Passenger-centric robust timetabling in railways

A case study for the Eindhoven-Den Bosch-Tilburg network

TIL Master Thesis

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by

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Project Period:	April 2024 - August 2024
Defence Date:	28 August 2024
Faculty:	Civil Engineering and Geoscience

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Contents

List of Figures

List of Tables

1 Introduction

1.1. Background

Rail transportation is pivotal in the logistics and passenger transport sectors in many countries globally. In 2018, approximately 4% of the total freight mass in the Netherlands was transported by rail. Additionally, trains accounted for 13% of the total distance travelled by individuals during the same period, making it the second most commonly used mode of transportation by distance. (ProRail, 2021)

The railway offers many benefits that make it a highly preferred transportation option today. One of the most important benefits is its high energy efficiency. Railway transportation consumes significantly less energy than its main competitor, road transportation. Recent technological innovations, such as regenerating braking energy and driver advisory systems, have enhanced rail's energy efficiency (Forward, n.d.). In addition, rail transport can efficiently move large numbers of people and goods compared to road-based transportation.

From the perspective of passenger services, the fixed train schedules and the predictability of travel times make rail transport a favoured option for medium- to long-distance journeys. However, the punctuality of railway transportation, a significant concern for passengers, frequently faces threats. This problem becomes more pronounced when various potential disturbances occur, such as extreme weather, mechanical failures, or other unforeseen events, leading to propagated delays. The largest railway operator in the Netherlands, Nederlandse Spoorwegen (NS), transported about 1 million passengers daily in 2023. During the same period, approximately 5,500 disturbances and disruptions occurred, equivalent to an average of 15 per day[1]. Some organisations and individuals are also actively campaigning to set punctuality standards and strengthen regulations to urge railway operators to focus more on punctuality(Banverket, 2005), but these efforts have yielded little success.

In the academic field, the punctuality problem in the railway system is interpreted more thoroughly. Palmqvist and Kristoffersson, 2022 points out the frequency and the severity of running time and dwelling time delays are directly related to punctuality. On the one hand, railway operation companies and staff need to apply a punctuality improvement method system to reduce the delay of trains (Aquilani et al., 2017, Veiseth et al., 2011). From the planning perspective, the railway system's robustness directly impacts its ability to handle delays, affecting train operations' punctuality. For passenger transportation, robustness is defined by Dewilde, Sels, Cattrysse, and Vansteenwegen, 2011 as *A railway system that is robust against the daily occurring small disturbances minimises the real weighted travel time of the passengers.* In this regard, Magnanti and Wong, 1984, García-Archilla et al., 2013, and Friesen et al., 2023 approach the problem from a strategic level, investigating the problems of robust network design for railway infrastructure under capacity constraints and uncertain timetabling. At the tactical level, Fioole et al., 2006, Hoogervorst et al., 2020, and Grafe et al., 2022 consider the problems of robustness and passenger delay management from the perspective of rescheduling rolling stock. Many researchers have focused their studies on robust timetabling in railways. This may be at-

[1] https://www.rijdendetreinen.nl/statistieken/2023

tributed to timetabling being positioned at an appropriate planning stage, which is neither as remote from actual operations as network design nor as constrained in scope for changes like rolling stock and crew scheduling, where the potential for adjustments is quite limited. (Lusby et al., 2018)

Generally, robust railway timetabling research can be categorised into two main types: operator-centric timetabling and passenger-centric timetabling. Kroon et al., 2008 and Högdahl and Bohlin, 2023a consider the problem from a macroscopic level and design a robust timetable to minimise the impact of weighted train delay during exceptional events. Solinen et al., 2017a and Solinen et al., 2017b evaluate the robustness in the micro way. Further on, Bešinović et al., 2016 describes an integrated iterative micro-macro approach to computing a conflict-free, stable, and robust railway timetable. However, research on the problem of passenger-centric robust timetabling is quite scarce. Sels et al., 2016 considers robustness one of the evaluation criteria in designing railway timetables with minimised passenger travel time. Still, robustness has not been the point of attention in the research. As Cacchiani and Toth, 2012 has noted, transport efficiency is the primary concern of both operators and passengers; however, the robustness cannot be overlooked. Especially for the served passengers, the delays they experience are more intuitive compared to operational delays. Therefore, this study will focus on the problem of passenger-centric robust railway timetable design, aiming to fill the current research gap.

Inserting appropriate time supplements and buffer times into the original timetable to reduce delays and their propagation can improve punctuality. However, appropriately distributing the available supplement and buffer time is a challenging task, especially in real-world situations, because a very robust timetable may not necessarily be a good choice, as ideal punctuality might come at the cost of large time slacks.(Lee et al., 2017) The trade-off between the robustness and efficiency of the timetable cannot be ignored. Therefore, this research will analyse the efficiency of the newly designed timetable in a case study.

1.2. Problem statement

The research aims to design a periodic robust timetable in railway networks and encourage operators to give greater consideration to passenger delays when disturbances occur and propagated delays appear. This can be achieved by answering the following main research question:

How to design a high-quality passenger-centric robust periodic timetable for railways?

This research question can be decomposed into the following sub-questions:

1. How to define the robustness of a timetable in a passenger-centric way?
2. How to incorporate passenger behaviour into a robust timetabling method?
3. How to evaluate the quality of the new-designed timetable?

The research overview in Figure 1.1 is designed to address the aforementioned research question and sub-questions. The research plan can be divided into 4 phases and begins with the literature review. In terms of models, laying the foundation for subsequent modelling of trains and passengers is essential.

In the second phase, some data preparation will be done. Firstly, the original timetable and passenger demand data need to be structured into an event-activity and network format, which helps to represent the network in modelling. Furthermore, the structured data can be closely linked to passenger path set generation. It is defined with all possible paths between target origin-destination(OD) pair without foolish transfer (defined in subsection 4.2.2), ensuring the model's applicability and the results' quality in robust timetabling. This preprocessing undertakes various critical roles throughout the research.

The core modelling phase is structured as follows: the train timetabling model lays the foundation of the whole process, ensuring the final result can meet the basic requirements of safe railway operation. In the real world, passengers with different destinations and arrive-at-origin times may choose different travel paths. The passenger model helps to determine their choices and assign different passenger groups to the network based on the characteristics of each path. The robust optimization model consists of train and passenger simulation models. The former gives the realisation of train delay propagation under disturbance scenarios. At the same time, the passenger simulation contributed to evaluating the performance of the newly designed timetable in these delay scenarios and determining the best result under the setting objective.

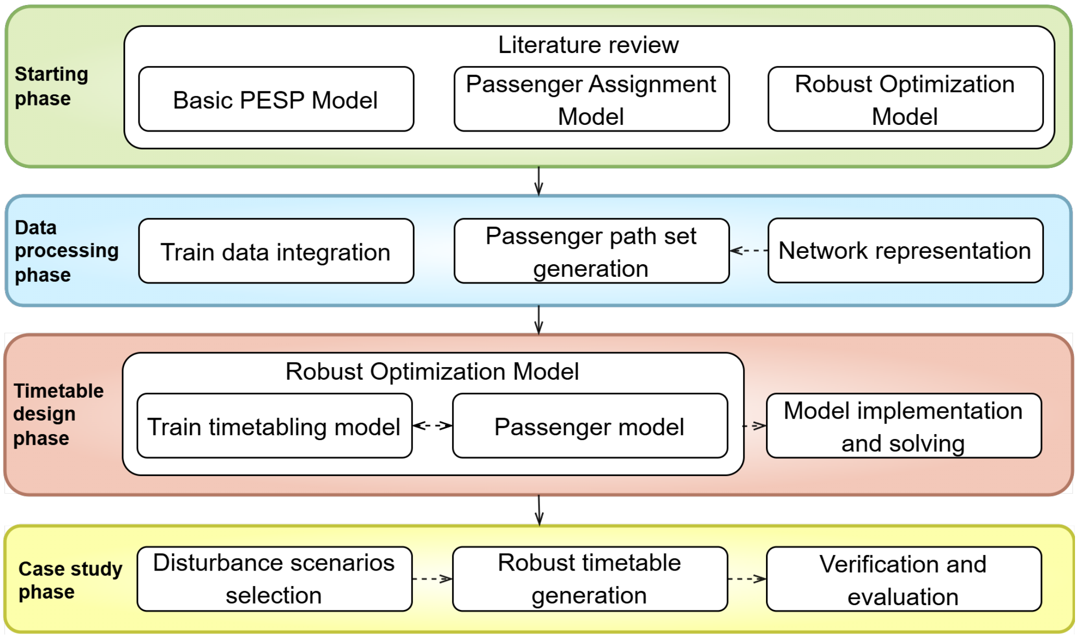

Figure 1.1: Research Overview

Finally, a case study will be conducted focusing on the Eindhoven-Den Bosch-Tilburg network in the Dutch railway system. This study will incorporate network data, scheduled timetable information, and passenger travel demand data into the model. In section 6.3, the designed passenger-centric robust timetable (PRT) generated by this model will then be thoroughly evaluated and compared with the evaluation results of the original timetable (OT) and operator-centric robust timetable(ORT). Another study (section 6.4) discusses the model's performance under disturbance scenarios with different intensities.

1.3. Definition of robust timetable

Timetable robustness is the ability of a timetable to withstand design errors, parameter variations, and changing operational conditions (Bešinović et al., 2016). According to Lusby et al., 2018, robustness, in general, is the capacity of some systems to absorb or resist changes. In this research, the robustness of a railway timetable can be defined as the ability of a timetable to handle small disturbances in the railway system. In addition, in railway robust timetabling, the word 'disturbance' is formally defined as human mistakes, malfunctions, or deviating conditions in the railway system or abnormal operating environments that may influence railway traffic. (Shafia et al., 2012) In this section, the evaluation indicator of timetable robustness, especially in the passenger-centric way mentioned in section 1.2, will be discussed. This is crucial as it will determine the objective function of the final robust timetabling model.

In the operator-centric robust timetabling problem, Kroon et al., 2008 tells that the robustness improves by reallocating the slack in the timetable to minimise the average delay of trains. In this definition, the minimal delay of a train is associated with robustness. Shafia et al., 2012 uses minimising the impact of uncertainties in train occupation times for the block sections as the objective function for robust optimisation at the microscopic level. All these cases, considering an operator-centric objective function, admit that an empty train that reaches its terminus with a 7-minute delay is worse than the situation where a crowded train that only has three 3-minute delays causes half of its passengers to miss a connection. But in fact, from the perspective of transportation services, the passenger is the 'king'. Passenger delay should be heavily considered as the evaluation standard for timetable robustness when disturbances occur.

When considering robustness from a passenger's point of view, it is better to think about the average delays of the passengers instead of trains.(Dewilde, Sels, Cattrysse, and Vansteenwegen, 2011) Furthermore, passengers waiting for trains at the station or sitting inside the carriages do not clearly know the reason for delays. Therefore, a better approach is to focus on the total delay experienced by passengers during the completion of their journey. Dewilde, Sels, Cattrysse, and Vansteenwegen, 2011 proposes an indicator called Perceived Extra Waiting Cost (PEWC) to more accurately depict passengers' varying perceptions of time in scenarios of waiting for transfers, waiting at stops, and failed transfers, and assigns different weights accordingly. Dewilde et al., 2014 considers a railway system that is robust against the daily occurring small disturbances, which minimises the real generalised travel time of the passengers. Sels et al., 2016 uses passenger generalised travel time as the main goal in the automated construction of a robust railway timetable. This provides a reference for the definition of passenger-centric robustness and the final optimisation objective in this paper.

In summary, in this research, a railway timetable is robust when *it can help mitigate the additional generalised travel time experienced by passengers when small disturbances occur*.

1.4. Model structure

This thesis aims to design a PRT, and the mathematical model will be based on a simulation model. In chapter 2, the relevant literature will be listed and discussed. The structure of the model can be divided into three core parts (chapter 3-chapter 5), Train timetabling model, Passenger model, and Simulation model, as shown in Figure 1.2.

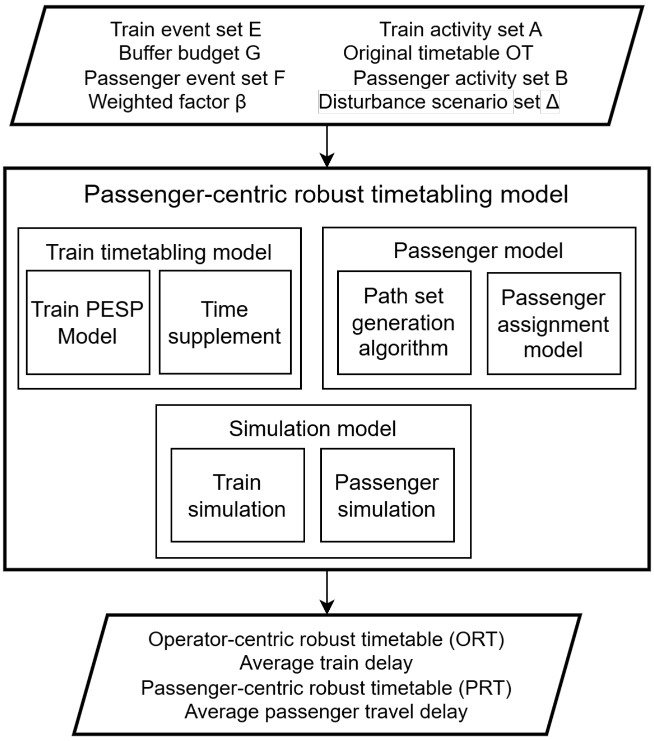

Figure 1.2: Model structure

The train timetabling model utilises a train Periodic Event Scheduling Problem (PESP) scheme to ensure train timetable feasibility, such as restricting the upper and lower bounds of train activity. It will be discussed in chapter 3. Besides, constraints on the time supplements budget and adjustment window compared to the original timetable will be introduced in section 3.3. Another model, shown in chapter 4, is about generalised travel time defining and calculation and passenger assignment, and it is more complex than the PESP model. The simulation model described in chapter 5 consists of two sub-models. One sub-model is used for the train, and the other is used for the passengers. These sub-models help to evaluate the robustness of the newly designed timetable under different disturbance scenarios. The input and possible output are given in Figure 1.2.

Although the different components of this model operate relatively independently, they are clearly defined and interconnected in order to achieve the goal of optimisation. The train timetabling model continuously proposes new feasible timetables. The passenger model calculates the generalised travel time for passengers based on the new timetable and assigns passengers to the railway network. Within the simulation model, the train simulation component simulates train delays and delay propagation in the new timetable under the disturbance scenarios. The delay information from train events is then translated into realised generalised travel time through the passenger simulation component, combining the planned passenger assignment results and travel time to obtain the passenger delay. This final average passenger delay serves as the indicator to evaluate the quality(passenger-centric robustness) of the new timetable generated by the train timetabling model. It continuously feeds results back to the model, encouraging the generation of timetables with smaller average passenger delays.

Table 1.1: Possible combination of sub-models

Input	**Model combination**	**Output**
Passenger event set F Passenger activity set B Original timetable OT Weighted factor β Disturbance scenario set Δ	Passenger model Simulation model	Realised selected path under OT Generalised passenger travel time under OT Average passenger delay under OT ......
Train event set E Train activity set A Time supplements budget G Original timetable OT Disturbance scenario set Δ	Train timetabling model Simulation model	Operator-centric robust timetable (ORT) Realised selected path under ORT Generalised passenger travel time under ORT Average passenger delay under ORT Average train arrival delay under ORT ......

In Table 1.1, the sub-model combinations required for evaluating the original timetable and generating ORT and each model's potential inputs and outputs are displayed. These two processes are designated as validation cases and will be discussed in detail in section 6.3. The original timetable already exists in reality; thus, it meets all the constraints of the PESP, and all train and passenger event times are fixed. Consequently, real-world information is fed into the passenger-centric model to generate path sets, define passenger generalised travel times, etc. These results are then utilised as new inputs in the simulation model to determine the actual chosen path for each group of passengers, generalised travel times, and average passenger delays in the event of disruptions. According to Kroon et al., 2008, the train PESP model and the train-related components of the simulation model will be used to generate ORT. The simulation part aims to minimise the average train arrival delay. The train arrival delay is the difference in delay between the realised and scheduled train arrival events within the railway network across all scenarios. The generated ORT will be analysed using the same process as the original timetable, which can yield corresponding analytical results. Detailed applications of the models will be further elaborated in chapter 6.

2 Literature study

Based on the research questions outlined in section 1.2 and the subsequent overview, the primary challenges revolve around the PESP Model, Passenger Assignment Model, and Robust Optimization Model. These are crucial components in creating a resilient timetable focused on passengers. Many researchers have extensively studied these topics from different angles. This chapter will introduce the study's contributions and summarize the methods and insights that can be used. Section 2.2 introduces the development of the PESP model and its applications. Section 2.2 discusses various approaches to assigning passengers to transport networks. Section 2.3 compares the robust optimization model.

2.1. Periodic Event Scheduling Problem Model

Serafini and Ukovich, 1989 first proposes a general framework for solving PESP and applies this concept to address issues in traffic light scheduling. The objective of the PESP model is to identify a feasible event timetable within a given cycle T and a series of events E, under the constraints A. Inspired by this model, since the 1990s, transportation scholars (Schrijver and Steenbeek, 1993, Nachtigall, 1994, Odijk, 1996) have started to preliminarily apply it to the formulation of periodic railway timetables and explore viable solution approaches.

Subsequent research in the railway sector refines and expands the model further, attempting to apply it to various scenarios. L. G. Kroon and Peeters, 2003 introduces an extension for variable travel times, significantly enlarging the solution space of the model because small deviations from the fixed trip times are allowed. Liebchen and Möhring, 2007 elaborates on the PESP model by more precisely defining primary safety requirements and more complex constraint settings, such as fixed events, bundling of lines, and train coupling. The capacity and limitations of the model are also detailed in his article. L. G. Kroon et al., 2014 introduces a dynamic passenger activity extension to the PESP model, allowing the model to choose which trains should connect in terms of rolling stock or passenger connections.

The PESP model has found applications beyond optimizing railway travel services in recent years. Wang et al., 2022 improves regenerative energy and passenger satisfaction on the basis of solving PESP. Masing et al., 2023 expands the PESP model's headway constraints to separate activities instead of events to study track choice under rail construction scenarios. Besides, Huang et al., 2023 establishes a generalised cyclic railway timetabling and rolling stock circulation planning model considering passenger demand for a Chinese suburban railway line.

2.2. Passenger Assignment Model

Initially, grounded in utility theory, Ben-Akiva, 1985 introduces the Random Utility Model (RUM), which has been extensively applied to transportation models for infrastructure investment and operational planning to quantitatively forecast passengers' mode choices and assign the passenger to the transportation network.

Passenger route selection is typically determined by the least generalised cost, incorporating factors such as in-vehicle time, waiting time, walking duration, the number of transfers, and ticket price. Based on this reality and assuming that the common lines problem (Chriqui and Robillard, 1975) is not taken into account, Spiess and Florian, 1989 develops a label-setting algorithm to solve the passenger assignment problem where passengers' waiting times at stations depend solely on the frequency of transit service combinations within polynomial time, and Tong and Wong, 1999 employs a Monte Carlo approach to address the issue of heterogeneous passenger dynamic assignment.

As modern transportation network structures and passenger compositions become increasingly complex, scholars attempt to integrate additional constraints and real-world scenarios into existing passenger assignment models while refining solution methods. Nguyen et al., 2001 introduce the concept of available capacity and consider the implicit first-in-first-out(FIFO) mechanism of utilizing transit lines. This approach incorporates the flow on each route when addressing the passenger assignment problem. Hamdouch et al., 2004 recommend employing strategies to characterize user behaviour, whereby at each boarding node, passengers rank transit lines by their preference in descending order and board the first line on this list with available capacity. In this case, Hamdouch and Lawphongpanich, 2008 introduces a methodology that utilizes successive averages for its iterations, and during each iteration, it formulates strategies by solving a dynamic program.

The evolution of the foundational model has enabled its integration with other pertinent issues, with Gentile and Nökel, 2016 categorizing passenger assignment models for transit systems into schedule-based and frequency-based models. The distinction lies in whether passengers rely on timetables for path selection. Frequency-based models are more commonly applied to urban transit networks like subways and buses with fixed frequencies, whereas schedule-based models align more closely with the focus of this study. Zhu and Goverde, 2017 summarizes and analyzes the scheduled-based passenger assignment model during railway system disruptions and, based on this, examines the effects of information interventions on the passenger distribution process (Zhu and Goverde, 2019). Furthermore, Wang et al., 2022 considers the passenger assignment problem under the strategy of incorporating regenerative energy.

2.3. Robust Optimization Model

In the field of transportation, methods used for studying the robustness of timetables include the Max-plus Model (De Kort, 2000, Goverde, 2010) and analytical models (Higgins and Kozan, 1998, Huisman and Boucherie, 2001), which are known for their efficiency and interpretability respectively. However, their essence lies in being evaluation models, meaning that optimization of timetable robustness can only be achieved through trial and error. L. G. Kroon et al., 2007 describes a stochastic optimization model capable of modifying a given periodic timetable and evaluating the modified timetable by simulating many realizations of the trains in the timetable. Furthermore, Kroon et al., 2008 introduces a method for achieving robust optimization. On this basis, Khan and Zhou, 2010 develops a stochastic optimization formulation for incorporating segment travel-time uncertainty and dispatching policies into a medium-term train-timetabling process.

As research into railway robust optimization models has progressed, models such as Light Robustness (LR) (Fischetti et al., 2009), Delay Resistant Timetabling (DRT) (Liebchen et al., 2010), Event Flexibility (EF) (Caimi et al., 2011) have been successively proposed. Compared to the Stochastic Optimization Model, these models struggle to simultaneously address Stochasticity and deal with an increase in complexity (Caimi et al., 2017). Consequently, the Stochastic Optimization Model has naturally become the core model of this study.

Högdahl and Bohlin, 2023b proposes a combined simulation-optimization approach for double-track lines, which generalizes previous work to allow full flexibility in the order of trains by including a new and more generic model to predict delays. From the energy-saving perspective, Xie et al., 2021 and Ji et al., 2024 consider the variated passenger demand and maintenance window as different scenarios accordingly and apply the stochastic optimization model to design and adjust the timetable. Gemander et al., 2023 considers and extends the model under uncertain dwell and running times. Despite these studies not considering robust timetabling research, they can provide a reference for multi-objective optimization modelling based on the Stochastic Optimization Model.

2.4. Conclusion

Through the review of the literature, it can be understood that the PESP model is a crucial scheme for railway timetabling, especially within the context of the Dutch periodic railway system. This model has undergone several generations of transformation from its proposal to improvement and subsequent practical application. It supports linking train events and activities with those of passengers and facilitates the modelling of passenger-related parameters and properties. Notably, the connection between passenger and train activities has been achieved in L. G. Kroon et al., 2014. In terms of the passenger assignment model, the passenger path choice model often relies on the Random Utility Model (RUM) based on Ben-Akiva, 1985, with the utility's influencing factors typically including in-vehicle time, waiting time, walking duration and the number of transfers. This model can determine the probabilities of passengers choosing various path combinations during their travels. This project will use the minimization of generalised travel time to simplify the passenger path selection process. It is assumed that the path choice of a homogeneous group of passengers is unique and fixed, opting for the path combination with the minimum generalised travel time. Compared with the frequency-based model, the schedule-based model is more suitable for the timetable formulation of Dutch railway transportation services, where departure frequencies are variable. In studies related to multi-objective timetable optimization for passenger allocation, researchers tend to adopt such methods to preliminarily determine a set of passenger paths and assign passengers based on the variation of the factors in the optimization model. The Robust Optimization Model is proposed by Kroon et al., 2008 and applied to the design of robust railway timetables. This approach has been esteemed in subsequent research due to its generality in modelling, especially in studies of timetable performance reactions to specific indicators under various scenarios.

According to current research findings, studies focusing on designing robust railway timetables from the passengers' perspective are relatively scarce. Most existing studies approach the concept of a PRT in an evaluative way. In summary, integrating the three models is feasible to address the central issue of this study, but this research requires adjustments and improvements to the existing models.

3 Train timetabling model

From the perspective of railway operators, running, dwelling, and turnaround activities are the main activities in daily railway operations. At a macroscopic level, the headway constraint between adjacent trains with the same infrastructure elements is one of the essential safeguards for safe train running. Modelling the railway network in an event-activity format to describe these activities provides an efficient and convenient method for addressing timetabling and rescheduling problems.

In chapter 3, before starting modelling, section 3.1 gives some assumptions on the train timetabling model. Section 3.2 introduces the train events and activities in this research. This section details the specific actions of trains, such as departures, arrivals, and their associated operational activities. Furthermore, section 3.3 discusses the corresponding variables and constraints in train operating feasibility.

3.1. Model assumptions

This research aims to modify and improve the current timetable rather than creating an entirely new timetable. This is because the design process needs to consider various real-world factors, such as railway and station capacity, the size of the train fleet, crew scheduling issues, and time allowances allocations. Given these considerations, certain assumptions have been incorporated into the design:

1. No train will be cancelled, even though a huge delay happens during its operation.
2. The train order remains consistent in a cycle, and no overtaking is allowed in the robust timetable.
3. Every train has enough capacity to accommodate all passengers without considering the issues of train capacity and passenger priority.
4. The railway network is formulated at the macroscopic level. The model's scope in this thesis does not address micro-level aspects, such as constraints within block sections.

Due to the assumptions, subsequent trains will experience secondary delays propagated from the preceding delayed train unless the supplement mitigates these delays. The delayed process will be introduced in chapter 5 by the train simulation model. For the second assumption, in a robust timetable, the sequence of train events within a cycle remains unchanged relative to the original timetable. This means that events at the end of the cycle cannot have their scheduled times moved to the beginning of the cycle by delaying their event times, and vice versa.

3.2. Train network representation

From the perspective of trains, the operational activities within a given railway network primarily occur between stations and on existing railway lines. In the mathematical model, these activities connect the events of a train arriving at and departing from a specific station. Indeed, dwelling and turnaround activities are linked to the arrival and departure events within the station. In this section, 3.2.1 and 3.2.2 introduce the sets of train events and activities in the network representation, respectively.

3.2.1. Train events

The event set for trains is denoted by E, and two adjacent connected events are denoted by i and j. There are mainly two kinds of events: departure and arrival. In addition, besides the passenger stations, there are junctions between certain stations where infrastructure constraints also exist. Therefore, to better explain and illustrate the infrastructure constraints, these two types of events are further subdivided into arrival and departure events at stations, which are contained in sets Ea and Ed. On the other hand, entry and exit events at non-station junctions are contained in sets Ea' and Ed', respectively. So, it has $E = Ea + Ed + Ea' + Ed'$.

For each event i, o_i is the original scheduled time, and x_i is the new-designed event time. Other characteristics of events, such as event ID, train ID, station, and direction, are also given in the event set, which can be used to map the connection between events and activities. Additionally, the train set is denoted as N, and the train serving a particular event can be identified by $t(i)$. Each train with a different itinerary is an individual element in the set, and two trains running on the same route but in opposite directions are also considered different trains.

3.2.2. Train activities

Train activities are the bridges between 2 connected train events $i, j \in E$, denoted by $(i, j) \in A$. This research considers four kinds of activities related to trains. They are running activity Ar, dwelling activity Ad, headway activity Ah and turnaround activity At.

Running and dwelling activities are derived from the existing operating plan. The former explains the train running from one station to the next station, and dwelling activity represents the train staying in the station and waiting for passengers to board and alight. Sometimes, not all trains stop at every station along their route, like the Intercity (IC) train in the Netherlands. In these cases, there are no scheduled events for this train at the non-stop stations. Also, some trains may pass through junctions where one end of the running activity is connected to a non-station arrival or departure event within the junction. When inside the junction, the train naturally does not make any stops, so no corresponding scheduled activities are generated for them. Note that running and dwelling activities connect the events with the same train ID.

Headway constraint ensures the adjacent trains have suitable headway and the system can operate safely. The headway activity only connects events of the same type (departure or arrival) at the same station but with different trains. Sometimes, trains travelling in different directions can create headway conflicts, especially when conflicting trains use the same switches or tracks. This usually results in non-overlapping time windows for trains in this junction section, with a few minutes of separation between them. As a result, there may be headway constraints between events in the non-station junction event set Ea' and Ed', which result from infrastructure conflicts involving these conflicting trains within the conflict area.

The same train can be assigned to different routes and directions in the railway system. The turnaround activity represents the process of preparing a set of trains and moving it in reverse direction after completing one itinerary before starting the return journey on the same route. For simplicity, this thesis does not discuss potential train couplings that may occur if the train is reassigned to other routes for operational tasks.

For each activity, $(i, j) \in A$, their minimum and maximum duration should be set as parameters, denoted by l_{ij} and u_{ij}, and employed as constraints to ensure the designed timetable follows the basic rule of train operation. Assume that all the activity durations are less than the cycle length, that is, l_{ij} and u_{ij} should fall in the range $[0, T)$ within one cycle.

3.3. Train timetabling constraints

The most crucial variable is x_i, which represents the event time of $i \in E$ in the passenger-centric robust timetable. As stated in subsection 3.2.2, the duration of each activity must fit within the interval defined by its lower and upper bounds.

$$l_{ij} \leq x_j - x_i + q_{ij} \cdot T \leq u_{ij} \quad \forall (i, j) \in A \tag{3.1}$$

In Equation 3.1, a 0-1 parameter q_{ij} is introduced for all train activities to indicate whether the activity $(i, j) \in A$ will cross the cycle in the original timetable, where T represents the cycle length. In other words, the value of q_{ij} can also indicate the order of events i and j within a cycle; when $q_{ij} = 0$, it means that event i occurs before j, and when $q_{ij} = 1$, it means the reverse is true. According to section 3.1, it is assumed that the new robust timetable does not change the order of events within a cycle. So, the value of q_{ij} is maintained in the design of a robust timetable. l_{ij} and u_{ij} are the lower and upper bounds of the duration of the train activity (i, j). Therefore, Equation 3.1 restricts the duration of each train activity to fall within a reasonable range. Infrastructure constraints are also included in Equation 3.1. Accordingly, there can be a minimum interval l_{ij} for two conflicting trains passing through the same junction. If no maximum time is specified, then u_{ij} can take the value T.

$$0 \leq x_i < T \quad \forall i \in E \tag{3.2}$$

Equation 3.2 ensures that the designed event times x_i are within the cycle. These two constraints (Equation 3.1 and Equation 3.2) are crucial in solving traditional PESP.

$$-m_i \leq x_i - o_i \leq m_i \quad \forall i \in E \tag{3.3}$$

Equation 3.3 allows but restricts the adjustment window of event time based on the original timetable. o_i is the event time for event i in original timetable and m_i is the maximum shifting step for event i.

To prevent potential delays from the scheduled times, extra time allowance is added to the activity durations, e.g., running times, as time supplements and to the interval between successive trains, e.g., minimum headway, as buffer times. (Sahin, 2017, Zieger et al., 2018) In this robust timetabling problem, we have constrained the time supplement for each train's running and dwelling activities. However, there is no such restriction on the turnaround activity at the terminal station, and the headway between trains does not consider buffer budget constraints.

$$\sum_{(i,j) \in Ar \cup Ad \wedge t(i), t(j) = n} (x_j - x_i + q_{ij} \cdot T - l_{ij}) \leq Z_n \quad \forall n \in N \tag{3.4}$$

Equation 3.4 indicates that the total time supplement for running (Ar) and dwelling (Ad) activities should not exceed the budget Z_n for each train $n \in N$. As introduced in subsection 3.2.1, each train with a different itinerary is an individual element n in set N. Therefore, each train over a line in one direction will have its corresponding supplement budget. The time supplement budget of the timetable is determined by factors such as operating routes, train types, and infrastructure capabilities. One method involves calculating the time supplement budget for each train in the original timetable and then applying the same budget to each train when creating the new timetable. In chapter 6, this approach will be used to determine the time supplement. The function $t(i)$ helps identify the train ID for event i and sorts out all the running and dwelling activities for train n.

3.4. Conclusion

Chapter 3 introduces the assumptions related to train operations and provides a detailed explanation of the types of train events and activities, along with their associated parameters and variables. This chapter also takes into account infrastructure conflicts when considering train events and activities. The sets Ed, Ea, and Ed', Ea' are used to distinguish between the station and the non-station events, making it convenient to reference the events within these sets when writing constraints and setting objectives.

The PESP model forms the basis of the modelling aspect of this research, ensuring that the new timetable is feasible. Its structured approach enables the systematic integration of different operational constraints, such as maximum shifting step and time supplement budget, which in turn facilitates the generation of optimised schedules.

The multi-part composition of the model in this research makes it challenging to implement and verify after completing the big model. However, the flexible structure of the train timetabling model in this chapter allows it to be used as a base model. It can be combined with other sub-models to test their feasibility, debug, and make necessary adjustments.

4 Passenger model

Modelling human behaviour is complex and challenging because humans possess subjective initiative; they make decisions based on their judgment to choose the most beneficial actions for themselves in different situations. This is also true for passengers selecting their modes of transportation. Passengers make their choices based on factors such as distance, fare, travel time, and other considerations. Even when they choose the train as their travel mode, passengers evaluate various paths based on their perceptions of waiting-at-origin time, number of transfers, in-vehicle time, cost, and other attributes, ultimately selecting the most 'cost-effective' path.

This chapter will make reasonable assumptions about passenger behaviour (section 4.1) and construct a passenger event-activity graph in conjunction with the train modelling described in section 3.2 (section 4.2). Section 4.3 introduces the process for generating the set of passenger paths. Section 4.4 focuses on developing a generalised travel time based on four key attributes: waiting-at-origin time, in-vehicle time, transfer time, and the number of transfers. This generalised travel time will be used as the basis for passengers' path choices, and passengers will be assigned within the network accordingly.

4.1. Model assumptions

Compared to the train modelling described in section 3.1, passenger modelling tends to exhibit more complexity. In the real world, individuals show significant heterogeneity; factors such as age, gender, educational level, etc., can influence their evaluations of specific matters, leading to different choices in the same situations. Passenger behaviour research is a crucial area in transportation system studies, aiming to understand better passengers' decision-making processes regarding transportation and travel activities. Knowledge of passenger behaviour can help improve the quality and efficiency of planning and timetabling in public transportation (Hafezi and Ismail, 2011).

However, the time and effort to complete this study are limited, so it is unrealistic to cover all possible passenger behaviours in this thesis comprehensively. Therefore, some reasonable assumptions should be made to simplify the model's complexity. These assumptions will help us focus on studying the most representative travel time and transfer-related factors and avoid encountering too many variables and uncertainties during the result analysis and case study.

1. Passengers are aware of the realized timetable. That means the passengers can evaluate the attributes of each path accurately.
2. The primary disturbances and passenger arrival time are assumed to be independent of the details of the timetable, and passenger arrival time follows a uniform distribution over an hour, even though passengers know the train they want to catch will experience a severe secondary delay.
3. Passengers will make a choice considering only waiting-at-origin time, in-vehicle time, transferring time and number of transfers when choosing their paths. The path choice of a homogeneous group of passengers is unique and fixed based on the generalised travel time.

4.2. Passenger network representation

For the passenger side, passenger activities have a strong link with train activities because passengers can interact with the trains only when the train comes into a dwelling. Besides, to more easily integrate the train model and the passenger model, the nodes in the passenger network representation are the passenger boarding and alighting events, which are assumed to be in line with the departure and arrival events, respectively. Section 4.2.1 details the passenger event sets and corresponding parameters and characteristics. Section 4.2.2 will explain the waiting-at-origin, in-vehicle and transfer activity. In the last section (subsection 4.2.3), a railway network example is presented, which will help in understanding the passenger event-activity graph and its relationship with train events and activities.

4.2.1. Passenger events

Passenger event set F is composed of passenger arrive-at-origin, boarding and alighting events. They form three subsets of F: Fs, Fb and Fa.In the real world, the boarding and alighting times are varied and follow the queuing theory in which the carriage doors are the service desks, and different groups of passengers have different service times; thus also, for simplification, all the passengers in the same passenger group share the same boarding event time, that is the departure time of that train. Alighting events are recognized as arrival events at the transfer or destination station. Based on this assumption, for all the passenger boarding or alighting events, there can be a mapped departure or arrival event in the train event set E accordingly. In this way, i and j can also represent the passenger event-activity graph.

There is one exception: the arrive-at-origin event. Firstly, the whole passenger set should be divided into several small groups $p \in P$, and the passengers in the groups share the same OD $k = (o, d)$ and the arrive-at-origin event time s.

4.2.2. Passenger activities

Like the train activities, passenger activities are the links between 2 connected passenger events $i, j \in F$ and the activity set is denoted by B and $(i, j) \in B$. This research discusses three types of activities, including waiting-at-origin activity Bw, in-vehicle activity Bi and transfer activity Bt.

The waiting-at-origin activity starts when the passengers arrive at their original stations and ends when they get aboard, which connects the arrive-at-origin event c_p for passenger group p with the first boarding event in path π, e_π^0, i.e., (c_p, e_π^0).

The in-vehicle activity describes the process where passengers aboard the train and are transported by the train from a starting station to an end station. Corresponding to the train activities, the train running and dwelling activities constitute this part. In real-life scenarios, passengers in the carriage perceive the train's running and dwelling times differently (Dewilde, Sels, Cattrysse, Vansteenwegen, et al., 2011). This thesis does not delve deeper into these perceptions for simplicity in the computational process.

Transfer activity refers to the process in which passengers transfer from one train to another at an interchange station. This activity includes the walking time for passengers to move from one platform to another within the station, the waiting time on the platform for the next train to arrive, and potentially the time spent using other facilities in the station. Therefore, a lower bound exists for transfer activity. Suppose the interval between the arrival and departure event times of the two trains is less than the minimum time required for passenger transfer activities. In that case, it is considered that passengers cannot make the transfer, resulting in a missed transfer, and they have to wait for the train in the next cycle if they insist on this path. The thesis defines certain transfer activities as foolish transfers, such as when passengers transfer to a train going in the opposite direction or when passengers transfer between trains going in the same direction but with the same stops. These types of transfers are considered invalid and are excluded when generating transfer activities. The final set of transfer activities will include all possible transfers within train dwelling stations, except for foolish transfers. Some of these transfer activities in the set may take a long time, but they are still included in the set because they could be potentially chosen under the new timetable.

4.2.3. Passenger event-activity graph

To better explain passenger events and activities and to illustrate their relationship with train events and activities, this section provides a small example and constructs a passenger event-activity graph.

As shown in Figure 4.1, this is a small example railway network consisting of four trains: `IC001`, `SP001`, `SP002`, and `SP003`. `IC001` is an Intercity (`IC`) train, traveling from `Station1` to `Station5`, with a stop at `Station3`. Besides, `SP001`, `SP002`, and `SP003` are three regional (`SPR`) trains, meaning they need to dwell at all stations. Their directions are distinguished by the odd or even train ID: `SP001` and `SP003` travel from `Station1` to `Station5`, while `SP002` travels from `Station5` to `Station1`. There is a group of passengers with $k = (\texttt{Station1}, \texttt{Station4})$. The different coloured arrows in the figure represent the passenger activities experienced on the possible paths chosen by the passengers. Noticeably, $(4, 21)$ and $(4, 13)$ are examples of two types of foolish transfer behaviours mentioned in subsection 4.2.2. In the transfer activity $(4, 21)$, passengers will transfer from one `SPR` to another `SPR` running in the opposite direction with the same stops. This indicates that the passengers have either missed their stop or are going in circles. While $(4, 13)$ refers to passengers transferring from one `SPR` to another `SPR` going in the same direction with the same stops. However, in reality, they could continue on the original train without needing to transfer to reach their destination station. These kinds of activities will not be included in the transfer activities for passengers.

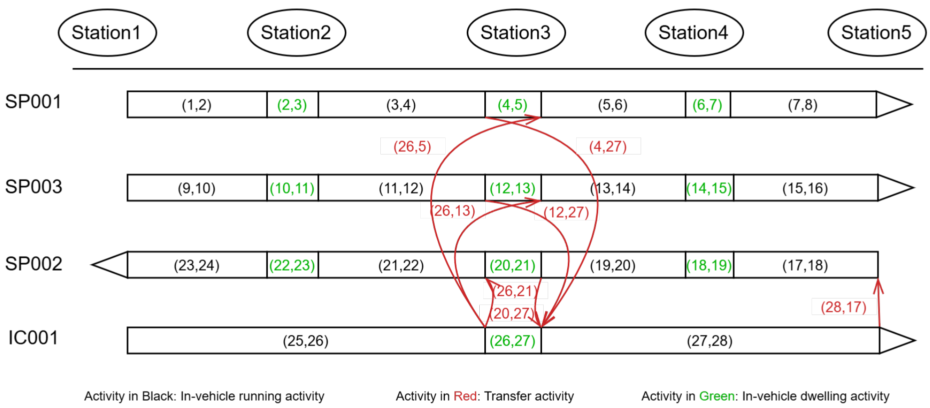

Figure 4.1: A small example rail network

A group of passengers plans to leave from `Station1`, board `IC001` to `Station3`, transfer to `SP001`, and finally reach `Station4`. They will arrive at `Station1` at time s (arrive-at-origin event c_p). Events and activities of the trains and passengers along this path are illustrated in Figure 4.2. Please note that infrastructure constraints of the trains, headway activities, and turnaround activities are not indicated in the Figure 4.2.In passenger events, according to the definition of the start and end events of passenger activities in subsection 4.2.1, each train event can give rise to corresponding potential passenger events. However, to keep the graph clearer and simpler, not all possible passenger events and activities are marked. The red arrows indicate the flow of passenger activities for this group of passengers along the chosen path from the starting point to the destination. First, they arrive at `Station1` at time s. The departure event of train `IC001` from `Station1` marks the end of the passenger waiting-at-origin activity and the start of their in-vehicle activity. The passenger transfer activity between different trains starts with the arrival of the preceding train at the transfer station and ends with the departure of the subsequent train from the station. Finally, after experiencing an in-vehicle activity on `SP001`, they arrive at their destination, `Station4`.

Constructing the passenger event-activity graph is a critical step in creating the passenger path set. The following section will explain how to generate all possible paths for each OD pair with a known passenger event-activity graph.

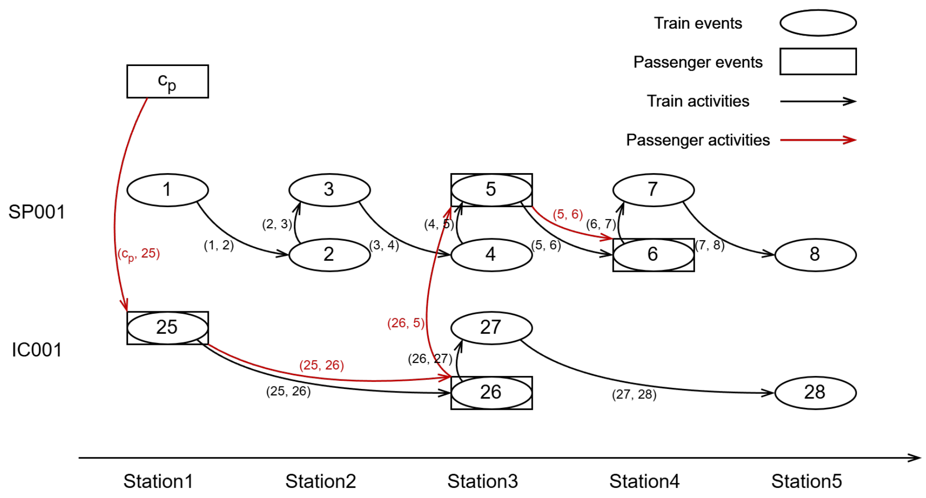

Figure 4.2: Passenger event-activity graph in the small example network

4.3. Passenger path set generation

Before conducting passenger assignment modelling, the set of possible passenger paths needs to be defined. This section will detail how to find all possible paths for each OD pair with the railway network graph and passenger activities.

4.3.1. Depth-First Search algorithm and Breadth-First Search algorithm

In this section, we will explore two fundamental algorithms for graph traversal: Depth-First Search (DFS) and Breadth-First Search (BFS). These algorithms are essential for various applications in pathfinding and network analysis.

DFS algorithm

DFS, proposed by Tarjan, 1972, is a traversal algorithm that starts at a given node (often called the ”root” in tree structures) and explores as far as possible along each branch before backtracking. This approach dives deep into the graph, reaching the leaf nodes before exploring sibling nodes.

The algorithm begins at the root or starting node and moves to an unvisited adjacent node, continuing this process until it reaches a dead end. After that, the algorithm backtracks to the previous parent node to explore other unvisited paths. This process repeats until all nodes have been visited or all paths from the starting node to the destination node have been explored.

BFS algorithm

BFS is a traversal algorithm that starts at a given node and explores all its neighbours at the present depth level before moving on to nodes at the next depth level. This approach explores the graph layer by layer and is widely used in searching the shortest path (Banerjee et al., 2018).

First, the root node is placed into the *Queue*. The traversal of the algorithm continues as long as there are elements in the *Queue*. During each iteration, the number of elements currently in the *Queue* is calculated, representing the number of elements at the current level of the tree. This number is denoted as *Size*. Next, in this *Size*, a number of elements are removed from the *Queue*, and operations specific to the problem are performed on them. While removing each node, its child nodes are added to the *Queue*. As long as elements remain in the *Queue*, indicating that there is another level to process, *Queue* calculation and *Size* removing is repeated to handle the next level.

Comparison

DFS offers several advantages, including memory efficiency and implementation simplicity. It uses less memory compared to BFS, as it only needs to store the current path in the stack. Therefore, DFS is suitable for exhaustive search scenarios, such as puzzle solving and maze navigation, where a comprehensive exploration of all possible paths is required.

For BFS, it guarantees finding the shortest path in an unweighted graph, making it ideal for pathfinding tasks. It processes nodes level by level, which can be beneficial for certain types of analyses, such as social network studies. However, BFS requires more memory than DFS, as it needs to store all nodes at the current level in the Queue. It can also be slower for graphs with many levels, as it explores nodes layer by layer. BFS is particularly suitable for cases where the shortest path between nodes is required.

In our case, the request is to input the OD pair and the network graph with passenger events and activities and find out all possible paths in the whole network. For this purpose, DFS is more suitable because it explores each path from the origin to the destination exhaustively before backtracking, ensuring that all possible paths are considered. DFS is particularly efficient for this task as it can be implemented with recursion and uses less memory than BFS. It is beneficial for handling potentially large networks with numerous paths.

4.3.2. Algorithm implementation

In this section, the DFS algorithm is introduced to find all the paths for the passengers with OD k. The network graph is constructed with station nodes and passenger in-vehicle and transfer activities. The station nodes connect to each other through a series of passenger activities, which are the edges of the graph. As shown in Figure 4.3, we break down the passenger in-vehicle activity into in-vehicle running activity and in-vehicle dwelling activity to provide a more detailed explanation of the DFS process in this chapter. Additionally, we combine the passenger in-vehicle dwelling activity and transfer activity with the passenger in-station activity.

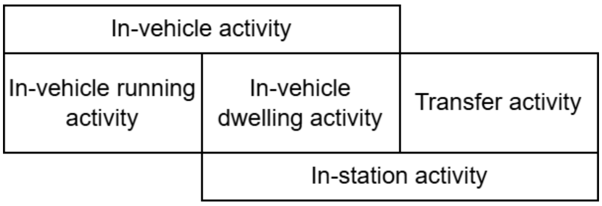

Figure 4.3: The definition of passenger in-vehicle activity and in-station activity

The path-searching process consists of two parts: The first step involves searching for all possible passenger in-vehicle running activities to connect the passing station between the origin and destination. It is not necessary to consider whether these activities are from the same train or whether transfers can be facilitated. The second part helps to complete the generated paths by adding connected in-station activities between the previous to-events and the following from-events in adjacent in-vehicle running activities. If it fails to find such an activity, the path will be marked as infeasible and removed from the path set. Below, the same example railway network in Figure 4.1 is employed to explain the exact process of finding all feasible paths.

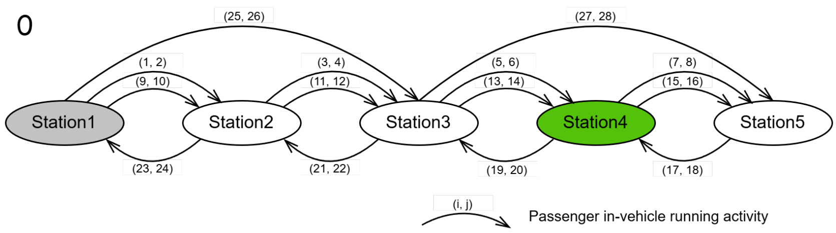

Figure 4.4: Graph representation of the example rail network

There is also a group of passengers who want to travel from `Station1` to `Station4`. They want to know all possible paths so that they can make a comparison among them and pick the shortest path. To find all the paths from `Station1` to `Station4` in the small example railway network, as structured in Figure 4.4, stations are the nodes in the graph because passengers can only change their activity state at train stops. The in-vehicle running activity links the stations, while the in-station activity (not shown in the graph) is responsible for connecting the passenger events within the station. However, the edges representing the in-vehicle running activities completed by different trains are not the same, even for activities between the same two stations in the same direction with similar routes. For instance, in Figure 4.1, $(1, 2)$ for `SP001` and $(9, 10)$ for `SP003` represent different activities when passengers travel to `Station2` on these two trains. These activities must be distinguished because they connect different events in the event-activity graph, and these events correspond to different event times when modelling. To facilitate their differentiation, we will define the edges as connections between station1 and station2 via activities $(1, 2)$ and $(9, 10)$ respectively.

The inputs of DFS are the passenger OD $k = (o, d)$ and the railway network graph with event set F and in-vehicle running activity (Figure 4.4). The goal is to find all potential passenger in-vehicle running paths between station nodes for each k.

Firstly, for each $k = (o, d)$, we will initialize a list Π_k^* to store the paths. Then, a DFS recursion function will take in the current searching node, the set of visited nodes, and the set of passenger in-vehicle running activities in the current path as inputs. Next, the algorithm will first check if the current searching node is the destination d that the passenger wants to reach. If it is, this indicates that a path has been found, and the current path will be added to Π_k^*, signifying that a path has been identified.

Of course, if the current node is not d, the algorithm will find all unvisited neighbouring nodes that can be accessed via edges in the network graph and record the connecting edges. Then, the algorithm will loop again, taking one of these neighbouring nodes and its connecting edge as new inputs for the recursion function. Note that when a path is found, that is, when the current node is the destination d, the function will exit because the destination d has already been visited and if we continue to traverse its neighbouring nodes, it will not be possible to visit the destination d again. Consequently, after breaking this iteration, the process will naturally return to the step of traversing neighbouring nodes in the previous recursion function.

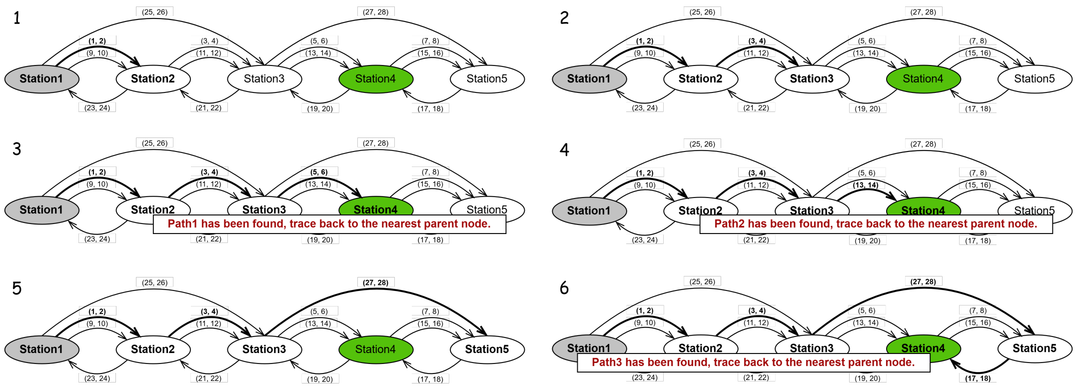

Figure 4.5: Path generation process for the first branch starting with $(1, 2), (3, 4)$ in a small example network

This recursion process is very complex and difficult to describe, thus, Figure 4.5 provides example steps of the algorithm applied to the small network shown in Figure 4.1, where o is `Station1` and d is `Station4`. In Iteration 1, the input is the starting point `Station1`, with the sets of visited nodes and activities both being empty. Then, it searches for its neighbour nodes and the corresponding edges, finding three pairs: `Station2`$(1, 2)$, `Station2`$(9, 10)$, and `Station3`$(25, 26)$ (`Station2`$(1, 2)$ means connecting to the neighbour node `Station2` with edge $(1, 2)$). Each pair of neighbour nodes will be traversed, and each traversal will end either upon reaching the destination or after all child nodes have been explored. For example, if we choose the pair `Station2`$(1, 2)$ and start Iteration 2. Obviously, `Station2` is not the desired destination d, the process will be repeated. Starting from `Station2`$(1, 2)$, we continue to search for child nodes of `Station2`, resulting in two possibilities: `Station3`$(3, 4)$ and `Station3`$(11, 12)$. Note that the pair `Station1`$(23, 24)$ is not considered, as `Station1` is already in the set of visited nodes. This loop will only unwind after all neighbour nodes of the next child node have been searched. Therefore, this looping process is not a horizontal search, but a vertical search algorithm that explores one branch of parent nodes completely before returning to a previous level for further search.

When considering the first branch (starting with activity $(1, 2), (3, 4)$), it finds one of the neighbour nodes for `Station3` that is `Station4` with edge $(5, 6)$ (Sub-figure 3) and luckily it reaches the destination

d. Therefore, the first potential path from `Station1` to `Station4` is found and saved in the path set Π_k^*. Subsequently, the algorithm backtracks to the closest parent node (`Station3`$(5,6)$) and requests another connecting edge, referred to as $(13,14)$. Similarly, starting from the parent node `Station3`, the last available path that can be found is $(27,28),(17,18)$ (Sub-figure `5,6`).

The branch starting from activity $(1,2),(3,4)$ has three paths:

1. $(1,2),(3,4),(5,6)$
2. $(1,2),(3,4),(13,14)$
3. $(1,2),(3,4),(27,28),(17,18)$

After completing the search on this branch, the algorithm will continue to backtrack to the higher-level parent nodes. From these parent nodes, the search will proceed again with similar steps, continuing until all possibilities have been traversed.

The first step involves identifying the edges between stations in the network. This entails finding the connections for passenger in-vehicle running activities between origin and destination stations. However, it's important to note that not all of these paths will have in-station activity connections. This is because, as mentioned in subsection 4.2.2, foolish transfer activities are not considered within the scope of this thesis. Thus, we need the second step, which is to check the feasibility of the paths in the finding path set. As previously explained, at intermediate stations for the paths (i.e., stations other than the origin and destination), the `to event` of the preceding edge must be connected to the `from event` of the subsequent edge via a passenger in-station activity (in-vehicle activity or transfer activity). Therefore, after completing the initial pathfinding, it is necessary to identify in-station activity that links the adjacent in-vehicle running activities. The path is deemed infeasible if such a connecting activity cannot be found.

With the initial path set Π_k^* obtained from the first step and all the in-station activity, the final output of the second step will be the passenger path set Π_k of OD k in the form of continuous passenger activities.

In the next step, we will revisit the previous example to analyse and refine the paths starting with activities $(1,2)$ and $(3,4)$, as illustrated in Figure 4.5. In order to visualize the distribution of passenger in-station activities more effectively, Figure 4.6 organizes all passenger activities based on the trains responsible for the operation. The dashed boxes correspond to the same station and are equivalent to the ellipses used to represent stations in Figure 4.5. The green lines indicate in-vehicle dwelling activities, while the red lines represent feasible transfer activities between different trains at the same station.

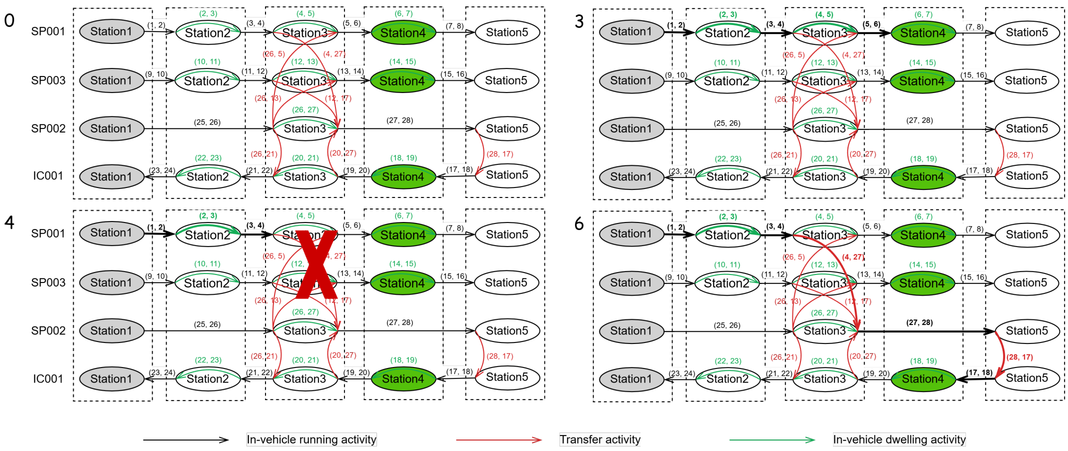

Figure 4.6: Path feasibility check for the first branch in a small example network

As shown in Figure 4.6, the in-vehicle running activity between `Station1` and `Station2`, and between `Station2` and `Station3` can be achieved through the in-vehicle dwelling activity $(2,3)$. Besides, in-vehicle dwelling activity $(4,5)$ can fill the connection in `Station3`. Each of the above three paths is examined in turn. It is found that, in the second path, the connection of in-vehicle running activities of different trains should be undertaken by the transfer activity at `Station3`, but there can't find such activity because `SP001` and `SP003` are trains having the same dwelling and running mode in the same direction. This transfer activity is recognized as a foolish transfer. Thus, this path will be discarded.

Therefore, the updated set of paths is as follows:

1. $(1,2),(2,3),(3,4),(4,5),(5,6)$
2. $(1,2),(2,3),(3,4),(4,27),(27,28),(28,17),(17,18)$

After getting all the possible paths for each passenger OD k, the number of transfer ϵ_π for each single path can be calculated by counting the number of transfer activities.

4.4. Passenger assignment constraints

In passenger assignment modelling, travel time and number of transfers determine a passenger's path choice in this thesis. For the travel time part, three types of time must be defined: waiting-at-origin time, in-vehicle time, and transfer time, as shown in Equation 4.1-4.7. Here, P represents the set of passenger groups, k and s are the OD, and the time this group of passengers p arrives at the original station is in a periodic form, respectively.

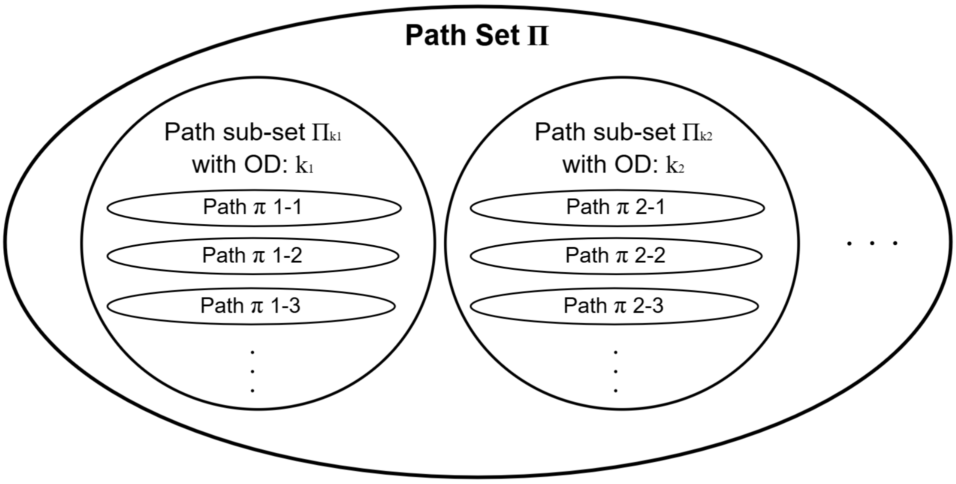

Figure 4.7: Path set structure

As shown in Figure 4.7, Π_k is a sub-set of the whole path set Π and represents the set of all paths for $k = (o, d)$, and π gives the path element in the path set. The first boarding event in each path π is denoted by e^0_π.

$$\omega_{p,\pi} = x_{e^0_\pi} - s + a_{p,\pi} \cdot T \quad s \in p, \forall p \in P, \forall \pi \in \Pi_k \tag{4.1}$$

$$\omega_{p,\pi} \geq 0 \quad \forall p \in P, \forall \pi \in \Pi_k \tag{4.2}$$

$$a_{p,\pi} \in \{0,1\} \quad \forall p \in P, \forall \pi \in \Pi_k \tag{4.3}$$

Equation 4.1 is used to decide the waiting time at the origin station for each passenger group p in path π, in which a binary variable $a_{p,\pi}$ is introduced to determine the sequence of passenger arrive-at-origin event c_p and first boarding event e^0_π in one cycle. When $a_{p,\pi} = 0$, arrive-at-origin event c_p comes earlier than the first event in path π within the cycle, otherwise, arrive-at-origin event c_p comes later. $x_{e^0_\pi}$ represents the time of the first boarding event in the path π, and s represents the corresponding event time for the arrive-at-origin event c_p. Equation 4.2 tells that passengers cannot board a train that has already departed unless they are willing to wait for that train the next cycle. In this way, the binary variable $a_{p,\pi}$ is restricted.

$$\phi_{p,\pi} = \sum_{(i,j)\in\pi\wedge(i,j)\in Bi} (x_j - x_i + q_{ij} \cdot T) \quad \forall p \in P, \forall \pi \in \Pi_k \tag{4.4}$$

Passenger in-vehicle time for the path π is defined by Equation 4.4. It is the duration summation of all the passenger in-vehicle activities Bi. The passenger activities share the same start and end times as trains' as assumed in section 4.1. Thus, q_{ij} is the same binary parameter discussed in train modelling and has already been defined in Equation 3.1.

$$\gamma_{p,\pi} = \sum_{(i,j)\in\pi\wedge(i,j)\in Bt} (x_j - x_i + b_{ij} \cdot T) \quad \forall p \in P, \forall \pi \in \Pi_k \tag{4.5}$$

$$\bar{l}_{ij} \leq x_j - x_i + b_{ij} \cdot T < T + \bar{l}_{ij} \quad \forall (i,j) \in Bt \tag{4.6}$$

$$b_{ij} \in \{0,1\} \quad \forall (i,j) \in Bt \tag{4.7}$$

Equation 4.5-4.7 illustrate the definition of transfer time for (i,j) in transfer activity set Bt, which has a similar structure as that of in-vehicle time. Its lower and upper bound $\bar{l}_{ij}$ can be determined by factors like station size, service infrastructure in the station, etc. b_{ij} is a binary variable to judge if the passenger can transfer the connected train in the same cycle from event i to event j. If $b_{ij} = 0$, the order is that event i comes first to the event j in one cycle, and passengers have enough time to transfer from event i to j; otherwise, the passenger has to wait for the connected train with event j in the next cycle. Although the activity sequence remains the same as the original timetable due to the assumption, b_{ij} still needs to be a binary variable rather than parameter q_{ij}, because there is a minimum transfer time $\bar{l}_{ij}$ given in Equation 4.6. If the transfer time after the schedule shifting is less than $\bar{l}_{ij}$, b_{ij} will change from 0 to 1.

$$\bar{\xi}_{p,\pi} = \phi_{p,\pi} + \beta_w \cdot \omega_{p,\pi} + \beta_t \cdot \gamma_{p,\pi} + \beta_n \cdot \epsilon_\pi \quad \forall p \in P, \forall \pi \in \Pi_k \tag{4.8}$$

Equation 4.8 provides a representation of generalised travel time $\bar{\xi}_{p,\pi}$ for passenger group p travel on path π, where β_w, β_t, and β_n represent the weights of waiting-at-origin time, transfer time and number of transfers relative to in-vehicle time, respectively. These values can be found in the literature (Robenek et al., 2016, Binder et al., 2017). ϵ_π is the parameter for the number of transfers in the path π, and it can be directly calculated when the path is generated. Passengers will determine their final travel plan based on each path's generalised travel time $\bar{\xi}_{p,\pi}$.

Equation 4.9-4.12 demonstrate the mathematical expression of how each group of passengers selects the path with the minimum generalised travel time.

$$\sum_{\forall\pi\in\Pi_k} \alpha_{p,\pi} = 1 \quad \forall p \in P \tag{4.9}$$

$$\alpha_{p,\pi} \in \{0,1\} \quad \forall p \in P, \forall \pi \in \Pi_k \tag{4.10}$$

Equation 4.9 and Equation 4.10 define a binary variable $\alpha_{p,\pi}$. When $\alpha_{p,\pi} = 1$, it indicates that the passenger group p with OD k and arrive-at-origin time s will choose path π as their final travel plan. At the same time, any other path in Π_k will marked as 'Not chosen' to passenger p ($\alpha_{p,\pi} = 0$).

$$-M \cdot (1 - \alpha_{p,\pi}) \leq \xi_p - \bar{\xi}_{p,\pi} \leq M \cdot (1 - \alpha_{p,\pi}) \quad \forall p \in P, \forall \pi \in \Pi_k \tag{4.11}$$

$$\xi_p - \overline{\xi}_{p,\pi} \leq M \cdot \alpha_{p,\pi} \quad \forall p \in P, \forall \pi \in \Pi_k \tag{4.12}$$

Equation 4.11 and Equation 4.12 are the constraints for selecting the shortest path. Here, the Big M method is used, and M is introduced as a large enough number. ξ_p gives the minimum generalised travel time passenger group p with OD k and arrive-at-origin time s. Only when $\alpha_{p,\pi} = 1$, which means the path π is selected by this group of passengers, Equation 4.11 will restrict $\xi_p = \overline{\xi}_{p,\pi}$. Besides, for Equation 4.12, it is employed to ensure ξ_p should be less or equal to the generalised travel time of any path in the set Π_k.

4.5. Conclusion

The passenger-related problem is the most complex and variable part of the passenger-centric robust timetabling problem discussed in this thesis. Therefore, at the beginning of this chapter, a series of assumptions are made before the passenger assignment model is introduced. Additionally, passenger events and activities are defined in detail, and they are closely related to the train's event-activity graph. Waiting-at-origin and transfer activities are introduced as additional passenger activities in constructing the passenger event-activity graph. Furthermore, in section 4.3, the DFS algorithm demonstrates more significant advantages in pathfinding problems compared to BFS, with all possible passenger paths generated and refined through a two-step DFS. The final part focuses on passenger assignment modelling, mainly defining various travel times, calculating generalised travel times, and finding the path with the minimal generalised travel time for each passenger group. These selected paths are affected by changes in the new timetable and are also a crucial part of optimising train event times. This modelling approach will also be addressed in the subsequent passenger simulation section.

5
Simulation model

This chapter will introduce how to build models simulating train operations and passenger assignments under disturbance scenarios. These results will be used to assess the passenger-centric robustness of the current train timetable. The robust timetabling problem aims to create schedules that can withstand disturbances, ensuring reliable and efficient railway operations. This thesis will use the average passenger delay as the primary evaluation factor for customising robust timetables. Therefore, the simulation section needs to discuss the train and the passenger model. This chapter will be divided into three main parts: in section 5.1, all relevant used parameters will be introduced. Section 5.2 and section 5.3 will then respectively introduce the constraints related to the delay propagation through trains and the evaluation process from the passenger perspective.

5.1. Scenario parameters

For the simulation model, the selection of scenarios will directly impact the final output results. The method of designing scenarios is crucial for addressing specific problems. The considerations for designing scenarios will be explained in more detail in the specific context of chapter 6. This section will introduce the parameters that may included in each scenario, laying the basis for the subsequent case study. ;

Number of cycle $h \in H$
The number of cycles indicates how long the scenario will last. A longer scenario will definitely increase the complexity of the model and calculation time. A scenario with more cycles helps consider situations where a specific activity always experiences disturbances, such as transfer stations, which often lead to additional dwell time.

Initial delayed time δ_{ijrh}
As an adjustable parameter, the initial delayed time is denoted as δ_{ijrh} for disturbed activity (i, j) in cycle h in each scenario h. In real-world situations, the initial delayed time of a specific activity may follow a certain distribution, such as a normal distribution, over a given period. However, due to the limitations of computational capabilities, this thesis only considers a fixed initial delayed time.

Disturbed activity $((i, j), r, h) \in \Delta$
Disturbed activity indicates which train activities in the scenario r have experienced disturbances, i.e. they require longer activity duration. In each scenario, multiple disturbed activities can be defined simultaneously. This refers to real-world scenarios where a small disturbance affects two-way train traffic or when a set of highly correlated activities simultaneously encounters disturbances.

5.2. Train simulation constraints

In this section, a realised event time for train event i and j is denoted by v_{irh} or v_{jrh}, in which r is the scenario ID included in the scenario set R discussed in the case and h is the realised cycle that the event i or j happens.

The delay propagation process is simulated when the initial delay appears through Equation 5.1 and Equation 5.2.

$$v_{jr(h+q_{ij})} - v_{irh} \geq l_{ij} + \delta_{ijrh} \quad \forall (i,j) \in A, \forall r \in R, \forall h \in H, ((i,j),r,h) \in \Delta \tag{5.1}$$

$$v_{jr(h+q_{ij})} - v_{irh} \geq l_{ij} \quad \forall (i,j) \in A, \forall r \in R, \forall h \in H, ((i,j),r,h) \notin \Delta \tag{5.2}$$

Note that Equation 5.1 describes the duration bound of activities experienced initial disturbance. The disturbance parameter for activity (i,j) in scenario r in cycle h is given by δ_{ijrh}, which is added to the lower (l_{ij}) bound of the duration of (i,j) and h should be a natural number $\mathbb{N}$. This activity has to process an extra δ_{ijrh} upon the lower bound. The binary parameter q_{ij} illustrates whether activity (i,j) crosses the cycle in the original timetable. Besides, in Equation 5.2, the constraint for other activities not in disturbance set Δ is presented. By these constraints, the delay will be propagated along the activity chain influenced by the initial delay δ_{ijrh} if there is no sufficient supplement to mitigate the previous delay. Noticeably, v is a linear variable, so the realised time v_{irh} may not exactly fall in the cycle h. Sometimes, v_{irh} will postpone to the next cycle $h+1$ due to the initial delay, but it still retains the original index.

$$x_i + h \cdot T \leq v_{irh} \quad \forall i \in Ed, \forall r \in R, \forall h \in H \tag{5.3}$$

In real-world operation, a train cannot depart earlier than the scheduled departure time under normal cases, even if it has finished its dwell activity ahead of time. Early departure can cause passengers who arrive at the station just in time to miss the train or result in failed transfers for transfer passengers. Thus, we address this problem with a constraint in Equation 5.3. $x_i + h \cdot T$ is the scheduled event time for event i in cycle h and v_{irh} is the realized event time in linear form.

5.3. Passenger simulation constraints

This section will focus on how passengers choose their paths and how the generalised travel time is calculated when delays occur under disturbance scenarios. Based on the first assumption in section 4.1, passengers are aware of the realised timetable and can accurately know the travel time they will experience in each path. Therefore, passengers can be assigned to the rail network using the methods outlined in section 4.4. In cycle h of scenario r, the realized waiting-at-origin time, in-vehicle time and transfer time for each path π for each passenger group p are denoted by $\omega^*_{p,\pi,r,h}$, $\phi^*_{p,\pi,r,h}$ and $\gamma^*_{p,\pi,r,h}$ respectively. Noticeably, variables with a superscript * indicate the corresponding passenger-related variables under the realised timetable as described in chapter 4. The subsequent constraints will all follow this notation.

$$\omega^*_{p,\pi,r,h} = v_{e^0_\pi rh} - (s_p + h \cdot T) + a^*_{p,\pi,r,h} \cdot T \quad \forall p \in P, \forall \pi \in \Pi_k, \forall r \in R, \forall h \in H \tag{5.4}$$

$$\omega^*_{p,\pi,r,h} \geq 0 \quad \forall p \in P, \forall \pi \in \Pi_k, \forall r \in R, \forall h \in H \tag{5.5}$$

$$a^*_{p,\pi,r,h} \in \{0,1\} \quad \forall p \in P, \forall \pi \in \Pi_k, \forall r \in R, \forall h \in H \tag{5.6}$$

Equation 5.4-5.6 define the passenger waiting-at-origin time for the passenger group p with OD k and arrive-at-origin time s in scenario r in cycle h. The binary variable $a^*_{p,\pi,r,h}$ is applied to determine the chronological order of the passenger arrive-at-origin event and boarding event of the target train in the cycle h. If the arrive-at-origin event occurs before the boarding event, then $a^*_{p,\pi,r,h} = 0$; otherwise, $a^*_{p,\pi,r,h} = 1$. Besides, $s + h \cdot T$ is the expression of the passenger arrive-at-origin time in a linear form. Equation 5.5 tells that passengers cannot board a train that has already departed unless they are willing to wait for that train the next cycle ($a^*_{p,\pi,r,h} = 1$).

$$\phi^*_{p,\pi,r,h} = \sum_{(i,j)\in\pi\wedge(i,j)\in Bi} (v_{jr(h+Q_{j,\pi})} - v_{ir(h+Q_{i,\pi})}) \quad \forall p \in P, \forall\pi \in \Pi_k, \forall r \in R, \forall h \in H \tag{5.7}$$

Equation 5.7 specifies the in-vehicle time for the passenger group with p under the realized timetable. Since v_{irh} is a linear variable, it is necessary to determine in which cycle the passenger activity $(i,j) \in \pi$ occurs during the journey on this path. Therefore, a new integer parameter $Q_{i,\pi}$ will be introduced, and it helps to record the number of cycles experienced by this route up to event i.

As shown in Figure 5.1, we assume that $\{1,2,3,4,5\}$ are passenger events in the event set B. The position of the triangles in the Figure 5.1 represents the event times of these events in the newly designed timetable. There is a given path π, $\pi = \{(1,2),(2,3),(3,4),(4,5)\}$, and to obtain $Q_{\hat{i},\pi}$, we should sum all the q_{ij} for passenger activities before the event $\hat{i}$. For example, to calculate $Q_{3,\pi}$, sum the q values of passenger activities up to event 3 ($q_{1,2}$, $q_{2,3}$), resulting in $Q_{3,\pi} = 0$. To calculate $Q_{5,\pi}$, sum the q values up to event 5 ($q_{1,2}$, $q_{2,3}$, $q_{3,4}$, $q_{4,5}$), resulting in a sum of 1. Note that the model determines the specific event times in the newly designed timetable, and all these event times are variables, but the order of events within a cycle remains the same as in the original timetable. Therefore, the $Q_{i,\pi}$ values are equal to that of the original timetable, making $Q_{i,\pi}$ a fixed value for each event i in each path π.

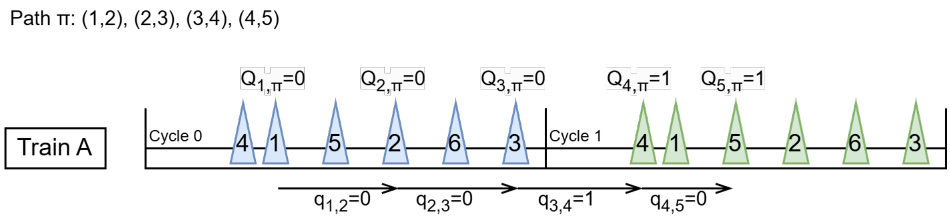

Figure 5.1: The relationship between $q_{\hat{i}j}$ and $Q_{\hat{i},\pi}$ in path π

$$\gamma^*_{p,\pi,r,h} = \sum_{(i,j)\in\pi\wedge(i,j)\in Bt} (v_{jr(h+Q_{j,\pi})} - v_{ir(h+Q_{i,\pi})} + b^*_{ijrh}\cdot T) \quad \forall p \in P, \forall\pi \in \Pi_k, \forall r \in R, \forall h \in H \tag{5.8}$$

$$\bar{l}_{ij} \leq v_{jr(h+Q_{j,\pi})} - v_{ir(h+Q_{i,\pi})} + b^*_{ijrh}\cdot T < \bar{l}_{ij} + T \quad \forall (i,j) \in Bt, \forall r \in R, \forall h \in H \tag{5.9}$$

$$b^*_{ijrh} \in \{0,1\} \quad \forall (i,j) \in Bt, \forall r \in R, \forall h \in H \tag{5.10}$$

Equation 5.8-5.10 are combined to restrict the realized transfer time. b^*_{ijrh} is a binary variable to judge whether the previous train arrival event i and the following train departure event j in the realised timetable is still in the same sequence as the designed robust timetable. Note that the sequence of events in the new timetable is fixed because, in section 3.1, we assumed this sequence is the same as the original timetable. However, in the realised timetable, this order can be changed due to the disturbance in the previous train. If the departure event j goes first in the realised timetable in scenario r in cycle h, b^*_{ijrh} will be 1. Although these passengers can wait for the same train in the next cycle, they must experience a much longer transfer time.

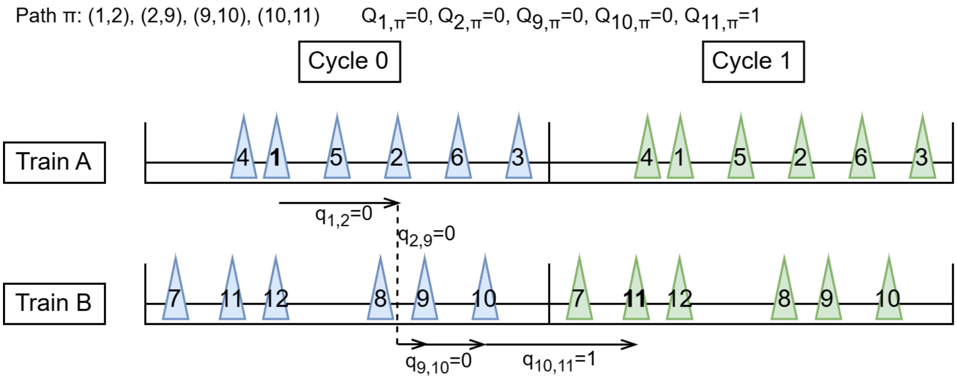

Figure 5.2: Example path π with a transfer activity under newly designed timetable

Now, here is an example of a path involving a transfer activity under a newly designed timetable. As shown in the Figure 5.2, this path in the example involves two trains. Train A's activity order is (1,2),(2,3),(3,4),(4,5),(5,6), where all q values are all 0 except for $q_{3,4}$ which is 1. Activity order for Train B is (7,8),(8,9),(9,10),(10,11),(11,12), where all q values are 0 except for $q_{10,11}$. Additionally, Train A's Event 2 and Train B's Event 9 are the arrival and departure events at `Station 1`, respectively. Passengers have a path π of (1,2), (2,9), (9,10), (10,11), which includes a transfer activity (2,9). In the newly designed timetable, this transfer is feasible within cycle 0 (as indicated by the dashed line in Figure 5.2). Suppose this path does not experience disturbances and ignores the waiting activity at the origin. In that case, the generalised travel time consists of three segments of in-vehicle time and one segment of transfer time, with the transfer time being the difference between the realised event times: $v_{9,r,0} - v_{2,r,0}$. At this point, $b^*_{2,9,r,0} = 0$.

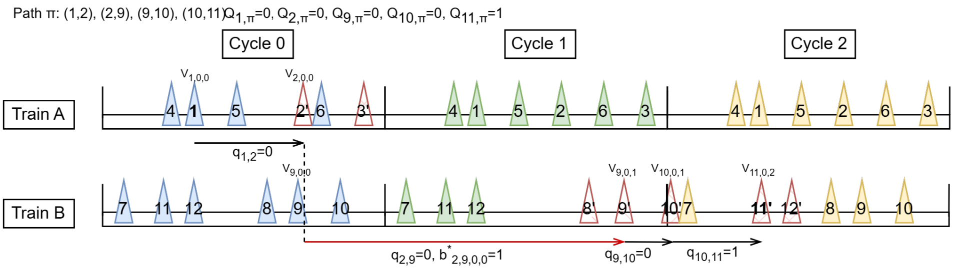

Figure 5.3: Example path π with a transfer activity in realised timetable under scenario 0

$$\gamma^*_{p,\pi,r,h} = \sum_{(i,j)\in\pi\wedge(i,j)\in Bt} \left(v_{jr(h+Q_{j,\pi}+b^*_{ijrh})} - v_{ir(h+Q_{i,\pi})}\right) \quad \forall p \in P, \forall \pi \in \Pi_k, \forall r \in R, \forall h \in H \tag{5.11}$$

Figure 5.3 illustrates a delay scenario, denoted as scenario 0. Unfortunately, Train A's activity (1,2) is delayed, making the transfer (2,9) in this path infeasible within cycle 0. That means this transfer is not possible in the scheduled cycle. Train B also experiences delays in cycle 1. Nevertheless, this path π is still feasible. As shown in the Figure 5.3, passengers need to wait for Train B's Event 9 in the next cycle to continue using this path. The transfer time is $v_{9,0,1} - v_{2,0,0}$, and $b^*_{2,9,0,0} = 1$. To accurately express this relationship, Equation 5.11 needs to be applied to replace Equation 5.8. However, the problem is that the index of realised event time v_{irh} includes a variable b^*_{ijrh}, which means this expression is not linear. To simplify the constraint, the model proposed (Equation 5.8-5.10) in this thesis approximates this issue by adding $b^*_{ijrh} \cdot T$ as an additional transfer time. The trade-off and limitation for this approach is that if subsequent train activities on this path experience delays resulting in the realised event time differing from the scheduled event time, this value will be inaccurate. As shown in Figure 5.3, the actual transfer time from Event 2' in cycle 0 to Event 9 in cycle 1 is $v_{9,0,1} - v_{2,0,0}$, but using Equation 5.8, the transfer time is $v_{9,0,0} - v_{2,0,0} + T$. Since Event 9 in cycle 1 is delayed, these two values are obviously unequal. Using the same method to check subsequent events, such as the in-vehicle time for (9,10) in cycle 1, the time might also differ from that in cycle 0. This paper refers to this phenomenon as cycle misalignment.

If a delay in the preceding train causes passengers to miss the transfer to the subsequent train in the scheduled cycle (i.e. $b^*_{ijrh} = 1$), it may result in a cycle misalignment. This can affect all passenger activities after the cycle misalignment occurs. The duration of these passenger activities may vary depending on the difference between values $v_{jr(h+Q_{j,\pi}+b^*_{ijrh})} - v_{ir(h+Q_{i,\pi})}$ and $v_{jr(h+Q_{j,\pi})} - v_{ir(h+Q_{i,\pi})} + b^*_{ijrh} \cdot T$.

As explained in this example, it must be admitted that if b^*_{ijrh} takes the value 1, this may cause a cycle misalignment in $Q_{i,\pi}$ corresponding to event i after missing the transfer in path π. This misalignment may lead to a deviation in calculating the passenger travel time. However, because the missed transfer significantly increases the transfer time and there are still many other paths to choose from, this path with the missed transfer will certainly not be the shortest. Therefore, these errors may not affect the final optimisation result in most cases. However, when there are few alternative paths available to passengers and the disturbance is intense, this limitation will be magnified. The path that deviates from the realised generalised travel time due to cycle misalignment will become the only option for these passengers, inevitably affecting the accuracy of the model and the final results.

$$\overline{\xi}^*_{p,\pi,r,h} = \phi^*_{p,\pi,r,h} + \beta_w \cdot \omega^*_{p,\pi,r,h} + \beta_t \cdot \gamma^*_{p,\pi,r,h} + \beta_n \cdot \epsilon_\pi \quad \forall p \in P, \forall \pi \in \Pi_k, \forall r \in R, \forall h \in H \tag{5.12}$$

Equation 5.12 calculates the generalised realized travel time $\overline{\xi}^*_{p,\pi,r,h}$ for each path for each group of passengers in scenario r in cycle h. Note that ϵ_π is the number of transfers in path π independent of the event times and passenger groups.

$$\sum_{\pi \in \Pi_k} \alpha^*_{p,\pi,r,h} = 1 \quad \forall p \in P, \forall r \in R, \forall h \in H \tag{5.13}$$

$$\alpha^*_{p,\pi,r,h} \in \{0,1\} \quad \forall p \in P, \forall \pi \in \Pi_k, \forall r \in R, \forall h \in H \tag{5.14}$$

$$-M \cdot (1 - \alpha^*_{p,\pi,r,h}) \leq \xi^*_{p,r,h} - \overline{\xi}^*_{p,\pi,r,h} \leq M \cdot (1 - \alpha^*_{p,\pi,r,h}) \quad \forall p \in P, \forall \pi \in \Pi_k, \forall r \in R, \forall h \in H \tag{5.15}$$

$$\xi^*_{p,r,h} - \overline{\xi}^*_{p,\pi,r,h} \leq M \cdot \alpha^*_{p,\pi,r,h} \quad \forall p \in P, \forall \pi \in \Pi_k, \forall r \in R, \forall h \in H \tag{5.16}$$

Equation 5.13-5.16 compare the generalised travel time for each group of passengers in each cycle under each scenario and assign the minimum generalised travel time to $\xi^*_{p,r,h}$. This structure is similar to the passenger assignment model under scheduled timetable in section 4.4.

5.4. Objective function

As discussed in section 1.3, the optimisation goal of robust timetabling in this study is to minimise the additional average generalised travel time experienced by passengers under small disturbances, i.e. average passenger travel delay.

$$\xi^*_{p,r,h} - \xi_p \leq d_{p,r,h} \quad \forall p \in P, \forall r \in R, \forall h \in H \tag{5.17}$$

Therefore, Equation 5.17 tells the calculation formulation of the travel delay $d_{p,r,h}$ for each passenger group p in scenario r in cycle h, where ξ_p and $\xi^*_{p,r,h}$ represent the scheduled travel time and realised travel time under the new robust timetable respectively.

$$Min \sum_{p \in P} \sum_{r \in R} \sum_{h \in H} d_{p,r,h} \cdot g_{p,h} / |G| \tag{5.18}$$

In the last, the objective function, as shown in Equation 5.18, minimises the average passenger travel delay for all passenger groups P in all scenarios R in all cycle H. The average passenger travel delay is derived by taking the product of the additional delay $d_{p,r,h}$ for each passenger group p and the number of passengers $g_{p,h}$ in the corresponding group, and then dividing by the total number of passengers $|G|$.

5.5. Conclusion

This chapter has comprehensively discussed the application of models to simulate the passenger generalised travel time under disturbance scenarios so as to help the optimisation of railway passenger-centric robust timetabling by focusing on the average passenger delay.

In section 5.1, the critical parameters forming the foundation of the simulation model are introduced. Secondly, section 5.2 details the constraints related to train simulation. These constraints ensure that delay propagation is accurately modelled and that trains adhere to realistic operational limits. They also prevent trains from departing earlier than scheduled, thereby maintaining operational reliability. In section 5.3, the constraints governing passenger behaviour and their impact on generalised travel time are explored. The model reflects passengers' responses to delays and their subsequent path choices, allowing for the calculation of additional generalised travel time experienced by passengers under various disturbance scenarios. However, the model has limitations in calculating the passenger realised transfer time. If a delay in the preceding train causes passengers to miss the transfer to the subsequent train in the scheduled cycle, it may result in a cycle misalignment. This will cause deviations between the activity duration in subsequent journeys and the actual duration. Luckily, in real-world networks, passengers have many paths to choose from, and paths with deviations, often due to long transfer times, tend not to be the passengers' final choice. Thus, this limitation may not affect the final results. Seeking improvement measures for this limitation will also be one of the subsequent research topics.

This chapter lays the groundwork for robust timetable optimisation by integrating detailed train and passenger simulation models. The methodologies and constraints introduced are applied to evaluate the average passenger delay on a timetable that provides feedback to the PESP model.

6 Case study

The Eindhoven-Den Bosch-Tilburg region, located in the southern Netherlands in the province of North Brabant, is one of the most important economic and cultural centres. Eindhoven, the fifth largest city in the Netherlands, is also the centre for innovation and technology. Besides, Den Bosch, the capital of North Brabant, experiences significant daily traffic due to its administrative importance. Consequently, the railway system in this region is one of the most heavily used in the province. This chapter will apply the models mentioned in the previous chapters to this region to evaluate the performance of the methodology.

6.1. Railway network

Figure 6.1 shows the layout of the railway network within the case study area. In this network, Eindhoven (`Ehv`), Den Bosch (`Ht`), and Tilburg (`Tb`) are the major interchange hubs, which are connected by Intercity and Sprinter (regional) trains. Additionally, the network includes four secondary stations: Vught (`Vg`), Boxtel (`Btl`), Best (`Bet`), and Eindhoven Strijp-S (`Ehs`), where only Sprinter trains dwell. Overall, there are three pairs of Sprinter trains operating in this network: the Eindhoven-Den Bosch route (`SP9640`, `SP9641`), the Den Bosch-Tilburg route (`SP6640`, `SP6641`), and the Tilburg-Eindhoven route (`SP6441`, `SP6444`).

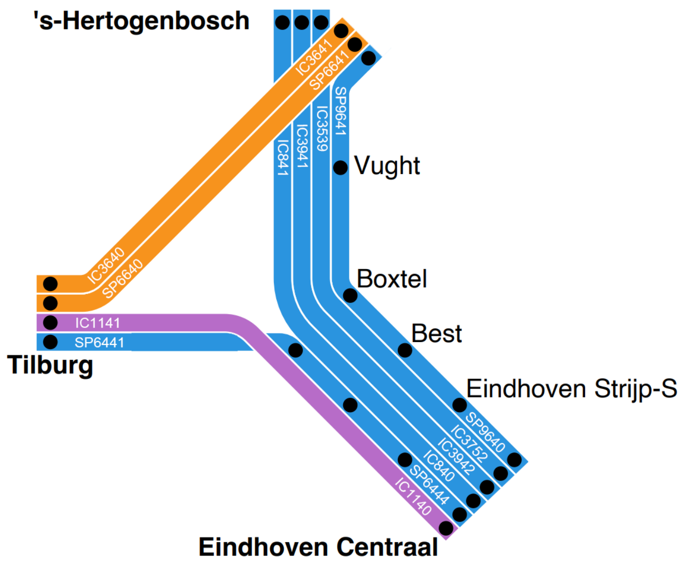

Figure 6.1: Case study region: Eindhoven-Den Bosch-Tilburg

According to the latest infrastructure layout from sporenplan in the case area (Figure 6.2), trains from Tilburg to Den Bosch (`SP6640`,`IC3640`) have to use the same infrastructure groups in the junction with the train from Eindhoven to Tilburg (`SP6444`, `IC1140`), as well as trains from Tilburg to Den Bosch (`SP6640`, `IC3640`) and from Tilburg to Eindhoven (`SP6441`, `IC1141`). This junction is named Tilburg aansl. (`Tba`). Similarly, there is a junction `Vga` in the northern of Station `Vg`. Thus, besides the headway constraints in the stations, junction infrastructure constraints should be added in `Tba` and `Vga`, in which the junction arrival and departure events with the same event times are inserted, and the lower bound l_{ij} between the conflicting passing train is 1 min and the upper bound u_{ij} is set as cycle length (30 min).

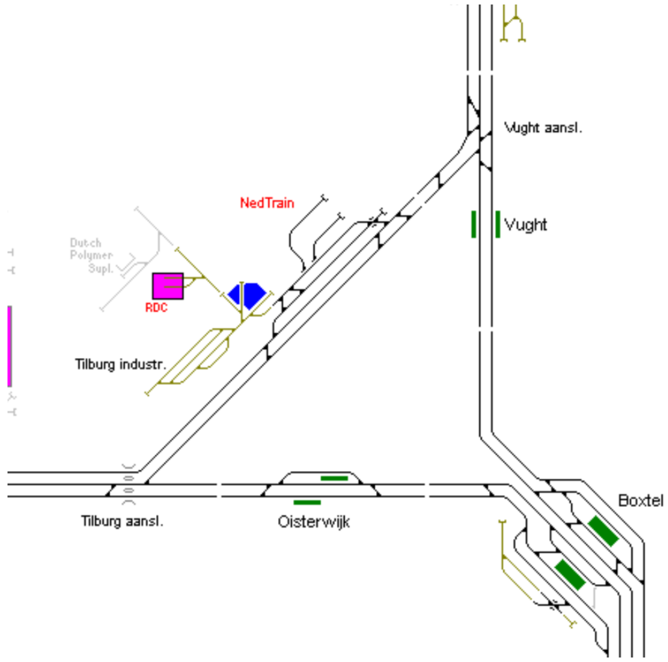

Figure 6.2: Infrastructure layout at `Btl`-`Tba`-`Vga` region

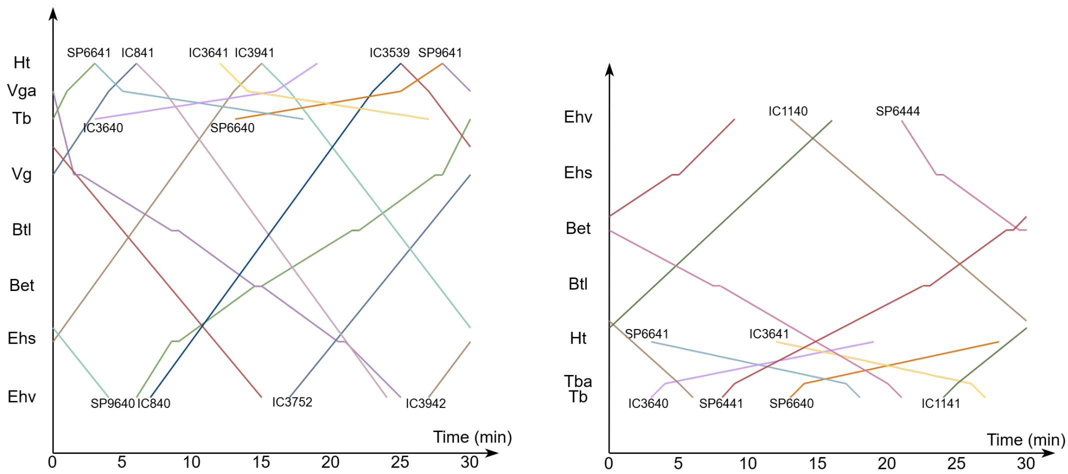

Figure 6.3: Case study timetable in 2019

In this thesis, we use the railway timetable in 2019 (Figure 6.3) from NS as the original timetable. This timetable is a periodic timetable with a cycle of 30 minutes, where each train is sent from the departure station once within the cycle. Besides, the minimum running time and headway for l_{ij} $(\forall(i,j) \in Ar \cup Ah)$ are provided, and the upper bound of the running time is set at 1.3 times the minimum running time. The dwell time at stations is assumed to be 0.5 min. The optimisation model allows 3 min shift in departure or arrival event times, i.e. $m_i = 3$ for all train events $i \in E$.(Wang et al., 2022)

6.2. Passenger groups and passenger parameters

6.2.1. Passenger groups

The passenger OD demand is calculated based on the average morning peak passenger flow in the study region in 2019. To avoid excessively long computation times due to large data volumes, the case study ultimately selects the passenger flow data for one hour, from 7:00 to 8:00 (covering two cycles), as the passenger OD demand data in the case study. During this period, a total of 20,081.5 passengers are included. The OD demand distribution is shown in Figure 6.4, and there is significant passenger traffic between core interchange stations.

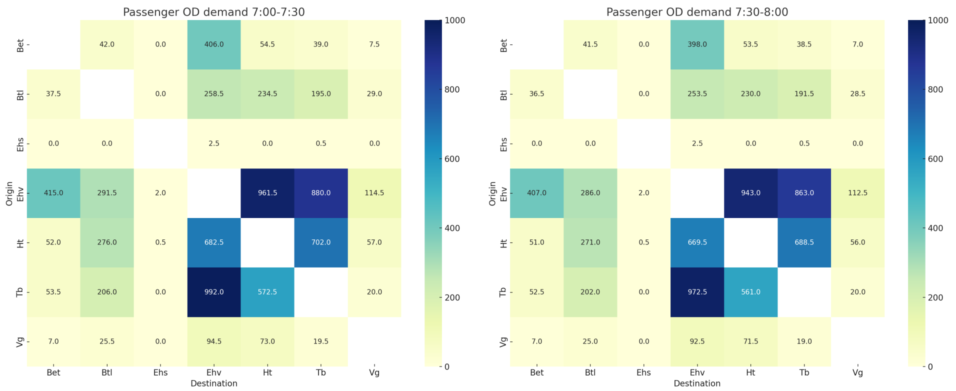

Figure 6.4: Passenger origin and destination distribution

With this passenger set, we need to create passenger groups according to the following assumptions:

1. The passenger flow distribution within one hour follows a uniform distribution.
2. Passengers are divided into five groups per OD with 6-minute intervals within one cycle.

Therefore, within the study period, 20,081.5 passengers are divided into 420 groups based on their OD k and arrive-at-origin time s. An example of passenger groups is given in Table 6.1.

Table 6.1: Passenger group example

Group ID	Origin	Destination	Arrive-at-origin time (min)	Flow	Cycle
0	Ht	Ehv	0	137	0
1	Ht	Ehv	6	137	0
2	Ht	Ehv	12	137	0
3	Ht	Ehv	18	137	0
4	Ht	Ehv	24	137	0
5	Ht	Vg	0	11	0
6	Ht	Vg	6	11	0
7	Ht	Vg	12	11	0
8	Ht	Vg	18	11	0
9	Ht	Vg	24	11	0

Noticeably, only the OD demand within this small area is considered in this case study. However, in real-world operations, some of the lines in the network shown in Figure 6.1 extend to other cities, such as Breda, Utrecht, and even The Hague. This means that when considering passenger delays, many passengers whose journey starts or ends outside of this region are overlooked. Consequently, during actual operations, the total delay experienced by passengers under disturbance scenarios would be greater.

6.2.2. Passenger path set

This section will use the DFS algorithm mentioned in section 4.3 to generate a set containing all possible paths between OD pairs. By inputting the network information and passenger activities B, we obtain the path set Π_k for each OD pair k represented in the form of passenger activity sequences. The exact path-finding process can be found in section 4.3.

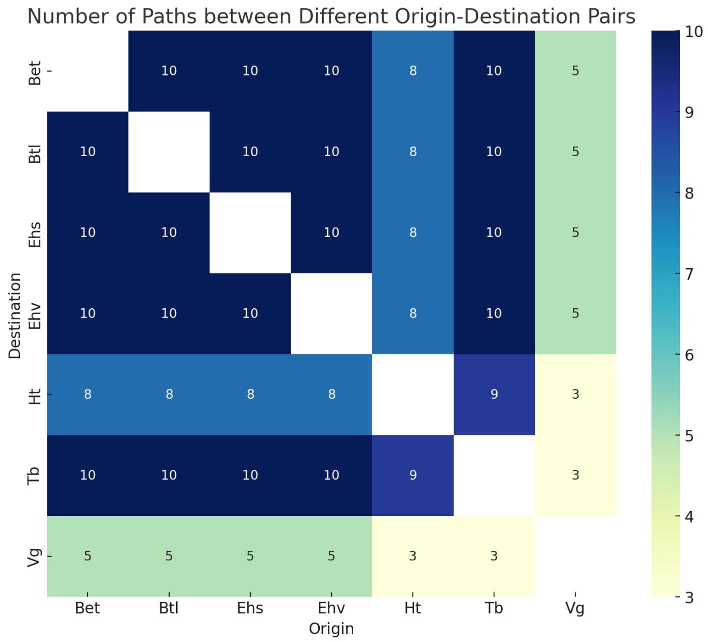

Figure 6.5: Number of possible paths for each OD pair

Figure 6.5 illustrates the number of possible paths between each OD pair after the path generation process. It can be observed that the path sets between OD pairs contain up to 10 paths. All feasible paths are considered in the model because both the newly designed timetable and the realized timetable are determined by variables within the model. This indicates that the paths chosen by passengers may differ in various scenarios.

Table 6.2 shows some of the paths from `Ht` to `Tb`, which are composed of consecutive passenger activities. The lists of passenger events and activities from the case study can be found in Appendix B and Appendix C.

Table 6.2: Some possible paths from `Ht` to `Tb`

Origin	Destination	Paths
`Ht`	`Tb`	Path 1: (20, 21), (21, 152), (152, 153), (153, 154), (154, 155), (155, 156), (156, 157), (157, 158), (158, 159) Path 2: (20, 21), (21, 160), (160, 161) Path 3: (26, 27), (27, 152), (152, 153), (153, 154), (154, 155), (155, 156), (156, 157), (157, 158), (158, 159) ... Path 9: ...

Additionally, the feasibility of transfer connections needs to be emphasized. Since the timetables studied in this thesis are periodic, all connections are guaranteed to be feasible during both modelling and actual execution. Even if passengers miss a connecting train, they can still choose to take the same train in the next period, albeit with a longer transfer time.

6.2.3. Passenger generalised travel time

The weight factors in Equation 4.8 and Equation 4.9 are $\beta_w = 2.5$, $\beta_t = 2.5$, and $\beta_n = 10$ (Robenek et al., 2016, Binder et al., 2017). Specifically, β_w represents the weight for waiting times at origin stations, β_t denotes the weight for transfer times, and β_n signifies the penalty per transfer. For passenger transfer activities, the minimum transfer time is 3 minutes, while the maximum transfer time is 33 minutes. (Wang et al., 2022)

6.2.4. Train sets and passenger sets

Table 6.3 lists the sizes of the sets that are used in this case study. In subsection 4.2.1, the train arrival time is assumed to be the alighting time for passengers arriving at the station, and the departure time corresponds to the boarding time. Therefore, there is a clear correlation between the size of the sets of passenger events and train events and the passenger in-vehicle running activity and in-vehicle dwelling activity, which correspond one-to-one with train running and dwelling activities.

In Table 6.3, the train headway activity includes 72 headway activities for trains at stations, as well as 24 infrastructure headway activities for trains at junctions. In this small network, not all trains originate or terminate at the stations within this region; only four turnaround activities occur at `Ht` and `Ehv`. From the passenger's perspective, the size of the arrive-at-origin event set and the waiting-at-station activity set is equal to the size of the passenger group. Besides, the transfer activity set does not include the foolish transfer defined in subsection 4.2.2.

Table 6.3: The size of sets in the case study

		Train events		
Set	Arrival	Departure	Junction arrival	Junction departure
Size	30	30	20	20
		Train activities		
Set	Running	Dwelling	Headway	Turnaround
Size	30	14	72+24	4
		Passenger events		
Set	Arrive-at-origin		Boarding	Alighting
Size	420		30	30
		Passenger activities		
Set	Waiting-at-station	In-vehicle running	In-vehicle dwelling	Transfer
Size	420	30	14	40

6.3. Case A: model verification and validation case

To test if the passenger-centric robust timetabling model is constructed correctly and if it can perform its primary functionality, model verification and validation will be conducted in this section. Two simple scenarios will be randomly generated in section 6.3.1, and the train and passenger data will follow the data set introduced in section 6.1 and 6.2. During this process, the integrated optimisation model will be split and combined to verify each sub-model's functionalities. And its performance can be validated by its final objective value.

Section 6.3.2 describes using the passenger model and simulation model to generate the average passenger delay of passenger groups and train delay after experiencing disturbances under the OT. In section 6.3.3, the PESP model and train simulation are used to generate an operator-centric robust timetabling model to minimise the average train arrival delay. The generated timetable is then applied to the passenger model to calculate passenger-related indicators. The results produced by the final integrated optimisation model in this verification and validation case will be reported in section 6.3.4. This final model will be a PRT with the objective function of minimizing the average passenger delay.

6.3.1. Disturbance scenario generation

In the verification and validation case A, two random disturbance scenarios are generated, as shown in Table 6.4. In Scenario 0, 4 activities departing from `Tb` are considered to have initial delays, with delay values of either 3 or 5 (min). In Scenario 1, the activities with initial delays are all train running activities arriving at Tb. Generally, all initial delays occur in the first cycle (cycle 0).

Table 6.4: Disturbance scenario for verification and validation case

Scenario	Train ID	From_station & To_station	Activity Type	Initial Delay	h
0	SP6441	(Tb, Btl)	running	3	0
	SP6640	(Tb, Ht)	running	3	0
	IC1141	(Tb, Ehv)	running	5	0
	IC3640	(Tb, Ht)	running	5	0
1	IC3641	(Ht, Tb)	running	3	0
	SP6641	(Ht, Tb)	running	5	0
	IC1140	(Ehv, Tb)	running	3	0
	SP6444	(Btl, Tb)	running	5	0

The above disturbance scenario might have practical significance in situations where the railway near Tilburg station frequently undergoes brief maintenance or is subject to external disturbances. However, this case study is primarily intended to verify the model's functionality. Therefore, we will not explore its meaning, and it may not be associated with any actual events or data.

6.3.2. Verification and validation of passenger-centric model

In this section, we will use the OT provided by train operator NS to verify the passenger modelling part, which includes the passenger assignment model and the simulation model.

Input

The OT is shown in the event-activity graph with original event time $o_i \in O$; passenger set $p \in P$, passenger path set $\Pi_k \in \Pi$.

Model

In this case, since the timetable has been fixed as the OT, the PESP model is not used.

For the definition of passenger generalised travel time, the constraints structure can follow Equation 4.1-4.12 in section 4.4.

Equation 5.1-5.17 (except Equation 5.11) are used to simulate the delay propagation and the passenger path choice in the realized scenarios. The objective function of the model (Equation 5.18) is to get the minimum average passenger generalised travel time so as to figure out the realized selected path in the scenarios.

Additionally, if only the train simulation part (Equation 5.1-5.3) is considered in the model and the objective function (Equation 6.1) minimizes the average train arrival delay, we can calculate the average train delay from the operator's perspective. $D_{i,r,h}$ denotes the train arrival delay for an arrival event i in scenario r in cycle h and is calculated by the difference between the realized arrival time v_{irh} and scheduled arrival time x_i. $|N|$ is a parameter recording the total number of trains in the case study rail network.

$$\textit{Min} \sum_{i \in Ea} \sum_{r \in R} \sum_{h \in H} D_{i,r,h}/|N| \tag{6.1}$$

Output

In the case where Equation 5.18 is used as the objective function, after solving the aforementioned model, the optimal solution obtained is 2.98, which means that passengers will experience an average delay of 2.98 minutes. Figure 6.6 shows the average delay calculated for different passenger groups with the same OD. Since, in this case, all initial train delays are set within the period of 7:00-7:30, all passenger delays occur within this period. This indicates that the initial delay starting in the first cycle does not propagate to the next cycle, affecting passenger travel activities. Moreover, it is clear that passengers departing from station `Tb` in Scenario 0 and those arriving at `Tb` in Scenario 1 have experienced relatively more severe delays. This phenomenon in Scenario 0 is due to the initial delays of four trains departing from `Tb`, and all trains from `Tb` to `Ht` experienced initial delays within this cycle. The reasons for the huge delay around Station `Tb` are similar in Scenario 1. Generally speaking, refer to Figure 6.4 and Figure 6.6, it can be seen that the OD pairs with high delays have a large passenger flow (`Tb`-`Ht`), which corresponds to a higher average passenger delay.

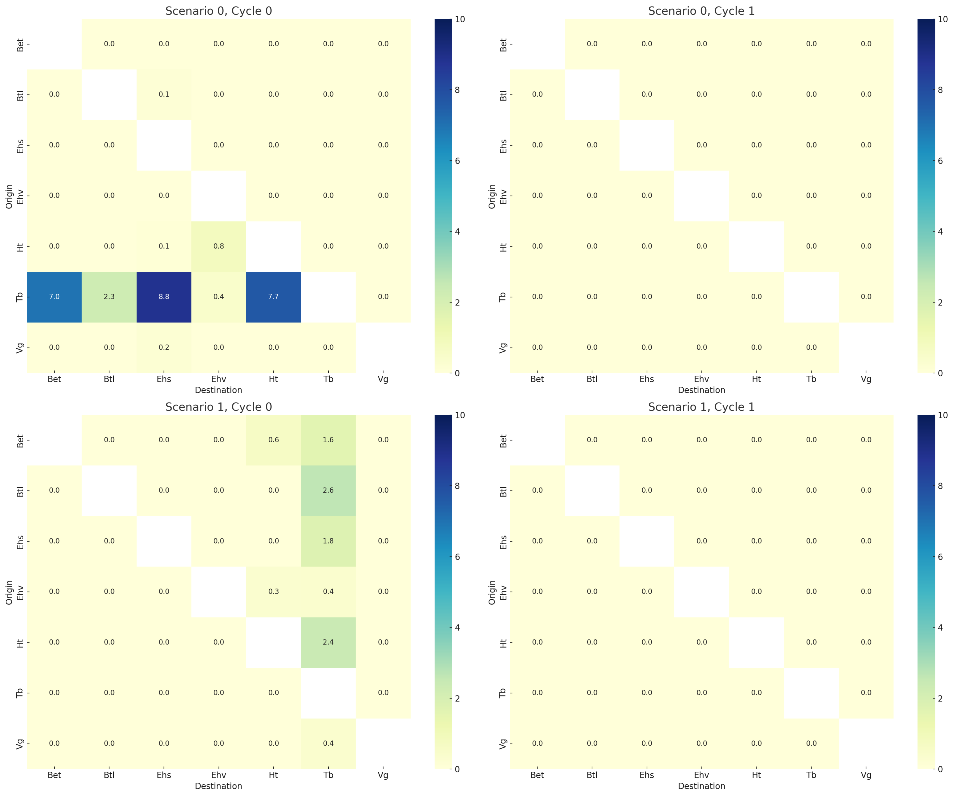

Figure 6.6: Average passenger delay in minutes for each OD pair in OT

Figure 6.7 shows the results generated by applying the train simulation model, which describes the arrival delays of trains at their dwell stations for each cycle in each scenario. If there is no data for a specific train at a particular station, it indicates that the train does not stop at that station. Overall, most delays are not propagated. In Scenario 0, `Ehv` and `Ht`, which are directly connected to the delayed running activity, experiences train delays of 3.9 minutes. In Scenario 1, the delays occur in the running activities of trains arriving at `Tb`, resulting in particularly severe train delays at station `Tb`, which is consistent with the analytical solutions. In summary, the total train arrival delay amounts to 10.20 minutes, and most of the disturbances can be mitigated by the robustness of OT.

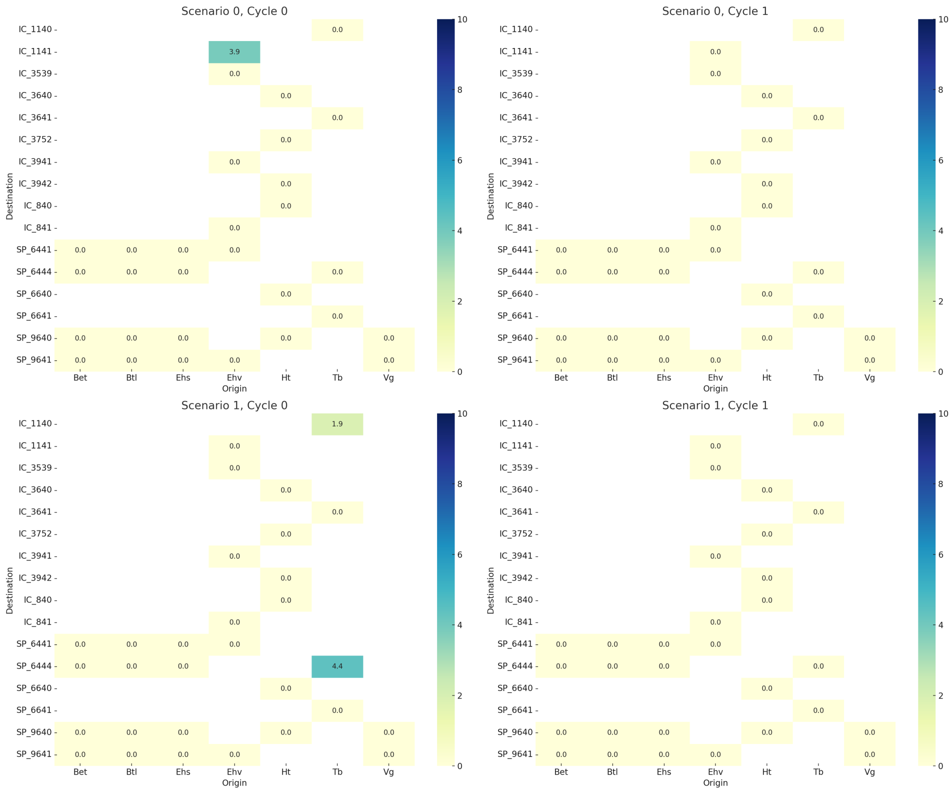

Figure 6.7: Train arrival delay in minutes in OT

Efficiency is also an essential element of train operations. Figure 6.8 presents the average passenger generalised travel time for passengers completing their journeys between different ODs in normal cases (without disturbance). Overall, journeys that require transfers, such as those between `Tb` and `Vg`, tend to have a higher passenger generalised travel time. In the end, it is calculated that the average generalised travel time for each OD pair is 46.13 minutes in the OT.

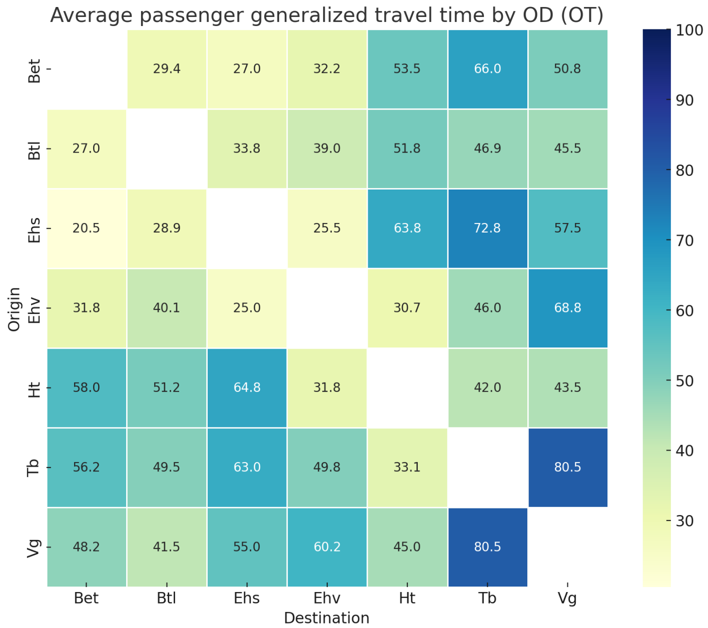

Figure 6.8: Average passenger generalised travel time in minutes in OT

6.3.3. Verification and validation of operator-centric model

This section will attempt to generate an operator-centric timetable based on the model proposed in chapter 3. The results of the generated timetable are analysed, and their rationality is examined.

Input

The OT is shown in the event-activity graph with original event time $o_i \in O$.

Model

In this case, an ORT will be generated. Therefore, the PESP model (Equation 3.1-3.4) has to be used to ensure the feasibility of the generated timetable. Note that under the supplement budget constraints, the method of allocating supplement budgets according to each train is adopted, which respects the supplement budget allocated to each train in the OT. Thus, Equation 3.4 is used as the supplement budget constraint. Additionally, the train simulation parts (Equation 5.1-5.3) need to be adopted to simulate the delay propagation of the currently generated timetable under the two disturbance scenarios.

According to the definition of robustness mentioned in section 1.3, as shown in Equation 6.2, the optimisation objective in this verification and validation model is to minimize the total train arrival delay within the experimental railway network.

$$\mathit{Min} \sum_{i \in Ea} \sum_{r \in R} \sum_{h \in H} D_{i,r,h} \tag{6.2}$$

Output

Figure 6.9 shows the optimise operator-centric timetable. Each coloured line represents a train service within a cycle, while the corresponding dotted lines indicate the train service in the original timetable. In the new ORT, the operational service lines of the trains do not intersect with the dotted lines indicating the original service. Therefore, there will be no cases where some events for the same train in the new timetable are delayed while others are postponed compared to the original timetable.

The diagram clearly shows that there is at least a one-minute interval in the arrival and passing times of the conflicting trains at the `Vga` and `Tba` junctions, which is a result of infrastructure constraints. Additionally, in the left subfigure of Figure 6.9, the `SP9641` (`Ht-Ehv`), represented by a purple line, has been shifted approximately 2 minutes to the right. This means that, compared to the OT, this train is required to operate about two minutes later. It is worth noting that its departure time is at the end of the cycle (29.9 min), which is highly likely because, the model has fixed the sequence of train events when designing the timetable in the cycle, preventing the departure event at `Ht` from being moved to the beginning of the cycle.

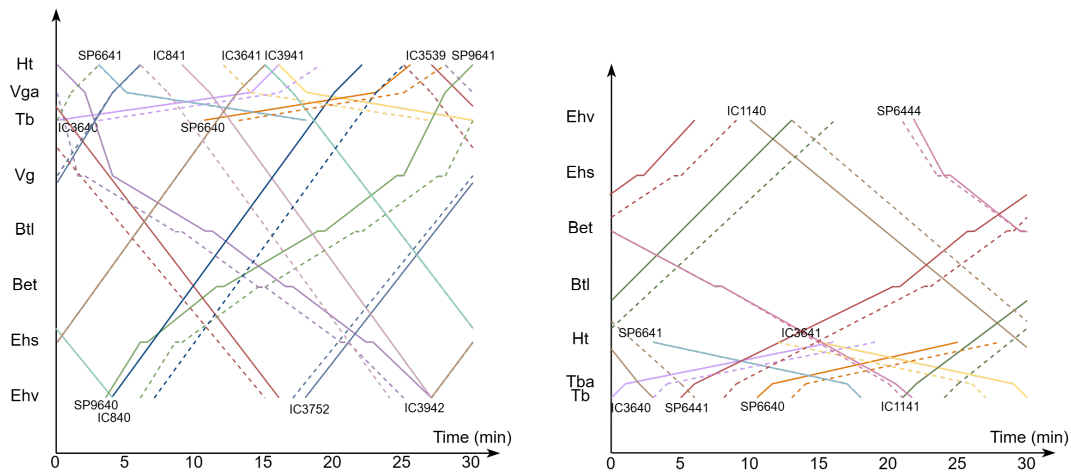

Figure 6.9: Comparison of train service shifts between ORT and OT

Figure 6.10 shows the average amount of time that event times for each train in the ORT have been adjusted compared to the OT. The orange and green bars represent trains that experienced initial delays of 3 and 5 minutes during their operation, while the light blue bars represent trains without any initial disturbance. More than half of the trains have been adjusted earlier by approximately 3 minutes. These adjustments have no obvious regularities and may be influenced by factors such as headway, infrastructure constraints, and the predefined order of events in the timetable creation process. The interaction between trains makes the timetable design process more complicated.

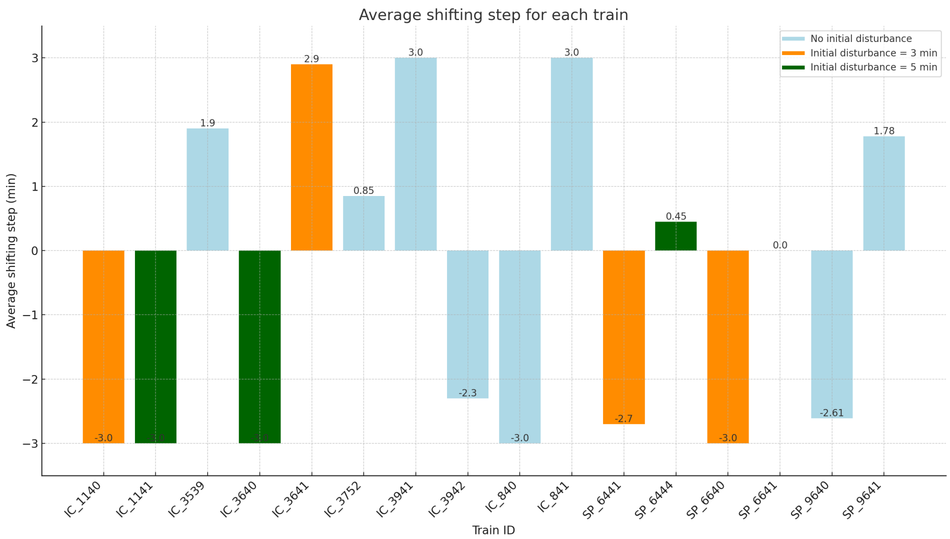

Figure 6.10: Average shifting step for each train in ORT

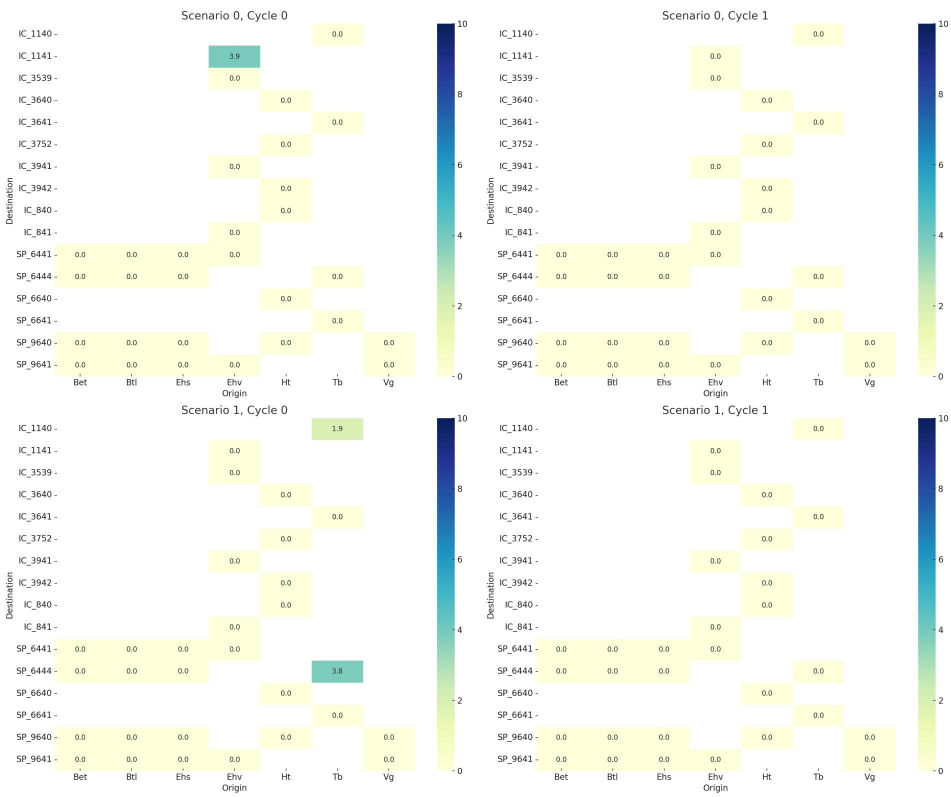

Figure 6.11: Train arrival delay in minutes in ORT

The optimised objective function value is 9.60, representing the total train arrival delay at the stations measured in minutes. Their distribution is shown in Figure 6.11. In comparison to Figure 6.7, it can be observed that in Scenario 1, the delay for `SP6444` arriving at `Tb` station has been reduced. This improvement is likely the result of optimising the supplement allocation for each train.

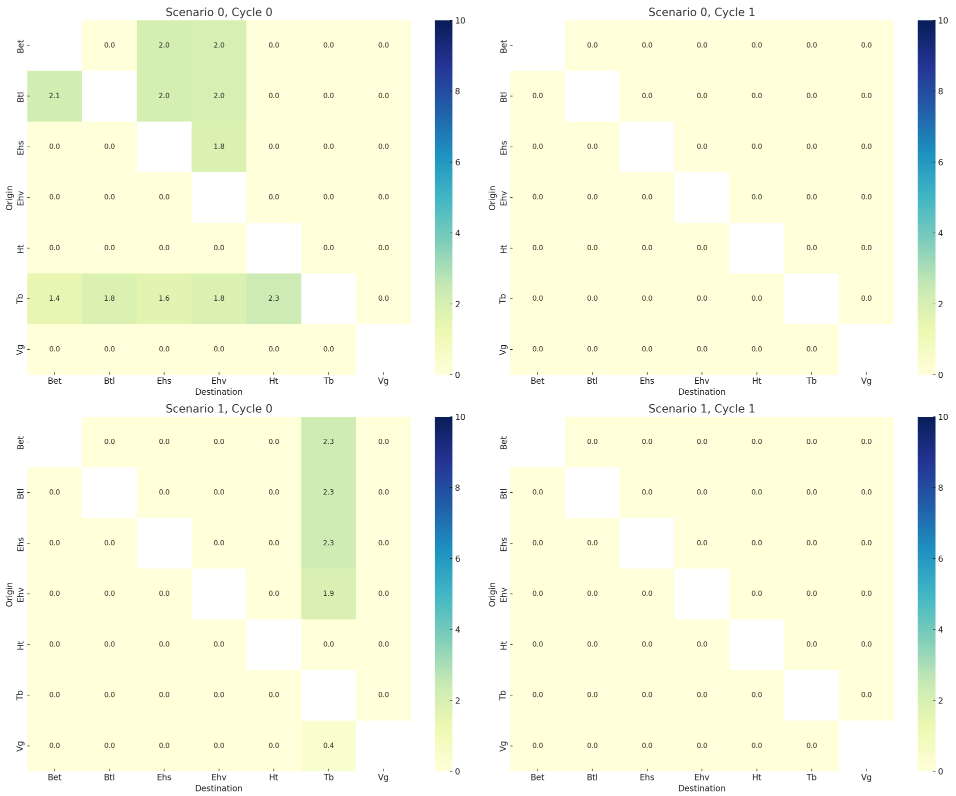

Figure 6.12: Average passenger delay in minutes for each OD pair in ORT

Suppose passenger-related information is input, and the model described in the previous section is applied. In that case, the average passenger delay in the ORT can be obtained as 2.33 min. This shows a slight improvement compared to the OT. This indicates that the operator-centric model has limited ability to reduce passenger delays in disturbance scenarios. From Figure 6.12, the delayed passengers are often distributed at `Tb` station and nearby stations, which corresponds to the distribution of train delays.

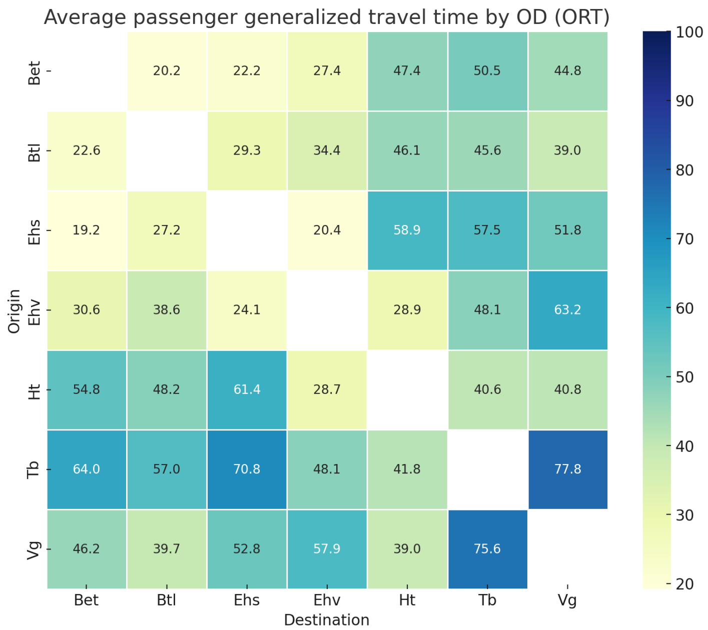

Figure 6.13: Average passenger in minutes generalised travel time in ORT

Regarding the efficiency of the train timetable, the average generalised travel time between each OD pair is even smaller, reaching 44.50 min, as shown in Figure 6.13. This is explainable because, in this verification and validation case, only a very limited disturbance scenario has been considered. In contrast, when designing the OT, it is possible that the designer did not consider the robustness of the timetable and the passenger travel efficiency but rather randomly allocated supplements to train activities.

6.3.4. Verification and validation of passenger-centric robust timetabling model

This section will verify the most important model proposed in this thesis, which is also the final contribution. This requires using all previously mentioned train, passenger and simulation models, resulting in a larger computational workload.

Input

The OT is represented in the event-activity graph with original event time $o_i \in O$, passenger set $k, s \in P$, and passenger path set $\Pi_k \in \Pi$.

Model

The constraints introduced in chapter 3, chapter 4, and chapter 5 will be applied. Similarly to section 6.3.3, the supplement budget is also allocated according to the trains. The optimisation objective function is set as the same as Equation 5.18.

Output

Figure 6.14 illustrates the PRT, which results from slight adjustments based on the original train timetable.

The Figure 6.14 clearly demonstrates the application of infrastructure constraints. Like the ORT, the trains' operational trajectories do not intersect with the OT's operational trajectories for the same train.

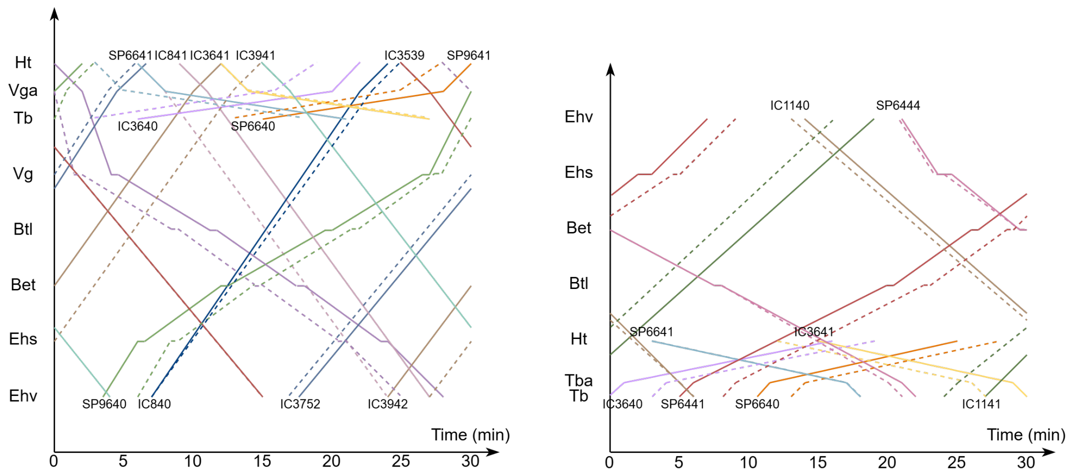

Figure 6.14: Comparison of train service shifts between PRT and OT

As shown in Figure 6.15, the average shifting steps for trains from OT to PRT are much smaller than those for ORT. Additionally, more train schedules have been delayed, likely to ensure that passengers do not miss their departure times or transfer connections.

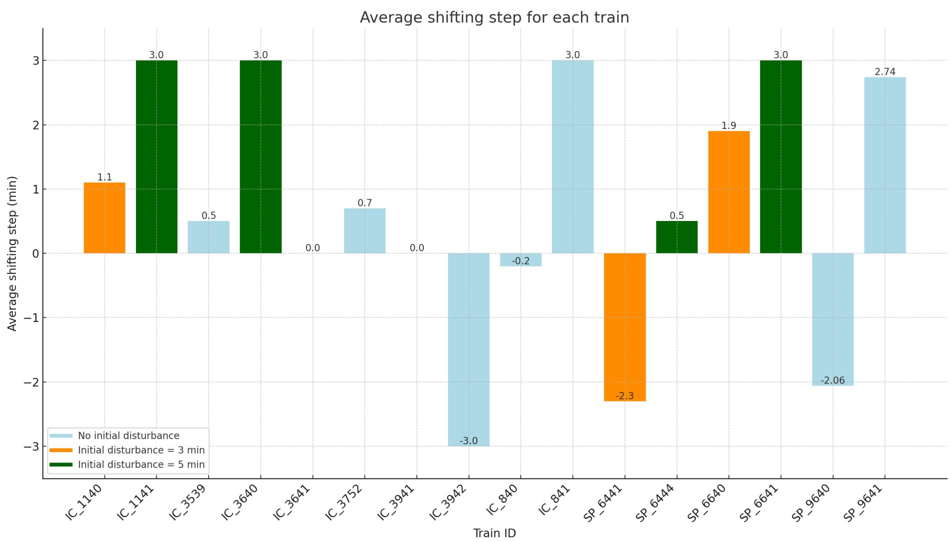

Figure 6.15: Average shifting step for each train in PRT

Under this timetable, the average passenger delay in the two scenarios reached 1.04 minutes (Figure 6.16). When referring to Figure 6.4, we notice that the passenger-centric robust model tries to aggregate the delays to the OD with lower passenger demand, such as `Ehs-Bet` and `Tb-Ehs`. This indicates that the PRT established using the model proposed in this thesis can reduce the passenger generalised travel time in specific delay scenarios. From the operator's perspective, the train arrival delay is higher with this timetable (10.20 minutes), and its distribution is shown in Figure 6.17.

In the passenger-centric model, applying more delays to OD pairs with smaller passenger flows significantly improves the optimisation objective, which may naturally sacrifice a certain amount of train arrival delay. However, in this small verification and validation case, the disturbances are not so severe that the average train arrival delay remains the same as that of OT.

The average passenger generalised travel time for each OD pair is 44.48min in this case (Figure 6.18), which is similar to the results of the other two timetables.

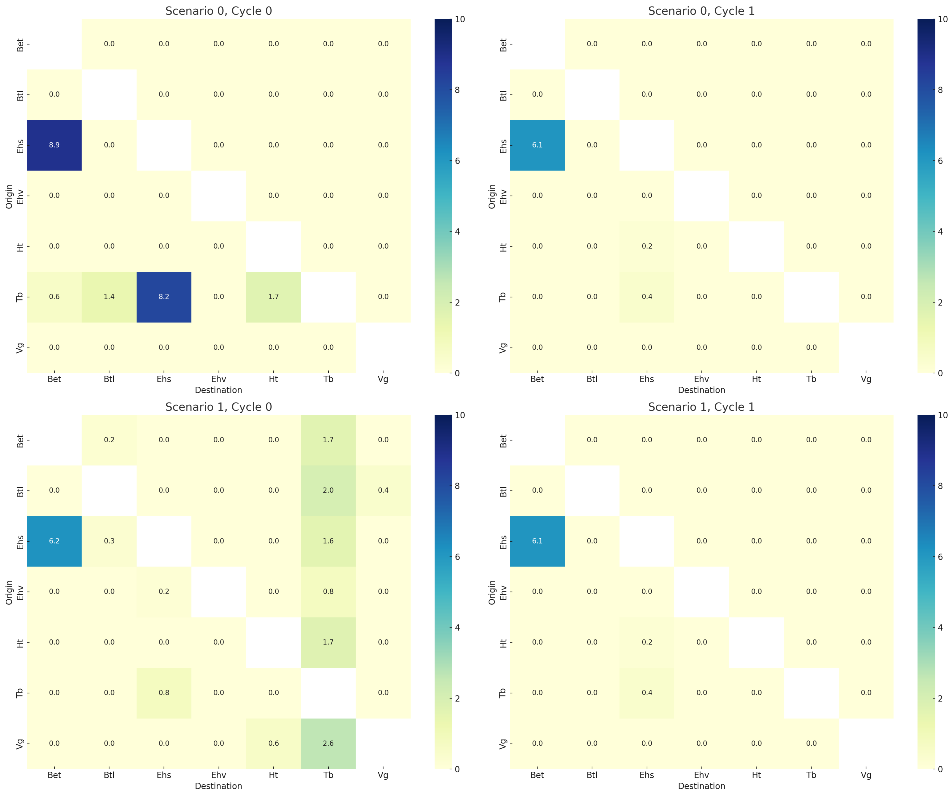

Figure 6.16: Average passenger delay in minutes for each OD pair in PRT

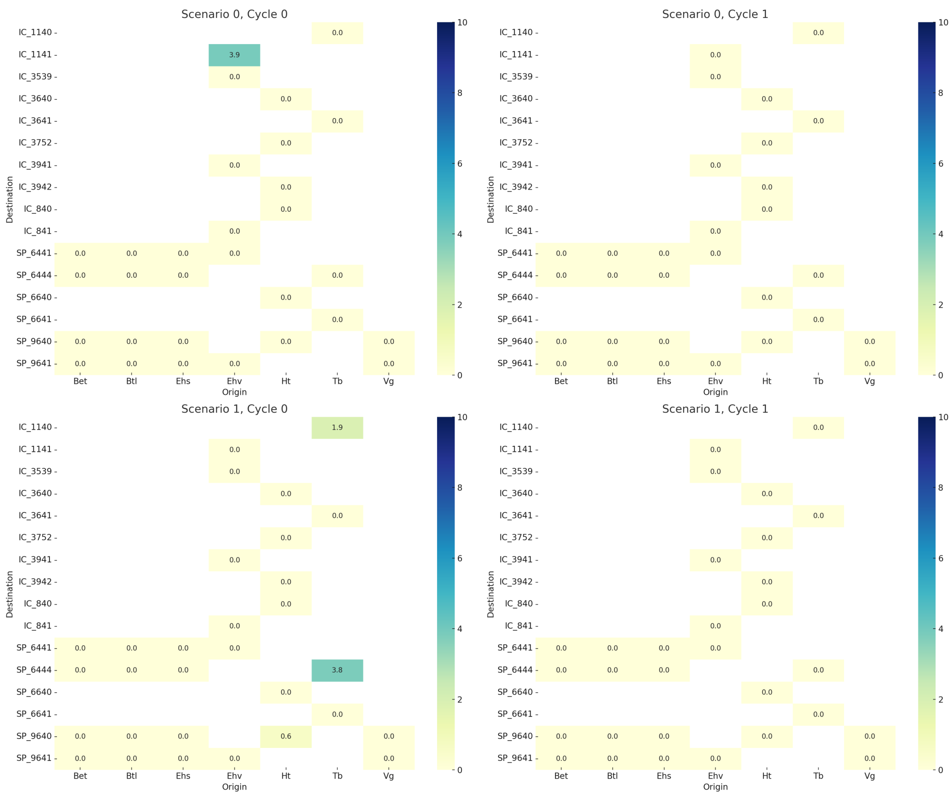

Figure 6.17: Train arrival delay in PRT

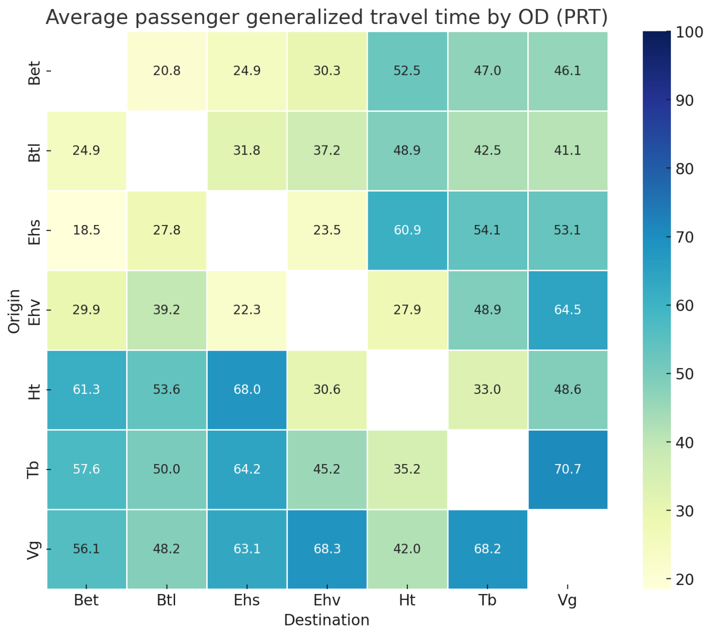

Figure 6.18: Average passenger generalised travel time in minutes in PRT

6.3.5. Conclusion

Case A divides the model into four parts and then reintegrates them, achieving different functionalities. These designed experiments are used to verify and validate each part of the model. Based on the output results from the two combined models and the final model, the models are correctly constructed. For example, in the PRT, ODs with lower passenger flows experience greater delays (`Ehs`-`Bet`). In contrast, in the ORT, all train delays are treated equally, leading to a more even distribution of passenger delays. This outcome is determined by the results of the passenger path allocation and the optimisation objective.

Based on the outputs(Table 6.5), all the results are consistent with the analytical solutions and aligned with the previous expectations. Among the three timetables, OT performs the worst in handling the two disturbance scenarios in this case. Even in normal cases without disturbances, the average generalised travel time for each OD pair is the longest. For the ORT, the total train arrival delay is the smallest, as this is the primary optimisation objective. An unexpected benefit is the improvement in timetable efficiency compared to OT. PRT, on the other hand, focuses on reducing average passenger delay during disturbances, successfully reducing delays to about one-third of the original level. Therefore, it is concluded that the passenger-centric robust timetabling model works as intended and can achieve the functionality of the model.

Table 6.5: Comparison among OT, ORT and PRT in case study A

Timetable	Average passenger delay (min)	Total train arrival delay (min)	Average generalised travel time (min)
OT	2.98	10.20	46.13
ORT	2.33	9.60	44.50
PRT	1.04	10.20	44.48

6.4. Case B: impact of disturbance intensity on optimisation results

Due to the passenger path allocation in each scenario, the number of constraints and variables involved is enormous. Based on practical testing, it is known that when considering 30 scenarios, the model will include over one million variables, making solving it extremely challenging. If we want to apply this model to larger networks and longer operational periods in real-world applications, it will be very costly. Additionally, more constraints need to be considered in practical scenarios. Therefore, we must consider whether this model has significant practical application or discuss the improvement of robust timetables derived from this model compared to the OT in different scenarios.

This chapter will explore the effectiveness of the passenger-centric robust timetabling model in constructing timetables under various disturbance scenarios. It will compare average passenger delay and efficiency indicators with the initial timetable. Section 6.4.1 explains the generation and selection of disturbance scenarios in this case. In contrast, sections 6.4.2 and 6.4.3 discuss the results generated under scenarios with different disturbance frequencies and severities, respectively.

6.4.1. Disturbance scenarios generation

This section will introduce the scenario generation process, and the parameter settings for each scenario vary according to the disturbance intensity, including the frequency and severity of disturbances and the specific scenario generation process.

As previously mentioned, solving the passenger-centric robust timetabling model requires substantial computational effort, so the number of scenarios selected must take into account the complexity of solving the model. Even worse, in this case study, very little information is available regarding the disturbance frequency or historical data of the trains within this network. Consequently, we have to generate the scenarios by inserting some disturbance values into specific train activities following a certain regulation or distribution. However, under these conditions, having too few scenarios will result in model outcomes that are not representative and lack statistical significance, thereby affecting the accuracy of the results analysis. Therefore, after multiple preliminary trials, we choose a set of 15 disturbance scenarios as the foundation for developing a PRT. The generation of these disturbance scenarios follows the regulations outlined below:

1. Only the train running activities will be disturbed in this case, and it does not consider the delays in dwelling, headway or turnaround activities.
2. The possibilities of primary delay in each running activity are the same with a mean of λ.
3. The amount of the primary delay lies in the range from 1 min to 5 min, and it keeps the same amount of delay in one timetabling process.

Nine sets of scenarios (Table 6.6) are designed to test the performance of the passenger-centric robust timetabling model under different frequencies and severity levels of scenarios, following the controlled variable method. The controlled variable method involves keeping all variables constant except for one, which is deliberately changed to observe its effect. Based on these scenarios, nine PRTs are developed in total. The optimised results are compared with the average passenger delay when adhering to the OT. Additionally, the efficiency of the robust timetable is also compared.

Table 6.6: Experiments design in case study B

Group	Experiment ID	Frequency(λ)	Severity(min)	Number of scenarios
A	1	0.10	5	15
	2	0.15	5	15
	3	0.20	5	15
	4	0.25	5	15
A/B	5	0.30	5	15
B	6	0.30	4	15
	7	0.30	3	15
	8	0.30	2	15
	9	0.30	1	15

It should be noted that, in the Eindhoven-Den Bosch-Tilburg case discussed in this paper, the cycle of the train schedule is 30 minutes.

After designing the disturbance frequency and severity for each scenario within each experiment, we can assign a random number between 0 and 1 to each running activity that can be disturbed. This number is then compared to the frequency value of that experiment. If the random number is less than the frequency value, then there is an initial delay for that activity in this scenario. The value of this delay corresponds to the severity designated for that experiment. After generating 15 scenarios for an experiment using this method, these scenarios must be recorded and then used as disturbance scenarios in the passenger assignment and simulation model that runs under OT.

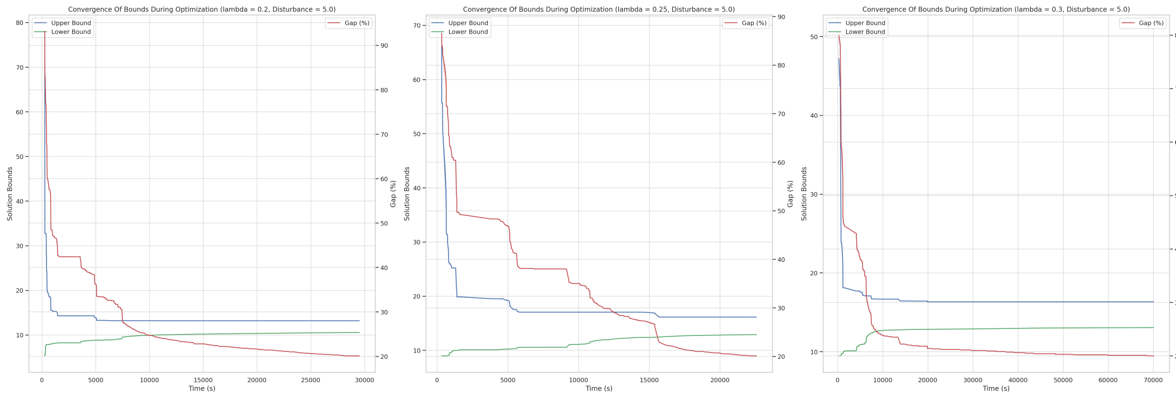

Figure 6.19: Convergence of the solution bounds during the optimisation process in three experiments

Finally, in addition to generating experimental scenarios, the target gap between the upper and lower bounds in the feasible region needs to be discussed. As the optimisation model approaches convergence during its operation, spending more time results in a relatively small improvement in the optimised result, as shown by the lower bound continuously approaching the upper bound. Therefore, a target gap needs to be defined. After multiple tests and trials, as shown in Figure 6.19, the rate of change in the gap (red line) decreases over time as it approaches 20%, and the upper bound (blue line) remains almost unchanged after the gap drops below 30%. This indicates that spending more time does not significantly improve the quality of the solution. Consequently, this gap is ultimately set at 20%. With this gap, the results obtained will not differ significantly from the potential optimal results, nor will they consume too much computation time.

6.4.2. Experiments on disturbance frequency

This section will conduct an experiment on Group A as shown in Table 6.6, analyzing the performance of PRTs under different disturbance frequencies and their improvements compared to the OT.

These five experiments utilize 15 scenarios and set the target gap between the upper bound and lower bound at 20%. Each disturbance lasts 5 minutes, but the frequency of disturbances varies. By applying the passenger-centric robust timetabling model, new timetables and average passenger delays can be obtained within a computation time ranging from 8 to 24 hours.

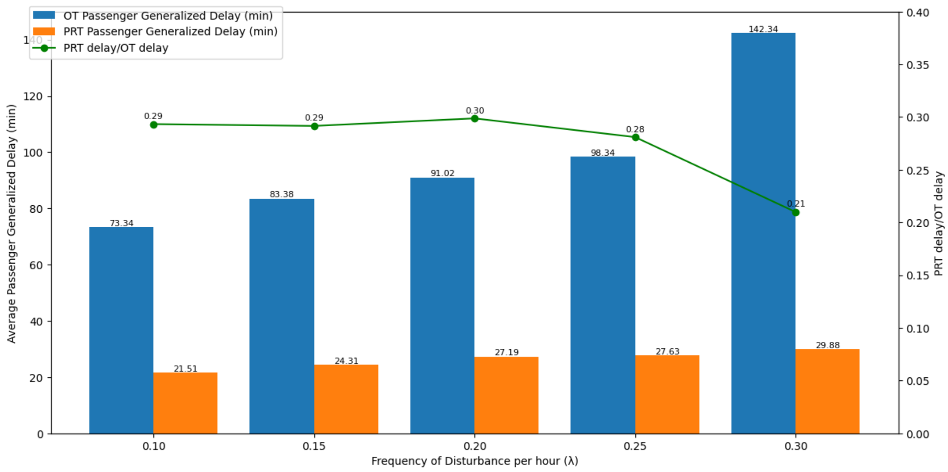

Figure 6.20: Comparison on average passenger delay between OT and PRT in frequency experiment

Accordingly, the same disturbance scenarios will be put in the OT and output the average passenger delays. Figure 6.20 tells the results of comparing the indicator of OT and PRT. As the probability of disturbances increases, the average passenger delay shows a noticeable upward trend, which is more pronounced under the OT. At a disturbance frequency of 0.30 interruptions per hour, the total sum of average passenger delays across 15 scenarios reaches 142.34 min, meaning that each passenger experiences an average delay of about 10 minutes per scenario. In contrast, under the five PRTs, the average total delay per passenger in the corresponding experiments is reduced to less than 30 min, indicating a significant improvement in the optimisation target.

The green line represents the ratio of the target indicator between PRT and OT. The smaller this ratio, the greater the improvement in the target value after applying this model. Overall, this ratio shows a decreasing trend. Particularly, when the disturbance frequency increases from 0.25 to 0.30, the passenger delay under OT shows a significant increase. However, the PRT obtained through the passenger-centric robust timetabling model can control this value to below 30 min.

In summary, it can be inferred that the model proposed in this paper significantly improves passenger delay, especially when disturbances are frequent.

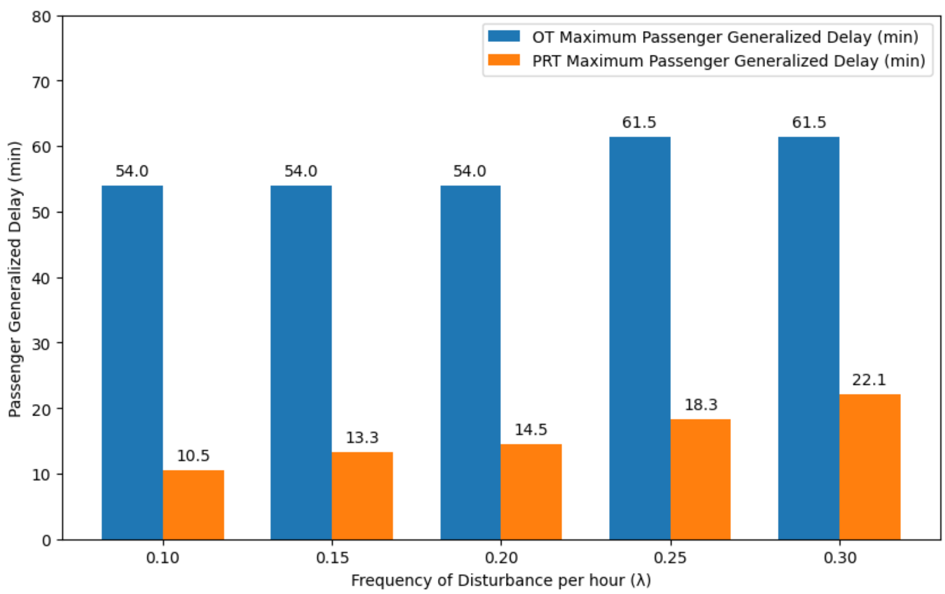

Figure 6.21: Comparison on maximum passenger delay between OT and PRT in frequency experiment

Figure 6.21 gives the maximum passenger delay. For OT, the maximum delay for an OD pair is 54 minutes at frequencies between 0.10 and 0.20 and 61.5 min at frequencies of 0.25 and 0.30, with starting and ending points respectively at `Tb` and `Vg`. This is related to additional waiting times caused by missed transfer connections. The newly designed PRT, however, can effectively reduce the maximum passenger delay, keeping it around 20 minutes. Even for the unluckiest passengers, this delay amount is more manageable than that under OT.

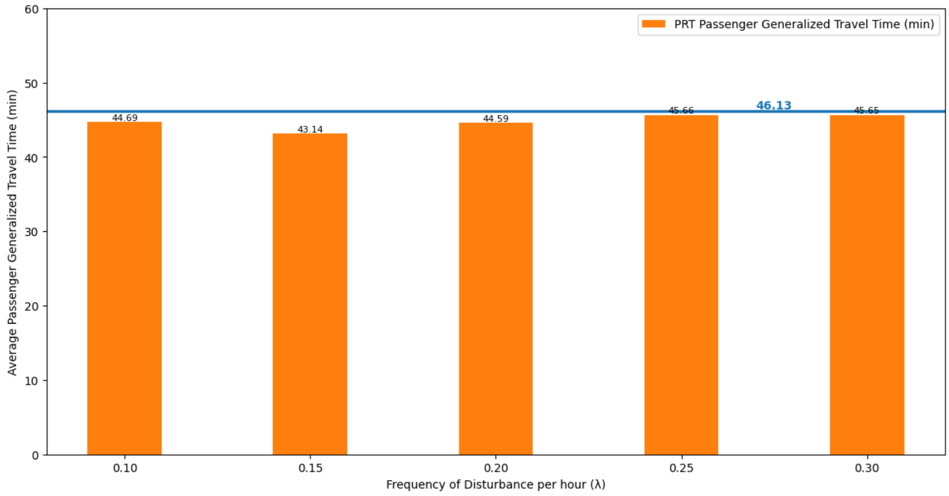

Figure 6.22: Average passenger generalised travel time in frequency experiment

Figure 6.22 reflects the average passenger generalised travel time for all OD pairs under OT (46.13min) and PRT, without any disturbances. Overall, the efficiencies of all PRTs are better compared to OT. This may be due to differences between the disturbance scenarios considered in the design of OT and those we considered in this case, or it may take into account other operator-related factors. Additionally, the orange bars in Figure 6.22 indicate that the differences between all timetables are not significant. This is likely because, as restricted in chapter 3, the adjustment window of the new PRT relative to the OT is limited, and each train's supplement budget is fixed.

6.4.3. Experiments on disturbance severity

This chapter uses the same methods and procedures as referenced in subsection 6.4.2 to analyze several experiments in Group B. The experiments in Group B will utilize different amounts of delay while keeping other parameters consistent to explore the performance of the model and the quality of the corresponding PRT under various severity levels.

The experiment examines five levels of severity on different activities, ranging from 1 min to 5 min, and Figure 6.23 compares the average passenger delays with OT and PRTs.

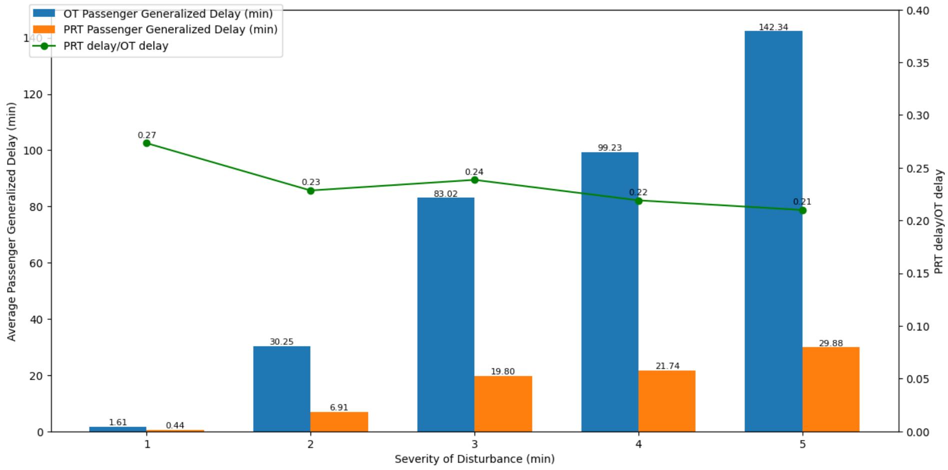

Figure 6.23: Comparison on average passenger delay between OT and PRT in severity experiment

The blue bars represent the OT passenger delay in minutes for each severity level. As the severity of the disturbance increases, the delay also increases significantly, particularly noticeable in the 5-minute disturbance experiment, with an average delay of approximately 142 minutes through 15 scenarios. However, when the trains run under PRTs the passenger delays go down. The green line represents the ratio of PRT delay to OT delay. This ratio decreases as the severity increases, demonstrating that the relative effectiveness of PRT increases with higher levels of disturbance.

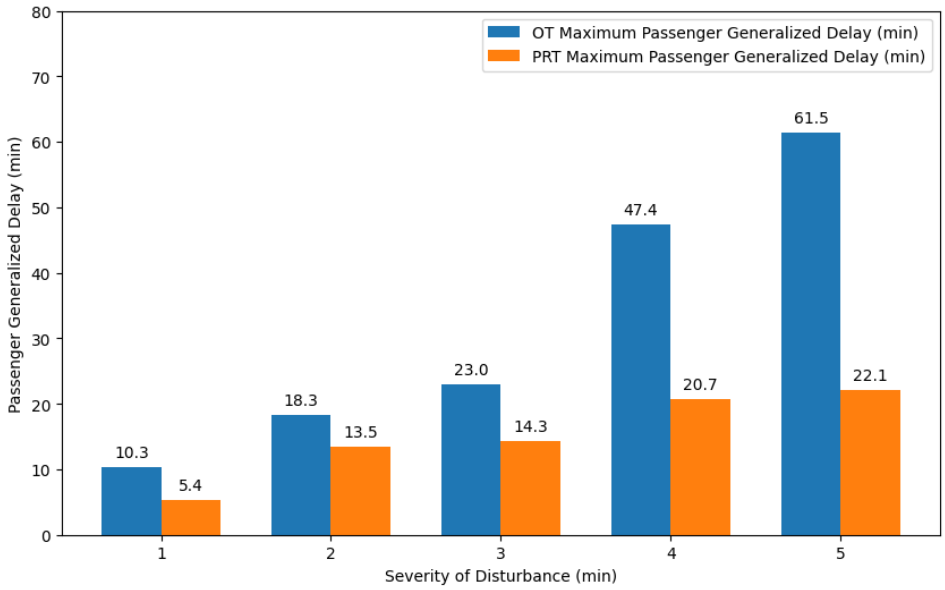

Figure 6.24: Comparison on maximum passenger delay between OT and PRT in severity experiment

From the perspective of each group of passengers, whenever a disturbance occurs, the greater the delay, the greater the maximum delay experienced by the passenger group (Figure 6.24). The implementation of PRT can significantly alleviate the worst-case passenger delay situations accordingly.

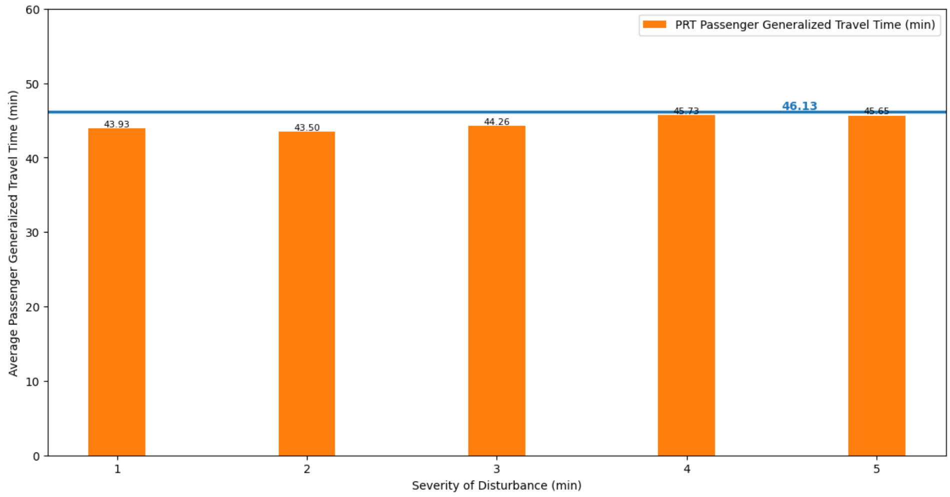

Figure 6.25: Average passenger generalised travel time in severity experiment

The patterns shown in Figure 6.25 and Figure 6.22 are similar, with all PRTs generated through a PRT exhibiting higher efficiency than OT, and all are very close in performance. Additionally, when dealing with cases with higher levels of severity, the efficiency of PRT will decrease slightly. However, this seems to be a negligible pattern.

6.4.4. Conclusion

This case study explores the effectiveness of the passenger-centric robust timetabling model proposed in this thesis under scenarios with different levels of disturbance frequency and severity and evaluates the performance of the corresponding generated PRTs. By comparing nine experiments divided into two groups, we learn that all PRTs have an absolute advantage over OT, which is more pronounced at higher frequencies and with more severe delays. Additionally, in Case B, the maximum passenger delay and the average passenger generalised travel time among all OD pairs are discussed to examine the distribution of passenger delays and the efficiency of train timetables to some extent. These indicators show significant advantages over those of OT. Based on the delay ratio of PRT to OT under different experiments, it can be seen that as disturbances become more intense, the effectiveness of PRT improves.

However, the limitations of Case B are also quite evident. Due to the large computational scale (50720 continuous variables, 210360 integer variables and 44920 binary variables), there are only 15 scenarios, but it takes nearly 20 hours to obtain PRT. Too few scenarios under the random disturbance scenario generation method can lead to deviations from expected outcomes. This is also a limitation of the model, as it struggles to consider more scenarios to ensure that the timetable possesses the desired robustness.

6.5. Conclusion

This chapter introduces two case studies conducted using the passenger-centric robust timetabling model proposed in this paper. Both studies are based on the railway network data and passenger flow data of the Eindhoven-Den Bosch-Tilburg region in the Netherlands.

Section 6.1 primarily describes the study area, which includes seven train stations and 16 lines, and also takes into consideration the headway constraint issue at Tilburg junction (`Tba`) and Vught junction(`Vga`). For passengers (section 6.2), all passengers are divided into 420 passenger groups based on OD and their arrival times at the origin, and parameters for passenger generalised travel time denoted as β are defined.

Case A (section 6.3) serves as the verification and validation case for the model, dividing the model into four parts and then reintegrating them to achieve different functionalities. These designed experiments are used to verify each part of the model. It considers two simple disturbance scenarios and compares indicators such as average passenger delay, total train arrival delay, and average generalised travel time under these scenarios. Ultimately, this model is deemed to be consistent with expectations.

The purpose of section 6.4 is to explore under which scenarios the model proposed in this paper shows more improvements relative to OT, and to test the model under more random conditions. However, due to limited computing power, this section only randomly generated 15 scenarios for each experiment and conducted the experiments from both frequency and severity perspectives. The results indicate that, based on the three indicators of average passenger delay, maximum passenger delay, and average passenger generalised travel time, PRTs consistently performed better than OT.

Overall, the PRT proposed in this thesis is considered to have been correctly verified and can be used to improve the train timetables of the Eindhoven-Den Bosch-Tilburg region for 2019 under the given nine experimental scenarios groups. This will make the model potentially valuable for practical applications in larger and more varied scenarios.

7 Conclusions

In this thesis, we have developed a robust timetabling model that focuses on the delay of passengers. The model is based on stochastic optimization and aims to minimize the average generalised travel delay for each group of passengers under specific disturbance scenarios. The conclusions can be found in section 7.1, which addresses the research questions. In section 7.2, we discuss the limitations of this model and the case study. Following that, we propose some recommendations and possible future works.

7.1. Conclusion

Reviewing section 1.2, three sub-questions have been proposed for the main research question of this paper. This section will address these three questions one by one.

How to define the robustness of a timetable in a passenger-centric way?

In this thesis, a railway timetable is considered robust if it can help mitigate the additional generalised travel time experienced by passengers when small disturbances occur. From the passengers' perspective, generalised travel time consists of four factors: waiting-at-station time, in-vehicle time, transfer time, and the number of transfers. Generalised travel time is the weighted sum of these four factors, and the passenger assignment process will follow the generalised travel time of each path in the generated path set.

How to incorporate the passenger behaviour into a robust timetabling scheme?

First, before the mathematical modelling, a literature review reveals that operator-centric robust timetabling already has a mature system and can be implemented. These modelling methods often use the approach of converting train timetables into an event-activity graph. Although passenger activities are more complex than train activities, in fact, passenger activities are closely tied to the train's dwelling activity. First, we divide passengers into a series of groups based on OD and arrive-at-origin time. All passengers in the same passenger group share the same boarding event time, which is the departure time of that train. Alighting events are recognized as arrival events at the transfer or destination station. Based on this assumption, a corresponding departure or arrival event can be found in the train event set E for all the passenger boarding or alighting events. In this way, i and j can also be used to represent the passenger event-activity graph. Moreover, these events are strongly associated with the train event-activity graph.

How to evaluate the quality of the new-designed timetable?

Regarding the problem of the quality of timetable creation, this thesis considers robustness and efficiency essential indicators for evaluating timetables, even though they are often challenging to balance. Therefore, the evaluation will primarily focus on these two aspects. The robustness evaluation follows the definition of passenger-centric robustness, focusing on analyzing the average passenger delay in disturbance scenarios. Additionally, in case study B, the distribution of passenger delays is briefly considered, comparing the maximum passenger delay under disturbance scenarios. The efficiency evaluation indicator is the shortest generalised travel time for passengers to complete their travel between OD according to the timetable in the normal case.

Considering this, an answer to the main question *How to design a high-quality passenger-centric robust periodic timetable?* can be formed by integrating the findings from the three sub-questions discussed above. Based on the performance of the passenger-centric robust timetabling model in Case Study A, it shows a significant improvement in the robustness evaluation indicator proposed in this thesis, average passenger delay, compared to other timetables. Although passenger transport efficiency decreases under the new timetable, it remains within an acceptable range.

7.2. Discussion

In this section, the thesis will be discussed. The discussion is split into two parts: subsection 7.2.1 introduces the limitations of the models and the limitations of the results found for the case study. In contrast, subsection 7.2.2 proposes a group of potential recommendations and future research related to this work.

7.2.1. Limitations

This thesis presents a passenger-centric robust timetabling model, which builds upon Kroon's operator-centric model (Kroon et al., 2008) by using average passenger delays under disturbance scenarios to measure timetable robustness. Although this is a novel idea and is well-articulated in this thesis, it still has many limitations due to constraints such as computational power and complexity.

Fixed order of train in PRT

In section 4.1, before establishing the train timetabling model, it is assumed that the trains follow the same sequence as the OT in the newly designed PRT. This means there will be no overtaking or changes in train departure order within the design of the timetable, significantly limiting the potential for timetable optimization.

Homogeneous passenger travelling preferences

In section 4.1, during the passenger assignment step, we assume that passengers follow the same preferences and rules to choose their paths, which may not fully align with real-world behaviour. Additionally, all passenger behaviours are simplified when calculating the generalised travel time, sharing the same β. This could lead to discrepancies between the passenger delays predicted during timetable design and the actual passenger delays, potentially failing to reflect passenger preferences accurately.

Fixed order of train in realizations

In chapter 5, it is mentioned that the simulation part of this model fixes the train running order and does not account for train cancellations. This means subsequent trains cannot pass any node ahead of preceding trains, even if the preceding train has experienced severe delays, inevitably leading to the propagation of delays. However, we clearly recognize that altering the train running order or even cancelling trains when they experience significant delays could mitigate the loss in objective value caused by severe delays. These dispatching methods might be applied in practical operations but are not considered in the model proposed in this thesis.

Cycle misalignment

In section 5.3, there is an issue with cycle misalignment in the calculation of realized transfer time in the current model. This misalignment causes discrepancies between the values calculated by formulas Equation 5.8 and Equation 5.11 when a delay in the preceding train causes passengers to miss the transfer to the subsequent train in the scheduled cycle. These discrepancies may also propagate to the subsequent calculation of passenger travel time. In this thesis, cycle misalignment does not affect the final results because passengers who miss transfers would have to wait longer to maintain their original travel path; if a shorter path is available, it would naturally be chosen instead. However, it cannot be denied that the impact of cycle misalignment would be amplified in cases where there is only one available path or very few alternative paths.

Limited scenario size

In section 6.4, we only generated 15 scenarios for each experiment when studying the model's performance under different intensities. While these results can indicate the trend of changes in the average objective value of passenger delay compared to on-time results to some extent, they may not be broadly applicable. Increasing the number of scenarios would make the model more complex to solve. Therefore, in practical applications, historical disturbance data can be used to generate disturbance scenarios selectively. However, we could not use this method in Case B due to data limitations.

7.2.2. Recommendations and future works

In this section, several recommendations are given for future research.

Flexible train order extension

The timetable design discussed in this thesis assumes a fixed train order and allows for slight adjustments to event times based on the existing timetable. This means that the timetable design process follows several restrictions. However, in actual practice, if we overlook the limitations of infrastructure and rolling stock and prioritize improved passenger reliability, these assumptions could be loosened. Nevertheless, it's possible that relaxing these constraints could broaden the model's range of solutions.

In the realized timetable under disturbance scenarios, the thesis also assumes that adjustments to the running order of trains are not permitted. This assumption is based on the idea that any disturbances in these cases are expected to be minor, not exceeding five minutes. However, in the event of significant delays, especially when disruptions occur in specific areas, the optimization of results could be greatly improved through train cancellations or overtakes. I also think implementing a passenger-centric approach to train dispatching could be an intriguing topic.

Passenger travel behaviour

This thesis assumes that passengers' travel behaviour and path choice behaviour remain consistent when faced with normal and delayed timetables. However, passengers' attitudes towards path selection can change after seeing or experiencing train delays. For example, passengers might prefer a train with a longer generalised travel time but no delays over a train that has already been delayed due to concerns about further delays. I believe this would be an interesting topic for social science research.

Solving cycle misalignment

As mentioned in subsection 7.2.1, the realized passenger assignment model proposed in this thesis is not entirely accurate, as it still faces the issue of cycle misalignment. This issue needs to be addressed or reasonably avoided in the future.

Efficient solving methods

The main factor limiting the quality of the results in the case study discussed in this thesis is the slow solving speed, particularly when dealing with many scenarios. Besides, the current solving speed limits its potential application to larger railway networks. In future research, a vital issue will be using heuristic algorithms to find locally optimal or high-quality solutions quickly. Additionally, improvements can be made by identifying more concise and efficient modelling methods. Once the solving speed is significantly improved, we can consider applying these methods to broader and more complex railway networks, which will significantly enhance their practical value and effectiveness.

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A

Sets, variables and parameters in model

Set

- A: The activity set of train, represented by (i, j). $A = Ar, Ah, Ad, At$. Ar: Running activity of train. Ah: Headway activity of train. Ad: Dwell activity of train. At: Turnaround activity of train.
- B: The passenger activity set of passenger groups, represented by (i, j). $B = \{Bw, Bi, Bt\}$. Bw: Waiting-at-origin activity of passengers. Bi: In-vehicle activity of passengers. Bt: Transfer activity of passengers.
- Δ: The disturbance scenario set with disturbance elements δ_{ijrh}.
- E: The event set of the train, represented by i. $E = \{Ed, Ea, Ed', Ea'\}$. Ed: Departure event of the train at the station. Ea: Arrival event of the train at the station. Ed': Departure event of the train at the junction. Ea': Arrival event of the train at the junction.
- G: The passenger set in the research period H.
- H: The set for cycle h in Δ.
- N: The train set in the research network, with t_i indicating the train ID of event i.
- P: The passenger group set with elements denoted by p.
- R: The set for scenario in Δ, with elements denoted by r.
- Π: The whole passenger path set with $\Pi_k \in \Pi$.
- Π_k: The passenger set for passenger group with OD k with elements π. The first node in π is denoted by e_π^0.

Variables

- $a_{p,\pi}$: The binary variable to determine whether the passenger waiting-at-origin activity of passenger group p following path π crosses the cycle under the newly designed timetable. When $a_{p,\pi} = 0$, the waiting-at-origin activity falls in the same cycle as the passenger boarding activity, and vice versa.
- $a^*_{p,\pi,r,h}$: The binary variable to determine whether the passengers in group p have missed the boarding event in the newly designed timetable under disturbance scenarios. When $a^*_{p,\pi,r,h} = 1$, it means that if passengers want to follow path π, they have to wait another cycle compared to the designed timetable in scenario r in cycle h.
- $\alpha_{p,\pi}$: The binary variable to judge if passenger group p will choose the path π under newly designed timetable.
- $\alpha^*_{p,\pi,r,h}$: The binary variable to judge if passenger group p will choose the path π in scenario r in cycle h under realized timetable.

- b_{ij}: The binary variable to determine whether the passenger transfer activity $(i,j) \in Bt$ crosses the cycle. When $b_{ij} = 0$, the endpoint of the previous leg of the journey falls in the same cycle as the following one, and vice versa.
- b^*_{ijrh}: The binary variable to determine whether the passenger transfer activity $(i,j) \in Bt$ is moved to the next cycle in the realized timetable compared to the designed timetable in scenario r in cycle h.
- $d_{p,r,h}$: Passenger generalized travel delay for passenger group p in scenario r in cycle h under realized timetable.
- D_{irh}: Delay for the arrival event in scenario r in cycle h under realized timetable. Note that i is in the arrival event set as Ar.
- $\gamma_{p,\pi}$: Passenger transfer time for passenger group $p \in P$ selecting path π under newly designed timetable.
- $\gamma^*_{p,\pi,r,h}$: Passenger transfer time for passenger group p selecting path π in scenario r in cycle h under realized timetable.
- $\phi_{p,\pi}$: Passenger in-vehicle time for passenger group p selecting path π under newly designed timetable.
- $\phi^*_{p,\pi,r,h}$: Passenger in-vehicle time for passenger group p selecting path π in scenario r in cycle h under realized timetable.
- $\omega_{p,\pi}$: Passenger waiting-at-origin time for passenger group p selecting path π under newly designed timetable.
- $\omega^*_{p,\pi,r,h}$: Passenger waiting-at-origin time for passenger group p selecting path π in scenario r in cycle h under realized timetable.
- v_{irh}: The realized time in disturbance scenario scenario r in cycle h.
- x_i: The newly designed robust timetable, where x_i indicates the event time of i. Decision variable.
- ξ_p: The shortest passenger generalized travel time for passenger group p under newly designed timetable.
- $\xi^*_{p,r,h}$: The shortest passenger generalized travel time for passenger group p in scenario r in cycle h under realized timetable.
- $\overline{\xi}_{p,\pi}$: Passenger generalized travel time for p selecting path π under newly designed timetable.
- $\overline{\xi}^*_{p,\pi,r,h}$: Passenger generalized travel time for p selecting path π in scenario r in cycle h under realized timetable.

Parameters

- δ_{ijrh}: The disturbance (delay) of activity (i,j) in scenario r in cycle h.
- ϵ_π: Number of transfers for path π.
- $g_{p,h}$: The number of passengers in passenger group p in cycle h.
- l_{ij} & u_{ij}: The lower and upper bound of the duration of train activity $(i,j) \in A$.
- $\overline{l}_{ij}$ & $\overline{u}_{ij}$: The lower and upper bound of the duration of passenger activity $(i,j) \in B$.
- m_i: The adjustment window for each event $i \in E$.
- o_i: The event time in the original timetable for event i.
- q_{ij}: The parameter to judge the sequence activity i, j in one cycle in the original timetable. When $q_{ij} = 0$, it means the event i occurs before j, and vice versa.
- $Q_{i,\pi}$: An integer parameter used to record the number of periods spanned by event i in path π relative to its first boarding event e^0_π.
- T: The length of the cycle.
- Z_n: The buffer budget for the train $n \in N$.

B

Passenger Events

Event id	Event type	Train id	from station	to station
20	boarding	IC 3539	Ht	Ehv
21	alighting	IC 3539	Ht	Ehv
26	boarding	IC 841	Ht	Ehv
27	alighting	IC 841	Ht	Ehv
48	boarding	IC 3941	Ht	Ehv
49	alighting	IC 3941	Ht	Ehv
54	boarding	IC 3641	Ht	Tb
55	alighting	IC 3641	Ht	Tb
56	boarding	SP 6441	Tb	Btl
57	alighting	SP 6441	Tb	Btl
58	boarding	SP 6441	Btl	Bet
59	alighting	SP 6441	Btl	Bet
60	boarding	SP 6441	Bet	Ehs
61	alighting	SP 6441	Bet	Ehs
62	boarding	SP 6441	Ehs	Ehv
63	alighting	SP 6441	Ehs	Ehv
64	boarding	IC 1141	Tb	Ehv
65	alighting	IC 1141	Tb	Ehv
66	boarding	SP 9641	Ht	Vg
67	alighting	SP 9641	Ht	Vg
68	boarding	SP 9641	Vg	Btl
69	alighting	SP 9641	Vg	Btl
70	boarding	SP 9641	Btl	Bet
71	alighting	SP 9641	Btl	Bet
72	boarding	SP 9641	Bet	Ehs
73	alighting	SP 9641	Bet	Ehs
74	boarding	SP 9641	Ehs	Ehv
75	alighting	SP 9641	Ehs	Ehv
94	boarding	SP 6641	Ht	Tb
95	alighting	SP 6641	Ht	Tb
128	boarding	IC 840	Ehv	Ht
129	alighting	IC 840	Ehv	Ht
134	boarding	IC 3942	Ehv	Ht
135	alighting	IC 3942	Ehv	Ht
140	boarding	IC 3752	Ehv	Ht

Continued on next page

Event id	Event type	Train id	from station	to station
141	alighting	IC 3752	Ehv	Ht
146	boarding	IC 3640	Tb	Ht
147	alighting	IC 3640	Tb	Ht
152	boarding	SP 6444	Ehv	Ehs
153	alighting	SP 6444	Ehv	Ehs
154	boarding	SP 6444	Ehs	Bet
155	alighting	SP 6444	Ehs	Bet
156	boarding	SP 6444	Bet	Btl
157	alighting	SP 6444	Bet	Btl
158	boarding	SP 6444	Btl	Tb
159	alighting	SP 6444	Btl	Tb
160	boarding	IC 1140	Ehv	Tb
161	alighting	IC 1140	Ehv	Tb
162	boarding	SP 9640	Ehv	Ehs
163	alighting	SP 9640	Ehv	Ehs
164	boarding	SP 9640	Ehs	Bet
165	alighting	SP 9640	Ehs	Bet
166	boarding	SP 9640	Bet	Btl
167	alighting	SP 9640	Bet	Btl
168	boarding	SP 9640	Btl	Vg
169	alighting	SP 9640	Btl	Vg
170	boarding	SP 9640	Vg	Ht
171	alighting	SP 9640	Vg	Ht
172	boarding	SP 6640	Tb	Ht
173	alighting	SP 6640	Tb	Ht

C

Passenger Activities

Activity id	Activity type	Train id	to train id	from event	to event	from station	to station	min duration	max duration
0	run	IC 3539	[]	20	21	Ht	Ehv	18	24
1	run	IC 841	[]	26	27	Ht	Ehv	17	22
2	run	IC 3941	[]	48	49	Ht	Ehv	18	24
3	run	IC 3641	[]	54	55	Ht	Tb	14	18
4	run	SP 6441	[]	56	57	Tb	Btl	14	18
5	dwell	SP 6441	[]	57	58	Btl	Btl	0.5	1
6	run	SP 6441	[]	58	59	Btl	Bet	5	7
7	dwell	SP 6441	[]	59	60	Bet	Bet	0.5	1
8	run	SP 6441	[]	60	61	Bet	Ehs	5	7
9	dwell	SP 6441	[]	61	62	Ehs	Ehs	0.5	1
10	run	SP 6441	[]	62	63	Ehs	Ehv	4	5
11	run	IC 1141	[]	64	65	Tb	Ehv	21	27
12	run	SP 9641	[]	66	67	Ht	Vg	3	4
13	dwell	SP 9641	[]	67	68	Vg	Vg	0.5	1
14	run	SP 9641	[]	68	69	Vg	Btl	6	8
15	dwell	SP 9641	[]	69	70	Btl	Btl	0.5	1

Continued on next page

Activity id	Activity type	Train id	to train id	from event	to event	from station	to station	min duration	max duration
16	run	SP 9641	[]	70	71	Btl	Bet	5	7
17	dwell	SP 9641	[]	71	72	Bet	Bet	0.5	1
18	run	SP 9641	[]	72	73	Bet	Ehs	5	7
19	dwell	SP 9641	[]	73	74	Ehs	Ehs	0.5	1
20	run	SP 9641	[]	74	75	Ehs	Ehv	4	5
21	run	SP 6641	[]	94	95	Ht	Tb	14	18
22	run	IC 840	[]	128	129	Ehv	Ht	17	22
23	run	IC 3942	[]	134	135	Ehv	Ht	17	22
24	run	IC 3752	[]	140	141	Ehv	Ht	18	24
25	run	IC 3640	[]	146	147	Tb	Ht	15	20
26	run	SP 6444	[]	152	153	Ehv	Ehs	2	3
27	dwell	SP 6444	[]	153	154	Ehs	Ehs	0.5	1
28	run	SP 6444	[]	154	155	Ehs	Bet	5	7
29	dwell	SP 6444	[]	155	156	Bet	Bet	0.5	1
30	run	SP 6444	[]	156	157	Bet	Btl	7	9
31	dwell	SP 6444	[]	157	158	Btl	Btl	0.5	1
32	run	SP 6444	[]	158	159	Btl	Tb	12	16
33	run	IC 1140	[]	160	161	Ehv	Tb	21	28
34	run	SP 9640	[]	162	163	Ehv	Ehs	2	3
35	dwell	SP 9640	[]	163	164	Ehs	Ehs	0.5	1
36	run	SP 9640	[]	164	165	Ehs	Bet	5	7
37	dwell	SP 9640	[]	165	166	Bet	Bet	0.5	1
38	run	SP 9640	[]	166	167	Bet	Btl	6	8
39	dwell	SP 9640	[]	167	168	Btl	Btl	0.5	1
40	run	SP 9640	[]	168	169	Btl	Vg	5	7
41	dwell	SP 9640	[]	169	170	Vg	Vg	0.5	1
42	run	SP 9640	[]	170	171	Vg	Ht	5	6
43	run	SP 6640	[]	172	173	Tb	Ht	14	18
44	transfer	IC 3539	SP 6444	21	152	Ehv	Ehv	3	33
45	transfer	IC 3539	IC 1140	21	160	Ehv	Ehv	3	33
46	transfer	IC 841	SP 6444	27	152	Ehv	Ehv	3	33
47	transfer	IC 841	IC 1140	27	160	Ehv	Ehv	3	33
48	transfer	IC 3941	SP 6444	49	152	Ehv	Ehv	3	33
49	transfer	IC 3941	IC 1140	49	160	Ehv	Ehv	3	33

Continued on next page

Activity id	Activity type	Train id	to train id	from event	to event	from station	to station	min duration	max duration
50	transfer	IC 3641	SP 6441	55	56	Tb	Tb	3	33
51	transfer	IC 3641	IC 1141	55	64	Tb	Tb	3	33
52	transfer	SP 6441	IC 840	63	128	Ehv	Ehv	3	33
53	transfer	SP 6441	IC 3942	63	134	Ehv	Ehv	3	33
54	transfer	SP 6441	IC 3752	63	140	Ehv	Ehv	3	33
55	transfer	SP 6441	SP 9640	63	162	Ehv	Ehv	3	33
56	transfer	IC 1141	IC 840	65	128	Ehv	Ehv	3	33
57	transfer	IC 1141	IC 3942	65	134	Ehv	Ehv	3	33
58	transfer	IC 1141	IC 3752	65	140	Ehv	Ehv	3	33
59	transfer	IC 1141	SP 9640	65	162	Ehv	Ehv	3	33
60	transfer	SP 9641	SP 6444	75	152	Ehv	Ehv	3	33
61	transfer	SP 9641	IC 1140	75	160	Ehv	Ehv	3	33
62	transfer	SP 6641	SP 6441	95	56	Tb	Tb	3	33
63	transfer	SP 6641	IC 1141	95	64	Tb	Tb	3	33
64	transfer	IC 840	IC 3641	129	54	Ht	Ht	3	33
65	transfer	IC 840	SP 6641	129	94	Ht	Ht	3	33
66	transfer	IC 3942	IC 3641	135	54	Ht	Ht	3	33
67	transfer	IC 3942	SP 6641	135	94	Ht	Ht	3	33
68	transfer	IC 3752	IC 3641	141	54	Ht	Ht	3	33
69	transfer	IC 3752	SP 6641	141	94	Ht	Ht	3	33
70	transfer	IC 3640	IC 3539	147	20	Ht	Ht	3	33
71	transfer	IC 3640	IC 841	147	26	Ht	Ht	3	33
72	transfer	IC 3640	IC 3941	147	48	Ht	Ht	3	33
73	transfer	IC 3640	SP 9641	147	66	Ht	Ht	3	33
74	transfer	SP 6444	IC 3640	159	146	Tb	Tb	3	33
75	transfer	SP 6444	SP 6640	159	172	Tb	Tb	3	33
76	transfer	IC 1140	IC 3640	161	146	Tb	Tb	3	33
77	transfer	IC 1140	SP 6640	161	172	Tb	Tb	3	33
78	transfer	SP 9640	IC 3641	171	54	Ht	Ht	3	33
79	transfer	SP 9640	SP 6641	171	94	Ht	Ht	3	33
80	transfer	SP 6640	IC 3539	173	20	Ht	Ht	3	33
81	transfer	SP 6640	IC 841	173	26	Ht	Ht	3	33
82	transfer	SP 6640	IC 3941	173	48	Ht	Ht	3	33
83	transfer	SP 6640	SP 9641	173	66	Ht	Ht	3	33

D
Scientific paper

This appendix presents the scientific paper version of the thesis.

Passenger-centric robust timetabling in railways

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***Abstract*—Punctuality in railway transport is a critical concern for passengers, especially during unavoidable disturbances. Therefore, a sufficiently robust timetable is necessary. This thesis proposes a new timetabling method that minimizes passenger delay as the optimization objective, aiming to create a passenger-centric robust timetable. The model is implemented in the Dutch railway network in the Eindhoven-Den Bosch-Tilburg area to verify and validate its correctness and functionality. Experimental results demonstrate that the proposed model significantly reduces passenger delays when dealing with specific disturbances.**

Keywords—Railway timetabling, robustness, generalised passenger delay

I. Introduction

Rail transportation is pivotal in the logistics and passenger transport sectors in many countries globally. In 2018, approximately 4% of the total freight mass in the Netherlands was transported by rail. Additionally, trains accounted for 13% of the total distance travelled by individuals during the same period, making it the second most commonly used mode of transportation by distance[1].

From the perspective of passenger services, the fixed train schedules and the predictability of travel times make rail transport a favoured option for medium- to long-distance journeys. However, the punctuality of railway transportation, a significant concern for passengers, frequently faces threats. This problem becomes more pronounced when various potential disturbances occur, such as extreme weather, mechanical failures, or other unforeseen events, leading to propagated delays. The largest railway operator in the Netherlands, Nederlandse Spoorwegen (NS), transported about 1 million passengers daily in 2023. Approximately 5,500 disturbances and disruptions occurred during the same period, equivalent to an average of 15 per day[1].

In the academic field, the punctuality problem in the railway system is interpreted thoroughly. Palmqvist and Kristoffersson[2] point out that the frequency and severity of running and dwelling time delays are directly related to punctuality. On the one hand, railway operation companies and staff need to apply a punctuality improvement method system to reduce the delay of trains[3][4]. From the planning perspective, the railway system's robustness directly impacts its ability to handle delays, affecting train operations' punctuality. For passenger transportation, robustness is defined by Dewilde et al.[5] as "*A railway system that is robust against the daily occurring small disturbances and minimises the real weighted travel time of the passengers*". In this regard, García-Archilla et al.[6] and Friesen et al.[7] approach the problem from a strategic level, investigating the problems of robust network design for railway infrastructure under capacity constraints and uncertain timetabling. At the tactical level, Fioole et al.[8], Hoogervorst et al.[9], and Grafe et al.[10] consider the problems of robustness and passenger delay management from the perspective of rescheduling rolling stock. Many researchers have focused their studies on robust timetabling in railways. This may be attributed to timetabling being positioned at an appropriate planning stage, which is neither as remote from actual operations as network design nor as constrained in scope for changes like rolling stock and crew scheduling, where the potential for adjustments is quite limited.[11]

Generally, robust railway timetabling research can be categorised into two main types: operator-centric timetabling and passenger-centric timetabling. Kroon et al.[12] and Högdahl and Bohlin[13] consider the problem from a macroscopic level and design a robust timetable to minimise the impact of weighted train delay during exceptional events. Solinen et al.[14] evaluates the robustness in the micro way. Further on, Bešinović et al.[15] describes an integrated iterative micro-macro approach to computing a conflict-free, stable, and robust railway timetable. However, research on the problem of passenger-centric robust timetabling is quite scarce. Sels et al.[16] considers robustness one of the evaluation criteria in designing railway timetables with minimised passenger travel time. Still, robustness has not been the point of attention in the research. As Cacchiani and Tothhas[17] noted, transport efficiency is the primary concern of both operators and passengers; however, the robustness cannot be overlooked. Especially for the served passengers, the delays they experience are more intuitive compared to operational delays. Therefore, this paper will focus on the problem of passenger-

[1] Passenger Travel Statistics in 2023, NS

centric robust railway timetable design, aiming to fill the current research gap, and it defines that *a railway timetable is robust when it can help mitigate the additional generalised travel time experienced by passengers when small disturbances occur.*

II. Assumptions and Network Representation

A. Model assumptions

This paper aims to modify and improve the current timetable rather than creating an entirely new timetable. This is because the design process needs to consider various real-world factors, such as railway and station capacity, the size of the train fleet, crew scheduling issues, and time allowances. Given these considerations, certain assumptions on the train timetabling model have been incorporated into the design:

- No train will be cancelled, even though a huge delay happens during its operation.
- The train order remains consistent in a cycle, and no overtaking is allowed in the robust timetable.
- Every train has enough capacity to accommodate all passengers without considering the train capacity and passenger priority.
- The railway network is formulated at the macroscopic level. The model's scope in this thesis does not address micro-level aspects, such as constraints within block sections.

For the passenger model, it tends to exhibit more complexity. In the real world, individuals show significant heterogeneity; factors such as age, gender, educational level, etc., can influence their evaluations of specific matters, leading to different choices in the same situations. However, the time and effort to complete this study are limited. Therefore, some reasonable assumptions should be made to simplify the model's complexity.

- Passengers are aware of the realized timetable. That means the passengers can evaluate the attributes of each path accurately.
- The primary disturbances and passenger arrival time are assumed to be independent of the details of the timetable, and passenger arrival time follows a uniform distribution over an hour, even though passengers know the train they want to catch will experience a severe secondary delay.
- Passengers will make a choice based only on waiting-at-origin time, in-vehicle time, transferring time, and number of transfers when choosing their paths. The path choice of a homogeneous group of passengers is unique and fixed.

B. Network representation

The event set for trains is denoted by E, and two adjacent connected events are denoted by i and j. There are mainly two kinds of events: departure and arrival. In addition, besides the passenger stations, there are junctions between certain stations where infrastructure constraints also exist. Therefore, to better explain and illustrate the infrastructure constraints, these two types of events are further subdivided into arrival and departure events at stations, which are contained in Ea and Ed. On the other hand, entry and exit events at non-station junctions are contained in sets Ea' and Ed', respectively. So, it has $E = Ea + Ed + Ea' + Ed'$.

Train activities are the bridges between 2 connected train events $i, j \in E$, denoted by $(i, j) \in A$. This paper considers four kinds of activities related to trains: running activity Ar, dwelling activity Ad, headway activity Ah and turnaround activity At.

Running and dwelling activities are derived from the existing operating plan. The former explains the train running from one station to the next station, and dwelling activity represents the train staying in the station and waiting for passengers to board and alight. Headway constraint ensures the adjacent trains have suitable headway and the system can operate safely. Sometimes, trains travelling in different directions can create headway conflicts, especially when conflicting trains use the same switches or tracks. This usually results in non-overlapping time windows for trains in this junction section, with a few minutes of separation between them. The same train can be assigned to different routes and directions in the railway system. The turnaround activity represents the process of preparing a set of trains and moving it in reverse direction after completing one itinerary before starting the return journey on the same route. For simplicity, this thesis does not discuss potential train couplings that may occur if the train is reassigned to other routes for operational tasks.

Passenger event set F is composed of passenger arrive-at-origin, boarding and alighting events. They form three subsets of F : Fs, Fb and Fa. In the real world, the boarding and alighting times are varied and follow the queuing theory in which the carriage doors are the service desks, and different groups of passengers have different service times; thus also, for simplification, all the passengers in the same passenger group share the same boarding event time, that is the departure time of that train. Alighting events are recognized as arrival events at the transfer or destination station. Based on this assumption, for all the passenger boarding or alighting events, there can be a mapped departure or arrival event in the train event set E accordingly. In this way, i and j can also represent the passenger event-activity graph.

There is one exception: the arrive-at-origin event. Firstly, the whole passenger set should be divided into several small groups $p \in P$, and the passengers in the groups share the same OD $k = (o, d)$ and the arrive-at-origin event time s.

Like the train activities, passenger activities are the links between 2 connected passenger events $i, j \in F$ and the activity set is denoted by B and $(i, j) \in B$. This research discusses three types of activities, including waiting-at-origin activity Bw, in-vehicle activity Bi and transfer activity Bt.

The waiting-at-origin activity starts when the passengers arrive at their original stations and ends when they get aboard, which connects the arrive-at-origin event c_p for passenger group p with the first boarding event in path π, e_{π}^{0}. The in-vehicle activity describes the process where passengers aboard the train and are transported by the train from a starting station to an end station. Transfer activity refers to the process in which passengers transfer from one train to another at an interchange station. Notice that this paper defines certain transfer activities as foolish transfers, such as when passengers transfer to a train going in the opposite direction or when passengers transfer between trains going in the same direction but with the same stops. These types of transfers are considered invalid and are excluded when generating transfer activities.

This section provides a small example and constructs a passenger event-activity graph to better explain passenger events and activities and illustrate their relationship with train events and activities.

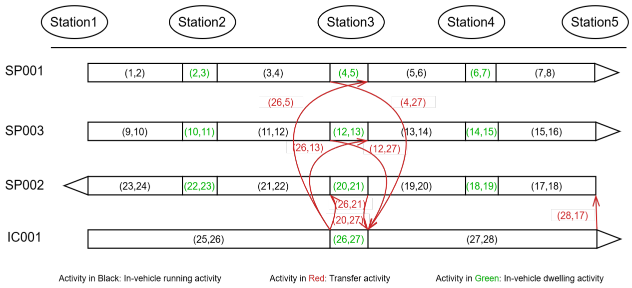

Fig. 1. A small example rail network

As shown in Fig. 1, this is a small example railway network consisting of four trains: *IC001*, *SP001*, *SP002*, and *SP003*. *IC001* is an Intercity (*IC*) train, traveling from *Station1* to *Station5*, with a stop at *Station3*. Besides, *SP001*, *SP002* and *SP003* are three regional (*SPR*) trains, meaning they need to dwell at all stations.

There is a group of passengers with $k = (Station1, Station4)$. The different coloured arrows in the figure represent the passenger activities experienced on the possible paths chosen by the passengers. Noticeably, (4, 21) and (4, 13) are examples of two types of foolish transfer behaviours mentioned above.

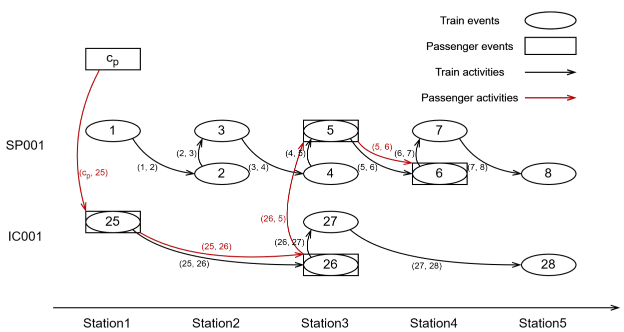

Fig. 2. Passenger event-activity graph in the small example network

This group of passengers plans to leave from *Station1*, board *IC001* to *Station3*, transfer to *SP001*, and finally reach *Station4*. They will arrive at *Station1* at time s. Events and activities of the trains and passengers along this path are illustrated in Fig. 2. Please note that infrastructure constraints of the trains, headway activities, and turnaround activities are not indicated in the Fig. 2. In passenger events, according to the definition of the start and end events of passenger activities, each train event can give rise to corresponding potential passenger events. However, to keep the graph clearer and simpler, not all possible passenger events and activities are marked. The red arrows indicate the flow of passenger activities for this group of passengers along the chosen path from the starting station to the destination.

C. Passenger path set generation

Before conducting passenger assignment modelling, the set of possible passenger paths needs to be defined. Depth-First Search algorithm (DFS) is a traversal algorithm that starts at a given node and explores as far as possible along each branch before backtracking[18]. This approach is widely used in path searching and so employed to find all the paths for the passengers with OD k.

We define network graph with station nodes and passenger in-vehicle and transfer activities. The station nodes connect to each other through a series of passenger activities, which are the edges in the graph. Besides, the passenger in-vehicle activity is broken into in-vehicle running activity and in-vehicle dwelling activity to provide a more detailed explanation of the DFS process. Additionally, we combine the passenger in-vehicle dwelling activity and transfer activity with the passenger in-station activity.

The path-searching process consists of two parts: The first step involves searching for all possible passenger in-vehicle running activities to connect the passing station between the origin and destination. The second part helps to complete the generated paths by adding connected in-station activities between the previous to-events and the following from-events in adjacent in-vehicle running activities. If it fails to find such an activity, the path will be marked as infeasible and removed from the path set. Below, the same example railway network in Fig. 2. is employed to explain the exact process of finding all feasible paths.

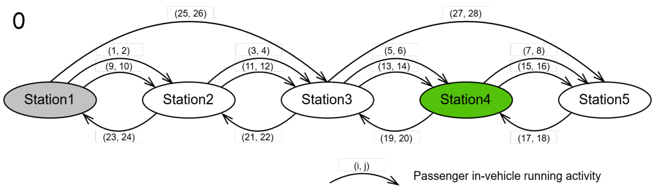

Fig. 3. Graph representation of the example rail network

There is also a group of passengers who want to travel from *Station1* to *Station4*. They want to know all possible paths so that they can make a comparison among them and pick the shortest path. To find all the paths from *Station1* to *Station4* in the small example railway network, as structured in Fig. 3., stations are the nodes in the graph because passengers can only change their activity state at train stops. The in-vehicle running activity links the stations, while the in-station activity (not shown in the graph) is responsible for connecting the passenger events within the station. However, the edges representing the in-vehicle running activities completed by different trains are not the same, even for activities between the same two stations in the same direction with similar routes, because they connect different events in the event-activity graph, and these events correspond to different event times when modelling.

The inputs of DFS are the passenger OD $k=(o,d)$ and the railway network graph with event set F and in-vehicle running activity (Fig. 3.). The goal is to find all potential passenger in-vehicle running paths between station nodes for each k.

Firstly, for each $k=(o,d)$, we will initialize a list Π_k^* to store the paths. Then, a DFS recursion function will take in the current searching node, the set of visited nodes, and the set of passenger in-vehicle running activities in the current path as inputs. Next, the algorithm will first check if the current searching node is the destination d that the passenger wants to reach. If it is, this indicates that a path has been found, and the current path will be added to Π_k^*, signifying that a path has been identified.

Of course, if the current node is not d, the algorithm will find all unvisited neighbouring nodes that can be accessed via edges in the network graph and record the connecting edges. Then, the algorithm will loop again, taking one of these neighbouring nodes and its connecting edge as new inputs for the recursion function. Note that when a path is found, that is, when the current node is the destination d, the function will exit because the destination d has already been visited and if we continue to traverse its neighbouring nodes, it will not be possible to visit the destination d again. Consequently, after breaking this iteration, the process will naturally return to the step of traversing neighbouring nodes in the previous recursion function.

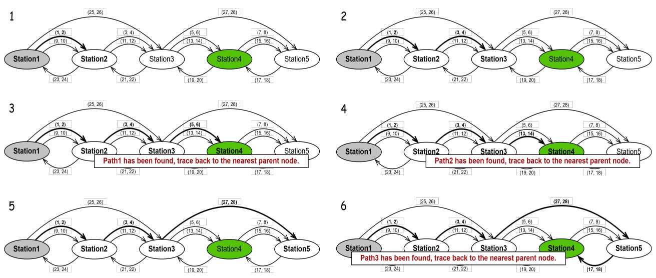

Fig. 4. Passenger event-activity graph in the small example network

This recursion process is very complex and difficult to describe, thus, Fig. 4. provides example steps of the algorithm applied to the small network shown in Fig. 1., where o is *Station1* and d is *Station4*. In Iteration 1, the input is the starting point *Station1*, with the sets of visited nodes and activities both being empty. Then, it searches for its neighbour nodes and the corresponding edges, finding three pairs: *Station2* (1,2), *Station2* (9,10), and *Station3* (25,26) (*Station2* (1,2) means connecting to the neighbour node *Station2* with edge (1,2)). Each pair of neighbour nodes will be traversed, and each traversal will end either upon reaching the destination or after all child nodes have been explored. For example, if we choose the pair *Station2* (1,2) and start Iteration 2. Obviously, *Station2* is not the desired destination d, the process will be repeated. Starting from *Station2*(1,2), we continue to search for child nodes of *Station2*, resulting in two possibilities: *Station3* (3,4) and *Station3* (11,12). Note that the pair *Station1* (23,24) is not considered, as *Station1* is already in the set of visited nodes. This loop will only unwind after all neighbour nodes of the next child node have been searched. Therefore, this looping process is not a horizontal search, but a vertical search algorithm that explores one branch of parent nodes completely before returning to a previous level for further search.

When considering the first branch (starting with activity (1,2), (3,4)), it finds one of the neighbour nodes for *Station3* that is *Station4* with edge (5,6) (Sub-figure 3) and luckily it reaches the destination d. Therefore, the first potential path from *Station1* to *Station4* is found and saved in the path set Π_k^*. Subsequently, the algorithm backtracks to the closest parent node (*Station3* (5,6)) and requests another connecting edge, referred to as (13,14). Similarly, starting from the parent node *Station3*, the last available path that can be found is (27,28), (17,18) (Sub-figure 5, 6).

The branch starting from activity (1,2), (3,4) has three paths:

- (1, 2), (3, 4), (5, 6)
- (1, 2), (3, 4), (13, 14)
- (1, 2), (3, 4), (27, 28), (17, 18)

After completing the search on this branch, the algorithm will continue to backtrack to the higher-level parent nodes. From these parent nodes, the search will proceed again with similar steps, continuing until all possibilities have been traversed.

The first step involves identifying the edges between stations in the network. This entails finding the connections for passenger in-vehicle running activities between origin and destination stations. However, it's important to note that not all of these paths will have in-station activity connections. This is because foolish transfer activities are not considered within the scope of this paper. Thus, we need the second step, which is to check the feasibility of the paths in the finding path set. As previously explained, at intermediate stations for the paths, the "to event" of the preceding edge must be connected to the "from event" of the subsequent edge via a passenger in-station activity.

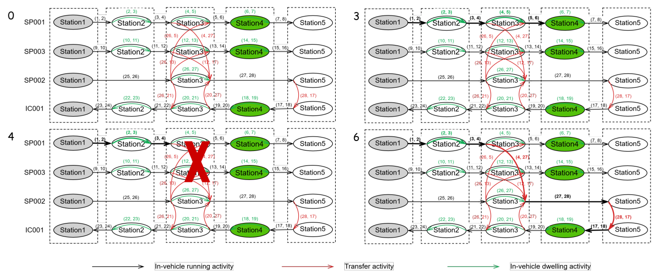

Fig. 5. Passenger event-activity graph in the small example network

With the initial path set Π_k^* obtained from the first step and all the in-station activity, the final output of the second step will be the passenger path set Π_k^* of OD k in the form of continuous passenger activities.

In the next step, we will revisit the previous example to analyse and refine the paths starting with activities (1, 2) and (3, 4), as illustrated in Fig. 4. In order to visualize the distribution of passenger in-station activities more effectively, Fig. 5. organizes all passenger activities based on the trains responsible for the operation. According to Fig. 5. the in-vehicle running activity between *Station1* and *Station2*, and between *Station2* and *Station3* can be achieved through the in-vehicle dwelling activity (2, 3). Besides, in-vehicle dwelling activity (4, 5) can fill the connection in *Station3*. Each of the above three paths is examined in turn. It is found that, in the second path, the connection of in-vehicle running activities of different trains should be undertaken by the transfer activity at *Station3*, but there can't find such activity because *SP001* and *SP003* are trains having the same dwelling and running mode in the same direction. This transfer activity is recognized as a foolish transfer. Thus, this path will be discarded. Therefore, the updated set of paths is as follows:

- (1, 2), (2, 3), (3, 4), (4, 5), (5, 6)
- (1, 2), (2, 3), (3, 4), (4, 27), (27, 28), (28, 17), (17, 18)

III. Timtabling Model

A. Train timetabling constraints

x_i represents the event time of $i \in E$ in the passenger-centric robust timetable. The duration of each activity must fit within the interval defined by its lower and upper bounds l_{ij} and u_{ij}.

$$l_{ij} \le x_j - x_i + q_{ij} \cdot T \le u_{ij} \quad \forall (i,j) \in A \tag{1}$$

In Eq.(1), a 0-1 parameter q_{ij} is introduced for all train activities to indicate whether the activity $(i,j) \in A$ will cross the cycle in the original timetable, where T represents the cycle length. In other words, the value of q_{ij} can also indicate the order of events i and j within a cycle; when $q_{ij} = 0$, it means that event i occurs before j, and when $q_{ij} = 1$, it means the reverse is true. According to II.A, it is assumed that the new robust timetable does not change the order of events within a cycle. So, the value of q_{ij} is maintained in the design of a robust timetable. Therefore, Eq.(1) restricts the duration of each train activity to fall within a reasonable range.

$$0 \le x_i < T \quad \forall i \in E \tag{2}$$

Eq.(2) ensures that the designed event times x_i are within the cycle.

$$-m_i \le x_i - o_i \le m_i \quad \forall i \in E \tag{3}$$

Eq.(3) allows but restricts the adjustment window of event time based on the original timetable. o_i is the event time for event i in original timetable and m_i is the maximum shifting step for event i.

To prevent potential delays from the scheduled times, extra time allowance is added to the activity durations, e.g., running times, as time supplements and to the interval between successive trains, e.g., minimum headway, as buffer times[19][20]. In this robust timetabling problem, we have constrained the time supplement for each train's running and dwelling activities.

$$\sum_{(i,j)\in Ar \cup Ad \wedge t(i),t(j)=n} (x_j - x_i + q_{ij} \cdot T - l_{ij}) \le Z_n \quad \forall n \in N \tag{4}$$

Eq.(4) indicates that the total time supplement for running and dwelling activities should not exceed the budget Z_n for each train $n \in N$. Each train with a different itinerary is an individual element n in set N. Therefore, each train over a line in one direction will have its corresponding supplement budget. The time supplement budget of the timetable is determined by factors such as operating routes, train types, and infrastructure capabilities. One method involves calculating the time supplement budget for each train in the original timetable and then applying the same budget to each train when creating the new timetable. In IV, this approach will be used to determine the time supplement. The function $t(i)$ helps identify the train ID for event i and sorts out all the running and dwelling activities for train n.

B. Passenger assignment constraints

In passenger assignment modelling, travel time and number of transfers determine a passenger's path choice in this paper. For the travel time part, three types of time must be defined: waiting-at-origin time, in-vehicle time, and transfer time, as shown in Eq.(5)-Eq.(11). Here, P represents the set of passenger groups, k and s are the OD, and the time this group of passengers p arrives at the original station is in a periodic form, respectively. Π_k is a sub-set of the whole path set Π and represents the set of all paths for $k = (o,d)$, and π gives the path element in the path set. The first boarding event in each path π is denoted by e_π^0.

$$\omega_{p,\pi} = x_{e_\pi^0} - s + a_{p,\pi} \cdot T \quad s \in p, \forall p \in P, \forall \pi \in \Pi_k \tag{5}$$

$$\omega_{p,\pi} \ge 0 \quad \forall p \in P, \forall \pi \in \Pi_k \tag{6}$$

$$a_{p,\pi} \in \{0,1\} \quad \forall p \in P, \forall \pi \in \Pi_k \tag{7}$$

Eq.(5) is used to decide the waiting time at the origin station for each passenger group p in path π, in which a binary variable $a_{p,\pi}$ is introduced to determine the sequence of passenger arrive-at-origin event c_p and first boarding event e_π^0 in one cycle. When $a_{p,\pi} = 0$, arrive-at-origin event c_p comes earlier than the first event in path π within the cycle, otherwise, arrive-at-origin event c_p comes later. $x_{e_\pi^0}$ represents the time of the first boarding event in the path π, and s represents the corresponding event time for the arrive-at-origin event c_p. Eq.(6) tells that passengers cannot board a train that has already departed unless they are willing to wait for that train the next cycle. In this way, the binary variable $a_{p,\pi}$ is restricted.

$$\phi_{p,\pi} = \sum_{(i,j)\in\pi\wedge(i,j)\in Bi} (x_j - x_i + q_{ij}\cdot T) \quad \forall p\in P, \forall \pi \in \Pi_k \quad (8)$$

Passenger in-vehicle time for the path π is defined by Eq.(8). It is the duration summation of all the passenger in-vehicle activities Bi. The passenger activities share the same start and end times as trains' as assumed in II.A. Thus, q_{ij} is the same binary parameter discussed in train modelling in Eq.(1).

$$\gamma_{p,\pi} = \sum_{(i,j)\in\pi\wedge(i,j)\in Bt} (x_j - x_i + b_{ij}\cdot T) \quad \forall p\in P, \forall \pi \in \Pi_k \quad (9)$$

$$\overline{l_{ij}} \le x_j - x_i + b_{ij}\cdot T < T + \overline{l_{ij}} \quad \forall (i,j)\in Bt \quad (10)$$

$$b_{ij} \in \{0,1\} \quad \forall (i,j)\in Bt \quad (11)$$

Eq.(9)-Eq.(11) illustrate the definition of transfer time for (i,j) in transfer activity set Bt, which has a similar structure as that of in-vehicle time. Its lower and upper bound $\overline{l_{ij}}$ can be determined by factors like station size, service infrastructure in the station, etc. b_{ij} is a binary variable to judge if the passenger can transfer the connected train in the same cycle from event i to event j. If $b_{ij}=0$, the order is that event i comes first to the event j in one cycle, and passengers have enough time to transfer from event i to j; otherwise, the passenger has to wait for the connected train with event j in the next cycle. Although the activity sequence remains the same as the original timetable due to the assumption, b_{ij} still needs to be a binary variable rather than parameter q_{ij}, because there is a minimum transfer time $\overline{l_{ij}}$ given in Eq.(11). If the transfer time after the schedule shifting is less than $\overline{l_{ij}}$, b_{ij} will change from 0 to 1.

$$\overline{\xi}_{p,\pi} = \phi_{p,\pi} + \beta_w\cdot\omega_{p,\pi} + \beta_t\cdot\gamma_{p,\pi} + \beta_n\cdot\epsilon_\pi \quad \forall p\in P, \forall \pi \in \Pi_k \quad (12)$$

Eq.(12) provides a representation of generalised travel time $\overline{\xi}_{p,\pi}$ for passenger group p travel on path π, where β_w, β_t, and β_n represent the weights of waiting-at-origin time, transfer time and number of transfers relative to in-vehicle time, respectively. These values can be found in the literature[21][22]. ϵ_π is the parameter for the number of transfers in the path π, and it can be directly calculated when the path is generated. Passengers will determine their final travel plan based on each path's generalised travel time $\overline{\xi}_{p,\pi}$.

$$\sum_{\forall\pi\in\Pi_k} \alpha_{p,\pi} = 1 \quad \forall p\in P \quad (13)$$

$$\alpha_{p,\pi} \in \{0,1\} \quad \forall p\in P, \forall \pi \in \Pi_k \quad (14)$$

Eq.(13) and Eq.(14) define a binary variable $\alpha_{p,\pi}$. When $\alpha_{p,\pi}=1$, it indicates that the passenger group p with OD k and arrive-at-origin time s will choose path π as their final travel plan.

$$-M\cdot(1-\alpha_{p,\pi}) \le \xi_p - \overline{\xi}_{p,\pi} \le M\cdot(1-\alpha_{p,\pi}) \quad \forall p\in P, \forall \pi \in \Pi_k \quad (15)$$

$$\xi_p - \overline{\xi}_{p,\pi} \le M\cdot\alpha_{p,\pi} \quad \forall p\in P, \forall \pi \in \Pi_k \quad (16)$$

Eq.(15) and Eq.(16) are the constraints for selecting the shortest path. Here, the Big M method is used, and M is introduced as a large enough number. ξ_p gives the minimum generalised travel time passenger group p with OD k and arrive-at-origin time s. Only when $\alpha_{p,\pi}=1$, which means the path π is selected by this group of passengers, Eq.(15) will restrict $\xi_p = \overline{\xi}_{p,\pi}$. Besides, for Eq.(16), it is employed to ensure ξ_p should be less or equal to the generalised travel time of any path in the set Π_k.

C. Train simulation constraints

In this section, a realised event time for train event i and j is denoted by v_{irh} or v_{jrh}, in which r is the scenario ID included in the scenario set R discussed in the case and \$h\$ is the realised cycle that the event i or j happens. The delay propagation process is simulated when the initial delay appears through Eq.(17) and Eq.(18).

$$v_{jr(h+q_{ij})} - v_{irh} \ge l_{ij} + \delta_{ijrh} \quad \forall (i,j)\in A, \forall r\in R, \forall h\in H, ((i,j),r,h)\in\Delta \quad (17)$$

$$v_{jr(h+q_{ij})} - v_{irh} \ge l_{ij} \quad \forall (i,j)\in A, \forall r\in R, \forall h\in H, ((i,j),r,h)\notin\Delta \quad (18)$$

Note that Eq.(17) describes the duration bound of activities experienced initial disturbance. The disturbance parameter for activity (i,j) in scenario r in cycle h is given by δ_{ijrh}, which is added to the lower bound l_{ij} of the duration of (i,j) and h should be a natural number $\mathbb{N}$. This activity has to process an extra δ_{ijrh} upon the lower bound. The binary parameter q_{ij} illustrates whether activity (i,j) crosses the cycle in the original timetable. Besides, in Eq.(18), the constraint for other activities not in disturbance set Δ is presented. By these constraints, the delay will be propagated along the activity chain influenced by the initial delay δ_{ijrh} if there is no sufficient supplement to mitigate the previous delay. Noticeably, v is a linear variable, so the realised time v_{irh} may not exactly fall in the cycle h. Sometimes, v_{irh} will postpone to the next cycle $h+1$ due to the initial delay, but it still retains the original index.

$$x_i + h\cdot T \le v_{irh} \quad \forall i\in Ed, \forall r\in R, \forall h\in H \quad (19)$$

In real-world operation, a train cannot depart earlier than the scheduled departure time under normal cases, even if it has finished its dwell activity ahead of time. Early departure can cause passengers who arrive at the station just in time to miss the train or result in failed transfers for transfer passengers. Thus, we address this problem with a constraint in Eq.(19).

$x_i + h\cdot T$ is the scheduled event time for event i in cycle h and v_{irh} is the realized event time in linear form.

D. Passenger simulation constraints

This section will focus on how passengers choose their paths and how the generalised travel time is calculated when delays occur under disturbance scenarios. Based on the first assumption in II.A, passengers are aware of the realised timetable and can accurately know the travel time they will experience in each path. Therefore, passengers can be assigned to the rail network using the methods outlined in II.B. Noticeably, variables with a superscript "*" indicate the corresponding passenger-related variables under the realised timetable as described in II.B. The subsequent constraints will all follow this notation.

$$\omega^*_{p,\pi,r,h} = v_{e^0_\pi rh} - (s_p + h\cdot T) + a^*_{p,\pi,r,h}\cdot T \quad \forall p\in P, \forall \pi\in\Pi_k, \forall r\in R, \forall h\in H \tag{20}$$

$$\omega^*_{p,\pi,r,h} \ge 0 \quad \forall p\in P, \forall \pi\in\Pi_k, \forall r\in R, \forall h\in H \tag{21}$$

$$a^*_{p,\pi,r,h} \in \{0,1\} \quad \forall p\in P, \forall \pi\in\Pi_k, \forall r\in R, \forall h\in H \tag{22}$$

Eq.(20)-Eq.(22) define the passenger waiting-at-origin time for the passenger group p with OD k and arrive-at-origin time s in scenario r in cycle h. The binary variable $a^*_{p,\pi,r,h}$ is applied to determine the chronological order of the passenger arrive-at-origin event and boarding event of the target train in the cycle h. If the arrive-at-origin event occurs before the boarding event, then $a^*_{p,\pi,r,h} = 0$; otherwise, $a^*_{p,\pi,r,h} = 1$. Eq.(20) tells that passengers cannot board a train that has already departed unless they are willing to wait for that train the next cycle ($a^*_{p,\pi,r,h} = 1$).

$$\phi^*_{p,\pi,r,h} = \sum_{(i,j)\in\pi\wedge(i,j)\in Bi} (v_{jr(h+Q_{j,\pi})} - v_{ir(h+Q_{i,\pi})}) \quad \forall p\in P, \forall \pi\in\Pi_k, \forall r\in R, \forall h\in H \tag{23}$$

Eq.(23) specifies the in-vehicle time for the passenger group with p under the realized timetable. Since v_{irh} is a linear variable, it is necessary to determine in which cycle the passenger activity $(i,j)\in\pi$ occurs during the journey on this path. Therefore, a new integer parameter $Q_{i,\pi}$ will be introduced, and it helps to record the number of cycles experienced by this route up to event i.

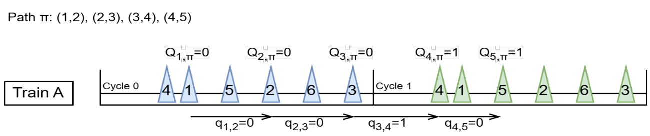

Fig. 6. The relationship between $q_{\hat{i}j}$ and $Q_{\hat{i},\pi}$ in path π

As shown in Fig. 6., we assume that $\{1,2,3,4,5\}$ are passenger events in the event set B. The position of the triangles in the Fig. 6. represents the event times of these events in the newly designed timetable. There is a given path π, $\pi = \{(1,2),(2,3),(3,4),(4,5)\}$, and to obtain $Q_{\hat{i},\pi}$, we should sum all the q_{ij} for passenger activities before the event $\hat{i}$. For example, to calculate $Q_{3,\pi}$, sum the q values of passenger activities up to event 3 ($q_{1,2}$, $q_{2,3}$), resulting in $Q_{3,\pi} = 0$. To calculate $Q_{5,\pi}$, sum the q values up to event 5 ($q_{1,2}, q_{2,3}, q_{3,4}, q_{4,5}$), resulting in a sum of 1. Note that the model determines the specific event times in the newly designed timetable, and all these event times are variables, but the order of events within a cycle remains the same as in the original timetable. Therefore, the $Q_{i,\pi}$ values are equal to that of the original timetable, making $Q_{i,\pi}$ a fixed value for each event i in each path π.

$$\gamma^*_{p,\pi,r,h} = \sum_{(i,j)\in\pi\wedge(i,j)\in Bt} (v_{jr(h+Q_{j,\pi})} - v_{ir(h+Q_{i,\pi})} + b^*_{ijrh}\cdot T) \quad \forall p\in P, \forall \pi\in\Pi_k, \forall r\in R, \forall h\in H \tag{24}$$

$$\bar{l}_{ij} \le v_{jr(h+Q_{j,\pi})} - v_{ir(h+Q_{i,\pi})} + b^*_{ijrh}\cdot T < \bar{l}_{ij} + T \quad \forall (i,j)\in Bt, \forall r\in R, \forall h\in H \tag{25}$$

$$b^*_{ijrh} \in \{0,1\} \quad \forall (i,j)\in Bt, \forall r\in R, \forall h\in H \tag{26}$$

Eq.(24)-Eq.(26) are combined to restrict the realized transfer time. b^*_{ijrh} is a binary variable to judge whether the previous train arrival event i and the following train departure event j in the realised timetable is still in the same sequence as the designed robust timetable. Although we assumed this sequence is the same as the original timetable, in the realised timetable, this order can be changed due to the disturbance in the previous train. If the departure event j goes first in the realised timetable in scenario r in cycle h, b^*_{ijrh} will be 1. Although these passengers can wait for the same train in the next cycle, they must experience a much longer transfer time.

However, this method has defect. If a delay in the preceding train causes passengers to miss the transfer to the subsequent train in the scheduled cycle (i.e. $b^*_{ijrh} = 1$), it may result in a cycle misalignment. This can affect all passenger activities after the cycle misalignment occurs. The duration of these passenger activities may vary depending on the difference between values $v_{jr(h+Q_{j,\pi}+b^*_{ijrh})} - v_{ir(h+Q_{i,\pi})}$ and $v_{jr(h+Q_{j,\pi})} - v_{ir(h+Q_{i,\pi})} + b^*_{ijrh}\cdot T$.

$$\bar{\xi}^*_{p,\pi,r,h} = \phi^*_{p,\pi,r,h} + \beta_w\cdot\omega^*_{p,\pi,r,h} + \beta_t\cdot\gamma^*_{p,\pi,r,h} + \beta_n\cdot\epsilon_\pi \quad \forall p\in P, \forall \pi\in\Pi_k, \forall r\in R, \forall h\in H \tag{27}$$

Eq.(27) calculates the generalised realized travel time $\bar{\xi}^*_{p,\pi,r,h}$ for each path for each group of passengers in scenario r in cycle h. Note that ϵ_π is the number of transfers in path π independent of the event times and passenger groups.

$$\sum_{\pi \in \Pi_k} \alpha^*_{p,\pi,r,h} = 1 \quad \forall p \in P, \forall r \in R, \forall h \in H \tag{28}$$

$$\alpha^*_{p,\pi,r,h} \in \{0,1\} \quad \forall p \in P, \forall \pi \in \Pi_k, \forall r \in R, \forall h \in H \tag{29}$$

$$-M \cdot (1-\alpha^*_{p,\pi,r,h}) \le \xi^*_{p,r,h} - \overline{\xi}^*_{p,\pi,r,h} \le M \cdot (1-\alpha^*_{p,\pi,r,h}) \quad \forall p \in P, \forall \pi \in \Pi_k, \forall r \in R, \forall h \in H \tag{30}$$

$$\xi^*_{p,r,h} - \overline{\xi}^*_{p,\pi,r,h} \le M \cdot \alpha^*_{p,\pi,r,h} \quad \forall p \in P, \forall \pi \in \Pi_k, \forall r \in R, \forall h \in H \tag{31}$$

Eq.(28)-Eq.(31) compare the generalised travel time for each group of passengers in each cycle under each scenario and assign the minimum generalised travel time to $\xi^*_{p,r,h}$. This structure is similar to the passenger assignment model under scheduled timetable in III.C.

E. Objective function

As discussed in I, the optimisation goal of robust timetabling in this study is to minimise the additional average generalised travel time experienced by passengers under small disturbances, i.e. average passenger travel delay.

$$\xi^*_{p,r,h} - \xi_p \le d_{p,r,h} \quad \forall p \in P, \forall r \in R, \forall h \in H \tag{32}$$

Therefore, Eq.(32) tells the calculation formulation of the travel delay $d_{p,r,h}$ for each passenger group p in scenario r in cycle h, where ξ_p and $\xi^*_{p,r,h}$ represent the scheduled travel time and realised travel time under the new robust timetable respectively.

$$Min \sum_{p \in P} \sum_{r \in R} \sum_{h \in H} d_{p,r,h} \cdot g_{p,h} / |G| \tag{33}$$

In the last, the objective function, as shown in Eq.(33), minimises the average passenger travel delay for all passenger groups P in all scenarios R in all cycle H. The average passenger travel delay is derived by taking the product of the additional delay $d_{p,r,h}$ for each passenger group p and the number of passengers $g_{p,h}$ in the corresponding group, and then dividing by the total number of passengers $|G|$.

IV. Case Study

A. Case area introduction

The Eindhoven-Den Bosch-Tilburg region, located in the southern Netherlands in the province of North Brabant, is one of the most important economic and cultural centres. Eindhoven, the fifth largest city in the Netherlands, is also the centre for innovation and technology. This chapter will apply the models mentioned in the previous chapters to this region to evaluate the performance of the methodology. Fig. 7. shows the layout of the railway network within the case study area.

In this thesis, we use the railway timetable in 2019 (Fug. 8) from NS as the original timetable. This timetable is a periodic timetable with a cycle of 30 minutes, where each train is sent from the departure station once within the cycle. Besides, the minimum running time and headway for $l_{ij} (\forall (i,j) \in Ar \cup Ah)$ are provided, and the upper bound of the running time is set at 1.3 times the minimum running time. The dwell time at stations is assumed to be 0.5 min. The optimization model allows 3 min shift in departure or arrival event times, i.e. $m_i = 3$ for all train events $i \in E$.[23]

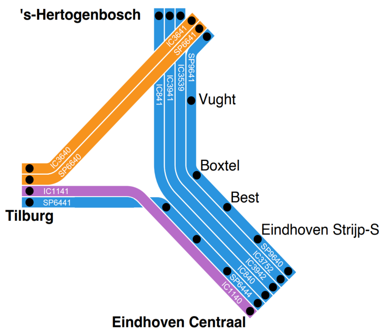

Fig. 7. Case study region: Eindhoven-Den Bosch-Tilburg

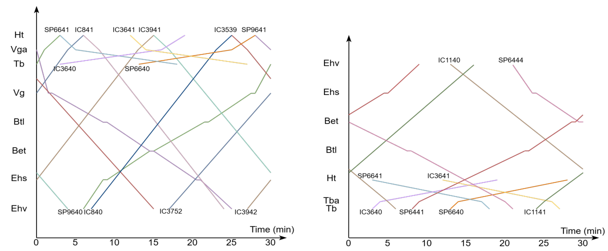

Fig. 8. Case study timetable in 2019

The passenger OD demand is calculated based on the average morning peak passenger flow in the study region in 2019. To avoid excessively long computation times due to large data volumes, the case study ultimately selects the passenger flow data for one hour, from 7:00 to 8:00 (covering two cycles), as the passenger OD demand data in the case study. During this period, a total of 20,081.5 passengers are included. The OD demand distribution is shown in Fig. 9. and there is significant passenger traffic between core interchange stations.

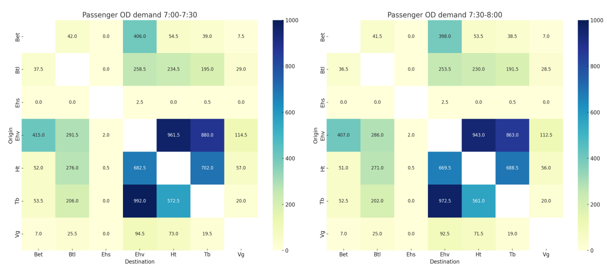

Fig. 9. Passenger origin and destination distribution

With this passenger set, we need to create passenger groups according to the following assumptions:

- The passenger flow distribution within one hour follows a uniform distribution.

- Passengers are divided into five groups per OD with 6-minute intervals within one cycle.

Therefore, within the study period, 20,081.5 passengers are divided into 420 groups based on their OD k and arrive-at-origin time s. The weight factors in Eq.(12) and Eq.(27) are $\beta_w = 2.5$, $\beta_t = 2.5$, and $\beta_n = 10$.[24][25] Specifically, β_w represents the weight for waiting times at origin stations, β_t denotes the weight for transfer times, and β_n signifies the penalty per transfer. For passenger transfer activities, the minimum transfer time is 3 minutes, while the maximum transfer time is 33 minutes.[23]

B. Model verification and validation case

In the verification and validation case A, two random disturbance scenarios are generated, as shown in TABLE I. In Scenario 0, 4 activities departing from *Tb* are considered to have initial delays, with delay values of either 3 or 5 (min). In Scenario 1, the activities with initial delays are all train running activities arriving at *Tb*. Generally, all initial delays occur in the first cycle (cycle 0).

TABLE I. DISTURBANCE SCENARIO FOR VERIFICATION AND VALIDATION CASE

Scenario	Train ID	From_station & To_station	Activity Type	Initial Delay	h
0	SP6441	(Tb, Btl)	running	3	0
	SP6640	(Tb, Ht)	running	3	0
	IC1141	(Tb, Ehv)	running	5	0
	IC3640	(Tb, Ht)	running	5	0
1	IC3641	(Ht, Tb)	running	3	0
	SP6641	(Ht, Tb)	running	5	0
	IC1140	(Ehv, Tb)	running	3	0
	SP6444	(Btl, Tb)	running	5	0

1) Verification and validation of passenger-centric model:

In this section, we will use the original timetable (OT) provided by train operator NS to verify the passenger modelling part, which includes the passenger assignment model and the simulation model. The model input will be the OT shown in the event-activity graph with original event time $o_i \in O$; passenger set $p \in P$, passenger path set $\Pi_k \in \Pi$.

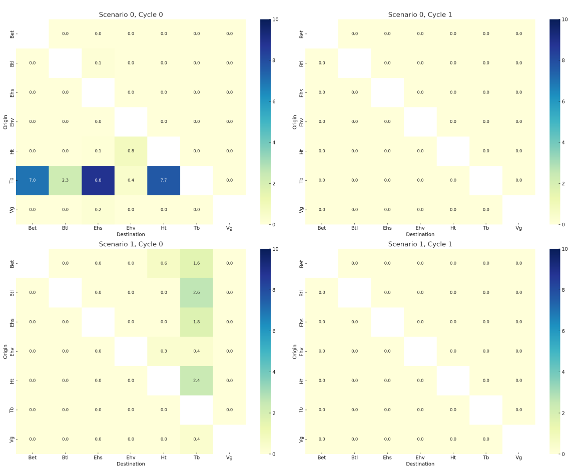

Fig. 10. Average passenger delay for each OD pair in OT

In the case where Eq.(33) is used as the objective function, after solving the aforementioned model, the optimal solution obtained is 2.98, which means that passengers will experience an average delay of 2.98 minutes. Fig. 10. shows the average delay calculated for different passenger groups with the same OD. Since, in this case, all initial train delays are set within the period of 7:00-7:30; all passenger delays occur within this period. This indicates that the initial delay starting in the first cycle does not propagate to the next cycle, affecting passenger travel activities. Moreover, passengers departing from station *Tb* in Scenario 0 and those arriving at *Tb* in Scenario 1 have experienced relatively more severe delays. This phenomenon in Scenario 0 is due to the initial delays of four trains departing from *Tb*, and all trains from *Tb* to *Ht* experienced initial delays within this cycle. The reasons for the huge delay around Station *Tb* are similar in Scenario 1. According to Fig. 9., the OD pairs with high delays have a large passenger flow (*Tb-Ht*), which corresponds to a higher average passenger delay.

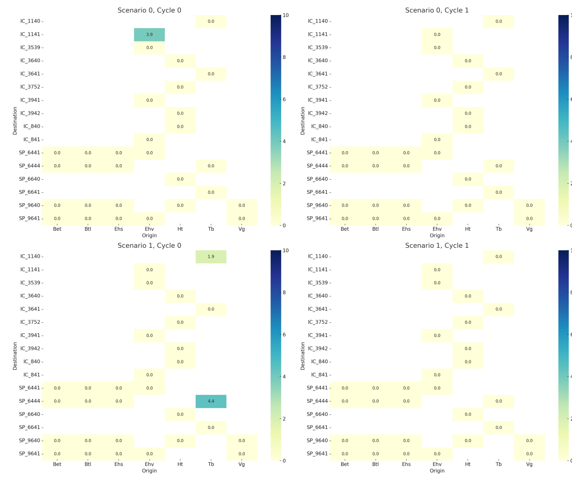

Fig. 11. Train arrival delay in OT

Fig. 11. shows the results generated by applying the train simulation model, which describes the arrival delays of trains at their dwell stations for each cycle in each scenario. If there is no data for a specific train at a particular station, it indicates that the train does not stop at that station. Overall, most delays need to be propagated. In Scenario 0, *Ehv* and *Ht*, which are directly connected to the delayed running activity, experiences train delays of 3.9 minutes. In Scenario 1, the delays occur in the running activities of trains arriving at *Tb*, resulting in particularly severe train delays at station *Tb*, which is consistent with the analytical solutions. In summary, the total train arrival delay amounts to 10.20 minutes, and most of the disturbances can be mitigated by the robustness of OT.

Efficiency is also an essential element of train operations. Fig. 12. presents the average passenger generalised travel time for passengers completing their journeys between different ODs in normal cases (without disturbance). Overall, journeys that require transfers, such as those between *Tb* and *Vg*, tend to have a higher passenger generalised travel time. In the end, it is calculated that the average generalised travel time for each OD pair is 46.13 minutes in the OT.

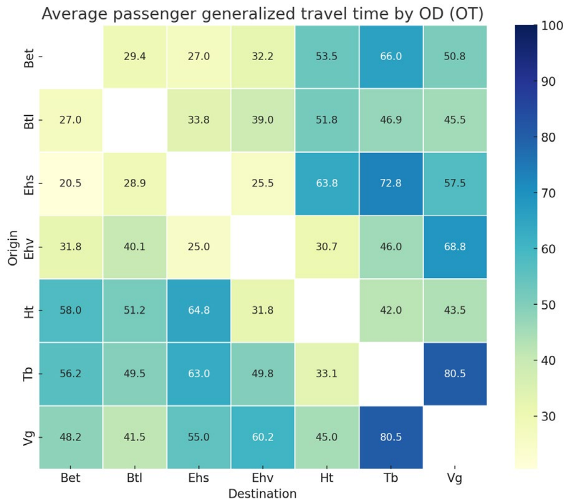

Fig. 12. Average passenger delay for each OD pair in OT

2) Verification and validation of operator-centric model:

This section will attempt to generate an operator-centric timetable based on the model proposed in III.A. The model input will be the OT shown in the event-activity graph with the original event time $o_i \in O$.

According to the definition of robustness mentioned in I, as shown in Eq.(34), the optimisation objective in this verification and validation model is to minimise the total train arrival delay within the experimental railway network.

$$Min \sum_{i \in Ea} \sum_{r \in R} \sum_{h \in H} D_{i,r,h} \tag{34}$$

Fig. 13. shows the optimised operator-centric timetable. Each coloured line represents a train service within a cycle, while the corresponding dotted lines indicate the train service in the original timetable. In the new ORT, the operational service lines of the trains do not intersect with the dotted lines indicating the original service. Therefore, there will be no cases where some events for the same train in the new timetable are delayed while others are postponed compared to the original timetable.

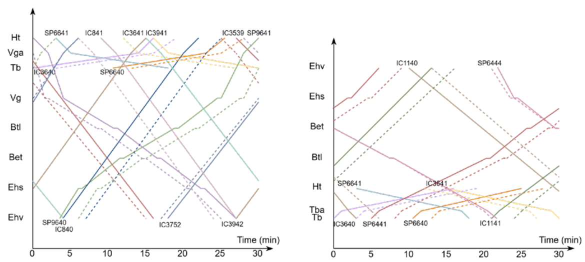

Fig. 13. Average passenger delay for each OD pair in OT

Fig. 14. shows the average amount of time that event times for each train in the ORT have been adjusted compared to the OT. More than half of the trains have been adjusted earlier by approximately 3 minutes. These adjustments have no obvious regularities and may be influenced by factors such as headway, infrastructure constraints, and the predefined order of events in the timetable creation process. The interaction between trains makes the timetable design process more complicated.

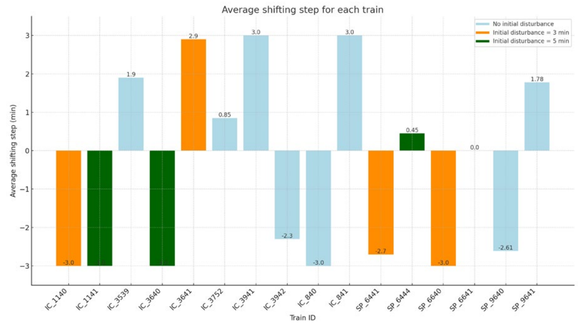

Fig. 14. Average shifting step for each train in ORT

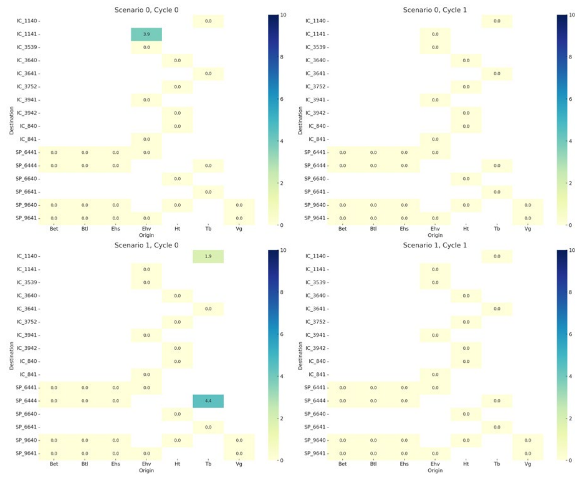

Fig. 15. Train arrival delay in ORT

The optimised objective function value is 9.60, representing the total train arrival delay at the stations measured in minutes. Their distribution is shown in Fig. 15. In comparison to Fig. 12, it can be observed that in Scenario 1, the delay for *SP6444* arriving at *Tb* has been reduced. This improvement is likely the result of optimising the supplement allocation for each train.

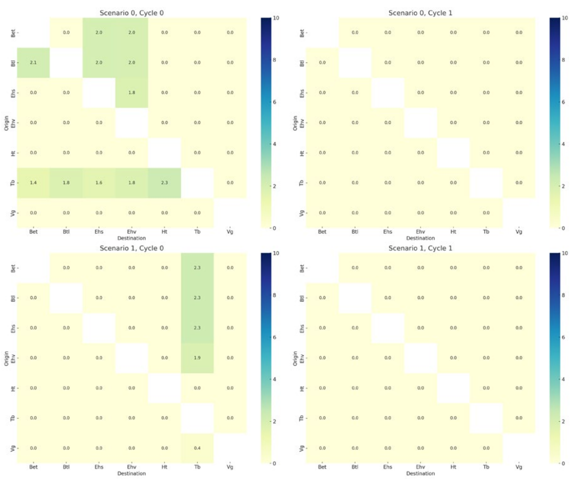

Fig. 16. Average passenger delay for each OD pair in ORT

Suppose passenger-related information is input, and the model described in the previous section is applied. In that case, the average passenger delay in the ORT can be obtained as 2.33 min. This shows a slight improvement compared to the OT. This indicates that the operator-centric model has limited ability to reduce passenger delays in disturbance scenarios. From Fig. 16., the delayed passengers are often distributed at Tb station and nearby stations, corresponding to the distribution of train delays.

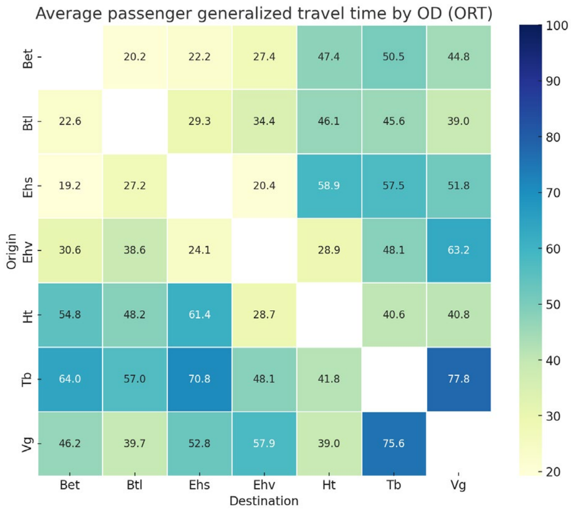

Fig. 17. Average passenger generalised travel time in ORT

Regarding the efficiency of the train timetable, the average generalised travel time between each OD pair is even smaller, reaching 44.50 min, as shown in Fig. 17. This is explainable because, in this verification and validation case, only a very limited disturbance scenario has been considered. In contrast, when designing the OT, more supplement time is allocated to the train activities that significantly impact passenger travel efficiency due to the need to ensure timetable robustness.

3) Verification and validation of passenger-centric robust timetabling model:

This section will verify the most important model proposed in this paper, which is also the final contribution. This requires using all previously mentioned train, passenger and simulation models, resulting in a larger computational workload. The model input will be the OT with original event time $o_i \in O$, passenger set $p \in P$, passenger path set $\Pi_k \in \Pi$. All the constraints introduced in III will be applied. The supplement budget is also allocated according to the trains.

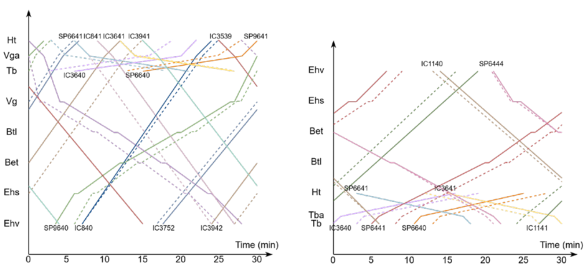

Fig. 18. Comparison of train service shifts between PRT and OT

Fig.18. illustrates the PRT, which results from slight adjustments based on the original train timetable. The figure clearly demonstrates the application of infrastructure constraints.

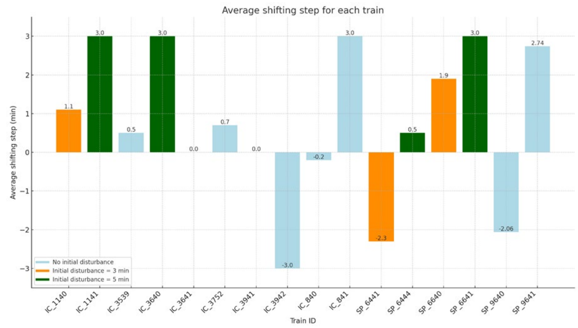

Fig. 19. Average shifting step for each train in PRT

As shown in Fig. 19, the average shifting steps for trains from OT to PRT are much smaller than those for ORT. Additionally, more train schedules have been delayed, likely to ensure that passengers do not miss their departure times or transfer connections.

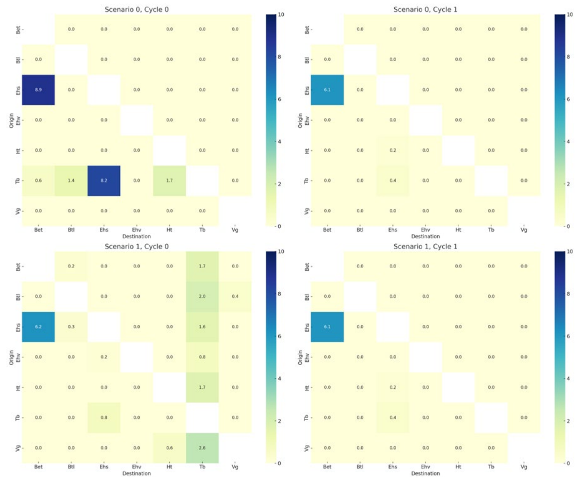

Fig. 20. Average passenger delay for each OD pair in PRT

Under this timetable, the average passenger delay in the two scenarios reached 1.04 minutes (Fig. 20.). When referring to Fig. 9, we notice that the passenger-centric robust model tries to aggregate the delays to the OD with lower passenger demand, such as *Ehs-Bet* and *Tb-Ehs*. This indicates that the PRT established using the model proposed in this thesis can reduce the passenger generalised travel time in specific delay scenarios. From the operator's perspective, the train arrival delay is higher with this timetable (10.20 minutes), and its distribution is shown in Fig. 21.

In the passenger-centric model, applying more delays to OD pairs with smaller passenger flows significantly improves the optimisation objective, which may naturally sacrifice a certain amount of train arrival delay. However, in this small verification and validation case, the disturbances are not so severe that the average train arrival delay remains the same as that of OT.

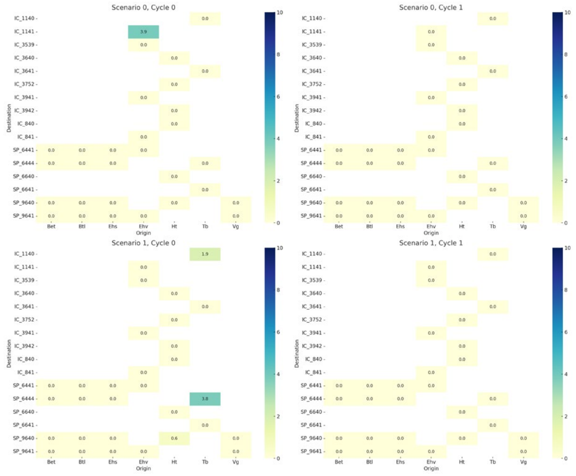

Fig. 21. Train arrival delay in PRT

The average passenger generalised travel time for each OD pair is 44.48min in this case (Fig. 22.), which is similar to the results of the other two timetables.

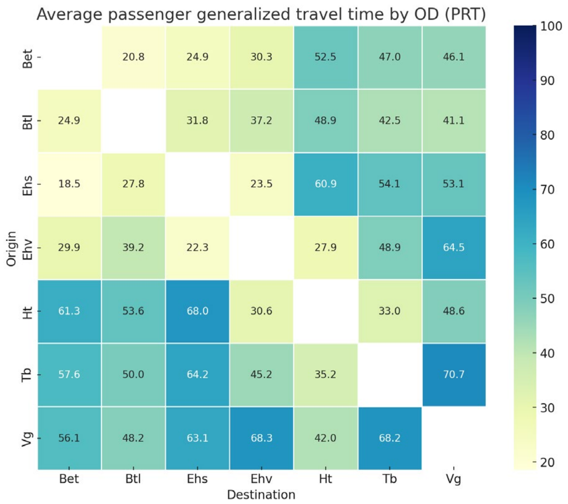

Fig. 22. Average passenger generalised travel time in PRT

This case divides the model into four parts and then reintegrates them, achieving different functionalities. These designed experiments are used to verify and validate each part of the model. Based on the output results from the two combined models and the final model, the models are correctly constructed. For example, in the PRT, ODs with lower passenger flows experience greater delays (*Ehs-Bet*). In contrast, in the ORT, all train delays are treated equally, leading to a more even distribution of passenger delays. This outcome is determined by the results of the passenger path allocation and the optimisation objective.

TABLE II. COMPARISON AMONG OT, ORT AND PRT IN CASE STUDY

Timetable	Average passenger delay (min)	Total train arrival delay (min)	Average generalized travel time (min)
OT	2.98	10.20	46.13
ORT	2.33	9.60	44.50
PRT	1.04	10.20	44.48

Based on the outputs in TABLE II, all the results are consistent with the analytical solutions and aligned with the previous expectations. Among the three timetables, OT performs the worst in handling the two disturbance scenarios in this case. Even in normal cases without disturbances, the average generalised travel time for each OD pair is the longest. For the ORT, the total train arrival delay is the smallest, as this is the primary optimisation objective. An unexpected benefit is the improvement in timetable efficiency compared to OT. PRT, on the other hand, focuses on reducing average passenger delay during disturbances, successfully reducing delays to about one-third of the original level. Therefore, it is concluded that the passenger-centric robust timetabling model works as intended and can achieve the functionality of the model.

V. CONCLUSION

This paper presents a passenger-centric timetabling model aimed at enhancing the robustness of current train timetable in mitigating the impact of disturbances on passengers. The study adopts the generalised passenger delay as the optimization objective of the model, seeking to minimize the additional delay experienced by passengers during disturbances compared to their travel time according to the normal scheduled time. The model is primarily divided into three components. The train timetabling model utilises a train PESP scheme to ensure train timetable feasibility, such as restricting the upper and lower bounds of train activity. Besides, constraints on the time supplements budget and adjustment window compared to the original timetable has been introduced. Another model is about generalised travel time defining and calculation and passenger assignment. The simulation model consists of two sub-models. One sub-model is used for the train, and the other is used for the passengers. These sub-models help to evaluate the robustness of the newly designed timetable under different disturbance scenarios.

After establishing the model, we conduct a case study using data from the Eindhoven-Den Bosch-Tilburg region in the Dutch railway system. The primary aim is to verify and validate the proposed model. The analysis focuses on several key metrics, including average shifting step, average passenger delay for each OD pair, train arrival delay, and average passenger generalized travel time under normal conditions. The results suggest that the proposed model is effective in reducing passenger delays under specific disturbance scenarios, thereby affirming its validity.

However, the model also has certain limitations. First, the train operation sequence is fixed during the timetable design process, and all passengers are treated as a homogeneous group with identical travel preferences. Second, when calculating passengers' realized travel time, cycle misalignment may occur, potentially leading to inaccuracies in subsequent calculations. Nevertheless, this issue does not significantly affect the results in scenarios where passengers have multiple route options. Finally, the model is computationally intensive, requiring approximately 20 hours to obtain a near-optimal solution for 15 disturbance scenarios.

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