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Low-Complexity Pilot-Independent Time Synchronization for Zero-Padding-Based IoT Devices

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Abstract—Many Internet of Things (IoT) devices are small, battery-powered systems, requiring energy-efficient solutions to extend their operational life. To reduce communication energy consumption, we suggest zero-padding (ZP)-orthogonal frequency-division multiplexing (OFDM) as a more efficient alternative to its cyclic-prefix (CP)-OFDM counterpart. However, ZP-OFDM typically relies on pilot data for time synchronization, which can be prohibitively expensive for resource-constrained IoT devices. This article introduces a low-complexity, pilot-independent time synchronization method for ZP-OFDM. Thus, the proposed approach eliminates the need for pilot data which further enhances the energy efficiency of ZP-based signaling for IoT applications. Simulation results demonstrate that the proposed estimators achieve a high lock-in probability even in low SNR with low computational complexity, enabling the practical deployment of ZP signaling in IoT systems. Although the proposed methods cannot be directly integrated to the existing IoT technologies, such as ZigBee for physical implementation, they pave the way for practical implementations in future IoT technologies.

Index Terms—Internet of Things (IoT), low signal-to-noise ratio (SNR), low-complexity, pilot-independent, time synchronization, zero-padding (ZP).

I. INTRODUCTION

INTERNET of Things (IoT) devices are used in various applications, including but not limited to environmental and disaster monitoring, asset tracking and management in inventories, industrial IoT for machinery maintenance, wearable health monitoring, and precision agriculture. Such devices are usually small and battery-powered, and hence, a lot of efforts are put into minimizing their energy consumption and increasing their lifetime. Thus, various solutions, such as data compression, data aggregation, and data prediction are proposed to reduce their communication energy consumption [1].

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Orthogonal frequency-division multiplexing (OFDM)-based signaling, and in particular cyclic prefix (CP)-OFDM, have been widely used in various Internet of Things (IoT) devices, including but not limited to narrowband IoT devices, IEEE 802.11ah (Wi-Fi HaLow) devices, and other OFDM-based wireless sensor networks [2]. However, such signaling suffers from high energy consumption, which hinders its implementation in many practical scenarios.

Zero-padded signaling, such as zero padding (ZP)-OFDM, can offer a more energy efficient solution, compared to CP-OFDM, for IoT devices. More specifically, ZP-OFDM consumes less power per transmission, compared to CP-OFDM, as it does not transmit the repeated information, i.e., CP [3], which could potentially account for about 17% percent of the OFDM symbol duration. Additionally, the zero-filled guard interval of ZP-OFDM can be also used for sensing applications, achieving joint communication and sensing [4], [5], [6] which may be crucial for many IoT applications.

Recently, there has been a lot of efforts to further reduce the communication energy consumption of ZP-OFDM via either minimizing or removing pilots [7], [8].¹ As the number of connected IoT devices grows exponentially, the pilot overhead, too, can grow exponentially. This, in turn, increases the energy consumption of communication and reduces the lifetime of IoT devices. Hence, researchers have tried to remove it. However, ZP-based waveforms, such as ZP-OFDM, heavily rely on the pilots for time synchronization as they do not have a CP for time synchronization purposes.

In general, there are two approaches in order to estimate timing offset (TO) in OFDM signals. In the first approach, called data-aided (DA) time synchronization, a series of training sequences (pilots) are used to estimate TO. The second approach, referred to as nondata-aided (NDA) time synchronization, relies on the statistical properties of the transmitted data sequence. CP-OFDM can use the CP to perform correlation-based NDA time synchronization methods. However, ZP-OFDM cannot use such methods due to the lack of CP. Hence, lacking a robust NDA time synchronization method for ZP-OFDM hinders its adoption in many IoT devices.

While there exist some NDA time synchronization algorithms for ZP-OFDM in the literature, the proposed algorithms fail to achieve either a high lock-in probability, or practical

¹Used for synchronization purposes.

complexity for meeting the low-complexity requirements of IoT applications. Hence, an accurate yet low-complexity NDA time synchronization algorithm for ZP-OFDM is needed. The goal of this article is to fill this existing gap by proposing low-complexity NDA time synchronization methods for ZP-based waveforms, such as ZP-OFDM systems. Furthermore, our proposed solution can achieve very high lock-in probability even in low signal-to-noise ratio (SNR) which is a common scenario in IoT devices.

Our approach can be also potentially applied to duty-cycled IoT devices that sleep and wake up periodically in order to save energy. More specifically, the zero-filled guard interval of ZP-OFDM models the sleep period of such devices in which they do not transmit any data. For instance, a CP-OFDM based IoT device, such as narrowband IoT devices, IEEE 802.11ah (Wi-Fi HaLow) devices, and other OFDM-based wireless sensor networks, which sleeps periodically for energy saving, follows similar equations presented in this article.

A. Related Work

The time synchronization problem for various waveforms and under various scenarios has been investigated in [9], [10], [11], [12], [13], [14], [15], [16], [17], and [18]. For instance, Sun et al. [9], proposed a pilot-aided joint time synchronization and channel estimation algorithm based on maximum length sequence for timing offset estimation in orthogonal time frequency space (OTFS) systems. Moreover, a minimum variance time synchronization algorithm using an output feedback consensus control scheme for time-division multiple access systems is introduced in [10]. Another time synchronization method based on sensing that leverages the echoed signals is proposed for UAV-assisted OFDM systems in [11]. Scazzoli [13] proposed a correlation-based technique via a novel synchronization-focused waveform in integrated sensing and communication scenarios.

The data-aided (DA) time synchronization for ZP-OFDM has been studied in [19] and [12], where a highly correlated training sequence (pilot) is employed in order to increase the auto-correlation of the received signal, which is then used for estimating the TO. A pilot-aided artificial intelligence approach via cascaded 1-D convolutional neural network (1-D CNN)-based time synchronization method for ZP-OFDM systems is proposed in [12]. Such a pilot-based time synchronization algorithm can achieve reliable performance while having a reasonable complexity [19]. However, transmitting pilots can be costly for small and battery-limited IoT devices.

For nondata-aided (NDA) time synchronization in ZP-OFDM, however, a low-complexity TO estimation algorithm with an accuracy comparable to that of CP-OFDM counterparts does not exist. Existing NDA time synchronization algorithms for ZP-OFDM, such as transition metric (TM) in [20] or the estimator in [21] mainly detect the jumps in the energy of the signal. Such methods are heuristics that detect the point that the energy² of the samples in the window changes significantly. However, jump-based techniques greatly

suffer from the natural randomness of the received samples; thus, exhibit poor performance in terms of probability of lock-in, i.e., correct TO estimation.

A mathematical approach toward NDA TO estimation for ZP-OFDM systems has been proposed in [23]. Roshandeh et al. [23] proposed a maximum likelihood (ML) TO estimator for ZP-OFDM under a frequency selective channel. However, the algorithm in [23] is highly complex which hinders its implementation in low-power low-complexity IoT devices.

B. Contributions

In this article, we first propose a moment-based NDA TO estimator for multiple-input-multiple-output (MIMO) ZP-OFDM. In general, moment estimators are derived via solving equations involving the theoretical moments of the received samples and their natural moment estimators [24]. In this article, we choose the Second order Moment (SoM) in order to keep the complexity of the estimator very low. Later, we propose a weighted version of the moment estimator to further improve the the probability of lock-in. The contribution of the article is summarized as follows.

- 1) An NDA TO estimator based on method of moments (MoM) for ZP-OFDM systems in highly selective channels is proposed. This algorithm
 - a) Achieves high lock-in probability.
 - b) Has significantly lower complexity compared to [23]; thus, it can be employed in low-complexity low-power IoT devices.
 - c) Is suitable for deployment in very low SNRs in contrast to [23] and TM [20], making it suitable for IoT applications.
- 2) A weighted moment-based NDA TO estimator, i.e., Weighted Method of Moments (WMoM), for ZP-OFDM systems in highly selective channels is proposed. This algorithm has all the benefits of the first estimator while significantly improving its lock-in probability. This algorithm asymptotically reaches the ML estimator's performance, while having significantly lower computational complexity.

This article is organized as follows. The system model is presented in Section II. The main ideas, the proposed estimators, and the complexity of the estimators are discussed in Sections III. Simulation results and conclusions are given in Sections IV and V, respectively.

Notations: Column vectors are denoted by bold lower case letters. Random variables are indicated by uppercase letters. Matrices are denoted by bold uppercase letters. Conjugate, absolute value, transpose, and the expected value are indicated by $(\cdot)^*$, $|\cdot|$, $(\cdot)^T$, and $\mathbb{E}\{\cdot\}$, respectively. Floor function is denoted by $\lfloor \cdot \rfloor$. Brackets, e.g., $\mathbf{a}[k]$, are used for discrete indexing of a vector $\mathbf{a}$. Natural estimator of an observation sample is defined as

$$\mathbb{E}\{|x[i]|^n\} = \frac{1}{m} \sum_{i=1}^m |x[i]|^n.$$

²Sometimes two sliding windows are used and the change in the energy ratio of these two windows is tracked.

II. SYSTEM MODEL

We consider a MIMO-OFDM [25], [26] wireless system with m_t and m_r transmit and receive antennas, respectively. This system uses ZP-OFDM technique for communication over a frequency selective Rayleigh fading channel. Let $\{x_i^{(n,k)}\}_{k=0}^{n_x-1}$ with $\mathbb{E}\{|x_i^{(n,k)}|^2\} = \sigma_x^2$, denote the n_x complex data samples from the n th OFDM block to be transmitted from the i th transmit antenna. The corresponding OFDM signal can be expressed as [23]

$$x_i^{(n)}(t) = \sum_{k=0}^{n_x-1} x_i^{(n,k)} e^{\frac{j2\pi kt}{T_x}} \quad 0 \leq t \leq T_x \quad (1)$$

where T_x denotes the duration of the data signal. By adding a zero-padding guard interval of length T_z to (1), the n th transmitted ZP-OFDM block from the i th transmit antenna is given as

$$s_i^{(n)}(t) = \begin{cases} x_i^{(n)}(t), & 0 \leq t \leq T_x \\ 0, & T_x < t \leq T_s \end{cases} \quad (2)$$

where T_s denote the symbol duration, and $T_s = T_x + T_z$.

Let $f_s = 1/T_{sa} = n/T_x$ denote the sampling rate at the receiver. In the absence of synchronization error, the discrete received baseband vector of the n th OFDM block is expressed as [27] and [28]

$$\mathbf{y}^{(n)} = \begin{cases} \mathbf{H}\mathbf{s}^{(n)} + \mathbf{w}^{(n)}, & n \geq 0 \\ \mathbf{w}^{(n)}, & n < 0 \end{cases} \quad (3)$$

where $\mathbf{H}$ denotes the discrete channel matrix, and is defined as

$$\mathbf{H} = \begin{pmatrix} \mathbf{H}_{11} & \mathbf{H}_{12} & \cdots & \mathbf{H}_{1m_t} \\ \mathbf{H}_{21} & \mathbf{H}_{22} & \cdots & \mathbf{H}_{2m_t} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{H}_{m_r1} & \mathbf{H}_{m_r2} & \cdots & \mathbf{H}_{m_rm_t} \end{pmatrix} \quad (4)$$

where $\mathbf{H}_{ji}$ is the $n_s \times n_s$ lower triangular Toeplitz channel matrix between the transmit antenna i and the received antenna j , with first column $[h_{ji}[k, 0] \ h_{ji}[k, 1] \ \cdots \ h_{ji}[k, n_h - 1] \ 0 \ \cdots \ 0]^T$ where $0 \leq k \leq n_s - 1$, and $\{h_{ji}[k, l]\}_{l=0}^{n_h-1}$ denote the n_h channel taps between the transmit antenna i and the received antenna j at time k and delay l [29]. The channel is considered to be Rayleigh fading, and the channel taps are assumed to be statistically independent and are modeled by zero-mean complex Gaussian random variables with the autocorrelation function

$$\mathbb{E}\{h_{j_1 i_1}[k_1, l] h_{j_2 i_2}^*[k_2, l - m]\} = \sigma_{h_l}^2 R[k_1 - k_2] \delta[m] \delta[j_1 - j_2] \delta[i_1 - i_2] \quad (5)$$

$l = 0, 1, \dots, n_h - 1$, $0 \leq k_1, k_2 \leq n_s - 1$, $1 \leq i_1, i_2 \leq m_t$, and $1 \leq j_1, j_2 \leq m_r$. In (5), $R[k_1 - k_2]$ is an arbitrary function with $R[0] = 1$, where it describes the interdependence of two channel taps at different times k_1 and k_2 . Without loss of generality and in this article, we consider $R[k] = \delta[k]$. However, the mathematical derivations in this article, are applicable to the general case where $R[k]$ is given for different values of k . As the relative speed of the transmitter and receiver increases, $R[k_1 - k_2]$ approaches $\delta[k_1 - k_2]$; thus, the power delay profile (PDP) of the channel is modeled as

$$\mathbb{E}\{h_{j_1 i_1}[k_1, l] h_{j_2 i_2}^*[k_2, l - m]\} = \sigma_{h_l}^2 \delta[k_1 - k_2] \delta[m] \delta[j_1 - j_2] \delta[i_1 - i_2]. \quad (6)$$

The PDP of the channel, i.e., $\sigma_{h_0}^2, \sigma_{h_1}^2 \dots \sigma_{h_{n_h-1}}^2$, is assumed to be known at the receiver. The PDP of an environment, can be obtained through channel sounding procedure prior to any data transmission, and hence is independent of such transmission. It can be obtained through transmitting a short pulse (wide-band) and measuring the power of the received signal at various delays, averaged over spatial locations. The vectors $\mathbf{s}^{(n)}$, $\mathbf{y}^{(n)}$, and $\mathbf{w}^{(n)}$ in (3) are given as

$$\mathbf{s}^{(n)} = \begin{pmatrix} \mathbf{s}_1^{(n)} \\ \mathbf{s}_2^{(n)} \\ \vdots \\ \mathbf{s}_{m_t}^{(n)} \end{pmatrix}, \quad \mathbf{y}^{(n)} \triangleq \begin{pmatrix} \mathbf{y}_1^{(n)} \\ \mathbf{y}_2^{(n)} \\ \vdots \\ \mathbf{y}_{m_r}^{(n)} \end{pmatrix}, \quad \mathbf{w}^{(n)} \triangleq \begin{pmatrix} \mathbf{w}_1^{(n)} \\ \mathbf{w}_2^{(n)} \\ \vdots \\ \mathbf{w}_{m_r}^{(n)} \end{pmatrix} \quad (7)$$

where $\mathbf{s}_i^{(n)}$, $\mathbf{y}_j^{(n)}$, and $\mathbf{w}_j^{(n)}$ denote the n th transmitted ZP-OFDM block from the i th transmit antenna, the corresponding received vector at the j th receive antenna, and the noise vector at the j th receive antenna, respectively, and are defined as

$$\mathbf{y}_j^{(n)} \triangleq [y_j^{(n)}[0] \ y_j^{(n)}[1] \ \cdots \ y_j^{(n)}[n_s - 1]]^T \quad (8a)$$

$$\mathbf{w}_j^{(n)} \triangleq [w_j^{(n)}[0] \ w_j^{(n)}[1] \ \cdots \ w_j^{(n)}[n_s - 1]]^T \quad (8b)$$

$$\mathbf{s}_i^{(n)} \triangleq [s_i^{(n)}[0] \ s_i^{(n)}[1] \ \cdots \ s_i^{(n)}[n_s - 1]]^T \quad (8c)$$

$$= [x_i^{(n)}(0) \ x_i^{(n)}(T_{sa}) \ \cdots \ x_i^{(n)}((n_x - 1)T_{sa}) \ \mathbf{0}_{n_z}^T]^T$$

where $n_s \triangleq \lfloor T_s/T_{sa} \rfloor$, $n_x \triangleq \lfloor T_x/T_{sa} \rfloor$, and $n_z \triangleq \lfloor T_z/T_{sa} \rfloor$ denote the total number samples, number of data samples, and the number of zero-padded samples per OFDM block, respectively, and $n_s = n_x + n_z$.

In order to avoid inter symbol interference (ISI), the length of the zero-padding should be greater than or equal to the number of channel taps, i.e., $n_z \geq n_h$. This assumption holds throughout our analysis in this article. Noise samples are assumed to follow zero-mean complex Gaussian random variable with correlation function as

$$\mathbb{E}\{w_j^{(m)}[k] * w_j^{(m')}[k']\} = \sigma_n^2 \delta[m - m'] \delta[j - j'] \delta[k - k'] \quad (9)$$

According to the central limit theorem (CLT) [30], the transmitted OFDM samples, i.e., $s_i^{(n)}[k] = x_i^{(n)}(kT_{sa}) \ \forall k \in \{0, 1, \dots, n_x - 1\}$, can be modeled as independent and identically distributed (i.i.d) zero-mean complex Gaussian random variables. Hence

$$s_i^{(n)}[k] \text{ or } x_i^{(n)}(kT_{sa}) \sim \mathcal{CN}(0, \sigma_x^2) \quad \forall k \in \{0, 1, \dots, n_x - 1\} \quad (10)$$

where

$$\mathbb{E}\{s_i^{(n)}[k] s_p^{(n)}[k']^*\} = \mathbb{E}\{x_i^{(n)}(kT_{sa}) x_p^{(n)}(k'T_{sa})^*\} \quad (11)$$

$$= \sigma_x^2 \delta[k - k'] \delta[i - p] \quad (12)$$

$$\forall i, p \in \{1, 2, \dots, m_t\}$$

$$\forall k, k' \in \{0, 1, \dots, n_x - 1\}.$$

Now, assume that there is a TO $\tau \triangleq dT_{sa} + \epsilon$ between the transmitter and the receiver, where d and ϵ represent the integer and fractional part of the TO, respectively. Since the fractional part of TO, ϵ , can be corrected through channel equalization and carrier frequency offset estimation [31], it suffices to estimate the integer part of TO. In fact, it is common in practice to model the TO as a multiple of the sampling period, and consider the remaining fractional error as part of the channel impulse response. To this end, we focus on estimating the integer part of the TO, d , which is essential in order to perform the fast Fourier transform (FFT) operation at the receiver, and decode the data in subsequent steps.

The next section proposes the MoM estimators for estimating d . In order to simplify the notations and derivations, we first consider a single-input–single-output (SISO) system, and then extend our discussions to MIMO devices.

III. MOMENT-BASED ESTIMATORS FOR ZP-OFDM

The MoM and its weighted version, i.e., WMoM, are mostly used in practical systems where it is either not possible to derive the ML estimator or its complexity is prohibitive for low-complexity low-power devices. In such scenarios, MoM and WMoM can potentially achieve high estimation accuracy while having low complexity.

The main idea behind MoM is to use the *moments* of the observation samples instead of their probability density function (PDF), used in methods like ML. The parameter of interest is then estimated by equating the practical and theoretical moments of the observation samples and solving it for the mentioned parameter. WMoM builds upon this method by additionally taking into account a weight for each moment. This weight is proportional to the inverse of the variance of the moment as a measure of confidence in the calculated moment. In this article, we will use these ideas to derive our estimators.

For the SISO system, the received signal in (3) can be rewritten as

$$\mathbf{y}^{(n)} = \begin{cases} \mathbf{H}\mathbf{s}^{(n)} + \mathbf{w}^{(n)} \triangleq \mathbf{v}^{(n)} + \mathbf{w}^{(n)}, & n \geq 0 \\ \mathbf{w}^{(n)}, & n < 0 \end{cases} \quad (13)$$

where $\mathbf{H} \triangleq \mathbf{H}_{11}$ and $\mathbf{v}^{(n)} \triangleq \mathbf{H}\mathbf{s}^{(n)}$.

We allow the integer part of the TO, d , takes values from a set $\mathcal{D} = \{-n_s + 1, \dots, -1, 0, 1, \dots, n_s - 1\}$. Note that the positive values of the delay, d , corresponds to situations when the receiver starts early to receive samples. That is, for $d > 0$, the receiver receives d noise samples, and then receives the transmitted OFDM samples starting from the $(d+1)$ th sample. Similarly, when $d \leq 0$, the receiver misses the first $|d|$ samples from the transmitted OFDM samples. Allowing d to take both negative and positive values enables the final estimator to be employed for both frame and symbol synchronization.

The problem of TO estimation can be formulated as a multiple hypothesis testing problem. Let H_d denote the hypothesis corresponding to TO $d \in \mathcal{D} = \{-n_s + 1, \dots, n_s - 1\}$. For $d = 0$, $\mathbf{y}^{(n)}$ in (13) is cyclostationary with period n_s . Cyclostationary here means that the PDF of the individual received samples repeats itself after certain number of samples. This means that the moments of the samples are also periodic

with period n_s . Hence, when $d = 0$, we only need to derive the moments of the samples for the first n_s samples and repeat them in the same order to obtain the moments of subsequent samples.

The periodic pattern in the moments of $\mathbf{y}^{(n)}$ given H_0 makes the MoM an efficient way for TO estimation. This is because the periodicity can give a better estimate of the true moments. The main idea behind the MoM is to derive a theoretical expression for the moment of the received signal given different hypotheses and compare it with the sample mean estimate of the moment.

The First-order Moment (FoM) of the received samples in (13) is zero, i.e., $\mathbb{E}\{\mathbf{y}^{(n)}\} = 0$, since the expected value of the channel taps, the OFDM signal samples, and the noise samples are all zero. Hence, the FoM of the received samples cannot be used for TO estimation. Therefore, we aim at obtaining the SoM of the received samples. Theorem 1 derives the SoM of the received samples for a single OFDM block, i.e., samples with indices from 0 to $n_s - 1$ given H_0 .

Theorem 1: The SoM of the received OFDM samples given hypothesis H_0 are given by

$$M_0[k] = M_0[k + nn_s] \triangleq \mathbb{E}\{|\mathbf{y}^{(n)}[k]|^2 | H_0\} = \sigma_s^2 \sum_{l=a_k}^{b_k} \sigma_{h_l}^2 + \sigma_n^2 \quad (14)$$

where $n \in \mathbb{Z}_{\geq 0}$, $k = 0, 1, \dots, n_s - 1$, and

$$(a_k, b_k) = \begin{cases} (0, k), & 0 \leq k \leq n_h - 2 \\ (0, n_h - 1), & n_h - 1 \leq k \leq n_x - 1 \\ (k - n_x + 1, n_h - 1), & n_x \leq k \leq n_x + n_h - 2 \\ (\text{NaN}, \text{NaN}), & n_x + n_h - 1 \leq k \leq n_s - 1. \end{cases} \quad (15)$$

with $n_s = n_x + n_z - 1$ and $n_h \leq n_z$.

Proof: Please see Appendix A. ■

Note that the right hand side of (14) is independent of n , and only depends on k . Hence, $M_0[k] \forall k \in \mathbb{N}_{\geq 0}$ is periodic with period n_s .

Now, we have the theoretical expression for the $n_s = n_x + n_h - 1$ SoMs of an OFDM block. In order to estimate the TO, d , we need to obtain the corresponding sample mean estimate of the SoMs and compare them with the corresponding theoretical values in (1). In the following, we first obtain the sample mean estimate of the SoMs of an OFDM block and then formulate the TO estimation as a least squares (LS) minimization problem.

A. MoM Estimator for SISO-OFDM

Let us consider an observation window of length L and define the observation vector $\mathbf{y}_{\text{obs}}$ as

$$\mathbf{y}_{\text{obs}} \triangleq [\mathbf{y}_{\text{obs}}[0] \ \mathbf{y}_{\text{obs}}[1] \ \dots \ \mathbf{y}_{\text{obs}}[L-1]]^T. \quad (16)$$

In order to estimate the TO given $\mathbf{y}_{\text{obs}}$ by using the MoM, we need to define the sample mean estimate of two conditional SoMs.

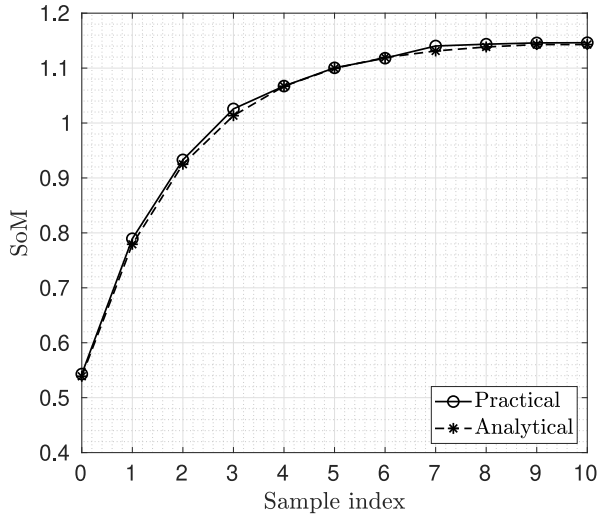


Fig. 1. Practical and analytical SoM of the starting samples of an OFDM symbol.

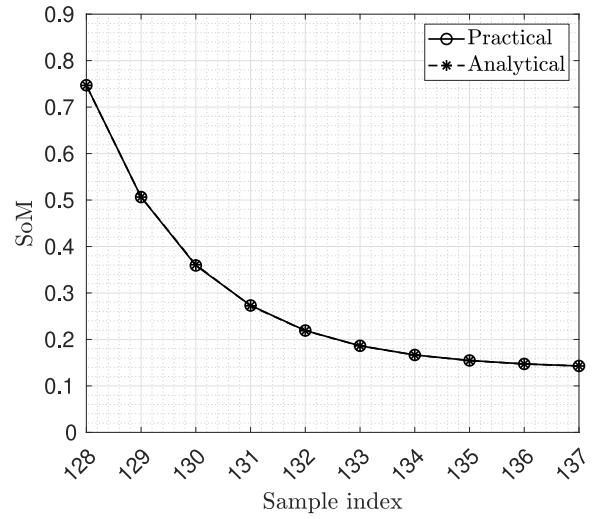


Fig. 2. Practical and analytical SoM of the trailing samples of an OFDM symbol.

Because $M_0[k]$ is cyclostationary, the sample mean estimate of $\mathbb{E}\{|y^{(n)}[k+d]|^2|H_d\} = M_0[k]$ for $k = 0, 1, \dots, n_s - 1$ and $d > 0$ is given by

$$\hat{M}_{d,L}^+[k] \triangleq \frac{1}{\lfloor (L-k-d+1)/n_s \rfloor} \times \sum_{r=0}^{\lfloor (L-k-d+1)/n_s \rfloor - 1} |y_{\text{obs}}[k+d+m_s]|^2 \quad \forall k \in \{0, 1, \dots, n_s - 1\} \quad (17)$$

where $+$ superscript denotes $d > 0$. Given hypothesis H_d , $d > 0$, the noise variance can be estimated as

$$\hat{M}_d^{\text{noise}} = \frac{1}{d} \sum_{r=0}^{d-1} |y_{\text{obs}}[r]|^2. \quad (18)$$

Similarly, the sample mean estimate of $\mathbb{E}\{|y^{(n)}[k]|^2|H_d\} = M_0[k + |d|]$ for $k = 0, 1, \dots, n_s - 1$ and $d \leq 0$ is given by

$$\begin{aligned} \hat{M}_{d,L}^-[k] &= \\ &= \frac{1}{\lfloor (L-k+1)/n_s \rfloor} \sum_{r=0}^{\lfloor (L-k+1)/n_s \rfloor - 1} |y_{\text{obs}}[k+m_s]|^2 \end{aligned} \quad \forall k \in \{0, 1, \dots, n_s - 1\} \quad (19)$$

where superscript $-$ denotes $d \leq 0$.

In the asymptotic case that $L \rightarrow +\infty$, we have

$$\lim_{L \rightarrow \infty} \hat{M}_{d,L}^+[k] \mapsto M_0[k] \quad \forall k \in \{0, 1, \dots, n_s - 1\} \quad (20)$$

and

$$\lim_{L \rightarrow \infty} \hat{M}_{d,L}^-[k] \mapsto M_0[k + |d|] \quad \forall k \in \{0, 1, \dots, n_s - 1\}. \quad (21)$$

Figs. 1 and 2 show the asymptotic convergence of the SoMs versus the index of the samples for an OFDM symbol over 10^5 realizations. Having defined the sample mean estimate of

two conditional SoMs in this section, we are now ready to propose the MoM estimator.

Using Theorem 1, and (20) and (21), we can formulate the TO estimation as

$$\begin{aligned} \hat{d} = \operatorname{argmin}_{d \in \mathcal{D}} \sum_{k=0}^{n_s-1} & \left| \hat{M}_{d,L}^-[k] - M_0[k + |d|] \right| u[-d] \\ & + \left| \hat{M}_{d,L}^+[k] - M_0[k] \right| u[d-1] + \left| \hat{M}_d^{\text{noise}} - \sigma_n^2 \right| u[d-1] \end{aligned} \quad (22)$$

where $u[d]$ is the discrete unit step function, defined as

$$u[k] = \begin{cases} 1, & k \geq 0 \\ 0, & k < 0. \end{cases} \quad (23)$$

From (22), it is clear that the accuracy of the estimator depends on how large the number of the received samples L is. This is a limiting factor in resource-limited devices where a large number of OFDM symbols cannot be loaded into memory. Hence, affecting the accuracy of the estimation. In order to address this issue and achieve higher accuracy, we will improve the estimator in the next section.

B. Weighted MoM Estimator for SISO-OFDM

In this section, we propose and develop the WMoM estimator that offers a higher probability of lock-in. The main idea of WMoM estimator is to give each sample's moment a weight which describes the confidence on the closeness of the analytical and practical means.

The variance of a random variable approximately shows how close on average is the realization value of a random value to its mean value. In our case, the mean value is the second moments that we derived in Theorem 1, and the variance of those second moments shows how close on average the second moments of the *received* samples, i.e., the left hand sides in (20) and (21) would be to their mean value, i.e., the right hand sides in (20) and (21). That is, the smaller the variance, the closer the second moments of the received samples, i.e., the

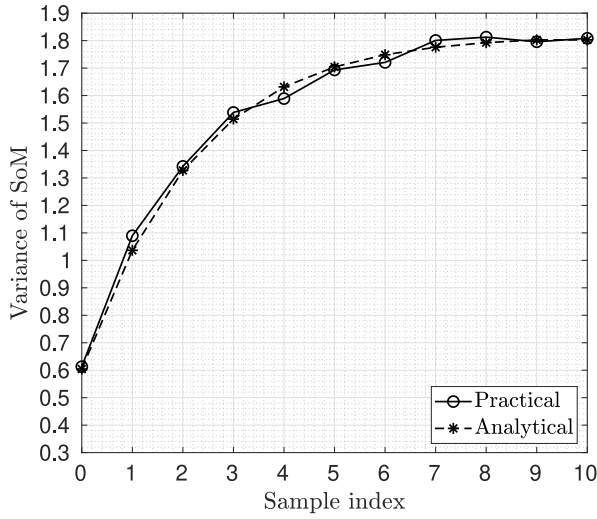


Fig. 3. Practical and analytical variance of SoM of the starting samples for an OFDM symbol.

left hand sides in (20) and (21), would be to their mean value, i.e., the right hand sides in (20) and (21). Hence, the inverse of the variance of $|y^{(n)}[k]|^2$, $k = 1, \dots, n_s$ shows our confidence in how close the left and right hand sides of (20) and (21) are. Next Theorem provides the variance of the SoM of the received samples

Theorem 2: The variance of the SoMs of different received OFDM samples are given as follows:

$$\sigma_{|y^{(n)}[k]|^2|H_0}^2 = \frac{13}{4}\sigma_s^4 \sum_{l=a_k}^{b_k} \sigma_{h_l}^4 + \frac{5}{2}\sigma_s^4 \sum_{r=a_k}^{r=b_k} \sum_{l=a_k, l \neq r}^{l=b_k} \sigma_{h_l}^2 \sigma_{h_r}^2 + 2\sigma_n^2 \sigma_s^2 \sum_{l=a_k}^{b_k} \sigma_{h_l}^2 + \sigma_n^4 - \frac{1}{4}\sigma_s^2 \left(\sum_{l=a_k}^{b_k} \sigma_{h_l}^2 \right)^2 \quad (24)$$

where $n \in \mathbb{Z}_{\geq 0}$

$$(a_k, b_k) = \begin{cases} (0, k), & 0 \leq k \leq n_h - 2 \\ (0, n_h - 1), & n_h - 1 \leq k \leq n_x - 1 \\ (k - n_x + 1, n_h - 1), & n_x \leq k \leq n_x + n_h - 2 \end{cases} \quad (25)$$

and it is equal to $2\sigma_n^2$ for $n_x + n_h - 1 \leq k \leq n_s - 1$.

Proof: See Appendix B. ■

Figs. 3 and 4 show the asymptotic convergence of the variance of the SoMs of the starting and trailing samples for an OFDM symbol with 10^5 Monte Carlo realizations.

Now, we use the inverse of the variance of the SoMs, given in Theorem 2, as weights in (22) to improve the accuracy of the estimator. Let us define

$$\begin{aligned} F[k + nn_s] &\triangleq \sigma_{|y^{(n)}[k]|^2|H_0}^2 \\ F^{\text{noise}} &\triangleq 2\sigma_n^2 \\ &\forall k \in \{0, 1, \dots, n_s - 1\} \\ &\forall n \in \mathbb{Z}_{\geq 0}. \end{aligned} \quad (26)$$

Clearly, each $F[i]$ maps to $M_0[i]$ defined in Theorem 1, hence, one can write the weighted version of the estimator [24] in (22) as

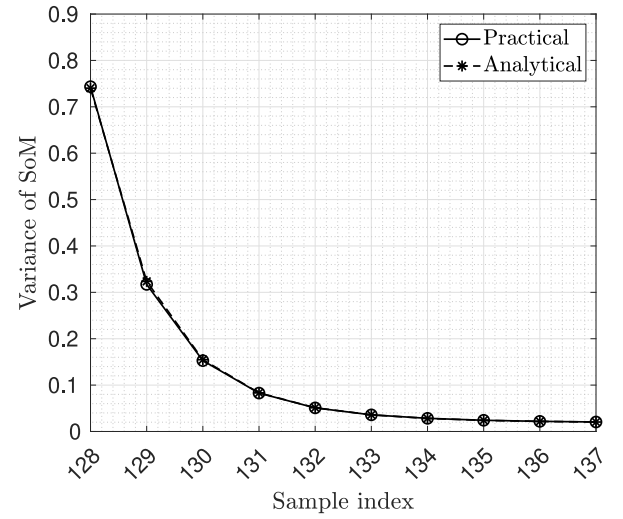


Fig. 4. Practical and analytical variance of SoM of the trailing samples for an OFDM symbol.

$$\begin{aligned} \hat{d} = \min_{d \in \mathcal{D}} \sum_{k=0}^{n_s-1} & \frac{1}{F[k + |d|]} \left| \hat{M}_{d,L}^-[k] - M_0[k + |d|] \right| u[-d] \\ & + \frac{1}{F[k]} \left| \hat{M}_{d,L}^+[k] - M_0[k] \right| u[d - 1] \\ & + \frac{1}{F^{\text{noise}}} \left| \hat{M}_d^{\text{noise}} - \sigma_n^2 \right| u[d - 1] \end{aligned} \quad (27)$$

This completes our derivation of the WMoM estimator for SISO systems. Next, we generalize the derived estimators in (22) and (27) to MIMO systems.

C. Extension to MIMO Systems

In this section, we assume both the transmitter and the receiver have multiple antennas. We keep the total power transmitted through multiple transmit antennas equal to that of single antenna transmitter in SISO systems. We also assume the delay at different receive antennas are all equal to d .

In order to extend the proposed estimators to MIMO devices, we need to find the accumulative SoM of the received samples at different receive antennas. Let us assume the transmit antennas at the transmitter are used for spatial multiplexing. That is, different signals (information) are sent through different antennas at the transmitter. This means that the SoM of the received samples at the receiver are simply the sum of the SoMs of the received samples coming from each transmit antenna. Using (3) and (4), we can write

$$\begin{aligned} \mathbb{E}\{|y_j^{(n)}[k]|^2|H_0\} &= \left(\sum_{i=1}^{m_t} \mathbf{H}_{ji} \mathbf{s}_i^{(n)} \right) [k] + \mathbf{w}_j^{(n)} [k] \\ &= \sum_{i=1}^{m_t} \sum_{l=a_k}^{b_k} \sigma_{s_i}^2 \sigma_{h_l}^2 + \sigma_n^2 \\ &= \sum_{i=1}^{m_t} \sigma_{s_i}^2 \sum_{l=a_k}^{b_k} \sigma_{h_l}^2 + \sigma_n^2 \\ &= \sigma_s^2 \sum_{l=a_k}^{b_k} \sigma_{h_l}^2 + \sigma_n^2 \end{aligned} \quad (28)$$

TABLE I
ESTIMATION TIME RATIO VERSUS n_x

Estimation time ratio $\backslash n_x$	64	128	256	512	1024
proposed / MLE [23]	0.5684e-03	0.3337e-03	0.3216e-03	0.1151e-03	0.0856e-03
proposed-weighted / MLE [23]	0.6201e-03	0.3616e-03	0.3642e-03	0.1425e-03	0.1040e-03
proposed / TM [20]	2.4730	2.1463	2.0151	1.8118	1.6226
proposed-weighted / TM [20]	2.6983	2.3253	2.2821	2.2424	1.9709

where the last equality comes from the fact that $\sum_{i=1}^{m_t} \sigma_{s_i}^2 = \sigma_s^2$, and a_k and b_k are given in (15). Hence, the theoretical SoMs for each received sample at each antenna j remain the same as the one in Theorem 1.

Similar to our discussions for SISO systems, the empirical SoMs of the received samples can be derived as

$$B_{d,L}^-[k] \triangleq \frac{1}{m_r \lfloor (L-k+1)/n_s \rfloor} \times \sum_{j=0}^{m_r-1} \sum_{r=0}^{\lfloor (L-k+1)/n_s \rfloor - 1} |y_{\text{rec}}^j[k + rn_s]|^2$$

$$\forall k \in \{0, 1, \dots, n_s - 1\}.$$

where y_{rec}^j denotes the vector of the received samples at the j th receive antenna. Similarly, for $d > 0$, one can write

$$B_d^{\text{noise}} = \frac{1}{d} \sum_{j=0}^{m_r-1} \sum_{r=0}^{d-1} |y_{\text{rec}}^j[r]|^2. \quad (29)$$

and

$$B_{d,L}^+[k] = \frac{1}{m_r \lfloor (L-k+1)/n_s \rfloor} \times \sum_{j=0}^{m_r-1} \sum_{r=0}^{\lfloor (L-k-d+1)/n_s \rfloor - 1} |y_{\text{rec}}^j[k + d + rn_s]|^2$$

$$\forall k \in \{0, 1, \dots, n_s - 1\}. \quad (30)$$

One can then write the TO estimator for the MIMO systems as

$$\hat{d} = \underset{d \in \mathcal{D}}{\text{argmin}} \sum_{k=0}^{n_s-1} \frac{1}{F[k + |d|]} |B_{d,L}^-[k] - M_0[k + |d|]| u[-d]$$

$$+ \left(\frac{1}{F[k]} |B_{d,L}^+[k] - M_0[k]| + \frac{1}{F^{\text{noise}}} |B_d^{\text{noise}} - \sigma_n^2| \right) u[d-1] \quad (31)$$

One can set the values of $F[i]$ s and F^{noise} s to 1 to obtain the unweighted estimator for MIMO devices. Equation (31) as the most general form of the proposed TO estimators for MIMO devices is expanded in (34) in Appendix.

Deriving an upper bound for the complexity of the TO estimator proposed in (31) is very easy. It should be noted that the weights $F[i]$ s and F^{noise} s can be calculated once before any signal reception; hence, the complexity of the SoM estimator in (31) is equal to $\mathcal{O}(m_r L)$ which is significantly lower than the one in [23]. Table I shows the ratio of estimation times, i.e., the time it takes to estimate the delay, for the proposed estimators compared to that of MLE [23] and TM [20] for different values of n_x . As can be seen, our proposed estimators offer orders

of magnitude lower computational complexity compared to the best NDA estimator in the literature, MLE [23], while achieving similar lock-in probability, shown in simulation results section, which confirms the energy-savings at the receiver side. On the other hand, the proposed estimators offer the same order of complexity to that of TM [20], while achieving significantly higher lock-in probability as confirmed in simulation results section.

IV. SIMULATION RESULTS

In this section, we investigate the performance of the proposed MoM and its weighted version, i.e., WMoM, through extensive simulations under various scenarios.

A. Simulation Setup

We consider a ZP-OFDM device in a highly-selective (frequency and time) Rayleigh channel. For data transmission, 128-QAM modulation scheme is used. If otherwise not specified, the number of data symbols is $n_x = 128$ and the number of zero-padding samples are $n_z = 12$. The number of received OFDM symbols, i.e., N , used for estimation is 10. The transmit power is set to $\sigma_x^2 = 1$.

The sampling frequency at the receiver is set to $f_{sa} = 10^9$ samples per second. A multipath fading channel with maximum delay spread of $\tau_{\text{max}} = 10$ ns, equivalent to $n_h = 10$ taps, is assumed. An exponential decaying function, i.e., $\sigma_{h_l}^2 = \beta_1 \exp(-\beta_2 l)$, $l = 0, 1, \dots, n_h - 1$, where $p_h = \sum_{l=0}^{n_h-1} \sigma_{h_l}^2 = 1$ is considered for the delay profile of the fading channel. The maximum Doppler spread of the channel is assumed to be $f_D = 150$ Hz. Depending on the choice of the carrier frequency, the given maximum Doppler shift, and the given number of channel taps, the described channel could be a device in hands of a pedestrian in an urban environment or a sensor attached to a very high speed vehicle.

An additive white Gaussian noise (AWGN) is considered and modeled as a zero-mean complex Gaussian random variable with variance σ_w^2 derived based on the value of E_b/N_0 . The TO affecting the system is modeled as an integer random variable uniformly distributed in the range $d \in [-30, 30]$. The number of Monte Carlo realization is set to 10^4 for all simulation setups. A strict performance measure, i.e., lock-in probability, is used to measure the effectiveness of the proposed estimators. Lock-in probability here refers the probability that the estimated TO is equal to the actual TO. Hence, any nonzero estimation error is considered a missed estimation.

TABLE II
LOCK-IN PROBABILITY OF THE PROPOSED ESTIMATORS UNDER VARIOUS SPEEDS

Estimation method \ Scenario (km/h)	Pedestrian (≈ 1.5)	high-speed car (≈ 113.3)	light airplane (≈ 383.1)
proposed-weighted	0.9536	0.9529	0.9601
proposed	0.6875	0.6847	0.6885

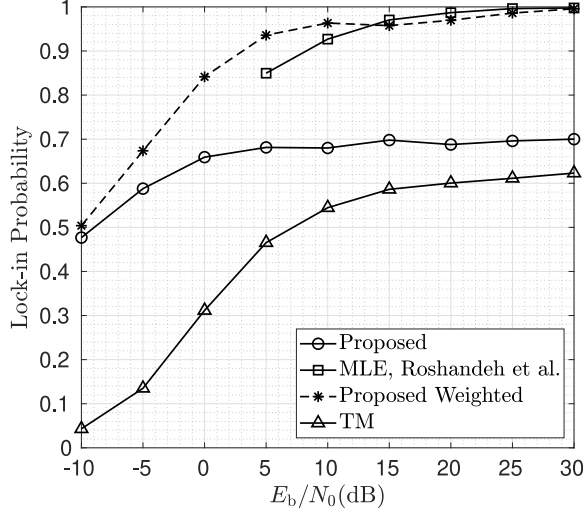


Fig. 5. Lock-in probability versus E_b/N_0 .

B. Fading Channel Models

There are various practical fading channel models, such as extended pedestrian A (EPA), extended vehicular A (EVA), IEEE standard 802.11 models and more [32], [33]. Such models mainly differ in the number of channel taps, maximum Doppler shift (relative speed), power profile, and others. Instead of simulating each model separately, we study the effect of fundamental parameters, such as Doppler shift, number of taps and others on the performance of the proposed estimators in various simulation figures. For example, for EVA, one of the main differences is the maximum Doppler shift which affects the relative speed of the transmitter and receiver. The effect of maximum Doppler shift is considered in Fig. 6. Another major difference among these models is the delay spread and the number of channel taps in order to account for a richer fading environment. The effect of such parameter is considered in Fig. 8.

C. Simulation Results

We compare the performance of the proposed MoM estimator and its improved weighted version, WMoM, versus the current state-of-the-art maximum likelihood estimator proposed in [23], i.e., MLE, and another estimator in [20], i.e., transition metric TM. The lock-in probability of the proposed estimators versus MLE and TM for different values of E_b/N_0 are shown in Fig. 5. In this figure and for comparison purposes only, we use the simulation setup in [23], and set $f_{sa} = 10^6$ and $n_z = 15$. As seen, the proposed weighted estimator outperforms the current state-of-the-art maximum likelihood estimator for low SNR region while having significantly lower computational complexity. Moreover, the proposed weighted

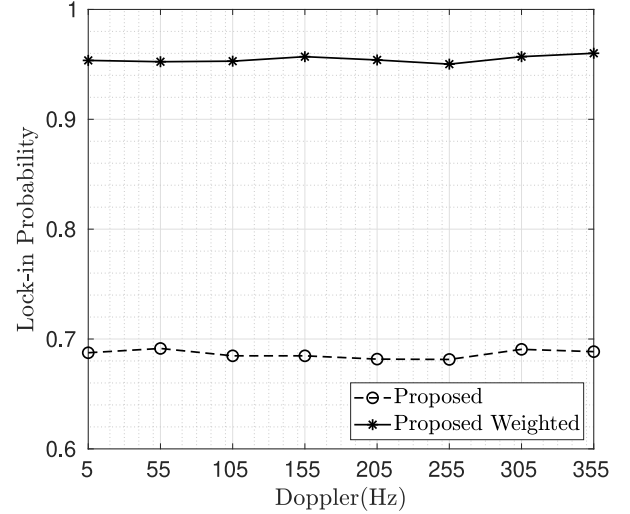


Fig. 6. Lock-in probability versus maximum Doppler spread of the fading channel at 15 dB E_b/N_0 .

estimator asymptotically reaches the same lock-in probability as of MLE's, in high SNRs. As shown, there is a performance gap between the proposed estimators and TM while WMoM having the highest probability of lock-in amongst them. This is due to the fact that WMoM takes advantage of the statistical information of the channel. The performance of the WMoM improves as E_b/N_0 increases because our confidence, modeled as the weights, in the derived theoretical second-order moment, and its closeness to the actual second-order moment of each received sample increases. However, the importance of the proposed estimators manifest itself in lower SNRs, i.e., 0 to -10 dB, where the lock-in probability is still at or above 50% compared to TM with negligible lock-in probability. This figure confirms our intuition that a weighted estimator leveraging the variance of the second order moments can bring the lock-in probability closer to that of MLE.

The effect of the maximum Doppler spread in Hz (speed of mobility) on the performance of the proposed estimators are shown in Fig. 6. As seen, the proposed estimators are relatively independent of the maximum Doppler shift (movement speed).³ This is mainly due to the fact that unlike [23] where the joint PDF of the received samples is approximated via independency assumption between channel taps, the proposed estimators do not heavily rely on the independency of different channel taps. Being relatively independent of the movement speed, makes the proposed estimators an appealing solution for very low to very high speed applications. A few of such scenarios are given in Table II.

³The trend remains similar even when the Doppler shift increases to larger values.

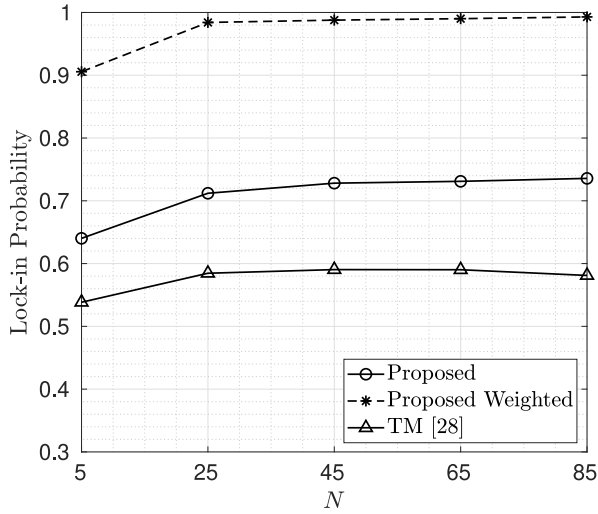


Fig. 7. Lock-in probability versus the number of observation vectors N at 15 dB E_b/N_0 .

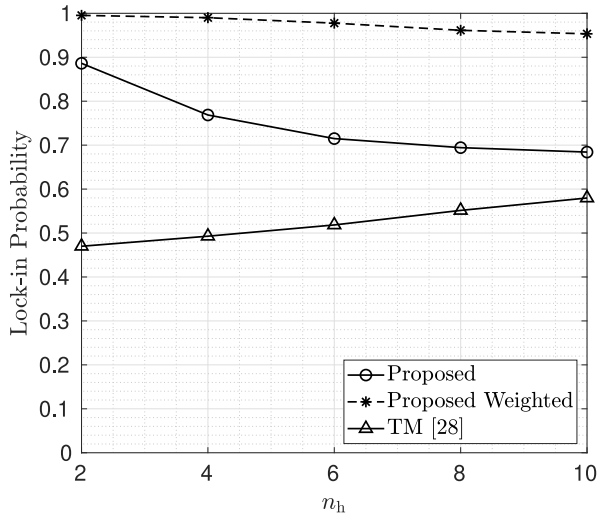


Fig. 8. Lock-in probability versus the number of channel taps n_h at 15 dB E_b/N_0 .

Fig. 7 illustrates the effect of the number of the observed OFDM symbols used for estimation, on the performance of the proposed estimators. As expected, the probability of lock-in increases as the number of observation samples used for estimation increases. This is due to the fact that the practical second-order moment becomes closer to its theoretical counterpart. The improvements, however, decreases as the number of observation symbols increases. This is because the new information that each new sample adds to the second-order moment and its variance decreases as the number of samples grows.

The performance of the proposed estimators and TM for different values of the number of channel taps n_h , is shown in Fig. 8. The lock-in probability of the proposed estimators decreases as the number of channel taps increases. This is because the deviation of the theoretical SoM in (14) from the actual value increases as the number of channel taps increases. As expected, this decrease in lock-in probability is heavier in unweighted version where the only information

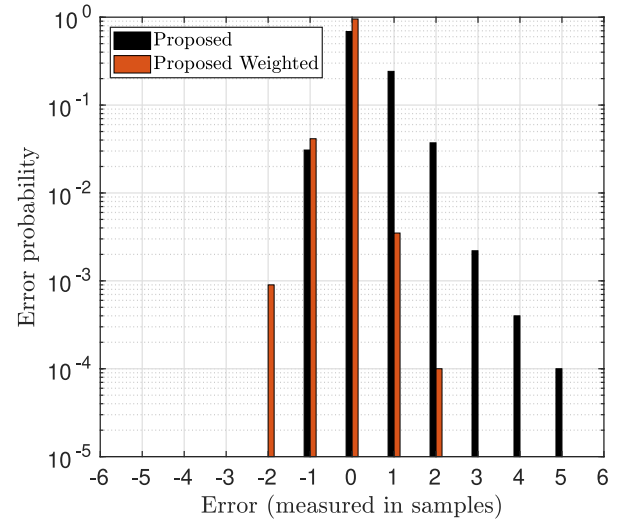


Fig. 9. PMF of the synchronization error at 15 dB E_b/N_0 .

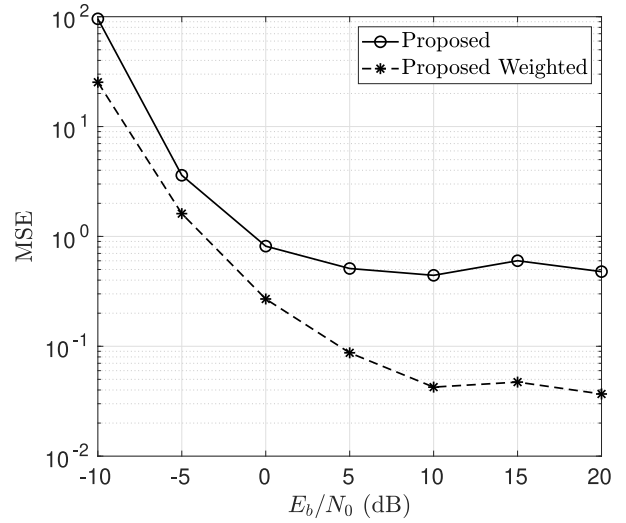


Fig. 10. MSE versus E_b/N_0 .

is used for estimation is (14), unlike WMoM that leverages extra statistical information, i.e., the variance of second-order moments.

The empirical probability mass function (PMF) of the synchronization error for the proposed estimators are shown in Fig. 9. As seen, the PMF of the error for WMoM estimator is relatively unbiased, while the MoM estimator is biased toward positive values. Notice, however, that the estimation error for WMoM is limited to a range of maximum two neighbouring samples from each side which results in low mean-squared errors (MSE) as shown in Fig. 10. Fig. 10 also shows an average estimation error of less than two for very low SNR, i.e., -5 dB, for the WMoM estimator. Hence, modified versions of the WMoM where neighbouring samples of the estimated TO are also checked can be devised to achieve 100% lock-in probability.

We have depicted the performance of the proposed estimators versus that of MLE and TM in Fig. 11 for a Rician channel model, with 18 channel taps and K-factor equal

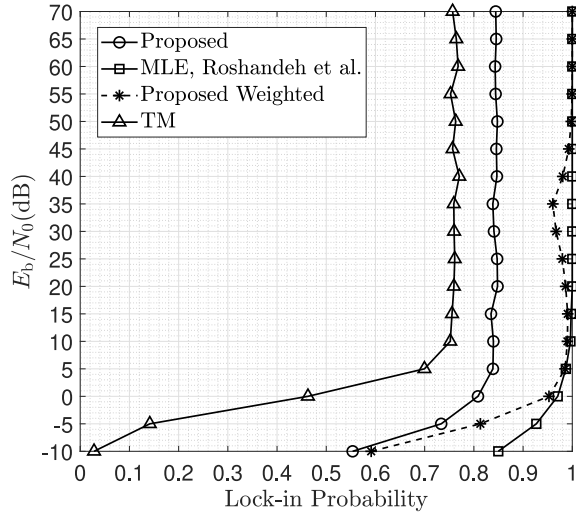


Fig. 11. E_b/N_0 versus Lock-in probability for a Rician channel.

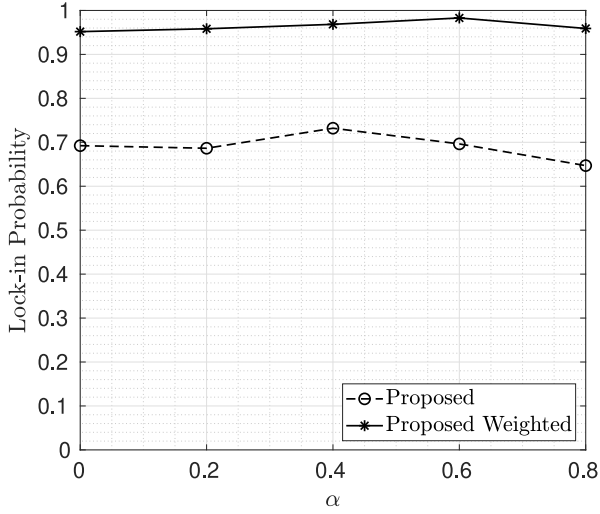


Fig. 12. Sensitivity of the proposed synchronization algorithms to PDP estimation error for $N = 10$ at 15 dB E_b/N_0 with uniform error distribution.

to 6, which resembles IEEE standard 802.11ah models E and F for indoor IoT scenarios, such as large offices and warehouses. As can be seen, compared to TM, the proposed estimators can achieve the lock-in probability of 0.7 while allowing the transmitter to save more than 10 dB in transmit power.

Fig. 12 shows the effect of PDP estimation error on the lock-in probability. The estimated PDP used for estimation is randomly uniform set to be either $\sigma_{h_k}^2 - \alpha\sigma_{h_k}^2$ or $\sigma_{h_k}^2 + \alpha\sigma_{h_k}^2$ where $\sigma_{h_k}^2$, $k = 0, 1, \dots, n_h - 1$, is the true PDP, and $\alpha \in [0, 0.8]$ denotes the PDP estimation error. As seen, the proposed estimators are relatively robust even for PDP estimation errors larger than 50%. Note, however, that the lock-in probability of the WMoM estimator remains above 90% even when the the PDP estimation error is about 80%. Fig. 13 shows the effect of the PDP estimation error in which the distribution of the error follows a normal distribution with variance equal to 0.2 and varying mean, μ , values. As seen, only a large error value, e.g., 80%, can affect the performance.

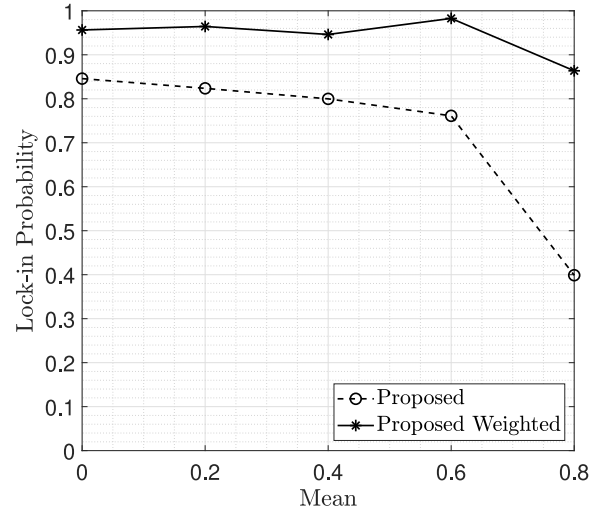


Fig. 13. Sensitivity of the proposed estimators to PDP estimation error for $N = 10$ at 15 dB E_b/N_0 with normal error distribution.

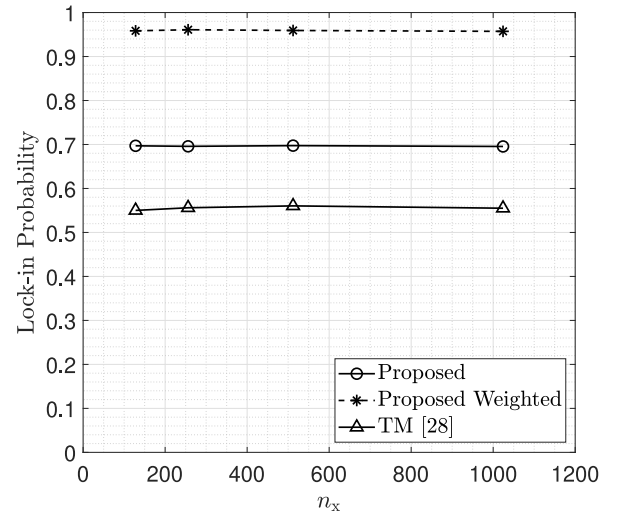


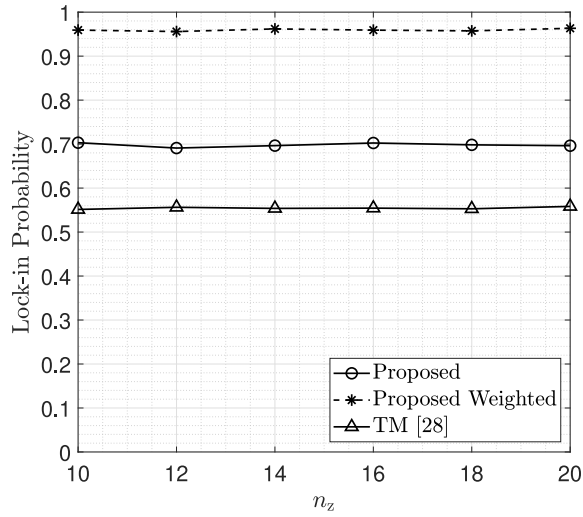
Fig. 14. Sensitivity of the proposed estimators to n_x .

Hence, the proposed estimators remain relatively robust even under this error distribution.

The effect of n_x and n_z on the performance of the proposed estimators are considered in Figs. 14 and 15, respectively. As seen, the proposed estimators perform robustly under various values of n_x and n_z .

V. CONCLUSION

In this article, we proposed two low-complexity NDA TO estimators based on MoM for ZP-based signaling in IoT devices. Unlike other estimators in the literature, the proposed estimators enjoy very low complexity, are feasible for very low SNRs, and have very high lock-in probability. Moreover, the proposed WMoM asymptotically reaches the performance of the state-of-the-art maximum likelihood estimator in the literature, proposed in [23], while having significantly lower complexity. Simulation results verify our analysis and demonstrate very high probability of lock-in for the proposed estimators as well as noticeable energy savings

Fig. 15. Sensitivity of the proposed estimators to n_z .

at both transmitter and receiver, making them suitable for implementation in low-power low-complexity IoT devices. Our approach can be also potentially applied to duty-cycled IoT devices that sleep and wake up periodically in order to save energy. We hope this work motivates practitioners to implement the proposed methods for future IoT technologies.

APPENDIX A PROOF OF THEOREM 1

Proof: Considering the fact that signal and noise are independent and by using (13) for $n \geq 0$, we have

$$\mathbb{E}\{|y^{(n)}[k]|^2 | H_0\} = \mathbb{E}\{|v^{(n)}[k]|^2\} + \mathbb{E}\{|w^{(n)}[k]|^2\}. \quad (32)$$

The expansion of $v^{(n)} = Hs^{(n)}$ is given in (33), shown at the bottom of the page. Taking the expected value of (33) and substituting the result in (32) yields (14) and (15). ■

APPENDIX B PROOF OF THEOREM 2

Proof: For notation simplicity and without loss of generality, we omit the conditioning on H_0 from equations. For mathematical simplicity, we consider $d = 0$ because nonzero delays are easily derived, mostly via a shift, from zero delay case. We have $\sigma_{|y_n[k]|^2}^2 = \mathbb{E}\{|y_n[k]|^4\} - \mathbb{E}\{|y_n[k]|^2\}^2$. The second term, $\mathbb{E}\{|y_n[k]|^2\}$ has been derived before, and so we need to only focus on $\mathbb{E}\{|y_n[k]|^4\}$. Note that

$$\begin{aligned} \mathbb{E}\{|y_n[k]|^4\} &= \mathbb{E}\left\{ (v^{(n)}[k] + w^{(n)}[k])(v^{(n)}[k]^* + w^{(n)}[k]^*) \right. \\ &\quad \left. (v^{(n)}[k] + w^{(n)}[k])(v^{(n)}[k]^* + w^{(n)}[k]^*) \right\} \\ &= \mathbb{E}\left\{ \left(|v^{(n)}[k]|^2 + v^{(n)}[k]w^{(n)}[k]^* + \right. \right. \\ &\quad \left. \left. v^{(n)}[k]^*w^{(n)}[k] + |w^{(n)}[k]|^2 \right)^2 \right\} \\ &\stackrel{a}{=} \mathbb{E}\left\{ |v^{(n)}[k]|^4 + 4|v^{(n)}[k]|^2|w^{(n)}[k]|^2 + \right. \\ &\quad \left. |w^{(n)}[k]|^4 + v^{(n)}[k]^2w^{(n)}[k]^{*2} + \right. \\ &\quad \left. v^{(n)}[k]^{*2}w^{(n)}[k]^2 \right\} \\ &\stackrel{b}{=} \mathbb{E}\{|v^{(n)}[k]|^4\} + 4\mathbb{E}\{|v^{(n)}[k]|^2\}\mathbb{E}\{|w^{(n)}[k]|^2\} \\ &\quad + \mathbb{E}\{|w^{(n)}[k]|^4\} \end{aligned} \quad (35)$$

where (a) comes from the fact that $v^{(n)}[k]$ and $w^{(n)}[k]$ are independent complex random variables, and $\mathbb{E}\{v^{(n)}[k]\} = \mathbb{E}\{w^{(n)}[k]\} = 0$ and (b) is easy to show that $\mathbb{E}\{v^{(n)}[k]^2w^{(n)}[k]^{*2} + v^{(n)}[k]^{*2}w^{(n)}[k]^2\} = 0$. We have derived $\mathbb{E}\{|v^{(n)}[k]|^2\}$ in previous sections. Also, since $w^{(n)}[k]$ is a complex random variable, we have $\mathbb{E}\{|w^{(n)}[k]|^2\} = \sigma_w^2$ and $\mathbb{E}\{|w^{(n)}[k]|^4\} = 2\sigma_w^4$. Hence, the problem further boils down to deriving $\mathbb{E}\{|v^{(n)}[k]|^4\}$. After some mathematical manipulation, one can show that

$$v_I^{(n)}[k] = \begin{cases} \sum_{u=0}^m h_I[k, u]s_I^{(n)}[k-u] - h_Q[k, u]s_Q^{(n)}[k-u], & 0 \leq k < n_h - 2 \\ \sum_{u=0}^{n_h-1} h_I[k, u]s_I^{(n)}[k-u] - h_Q[k, u]s_Q^{(n)}[k-u], & n_h - 1 \leq k \leq n_x - 1 \\ \sum_{u=m-n_x+1}^{n_h-1} h_I[k, u]s_I^{(n)}[k-u] - h_Q[k, u]s_Q^{(n)}[k-u], & n_x \leq k \leq n_x + n_h - 2 \\ 0, & n_x + n_h - 1 \leq k \leq n_s - 1 \end{cases} \quad (33)$$

$$\begin{aligned} \hat{d} = \operatorname{argmin}_{d \in \mathcal{D}} \sum_{k=0}^{n_s-1} & \frac{\left| \frac{1}{m_r \lfloor (L-k+1)/n_s \rfloor} \sum_{j=0}^{m_r-1} \sum_{r=0}^{\lfloor (L-k+1)/n_s \rfloor - 1} |y_{\text{rec}}^j[k + m_s]|^2 - \sigma_s^2 \sum_{l=a_k+|d|}^{b_k+|d|} \sigma_{h_l}^2 - \sigma_n^2 \right| u[d]}{\frac{13}{4}\sigma_s^4 \sum_{l=a_k+|d|}^{b_k+|d|} \sigma_{h_l}^4 + \frac{5}{2}\sigma_s^4 \sum_{r=a_k+|d|}^{r=b_k+|d|} \sum_{l=a_k+|d|, l \neq r}^{l=b_k+|d|} \sigma_{h_l}^2 \sigma_{h_r}^2 + 2\sigma_n^2 \sigma_s^2 \sum_{l=a_k+|d|}^{b_k+|d|} \sigma_{h_l}^2 + \sigma_n^4 - \frac{1}{4}\sigma_s^2 \left(\sum_{l=a_k+|d|}^{b_k+|d|} \sigma_{h_l}^2 \right)^2} \\ & + \frac{\left| \frac{1}{m_r \lfloor (L-k+1)/n_s \rfloor} \sum_{j=0}^{m_r-1} \sum_{r=0}^{\lfloor (L-k-d+1)/n_s \rfloor - 1} |y_{\text{rec}}^j[k + d + m_s]|^2 - \sigma_s^2 \sum_{l=a_k}^{b_k} \sigma_{h_l}^2 - \sigma_n^2 \right| u[d-1]}{\frac{13}{4}\sigma_s^4 \sum_{l=a_k}^{b_k} \sigma_{h_l}^4 + \frac{5}{2}\sigma_s^4 \sum_{r=a_k}^{r=b_k} \sum_{l=a_k, l \neq r}^{l=b_k} \sigma_{h_l}^2 \sigma_{h_r}^2 + 2\sigma_n^2 \sigma_s^2 \sum_{l=a_k}^{b_k} \sigma_{h_l}^2 + \sigma_n^4 - \frac{1}{4}\sigma_s^2 \left(\sum_{l=a_k}^{b_k} \sigma_{h_l}^2 \right)^2} \\ & + \frac{1}{2\sigma_n^2} \left| \frac{1}{d} \sum_{j=0}^{m_r-1} \sum_{r=0}^{d-1} |y_{\text{rec}}^j[r]|^2 - \sigma_n^2 \right| u[d-1] \end{aligned} \quad (34)$$

$$\begin{aligned}
& \mathbb{E}\{|v^{(n)}[k]|^4\} \\
&= \mathbb{E}\left\{\left(\left(\sum_{u=a}^b h_I[k, u]s_I^{(n)}[k-u] - h_Q[k, u]s_Q^{(n)}[k-u]\right)^2 + \left(\sum_{u=a}^b h_I[k, u]s_Q^{(n)}[k-u] + h_Q[k, u]s_I^{(n)}[k-u]\right)^2\right)^2\right\} \\
&= \mathbb{E}\left\{\left(\sum_{u=a}^b h_I[k, u]s_I^{(n)}[k-u] - h_Q[k, u]s_Q^{(n)}[k-u]\right)^4\right\} \\
&\quad + \mathbb{E}\left\{\left(\sum_{u=a}^b h_I[k, u]s_Q^{(n)}[k-u] + h_Q[k, u]s_I^{(n)}[k-u]\right)^4\right\} \\
&\quad + 2 \mathbb{E}\left\{\left(\sum_{u=a}^b h_I[k, u]s_I^{(n)}[k-u] - h_Q[k, u]s_Q^{(n)}[k-u]\right)^2\right. \\
&\quad \quad \left.\left(\sum_{u=a}^b h_I[k, u]s_Q^{(n)}[k-u] + h_Q[k, u]s_I^{(n)}[k-u]\right)^2\right\} \\
&\stackrel{(i)}{=} 2 \mathbb{E}\left\{\left(\sum_{u=a}^b h_I[k, u]s_I^{(n)}[k-u] - h_Q[k, u]s_Q^{(n)}[k-u]\right)^4\right\} \\
&\quad + 2 \mathbb{E}\left\{\left(\sum_{u=a}^b h_I[k, u]s_I^{(n)}[k-u] - h_Q[k, u]s_Q^{(n)}[k-u]\right)^2\right. \\
&\quad \quad \left.\left(\sum_{u=a}^b h_I[k, u]s_Q^{(n)}[k-u] + h_Q[k, u]s_I^{(n)}[k-u]\right)^2\right\} \\
&\quad \left.\left(\sum_{u=a}^b h_I[k, u]s_Q^{(n)}[k-u] + h_Q[k, u]s_I^{(n)}[k-u]\right)^4\right\} \quad (36)
\end{aligned}$$

where (i) comes from the fact that $-h_Q[k, u]$ and $h_Q[k, u]$, i.e., the quadrature components, have the same distribution, i.e., both are Gaussian random variables with zero mean and variance of $(\sigma_{h_u}^2/2)$. After some mathematical manipulations and using the facts that (i) in-phase and quadrature components are independent (ii) channel taps with different delays are independent, one can show that

$$\begin{aligned}
& \mathbb{E}\left\{\left(\sum_{u=a}^b h_I[k, u]s_I^{(n)}[k-u] - h_Q[k, u]s_Q^{(n)}[k-u]\right)^4\right\} \\
&= 2 \mathbb{E}\{s_I^{(n)}[k-u]^4\} \sum_{u=a}^b \mathbb{E}\{h_I[k, u]^4\} + \\
&\quad 12 \mathbb{E}\{s_I^{(n)}[k-u]^2\}^2 \sum_{r=a}^{r=b} \sum_{l=a, l \neq r}^{l=b} \mathbb{E}\{h_I[k, l]^2\} \mathbb{E}\{h_I[k, r]^2\} \\
&\quad + 6 \mathbb{E}\{s_I^{(n)}[k-u]^2\}^2 \left(\mathbb{E}\{h_I[k, u]^2\}\right)^2 \\
&= \frac{9}{8} \sigma_s^4 \sum_{u=a}^b \sigma_{h_u}^4 + \frac{3}{4} \sigma_s^4 \sum_{r=a}^{r=b} \sum_{l=a, l \neq r}^{l=b} \sigma_{h_l}^2 \sigma_{h_r}^2 + \frac{3}{8} \sigma_s^4 \left(\sum_{l=a}^b \sigma_{h_l}^2\right)^2 \quad (37)
\end{aligned}$$

Similarly, one can verify that

$$\begin{aligned}
& \mathbb{E}\left\{\left(\sum_{u=a}^b h_I[k, u]s_I^{(n)}[k-u] - h_Q[k, u]s_Q^{(n)}[k-u]\right)^2\right. \\
&\quad \left.\left(\sum_{u=a}^b h_I[k, u]s_Q^{(n)}[k-u] + h_Q[k, u]s_I^{(n)}[k-u]\right)^2\right\}
\end{aligned}$$

$$\begin{aligned}
&= 4 \mathbb{E}\{s_I^{(n)}[k-u]^2\}^2 \sum_{l=a}^{l=b} \mathbb{E}\{h_I[k, l]^4\} + \\
&\quad 16 \mathbb{E}\{s_I^{(n)}[k-u]^2\}^2 \sum_{r=a}^{r=b} \sum_{l=a, l \neq r}^{l=b} \mathbb{E}\{h_I[k, l]^2\} \mathbb{E}\{h_I[k, r]^2\} \\
&\quad + \left(4 \mathbb{E}\{s_I^{(n)}[k-u]^4\} - 8 \mathbb{E}\{s_I^{(n)}[k-u]^2\}^2\right) \\
&\quad \times \sum_{l=a}^{l=b} \left(\mathbb{E}\{h_I[k, l]^2\}\right)^2 \\
&= \sigma_s^4 \left(\sum_{u=a}^b \sigma_{h_u}^4 + \sum_{r=a}^{r=b} \sum_{l=a, l \neq r}^{l=b} \sigma_{h_l}^2 \sigma_{h_r}^2\right) \quad (38)
\end{aligned}$$

Substituting (37) and (38) in (36) and then in (35) yields (24). One can easily show that the variance of the SoM of a complex Gaussian random variable w with zero mean and variance σ_n^2 is

$$\sigma_{|w|^2}^2 = 2\sigma_n^2. \quad (39)$$

This completes the proof. ■

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