.17 feet; in addition, wave meters were mounted at each end of the towing tank for the latter tests.

A flood light was installed within the box to light the water surface. A 35 mm Bell & Howell movie camera was installed in the camera pit (at about the center of the tank) to photograph the box and the water gravity waves as they passed the glass wall section of the tank. The camera operation was synchronized with the records from the wave meters by means of an event marker on the Brush penmotor which was used to record the output of the wave meters.

LABORATORY PROCEDURE

The automatic carriage control unit was set for a chosen carriage speed and the motor started. As soon as the carriage had come up to its steady speed the fan was started. The water-surface recorders were started prior to starting the fan so that a complete water surface time history would be available. As the carriage moved past the camera pit, 35 mm moving pictures were taken. A sweep second-hand clock was in the field of vision of the cameras. The starting and stopping of the camera was recorded on the water-surface recorder chart by means of the event marker. When the carriage had run a sufficient distance the carriage and fan were stopped simultaneously. An electric braking system was used so that the carriage could be stopped within a distance of two or three feet. The recorders were left running until the wave system had reflected from the far end of the tank and passed the recording system.

Tests with all three boxes were made with water depths of 4.07 ft., 3.08 ft., 2.66 ft. and 1.06 ft. In addition, tests with the 6 ft. box were

made with water depths of 0.50 ft. and 0.17 ft.

RESULTS AND DISCUSSION

One series of tests was run with the fan turned on when the carriage was stationary. The only waves formed by the stationary low pressure area and the associated "winds" were of a very small amplitude and length: compared with the waves formed by the moving system they were considered to be "noise". The effect of starting and stopping the fan was unnoticeable. Another series of test consisted of running the carriage without the fan operating. Records taken of the water surface again showed only "noise". From the two sets of tests it was evident that the waves discussed in this report were associated only with a moving low pressure area.

Using the moving pictures taken of the experiments, together with the records of the wave recorders, it was possible to obtain the water-surface profiles under moving boxes. These profiles were not always constant with time; however, their general characteristics with respect to box velocity can be seen in Figure 6.

The main feature of the water gravity waves generated by the pressure area moving over the water surface was that the waves were essentially of constant period, except in very shallow water (Figure 7). Thus the wave group did not resemble the group associated with an impulsive source. The wave group was dispersive as can be seen in the sample records on the right hand side of Figure 7 which shows the wave trains after they have travelled about two hundred feet from the point at which the moving pressure area had been stopped. The group of waves, where there was a group, travelled with group velocity. As the group velocity is less than its phase velocity for gravity waves the group followed the low pressure area. The phenomenon was similar to that reported by Press and Oliver (1955).

In Figure 8 the measured wave periods have been plotted as a function of the ratio of the low pressure area velocity to $\sqrt{gh}$ and compared with the theoretical relationship between wave period and velocity for freely running waves. Except for relatively shallow water the period of the waves was such that the phase velocity was the same as the velocity of the moving low pressure area where the phase velocity is given by the set of equations

$$C = \frac{gT}{2\pi} \tanh \frac{2\pi h}{\lambda}$$

$$\lambda = CT$$

(where C = phase velocity, g = acceleration of gravity, T = wave period, h = water depth, λ = wave length computed from $\lambda = \left(\frac{gT^2}{2\pi}\right) \cdot \tanh \frac{2\pi h}{\lambda}$. There was no apparent effect of the length of low pressure area on the wave velocity.

One trend was evident. The smaller the water depth the greater was the value of $C/\sqrt{gh}$ for which measured values of the forced waves compared closely with theory. When considering relatively shallow water it appears that a forced wave moving with a given velocity has a smaller period than a freely running wave with the same velocity.

When the velocity of the moving low pressure area was in the immediate vicinity of $\sqrt{gh}$ a different phenomenon was observed. The first wave in the series was of a "solitary" (actually, probably a "cnoidal") nature: this wave was followed by a series of waves which were dispersive (Figures 9 and 10). After the pressure area movement was stopped the first wave would move ahead of the tail of the disturbance, the distance between the two phenomenon increasing with increasing distance travelled by the waves. It can be seen in several of the sample wave records that the trough preceding the major crest disappears after the waves have travelled a ways. It appears that this

"negative wave" becomes a part of the dispersive train to the rear of the first crest (see Run 2-3, Figure 9 for example).

For certain combinations of initial conditions several waves were formed which had the superficial appearance, at least, of cnoidal waves (see Run 2-34 and 2-37 of Figure 10).

In Run 2-44 of Figure 10 another phenomenon can be observed. This is a large negative wave which is dispersive, but which travels at a velocity considerably in excess of the dispersive "periodic" wave train which follows. It is believed that this wave was associated with the displacement of the water surface under the moving pressure area and is related to the dimensions of the pressure area.

The amplitudes of the waves were plotted in various manners. The most significant was found to be in the manner of Havelock (1922). This consisted of plotting the wave height as a function of the ratio of the velocity of the moving low pressure area to $\sqrt{g\,l/2}$ where l is the length of the low pressure area (i.e., 2 ft., 4 ft., or 6 ft.) (Figure 2). These data have been shown in Figures 11 and 12. As Havelock predicted, in shallow water the effect of $\sqrt{g\,l/2}$ becomes small compared with the effect of $\sqrt{gh}$. This can be seen for the water depths of 0.50 and 0.17 feet for the 6 ft. box.

Data for the 2 ft. box moving over water 1.06 ft. deep have been plotted in another manner in Figure 1 for comparisons with Havelock's theory of the waves generated by a moving diffuse line pressure area (Havelock, 1909). The theory compares rather well with the data until a value of the ratio of moving pressure area velocity to $\sqrt{gh}$ of 0.86 is reached, at which point the measured height starts to decrease and continues to decrease with increasing pressure area velocity, as was expected.

APPLICATION TO HURRICANE WAVES

It is evident from these tests that waves can be formed in shoal water with relatively long periods. It is necessary to determine whether hurricanes do move with the necessary velocity over shoal water to generate such waves.

In Figure 13 is a reproduction in reduced size of a section of U. S. C&GS Chart No. 1000 (Cape Sable to Cape Hatteras) showing the water depths offshore the East Coast of the U. S. The path of the center of Hurricane Carol (30, 31 August 1954 is plotted on this chart. It travels over shallow water for an appreciable distance with a forward speed of approximately thirty to thirty-five knots (between Cape Hatteras, North Carolina and Long Island, N. Y.). The shallow water wave velocities ($\sqrt{gh}$) associated with water depths of 5, 10, 15, 20 and 25 fathoms are 18.4, 26.1, 31.9, 36.8 and 41.2 knots. It is evident that Hurricane Carol should have been able to generate the type of long wave that would be known as a surge, especially in the vicinity of Long Island. This is in agreement with the data presented by Harris (1956) which has been shown in Figure 14.

The forward speed of Hurricane Carol was not unusually great for the coastal area north of Cape Hatteras as can be seen from Table I.

APPLICATION TO WIND GENERATED WAVES

The moving low pressure area which was studied can be considered to be a part of a gust of low pressure in a wind blowing over the surface of deep water. The gust may last only for a minute or so at the air-sea boundary; however, it would be able to generate a train of waves with periods associated with the wave velocity identical to the forward velocity of the gust. Thus the mechanism assumed by Eckart (1953) has been shown to raise waves of the periods observed in the ocean. If, as Eckart assumes, the gusts

on the average move with the mean wind speed this mechanism would explain the trend of increase of maximum wave period in a spectral analysis of a wave record with increasing wind speed as observed by Barber and Ursell (1948). The fact that the speeds of such gusts (being large scale turbulence) have a spectrum about the mean value would explain the spectrum of wave periods about a mean value. If the speed of advance of low pressure area is plotted as a function of the generated wave period in deep water one gets nearly the identical curve drawn by Darbyshire (1952) in his plot of observed maximum wave periods in the ocean versus maximum wind speed.

CONCLUSIONS

Water gravity waves, which were not edge waves nor wind waves, were generated by a low pressure area moving over a water surface in a towing tank.

In the deeper water many of the characteristics, such as wave period, amplitude and group velocity, could be predicted from a theory presented by Havelock (1909, 1922). If one considers the moving low pressure area as part of a gust, then a mechanism for producing "wind waves" of the periods observed in the ocean has been demonstrated.

In shallow water, defined as the case of the low pressure area moving at approximately $\sqrt{gh}$, the characteristics of the long waves were similar to those observed in conjunction with certain hurricanes. An examination of the path of Hurricane Carol (30, 31 August 1954) in relation to the water depth along the East Coast of the U. S. indicated that the hurricane moved with a sufficient speed over water which was shallow enough that the type of wave observed could be predicted from these studies.

ACKNOWLEDGEMENTS

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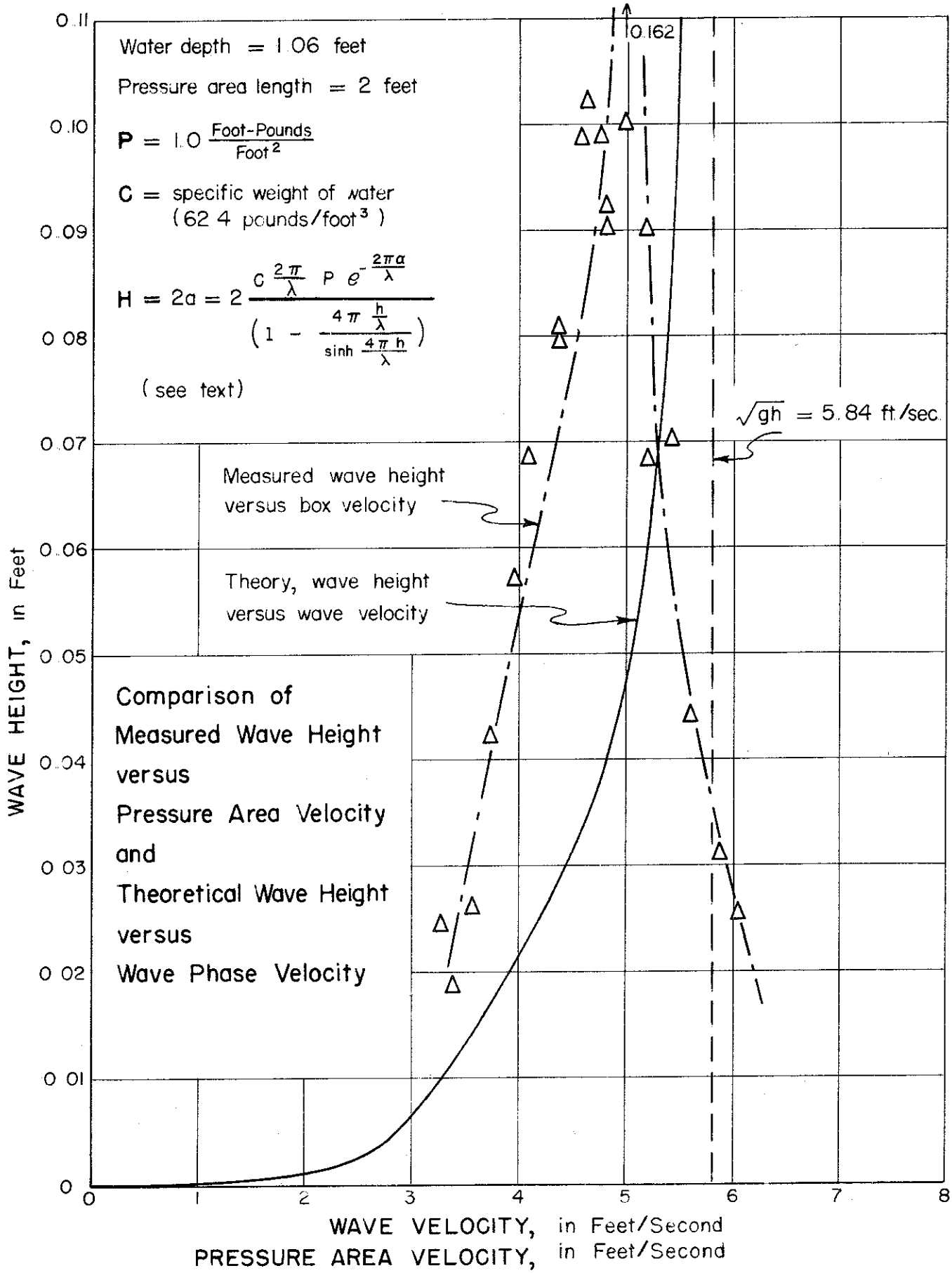
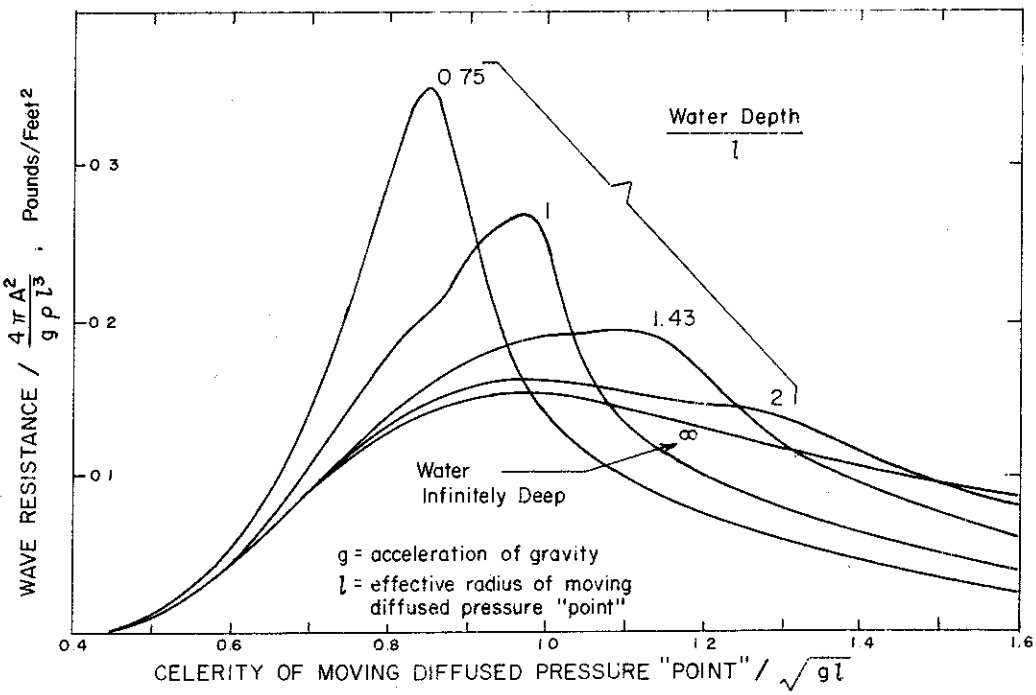


HYD-7688

FIGURE 1



RELATIONSHIP BETWEEN WAVE RESISTANCE AND RATIO OF CELERITY OF MOVING DIFFUSED PRESSURE "POINT" TO $\sqrt{gl}$

(after Havelock (1922))

HYD-7689

FIGURE 2

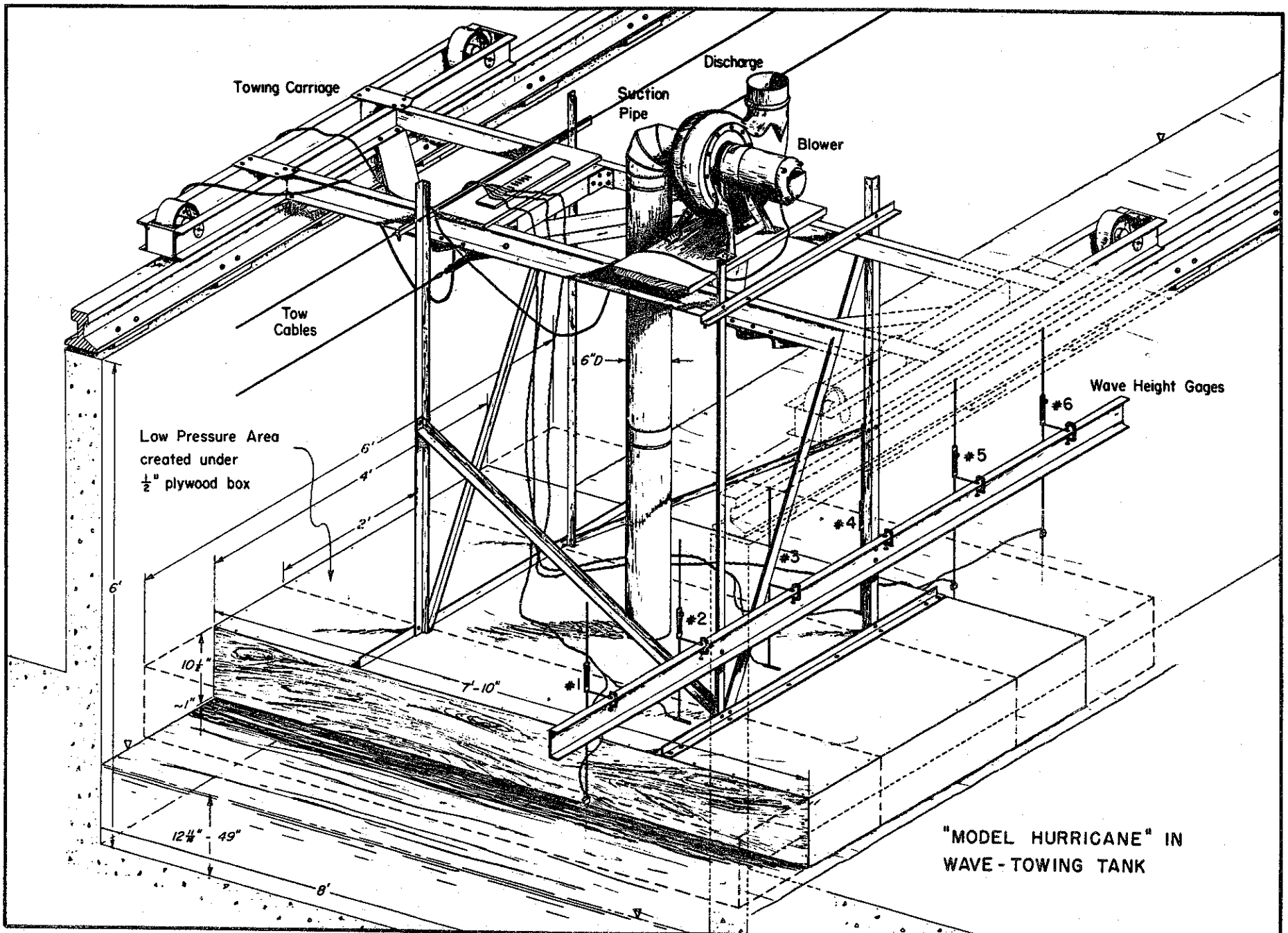
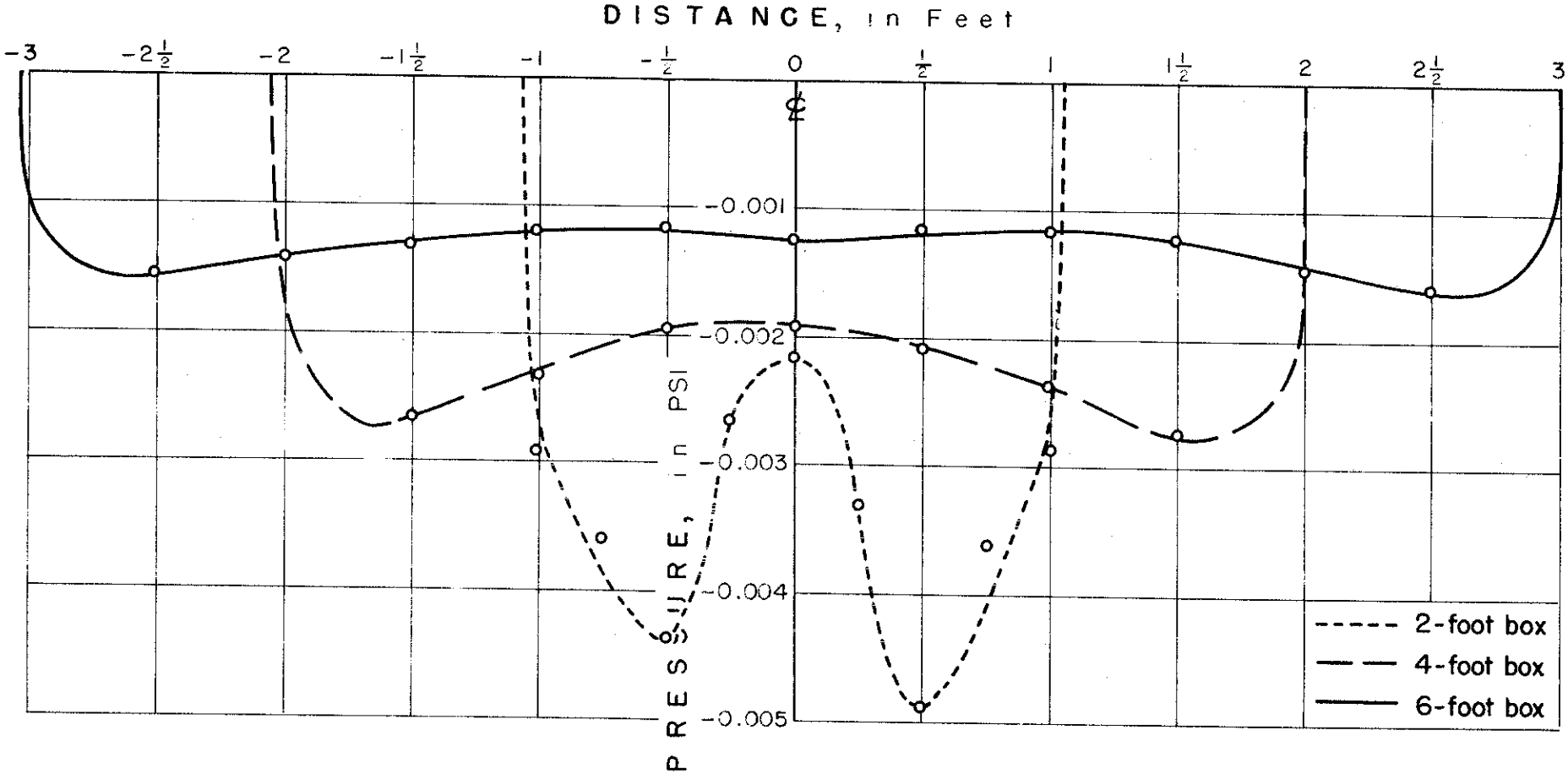

Towing Carriage
Discharge
Suction Pipe
Blower
Tow Cables
6"D
Wave Height Gages
#6
#5
#4
#3
#2
#1
Low Pressure Area created under ½" plywood box
6'
4'
2'
6'
10½"
~1"
7'-10"
12¼" - 49"
8'
"MODEL HURRICANE" IN WAVE-TOWING TANK

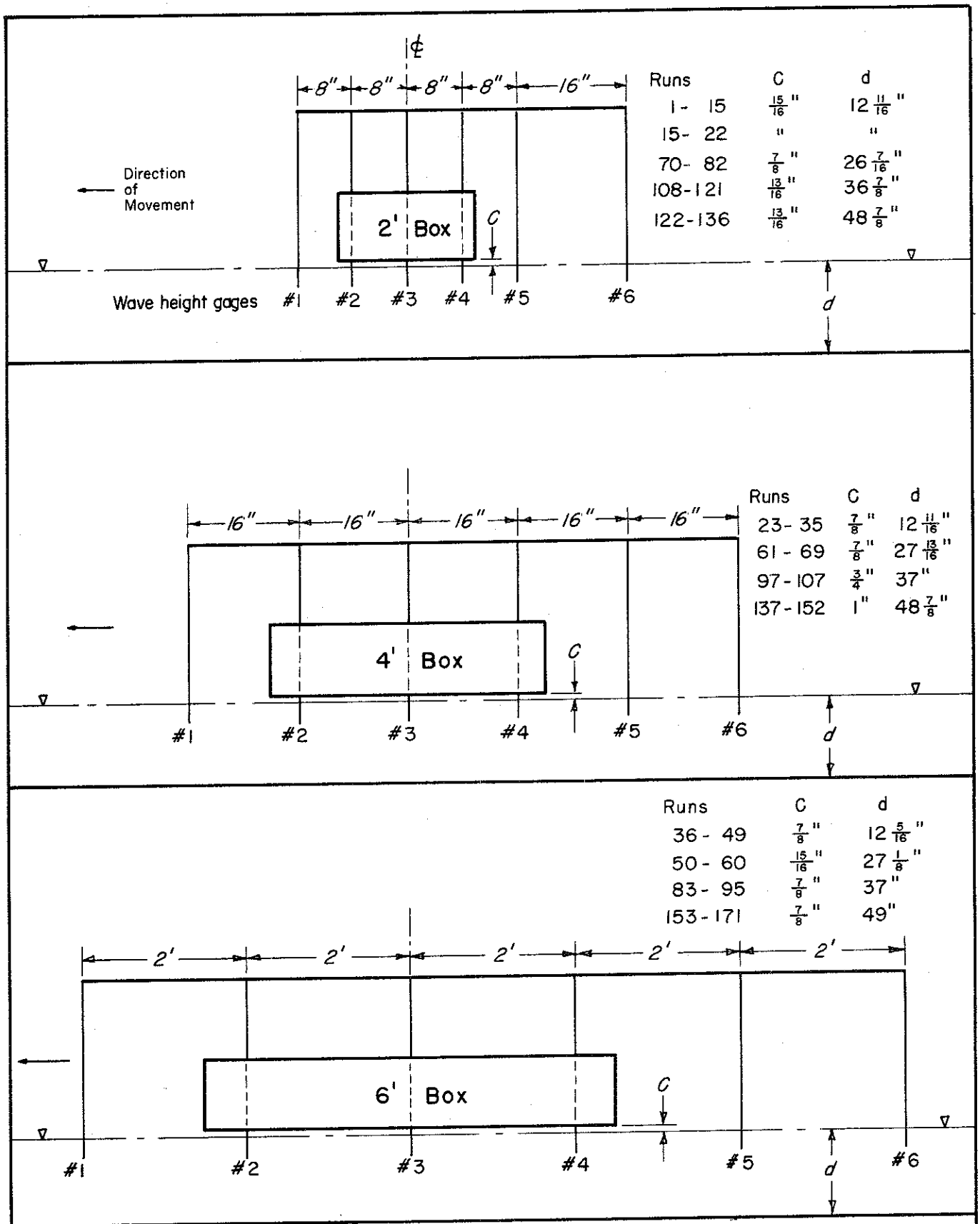
FIGURE 3



PRESSURE DISTRIBUTION IN MODEL HURRICANE TEST BOXES

HYD-7690

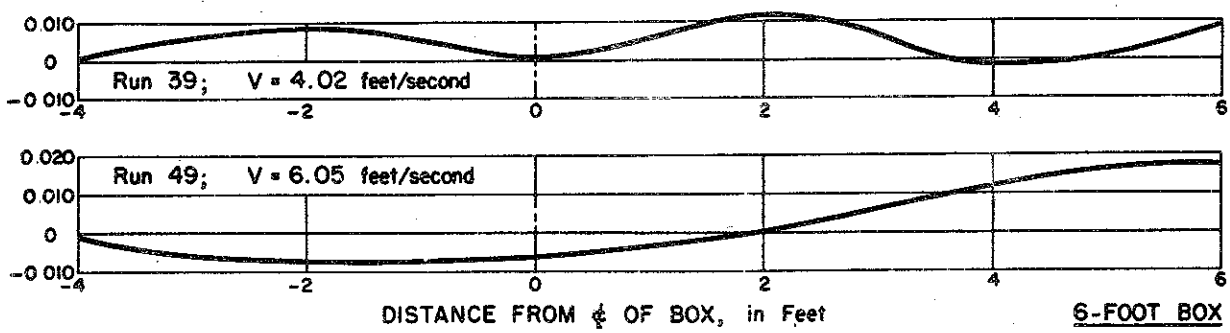
FIGURE 4

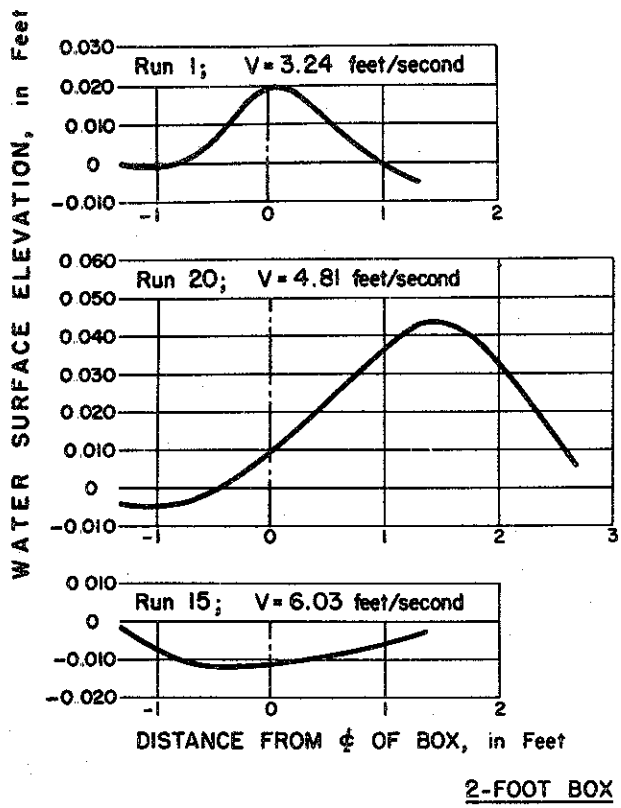


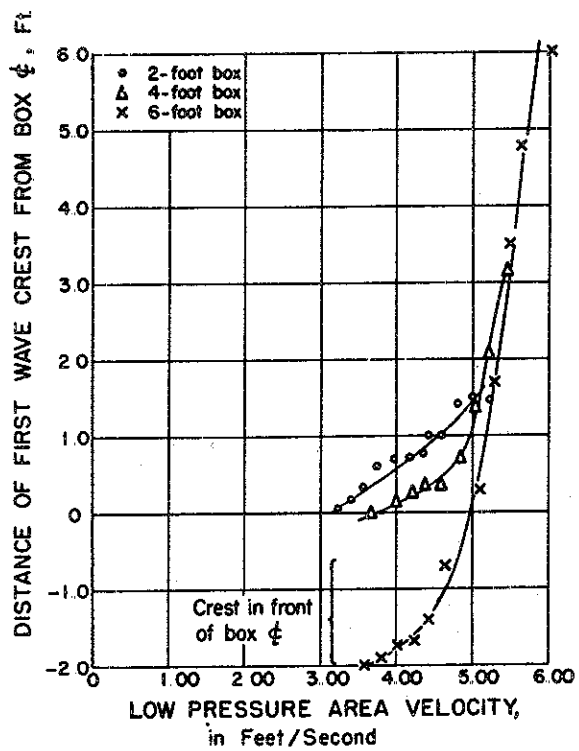
Locations of wave meters on model hurricane for experiments in water depths of 1.06, 2.26, 3.08 and 4.07 feet

HYD-769I

FIGURE 5





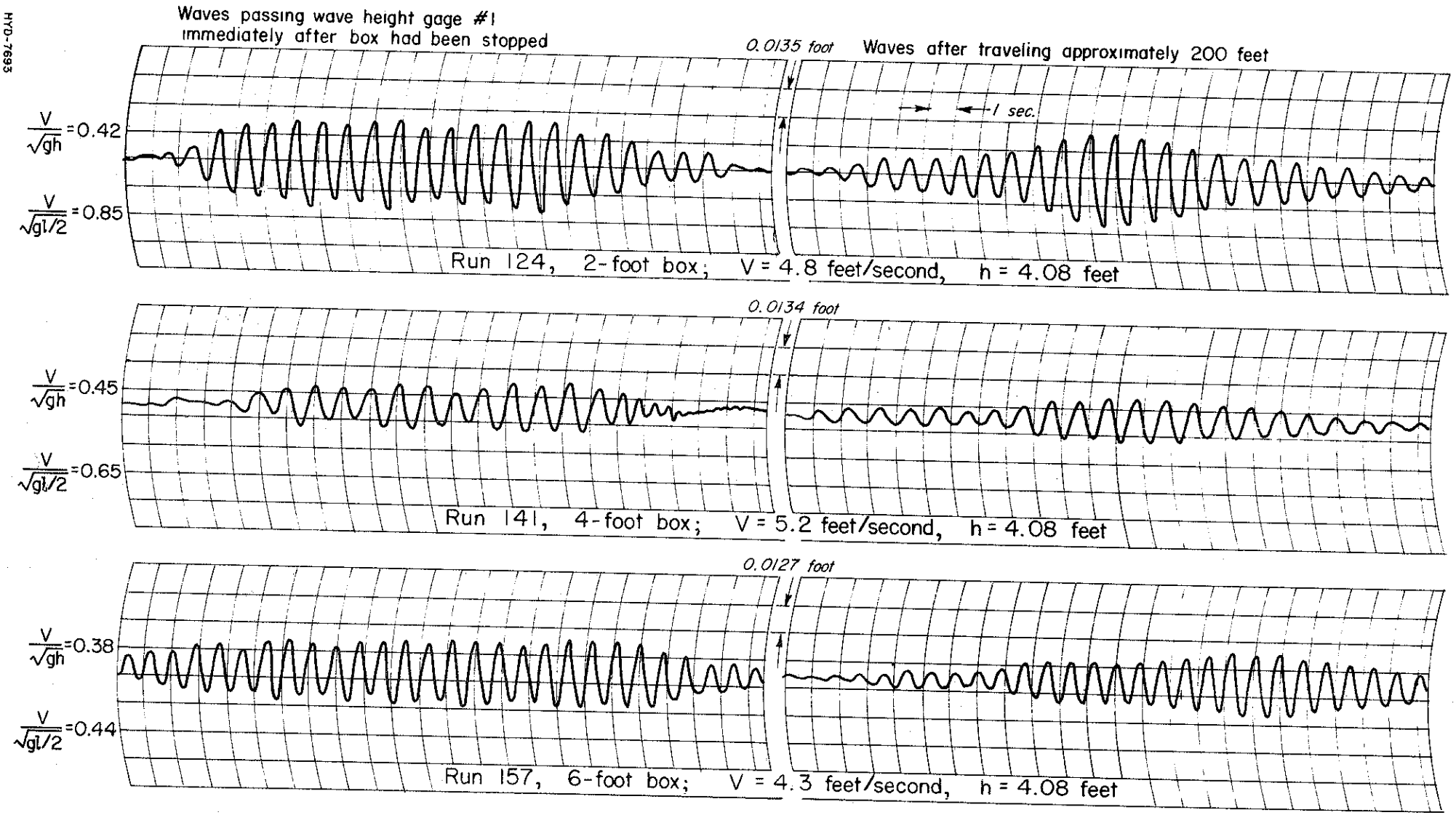


Relationships between Water Surface Profiles Generated by Low Pressure Area and Velocity of Low Pressure Area

HYD-7592

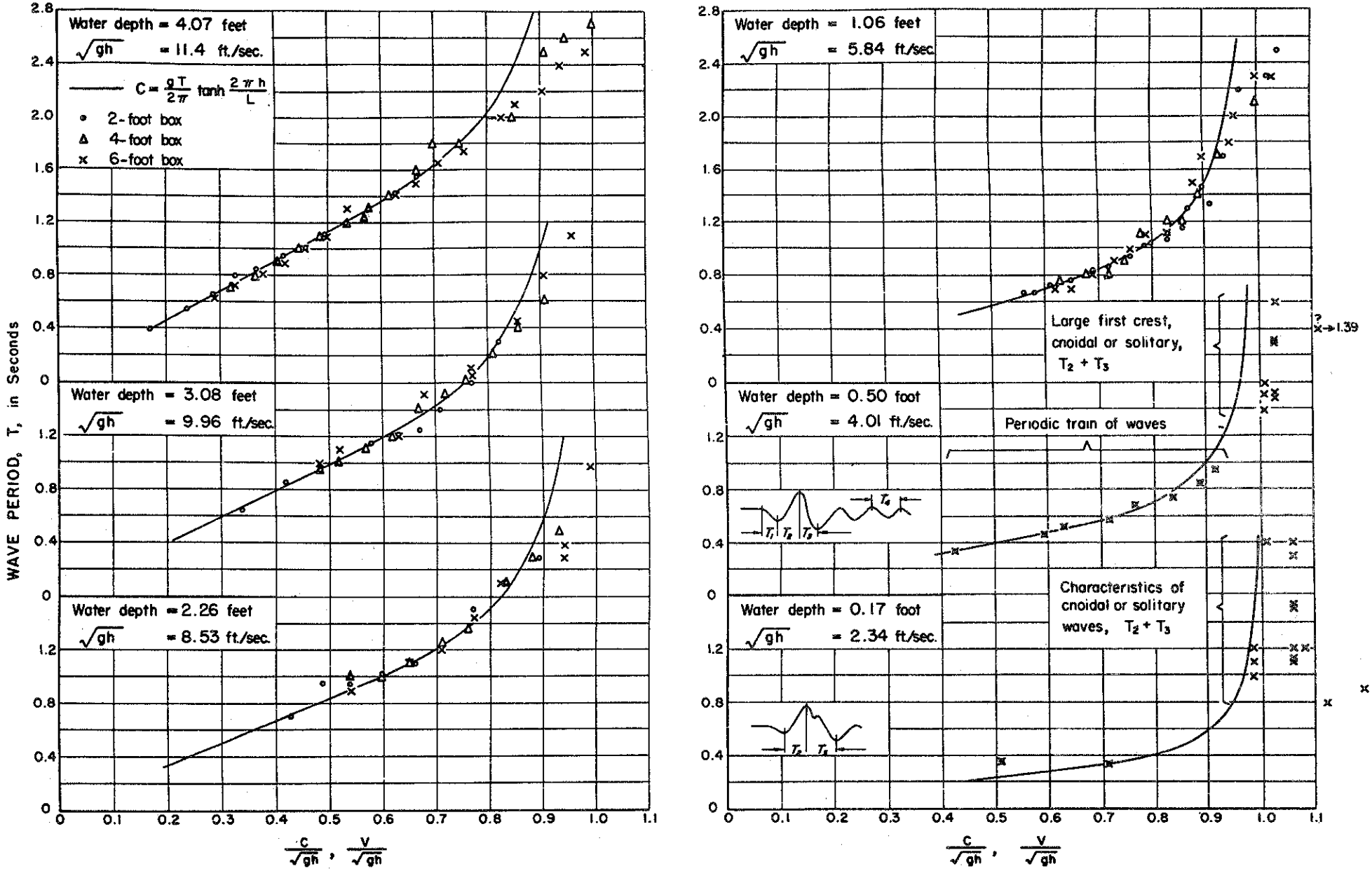
FIGURE 6

HYD-7693



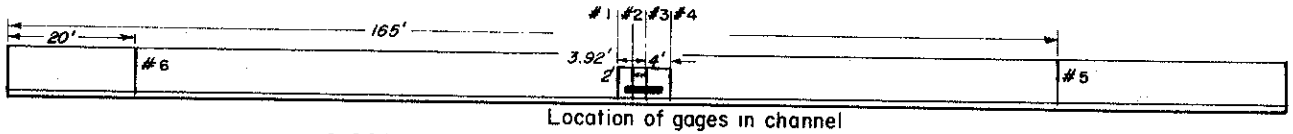
Sample records of waves generated by diffused low pressure area moving with constant velocity over water.

FIGURE 7

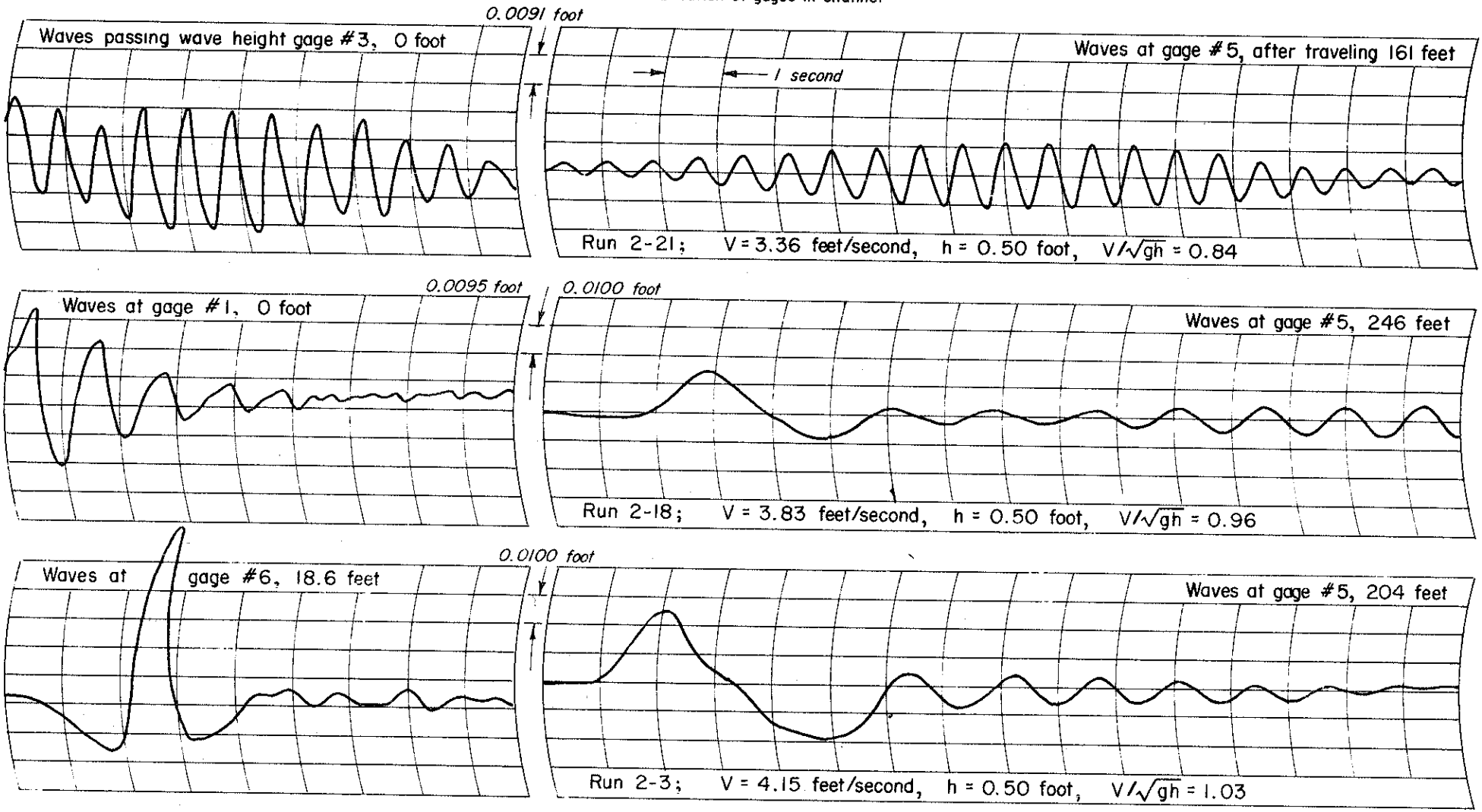


Wave Period as a Function of Pressure Area Velocity Compared with Linear Theory Relationship between Wave Phase Velocity and Wave Period

FIGURE 8



Location of gages in channel



Surface time histories of shallow water waves generated by the 6-foot low pressure area for a water depth of 0.50 foot.

HYD-7695

FIGURE 9

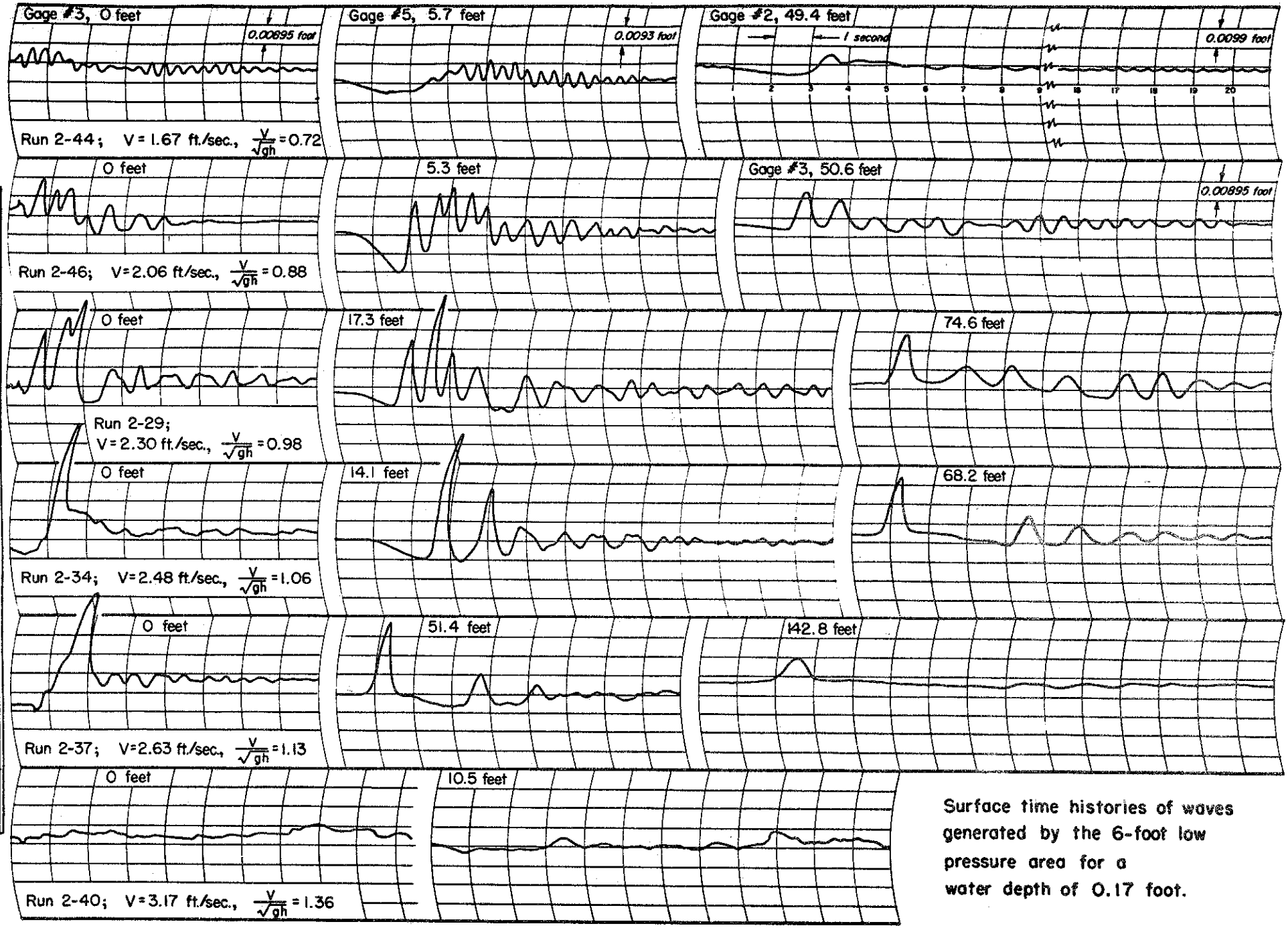


FIGURE 10

Surface time histories of waves generated by the 6-foot low pressure area for a water depth of 0.17 foot.

HYD-7697

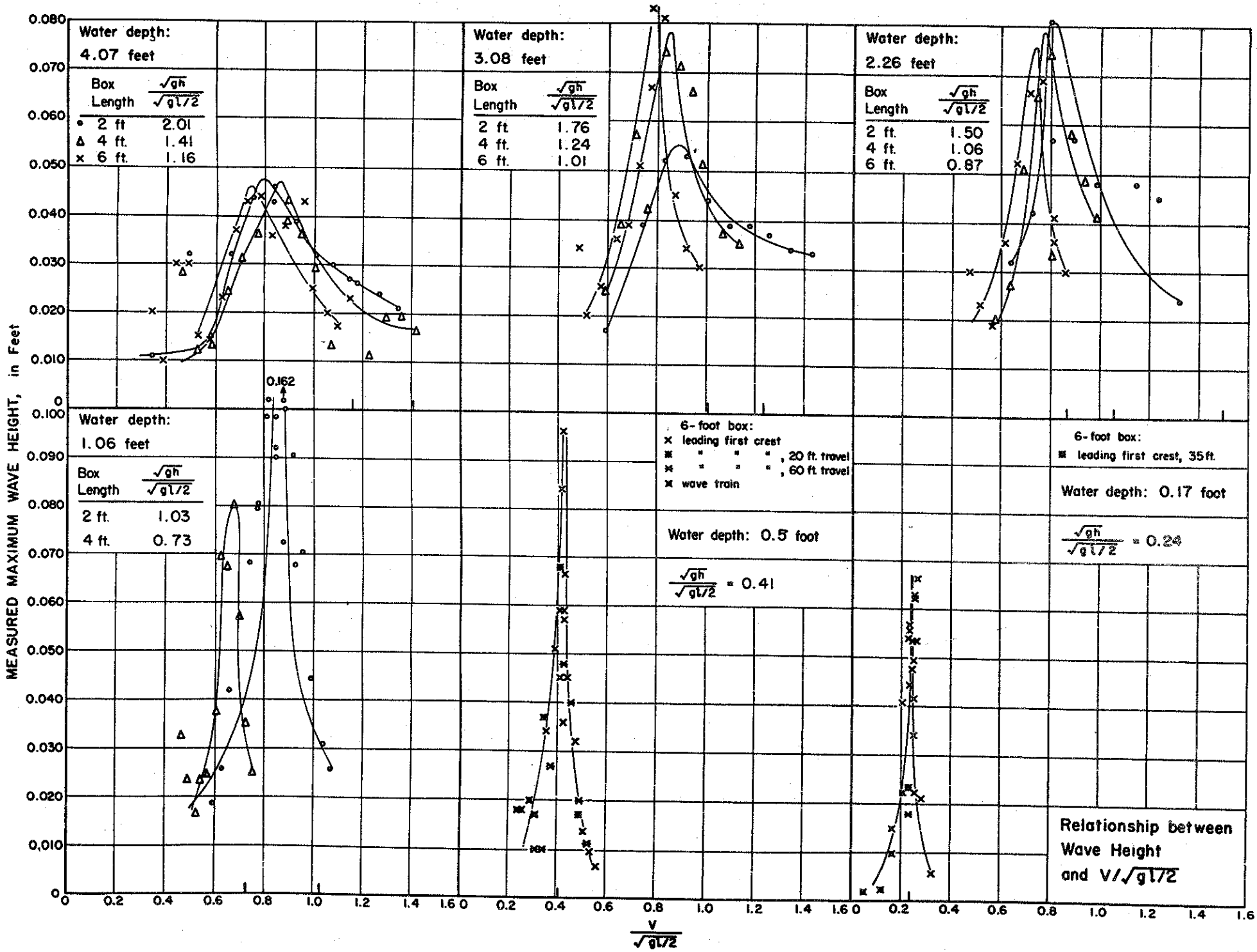
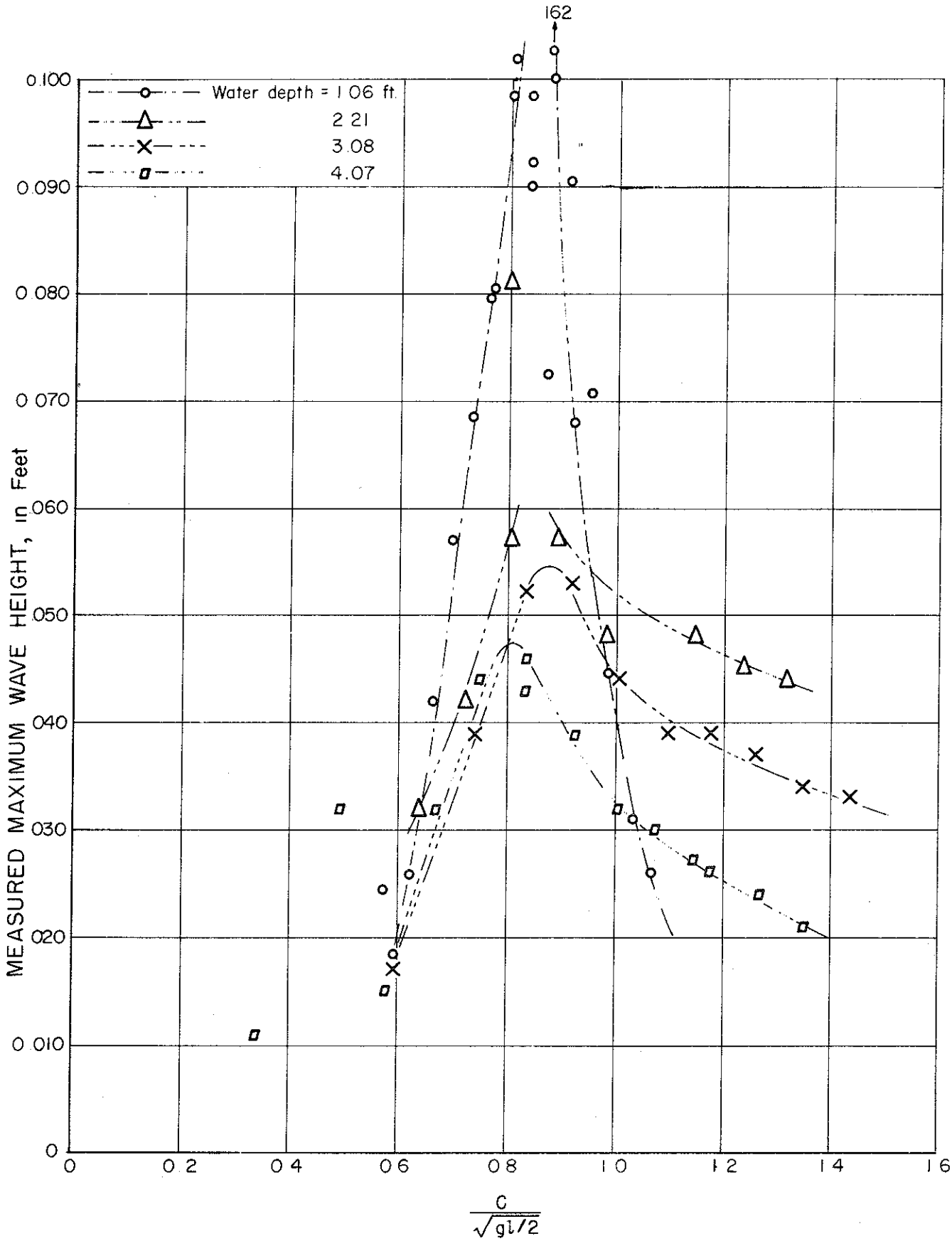


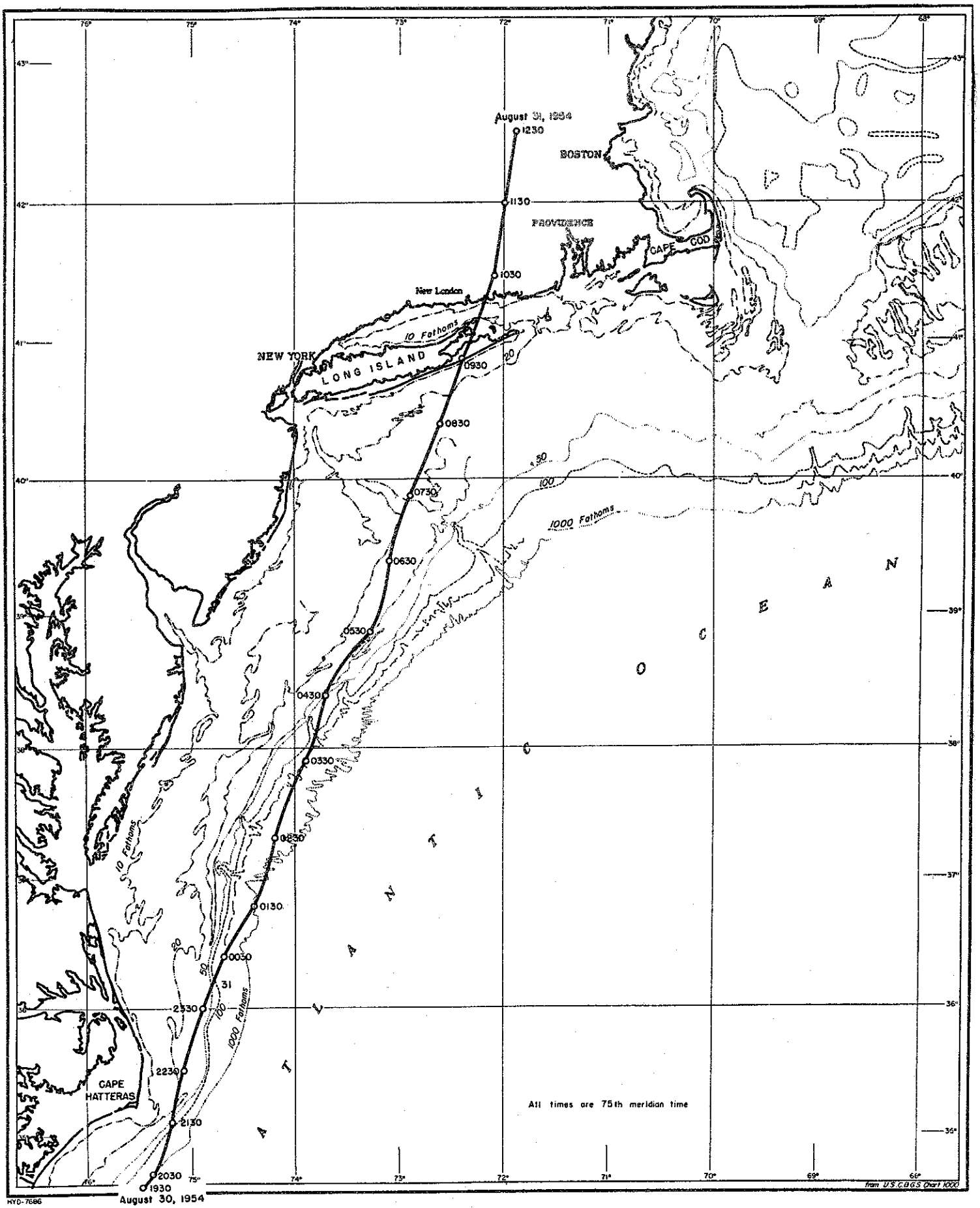
FIGURE 11



Wave Height versus $\frac{C}{\sqrt{gl/2}}$ for the 2-Foot Box and Various Depths of Water

HYD-7698

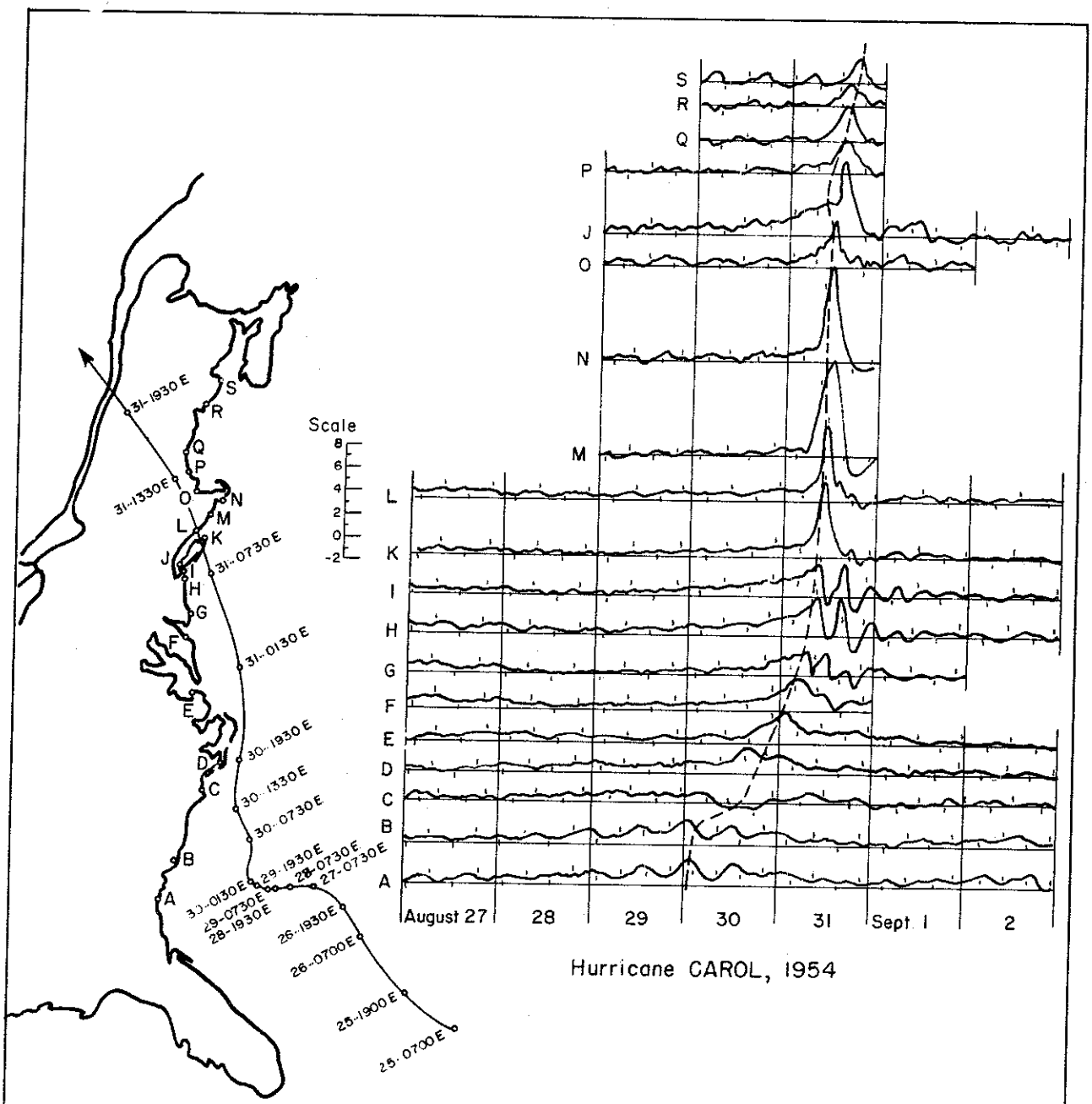
FIGURE 12



Path of center of hurricane CAROL, 30-31 August 1954

(after Harris, 1956)

FIGURE 13



STATIONS:

A Fort Pulaski, Ga
B Charleston, S. C
C Wilmington, N. C
D Morehead City, N. C
E Hampton Roads, Va
F Breakwater Harbor, Del.
G Atlantic City, N. J.
H Sandy Hook, N. J.
I The Battery, New York City, N. Y
J Willets Point, N. Y.
K Montauk Point, N. Y.
L New London, Conn.
M Newport, R. I.
N Woods Hole, Mass
O Boston, Mass
P Portsmouth, N. H.
Q Portland, Me.
R Bar Harbor, Me.
S Eastport, Me.

Hourly storm surge height (observed minus predicted sea level), Atlantic coast tide stations, August 27 - September 2, 1954. (after Harris, 1956)

HYD-7699

FIGURE 14