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# **A Distributed Predictive Control Strategy for a Two-Route Public Transport System with a Transfer Station**

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## **1. Introduction**

In the context of transit systems' management, uncertainty is present in different inputs required for the operator to make dispatch decisions properly. At the operational level, traffic congestion, passenger demand, non-recurrent congestion due to incidents, among multiple possible facts, make these systems very difficult to control without efficient tools or systematic rules to ameliorate the effect on the level of service caused by unexpected randomness in these features of the real time-operation. One efficient way to deal with such situations is to impose real-time control strategies to set the service frequencies trying to minimize the effects of the different disturbances in the operation of such transit systems. In this sense, the literature is extensive in developing flexible control strategies, depending on the specific features of the problem. The most studied strategy is holding (Eberlein, 2001; Yu and Yang, 2007), in which vehicles are held at specific stations for a certain time, mostly trying to keep the headway between successive buses as close as possible to a predefined value. Moreover, and as a complement to holding-type strategies, rules based on expressing or stop-skipping are formulated in order to somehow speed up vehicles instead of delay them as in holding (Fu et al., 2003). In addition, if the dynamic control of the systems involves synchronization of routes at specific stations in order to allow travelers to transfer from one line to another, the resulting optimization problem becomes more complicated and hard to implement in a real setting (Nesheli and Ceder, 2015; Kim and Schonfeld, 2014).

In the literature, centralized model predictive control (MPC) strategies have been proposed to control single-route public transport systems (Sáez et al., 2010; Cortés et al., 2012; Koehler et al., 2011) based on holding and stop skipping. The strategy can find the global optimal solution; however, the main drawback is the expensive computational effort in solving the related optimization problem. In this paper a distributed model predictive control strategy is proposed to minimize the waiting time of the passengers in a two-route public transportation system with a transfer station. In this strategy, the optimization problem of the two routes is solved in a decoupled

way (distributed) with the possibility to communicate the control decisions in a synchronise/coordinated manner. Distributed strategies can outperform the case of totally decentralised strategies, in which each route is operated independently without communication.

## 2. Proposed Distributed Predictive Control Strategy

Figure 1 depicts the proposed distributed predictive control (DMPC) strategy. Each route has its own local hybrid predictive controller, to determine holding  $h_i(k)$  and/or stop-skipping  $Su_i(k)$  at each bus  $i$ . Particularly, for the transfer stop, the local controllers interact with each other to coordinate the arrival of buses coming from both routes. The local controllers require information coming from a demand estimator module and the measurable state vector of the public transport system. Given the measured demand at bus-stop  $p$  of route 1,  $\Gamma_p(k)$  ( $\Gamma'_p(k)$  for route 2), the estimator block provide the expected demand value  $\hat{\Gamma}_p(k+1)$  ( $\hat{\Gamma}'_p(k+1)$  for route 2). Regarding the estimation of the state vector, this includes at time step  $k+1$ , the positions of buses  $x_i(k+1)$ ,  $i=1,2, \dots, N$  (for a fleet of  $N$  buses), their free-capacity  $L_i(k+1)$ , and their departure time from the last stop  $Td_i(k+1)$  ( $x'_i(k+1)$ ,  $L'_i(k+1)$ , and  $Td'_i(k+1)$  for route 2). The free-capacity of the buses and the estimated demands at the transfer station are communicated between the local controllers, so they can decide whether or not synchronize their operations.

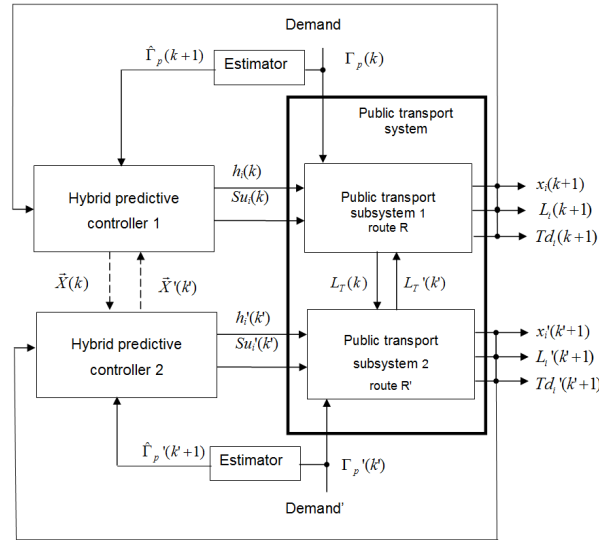


Figure 1. Proposed distributed predictive control strategy for the control of a two-route public transport system

The local hybrid predictive controllers decide the holding and stop-skipping actions by minimizing the travel time, the waiting time, the deviation of the headway between two consecutive buses from its design value. The local optimization problem solved by local controller of route 1 is the following (local controller of route 2 has the same formulation):

$$\begin{aligned} \min_{h_i(k), Su_i(k)} \quad & w_1 \hat{H}_i(k+1) \hat{\Gamma}_p(k+1) + w_2 (\hat{H}_i(k+1) - H_d)^2 + w_3 L_i(k+1) h_i(k) Su_i(k) \\ & + w_4 \hat{\Gamma}_p(k+1) \hat{H}_{i+1} (1 - Su_i(k)) + w_5 D_T (L_T(k+1), L'_T(k+1), \hat{\Gamma}_T(k+1), \hat{\Gamma}'_T(k+1), \tau(k+1), \tau'(k+1)) \end{aligned} \quad (1)$$

$$\begin{aligned}
\text{s.t} \quad & \hat{H}_i(k+1) = x_i(k+1) - x_{i-1}(k+1) \\
& h_i(k)Su_i(k) = 0 \\
& h_i(k) \text{ discrete,} \quad Su_i(k) \in \{0,1\}
\end{aligned}$$

where  $w_1, \dots, w_5$  are positive weighting factors,  $H_d$  is the desired headway between two consecutive buses, and  $D_T(L_T(k+1), L'_T(k+1), \hat{\Gamma}_T(k+1), \hat{\Gamma}'_T(k+1), \tau(k+1), \tau'(k+1))$  is a function that relates the free-capacity of the route 1 bus closest to the transfer station,  $L_T(k+1)$ , the free-capacity of route 2 bus closest to the transfer station,  $L'_T(k+1)$ , the estimated demand, computed by each estimator, in the transfer station  $\hat{\Gamma}_T(k+1)$  and  $\hat{\Gamma}'_T(k+1)$ , and the arrival of route 1 and route 2 closest buses time to the transfer station  $\tau(k+1)$  and  $\tau'(k+1)$  respectively, and allows each controller determining whether or not synchronise the arrival of the closest buses to the transfer station. Indeed, based on the values of  $\tau(k+1)$  and  $\tau'(k+1)$ ; and on the values of  $L_T(k)$  and  $L'_T(k)$  the arrival times of the nearest buses to the transfer station are estimated, and holding  $h(k)$  and stop-skipping actions  $Su(k)$  are assigned to these buses to synchronise both routes if necessary.

### 3. Results

The two-route system consists of two corridors of 4 km long with a fleet of 6 buses with a capacity of 72 passengers each. In total 10 bus-stops are considered, one of which is common to both routes, and allows passenger transference. At each bus-stop, the arrival of passengers were modelled as a Poisson process with different demand rates, and the time taken by the passengers to get on and get off the bus were 2 s/person and 1.5 s/person respectively. Furthermore, it was assumed that each bus moved between stations at a constant speed of 25 km/h. Simulations were performed over 30 days with different transfer rates and considering an average historical demand.

Table I presents the results obtained with the proposed distributed predictive control strategy, and compares them with the results obtained with a decentralized strategy in which each route is independently operated using the controller in Sáez et al. (2010) and Cortés et al. (2012). Average total time spend by the passengers was considered for comparison purposes. Specifically, the relative difference between average total times spends by passenger with both strategies was reported. Average total time spend was computed as the sum of travel time and waiting time, both also reported in Table I. Whereas, relative difference was computed considering the results with the decentralised controller as ground-truth. Independently of the transference percentage, distributed predictive control allowed reaching a better performance in terms of average total time spend than the decentralised control strategy. As the transference rate increased, the relative difference between both control strategies also increased, showing the importance of coordination mechanism.

Note that, under the simulated conditions, passengers average travel time was almost the same for both controllers. A striking difference between two assessed control strategies was only evidenced on the passengers average waiting time, being 13.62% when 85% transference rate was considered. These results also imply that high coordination was achieved between two routes with the proposed controller. But such coordination had less effect on the waiting time as the transference rate

decreases, and explains the direct relationship between transference rate and relative difference in the results reported in Table I.

Table I. Comparison of the results obtained with a decentralised and the distributed predictive control strategies.

Control strategy	Transference	Travel time [min]	Waiting time [min]	Difference [%]
Decentralised	85%	20.34	9.91	-
Distributed		20.40	8.56	4.26
Decentralised	50%	14.18	8.57	-
Distributed		14.22	7.59	4.13
Decentralised	15%	10.31	6.44	-
Distributed		10.29	5.85	3.64

#### 4. Concluding Remarks

Since centralised predictive control strategies might become unsuitable for large-scale applications, in this paper distributed predictive control is proposed to provide an alternative for the optimal control of public transportation systems with two routes. Under the simulated conditions, distributed predictive control allowed improving the quality of service of the system, measured as reduction of the waiting time and reduction of the total time spend by passengers. Although the formulation presented in this paper considered only two routes, it can be easily extended to public transport systems with more than two routes and even to systems that combines buses with other forms of massive public transport. Further research is required to determine how the distribution of the headways of the buses throughout the corridors can be incorporated in the distributed scheme.

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