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















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The impact of lunar topography on the 21-cm power spectrum for grid-based arrays: Insights for the Dark-ages EXplorer (DEX)

S. Ghosh ^{1,★}, L. V. E. Koopmans ¹, C. Brinkerink ², A. R. Offringa ^{1,3}, A. J. Boonstra ^{3,†}, S. A. Brackenhoff ^{1,4}, E. Ceccotti ^{1,5}, J. K. Chege ¹, L. Y. Gao ^{1,6}, B. K. Gehlot ¹, L. I. Gurvits ^{7,8}, C. Höfer ¹, F. G. Mertens ^{1,9}, M. Mevius ³, S. Munshi ¹, A. Saxena ¹⁰, J. A. Tauber ¹¹, H. Vedantham ^{1,3}, S. Yatawatta ³ and S. Zaroubi ^{1,12}

¹Kapteyn Astronomical Institute, University of Groningen, PO Box 800, NL-9700 AV Groningen, the Netherlands

²Radboud Radio Lab, Radboud University, Heyendaalseweg 135, NL-6525 AJ Nijmegen, the Netherlands

³Netherlands Institute for Radio Astronomy (ASTRON), PO Box 2, NL-7990 AA Dwingeloo, the Netherlands

⁴Faculty of Electrical Engineering, Mathematics and Computer Science, Delft University of Technology, Mekelweg 4, NL-2628 CD Delft, the Netherlands

⁵INAF – Istituto di Radioastronomia, Via P. Gobetti 101, I-40129 Bologna, Italy

⁶Liaoning Key Laboratory of Cosmology and Astrophysics, College of Sciences, Northeastern University, Shenyang 110819, China

⁷Faculty of Aerospace Engineering, Delft University of Technology, Kluwerweg 1, NL-2629 HS Delft, the Netherlands

⁸Joint Institute for VLBI ERIC, Oude Hoogeveensedijk 4, NL-7991 PD Dwingeloo, the Netherlands

⁹LUX, Observatoire de Paris, PSL Research University, CNRS, Sorbonne Université, F-75014 Paris, France

¹⁰Kavli Institute for Cosmology, University of Cambridge, Madingley Road, CB3 0HA Cambridge, UK

¹¹Leiden Observatory, Leiden University, P.O. Box 9513, NL-2300 RA Leiden, the Netherlands

¹²ARCO (Astrophysics Research Center), Department of Natural Sciences, The Open University of Israel, 1 University Road, PO Box 808, Ra'anana 4353701, Israel

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ABSTRACT

The Dark Ages (DA) provides a crucial window into the physics of the infant Universe, with the 21-cm signal offering the only direct probe for mapping out the three-dimensional distribution of matter at this epoch. To measure this cosmological signal, the Dark-ages EXplorer (DEX) has been proposed as a compact, grid-based radio array on the lunar farside. The minimal design consists of a 32×32 array of 3-m dipole antennas, operating in the 7–50 MHz band. A practical challenge on the lunar surface is that the antennas may get displaced from their intended positions due to deployment imprecisions and non-coplanarity arising from local surface undulations. We present, for the first time, an end-to-end simulation pipeline, called SPADE-21 cm, that integrates a sky model with a DA 21-cm signal model simulated in the lunar frame and incorporating lunar topography data. We study the effects of both lateral (xy) and vertical (z) offsets on the two-dimensional power spectra across the 7–12 and 30–35 MHz spectral windows, with tolerance thresholds derived only for the latter. Our results show that positional offsets bias the power spectrum by 10–30 per cent relative to the expected 21-cm power spectrum during DA. Lateral offsets within $\sigma_{xy}/\lambda \lesssim 0.027$ (at 32.5 MHz) keep the fraction of Fourier modes with strong contamination (> 50 per cent of the signal) to less than 1 per cent, whereas vertical height offsets affect a larger fraction. This conclusion holds for the 21-cm window with $k_{\parallel} > 0.5 h \text{ cMpc}^{-1}$ over the range of $k_{\perp} = 0.003\text{--}0.009 h \text{ cMpc}^{-1}$.

Key words: methods: analytical – techniques: interferometric – software: simulations – Moon – (cosmology:) dark ages, reionization, first stars.

1 INTRODUCTION

The launch of the *James Webb Space Telescope* (*JWST*; J. P. Gardner et al. 2006) has enabled us to push the redshift limit of direct observations. With reports of high redshift candidates at $z \sim 20$

(H. Yan et al. 2022) to spectroscopically confirmed galaxies at redshift $z \sim 13$ (E. Curtis-Lake et al. 2023; B. Robertson et al. 2023), *JWST* has exceeded the observational capabilities of its predecessor, the *Hubble Space Telescope* (*HST*). Despite these observations, the infant Universe remains largely unexplored, including the crucial periods of the Dark Ages (DA; $200 \gtrsim z \gtrsim 30$) and the Cosmic Dawn (CD; $30 \gtrsim z \gtrsim 15$). The DA defines the period of cosmic history of the Universe before the first stars ‘turned on’ (D. Varshavovich & V. Khersonskii 1977; M. J. Rees 2000). During

* E-mail: soniaghosh@astro.rug.nl

† Deceased

this time, the underlying physics of the Universe was believed to be relatively simple and described by the physics of the standard Λ Cold Dark Matter (Λ CDM) model. This era was followed by the CD, a period marked by the emergence of the first sources of radiation invoking structural complexity of the Universe (J. R. Pritchard & A. Loeb 2010).

Even with the capabilities of *JWST*, observing sources beyond the early CD is not feasible. The very first luminous objects, although intrinsically massive and extremely bright, either appear faint owing to their large cosmological distances or they are obscured by the intergalactic medium (IGM). Moreover, probing the DA remains entirely unattainable due to the lack of any known luminous sources.

Currently, the spin-flip, hyperfine transition of neutral hydrogen (HI), with a wavelength of 21 cm in the rest frame, is the only known direct probe for studying the properties of the DA. It also provides one of the most comprehensive probes of the subsequent CD and the Epoch of Reionization (EoR) eras (R. Barkana & A. Loeb 2005; J. R. Pritchard & S. R. Furlanetto 2007). The spatial distribution of HI is measured indirectly by radio interferometers through its differential brightness temperature (δT_b) against the Cosmic Microwave Background (CMB) radiation. This signal not only depends on the density and velocity fields of HI, but also on the spin temperature and ionization fraction. By mapping δT_b across the sky and over frequency (a proxy of redshift and co-moving distance), we can reconstruct a three-dimensional (3D) map of HI.

During the DA, baryonic matter consists mostly of HI and helium, providing a relatively clean measurement window of cosmology compared to complex astrophysical probes of the present-day Universe. The Universe is also believed to be nearly homogeneous and isotropic during this period (L. V. Koopmans et al. 2021). Hence, the small spatial fluctuations of the 21-cm signal from the DA are predominantly influenced by the dark-matter field power spectrum and remain well within the linear regime, even at very small scales (see A. Liu & J. R. Shaw 2020). This implies that modelling and interpreting them does not require addressing the complex non-linearities and physical processes encountered in the more recent Universe. The 21-cm signal observations from the DA can thus provide a unique way to test the standard Λ CDM model. Any deviations from this well-established model could provide valuable insights into the physics of structure formation, by providing clues about the nature of dark matter (T. R. Slatyer 2013; H. Tashiro, K. Kadota & J. Silk 2014; T. R. Slatyer 2016a, b; K. Short et al. 2020; N. Hiroshima et al. 2021; R. Mondal, R. Barkana & A. Fialkov 2024), early dark energy (J. C. Hill & E. J. Baxter 2018), or any exotic physics (S. J. Clark et al. 2018; R. Thériault, J. T. Mirocha & R. Brandenberger 2021; Y. Yang 2022). The large number of linear modes measured by the 21-cm signal from the DA could also enable us to test the primordial non-Gaussianity present in the initial density field (A. Cooray 2006; J. B. Munoz, Y. Ali-Haïmoud & M. Kamionkowski 2015; P. D. Meerburg et al. 2017; T. Flöss et al. 2022), and even reveal the signatures of primordial gravitational waves (L. Book, M. Kamionkowski & F. Schmidt 2012; F. Schmidt, E. Pajer & M. Zaldarriaga 2014; K. W. Masui, U.-L. Pen & N. Turok 2017; R. Ansari et al. 2018).

Given the enormous potential of 21-cm observations to inform us about the physical processes in the infant Universe, constrain or estimate cosmological parameters, and distinguish between various theoretical reionization models, there have been worldwide efforts to design either low-frequency radio arrays mea-

suring spatial fluctuations of the 21-cm signal, or single radio antenna experiments measuring the spatially averaged 21-cm signal. Most of these current ground-based radio interferometers primarily focus on EoR, placing increasingly tighter constraints on the 21-cm signal power spectrum. However, a confident detection remains elusive. Although the EDGES experiment (J. D. Bowman et al. 2018) claimed a detection, this has been refuted by SARAS2 experiment with 95 per cent confidence (S. Singh et al. 2022). While the first generation of radio instruments are making substantial progress, the next-generation ground-based radio interferometers such as SKA¹ (Square Kilometre Array, P. E. Dewdney et al. 2009) are projected to achieve significantly higher sensitivity at these redshifts (CD and EoR), particularly through its SKA-Low component, thus offering better constraints on astrophysical models and making tomographic imaging possible (L. Koopmans et al. 2015).

One of the primary challenges to an accurate measurement of 21-cm fluctuations, irrespective of it being observed with ground-based or space-based experiments, is the synchrotron and free-free emission from galactic and extragalactic sources. This foreground emission exceeds the 21-cm signal from the EoR by 4–5 orders of magnitude in intensity (S. R. Furlanetto, S. P. Oh & F. H. Briggs 2006). The difficulty of foreground emission is further increased during the CD and DA, as the foreground emission surpasses the 21-cm signal by 6–7 orders of magnitude. This challenge is further compounded by the chromaticity of the instrument and the higher thermal noise at low frequencies.

In addition to these, ground-based experiments must also overcome other difficulties such as the Earth’s ionosphere and terrestrial radio frequency interference (RFI). The frequency-dependent, direction-dependent, and time-varying effects of the Earth’s ionosphere, which by means of refraction, diffraction, absorption, and through its own thermal emission, can potentially introduce chromatic leakage of the bright foregrounds into the 21-cm signal (L. Koopmans 2010; M. Sokolowski et al. 2015; A. Datta et al. 2016; M. Mevius et al. 2016; H. Vedantham & L. Koopmans 2016; E. Shen et al. 2021). Also, below the plasma frequency (~ 10 MHz) of the ionosphere’s F-layer, which generally depends on the electron density, the ionosphere is effectively opaque to radio waves. As a result, measuring the highly redshifted 21-cm signal from DA at a few tens of MHz and below is not feasible from the ground.

Furthermore, human-generated RFI poses an increasingly larger challenge for ground-based 21-cm experiments. RFI can originate from a range of sources such as FM towers, digital TV, audio broadcasts, satellite and aircraft communications. Even infrastructures such as wind and solar farms, transformers add to the interference (A. Offringa et al. 2013, 2015; A. Offringa, F. Mertens & L. Koopmans 2019; L. R. Whitley, A. Beardsley & D. Jacobs 2019; B. Gehlot et al. 2024; S. Munshi et al. 2025b). Of growing concern is also the continuing deployment of satellite constellations such as Starlink, which have been found to emit unintended radiation in the frequency band of interest (F. Di Vruno et al. 2023; D. Grigg et al. 2023; C. Bassa et al. 2024; X. Zhang et al. 2025). This issue is becoming increasingly severe and is expected to worsen in the coming years. Without effective mitigation strategies, such emissions could lead to substantial data loss or even the loss of entire frequency ranges.

¹<http://www.skatelescope.org>

This compounded set of challenges motivates us to investigate space-based or lunar-based 21-cm DA driven experiments that would eliminate the detrimental impact of the ionosphere and offer a more pristine radio-environment. The idea of deploying radio telescopes on the Moon (J. N. Douglas & H. J. Smith 1988), or in lunar orbit (H. U. Schzcerch & M. Hedgepeth 1968) has been under consideration for many decades now. In recent years, however, there has been renewed interest in returning to the Moon, driven by both scientific and technological motivations, as well as economic and strategic reasons. This has led to a wide range of proposals for lunar missions that aim to measure the radio emission sky (and sometimes subsurface), including experiments specifically focused on 21-cm cosmology. Among them are DARE (Dark Ages Reionization Explorer; J. O. Burns et al. 2012), LuSEE-Night² (Lunar Surface Electromagnetics Experiment; S. D. Bale et al. 2023), DAPPER (Dark Ages Polarimeter Pathfinder; J. O. Burns, S. Bale & R. F. Bradley 2019), LCRT³ (Lunar Crater Radio Telescope; S. Bandyopadhyay et al. 2021), FARSIDE (Farside Array for Radio Science Investigations of the Dark ages and Exoplanets, J. Burns et al. 2019), FarView (R. S. Polidan et al. 2024), DSL (Discovering the Sky at Longest wavelength mission; X. Chen et al. 2020), PRATUSH⁴ (Probing Reionization of the Universe using Signal from Hydrogen; M. Sathyanarayana Rao et al. 2023), LARAF (Large-scale Array for Radio Astronomy on the Farside; X. Chen et al. 2024), and TSUKUY-OMI (T. Matsumoto et al. 2024). Another notable space-based low-frequency mission concept is SEAMS (Space Electric and Magnetic Sensor; H. A. Tanti et al. 2023), aiming to explore the radio sky at ultra-long wavelengths. At the moment of writing, NCLE (Netherlands-China Low-Frequency Explorer; L. Chen et al. 2020), LFRS (Low Frequency Radio Spectrometer; X.-Y. Zhu et al. 2021), and ROLSES⁵ (Radio wave Observations at the Lunar Surface of the photoElectron Sheath; J. O. Burns et al. 2021; J. J. Hibbard et al. 2025) are the only missions to have conducted low-frequency radio observations from the Earth-Moon L2 point, and from the farside and near side of the Moon, respectively. These missions have operated in the frequency ranges of 80 kHz–80 MHz (NCLE), 100 kHz–40 MHz (LFRS), and 5 kHz–30 MHz (ROLSSES).

To explore the concept of a lunar-based low-frequency radio telescope, the European Space Agency (ESA) formed the Topical Team for an Astronomical Lunar Observatory (ALO) (ESA 2021; M. Klein Wolt, H. Falcke & L. Koopmans 2024), as part of ESA’s European Large Logistics Lander (EL3) programme. The topical team proposed the deployment of the Dark-ages EXplorer (DEX), an early-stage concept to build a compact, highly scalable, grid-based radio array on the lunar farside to study the early CD and DA using the 21-cm signal over a frequency range of 7–50 MHz (C. Brinkerink et al. 2025).

While compelling due to its shielded and almost ionosphere-free environment, a 21-cm experiment on the lunar farside (either on the surface or in orbit) comes with its own set of challenges. It is important to note that for any experiment, whether on Earth or

(around) the Moon, the received signal is often compromised by additional spectral features that can leak power into the modes that would otherwise be dominated by 21-cm signal. These artefacts can arise from several sources, including impedance mismatches in the analogue signal chain (A. Beardsley et al. 2016; A. Ewall-Wice et al. 2016; O. S. O’Hara et al. 2024), mutual coupling between antennas (N. S. Kern et al. 2019, 2020; A. T. Josaitis et al. 2022; E. Rath et al. 2024; O. S. O’Hara et al. 2025), digital processing artefacts (T. Prabu et al. 2015; N. Barry et al. 2019), incomplete sky model and beam modelling inaccuracies (N. Barry et al. 2016; A. Ewall-Wice et al. 2017; R. Byrne et al. 2019; N. Barry & A. Chokshi 2022; H. Gan et al. 2022; S. Munshi et al. 2024; S. A. Brackenhoff et al. 2025), and polarization leakage (D. F. Moore et al. 2013; K. Asad et al. 2016; S. Kohn et al. 2016; K. Asad et al. 2018). In addition to this, the ‘coupling’ of antenna properties to the medium over which it resides, especially if no ground plane is present, adds to the complication, regardless of the location of observation (R. F. Bradley et al. 2019; N. Mahesh et al. 2021; M. Spinelli et al. 2022; R. A. Monsalve et al. 2024; I. Hendricksen et al. 2025).

The development of radio interferometers on the lunar farside will be implemented in stages, starting with a few-metre scale single-element system to a few-kilometre scale array. The site selection for any array of radio antennas, especially those focused on 21-cm signal observations, is a crucial first step. For example, ensuring coplanarity (antenna elements in the same geometric plane) significantly reduces data processing costs by eliminating the need to account for w -term effects during imaging. To maintain this, the array requires a relatively flat surface, preferably having perpendicular deviations from the plane well below the observed wavelength. Therefore, a potential site depends on a number of factors, including metre-to-decametre scale roughness and gradients of the surface.

In addition, most of the rovers used for current Mars and Moon missions by NASA (National Aeronautics and Space Administration) have a maximum safety incline limit of 30° with engineering constraints on the wheel size and speed limit (Z. A. Le Conte, M. Elvis & P. A. Gläser 2023). These considerations are critical to ensure the optimal deployment and operation of the radio array. Since DEX is designed as a regular array on the lunar farside, its deployment could result in deviations in antenna positions and orientation from a regular coplanar grid, either due to local undulations, gradient of the surface or deviations in the rover trajectory. This poses a potential issue for calibration and imaging if the relative locations of the antennas are not determined with sufficient precision (i.e. if each antenna’s position is not known within a small fraction of a wavelength or if their orientations are uncertain). Furthermore, the resulting irregular array may prohibit the use of FFT-based techniques for correlation and imaging, which are essential for reducing both computational costs and the energy usage of the array on the Moon.

For example, lessons from ground-based radio instruments have shown that antenna feeds can deviate from their intended positions due to misalignments, rotations, or positional displacements (R. C. Joseph, C. M. Trott & R. B. Wayth 2018; N. Orosz et al. 2019). Additionally, slight variations in electronic gains, mechanical deformations, or even the surrounding environment can introduce non-uniform primary beams (T. Ansah-Narh et al. 2018; S. Choudhuri, P. Bull & H. Garsden 2021; H. Kim et al. 2022, 2023). By inducing unwanted spectral structure, these perturbations contaminate the Fourier modes used for 21-cm signal

²<https://www.colorado.edu/ness/projects/lunar-surface-electromagnetics-experiment-night-lusee-night>

³<https://www.nasa.gov/general/lunar-crater-radio-telescope-lcrt-on-the-far-side-of-the-moon>

⁴<https://www.rri.res.in/DISTORTION/pratush.html>

⁵<https://www.colorado.edu/ness/projects/radiowave-observations-lunar-surface-photoelectron-sheath-rolses>

measurements, thereby impacting the extraction of the desired signal.

In this paper, we focus on antenna position offsets, both lateral and vertical height, that may arise in lunar-based array during the practical construction and deployment of radio antennas on the surface of the Moon. By systematically introducing controlled perturbations, we assess how these impact the power spectrum estimation. We present, for the first time, a science-driven forward simulation for DEX (and any lunar surface-based array) that incorporates a sky model, a cosmological 21-cm signal model from the DA, as well as a framework for coordinate transformation to a lunar topocentric reference frame, and lunar topography data from the Lunar Reconnaissance Orbiter (LRO). This work presents an initial step towards understanding this particular systematic effect among the many challenges associated with constructing a large-scale radio array on the farside of the Moon. The findings will inform and guide decisions regarding the array configuration, deployment strategies, and selection of sites on the lunar surface.

This paper is organized as follows. In Section 2, we present an analysis of the lunar topography data obtained from LRO, relevant for a radio array. Section 3 presents an overview of DEX and its design consideration. The different cases of simulated antenna position errors are described in detail in Section 4. In Section 5, we describe our forward simulation pipeline. We focus on the impact of antenna position errors on the power spectrum for DEX across two spectral windows, 7–12 MHz ($z = 148.7$, Z148 hereafter) and 30–35 MHz ($z = 42.5$, Z42 hereafter), with results primarily focusing on Z42 in Section 6. Finally, Section 7 presents our summary and main conclusions derived from this study. Throughout this study, we adopt cosmological parameters from the Planck Collaboration 2016: $\Omega_m = 0.315$, $\Omega_b = 0.049$, and $H_0 = 67.7 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

2 LUNAR TOPOGRAPHY

With a diameter of ~ 3474 km, the Moon is the only natural satellite of Earth orbiting at an average distance of $\sim 384\,400$ km, tidally locked so that the same face always points towards us. Unlike Earth, it is enveloped only by an extremely tenuous ionosphere with electron densities $n_e \sim 0.1 - 1 \text{ cm}^{-3}$ (J. Halekas et al. 2018; H.-W. Shen, J. S. Halekas & A. R. Poppe 2023), giving a plasma cut-off frequency ≤ 30 kHz. Therefore, it is transparent to the long wavelengths needed to probe the 21-cm signal from the DA. The farside of the Moon also provides the best possible natural radio-quiet environment in the inner solar system. This was observed decades ago at low radio frequencies by Radio Astronomy Explorer 2 (RAE-2; J. K. Alexander et al. 1975), the WAVES instrument (J. L. Bougeret et al. 1995) and very recently by the Low-Frequency Interferometer and Spectrometer (LFIS) aboard the satellites Longjiang-1 and Longjiang-2 (J. Yan et al. 2023). Electrodynamical simulations have also demonstrated that terrestrial RFI is attenuated by at least 80 dB (N. Bassett et al. 2020), required to suppress RFI from Earth below the 21-cm signal, depending on the frequency and altitude of observation on the farside of the Moon.

To deploy low-frequency radio arrays such as DEX on the Moon, it is important to assess the physical characteristics of the lunar surface for rover traversability, safe landings, and stable antenna deployment. Beyond these engineering considerations, the primary motivation for characterising surface roughness is its potential to introduce systematic effects, thereby impacting the

accuracy of the measurement of the DA 21-cm signal. A critical aspect of this is the scale-dependent surface roughness, which affects both engineering design and the scientific return of the instrument. The surface of the Moon is broadly divided into two distinct types. Maria, which are basaltic plains formed by volcanic activity, and highlands, which are older, elevated, and more heavily cratered regions. Investigation of the topographic roughness of the lunar surface at scales from metres to tens of kilometres using the LRO Lunar Orbiter Laser Altimeter (LOLA; D. E. Smith et al. 2010), and Kaguya Laser Altimeter (H. Noda et al. 2009) digital elevation models (DEMs) has shown that the lunar highlands are generally much rougher than the maria, and that young, large impact craters exhibit the steepest local slopes (M. A. Rosenburg et al. 2011; W. Cao, Z. Cai & Z. Tang 2015; M. A. Kreslavsky & J. W. Head 2016; Y. Cai & W. Fa 2020). However, the small-scale roughness characteristics remain poorly understood, highlighting the need for further high-resolution studies to support the design of lunar surface infrastructure, and identify deployment sites suitable for a DA 21-cm signal detection experiment.

2.1 Analysis of the surface topography

To quantify the surface topography of the Moon, we have mainly used the data taken by LRO, a NASA mission which has been collecting data since 2009. LRO carries a range of scientific instruments, and has been instrumental in supporting lunar landing site selection, including for the ongoing Artemis program, by providing detailed measurements of lunar topography, surface morphology, illumination conditions, thermal environment, and surface roughness. Notably, the Lunar Reconnaissance Orbiter Camera (LROC) enables production of high-resolution (about $0.5\text{--}2 \text{ m pixel}^{-1}$) digital terrain models (DTMs) using two Narrow Angle Cameras (NACs) that produce stereo observations (M. S. Robinson et al. 2010). This allows for detailed characterization of surface roughness on metre-scales, which is below the scale of the planned antenna elements for DEX. In this study, we select Mare Ingenii, an impact basin in the northwestern part of the South Pole-Aitken basin (SPA), as a representative site for the deployment of DEX. This mare comprises a largely flat surface with slopes less than 5° , bounded by steep crater walls exceeding 25° in inclination. Flat passageways between these walls offer navigable routes for rover traversability and, as concluded by Z. A. Le Conte et al. (2023), could feasibly host a 200 km radio array. However, the final site selection will depend on future analyses using higher-resolution topography data and a broader range of engineering and scientific criteria. A detailed discussion of the selection criteria is presented in Section 3.2.3.

2.1.1 Topography dataset

We use the LROC NAC DTM products of Mare Ingenii for first-order characterization of surface roughness. The generated DTMs are archived by Arizona State University (ASU)⁶ within NASA's Planetary Data System (PDS), where they are provided in the Gridded Data Record (GDR) format and projected using an equirectangular coordinate system. The spatial resolution of the data is 2 m, covering $\sim 0.2^\circ$ in the longitude direction, and $\sim 1.2^\circ$ in the latitude direction with total surface area covered by this

⁶Available at: https://data.lroc.im-ldi.com/lroc/rdr_product_select

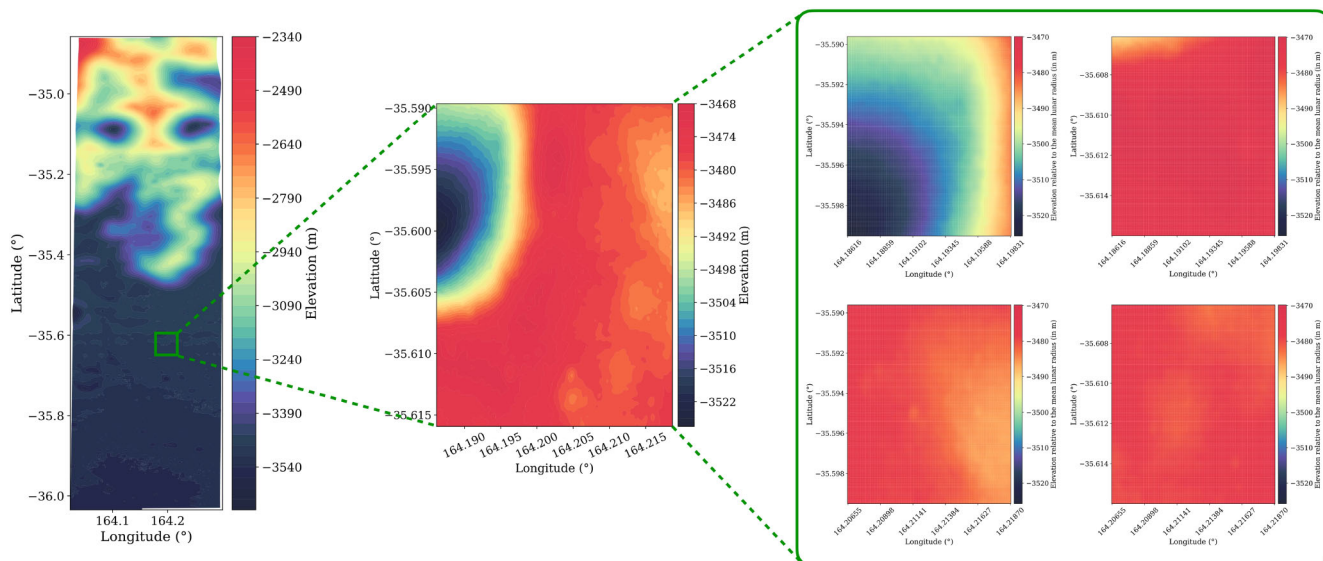


Figure 1. Left panel: LROC NAC DTM of the Mare Ingenii, with a spatial resolution of 2 m. The colourbar indicates elevation in metres measured relative to the mean lunar radius (~ 1737.4 km). Middle Panel: The region selected from the full DTM for this work. Right panel: The selected region in the middle panel is further divided into four sub-regions (Surface 1: top left, Surface 2: top right, Surface 3: bottom left, Surface 4: bottom right), each spanning an area ~ 0.09 km².

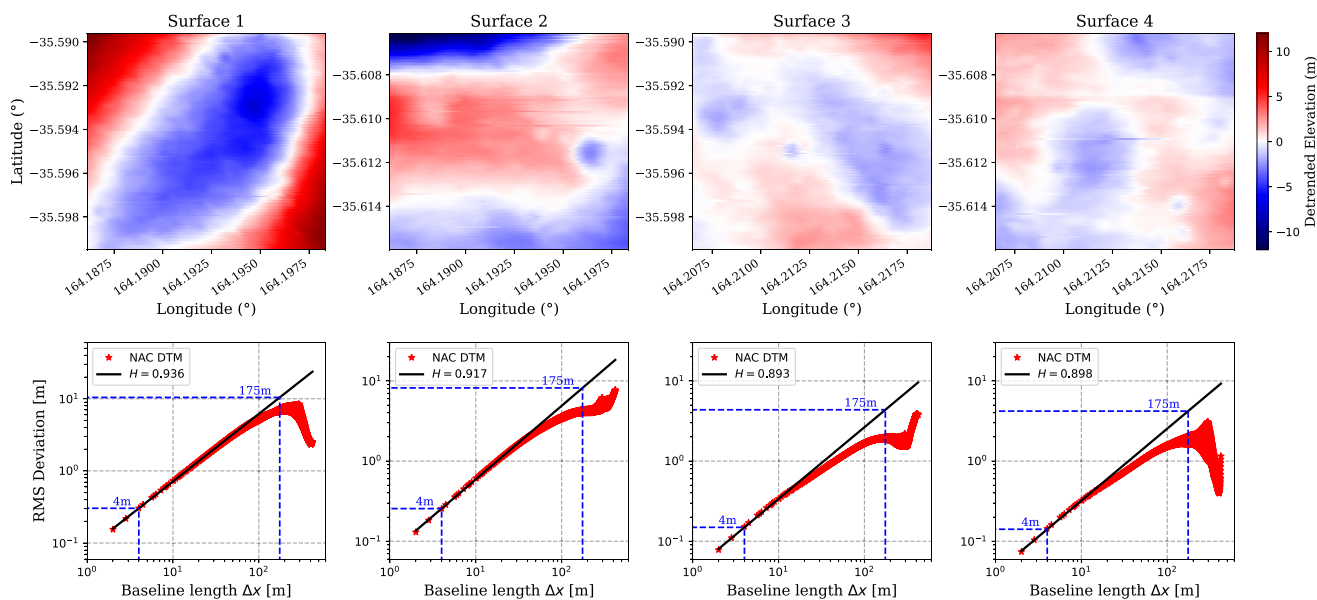


Figure 2. Top row: The four sub-surfaces that have been detrended by fitting a 2D plane such that the small-scale topographic features are highlighted. Bottom row: The observed structure functions (or devigrams) of the detrended surfaces. The derived Hurst exponent, H ranges from 0.898 to 0.936, indicating persistent and self-affine behaviour. It is observed that at 4 m scale, the RMS deviation of the elevation lies between 0.14 m and 0.30 m, while at ~ 175 m scale, they reach up to 7 m. Here, the NAC DTM data points are shown in red, and the fitted power-law model in black.

dataset being 242.4 km². However, for the purpose of this study, we have visually selected a smooth region spanning 0.64 km² from this dataset. This region was further subdivided into four sub-regions as shown in Fig. 1, each with an area ~ 0.09 km², for a comparative analysis of their surface roughness. We note that the process of selecting ‘smooth’ sites can be automated and extended to higher resolution datasets, but that is beyond the scope of this work.

2.1.2 Topography roughness parameter

Prior to the analysis, the topographic data are detrended by fitting and subtracting a plane from each of the subregions, such that they have a mean value of zero. This step is essential to isolate local surface undulations from broader regional trends, thereby allowing a more accurate characterization of small-scale roughness. The top row of Fig. 2 shows the detrended elevation maps of the four surfaces.

Natural surfaces are often modelled as stationary Gaussian random fields. The root-mean-square (RMS) difference in elevation (height) between two points separated by distance r can often be well-described by a power law r^H , where $H \in [0,1]$ is the Hurst exponent (M. K. Shepard et al. 2001). We calculate the RMS height difference (also known as RMS deviation) as a function of separation r as follows:

$$\sigma_H(r) = \sigma_s \left(\frac{r}{r_s} \right)^H, \quad (1)$$

where σ_s is the RMS deviation at a reference distance r_s between two points on the surface. We then analyse the resulting de-viogram or structure function to characterise the scale-dependent surface roughness. To limit sample variance and edge effects, it is generally recommended that the maximum baseline (spatial separation between two points on the surface) in the structure function be restricted to less than one-tenth of the total profile length (M. K. Shepard et al. 2001). The Hurst exponent is then estimated as the slope of a linear fit to the log–log plot (see the bottom row of Fig. 2). A larger Hurst exponent, for a fixed r_s , implies higher degree of spatial correlations across scales, i.e. nearby points remain similar in elevation over longer distances. M. K. Shepard et al. (2001); M. A. Rosenburg et al. (2011) note that the scaling behaviour often deviates from linearity at larger scales, therefore the linear fit is applied from the sampling interval (i.e. the resolution of the data) up to an outer scale, beyond which the scaling behaviour no longer follows a power-law but in general flattens off. This outer scale maybe interpreted as the characteristic spatial scale at which dominant processes that form or modify the surface undergo a transition (M. K. Shepard et al. 2001). To quantify the goodness of fit, we use the Pearson correlation coefficient with a threshold value of 0.999 to identify the valid linear regime following Y. Cai & W. Fa (2020).

2.1.3 Topography roughness results

We compute the value of H for the four detrended surfaces, and find no discernible difference among them, with values ranging from 0.89 to 0.93. These values are consistent with those reported in the bottom row of Fig. 3 of Y. Cai & W. Fa 2020, and indicate that the surfaces have a similar and strong spatial correlation. Over small geographic separations, as in the present study, this is expected due to the shared geological and morphological context. However, it is well established that different (widely separated) regions on the Moon can exhibit significantly different scaling behaviours of H (M. A. Rosenburg et al. 2011; Y. Cai & W. Fa 2020; M. K. Barker et al. 2025). We observe that Surfaces 1 and 2, show slightly higher values of H , suggesting a marginally greater degree of correlation in their scaling behaviour compared to Surfaces 3 and 4. In the RMS deviation plots (or structure function) shown in Fig. 2, we observe that towards larger scale, the fitted power-law model (in black) begins to diverge from the NAC DTM data points (in red). This divergence indicates the outer scale beyond which the surface no longer exhibits self-affine fractal behaviour, and may instead flatten, saturate, or exhibit more complex topographic behaviour, indicating a change in the nature of surface variation. A self-affine surface is one whose statistical properties remain invariant under anisotropic scaling. The outer scale varies from approximately 100 to 80 m for Surfaces 1 and 2, and 20 m for Surfaces 3 and 4, respectively. If the structure function flattens at the outer scale, then baselines longer than this

Table 1. Fractal roughness characterization of the four surfaces, showing the Hurst exponent H , and RMS deviation at reference scale of 4 m ($\sigma_{4\text{m}}$), and 175 m ($\sigma_{175\text{m}}$).

| Surface | H | $\sigma_{4\text{m}}(\text{m})$ | $\sigma_{175\text{m}}(\text{m})$ |
|---------|-------|--------------------------------|----------------------------------|
| 1 | 0.936 | 0.30 | 7.02 |
| 2 | 0.917 | 0.26 | 4.12 |
| 3 | 0.893 | 0.15 | 1.91 |
| 4 | 0.898 | 0.14 | 1.81 |

Table 2. Simulated properties of DEX.

| Parameter | Value |
|------------------------------|-------------------------------------|
| Phase centre | RA = 23.052060°, Dec. = −25.696129° |
| Bandwidth [in consideration] | 7–12 MHz [Z148] and 30–35 MHz [Z42] |
| Total frequency range | 7–50 MHz |
| Frequency resolution | 50 kHz |
| Number of channels | 100 |
| Total observation time | 5 min [snapshot] |
| Number of elements | 1024 |
| Antenna configuration | 32×32 regular square grid |
| Antenna length | 3 m [tip-to-tip] |
| Longest baseline | 175 m |
| Shortest baseline | 4 m |
| Location | Mare Ingenii [33° S, 163.5° E] |

scale sample approximately baseline-independent RMS height differences. However, if the surface shows long-range correlations or a different scaling law beyond the outer scale, the RMS height difference may continue to grow with baseline length, reflecting more complex topographic structure.

Next, we analyse σ_s at the scale of the smallest baseline ~ 4 m and at the scale of the array’s longest baseline ~ 175 m (see Table 1). Within the self-affine regime, σ_s varies from 0.14 m to 0.30 m at a 4 m length-scale. However, at larger scales (~ 175 m, longest baseline), the measured σ_s falls below the power-law extrapolation. We observe that across our scale of interest, σ_s of Surfaces 3 and 4 remain consistently lower than those of Surfaces 1 and 2. This implies that, despite their marginally lower H , Surfaces 3 and 4 are comparatively flatter in terms of absolute variations of elevation, and are thus more favourable for deployment. The values of H and σ_s in our analysis are first-order diagnostics that capture the distinct aspects of surface roughness, sufficient for the purposes of this study.

An optimal deployment site would, therefore, ideally combine both characteristics, namely, σ_s (setting absolute height variations) at a reference distance r_s and H (defining spatial scaling). The influence of H depends on the baseline distribution of the array. For experiments with sensitivity concentrated at small baselines, such as those targeting 21-cm fluctuations like DEX, higher values of H are favourable as they suppress variance on the smaller baselines and reduce the difference in height between neighbouring antennas. Lower H corresponds to larger fluctuations at short length-scales, which can increase biases on the small baselines. For long-baseline arrays designed for high-resolution imaging, relatively lower H is favourable because the power spectrum of the surface tends towards baseline-independence, so the error contribution is nearly uniform across all scales. At all times, σ_s , must be low to ensure that the induced bias remains within tolerance across the baseline length

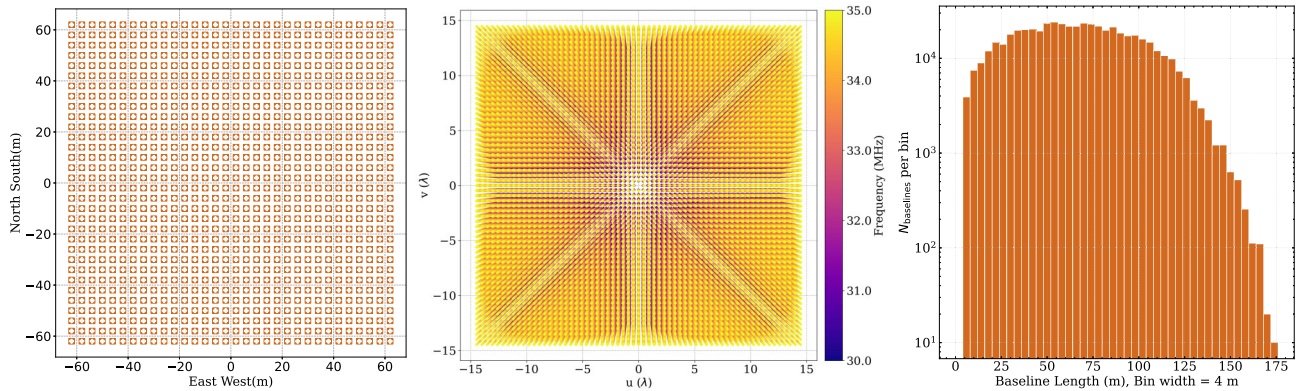


Figure 3. Left panel: Array layout of the DEX configuration used for the simulation. DEX is planned to be a compact array with 1024 antenna elements, arranged on a regular grid, on the farside of the Moon. Middle panel: The corresponding instantaneous uv coverage of the array for a simulated snapshot of 5 min within the Z42, demonstrating significant redundancy of the array in uv sampling. A colour gradient is used to indicate the distribution of independent uv samples across the full spectral window. The observation was simulated from Mare Ingenii at RA = 23.052060°, Dec. = -25.696129°, corresponding to an observation time of 16:22:30 UTC on 2040 July 30. Right panel: A histogram of the baseline lengths ($|u| = \sqrt{u^2 + v^2}$) distribution (in m) with a bin width of 4 m.

of interest. This emphasizes that both the amplitude and scale-dependence of roughness must be considered when evaluating the feasibility of a surface for hosting low-frequency lunar arrays such as DEX. We do not discuss the geological processes responsible for affecting metre-scale surface roughness, as such analysis lies beyond the scope of this work. Note that in applying a fractal structure function description, we assume the absence of large discrete features such as boulders, which may violate the underlying scaling behaviour.

3 THE DARK AGES EXPLORER

The 21-cm signal from the DA is expected to be extremely faint, with brightness temperature fluctuations of the order of only a few mK^2 at $k \sim 0.1\text{Mpc}^{-1}$. To reach the desired sensitivity level in the presence of very bright polarized foregrounds, it is estimated that a minimum of 128×128 , or more dipoles are necessary for any radio array targeting detection via power-spectrum measurement, as analysed by L. V. Koopmans et al. (2021). Currently in the early conceptual phase, DEX is envisioned as a compact, grid array of zenith-pointing antennas with a near-unity filling factor, shown in Fig. 3, deployed on the farside of the Moon (ESA 2021). This array is primarily designed to observe the Universe in the ultra-long wavelength domain using the highly redshifted 21-cm signal. In its basic and minimal configuration, DEX currently consists of 32×32 cross-dipole antennas of length 3 m operating in the radio frequency band of 7–50 MHz. In future iterations of the array design, there should be scope for expansion in both the number of antennas and operational bandwidth (to 100 MHz), which would enable improved sensitivity and enhanced angular resolution. Consequently, the specific configuration parameters used in this work are subject to change as the project continues to mature.

3.1 Correlation architecture

The data rates increase quadratically with increasing number of antennas when computing the cross-correlated voltages or the visibilities using a standard FX (Fourier Transform followed by cross-multiplication) correlator. This requires considerable com-

puting resources, output data bandwidth, and power consumption. Addressing these challenges requires careful consideration, particularly for any lunar surface-based or space-based missions, where available resources are inherently limited. Therefore, it requires the development of novel and efficient architectures for correlation, data processing, transfer, storage, and minimal power usage, while ensuring that important performance factors such as resolution and sensitivity are not compromised (D. C. Price 2024). N. Thyagarajan (2025) provides a comparative analysis of the different imaging architectures based on their computational cost efficiency for various planned and proposed radio arrays such as SKA-low, SKA-low-core, LAMBDA-I, CASPA, and FarView-core. One of the key findings is that imaging architectures based on the Fast Fourier Transform (FFT; J. W. Cooley & J. W. Tukey 1965) offer much greater computational efficiency for large, densely packed arrays across a wide range of observational cadences. This class of architectures, broadly referred to as *Direct Imaging*, avoids the computationally expensive step of correlating all antenna pairs individually (T. Daishido et al. 1991; M. Tegmark & M. Zaldarriaga 2009, 2010). As a result, it eliminates the need to transport and process large volumes of correlation data for subsequent imaging, although access to individual visibility data is lost.

The classification of *Direct Imaging* architecture naturally leads to the exploration of FFT correlators. Unlike the computational cost of the traditional FX and XF (cross-multiplication followed by Fourier Transform) architectures, which scale as $\sim N^2$, the FFT correlator reduces this to $\sim N \log N$, where N is the number of antennas. This improvement is achieved by using the relationship between the electric field measured on the ground and its corresponding image in the sky through a spatial Fourier transform. In its simplest implementation, this approach requires the antennas to be placed on a regular grid. A recent advancement in the direction of this alternative imaging and correlator scheme is the development of E-field Parallel Imaging Correlator (EPIC; N. Thyagarajan et al. 2017). EPIC uses the Modular Optimal Frequency Fourier (MOFF) algorithm (M. F. Morales 2011), eliminating restrictions on the antenna being physically placed on a nearly perfect grid. Instead, the voltages are electronically gridded based on the aperture illumination pattern of the individual

antennas, similar to visibility gridding in traditional correlation and imaging steps. We note that this method performs optimally when the array has a near-unity filling factor and closely approximates a grid layout.

In recent years, EPIC has been tested and commissioned at the LWA-SV station (J. Kent et al. 2019; H. Krishnan et al. 2023). Although EPIC is computationally efficient when applied to real data, a full science-driven forward simulation can be computationally demanding for DEX-like arrays. For example, generating stochastic electric field realizations at each antenna by coherently summing contributions from all sky directions across a wide frequency band can be computationally intensive, particularly for large- N arrays.

As a practical workaround for spatially uncorrelated stationary electric fields (and voltages), we adopted a visibility-based approach (as for FX correlator) for this work. We can do this, without loss of generality, because the images produced by a FFT correlator (as in EPIC) are mathematically identical, to a first order, to those produced from a standard FX correlator via the convolution theorem of Fourier Transform. This approach also ensures compatibility with established data formats and facilitates the use of well-tested software tools within the radio astronomy community.

3.2 Design consideration

The main factors influencing the observational capabilities of a radio interferometer are the field of view (FoV) of each antenna element (also called the primary beam), total effective collecting area and filling factor, observing frequency range and spectral resolution, the number and arrangement of the antenna elements forming baselines (G. Mellema et al. 2013; L. Koopmans et al. 2015). For DEX, these technical considerations are closely linked to the need for extremely high sensitivity to enable making a statistically significant detection of the extremely faint 21-cm signal from the DA at millikelvin (mK) levels.

3.2.1 Array configuration and spatial resolution

Compact, centrally concentrated configurations improve sensitivity by increasing sampling density and redundancy, as they provide a large number of visibility measurements per uv -cell within a given integration time. This principle has driven the design of interferometers aimed at detecting the 21-cm signal (J. D. Bowman, M. F. Morales & J. N. Hewitt 2006; A. Parsons et al. 2012; L. Koopmans et al. 2015). A high-filling-factor array, consisting of 1024 antenna elements, is considered the minimally useful configuration for DEX for 21-cm observations up to the early CD. Although this configuration defines the lower bound for 21-cm cosmology, smaller arrays may still support a range of complementary science cases. We also limit ourselves to this array size because simulating cross-correlated visibilities for larger arrays, such as 64×64 , 128×128 , 512×512 , is beyond our current computational capabilities.

L. V. Koopmans et al. (2021); R. Mondal & R. Barkana (2023); R. S. Polidan et al. (2024) predicted that an array spanning over 100 km^2 , with $\sim 10^6$ antenna elements, will be necessary to probe angular scales down to $k \sim 0.1 \text{ Mpc}^{-1}$ at frequencies corresponding to the redshifts ($z \sim 50$) of the DA. Such a large array on the Moon can only be realized through multiple stages of development and deployment. It is believed that although the current

configuration of DEX may not provide access to the DA redshifts assuming standard 21-cm signal models, it can still partially probe the early CD, allowing testing of various early star formation and cosmological models that predict the power spectrum for this epoch. Also, such a pathfinder array helps in investigating various aspects, such as the impact of systematics on the science case, antenna deployment techniques, and the implementation of the FFT correlator.

Based on electromagnetic simulations of several cross-dipole antenna models for DEX (Arts, private communication), we find that within the frequency range of the DA, a relatively short dipole of 3 m behaves spectrally better compared to a longer 5 m dipole, assumed earlier, which shows sharper features in its bandpass. Accordingly, in this work, we assume a simple 3 m cross-dipole (measured tip-to-tip). We set a minimum baseline of 4 m when measured from antenna centre to centre (1 m edge to edge). To isolate the effects of positional perturbations, we do not include a direction-dependent beam model in the visibility simulation. Instead, we adopt a first-order approximation of the primary beam effect by multiplying a frequency-independent $\cos^2 \theta$ beam model, where θ is the angular distance from the zenith, with the simulated sky map. The chromatic effects of the primary beam and mutual coupling between antenna elements will be addressed in detail in a subsequent paper. A more complete treatment that incorporates beam patterns may require revisiting the minimum baseline, as closely spaced elements can alter the effective beam response and impact the scientific performance of the array (G. Virone et al. 2018; N. S. Kern et al. 2019; B. McKinley et al. 2020; P. Bolli et al. 2022; D. Anstey et al. 2024). The maximum baseline for DEX is given by the diagonal which is $\sim 175 \text{ m}$.

The angular resolutions, characterized by the full width at half-maximum (FWHM) of the synthesized beam, are $\sim 14^\circ$ at 7 MHz and $\sim 1.9^\circ$ at 50 MHz. This relatively low resolution is a natural drawback of a compact array at such low frequencies, along with the presence of strong grating lobes in the point spread function (PSF) due to the regularity of the array. These shortcomings can be mitigated by adding ‘outrigger’ antennas, which improve angular resolution and enable instantaneous wide-field imaging by improving uv coverage (J. S. Dillon & A. R. Parsons 2016). However, these constraints do not fundamentally limit 21-cm power spectrum measurements since the power spectrum is estimated in Fourier space. Furthermore, compact arrays offer enhanced sensitivity and support redundant calibration strategies, which may help to achieve robust statistical measurements.

3.2.2 Bandwidth and spectral resolution

DEX is currently planned to operate over a broad frequency range spanning 7–50 MHz, thereby covering the eras of the DA ($z \sim 200$) and the early CD ($z \sim 27$). As discussed earlier, this frequency range may be extended to higher frequencies to overlap with current ground-based experiments, allowing for complementary measurements across different redshift regimes and improved cross-validation.

The frequency axis of 21-cm observation captures the spatial fluctuations of the 21-cm field along the line of sight (LOS). In a cylindrical two-dimensional (2D) power spectrum, which estimates the variance of the field, the spatial modes along the LOS are represented by the Fourier wavenumber, k_{\parallel} . The maximum measurable Fourier wavenumber, $k_{\parallel, \text{max}}$ is limited by the spectral

resolution, whereas the total bandwidth establishes the $k_{\parallel, \min}$, which determines the largest LOS scale of the 3D volume being probed.

The redshift evolution of the cosmological signal is strongly dependent on frequency. When calculating power spectra from observational data, the full bandwidth of the instrument is typically divided into narrower sub-bands. Therefore, the appropriate sub-band width must be chosen carefully, to ensure that the redshift evolution remains negligible within each sub-band. The evolution of the 21-cm signal over a fixed bandwidth during the DA is significantly more pronounced than during EoR, as the fluctuations trace the matter power spectrum (S. R. Furlanetto et al. 2006; W. Smith & J. C. Pober 2025). The 21-cm power spectrum during the DA is fully specified by the Λ CDM cosmological model. For a compact array like DEX, achieving high k values can only be reached by accessing higher k_{\parallel} values because the short baselines limit sensitivity to only the smallest k_{\perp} , the Fourier modes perpendicular to the LOS. As a result, sensitivity at large k values is dominated by LOS modes, which also provide a cleaner 21-cm window and enable improved calibration by allowing systematics to be distinguished from foregrounds. Based on the above arguments, we choose a spectral resolution of 50 kHz for this study, which balances sensitivity and computational cost.

We choose 5 MHz spectral windows with 100 channels, following R. Mondal & R. Barkana 2023, who showed that excluding the light-cone effect in the analysis causes a small increase in the error of the power spectrum for a 5 MHz bandwidth during DA. The two spectral windows considered, Z148 and Z42, are 7–12 MHz ($z \sim 201$ –117), and 30–35 MHz ($z \sim 45$ –39), respectively. Although Z148 corresponds to a very early epoch of the Universe and is of great scientific interest, it presents severe observational challenges. These include stronger foreground signals, increased thermal noise, Galactic self-absorption, and the requirement for a large array footprint to achieve the required signal-to-noise ratio at such longer wavelengths (or higher redshifts such as $z > 50$) (L. V. Koopmans et al. 2021). On the other hand, Z42 lies within a range that may be more observationally accessible.

3.2.3 Location

At mid-latitudes, row orientation of an array has minimal impact on the resulting uv coverage as the sky drifts at an angle relative to the local horizon. This angular drift ensures both east–west and north–south baselines sample a range of projected baselines over time. In contrast, near the equator, east–west rows enhance sensitivity via rotational synthesis, but north–south baselines experience minimal change in projected length. This produces slower fringe rates, which might be prone to systematics. We therefore prefer sites at mid-latitude. This choice is preferable for several other reasons: (i) locations near the poles are more prone to terrestrial RFI that can diffract around the lunar limb and reach high-latitude sites (N. Bassett et al. 2020), (ii) flat-mounted solar panels are less efficient near the lunar poles due to the low solar elevation which reduces direct solar incidence on horizontal panels, and (iii) at near-polar latitudes, reduced sky coverage limits the number of independent modes, making power-spectrum measurements increasingly cosmic-variance limited. Note that the rotation period of the Moon is about 27.3 d, the apparent motion of the sky and hence the timescale for rotation synthesis is roughly 28 times slower than on Earth.

Site selection must also consider additional factors such as relatively flat terrain, minimal physical hazards like boulders, craters, and ridges, as discussed in Section 2. The site selected for this study, Mare Ingenii, is located 33° south of the lunar equator and 163.5° east of the prime meridian (the meridian that points towards Earth in the Moon Mean Earth/Polar Axis, ME frame). The Moon ME frame has the Sub-Earth point, i.e. the location on the lunar surface directly facing Earth, at Longitude 0° and Latitude 0° (see Fig. A1). We emphasize that the selected site is chosen solely for demonstrative purposes, as this particular mare has been surveyed at high spatial resolution by the LRO in a localized region. A site located in the lunar southern hemisphere is exposed to the bright Galactic Centre during a significant portion of the lunar day. For operational deployment, a site in the lunar northern hemisphere would, therefore, be preferable, assuming that a sufficiently flat site can be identified that meets the engineering and scientific requirements of the array.

4 ANTENNA POSITION ERROR

In practice, even with high-resolution surface mapping of a given deployment site, there is always a risk that the antennas will deviate from their intended positions due to unresolved surface features. Positioning inaccuracies arising from mechanical tolerances and effects such as slippage during deployment by rovers further contribute to the deviation (R. Li et al. 2008; R. Gonzalez & K. Iagnemma 2018). These positional offsets might perturb the projected baselines of the array, thereby changing the geometric phase as a function of baseline and angular direction in the sky. To model the errors, we categorize the offsets into (i) the horizontal or xy direction (primarily arising from inaccuracies along rover trajectory and potential small obstacles); (ii) the vertical or z direction (primarily arising from roughness of local lunar surface). These perturbations are treated independently to isolate their individual impact on the performance of a grid-based array such as DEX.

In this section, we derive an analytical expression for the visibility error introduced by antenna position offsets, expressed in terms of baseline geometry and source direction. This derivation, when used in conjunction, supports the trends observed in our simulations in Section 5, and forms the basis for understanding how such errors propagate into the cylindrically averaged 21-cm power spectrum.

4.1 Analytical treatment

Considering a 3D coordinate system, with the baseline coordinates given by $\mathbf{b} = (u, v, w)$ in wavelength units, and the unit vector pointing towards the source is given by $\mathbf{s} = (l, m, n)$ with $n = \sqrt{1 - l^2 - m^2}$, the complex visibility function is given by

$$V(v, \mathbf{b}) = \iint A(\mathbf{s}, \nu) I(\mathbf{s}, \nu) \exp[-2\pi i \varphi] \frac{dl dm}{n}, \quad (2)$$

$$\varphi(v, \mathbf{b}, \mathbf{s}) = ul + vm + w(n - 1) \quad (3)$$

where $I(\mathbf{s}, \nu)$ is the sky brightness distribution as a function of direction cosines (l, m) and frequency ν , and $A(\mathbf{s}, \nu)$ is the primary beam response of the antenna. This expression assumes that the data have been phased to the phase centre $\hat{\mathbf{p}} = (0, 0, 1)$, such that the phase contribution from $\hat{\mathbf{p}}$ is zero for all baselines. Equation (2) forms the basis of the Radio Interferometric Measurement Equation (RIME; J. Hamaker, J. Bregman & R. Sault

1996; O. M. Smirnov 2011), which provides a general framework for modelling the response of an interferometer to the sky brightness distribution. In this study, visibilities are simulated using a perturbed array, while the analysis assumes a regular array configuration. The resulting inconsistency in the array configuration captures the effect of antenna position offsets, introducing perturbations in the baseline coordinates given by $\mathbf{b}' = (\Delta u, \Delta v, \Delta w)$ that may not be accounted for during calibration or imaging. The perturbed visibility function is then given by

$$V'(v, \mathbf{b}) = \iint A(\mathbf{s}, \nu) I(\mathbf{s}, \nu) \exp[-2\pi i \varphi'] \frac{d\Omega}{n}, \quad (4)$$

$$\begin{aligned} \varphi'(v, \mathbf{b}, \mathbf{s}) &= (u + \Delta u)l + (v + \Delta v)m + (w + \Delta w)(n - 1) \\ &= \varphi + \Delta\varphi, \end{aligned} \quad (5)$$

where $\Delta\varphi(v, \mathbf{b}, \mathbf{s}) \equiv \Delta u l + \Delta v m + \Delta w(n - 1)$ represents the geometric phase error due to positional offsets. For observations when the phase centre is at zenith and under the assumption of a coplanar array, such that a single delay correction can be applied, the w term and its associated geometric phase error reduce to zero for sources at zenith. In this case, the antenna position errors do not project along the line of sight and therefore do not contribute to visibility phase errors.

In our simulation, first, we define the antenna positions in the local East-North-Up (ENU) coordinate system, a topocentric 3D coordinate system centred at the array location. In this frame, the E -axis points east, the N -axis points north, and the U -axis points upward towards the local zenith. Next, we transform this offset to the Moon-Centred, Moon-Fixed (MCMF) (analogous to the International Terrestrial Reference System, ITRS) coordinate system with its origin at the Moon's centre. The Z -axis is aligned with the mean rotation axis of the Moon towards the geographic North Pole. The X -axis lies in the equatorial plane along the prime meridian (intersection of the lunar equator and the mean direction to the centre of the Earth), and the Y -axis points towards 90° east longitude along the equator such that XYZ is a right-handed system. For reference, Fig. A1 in Appendix A illustrates both the coordinate systems.

The final transformation projects the antenna offset into the uvw coordinate system, which is defined relative to the phase centre of the observation with right ascension α_0 and declination δ_0 . The uvw frame is a right-handed orthonormal basis where the w axis points towards the phase centre, the u axis points along decreasing hour angle (i.e. east on the sky), and the v axis points northward (increasing declination). A schematic of this coordinate system is presented in Fig. A2 in Appendix A.

The antenna position offset in the local ENU coordinate system is given by

$$\Delta \mathbf{r}_{\text{enu}} = (\Delta E, \Delta N, \Delta U)^T, \quad (6)$$

expressed in metres.

The offset in uvw coordinate system is then:

$$\begin{pmatrix} \Delta u \\ \Delta v \\ \Delta w \end{pmatrix} = \frac{1}{\lambda} [\mathbf{R}_{uvw} \cdot \mathbf{R}_{\text{MCMF}} \cdot \Delta \mathbf{r}_{\text{enu}}], \quad (7)$$

where \mathbf{R}_{uvw} , \mathbf{R}_{MCMF} are the rotation matrices mapping the MCMF frame to uvw coordinate system, and the local ENU coordinate system to MCMF frame, respectively. For readability, we defer the detailed derivation to Appendix A, and present only the key equations. Expanding the matrix multiplication in equation

(7) gives

$$\begin{pmatrix} \Delta u \\ \Delta v \\ \Delta w \end{pmatrix} = \frac{1}{\lambda} \mathbf{A} \begin{pmatrix} \Delta E \\ \Delta N \\ \Delta U \end{pmatrix}, \quad \mathbf{A} = \begin{pmatrix} a_E & a_N & a_U \\ b_E & b_N & b_U \\ c_E & c_N & c_U \end{pmatrix}. \quad (8)$$

where

$$\begin{aligned} &= [\cos H_0, -\sin H_0 \sin \phi, \sin H_0 \cos \phi], \\ [b_E, b_N, b_U] &= [\sin \delta_0 \sin H_0, \\ &\quad (\cos \delta_0 \cos \phi + \sin \delta_0 \cos H_0 \sin \phi), \\ &\quad (\cos \delta_0 \sin \phi - \sin \delta_0 \cos H_0 \cos \phi)], \\ [c_E, c_N, c_U] &= [-\cos \delta_0 \sin H_0, \\ &\quad (\sin \delta_0 \cos \phi - \cos \delta_0 \cos H_0 \sin \phi), \\ &\quad (\sin \delta_0 \sin \phi + \cos \delta_0 \cos H_0 \cos \phi)]. \end{aligned} \quad (9)$$

Here, λ is the observing wavelength in metres, H_0 is the local hour angle, and ϕ is the latitude of the site. We now define a covariance matrix in the uvw coordinate system to capture the induced correlated uncertainties after coordinate transformation as

$$\Sigma_{\Delta} = \begin{pmatrix} \sigma_{uu} & \sigma_{uv} & \sigma_{uw} \\ \sigma_{uv} & \sigma_{vv} & \sigma_{vw} \\ \sigma_{uw} & \sigma_{vw} & \sigma_{ww} \end{pmatrix} = \text{Cov}(\Delta u, \Delta v, \Delta w), \quad (10)$$

where σ_{uu} , σ_{vv} and σ_{ww} are the variances of the baseline errors along the u , v , w axes, respectively, and σ_{uv} , σ_{uw} and σ_{vw} are the covariances between the corresponding pairs of axes. The variance in the phase error can then be written in quadratic form as

$$\text{Var}(\Delta\varphi) = \mathbf{x}^T \Sigma_{\Delta} \mathbf{x}, \quad (11)$$

where $\mathbf{x} \equiv (l, m, n - 1)^T$, since the phase error is a linear combination of the baseline coordinate offsets introduced by errors in the antenna position. From equation (9), we note that $\Delta\varphi$ is now $\Delta\varphi = \Delta\varphi(v, \mathbf{b}, \mathbf{s}; H_0, \delta_0, \phi)$. These equations are equally valid for an interferometer on the Earth. Note that these equations hold for a fixed array site, using longitude $\Lambda = 0$ in a local coordinate frame. However, for analysis involving multiple sites or spatially extended arrays that span a significant range in longitude, it is necessary to retain the general expression (see equations (A11) and A15 in Section A) to correctly account for the longitudinal dependence. Note that the ENU and MCMF coordinate systems are fixed to the array location and remain identical for both tracking and drift-scan arrays. In contrast, the uvw frame follows the moving phase centre in a tracking array, while in a drift-scan array it is fixed, causing sources to drift through the FoV. In the special case when zenith is chosen as the phase centre, the transformation further simplifies since the ENU and uvw axes align as $H_0 = 0$ and $\delta_0 = \phi$. We now examine how the different types of positional offsets, i.e. along the xy direction or z direction appear in the visibilities.

4.1.1 Case 1: Perturbation along xy directions

To evaluate the antenna position deviations on the xy plane, the positional offsets are modelled using a Gaussian distribution with mean $\mu = 0$, and standard deviation σ_{xy} , which parameterizes the magnitude of lateral perturbation. The direction of each displacement is drawn uniformly over azimuth, meaning that the lateral offsets have no preferred direction. We vary $\sigma_{xy} = [0.05, 0.15, 0.25, 0.55]$ m. Recent lunar rover navigation studies suggest that while

challenging, absolute positional accuracies of a few metres are feasible (M. Cortinovis, T. Mina & G. Gao 2024; M. Sabatini et al. 2025). In addition, relative trajectory accuracy on the scales of centimetres is achievable within locally mapped areas (L. Ding et al. 2024). We therefore span perturbation levels from optimistic to conservative bounds to cover both best-case and realistically attainable scenarios.

For perturbations along xy direction, $\Delta E, \Delta N \sim \mathcal{N}(0, \sigma_{xy}^2)$, whereas $\Delta U = 0$. The covariance matrix for the xy offsets becomes

$$\Sigma_{\Delta}^{xy} = \frac{2\sigma_{xy}^2}{\lambda^2} A_{xy} A_{xy}^{\top}, \quad A_{xy} = \begin{pmatrix} a_E & a_N \\ b_E & b_N \\ c_E & c_N \end{pmatrix}. \quad (12)$$

Here, $A_{xy} A_{xy}^{\top}$ is a Gram matrix and therefore symmetric and positive semi-definite. The factor of 2 appears because we introduce per-antenna horizontal offsets, and the error in a baseline is therefore the difference of the offsets of its two antennas. By the Gram-quadratic-form identity, equation (11) can be expressed as

$$\text{Var}(\Delta\varphi_{xy}) = \mathbf{x}^{\top} \Sigma_{\Delta}^{xy} \mathbf{x} = \frac{2\sigma_{xy}^2}{\lambda^2} \mathbf{x}^{\top} A_{xy} A_{xy}^{\top} \mathbf{x} = \frac{2\sigma_{xy}^2}{\lambda^2} \|A_{xy}^{\top} \mathbf{x}\|^2, \quad (13)$$

where $\|\cdot\|$ denotes the Euclidean norm.

Following the characteristic function of a zero-mean Gaussian random variable, the ensemble-averaged perturbed visibility for a single baseline takes the form

$$\langle V'(v, \mathbf{b}) \rangle = \iint K(\mathbf{s}, v) \exp[-2\pi i \varphi(v, \mathbf{b}, \mathbf{s})] P_{xy} \frac{dl dm}{n} \quad (14)$$

with

$$K(\mathbf{s}, v) \equiv A(\mathbf{s}, v) I(\mathbf{s}, v), \quad (15)$$

$$P_{xy}(\mathbf{s}, v, H_0, \delta_0, \phi) \equiv \exp[-2\pi^2 \text{Var}(\Delta\varphi_{xy})] \in (0, 1]. \quad (16)$$

Here, P_{xy} is the additional phase coherence factor introduced by ensemble averaging over random antenna position errors along xy direction. The equation shows that this term acts like a direction-dependent (DD), frequency-dependent, multiplicative attenuation kernel in the image domain. Due to the assumption of independent and identically distributed horizontal offsets, the phase factor is baseline-independent but time-dependent (via H_0), and its amplitude scales with the σ_{xy} of the offsets in the lateral position. In addition to these, P_{xy} also depends on the latitude of the array site, and the phase centre. Generally, we average all the visibilities that contribute to an uv cell or average all uv cells within annuli of constant $|\mathbf{u}| (= \sqrt{u^2 + v^2})$. Let $\{b_i\}_{i=1}^{N_{\text{samp}}}$ index all measured visibility samples that fall in a given uv cell, G be the gridding kernel, and w_{b_i} be the weights. The gridded, ensemble-averaged visibility, under the small perturbation approximation, can then be written as

$$\langle V'_g(v, \mathbf{b}) \rangle = \frac{\sum_{i=1}^{N_{\text{samp}}} G(u - u_{b_i}, v - v_{b_i}) w_{b_i} \langle V'_{b_i}(v) \rangle}{\sum_{i=1}^{N_{\text{samp}}} G(u - u_{b_i}, v - v_{b_i}) w_{b_i}}, \quad (17)$$

where $\langle V'_{b_i}(v) \rangle$ is the ensemble-averaged, perturbed visibility for baseline b_i . Now we see two forms of decorrelation arising from antenna position offsets. The first is the random (stochastic) loss of phase coherence by the phase factor P_{xy} for a single baseline. The second is a decorrelation introduced when visibilities from multiple baselines with slightly different geometric phases or attenuation factors are averaged within a given uv cell or across annuli. This averaging leads to an additional smearing in the

uv plane whose strength grows with baseline length, frequency, and the lunar rotation synthesis. Therefore, a closed-form analytical expression after gridding or annular averaging is not straightforward.

Note that even when antenna position errors are along the horizontal plane ($\Delta U = 0$), they can induce non-zero Δw components due to the projection of $(\Delta E, \Delta N)$ into the line-of-sight component. These projected Δw terms contribute to the total phase error through the $w(n-1)$ term, which becomes increasingly large for sources away from the phase centre ($l^2 + m^2 \gg 0$).

From equations (14)–(17), we see that in the image domain, the additional phase factor P_{xy} acts as a multiplicative attenuation kernel, whose Fourier transform sets the width of the convolution kernel in uv domain. When P_{xy} is constant across the image domain, its Fourier transform is equivalent to a δ -function and the kernel effectively reduces to a constant rescaling of the visibilities, i.e. negligible smoothing/smearing in uv domain. To quantify the acceptable level of phase error required for the smoothing of uv modes to remain negligible (i.e. equivalent to multiplication by an effectively constant kernel), we derive an analytical estimate in Appendix D.

Each visibility in the uv domain is smeared by the same baseline-independent kernel that arises directly from the assumption that horizontal offsets are independent, zero-mean Gaussian perturbations applied to each antenna. Fig. 4 shows the fractional difference of the absolute visibilities on the uv plane, averaged along frequency for a simulated observation as discussed in Section 5 for the antenna offsets along xy direction. Although the kernel is the same for all baselines for xy position offsets, we see that the uv plane shows a slightly larger fractional difference towards longer baselines. The convolution mixes power between neighbouring modes, and its effect is determined by the slope of the intrinsic sky power spectrum. Since the power spectrum of diffuse sky emission decreases rapidly with baseline length, short baselines that probe the large-scale modes are dominated by higher intrinsic power, thus, leading to leaking power into adjacent modes, contaminating slightly longer baselines. At longer baselines, which probe higher spatial frequencies (smaller angular scales) where the intrinsic sky power is lower, the convolution averages over neighbouring modes of smaller amplitude, leading to a larger fractional suppression of the visibility amplitude.

For a fixed array configuration, the projected baseline length in wavelength units scale linearly with frequency, so naturally the extent of uv coverage of Z42 is larger than that of the Z148. This also means that Z148 has fewer independent Fourier modes but a high filling factor compared to Z42, which may affect sensitivity to large-scale cosmological modes but improve the robustness of redundant calibration at low frequencies. Comparing the two spectral windows, the fractional differences are observed to increase with frequency (or equivalently, shorter wavelengths), in agreement with the analytical prediction. This is because positional offsets lead to larger phase deviations at higher frequencies. For a fixed spectral window, increasing the perturbation amplitude σ_{xy} , increases the visibility suppression across the uv plane. Even in the case of the largest σ_{xy} considered, the fractional difference remains well below 5 per cent for Z42, and for the same σ_{xy} , the variation in fractional difference for Z148 is less than 2 per cent.

The time-dependence of the factor through H_0 arises from rotational synthesis, which changes the projection of xy positional offsets in the uvw coordinates, changing the DD phase errors with LST. Although our simulation is for a fixed LST, this implies that

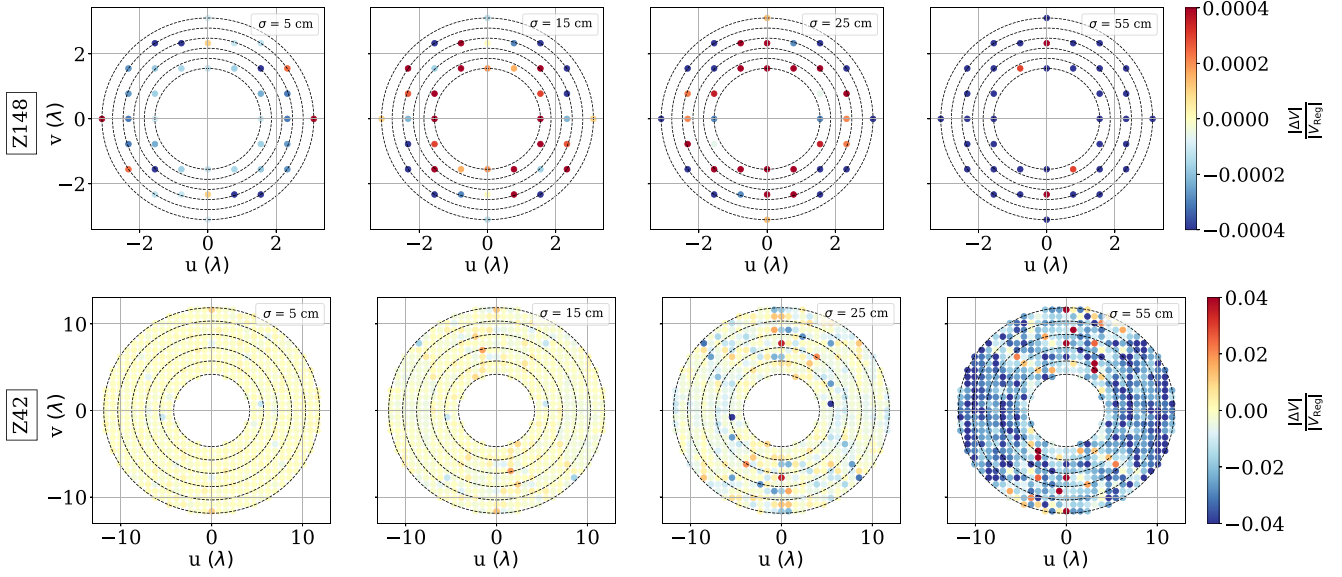


Figure 4. The fractional difference of the frequency-averaged absolute visibilities on the uv plane, showing the effect of antenna position perturbations along the xy direction relative to the unperturbed array. Results are shown for two spectral windows (rows) and increasing perturbation (columns). Concentric circles indicating constant baseline length are drawn on the uv plane for illustrative purposes only.

full LST coverage could modulate the effective contamination in wide-FoV observations, motivating time-dependent models for calibration.

4.1.2 Case 2: Perturbation along z direction

We study the effect of array's non-coplanarity while accounting for a realistic 3D lunar topography. To achieve this, we use the LROC NACs stereo observations yielding DTMs at scales as fine as 2 m, as discussed in Section 2. Unlike horizontal position errors, which are assumed uncorrelated between antennas, the offsets along z direction arise from the underlying lunar surface and are highly correlated between antennas. These height variations are directly sampled from the DTM and reflect the statistical properties of lunar topography, including its local roughness and spatial correlations.

For perturbations along the z direction, the variance of ΔU follows from equation (1),

$$\text{Var}(\Delta U) = \sigma_s^2 \left(\frac{r}{r_s} \right)^{2H}, \quad (18)$$

while we keep $\Delta E = \Delta N = 0$. Equation 18 gives the variance of the height difference between two antennas separated by a distance r , so it already accounts for the pair of antennas. Hence, here no additional factor of 2 is introduced. Therefore, for vertical height offsets of the antennas, equation (11) becomes

$$\text{Var}(\Delta \varphi_z) = x^\top \Sigma_\Delta x = \frac{\sigma_s^2}{\lambda^2} \left(\frac{r}{r_s} \right)^{2H} \|A_z^\top x\|^2, \quad (19)$$

$$A_z = \begin{pmatrix} a_U \\ b_U \\ c_U \end{pmatrix}.$$

The corresponding ensemble-averaged perturbed visibility can be written as

$$\langle V'(\nu, \mathbf{b}) \rangle = \iint K(\mathbf{s}, \nu) \exp[-2\pi i \varphi(\nu, \mathbf{b}, \mathbf{s})] P_z \frac{dl dm}{n}, \quad (20)$$

with

$$K(\mathbf{s}, \nu) \equiv A(\mathbf{s}, \nu) I(\mathbf{s}, \nu), \quad (21)$$

$$P_z(\mathbf{s}, \nu, \mathbf{b}, H_0, \delta_0, \phi) \equiv \exp[-2\pi^2 \text{Var}(\Delta \varphi_z)] \in (0, 1]. \quad (22)$$

Equations (14) and (20) have a similar functional form and dependencies when ensemble-averaged over perturbed realizations for a single baseline. Also, equation (17) retains the same functional form for vertical height offsets, with the additional phase factor now being replaced by P_z . The key difference is that, unlike the uncorrelated lateral offsets, which induce a baseline-independent factor (uniform mode mixing across all baselines), offsets along z direction arising from correlated surface topography introduce baseline dependence through the r^{2H} scaling in equation (18). This difference follows directly from the statistical model assumed for the different types of positional perturbations. The suppression strength of the phase factor increases with increasing angular distance from the phase centre, frequency, and baseline length modulated by the Hurst exponent of the surface, H . For the four surfaces considered in this study, Surface 1 shows the largest height fluctuations, while Surface 4 remains the smoothest, with the lowest height difference across all scales (see Table 1). Our analytical expression therefore, predicts that Surface 4 will produce the least visibility decorrelation due to height-induced phase errors, whereas Surface 1 will produce the strongest suppression of small-scale power due to baseline-dependent phase variance.

Fig. 5 is analogous to Fig. 4, but corresponds to antenna perturbations along the z direction. The fractional difference in visibility amplitudes across the uv plane shows a frequency scaling similar to that seen for lateral offsets. The effective width of the smearing due to the convolution in the uv domain for height offsets is

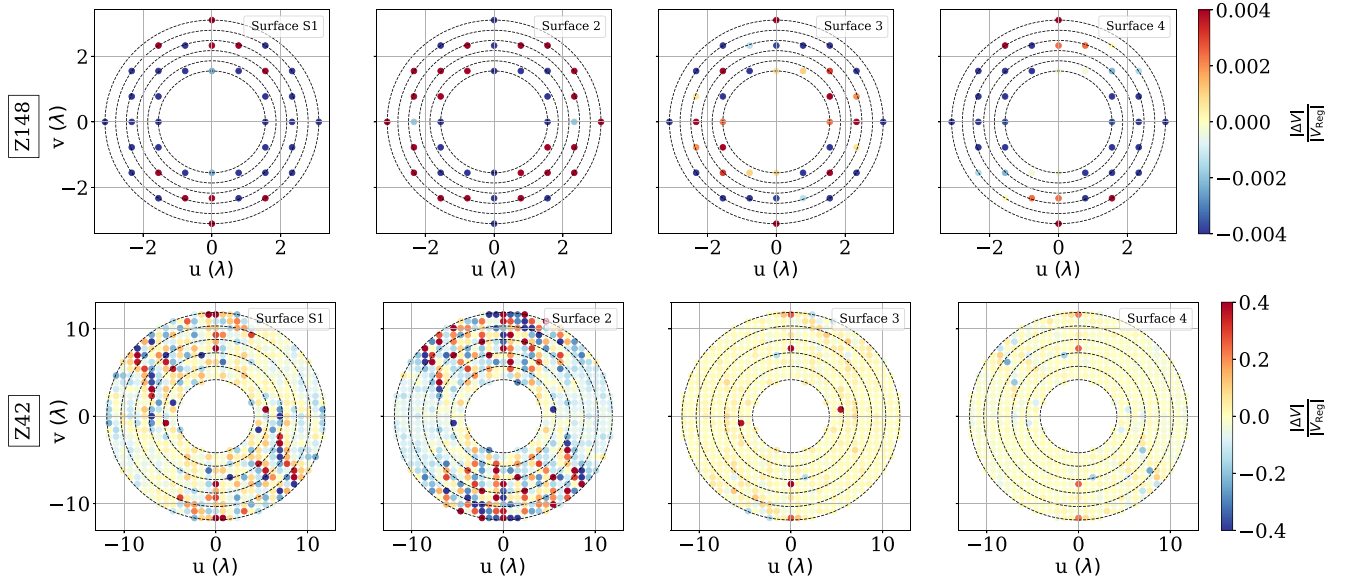


Figure 5. The fractional difference of the frequency-averaged absolute visibilities on the uv plane, showing the effect of antenna position offsets for the four different surfaces (columns). Results are shown for two spectral windows (rows).

controlled by the values of H and σ_s . This implies that smooth surfaces will show slowly varying phase decoherence, due to a compact kernel. In contrast, rougher surfaces, characterized by smaller H and/or larger σ_s , introduce rapid phase decoherence, even on short baselines, due to a broader convolution kernel in the uv domain.

The anisotropy observed in the uv plane can be directly attributed to anisotropy in the underlying surface height statistics, as evident in the detrended elevation maps in Fig. 2, where the preferred axes of maximum roughness are apparent. This leads to greater phase decoherence for baselines aligned with that axis compared to those aligned perpendicular to it. This can be seen from the different azimuthal structure of the convolution kernel for the different surfaces (columns) in the uv map. A secondary contribution to the anisotropy may come from the projection effect, where the projected baseline length varies depending on its orientation relative to the source direction.

The positive residuals (red) in the fractional difference plots correspond to uv cells where the perturbed visibility amplitude is higher than that of the unperturbed visibility. For offsets in the xy plane, we saw that the effect can be understood as a baseline-independent convolution in the uv domain. This convolution smooths the visibilities by averaging over neighbouring uv cells. Positive residuals may occur near steep gradients (e.g. the regions where the amplitude of visibility is very high from bright sources or primary beam edges). For perturbations along the z direction, the resulting kernel becomes both baseline-dependent and direction-dependent, driven by the fractal nature of lunar topography. This causes anisotropic smearing, with positive residuals amplified along directions where the surface height difference is largest.

Some of the positive residuals are also likely to be realization-specific and strongly driven by sample variance, owing to the limited number of independent baselines in the array. Since visibilities are formed by adding many complex phasors across the sky, small changes in phase from one realization or one LST to another can change the sign of the residual in a given uv cell.

However, the overall trend due to antenna position offsets remains statistically robust when averaged in annuli.

We note that the impact of surface roughness can be viewed, in a statistical sense, as analogous to a thin ionospheric phase screen. In both cases, if the phase fluctuations are assumed to be zero mean, Gaussian, and stationary, the ensemble-averaged visibilities take the form $\langle V \rangle = \exp[-D_\phi/2] V_{\text{true}}$, where D_ϕ is the phase structure function and V_{true} is the uncorrupted visibility.

For a fractal surface with Hurst exponent H , the topographic phase structure function is $D_{\phi,\text{surf}} \propto \sigma_s^2 (r/r_s)^{2H} v^2$ scales with baseline r and frequency ν , while the ionospheric phase structure function for a Kolmogorov screen in compact arrays $D_{\phi,\text{iono}} \propto r^{5/3} v^{-2}$ (H. Vedantham & L. Koopmans 2016). With $2H \approx 5/3$, the effect of phase decoherence will be similar, though the absolute frequency scalings differ (surface roughness $\propto v^2$; ionosphere $\propto v^{-2}$).

A key distinction is that the phase errors due to surface roughness are anisotropic and quasi-static, set by the local topography and evolving slowly with LST as the baseline projection changes. However, ionospheric phase errors are assumed to be isotropic and time-variable, often on minute scales. This leads to different diffractive scales, $r_{\text{diff, surf}} \propto v^{-1/H}$ and $r_{\text{diff, iono}} \propto v^{-6/5}$. Therefore, the analogy is helpful for understanding coherence loss but highlights the distinct calibration requirements for lunar arrays (or non-coplanar arrays).

5 END-TO-END SIMULATION PIPELINE

To quantify the effect of the uncorrected antenna position deviations in the optimal performance of DEX, we develop a forward simulation pipeline, called SPADE-21cm (Simulation Pipeline for Analysing Dark agEs using 21-cm). The full-sky simulations will provide insights into first-order tolerance levels of both lateral and vertical antenna position offsets. This pipeline includes the components necessary to generate a realistic scenario that closely resembles actual observations. Fig. 6 presents a schematic of the end-to-end simulation pipeline.

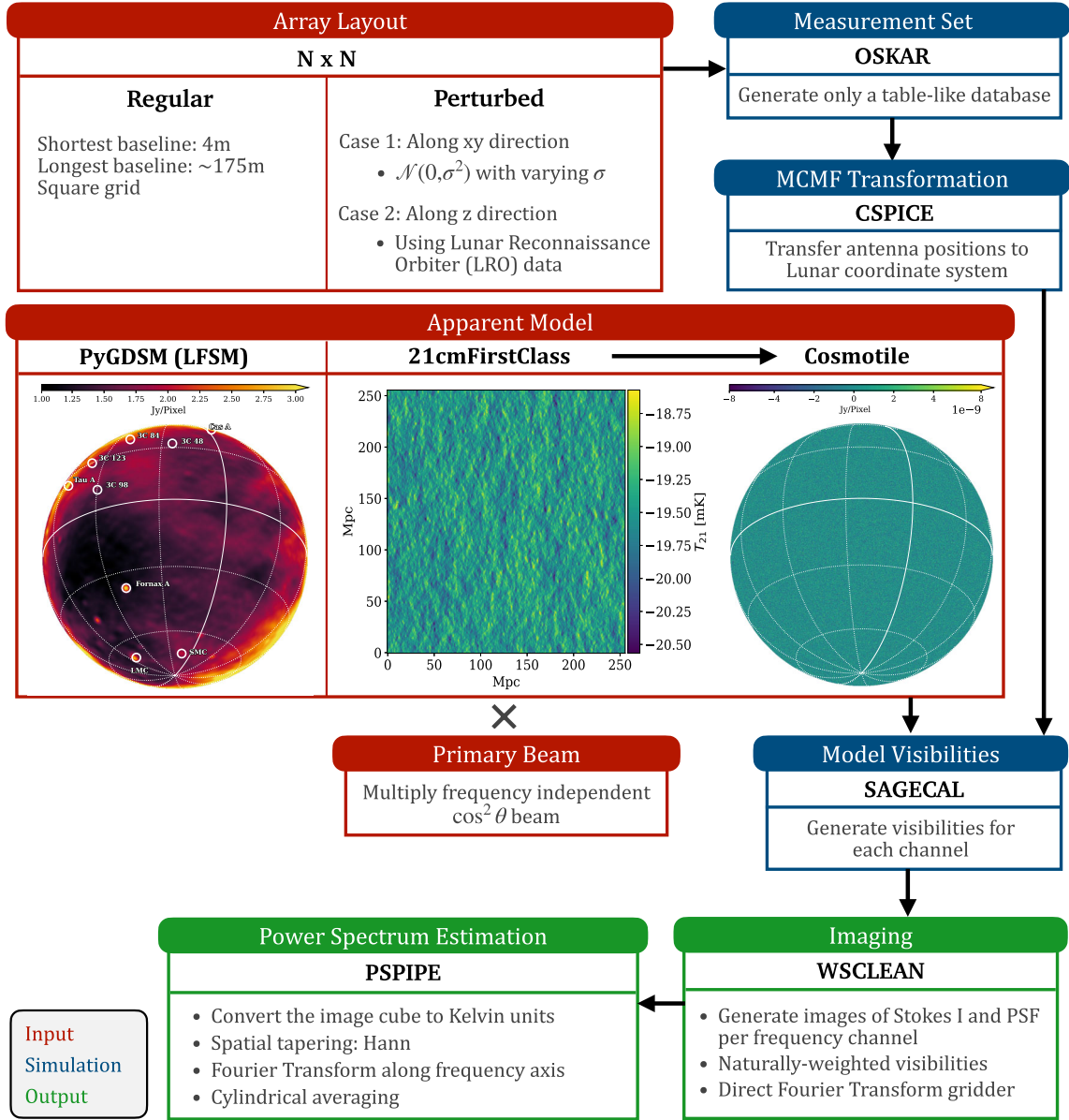


Figure 6. A flowchart of SPADE-21cm.

5.1 Simulation pipeline

We simulate a zenith-pointed snapshot observation of 5 min centred at RA = 23.052060°, Dec. = −25.696129° from the Moon. The observations, Z148 and Z42, each consist of 100 channels with a resolution of 50 kHz. Only a single snapshot observation is considered in this study for two primary reasons. First, due to the near-unity filling factor of the array, the uv coverage does not evolve significantly over the course of a full lunar synthesis. As a result, conclusions drawn from a single snapshot are expected to remain valid to good precision over longer integrations. Second, simulating a full synthesis for a 1024 antenna element array, across 100 frequency channels, incorporating both a full foreground model and a 21-cm signal model, is computationally intensive, and therefore, a full synthesis is currently not feasible. For our simulations, we select the observation time to be at 16:22:30 UTC on July 30, 2040, when the centre of the Galaxy

is located along the horizon, far from the phase-centre, but not entirely absent. The fiducial parameters chosen for the simulation are summarised in Table 2. Below we provide a detailed description of each stage of the pipeline.

5.1.1 Foreground model

We adopt the Low Frequency Sky Model (LFSM) (J. Dowell et al. 2017) from the PyGDSM (D. C. Price 2016) package as our diffuse sky model. At the angular resolution of DEX, the diffuse Galactic emission dominates the sky brightness, with point sources implicitly incorporated within the model, albeit at lower angular resolution, where they are blended with the large-scale emission and contribute to the overall confusion noise. The LFSM is based on a principle component analysis of data, a similar approach to that of the Global Sky Model (GSM) (A. Oliveira-Costa et al. 2008;

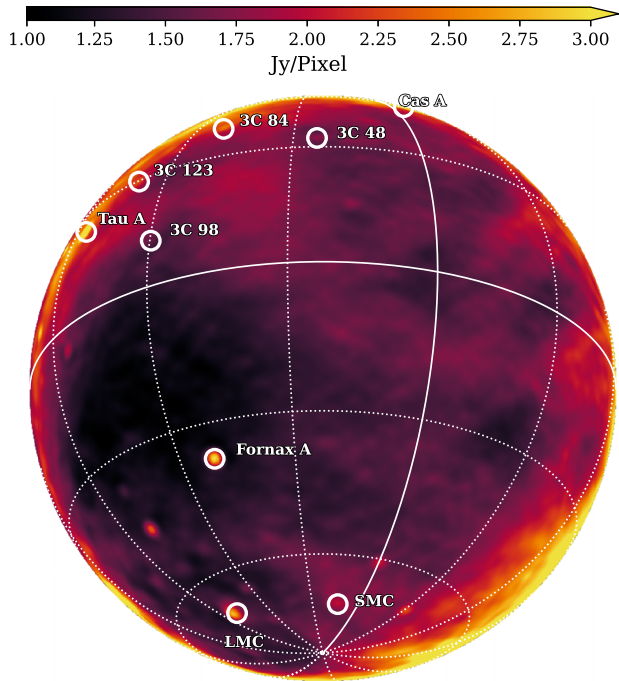


Figure 7. Orthographic projection of the diffuse sky model from LFSM at a reference frequency of 45 MHz, corresponding to 16:22:30 UTC on 2040 July 30, as seen from Mare Ingenii when the Galactic Centre lies along the horizon. To obtain the apparent sky brightness distribution, this model is multiplied by a frequency-independent $\cos^2 \theta$ beam.

H. Zheng et al. 2017). However, a key distinction is that LFSM incorporates additional observations at 40, 50, 60, 70, and 80 MHz from the LWA1 Low Frequency Sky Survey, extending beyond the datasets used in GSM. This allows for improved interpolation of the sky towards lower frequencies. In this work, we neglect free-free absorption, which can become increasingly important at frequencies below ~ 7 MHz, where it significantly alters the spectral structure of the diffuse Galactic foreground and similarly suppresses the 21-cm signal. To account for this effect, sky models such as the Ultralong-wavelength Sky Model with Absorption (ULSA) (Y. Cong et al. 2021) can be used. In this study, we are primarily interested in isolating the relative effects of perturbations in antenna positions on the power spectrum estimation, therefore, the LFSM skymodel is a sufficient approximation of the true radio sky.

We generate our reference diffuse foreground map at 45 MHz, in the form of a HEALPIX (K. M. Gorski et al. 2005; A. Zonca et al. 2019) map as shown in Fig. 7. This allows us to convert the foreground brightness distribution into pixelized cells with units of flux density in units of Jansky. Insufficient resolution of the HEALPIX map can result in undersampling of the (l, m) space, leading to resampling errors when simulating visibilities, especially for long baselines. To mitigate such effects, it is recommended that the diffuse sky model be sampled well beyond the Nyquist rate (see Section 5.3 in P. Kittiwisit et al. 2025). We have used $N_{\text{side}} = 512$ that gives a HEALPIX pixel resolution of $\sim 0.11^\circ$. This is ~ 17 times finer than that the FWHM of the synthesized beam corresponding to the longest baseline of our array, which is $\sim 1.9^\circ$. The pixels in the HEALPIX map are treated as unresolved point sources. So for $N_{\text{side}} = 512$, the number of pixels that still remain after removing the pixels below the horizon can

be as large as $\sim 10^6$. This remains computationally demanding, particularly when simulating observations for 523 776 baselines. The final maps derived from the LFSM are corrected for their missing ‘zero-spacing’ data using the total power data of LEDA (see Section 2.4 in J. Dowell et al. 2017). We note that in an FFT-based instrument such as DEX, autocorrelations are inherently included, resulting in a sky map with a non-zero mean that also includes contributions from receiver noise and other systematics. In practice, we can subtract the mean intensity, effectively removing the zero-spacing component. This is an important distinction compared to FX-correlated instruments, which in general do not include total power.

To simulate the sky as observed from the lunar surface, we use `lunarsky`⁷, an extension of `astropy` that provides selenocentric and topocentric reference frames for the Moon. The sky map is phased to the zenith of the array and multiplied with a frequency-independent $\cos^2 \theta$ beam model to create an apparent (instantaneous) foreground sky model.

5.1.2 21-cm signal model

To simulate the 21-cm signal from DA, we use `21cmFirstCLASS`⁸ (J. Flitter & E. D. Kovetz 2023a, b) which is a merger of two widely used popular codes, `21cmFAST`⁹ (A. Mesinger, S. Furlanetto & R. Cen 2011; J. B. Munoz et al. 2022) and `CLASS`¹⁰ (D. Blas, J. Lesgourgues & T. Tram 2011). `21cmFirstCLASS` begins its calculation from initial conditions at recombination (via `CLASS`) and evolves the 21-cm signal. During the DA, the density fluctuations of baryons (δ_b) and cold dark matter (δ_c) evolved differently (J. Flitter, S. Libanore & E. D. Kovetz 2024). In contrast, at lower redshifts, these differences become negligible as sufficient time has elapsed for δ_b and δ_c to evolve similarly, with their initial conditions no longer significantly influencing their dynamics. This assumption underlies in the `21cmFAST` making it less suitable for modelling 21-cm signal from the DA.

We simulate a box of $[500 \text{ cMpc}]^3$ on a 256^3 grid giving a spatial resolution of ~ 1.953 Mpc per voxel. `21cmFirstCLASS` generates a lightcone, from which the coeval boxes can be extracted. While running, `21cmFirstCLASS` uses the closest redshift given as input to generate the co-eval boxes. As a result, the redshifts of the output coeval boxes may not always exactly match those specified in the input. We use a quadratic scheme to perform pixel-wise interpolation of the boxes to the desired redshifts. The boxes are generated for the two spectral windows mentioned in Section 3.2.2 with a step-size of 0.05 MHz.

Given that DEX is sensitive to spatial modes exceeding the size of a single co-eval box, we generate a larger volume in comoving space by tiling the co-eval boxes using `Cosmotile`¹¹ (P. Kittiwisit et al. 2018). In addition to the tiling of the coeval boxes, `Cosmotile` is used to project the resulting tiled volume into angular sky coordinates. Specifically, the `make_healpix_lightcone_slice` function in `Cosmotile` takes the tiled 3D Cartesian comoving cubes, selects the appropriate comoving radius corresponding to the desired redshift (i.e. the lightcone shell), and projects the data onto full-

⁷<https://github.com/aelanman/lunarsky>

⁸<https://github.com/jordanflitter/21cmFirstCLASS>

⁹<https://github.com/21cmfast/21cmFAST/tree/master>

¹⁰https://github.com/lesgourg/class_public

¹¹<https://github.com/steven-murray/cosmotile>

sky HEALPIX maps. These angular-frequency 21-cm maps then serve as inputs to the subsequent step for visibility simulation.

However, we note that while Cosmotile enables the construction of larger comoving volumes, it does not produce intensity modes on scales exceeding the size of the original co-eval box. As a result, spatial k -modes from the 21-cm signal with $k_{\perp} \lesssim 2\pi/(500 \text{ cMpc}) \approx 0.01 \text{ cMpc}^{-1}$ are absent from our simulation. For baselines corresponding to $k < 0.01 \text{ cMpc}^{-1}$, only k_{\parallel} modes are sampled, which are averaged over multiple stacked simulation boxes along the LOS. However, since the minimum accessible k_{\parallel} in our setup is set by the finite 5 MHz bandwidth and exceeds 0.01 cMpc^{-1} , the absence of transverse 21-cm signal modes on scales $k < 0.01 \text{ cMpc}^{-1}$ does not impact our results.

To mitigate the edge effect at the boundaries that might arise when periodic boxes with different absolute mean levels are stitched together, the mean brightness-temperature value of each co-eval box is subtracted prior to tiling. This step effectively removes the $k = 0$ mode which basically mimics the absence of the zero-spacing (autocorrelation) measurement in real interferometric observations. A FFT-based correlator will retain the zero-spacing mode but including it in our simulations would introduce sharp discontinuities at the boundaries, leading to spurious artefacts in the power spectrum. Finally, the 21-cm HEALPIX maps are multiplied with the same beam model to ensure consistency with the apparent foreground sky model.

5.1.3 Measurement Set generation

Measurement sets (MS) are created using OSKAR¹² (F. Dulwich 2020), a GPU-accelerated visibility simulator specifically designed for the SKA. For the purpose of this paper, we use OSKAR only to generate the template that contains the properties of the telescope array, such as antenna positions, uv -coverage, pointing directions, and frequency channels. However, we do not use the visibilities simulated by OSKAR (see Section 5.1.4).

Once the MSs are generated for each of the frequency channels, we update the antenna positions in the MS to MCMF coordinates using `uvutils.ECEF_from_ENU(frame = 'MCMF')`. The MCMF coordinate system is often defined using two closely related coordinate systems, the Mean Earth (ME)/Polar Frame coordinate system, and the other is the Principal Axis (PA) coordinate system (NASA 2008). In this work, we choose the ME frame, with the origin located at the barycenter of the Moon (see Fig. A1).

Finally, the uvw positions of the array are also updated with respect to the MCMF frame. This is done by using a new functionality in SAGECal-CO¹³ (S. Yatawatta 2015), the standard visibility prediction and calibration software of LOFAR-EoR data processing pipeline, that incorporates CSPICE¹⁴. CSPICE is the C component of the SPICE Toolkit, a library developed by the Navigation and Ancillary Information Facility of the Jet Propulsion Laboratory (JPL) to provide access to planetary and spacecraft ephemerides and other functions for engineering computations. With this extension, the simulated observations can use the precise information on the motion of the DEX site on the lunar surface at any epoch of time.

¹²<https://github.com/OxfordSKA/OSKAR>

¹³<https://github.com/nlesc-dirac/sagecal>

¹⁴<https://naif.jpl.nasa.gov/naif/toolkit.html>

5.1.4 Visibility prediction

The visibilities are predicted using SAGECal-CO following the RIME framework. The model visibility prediction can be computationally demanding for 1024 antenna elements. Upon comparison, a difference was observed in the wall time required for visibility prediction, with SAGECal-CO demonstrating quicker prediction in our case compared to OSKAR. However, we note that either of these can be utilized to predict model visibilities based on the availability of computing resources. With the integration of CSPICE into SAGECal-CO, the software gains the capability to forward predict visibilities in the lunar frame, enabling more accurate simulations for lunar-based interferometry. We simulate the following two data sets:

(i) *Regular visibilities* (V_{reg}): The model visibilities of the sky and 21-cm signal for the ‘ideal case’ when the antenna positions are perfectly regular, hence unperturbed.

(ii) *Perturbed visibilities* (V_{pert}): This set of visibilities is generated by introducing antenna offsets along the xy direction or the z direction independently.

5.1.5 Imaging and power-spectrum estimation

The visibilities are imaged with WSCLEAN¹⁵ (A. Offringa et al. 2014). A ‘natural’ weighting scheme is used to generate full-sky ‘dirty’ Stokes I images (in Jy PSF^{-1} , where PSF denotes the point spread function), and PSF images for each of the frequency channels. No deconvolution is performed on the images. We used the direct-ft algorithm in WSCLEAN which evaluates the visibilities at their native baseline coordinate (u, v, w) . Consequently, this helps to avoid gridding related artefacts and provides more accurate imaging, albeit at a significantly higher computational cost. In addition, the direct-ft can account for the w -term naturally (R. A. Perley 1999). This is important for wide-field or non-coplanar array imaging. Once the ‘dirty’ image per channel is generated, we apply a multiplicative factor of $\sqrt{1 - l^2 - m^2}$ to each pixel to obtain accurate temperature units across the FoV.¹⁶

Prior to power-spectrum estimation, these image cubes for each channel are converted to units of Kelvin. We subsequently construct the cylindrically averaged power spectrum using `pspipe`¹⁷ following the standard definitions of cosmological power spectra in M. F. Morales & J. Hewitt 2004; M. McQuinn et al. 2006. Following the notation as in F. G. Mertens et al. 2020, the 21-cm brightness temperature field, $T(\mathbf{r})$, is defined over spatial coordinates \mathbf{r} . The corresponding power spectrum $P(\mathbf{k})$, as a function of the wavevector \mathbf{k} , is given by

$$P(\mathbf{k}) = V_c |\tilde{T}(\mathbf{k})|^2, \quad (23)$$

where $\tilde{T}(\mathbf{k})$ is the discrete Fourier transform of the temperature field, computed as

$$\tilde{T}(\mathbf{k}) = \frac{1}{N_l N_m N_v} \sum_{\mathbf{r}} T(\mathbf{r}) e^{-2i\pi \mathbf{k} \cdot \mathbf{r}}. \quad (24)$$

¹⁵<https://wscclean.readthedocs.io/en/latest/>

¹⁶This factor enters the conversion from Jy PSF^{-1} to Kelvin, as it accounts for the projection from direction cosines to solid angle. The PSF in non-deconvolved images is direction-dependent, and this correction ensures to properly account for the spatial variation of the PSF when converting to brightness temperature.

¹⁷<https://gitlab.com/flomertens/pspipe>

Here, N_l , N_m , and N_ν are number of pixels in l , m , and ν direction. The observed comoving volume V_c depends on the primary beam of the instrument $A_{pb}(l, m)$, the spatial window function $A_w(l, m)$ and the spectral window function $B_w(\nu)$ applied to the image cube before the Fourier transform. It is estimated as follows:

$$V_c = \frac{(N_l N_m N_\nu dl dm d\nu) D_M(z)^2 \Delta D}{A_{\text{eff}} B_{\text{eff}}}, \quad (25)$$

where dl and dm are angular pixel sizes, $d\nu$ is the channel width, $D_M(z)$ is the transverse comoving distance at redshift z and ΔD represents the comoving distance corresponding to the observed frequency range. A_{eff} and B_{eff} are the effective area on the sky and effective bandwidth, respectively, defined as

$$A_{\text{eff}} = \langle A_{pb}(l, m)^2 A_w(l, m)^2 \rangle, \quad (26)$$

$$B_{\text{eff}} = \langle B_w(\nu)^2 \rangle. \quad (27)$$

In equation (26), $\langle \rangle$ represent an average over spatial coordinates (the image plane), with weighting from the primary beam and spatial tapering window. On the other hand, in equation (27) they represent an average over frequency channels, with weighting from the spectral tapering window. A Hann-filter with a width of 80° is applied as a spatial taper and the choice of spectral window function is discussed in Appendix B.

Assuming the cosmological 21-cm signal is isotropic, the power spectrum $P(\mathbf{k})$ can be cylindrically averaged, which preserves the separation between transverse and LOS components in Fourier space. It is defined as

$$P(k_\perp, k_\parallel) = \langle P(\mathbf{k}) \rangle_{k_\perp, k_\parallel}, \quad (28)$$

and is widely used as a diagnostic tool.

6 RESULTS

In this section, we analyse the effects of antenna position offsets on the simulated data described in Section 5. We first present the fiducial cylindrically averaged power spectrum for the different signal components in the case of an unperturbed array in Section 6.1. We then look into how errors arising from antenna position offsets appear in the cylindrically averaged power spectrum for offsets along the xy direction in Section 6.2.1, and along the z direction in Section 6.2.2 independently. Finally, in Section 6.2.3, we quantify the impact of the positional offsets on the 21-cm DA power spectrum.

6.1 Power spectrum characteristics (unperturbed case)

The cylindrically averaged power spectrum (or the 2D power spectrum) in (k_\perp, k_\parallel) space is one of the most widely used statistical metrics for analysing the impact of foreground contamination and systematic biases. While the 21-cm signal is expected to be statistically isotropic to first order, with power distributed in nearly all k -modes, the foregrounds are highly anisotropic and, owing to their spectral smoothness, are largely confined to low k_\parallel modes. The inherent chromaticity of interferometers results in a leakage of foregrounds to higher k_\parallel with increasing k_\perp , also called ‘mode-mixing’ (M. F. Morales et al. 2012), leading to a region often referred to as the ‘foreground wedge’. The spectrally smooth foregrounds are typically confined within this region. The largest possible extent of the wedge region is determined by the maximum geometric delay between two antennas in a baseline,

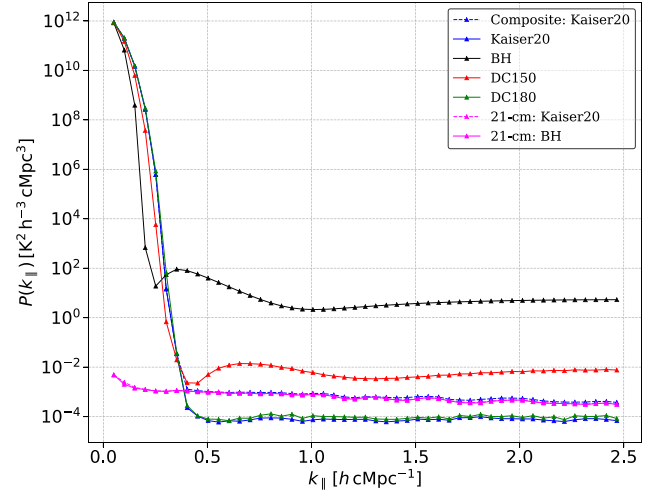


Figure 8. The cylindrically averaged power spectra as a function of k_\parallel (averaged over k_\perp) are shown for different spectral window functions: Blackman–Harris (BH), Dolph–Chebyshev with attenuation levels set to 150 dB (DC150) and 180 dB (DC180), and Kaiser with $\beta = 20$ (Kaiser20). The coloured curves correspond to the foregrounds only simulations. For comparison, the magenta curves represent the 21-cm signal only at $z \sim 42.5$. The dashed blue line shows the composite sky when applying the Kaiser20 window. Results are shown here for Z42.

which is achieved when the source is positioned at the horizon, for a phase centre at zenith.

Generally, in any 2D power spectrum for 21-cm experiments, a window function is applied along the LOS axis to taper the additional spectral components originating as a result of finite bandwidth. This reduces the effective bandwidth, broadens the main lobe of the window function in delay space, and increases spillover beyond the horizon. For DA experiments, the dynamic range required to suppress foreground contamination in the 21-cm signal window is at least 11–15 orders of magnitude in power, implying a trade-off between the level of foreground sidelobe level (SLL) suppression and the extent of low k_\parallel modes lost due to the leakage. We find that applying a Kaiser window function (see Appendix B) with $\beta = 20$ (hereafter denoted as Kaiser20) achieves ~ 164 dB SLL suppression, compared to the commonly used 4-term Blackman-Harris which gives a SLL suppression of ~ 92 dB when comparing foreground-only power outside the wedge with the peak foreground power inside the wedge. In the Kaiser window function, β is a shape parameter that controls the trade-off between the width of the main lobe and the suppression of the SLL. Although this improvement comes at the expense of k_\parallel resolution, any leakage into LOS modes is now governed mostly by the inherent frequency dependence of the foregrounds, rather than by windowing artefacts. A comparison of the cylindrically averaged power spectra (averaged over all baselines) with different window functions is illustrated in Fig. 8. Note that although the foreground-only cylindrically averaged power spectrum (solid blue) in Fig. 8 may lie below the 21-cm signal only power spectrum (dashed magenta) after applying the Kaiser20 window function, the composite power spectrum (dashed blue) cannot fall below the 21-cm signal. This is because it includes the intrinsic power of the 21-cm signal, and therefore sets a nonzero lower bound (or signal floor) on the composite power spectrum.

For this study, we first apply a third-order polynomial fit to the gridded visibility data cube to remove most of the spectrally

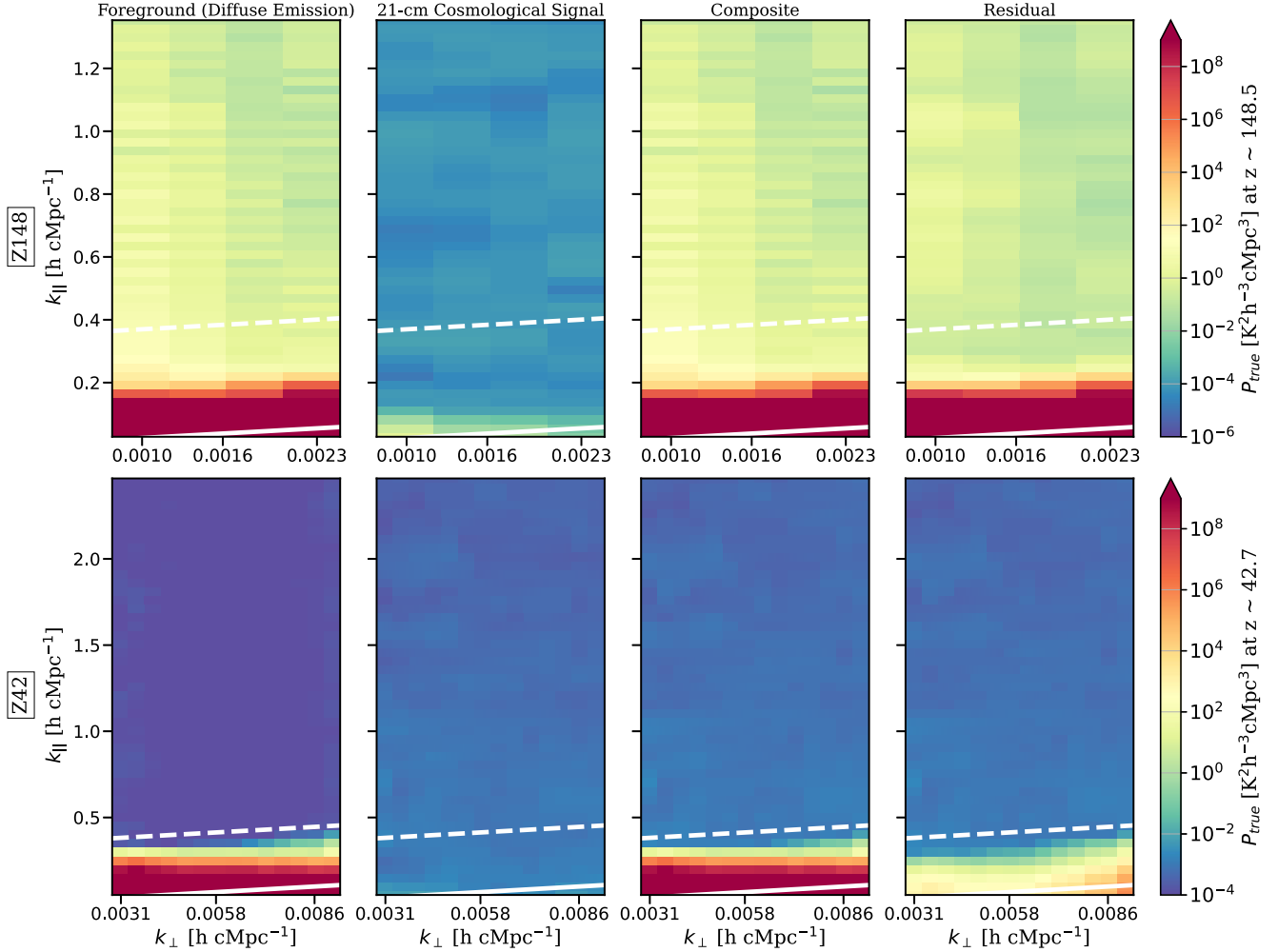


Figure 9. (From left to right panel): The 2D cylindrically averaged power spectra of simulated data for DEX from a diffuse Galactic emission model, 21-cm cosmological signal, a composite model prior to polynomial subtraction, and the residuals after applying a third-order polynomial fit, as described in Section 5 for Z148 (top row), and Z42 (bottom row). The solid white line represents the horizon limit, and the white dashed line represents the horizon buffer limit (for Kaiser $\beta = 20$) equals to 346 nano seconds. The lines shown here are computed based on the improved delay formalism by S. Munshi et al. (2025a), which accounts for sky curvature. These power spectrums represent the case when all the antenna elements are perfectly on a regular grid.

smooth foreground power, followed by Kaiser20 to approximate the level of spectral suppression typically implemented in real data analysis pipelines. Prior to power spectrum estimation, we remove the longer baselines with poor uv coverage to avoid sampling artefacts. We show the 2D cylindrically averaged power spectra of the foregrounds, 21-cm signal, a composite model prior to polynomial subtraction, and the residuals after applying a third-order polynomial fit in the ideal scenario for both Z148 and Z42 (rows), in Fig. 9. Since the primary beam considered in this work is frequency-independent, the observed wedge in the foreground emission panels arises due to the interaction of the frequency-dependent PSF with the otherwise spectrally smooth foreground emission. As expected, the amplitude of the foreground power decreases with increasing frequency due to the steep synchrotron spectral index $T_b \propto \nu^\beta$, $\beta \sim -2.5$.

Spectral tapering can minimize leakage from finite-bandwidth effects, but the extent of recoverable 21-cm window is also strongly redshift-dependent. At Z148, the foreground is almost an order of magnitude higher than at Z42, while the strength of the 21-cm signal is intrinsically weaker in most theoretical models

at these redshifts. Also, since the slope of the wedge is redshift-dependent, at lower frequencies (or higher redshifts), it becomes steeper, occupying a large portion of the 2D k -space, thereby narrowing the 21-cm DA window (J. C. Pober & W. Smith 2025). Therefore, the simple foreground treatment adopted here is insufficient to suppress the bright, spectrally smooth foregrounds below the expected 21-cm signal level for Z148 inside the 21-cm window. For this reason, in the remainder of this paper, we focus our analysis on Z42, where a sufficiently clean 21-cm window is achievable. To isolate the impact of position offsets on the 21-cm power spectrum, the same spectral and spatial window functions, along with polynomial fitting, are applied to both unperturbed and perturbed power spectra. As the positional perturbations considered here primarily introduce smooth spectral features (see Section 6.2), we expect that foreground mitigation techniques using Gaussian Process Regression (GPR; F. Mertens, A. Ghosh & L. Koopmans 2018; F. G. Mertens, J. Bobin & I. P. Carucci 2024) would perform well. The development and testing of this technique to determine whether it enables less foreground leakage in the 21-cm window for Z148 is left for future work.

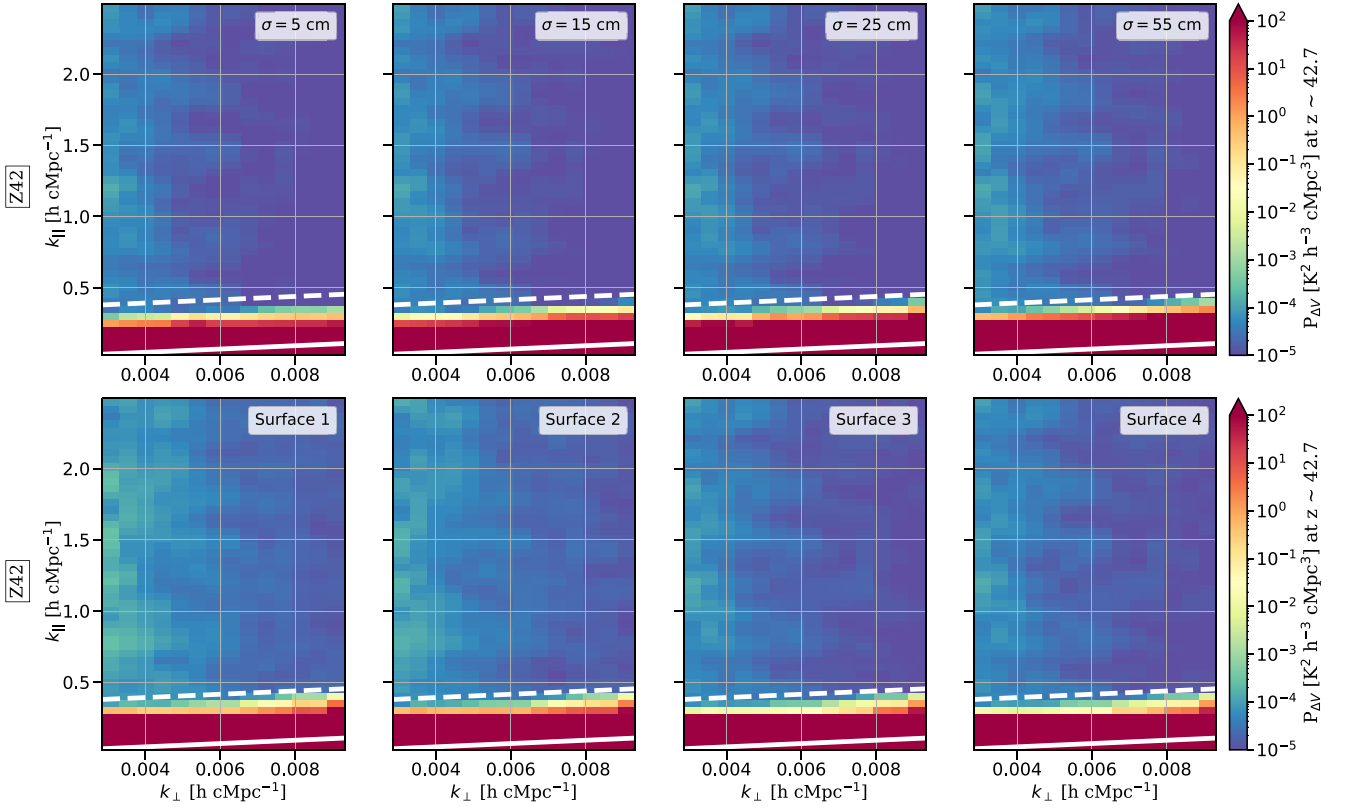


Figure 10. The cylindrically averaged power spectra of the difference in visibilities between the composite model with unperturbed antenna elements and those with perturbations on the xy plane (top row) and those induced due to vertical height differences (bottom row). The offsets along xy directions are drawn from a 2D normal distribution $\mathcal{N}(0, \sigma^2)$ with a fixed random seed of varying standard deviation (increasing towards the right). The different columns in the bottom row represent the four surfaces, each characterized by a different intrinsic surface height variance. Results are shown for Z42. The solid white line represents the horizon limit, and the white dashed line represents the horizon buffer limit.

6.2 Effect of antenna position errors

In this section, we demonstrate the impact of deviation in the antenna position in both the lateral direction and the vertical direction independently on the cylindrically averaged power spectrum. We take a difference of the *Regular visibilities* and *Perturbed visibilities* where $\Delta V \equiv V_{\text{pert}} - V_{\text{reg}}$, and create power spectra of this difference denoted by $P_{\Delta V}(k)$.

6.2.1 Along xy direction

The top row of Fig. 10 shows the cylindrically averaged power spectra of the visibility difference for antenna position offsets along xy direction with amplitudes $\sigma_{xy} = [0.05, 0.15, 0.25, 0.55]$ m (from left to right). The results for Z42 are shown here, while the results for the Z148 are shown in the top row of Fig. B1 in the Appendix C. The change between these columns quantifies the spatial-spectral variance introduced by the antenna position errors on the xy plane. We observe a systematic increase in the variance with the perturbation amplitude σ_{xy} , both within the foreground wedge and in the 21-cm window. To illustrate this more clearly, in Fig. 11, we show the cylindrically averaged power spectra of the difference in visibilities as a function of k_{\parallel} (top row) and k_{\perp} (bottom row), for regions within the foreground wedge ($k_{\parallel} < 0.5$ h cMpc $^{-1}$) and the cosmological window ($k_{\parallel} > 0.5$ h cMpc $^{-1}$) for xy offsets (solid). Inside the wedge, we are dominated by the foreground power at such low frequencies (see top

left panel in Fig. 11). We see that the power spectra decrease with k_{\parallel} because the smooth foregrounds concentrate near low k_{\parallel} , and the Kaiser20 spectral window further suppresses the higher k_{\parallel} modes. Any spectral feature introduced by the perturbation remains subdominant within the wedge. This is because the spectral shape there is set by the smooth foregrounds convolved with the main lobe of the chosen spectral window. On the other hand, we see that in the cosmological window, the power spectra show a weak but coherent oscillatory pattern as a function of k_{\parallel} that preserves its shape across all perturbation amplitudes, with the absolute amplitude scaling with σ_{xy} (see top right panel in Fig. 11). We attribute these features to the Fourier response of the Kaiser20 window, modulated by the additional phase factor introduced by the lateral offsets of the antenna position.

Now we see in the bottom left panel of Fig. 11 that inside the wedge, the perturbations act, to first order, as multiplicative scaling on the bright, spectrally smooth foreground. Any structure from phase error remains small and largely averaged out, leaving a nearly flat response in k_{\perp} . However, in the cosmological window, the power spectra decrease with k_{\perp} because the residuals are mainly dominated by the leakage of the foregrounds through the sidelobes of the spectral window, with not much modulation induced by the positional offsets. Since diffuse foregrounds are strongest on small baselines and decrease with k_{\perp} , the magnitude of the leakage therefore inherits this k_{\perp} trend.

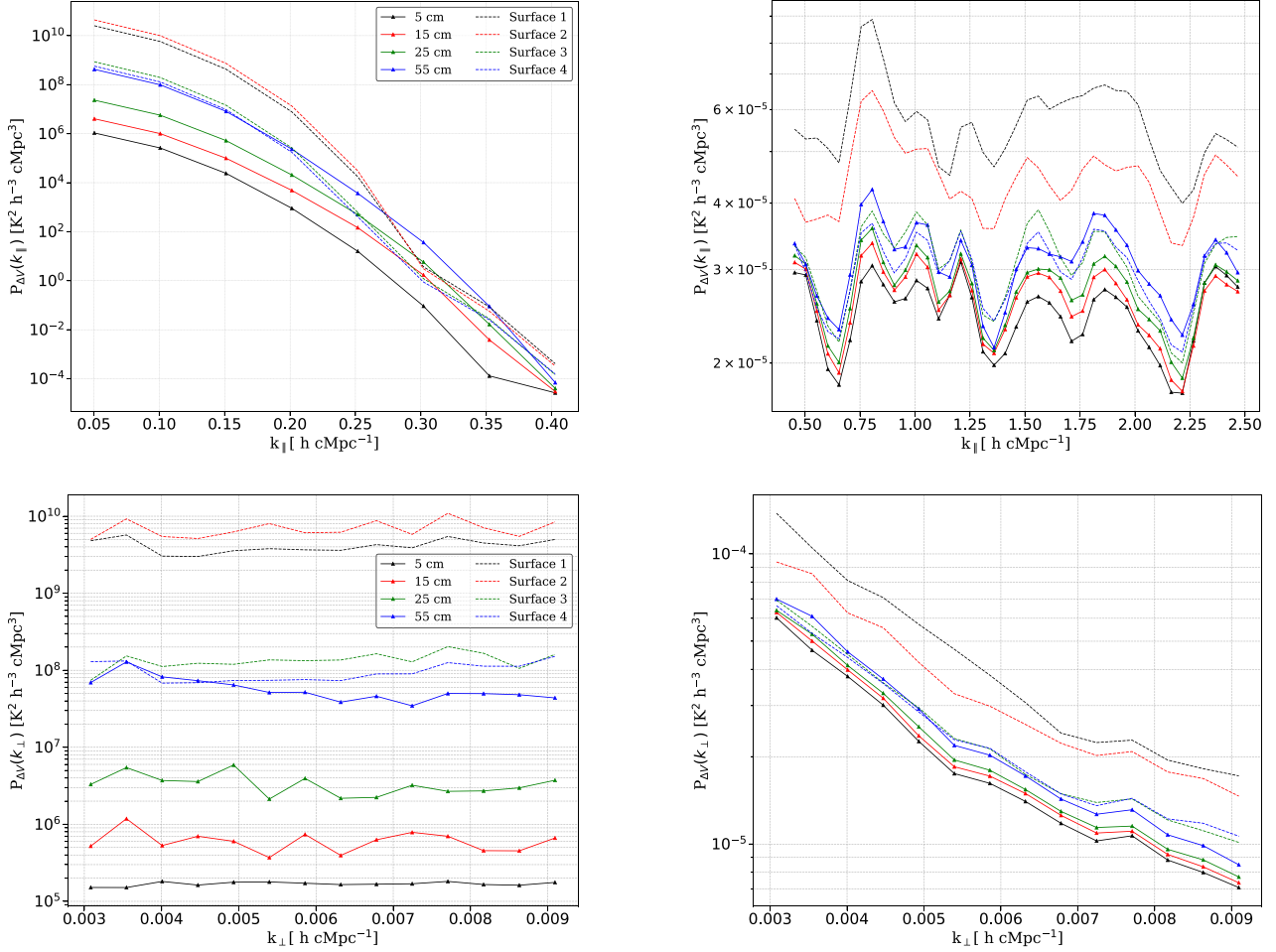


Figure 11. The top row shows the cylindrically averaged power spectra of the difference in visibilities as a function of k_{\parallel} (averaged over k_{\perp}), while the bottom row shows the same as a function of k_{\perp} (averaged over k_{\parallel}). Two regions of $(k_{\perp}, k_{\parallel})$ space are analysed: the cosmological window for $k_{\parallel} > 0.5$ h cMpc $^{-1}$ in the right column and the region inside the foreground wedge for $k_{\parallel} < 0.5$ h cMpc $^{-1}$ in the left column. Different colours represent the perturbation amplitude along xy direction (solid lines) and the four different surfaces (dashed lines) considered in this study. The antenna position offsets themselves change the visibilities only at the few per cent level in amplitude for the perturbation levels considered (Section 4.1), so the difference visibilities $\Delta V = V_{\text{pert}} - V_{\text{reg}}$ are correspondingly small. The oscillatory features in the cosmological window therefore represent very small absolute changes to an already foreground-suppressed region, governed by the Fourier response of the Kaiser20 window function and modulated by the small phase errors from the positional offsets.

6.2.2 Along z direction

The bottom row of Fig. 10 shows the cylindrically averaged power spectra of the visibility differences obtained when antenna position offsets are introduced along the z direction, corresponding to the variation in the height of Surfaces 1, 2, 3, and 4 (see Section 2.1.3), from left to right. Here we present the results for Z42, and the results for the Z148 are shown in the bottom row of Fig. B1 in the Appendix C. Although all four surfaces show similar spatial correlation, the RMS difference between vertical heights was seen to vary across the array scale (see Table 1). Specifically, Surface 1 and 2 show a higher RMS height deviation than Surface 3 and 4.

For correlated height errors arising from surface roughness, the additional phase factor scales as $\frac{\sigma_z^2}{\lambda^2} \left(\frac{r}{r_s}\right)^{2H}$ (see equations 18 and 20). This implies that small k_{\perp} modes are only weakly affected for all surfaces, while long baselines experience the strongest suppression in the image domain. Surfaces 1 and 2, with higher σ_s , lead to broader convolution kernels in the uv domain, causing significant smearing across uv cells. In the power spectra

of the visibility difference, this appears as increase in variance towards larger k_{\perp} modes and also leaks power into the 21-cm window. Surfaces 3 and 4 will have a comparatively narrower convolution kernel, even for longer baselines, thus minimizing mode mixing. As a result, the total variance introduced in both the wedge and the 21-cm window by these surfaces remains low. This is also evident in Fig. 11, as indicated by the dashed lines. The weak oscillatory pattern in the power spectra is the same for the different surfaces as for the lateral offsets, as discussed above. Clearly, rough surfaces introduce stronger chromatic distortions compared to the largest lateral offsets considered in this study.

For non-coplanar arrays, the projection effect further amplifies phase decoherence for near horizon sources. As the sky rotates, projected baseline lengths vary with orientation, and for sources towards the horizon, the baselines experience the maximum change. Therefore, these baselines are more sensitive to small phase errors, resulting in a brighter wedge with leakage into the higher k_{\parallel} modes. This geometric projection effect, in addition to the surface-induced kernel broadening in the uv domain can

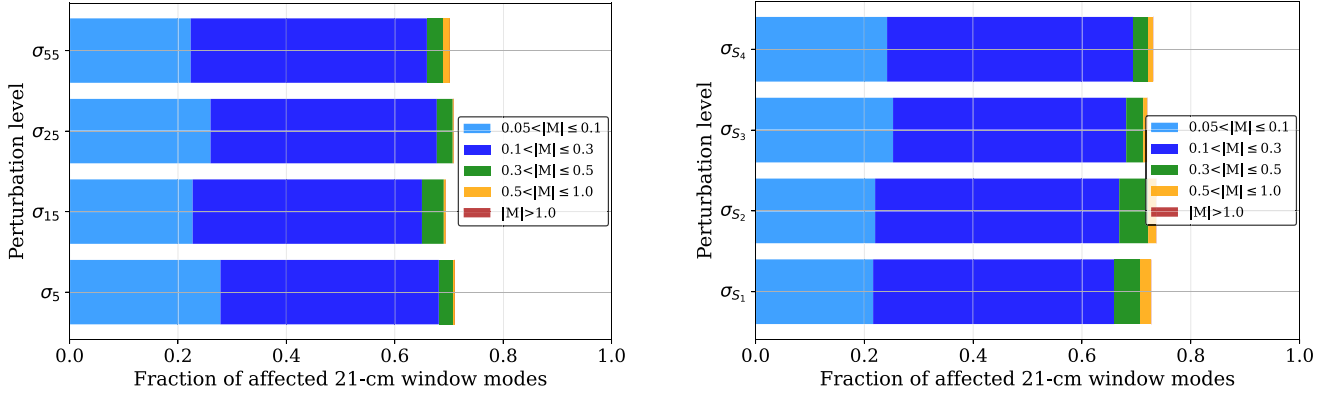


Figure 12. Fraction of contaminated modes in 21-cm window for perturbations along xy direction (left column) and along z direction (right column). Colours distinguish increasing levels of contamination: light blue ($0.05 < |M| \leq 0.1$), dark blue ($0.1 < |M| \leq 0.3$), green ($0.3 < |M| \leq 0.5$), orange ($0.5 < |M| \leq 1.0$), and red ($|M| > 1.0$). The comparison highlights that the majority of affected modes (~ 60 per cent) fall in the range ($0.1 < |M| \leq 0.3$) i.e. contaminated at 10–30 per cent level. However, antenna offsets induced by height differences of the surfaces produce a broader distribution towards higher contamination, with a slightly larger fraction of modes exceeding ($|M| > 0.3$) compared to offsets along xy direction.

be a problem for wide FoV instruments such as DEX, but could be mitigated by choosing an appropriate spatial window function to suppress the larger errors near the horizon.

6.2.3 Impact on the Dark Ages 21-cm power spectrum

In Sections 6.2.1 and 6.2.2, we have characterized the impact of antenna position errors using the power spectrum of visibility differences. In addition, we now define a dimensionless metric in the 21-cm window ($k_{\parallel} > 0.5 \text{ h cMpc}^{-1}$),

$$|M(k_{\perp}, k_{\parallel})| \equiv \frac{|P_{\text{pert}}(k_{\perp}, k_{\parallel})| - |P_{\text{reg}}(k_{\perp}, k_{\parallel})|}{|P_{21}(k_{\perp}, k_{\parallel})|}, \quad (29)$$

where P_{reg} and P_{pert} are the cylindrically averaged power spectra of the composite signal for the regular and perturbed arrays, respectively, and P_{21} is the cylindrically averaged 21-cm signal power spectrum. This metric provides an answer to the question: ‘How large is the error introduced by the antenna position offsets compared to the cosmological signal we want to detect?’ Therefore, equation (29) provides a quantitative measure of the fraction of Fourier modes that are contaminated at different levels relative to the DA 21-cm signal. As seen in the bottom panel of Fig. 9 for Z42, within the 21-cm window, the foreground is more than a factor of ten lower than the DA 21-cm signal, so the numerator in equation (29) is dominated by the signal. A conservative threshold of $|M| < 0.05$ is acceptable in our case. We believe that contamination below this level is small compared to other systematic uncertainties (e.g. calibration errors, thermal noise). Modes exceeding this threshold are further subdivided into bins corresponding to increasing levels of contamination.

The left panel of Fig. 12 summarizes the impact of antenna position errors along the xy direction on the 21-cm window for Z42. For all the perturbation cases considered in this study, the majority of the contaminated modes (≈ 57 – 62 per cent) depending on the perturbation level fall within the range $0.1 < |M| \leq 0.3$ i.e. they have a bias between 10 per cent and 30 per cent relative to the 21-cm signal. A smaller but still significant fraction of modes (≈ 32 – 39 per cent) lie in the mild contamination bin $0.05 < |M| \leq 0.1$. Only a few per cent of modes are more strongly affected, with $0.3 < |M| \leq 0.5$ contributing ≈ 3 – 6 per cent depending on the perturbation level. Modes with $|M| > 0.5$, where perturbations

begin to dominate over the 21-cm signal itself, are exceedingly rare (≤ 2 per cent) and no modes were found with $|M| > 1$ across the tested cases.

For the four representative surfaces, the impact of antenna position errors along the z direction on the 21-cm window for Z42 is shown in the right panel of Fig. 12. As before, for almost all cases, the largest affected modes (≈ 60 per cent) fall within the range $0.1 < |M| \leq 0.3$. Between 30 and 35 per cent of contaminated modes lie in the $0.05 < |M| \leq 0.1$ range, independent of the surfaces. Only 4–7 per cent of modes exceed the $0.3 < |M| \leq 0.5$ threshold, and fewer than 3 per cent lie in the $0.5 < |M| \leq 1.0$ range. No modes with $|M| > 1$ were observed.

We reiterate for the readers that the cylindrically averaged power spectrum of visibility differences for the composite sky (foreground + 21-cm cosmological signal) shown in Fig. 10 and the quantity defined in equation (29) measure fundamentally different aspects of the perturbation. The power spectrum of visibility differences, $P_{\Delta V} \propto |\Delta V|^2$, isolates only the perturbation-induced component and therefore remains small, at the level of $\sim 10^{-4}$ – 10^{-5} in the cosmological window for $k_{\parallel} > 0.5 \text{ h cMpc}^{-1}$ (as seen in Fig. 11). In contrast, the numerator of equation (29) is the difference between the power spectra of the composite sky which gives the fractional error relative to the 21-cm signal only power spectrum. Because the 21-cm signal is intrinsically weak, even a very small absolute change in the visibilities can translate into a comparatively large fractional difference. Consequently, values within the range $0.1 < |M| \leq 0.3$ reflect small absolute perturbations to a foreground-dominated quantity when expressed in units of the DA 21-cm signal.

Additionally, for the very low frequencies considered here, the foreground power is extremely bright compared to the 21-cm signal. Consequently, the dominant source of leakage into the 21-cm window is the shape of the spectral window function, in our case the Kaiser20. Under these conditions, errors from antenna position offsets remain subdominant compared to the spectral correlations imposed by this window and thus do not introduce significant additional contamination. Therefore, in all cases studied, the impact of deviation from a regular, coplanar grid lies below the 21-cm power spectrum within the 21-cm window for $k_{\parallel} > 0.5 \text{ h cMpc}^{-1}$, given a Kaiser20 window function applied to a 5 MHz bandwidth with 100 channels at Z42.

However, an important difference arises between the two perturbation cases. The offsets along xy direction produces fewer modes with higher contamination levels, i.e. $|M| > 0.3$ than height perturbations. This difference is most evident for Surfaces 1 and 2 that produce the largest fraction of modes with $|M| > 0.3$.

For offsets along xy direction, we can directly frame the tolerances in terms of the dimensionless ratio $\frac{\sigma}{\lambda}$. Adopting a conservative approach to ensure that contaminated modes ($|M| > 0.5$) remain below the 1 per cent level, we find from Fig. 12 that this occurs when $\sigma_{xy} = 0.25$ m. This is roughly 3 per cent of the observing wavelength ($\lambda \simeq 9.23$ m), corresponding to $\frac{\sigma_{xy}}{\lambda} \leq 0.027$. This implies that as long as deployment accuracy in the lateral direction is controlled to within 3 per cent of a wavelength, strongly contaminated modes occur only at sub-per cent levels in the 21-cm for $k_{\parallel} > 0.5$ h cMpc $^{-1}$.

For vertical offsets, tolerances cannot be expressed through a single $\frac{\sigma}{\lambda}$ ratio, since the perturbations arise from the roughness of a fractal surface that depends on the reference length scale, r_s . Table 1 summarizes H and σ_s for the four representative lunar terrains, evaluated at two characteristic length scales: $L = 4$ m and $L = 175$ m. At the smaller scale ($L = 4$ m), $\frac{\sigma_{4m}}{\lambda}$ is 0.032, 0.028, 0.016, and 0.015 for Surfaces 1–4, respectively, at 32.5 MHz. Clearly, the ratio for Surfaces 3 and 4 are well within 3 per cent of a wavelength. This level is comparable to acceptable offsets along xy direction and is expected to place most affected modes well below the threshold $|M| > 0.5$. Therefore, they are unlikely to cause significant phase decoherence in the visibilities. However, at longer scales ($L = 175$ m), $\frac{\sigma_{175m}}{\lambda}$ is 0.760, 0.446, 0.206, and 0.196. At 32.5 MHz, this corresponds to 20–76 per cent of a wavelength, which will produce phase errors sufficient for a significant fraction of modes to exceed the $|M| > 0.5$ threshold. Surfaces 3 and 4, although smoother on small scales and therefore less problematic for smaller baselines, show height differences that is a significant fraction of a wavelength at larger scales. Therefore, while such surfaces are preferred for the deployment of compact radio arrays, careful calibration will still be required to correct for the phase errors introduced by height variations at large scale. Thus, lateral offsets can be controlled through deployment tolerances ($\frac{\sigma}{\lambda} \lesssim 0.027$), but vertical offsets must be mitigated primarily through the choice of deployment site.

7 SUMMARY AND CONCLUSIONS

DEX is a proposed low frequency radio interferometer concept for deployment on the lunar farside, designed to probe the DA using 21-cm signal. The current minimum baseline configuration consists of a compact, regular 32×32 grid of zenith-pointing, co-located cross-dipole antennas of 3 m length with near-unity filling factor, operating in the 7–50 MHz. band. A key feature of DEX is the intention to use an FFT correlator, which allows efficient data processing with reduced power consumption, an essential advantage for large arrays operating under the resource constraints of a lunar surface mission.

However, the effectiveness of such a correlation architecture is dependent on the assumption of a perfectly regular grid. Even small deviations from this geometry can have important consequences. First, position errors will break the redundancy required for FFT correlation, potentially prohibiting its use, and thereby greatly increasing both computational and energy costs. This is an important limitation for large arrays based on the lunar surface. Secondly, such offsets lead to decoherence of the visibilities, lead-

ing to spectral contamination, and thereby biasing the Fourier modes used for 21-cm signal extraction. Finally, they complicate calibration, since the relative positions and orientations of antennas must be known to within a small fraction of a wavelength.

In this work, we focus on lateral and vertical position offsets independently, which are the most likely errors to arise from rover deployment and local topographic undulations on the lunar surface. To do so, we adopt a visibility-based approach. This is a practical consideration because full forward simulations at the electric field level for DEX-like arrays would be computationally intensive.

To quantify the impact of such perturbations on the 21-cm power spectrum for DEX, we first derive and generalize the equations describing the perturbed visibilities, without taking a flat-sky approximation and thereby enabling a wide-field treatment. Additionally, these equations provide a theoretical basis for interpreting the trends observed in the simulations obtained with the SPADE-21cm (Simulation Pipeline for Analysing Dark agEs using 21 cm). This end-to-end pipeline incorporates a cosmological signal model, a realistic sky, lunar topography data, and a complete lunar topocentric coordinate system. Mare Ingenii, an impact basin located in the lunar southern hemisphere on the farside of the Moon, is selected as the representative deployment site for our simulations. To our knowledge, this constitutes the first systematic assessment of antenna position errors for a lunar surface array, providing practical guidance for array design, deployment strategy, and site selection. Our main conclusions are summarized below:

Criteria for reliable deployment site: A preferred deployment site should ideally depend on RMS deviation in height differences σ_s , Hurst exponent H . These quantities are coupled and degenerate, and their interpretation also depends on the choice of a reference distance r_s between two points on the surface. At all times, σ_s must remain low across all baseline lengths of interest such that the phase errors due to the height differences are minimized. The impact of H depends on array configuration: higher H is preferable for compact arrays as it introduces least possible variance on small baselines, whereas lower H is advantageous for longer baselines.

Impact of antenna position offsets on the $uw1$ plane: Random antenna position errors can be described as multiplicative attenuation kernels in the image domain that act as an additional window function, equivalent to convolutional kernels in the uw domain. For offsets along xy direction, the ensemble-averaged kernel is baseline-independent, producing relatively uniform mode mixing across all baselines. However, projection effects generate non-zero Δw terms that increase suppression at larger angular distances from the phase centre, particularly in wide-field observations with maximum impact towards the horizon.

In contrast, offsets along z direction introduce baseline-dependent suppression due to the statistical model of correlated surface topography, where the height-difference scales as r^{2H} . The kernel width and degree of attenuation therefore increase with baseline length, observing frequency, and angular distance from the phase centre, and are further modulated by the Hurst exponent and RMS surface height deviation. Anisotropy in the uw plane also directly reflects anisotropy in the underlying surface roughness, with stronger phase decorrelation for baselines aligned along axes of maximum roughness. Thus, while xy and z offsets can both be described within the same analytical framework, their different statistical models give rise to a baseline-

independent kernel and baseline-dependent kernel, in the image and uv domains, respectively.

Impact of antenna position offsets on the power spectrum: For both the perturbation scenarios, we see the power to increase with increasing perturbation levels in both the wedge and in the 21-cm window, and dominated fully by residual leakage due to the chosen spectral window function (Kaiser20 in our case). For the offsets along xy direction, the increment remains modest up to $\sigma \leq 0.25$ m, but becomes measurable beyond this level. For positional perturbations along the z direction, the phase decoherence is governed by the distinct surface topography, characterized by σ_s and H . In this study, all the four surfaces have similar values of H , indicating comparable spatial correlation. However, Surfaces 1 and 2 show larger σ_s across all scales, leading to larger phase errors, which scale as $\sigma_s^2 (r/r_s)^{2H}$. In contrast, Surfaces 3 and 4 are comparatively smoother, with lower σ_s . Consequently, Surfaces 1 and 2 show increased leakage into the 21-cm window, while Surfaces 3 and 4 show leakage at a reduced level.

Our simplified treatment provides a first-order estimate of the impact of the positional offsets along xy direction and along z direction independently. More realistic scenarios with correlated offsets could be incorporated in future, with more detailed simulations and more complex spatial error patterns. But as long as the level of the perturbations remains comparable to the values explored in this study, we do not expect a larger impact on the power spectrum. However, we do note that the vertical height offsets are expected to dominate the resulting phase errors and hence distort the power spectrum.

Position induced errors in the 21-cm DA power spectrum: Antenna position errors typically bias the 21-cm signal at the 10–30 per cent level, with about one-third of modes affected at 5–10 per cent and only a few per cent exceeding 30 per cent. Modes where the error is larger than 0.5 times the 21-cm signal ($|M| > 0.5$) are rare (≤ 2 per cent), and no modes were found with errors larger than the 21-cm signal itself ($|M| > 1$). For lateral offsets, keeping ($\frac{\sigma_{xy}}{\lambda} \lesssim 0.027$) (corresponding to $\sigma_{xy} = 0.25$ metres at 32.5 MHz) limits the fraction of modes with $|M| > 0.5$ to less than 1 per cent. For vertical offsets, the impact depends on the surface roughness. The small-scale variations correspond to 1–3 per cent of a wavelength, comparable to xy offsets. But long-scale roughness corresponds to 20–76 per cent of a wavelength, producing a large fraction of modes $|M| > 0.3$. This conclusion holds for the 21-cm window with $k_{\parallel} > 0.5$ h cMpc $^{-1}$ over the range of $k_{\perp} = 0.003$ – 0.009 h cMpc $^{-1}$ while applying a Kaiser spectral window with $\beta = 20$.

Thus, errors from lateral offsets can be reduced by maintaining placement accuracy within the derived tolerance limits, whereas errors from vertical offsets introduce more severe phase errors and must be taken care through careful site selection.

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DATA AVAILABILITY

The data underlying this article will be shared on reasonable request to the corresponding author.

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SUPPORTING INFORMATION

Supplementary data are available at *MNRAS* online.

Figure_1.png

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APPENDIX A: ROTATION MATRICES FOR ENU TO MCMF TO UVW

Consider a point on the lunar surface defined by selenographic latitude ϕ and longitude Λ , with Λ increasing westward. The unit position vector in the MCMF frame is given by

$$\hat{\mathbf{r}} = \begin{pmatrix} \cos \phi \cos \Lambda \\ -\cos \phi \sin \Lambda \\ \sin \phi \end{pmatrix}. \quad (\text{A1})$$

The outward radial unit vector $\hat{\mathbf{u}}$, is identical to the position vector. The $\hat{\mathbf{n}}$ points along the meridian towards the North Pole, obtained by projecting the $\hat{\mathbf{Z}}_{\text{MCMF}}$ represented by $[0, 0, 1]^T$ onto the tangent plane perpendicular to $\hat{\mathbf{u}}$. The projection is

$$\hat{\mathbf{n}} = \frac{[0, 0, 1]^T - (\hat{\mathbf{u}} \cdot [0, 0, 1]^T)\hat{\mathbf{u}}}{\|[0, 0, 1]^T - (\hat{\mathbf{u}} \cdot [0, 0, 1]^T)\hat{\mathbf{u}}\|} \quad (\text{A2})$$

where $\hat{\mathbf{u}} \cdot [0, 0, 1]^T = \sin \phi$. Solving the numerator in equation (A2), we get

$$[0, 0, 1]^T - \sin \phi \hat{\mathbf{u}} \quad (\text{A3})$$

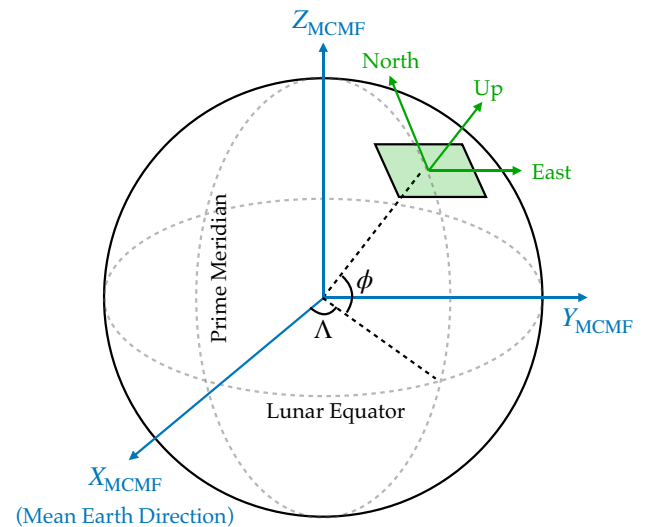


Figure A1. The MCMF and lunar topocentric frames.

$$\begin{aligned}
 &= [0, 0, 1]^T - \sin \phi [\cos \phi \cos \Lambda, -\cos \phi \sin \Lambda, \sin \phi]^T \\
 &= [-\sin \phi \cos \phi \cos \Lambda, \sin \phi \cos \phi \sin \Lambda, 1 - \sin^2 \phi]^T \\
 &= [\sin \phi \cos \phi (-\cos \Lambda), \sin \phi \cos \phi \sin \Lambda, \cos^2 \phi]^T. \quad (\text{A4})
 \end{aligned}$$

The denominator in equation (A2) is simply the norm given by

$$\|[-\sin \phi \cos \phi \cos \Lambda, \sin \phi \cos \phi \sin \Lambda, \cos^2 \phi]^T\| \quad (\text{A5})$$

$$\begin{aligned}
 &= \sqrt{(\sin \phi \cos \phi)^2 (\cos^2 \Lambda + \sin^2 \Lambda) + \cos^4 \phi} \\
 &= \sqrt{\sin^2 \phi \cos^2 \phi + \cos^4 \phi} = \cos \phi. \quad (\text{A6})
 \end{aligned}$$

Finally, we get

$$\hat{\mathbf{n}} = \frac{1}{\cos \phi} [\sin \phi \cos \phi (-\cos \Lambda), \sin \phi \cos \phi \sin \Lambda, \cos^2 \phi]^T \quad (\text{A7})$$

$$= [-\sin \phi \cos \Lambda, \sin \phi \sin \Lambda, \cos \phi]^T. \quad (\text{A8})$$

The $\hat{\mathbf{e}}$, perpendicular to $\hat{\mathbf{u}}$ and $\hat{\mathbf{n}}$, lies along the parallel (constant ϕ) and points eastward (towards decreasing Λ). Using the right-hand rule,

$$\hat{\mathbf{e}} = \hat{\mathbf{n}} \times \hat{\mathbf{u}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ -\sin \phi \cos \Lambda & \sin \phi \sin \Lambda & \cos \phi \\ \cos \phi \cos \Lambda & -\cos \phi \sin \Lambda & \sin \phi \end{vmatrix}. \quad (\text{A9})$$

Expanding the determinant:

$$\begin{aligned}
 &= \hat{\mathbf{i}}[(\sin \phi \sin \Lambda)(\sin \phi) - (\cos \phi)(-\cos \phi \sin \Lambda)] \\
 &\quad - \hat{\mathbf{j}}[(-\sin \phi \cos \Lambda)(\sin \phi) - (\cos \phi)(\cos \phi \cos \Lambda)] \\
 &\quad + \hat{\mathbf{k}}[(-\sin \phi \cos \Lambda)(-\cos \phi \sin \Lambda) - (\sin \phi \sin \Lambda)(\cos \phi \cos \Lambda)] \\
 &= \hat{\mathbf{i}}[\sin \Lambda] + \hat{\mathbf{j}}[\cos \Lambda] + \hat{\mathbf{k}}[0] = [\sin \Lambda, \cos \Lambda, 0]^T. \quad (\text{A10})
 \end{aligned}$$

We note that the unit vectors ($\hat{\mathbf{e}}, \hat{\mathbf{n}}, \hat{\mathbf{u}}$) corresponds directly to the unit vectors in standard spherical-polar coordinate system. The vector $\hat{\mathbf{u}}$ is identical to the radial unit vector $\hat{\mathbf{r}}$. The eastward tangent vector $\hat{\mathbf{e}}$, along a line of constant latitude, coincides with the azimuthal unit vector $\hat{\phi}$. The northward tangent vector $\hat{\mathbf{n}}$ points towards increasing latitude and is therefore opposite to the usual polar unit vector $\hat{\theta}$, which increases towards the south. Thus, $\hat{\mathbf{u}} = \hat{\mathbf{r}}$, $\hat{\mathbf{e}} = \hat{\phi}$, and $\hat{\mathbf{n}} = -\hat{\theta}$. The rotation matrix \mathbf{R}_{MCMF} is finally

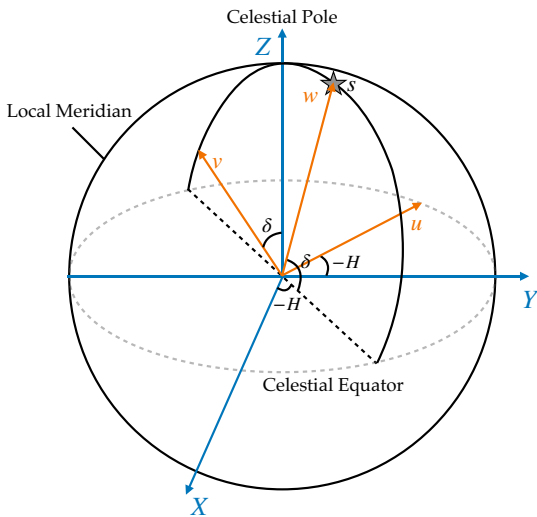


Figure A2. Geometric relation between the celestial and interferometric coordinate systems.

expressed as

$$\mathbf{R}_{\text{MCMF}} = \begin{pmatrix} \hat{\mathbf{e}} \\ \hat{\mathbf{n}} \\ \hat{\mathbf{u}} \end{pmatrix} = \begin{pmatrix} \sin \Lambda & -\sin \phi \cos \Lambda & \cos \phi \cos \Lambda \\ \cos \Lambda & \sin \phi \sin \Lambda & -\cos \phi \sin \Lambda \\ 0 & \cos \phi & \sin \phi \end{pmatrix}. \quad (\text{A11})$$

We now define the direction to a source \mathbf{s} in the MCMF coordinate system. A source located at hour angle H (west positive) and declination δ has unit vector

$$\hat{\mathbf{s}} = \hat{\mathbf{w}}(H, \delta) = \begin{pmatrix} \cos \delta \cos H \\ -\cos \delta \sin H \\ \sin \delta \end{pmatrix}. \quad (\text{A12})$$

The tangent plane at the phase centre is perpendicular to $\hat{\mathbf{w}}$. The $\hat{\mathbf{v}}$ point towards the Celestial pole. The $\hat{\mathbf{v}}$ is obtained by projecting the $\hat{\mathbf{Z}}_{\text{MCMF}} = [0, 0, 1]^T$ on to this plane and normalizing

$$\hat{\mathbf{v}}(H, \delta) = \begin{pmatrix} -\sin \delta \cos H \\ \sin \delta \sin H \\ \cos \delta \end{pmatrix}. \quad (\text{A13})$$

The $\hat{\mathbf{u}}$ lies in the tangent plane, orthogonal to $\hat{\mathbf{w}}$ and $\hat{\mathbf{v}}$. By the right-hand convention,

$$\hat{\mathbf{u}} = \hat{\mathbf{v}} \times \hat{\mathbf{w}} = \begin{pmatrix} \sin H \\ \cos H \\ 0 \end{pmatrix}. \quad (\text{A14})$$

The rotation matrix from MCMF to uvw coordinate system is then given by

$$\mathbf{R}_{uvw} = \begin{pmatrix} \hat{\mathbf{u}}^T \\ \hat{\mathbf{v}}^T \\ \hat{\mathbf{w}}^T \end{pmatrix} = \begin{pmatrix} \sin H & \cos H & 0 \\ -\sin \delta \cos H & \sin \delta \sin H & \cos \delta \\ \cos \delta \cos H & -\cos \delta \sin H & \sin \delta \end{pmatrix}. \quad (\text{A15})$$

For a fixed array at longitude Λ , observing a phase centre (α_0, δ_0), we absorb Λ by setting $\Lambda = 0$ in the local frame and define the local hour angle $H_0 \equiv \text{LST}(t, \Lambda) - \alpha_0$. The equations (A11) and A15 then become

$$\mathbf{R}_{\text{MCMF}\Lambda=0} = \begin{pmatrix} 0 & -\sin \phi \cos \phi \\ 1 & 0 & 0 \\ 0 & \cos \phi & \sin \phi \end{pmatrix}, \quad (\text{A16})$$

$$\mathbf{R}_{uvw} = \begin{pmatrix} \sin H_0 & \cos H_0 & 0 \\ -\sin \delta_0 \cos H_0 & \sin \delta_0 \sin H_0 & \cos \delta_0 \\ \cos \delta_0 \cos H_0 & -\cos \delta_0 \sin H_0 & \sin \delta_0 \end{pmatrix}. \quad (\text{A17})$$

Let $\Delta \mathbf{r}_{\text{ENU}} = (\Delta E, \Delta N, \Delta U)^T$ denote antenna position offsets in metres. From the main text (equations 6 and 7)

$$\begin{pmatrix} \Delta u \\ \Delta v \\ \Delta w \end{pmatrix} = \frac{1}{\lambda} \mathbf{R}_{uvw} \mathbf{R}_{\text{MCMF}} \begin{pmatrix} \Delta E \\ \Delta N \\ \Delta U \end{pmatrix}. \quad (\text{A18})$$

APPENDIX B: IMPACT OF WINDOW FUNCTIONS

The choice of spectral window functions in the 21-cm data analysis is closely related to the foreground mitigation strategy adopted, as it shapes the instrument's response along the k_{\parallel} direction in Fourier space.

In this paper, we test four spectral window functions: the 4-term Blackman-Harris (BH), Dolph-Chebyshev with 150 and 180 dB sidelobe suppression (DC150, DC180), and the Kaiser window with $\beta = 20$ (Kaiser20). Each offers distinct trade-offs between

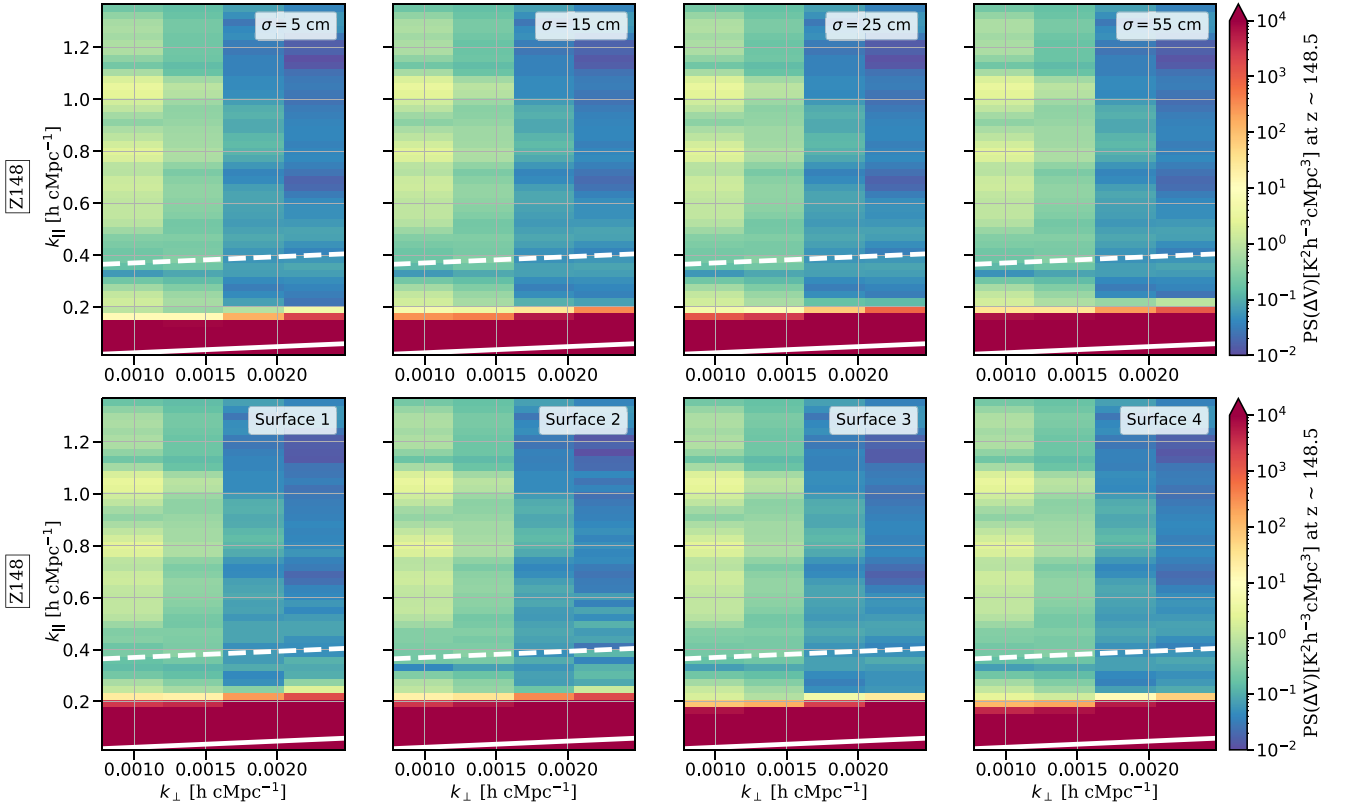


Figure B1. The cylindrically averaged power spectra of the difference in visibilities between the composite model with unperturbed antenna elements and those with perturbations on the xy plane (top row) and those induced due to the surface irregularities (bottom row). The offsets along xy directions are drawn from a 2D normal distribution $\mathcal{N}(0, \sigma^2)$ with a fixed random seed of varying standard deviation (increasing towards right). The different columns in the bottom row represents the variance in the power spectra for the different surfaces. Results are shown for Z148. The solid white line represents the horizon limit, and the white dashed line represents the horizon buffer limit.

spectral leakage suppression and resolution along k_{\parallel} in the noiseless scenario. The equivalent-noise bandwidth (ENBW) of a window is defined as

$$\text{ENBW} = N \frac{\sum_n w_n^2}{(\sum_n w_n)^2}, \quad (\text{B1})$$

where N is the number of frequency channels and w_n are the coefficients of the chosen spectral window function. Therefore, this ratio gives the effective width of the main lobe of the window in units of frequency bins, and provides a measure of the spectral resolution along k_{\parallel} . Narrower ENBW values correspond to sharper resolution in k_{\parallel} , but that comes with weaker suppression of SLL, whereas wider ENBW values provide stronger suppression of SLL at the cost of coarser k_{\parallel} resolution.

Fig. 8 shows the cylindrically averaged power spectra for Z42, averaged over all baselines, comparing the effect of different window functions. The BH window gives the narrowest ENBW (≈ 2 bins), providing the sharpest k_{\parallel} resolution. However, with a SLL of ≈ 92 dB, it lacks the required dynamic range to suppress foregrounds at the levels required for DA experiments. By contrast, the DC150, DC180, and Kaiser20 windows achieve a SLL of $\gtrsim 150$ dB. The DC150 window has an ENBW of 2.38 bins and provides strong SLL suppression relative to BH. The Kaiser20 window and DC180 window have nearly identical ENBW of 2.59 bins, and a SLL suppression sufficient to reach the sensitivity beyond the expected 21-cm level at $z \sim 42.5$ in the noiseless

regime. Although their ENBWs are similar, they show different spectral behaviour. The DC180 window produces an equiripple sidelobe, whereas the Kaiser20 window shows a monotonic sidelobe roll-off that continues to decrease towards higher k_{\parallel} modes. For foreground-dominated measurements during DA, Kaiser20 is preferable as it minimizes the risk of isolated contamination peaks and provides a smooth roll-off at higher delays. The solid magenta and dashed lines in Fig. 8 represent the cylindrically averaged power spectra of the 21-cm signal only using BH and DC180, respectively. The two curves are in close agreement with each other, showing that the choice of window functions does not affect the intrinsic spectral features of the 21-cm signal, at least in the absence of thermal noise and foregrounds.

We note that the most suitable window function will depend on the approach taken for foreground mitigation. Since our aim in this paper is to quantify the impact of antenna position offset on the 21-cm power spectrum, the Kaiser20 window is found to be sufficient for our purposes. It is also important to note that when strongly tapered windows are used, a sufficient number of frequency channels (≥ 100) are used for the window to work effectively. This is because such windows require a certain minimum width in the spectral domain to fully realize their sidelobe suppression characteristics and avoid numerical artefacts such as poor frequency response or incomplete tapering near the window edges (F. J. Harris 2005).

APPENDIX C: IMPACT OF POSITION ERROR FOR Z148

The top row of Fig. B1 presents the cylindrically averaged power spectra of the visibility difference for antenna position offsets along xy direction with amplitudes $\sigma_{xy} = [0.05, 0.15, 0.25, 0.55]$ m (from left to right). The bottom row of Fig. B1 shows the corresponding results for vertical height offsets, where the variation in the antenna height is determined by Surfaces 1, 2, 3 and 4 (see Section 2.1.3), from left to right. The results for Z148 are shown here.

APPENDIX D: PHASE-ERROR THRESHOLD FOR LATERAL OFFSET

In this section, we provide an asymptotic estimate of the phase accuracy required for the smoothing of uv modes to remain small in effect. To quantify this, we Taylor expand the exponential for small phase variance using $\exp(-x) = 1 - x$ for $|x| \ll 1$, then equation (16) can be written as

$$P_{xy} \simeq 1 - 2\pi^2 \text{Var}(\Delta\phi_{xy}). \quad (\text{D1})$$

If there are no positional offsets, then $P_{xy} = 1$. Therefore, the fractional difference from 1 is

$$\frac{1 - P_{xy}}{P_{xy}} \simeq 2\pi^2 \text{Var}(\Delta\phi_{xy}).$$

We define ‘negligible’ smoothing as the regime where P_{xy} deviates from 1 by less than about 1 per cent across the finite field of view, i.e. $|1 - P_{xy}/P_{xy}| \lesssim 0.01$. This implies

$$2\pi^2 \text{Var}(\Delta\phi_{xy}) \lesssim 0.01.$$

Therefore,

$$\text{Var}(\Delta\phi_{xy}) \lesssim \frac{0.01}{2\pi^2} \approx 5 \times 10^{-4},$$

corresponding to an rms phase error $\lesssim 0.02$ rad $\approx 1^\circ$.

Using equation (13), and assuming that $\|\mathbf{A}_{xy}^\top \mathbf{x}\|$ is ~ 1 over the finite field of view, this translates to $\frac{\sigma_{xy}}{\lambda} \lesssim 0.02$.

Thus, if the rms positional offset along xy direction is below a few per cent of the wavelength, P_{xy} is flat at the 1 per cent level and the associated smoothing in uv domain is negligible. For e.g. at $\nu = 32.5$ MHz, we have $\lambda \simeq 9.2$ m, then for our largest perturbation along xy direction, $\sigma_{xy} = 0.55$ m, σ_{xy}/λ corresponds to 0.06, with an rms phase error of $\approx 4.8^\circ$ and $P_{xy} \approx 0.87$ (a ~ 13 per cent deviation). This test case therefore lies outside the ‘negligible smoothing’ regime by our 1 per cent criterion.

Now let us consider $\sigma_{xy} = 0.25$ m. At the same observing frequency, this corresponds to $\frac{\sigma_{xy}}{\lambda} = \frac{0.25}{9.2} \approx 0.027$. Therefore,

$$\text{Var}(\Delta\phi_{xy}) \simeq (0.027)^2 \approx 7.3 \times 10^{-4}.$$

Substituting in equation (D1) gives,

$$P_{xy} \simeq 1 - 2\pi^2(7.3 \times 10^{-4}) \approx 1 - 0.014 \approx 0.986.$$

This corresponds to a deviation of ~ 1.4 per cent, only slightly above the 1 per cent threshold. Therefore, a positional offset of $\sigma_{xy} = 0.25$ m along xy direction produces a small level of smoothing of uv modes and lies close to the boundary of the ‘negligible smoothing’ regime as defined above.

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