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van Erp, H.R.N.; Orcesi, André D.

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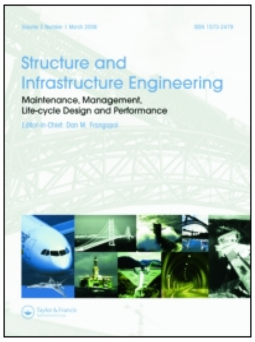

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The use of nested sampling for prediction of infrastructure degradation under uncertainty

H. R. Noël Van Erp[a] and André D. Orcesi[b]

[a]Faculty of Technology, Policy and Management, TU Delft Safety and Security Institute, Delft, The Netherlands; [b]University Paris-Est, IFSTTAR, Marne-la-Vallé, France

ABSTRACT
Because of the competing demands for scarce resources (funds, manpower, etc) national road owners are required to monitor the condition and performance of infrastructure elements through an effective inspection and assessment regime as part of an overall asset management strategy, the primary aim being to keep the asset in service at minimum cost. A considerable amount of information is then already available through existing databases and other information sources. Various analyses have been carried out to identify the different forms of deterioration affecting infrastructures, to investigate the parameters controlling their susceptibility to, and rate of, deterioration. This paper proposes such an approach by building a transition matrix directly from the condition scores. The Markov assumption is used stating that the condition of a facility at one inspection only depends on the condition at the previous inspection. With this assumption, the present score is the only one which is taken into account to determine the future of the facility. The objective is then to combine nested sampling with a Markov-based estimation of the condition rating of infrastructure elements to put some confidence bounds on Markov transition matrices, and ultimately on corresponding maintenance costs.

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Introduction

Infrastructure owners and managers throughout the world are facing increasing demands to ensure that the asset for which they are responsible are safe for the users and economic in terms of maintenance and repair requirements. This is particularly true for old structures (bridges, steep embankments, slopes, etc) that may have been designed using outdated design methods, loading and detailing standards. For example, because of the ever-increasing volume and weight of traffic, the live loading specified by national standards for bridges have increased many times over the last few decades in Europe.

In addition, durability issues are given a prominent place and the goal is to make decisions that balance the increasing demands for better performance with restrained financial resources and budget allocation (Gervásio, Simões da Silva, Perdigão, Orcesi, & Andersen, 2015). Being able to address these issues requires the development of management tools, which can be used as a basis for discussion on sustainability between technicians/engineers and managers/decision-makers. Such management tools need to take into account the multi-scale, multi-actor, multi-criteria aspects with different time horizons (infinite or finite with the concept of short-term, medium-term or long-term planning), uncertainties, hazards (environmental, exposure to risk) and economic, social and politic aspects.

One of the objectives of sustainable management of infrastructures is the minimisation of consequences, should they be societal, environmental or economic, caused by inadequate functioning or by unexpected structural failure, of a component, a system or equipment (BRIME, 2001). The implementation of a suitable maintenance management strategy should help reach this objective by managing the lack and heterogeneity of available information and the reliability of data sources (Fwa & Farhan, 2012; NCHRP, 2007; Shepard & Johnson, 2001; Thompson, 2000).

The proposed methodology should have the ability to include not only engineering aspects but also concepts from humanities and social sciences (economics, management sciences, …) and computer science as well (PIARC, 2003). Sustainable indicators, economic and social based, should be able to capture, based on the technical performance of a structure, additional aspects that may influence the decision process and typically represent the discounted (accumulated) direct or indirect costs associated with construction and maintenance. Summed up over the full lifetime, they represent part of or the full life-cycle impacts.

For highway infrastructures, the objective is to provide owners with tools predicting in a quantitative way the future degradation of elements as well as the associated uncertainties, depending on the available information and on the means devoted to appraisal campaigns (Sánchez-Silva & Klutke, 2016).

CONTACT André D. Orcesi andre.orcesi@ifsttar.fr

Several previous research projects contributed to the development of degradation models for structures (corrosion of reinforced concrete, and steel structures, alkali silica reaction and delayed ettringite formation in concrete, fatigue of welded details, fatigue performance of asphalt concrete pavements, durability of treated soils, etc). Nevertheless, such models are sometimes hardly applicable to structures in a global way and even less at the level of the overall stock (Orcesi & Cremona, 2010). This difficulty remains a significant scientific obstacle for the implementation of a relevant management strategy. In this context, the objective is to strengthen the existing knowledge by extrapolating prediction at a macroscopic scale and by including not only the degradation kinetics but also the decrease of functionality over time. More precisely, one main goal is to build a prediction model based on a discrete scoring system such as a visual inspection condition rating where information is easily available for road owners.

Several methods have been recently considered, based on Markov assumption stating that the condition of a facility at one inspection only depends on the condition at the previous inspection. With such an assumption, the present score is the only one which is taken into account to determine the future condition of the facility. Those methods include Markov decision process (MDP) for which the distribution of a waiting time until a certain event does not depend on how much time has elapsed already (memorylessness), semi-Markov decision process (semi-MDP) that includes the concept of the time spent in a given state, namely sojourn time, to define the transition among states, and partially observable MDP (POMDP) when inspection techniques and observations do not reveal the true state of the system with certainty (Memarzadeh & Pozzi, 2016; Papakonstantinou & Shinozuka, 2014a, 2014b; Schöbi & Chatzi, 2016). One should also mention the hidden Markov models that allow the unobserved condition state to be captured, eliminating the noise and bias associated with inspection/monitoring data (Kobayashi, Kaito, & Lethanh, 2012a).

Nested sampling, first introduced by John Skilling in 2004 for general Bayesian computation, and directly estimates how the likelihood function relates to prior mass. This method relies on sampling within a hard constraint on likelihood value, as opposed to the softened likelihood of annealing methods. The corresponding algorithm has caught a lot of attention because of its robustness, broad applicability, power on dealing with difficult posterior distributions, and little requirement for manual tuning. The key technical requirement of nested sampling is an ability to draw samples uniformly from prior distribution with restriction that the likelihoods of samples need to be larger than certain value.

The objective in this paper is to combine nested sampling with a Markov-based estimation of the condition rating of infrastructure elements to put some confidence bounds on Markov transition matrices, and ultimately on corresponding maintenance costs. The paper is organised as follows: first, the Nested sampling theory is introduced and its basic philosophy is exemplified with a simple numerical example. Second, the combination of the nested sampling approach with a homogeneous Markovian process is detailed. Factors unique to highway infrastructures such as relatively small number of discrete condition states and long service life make discrete time-homogenous Markov chains an attractive model for condition development (Mašović, Stošić, & Hajdin, 2015). Moreover, nested sampling enables the evaluation of the integrals of multivariate functions, particularly interesting when considering the probability of some data D conditional on the value of parameter θ and some likelihood model. Combining both approaches should enable efficient condition and cost predictions while considering inherent uncertainties. The proposed concepts are illustrated by considering some transitions observed on a virtual road highway infrastructure asset.

The method to determine the degradation process is formulated so that any infrastructure managers can determine their own deterioration processes based on the inventory and condition assessment of their stock. This approach is developed in the project RE-GEN (Risk assEssment of aGEing iNfrastructure) funded through the CEDR Transnational Road Research Programme Call 2013 'Ageing Infrastructure'.

Nested sampling

Nested sampling theory

Bayesian statistics has four fundamental constructs, namely, the prior, the likelihood, the posterior, and the evidence. These constructs are related in the following manner:

$$\text{posterior} = \frac{\text{prior} \times \text{likelihood}}{\text{evidence}} \tag{1}$$

The concepts of a prior, likelihood, and posterior are generally well known, which is not the case for that related to evidence. A possible explanation for this is that one often comes to Bayesianity by way of the more compact relationship:

$$\text{posterior} \propto \text{prior} \times \text{likelihood} \tag{2}$$

which does not make any explicit mention of the evidence construct (Zellner, 1971).

In this paper, the goal is to focus the analysis on the correct, though notationally more cumbersome, Equation (1), and forego of the more compact, but incomplete, Bayesian shorthand (Equation (2)). This allows providing some feeling for the evidence construct, and how this construct relates to the other three Bayesian constructs of prior, likelihood, and posterior. In particular, the nested sampling theory, introduced by Skilling (2004, 2012) aims to provide a Monte Carlo framework in link with Bayesian theory. Let $p(\theta, D)$ be the product of some prior $p(\theta)$ of some unknown set of parameters θ and the likelihood function $L(\theta, D)$ of a data-set D:

$$p(\theta, D) = p(\theta)L(\theta, D) \tag{3}$$

Then, the integral:

$$Z = \int p(\theta, D)d\theta \tag{4}$$

is the evidence measure which may be used to differentiate between competing models, by way of Bayesian model selection. Furthermore, the posterior of the parameters θ, given the data D, that is, $p(\theta, D)$, is given as:

$$p(\theta|D) = \frac{p(\theta, D)}{Z} \tag{5}$$

The nested sampling algorithm is specifically designed to evaluate the integral in Equation (4), giving us an estimate of the evidence Z. Furthermore, it also provides us with a set of representative samples from the posterior (Equation (5)), which may function as a proxy for that posterior. For those cases where the integral (Equation (4)) may be evaluated analytically, one will have no need for the nested sampling algorithm. However, for those problems where the integral (Equation (4)) is both intractable and highly dimensional, there one will have to take his recourse to Nested Sampling, in order to be able to evaluate the evidence (Equation (4)) and obtain a set of representative samples from the desired posterior (Equation (5)). The following section illustrates the nested sampling framework with a basic numerical example.

Univariate representation of multivariate pdfs

By reducing any k-variate function f to a corresponding monotonic descending univariate function g, and by using order statistics, the integral of any k-variate function f may be evaluated using a Monte Carlo sampling scheme called Nested Sampling. In the special case where the function f is either a likelihood function or a probability distribution, a representative probability weighted set of random samples may be determined to represent f (Skilling, 2004).

For illustration purposes, the following bivariate probability distribution $f(x, y)$ is examined numerically:

$$f(x, y) = \frac{\sqrt{0.51}}{2\pi} \exp\left[-\frac{1}{2}\left(x^2 + 1.4xy + y^2\right)\right] \tag{6}$$

where $-5 \le x \le 5$, $-5 < y < 5$ (Figure 1). The total volume under the curve $f(x, y)$ on this restricted domain is given by the integral:

$$\int_{-5}^{5}\int_{-5}^{5} f(x, y)\, dx\, dy = 0.9993 \tag{7}$$

This integral can be evaluated through brute force by considering a partition of the x, y-plane in little squares with area $dx_j\, dy_k$, $j = 1, \ldots, 20$, $k = 1, \ldots, 20$, then define the centre of these areas as $\left(\tilde{x}_j, \tilde{y}_k\right)$, and compute the strips of volume V_{jk} as:

$$V_{jk} = f\left(\tilde{x}_j, \tilde{y}_k\right) dx_j\, dy_k \tag{8}$$

In Figure 2 the volume elements V_{jk} are all given together. The total volume under the curve $f(x, y)$ may be approximated as:

$$\text{volume} \approx \sum_{j=1}^{20}\sum_{k=1}^{20} V_{jk} = 0.9994 \tag{9}$$

Then, these 3-dimensional volume elements V_{jk} are mapped to corresponding 2-dimensional area elements A_i. This is easily done by introducing the following notation:

$$dw_i = dx_j\, dy_k,\ f\left[\left(\tilde{x}, \tilde{y}\right)_i\right] = f\left(\tilde{x}_j, \tilde{y}_k\right) \tag{10}$$

where index i is a function of the indices j and k:

$$i \equiv (j - 1)20 + k \tag{11}$$

and $i = 1, \ldots, 400$. Using Equation (10), Equation (8) can be rewritten as:

$$A_i = f\left[\left(\tilde{x}, \tilde{y}\right)_i\right] dw_i \tag{12}$$

In Figure 3 the 400 elements A_i are given together. Since Equation (12) is equivalent to Equation (8), the mapping of the 3-dimensional volume elements V_{jk} to their corresponding 2-dimensional area elements A_i has not led to any loss of information; that is:

$$\text{area} = \sum_{i=1}^{400} A_i = \sum_{j=1}^{20}\sum_{k=1}^{20} V_{jk} = \text{volume} \tag{13}$$

The elements A_i in Figure 3 are now rearranged in descending order. It is noted that the horizontal axis of Figure 4 is non-dimensional since the collection of rectangular area elements is ordered in one of many possible configurations. All these rectangular elements have a base of $dw = dx\, dy = 0.25$, being that there are 400 area elements, Figure 4 might be considered as a representation of some monotonic descending function $g(w)$, where $0 \le w \le 100$.

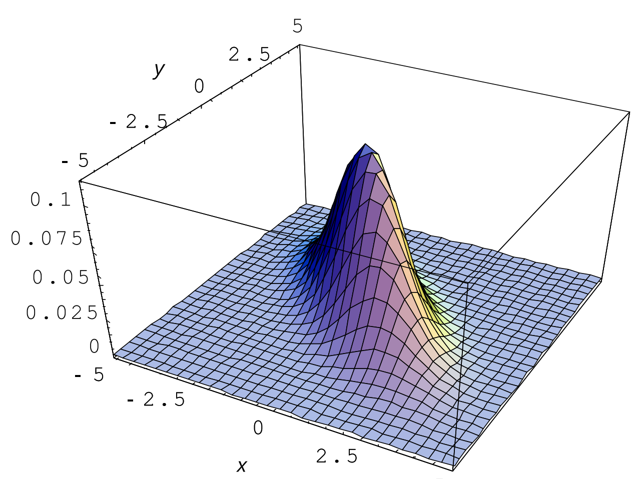

Figure 1. Plot of function *f*.

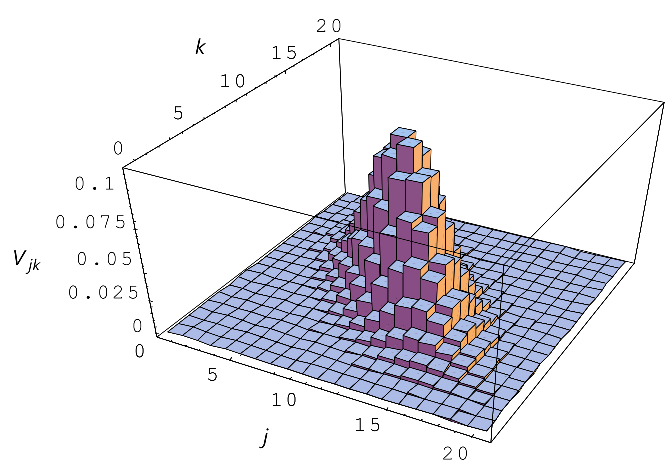

Figure 2. Volume elements *V* of function *f*.

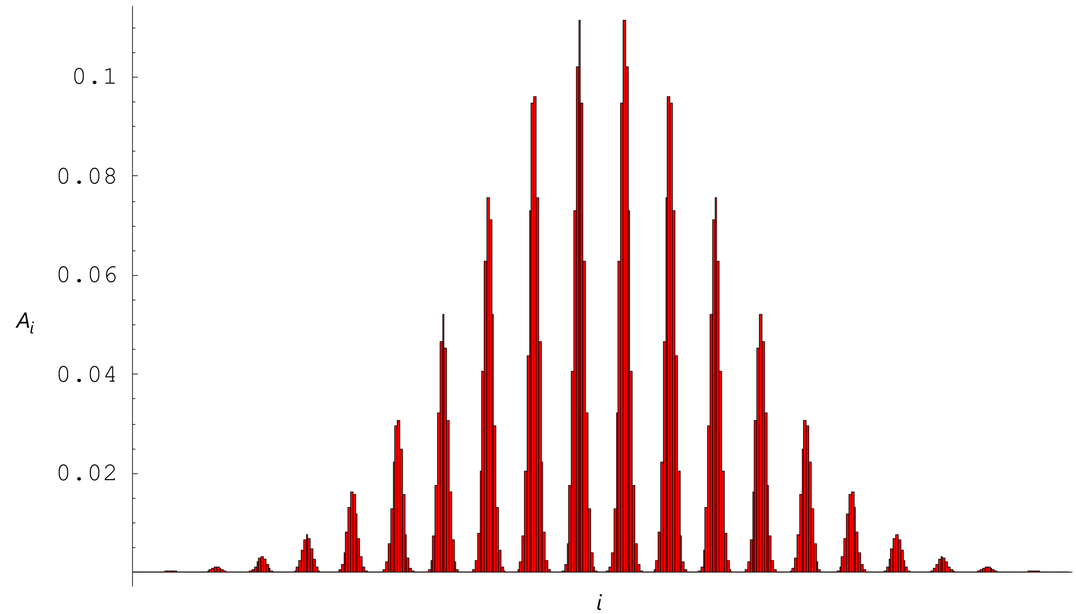

Figure 3. Area elements A of function f.

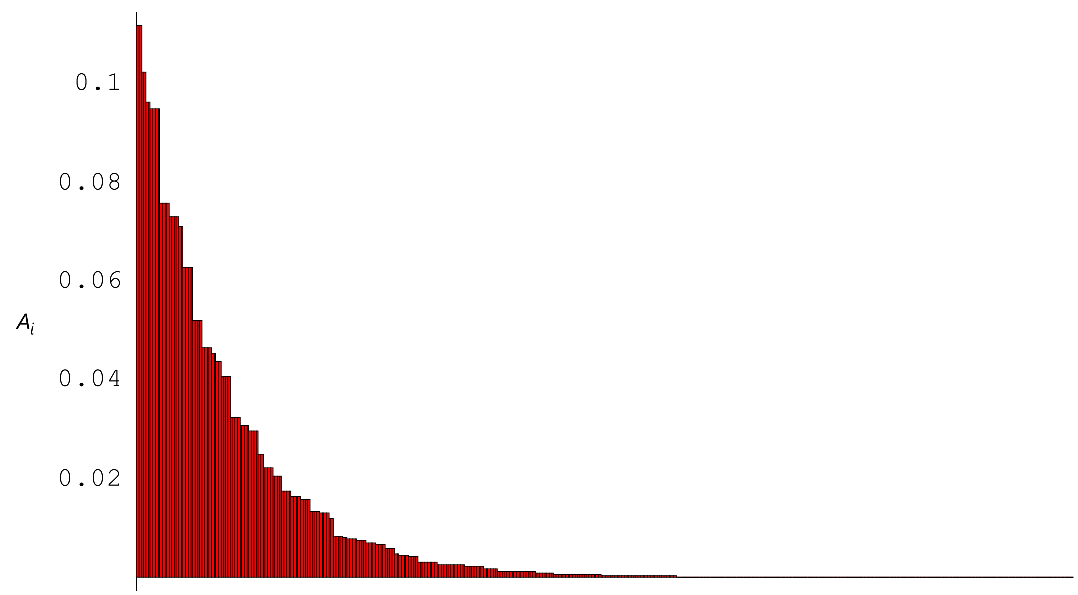

Figure 4. Ordered area elements A of function f.

What has been accomplished is a mapping of 3-dimensional volume elements (Figure 2) to 2-dimensional area elements (Figure 3) then rearranged (Figure 4) so as to get a monotonic descending 'function' $g(w)$ (Figure 5). The univariate function $g(w)$ might now be integrated and, again, get the volume one is looking for. In this manner, any k-variate function may be reduced to a corresponding monotonic descending univariate function $g(w)$. It will be shown in the following how the procedure of nested sampling is based upon the equivalence between any k-variate function and its corresponding $g(w)$.

Sampling abscissas

If a value of $g(w)$ is considered, without knowing the value of w, the only knowledge about w is that it must lie somewhere in the region $0 \leq w \leq W$, where W is the area for which the k-variate function is defined (see for example, the area for which (1) is defined and the w-axis of Figure 5). Hence, it is derived that w is univariate uniformly distributed, $w \sim U(0, W)$, with mean and standard deviation of:

$$E(w) = \frac{W}{2}, \text{std}(w) = \frac{W}{2\sqrt{3}} \tag{14}$$

It is then supposed to have sampled N values of $g(w)$, which corresponds to having sampled $g(w_1), \ldots, g(w_N)$. Though the values of $w_1, \ldots, w_N$ are still unknown, the one thing which is known is that the smallest realisation of $g(w)$ must correspond with the greatest value of w. This is because function $g(w)$ is a monotonic descending function. It follows

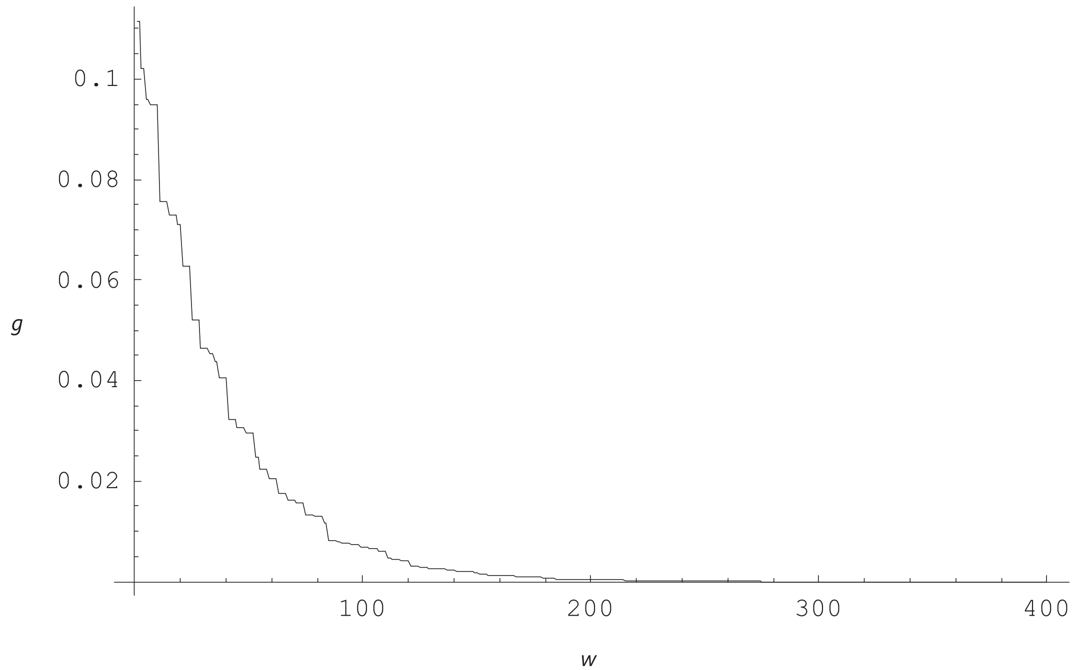

Figure 5. Plot of function *g*.

that one may use an order distribution for the unknown value w_{max}:

$$p(w_{max}) = N\left(\frac{w}{W}\right)^{N-1}\frac{1}{W} \tag{15}$$

with mean and standard deviation of:

$$E(w_{max}) = W - \frac{1}{N+1}W \tag{16}$$

while:

$$\text{std}(w_{max}) = W\sqrt{\frac{N}{(N+1)^2(N+2)}} \tag{17}$$

and where both the values of N and W are known to us. It comes that the standard deviation, that is, our uncertainty regarding the unknown value of w_{max}, falls of with a factor N. Actually, Equations (16) and (17) form the backbone of the nested sampling algorithm.

The naïve nested sampling algorithm

The steps of the nested sampling algorithm (which is naïve in that issues of under- and overflow are neglected) are provided in the sequence:

Step 1: Set $W^{(1)}$ to be the area/volume/hyper-volume of the domain of the function $f(\text{x})$.

Step 2: Sample N realisations of f uniformly over the domain of f, Figure 1:

$$S^{(1)} = \{f_1, \ldots, f_N\} \tag{18}$$

which by construction is equivalent to sampling N realisations of g uniformly over the w-axis of the univariate representation g, Figure 5.

Step 3: The realisation with the smallest value f (see Figure 1 and Equation (18)):

$$f_{\min}^{(1)} = \min S^{(1)} \tag{19}$$

corresponds with the evaluation on g (see Figure 5), that lies farthest to the right on the w-axis; as g is a monotonic descending function. For these N evaluations, the unknown abscissa w of the known ordinate $g^{(1)} = f_{\min}$ is then set as in (11):

$$w^{(1)} = E(w_{max}) = \left(\frac{N}{N+1}\right)W^{(1)} \tag{20}$$

which gives us as a first approximate point of g the coordinate:

$$\left(w^{(1)}, g^{(1)}\right) = \left[\left(\frac{N}{N+1}\right)W^{(1)}, f_{\min}^{(1)}\right] \tag{21}$$

Step 4: Store the domain point x_n which gave $f(\text{x}_n) = f_{\min}^{(1)}$ as:

$$\text{x}^{(1)} = \text{x}_n, \text{ where } 1 \le n \le N \tag{22}$$

The likelihood weight which is associated with this domain point of f is:

$$A^{(1)} = \frac{W^{(1)}}{N+1}f_{\min}^{(1)} \tag{23}$$

Step 5: Set $W^{(2)}$ equal to $w^{(1)}$:

$$W^{(2)} = \frac{N}{N+1}W^{(1)} \tag{24}$$

All the proposal values $g^{(2)}$ are constrained to be greater than $g^{(1)} = f_{\min}$, thus respecting the monotonic descending character of g, as presented in Figure 5.

Step 6: One now sets the constraint that all the realisations N in Step 2 should have all have values greater than $f_{\min}^{(1)}$. If one drops $f_{\min}^{(1)}$ from $S^{(1)}$, (Equation (18)), and produces a new realisation $f > f_{\min}^{(1)}$ which is added to $S^{(1)}$, then the new set of realisations is produced:

$$S^{(2)} = \{f_1, \ldots, f_N\}, \text{where} f_n > f_{\min}^{(1)} \text{ for } n = 1, \ldots, N \quad (25)$$

Step 7: Set:

$$f_{\min}^{(2)} = \min S^{(2)} \quad (26)$$

and set the unknown abscissa of this known ordinate as (Equation (20)):

$$w^{(2)} = \left(\frac{N}{N+1}\right) W^{(2)} = \left(\frac{N}{N+1}\right)^2 W^{(1)} \quad (27)$$

The second approximate point of g then is the coordinate:

$$\left(w^{(2)}, g^{(2)}\right) = \left[\left(\frac{N}{N+1}\right)^2 W^{(1)}, f_{\min}^{(2)}\right] \quad (28)$$

Step 8: Store the domain point x_n which gave $f\left(x_n\right) = f_{\min}^{(2)}$ as:

$$x^{(2)} = x_n, \text{ where } 1 \le n \le N \quad (29)$$

The likelihood weight which is associated with this domain point of f is:

$$A^{(2)} = \frac{W^{(2)}}{N+1} f_{\min}^{(2)} = \left(\frac{N}{N+1}\right) \frac{W^{(1)}}{N+1} f_{\min}^{(2)} \quad (30)$$

Step 9: Set:

$$S^{(r)} = \{f_1, \ldots, f_N\}, \text{where} f_n > f_{\min}^{(r-1)} \quad (31)$$

for $n = 1, \ldots, N$, and

$$f_{\min}^{(r)} = \min S^{(r)} \quad (32)$$

Then the rth approximate point is:

$$\left(w^{(r)}, g^{(r)}\right) = \left[\left(\frac{N}{N+1}\right)^r W^{(1)}, f_{\min}^{(r)}\right] \quad (33)$$

Step 10: Store the domain point x_n which gave $f\left(x_n\right) = f_{\min}^{(r)}$ as:

$$x^{(r)} = x_n, \text{where } 1 \le n \le N \quad (34)$$

The likelihood weight which is associated with this domain point of f is:

$$A^{(r)} = \left(\frac{N}{N+1}\right)^{r-1} \frac{W^{(1)}}{N+1} f_{\min}^{(r)} \quad (35)$$

Step 11: Because of the factor $\left[N/(N+1)\right]^{r-1}$ in Equation (35), $A^{(r)}$ will tend to 0 as $r \to \infty$, even as $f_{\min}^{(r)}$ increases. Consequently, the algorithm may be terminated if for instance (Equation (35)):

$$A^{(r)} < \frac{\sum_{i=1}^{r-1} A^{(i)}}{N^2} \quad (36)$$

However, it should be noted that there has been no rigorous criterion developed so far to ensure the validity of the above terminating condition; as there is always the probability that some high likelihood remains for which the dropping off of the factor $\left[N/(N+1)\right]^{r-1}$ is compensated by the increase in the factor $f_{\min}^{(r)}$ (Equation (35)). Therefore, termination still remains a matter of user judgement (Skilling, 2006). Let T be the termination step. Then, the nested sampling output consists of a collection of domain points $x^{(r)}$, for $r = 1, \ldots, T$, with corresponding probability weights:

$$P_r = \frac{A^{(r)}}{\sum_{i=1}^{T} A^{(i)}} \quad (37)$$

In summary, if one can sample within the nested likelihood constraints (Equations (18), (25), (31)), then one may use the laws of probability theory, that is, the order statistic (Equation (16)), to construct the unknown univariate representation g, Figure 5, of f, Figure 1, by way of the approximate points (Equations (21), (28), and (33)). This provides a set of probability weighted domain points (Equations (34), (35) and (37)). These probability weighted domain points form a representative set of random samples from f (Skilling, 2004).

The sophisticated nested sampling algorithm

The original nested sampling algorithm is sophisticated in that it guards the user against the almost inevitable under- and overflow that will arise in non-trivial data analysis problems, by making in Figure 5 a change of variable from w to $u = \log w$, and by taking the log of the ordinate, that is, by sampling log g's rather than g's (Skilling, 2004).

The reason why the naïve algorithm was discussed herein over the original one is because it is believed that such a naïve approach better demonstrates the elegance of the nested sampling framework, as issues of optimal implementation may obfuscate the intimate link between the well-known order statistic (Equation (16)) and this relatively new Monte Carlo sampling framework.

For non-trivial data analysis problems the reader is referred most emphatically to the original nested sampling algorithm (Skilling, 2004). For an explicit link between this original nested sampling algorithm and the here given naïve discussion, the interested reader can refer to (Van Erp & Van Gelder, 2009).

Generating nested sampling samples

Let f be a function defined on a highly multivariate parameter space x. Then nested sampling is a Monte Carlo framework by which this multivariate f may be evaluated. The nested sampling framework needs uniformly sampled realisations of f within the multivariate geometry of some constraint f^* in order for it to work. However, this framework does not tell us how to obtain these samples, that is, its optimal implementation is an open-ended research question.

The nested sampling evaluations of the Dirichlet distributions (Equation (40)) have been implemented by way of the Inner nested sampling algorithm (Van Erp, Linger, & Van Gelder, 2017), which obtains uniform samples within some constraint f^*. The idea behind Inner nested sampling is to obtain a set of differentials of the multivariate geometry of the initial constraint

f^* at iteration step $t = 0$ of nested sampling proper. These differentials are defined by a direction e and a radius $R(\mathrm{e})$ and serve as a proxy for the actual geometry. This proxy geometry has the nice property that it is extremely amenable to uniform sampling. Furthermore as with each iteration step t the geometry defined by the f^* constraint will shrink, the radii $R(\mathrm{e})$ may be updated so as to reflect this shrinkage. This then allows us to continue the uniform sampling of these differentials and, by proxy, the likelihood geometry of interest, with a minimum of rejections.

Prediction of maintenance costs considering inspection database

Use of inspection database

As mentioned in the introduction, the objective of the proposed framework is to deliver an asset management framework based on the inventory of an asset and condition assessment. The goal is to determine some degradation profiles for infrastructure components or infrastructures as a whole. Once the degradation profiles are determined, they can be used to characterise how the degradation of infrastructures evolves with time. For this purpose, a stochastic Markov chain approach is used for predicting the performance of infrastructure components and combined with the nested sampling framework presented in the previous section.

The following case study considers transition sequences in an inspection database observed during a certain period of time. For a five-stage scoring system (rates 1–5 modelling good to poor conditions), it is supposed to observe transitions of Table 1 during the reference period. Such scenario is noted Sc1. Two alternative scenarios are then considered to model ageing and deterioration of these components exacerbated either by climate change or increasing traffic intensities and loads (scenarios Sc2 and Sc3 explicited in Tables 2 and 3, respectively). For scenarios Sc2 and Sc3 and compared to scenario Sc1, it is assumed to see 10% more of infrastructures in i moving in $i + 1$ (scenario Sc2), or 10% more of infrastructures in i moving in $i + 2$ (scenario Sc3). The transitions observed in each case are illustrated in Figure 6 for each initial condition state.

Table 1. Number of transitions between condition states (CS) for Scenario Sc1.

	1	2	3	4	5
1	117	31	8	0	0
2	14	315	22	2	0
3	2	18	325	6	1
4	0	4	3	86	1
5	1	1	1	0	27

Table 2. Number of transitions between condition states (CS) for Scenario Sc2.

	1	2	3	4	5
1	106	42	8	0	0
2	14	284	53	2	0
3	2	18	293	38	1
4	0	4	3	78	9
5	1	1	1	0	27

Table 3. Number of transitions between condition states (CS) for Scenario Sc3.

	1	2	3	4	5
1	106	31	19	0	0
2	14	284	22	33	0
3	2	18	293	6	33
4	0	4	3	78	9
5	1	1	1	0	27

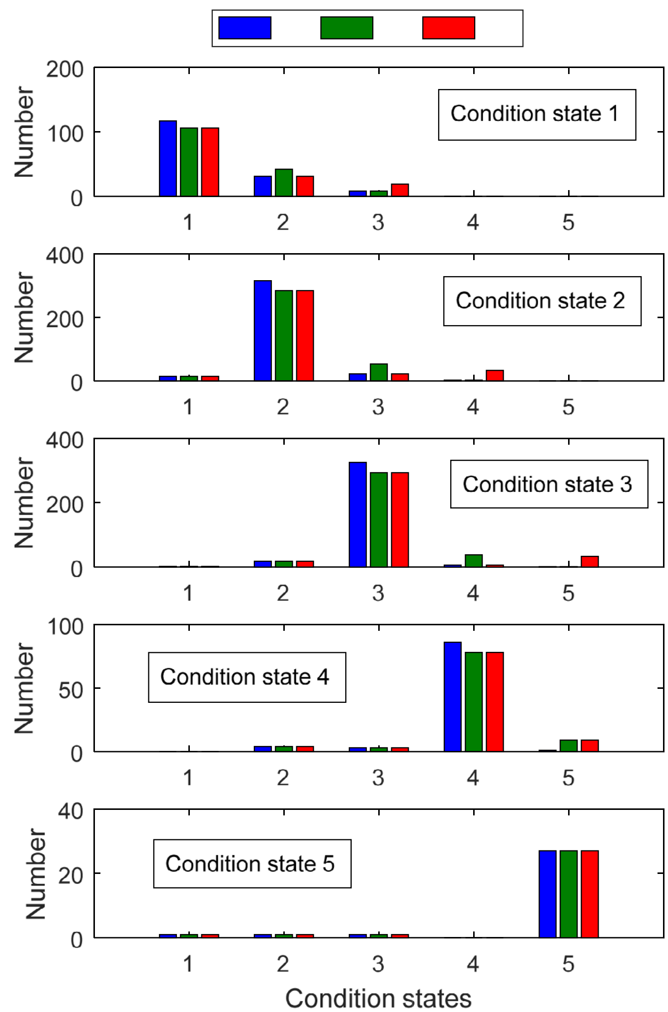

Figure 6. Number of transitions observed in the database for each condition state (original state is associated with each subplot and destination state is associated with y-axis value) for scenarios Sc1, Sc2 and Sc3.

Combination of nested sampling with the Markov assumption

Once transition probabilities are determined, the performance of each bridge/retaining wall component through the use of an adequate lifetime indicator. This indicator is determined herein by the probability for a component to be scored in a certain condition with time. If (i) the probability of a component b to be quoted in any score is known at year i (for example, after a visual inspection of the bridge) and stored in a vector q_b^i and (ii) the associated homogeneous Markov chain, associated with a transition matrix P_b, is determined, the probability at year $i + 1$ is given by the following equation:

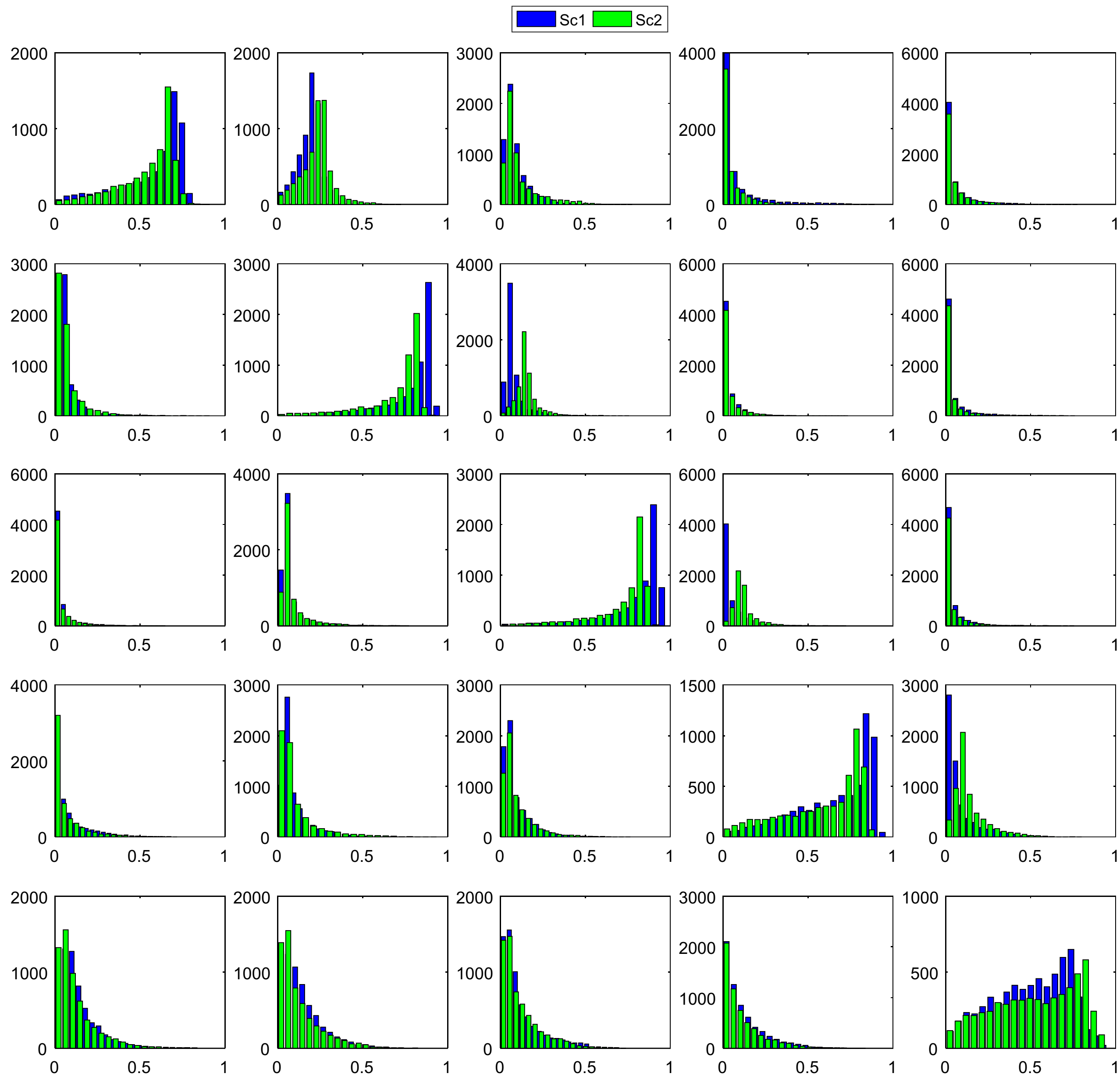

Figure 7. Comparison of terms in matrix $M\left(\theta^{(r)}\right)$ between scenarios Sc1 and Sc2.

$$q_b^{i+1} = q_b^i \, P_b \tag{38}$$

Assuming a homogeneous Markovian process, the scoring probability can then be forecasted if the transition matrix and the initial probability vector are known. If the costs of each degradation state are put in the column cost vector c, then the total cost C at time step $i+1$ of the state q_b^{i+1} is given as the inner vector product:

$$C^{i+1} = q_b^{i+1} \cdot c = \left\langle q_b^{i+1}, c \right\rangle \tag{39}$$

In order to put confidence bounds around the Markovian estimate (Equation (38)), the entries in Tables 1 through 3 are inputted into corresponding Dirichlet probability distributions of the unknown vector of probabilities θ:

$$p(\theta|D) \propto \theta_1^{r_1 - 1/n} \cdots \theta_n^{r_n - 1/n} \tag{40}$$

for $n = 25$, where (Equation (38)):

$$P_b = M(\theta) = reshape(\theta) \tag{41}$$

where M is some vector-to-matrix reshaping function like MATLAB's *reshape* to move from the vector of probabilities to a 5×5 transition matrix. If one sets (Equation (38)):

$$f(\theta) = p(\theta|D) \tag{42}$$

then, the nested sampling framework gives us a collection of vectors of probabilities $\theta^{(r)}$, for $r = 1, \ldots, T$, with corresponding probability weights P_r.

Figure 7 (respectively, Figure 8) illustrates a comparison of terms in $M\left(\theta^{(r)}\right)$ between scenarios Sc1 and Sc2 (respectively between scenarios Sc1 and Sc3). It can be observed how the distribution of probabilities to move from state i to $i+1$ (respectively, from state i to $i+2$) shifts to the right between scenarios

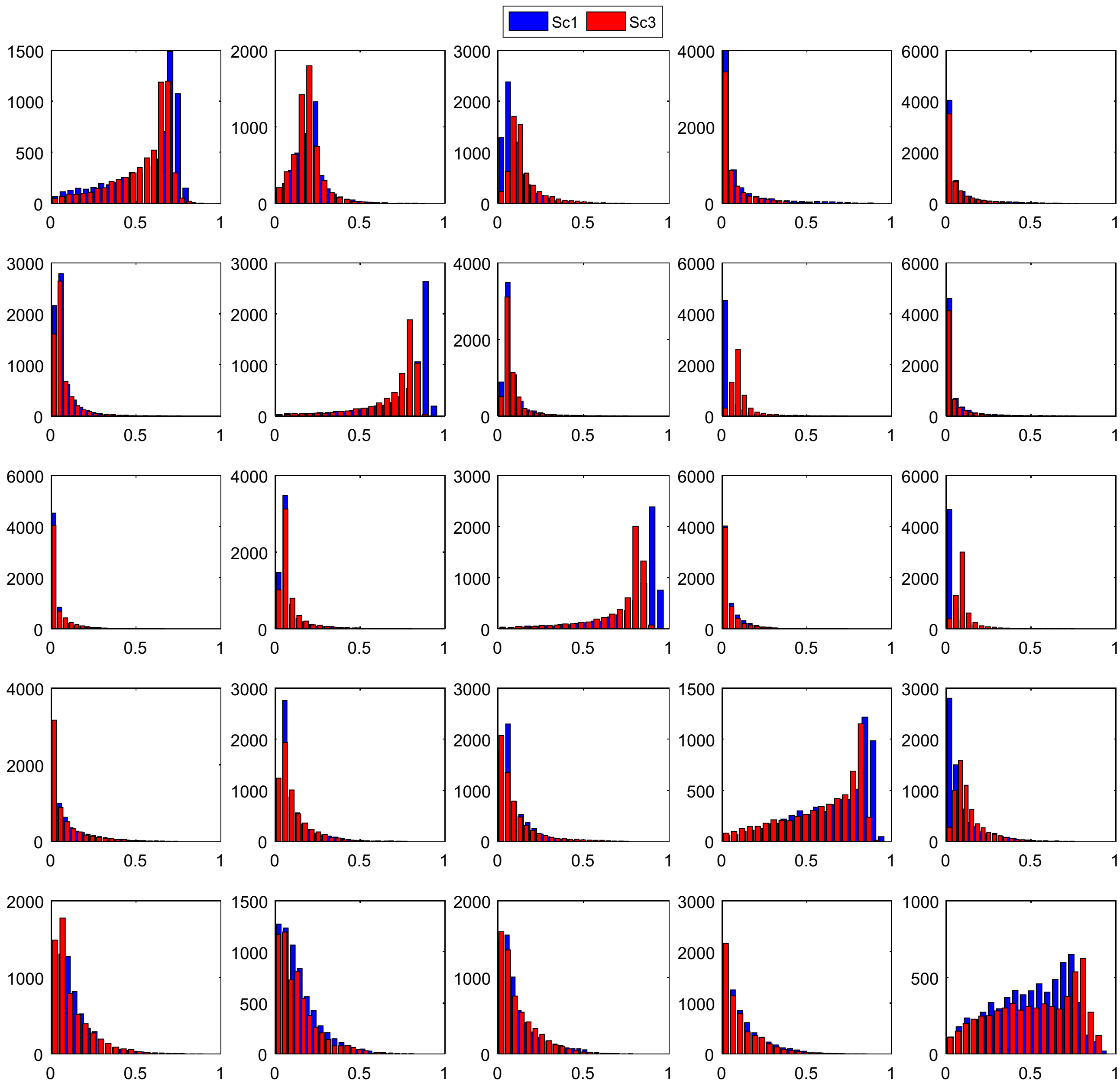

Figure 8. Comparison of terms in matrix $M\left(\theta^{(r)}\right)$ between scenarios Sc1 and Sc3.

Sc1 and Sc2 (respectively, between scenarios Sc1 and Sc3). It comes that (Lindgren, 1993):

$$E\left[C^{i+1}\right] = \sum_{r=1}^{T} P_r \left\langle \mathrm{q}_b^i M\left(\theta^{(r)}\right), \mathrm{c}\right\rangle \tag{43}$$

and:

$$E\left[\left(C^{i+1}\right)^2\right] = \sum_{r=1}^{T} P_r \left\langle \mathrm{q}_b^i M\left(\theta^{(r)}\right), \mathrm{c}\right\rangle^2 \tag{44}$$

The mean cost at time step $i + 1$ then is given as:

$$\mu_C^{i+1} = E\left[C^{i+1}\right] \tag{45}$$

while the standard deviation at time step $i + 1$ then is given as:

$$\sigma_C^{i+1} = \sqrt{E\left[\left(C^{i+1}\right)^2\right] - \left(E\left[C^{i+1}\right]\right)^2} \tag{46}$$

The outputs (Equations (45) and (46)) are then used to compute the confidence bounds of the total costs for the various time steps, as displayed in Figure 9. In this figure, it is supposed to have $\mathbf{c} = \begin{pmatrix} 0 & 5 & 10 & 40 & 50 \end{pmatrix} \times 10^3$€ and to consider an asset initially with 30 elements in state 1, 15 in state 2, 20 in state 3, 5 in state 4 and 6 in state 5. This case study shows how the combination of nested sampling theory with a Markov-based model enables to build lifetime indicators (herein some maintenance cost associated with the condition of the asset). One can clearly see the impact of changes in the number of transitions observed in scenarios Sc1, Sc2 and Sc3, on the profile of the degradation cost with time.

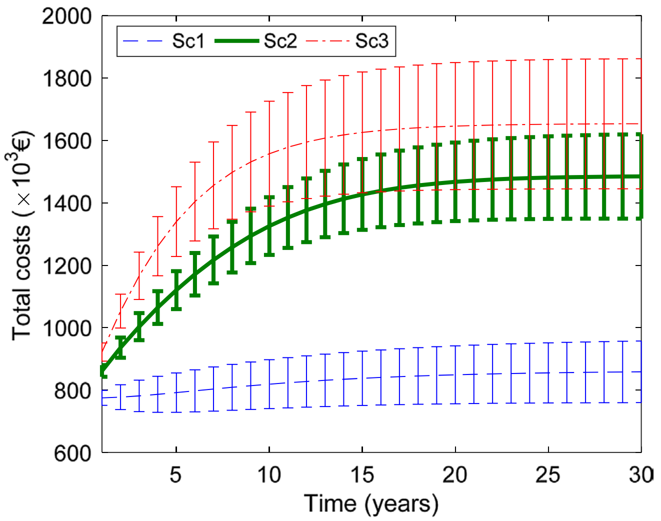

Figure 9. Prediction of maintenance costs for scenarios Sc1, Sc2 and Sc3.

Conclusions

Bridges, retaining structures, and steep embankments are significant critical infrastructure components in terms of safety and functionality for the whole transportation infrastructure. Decisions about the replacement or repair of these infrastructures, as well as when and how to repair each infrastructure are common and among difficult management issues for asset managers. In particular, asset management systems are employed to manage transportation infrastructure and help guide policy-makers.

The proposed framework is based on visual inspections (e.g., condition rating) and enables to forecast condition of road infrastructures. More specifically, using as input the inventory of the asset and condition assessment, the proposed method combines the nested sampling theory with the Markovian approach to determine cost confidence bounds. Different types of hazards (due to climate change, traffic growing …) can then be considered. The next steps of this research work are indicated below:

(1) Combine nested sampling theory with advanced Markov based approaches such as semi-MDP or POMDPs to better take into account the complexity of the degradation phenomena, the lack and heterogeneity of available information and the reliability of data sources. Indeed, the proposed homogeneous Markov-based model to determine transition probabilities among states still suffers from some limitations among which the following ones, identified by Mishalani and Madanat (2002): it does not explicitly capture the effect of various explanatory variables; the possible nonhomogeneity (i.e., time dependence) of the deterioration process can only be captured indirectly through ad hoc time segmentation; the presence of an underlying continuous deterioration level is not recognised.

(2) Compare the confidence intervals obtained with the nested sampling theory with those using other sampling methods like the Gibbs sampling (Gamerman & Lopes, 2006; Kobayashi, Kaito, & Lethanh, 2012b).

(3) Investigate how the global prediction of the system condition (e.g., bridge) can relate to its individual components (deck, joints, piers, bearings), under uncertainty. Indeed, an infrastructure consists of several components, and each component has its own failure probability; the interaction between components determining the overall failure probability of an infrastructure. Elaborating a performance indicator at a system level requires considering interactions between infrastructure components.

(4) Translate condition profiles in a risk analysis and determine optimal maintenance/repair actions (Memarzadeh & Pozzi, 2016) under uncertainty and limited funds. Optimal parameters are those that minimise the overall risk while minimising the maintenance costs. The ultimate goal is to allow stakeholders to assess the necessary additional effort to satisfy performance constraints under different scenarios (e.g. traffic growth or exacerbated degradation due to climate change effects).

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Disclosure statement

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