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Comments on "Analytical Design Equations for Class-E Power Amplifier"

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***Abstract*— In their seminal work, Acar et al. (2007) proposed analytical design equations for Class E power amplifiers, which have significantly influenced subsequent research in this field. However, their analysis contains calculation errors in the evaluation of certain expressions, leading to inaccuracies in the derived design equations. This error results in significantly incorrect values for the design parameters, which, in turn, affect the accuracy of the overall design set. This work addresses these errors, providing a corrected set of design equations for Class E PAs, further supported by supplementary Python code, enabling researchers to readily explore and verify the corrected Class E design framework.**

***Index Terms*— Class E, analytical solution, integration.**

I. Introduction

In the above article [1], Acar et al. design equations of Class E power amplifiers (PAs) have been explored in depth by various researchers since their inception. A notable contribution in this field is the paper by Acar et al. [1] and Ph.D. thesis [2], which investigate the underlying principles of infinitely many solutions for Class E amplifiers. Based on analytical solutions, the paper presents a coherent, non-iterative procedure for selecting circuit parameters for Class E PAs with arbitrary duty cycles and finite DC-feed inductances.

The analysis provided in [1] unifies all known Class E PA design equations while also introducing new ones, offering greater flexibility to engineers by expanding the design space and providing more degrees of freedom for optimization. The analysis of Class E Amplifiers begins by selecting the free design parameters q and d, as defined in [1] and [2]. These parameters are subsequently used to determine p and φ, which, in turn, appear in the final calculation of the design set $\{K_X, K_C, K_L, K_P\}$.

However, some unfortunate evaluations of integrals and occasional typographical errors introduce confusion in interpreting the design equations.

This work addresses these issues by providing the correct expressions for the design parameters. Additionally, the paper includes Python code for obtaining numerical values for the design set, as well as for generating voltage and current waveforms for any given d and q, except for $q = 1$, which is a special case explained in detail in [3].

II. Corrections

A. Corrected Expression for p

The expression for p is defined in terms of $a_1, a_2, b_1, b_2, c_1, c_2$, which are themselves functions of the given design parameters q and d. In both references [1] and [2], the original expression for p contains an erroneous numerator and omits the absolute value in the denominator. The corrected analytical expression for p should be given by Eq. (1). The absolute value is taken in the denominator to ensure that p has only positive values.

$$p = \frac{\sqrt{(a_1^2 + b_1^2)c_2^2 + (a_2^2 + b_2^2)c_1^2 - 2(a_1 a_2 + b_1 b_2)c_1 c_2}}{|a_1 b_2 - a_2 b_1|} \tag{1}$$

B. Corrected Expression for K_X

$K_X(q, d)$ is defined as the ratio of V_X and V_R, which are defined in [1] and [2]. However, the final analytical expression for K_X has some unfortunate errors.

The corrected analytical value can be determined using the constants C_1 and C_2 as defined in [1] and [2], along with an additional set of parameters T_n (for $n = 1$ to 6) introduced in this work. The expressions for the parameters T_n (for $n = 1$ to 6) are defined by Eqs. (2) to (4) and (6) to (8).

Using these constants and parameters, the values of V_X and V_R can be given by Eqs. (5) and (9), respectively. Finally, the corrected analytical expression for K_X is given by Eq. (10).

$$T_1 = \frac{C_1}{2}\left\{\frac{\sin(2\pi q+\varphi)-\sin((q+1)d\pi+\varphi)}{q+1} + \frac{\sin(2\pi q-\varphi)-\sin((q-1)d\pi-\varphi)}{q-1}\right\} \tag{2}$$

$$T_2 = \frac{C_2}{2}\left\{\frac{-\cos(2\pi q+\varphi)+\cos((q+1)d\pi+\varphi)}{q+1} + \frac{-\cos(2\pi q-\varphi)+\cos((q-1)d\pi-\varphi)}{q-1}\right\} \tag{3}$$

$$T_3 = V_{\mathrm{DD}}\left\{(\sin(\varphi) - \sin(d\pi + \varphi)) - \frac{q^2}{1-q^2} p\pi\left(1 - \frac{d}{2}\right) - \frac{q^2}{1-q^2} p\,\frac{\sin(2\varphi)-\sin(2(d\pi+\varphi))}{4}\right\} \tag{4}$$

$$V_X = \sum_{n=1}^{3} T_n \tag{5}$$

$$T_4 = \frac{C_1}{2}\left\{\frac{-\cos(2\pi q+\varphi)+\cos((q+1)d\pi+\varphi)}{q+1} + \frac{-\cos(2\pi q-\varphi)+\cos((q-1)d\pi-\varphi)}{1-q}\right\} \tag{6}$$

$$T_5 = \frac{C_2}{2}\left\{\frac{\sin(2\pi q-\varphi)-\sin((q-1)d\pi-\varphi)}{q-1} + \frac{-\sin(2\pi q+\varphi)+\sin((q+1)d\pi+\varphi)}{1+q}\right\} \tag{7}$$

$$T_6 = V_{DD}\left\{\frac{q^2}{1-q^2} p\,\frac{\cos(2\varphi)-\cos(2d\pi+2\varphi)}{4} + \cos(d\pi + \varphi) - \cos(\varphi)\right\} \tag{8}$$

$$V_R = \sum_{n=4}^{6} T_n \tag{9}$$

$$K_X = \frac{V_X}{V_R} \tag{10}$$

C. Corrected Expression for K_L

Due to an unfortunate evaluation of an integral, the expression for $K_L(q, d)$ reported in [1] is incorrect. The correct expression of

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TABLE I
COMPARISON OF CORRECTED EQUATIONS WITH CITED WORK

Parameter	$q = 0.5$			$q = 1.412$			$q = 2$		
	This work	**[1] & [2]**		**This work**	**[1] & [2]**		**This work**	**[1] & [2]**	
p	21.3126	21.1537		1.2106	1.1015		1.1781	1.1781	
φ	-30.1742°			15.1238°			90°		
K_P	0.6347	2.2116	0.6353	1.3632	39.2285	1.5838	0.0556	29.0057	0.0556
K_L	18.9161	10.0582	18.7658	0.7332	0.1244	0.6189	3.5343	0.1547	3.5343
K_C	0.2115	0.3977	0.2132	0.6841	4.0334	0.8104	0.0707	1.6163	0.0707
K_X	1.0580	-0.9655		-0.0002	0.1930		-4.9027	7.4492	

$K_L(q, d)$ is repeated from [2] and is given by Eq. (11). Furthermore, since the numerical values of $K_C(q, d)$ and $K_P(q, d)$ depend on $K_L(q, d)$ this correction ensures the accuracy of the other two design parameters.

$$K_L(q, d) = \frac{p}{\frac{d^2\pi}{2p} - \frac{1}{\pi}(\cos(d\pi + \varphi) - \cos(\varphi)) - d\sin(\varphi)} \tag{11}$$

III. NUMERICAL EXAMPLE

Table I compares the numerical values for $q = 0.5$, 1.412 and 2 with $d = 1$. A Python routine is also provided to assist researchers [4], using the corrected analytical expressions given in this paper. The routine takes the design parameters q and d as input and generates the complete design set $\{K_X, K_C, K_L, K_P\}$, as well as the voltage and current waveforms.

IV. CONCLUSION

This work provides a corrected set of design equations for Class E PAs and addresses the previously identified errors. To facilitate further research and practical application, we have included a Python program as supplementary material. These resources allow researchers to explore, verify, and apply the updated Class E design framework, thereby advancing both its theoretical understanding and practical uses.

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REFERENCES

[1] M. Acar, A. J. Annema, and B. Nauta, “Analytical design equations for class-E power amplifiers,” *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 54, no. 12, pp. 2706–2717, Dec. 2007.

[2] M. Acar, “Power amplifiers in CMOS technology: A contribution to power amplifier theory and techniques,” Ph.D. thesis, Fac. Elect. Eng., Math. Comput. Sci., Res. UT, Graduation UT, Univ. Twente, Enschede, The Netherlands, Feb. 2011, doi: 10.3990/1.9789036531382.

[3] A. Grebennikov and M. J. Franco, *Switchmode RF and Microwave Power Amplifiers*. New York, NY, USA: Academic, 2012.

[4] A. Sharma, S. Stefanescu, and P. K. Gogoi, “Revised Python code for comments on ‘analytical design equations for class-E power amplifier,”’ Tech. Rep., Dec. 2024, doi: 10.5281/zenodo.14254000.