Relay Selection and Resource Allocation in Cooperative Wireless Communication Networks

MSc. Thesis

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Abstract

Cooperative communication is a technique to use nodes in the wireless network as relays to forward information to the destination. Cooperative communication technique relies on the relay selection and power allocation process to optimize its performance since not all available nodes are beneficial to use for forwarding. Game theory is a branch of mathematics theory which tries to model the behavior of a player in strategic situation, or games, when dealing with other players' choices. In the cooperative communication networks, the game theoretical approach has been shown to be a good means to solve the relay selection and resource allocation problems which have multiple objectives.

In this thesis we analyze the relay selection and resource allocation using pricing game in a cooperative wireless communication network with interference. Using simulation results we show that interference affects the relay selection and resource allocation process. From the relay selection result we also find that the number of selected relay nodes influences the optimization process and when the number of selected nodes is large, the source node significantly suffers from payment to relay nodes. We propose an algorithm to limit the number of selected relay nodes and to keep the benefit of the source node.

We also propose an evolutionary game approach for resource allocation in cooperative wireless networks. The method has flexibility in optimizing the system based on preferred objectives. The evolutionary game approach is based on evolutionary programming, where the behavior of each node is modeled to mimic the biological evolution process.

Results of this study can be used in a wide range of wireless sensor networks where power, bandwidth, and other resource constraints are present. The proposed scheme provides a flexible relay node selection given the resource constraints of the wireless network.

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List of Major Symbols

N	-	Number of available relay nodes
L	-	Number of selected relay nodes
P_s	-	Transmit power of the source node
P_{r_i}	-	Transmit power of relay node i
p_i	-	Price of relay node i
c_i	-	Cost of relay node i
$R_{s,r_i,d}$	-	Capacity of the transmission in a cooperative wireless communication network
U_s	-	Utility function of the source node
U_{r_i}	-	Utility function of relay node i
$G_{s,d}$	-	Channel coefficient between the source node and the destination node
G_{s,r_i}	-	Channel coefficient between the source node and relay node i
$G_{r_i,d}$	-	Channel coefficient between relay node i and destination node

1 Introduction

As people grow to depend on wireless technology as their main medium for connectivity, demand for high data rate in wireless communication grows even more. This issue is caused not only because of increase in the number of wireless communication users, but also because of the fact that the information which has to be transported has also grown significantly. Although development in the wireless technology has been rapid, certain physical parameters are still limiting the utility of the wireless communication technology. In many cases, limited frequency band, battery life, and severe fading channel are factors which have become challenges for researchers to overcome.

Cooperative communication has become one of the popular research topics as the solution to the battery life problem and increasing the transmission capacity and performance. The idea of using relay for communication has been there since the work of Cover and El Gamal in [1], but not until the work of Sendonaris *et al*. [2] [3] the cooperative communication scheme and system description are formulated in detail. The basic principle of cooperative communication is using other communication devices to relay transmission. As shown in Figure 1-1, the source node broadcasts information to both the relay node and the destination node. The relay node then forwards the transmission to the destination node. The source node regards the relay node as a virtual antenna, enabling MIMO systems to be used without having to add physical antenna.

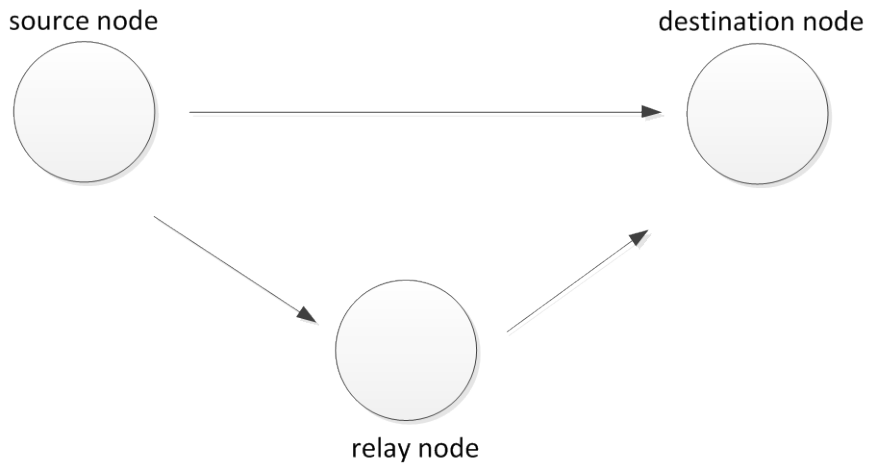

Figure 1-1 Cooperative communication scheme

The cooperative communication scheme is not only capable of increasing the capacity performance of the system. Suppose that the channel between the source node and the destination node suffers from severe fading channel, and then direct transmission from the source node to the destination node will have low performance. By using the cooperative communication scheme, the source node can find relay nodes which have good channels to the destination node, use it to relay the transmission to the destination node, and increasing the reliability of the whole transmission. By selecting relay nodes closer to it, the source node can also save battery power, since it does not have to transmit at high power and can use the relay devices' power to perform the transmission instead.

Since the source node in the cooperative communication scheme depends on the relay nodes to forward the transmission, relay selection and resource allocation for the relay nodes become important in order to obtain optimal performance of the cooperative communication system. By choosing the right nodes to relay the transmission, the system can achieve higher capacity by using lower resources. In this work, optimal relay selection and resource allocation issue is addressed.

1-1 Research background of Thesis Work

In the cooperative communication network, how to get relays to join cooperation and select relays to cooperate with others are the problems that have been arisen since cooperative communication scheme was proposed. As the communication nodes are not cooperative by nature, incentive is needed to encourage nodes to participate in the cooperative communication networks as forwarding nodes. In work by Ileri *et al.* [4], pricing is introduced to encourage the relay nodes to join the cooperation and to match practical application. Relay nodes are given incentives in a form of payment for consumption of the resource they spent on forwarding the information for other nodes.

Many works have been done in the field of relay selection for cooperative communication networks. In [5], the author proposed a relay selection based on the location of the relay nodes. Relay selection based on instantaneous link with interference is done in [6]. Distributed relay selection using game theory [7] focuses on the capacity of the total transmission. After the relay selection, the resource allocation and optimization is the next problems in the cooperative communication network. Allocating optimal resource to each relay node with different objectives between the nodes in the system is required to obtain best performance from the cooperative communication network. A cooperative communication network with both relay nodes and source node want to maximize their own gain through resource allocation and optimization will have different resource allocation and optimization process than where the objective is only to maximize the source node gain.

Game theoretical approach in solving problems for wireless communication has been gaining a lot of attention lately. In the field of cooperative communication, game theoretical approach has been used in many aspects, especially in resource optimization and managing the relay nodes behavior. Game theoretical approach is used because it can model the behavior of nodes in real world situation, and perform multi objective optimization.

This thesis work is an attempt to formulate the relay selection criterion and resource allocation and optimization which fulfills the practical situation and obtains optimal result in the system performance using game theoretical approach. The system proposed in this thesis work will provide an approach for solving the relay selection and resource allocation and optimization problems with flexibility to adjust to different situation and solution which approaches the optimal results.

1-2 Objectives of Thesis Work

The primary objective of the project work is to build a cooperative communication network model with relay selection and resource allocation system which has optimal performance in terms of resource allocation and relatively easy to implement. In order to achieve this objective, we work step-by-step in smaller objective groups, which consist of–

- To model game theoretical based relay selection criterion and resource allocation for the cooperative communication scheme and proof the formulation analytically
- To established a computer simulation setup for the cooperative communication system
- To evaluate the performance of the proposed system model by confirming it with literature

The workflow for the objectives of the thesis work has been pictorially represented in Figure 1-2. We define the relay selection and resource allocation problem; perform literature study for related works; build system model and computer simulation setup based on the system model; perform the simulation and analyze the result using the available literature study as comparison.

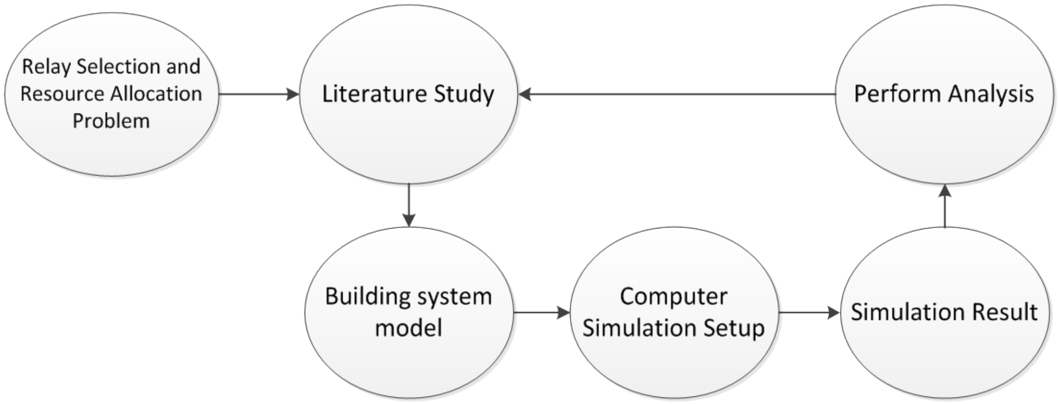

Figure 1-2 Workflow diagram for objectives of the thesis work

1-3 Main Contribution

In this thesis work, we have done some contribution to the cooperative wireless communication networks research area, especially in relay selection and resource allocation and optimization. First, we propose a pricing game model based on Stackelberg game for relay selection and resource allocation and optimization in cooperative wireless communication networks with interference. Stackelberg game is chosen because it can model the behavior of player which moves in turn and it has equilibrium. We show that interference influences the relay selection and resource allocation process and therefore should not be neglected from calculations. Our next contribution is an algorithm to limit the number of relay nodes in the relay selection process. The algorithm can mitigate the effect of number of relay nodes to the resource allocation process. The final contribution is the evolutionary game based resource allocation and optimization for the cooperative wireless communication network. The evolutionary game approach provides flexibility to resource allocation and optimization scheme to solve different objectives.

1-4 Organization of Thesis Work

The thesis work is organized as follows (Figure 1-3). In Chapter 2, cooperative communication principles and system description are explained in detailed manner. Chapter 2 also covers previous works and problems in the field. Chapter 3 contains the basics of game theory and its application in wireless communication system, with focus on cooperative communications. We define the system model, formulate the problem, and analyze the proposed model for relay selection and resource allocation in cooperative communication network in Chapter 4. The simulation result and its analysis for the system model described in Chapter 4 are given in Chapter 5. Finally, the conclusion of the research work and the recommendations for further studies in this field are listed in Chapter 6.

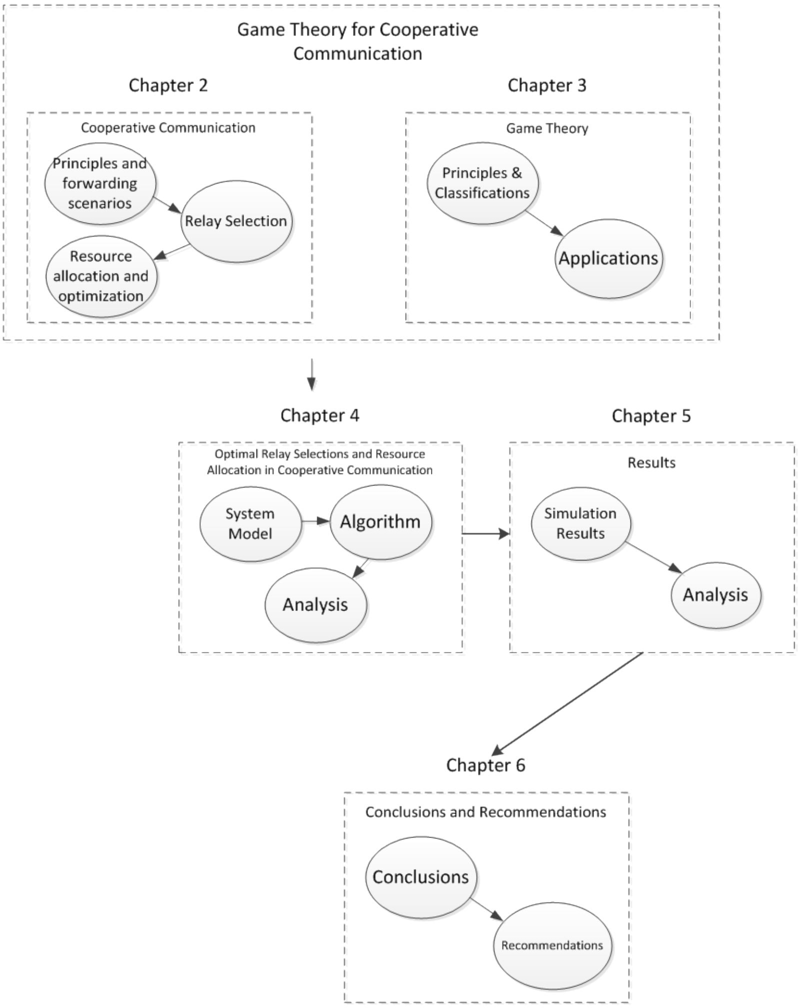

Figure 1-3 Organization of thesis work

2 Cooperative Communication

The idea of cooperative communication which uses the relay nodes to assist the transmission by forwarding the information starts from the work of Cover and El Gamal [1]. In this work, the relaying strategies and the outage performance upper bound is provided. They also provided an expression for the capacity of the degraded relay channel, in which the communication channel between the source and the relay is physically better than the source-destination link. This work provided the cornerstone for relay communication, although in development it was used for multi-hop communication.

Then in 2004, Sendonaris *et al* [2] [3] formulated the cooperative communication scheme and system description in details. At the same year, Laneman *et al* [8] also provided the capacity and outage behavior of the cooperative communication network. This two works have been used as a basis for research in the cooperative communication field. Since then, there have been many works in the area of cooperative communication.

The basic idea of using relay nodes to aid transmission by forwarding information was supported by the existing concept of wireless ad-hoc communication. In wireless ad-hoc network, the nodes in the network can instantly connect to each other, and connecting to nodes which lie outside the transmission coverage of a node is possible through relaying or multi-hop communication. The cooperative communication idea combines the concept of ad-hoc relaying network and the MIMO system concept to build a network with has spatial diversity without having to add physical antennas to the transmitter.

In this chapter, the underlying technology which enables cooperative communication, such as the ad hoc network and MIMO system will be discussed. Then, descriptions about how cooperative communication network work will be provided. The last part of this chapter will talk about the current works in the area of cooperative communication, especially in the field of relay selection and resource allocation.

2-1 Wireless Ad Hoc Communication Network

Ad hoc communication is defined as communication between two or more devices without preexisting infrastructure. Ad hoc communications originally came from packet radio network [9]. In 1970, DARPA initiated a project called packet radio, which enables several wireless radios to communicate with each other. From then, the packet radio concept is extended to the packet network, which in turn evolved into the broadcast radio network.

Ad hoc wireless network has the self-organizing and adaptive properties. These properties mean that the network created in ad hoc manner has to be able to makes change (even creating or terminating the network itself) on-the-fly without having to be controlled by any system administration. As shown in Figure 2-1, ad hoc networks also have to be able to support different devices, since interconnectivity is become more important in infrastructureless network such as the ad hoc network.

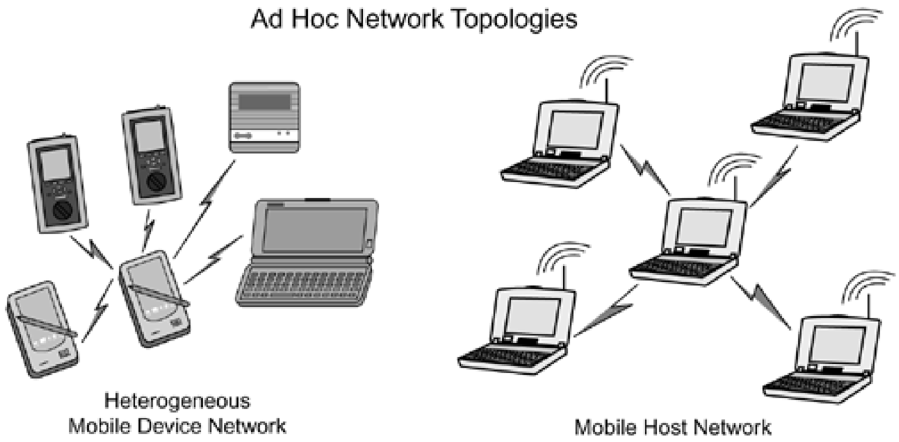

Figure 2-1 Topologies of heterogeneous ad-hoc network [9]

The ad hoc network is an important property of wireless network which enables the cooperative communication network. The source node has to be able to communicate with the destination node with certain transmission performance using relay nodes as forwarding nodes, thus the source node is required to be able to quickly connect to the relay nodes, and also for the relay nodes to be able to quickly connect to the destination node. These criteria are fulfilled by the ad hoc network, since the ad hoc network can be quickly deployed with minimum configuration.

Wireless ad hoc network is classified according to the applications, which have different routing protocols, auto configuration protocols, and nodes behavior. The most common types of ad hoc networks in the wireless network are the mobile ad hoc network (MANET) and wireless sensor network (WSN).

2-3-1. Mobile Ad Hoc Network

Mobile ad-hoc network is a self-configuring infrastructureless network of mobile devices connected by wireless links devices. Since each node in the mobile ad-hoc network is a mobile device, this type of network has dynamic topology, instant deployment requirement, and resource constraint such as bandwidth, battery life, and computing power. However, compared to wireless sensor networks, mobile ad-hoc networks usually have more battery life to spare for the transmission. Bandwidth constraint is a common issue especially in ad-hoc network with a large number of nodes. Since each node occupies certain bandwidth in the frequency band, managing how many bandwidth each node get has been an area for research in resource allocation and optimization.

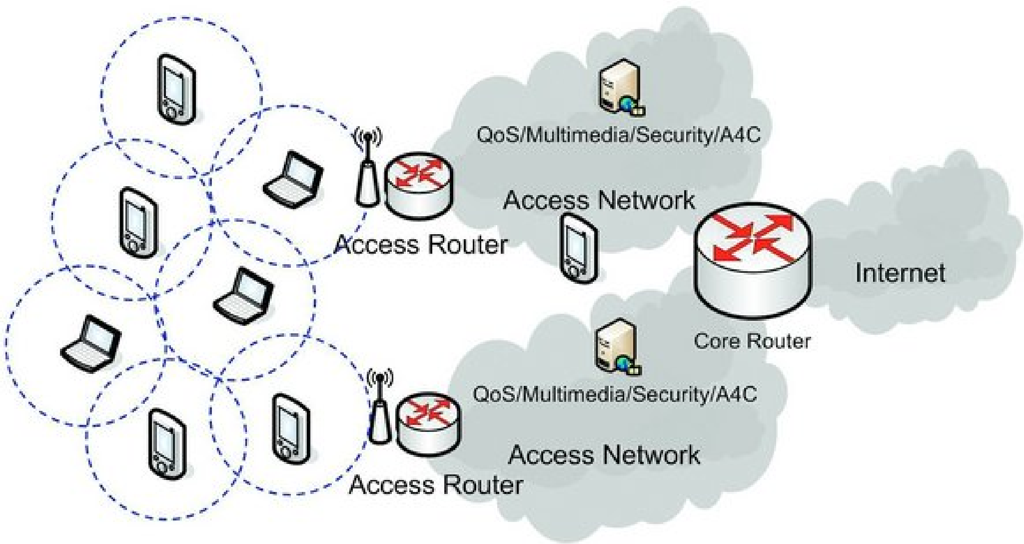

Figure 2-2 Topology of mobile ad hoc networks

2-3-2. Wireless Sensor Networks

Wireless sensors network is a network of distributed sensors which monitor the condition of the surrounding environment. The data acquired from these sensors are transmitted through the network to a gateway node, where the data will be stored or used as shown in Figure 2-3. Wireless sensor network has been used for many applications, ranging from area monitoring such as air pollution monitoring and forest fire detection, industrial monitoring, and agriculture.

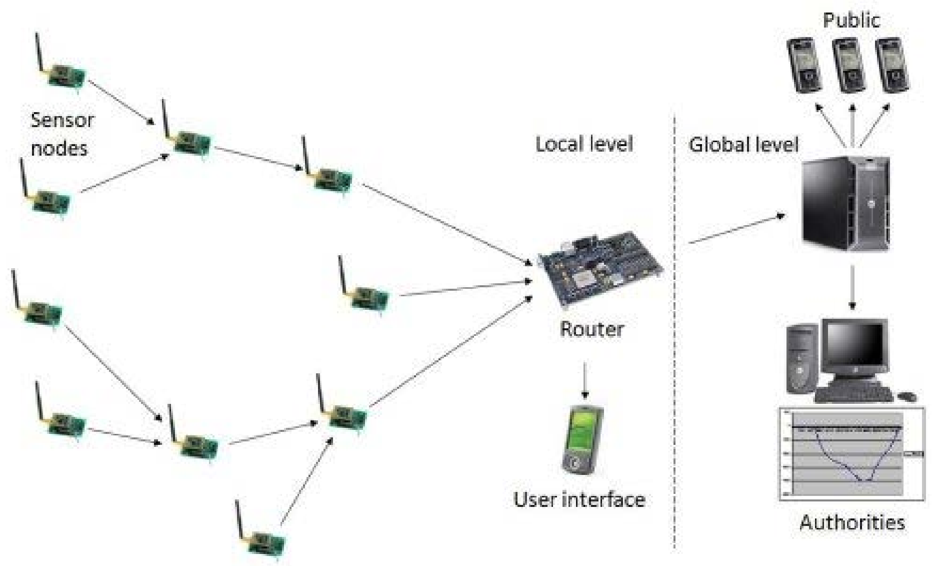

Figure 2-3 Sensor network architecture [10]

The characteristics of wireless sensor network highly depend on the applications, but usually sensor networks have limited battery life and limited computational power. Thus, sensor network usually have power constraint for the transmission, while the computation of complex problems is usually performed in centralized manner.

2-2 MIMO System

MIMO system adapts multiple antennas in the transmit side and/or receive side to exploit spatial diversity. As shown in Figure 2-4, a signal transmitted by an antenna will propagate through different channel. This was called spatial multiplexing, and this technique is used to increase channel capacity by increasing the spectral efficiency of the transmission. In MIMO system with spatial multiplexing, a signal with high rate is split into multiple lower rate streams and transmitted using different antenna in the same frequency. The channels are assumed to be independent or having low correlation through antenna separation, hence the streams can be separated into parallel channel at the receiver.

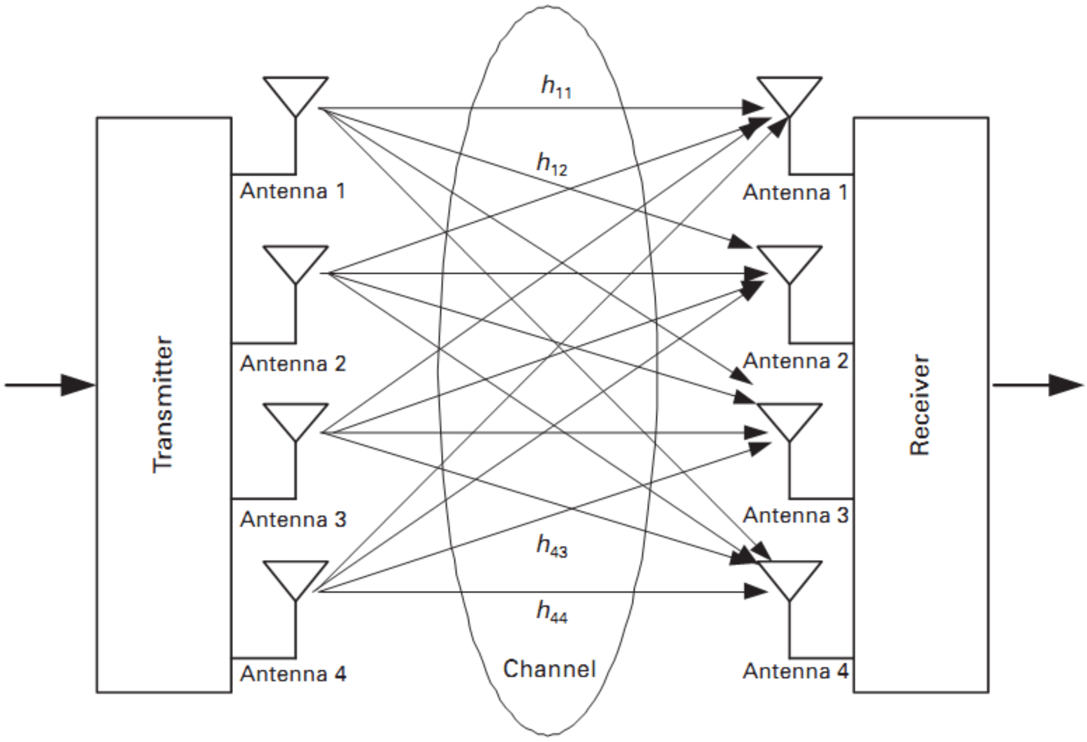

Figure 2-4 A 4-by-4 MIMO system [11]

In a MIMO system with M transmit antennas and N receiver antennas, the capacity of the MIMO system is

$$C = \log_2\left(\det\left(\mathbf{I}_M + \frac{\gamma}{M} HH^*\right)\right) = \log_2\left(\det\left(\mathbf{I}_N + \frac{\gamma}{N} HH^*\right)\right) \qquad (2\text{-}1)$$

where * denotes the complex conjugate transpose operation, H is the $N \times M$ channel matrix, γ is the SNR at the transmitter, $\mathbf{I}_M$ and $\mathbf{I}_N$ are the identity matrix for the transmit antennas and receive antennas [12]. The capacity in equation (2-1) depends on the minimum number of antennas between the transmit side and the receiver side.

The MIMO system increases the capacity of the transmission according to the minimum number of antennas in the transmit side or the receive side. In some cases, providing more antennas to the receive side or the transmit side might not be possible because of hardware limitation such as in sensor networks. Cooperative communication could become a virtual MIMO system by using relay nodes as virtual antennas, thus allowing the application of MIMO without having to add multiple antennas to a device.

2-3 Cooperative Communication using Relay Nodes

2-3-1. Forwarding Scenario

The cooperative communication strategy uses relays to increase the performance of the transmission. This can be achieved through diversity which is gained from the independent paths the source node gets by using the relay nodes. The common set up for cooperative communication networks is shown in Figure 2-5.

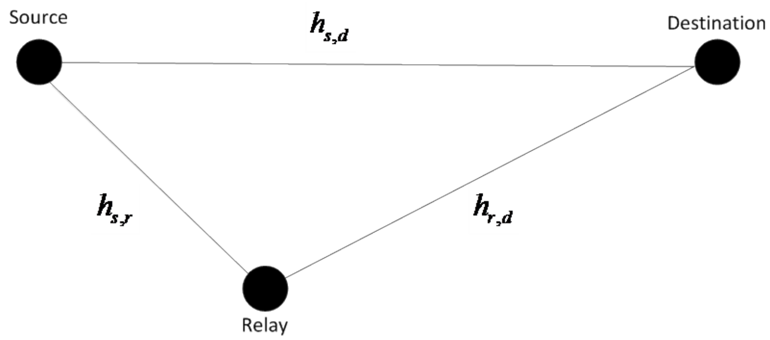

Figure 2-5 Common cooperative communication network model

The source node transmits signal to both relay node and destination node. The received signals at the destination node $y_{s,d}$ and the relay node $y_{s,r}$ can be denoted as

$$y_{s,d} = \sqrt{P_s}\, h_{s,d}\, x + \eta_{s,d} \tag{2-2}$$

$$y_{s,r} = \sqrt{P_s}\, h_{s,r}\, x + \eta_{s,r} \tag{2-3}$$

where P_s is the transmit power at the source node, x is the transmitted information symbol, $h_{s,d}$ is the channel coefficient between the source node and the destination node, $h_{s,r}$ is the channel coefficient between the source node and the relay node, $\eta_{s,d}$ and $\eta_{s,r}$ are the additive noise, modeled as zero-mean complex Gaussian random variables with variance σ^2. The transmission impedance is assumed to be $1\,\Omega$ to simplify the calculation. The received signal at the relay node then will be forwarded to the destination node according to the forwarding scenario.

In the destination side, the signals received from the source node from direct transmission and from the relay nodes are processed according to the forwarding scenario used. The forwarding scenario can be classified into two categories: fixed relaying and adaptive relaying.

Fixed relaying is the simplest forwarding scenario, the relay nodes are in a fixed strategy to forward the transmission coming from the source node to the destination node. Since computation of resource sharing between relay's own data and forwarded data is not needed in the relay nodes, complexity of this scenario can be kept to minimum.

There are two common strategy used in fixed relaying, the amplify-and-forward (AF) scenario and the decode-and-forward (DF) scenario.

i. Fixed Amplify and Forward Scenario

In cooperative communication network with AF scenario, the relay nodes forward amplified version of the received signal from the source node. In the general model of the cooperative communication network as shown in Figure 2-5, the source node transmits information to both destination node and the relay node. The received signals at the destination node and relay node can be expressed as Equation (2-2) and (2-3) respectively. For the AF scenario, received signal at the destination node from the relay node can be written as

$$y_{r,d} = \sqrt{P_r}\, h_{r,d}\, x' + \eta_{r,d} \qquad \text{(2-4)}$$

where

$$x' = \frac{y_{s,r}}{\left|y_{s,r}\right|} \qquad \text{(2-5)}$$

is the normalized transmitted signal from relay node to the destination node, P_r is the transmitted power from the relay node, $h_{r,d}$ is the channel coefficient between the relay node and the destination node, and $\eta_{r,d}$ is additive noise, modeled as zero-mean complex Gaussian random variables with variance σ^2. The transmission impedance is assumed to be 1Ω to simplify the calculation.

In the destination node, the received signals expressed by Equation (2-2) and (2-4) are combined to obtain the information sent over the transmission. The most optimal technique used at the receiver of the destination node for AF scenario is the maximal ratio combining (MRC) [11]. Maximal ratio combining is a technique to linearly combine the signals so that the signal-to-noise ratio (SNR) is maximized at the resulting signal.

ii. *Fixed Decode and Forward Scenario*

In decode-and-forward scenario, the relay node first decodes the signal received from the source node, re-encodes the information, and forwards it to the destination node. In the general application of cooperative communication networks as shown in Figure 2-5, the information received at the relay nodes after the decoding process can be denotes as $\hat{x}$. Then, the received signal at the destination node from the relay node can be written as

$$y_{s,d} = \sqrt{P_r}\, h_{r,d}\, \hat{x} + \eta_{r,d}. \quad (2\text{-}6)$$

In the full decoding DF scenario, the noise from the relay node will not be amplified and forwarded to the destination node as in the AF case, but the relay node can decode the signal incorrectly, thus the relay node will forwards an error to the destination node as well, degrading the performance of the transmission. In this scenario, the performance of the transmission will depends on the weakest link between the source-relay node and relay-destination node.

iii. *Adaptive Relaying Scenario*

Fixed relaying simplified the implementation of the cooperative communication network since the relay nodes always forward the information to the destination node in deterministic manner. However, this scenario does not possess high efficiency, since the received signals at relay nodes may not be good enough to be forwarded to the destination, which is the usual case due to the bad channel condition or high noise level. Adaptive relaying scenario is a means to mitigate the inefficiencies of fixed relaying, by employing different scenarios in forwarding the information.

Selective relaying [8] is one form of adaptive relaying. In the DF scenario, it is noted that the transmission reliability is limited not only by the relay-destination node link but also by the link between the source node and the relay node. Selective relaying takes advantage of the SNR information on the relay node to make decision whether to transmit to the destination or not. If the SNR of the received signal at the relay node falls below a certain threshold, the probability of correct decoding at the relay node will be lower, and it is better not to forward the transmission since it will have high probability to cause error in the destination node.

The other form of adaptive relaying, the incremental relaying scenario, imitates the Automatic Repeat re-Quest (ARQ) protocol to prevent the relay nodes to transmit erroneous transmission to the relay nodes. In this scenario, the source node broadcasts the information to the destination and to the relay nodes. The destination node then checks the received signal. If the received signal fulfills the required SNR for the receiver, the destination node sends an acknowledgement to the source node. If the received signal has SNR below the requirement, the destination node can ask the relay nodes to forward the information to itself, and combines the signals from both source node and relay nodes.

2-3-2. Capacity of Cooperative Communication Network

The cooperative communication network helps to increase the transmission reliability by using the relay nodes to forward information. One of the parameters in measuring the transmission reliability is the transmission capacity, which can be measured by the mutual information. The mutual information of the cooperative communication network can be compared to the mutual information of direct transmission from the source node to the destination node, which is given by

$$I_D = \log\left(1+\gamma_{s,d}\right) \quad (2\text{-}7)$$

where

$$\gamma_{s,d} = \frac{P_s \left|h_{s,d}\right|^2}{N_0} \quad (2\text{-}8)$$

is the SNR at the destination from direct transmission. As mentioned in Section 2-12-3-1, P_s is the transmit power of the source node, $h_{s,d}$ is the channel coefficient from the source node to the destination node, and N_0 is the variance for the power of the additive white Gaussian noise with zero mean at the destination node. For the cooperative communication network with AF scenario, the maximum average mutual information with maximal ratio combining at the output is given by

$$I_{AF} = \frac{1}{2}\log\left(1+\gamma_{s,d}+\gamma_{s,r,d}\right) \quad (2\text{-}9)$$

where

$$\gamma_{s,r,d} = \frac{1}{N_0} \frac{P_s P_r \left|h_{s,r}\right|^2 \left|h_{r,d}\right|^2}{P_s \left|h_{s,r}\right|^2 + P_r \left|h_{r,d}\right|^2 + N_0} \quad (2\text{-}10)$$

is the SNR at the destination from relay nodes' transmission. P_r is the transmit power for the relay node and $h_{s,r}$ and $h_{r,d}$ denotes the channel coefficient for the source-relay node and relay-destination node links, respectively. The mutual information for DF scenario in cooperative communication requires correct decoding of the received signals at the relay nodes. If the relay nodes can decode correctly the information, the maximum average mutual information for the DF scenario is

$$I_{DF} = \frac{1}{2} \min\left\{\log\left(1+\gamma_{s,r}\right), \log\left(1+\gamma_{s,d}+\gamma_{r,d}\right)\right\}. \quad (2\text{-}11)$$

The capacity of the DF scenario is limited by the received signal at the relay nodes, which is represented by the first term of equation (2-11).

The selective relaying scenario attempts to optimize the DF scenario by applying the threshold for information to forwards to the destination node. The mutual information for the selective relaying is shown by

$$I_{SDF} = \begin{cases} \frac{1}{2}\log\left(1+2\gamma_{s,d}\right), & \left|h_{s,r}\right|^2 < g(\gamma) \\ \frac{1}{2}\log\left(1+\gamma_{s,d}+\gamma_{r,d}\right), & \left|h_{s,r}\right|^2 \geq g(\gamma) \end{cases} \quad (2\text{-}12)$$

where $g(\gamma) = \left[2^{2R}-1\right] N_0 / P$ is the threshold at the relay nodes and $R = 2r/W$ is the spectral efficiency of the transmission. These expressions, which used as the basis for many researches in the cooperative communication field, was given in the work by Laneman *et al.* [8].

2-3-4. Relay Selection Criterion and Resource Allocation

Relay selection in cooperative communication is a mean to improve the performance of the cooperative communication networks by selecting nodes which relatively 'better' compared to other nodes available in the system. In a cooperative communication network with amplify-and-forward scenario and identical transmit power at all relay nodes, improving transmission performance can be done by choosing relay nodes with better channel coefficient. In a cooperative network where the source node have limited power, relay nodes which is closer to the source node, or have better source-relay link might be chosen to increase the performance of the transmission. Resource allocation and optimization is a process to allocate power, timeslot, bandwidth, or any form of resources to each node in the network to improve the performance of the system with different objectives. The objective in the system can be the BER of the system, the transmission capacity, or even the power consumption.

Up to now, there have been many works in relay selection for cooperative communication networks, using different parameters constraint and different approach. In the work by Zhao and Valenti [5], the source node is assumed to have *a priori* knowledge about the channel model and the location of other nodes using GPS. This knowledge is then used to estimate the SNR between the links, and the relay nodes closer to the destination will be chosen to forward the information. The simulation results in this work compare the geographical location based relay selection, the relay selection based on instantaneous SNR at the destination, multi-hop relay network, and direct transmission. In the simulation result, it is shown that the geographical location based relay selection is better than random relay selection or relay selection by the instantaneous SNR at the destination. The novelty in this works though, is not the relay selection criterion, but the application of hybrid-ARQ in the cooperative communication. The relay nodes in this scenario keep the information which they forward to the destination node until they reach the acknowledgement (ACK) from the destination node. If the destination failed to obtain the information due to errors, the relay nodes can re-send the information without having the source node to re-transmit the information.

Zhao and Su [13] also did some work in location based relay selection. A scheme where the relays are placed in a uniform distributed (all the relays have same distance to the source) and a scheme where the source is assumed to know the location of the relays and choose a small subset of optimal relays to forward the data is compared. Power allocation to the relay selection problem is also introduced here. The power allocation is derived from estimation of the outage probability at the destination. Simulation result shows that the distributed scheme has better outage probability with a large number of users. The optimal power allocation for distributed scheme provided in this work is by using half the power to transmit to the destination, and half of the power is used to transmit to the relay nodes.

The multi-carrier system properties such as OFDM can also be exploited as relay selection criteria. Kaneko *et al* [14] proposed a relay selection based on sub-carrier allocation in a cooperative communication network with OFDM. In this scheme, each node is transmitting in multiple OFDM subcarriers. The source node broadcasts information to all the available relay nodes. Then, different relay selection scenarios based on multiple carriers was applied. In the first scenario, the All Participate All Subcarrier (APN), all relay nodes forward the information received from the source node to the destination sequentially. In the second scenario, All

Participate Rate Splitting (AP-RS), the relay nodes are dividing the information they need to forward into the number of relay nodes, before transmitting each part to the destination node. The third scenario, the Average Best Relay Selection Scheme (AvgBRS), is using the relay with the best average SNR from all the subcarriers to forwards the information. In the PerSubcarrier Best Relay Selection Scheme (NBRS), the relay nodes only transmitted the subcarriers with the best SNR. The last scenario, the Random Based Selection (RBS), selects a random relay to forward information to the destination. The RBS is used as a reference to other relay selection scenario. The simulation results provided in the work show that the NBRS is the best scenario for relay selection, but this scenario is not practical for implementation since we need complete channel state information (CSI) on the SNR for each subcarriers. The AvgBRS on the other hand, provides result approaching to the NRBS result with less complicated implementation. Figure 2-6 shows the illustration of how the APN and the NRBS scheme work in the time frequency scale.

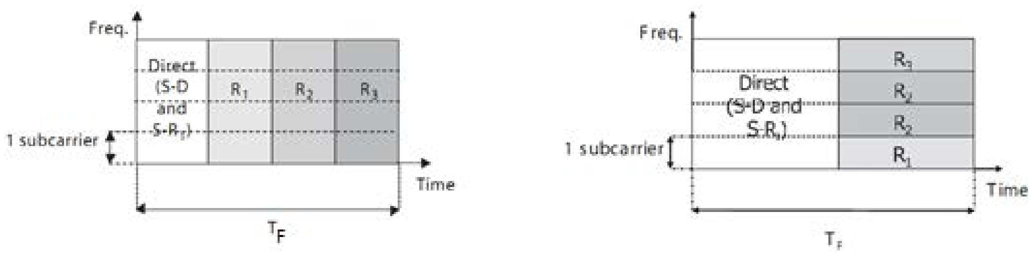

Figure 2-6 Illustration of APN and NRBS scheme. The left figure shows APN scheme with each transmission performed sequentially in time. In NRBS scheme, only subcarriers with best SNR are allowed to forward the information. [14]

The relay selection criterion can also be based on the interference. Selecting relay nodes based on the instantaneous SNR calculation in the destination node or in the relay node cannot guarantee the transmission performance since the effect of interference from other nodes might affect the system performance. In work done by Krikidis *et al* [6], the relay nodes are considered to receive interference, which change the result of the relay selection criterion. In this work, the nodes in the network are grouped into clusters. Each cluster consists of set of source node, destination node, and a number of relay nodes like shown in Figure 2-7.

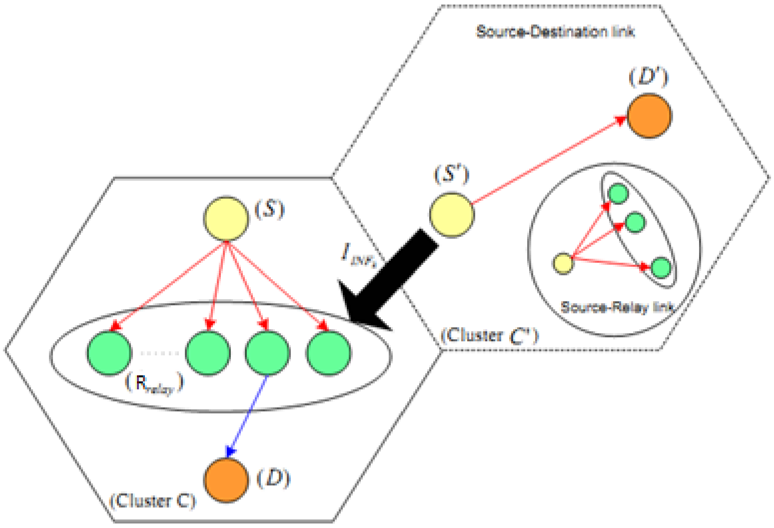

Figure 2-7 Scheme of cooperative communication network with interference[6]

The relay nodes received interference from the source node from other cluster. The received signal at the relay nodes then changed to

$$y_{s,r} = \sqrt{P_s} h_{s,r} x + \sqrt{P_{s'}} h_{s',r} \hat{x} + \eta_{s,r} \quad (2\text{-}13)$$

where $P_{s'}$ is the transmit power of the source node from other cluster, $h_{s',r}$ is the channel coefficient from the source node of the other cluster to the relay nodes, and $\hat{x}$ is the information sent from the source node of the other cluster. This interference became additive component to the calculation of signal to interference noise ratio (SINR) and the SINR became dependent to the statistical description of the interference.

In this paper, the relay selection is divided into four different scenarios with different assumptions. In the first scenario, the assumption is the system works with low SNR, and the relay selection for this scenario is

$$k = \arg\left\{\max_{k\in L}\left\{\min\left\{\gamma_{s,r_k}, \gamma_{r_k,d}\right\}\right\}\right\} \quad (2\text{-}14)$$

where $\gamma_{s,r}$ is the SNR at the relay node from the source node transmission and $\gamma_{r,d}$ is the SNR at the destination from the relay nodes transmission. This relay selection criterion ensures that the relay nodes with the best link between the source-relay nodes and relay-destination node are selected and provides the diversity gain according to the number of relay nodes selected [15] as well. The upper limit definition for low

SNR given on this work is $\gamma_{low} = 2\frac{\gamma}{\gamma_{INF}}$, where γ is the average SNR for each hop in the cluster of interest and γ_{INF} is the average interference-to-noise ratio at the relay nodes.

The next scenario uses the assumption that the system works at high SNR region with interference. When the system works at high SNR, the SINR equation at the destination node can be changed from the form

$$\gamma_{s,r,d} = \frac{\gamma_{s,r}\gamma_{r,d}}{\gamma_{\text{INF}}\left(\gamma_{r,d}+1\right)+\gamma_{s,r}+\gamma_{r,d}+1} \tag{2-15}$$

into the simplified form

$$\gamma_{s,r,d} \approx \frac{\gamma_{s,r}\gamma_{r,d}}{\gamma_{INF}\gamma_{r,d}} = \frac{\gamma_{s,r}}{\gamma_{INF}}. \tag{2-16}$$

This equation can be simplified since at high SNR region $(\text{SNR} \to \infty)$, the denominator at Equation (2-15) can be simplified from $\gamma_{\text{INF}}\left(\gamma_{r,d}+1\right)+\gamma_{s,r}+\gamma_{r,d}+1$ into only $\gamma_{INF}\gamma_{r,d}$. This simplification in turn changes the relay selection criterion to

$$k = \arg\max_{k\in L}\left\{\frac{\gamma_{s,r}}{\gamma_{INF}}\right\}. \tag{2-17}$$

This relay selection criterion selects relays which have the best SINR at the relay nodes since the SINR value at the destination node depends highly to this ratio, as shown in equation (2-16). In this work, the lower bound of the high SNR region is given by

$$\gamma_{high} = \frac{2\left(2^{2R_0}-1\right)\left(U_1-1\right)}{U_1-U_2}, \tag{2-18}$$

where $U_1 = \left(\frac{2^{2R_0}-1}{L+2^{2R_0}-1}\right)^K, U_2 = \left(\frac{2\left(2^{2R_0}-1\right)}{L+2\left(2^{2R_0}-1\right)}\right)^K$, and R_0 is the outage probability of the receiver.

The third relay selection criterion uses the assumption that the system works at "intermediate SNR". Intermediate SNR range falls between $\left[\gamma_{low}\ \ \gamma_{high}\right]$. The relay selection criterion for this scenario is

$$k = \arg\max_{k \in L} \left\{ \frac{\min\{\gamma_{s,r}\gamma_{r,d}\}}{\gamma_{INF}} \right\} \quad (2\text{-}19)$$

The fourth relay selection criterion is basically the same with the first one, but includes the interference in the calculations. In this scenario, the relay selection criterion is

$$k = \arg\max_{k \in L} \min \left\{ \frac{\gamma_{s,r}}{\gamma_{INF}}, \gamma_{r,d} \right\}. \quad (2\text{-}20)$$

The simulation results provided in this work show that at low SNR, the first criterion has the best results, the second criterion is better at high SNR, and the third criterion is better in the SNR area between the first criterion and the second criterion. The last relay selection criterion however, yields the best results at all SNR region.

From the literature study, it is shown that in the field of cooperative communication, relay selection and also resource allocation have become promising research areas. In the next chapter, game theory basics and classification will be discussed as an introduction to its application in cooperative wireless communication. Game theory has become an important tool in cooperative communication network since the nodes in this network have different objectives. After the introduction, some literature study for game theoretical approach in cooperative communication network will be provided.

3 Game Theory in Cooperative Communication

Game theory is a mathematical model that analyzes how different players with different strategies interact with each other. It provides the necessary tool to model the situation and predict the output of the interaction. The concept of game theory can be traced back to Mile Borel's researches in his 1938 book Applications aux Jeux des Hazard, followed by the book Theory of Games and Economic Behavior in 1994 by J. von Neumann and O. Morgenstern. The book by J. von Neumann and O. Morgenstern laid the foundation of game theory by providing the method for finding mutually consistent solutions for two-person zero-sum games. J. Nash developed a new concept known as Nash equilibrium in his research between 1950 and 1953. It marks an important development in the game theory, by providing a method that can be applied to non-zero-sum games. The research in game theory continued, covering the cooperative game, various kinds of equilibriums, and many game forms.

In wireless communication, especially cooperative communication, game theory is used to solve multi objective problems. When transmitting information, the source node and the relay nodes might have different objectives. The source node may only care about the signal to noise ratio of the transmitted signal, and requires its value to be higher, but the relay nodes may only care about low power consumption. This different objectives problem can be solved using game theoretical approach by allowing different players with different objectives to interact with each other in the objective optimization process until some equilibrium is reached.

Thus, the game theory became a popular tool in solving multi objective problems in wireless communication, and sometimes it can also be used to model the behavior of nodes in a network. In this chapter, game theory basics and principles such as components, types of games, and equilibrium definition will be provided. The applications of game theory in the wireless communication and cooperative communication are also provided to give clearer view of game theory in cooperative communication.

3-1 Game Theory Basics

A game in game theoretical point of view is composed by three elements: a set of players, a set of actions for each player, and payoff/utility functions. Players in a game are defined as the decision maker, individuals or groups of individuals which have the capabilities to make decision over a choice of actions. Players interact with each other using a set of actions available to them, resulting in payoff/utility function for each player.

In game theoretical modeling, there are assumptions about the characteristic of the players. The most important is the assumption that the players are rational, in sense that each player understands the goal of the game, the consequences of every action it took towards the goal, and ability to choose the action over optimization processes.

In short, a player i with rational decision capability will choose action a_i^* from A_i action space which will give $u\left(a_i^*\right) \geq u\left(a_{-i}\right)$ where $a_{-i} \in A_i$ is the complement of a_i^* in the A_i action space. Sometimes in the game theoretical modal, the players will have to make decision under uncertainty condition, caused by imperfect information between players, uncertainty from the environment, and non-deterministic action of other players in the game. In this case, decision making are done based on probabilistic approach, and the players will have to maximize the expected value of the objective functions.

3-2 Game Theory Classifications

Game theory basically can be divided into two main groups, the non-cooperative game theory and the cooperative game theory. Non-cooperative game theory has game models where the players in the game are individuals which competing against each other. In cooperative game theory, the players are groups of individuals. The actions in the game are taken as a group, and each group are competing on the objective functions. Most game model is based on the non-cooperative game group, although lately cooperative game theory has also gain popularity in research. Figure 3-1 provides a diagram of game theory classification.

3-2-1. Strategic Games

The simplest and the earliest model of game theory are the strategic games. In strategic games, each player chooses action once and simultaneously. Every player is also chasing their own benefits, using the optimal strategy to maximize their own utility. A strategic game is composed by three elements of game theory

- a finite set of player, denoted by N ;
- a set of action for each player i, denoted by A_i;
- payoff/utility functions, denoted by $u_i : A_i \rightarrow \Re$, which measures the outcome for player i determined by actions of all players, $A = \times_{i \in N} A_i$,

and a strategic game form can be written as $\langle N, (A_i), (u_i) \rangle$.

This strategic game can be used to model a situation in cooperative communication where the source node also acted as a relay node. In this situation, there are two nodes trying to cooperate to increase diversity. In the first time slot, the first node transmit the information to the destination node, some of the information is also broadcasted to node second. In the second time slot, the second node will forward the information from the first node together with its own information to the destination. The second node also uses the first node in the same manner, and this scenario provides cooperation diversity to both nodes in the transmission process.

Since forwarding information for other node will occupy some of the timeslot, the node might refuse to participate in the forwarding scenario. Hence the cooperative strategy will not work out. The more selfish node will try to use another node to forward its own information, and deny to forward information for another node. This will keep the throughput of its own node high by getting cooperation diversity from other relay. If both nodes become selfish and they both deny cooperation, the situation becomes a loss to both players, since they will keep broadcasting the information to the other node without getting the information forwarded to the destination. If the payoff from situation can be represented by numbers, the higher the more throughputs each node will get, and the action from each node can be represented by C for cooperating and D for denying cooperation, the strategic game which represents this situation can be shown in Table 3-1

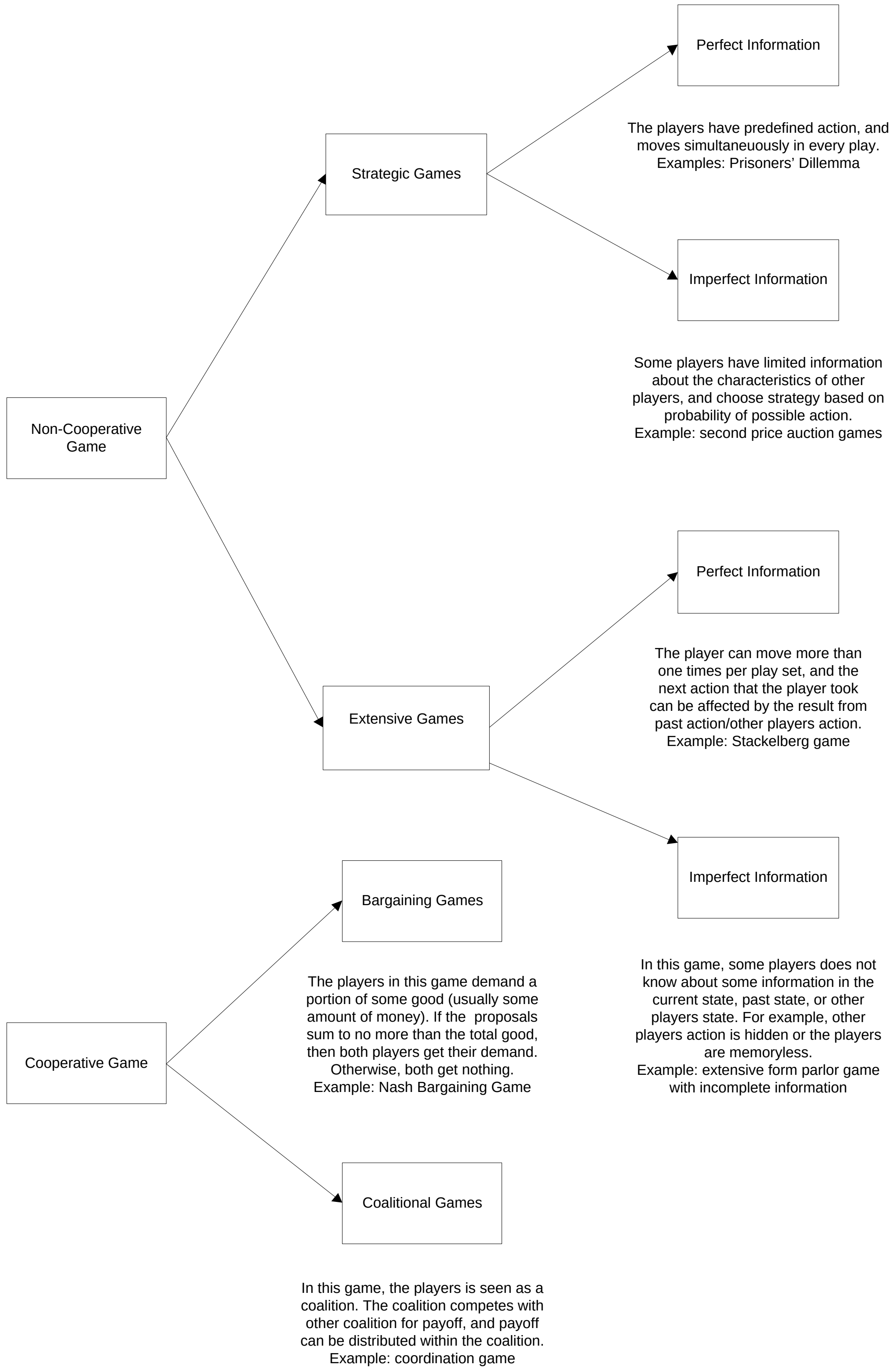

Figure 3-1 Game Theory Classification

Table 3-1 A simple cooperative communication game

	C	D
C	(4,4)	(3,5)
D	(5,3)	(2,2)

As shown in Table 3-1, if a node chooses to deny cooperation while the other node is cooperating, the node denies cooperation will gain more throughputs, while leaving the other node suffers loss in the throughput. However, if both relays decide to deny cooperation, both will suffer loss in throughput. In this case, the best strategy in this situation is for both nodes to cooperate. The option to cooperate is the steady state condition, since neither player can increase their throughput without having other player to change their action. This condition is known as the Nash Equilibrium of a game.

The Nash Equilibrium is defined as a profile $a^* \in A$ of actions where for each player $i \in N$, the game result is

$$u_i(a_i^*, a_{-i}^*) \geq u_i(a_i, a_{-i}^*) \qquad (3\text{-}1)$$

for all $a_i \in A_i$. From the example, it is clear that Nash Equilibrium is the solution to the strategic game, but Nash equilibrium does not have to exist in every game. In an example of Matching Pennies game, each of two people chooses either Head or Tail. If the choices are different, person A pays person B a dollar; if they are the same, person B pays person A a dollar. Each person cares only about the amount of money that he receives. The game that models this situation is shown in Table 3-2.

Table 3-2 Matching Pennies

	Head	Tail
Head	1,-1	-1,1
Tail	-1,1	1,-1

As seen from Table 3-2, the game of Matching Pennies is a zero sum game and has no Nash equilibrium. From this example, it is proven that Nash equilibrium does not have to be exist in a game. But it is rare to find a zero sum game in the case of wireless communication, so those games without Nash Equilibrium are not concerning. In [16], it has been shown using fixed point theorem that a game which has Nash Equilibrium will follow a criterion shown in Theorem I below.

Theorem I

A strategic game $\langle N,(A_i),(u_i)\rangle$ *has Nash equilibrium if, for all* $i \in N$, *the action set* A_i *of player* i *is a non-empty compact convex subset of a Euclidian space, and the payoff function* u_i *is continuous and quasi-concave on* A_i.

By definition, a function f(x) mapping from the reals to the reals is quasiconcave if it is nondecreasing for all values of x below some x_0 *and nonincreasing for all values of x above* x_0. x_0 *can be infinity or negative infinity: that is, a function that is everywhere nonincreasing or nondecreasing is quasiconcave.*

The theorem above stated that every game that fulfills the condition above have at least one Nash equilibrium.

3-2-2. Bayesian Game

Normally, in strategic game all players have the information of other players in the game. Then they can make decisions based on the preference of the other players and predict the outcome of their choices. In Bayesian game, some of the players in the game do not have all the information regarding the characteristics of other players.

In Bayesian Game, besides the three components of the game, the players, the set of actions, and the utility functions, there is a probability distribution over the available set of action which used to describe the uncertainty in the state of the game. Each player has their different views of the probability distribution over the states of the nature. In the game, they never know the exact state of the nature because of the imperfect information in the game.

3-2-3. Extensive Games

An extensive game is a form of game where all players will have to make decisions sequentially to solve a problem and approach their objectives. In an extensive game with perfect information, each player is perfectly informed of all the events that have previously occurred, and use the information when to make the decisions over the choice of actions. An extensive game with perfect information has the following component.

- A set N (the set of players).
- A set H of sequences (finite or infinite)

(Each member of H is a history; each component of a history is an action taken by a player.) A history $(a^k)_{k=1,\ldots,K} \in H$ is terminal if it is infinite or if there is no a^{K+1} such that $(a^k)_{k=1,\ldots,K} \in H$. The set of terminal histories is denoted by Z.

- A function P that assigns to each nonterminal history (each member of $H \setminus Z$) a member of N. (P is the player function, $P(h)$ being the player who takes an action after the history h.)
- For each player $i \in N$ a payoff/utility function u_i on Z

Most of multi objectives optimization problems, which need a game to be played over a number of iterations before equilibrium is reached, are extensive game with perfect information of imperfect information. A good example of extensive game in the wireless communication game is the pricing game for power allocation. When a node is transmitting information to a mobile device, a certain level of power is used. In order to increase the transmission throughput, increasing the transmit power is a choice, but since the battery power of a mobile device is limited and the battery life consumption is not a linear function, the higher transmit power used; the price for transmission will become higher. The optimization process for allocating the amount of power requires iteration of power level and pricing game until it reaches an equilibrium.

3-2-4. Bargaining Games

In this game, the individual players in the game have their own goals to achieve, and conflict with each other. The players involved in this game are trying to reach an agreement for each payoff. If all players do not reach an agreement, they will get nothing at all. Let us consider a two-player bargaining game $N = \{1, 2\}$, with u_1 and u_2 are respectively the payoff for each player when they reach an agreement and u_1^0 and u_2^0 are respectively the payoff when they did not reach an agreement. The set of all possible utility pairs is the feasible set denotes by U.

A two player bargaining problem is a pair $\langle U, (u_1^0, u_2^0) \rangle$, where $U \subset \Re^2$ is a compact and convex set, and there exists at least one utility pair $(u_1, u_2) \in U$ such that $u_1 > u_1^0$ and $u_2 > u_2^0$. A bargaining solution is a function $(u_1^*, u_2) = f(U, u_1^0, u_2^0)$ that assigns a bargaining problem $\langle U, (u_1^0, u_2^0) \rangle$ to a unique element of U

3-2-5. Coalitional Games

One component of this model is the sets of joint actions that each group of players (coalition) can take independently without being affected by the remaining players. An outcome of a coalitional game is the coalition that forms and the joint action it takes. The other parameter of the model of a coalitional game is the profile of the players' preferences over the set of all possible outcomes. Thus although actions are taken by coalitions, the theory is based on the individuals' preferences. In this sense, while players are forming coalition to reach certain outcome, they are still competing within the coalition to fulfill their own objectives.

A coalitional model is distinguished from a non-cooperative model primarily by its focus on what groups of players can achieve rather than on what individual players can do and by the fact that it does not consider the details of how groups of players function internally.

A coalitional game with transferrable payoff consist of a finite set N (the set of players) and a function v that associates with every nonempty subset S of N (a coalition) a real number $v(S)$ (the worth of S). For each coalition S the number $v(S)$ is the total payoff that is available for division among the members of S . That is, the set of joint actions that the coalition S can take consists of all possible divisions of $v(S)$ among the members of S .

The coalitional game does not always have transferrable payoff. A coalitional game without payoff to each individual player in the coalition is similar to non-cooperative game with coalition as the players.

3-3 Evolutionary Game Theory

Evolutionary game theory is an application of game theory where the strategy interaction is changing through the evolution process. The evolutionary game theory is based on the theory of evolutionary programming, a branch global search and optimization algorithm.

In the evolutionary algorithm, an individual in a population is an encoded solution to some problems. The individual is represented by a string corresponding to a *genotype* in the biological aspect. Each string contains chromosomes, which represents the strategies of each individual to contribute to the solution of the problems. Inside the population, each individual interact to generate better solutions to solve the problems. The result of this solution is quantified in a fitness valuation. A fitness valuation shows how close a solution from an individual to solve the problems.

To generate better solutions to solve the problems, evolutionary algorithm uses evolutionary operators. There are three major evolutionary operators usually used in the evolutionary algorithm: mutation, recombination, and selection. In the mutation process, the chromosomes an individual change its value by a random distribution. In the recombination, a set of chromosomes from an individual crossovers to another set of chromosomes from different individual. The last, the selection process is a way to pass down chromosomes which gives higher fitness valuation. Figure 3-2 provides the explanation of basic terms in evolutionary algorithm.

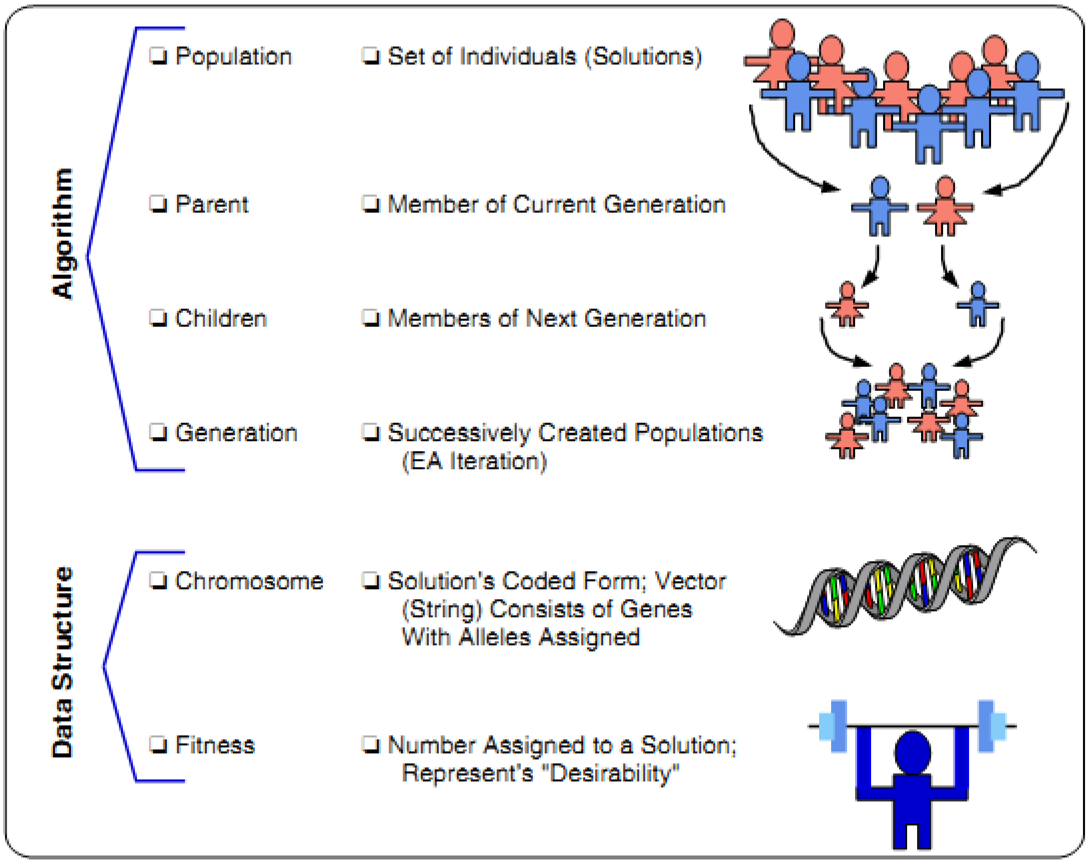

Figure 3-2 The basic concept of evolutionary algorithm[20]

3-4 Applications in Cooperative Communications

In the field of cooperative communication, game theoretical approach has been used to model the nodes behavior in the cooperative communication network and to solve the relay selection and resource allocation problems.

One of the earliest work is the paper by Ileri *et al.* [4], which introduces pricing in the cooperative communication network to encourage cooperation and to solve the resource allocation problems. From game theoretical approach, relay nodes which want to optimize their utility functions might not join the cooperation without some benefit. By introducing pricing, the relay nodes will gain some benefit by joining the cooperative communication scenario, which encourages relay nodes to forward the information.

In this scenario, the user is trying to send data to the Access Point (AP). The AP charges users for using the resources (time slot) when sending the data. The user can also use other users to forward the data to the AP, and the relaying users will get reimbursement from the AP for forwarding the data. The nodes are trying to maximize the network revenue ρ for the AP which is defined by

$$\rho = \sum_{i=1}^{N} \lambda T_i - \sum_{j=1}^{N} \mu T_j \qquad (3\text{-}2)$$

where N is the number of user in the network, λ is the price the AP charged to the users for using the resource (time slot), T_i is the throughput of the data sent to the AP by user i, μ is the reimbursement given to user j for forwarding T_j data. The user i (the non-forwarding user) net utility function is defined as

$$U_i = \lambda T_i \qquad (3\text{-}3)$$

and the user j (the forwarding user) net utility function, defined as

$$U_j = \lambda T_i + \mu T_j. \qquad (3\text{-}4)$$

In this scheme, the user i sent $(1-l)$ part of his/her data directly to the AP, and send l part of the data to the forwarding user j, which will forward the data in its k part of transmission, using $(1-k)$ part to send his/her own data. The scenario can be illustrated in Figure 3-3.

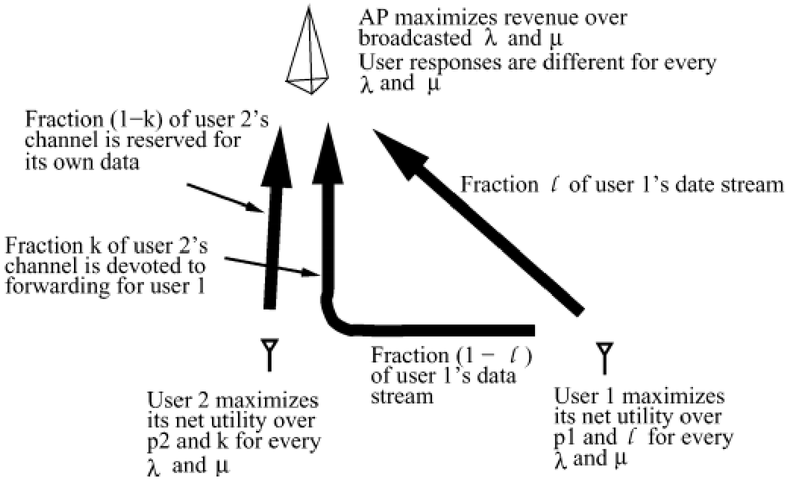

Figure 3-3 Illustration of pricing based cooperative communication[17]

At first the AP set the prices λ and μ, then the users (both the forwarding and the non-forwarding) is interacting in a non-cooperative game manners to maximize net utilities by changing the power and bandwidth (which change the l and k). After reaching Nash equilibrium which fulfill the Pareto optimality, the AP calculates its revenue and changes λ and μ to maximize the network revenue. The simulation result also show only users which is close to the AP and to the non-forwarding users, thus the location of the users is important in deciding the relay selection criteria.

The work of Shastry *et al.* [18] basically extends the work of Ileri *et al*. The previous work is mainly focusing on the network revenue, and in some conditions, the relay utility function for forwarding is lower than the non-forwarding case. This can discourage forwarding in the cooperative communication network since joining cooperation is not beneficial for the relay nodes. In this paper, the author suggests that the price for reimbursement should be decided by the source. Hence, the source will pay reimbursement to the forwarding user. The forwarding user will still pay to the AP for using the resource when forwarding the data but at the same price with sending its own data. The relay will gain profit from the difference between the price that AP set and the reimbursement price from the source. The illustration for this scenario can be seen in Figure 3-4.

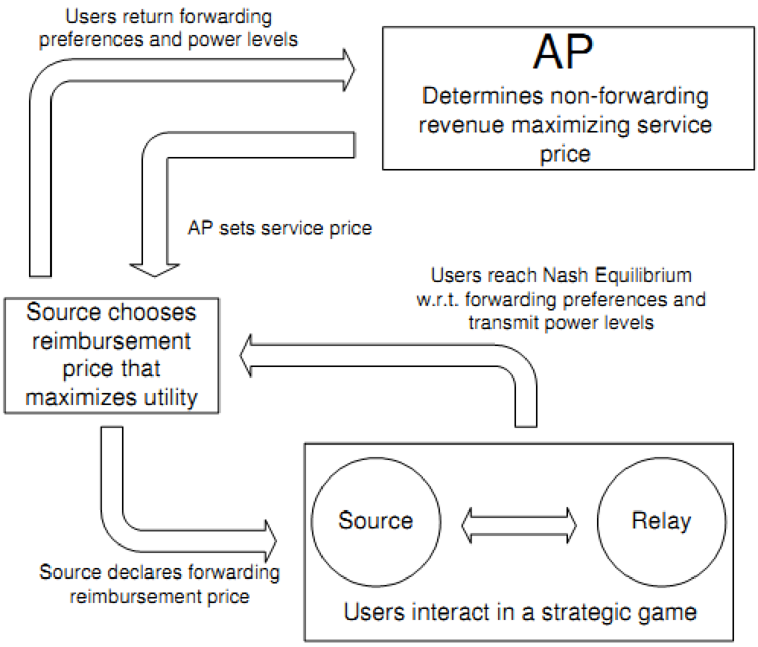

Figure 3-4 Illustration for user based pricing scheme [18]

The source and the relay will still have to reach Nash equilibrium by changing the power level and bandwidth, and then the result will be used by source to recalculate the reimbursement price for the relay and also used by the AP to recalculate the price for transmitting data to AP to maximize its own utility.

In Wang *et al.* work [7], the scenario is that the nodes (both the source and relaying nodes) are trying to maximize their own utility function. The source and the relay did not have to pay a price to the destination for sending of forwarding the data. The source node only pays for the power that it uses from the relay to forward its own data to the destination. The equilibrium is only controlled by the how much power from the relay the source will use and at what price it will buy it from the relay. The price is set by the relay according to the channel quality, and the source can choose which relay it wants to use depending on the price. In the simulation, the result shows that the locations of the relays affect the channel quality, hence affecting the price. By removing the AP control in the optimization process, optimization in distributed network can reach convergence more quickly. Comparison between proposed protocol and a centralized approach shows that the proposed protocol could reach the same performance as the centralized approach.

Game theory also used in work by Shi *et al.* [19] to optimized power control in distributed manner in a cooperative communication network with interference. In this scenario, there are two source nodes which want to transmit information to their own destination nodes and two relay nodes which help forward the information. Each source node will cause interference to the other relay nodes, and the relay nodes will cause interference to the destination nodes. Interference comes because of side lobe leakage in transmission beam width. The scenario in this work can be described by Figure 3-5.

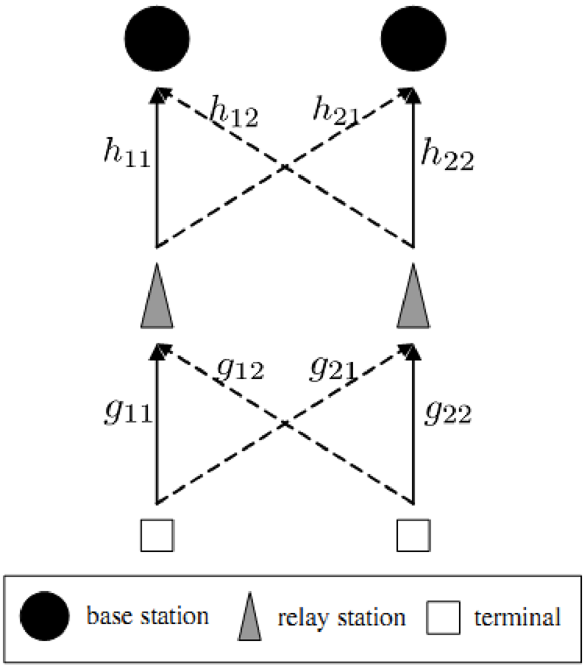

Figure 3-5 Relay network with interference [19]

In this scenario, DF forwarding is used, and the received signals at the relay nodes and the source node are defined by

$$y_{R_1} = h_{11}\sqrt{p_{11}}x_1 + h_{21}\sqrt{p_{21}}x_2 + n_{11}, \tag{3-5}$$

$$y_{R_2} = h_{22}\sqrt{p_{22}}x_2 + h_{12}\sqrt{p_{21}}x_1 + n_{21}, \tag{3-6}$$

$$y_{D_1} = g_{11}\sqrt{p_{12}}x_1 + g_{21}\sqrt{p_{22}}x_2 + n_{12}, \tag{3-7}$$

$$y_{D_2} = g_{22}\sqrt{p_{22}}x_2 + g_{12}\sqrt{p_{12}}x_1 + n_{22}, \tag{3-8}$$

where y_{R_1} and y_{R_2} are the received signal at each relay node, y_{D_1} and y_{D_2} are the received signal at each destination node, h_{11}, h_{22}, h_{12}, and h_{21} are the channel coefficient between each source node and the relay nodes, g_{11}, g_{22}, g_{12}, and g_{21} are the channel coefficient between each relay node and the destination nodes, p_{11}, p_{22}, p_{12}, and p_{21} are the transmit power for each node, x_1 and x_2 are information which the source nodes want to transmit, and finally n_{11}, n_{22}, n_{12}, and n_{21} are the additive white Gaussian noise at each relay node and destination node with variance σ_{ij}^2.

Since the scenario uses DF forwarding, the maximum capacity for each link is defined by

$$I_1 = \frac{1}{2}\min\left\{\log_2\left(1+\frac{p_{11}}{p_{21}\alpha_{21}+\omega_{11}}\right), \log_2\left(1+\frac{p_{12}}{p_{22}\beta_{21}+\omega_{12}}\right)\right\} \quad (3\text{-}9)$$

$$I_2 = \frac{1}{2}\min\left\{\log_2\left(1+\frac{p_{21}}{p_{11}\alpha_{12}+\omega_{21}}\right), \log_2\left(1+\frac{p_{22}}{p_{12}\beta_{12}+\omega_{22}}\right)\right\} \quad (3\text{-}10)$$

where $\omega_{i1} = \sigma_{i1}^2 / |h_{ii}|^2$ and $\omega_{i2} = \sigma_{i2}^2 / |g_{ii}|^2$ are the normalized noise power at the relay nodes and the destination nodes. $\alpha_{ij} = |h_{ij}|^2/|h_{jj}|^2$ and $\beta_{ij} = |g_{ij}|^2/|g_{jj}|^2$ are the normalized interference coefficient at the relay nodes and the destination nodes.

The game in this scenario is subjected to solve the maximization of the maximum capacity given by (3-9) and (3-10) with a power constraint. The power constraints are defined by P_1 and P_2 for each source node. The game then can be formulated as

$$\begin{array}{ll}\text{maximize} & I_1 + I_2 \\ \text{subject to} & P_i\end{array}. \quad (3\text{-}11)$$

The solution to this problem, which is the Nash Equilibrium of the game, are given by

$$\frac{p_{11}^*}{p_{21}^*\alpha_{21}+\omega_{11}} = \frac{p_{12}^*}{p_{22}^*\beta_{21}+\omega_{12}}, \quad (3\text{-}12)$$

$$\frac{p_{21}^*}{p_{11}^*\alpha_{12}+\omega_{21}} = \frac{p_{22}^*}{p_{12}^*\beta_{12}+\omega_{22}}, \quad (3\text{-}13)$$

with power constraint

$$p_{11}^{*} + p_{12}^{*} = P_1, \tag{3-14}$$

$$p_{21}^{*} + p_{22}^{*} = P_2 \tag{3-15}$$

The simulation results in this work show that the optimization based on the game theoretical scenario approaches the optimal solution.

In cooperative communication networks, we have nodes with different objectives. A source node wants to maximize its throughput but the relay nodes might want to reserve its resources. Through literature study, the game theory has been shown to be a useful tool to solve multi-objective optimization, especially in the field of relay selection and resource allocation for cooperative communication. In the next chapter, we will use game theoretical approach to solve relay selection and resource allocation problems.

4 Relay Selection and Resource Allocation using Game Theoretical Approach in Cooperative Wireless Communication

In Chapter 3, some literature studies in relay selection for cooperative communication have been provided. In those works, many relay selection models are proposed with different parameters as the relay selection criterion. Each models and parameters have different objectives and different assumptions for the environment condition. Accordingly, the proposed models for resource allocation and optimization process also have different objectives and different assumptions for the environment condition.

In this chapter, we will propose a novel resource allocation and optimization based on evolutionary game theory. First, we will describe a cooperative communication system with interference. Then, we formulate a pricing game model for the cooperative communication system with interference, and optimize the relay selection criterion. Afterwards, we will define the evolutionary game model for resource allocation and optimization which can be used for different objectives and different assumptions without having to make a lot of modification to the system models.

4-1 System Description

The work of Wang *et al.* [7] provides game theoretical approach in relay selection and resource allocation in cooperative communication. Unfortunately, this system does not provide the effects of interference in the system to the relay selection and resource allocation process. Here, we will model the system with disturbance from interference and shows the effects of interference to the relay selection and resource allocation process.

The nodes we have in the system are configured as in Figure 4-1. The nodes are divided into clusters as in [6], and each cluster consists of a source node, N number of relay nodes, and a destination node. The source node wants to transmit information to the destination node, through direct transmission and using the relay nodes to forward the information. Due to the broadcasting nature of the wireless nodes, the relay nodes and the destination node at a cluster might receive interferences from other clusters. We first denote the received signal at the destination from direct transmission as

$$y_{s,d} = \sqrt{P_s G_{s,d}}\, x + \eta_{s,d} + \omega_d \ , \qquad (4\text{-}1)$$

where P_s is the transmit power of the source node, $G_{s,d}$ represents the channel gain between the source node and the destination node, x is the unit power of information which the source node wants to transmit, $\eta_{s,d}$ is the independent additive white Gaussian noise (AWGN) with zero mean and variance σ^2 at the source-destination node link, and ω_d denote the interference from the neighboring cooperative communication clusters to the destination node.

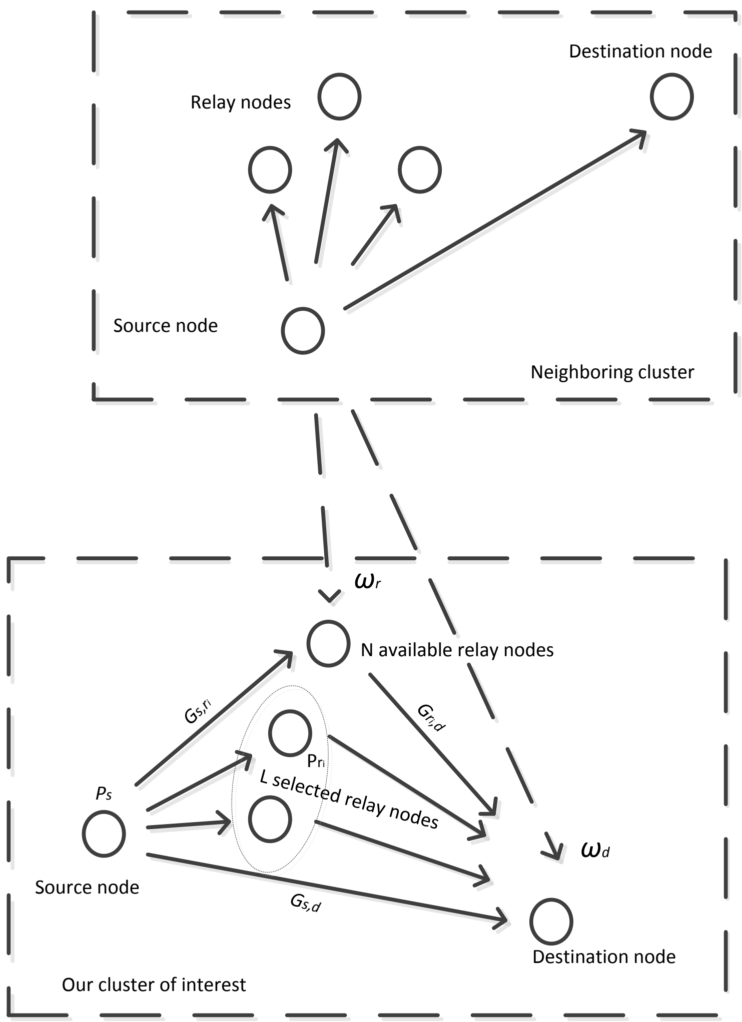

Figure 4-1 Cooperative communication network with interference from neighboring cluster

Due to the broadcasting nature, the relay nodes also received the signal transmitted by source node. To increase the capacity of the transmission, the relay nodes help forward the received signals using amplify–and–forward scenario. The received signal at relay node i is

$$y_{s,r_i} = \sqrt{P_s G_{s,r_i}}\, x + \eta_{s,r_i} + \omega_{r_i} \quad i \in N, \tag{4-2}$$

where N is the number of available relay nodes, G_{s,r_i} is the channel gain between the source node and relay node i, η_{s,r_i} is the independent additive white Gaussian noise (AWGN) with zero mean and variance σ^2 between the source node and relay node i, and ω_{r_i} denotes the interference from the neighboring cooperative communication clusters to the relay node i.

After receive the signals from the source node, the relay nodes amplify the signals and then re-transmit the information to the destination node. The received signal from relay node i at the destination is denoted as

$$y_{r,d} = \sqrt{P_{r_i} G_{r_i,d}}\, x' + \eta_{r,d} + \omega_d \tag{4-3}$$

where

$$x' = \frac{y_{s,r_i}}{\left| y_{s,r_i} \right|} \tag{4-4}$$

is the normalized transmitted information from relay node i to the destination node, P_{r_i} is the transmit power from relay node i, and $\eta_{r,d}$ is the independent additive white Gaussian noise (AWGN) with zero mean and variance σ^2 at the relay-destination node link.

If the interference is coming from a large number of nodes, according to the Central Limit Theorem, we can model it as a zero mean Gaussian distributed variable, with variance σ_{int}^2. Then, we can substitute (4-2) to (4-4) to make equation (4-3) become

$$y_{r_i,d} = \frac{\sqrt{P_{r_i} G_{r_i,d}} \left(\sqrt{P_s G_{s,r_i}}\, x + \eta_{s,r_i} + \omega_{r_i} \right)}{\sqrt{P_s G_{s,r_i} + \sigma_{\text{int}}^2 + \sigma^2}} + \eta_{r_i,d} + \omega_d . \tag{4-5}$$

We can calculate the signal-to-interference noise ratio (SINR) at the destination from direct transmission as

$$\gamma_{s,d} = \frac{P_s G_{s,d}}{\sigma^2 + \sigma_{\text{int}}^2} \tag{4-6}$$

and the SINR at the destination from relay node i transmission as

$$\gamma_{s,r_i,d} = \frac{P_{r_i} P_s G_{r_i,d} G_{s,r_i}}{\left(\sigma^2 + \sigma_{\text{int}}^2\right)\left(P_{r_i} G_{r_{i,d}} + P_s G_{s,r_i} + \sigma^2 + \sigma_{\text{int}}^2\right)}. \quad (4\text{-}7)$$

Using equation (2-7) $I_D = \log\left(1+\gamma_{s,d}\right)$, the capacity of the direct transmission can be calculated as

$$R_{s,d} = W \log_2\left(1+\gamma_{s,d}\right), \quad (4\text{-}8)$$

where W is the bandwidth of the transmission. Then, according to equation (2-9) $I_{AF} = \frac{1}{2}\log\left(1+\gamma_{s,d}+\gamma_{s,r,d}\right)$, the total capacity of the transmission using maximal-ratio-combining where each relay occupies W bandwidth is

$$R_{s,r,d} = W \log_2\left(1+\gamma_{s,d} + \sum \gamma_{s,r_i,d}\right). \quad (4\text{-}9)$$

After we use the relay selection process, then if the number of selected nodes is L, where $L \in N$, we can re-calculate the total capacity of the transmission as

$$R_{s,r,d} = W \log_2\left(1+\gamma_{s,d} + \sum_{i \in L} \gamma_{s,r_i,d}\right). \quad (4\text{-}10)$$

4-2 Game Theoretical Model

After the system description mentioned above, we continue with the game theoretical model of the system. We define a pricing game model based on Stackelberg game similar to [7]. Stackelberg game is a form of game where a player is chosen as a leader and chooses its strategy first in the game. The other players are the followers, and choose the strategy according the strategy of the leader. We choose the Stackelberg game model because at this game model, optimization process starts by assigning the optimal P_{r_i} for each relay by the source node, followed by assigning p_i for each relay by the relay nodes according to the P_{r_i}.

First, we define players in the game, which are the source node and the relay nodes. The source node wants to increase the capacity of the transmission by assigning P_{r_i} to each selected relay node. The relay nodes will be given compensation p_i for each unit power P_{r_i} used. In the pricing game, the source node wants to assign P_{r_i} to increase the transmission capacity, and the relay nodes want to assign p_i to increase the compensation. The game will be played between the source node and the relay nodes to determine the value of P_{r_i} and p_i.

We define the utility function of the source node as the gain source node gets through the transmission, subtracted by the payment to the relay nodes. The utility function of the source node can be denoted as

$$U_s = aR_{s,r_i,d} - \sum_{i \in L} P_{r_i} p_i \tag{4-11}$$

where a is the gain per unit rate at the receiver. Then, we define the utility function of the relay node as the compensation the relay node gets from the source node for forwarding the information subtracted by the fixed cost of relay nodes' power usage. We can write the utility function of relay node i as

$$U_{r_i} = p_i P_{r_i} - c_i P_{r_i}. \tag{4-12}$$

As mentioned before, the strategy of the source node is to assign the value of P_{r_i} to maximize the source node utility function

$$\max_{\{P_{r_i}\}} U_s = aR_{s,r_i,d} - \sum_{i \in L} P_{r_i} p_i, \quad P_{r_i} \geq 0 \tag{4-13}$$

and the strategy of the relay node i is to assign the value of p_i to maximize the utility function of relay node i

$$\max_{\{p_i\}} U_{r_i} = \left(p_i - c_i\right) P_{r_i}. \tag{4-14}$$

Before deriving the solution for the pricing game between the source node and the relay nodes, we first define the relay selection criterion for the cooperative communication network. We use the price of the relay nodes as the criterion for the selection. At the beginning of the game, the transmit power of the relay nodes have not been allocated and the initial p_i of the relay nodes are the same as its c_i. Then, we select relay nodes which has the following criterion

$$\frac{\partial U_s}{\partial P_{r_i}} = a\frac{\partial R_{s,r,d}}{\partial P_{r_i}} - p_i > 0, \quad i \in N. \tag{4-15}$$

After expansion, equation (4-15) becomes

$$\begin{aligned}\frac{\partial U_s}{\partial P_{r_i}} &= a\frac{\partial}{\partial P_{r_i}}\left\{W\log_2\left(1+\gamma_{s,d}+\sum_{i\in N}\gamma_{s,r_i,d}\right)\right\} - p_i > 0 \\ &= \frac{aW}{\ln 2}\frac{\partial}{\partial P_{r_i}}\left\{\ln\left(1+\frac{P_s G_{s,d}}{\sigma^2+\sigma_{\text{int}}^2}+\sum_{i\in N}\frac{P_{r_i}P_s G_{r_i,d}G_{s,r_i}}{\left(\sigma^2+\sigma_{\text{int}}^2\right)\left(P_{r_i}G_{r_{i,d}}+P_s G_{s,r_i}+\sigma^2+\sigma_{\text{int}}^2\right)}\right)\right\} - p > 0_i.\end{aligned} \tag{4-16}$$

If we define $A_i = \frac{P_s G_{s,r_i}}{\left(\sigma_{\text{int}}^2+\sigma^2+P_s G_{s,d}\right)}$ and $B_i = \frac{\left(P_s G_{s,r_i}+\sigma_{\text{int}}^2+\sigma^2\right)}{G_{r_i,d}}$ then equation (4-16) can be modified to

$$\begin{aligned}\frac{\partial U_s}{\partial P_{r_i}} &= \frac{aW}{\ln 2}\frac{\partial}{\partial P_{r_i}}\left\{\ln\left(1+\frac{P_s G_{s,d}}{\sigma^2+\sigma_{\text{int}}^2}\right)+\ln\left(1+\sum_{i\in N}\frac{P_{r_i}P_s G_{s,r_i}}{\left(\sigma^2+\sigma_{\text{int}}^2\right)\left(P_{r_i}G_{r_{i,d}}+P_s G_{s,r_i}+\sigma^2+\sigma_{\text{int}}^2\right)}\frac{\sigma^2+\sigma_{\text{int}}^2}{P_s G_{s,d}+\sigma^2+\sigma_{\text{int}}^2}\right)\right\} - p_i > 0 \\ &= \frac{aW}{\ln 2}\frac{\partial}{\partial P_{r_i}}\left\{\ln\left(1+\frac{P_s G_{s,d}}{\sigma^2+\sigma_{\text{int}}^2}\right)+\ln\left(1+\sum_{i\in N}\frac{A_i}{1+\frac{B_i}{P_{r_i}}}\right)\right\} - p_i > 0 \\ &= \frac{aW}{\ln 2}\frac{\partial}{\partial P_{r_i}}\left\{\ln\left(1+\frac{P_s G_{s,d}}{\sigma^2+\sigma_{\text{int}}^2}\right)+\ln\left(1+\sum_{i\in N}\frac{A_i P_{r_i}}{P_{r_i}+B_i}\right)\right\} - p_i > 0.\end{aligned} \tag{4-17}$$

After differentiation, equation (4-17) will become

$$\frac{\partial U_s}{\partial P_{r_i}} = \frac{aW}{\ln 2}\frac{A_i B_i}{\left(1+\sum_{k\in L}\left(\frac{A_k P_{r_k}}{P_{r_k}+B_k}\right)\right)\left(P_{r_i}+B_i\right)^2} - p_i > 0. \tag{4-18}$$

Since at the beginning of the game the power at each relay is not allocated, $P_{r_i} = 0, i \in N$ and, then we will have

$$\frac{\partial U_s}{\partial P_{r_i}} = \frac{aW}{\ln 2}\left(\frac{A_i}{B_i}\right) - p_i > 0, \quad i \in N, i \neq j. \tag{4-19}$$

The rationalization of this criterion is that we choose relay nodes whose initial price p_i is low enough that increasing P_{r_i} will let us increase U_s, thus we use the first-order differential of U_s to P_{r_i}.

After the relay selection process, the pricing game is played between the source node and the selected relay nodes. The source node assigns the value of P_{r_i} by using the first-order differential optimization as in

$$\frac{\partial U_s}{\partial P_{r_i}} = a\frac{\partial R_{s,r,d}}{\partial P_{r_i}} - p_i = 0, \quad i \in L. \tag{4-20}$$

In Appendix 1 it will be shown that the extreme point at $\frac{\partial U_s}{\partial P_{r_i}} = 0$ is a global maximum. We know from equation (4-10) that

$$aR_{s,r_i,d} = aW\log_2\left(1+\gamma_{s,d}+\sum_{i\in L}\gamma_{s,r_i,d}\right) \tag{4-21}$$

And from equation (4-18) we have

$$\frac{\partial U_s}{\partial P_{r_i}} = \frac{aW}{\ln 2}\frac{A_i B_i}{\left(1+\sum_{k\in L}\left(\frac{A_k P_{r_k}}{P_{r_k}+B_k}\right)\right)\left(P_{r_i}+B_i\right)^2} - p_i = 0 \tag{4-22}$$

which can be modified into

$$\frac{\frac{aW}{\ln 2}}{\left(1+\sum_{k\in L}\left(\frac{A_k P_{r_k}}{P_{r_k}+B_k}\right)\right)} = \frac{p_i}{A_i B_i}\left(P_{r_i}+B_i\right)^2 \tag{4-23}$$

As provided in [7], since the left-hand-side of equation (4-23) is the same for every relay j and relay i, we have

$$P_{r_j} = \sqrt{\frac{p_i A_j B_j}{p_j A_i B_i}}\left(P_{r_i}+B_i\right) - B_j \tag{4-24}$$

Using equation (4-24) we can modify $\frac{A_j}{1+\frac{B_j}{P_{r_j}}}$ from equation (4-17) for relay j into

$$\frac{A_j}{1+\frac{B_j}{P_{rj}}} = A_j - \sqrt{\frac{p_j A_i B_i}{p_i A_j B_j}} \frac{A_j B_j}{\left(P_{r_i} + B_i\right)}. \tag{4-25}$$

Solving equation (4-22) to P_{r_i} and with $X = 1 + \sum_{j \in L} A_j$ and $Y = \sum_{j \in L} \sqrt{p_j A_j B_j}$ we will have

$$P_{r_i}^* = \sqrt{\frac{A_i B_i}{p_i}} \frac{Y + \sqrt{Y^2 + 4XW'}}{2X} - B_i, \tag{4-26}$$

which is the allocated power for relay node i. After the transmit power for each selected relay node has been allocated, the relay nodes assign the new prices which will fulfill equation (4-14). We use first-order differential maximization of the utility function of relay node i

$$\frac{\partial U_{r_i}}{\partial p_i} = P_{r_i}^* + \left(p_i - c_i\right) \frac{\partial P_{r_i}^*}{\partial p_i} = 0. \tag{4-27}$$

Solving equation (4-27) for p_i, we will have the new price for relay node i as

$$p_i^* = c_i - \frac{P_{r_i}^*}{\partial P_{r_i}^* / \partial p_i} \tag{4-28}$$

where $\partial P_{r_i}^* / \partial p_i$ is the first-order differential of the optimal transmit power of the relay node i to the price of relay node i, and can be written down as

$$\frac{\partial P_{r_i}^*}{\partial p_i} = \sqrt{\frac{A_i B_i}{p_i}} \frac{Y + \sqrt{Y^2 + 4X \frac{aW}{\ln 2}}}{2X} \left(-\frac{1}{2p_i} \left(1 - \frac{\sqrt{p_i A_i B_i}}{\sqrt{Y^2 + 4X \frac{aW}{\ln 2}}} \right) \right) \tag{4-29}$$

The power and price updating function are the two strategies that each player plays in the pricing game. From equation (4-26) and equation (4-28) we can see that if P_{r_i} is changed, p_i will also change and vice versa.. The game is played until we reach a convergence point for both P_{r_i} and p_i values. The convergence point is also an equilibrium in the game, which is the optimal solution for both equation (4-13) and equation (4-14), which are the objectives of the optimization process. More about the existence of the equilibrium and proof about the optimal solution will be provided in Appendix 1.

From equations (4-26) and (4-28) we can see that the updating function of the transmit power $P_{r_i}^*$ and the price p_i depend also to other selected relay nodes beside relay node i, $i \in L$. This shows that the pricing game is not played only between the source node and relay node i, but also with other selected nodes. Thus, the number of selected nodes is affecting the resource allocation process. Because of this property, later in Chapter 5 we will show through simulation results that in some cases, the selected nodes might contribute to increases the source node utility function U_s at the beginning of the pricing game, but as the price increase through the updating function, the relay nodes will not contribute to increase the source node utility function U_s anymore.

We propose an algorithm to limit the number of selected relay nodes to mitigate the problem with the price updating function. First, we arrange the relay nodes based on their initial price by comparing each node price to all others. Then, we select the node with the lowest price, and perform the relay selection process and pricing at each iteration until the optimal U_s is reached. Then, we add the next node with lowest initial price to the calculation and compare the optimal U_s to the previous optimal U_s. If this U_s is higher, we continue the process until the optimal U_s stops increasing or starts decreasing. The maximum number of usable nodes is the number of the nodes when the maximum optimal U_s is reached. We can see the flowchart of the algorithm in Figure 4-2.

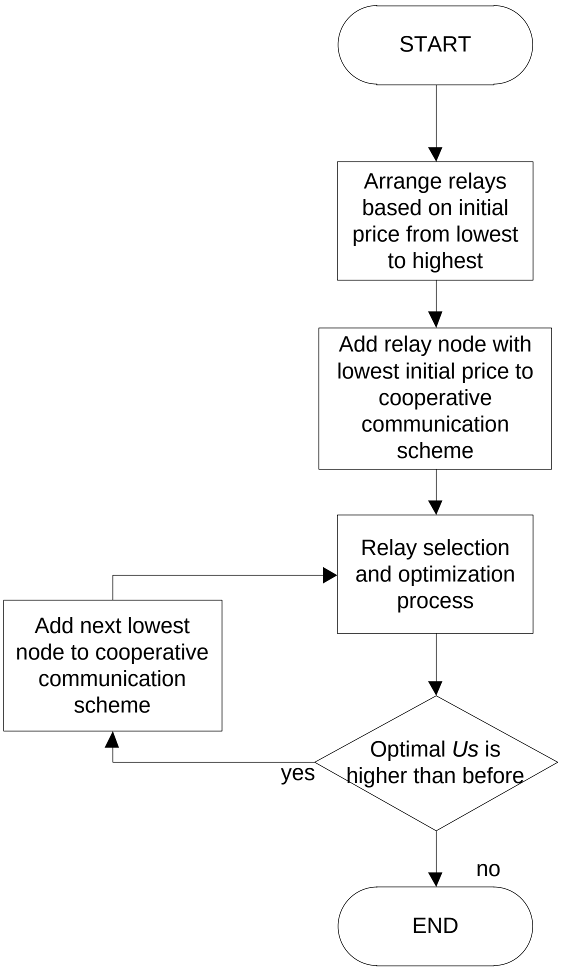

Figure 4-2 Relay selection algorithm for limiting the number of nodes

After we obtain the number of usable nodes, we select the nodes according to the relay selection criterion in equation (4-15), sort the selected nodes according to its price, and select the nodes according to the maximum number of usable nodes. Afterwards, we perform the pricing game to allocate the optimum power and price for each node.

As mentioned before, the equilibrium of the game is the optimal solution for solving the maximization of U_s and U_{r_i} problems. Equilibrium is reached by assigning P_{r_i} and p_i sequentially using equation (4-26) and equation (4-28) until convergence point is reached. In Figure 4-3 and Figure 4-4 the example of P_{r_i} and p_i in a pricing game can be seen. In the example, the power and the price of the relay node reach the equilibrium after five iterations.

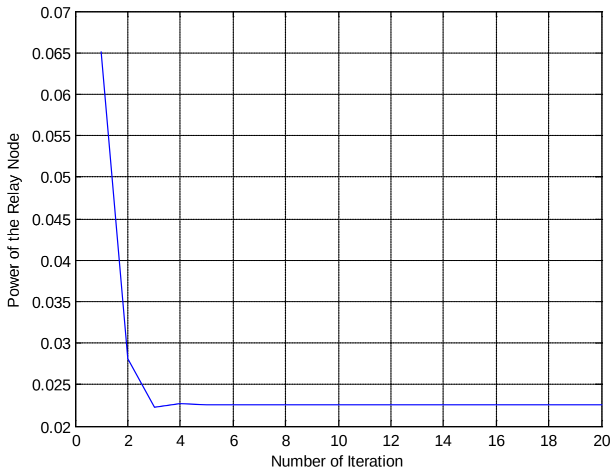

Figure 4-3 Power of the relay node in a pricing game

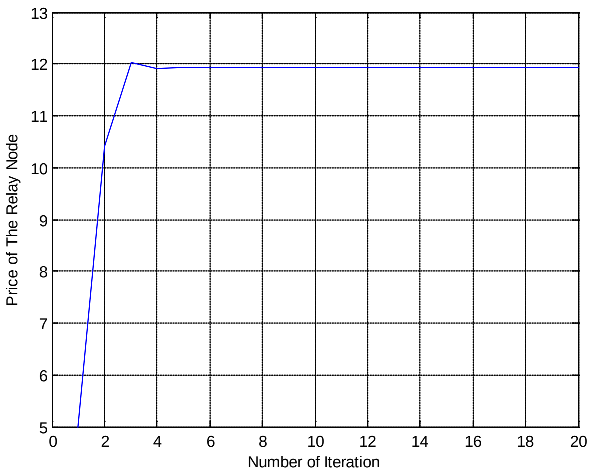

Figure 4-4 Price of the relay node in a pricing game

Since all selected relay nodes are competing to each other when P_{r_i} and p_i are assigned, the payment to the relay nodes does not increase linearly when the number of selected relay nodes is increased and the relay selection criterion in equation (4-15), is not fulfilled as the U_s is decreased when the number of selected nodes is high. Therefore, limiting the number of usable nodes is one solution to avoid decreasing in U_s when the selected number of relay nodes is high.

4-3 Evolutionary Game for Resource Allocation

The relay selection and resource allocation process described in previous section is mainly based on the price of the relay nodes by using the assumption that the objectives of the system are described in equation (4-13) and equation (4-14). Suppose that we have a system which has different objectives , for example to select the relay based on max-min optimization as in [6]. We select the relay based on

$$l = \arg\max_{l \in N} \min \left\{ \gamma_{s,r_i}, \gamma_{r_i,d} \right\}, \tag{4-30}$$

which means that we select the relay according to best source-relay-destination link, without considering the prices of the relay nodes. In this situation, the previous pricing game for resource allocation and optimization will not be sufficient, and we have to change the game model. In this section, we will propose an evolutionary game based resource allocation which can adapt to different objectives with acceptable performance.

The evolutionary game is based on the evolutionary programming which is a form of technique to solve global search and optimization problems [20]. Evolutionary programming was chosen since we have multi-objective problems and using the fitness valuation we can adjust the behavior of each player according to different objectives. The selection, crossover, and mutation process also makes sure that the objectives are reached faster compared to other search algorithm such as Monte Carlo or Random Search/Walk algorithm.

We have the system description as in Section 4-1, and we make the game model with the source node and the relay nodes as the players of the game, and both utility functions of the players as in equation (4-11) for the source node and equation (4-12) for relay node i. Unlike the previous model, the players in this game select their strategy simultaneously. The strategy for the source node is to assign P_{r_i} and the strategies of the relay nodes are to assign each p_i. The strategies for each player can be described in a stream of bits as shown in Figure 4-5. Each bit represents an instruction to the power and price updating function. In this case, bit "1" means that the transmit power or price of the relay is to be increased by a constant value and bit "0" means that the transmit power or the price of the relay is to be decreased by a constant value.

1 0 0 0 1 0 1 0 1 0 1 0 1 0 1 0 0

Figure 4-5 Example of a players' strategy in an evolutionary game

In this evolutionary game model, each player will have a set of strategy called population. The population consists of m strategies which will be played in the game between the source node and the relay nodes. Figure 4-6 shows the examples of a population in evolutionary game model.

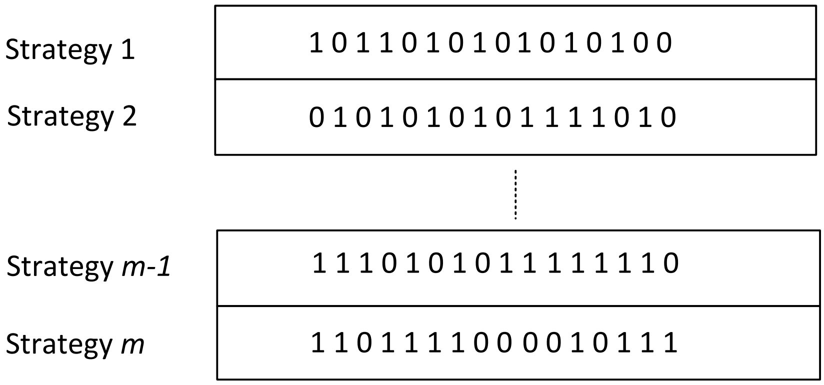

Figure 4-6 Example of a population in the evolutionary game model

The flowchart of evolutionary game for resource allocation is shown in Figure 4-7. We generate random stream of bits, and then assign it to the strategy for the source node and the relay nodes. The source node and the relay nodes run the game until all strategies in the population are played. After all strategies have been played, we obtain m number of P_{r_i} and p_i, and with these values we can obtain the responding $R_{s,r_i,d}$, U_s, and U_{r_i}. We then assign fitness valuation for each strategies from $R_{s,r_i,d}$, U_s, or U_{r_i} values, depend on our system objectives. The P_{r_i}, p_i, $R_{s,r_i,d}$, U_s, and U_{r_i} values are saved to an external memory for implementing the local search later.

After we applied the game between the source node and the relay nodes, we perform the selection, crossover, and mutation processes to the replaceable values of the population to produce a new generation of population. Selection, crossover, and mutation are evolutionary operators as explained in Chapter 3. Selection is a process to make a new generation of strategies where the strategy with higher fitness valuation will have higher probability to appear again in the next generation. Crossover is a process to cut a part of a strategy and recombine it to another strategy with a probability of P_c. The mutation is a process where a random part of the strategy changes its value with a probability of P_m.

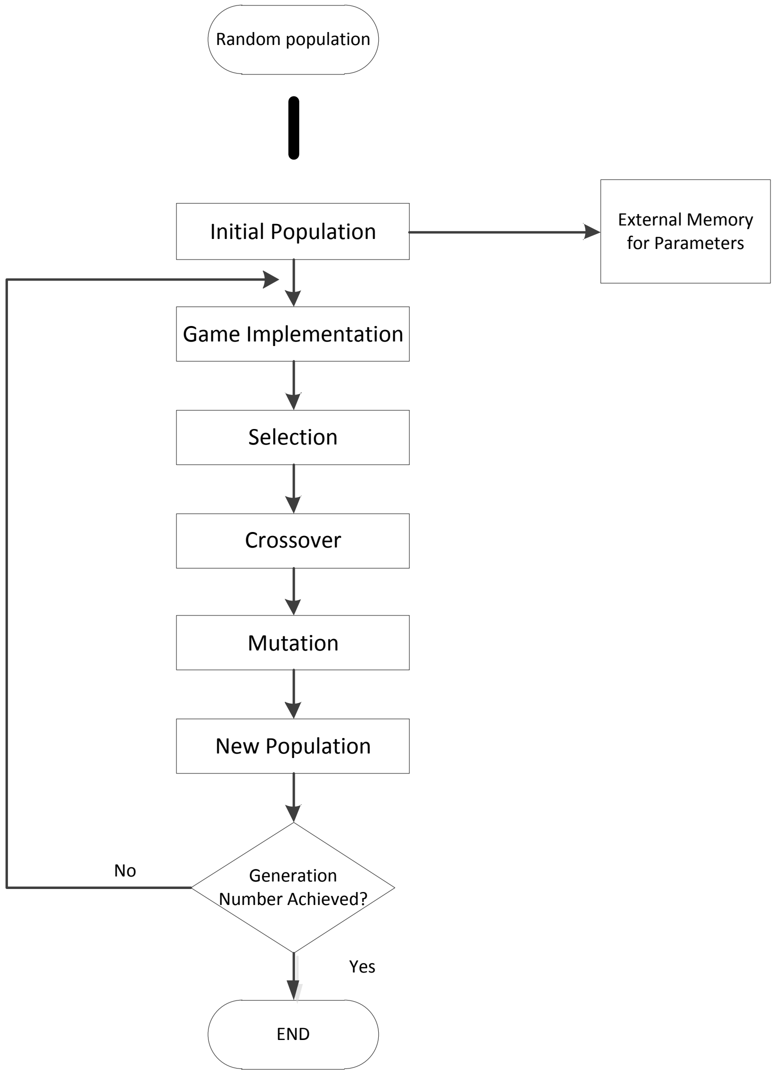

Figure 4-7 Evolutionary algorithm for resource allocation

After a new generation is produced by the evolutionary operators, the game is played between the source node and the relay nodes using the new set of strategies. The process is repeated until all k generations are achieved. We then have $k \times m$ number of P_{r_i}, p_i, $R_{s,r_i,d}$, U_s, and U_{r_i} values in the memory. Then, we perform the local search throughout the results from the evolutionary game process. For example, if we have multiple objectives as the previous game model, which are the equation (4-13) and (4-14), we will find the maximum value in the memory using aggregate objective function (AOF), which is defined by

$$AOF = w_1 U_s + w_2 \sum_{i \in L} U_{r_i} \tag{4-31}$$

where w_1 and w_2 are the weighting parameters which are defined by the user of the system and depend to the objectives of the system. In the case where we have multiple objectives as the previous game model, we select both w_1 and w_2 as 1.

Since this evolutionary game is a form of stochastic technique for global search and optimization approach, the final solution from the evolutionary game, which are the values of P_{r_i} and p_i are not guaranteed to be the optimal solution as in the previous game model in Section 4-2. The optimality of the solution depends on the number of generation (the higher the number, the higher the probability of the optimal solution is reached), the number of step size in increasing/decreasing the values of P_{r_i} and p_i, the length of each strategy in the population, and the probability of mutation and crossover.

This algorithm however, provides flexibility in adjusting the objectives of the system compared to the pricing game mentioned in Section 4-2. By adjusting the fitness valuation and the AOF, we can change the solution of the algorithm according to the objectives of the system. For example, if we want the system to only maximize $R_{s,r_i,d}$, we can assign its value to the fitness valuation and the AOF will perform the local search at $R_{s,r_i,d}$ in the memory. The pricing game can only optimize the system based on maximization of U_s and U_{r_i}. This algorithm also requires simpler calculation than the pricing game model in Section 4-2 which will lead to lower hardware complexity.

In the next chapter, we will give simulation results for the proposed system models in different scenarios and analysis for the simulation results to give a clearer understanding of the proposed system models.

5 Results and Analysis

In Chapter 4, we have discussed about the system model for the relay selection and resource allocation and optimization. Based on the model defined in Chapter 4, we set up a computer simulation to justify our theoretical analysis for the system model and give more clear understanding of the system. In this chapter we will present results from our computerized simulation model.

First, we will simulate the relay selection and resource allocation in the cooperative communication network with interference. Using system and game models from Section 4-1 and Section 4-2, we simulate the relay selection and resource allocation for cooperative communication network with interference.

We consider the system model with different signal-to-interference-noise ratio (SINR) ratios. We use fixed source node transmit power with different interference power level. The SINR at the destination can be calculated using equation (4-6) and (4-7). In this scenario, we have 20 available relay nodes with different c_i and channel coefficients. In Figure 5-1, we can see the number of selected nodes L for different SINR values. We can see that as at all SINR value the number of selected nodes. The number of selected nodes is constant since the average cost for relay nodes is constant.

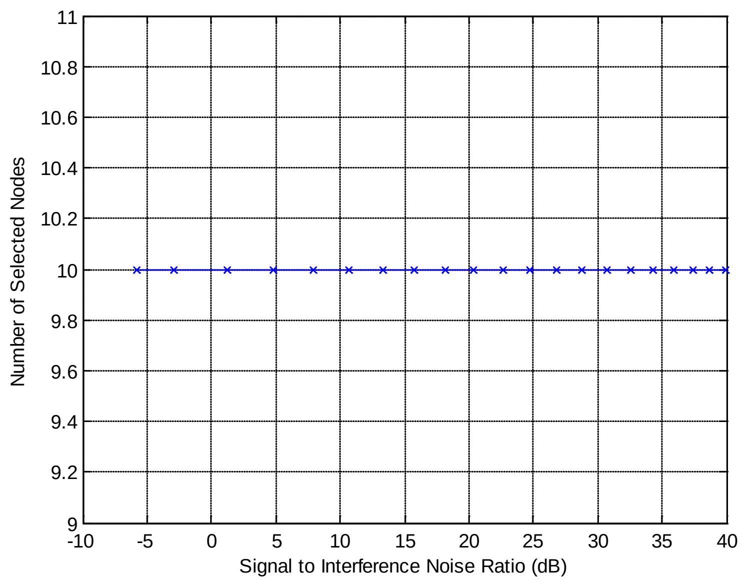

Figure 5-1 Number of selected nodes for different SINRs

To see the effect of interference to the relay selection, we simulate the relay selection process for cooperative wireless communication networks with interference and without interference with different relay cost.

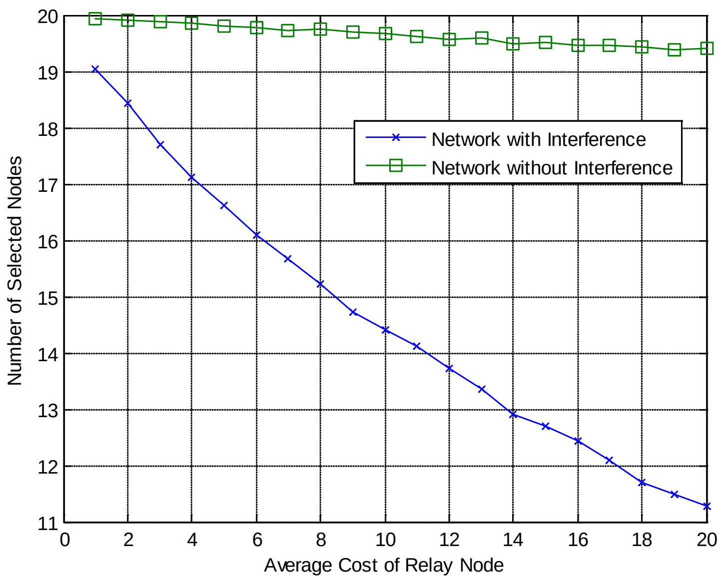

Figure 5-2 Number of selected nodes from relay selection to cooperative wireless communication network with and without interference for different relay cost

We then continue our simulation for resource allocation and optimization process using pricing game model. Here, we consider again the system model with different signal-to-interference-noise ratio (SINR) ratios. We use fixed source node transmit power with different interference power level.

From Figure 5-2 we can see that the number of selected nodes when interference is included in the network is lower than when the interference is excluded. This result confirm our relay selection criterion model according to equation (4-19). In equation (4-19) $\frac{\partial U_s}{\partial P_{r_i}} = \frac{aW}{\ln 2}\left(\frac{A_i}{B_i}\right) - p_i > 0, \quad i \in N, i \neq j$ with $A_i = \frac{P_s G_{s,r_i}}{\left(\sigma_{\text{int}}^2 + \sigma^2 + P_s G_{s,d}\right)}$ and $B_i = \frac{\left(P_s G_{s,r_i} + \sigma_{\text{int}}^2 + \sigma^2\right)}{G_{r_i,d}}$ we know that adding interference level will decrease $\frac{aW}{\ln 2}\left(\frac{A_i}{B_i}\right)$ and the relay node will be removed from the selection since $\frac{\partial U_s}{\partial P_{r_i}}$ will likely to be lower than zero.

In Figure 5-3 and Figure 5-4, we can see the total power consumption for the relay nodes $\sum_{i \in L} P_{r_i}$ and the total price of the relay nodes $\sum_{i \in L} p_i$. We can see that the total power consumption increases as the SINR increases. The total price also increases as the SINR increases and from SINR around -10 dB it increases dramatically. From equation (4-28) we know that the price of the relay node is affected by $\frac{\partial P_{r_i}^*}{\partial p_i} = \sqrt{\frac{A_i B_i}{p_i}} \frac{Y + \sqrt{Y^2 + 4X \frac{aW}{\ln 2}}}{2X} \left(-\frac{1}{2p_i} \left(1 - \frac{\sqrt{p_i A_i B_i}}{\sqrt{Y^2 + 4X \frac{aW}{\ln 2}}} \right) \right)$, and the interference level is affecting both A_i and B_i and also X and Y consequently, which causes the total price of the relay nodes to increase dramatically.

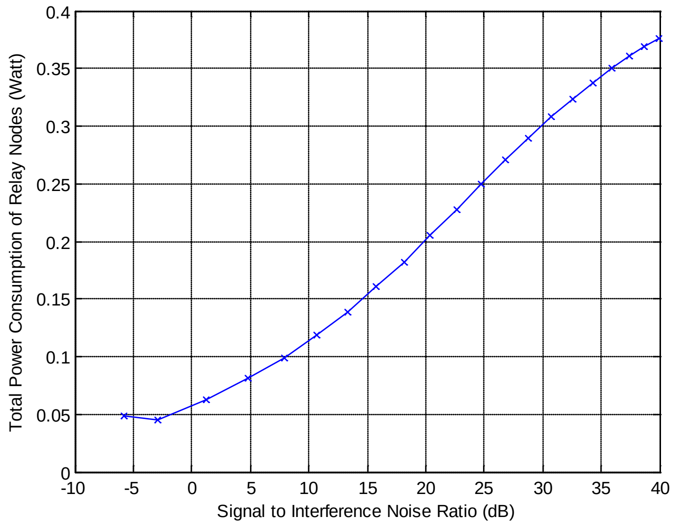

Figure 5-3 Total power consumption for different SINRs

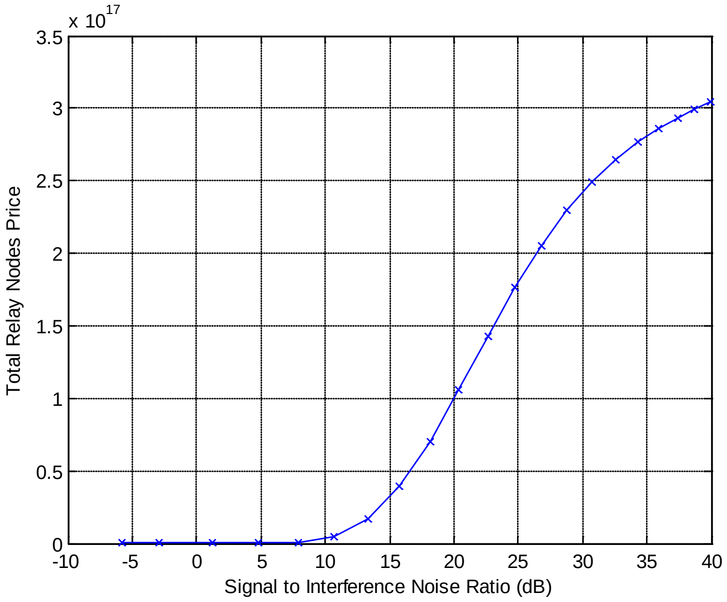

Figure 5-4 Total relay nodes price for different SINRs

The total relay node utility function $\sum_{i \in L} U_{r_i}$ increases with SINR as shown in Figure 5-5. The total utility function of the relay nodes $\sum_{i \in L} U_{r_i} = \sum_{i \in L} (p_i - c_i) P_{r_i}$ is increasing as the SINR increases, since both total price of the relay nodes $\sum_{i \in L} p_i$ and the total power consumption $\sum_{i \in L} P_{r_i}$ are increasing as the SINR increases.

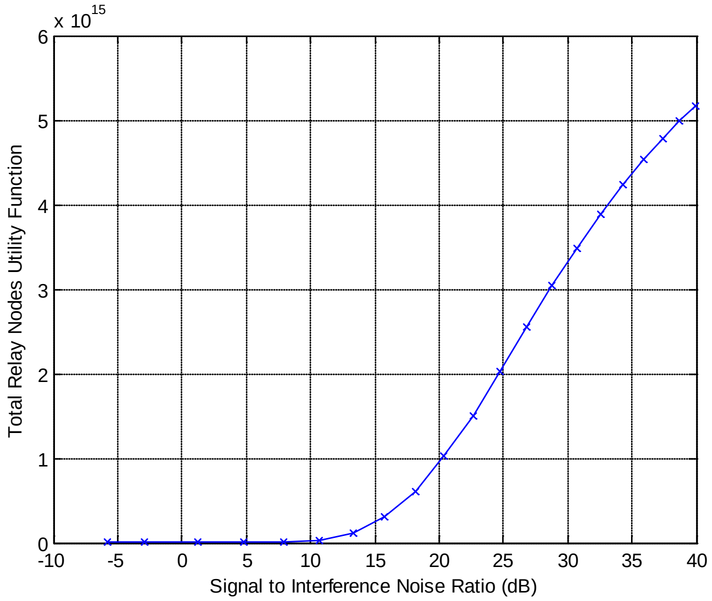

Figure 5-5 Total relay nodes utility function for different SINRs

The total transmission capacity $R_{s,r_i,d}$ for different SINRs is shown Figure 5-6. The transmission capacity is always increasing as SINR increases just like the total power consumption. From equation (4-10) we have $R_{s,r,d} = W \log_2\left(1+\gamma_{s,d}+\sum_{i\in L}\gamma_{s,r_i,d}\right)$, and from equation (4-6) $\gamma_{s,d} = \frac{P_s G_{s,d}}{\sigma^2+\sigma_{\text{int}}^2}$ and equation (4-7) $\gamma_{s,r_i,d} = \frac{P_{r_i} P_s G_{r_i,d} G_{s,r_i}}{\left(\sigma^2+\sigma_{\text{int}}^2\right)\left(P_{r_i} G_{r_{i,d}} + P_s G_{s,r_i} + \sigma^2 + \sigma_{\text{int}}^2\right)}$ we know that the total transmission capacity depends on the power consumption.

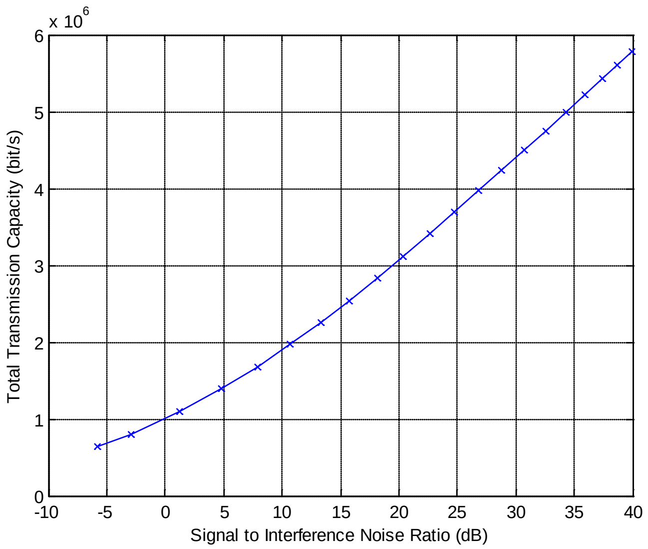

Figure 5-6 Total transmission capacity for different SINRs

In Figure 5-7, the source node utility function U_s for different SINRs is shown. Since according to equation (4-11) $U_s = aR_{s,r_i,d} - \sum_{i\in L} P_{r_i} p_i$, from the total transmission capacity $R_{s,r_i,d}$ shown in Figure 5-6 and the total price and total power in Figure 5-3 and Figure 5-4 we say that the U_s decreases as $\sum_{i\in L} p_i$ increases.

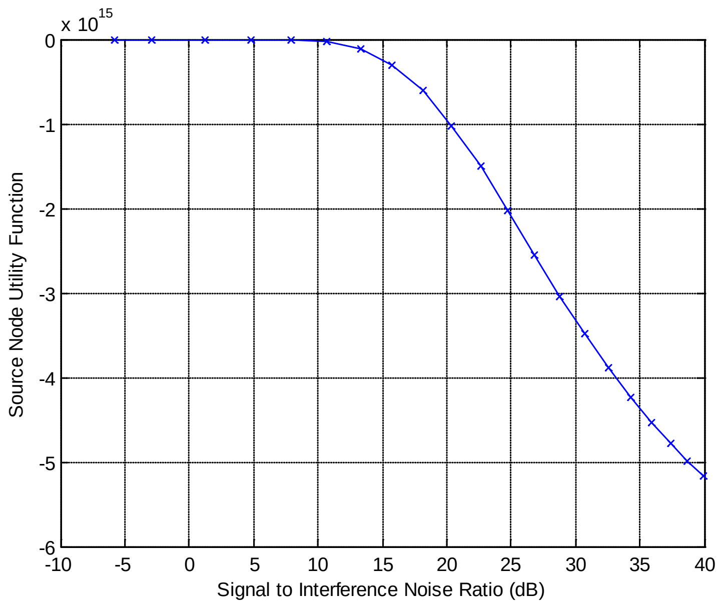

Figure 5-7 Utility function of the source node for different SINRs

We then apply the algorithm from Figure 4-2 to our system model. We consider two different *schemes* when simulating the algorithm to limit the number of selected relay nodes. In Scheme #1, we have 18 different relay nodes with price from 2 and increased linearly until 40. In scheme #2, all relay nodes have initial price distributed uniformly between 2 and 7. Then we consider three different *scenarios*. Our pricing game model provided in Section 4-2 is the scenario #1, our proposed algorithm for limiting the number of selected nodes is scenario #2, and scenario #X stands for an algorithm where the relay selection criterion in our pricing game model is performed after every iteration in the game. In Figure 5-8, we show that by using scenario #X, the number of relay nodes used decreases after certain number relays is available to the source node. This happens because the price of the relay nodes increases after each iteration according to the price updating function of the relay nodes shown by equation (4-27). The price updating function depends on $P_{r_i}^*$ and $\partial P_{r_i}^* / \partial p_i$ which in turn depends on the number of selected relay nodes. When the number of selected relays is high, the optimal price will become too high and the relay nodes become not beneficial anymore to the source node. In scenario #X, these relays are removed by the relay selection process. Thus, we know how much relay nodes that can be used.

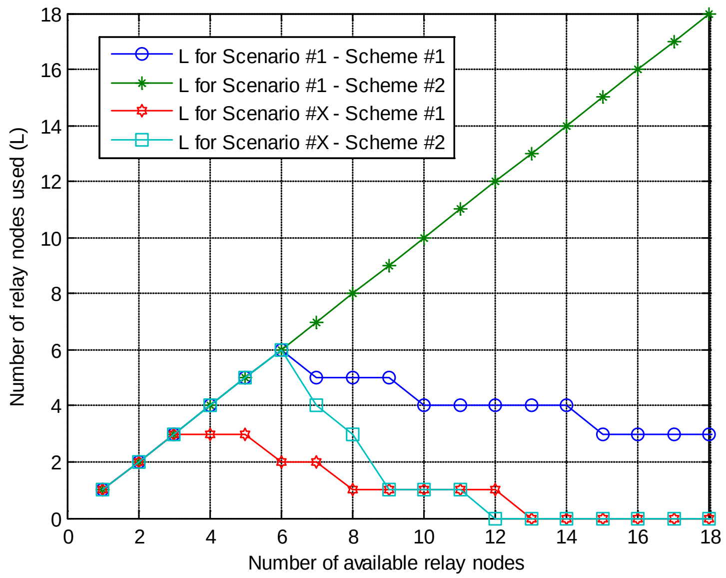

Figure 5-8 Number of relay nodes used versus the number of available nodes

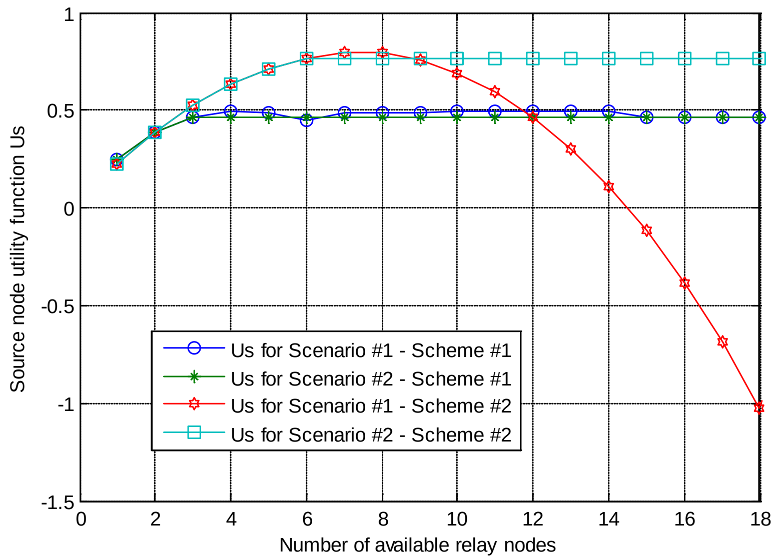

Figure 5-9 Utility function of the source node versus the number of available relay nodes.

In Figure 5-9 and Figure 5-10 we compare the performance of scenario #1 and scenario #2 in both scheme #1 and scheme #2. In Figure 5 we can see the optimum U_s for different numbers of available relay nodes. In scheme #1 the proposed method for limiting the number of selected relay nodes (scenario #2) has slightly lower U_s than the scenario #1 when the number of available relay nodes is between 3 and 15. In Figure 5-10 we can see that the total payment to the relay nodes in scheme #1 for the scenario #1 is higher than in scenario #2. For scheme #2, we can see in Figure 5-9 that the optimal U_s for scenario #1 starts decreasing when the available number of nodes is 9, and for scenario #2 it keeps steady until the number of available nodes is 20. This happens because when the number of relays is 9, the price of the relays increased more than the benefit that the source node gets. In scenario #2, the number of usable relay nodes is limited to 6, so it will keep a steady value even when the number of available nodes is larger. Scenario #2 is better since by limiting the number of the nodes, we can keep the source node utility function at a high level, which gives the source node more benefit. Lower payment also shows that resource usage is lower and the efficiency of the system is higher. These results show the usage of algorithm for limiting the number of selected relay nodes provided in Chapter 4-2.

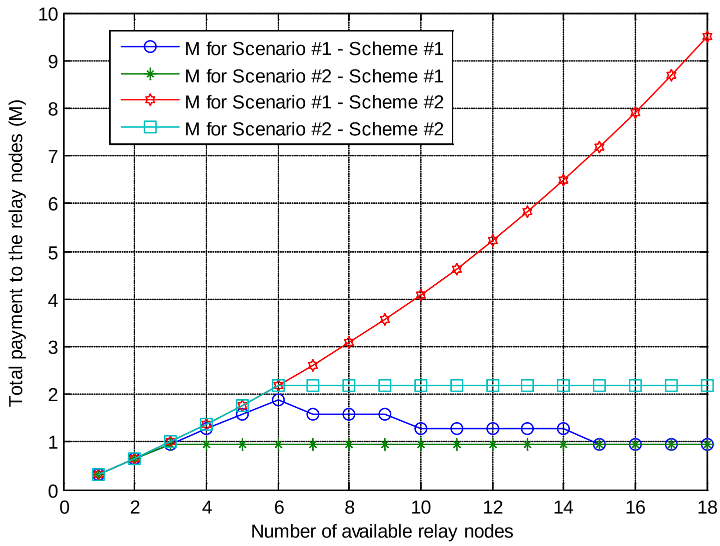

Figure 5-10 Total payment (M) to the relay nodes versus number of available relay nodes.

In the last part of the simulation, we simulate the evolutionary game optimization and compare it to the pricing game model that we have in Section 4-2. Here, we have six different scenarios. In all scenarios, we compare the values of source node utility function U_s, total relay nodes utility function $\sum_{i \in L} U_{r_i}$, total relay nodes power consumption $\sum_{i \in L} P_{r_i}$, total payment to the relay node M, and transmission capacity $R_{s,r_i,d}$ for different number of available relay nodes.

In the Scenario 1, we simulate the relay selection and resource allocation process based on game model we have on Section 4-2. Scenario 2 uses the same relay selection criterion, but uses evolutionary game for resource allocation. In Scenario 3-6, we use the same number of relays selected in the relay selection process using pricing game but we select relay nodes using SINR criterion, i.e. we select nodes with best SINR at the destination. In Scenario 3, the optimization process uses evolutionary game approach with maximization of transmission capacity $R_{s,r_i,d}$ as the objective. Scenario 4 uses maximization of both source node utility function U_s and relay node utility function U_{r_i} as the objectives of the optimization process. In Scenario 5 only U_s is used as the objective of the optimization process and in Scenario 6 only U_{r_i} is considered.

In Figure 5-11, we can see the source node utility function for Scenario 1. The source node utility function drops dramatically as the number of available nodes is high. In Figure 5-12 we can see the total relay nodes utility function. We can see from the figure that the total relay nodes utility function increases dramatically when the number of relay nodes is high. We already know that the price updating function of the relay nodes provided by equation (4-28) depends on the number nodes and from the analysis of the pricing game model we know that the price of the relay nodes dominates the U_s and U_{r_i} values. At high number of relay nodes, the price increases dramatically and affecting both U_s and U_{r_i} values.

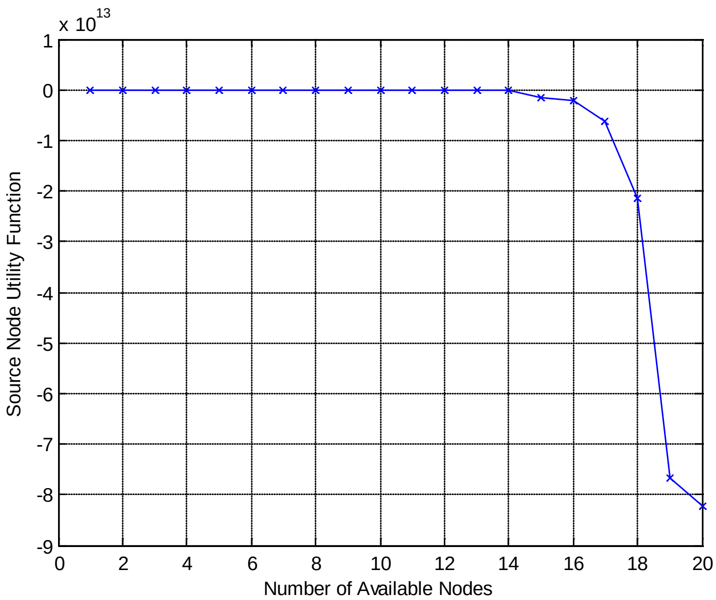

Figure 5-11 Source node utility function for Scenario 1

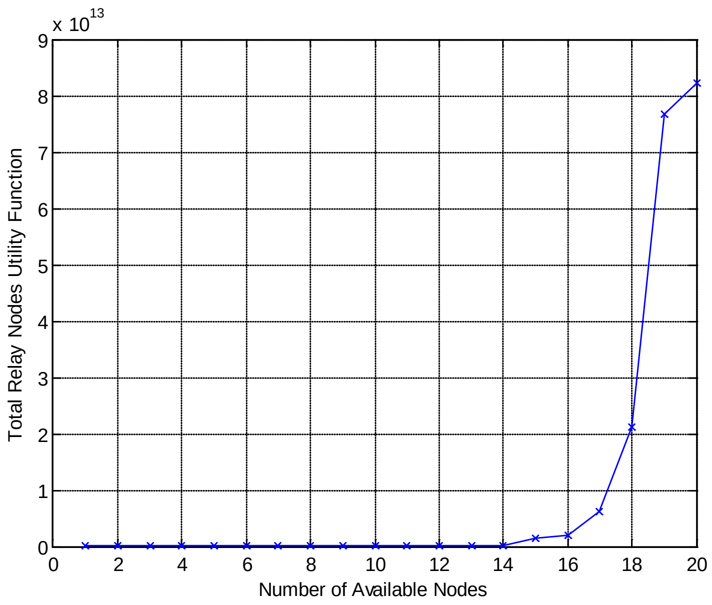

Figure 5-12 Total relay nodes utility function for Scenario 1

In Figure 5-13 we can see the source node utility function for different scenarios using the evolutionary game optimization and in Figure 5-14 we can see the total relay nodes utility function. As we can see from both figures, Scenario 4 which selects the relay nodes using SNR criterion and has U_s and U_{r_i} as the maximization constraints yields the highest value for the source node utility function but have the lowest value for U_{r_i}. Both Scenario 2 and Scenario 5 have the second highest value of U_s after Scenario 4. The optimization process with both utility functions as the constraint still yields higher result than the optimization process using only U_s as the constraint. We also can see that the scenario with only U_{r_i} as a constraint yields the highest value. These results show that the evolutionary game algorithm for resource allocation and optimization does not have the price of the relay nodes dominates the resource allocation process as in the pricing game model, so the relay nodes utility function is lower using this resource allocation and optimization process.

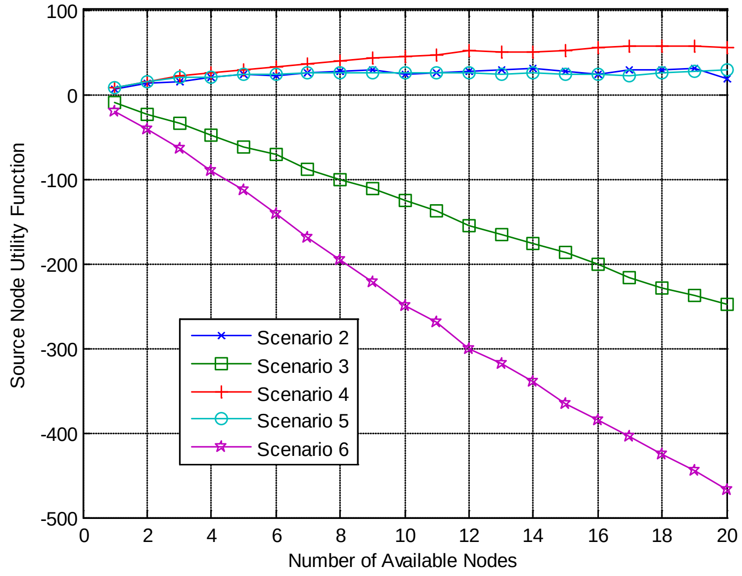

Figure 5-13 Source node utility function for different scenarios

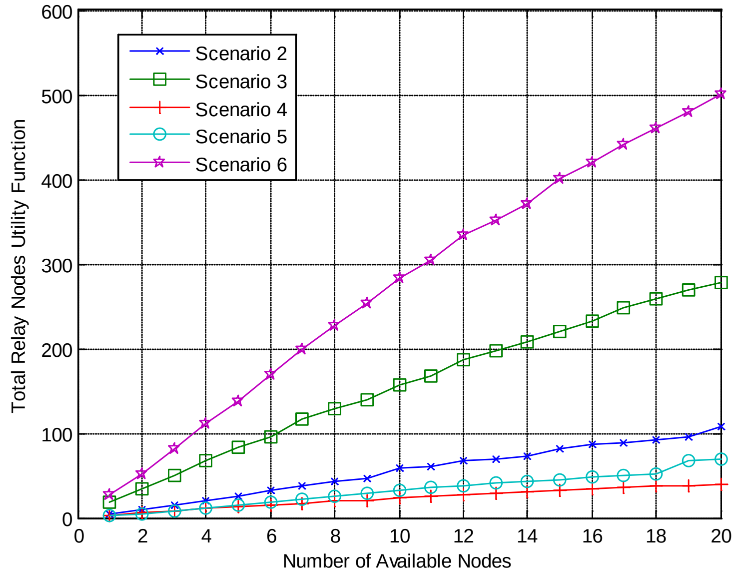

Figure 5-14 Total relay nodes utility function for different scenarios

In Figure 5-15 we can see the total power consumption for all scenarios and in Figure 5-16 we can see their corresponding transmit capacity. It can be seen from Figure 5-15 and Figure 5-16 that the pricing game model (Scenario 1 in both figures) consume lower power and have lower transmission capacity compared to Scenario 3 and Scenario 6 when the number of available relay nodes is low. When the number of available relay nodes is high, the power consumption and transmission capacity suddenly increased dramatically and become higher than other scenarios.

Scenario 2 yields an interesting result in these figures. We can see from Figure 5-15 that the total power consumption for Scenario 2 is lower than Scenario 3 and 6 at all number of available relay nodes, but in Figure 5-16 we can see that at high number of relay nodes, the transmission capacity also follows the Scenario 1 and become higher than Scenario 3 and 6. This result shows that the relay selection criterion using the game theoretical approach still yields better result than selecting the relay nodes based on SNRs even with different optimization process.

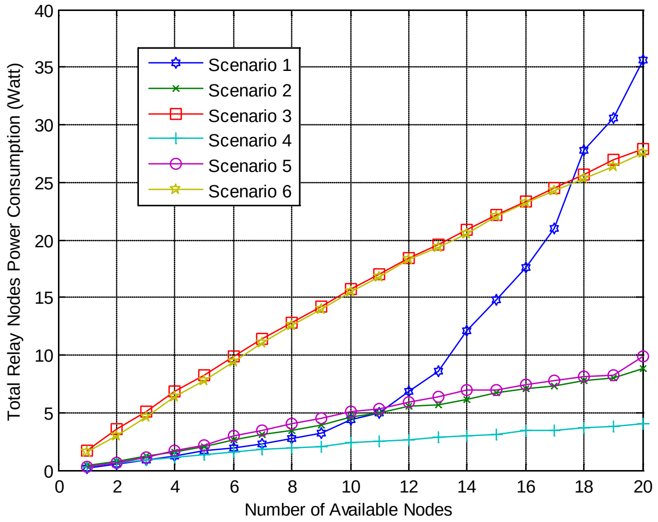

Figure 5-15 Total power consumption for relay nodes in different scenarios

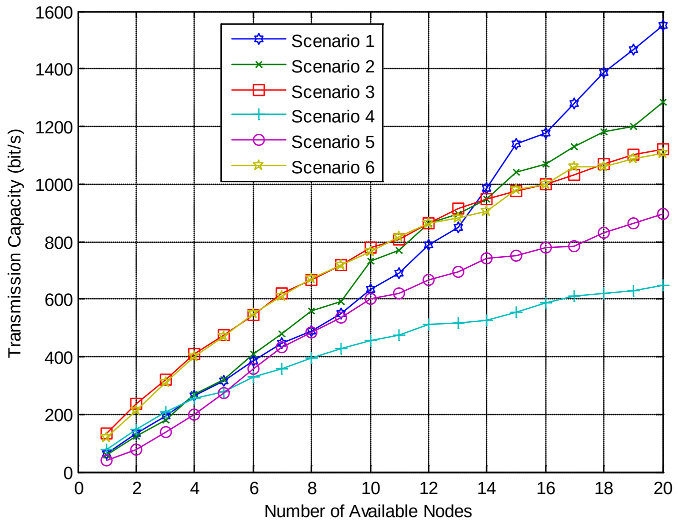

Figure 5-16 Total transmission capacity in different scenarios

From these simulation results, we have shown that the evolutionary game theory can be used for optimization process with comparable result to the game theoretical approach. At high number of relay nodes, the evolutionary game optimization even yields more fair result in the sense that the value of source node utility function U_s, the total relay nodes utility function $\sum_{i \in L} U_{r_i}$, and total payment M does not change drastically with the number of available relay nodes.

6 Conclusions and Future Research

6-1 Conclusions

In this thesis work, we proposed two methods for relay selection and resource allocation and optimization in cooperative wireless communication networks with interference with game theoretical approach. First, we proposed a pricing game for relay selection and resource allocation and optimization in cooperative communication networks with interference based on Stackelberg game. Simulation results show that interference in the cooperative communication network can change the relay selection and resource allocation and optimization result, and should not be neglected from the calculations so we can predict the behavior of the system in an environment closer to real world situation, compared to the case when only noise is considered in the system.

Then we found out that the pricing game for resource allocation and optimization, when the number of available nodes is high, can result in high payment to relay nodes which will highly reduce the source node utility function. Therefore, we proposed an algorithm to limit the number of selected relay nodes to mitigate this problem. Using this algorithm, we chose a number of relay nodes where the result of resource allocation and optimization is still beneficial for the value of source node utility function U_s.

We then proposed an evolutionary game based resource allocation and optimization for cooperative wireless communication networks with interference. Using this algorithm, we can easily change the objectives of the optimization process without having to change many parameters in the system. This game theoretical optimization process is also suitable for different relay selection criteria in the cooperative communication network. With the simulation efforts, we showed that the result from the evolutionary game based resource allocation and optimization are still in an acceptable range compared to the pricing game based on resource allocation and optimization at low number of available nodes. Through simulations, we also showed that the evolutionary based approach is better than the pricing game based resource allocation and optimization at high number of available nodes since the result from the evolutionary game based optimization when the number of available nodes changed from low to high is not changed drastically. We also showed in the simulation results that the relay selection criterion using the pricing game is comparably better than the relay selection based on best SNR at the destination.

6-2 Future Research

In the area of cooperative wireless communication, especially in relay selection and resource allocation and optimization using game theoretical approach, there are still many possibilities for further research. We would recommend designing a game model using destination node as the third player in the game since in the real world situation; the destination could have some interest in the relay selection and resource allocation process, and its objectives might be different from the source node and the relay nodes. The game will be played between the source node, the relay nodes, and the destination node, and relay selection criteria and resource allocation process will be based on their objectives.

Our second suggestion is to model a new relay selection method based on combination of different parameters which have not been exploited before such as bandwidth and power, price and multi-carrier, or time slot and location using the game theoretical approach. These relay selection criteria combined with the evolutionary game for resource allocation and optimization will possibly provide a closer solution to the real life problems in cooperative radio communication network.

Third, our suggestion is to perform research on the effects of parameters in the evolutionary game for resource allocation and optimization. In this thesis work, we use a constant value for price updating function when applying the strategy in the game part of the evolutionary game. We would recommend using a dynamic value to see its effects on the resource allocation and optimization process.

Another interesting research topic is the performance of the system in the CDMA network. In CDMA network, users can access the channel at the same timeslot by convoluting the signal with different codes. In this network, interference between users becomes factors which have to be calculated in power control. Game theoretical approach for power control in CDMA based cooperative communication could be a good research topic.

In this thesis work, we considered an AF scheme for the forwarding scenario in the cooperative wireless communication networks. The drawback of AF scheme is that the noise from source-relay node transmission is amplified at the relay node and then transmitted to the destination node. At low SNR, we will get a transmission error. Using DF scheme for forwarding might give better result at low SNR since the noise from source-relay node transmission is not forwarded to the destination node.

Last, we would like to suggest a research in the area of cognitive radio networks. In cognitive radio networks, cooperative spectrum sensing has been used to enhance the reliability of primary user detection. The game theoretical approach in relay selection and resource allocation of this thesis can be applied to the cooperative wireless communication systems in the spectrum sensing part of the cognitive radio networks.

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Based on this thesis research the following paper has been accepted for publication at European Microwave Conference 2011, October, Manchester, United Kingdom

W.A. Prasetyo, H. Lu, H. Nikookar, "Optimal Relay Selection and Power Allocation Using Game Theory fo Cooperative Wireless Networks with Interference", *to appear in EuMC 2011, Manchester, United Kingdom.*

Appendix 1

In Section 4-2, we provided a pricing game model based on Stackelberg game. In that chapter, we mentioned that the $P_{r_i}^*$ is the optimal solution for the source node utility function maximization problem and p_i^* is the optimal solution for the relay node utility function maximization problems. We also mentioned in Section 4-2 that the final solution from the optimization process is the equilibrium of the game. In this Appendix, we provide derivations which support our hypothesis in Section 4-2.

The $P_{r_i}^*$ provided in Section 4-2 which is the optimal solution for the equation

$$\frac{\partial U_s}{\partial P_{r_i}} = a\frac{\partial R_{s,r,d}}{\partial P_{r_i}} - p_i = 0, \quad i \in L \qquad \text{(A-1)}$$

requires U_s to be continuous and concave in P_{r_i}. Taking second order derivatives from the source node utility function U_s, provided that

$$\frac{\partial U_s}{\partial P_{r_i}} = \frac{aW}{\ln 2}\frac{A_i B_i}{\left(1+\sum_{k\in L}\left(\frac{A_k P_{r_k}}{P_{r_k}+B_k}\right)\right)\left(P_{r_i}+B_i\right)^2} - p_i, \qquad \text{(A-2)}$$

we can get

$$\frac{\partial^2 U_s}{\partial P_{r_i}^2} = -\frac{aW}{\ln 2}\frac{1}{\left(1+\sum_{k\in L}\left(\frac{A_k P_{r_k}}{P_{r_k}+B_k}\right)\right)^2}\left[\frac{A_i B_i}{\left(P_{r_i}+B_i\right)^2}\right]^2 - \frac{2aW}{\ln 2}\frac{A_i B_i}{\left(1+\sum_{k\in L}\left(\frac{A_k P_{r_k}}{P_{r_k}+B_k}\right)\right)\left(P_{r_i}+B_i\right)^3} \qquad \text{(A-3)}$$

since for each relay i, $W > 0$, $a > 0$, $A_i = \frac{P_s G_{s,r_i}}{\left(\sigma_{\text{int}}^2 + \sigma^2 + P_s G_{s,d}\right)} > 0$, $B_i = \frac{\left(P_s G_{s,r_i} + \sigma_{\text{int}}^2 + \sigma^2\right)}{G_{r_i,d}} > 0$,

and $P_{r_i} \geq 0$, we can safely assume that $\frac{\partial^2 U_s}{\partial P_{r_i}^2} < 0$. By definition, since the second derivative of the utility function U_s is always negative, we can say that U_s is concave in P_{r_i} . Since U_s is concave in P_{r_i} , the extreme point in U_s which solve the equation

$$\frac{\partial U_s}{\partial P_{r_i}} = \frac{aW}{\ln 2} \frac{A_i B_i}{\left(1 + \sum_{k \in L} \left(\frac{A_k P_{r_k}}{P_{r_k} + B_k}\right)\right)\left(P_{r_i} + B_i\right)^2} - p_i = 0, \quad \text{(A-4)}$$

is the maximum point. Thus, we can safely say that $P_{r_i}^*$ is the optimal solution for maximizing U_s .

Next we proof that the p_i^* in Section 4-2 is the optimal solution for the equation

$$\frac{\partial U_{r_i}}{\partial p_i} = P_{r_i}^* + (p_i - c_i)\frac{\partial P_{r_i}^*}{\partial p_i} = 0. \quad \text{(A-5)}$$

Since we have

$$\frac{\partial P_{r_i}^*}{\partial p_i} = \sqrt{\frac{A_i B_i}{p_i}} \frac{Y + \sqrt{Y^2 + 4X \frac{aW}{\ln 2}}}{2X} \left(-\frac{1}{2p_i} \left(1 - \frac{\sqrt{p_i A_i B_i}}{\sqrt{Y^2 + 4X \frac{aW}{\ln 2}}} \right) \right) \quad \text{(A-6)}$$

and

$$P_{r_i}^* = \sqrt{\frac{A_i B_i}{p_i}} \frac{Y + \sqrt{Y^2 + 4XW'}}{2X} - B_i \quad \text{(A-7)}$$

equation (A-5) can be expanded to

$$\frac{\partial U_{r_i}}{\partial p_i} = -B_i + \sqrt{\frac{A_i B_i}{p_i}} \frac{Y + \sqrt{Y^2 + 4X \frac{aW}{\ln 2}}}{2X} \times \left[1 - \frac{p_i - c_i}{2p_i} \left(1 - \frac{\sqrt{p_i A_i B_i}}{\sqrt{Y^2 + 4X \frac{aW}{\ln 2}}} \right) \right]. \quad \text{(A-8)}$$

Taking second order derivatives of (A-8) we will get

$$\frac{\partial^2 U_{r_i}}{\partial p_i^2} = \sqrt{\frac{A_i B_i}{p_i}} \frac{Y-\sqrt{p_i A_i B_i}}{2X}\left(1-\frac{\sqrt{p_i A_i B_i}}{\sqrt{Y^2+4X\frac{aW}{\ln 2}}}\right)\times\left(\frac{-p_i-3c_i}{4p_i^2}\right)+\frac{\sqrt{\frac{A_i B_i}{p_i}}}{8Xp_i^2\left(\sqrt{Y^2+4X\frac{aW}{\ln 2}}\right)^3}$$

$$\times\left[\left(\left(Y-\sqrt{p_i A_i B_i}\right)^2+2\left(Y-\sqrt{p_i A_i B_i}\right)\sqrt{p_i A_i B_i}+4X\frac{aW}{\ln 2}\right)^2\left(-p_i-3c_i\right)\right] \quad \text{(A-9)}$$

$$+\,p_i A_i B_i\left(\left(Y-\sqrt{p_i A_i B_i}\right)^2+2\left(Y-\sqrt{p_i A_i B_i}\right)\sqrt{p_i A_i B_i}\right)\left(-p_i-3c_i\right)$$

$$+\,p_i A_i B_i 4X\frac{aW}{\ln 2}\left(-4c_i\right).$$

and since $A_i > 0$, $B_i > 0$, $p_i > 0$, $c_i > 0$, $a > 0$, $W > 0$, $X = 1+\sum_{j\in L} A_j > 0$, and $Y = \sum_{j\in L}\sqrt{p_j A_j B_j} > 0$, then $\frac{\partial^2 U_{r_i}}{\partial p_i^2} < 0$. Then we can safely assume that U_{r_i} is concave in p_i, thus p_i^* which is the solution to

$$\frac{\partial U_{r_i}}{\partial p_i} = P_{r_i}^* + \left(p_i - c_i\right)\frac{\partial P_{r_i}^*}{\partial p_i} = 0$$

is the maximum point.

After we have both solution to maximize both U_s and U_{r_i}, we define the equilibrium of the game as $P_{r_i}^{eq}$ and p_i^{eq} which fulfill

$$U_s\left(\left\{P_{r_i}^{eq}\right\}\right) = \sup_{P_{r_i}\geq 0} U_s\left(\left\{P_{r_i}\right\}\right) \quad \text{(A-10)}$$

and

$$U_{r_i}\left(\left\{p_i^{eq}\right\}\right) = \sup_{p_i\geq 0} U_s\left(\left\{p_i\right\}\right). \quad \text{(A-11)}$$

The supremum of both utility functions mean that we have to find the value of P_{r_i} and p_i which is the least value for $P_{r_i}^*$ and p_i^* in the domain of optimal solution. At the convergence point of the game, where $P_{r_i}^*(t+1) = P_{r_i}^*(t)$ and $p_i^*(t+1) = p_i^*(t)$, t is the iteration number in the game, the $P_{r_i}^*$ and p_i^* is a point where neither the source node or the relay nodes can increase their utility function without having other players change their strategies. Thus, $P_{r_i}^{eq}$ and p_i^{eq} are the equilibrium of the game.