Solving liquids by discarding fluid dynamics

Predicting force feedback of liquids for haptic bilateral teleoperation

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Abstract

This paper presents a novel approach to simulating liquid interactions for haptic bilateral teleoperation without complex fluid dynamics simulations. We propose a model combining a simplified drag equation with a "position tail" mechanism to approximate force feedback for viscous and turbulent liquids. The model was evaluated through theoretical analysis, manual testing, and a user study. Results of the latter demonstrate the ability to convincingly simulate different liquid types, with participants correctly identifying viscous and turbulent liquid types. The study also revealed that visual feedback delays up to 125 ms did not significantly impact user experience. While promising, the system remains sensitive to noisy input data. This research contributes to the development of efficient and believable haptic simulations for teleoperation, paving the way for future developments.

1 Introduction

In recent years, there have been great and awe-inspiring strides in the fields of robotics, networking and control. We seem to be steadily getting closer to a world where automatons handle our most difficult tasks. For now, however, no technology can easily match the skill and accuracy of a human operator. Yet, with this in mind, we can already start deploying teleoperated robots that can help surgeons and rescue workers to deliver much-needed aid.

The greatest benefit of this approach by far is the application of human expertise over distances or in dangerous places [1]. In theory, a skilled surgeon could help people around the whole world with no travel time needed, while rescue workers could easily explore hazardous areas. Yet, these distances induce delays and inaccuracies that make even the simplest tasks much harder. For example, imagine moving a robotic arm that can relay touch feedback. If we tried to pick up an egg, it could be the case that the robot had already crushed it by the time we felt the egg's shell.

One of the solutions to this problem is the use of complex simulations on the side of the operator, yet this approach is not always ideal [2]. It necessitates the simulation of complex materials such as fluids, fabrics, aggregates etc. By limiting the scope of the simulation to only recreating the haptic feedback and replacing the visual feed with live video, we can greatly simplify the problem. It allows us to only focus on simulating the haptic feedback and ensuring that the camera image is transmitted quick enough for a comfortable experience.

Out of the materials mentioned previously, liquids often pose the biggest challenge. They can be seen in many real-world scenarios and modified liquid simulations can be used for other materials such as gases or soft bodies. Overcoming these problems would assist in underwater or harsh terrain rescue operations, help in certain surgical procedures and fundamentally broaden the horizon of bilateral teleoperation. To achieve this we must inquire how to approximate satisfactory liquid-induced force feedback in haptic bilateral teleoperation? Can this be achieved without relying on slow and complex fluid simulations?

The contributions contained within this paper are shown below:

- A fluid interaction model based on the Drag Equation that can facilitate interactions with static or constant flow liquids.
- An implementation of the model that can handle interactions with static viscous fluids.
- Augmentation of the model by the addition of a proposed *position tail*, which allows it to approximate certain interactions with turbulent liquids.
- Comprehensive theoretical evaluation of the solution's effectiveness based on simulated data, supplemented with a limited user study.
- Evaluation of maximum delay between visual feed and haptic feedback that allows for a comfortable interaction with the system.

To describe these contributions, the remainder of this paper is structured as follows. Firstly, in Section 2 we will overview the existing literature and place our research in a wider academic context. Section 3 provides background on the proposed solution and outlines the experimental setup. Then Section 4 describes our proposed model for handling liquid force feedback followed by test results in Section 5. Section 6 contains an additional interpretation of the results and outlines possible future work. Finally, Section 7 touches on possible ethical considerations with Section 8 summarizing our contributions.

2 Related Literature

The focus of this research is the simulation of liquids, however, the simulation approach should be explained first. Other solutions exist, but when dealing with delays and large distances this one offers the best qualities [3]. In this way, the operator can perform a task in a simulated environment, so that later their movements can be relayed back to the machine. However, this approach does have limitations, as it requires the simulation of a complex environment in which the machine operates which includes different types of surfaces, gases, liquids, cutting, moving through particles or any other possible physical interaction.

Fortunately, this can also be partially mitigated by only relaying haptic feedback, as shown in the yet-unpublished paper authored by our supervisor. Our goal is then reduced to creating force feedback that can convince a human operator and facilitate an accurate application of their expertise. This approach vastly reduces the required complexity of the simulation, making it possible to circumvent traditional physical simulations and is the basis of our proposed solution.

As for the liquids themselves, there exist a few approaches. The first one would be the use of *Computational Fluid Dynamics*, which delivers great accuracy, yet it requires large and expensive calculations [4]. This makes it unsuitable for real-time simulations. Another solution could be the *Navier-Stokes Equation*, based on Newton's conservation of momentum. It takes into account many parameters, not all of which are easily modelled, but as shown in engineering textbooks, by making certain assumptions, it can be simplified into the *Drag Equation* [5], which will be the equation evaluated in this paper.

This equation is rarely used in its basic form since it heavily relies on the *drag coefficient*, which is usually obtained through field measurements. Such evaluations are sometimes not possible in other fields and lead to the development of specific ways of numerical estimation [6; 7]. However, the values for common shapes and liquid types that our solution is dealing with are readily available and have been exhaustively tested [8]. The only remaining issue is the estimation of the flow generated by the moving body, yet this problem is handled by our solution as shown in Section 4.

3 Environment & Problem Description

3.1 Simulated Environment

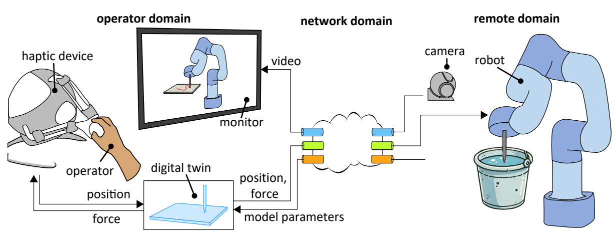

Figure 1: Overview of the whole environment. Our proposed solution for liquid simulation is located in the Operator Domain

The devised model will be working inside an already existing software solution. An unpublished paper is based on this environment, and thus we cannot attach any code snippets in this publication, however an overview of the system can be seen in Figure 1. The system provides the operator with a live camera feed of the interaction paired with simulated haptic feedback calculated in a digital copy of the environment.

The software allows for the creation of various objects and the control of a proxy existing in the environment via a pointer. Based on what is happening to the proxy, forces will be applied to the controller and we will focus on forces applied when the proxy is within liquids. In the future, the system could support actual force measurements, that could be used in a feedback loop to increase the accuracy of the feedback. Due to time limitations, and the still ongoing development of the software this was not pursued during our research.

A significant advantage of developing our solution within this preexisting software is the fact that it can already be used with actual haptic devices and remote robots. This means that our solution can be easily evaluated by connecting a haptic device. Currently, this is the Novint Falcon[1], a device that can rely haptic feedback and is intended operating device for the software.

[1]https://delfthapticslab.nl/device/novint-falcon/

3.2 Problem & Theoretical Approach

First, it should be noted that the chief goal of this research is to aid the creation of a believable and tactile teleoperation. The *Drag Equation* has various limitations, yet it is the simplest description of fluid dynamics (so simple, that it is practically unused for real applications). In the area of real-time simulations, this is a significant advantage as more advanced models are simply too costly to run in real-time at 1000 frames per second. Furthermore, it is crucial to understand we are not attempting to make a fully accurate model. Humans are not always able to accurately measure the forces they are applying, which allows for certain inaccuracies [9].

The proposed model relies on an equation, whose principles and derivation can be seen in engineering textbooks [5]. For now, we are assuming that the liquids in question are constrained by simple geometric shapes, such as large cubes or cylinders with no internal complications. Such objects can easily facilitate large bodies of water such as lakes and rivers or smaller ones such as containers or bowls. Furthermore, we are also only working with static liquids i.e. ones that do not have any flow changes and these changes do not significantly affect the predetermined drag coefficient. This leaves us with only one other parameter which has to be evaluated.

Velocity over liquid is the main contributor to the strength of our haptic feedback. If we were to make a simple assumption about its value, it would diminish the accuracy of our model too far. For example, assuming a certain value (or zero) would mean no feedback is ever felt after stopping or that feedback is always felt when not moving. A force value that diminishes with time might be considered, yet it cannot take into account previous movements or various force directions which can range in a full 3D space. This then leads to a *position tail*, a novel approach to this problem, that approximates all these interactions and is described further in Section 4.

3.3 Verification Methods

With many parameters and possible movement paths, it is often difficult to choose the right answers. As this is just an attempt at tackling a much more complicated challenge, we have chosen to focus on the basic ideas first. Our proxy was spherical to have the same drag coefficient in every direction. It omits the drag of the robot arm, however, the solution can be easily altered to accommodate this in the future. For now, two main types of liquids were considered as benchmarks, viscous honey which relies chiefly on the drag equation and turbulent water which uses the *position trail* to its full extent. The motion paths covered examples such as:

- simple linear movement
- circular movement, which resulted in the creation of simulated whirlpools
- movement and sudden stop, which showcases residual forces and their decay
- jagged movements that rapidly change direction

These assumptions were then used to iteratively improve the model, based on graphs that depicted the force feedback over time in repeatable programmed paths. Special emphasis was placed not on exact force values, but on correct overall responses. For example, viscous liquids should not have much of a turbulent response, while turbulent forces in water should decay and be properly directed. Afterwards, the model was tested by the researchers in simulations with live human control to once again iteratively improve and ensure correctness as the ultimate goal is fooling human perception. Finally, the solution was used in a small user study to verify its performance with non-researchers.

It must be noted that real-world tests with liquids were considered, but deemed not good enough to provide useful data. Measuring accurate forces in liquids is difficult, access to such tools is limited and the forces exerted in common situations are minuscule. For this reason, and because humans are not good at accurately measuring forces, a bigger focus was placed on replicating the viscous/turbulent behaviours and checking them in the ways mentioned above.

4 Force Feedback Model

The core of our solution is based on defining liquids as empty shapes that do not collide with other elements in the simulation. They are used to detect any objects moving within the liquids and to consequently apply adequate forces. Such an approach is beneficial as it can accommodate various shapes and sizes of liquid bodies while allowing for efficient detection of submerged objects.

4.1 Drag Equation Model

The implementation of the primary part of our solution relies only on the simplified drag equation with the assumptions outlined in Section 3. It is best suited for viscous liquids, as they do not create perturbations strong enough to noticeably alter the calculated force feedback. The speed of the object moving within the shape is measured and then used within the equation to calculate force feedback in the direction opposite to the movement.

The parameters of the equation can be easily changed and adjusted to accommodate different types of liquids and flows. For example, a strongly flowing river can be modeled with a velocity offset as it has no other noticeable turbulence, while for viscous fluids the velocity corresponds to the movements of the body.

4.2 Position Tail

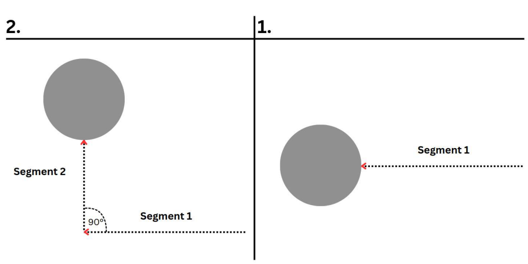

Figure 2: Movement of a body with a position tail in two steps. After a 90 degree turns, a new segment is created and the old one stays in place. Directions of each segment are highlighted in red.

To encompass a much wider range of fluid interactions, we have also devised a way of approximating the turbulent forces acting on bodies moving in liquids. This is achieved by keeping track of previous positions and velocities of the object, which will be referred to as the object's *position tail*. These position measurements are taken at every simulation step with a set minimal distance that can be adjusted or when the body is not moving at all.

These positions are combined into segments whose division is decided based on direction changes as shown in Figure 2. Their breaks are determined by a predetermined angle, which ensures that objects can change their directions (for example in a circular motion) but must do so slowly to maintain a high approximation of the liquid's velocity. To calculate the value that is used in the drag equation, all segments are summed as shown in eq. (1).

$$\mathbf{v}_{\text{tail}} = \sum_{i=1}^{n} \left(\frac{d_i \cdot m_i \cdot l_i}{S} \right) \tag{1}$$

where:

- $\mathbf{v}_{\text{tail}}$ is the resulting velocity of the tail.
- n is the number of segments into which the tail is divided.
- d_i is the normalized direction vector of the first element in a segment i.
- m_i is the average magnitude of elements in the segment i.
- l_i is the length of segment i.
- S is the scaling factor.

The parameters can be adjusted, resulting in different behaviours.

- The scaling factor is currently equal to the length of the tail for turbulent liquids, which means that the force feedback when moving in such a fluid cannot be beyond zero, and a sudden stop results in only as much force as could have been exerted on the liquid. Increasing it dampens the tail's effect.
- Changes in the angle that determines this division can lead to tails that always apply their force, or to tails that lose all of their momentum on the slightest changes.
- The resolution of the tail can be changed to increase its length without affecting the performance of the simulation. A single element is added at every simulation step if not stopped by the tail's resolution. This means that the tail itself has to be shortened if the goal is to reduce the tail beyond a certain point.

These adjustments and their effects will be evaluated in Section 5.

4.3 Other improvements

To slightly increase realism, at every simulation step, each singular element (not whole segment) of the tail is scaled by a fraction close to one. This ensures that when they are summed they do not reduce the overall drag force to zero unless the body is actively decreasing its speed. Additionally, the tail was not calculated within the Y axis to accommodate gravity effects that counteract such movements in real life.

Due to the scale of the simulated environment, speed measurements could rise dramatically with rapid movements. This caused undesirable jittering above certain speed values, as the exponential nature of the drag equation outputted large forces. To combat this effect, the speed value was artificially lowered after a certain value. Furthermore, to improve the accuracy of the feedback, the velocity was passed through an EWMA [2] filter. The effect of these changes are shown in Section 5.

5 Experimental Setup and Results

In this section, we present the performance of our model. Firstly, we will evaluate the model purely with simulated data in an ideal environment. Then we will show findings resulting from tests performed live by researchers and conclude with the results of a limited user study.

5.1 Theoretical Evaluation

The parameters used for the measurements were as follows if not indicated differently under a specific figure. Density of 1000 kg/m^3 (water) and a ball of roughly 68 mm. The simulated environment operates with already predetermined scales, and after initial tests by the researchers, these values were observed to provide adequate force feedback that could facilitate an effective user study later. The tail has a length of 5000 elements with the scaling factor equal to this length. Its effects are applied with a linear $y = x$ function and on every iteration an element is scaled by 0.999975 to ensure it doesn't completely negate the force feedback at constant speeds.

Note that due to the limitations of the software environment, the graphs contain random downward spikes lasting a single unit of time. They only appear when recording data and are not a result of the described solution.

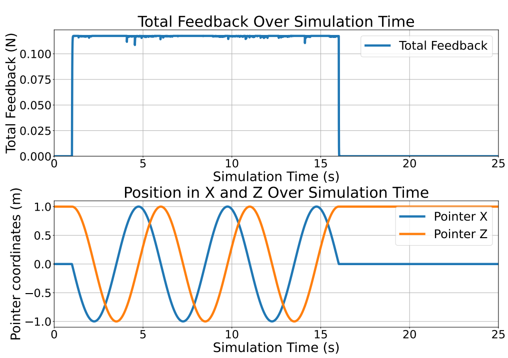

Figure 3: The body performs three perfect circles within a viscous liquid and is affected only be the drag equation.

In Figure 3 we can see the object performing three circles inside the liquid with constant velocity and then stopping. As a result of the drag equation, the resulting force remains constant and immediately drops to zero when the object stops.

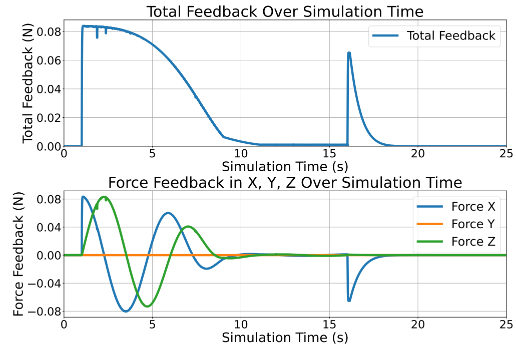

Figure 4: The body performs three perfect circles within a turbulent liquid and is affected by the drag equation supplemented with the position tail.

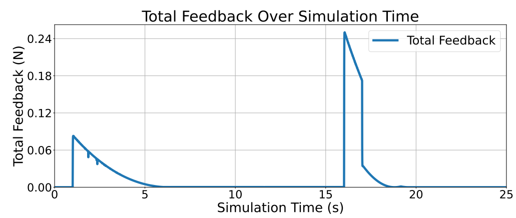

Figure 5: The body performs three perfect circles within a turbulent liquid, then moves directly against the flow for two seconds before stopping.

This demonstrates that the simplest case of viscous liquids can be easily dealt with.

In Figure 4 the object moves in the same way as in Figure 3, however, the force feedback is also affected by the position tail. This results in a linear decrease in feedback as the tail builds up. Once the object stops moving, the velocity of the liquid remains high and results in the illusion of water pushing on the object. Note that the force is slightly lower than the initial drag force due to the scaling mentioned in Section 4.

The aforementioned figure also showcases the accuracy of the position tail. Initially, the forces are directed against the direction of motion, while slowly increasing towards zero, as the tail builds up. Then once the body stops, an accurate spike of the X force vector simulates a splash of liquid acting upon the body.

In Figure 5 the object moves as in Figure 3, however at the end it moves against the build-up position trail. This results in a sudden force feedback hike, as long as the body moves in an opposite direction and falls linearly as the body stops.

In Figure 6 the object moves in small steps, periodically changing direction by 90° back and forth. This drastically decreases the effect of the tail on the resulting force. The steps alternate directions so the tail slowly builds up, yet it is limited. If the breaking angle was to be increased above 90° then the force feedback would decrease further as in previous figures.

All the above sections together, encompass the effects of the position tail. They showcase that the proposed model can deal with the problems described in Section 3 under ideal

[2] https://www.geogebra.org/m/tb88mqrm

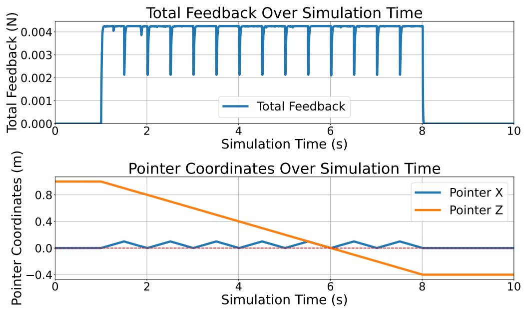

Figure 6: Body moves towards negative Z axis, alternating its direction in the X axis. Due to direction changes the tail cannot build up.

conditions. Furthermore as shown below it can be adjusted to fit the needs of different liquid types.

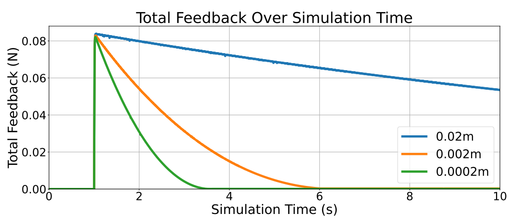

Figure 7: Three bodies move in a straight line with different tail resolutions.

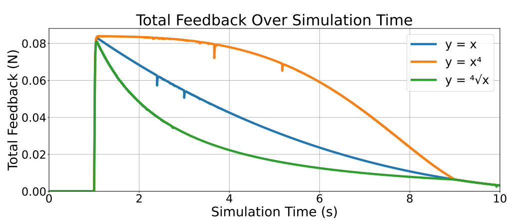

Figure 8: Three bodies move in a straight line with different tail functions.

In Figure 7 the body moves with a constant velocity in a single direction. It can be seen that changing the resolution of the tail, affects its length, with lower resolutions resulting in slower tail build-up and vice versa. It is important to note, that for tails that have high resolutions, increasing the resolution further might not decrease the tail's length due to the speed of the underlying simulation. This can be mitigated by simply shortening the tail itself.

Finally, Figure 8 shows how different functions can be used to control the tail's effects in detail. For example, the x^4 is used to initiate flow that slowly builds up to quickly give way near the end. Note that due to the scaling as mentioned in Section 4, the functions get closer together after 8s, and are not perfect representations.

5.2 Human Input Evaluation

These tests were done by researchers by connecting a Novint Falcon device and manually assessing the performance of the model. The immediate result of such tests was the need for noise filtering of velocity measurements. The α value of 0.1 was chosen as the most suitable for the EWMA filter. Furthermore, the velocity was also artificially lowered by squaring the part of the velocity magnitude that is above 5 m/s.

All the results of this evaluation were used to set up the liquid parameters for the subsequent user study.

5.3 User Study

Case	Density (kg/m^3)	Tail Length (m)	Scaling Factor	x^y
Water	1000.0	1700.0	1700.0	y = 0.75
Honey	1400.0	1.0	5000.0	y = 1.0
Light oil	4500.0	125.0	250.0	y = 0.25

Table 1: Liquid properties for different cases. Note the high density of light oil was counteracted by a very short tail

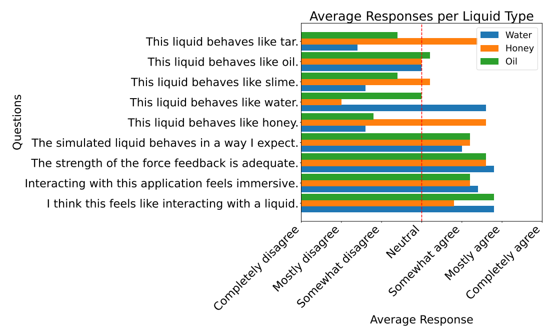

Figure 9: Responses of the participants divided by the liquid type they were interacting with

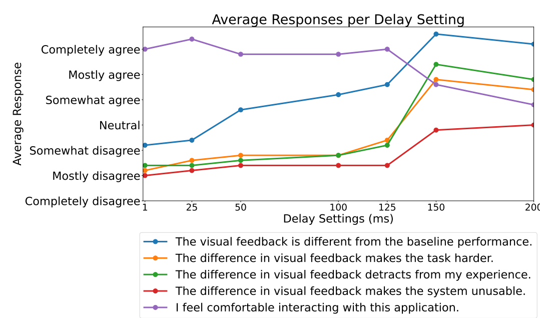

Figure 10: Changes in participants' responses over increasing visual feedback delay time

The study was performed with the setting outlined in Table 1 for the liquid types and the *Water* setting in delay tests. There were 5 participants with the exact form shown to them located in Appendix A.1. They were first asked to evaluate the types of three unknown liquids by haptic feedback alone and judging whether they agreed it was of a certain predetermined type. Then they were asked to move the proxy between two points within the liquid with variable and unknown delay

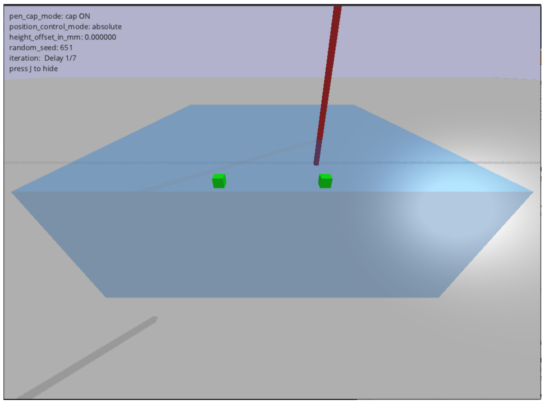

Figure 11: The simulated environment with one of the randomized delay settings as seen by a participant.

settings. An example of the described task can be seen in Figure 11.

As seen in Figure 9, most participants felt comfortable and immersed in the simulation, with their responses strongly above *Neutral*. For water and honey, the two opposite types of liquids evaluated have been usually correctly recognised with honey confused for tar, a similar type of viscous liquid. The *oil* type had a turbulent and low force feedback that was not correctly categorised by the majority.

Figure 10 shows the participant experience as the delay of visual feedback increases. The delay seems to not be noticeable before 125 ms, with a sharp increase afterwards. However, the overall satisfaction with the system remains high throughout the settings only slightly dropping after the effects become noticeable.

6 Discussion & Future Work

Before moving to the actual discussion, we would like to thank Koen Wösten for providing support with the preexisting software solution, which has been a great aid in developing this solution. Furthermore, his graph design was an inspiration for Figure 10. Furthermore, we would like to thank Kees Kroep, for providing help with, among other things, velocity filtering, which cleared the way for user tests.

6.1 Evaluation

The results shown in Section 5 outline the functionality of the proposed model and make it easier to evaluate its effectiveness in real-world applications. Firstly, let us examine the simplest drag equation, which seems to be effective for viscous fluids. With a constant velocity, it outputs a constant opposing force that is well within the expectations of a human operator.

For the more turbulent liquids, the position trail is the first stepping stone in tackling this issue without using complex fluid simulations. This solution is certainly less accurate, however, it is much more resource-efficient and as shown in the figures involving the position trail, can imitate certain liquid behaviours. As seen in graphs moving a single object within a liquid and with the correct parameters set for the tail can simulate a convincing experience. This means that after careful measurements in the real world, the parameters can be fine-tuned to roughly represent the feedback in static water containers or even certain ponds and lakes.

The user study supports these claims as users were able to correctly determine the viscous and turbulent liquids. When considering that in the intended scenario the participants would have seen footage of a real liquid, it seems likely they would have been immersed in the tactile experience. Additionally, the findings of the delay questionnaire further facilitate live camera feeds and should allow for delays of up to 125 ms.

On the other hand, the third oily liquid type was not correctly labelled by the participants. It is unclear whether this was caused by differing interpretations of oily liquids, differing lexical terms or small sample sizes. It could also be the case that with an actual camera feed, the participants would have felt immersed since they have marked the third option as behaving as expected of a liquid.

6.2 Impact of measurement accuracy

Due to the need for filtering of speed measurements, the impact and accuracy of the position tail are slightly diminished. In ideal, simulated scenarios it behaves exactly as expected, yet loses certain aspects when confronted with human input. This means that the proposed model performs well in scenarios with accurate velocity measurements, while it can become unusable with noisy data.

6.3 Future Work

This solution can be further improved in multiple directions and currently remains as mostly a proof of concept. Firstly, the solution should be reevaluated to smooth out any bugs and fix inconsistencies. These include rapidly moving away from a position after stopping with the tail in effect, implementing constant flows and height differences or any other assumptions mentioned in Section 3 and Section 4.

Then the model could be augmented with a feedback loop based on actual force measurements. These could be primarily used to reevaluate the *drag coefficient*, and if its value is certain to approximate other parameters for liquids that are unknown. The current state of the solution facilitates these real-time changes, but due to time constraints, they were not implemented. In our opinion, these could greatly increase the model's accuracy.

Lastly, all parameters mentioned in this paper, especially those tuned during manual testing by the researchers should be comprehensively rechecked. By its highly customizable nature, the position tail system in tandem with the drag equation is susceptible to parameter changes. Being able to easily measure and apply these values would be the element transforming the proposed solution into a fully usable system.

6.4 Limitations

This paper is a thesis submission for the Computer Science and Engineering bachelor at the TU Delft University. Any work related to this thesis was limited to a 10-week period, which with all other academic requirements limited the depth and scope of research. However, in the end, it did allow for the creation of a basic solution, that after customization can deliver adequate performance. Furthermore, a small user study was also performed to better gauge the effectiveness of the model. With more time, these actions could have been broader in scope, yet currently, they can be easily taken up by other researchers or students in the future.

It is also relevant to note that the existing simulation software in which the solution is situated is still a work in progress. This had a limiting effect on the progress of any work with software errors or undocumented behaviours, however, it is to be expected in such scenarios. The most relevant factor was the instability of speed measurements which was reinforced by instability in the simulations' frame rate. Most of the preparations for the user study were spent on devising ways of filtering the noise present in measurements and certain modifications had to be made in the presented solution to facilitate this.

7 Responsible Research

When doing any research it is important to evaluate and scrutinize its process. We have identified two main aspects of our research that necessitate a close inquiry, such as the use of volunteers to examine the model and the reproducibility of our findings.

7.1 User Study

In this study, we have only collected information regarding the model's performance and whether a given participant was pleased with the tactile experience. No other information was necessary, and thus not collected. The study was performed on participants who were either other researchers or willing volunteers found in the faculty's building. There was no financial or other incentive for them to participate and they were informed of the study's goal.

There were no known risks to participating in the study, as the volunteers had to move the pointer via a haptic device that delivered only up to 8 Newtons of force, which is not enough to cause harm. It is also important to note the small sample size of the study, which is not enough for fully comprehensive papers, yet good enough as a guideline.

7.2 Reproducibility

Being able to replicate findings is an irreplaceable part of the scientific process. The researchers have tried their best to provide any values and data necessary to evaluate any test results. This can be especially useful when verifying the accuracy of the drag equation calculations or the effects of the tail. Furthermore, the process of the user study was also explained with exact forms provided in Appendix A.1.

7.3 Large Language Models

These models were used during the development of the solution. They provided limited help and were not used frequently with any of the information obtained scrutinized and checked for errors. They were also used to rephrase or improve certain parts of the paper itself, yet once again any text was thoroughly evaluated by human researchers and often improved further.

8 Conclusions

The primary objective of this research was to emulate a convincing tactile expression for interacting with fluids, and we believe that this paper provides a solid cornerstone for further research. It shows that haptic simulations that allow for a certain degree of inaccuracy succeed in creating a believable experience for human operators.

Through a theoretical model and later its implementation, we have shown that the drag equation is an effective method of simulating viscous liquid interactions. For turbulent fluids, the position tail mechanism successfully approximates the various forces of such environments, capturing their essential characteristics. The simulated experiments indicate that the model is versatile and can adapt to various liquid types.

This is further reinforced by the user study, in which the participants were able to correctly identify viscous and turbulent liquid types. This was done without a live camera feed, which would have improved the perceived immersion, with the participants reporting the system to behave comfortably and within expectations. Furthermore, the simplified visual feedback could be delayed up to 125 ms without noticeable user discomfort.

However, many possibilities of improving the solution still remain. The system is susceptible to noisy data and is currently much more usable in environments with clear inputs. Additionally, future work should explore various extensions of the model such as height and flow variations, parameter tuning and force loops.

In conclusion, this research presents a promising step towards efficient and believable haptic simulations for teleoperation. By omitting the many complexities of traditional fluid simulations, it paves the way for modelling many new environments. We hope that our findings can inspire further advancements and ultimately facilitate more sophisticated and accessible teleoperation.

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A Appendix

A.1 User Study Response Form

Questionnaire

Seed:

1 Baseline Settings

In this part of the study, different liquid types are examined. Please evaluate the three possible settings.

Liquid setting 1	Completely disagree	Mostly disagree	Somewhat disagree	Neutral	Somewhat agree	Mostly agree	Completely agree
I think this feels like interacting with a liquid.	◯	◯	◯	◯	◯	◯	◯
Interacting with this application feels immersive.	◯	◯	◯	◯	◯	◯	◯
The strength of the force feedback is adequate.	◯	◯	◯	◯	◯	◯	◯
The simulated liquid behaves in a way I expect.	◯	◯	◯	◯	◯	◯	◯
This liquid behaves like honey	◯	◯	◯	◯	◯	◯	◯
This liquid behaves like water	◯	◯	◯	◯	◯	◯	◯
This liquid behaves like slime	◯	◯	◯	◯	◯	◯	◯
This liquid behaves like oil	◯	◯	◯	◯	◯	◯	◯
This liquid behaves like tar	◯	◯	◯	◯	◯	◯	◯

Liquid setting 2	Completely disagree	Mostly disagree	Somewhat disagree	Neutral	Somewhat agree	Mostly agree	Completely agree
I think this feels like interacting with a liquid.	◯	◯	◯	◯	◯	◯	◯
Interacting with this application feels immersive.	◯	◯	◯	◯	◯	◯	◯
The strength of the force feedback is adequate.	◯	◯	◯	◯	◯	◯	◯
The simulated liquid behaves in a way I expect.	◯	◯	◯	◯	◯	◯	◯
This liquid behaves like honey	◯	◯	◯	◯	◯	◯	◯
This liquid behaves like water	◯	◯	◯	◯	◯	◯	◯
This liquid behaves like slime	◯	◯	◯	◯	◯	◯	◯
This liquid behaves like oil	◯	◯	◯	◯	◯	◯	◯
This liquid behaves like tar	◯	◯	◯	◯	◯	◯	◯

Liquid setting 3	Completely disagree	Mostly disagree	Somewhat disagree	Neutral	Somewhat agree	Mostly agree	Completely agree
I think this feels like interacting with a liquid.	◯	◯	◯	◯	◯	◯	◯
Interacting with this application feels immersive.	◯	◯	◯	◯	◯	◯	◯
The strength of the force feedback is adequate.	◯	◯	◯	◯	◯	◯	◯
The simulated liquid behaves in a way I expect.	◯	◯	◯	◯	◯	◯	◯
This liquid behaves like honey	◯	◯	◯	◯	◯	◯	◯
This liquid behaves like water	◯	◯	◯	◯	◯	◯	◯
This liquid behaves like slime	◯	◯	◯	◯	◯	◯	◯
This liquid behaves like oil	◯	◯	◯	◯	◯	◯	◯
This liquid behaves like tar	◯	◯	◯	◯	◯	◯	◯

2 Delay Effect

In this part of the study, the effect of different delays is tested.

Delay setting 1	Completely disagree	Mostly disagree	Somewhat disagree	Neutral	Somewhat agree	Mostly agree	Completely agree
The visual feedback is different from the baseline performance.	◯	◯	◯	◯	◯	◯	◯
The difference in visual feedback makes the task harder.	◯	◯	◯	◯	◯	◯	◯
The difference in visual feedback detracts from my experience.	◯	◯	◯	◯	◯	◯	◯
The difference in visual feedback makes the system unusable.	◯	◯	◯	◯	◯	◯	◯
I feel comfortable interacting with this application.	◯	◯	◯	◯	◯	◯	◯

Delay setting 2	Completely disagree	Mostly disagree	Somewhat disagree	Neutral	Somewhat agree	Mostly agree	Completely agree
The visual feedback is different from the baseline performance.	◯	◯	◯	◯	◯	◯	◯
The difference in visual feedback makes the task harder.	◯	◯	◯	◯	◯	◯	◯
The difference in visual feedback detracts from my experience.	◯	◯	◯	◯	◯	◯	◯
The difference in visual feedback makes the system unusable.	◯	◯	◯	◯	◯	◯	◯
I feel comfortable interacting with this application.	◯	◯	◯	◯	◯	◯	◯

Delay setting 3	Completely disagree	Mostly disagree	Somewhat disagree	Neutral	Somewhat agree	Mostly agree	Completely agree
The visual feedback is different from the baseline performance.	◯	◯	◯	◯	◯	◯	◯
The difference in visual feedback makes the task harder.	◯	◯	◯	◯	◯	◯	◯
The difference in visual feedback detracts from my experience.	◯	◯	◯	◯	◯	◯	◯
The difference in visual feedback makes the system unusable.	◯	◯	◯	◯	◯	◯	◯
I feel comfortable interacting with this application.	◯	◯	◯	◯	◯	◯	◯

Delay setting 4	Completely disagree	Mostly disagree	Somewhat disagree	Neutral	Somewhat agree	Mostly agree	Completely agree
The visual feedback is different from the baseline performance.	◯	◯	◯	◯	◯	◯	◯
The difference in visual feedback makes the task harder.	◯	◯	◯	◯	◯	◯	◯
The difference in visual feedback detracts from my experience.	◯	◯	◯	◯	◯	◯	◯
The difference in visual feedback makes the system unusable.	◯	◯	◯	◯	◯	◯	◯
I feel comfortable interacting with this application.	◯	◯	◯	◯	◯	◯	◯

Delay setting 5	Completely disagree	Mostly disagree	Somewhat disagree	Neutral	Somewhat agree	Mostly agree	Completely agree
The visual feedback is different from the baseline performance.	◯	◯	◯	◯	◯	◯	◯
The difference in visual feedback makes the task harder.	◯	◯	◯	◯	◯	◯	◯
The difference in visual feedback detracts from my experience.	◯	◯	◯	◯	◯	◯	◯
The difference in visual feedback makes the system unusable.	◯	◯	◯	◯	◯	◯	◯
I feel comfortable interacting with this application.	◯	◯	◯	◯	◯	◯	◯

Delay setting 6

	Completely disagree	Mostly disagree	Somewhat disagree	Neutral	Somewhat agree	Mostly agree	Completely agree
The visual feedback is different from the baseline performance.	◯	◯	◯	◯	◯	◯	◯
The difference in visual feedback makes the task harder.	◯	◯	◯	◯	◯	◯	◯
The difference in visual feedback detracts from my experience.	◯	◯	◯	◯	◯	◯	◯
The difference in visual feedback makes the system unusable.	◯	◯	◯	◯	◯	◯	◯
I feel comfortable interacting with this application.	◯	◯	◯	◯	◯	◯	◯

Delay setting 7

	Completely disagree	Mostly disagree	Somewhat disagree	Neutral	Somewhat agree	Mostly agree	Completely agree
The visual feedback is different from the baseline performance.	◯	◯	◯	◯	◯	◯	◯
The difference in visual feedback makes the task harder.	◯	◯	◯	◯	◯	◯	◯
The difference in visual feedback detracts from my experience.	◯	◯	◯	◯	◯	◯	◯
The difference in visual feedback makes the system unusable.	◯	◯	◯	◯	◯	◯	◯
I feel comfortable interacting with this application.	◯	◯	◯	◯	◯	◯	◯