Robust Curvature Estimation by Finding Optimal Circular Segments in Orientation Space

M. van Ginkel, L.J. van Vliet, P.W. Verbeek

Pattern Recognition Group Department of Applied Physics, Delft University of Technology, Lorentzweg 1, 2628 CJ Delft, The Netherlands, {michael,lucas,piet}@ph.tn.tudelft.nl

Keywords: lines, edges, orientation, curvature

Abstract

We have developed a new, robust, curvature estimator. It is based on "fitting" circular segments to a curve using a generalised Radon transform. The key step in the approach is that the fitting operation is not done in the original image, but in a transformed version, called Orientation Space.

The original curve is transformed into another curve in Orientation Space. Apart from the original spatial dimensions, Orientation Space has one extra dimension: orientation. A given point on the curve in Orientation Space contains information both about the original position of the point, but also about the orientation of that point.

This explicit representation of orientation allows us to incorporate it as a contraint in the fitting stage, where curves corresponding to circular segments in the original image are fitted to the data. The estimated curvature is the reciprocal of the radius of the best fitting segment.

The method performs well on noisy (synthetic) data and should work correctly on intersecting curves as well. The performance of the estimator has not yet been compared to existing approaches in the litature.

The method is a demonstration of a novel, general, approach to incorporating constraints in the Radon Transform.

1 Introduction

Traditional curvature estimators are based on binary representations. A segmentation step to obtain a binary representation of the image is required. Vital information is lost in the segmentation step.

Curvature estimators that operate directly on a gray value image were introduced in [7, 1]. In a recent series of papers [8, 2, 3, 9] a number of new, robust, curvature estimators were introduced. These estimators estimate the curvature of a curved pattern, rather than of an individual curve. Prototypes of both image types are depicted in figure 1.



Figure 1: Left: a single curve. Right: a curved pattern.

In the current paper we extend the set of curvature estimators with one that deals with individual curves, while retaining much of the robustness of the estimators for curved patterns. The estimator is based on some of the same principles as the pattern based estimators. We start therefore with an overview of these estimators:

• The estimator in [9], which supercedes [8], locally optimises the local anisotropy by varying the parameters (ϕ, κ) of a local coordinate transformation. The anisotropy is maximal when the parameters of the transformation coincide with those of the pattern (ϕ_p, κ_p) . This estimator will be referred to as the MAC estimator.

in: L.J. van Vliet, J.W.J. Heijnsdijk, T. Kielman, P.M.W. Knijnenburg (eds.), Proc. ASCI 2000, 6th Annual Conf. of the Advanced School for Computing and Imaging (Lommel, Belgium, June 14-16), ASCI, Delft, 2000, 53-58.

- The estimator in [3] uses two stages to estimate curvature. In the first stage the local orientation is obtained using a robust orientation estimator, for instance the structure tensor [4]. In the second stage the curvature is estimated by measuring the change in orientation along a straight axis. We will refer to this estimator as the ODC estimator.
- The estimator in [2] is based on the same idea as the ODC estimator. Two stages are involved; in the first stage an Orientation Space is computed. Orientation Space is explained in more detail in the next section. Curved patterns manifest themselves as tilted response planes in Orientation Space. The tilt corresponds to the curvature of the pattern. The tilt can be estimated using an orientation estimator. The key difference with the previous method is its ability to deal with overlapping patterns. This estimator will be refered to as the OSC estimator.

There are two important reasons why these estimators are not applicable to individual curves:

- The MAC and ODC estimators gather (average) information over a rather large window. This is a desirable property for estimating the curvature of a pattern. In the case of an individual curve only a small part of the window contains useful information. Using a large window merely decreases the signal to noise ratio in this case.
- The ODC and OSC estimators compute curvature by measuring the change in orientation. This is possible when a complete, consistent, orientation field is available. In the case of a curve, orientation information is present only along the curve rather than throughout the analysis window.

Our proposed curvature estimator combines two essential properties of the MAC and OSC estimators. To make the estimator robust, the analysis window should be large, but as explained above, non-isotropic. The elongated filters used to compute Orientation Space are suitable for this purpose.

The next step is to adopt a model for the curve. A circular segment is used as the model. The model has two parameters; the curvature κ and the orientation ϕ of the normal vector of the curve. The next step is, just as for the MAC estimator, the optimisation of the parameters of the model so it best fits the data. The κ that optimises the model is an estimate for the curvature.

Parent and Zucker [5] describe a method for retrieving curves from an image. It consists of two stages: finding a rough estimate of the orientation (using three oriented filters) and of the curvature, followed by a regularisation procedure to find a smooth curve (discontinuities are allowed), using the orientation and curvature estimates as constraints. The main difference is that our method consists of a generic procedure to incorporate constraints for finding parameterised curves using an intuitive geometric approach, rather than a dedicated approach.

The structure of the paper is as follows. In section 2 we briefly explain the concept of Orientation Space. The details of finding the best fit of the model are given in section 3. We then proceed with some experiments and conclusions.

2 Orientation Space

An Orientation space is an "measurement" space for orientation. An measurement space $I^{[m]}(\vec{x}, \vec{f})$ is a space obtained by some transformation applied to the image to be analysed. Its dimensions are those of the input image $I(\vec{x})$ plus one extra dimension for each image feature we are interested in:

$$I(\vec{x}) \to I^{[m]}(\vec{x}, \vec{f}) \tag{1}$$

Where \vec{f} are the feature dimensions. The interpretation of $I^{[m]}$ is simple; for a given feature vector \vec{f} , the measurement space at a given location \vec{x} tells us how much evidence there is for that particular feature vector at that particular location.

In contrast, the usual way to extract features corresponds to a transformation of the following type:

$$I(\vec{x}) \to \vec{f}(\vec{x})$$
 (2)

In the current paper we are interested in a single feature, orientation. In this case the feature vector reduces to a scalar: the orientation ϕ . The orientation estimator in [4] is a good example of feature extraction of the type given in equation 2, i.e.:

$$I(x,y) \xrightarrow[\text{tensor}]{\text{structure}} \phi(x,y) \tag{3}$$

In general, there are two arguments for using Orientation Space rather than a conventional orientation estimator (the same arguments hold for any measurement space):

- The ability to deal with intersecting or overlapping structures. If these structures each have a distinct orientation they will no longer overlap in Orientation Space.
- Tracking a structure. Each added feature dimension constrains the search more, which results in a more pronounced "best track".

In the next section, we show how we use Orientation Space to track circular segments, and thereby obtain an estimate of the curvature.

Orientation Space $I^{[os]}$, an measurement space for orientation, is obtained by a transformation of the input image:

$$I(x,y) \to I^{[os]}(x,y,\phi) \tag{4}$$

The transformation of the input image is obtained by applying a filter bank to the image. The filter bank consists of rotated copies of a single, orientation sensitive, bandpass filter [2]. $I^{[os]}$ is periodical along the ϕ axis with period π (we do not distinguish between a structure and the same structure rotated over 180 degrees).

The Fourier transform over the spatial coordinates of the Orientation Space can be written as:

$$\widetilde{I}^{[os]}(\omega_x, \omega_y, \phi) = \widetilde{F}(\omega_x, \omega_y, \phi) \widetilde{I}(\omega_x, \omega_y) \qquad (5)$$

where $\widetilde{I}^{[os]}$ is the Fourier transform of the orientation space, \widetilde{F} the Fourier transform of the filter and \widetilde{I} the Fourier transform of the image. If the Fourier transform is written in cylindrical coordinates ω, θ , the following relation holds:

$$\widetilde{F}(\omega,\theta,\phi) = \widetilde{F}_r(\omega)F_a(\theta-\phi) \tag{6}$$

 F_r is the radial part of the filter. It selects a nonsymmetric frequency band "centered" at ω_c with width σ_r . It is given by:

$$\widetilde{F}_r(\omega;\omega_c,\sigma_r) = \left(\frac{\omega}{\omega_c}\right)^{\frac{\omega_c^2}{\sigma_r^2}} \exp\left(-\frac{\omega^2 - \omega_c^2}{2\sigma_r^2}\right) \quad (7)$$

Frequencies lie in the range $(-\pi, \pi)$. $\widetilde{F}_r(0, \omega_c, \sigma_r) = 0$ and F_r attains its maximum value of 1 for $\omega = \omega_c$.

The angular part of the filter, F_a , has a single parameter N. The shape of this filter is Gaussian with standard deviation $\sigma_a = \pi/N$:

$$F_a(\alpha; N) = \exp(-\frac{(N\alpha)^2}{2\pi^2}) \tag{8}$$



Figure 2: The solid line shows the curve under investigation. The dashed lines are candidate circular segments. For one of the candidates we show the full circle (dotted). The segment is traced by θ with centre point (x_1, y_1) .

Throughout the experiments we have set $\omega_c = 0.6$, $\sigma_r = 0.5$ and N = 8. The orientation selectivity N = 8 sufficiently contrains the tracking stage. The centre frequency, ω_c is tuned to the data. The σ_r parameter has been tuned to detect individual lines and edges, by setting $\omega_c/\sigma_r = 1.2$, contrary to the settings in [2], where the parameters were tuned to detect line bundles.

3 Finding the optimal circular segment

Our approach to estimating curvature is based on finding, at each spatial position, the circular segment that best describes the curve. The reciprocal of the radius of that segment is an estimate of the curvature. To estimate the curvature at (x, y) we consider all circular segments that go through this point:

$$\vec{c}_s(x, y, R, \phi, \theta) = \begin{pmatrix} x_c + R\cos(\phi + \pi + \theta) \\ y_c + R\sin(\phi + \pi + \theta) \end{pmatrix}$$
(9)

with

$$\begin{aligned} x_c &= x + R\cos\phi\\ y_c &= y + R\sin\phi \end{aligned} \tag{10}$$

where (x_c, y_c) is the centre of circular segment specified by R and ϕ . The model is illustrated in figure 2. There is a curve \vec{c}_{os} in Orientation Space that corresponds to \vec{c}_s in the original image space. We can obtain this curve by adding a third component to \vec{c}_s : the orientation as we walk along the circle segment. The orientation at a given point along the circle segment is simply $\phi + \pi + \theta$. Taken the periodicity of $I^{[os]}$ into account we obtain:

$$\vec{c}_{os}(x, y, R, \phi, \theta) = \begin{pmatrix} x_c + R\cos(\phi + \pi + \theta) \\ y_c + R\sin(\phi + \pi + \theta) \\ (\phi + \theta) \mod \pi \end{pmatrix}$$
(11)

We now seek the parameters for which the curve \vec{c}_{os} best fits the data. This can be done using a brute force search over all possible parameters. Our fit criterion is based on finding the parameters that maximises the integrated intensity along the model curve. In analogy to the Radon transform for straight lines, such a procedure is known as a generalised Radon transform, see [6] for example. At a given point (x, y) we seek the maximum of the function $M(R, \phi)$:

$$M(R,\phi) = \frac{1}{\beta 2\pi} \int_{-\beta\pi}^{\beta\pi} |I^{[os]}| (\vec{c}_{os}(x,y,R,\phi,\theta)) d\theta$$
(12)

with β a parameter that corresponds to the fraction of a full circle covered by the segment. In the experiments we have set it to 0.25. The maximum is found by evaluating $M(R, \phi)$ at a discrete set of points. This leads to a quantisation error in the estimated radius, and hence the curvature.

The scheme is rather computation intensive. If N_x , N_y , N_R and N_{ϕ} are the number of points along the x and y axis, and the number of evaluated for R and ϕ respectively, the computation time T_c required depends on these as:

$$T_c \propto N_x N_y N_R N_\phi \tag{13}$$

where we have neglected the fact that the time to evaluate $M(R, \phi)$ depends on R and ϕ .

We can reduce the amount of computation significantly by noting that we can get a good estimate of ϕ_{max} , the value of ϕ at which M reaches its maximum, directly from Orientation Space. The following holds:

$$I^{[os]}(x, y, \phi_{max}) = \max_{\phi} I^{[os]}(x, y, \phi)$$
 (14)

Orientation Space does not discriminate between an angle ϕ and an angle $\phi + \pi$. In our case this distinction is important, since the centre of the best fitting circular segment can lie on either side of the curve. Our estimate for R is now the R that maximises:

$$\max(M(R,\phi_{max}), M(R,\phi_{max}+\pi))$$
(15)

To compensate for small errors in ϕ_{max} the search should not be restricted to only ϕ_{max} and $\phi_{max} + \pi$, but rather to small bands around these values.

In the case of intersecting curves, $M(R, \phi)$ contains more than one maximum, each corresponding to one curve. Each maximum gives an estimate of the curvature of the corresponding curve. Note, however, that we cannot use the ϕ_{max} estimate to restrict the search in this case.

4 Experiments

To evaluate the method, we have tested it on synthetic data, both with and without noise. In the first experiment we have generated a disk with a radius of 53 pixels. The search was performed for radii 45, 46, \cdots , 70. The search over ϕ was restricted to ϕ_{max} and $\phi_{max} + \pi$. Figure 3 shows the results. To get a quantitative result, we have computed the average and standard deviation of the estimated radius over the pixels that lie on the edge of the disk. The resulting estimate for the radius of the disk as a whole is:

$$R_{disk} = (53.2 \pm 1.2)$$

The position of the edge pixels considered suffers from quantisation errors. We have therefore generated an image $I(x,y) = \sqrt{(x^2 + y^2)}$ and computed the average and standard deviation over the same points as above. This yields:

$$R_{quant} = (53.4 \pm 0.5)$$

These values are in good agreement with each other, although the standard deviation of the estimate is larger than that purely due to quantisation errors in the position of the edge pixels. To further investigate the effects of quantisation, we have taken an arbitrary point along the disk: (39.39, 35.46). If we optimise $M(R, \phi)$ at this point, the estimate for R is 54. The estimate at the closest grid point, (39, 35) is 56. This shows that the curvature should be computed at the true (sub-pixel) location of the curve, although this is usually impractical.

In the second experiment we have applied the approach to an image containing disks with various radii and additive, Gaussian noise with a standard deviation of 0.25. The disks have intensity 1, the background -1. The experiment was performed with the same parameters as in



Figure 3: Left: a disk with R = 53. Middle: the estimated radius. Right: The maximum of $M(R, \phi)$ at each (x, y) position.



Figure 4: Left: Various noisy disks. Middle: the estimated radii. Right: The maximum of $M(R, \phi)$ at each (x, y) position.

the first experiment, but the search is performed over radii $2, 3, \dots, 80$. The following table gives the true radius and the estimated radius of each of the disks:

true radius	estimated radius
65	66.4 ± 1.6
25	25.4 ± 1.1
10	8.7 ± 3.0
5	3.4 ± 1.4

The method seems to break down for the disks with smaller radii. There are two reason for this. First, there are less points to average over to reduce the noise. Secondly, the filters used to compute Orientation Space are straight, elongated, filters. We can expect these to perform better on larger disks, because their edges are relatively straight.

5 Conclusions

We have developed a new, robust, curvature estimator. A few simple experiments have been performed to verify the approach. Further testing on, for example, non-circular objects should be performed. Also, the method has not been tested on intersecting curves yet.

The method performs well, even when the

noise level is considerable. Further evaluation of the method in comparison with other methods, such as [1, 5], is necessary. We are preparing a paper in which the various estimators are evaluated.

The results in the second experiment show that the performance depends on the radius of the curve under analysis. This is partly due to the use of the Orientation Space filterbank. The performance can be improved by directly constructing an orientation/curvature space, rather than computing curvature indirectly through Orientation Space. Efficient methods to perform such an analysis are the topic of further research. A brute force implementation of a combined orientation/curvature space is currently prohibitively expensive.

Acknowledgements

This work was partially supported by the Rolling Grants program 94RG12 of the Netherlands Organization for Fundamental Research of Matter (FOM) and the Royal Netherlands Academy of Arts and Sciences (KNAW).

References

 H. Bårman, Hierarchical Curvature Estimation in Computer Vision, *PhD thesis*, Linköping University, Sweden, 1991.

- [2] M. van Ginkel, P.W. Verbeek and L.J. van Vliet, Curvature Estimation for Overlapping Curved Patterns using Orientation Space, in: B.M. ter Haar Romeny, D.H.J. Epema, J.F.M. Tonino, A.A. Wolters (eds.), Proceedings of the third annual conference of the Advanced School for Computing and Imaging, Lommel, Belgium, June 9-11, 1998, pp. 173-178.
- [3] M. van Ginkel, J. van de Weijer, L.J. van Vliet, and P.W. Verbeek, Curvature estimation from orientation fields, in: B.K. Ersboll, P. Johansen (eds.), *Proceedings of the 11th Scandinavian Conference on Image Analysis*, Kangerlussuaq, Greenland, June 7-11, Pattern Recognition Society of Denmark, Lyngby, 1999, 545-551.
- [4] M. Kass and A. Witkin, Analyzing Oriented Patterns, Computer Vision, Graphics and Image Processing, vol. 37, 1987, pp. 362-385.
- [5] P. Parent and S.W. Zucker, Trace Inference, Curvature Consistency, and Curve Detection, *IEEE transaction on Pattern Analysis and Machine Intelligence*, vol. 11, no. 8, August 1989, pp. 823-839
- [6] P. Toft, The Radon Transform Theory and Implementation, *Ph.D. thesis*, Department of Mathematical Modelling, Technical University of Denmark, June 1996. 326 pages.
- [7] P.W. Verbeek, A class of sampling-error free measures in oversampled band-limited images, *Pattern Recognition Letters*, vol. 3, 1985, pp. 287-292
- [8] P.W. Verbeek, L.J. van Vliet, and J. van de Weijer, Improved curvature and anisotropy estimation for curved line bundles, in: A.K. Jain, S. Venkatesh, B.C. Lovell (eds.), Proc. 14th Int. Conference on Pattern Recognition, Brisbane, Aug. 16-20, IEEE Computer Society Press, Los Alamitos, 1998, 528-533.
- [9] J. van de Weijer, L.J. van Vliet, P.W. Verbeek, and M. van Ginkel, Curvature Estimation in Line Bundle Images using Curvilinear Models applied to Gradient Vector Fields, *IEEE Trans*actions on Pattern Analysis and Machine Intelligence, submitted.