



EXPERIMENTAL AND NUMERICAL INVESTIGATION ON THE HEEL AND DRIFT INDUCED HYDRODYNAMIC LOADS OF A HIGH SPEED CRAFT

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ABSTRACT : In order to provide an insight into the manoeuvring of high speed crafts, an experimental study was undertaken at the towing tank of Delft University of Technology, using a rescue vessel of the Royal Netherlands Sea Rescue Institution (KNRM).

One of the primary objectives of this work is to validate a 3D time domain panel method in the manoeuvring loads calculation in calm water using the results of the experiments. The mathematical model was successfully implemented for the prediction of high speed craft vertical motions in waves. The application of this panel code to manoeuvrability problems is still in development and has to be further analysed and validated.

This paper focuses on the determination of the heel and drift hydrodynamic induced hydrodynamic loads. During the experimental campaign, the heel-sway, heel-yaw coupled linear coefficients (Y_ϕ , N_ϕ) and hydrodynamic heel moment (K_ϕ) were obtained using static heeled model measurements over a range of speeds. Then the side force, yaw and roll moment linear coefficients due to drift (Y_ν , N_ν , K_ν) were measured using static pure drift measurements.

The prediction of the hydrodynamic loads in the horizontal and transverse plane for high speed marine vehicles is fundamental. The estimation of these manoeuvring loads in calm water is the first step for future applications of the mathematical model to complex dynamic problems which are dominated by the manoeuvrability characteristics of high speed craft.

KEYWORDS : high speed craft, manoeuvrability, heel-sway-yaw hydrodynamic coupling, pure sway hydrodynamic loads, panel method, captive model tests

1 INTRODUCTION

The manoeuvrability of high speed craft is an interesting and challenging topic. Its importance has increased during the last decades as a result of the need for fast ships to be able to be controlled and manoeuvred in the safest manner, even in the most adverse sea conditions.

The current work investigates the manoeuvring of high speed planing mono-hull craft. A rescue vessel of the Royal Netherlands Sea Rescue Institution (KNRM) was chosen as the test case for this research study. The vessel is 18 meters long and has a maximum speed of 35 knots. It mainly operates in the pre-planing and planing regime. The vessel is the result of a joint project started in 2009 between KNRM, the Faculty of Marine Technology of Delft University of Technology, Damen Shipyards and Willem de Vries Lentsch Yacht Designers & Naval Architects. The aim of this work is to address the development of a new type of life-boats capable of dealing with any adverse sea conditions meeting the future regulations. NH-1816 has evolved from the previous concept of the Arie Visser life-boat. The vessel is based on the axe-bow concept, conceived by Keuning at TU Delft (Keuning 2006). The concept successfully demonstrates significant enhancements in the seakeeping capabilities in harsh sea states, reducing the vertical accelerations at the bow. Recent work (De Jong et al. 2013) investigated the small fast axe-bow vessels' behaviour in following and stern-quartering sea states. An insight into the manoeuvrability characteristics was given. Loss of course stability and reduced course keeping ability phenomena are in fact some of the most significant undesired consequences that affect the dynamic mechanisms of these vessels.

Historically the manoeuvrability of surface ships has been analysed considering the motions in the horizontal plane alone, i.e. surge, sway and yaw. Typically, sway force and yaw moment due to drifting and turning motions are considered as dominant in a manoeuvrability problem. However, the dynamics related to heeling and its effect on the manoeuvring features of the vessel are of particular importance for small high speed craft, for which the heel angle in a turn can be substantial (Tuite & Renilson 1995, Renilson & Tuite 1996, 1997, Renilson 2007, Renilson & Manwarring 2000 and Oltmann 1993). This happens in calm water in tight and fast turns, but also in waves. Unlike large displacement ships, planing and semi-planing small hulls are more subjected to relatively large transverse motions when sailing in a seaway which could affect the manoeuvrability of the vessel. Heel-sway-yaw coupling is still a not well understood

phenomenon in ship manoeuvrability, especially for small and fast vessels. This coupling effect has been noted on displacement and semi-displacement vessels, such as naval ships or container ships – see for example (Tuite and Renilson 1995), but not yet on small high speed craft.

This research study covers some of the main aspects of ship manoeuvrability: the sway, roll and yaw loads due to a drift motion; the sway, roll and yaw loads due to heeling. The forces and moments were evaluated over a large range of speeds, covering the pre-planing and planing regimes of the high speed craft being investigated. The study was carried out by means of experimental captive model tests performed at Delft University of Technology. The experimental results were used to validate a potential flow boundary element mathematical method for the prediction of the principal manoeuvring hydrodynamic loads.

The mathematical model considered is 3D time domain method based on the potential flow theory. A transient Green function is used to take into account the free surface effects. The model was developed in the past years as a reliable tool aimed at the prediction of motions and loads in waves (van Walree 1999, van Walree 2002, de Jong et al. 2007, de Jong & van Walree 2008, de Jong & van Walree 2009, de Jong 2011). The model was extensively applied to seakeeping problems in head seas and in the prediction of resistance and running vertical attitude of high speed craft. More recently it was implemented in following and stern-quartering seas investigations (Van Walree & de Jong 2011, de Jong et al. 2013, de Jong et al. 2015). The challenge was to extend the method to the prediction of large amplitude motions when the vessel is in a manoeuvre (Quadvlieg 2015).

Further studies on the prediction of hydrodynamic manoeuvring loads are still necessary, with the final scope of a proper description of complex dynamic phenomena dominated by the manoeuvrability characteristics of the vessel. This work tests the capabilities of the numerical model in the prediction of hydrodynamic loads in calm water. This investigation can be considered as a basic background for the future applications of high speed craft manoeuvring in waves situations.

2 MOTION EQUATIONS

Traditionally the manoeuvring of surface ships has been examined by considering only the equations in: surge, sway and yaw. Motions in: heave, pitch and roll are usually neglected. However, it is well known that for those ships which experience significant heel motions when turning the coupling between heel and

yaw/sway must be considered. For high speed craft the equations of motion must be extended to include the equation for heeling (Yasukawa & Yoshimura 2014, Yasukawa 2010), together with the relevant coupling terms, as shown in Equations 1-4 below. The equations are given in non-dimensional form. The axis system is given in Figure 1.

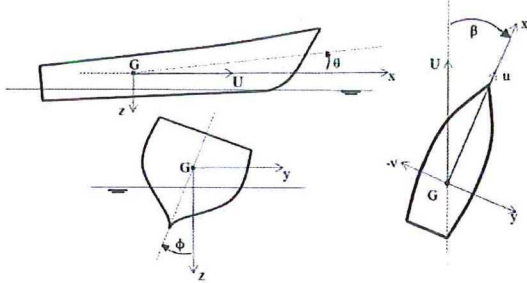


Figure 1 Coordinate system

One of the main difficulties when attempting to study the manoeuvring of high speed craft is in the determination of the values of the linear hydrodynamic coefficients due to heeling (Y'_ϕ , K'_ϕ and N'_ϕ), in addition to the drift dependent terms (Y'_v , K'_v and N'_v).

In the following only these hydrodynamic linear components of the motion equations will be analysed (right hand of the equations). The linear coefficients due to yaw rate, surge dynamics as well as non-linear terms (denoted with f') are not considered in the present work as these are covered extensively by other researchers (see for example (Ommani et al. 2012), (Ommani & Faltinsen 2014)).

$$\begin{aligned} (m' + m'_x)\dot{u}' - (m' + m'_y)v'r' \\ = -R'_T(u') \\ + f'_X(v', r', \delta, \phi, \dot{\phi}) \end{aligned} \quad (1)$$

$$\begin{aligned} (m' + m'_y)\dot{v}' - (m' + m'_x)u'r' \\ = Y'_v v' + Y'_r r' + Y'_\phi \phi \\ + f'_X(v', r', \delta, \phi, \dot{\phi}) \end{aligned} \quad (2)$$

$$\begin{aligned} (I'_x + J'_x)\ddot{\phi} - m'_y \alpha'_z \dot{v}' \\ = -\Delta' GM' \phi + K'_v v' + K'_r r' \\ + Y'_\phi \phi + f'_K(v', r', \delta, \phi, \dot{\phi}) \end{aligned} \quad (3)$$

$$\begin{aligned} (I'_z + J'_z)\dot{r}' - m'_y \alpha'_x \dot{v}' \\ = N'_v v' + N'_r r' + N'_\phi \phi \\ + f'_N(v', r', \delta, \phi, \dot{\phi}) \end{aligned} \quad (4)$$

3 MODEL EXPERIMENTS

3.1 The model

The experimental campaign was executed using a model of the rescue boat SAR NH-1816 of the Royal Netherlands Sea Rescue Institution (KNRM) (Bonci et al. 2017). The hull lines and the model particulars are showed in Figure 2 and Table 1 respectively.

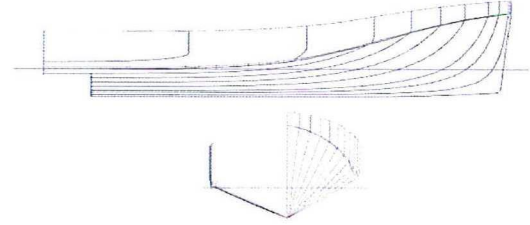


Figure 2 NH 1816 hull lines

This vessel was chosen for the present work for two main reasons. First, a great deal of experimental results, full scale direct observations and numerical data are available thanks to the long and detailed past research studies of the SAR NH-1816 project (de Jong et al. 2013). Second, further investigations on the manoeuvrability of axe-bow high speed crafts are still needed.

Table 1 NH1816 particulars

<i>NH-1816 model properties</i>	<i>SU</i>	<i>Values</i>
Model scale	[-]	10
Overall Length	[m]	1.93
Length between Perpendiculars	[m]	1.84
Overall Breadth	[m]	0.56
Draft	[m]	0.11
Weight	[kg]	26.28
Longitudinal Centre of Gravity (with respect to transom)	[m]	0.6
Wetted Surface at Zero Speed	[m ²]	0.78

3.2 Experimental set-up

Fully captive model experiments were conducted in the model towing tank at the Delft University of Technology. The experiments were divided in two main phases. In the first phase, the heel-sway, heel-yaw coupled linear coefficients (Y'_ϕ , N'_ϕ) and hydrodynamic heel moment (K'_ϕ) were obtained using static heeled model measurements over a range of speeds. The speed range investigated was between Froude number 0.4 – 1.3, corresponding to 10 – 35 knots full scale.

In the second phase the pure sway hydrodynamic coefficients for sway, roll and yaw (Y'_v , K'_v and N'_v) were obtained towing the model along the rectilinear tank with a drift angle. Heeling was set to zero in this phase. The speed range investigated was the same of Stage 1; the last two speeds series of pure drift tests are the ones carried out by a past study (de Jong et al. 2013). The tests particulars are summarized in Table 2.

During the experiment the model's rise and trim were constrained to the vessel equilibrium (planing or semi-planing) running conditions at speed. The model was towed in static conditions at different speeds: the rise of the model, trim and heel angles were fixed during each run. The rotations were set around the centre of gravity of the model. The rise and orientations were set by a 6DOF oscillator which is based on the principle of the Stewart platform (Stewart 1965), also known as Hexapod. The Hexapod can move in six degrees of freedom using six electrical actuators. The model was mechanically attached underneath the moving platform of the hexapod by a frame made up of two plates rigidly connected to each other by six strain gauges. The structure of the frame permitted the calculation of forces and moments in all directions.

Table 2 Experimental test summary

Stage	Fr [-]	Z_0^G [mm]	θ [deg]	ϕ [deg]	β [deg]
1	0.38	8.80	0.82	0 to 12	0
	0.48	13.0	2.63		
	0.57	7.40	3.38		
	0.67	-1.40	3.67		
	0.77	-11.0	3.82		
	0.86	-19.8	4.07		
	0.96	-28.8	4.27		
	1.05	-35.4	4.23		
	1.15	-39.8	4.08		
	1.25	-43.1	3.87		
	1.34	-45.8	3.65		
2	0.38	8.80	0.82	0	0 to 10
	0.48	13.0	2.63		
	0.57	7.40	3.38		
	0.67	-1.40	3.67		
	0.96	-28.8	4.27		
	1.34	-45.8	3.65		

When the vessel is heeled, the trim may change with respect to straight upward forward conditions, due to asymmetries in the submerged body, in the motion or in the disturbance loads. Likewise, the vessel can rise or sink, depending on the pressure distribution on the hull. These effects are not

considered in the present study: this change in running attitude is assumed to be small, and so the effect of heel and drift on the vertical motions is neglected. Figure 3 shows an example run.

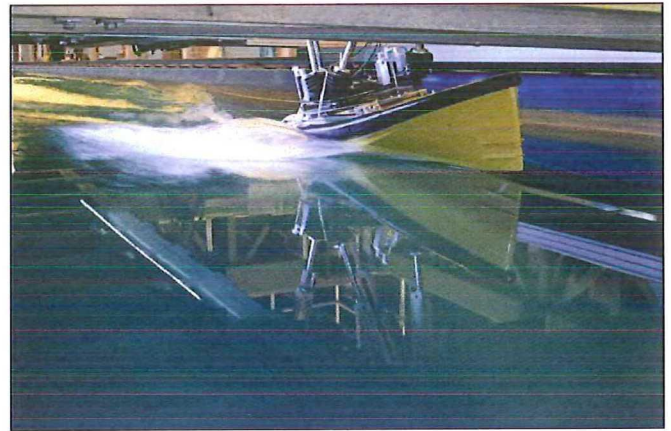


Figure 3 Model test run (Stage 1, $Fr = 1.05$, $\phi = 12$ deg)

3.3 Data post-processing

The objective of the experiments was to derive the hydrodynamic coefficients (for sway, roll and yaw) as functions of heel angle and sway speed. The hydrodynamic forces and moments were evaluated in two steps. Before each measurement at speed, the static loads were measured on the model at zero velocity in the same position and orientation of the upcoming run. At this point, only buoyancy and gravity loads were measured. These static forces and moments were subtracted from the total ones evaluated during each run at speed, resulting in the only hydrodynamic components. The resulting values comprised hydrodynamic pressure, viscous and disturbed wave loads. During the runs at speed, the calm waterline was deformed because of the disturbance of the moving vessel, and as a consequence the buoyancy changes. This is considered to be a hydrodynamic effect due to speed.

At each speed, the hydrodynamic loads were measured at different heel and drift angles (for example 0, 4, 8 and 12 degrees for heel angle, and 0, 2.5, 5, 7.5 and 10 degrees for drift angle). The curves obtained as functions of angles were fitted with a linear polynomial regression. An example of the regression fitting for sway force of one of the conditions of Stage 1 measurement is shown in Figure 4. Experimental sway force points and regression line are depicted against heel angles.

The linear coefficients of each polynomial were calculated using the Least-Square fitting method. In Stage 1, the polynomial is function of heel angle expressed in radians; in Stage 2 the polynomial is expressed as function of sway speed which is

derived from the drift angle chosen by means of Equation 5.

$$v = U \sin \beta \quad (5)$$

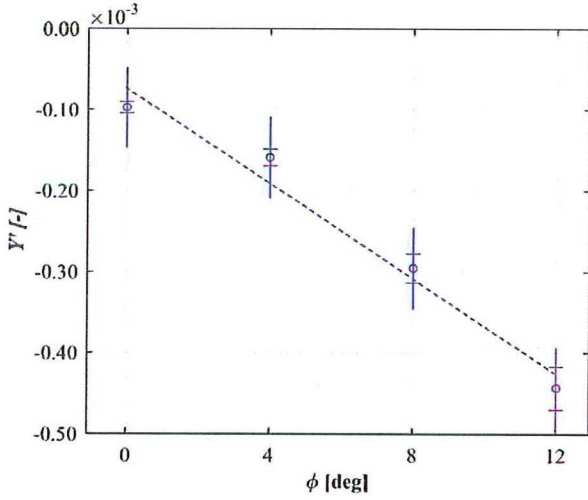


Figure 4 Sway force due to heel (Stage 1, $Fr = 1.05$)

Forces and moments are expressed in non-dimensional form; these values are derived in Equations 6 and 7 (forces and moments respectively), according to the usual nomenclature of ship manoeuvrability.

$$F' = \frac{F}{0.5\rho L^2 U^2} \quad (6)$$

$$M' = \frac{M}{0.5\rho L^3 U^2} \quad (7)$$

In general, all the other quantities are non-dimensionalized in the same way, i.e. with the proper combination of ship total speed U , water density ρ and ship length L .

The uncertainty of the measured experimental values was estimated following the ITTC Recommended Procedures and Guidelines no. 7.5-02 06-04 (*Force and Moments Uncertainty Analysis, Example for Planar Motion Mechanism Test*). The uncertainty estimation was made up of six elemental bias and of the precision limits. The elemental bias considered were related to inaccuracies in the determination of model length L , model draft at zero speed T_m , water density ρ , carriage speed U , model alignment with respect to the towing tank and strain gauge calibration factors. The precision limits were estimated due to $N = 10$ repeated runs carried out on different days of tests. In the following, the experimental measured values include uncertainty bars denoting the uncertainty boundaries estimated. For the results of the uncertainty analysis see (Bonci et al. 2017).

4 NUMERICAL METHOD

4.1 Background

The calm water captive model tests were simulated by means of a time domain potential flow boundary element method. The conditions of the experiments were repeated in the simulations. The linear hydrodynamic coefficients were then obtained as described in Section 3.3.

The mathematical model is based on fluid potential theory. A boundary value problem is solved to obtain the computation of the disturbed flow potential caused by the ship body moving on the free surface of the water. The problem domain is the body and the free water surface.

The fluid is assumed to be homogeneous, incompressible, inviscid, irrotational and without surface tension. The flow velocity potential Φ is composed by the incident and disturbances potentials, denoted by I and D superscripts respectively (Equation 8). The Laplace equation (Equation 9) is solved in the interior of the fluid assuming dynamic and kinematic conditions on the linearized free surface, and zero normal flow speed on the body surface. The free surface boundary conditions are combined in Equation 10. Disturbances potential is set to vanish at a great distance from the body surface.

$$\Phi = \Phi^I + \Phi^D \quad (8)$$

$$\nabla^2 \Phi = 0 \quad (9)$$

$$\frac{\partial^2 \Phi}{\partial t^2} + g \frac{\partial \Phi}{\partial z} = 0 \quad \forall x \in S_F \quad (10)$$

$$V_n = \frac{\partial \Phi}{\partial n} \quad \forall x \in S_H \quad (11)$$

The hull surface is discretized by planar quadrilateral panels; this geometry is cut at the flat waterline intersection, dividing the geometry into above water and below water panels. On each of the underwater panel a source singularity of unknown strength is defined. Potential dynamic pressures due to ship disturbance and radiation are evaluated only on the fixed below water geometry, where the transient Green function is specified (Equation 12). This function expresses the influence of the singularity p of a panel on the potential at each other point of the discretized domain q , and eliminates the need of the free surface discretization. The Green function satisfies the Laplace equation. Applying the Green second identity it is possible to derive a boundary integral formulation of the problem (Equation 13). The Laplace equation is integrated on the domain surfaces and in any time between $\tau = 0$ and $\tau = t$, in

order to account for the influence of the singularities on the moving body in the past (memory effect).

$$G(p, t, q, \tau) = \frac{1}{R} - \frac{1}{R_0} + \quad (12)$$

$$2 \int_0^\infty [\cos(1 - \sqrt{gk}(t - \tau))] e^{k(z+\zeta)} J_0(kr) dk$$

for $p \neq q, t > \tau$

$$\int_0^t \int_{S_{FH}(\tau)} (\Phi^D G_{\tau n} - G_\tau \Phi_n^D) dS d\tau = 0 \quad (13)$$

Solving Equation 13, it is then possible to calculate the strength of each singularity on the discretized domain. The source strength is set equal to the difference between the normal derivative of the disturbed potential of the inner (-) and outer (+) sides of the surface.

$$\frac{\partial \Phi^{D+}}{\partial n} = \frac{\partial \Phi^{D-}}{\partial n} \quad \forall q \in S_H \quad (14)$$

Once the disturbance potential is known, the hydrodynamic pressures acting on the submerged hull are calculated by means of the Bernoulli theorem.

Static and viscous pressures are kept in account at the actual submerged geometry defined by the diffracted and radiated wave elevation. Viscous forces are determined by semi-empirical formulations. Viscous effects do not affect significantly the sway and yaw loads acting on a high speed craft in static conditions. Moreover, the present investigation focuses only on the linear coefficients characterization, the non-linear terms are not considered.

5 HEEL INDUCED HYDRODYNAMIC LOADS

5.1 Procedure

In order to simulate the heeled vessel conditions of Stage 1, the mathematical model was extended to non-symmetric hull discretization (see Figure 5). In this way, the wetted hull considered is exactly the same as that for the heeled ship, improving the accuracy of the results.

The results are presented in terms of the linear hydrodynamic coefficients Y'_ϕ , K'_ϕ and N'_ϕ at the different speeds tested during the experiments.

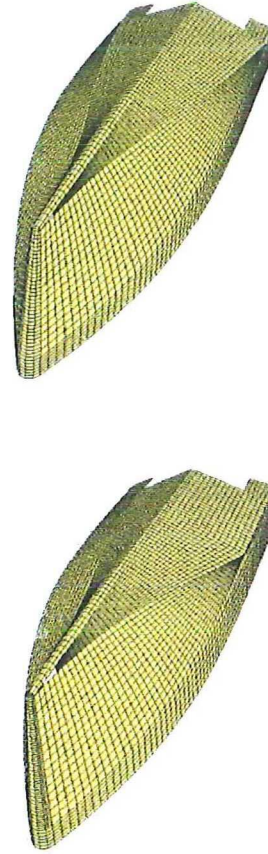


Figure 5 Hull discretization (Stage 1, Fr = 1.05). Symmetric (top) and non-symmetric $\phi = 8$ deg (bottom) panels distributions

5.2 Results

The coupling between heel and yaw is quantified in Equations 2 and 4 by the terms $Y'_\phi \phi$ and $N'_\phi \phi$ which represent the linear sway force in due to heel angle and the linear yaw moment due to heel angle respectively.

The values of the experimental non-dimensional linear coefficients Y'_ϕ and N'_ϕ are given in Figure 6 and 7 as functions of Froude number. For all the speeds, the values of the hydrodynamic coefficients are both negative. This means that when heeled to starboard, a hydrodynamic side force to port is generated by the heel angle. Also, the yaw moment turns the bow to port (the coordinate system is shown in Figure 1). The absolute values of both sway and yaw hydrodynamic coefficients are maximum around Froude number 0.5 – 0.6. The magnitudes of the coefficients are lower at higher speeds. One of the possible explanations of the smaller coefficients values at higher Froude numbers is that at high speed the lift on the hull bottom increases and raises the vessel out of the water.

Being less submerged, the heel angle does not have such a great effect on the side force and yaw turning moment, resulting in low hydrodynamic coefficients.

The predicted values of Y'_ϕ and N'_ϕ show the same slope with respect to Froude number as the measured values. However, the predicted values of the heel-sway coupling coefficient is lower than the measured values at lower speeds, while greater at higher speeds. The prediction of the heel-yaw coupling coefficient does not agree very well with the experimental results. The prediction of the yaw moment linear coefficients presents the same slope with respect to speed as the measured ones; however, it is under predicted at all the Froude numbers examined, except the lowest ones.

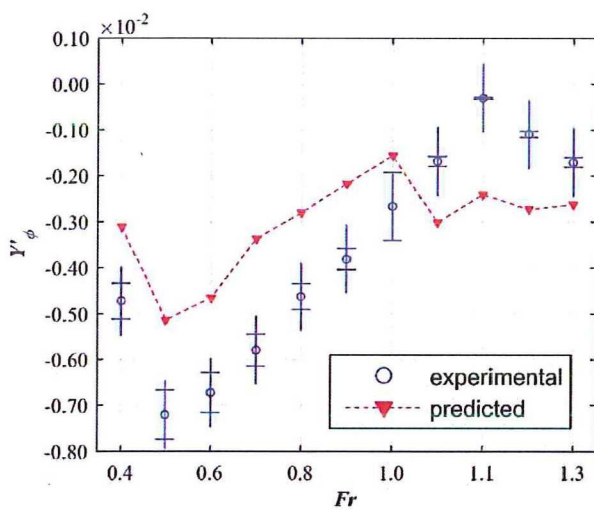


Figure 6 Sway hydrodynamic force coefficient due to heeling (Stage 1)

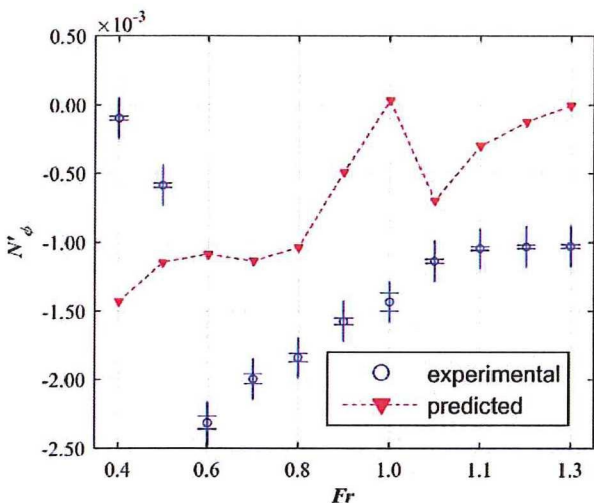


Figure 7 Yaw hydrodynamic moment coefficient due to heeling (Stage 1)

Roll hydrodynamic moment due to static heeling is shown in Figure 8 as a function of Froude number. The experiments revealed that at lower speeds the hydrodynamic roll coefficient K'_ϕ has a positive

value. This means that the hydrodynamic pressure distribution developed on the heeled hull creates a roll moment in the same direction as the vessel is heeled. This behaviour is partially reproduced by the mathematical model even if the values are relatively smaller than the measured ones. At higher speeds the predicted and measured coefficients match with low negative values.

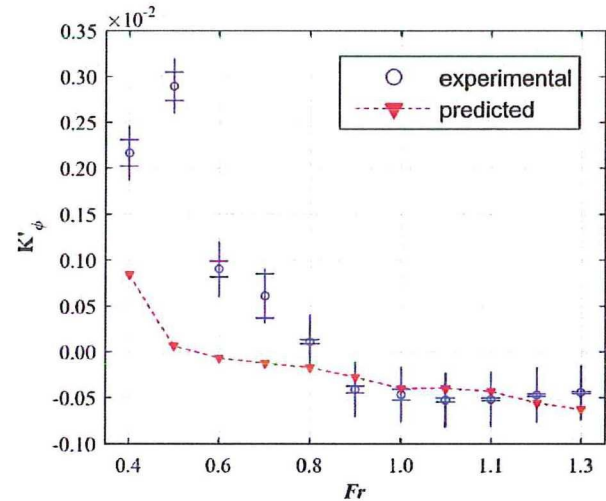


Figure 8 Roll hydrodynamic moment coefficient due to heeling (Stage 1)

6 DRIFT INDUCED HYDRODYNAMIC LOADS

6.1 Procedure

Regarding Stage 2, upright symmetric hull panel distributions were used. An instantaneous sway speed v was set as input for the pure drift simulations, together with a proper advance speed u in order to match the total carriage speed. As already described in the previous Section 5.2, measured and predicted hydrodynamic linear coefficients Y'_v , K'_v and N'_v are compared in the same plot with respect to Froude number.

6.2 Results

Figures 9 and 10 show the measured and predicted linear hydrodynamic coefficients Y'_v and N'_v as functions of Froude number. Both coefficients are negative, meaning that the vessel is subjected to a sway force opposite to the drift motion and to a yaw moment that increases the drift angle.

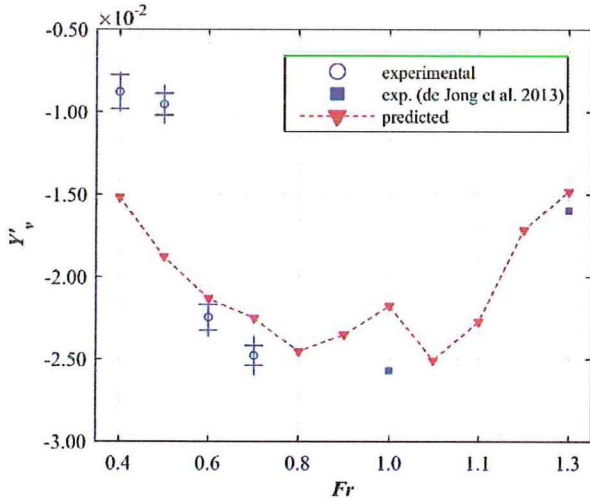


Figure 9 Sway hydrodynamic force coefficient due to drift (Stage 2)

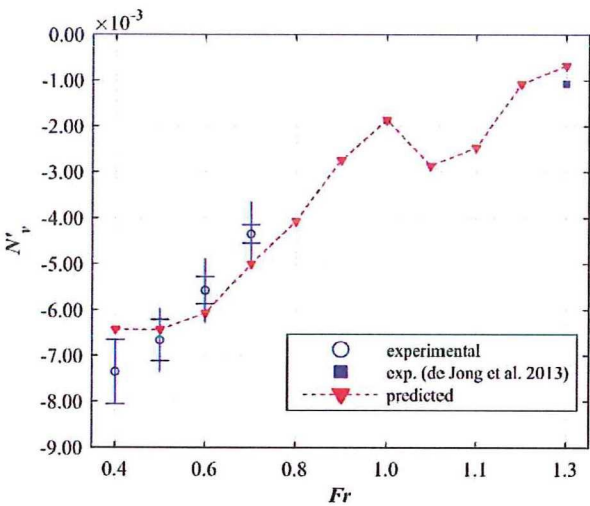


Figure 10 Yaw hydrodynamic moment coefficient due to drift (Stage 2)

The pure sway side force coefficient Y'_v reaches a maximum in absolute value between $Fr = 0.7 - 1.0$; at lower and higher speed its value decreases significantly. The predicted values show a good agreement with the measured values; larger discrepancies are limited to the range of $Fr = 0.4 - 0.5$.

The pure sway N'_v yaw hydrodynamic coefficient, N'_v shows the largest absolute values at the lowest speeds ($Fr = 0.4 - 0.7$). The coefficients are smaller in absolute value with higher Froude numbers. The predicted values show a good agreement with the measured ones.

The roll hydrodynamic moment linear coefficient is given in Figure 11 as a function of Fr . At low speeds K'_v is small and progressively becomes more negative with increasing speed. This means that a moment opposite to the direction of the drift motion is generated. The predicted values of the

coefficients match the experimentally measured ones quite well at low speeds, but are significantly lower than the measurements at high speeds.

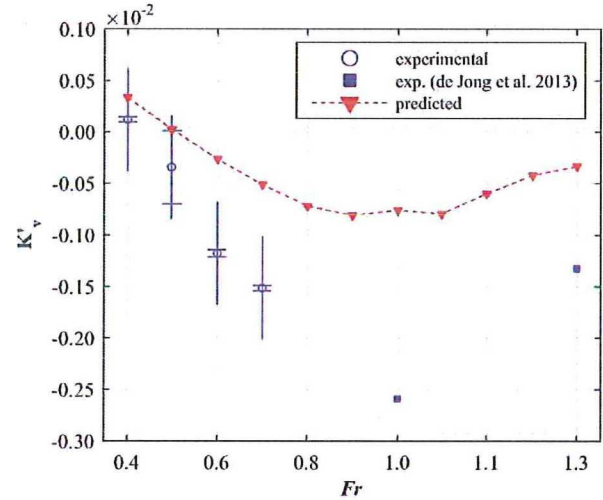


Figure 11 Roll hydrodynamic moment coefficient due to drift (Stage 2)

7 CONCLUDING REMARKS

The sway, roll and yaw hydrodynamic moments of a high speed rescue craft were investigated in two conditions: first, when the vessel had a non-zero heel angle; and second, when the vessel had a non-zero drift angle.

The hydrodynamic coefficients for heel () and drift () were obtained from captive model tests, and the results compared with predictions made using a potential flow boundary element method. The predictions give a good agreement with the coefficients obtained experimentally, proving the suitability of the numerical method in predicting the manoeuvring of high speed craft. Moreover, the boundary element method can be considered as a valuable and less time-consuming alternative to experimental techniques and more complex mathematical approaches, as RANS solvers.

The results showed a strong dependency of the non-dimensional coefficients on the speed, over the range investigated between Froude number 0.3 to 1.3, both for the drifting and heeling situations. The yaw hydrodynamic coefficients had a maximum value around the Froude number range of 0.4 to 0.6. This is of interest for the phenomena of non-oscillating dynamic instability in waves, typically occurring in the pre-planing regime. Future work will include an investigation into the manoeuvring aspects of the loss of dynamic stability in waves.

The results of this study can be used in many different ways. The mathematical model can be

applied for manoeuvring applications. The calculated and measured hydrodynamic coefficients can be implemented in practical modular mathematical formulations used to simulate the high speed craft behaviour during a manoeuvre.

In future work the authors are planning to apply the outcome of this work to the course stability assessment of high speed craft in calm water as first step, and then in waves, where they will investigate non-oscillating dynamic instability, including broaching.

NOMENCLATURE

Roman

f	Non-linear hydrodynamic loads	[N, Nm]
F	Linear hydrodynamic force	[N]
Fr	Froude number U/\sqrt{gL}	[-]
g	Gravity acceleration	[m/s ²]
GM	Metacentric height	[m]
I	Mass moment of inertia	[kgm ²]
J	Added mass moment of inertia	[kgm ²]
K_ϕ	Roll coefficient due to heel	[Nm/rad]
K_r	Roll coefficient due to yaw rate	[Nms/rad]
K_v	Roll coefficient due to sway	[Ns]
L	Ship length	[m]
m	Ship mass	[kg]
M	Linear hydrodynamic moment	[Nm]
m_x	Added mass in x-direction	[kg]
m_y	Added mass in y-direction	[kg]
n	Surface normal	[-]
N_ϕ	Yaw coefficient due to heel	[Nm/rad]
N_r	Yaw coefficient due to yaw rate	[Nms/rad]
N_v	Yaw coefficient due to sway	[Ns]
r	Yaw rate	[rad/s]
R_T	Resistance in upright conditions	[N]
t	Time	[s]
T_m	Ship mean draft at zero speed	[m]
u	Advance speed	[m/s]
U	Total ship speed	[m/s]
v	Sway speed	[m/s]
Y_ϕ	Sway coefficient due to heel	[N/rad]
Y_r	Sway coefficient due to yaw rate	[Ns/rad]
Y_v	Sway coefficient due to sway	[Ns/m]
Z_0^G	Rise of centre of gravity	[m]

Greek

α_x	x-location of added mass centre	[m]
α_z	z-location of added mass centre	[m]
β	Drift angle	[deg]
δ	Control devices deflection	[deg]
Δ	Ship displacement	[N]
ϕ	Heel angle	[deg]
Φ	Flow speed potential	[m ² /s]
θ	Pitch angle	[deg]
τ	Past time	[s]
ρ	Water density	[kg/m ³]

An over-dot denotes a time derivative. Primed symbols denote non-dimensional quantities.

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