An analysis of temperature changes in Dutch winters

by

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Preface

Before you lies the thesis *"An analysis of temperature changes in Dutch winters"*. It is written in order to obtain the degree of Bachelor of Science. The research has been conducted under supervision of F. J. Vermolen in the Numerical Methods department of the faculty of EEMCS at the TU Delft, but has a lot of common ground with statistics.

In this thesis I attempt to give a clearer picture of whether the winters in The Netherlands have changed significantly over the past century. In order to do so, I have analysed if there is a trend in the data of some meteorological parameters at different weather stations.

Finally, I would like to give special thanks to F. J. Vermolen for the support and supervision in the past quarter and for all the feedback. Moreover, I would also like to thank G. Jongbloed and J. G. Spandaw for taking place in the thesis committee and for all their help.

D. Sarkisian
Delft, June 2019

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Introduction

According to the older generation mild winters occur more and more often the last few decades and in the past people had to endure severe winters with temperatures frequently dropping below freezing. These statements are rather subjective though, since everyone could have memorised a winter differently. This thesis will attempt to answer the question *Are winters in The Netherlands becoming warmer*?

First we will visualise the data of various meteorological parameters like the average winter temperature, the Hellmann number (which is a measure of the amount of frost over a certain period, for instance over a whole winter season), the North Atlantic Ocean index and such like. We will analyse the development of those parameters for each weather station. Although these findings will not tell us whether the change is significant, it will give us more solid grounds that lead to our hypothesis that there might be a warming tendency in the winters.

It is suggested by Gilbert [5] and Goossens & Berger [7] that the Mann-Kendall test is a reliable method to identify trends in climate data, which fits very well in our research. To obtain a better understanding of the test, we will dig into its theoretical foundations.

Another statistical tool we came across together with the Mann-Kendall test is the so-called Sen's slope estimator. This is a non-parametric estimator, which looks at the median value of rankings. Often, when a significant trend is found in the data, Sen's slope is used to analyse the magnitude of this trend. Together with the Mann-Kendall test, this is frequently used to analyse the trend of climate data. Following the description of the Mann-Kendall test and the Sen's slope estimator, we will apply these principles to analyse the data. We will draw a conclusion regarding possible trends in the Dutch winters.

In addition, since the change of winter temperatures also directly affects (one of) the biggest sports event in The Netherlands, we will also analyse the likelihood of an Eleven cities tour (in Dutch: *Elfstedentocht*) taking place.

1

Data and Methods

In this chapter we will describe what data are used in our research and how it was processed into data that we can use for the statistical assessment. We will also introduce some notation, that is frequently used in this thesis. The data used in the analysis were mainly retrieved from the Royal Netherlands Meteorological Institute (in Dutch: *Koninklijk Nederlands Meteorologisch Instituut* or shorthand *KNMI*) and the Evironmental Data Compendium (in Dutch: *Compendium voor de Leefomgeving* or *CLO* in short).

1.1. Data of the meteorological parameters

The parameters that have been obtained from the KNMI database are the daily average temperatures, the daily minimum temperatures and the daily maximum temperatures in the period of 12-01 to 03-01 (the month March in the next year) from 1906-2019. Subsequently, we have calculated the average of each period for the average, minimum and maximum temperature in order to get the annual average, minimum and maximum winter temperatures.

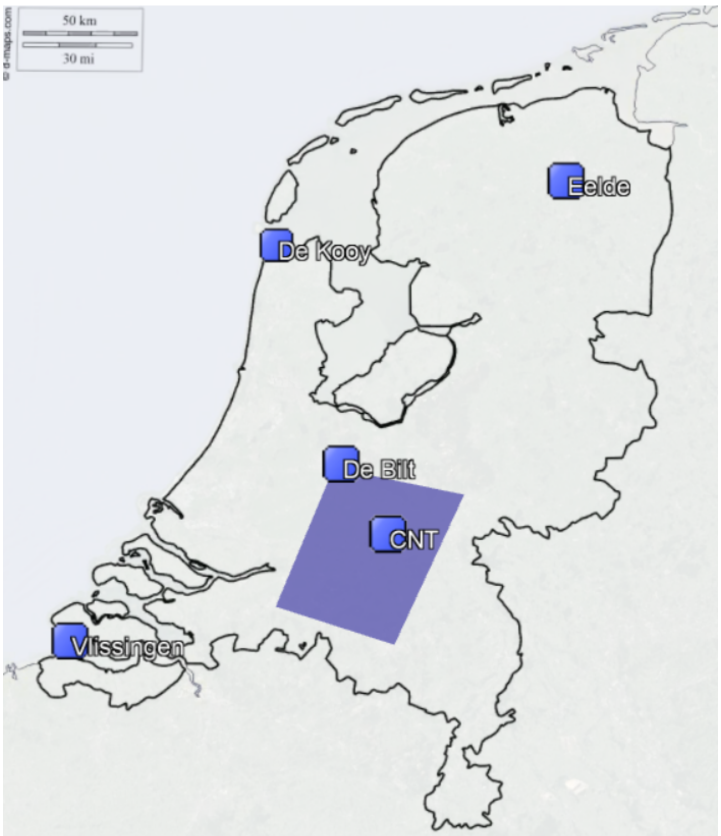

Figure 1.1: Map of The Netherlands with the weather stations and the area of the CNT

The data are obtained from four weather stations spread across The Netherlands, namely De Bilt, Eelde, De Kooy and Vlissingen as can be seen in figure 1.1. Choosing these stations has several reasons. The most important reason is that these data have been homogenised by the KNMI, which means that irregularities like a change of measuring instruments or a change of the environment have been corrected. This make the data reliable and useful for trend analysis. These are also stations where measurements have started the earliest, i.e. from 1901 (De Bilt) and 1906 (the others), so this gives us more data to use for the trend analysis. Unfortunately, these data of the temperatures miss some values around 1944 and 1945, but a work-around will be described in Chapter 5.

The Hellmann numbers are obtained from the CLO and are a measure of classifying winters on the amount of sub-zero temperatures. This number is often used in The Netherlands to compare different winters on how severe they have been. The Hellmann numbers are measured at De Bilt, so it only gives a picture of the winters in the central parts of the country. Nevertheless, it is still believed to be a good indicator, that can be used in our research. The Hellmann number will be explained more technically in Section 1.1.1.

The *coldest average period of 15 consecutive days* and The Central Netherlands Temperature (CNT) will also be analysed. In addition, the North Atlantic Oscillation index (NAO-index), which is believed to be a useful index, will be taken into account in our analysis as well. These indices will be explained in further detail in the coming sections.

1.1.1. Hellmann number

Since the Hellmann number is a bit more technical, we will give a proper definition on how the KNMI calculates this number.

Definition 1.1. *Let n be the number of days in a given period and let $\bar{T}_1, \ldots, \bar{T}_n$ be the average daily temperatures in Centigrade in that period. Then the Hellmann number of this period is defined as*

$$H = \sum_{i=1}^{n} |\min(0, \bar{T}_i)|. \tag{1.1}$$

Remark. Notice that our observed period is from December 1 till March 1 in the next year, so we have $n = 90$ or $n = 91$ depending on whether the observed winter falls in a leap year.

Winters in The Netherlands are usually classified by the KNMI by their Hellmann number as follows [2]:

Hellmann number H	Classification
$H > 300$	Severe
$160 < H \leq 300$	Very cold
$100 < H \leq 160$	Cold
$40 < H \leq 100$	Normal
$20 < H \leq 40$	Mild
$10 < H \leq 20$	Very mild
$0 < H \leq 10$	Unusually mild

1.1.2. Coldest average period of 15 consecutive days

The data of the coldest average period of 15 consecutive days have been obtained from the KNMI and are measured at De Bilt. This parameter is a good addition to our analysis, since the average temperatures of a whole winter can sometimes be somewhat misleading and one winter can have multiple extreme temperatures that average each other. However, when we look at the coldest 15 consecutive days, we take into account that winters often *feel* cold if there have actually been a few cold days right after each other, so this parameter might give more realistic insights in the behaviour of Dutch winters.

1.1.3. Central Netherlands Temperature

As the name *Central Netherlands Temperature* suggests, it is a benchmark for temperatures in the central parts of The Netherlands. The CNT is constructed by the KNMI for usage in climate models, that typically look at larger spatial scales than just one weather station. The construction of this quantity is based on the data of different weather stations that lie in the area that is spanned by Utrecht, Arnhem, Breda en Eindhoven [1]. This quantity is also reliable, since it has been corrected and homogenised by the KNMI.

Unfortunately, the CNT is less useful for other parts of the country like the northern provinces and coastal regions. For these regions we have to rely on the data of individual weather stations, but together with these stations it still contributes to understanding the overall picture.

These data have also been obtained from the KNMI. However, the data set was in a matrix form where each row is a year with twelve corresponding columns of the months. To use these data and store them as a time series, we have transposed each column and added the columns per row to obtain one vector (see Appendix A). Since we are mainly concerned with the variation in winter temperatures, we will only consider the data points of the months *November, December, January and February* for each year and take the average of those months.

1.1.4. North Atlantic Oscillation index

The North Atlantic Oscillation is a measure of the difference in average air pressure between weather stations in Reykyavik, Iceland and Lisbon, Portugal or Azores, Portugal. In these regions low and high pressure regions are present, called the Icelandic low and the Azores High, as illustrated in figure 1.2. These low/high pressure regions vary in their exact position and this variation is linked to temperatures, i.e. having a severe or mild winter, in Europe (Beranová and Kyselý [3]).

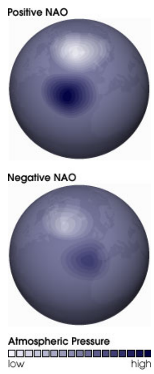

Figure 1.2: Difference in high and low air pressure regions (*Image by Robert Simmon*)

The difference in the air pressure between the aforementioned weather stations is the so-called NAO-index. In particular, a positive NAO-index (denoted by NAO+) is when there is a large difference between the air pressure at the stations and is often linked with milder winters in West-Europe and a negative NAO-index (denoted by NAO-) is when there is a smaller difference between the air pressure at the stations and is linked with colder winters in West-Europe. So, together with the other parameters we will analyse, this index can give us more insight in the change of winters in The Netherlands.

The data of the NAO-index have been retrieved from the National Center for Atmospheric Research (in short: *NCAR*). The NCAR has data of the NAO-index which is measured at weather stations Reykyavik, Iceland and Ponta Delgada, Azores. At these two stations the sea level pressures are measured and afterwards the data are normalised, that is, the data are adjusted to the long term mean of 1864-1983 [8]. For our research, we have used the annual data since 1865 collected by NCAR.

1.2. Used methods

In several studies (Zhao et al. [4], Gocic and Trajkovic [6]) we have found that the *Mann-Kendall* test was applied to identify a trend in a given data set and afterwards if such a trend is present, *Sen's slope estimator* was used to analyse the magnitude of that trend.

Since we came across this method of trend analysis so frequently and the researchers suggest that the Mann-Kendall test and Sen's slope estimator give a desired result in their analysis, we will explore the theory behind it and apply it to our own data in order to draw some conclusions about the trends in the winters in The Netherlands.

It is suggested by Gilbert [5] and Goossens & Berger [7] that the Mann-Kendall test is an appropriate way to detect trends in climate data and even to localise the spot where this trend begins. An advantage of this test is that it is non-parametric, so we do not have to know the distribution of the data beforehand and because it takes into account the ranks of the observations in a data set it is also insensible to outliers. A more in-depth analysis of the Mann-Kendall test will follow in Chapter 3. Additionally, Sen's slope estimator is also a non-parametric method to analyse trends in data. Since this method looks at different slopes between pairs of observations and the median value of these slopes, it is also less prone to outlying data.

2

Description of the data

Before we focus on the Mann-Kendall test and the theory, we first take a look at the observations by visualising the data to form a motivation on the following analysis. This will be done by looking at the data and their so-called *simple moving average* (SMA) to see if we can spot some changes. Then we will take a look at different periods of time in histograms of the average, minimum and maximum annual winter temperatures, the Hellmann number, the CNT and the NAO-index. This way, we will try to see whether the observations of the last two decades fall in the *warmer part* of the data, but we will not yet analyse whether these changes are significant in this chapter.

2.1. Simple moving average

When analysing data one often encounters observations with high variability and a lot of fluctuations. In order to see the underlying long-term trend, we look at the mean of consecutive periods of n time units. The simple moving average is a special type of the so-called moving average, which makes us able to see these long-term cycles. The simple moving average is defined as follows.

Definition 2.1 (Simple moving average)**.** *Let $y_1,...,y_n$ be observations, then the simple moving average (SMA) of a period of N time units is defined as*

$$\hat{y} = \frac{1}{N}\sum_{i=0}^{N-1} y_{n-i}, \tag{2.1}$$

which is called a N-SMA. By (2.1) it follows that the SMA of the previous period is given by

$$\hat{y}_{previous} = \hat{y} - \frac{y_n + y_{n-N}}{N} \tag{2.2}$$

Remark. The unit of time of all of our data is in years and the time span of *climate* is often taken to be 30 years. This means that we will look at the 30-SMA in the following sections, but we will simply refer to it as SMA.

2.2. Average winter temperature

In figure 2.1 we see the average winter temperatures for each weather station and the corresponding SMA. For each station we see that the SMA tends to increase in the last three decades. We also notice that the peaks, or extreme points, are larger from around 1980-2018 than from 1906-1980. This gives us a presumption that the average winter temperatures might have increased.

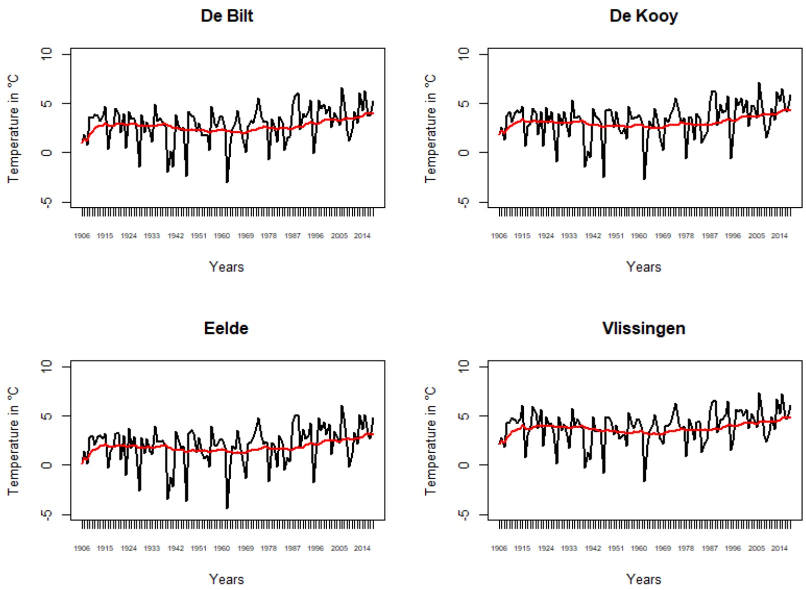

Figure 2.1: Simple moving average of the average winter temperature per station

When we take a closer look at the histograms of each station in figure 2.2 and compare the last thirty year period 1988-2018 with 1958-1987, we immediately see that warmer extreme years occur more often in 1988-2018 than in 1958-1988. This gives us an even stronger assumption that the winters in The Netherlands might have become warmer in the last few decades. Interestingly, it seems like the winters in Eelde from the period 1988-2018 are looking more like the winters in Vlissingen from the period 1958-1987, which lies on the coast.

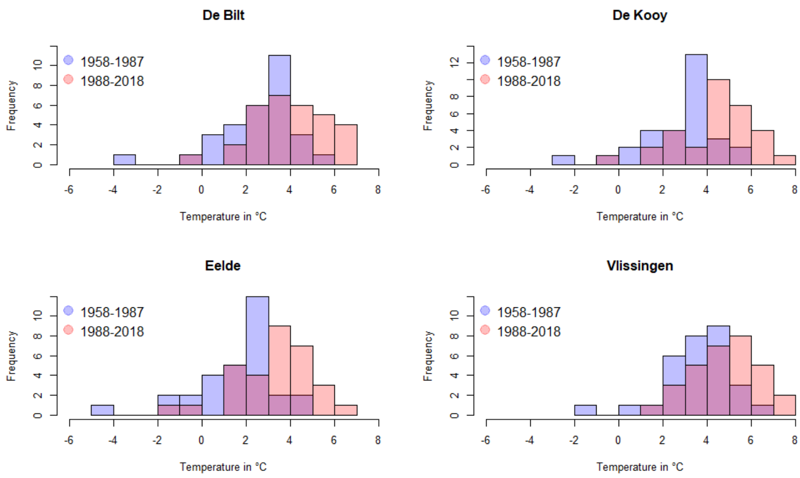

Figure 2.2: Histogram of the average winter temperature per station 1958-1987 and 1988-2018

2.3. Maximum winter temperature

In figure 2.3 the same pattern is seen as for the average winter temperatures, namely a light increase of the SMA from the 1980s on. This might indicate that the maximum temperatures in the winter reach higher values now than in 1958-1987.

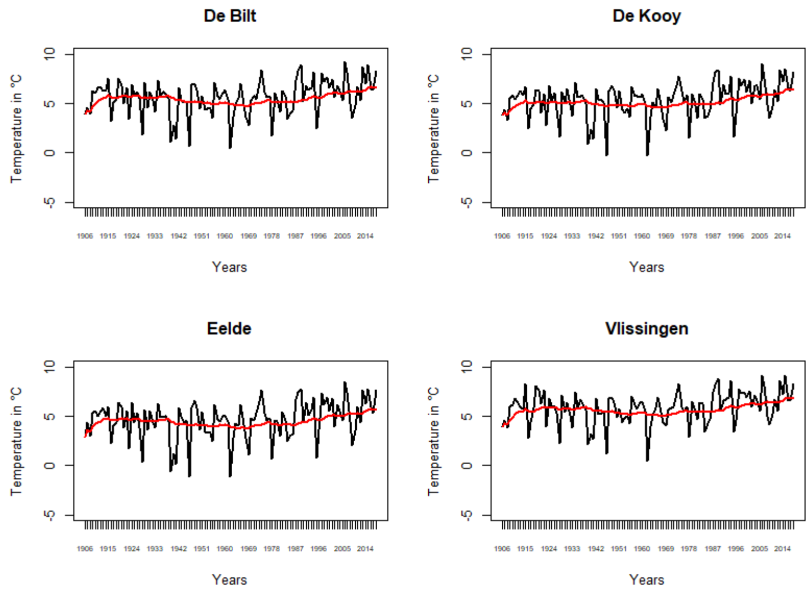

Figure 2.3: Simple moving average of the maximum winter temperature per station

In the histograms in figure 2.4 we see that high maximum temperatures also happen more often in the last three decades compared with 1958-1987, which strengthens our presumption that the temperatures are getting more extreme and higher.

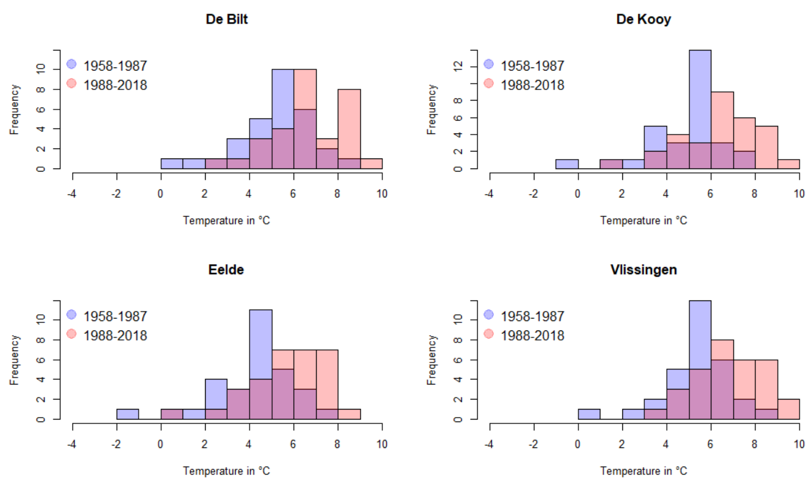

Figure 2.4: Histogram of the maximum winter temperature per station in 1958-1987 and 1988-2018

2.4. Minimum winter temperature

Looking at figure 2.5 we see the same happening with the SMA for the minimum winter temperatures as in the previous sections. This means that it seems like the minimum winter temperatures have shifted as well. It could be of interest to look at the range of the temperatures by looking at the difference between the maximum and minimum temperatures. However, since we already take into account many other parameters, we will not particularly focus on the range.

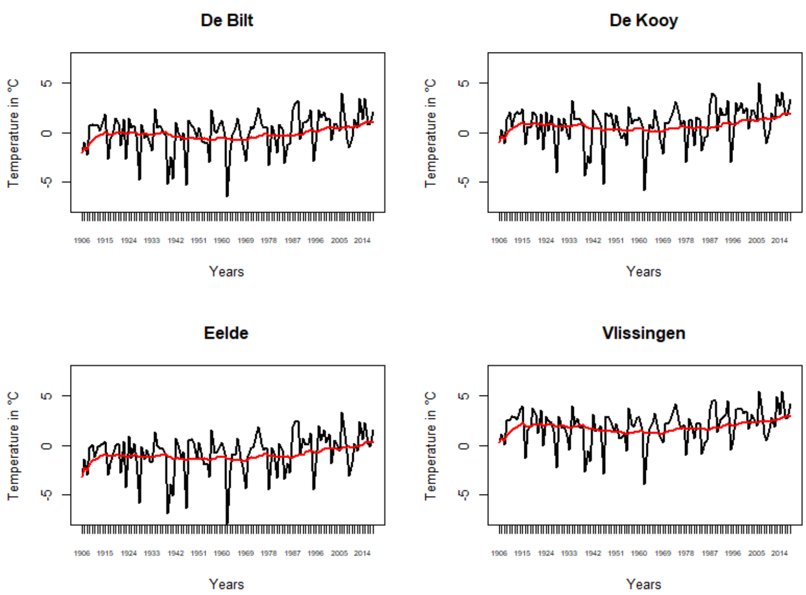

Figure 2.5: Simple moving average of the minimum winter temperature per station

In figure 2.6 we see that the same holds for the minimum winter temperatures as we have seen for the maximum and average temperatures. As mentioned earlier, we see together with our previous findings that not only the average temperature has increased, but also the extreme temperatures have increased. So the range where temperatures are located has shifted a bit towards higher values. This could be coincidental of course and we have to be careful with drawing such strong conclusions, since we have not analysed whether these results are significant.

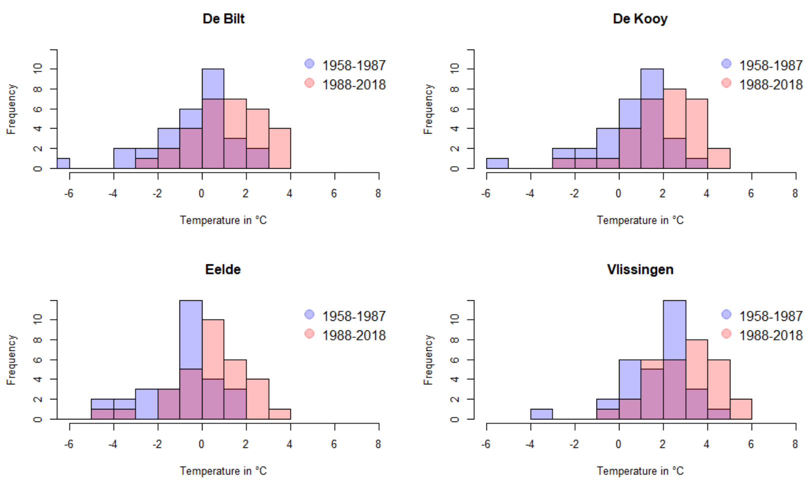

Figure 2.6: Histogram of the minimum winter temperature per station in 1958-1987 and 1988-2018

2.5. Coldest average period of 15 consecutive days

In figure 2.7 we notice that, like the previous parameters, there is a large increase in the first decade of the 20th century, which is followed by a colder mid-century period. The increase from the 1980s on seems to accelerate after the turn of the century, which makes us hypothesise that this parameter might have a significant trend over the whole period of time as well.

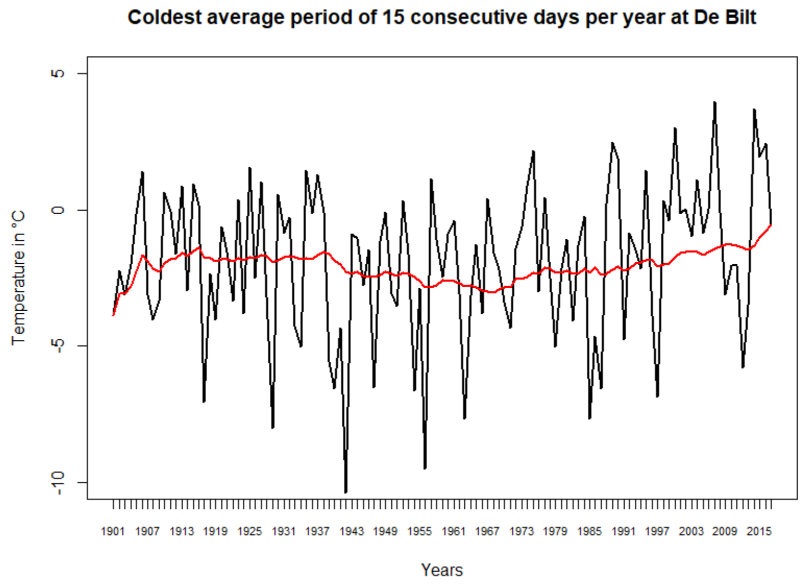

Figure 2.7: Simple moving average of the coldest average period of 15 consecutive days per year

In the histogram in figure 2.8 we can see that there is also a shift in temperatures when we compare the last two 30-year periods. However, the cold extreme values are still present, be it one or two Centigrade higher. Moreover, the temperatures from 1987-2017 are more concentrated just above 0°C, while the temperatures from 1957-1986 are more concentrated below 0°C. Altogether, it seems like the coldest average period of 15 consecutive days has become less cold over the years.

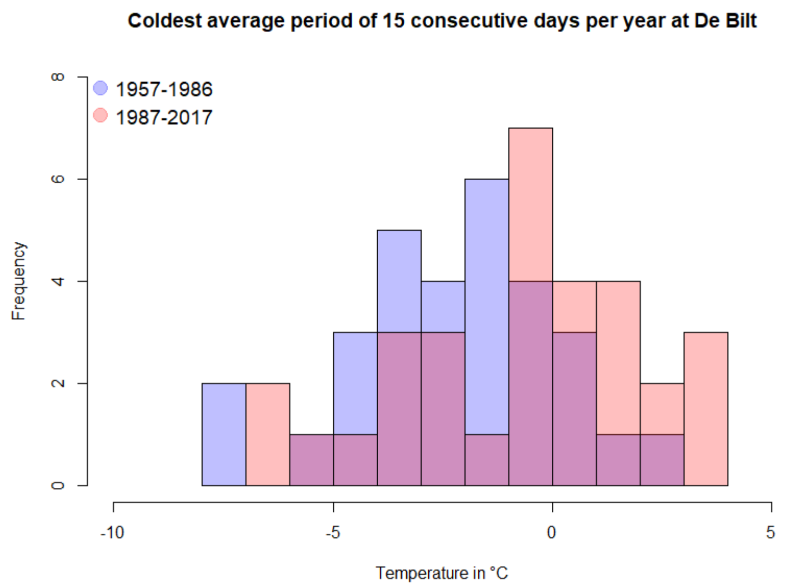

Figure 2.8: Histogram of the coldest average period of 15 consecutive days per year

2.6. CNT

In figure 2.9 a slight change can be seen. In particular, it seems like the SMA increases from the 1980s on. However, the increase does not seem to accelerate in the last two decade unlike the other parameters.

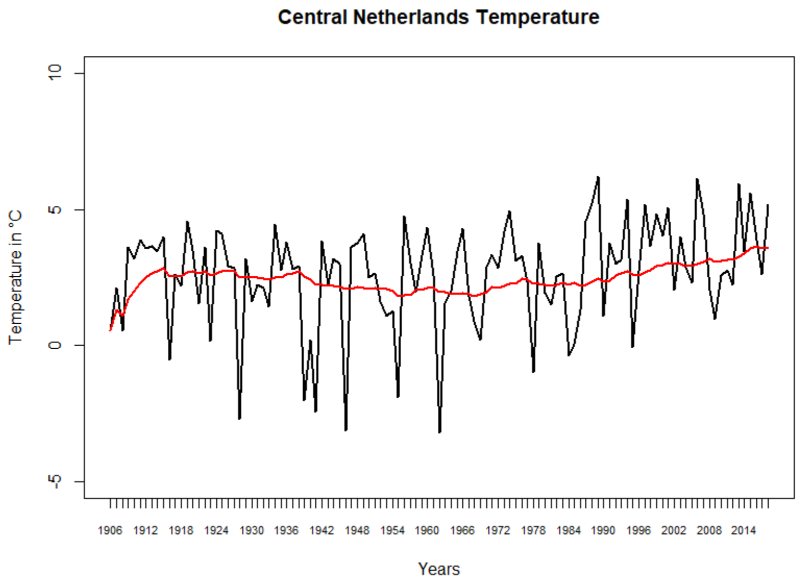

Figure 2.9: Simple moving average of the Central Netherlands Temperature

In figure 2.10 we can see that in 1988-2018 the lowest values occur three times around freezing point (between -1°C and 1°C), while most of the years the CNT is strongly concentrated towards 2°C to 5°C. Whereas compared to the period of 1958-1987 we see that most years have temperatures concentrated around 2°C and quite some observations were below 2°C, with one extreme year around -3°C. There clearly is some sort of shift happening, but whether this is significant we yet have to analyse.

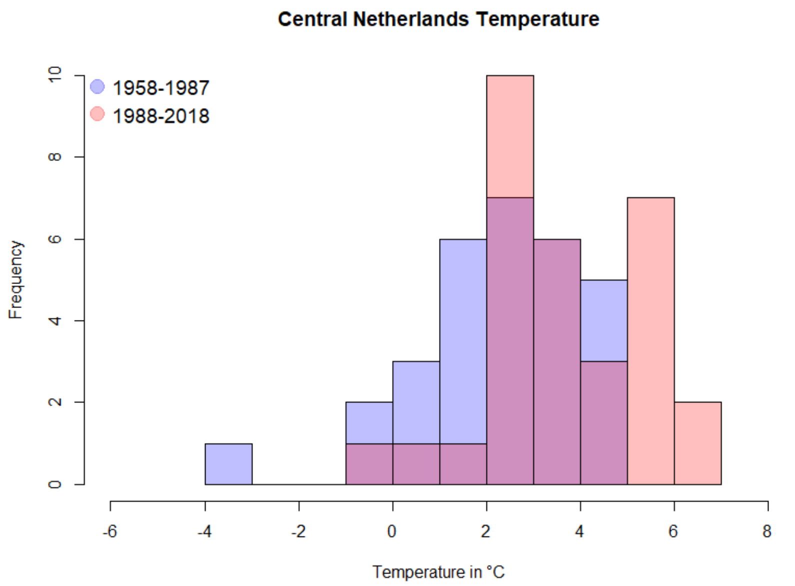

Figure 2.10: Histogram of the Central Netherlands Temperature in 1958-1987 and 1988-2018

2.7. Hellmann number

In figure 2.11 we can see two interesting things. The first being that large extreme values are occurring less often, i.e. the variance is lower in the last few decades than in the mid-19th century. However, in the beginning of the 19th century we see this as well. The second thing we notice, is that the SMA has decreased if we look at 1988-2018.

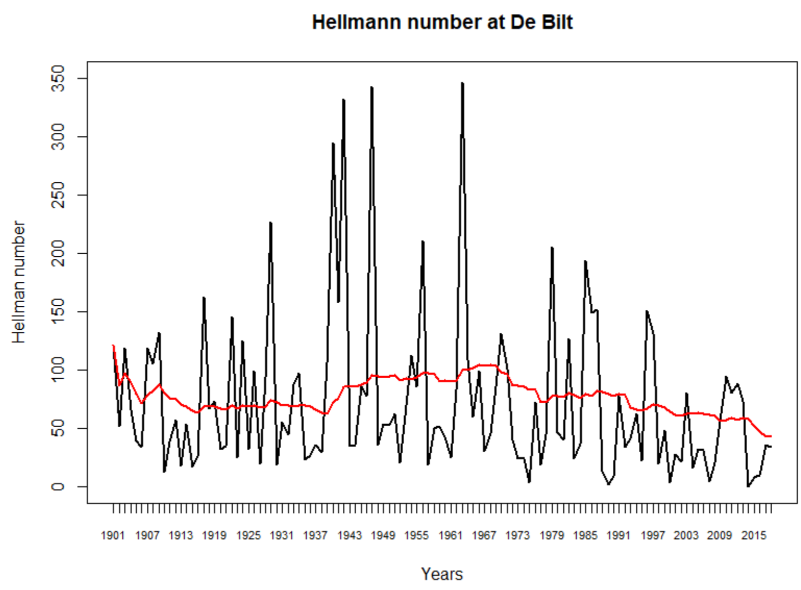

Figure 2.11: Simple moving average of the Hellmann number at De Bilt

In figure 2.12 we see that in 1988-2018 we have had (unusually) mild winters more often than in 1958-1987 and we also have not experienced cold or very cold [2] winters anymore since 1988. Another big change is that the amount of cold winters has decreased a lot in 1988-2018 and has made place for more normal winters.

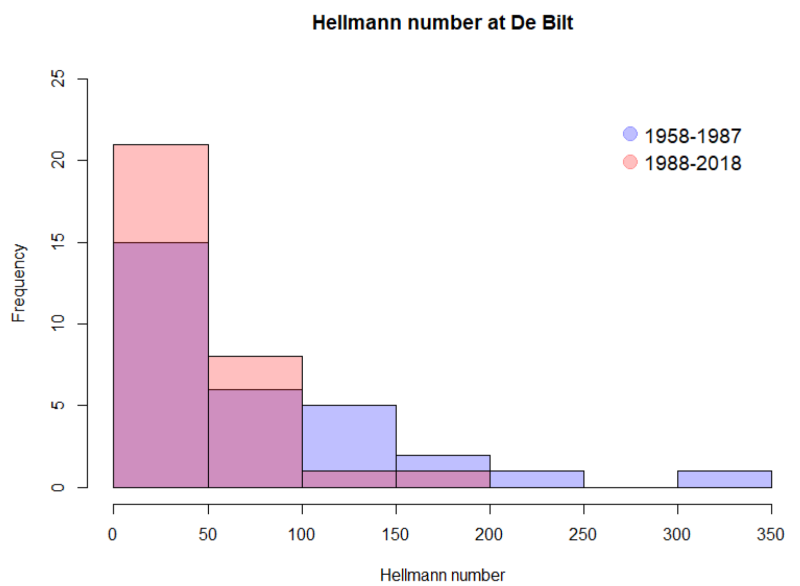

Figure 2.12: Histogram of the Hellmann number in De Bilt in 1958-1987 and 1988-2018

The results of this chapter so far give us a strong reason to believe that Dutch winters may indeed have become warmer and extremer in the past few decades, particularly since the turn of the century. As mentioned before, though, this does not have to mean anything and the results could just be coincidental outliers. In Chapter 5 we will attempt to see whether these changes are significant and whether we see a trend or not in Dutch winters.

2.8. NAO-index

At first glance, the SMA in figure 2.13 seems to follow the same path as we have seen in the graphs of the average, maximum and minimum temperatures. Around 1900 a quick increase can be seen together with values reaching a minimum around the 1960/70s to finally follow a sudden increase from the 1980s on. Overall, it looks like there is some periodicity between NAO+ and NAO- and some correlation between temperature and NAO-index.

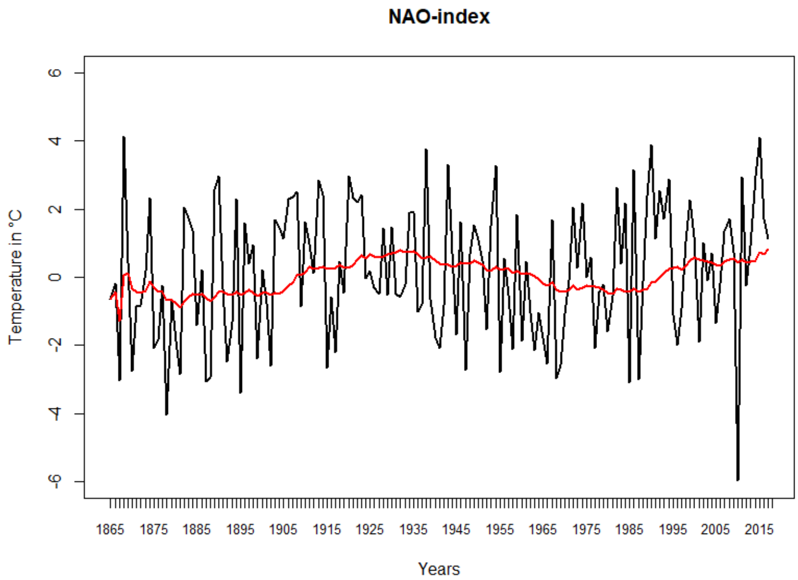

Figure 2.13: Simple moving average of the NAO-index

In figure 2.14, the histograms of four consecutive periods of 30 years to the most recent period of 1987-2017 are compared. It seems like the NAO-index has slightly shifted to the positive values. In particular, the values of the most recent period of 1987-2017 are concentrated towards the positive end of the NAO-index. However, as for now we cannot yet make any premature conclusions, but it does give us the impression that some interesting phenomena could be taking place and we will analyse the different correlations between the NAO-index and the other meteorological parameters in Chapter 5.

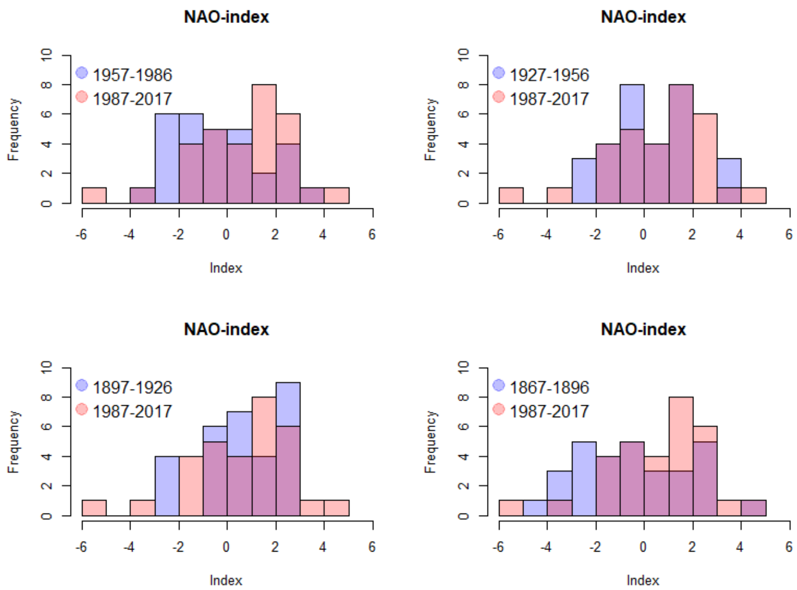

Figure 2.14: Histogram of the NAO-index in 1987-2017 compared with 1957-1986, 1927-1956, 1897-1928 and 1867-1896

2.9. Motivation

In this chapter we have described the data visually to see whether we can form any presumptions on Dutch winters. Looking at the graphs and the 30 years simple moving average we see that there is a decrease in temperature around the 1960s and from the 1980s on we see an increase in the temperatures. Interestingly, the Hellmann number seems to increase in the 1960s and to decrease after the 1980s, which seems consistent with the changes in temperature. Whether these changes are significant, we cannot yet answer, but it gives us a better first insight.

Subsequently, when we look at the histograms and compare different time intervals, we notice that some shift is occurring. We see that from 1958-1987 there were more outliers tending to low temperatures and high Hellmann numbers, whereas from 1988-2018 we see that most temperatures concentrate around higher values and the Hellmann numbers are in the low numbers more.

These results give us a strong reason to think that some changes are developing since the last few decades. In particular, we see that winter temperatures tend to shift towards higher values and cold extremes have become less frequent. With these observations in mind, we will analyse the data more thoroughly and try to conclude whether these changes are significant.

3
Mann-Kendall test

During the analysis of climate data, one wants to find out whether the observed changes are significant from a statistical point of view. One way that leads to an answer is the use of the Kolmogorov-Smirnov test, which is strongly based on the assumption that the data follow a normal distribution. In this chapter, we will analyse the data using a different test.

As mentioned earlier, in various studies it is attempted to find an answer to the aforementioned questions by using the so-called *Mann-Kendall* test. While carrying out this test, the actual values of the observations are ignored and instead the data are ranked. Obviously, taking this approach has its advantages and disadvantages. In this chapter we will explore how this test is developed by Kendall [9],(and described by Gilbert [5]) and on our way we will find out what aspects can be advantageous in analysing the ranks in order to find a trend and what aspects are not.

3.1. Rankings

In order to understand how the Mann-Kendall test works, we first have to dig into some underlying theory. Namely, ranking data on some quality. This could be anything, like temperature, grades, height or some other more abstract quality like your favourite singer. Ranking occurs very naturally.

Suppose there are three balls, i.e. ball 1, ball 2 and ball 3 and every ball is weighed by hand, then it could result in the third ball being the heaviest, followed by the second and then the first ball. The ranks of the balls would be 1, 2, 3 respectively, if the ranking is based on "heaviness". Anyhow, we will define a ranking on some quality as follows.

Definition 3.1 (Ranking)**.** *Let $y_1, y_2, ..., y_n$, $n \in \mathbb{N}$ be observations based on some quality Q and let $\phi : \mathbb{R}^n \to \mathbb{N}^n$ be a mapping. Then we can arrange these observations in a certain order by defining*

$$\phi(y_1, ..., y_n) = (r_1, ..., r_n) \text{ where } i, r_i \in [1, ..., n], \tag{3.1}$$

such that $r_i \le r_j$ whenever $y_i \le y_j$. We say that $(r_1, ..., r_n)$ is a ranking and the observation with index i has rank r_i.

Remark. Notice how ϕ is not required to be injective in the definition above. By not assuming this on the mapping it can happen that, for instance, two observations y_i and y_j have rank $r_i = r_j$. In this case we say that y_i and y_j have *tied ranks*. For now, we will only look at rankings without tied ranks, however, we will address the possibility of tied ranks later in Section 3.3.

Let us give an example to give a clearer picture. Suppose that we develop a ranking on the traveled distance on our bike of the last three days. Then we have three observations y_1, y_2, y_3 which we want to rank with respect to their distance. Using our phone, we see that on the first day we traveled 3.5 km, 3 km on the second day and 2.5 km on the third day. Hence we get the following.

$$\phi(y_1, y_2, y_3) = (3, 2, 1) \tag{3.2}$$

If we look at more qualities, however, then it is more convenient to look at the rankings in a table. So suppose after looking at the traveled distance on a bike, the number of hours we are active on our phone is measured for the same three days. The results are 2 hours on day 1, 3 hours on day 2 and 5 hours on day 3. Then we find the following.

	y_1	y_2	y_3
Traveled distance	3	2	1
Time on phone	1	2	3

The qualities and way of measuring can be chosen to be as abstract as in the beginning of this section, but as we continue our analysis on the change of temperatures we will only look at data that are measured and have specific values. However, by looking at rankings instead of the actual values, only the relative distances are observed, which gives a clear and general way of comparing qualities. Additionally, another advantage could be that outlying observations do not have as much effect as they would have if we look at the actual value. The disadvantage, however, is that the exact distance between various data points as well as their influence on the data cannot be examined.

3.2. Rank correlation

In our analysis and in the studied literature, one often observes two rankings. The first ranking is frequently some quality of time, like *years* or *months*. The second quality in our research will often be the *average winter temperature per year*, *Hellmann number per year*, or something of the like. A natural question that arises is, *how strong is the agreement between two rankings*?

To describe this *degree of agreement*, Kendall introduced a so-called rank correlation coefficient, which is coined as Kendall's tau. This tau can be calculated by looking at the ranks of pairs of elements in both rankings. Here we assume that the *order of reference* for all other rankings is $(1,2,...,n)$. We assign a *score* of $+1$ if the two ranks of a pair occur in the same order and a -1 is assigned if the ranks of a pair occur in the inverse order.

We will give an example to clarify this procedure. Suppose there are two qualities A and B and four observations y_1, y_2, y_3, y_4 with the following rankings.

	y_1	y_2	y_3	y_4
A	1	2	3	4
B	1	3	4	2

We see that the pair (y_1, y_2) has the ranks $(1,2)$ in the A-ranking and the ranks $(1,3)$ in the B-ranking. In both rankings the ranks of the pair (y_1, y_2) occur in the same order with respect to the order of reference, so a score $S_{i,j}$ of $+1$ is assigned to this pair. Continuing in this manner for all pairs (y_i, y_j) with $i < j$ we obtain the following scores.

Pair	Score $S_{i,j}$
$y_1 y_2$	1
$y_1 y_3$	1
$y_1 y_4$	1
$y_2 y_3$	1
$y_2 y_4$	-1
$y_3 y_4$	-1

The score S of the rankings is obtained by adding $S_{i,j}$ for $i < j$. So we obtain $S = 4 - 2 = 2$. Notice that $-6 \leq S \leq 6$ and in this case $S > 0$, so there is a slightly positive relation between the two rankings. In other words, the two rankings seem to positively agree with each other.

More generally, for S it holds that $-S_{max}(n) \leq S \leq S_{max}(n)$, where

$$S_{max}(n) = 1 + 2 + ... + (n-2) + (n-1) = \frac{1}{2} n(n-1), \tag{3.3}$$

In particular, if $S = S_{max}(n)$, then there is a strongly increasing trend. If $S = -S_{max}(n)$, then there is a strongly decreasing trend.

Now that we are familiar with the concept of the score of rankings, this example can be formalised in the definition of the score of rankings and Kendall's tau.

Definition 3.2. *Let $x_1,...,x_n$ and $y_1,...,y_n$ be two sets of observations. Let $r_1,...,r_n$ and $s_1,...,s_n$ be the rankings of some qualities A and B respectively, where we order the A-ranking like $r_1 < ... < r_n$. For each pair (y_i, y_j) define the pairwise score*

$$S_{i,j} = \begin{cases} 1 & \text{if } s_i < s_j \\ -1 & \text{if } s_i > s_j \end{cases}. \tag{3.4}$$

The score of the rankings of A and B is then defined by

$$S = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} S_{i,j} = P - N, \tag{3.5}$$

where $P = |\{S_{i,j} > 0 | i < j\}|$ and $N = |\{S_{i,j} < 0 | i < j\}|$.

However, the definition of S, which is often used in literature and as described by Gilbert [5], is

$$S = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \operatorname{sgn}(y_j - y_i), \tag{3.6}$$

where $y_1,...,y_n$ are observations, ranked on the time when they were measured. So if we look at data from 1900 till 2000, the data will be ranked as $y_{1900}, y_{1901},..., y_{1999}, y_{2000}$. If we look at the sign of the differences, we are essentially just counting the number of positive differences and subtracting the number of negative differences, so we may obtain

$$S = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \operatorname{sgn}(y_j - y_i) = P - N = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} S_{i,j}. \tag{3.7}$$

Now that we have more of a feeling of how to look at the degree of agreement of rankings, we are closer to defining a rank correlation, i.e. Kendall's tau.

Definition 3.3 (Kendall's tau)**.** *Let us have the same setting as in Definition 3.2. Let S be the score of the rankings A and B. Let S_{max} be the maximum value that S could attain for two rankings consisting of n elements. Then Kendall's tau is defined by*

$$\tau = \frac{S}{S_{max}} = \frac{S}{\frac{1}{2}n(n-1)}. \tag{3.8}$$

Here we used the result (3.3), that $S_{max} = \frac{1}{2}n(n-1)$, which is easy to see. Indeed, S_{max} is the case where all the possible pairs (two elements with indices $i < j$) would have a pairwise score of 1. In particular, we are looking at how we can arrange two elements in a total of n elements, which gives us

$$\begin{aligned} S_{max} &= \binom{n}{2}, \\ &= \frac{n!}{2!(n-2)!}, \\ &= \frac{n(n-1)(n-2)...}{2(n-2)(n-3)(n-4)...}, \\ &= \frac{n(n-1)}{2}. \end{aligned}$$

3.3. Tied ranks

Up till now, we have only looked at rankings without observations that could have resulted in tied ranks, in which the scores are the same, that is $y_i = y_j$, for $i < j$. Of course this is not realistic, since it can happen that certain observations have the same actual value, in particular if measurements are not significant, i.e. have no or a few decimals. In this case a so-called fractional ranking will be applied, as is suggested by Kendall [9]. So the observations are ranked, by mapping the observations to the ranks they would have had if the observations were not equal and then we take the average of these ranks.

Definition 3.4. *Let $(r_1, ..., r_n)$ be a ranking of the observations $y_1, ..., y_n$, $n \in \mathbb{N}$. If $y_{i_1} = ... = y_{i_m}$ for some $i, m \in [1, ..., n]$, then*

$$r_{i_1} = ... = r_{i_m} = r'_i = \frac{r_{i_1} + ... + r_{i_m}}{m} \tag{3.9}$$

This procedure will be clarified with an example. Suppose there are 10 observations ranked on some quality B, where the third and fourth ranks are tied and the sixth, seventh and eighth ranks are tied as well, then the ranking below would follow from the definition.

Observation	y_1	y_3	y_2	y_4	y_8	y_5	y_6	y_7	y_9	y_{10}
B	1	2	3.5	3.5	5	7	7	7	9	10

3.3.1. Tied ranks in one ranking

It is possible that two rankings have tied ranks. In our research we will, however, mainly look at the ranking of some meteorological parameter and a ranking of time. The ties will only occur in the former ranking, because there is only one observation per unit of time (for example, one observation per year from 1901-2018). This reduces the complexity of calculating Kendall's tau in the tied case. However, in Section 5.7 we will compare the NAO-index with the other parameters where tied ranks can occur in both rankings. This situation will be explained in slightly more detail, however, first we have to tweak the way we defined the score of rankings to work with tied ranks in one ranking.

Definition 3.5. *Let $x_1, ..., x_n$ and $y_1, ..., y_n$ be observations of some quality A and B. Let $(r_1, ..., r_n)$ and $(s_1, ..., s_n)$ be their rankings, where we order the A-ranking as $r_1 < ... < r_n$ and ties only occur in the B-ranking. Then the pairwise score $S_{i,j}$ is given by*

$$S_{i,j} = \begin{cases} 1 & \text{if } s_i < s_j \\ 0 & \text{if } s_i = s_j \\ -1 & \text{if } s_i > s_j \end{cases}. \tag{3.10}$$

The score of the rankings is then defined by

$$S = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} S_{i,j} \tag{3.11}$$

Now that the ties-adjusted score S is defined, a definition for an adjusted Kendall's tau can be given.

Definition 3.6. *Let us have the same setting as in Definition 3.5. Let S be the score of the rankings A and B, then the adjusted Kendall's tau is defined as*

$$\tau' = \frac{S}{\sqrt{\frac{1}{2}n(n-1) - \frac{1}{2}\sum_{j=1}^{T} t_j(t_j-1)}\sqrt{\frac{1}{2}n(n-1)}}, \tag{3.12}$$

where T is the amount of tied groups and t_j is the number of observations in the j-th tied group.

If we take a look at the previous example we see that there are two tied groups, namely in the first group y_2, y_4 and in the second group y_5, y_6, y_7. Suppose we have another ranking A and we obtain the following.

Observation	y_1	y_3	y_2	y_5	y_8	y_4	y_6	y_7	y_{10}	y_9
A	1	2	3	4	5	6	7	8	9	10
B	1	2	3.5	7	5	3.5	7	7	10	9

This results in $t_1 = 2$ and $t_2 = 3$ and by Definition 3.5 we obtain $S = 33$. Using Definition 3.6 we get

$$\tau' = \frac{S}{\sqrt{\frac{1}{2}n(n-1) - \frac{1}{2}\sum_{j=1}^{T} t_j(t_j-1)}\sqrt{\frac{1}{2}n(n-1)}} = \frac{33}{\sqrt{\frac{10*9}{2} - \frac{1}{2}(2*1+3*2)}\sqrt{\frac{10*9}{2}}} \approx 0.768.$$

Compared by the τ that is obtained if the tied ranks were not taken into account, we would get

$$\tau = \frac{S}{S_{max}} = \frac{33}{45} \approx 0.733.$$

So we see that the adjusted τ' gives a slightly higher rank correlation than the normal τ would have given.

3.3.2. Tied ranks in two rankings

When there are tied ranks in both rankings, we have to adjust the calculation of τ for both rankings. We obtain the following.

Definition 3.7. *Let $x_1, ..., x_n$ and $y_1, ..., y_n$ be observations of some quality A and B. Let $(r_1, ..., r_n)$ and $(s_1, ..., s_n)$ be their ranking. Order the A-ranking as $r_1 < ... < r_n$ and assume that ties can occur in the A-ranking and B-ranking as well. Let S be the same as in Definition 3.5. Then the adjusted Kendall's tau is defined as*

$$\tau'' = \frac{S}{\sqrt{\frac{1}{2}n(n-1) - \frac{1}{2}\sum_{j=1}^{T} t_j(t_j-1)}\sqrt{\frac{1}{2}n(n-1) - \frac{1}{2}\sum_{i=i}^{U} u_i(u_i-1)}},$$

where T and U are the amount of tied groups in ranking A and B and t_j and u_i are the numbers of observations in the the tied groups.

3.4. Testing significance

Now that the statistics that will be used in our analysis are defined, we want to know whether the values we obtain are significant. In order to test the significance, the following null-hypothesis and alternative hypothesis will be tested against each other.

$$\begin{cases} H_0 : (X_i)_{i\geq 1} \text{come from a population with independent realisations and are i.i.d.} \\ H_1 : (X_i)_{i\geq 1} \text{follow a monotonic trend} \end{cases} \tag{3.13}$$

Or, if there is an assumption based on the data that the trend will be upward or downward:

$$\begin{cases} H_0 : (X_i)_{i\geq 1} \text{come from a population with independent realisations and are i.i.d.} \\ H_1 : (X_i)_{i\geq 1} \text{follow an upward/downward monotonic trend} \end{cases} \tag{3.14}$$

In the Mann-Kendall test we will asses whether the score S deviates significantly and together with the value of Kendall's tau we will try to give an answer to whether there is any trend in the data of Dutch winters. In order to do so, the behaviour of S has to be investigated by looking at the so-called *frequency-distribution* as is done by Kendall.

3.4.1. Frequency-distribution of the score

The goal in this section is to have a better look at how can be determined whether a value of S is deviating a lot or not. A logical way to have a clearer picture of how much a score is deviating, is by looking at all the possible values that S can attain relative to the order of reference $(1, 2, ..., n)$ and how often those values are attained. This is called the *frequency-distribution* of S, which we denote by $\mathscr{F}(S)$.

Definition 3.8 (Frequency-distribution)**.** *Let $(1, 2, ..., n)$ be the order of reference for some $n \in \mathbb{N}$. Then the maximum value that S can attain is $N = \sum_{j=1}^{n-1} j$. If N is odd then there are $N+1$ different values that S can obtain and if N is even there are N different values that S can obtain. Each value has its frequency of how often it will occur. We call this the frequency-distribution of S, denoted by $\mathscr{F}(S)$.*

We will give an example for $n = 3$. There are $3! = 6$ ways to arrange $1, 2, 3$, namely $1, 2, 3$; $1, 3, 2$; $2, 1, 3$; $2, 3, 1$; $3, 1, 2$ and $3, 2, 1$. From this we can derive the scores S per arrangement and we can deduce the following distribution.

Score (S)	Frequency of S (f)
3	1
1	2
-1	2
-3	1

For $n = 4$ and $n = 8$ we refer to Kendall [9] (p.49-50), where this has already been calculated. The result of the 4! possible arrangements for $n = 4$ that can be found there is the following.

Score (S)	Frequency of S (f)
6	1
4	3
2	5
0	6
-2	5
-4	3
-6	1

Remark. Notice that depending on whether $\frac{1}{2}n(n-1)$ is even or odd we have maximum values of f at $S = 0$ or at $S = \pm 1$ respectively. As n tends to infinity we will see that along with this symmetry, the frequency tends to the normal distribution.

For n small the calculations above can be done manually by arranging the $n!$ possibilities against the order of reference, but as n gets larger this becomes a very tedious task. That is why will use the following result for big n, i.e. $n \geq 40$ (Gilbert, [5]).

Theorem 3.9. *Let $\mathscr{F}(S)$ be the frequency-distribution of S. Then we have, as $n \to \infty$, that*

$$\mathscr{F}(S) \to N(0, \sigma^2), \tag{3.15}$$

where $N(0, \sigma^2)$ is the normal distribution with $\sigma^2 = \frac{1}{18}n(n-1)(2n+5)$.

Proof. The proof can be found in *Rank Correlation Methods* by Kendall [9] (p. 69-74) □

3.4.2. Testing the hypotheses

An important observation is that $\mathscr{F}(S)$ is a discrete distribution and $N(0, \sigma^2)$ is a continuous distribution. As we have seen earlier, the frequencies of S lie two units away from each other. Hence, Kendall and Gilbert suggest a corrected S, where it is assumed that the values of the frequency of the score f lie uniformly between $S-1$ and $S+1$.

Definition 3.10. *Let S be the score of some ranking where ties may only occur in one ranking. Then the corrected statistic Z is given by*

$$Z = \begin{cases} \frac{S-1}{\sqrt{\operatorname{Var} S}} & \text{if } S > 0 \\ 0 & \text{if } S = 0 \\ \frac{S+1}{\sqrt{\operatorname{Var} S}} & \text{if } S < 0 \end{cases}, \tag{3.16}$$

where $\operatorname{Var} S = \frac{1}{18}\left[n(n-1)(2n+5) - \sum_{j=1}^{T} t_j(t_j-1)(2t_j+5)\right]$.

If there are tied ranks in both rankings, the value of $\operatorname{Var} S$ changes in the more complex definition.

$$\begin{aligned} \operatorname{Var} S = {} & \frac{1}{18}\left[n(n-1)(2n+5) - \sum_{j=1}^{T} t_j(t_j-1)(2t_j+5) - \sum_{i=1}^{U} u_i(u_i-1)(2u_i+5)\right] \\ & + \frac{1}{9n(n-1)(n-2)}\left[\sum_{j=1}^{T} t_j(t_j-1)(t_j-2) \sum_{i=1}^{U} u_i(u_i-1)(u_i-2)\right] \\ & + \frac{1}{2n(n-1)}\left[\sum_{j=1}^{T} t_j(t_j-1) \sum_{i=1}^{U} u_i(u_i-1)\right]. \end{aligned} \tag{3.17}$$

In our analysis of the meteorological parameters the statistic Z will be the one that is used most often, because there are clearly more than 40 observations per parameter and sometimes there are observations with tied ranks. In order to carry out the test we have to compare Z with a corresponding value of the standard normal distribution, which is defined as follows.

Definition 3.11 (Normal distribution)**.** *Let X be a random variable, with $X \sim N(0,1)$. Then we define the cumulative distribution function (CDF), $\Phi : \mathbb{R} \to [0,1]$ as*

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{\frac{-t^2}{2}} \, dt$$

We will use $\Phi^{-1}(x) = Z_x$ as shorthand notation for further analysis.

If we want to test whether we have to reject H_0 in the setting of 3.13 at significance level α we can compare the value of Z with the value of $Z_{1-\alpha}$. In this case, we reject H_0 in favour of H_1 if $|Z| > Z_{1-\alpha}$. Alternatively, it is possible that we want to test whether we have to reject H_0 in the setting of 3.14 at significance level α. In this case, we reject H_0 in favour of H_1 if $|Z| > Z_{1-\frac{\alpha}{2}}$. If $Z < 0$ significantly, then it indicates a downward trend. However, if $Z > 0$ significantly, then it indicates an upward trend.

4

Sen's slope estimator

In Chapter 3 the aim was to find a robust way of testing whether there is a trend in data. However, if a trend is present we also need a method to find how strong that trend is, i.e. *the magnitude of the trend*. In several studies we found the frequent use of *Sen's slope estimator* (Sen, [10]) after having applied the Mann-Kendall test, in order to find the slope of the (monotonic) trend.

4.1. Method of obtaining Sen's slope

The method is advantageous for its use of the earlier computed score S from Chapter 3 and its way of calculation. It is calculated by looking at slopes between observations, ranking the slopes and taking the median of these values, so it is less affected by outlying data. Moreover, this method is non-parametric so it does not assume any distribution on the data, which is convenient since finding a distribution in climate data is difficult.

If we accept the alternative hypothesis H_1 as in 3.13 or 3.14, then we assume that the data follow some monotonic trend. So we assume that the data follow some regression in the form of

$$\hat{y}_i = \alpha + \beta t_i,$$

where we want to estimate the value of β, say $\hat{\beta}$. This is done by first calculating all the *pairwise slopes* of the observations. Moreover, since our data set only contains data with one observation per unit of time and every time step is one year, we can assume that $t_j - t_i = j - i$.

Definition 4.1. *Let $y_1, ..., y_n$ be observations, then we define the pairwise slope $Q_{i,j}$ of the pair (y_i, y_j) as*

$$Q_{i,j} = \frac{y_j - y_i}{j - i}, \quad \text{for } i < j \tag{4.1}$$

where $N = \binom{n}{2} = \frac{1}{2}n(n-1)$ is the total number of combinations.

Subsequently, when all the pairwise slopes $Q_{i,j}$ are calculated, they are ranked from smallest to largest slope. The median value of those pairwise slopes is called Sen's slope. This can be formalised in the following definition.

Definition 4.2 (Sen's slope estimator)**.** *Let $y_1, ..., y_n$ be observations and let $Q_{i,j}$ be the pairwise slope as defined in Definition 4.1. Rank all the pairwise slopes for $i < j$ as $Q_{(1)} \leq ... \leq Q_{(N)}$. Then Sen's slope is given by*

$$\hat{\beta} = \begin{cases} Q_{\left(\frac{N+1}{2}\right)} & \text{if } N \text{ is odd} \\ \frac{1}{2}\left(Q_{\left(\frac{N}{2}\right)} + Q_{\left(\frac{N+2}{2}\right)}\right) & \text{if } N \text{ is even} \end{cases}. \tag{4.2}$$

Now that Sen's slope estimator is defined, we will give an example of the method to find Sen's slope as described above. Suppose the following observations with corresponding values are given.

Observation	y_1	y_2	y_3	y_4	y_5	y_6
Value	13	15	18	16	19	20

We have to proceed by computing the pairwise slopes $Q_{i,j}$ for each pair (y_i, y_j) with $i < j$ as defined in Definition 4.1 and find the following slopes.

Pair	Slope $Q_{i,j}$	Pair	Slope $Q_{i,j}$	Pair	Slope $Q_{i,j}$
$y_1 y_2$	2.0	$y_2 y_3$	3.0	$y_3 y_5$	0.5
$y_1 y_3$	2.5	$y_2 y_4$	0.5	$y_3 y_6$	0.667
$y_1 y_4$	1.0	$y_2 y_5$	1.333	$y_4 y_5$	3.0
$y_1 y_5$	1.5	$y_2 y_6$	1.25	$y_4 y_6$	2.0
$y_1 y_6$	1.4	$y_3 y_4$	-2.0	$y_5 y_6$	1.0

Now that the pairwise slopes are computed, they have to be ranked from small to large. For $Q_{(1)}, ..., Q_{(15)}$ we find

$$-2.0, 0.5, 0.5, 0.667, 1.0, 1.0, 1.25, 1.333, 1.4, 1.5, 2.0, 2.0, 2.5, 3.0, 3.0$$

There are fifteen pairs in total, so N is odd and we conclude that

$$\hat{\beta} = Q_{\left(\frac{N+1}{2}\right)} = Q_{(8)} = 1.333.$$

4.2. Confidence interval

Consequently, when an estimate $\hat{\beta}$ for the slope is found, we want to construct a $100(1-\alpha)\%$ confidence interval. The usual approach of assuming the data to follow a normal distribution could be used by calculating

$$\left(\bar{y} - Z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}\,,\ \bar{y} + Z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}\right),$$

where $\bar{x}$ is the sample mean, σ is the standard deviation of the sample and Φ as in Definition 3.11. However, a disadvantage of this method is the assumption that the data comes from a normal distribution, which in practice is a rather strong assumption. Especially, if we want to analyse data which do not seem to follow a normal distribution, then this assumption is inappropriate.

Another method, that is non-parametric, is developed by Sen [10]. The confidence interval is found by ranking the observations and calculating two specific ranks that are the lower and upper bound of the confidence interval. The confidence interval is defined as follows.

Definition 4.3. *Let $y_1, ..., y_n$ be observations. Let α be the level of significance and let S be the score as in Definition 3.5. Define the constant C_α as*

$$C_\alpha = Z_{1-\frac{\alpha}{2}} \sqrt{\operatorname{Var} S}$$

Furthermore, let L denote the lower rank and let U denote the upper rank, where L and U are defined as

$$L = \frac{N - C_\alpha}{2}$$
$$U = \frac{N + C_\alpha}{2}$$

Then the $100(1-\alpha)\%$ confidence interval is given by the interval

$$\left(Q_{(L)}, Q_{(U+1)}\right) \tag{4.3}$$

Remark. It can happen that $L, U \notin \mathbb{N}$, such that there is not one observation that can serve as the lower or upper bound. In this case, a (linear) interpolation has to be applied as suggested by Gilbert. So for $Q_{(r)}$, with $r \in \mathbb{R}$ we have

$$Q_{(r)} = Q_{(r)} + (r - \lfloor r \rfloor)\left(Q_{(r+1)} - Q_{(r)}\right),$$

where $\lfloor \cdot \rfloor$ is the floor function given by $\lfloor x \rfloor = \max\{k \in \mathbb{Z} | k \leq x\}$

In the previous section we found in the example that $\hat{\beta} = 1.333$. Now that the method of calculating the confidence interval in a non-parametric way is known, we can compute a confidence interval with level of significance $\alpha = 0.05$. Since there are no tied ranks in our example, we find that

$$\begin{aligned} C_\alpha &= Z_{1-\frac{\alpha}{2}} \sqrt{\operatorname{Var} S}, \\ &= Z_{1-\frac{\alpha}{2}} \sqrt{\tfrac{1}{18} n(n-1)(2n+5)}, \quad (\textit{Definition 3.10}) \\ &= 1.96 \sqrt{\tfrac{1}{18} * 6 * 5 * 17}, \\ &\approx 10.433. \end{aligned}$$

We continue by calculating the lower and upper rank L and U respectively for the 95% confidence interval.

$$\begin{aligned} L &= \frac{N - C_\alpha}{2}, \\ &\approx 2.284, \\ U &= \frac{N + C_\alpha}{2}, \\ &\approx 12.716. \end{aligned}$$

This means we have to take a look at $Q_{(2.284)}$ and $Q_{(13.716)}$. As mentioned in the remark after Definition 4.3, we have to interpolate. This gives us the following lower and upper bound.

$$\begin{aligned} Q_{(2.284)} &= Q_{(2)} + 0.284 \left(Q_{(3)} - Q_{(2)}\right) \\ &= 0.5 \\ Q_{(13.716)} &= Q_{(13)} + 0.716 \left(Q_{(14)} - Q_{(13)}\right) \\ &= 2.858. \end{aligned}$$

5

Results of Mann-Kendall and Sen's slope

In this chapter the average, maximum and minimum winter temperatures from 1906-2018, T_{avg}, T_{max} and T_{min} resp. and the Hellmann numbers H from 1901-2019 are analysed. This is done for the various weather stations that are mentioned in the first chapter. More specifically, the two-sided Mann-Kendall test as described in Chapter 3 will be used. This way, we can find whether there is a trend in these parameters for the stations De Bilt, Eelde, De Kooy and Vlissingen. The test results have been obtained with R and have been summarised in tables.

As mentioned in Chapter 1, some of the data sets miss values around 1944 and 1945. Since one of the functions in R that is required to carry out the test does not work with incomplete data sets, we will replace these missing values by the previous year. This way the data will contain a tied ranking which will not contribute anything to the score of the ranking. Therefore the result will be affected as minimal as possible.

5.1. Average winter temperature

The Mann-Kendall test is applied on the aforementioned weather stations with a level of significance of $\alpha = 0.05$ and the following results are found for the values of the statistic Z, the score S, the p-value p and Kendall's tau τ.

Station	Z	S	τ	p
De Bilt	2.7419	1106	0.175	0.0061094
De Kooy	2.9999	1210	0.191	0.0027002
Eelde	2.8709	1158	0.183	0.0040933
Vlissingen	2.5955	1047	0.165	0.0094464

We can see that the statistic Z for all stations is higher than the value of $Z_{1-\frac{\alpha}{2}} = Z_{0.975}$ and the p-values for all stations are far below $\alpha = 0.05$, so we reject the null-hypothesis as defined in 3.13. Even if we choose a stricter level of significance, say $\alpha = 0.01$ we could reject H_0 and accept the alternative hypothesis that there is a monotonic trend in the data of the average winter temperatures.

What is remarkable though, is that the τ values are not that high, so there is a weak rank correlation between the average winter temperature and the years. Keeping this in mind we suggest that even though we can accept the alternative hypothesis that there is a trend, the trend is not really as strong as we had expected.

If we take a look at Sen's slope estimator and the corresponding 95% confidence interval of the weather stations, we see that the estimated slopes $\hat{\beta}$ lie between 0.011 and 0.015. This means that on average, the average winter temperature has increased with 0.01182-0.01436°C per year from 1906-2018. Although this does not seem as much, this means that over the whole period we see an average increase of 1.3238-1.6083°C.

Station	$Q_{(L)}$	$\hat{\beta}$	$Q_{(U+1)}$
De Bilt	0.003614	0.01319	0.02248
De Kooy	0.005103	0.01416	0.02320
Eelde	0.004711	0.01436	0.02373
Vlissingen	0.003071	0.01180	0.02072

5.2. Maximum winter temperature

Using the same setting as in the previous section, we obtain the results below after applying the two-sided Mann-Kendall test. We notice again that the values of the statistic Z are higher than the values $Z_{0.975}$ of the standard normal distribution and we see that the p-values are very small, so we may reject H_0 and accept the alternative hypothesis. In particular, the p-values of De Kooy and Vlissingen are even lower than $\alpha = 0.005$. We also notice that the τ values are between 0.15 and 0.20, so even though the test suggests that there is a trend, it probably is not a very strong one.

Station	Z	S	τ	p
De Bilt	2.3498	948	0.150	0.018783
De Kooy	3.1736	1280	0.202	0.0015055
Eelde	2.5632	1034	0.163	0.010371
Vlissingen	2.7692	1117	0.177	0.0056201

If we look at Sen's slope estimator we see that on average the maximum winter temperature has increased with 0.01158-0.01457°C per year from 1906-2018. However, the maximum winter temperatures have increased slightly less fast at De Bilt, Eelde and Vlissingen than the average winter temperature. Whereas De Kooy has seen a faster increase in the maximum winter temperature. Consequently, we can see that over the whole period the maximum winter temperature has on average increased with 1.2970-1.6318°C.

Station	$Q_{(L)}$	$\hat{\beta}$	$Q_{(U+1)}$
De Bilt	0.002007	0.01158	0.02106
De Kooy	0.006024	0.01439	0.02355
Eelde	0.003159	0.01288	0.02226
Vlissingen	0.003733	0.01285	0.02171

5.3. Minimum winter temperature

The test gives a similar result for the minimum winter temperature as we have seen for the average and maximum winter temperature. The test statistic and the p-values are both such, that we can reject the null-hypothesis and therefore accept H_1 that there is a trend. Furthermore, we can see that the values of τ give us a similar result as in the previous sections.

Station	Z	S	τ	p
De Bilt	2.7791	1121	0.177	0.0054513
De Kooy	2.8659	1156	0.183	0.0041578
Eelde	2.9776	1201	0.190	0.0029052
Vlissingen	2.4714	997	0.158	0.013458

When we look at Sen's slope estimator we see that at De Bilt and Eelde, the magnitude of the trend is larger than the trend of the average winter temperature. For now we notice that the average and minimum winter temperature at De Bilt, De Kooy and Eelde increase slightly faster than the maximum winter temperature, whereas at Vlissingen the increase is the slowest. Moreover, the average increase of the minimum winter temperature is 0.01116-0.01460°C per year. Hence, the average increase over the whole period is 1.2499-1.6352°C.

Station	$Q_{(L)}$	$\hat{\beta}$	$Q_{(U+1)}$
De Bilt	0.003601	0.01333	0.02274
De Kooy	0.004145	0.01332	0.02289
Eelde	0.004552	0.01460	0.02435
Vlissingen	0.002538	0.01113	0.02033

5.4. Coldest average period of 15 consecutive days

If we look at the coldest average period of 15 consecutive days per year, which is denoted by T, then no significant trend can be found since $Z < 1.96$. Additionally, the τ value is also very low, so the rank correlation between T and the passed time is also very low. This result is in contrast with previous results, where a significant trend was present. Since we argued in Chapter 1 that T might be one of the most important parameters, this result gives extra weight on our conclusion.

Since the test concluded that there is no significant trend, we cannot give a Sen's slope estimation.

	Z	S	τ	p
T	1.47	625	0.0922	0.1416

5.5. CNT

If we carry out the test on the CNT we see that, surprisingly, we accept H_0, so we cannot conclude that there is a trend in the CNT. We see that the score is positive and that $Z < Z_{0.975}$ just by a little, but it is not significant. Nevertheless, this contributes to our results in the sense that we concluded in the previous sections that there is some trend in the data, but due to the low rank correlations the trends are not very strong. Hence, this result might not be as surprising as we thought and relativises previous findings like in Section 5.4.

	Z	S	τ	p
CNT	1.8635	752	0.119	0.062396

Because there is no significant trend present in the observations of the CNT, we cannot compute Sen's slope estimator.

5.6. Hellmann number

When we apply the Mann-Kendall test we see that the tendency of a changing weather in the winter continues. We see that $|Z| > Z_{0.975}$, so we reject H_0 and accept H_1. Notice that S is a negative value, indicating that there might be a negative monotonic trend. This translates to the situation that the winters have turned milder, which confirms our preliminary findings in Chapter 2.

	Z	S	τ	p
H	-2.2236	-957	-0.139	0.026176

When we look at Sen's slope estimator we see that on average the Hellmann number has decreased by 0.2371 points per year. This would mean that over the whole period we see a decrease of 26.5552 points. This confirms our previous findings that winters tend to be less (very) cold and more (very) mild.

Station	$Q_{(L)}$	$\hat{\beta}$	$Q_{(U+1)}$
H	-0.4481	-0.2371	-0.03645

5.7. NAO-index

First we have applied the Mann-Kendall test on the NAO-index and then on the NAO-index and average, maximum, minimum temperature, T, CNT and the Hellmann number respectively. This way we can analyse whether there is a trend in the NAO-index, but also whether there is any significant trend correlation between the NAO-index and the other mentioned parameters.

We see that for all the tests, except for the case of the NAO-index versus T, we have that $|Z| < Z_{0.975}$, so we accept the null hypotheses in almost all of the cases. This means that according to the Mann-Kendall test there is no significant trend in the NAO-index and between these parameters and the NAO-index. The τ values confirm this as well, because in all cases τ is close to zero and the p-values are rather high too.

	Z	S	τ	p
NAO	1.5871	1007	0.0866	0.1125
NAO/T_{avg}	1.1265		0.0719	0.2599
NAO/T_{max}	1.0620		0.0678	0.2882
NAO/T_{min}	1.0844		0.0692	0.2782
NAO/CNT	0.9578		0.0612	0.3382
NAO/T	3.0955		0.194	0.001965
NAO/H	-1.6328		-0.102	0.1025

Most interestingly however, is that there is a significant relation between the NAO-index and T, while there is none between the other parameters. Even with a level of significance of $\alpha = 0.005$, this trend correlation is still significant. In Chapter 1 we argued that T is an important indicator of how cold a winter is experienced. This means that if the NAO-index reaches NAO+ values more often, we might experience less cold spells in the future.

From these results we can conclude that the connection between the NAO-index and the other parameters is less strong than we might have expected, but still present with T. However, one should keep in mind that the NAO is a phenomenon stretching the whole northern part of the Atlantic Ocean and that it is a very complex climatological system. This makes it very hard to draw any strong conclusions and even if there were a trend, we should be careful. In addition, the NAO and winter weather have been linked in several studies, so our results give a subtle nuance to our earlier findings.

Since the result of the Mann-Kendall test of the NAO-index did not conclude a significant trend, we cannot use Sen's method to find the magnitude of a trend.

6

Likelihood of an Eleven cities tour

The temperature change of Dutch winters has been analysed in the previous chapters and the presence of a significant trend has been tested together with the magnitude of the trend. In this chapter the behaviour of the winters in a cultural sense is explored more by analysing the probability that an Eleven cities tour will happen. We will look at the *coldest average period of 15 consecutive days per year*, which is again denoted by T.

6.1. Organising the tour

The Eleven cities tour is an ice-skating event that takes place in Friesland, one of the northern provinces of The Netherlands. The path of the tour follows various rivers and ditches and visits the eleven cities that historically had city rights. The first organised tour was held in 1909 and the most recent one in 1997. Since then it has been 22 years that there has not been any tour organised and after our research on the winter temperatures it seems like in the future it will be more unlikely that the Dutch winters will offer the needed conditions for a tour. Particularly, we want to quantify this *unlikeliness*, i.e. have some probability that a tour can be held, and compare how this has changed over time. This will be done in Section 6.2.

In order to organise an Eleven cities tour it has to freeze in such extent, that the ice on the entire path is at least 15 cm thick. Such a thickness is usually the result of a cold spell. The parameter T is intuitively a good indicator whether a tour can be organised, because it contains information on a longer period of time instead of one day. This is also suggested by Visser and Petersen [12], who have found that T has the highest correlation (0.86) with the *maximum ice thickness*. However, since T has been measured at De Bilt, the data is unfortunately less representative for Friesland, but it still gives an indication of what can be expected.

6.2. Likelihood estimation

Inspired by the method of Vermolen [11] we will compute the maximum likelihood estimation that an Eleven cities tour will take place. However, instead of using the Hellmann number, the *coldest average period of 15 consecutive days* T will be used. Let E be the event that a marathon takes places, then by the law of total probability we obtain

$$\mathbb{P}(E) = \int_{-\infty}^{\infty} \mathbb{P}(E|T) f(T) dT, \tag{6.1}$$

where $f(T)$ is the probability density of T and $\mathbb{P}(E|T)$ is the probability of a tour taking place if the coldest average period of 15 consecutive days is equal to T. However, the data of T takes values that are not integers, which makes it difficult to compute a likelihood. To tackle this problem the values of T will be rounded to the nearest integers to make the data discrete. Despite the fact that this makes the final result somewhat less accurate, it gives an indication nonetheless. In the discrete case we obtain the following

$$\mathbb{P}(E) = \sum_{j=-\infty}^{\infty} \mathbb{P}(E|T=j) F(T=j) \Delta T. \tag{6.2}$$

Since the values have been rounded, we let $\Delta T = 1$. Moreover, E can be true for all $T \leq j$, so we have a Bernoulli process where we assume that

$$\hat{\mathbb{P}}(E|\hat{T} = j) = (1-p)^{j-1} p. \tag{6.3}$$

Here $\hat{T}$ is the rounded value of T and p is the probability that E is true for $\hat{T} = j$. If we combine this with the previous results, we find that

$$\hat{P} = \hat{\mathbb{P}}(E|\hat{T}) = \sum_{j=-\hat{T}}^{\hat{T}} \hat{\mathbb{P}}(E|\hat{T} = j) = \sum_{j=-\hat{T}}^{\hat{T}} (1-p)^{j-1} p = (1-(1-p)^{\hat{T}}). \tag{6.4}$$

Now we define $\mathscr{E}$ as the set of all E that are true and let $\mathscr{Y} = \mathscr{E} \cup \mathscr{E}^C$ denote the set of all years. Then we find the following expression of the maximum likelihood estimator.

$$\hat{P} = C \prod_{j\in\mathscr{E}} (1-p)^{\hat{T}_j - 1} p \prod_{j\notin\mathscr{E}} (1-p)^{\hat{T}_j}, \tag{6.5}$$

where $\hat{T}_j$ denotes the $\hat{T}$ of a year j. To compute the maximum likelihood estimation, we proceed with the so-called log-likelihood and obtain

$$\begin{aligned}
\log(\hat{P}) &= \log(C) + \sum_{j\in\mathscr{E}} \log(1-p)(\hat{T}_j - 1) + \log(p) + \sum_{j\notin\mathscr{E}} \hat{T}_j \log(1-p), && (6.6)\\
&= \log(C) + |\mathscr{E}| \log(p) + \log(1-p) \sum_{j\in\mathscr{E}} \hat{T}_j - 1 + \sum_{j\notin\mathscr{E}} \hat{T}_j \log(1-p), && (6.7)\\
&= \log(C) + |\mathscr{E}| \log(p) + \log(1-p) \Big(\sum_{j\in\mathscr{E}} \hat{T}_j - 1 + \sum_{j\notin\mathscr{E}} \hat{T}_j \Big). && (6.8)\\
& && (6.9)
\end{aligned}$$

In order to find the maximum value of the likelihood estimation, we differentiate $\log(\hat{P})$ with respect to p. Proceeding with the usual machinery, the following is obtained.

$$\begin{aligned}
\frac{d\log(\hat{P})}{dp} &= \frac{|\mathscr{E}|}{p} - \frac{\sum_{j\in\mathscr{E}} \hat{T}_j - 1 + \sum_{j\notin\mathscr{E}} \hat{T}_j}{1-p} = 0, && (6.10)\\
\frac{|\mathscr{E}|}{p} &= \frac{\sum_{j\in\mathscr{E}} \hat{T}_j - 1 + \sum_{j\notin\mathscr{E}} \hat{T}_j}{1-p}, && (6.11)\\
p &= \frac{|\mathscr{E}|}{\sum_{j\in\mathscr{E}} \hat{T}_j - 1 + \sum_{j\notin\mathscr{E}} \hat{T}_j + |\mathscr{E}|}, && (6.12)\\
p &= \frac{|\mathscr{E}|}{\sum_{j\in\mathscr{E}} \hat{T}_j - |\mathscr{E}| + \sum_{j\notin\mathscr{E}} \hat{T}_j + |\mathscr{E}|}, && (6.13)\\
p &= \frac{|\mathscr{E}|}{\sum_{j\in\mathscr{Y}} \hat{T}_j}. && (6.14)
\end{aligned}$$

6.3. Results

Now that the procedure of finding the maximum likelihood estimator is known, we can use data of T from 1901-2017. This results in the following estimation of the probability of an Eleven cities tour taking place.

$$\hat{p} = 0.0679.$$

This means that if weather conditions stay the same, we can expect a tour approximately once every 15 years. Furthermore, we have computed the maximum likelihood estimations for other periods of 30 years, except for 1901-1927 which spans 27 year. Some probabilities from literature ([11], [12]) are also added for comparison. Those results are put together in the following table.

	Probability	Once in every
1901-2017	0.0679	15 years
1987-2017	0.0385	26 years
1957-1987	0.0423	24 years
1927-1957	0.0941	11 years
1901-1927	0.0682	15 years
Vermolen	0.00175	571 years
Visser & Petersen	0.055	18 years

The maximum likelihood estimation of the most recent period is the smallest compared with the other periods. Compared with 1927-1957 the likelihood of 1987-2017 is even three times as small. However, the estimations of all time periods are still relatively large compared with the result obtained by Vermolen. Altogether, the likeliness of having an Eleven cities tour has become relatively small. The likeliness of organising a tour has changed from once every 15 years to once every 26 years. Together with the results of Chapter 5 it seems like the probability of having an Eleven cities tour is probably not going to increase in the following periods of time.

7 Conclusion

In this thesis we have tried to give an answer to whether winters in The Netherlands are becoming warmer. In order to have representative data, we have collected observations from the KNMI from various meteorological parameters at four weather stations spread across the country. We have analysed whether we see any significant trends in the data with methods that are suggested by different researchers in mathematics and geophysics.

We can conclude that the average, minimum and maximum winter temperatures have significantly increased over the last century at all four weather stations, but the CNT has not seen any significant change. So this suggests that we see a warming trend around the coast of The Netherlands, while the more inland regions see fewer changes in the winter temperatures. Furthermore, we see that three out of four of the Sen's slope estimates of the average and minimum temperature are above 0.013°C per year, whereas only one of the four estimates of the maximum temperature is above 0.013 per year. So the variability of the winter temperatures has slightly decreased, resulting in less colder extreme temperatures and more warmer extreme temperatures. This combination of less variability, which is concentrated in higher temperatures, might be one of the reasons why winters indeed *feel* warmer.

Another important parameter is the Hellmann number, which is frequently used in The Netherlands to classify winters. We have seen that the Hellmann number shows a significantly decreasing trend. Together with our findings of the minimum temperature, this result confirms the present situation that cold winters have evolved into mild winters as fewer and fewer extremely cold days occur.

As mentioned, the CNT and T do not show a significant trend, which gives more nuance to our results. The insignificant result of the trend of the CNT makes us hypothesise that changes in Dutch winters take place at a faster rate in coastal regions than inland regions. Moreover, T has been measured at De Bilt, which lies at the edge of the CNT area. So together with the results of the CNT, this strengthens the hypothesis of such a division even more. However, to make such strong statements one should investigate the observations in the inland regions more.

Furthermore, we hypothesised that the NAO-index might also play a significant role in the behaviour of winters in The Netherlands. Against our expectation, however, there did not seem to be any significant trend in the data of the NAO-index. Also, when we analysed the trend correlation between the NAO-index and the other parameters, there did not seem to be any significant relation for most of the parameters. Surprisingly though, there seems to be a significant relationship between the NAO-index and the coldest average period of 15 consecutive days. However, since there is no significant trend in both parameters, we cannot draw any conclusions other than that there is some significant connection between the two parameters.

Lastly, we stated that the change of Dutch winters does not only affect the country's ecosystems, but it also affects The Netherlands culturally. In chapter 6 we have found that the maximum likelihood estimation of having an Eleven cities tour is rather low and particularly when the most recent period of 30 years is compared with the others. However, our method used the parameter T, which did not show any significant trend, so we should be careful with this result. But despite this, together with the other findings, it still gives an idea of what the future may hold for such large scale winter events, which play a significant role in Dutch culture.

In short, our research gives a lot of results that indicate that there are indeed significant changes in Dutch winters, particularly in the coastal regions, while there are no significant changes in the inland regions. Furthermore, these changes might have a connection with the NAO. Altogether, the changes in Dutch winters will still have an impact on the nature as well as the culture of The Netherlands, but to know to what extent, more research has to be carried out.

8 Discussion

This thesis attempted to answer whether Dutch winters have become warmer. During the analysis of the different meteorological parameters, we have made assumptions on the data and we have made choices on subjects to analyse and subjects to leave out. In this chapter some of the limitations of this research will be discussed and we will give some suggestions for follow-up studies.

8.1. Data of the parameters

In our analysis we mainly looked at the weather stations in De Bilt, Eelde, De Kooy and Vlissingen. Although these stations are spread out across the country, De Kooy and Vlissingen are both located at the western coast of The Netherlands and De Bilt and Eelde only lie ca. 50 km and 30 km from the coast respectively. In order to give an overall view it would be better if some weather stations located in the inland regions like Limburg, Twente or North-Brabant had been taken into account. Particularly, since the CNT does not have a significant trend in the winter temperatures, other inland regions can give more insight on whether there is some division between coastal and inland regions in regard to the rate of warming. Nevertheless, our research gives some important clues on the situation of Dutch winters.

Moreover, the data used in our research only dates back to 1901. This is due to the fact that the KNMI started with measuring temperatures in 1901. However, these data are corrected over time and homogenised, so the data we used are reliable. There are some databases that contain data on Dutch temperatures dating back to the 18th century though, but these data are less reliable which make results less accurate.

Furthermore, in Chapter 1 we have mentioned that the temperature data miss values around 1944/45 which causes a problem with applying the Mann-Kendall test in R. In order to still use the data we described in Chapter 5 that we set the missing values equal to the preceding value. By doing so, the missing values do not contribute to the total score S. However, it still affects the variance of S, so for further research it is suggested that the missing values are not replaced.

8.2. Used parameters

We have mainly analysed the temperature of winters in The Netherlands, but there are more parameters that affect winters. It would also be interesting to analyse, for instance, precipitation or wind in order to find deeper connections between various parameters.

Interestingly, one could also analyse the difference between the maximum and minimum winter temperature in order to obtain results on the range of winter temperatures. Since the trends in these parameters are significant, we hypothesise that the range follows some decreasing trend making cold extreme temperatures less likely in the future.

Another parameter that is an interesting addition to our research is the NAO-index. Surprisingly, there does not seem to be any significant trend correlation between the NAO-index and other parameters, except for the coldest average period of 15 consecutive days. This connection would be a very interesting subject for further studies.

8.3. Used methods

The main reason for applying the Mann-Kendall test and Sen's slope estimator are that the properties of these methods suit our data well (non-parametric) and they are suggested in several studies. However, there are more ways of carrying out trend analysis, which we have not explored in further detail, so it can be worth to study more literature in order to find other methods.

8.4. Likelihood of an Eleven cities tour

In Chapter 6 we looked at the likelihood of an Eleven cities tour taking place. In order to quantify this likelihood we calculated the maximum likelihood estimator and compared estimations of different time periods. However, the quantity of tours being held $|\mathscr{E}|$ has a strong influence on the value of the estimation $\hat{p}$. Since safety regulations have changed over the years, it can be possible that the likeliness of a tour being held is also reduced due to such kind of regulations which results in a lower maximum likelihood estimation.

A

Data processing

A.1. Computing average temperatures

```
import pandas as pd
import matplotlib.pyplot as pl
import numpy as np
import math as m

#Import the dataset from a .xlsx file and store in a dataframe
data = pd.read_excel(r'C:\TG_stations.xlsx')
Mat = pd.DataFrame(data,
                   columns=["YEAR", "TGBILT", "TGKOOY", "TGEELD", "TGVLIS"])

#Define the lists where we store every weather station
bilt_lst = []
kooy_lst = []
eeld_lst = []
vlis_lst = []

glob_count = 0
tot_len = len(Mat.index)
while glob_count < tot_len:
    sombilt = 0
    somkooy = 0
    someeld = 0
    somvlis = 0
    count = 0
    while str(Mat.iat[glob_count + count, 0])[4:8] != "0301":
        count += 1
        sombilt += float(Mat.iat[glob_count + count, 1])
        somkooy += float(Mat.iat[glob_count + count, 2])
        someeld += float(Mat.iat[glob_count + count, 3])
        somvlis += float(Mat.iat[glob_count + count, 4])
    sombilt += float(Mat.iat[glob_count + count, 1])
    somkooy += float(Mat.iat[glob_count + count, 2])
    someeld += float(Mat.iat[glob_count + count, 3])
    somvlis += float(Mat.iat[glob_count + count, 4])
    count += 1
    avgbilt = float(sombilt)/(10*count)
    avgkooy = float(somkooy)/(10*count)
    avgeeld = float(someeld)/(10*count)
```

```
    avgvlis = float(somvlis)/(10*count)
    bilt_lst.append(avgbilt)
    kooy_lst.append(avgkooy)
    eeld_lst.append(avgeeld)
    vlis_lst.append(avgvlis)
    glob_count += count

#Make a corresponding list of years
year_lst = []
for y in range(1906,2019):
    year_lst.append(y)

#Converting back to dataframe
avtem = [('Year', year_lst), ('TGBILT', bilt_lst),
         ('TGKOOY', kooy_lst), ('TGEELD', eeld_lst), ('TGVLIS', vlis_lst)]
AvgTemp = pd.DataFrame.from_items(avtem)

#Export to a .xlsx file
AvgTemp.to_excel(r'C:\TG stations per jaar.xlsx', header=True)
```

A.2. Converting CNT data to time series

```
import pandas as pd
import numpy as np

data = pd.read_excel(r'C:\KNMI␣data␣van␣CNT.xlsx')
Mat_dum = pd.DataFrame(data,
                       columns=["YEAR", "Jan", "Feb", "Mar",
                                "Apr", "May", "Jun", "Jul",
                                "Aug", "Sep", "Oct", "Nov",
                                "Dec"])
zero_data = np.zeros(shape = (113, 13))
Mat = pd.DataFrame(zero_data,
                   columns=["YEAR", "Jan", "Feb", "Mar",
                            "Apr", "May", "Jun", "Jul",
                            "Aug", "Sep", "Oct", "Nov",
                            "Dec"])

#convert data to Centigrade
for i in range(0,113):
    for j in range(0,13):
        if j == 0:
            Mat.iat[i,j] = int(Mat_dum.iat[i,j])
        else:
            Mat.iat[i,j] = Mat_dum.iat[i,j]/100000

#make data suitable for Time Series
CNT = []
for k in range(0,113):
    for l in range(1,13):
        cnt_vec = Mat.iat[k, l]
        CNT.append(cnt_vec)
print(CNT)

#make dataframe of CNT
date_lst = []
for y in range(1906,2019):
    for m in range(1,13):
        date = str(y) + "-" + str(m)
        date_lst.append(date)
print(date_lst)

cntlist = [('Date', date_lst), ('CNT', CNT)]
CNT_Data = pd.DataFrame.from_items(cntlist)

#export to an excelfile
CNT_Data.to_excel(r'C:\cnt_timeseries.xlsx',header=True)
```

B
Applying tests

```
library(fpp2)
library(Kendall)
library(trend)
library(readxl)
library(pracma)

#Importing the datasets
nao <- read_excel("~/NAO_Hurrel.xlsx")
TG<- read_excel("~/TG_stations_per_jaar.xlsx")
TX <- read_excel("~/TX_stations_per_jaar.xlsx")
TN <- read_excel("~/TN_stations_per_jaar.xlsx")
H<- read_excel("~/koudegetallen_clo.xlsx")
cnt <- read_excel("~/cnt_timeseries_winter_per_year.xlsx")
T15 <- read_excel("~/T15_per_jaar.xlsx",
                  sheet = "Historie_van_Elfstedentocht",
                  col_types = c("text", "numeric", "numeric",
                                "skip", "skip", "skip", "skip",
                                "numeric"),
                  range = "A1:H118")

#putting the data in ts() (Time Series) format
#and saving as vector
yearnao<-ts(nao[[1]])
nao_val<-ts(nao[[2]])

yearhellm<-ts(H[[1]])
hellm<-ts(H[[2]])

yeart15<-ts(T15[[1]])
t15<-ts(T15[[2]])

yearcnt<-ts(cnt[[1]])
cnt_val<-ts(cnt[[2]])

yearTG<-ts(TG[[1]])
biltTG<-ts(TG[[2]])
kooyTG<-ts(TG[[3]])
eeldTG<-ts(TG[[4]])
vlisTG<-ts(TG[[5]])
```

```
kooyTG[39]<-kooyTG[38]
vlisTG[39]<-vlisTG[38]

###Carrying out the Mann-Kendall test and Sen's slope estimation
#TG can easily be changed to TX or TN
mk.test(biltTG)
MannKendall(biltTG)
MannKendall(biltTG)$S
sens.slope(biltTG, conf.level = 0.95)

mk.test(kooyTG)
MannKendall(kooyTG)
MannKendall(kooyTG)$S
sens.slope(kooyTG, conf.level = 0.95)

mk.test(eeldTG)
MannKendall(eeldTG)
MannKendall(eeldTG)$S
sens.slope(eeldTG, conf.level = 0.95)

mk.test(vlisTG)
MannKendall(vlisTG)
MannKendall(vlisTG)$S
sens.slope(vlisTG, conf.level = 0.95)

#Mann-Kendall test and Sen's slope estimation
#of T15, CNT, Hellman
mk.test(hellm)
MannKendall(hellm)
MannKendall(hellm)$S
sens.slope(hellm, conf.level = 0.95)

mk.test(t15)
MannKendall(t15)
MannKendall(t15)$S

mk.test(cnt_val)
MannKendall(cnt_val)
MannKendall(cnt_val)$S

#Mann-Kendall test of the NAO-index and NAO-index
#vs. other parameters
mk.test(nao_val)
MannKendall(nao_val)
MannKendall(nao_val)$S

cor.test(nao_val[41:153], cnt_val,
         method = "kendall",
         conf.level = 0.95,
         continuity = TRUE)

cor.test(nao_val[36:153], hellm,
         method = "kendall",
         conf.level = 0.95,
         continuity = TRUE)
```

```
cor.test(nao_val[37:153], t15,
         method = "kendall",
         conf.level = 0.95,
         continuity = TRUE)

cor.test(nao_val[41:153], biltTG,
         method = "kendall",
         conf.level = 0.95,
         continuity = TRUE)
```

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