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Analytical Approach for Maximum Power Point Calculation of Photovoltaic Modules: based on the Fixed Point Method

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***Abstract*—This paper introduces a novel detailed formulation for analytically determining the maximum power point (MPP) of a photovoltaic (PV) module. The approach involves an analytical approximation of a 4-parameter PV model, taking into account the series resistance, which serves as the initial value for the fixed-point method to enhance accuracy. Additionally, the paper includes a statistical analysis of convergence to highlight the method's limitations. The performance of the proposed method is evaluated against actual measurements of over one hundred thousand current-voltage (I-V) characteristic curves from eight different PV modules, as reported by the National Renewable Energy Laboratory (NREL), and is also compared with two existing literature approximations. Furthermore, a sensitivity analysis of the solution error under varying irradiance and temperature conditions is conducted. The method demonstrates high accuracy for engineering applications.**

***Index Terms*—Analytical solution, fixed-point method, maximum power point (MPP), photovoltaic module.**

I. Introduction

The growing number of green energy systems, particularly solar power equipment, linked to the electrical grid has altered the system's dynamics. Therefore, precise models are essential for accurately representing the performance of both existing and emerging technologies. Photovoltaic (PV) modules are extensively used among these systems because of their low operational and maintenance costs [1]. However, there is significant potential to enhance the efficiency of PV modules, opening up exciting possibilities for the future. These modules' output, current, and voltage are inherently nonlinear and fluctuate with changes in solar irradiance and temperature [2]. To achieve optimal efficiency, PV modules must operate at the voltage (V_{mpp}) and current (I_{mpp}) point where the maximum power output is achieved, known as the maximum power point (MPP). Directly obtaining this point from the module is challenging, mainly due to the nonlinearities present in the model. As a result, the precise results derived from an analytical formula to calculate the MPP in a simulation are highly beneficial for power sizing and quick comparison of different modules.

The problem was first tackled in [3], where an explicit expression describing the relationship between voltage and current for the 4-parameter single diode model (SDM) was provided. From this expression, formulas to calculate V_{mpp} and I_{mpp} were derived. Subsequently, in [4], expressions for MPP voltage and current were obtained through analytical methods based on the Mean Value Theorem applied to the implicit current equation. In [5], a formula for calculating V_{mpp} for an ideal cell was presented, utilizing the Lambert-W function and its characteristic parameters. In [6], an implicit formula for computing V_{mpp} was developed by aligning two I_{mpp} expressions. The first expression was a quadratic equation with respect to V_{mpp} derived implicitly from the current equation. At the same time, the second was an explicit current-voltage equation using the Lambert-W function, as introduced in [7]. The arguments of the Lambert-W function in the PV module's current equation were found to take small values, prompting an approximation for calculating V_{mpp} and I_{mpp} by substituting the Lambert-W function terms with their arguments in the explicit current drift in [8]. This resulting equation was then analytically solved using the Lambert-W function. Additionally, a formula for determining the series resistance was derived, and a sensitivity analysis of V_{mpp} concerning certain characteristic parameters of the module was performed in [8].

This paper introduces an analytical approximation method for calculating the maximum power point (MPP) in PV modules, considering the impact of series resistance. This approximation serves as the Fixed Point Method's initial value, enhancing the results' accuracy. The approach begins with the current equation as a function of voltage and the four parameters of the module and then derives a dimensionless version of this equation. This dimensionless form depends solely on two parameters, with its solution constrained between zero and one. A statistical analysis of the convergence is conducted to support the proposed method's evaluation. The solutions are rigorously validated using an extensive dataset of over one hundred thousand I-V curve measurements provided by the National Renewable Energy Laboratory (NREL), covering eight different PV module technologies. The use of this extensive dataset reassures the robustness of the method. A comprehensive sensitivity analysis of the error was also performed under varying irradiance and temperature conditions. The presented approximation is also compared with two models from previous literature: the ideal and 5-parameter SDM, across different irradiance and temperature scenarios.

The rest of the paper is structured as follows: Section II describes the used PV model. Section III, develops the proposed approximations for the calculation I_{mpp} and V_{mpp}. Section IV, validates the presented under extensive simulations. Finally, the conclusion is shown in section V.

II. Single Diode PV Module Model

In a single diode model of PV module with series resistance (R_s), its current is given by:

$$I = I_{\text{L}} - I_0 \left(\exp\left(\frac{V + IR_s}{AN_{\text{s}}V_{\text{th}}} \right) - 1 \right), \tag{1}$$

where:

$$V_{\text{th}} = \frac{kT}{q}, \tag{2}$$

is called thermovoltage, T is the cell temperature in K, N_{s} is the number of cells connected in series, k is the Boltzmann constant (1.381×10^{-23} J/K), q is the electron charge (1.602×10^{-19} C), A is the diode ideality factor, I_0 is the diode saturation current, I_{L} is the photocurrent and V is the shunt voltage at the terminals. In virtue of I_0 being transcendentally small compared to the rest of the parameters, (1) can be simplified to:

$$I = I_{\text{L}} - I_0 \exp\left(\frac{V + IR_s}{AN_{\text{s}}V_{\text{th}}} \right), \tag{3}$$

solving for V:

$$V = \ln\left(\left(\frac{I_{\text{L}} - I}{I_0} \right)^{AN_{\text{s}}V_{\text{th}}} \right) - IR_s. \tag{4}$$

The output power of the module is given by:

$$P = I \ln\left(\left(\frac{I_{\text{L}} - I}{I_0} \right)^{AN_{\text{s}}V_{\text{th}}} \right) - I^2 R_s. \tag{5}$$

Regarding the power-voltage P-V characteristics, the slope at the MPP is zero. Hence, the next expression must be satisfied at MPP:

$$\left(V + I\frac{dV}{dI} \right) \Big|_{I = I_{\text{mpp}}} = 0. \tag{6}$$

Therefore the derivative of the voltage w.r.t. the current is given by:

$$\frac{dV}{dI} = -\frac{AN_{\text{s}}V_{\text{th}}}{I_{\text{L}} - I} - R_s, \tag{7}$$

substituting (4) and (7) in (6), combining and simplifying alike terms yields a transcendental equation for the I_{mpp} as:

$$\ln(I_{\text{L}} - I_{\text{mpp}}) - \ln(I_0) - I_{\text{mpp}} \left(\frac{2R_s}{AN_{\text{s}}V_{\text{th}}} + \frac{1}{I_{\text{L}} - I_{\text{mpp}}} \right) = 0, \tag{8}$$

the resulting expression may be represented with the next variables:

$$a \equiv \left(\frac{2R_s I_{\text{L}}}{AN_{\text{s}}V_{\text{th}}} \right), \tag{9}$$

$$\gamma \equiv 1 - \ln\left(\frac{I_0}{I_{\text{L}}} \right), \tag{10}$$

$$u \equiv 1 - \frac{I_{\text{mpp}}}{I_{\text{L}}}, \tag{11}$$

therefore, the problem becomes dimensionless and equation (8) can be rewritten as:

$$\gamma = \frac{1}{u} - \ln(u) + a(1 - u). \tag{12}$$

In the MPP the next inequality is satisfied:

$$I_{\text{mpp}} \geq 0, \tag{13}$$

substituting (11) in (13) and simplifying it is found that

$$u \leq 1. \tag{14}$$

To guarantee that the solution of (12) resides in the real plane, u must be bounded by $0 < u \leq 1$. The value of I_{mpp} that solves (8) would be in the range of $0 < I_{\text{mpp}} < I_{\text{sc}}$, where I_{sc} is the short circuit current. This value depends on the PV module technology and weather conditions. The analysis in this section shows that the value of u that solves (12) has a well-defined boundary for constant values. In the parameter case $a \geq 0$ since I_{L}, I_0, R_s, V_{th}, $A > 0$, and $N_{\text{s}} \geq 1$ for any standard PV module. Also, based on the fact that $I_0 \ll I_{\text{L}}$, using (10) satisfied that $\gamma > 1$.

III. Maximum Power Point Approximation

The equation (12) is transcendental and cannot be solved for u. Nevertheless, analyzing the particular case where $\gamma \to \infty$ in (12), for the equality to be conserved, the right-hand side has to tend to infinity too and this occurs only when $u \to 0$. Therefore, the significative terms, the ones that contribute most to the solution, are the first two on the right-hand side. In this special case it is possible to neglect the au term causing a transcendentally small error, hence (12) has an exact solution given by:

$$u_0 \sim \frac{1}{W_0(\exp(\gamma - a))}, \gamma \to \infty \text{ and } u \to 0, \tag{15}$$

where $W_0(x)$ is the principal branch of Lambert-W function. Thus, the MPP is calculated by substituting (15) into (11) for I_{mpp} and in turn into (4) for V_{mpp}.

The MPP approximation can be improved by using fixed-point method. To this end, (12) can be represented in terms of the Lambert-W function as:

$$u = \frac{1}{W_0\left(\exp\left(a(u-1)+\gamma\right)\right)}. \tag{16}$$

The form of (16) makes it straightforward to apply the fixed-point method as follows:

$$u_n = \frac{1}{W_0\left(\exp(a(u_{n-1}-1)+\gamma)\right)}, \text{ for: } n = 1, 2, ... \tag{17}$$

The argument of the Lambert-W function of (17) is denoted by $\psi(u) = \exp\left(a(u-1)+\gamma\right)$. It is well known that the function $W_0(x)$ increase monotonically when $x \geq -\exp\left(-1\right)$ [10]. Moreover $\psi(u)$ is monotonically increasing w.r.t u, therefore $f(u) = W_0^{-1}(\psi(u))$ decreases monotonically to zero when $\psi(u) \to \infty$ and the greater value that can be taken is given by $f(0) = \frac{1}{W_0(\exp(\gamma-a))}$, which coincides exactly with (15). It can be concluded that $f(\psi(u))$ is bounded by $[0, u_0]$. In order to reach convergence in the iterative scheme (17), the following condition must be satisfied [11]:

$$|f'(u)| \leq k < 1, \forall u \in [A, B]. \tag{18}$$

In this case: $A = 0$, $B = 1$ and $|f'(u)|$ is:

$$|f'(u)| = \frac{a}{W_0\left(\exp\left(a(u-1)+\gamma\right)\right)} \cdot \frac{1}{\left(W_0\left(\exp\left(a(u-1)+\gamma\right)\right)+1\right)} \tag{19}$$

The expression (19) monotonically decrease; therefore, the maximum value can be taken when $u \to 0$, satisfying the following inequality:

$$|f'(u)| \leq \frac{a}{W_0\left(\exp\left(\gamma-a\right)\right)} \cdot \frac{1}{\left(W_0\left(\exp\left(\gamma-a\right)\right)+1\right)} < 1. \tag{20}$$

To guarantee the convergence of (17), the inequality (20) must be satisfied. It is shown in experimental tests of Section IV that this condition is indeed fulfilled for a large number of cases.

IV. Approach Validation

In order to validate the methodology, real measurements of the MPP performed by NREL and reported in [12] are used to be compared against those obtained by the proposed methods and two well-established methods in the literature.

A. *Comparison with experimental data*

Analytical methods are simple to implement and have a low computational burden, however, they have the disadvantage of presenting irregularities when negative or complex values are extracted from the model parameters [14]. In the present work, the cases with irregularities are excluded to maintain the statements ($R_s > 0$) made in section II.

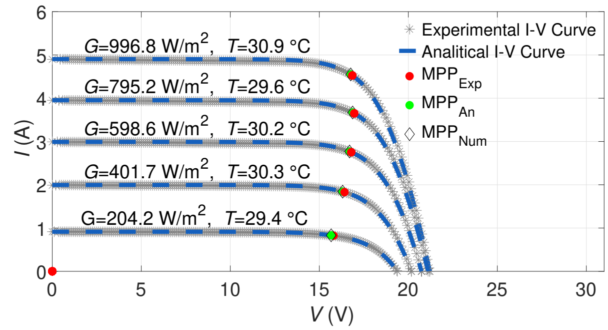

Fig. 1. The $I-V$ curve of the mSi0166 module for different irradiance levels.

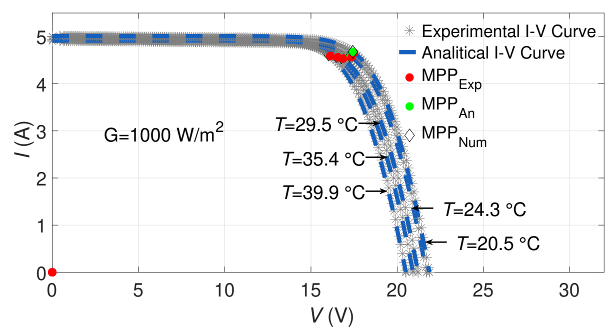

Fig. 2. The $I-V$ curve of the mSi0166 module for different temperature levels.

A total of 161,448 $I-V$ curve measurements were analyzed from eight modules with different technologies. The physical devices are located in Eugene City, Oregon, E.U., and the data were collected over about one year during the daytime. The comparison process was thorough, considering (17) as the sole first term of the sequence and (15) as the initial condition. The analytical method presented in [13] is employed to estimate the SDM parameters, and the $|u_n - u_{n-1}| < 1 \times 10^{-12}$ criteria is rigorously applied for the numerical solution. Fig. 1 and Fig. 2 present the experimental and analytical I-V curves of the mSi0166 module for different irradiances and temperature levels, respectively. The Figures also show the approximation of the MPP calculated by the experimental (MPP_{Exp}), the analytical proposed method (MPP_{An}) and the numerical (MPP_{Num}) method. The overlapping of the curves further validates the proposed approach.

From all measurements, Table I shows a statistical summary of the calculated values for (20). To ensure the convergence of the method, (20) must be less than 1. In the first four columns, the restriction is not reached when the HIT05667 and the aSiTriple28324 are evaluated. However, in the first case, according to the empirical rule reported in [15], the probability of a value greater than 1 is less than 0.00458%, and in the second case it is less than 0.05633%. This indicates that there is a small probability that the iteration scheme diverges. The next four columns of Table I show the main statistical data for the number of iterations used to calculate the MPP for each module. It shows that the maximum, mean, and dispersion of a number of iterations (bold text) occurs in the cases where (20) is greater than or near one, as in the case of modules CdTe75638 and aSiTriple28324. The foregoing indicates that the methodology presented is not suitable for these types of technology. Nevertheless, this does not apply to the HIT05667 module since it does not satisfy the $|f'(u)| < 1$ constraint but has the lowest mean and dispersion in the number of iterations. This may be caused by a measurement error and does not reflect the behaviour of the majority of the data since the greatest accuracy of the present methodology is in this technology. Fig. 3 shows a box plot illustrating the dispersion of the logarithm of (20) for each of the technologies.

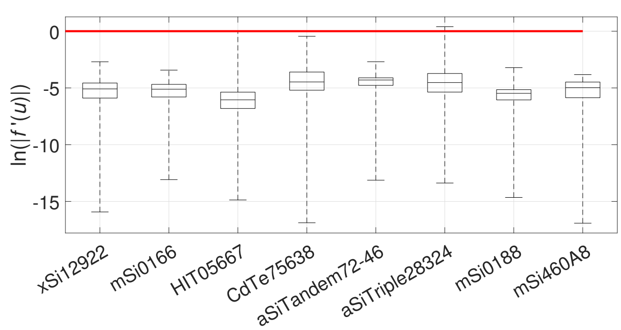

Fig. 3. Box plot of a left-hand side of (20).

Table II shows the Mean Absolute Percentage Error (MAPE) for different technologies. The second and third columns show the MAPE between the analytical and numerical solutions concerning the respective experimental measurements. The fourth column shows the MAPE of the analytical solution with respect to the numerical solution. It can be observed that the best performance of the 4-parameter SDM with respect to the experimental measurements is obtained with the HIT05667 module, while the worst is obtained with the ASiTandem72-46. On the other hand, the comparison of the analytical solution accuracy with respect to the numerical one highlights that in all cases, an error of less than 1.4×10^{-3} is obtained. These margins of error are sufficient for several practical applications.

B. Comparison with previous methodologies.

In order to validate the methodology under different environmental conditions, we have calculated the absolute percentage of error (APE) between experimental MPP measurements of aSiTandem72-46 and HIT05667 modules and two methodologies from previous literature. These APE values are shown in Table III, are crucial in determining the accuracy and reliability of the methodologies under varying environmental conditions. The first is the exact solution for the ideal case of the SDM (labelled $|\mathrm{E}_{\mathrm{X(3P)}}|$), initially presented in [5], whose parameters were calculated using [16]. The second one is an approximation for the 5-parameter SDM model (labelled $|\mathrm{E}_{\mathrm{X(5P)}}|$) using the mean value theorem recently published in [17], whose parameters were calculated with the methodology of [18]. Finally, the presented in this work is labelled as $|\mathrm{E}_{\mathrm{X(4P)}}|$. The first two columns show the different irradiance and temperature conditions covering a wide range of weather conditions. The last row shows the MAPE for each methodology. In general, it is observed that the 5-parameter model has the highest performance for both modules, which is to be expected since it represents more physical aspects of the module. On the other hand, the methodology presented in this work is superior in accuracy to the ideal model for the case of the HIT05667 module. Nevertheless, in the case of the MAPE of the aSiTandem72-46 module, the ideal model is more accurate, proving that the model presented is not suitable for calculating the MPP for this type of technology. It is also observed that for the case of the HIT05667 module considering levels below 200W/m^2 irradiance, as the temperature rises, the accuracy improves, and for levels above this irradiance value, as the temperature increases, the accuracy decreases.

V. Conclusion

This paper introduces a new analytical approximation for solving the MPP calculation problem by making the problem dimensionless and assuming that the photocurrent (I_{L}) is much greater than the diode saturation current (I_0). t is also established that this approximation represents the upper limit of the solution, providing a boundary for its search. This approximation was then used as the initial value for the iterative fixed-point method. A statistical analysis of the convergence of this iteration method shows that in most cases, the condition $|f'(u)| < 1$ is satisfied, ensuring convergence. However, convergence issues can arise when this condition is not met, sometimes requiring up to 30 iterations. The approximations obtained for the MPP calculation were compared to over one hundred thousand real measurements from eight different PV module technologies. The results indicated a MAPE ranging from 4.1% and 0.37%, which is useful for engineering applications. The comparison with the ideal and 5-parameter model approximations highlighted some limitations of the presented model. Future work aims to increase the number of curves analyzed to encompass a broader range of technologies and to investigate more complex models, such as the 5-parameter or double diode models, which offer greater accuracy. Additionally, the impact of inconsistencies (such as $R_s < 0$) on the calculation of model parameters will be explored.

References

[1] International Energy Agency, *Renewables 2022: Analysis and forecast to 2027*. OECD Publishing, Paris, 2022.

[2] A. Tuohy et al., “Solar Forecasting: Methods, Challenges, and Performance,” *IEEE Power and Energy Magazine*, vol. 13, no. 6, pp. 50–59, 2015.

[3] C. M. Singal, “Analytical expression for the series-resistance-dependent maximum power point and curve factor for solar cells,” *Solar Cells*, vol. 3, no. 2, pp. 163–177, 1981.

[4] C. Rodriguez and G. A. Amaratunga, “Analytic solution to the photovoltaic maximum power point problem,” *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 54, no. 9, , pp. 2054–2060, 2007.

TABLE I
STATISTICAL PARAMETERS OF THE CALCULATED VALUES FOR (20)

	$\lvert f'(u)\rvert$				**Iterations**			
Module	Min	Max	Mean	Std Dev	Min	Max	Mean	Std Dev
xSi12922	1.209×10^{-7}	6.734×10^{-2}	6.755×10^{-3}	4.647×10^{-3}	3	**8**	6.337	0.852
mSi0166	2.088×10^{-6}	3.239×10^{-2}	6.331×10^{-3}	$\mathbf{4.238}\times10^{-3}$	3	9	6.128	0.985
HIT05667	3.474×10^{-7}	1.008	$\mathbf{3.116}\times10^{-3}$	6.746×10^{-3}	3	30	**5.698**	**0.756**
CdTe75638	4.673×10^{-8}	0.637	$\mathbf{3.174}\times10^{-2}$	$\mathbf{4.811}\times10^{-2}$	3	32	**8.324**	**2.199**
aSiTandem72-46	1.982×10^{-6}	6.759×10^{-2}	1.225×10^{-2}	5.414×10^{-3}	3	11	6.958	0.923
aSiTriple28324	$\mathbf{1.544}\times10^{-6}$	**1.483**	1.586×10^{-2}	2.336×10^{-2}	3	**39**	7.428	1.479
mSi0188	4.340×10^{-7}	4.014×10^{-2}	5.343×10^{-3}	4.891×10^{-3}	3	10	6.096	0.864
mSi460A8	$\mathbf{4.487}\times10^{-8}$	$\mathbf{2.171}\times10^{-2}$	7.106×10^{-3}	4.592×10^{-3}	3	**8**	6.334	0.830

TABLE II
MAPE BETWEEN THE REAL $I-V$ CURVE FOUND ON [12] AND THE PRESENTED MODEL

PV code	**An-Exp**		**Num-Exp**		**An-Num**	
Module	I_{mpp}	V_{mpp}	I_{mpp}	V_{mpp}	I_{mpp}	V_{mpp}
xSi12922	0.8323	0.7800	0.8323	0.7800	1.0641×10^{-5}	1.0642×10^{-5}
mSi0166	1.3263	1.2171	1.3263	1.2171	$\mathbf{9.9310}\times10^{-6}$	$\mathbf{9.9309}\times10^{-6}$
HIT05667	**0.3889**	**0.3736**	**0.3890**	**0.3737**	6.6545×10^{-5}	6.5764×10^{-5}
CdTe75638	3.4275	2.9150	3.4298	2.9170	2.1569×10^{-3}	2.1526×10^{-3}
aSiTandem72-46	**4.0537**	**3.4416**	**4.0536**	**3.4416**	6.6855×10^{-5}	6.6855×10^{-5}
aSiTriple28324	3.4305	2.9641	3.4312	2.9648	$\mathbf{1.3200}\times10^{-3}$	$\mathbf{1.3084}\times10^{-3}$
mSi0188	0.5038	0.4582	1.3220	1.2023	8.9461×10^{-4}	8.9474×10^{-4}
mSi460A8	0.30647	0.29199	0.64514	0.61465	6.0333×10^{-4}	6.0337×10^{-4}

TABLE III
APE BETWEEN THE METHODOLOGIES PRESENTED IN [5], [17] AND THE METHODOLOGY DEVELOPED IN THE PRESENT WORK WITH RESPECT TO EXPERIMENTAL MEASUREMENTS

G	**T (°C)**	**aSiTandem72-46**						**HIT05667**					
(W/m^2)		$\lvert E_{I(5P)}\rvert$	$\lvert E_{V(5P)}\rvert$	$\lvert E_{I(4P)}\rvert$	$\lvert E_{V(4P)}\rvert$	$\lvert E_{I(3P)}\rvert$	$\lvert E_{V(3P)}\rvert$	$\lvert E_{I(5P)}\rvert$	$\lvert E_{V(5P)}\rvert$	$\lvert E_{I(4P)}\rvert$	$\lvert E_{V(4P)}\rvert$	$\lvert E_{I(3P)}\rvert$	$\lvert E_{V(3P)}\rvert$
100	15	1.956	0.757	4.540	3.804	6.748	5.146	0.006	0.139	0.698	0.661	2.365	1.953
100	25	1.638	0.821	4.073	3.461	5.598	4.419	0.072	0.213	0.562	0.536	1.114	1.015
200	15	1.283	0.807	3.191	2.781	4.088	3.390	0.002	0.095	0.735	0.694	2.884	2.283
200	25	1.427	0.741	3.865	3.298	5.771	4.526	0.055	0.145	0.539	0.515	1.476	1.297
400	25	1.238	0.854	3.282	2.855	4.036	3.352	0.098	0.202	0.462	0.444	0.705	0.661
400	50	0.773	0.839	2.993	2.625	3.654	3.076	0.161	0.319	0.576	0.550	0.363	0.352
600	25	1.312	0.935	3.300	2.873	3.593	3.022	0.152	0.267	0.412	0.397	0.048	0.048
600	50	0.745	0.956	2.850	2.515	2.815	2.437	0.211	0.367	0.521	0.499	0.126	0.128
600	65	0.538	1.036	2.729	2.420	2.252	1.981	0.245	0.449	0.630	0.600	0.163	0.166
800	25	1.351	1.105	3.196	2.800	2.502	2.150	0.209	0.334	0.383	0.369	0.485	0.507
800	50	0.801	1.087	2.883	2.547	2.272	1.986	0.264	0.436	0.505	0.485	0.541	0.566
800	65	0.580	1.127	2.783	2.467	1.974	1.738	0.323	0.535	0.585	0.559	0.726	0.768
1000	25	1.443	1.200	3.259	2.854	2.137	1.822	0.242	0.388	0.389	0.375	0.742	0.793
1000	50	0.880	1.183	2.973	2.622	2.012	1.751	0.266	0.482	0.556	0.532	0.557	0.582
1000	65	0.624	1.253	2.838	2.515	1.577	1.376	0.364	0.609	0.600	0.574	0.988	1.063
1100	25	1.521	1.235	3.339	2.920	2.093	1.770	0.175	0.390	0.508	0.487	0.213	0.217
1100	50	0.875	1.258	2.944	2.601	1.651	1.429	0.338	0.574	0.535	0.513	1.056	1.146
1100	65	0.581	1.353	2.769	2.462	1.091	0.929	0.333	0.638	0.681	0.648	0.825	0.875
MAPE		1.087	1.030	3.211	2.801	3.104	2.572	0.195	0.366	0.549	0.524	0.854	0.801

[5] G. Farivar, B. Asaei and M. A. Ali, "A novel analytical solution for the PV-arrays maximum power point tracking problem," in *2010 IEEE International Conference on Power and Energy.* IEEE, 2010, pp. 917-922.

[6] A. Laudani, G. M. Lozito, V. Lucaferri, M. Radicioni, and F. R. Fulginei, A. Salvini, S. Coco, "An analytical approach for maximum power point calculation for photovoltaic system," in *2017 European Conference on Circuit Theory and Design (ECCTD)* IEEE, 2017, pp. 1-4.

[7] A. Jain and A. Kapoor, "Exact analytical solutions of the parameters of real solar cells using Lambert W-function," *Solar Energy Materials and Solar Cells*, vol. 81, no. 2, pp. 269–277, 2004.

[8] A. S. Garcia and R. Strandberg, "Analytical Modeling of the Maximum Power Point with Series Resistance," *Applied Sciences*, vol. 11, no. 22, pp. 10952, 2021.

[9] O. Trejo and A. Ortiz-Conde, "A simple algorithm for high-accuracy maximum power point calculation of photovoltaic systems," *IEEE Journal of Photovoltaics*, vol. 10, no. 6, pp. 1839–1845, 2020.

[10] R. M. Corless, G. H. Gonnet, D. EG Hare, D. J. Jeffrey and D. E. Knuth, "On the Lambert W function," *Advances in Computational mathematics*, vol. 5, no. 1, pp. 329–359, 1996.

[11] R. L. Burden, J. D. Faires and A. M. Burden, *Numerical analysis*. Cengage learning, 2015.

[12] B Marion, A. Anderberg, C. Deline, J. Del Cueto, M. Muller, G. Perrin, J. Rodriguez, S. Rummel, T. J. Silverman, F. Vignola and others, "New data set for validating PV module performance models," in *2014 IEEE 40th Photovoltaic Specialist Conference (PVSC)*, IEEE, 2014, pp. 1362–

1366.
[13] B. Aldwane, “Modeling, simulation and parameters estimation for Photovoltaic module,” in *2014 First International Conference on Green Energy ICGE 2014.* IEEE, 2014, pp. 101–106.
[14] E. Batzelis, “Non-iterative methods for the extraction of the single-diode model parameters of photovoltaic modules: A review and comparative assessment,” *Energies*, vol. 12, no. 3, pp. 358, 2019.
[15] W. Mendenhall, R. J. Beaver and B. M. Beaver, *Introduction to probability and statistics*. Cengage Learning, 2012.
[16] E. Saloux, A. Teyssedou, and M. Sorin, “Explicit model of photovoltaic panels to determine voltages and currents at the maximum power point,” *Solar energy*, vol. 85, no. 5, , pp. 713–722, 2011.
[17] J. G. Tirado-Serrato, “Exact Solution to the Mean Value Theorem Applied to the Maximum Power Point Estimation,” *IEEE Journal of Photovoltaics*, vol. 13, no. 5, pp. 750-755, 2023.
[18] N. Femia, G. Petrone, G. Spagnuolo and M. Vitelli, *Power Electronics and Control Techniques for Maximum Energy Harvesting in Photovoltaic Systems*. CRC Press, 2012.