Quasiparticle tunneling and quasiparticle-pair interference in granular superconductors

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We work out the phase diagram of an array of Josephson junctions with quasiparticle tunneling, taking into account the quasiparticle-pair interference term (QPPI). Besides introducing interesting renormalization effects in the Josephson term, the QPPI shifts the critical resistance. At finite temperatures a proper treatment of tunneling effects leads to reentrant behavior.

I. INTRODUCTION

In the past few years, there has been a constant growth of interest in the physics of quantum dissipative systems at low temperature.[1,2] With modern lithographic techniques, it is possible to fabricate low-capacitance single junctions, chains, and two-dimensional arrays with both normal-metal and superconductor islands, in which quantum effects can be studied avoiding the complications due to disorder.

It is well known that quantum fluctuations due to the finite junction capacitances can suppress phase coherence of a Josephson-junction array even at $T=0$, when the electrostatic energy overcomes the energy gained in the formation of the coherent state. In the path-integral formulation, one studies the effective Euclidean action (units such that $\hbar=1$ will be used):

$$S_c[\phi(\tau)]+S_J[\phi(\tau)]=\int_0^\beta d\tau\left[\sum_{ij}\frac{1}{8e^2}C_{ij}\dot\phi_i(\tau)\dot\phi_j(\tau)-\sum_{\langle ij\rangle}E_J\cos\phi_{ij}(\tau)\right], \tag{1}$$

where ϕ_i is the phase of the superconductive order parameter of the ith island, $\phi_{ij}=\phi_i-\phi_j$, and $\langle ij\rangle$ means a summation over the lattice bonds. The first term accounts for the electrostatic interaction between the metallic islands through the capacitance matrix C_{ij}. The second is the Josephson interaction term, and E_J is the related energy. In the self-charging model ($C_{ij}=\delta_{ij}C$) (see, however, Ref. 3) one finds that phase coherence is suppressed at $T=0$ if[4] $zE_JC\lesssim 1$, z being the coordination number of the array, this threshold increasing at finite temperatures.[4,6]

The possibility of reentrant behavior has been envisaged by Simanek and Efetov and has raised an extended discussion in the literature.[4–8] By considering a capacitance matrix in which only diagonal and nearest-neighbor elements are nonvanishing, reentrant behavior has been found in a mean-field (MF) study,[7] whereas in the self-consistent harmonic approximation (SCHA), no reentrance was found.[8]

It is worth mentioning that, even in the self-charging limit, the physics described by Eq. (1) can be more complex; indeed, Monte Carlo studies have revealed the presence of two distinct superconducting phases,[9] and interesting features emerge when the fluctuations of the single-grain superconductive order-parameter modulus Δ are accounted for.[10]

Dissipation due to Ohmic shunts or quasiparticle tunneling is believed to be responsible for a phase transition, at $T=0$, at a critical resistance of the order of $R_0=h/4e^2\approx 6.5$ kΩ, and it has been accounted for in various ways.[11–15] In the experimental range of the parameters, a SCHA yields an universal critical resistance R_0/d (d is the dimensionality of the array) for dissipation due to an Ohmic shunt,[11] whereas, for quasiparticle (QP) dissipation, a MF approach indicates a weak dependence on the Josephson energy E_J.[12]

The effect of QP tunneling is studied starting from the Ambegaokar-Eckern-Schön (AES) Euclidean effective action, obtained from the microscopic model.[16] If the subgap conductance is negligible, the AES action can be studied in the adiabatic limit, which reads

$$S[\phi(\tau)]=S_C[\phi(\tau)]+S_J[\phi(\tau)]+\int_0^\beta d\tau\left[\sum_{\langle ij\rangle}\frac{1}{8e^2}\delta C_\alpha\dot\phi_{ij}^2(\tau)-\sum_{\langle ij\rangle}\frac{1}{24e^2}\delta C_\gamma\dot\phi_{ij}^2(\tau)\cos\phi_{ij}(\tau)\right]. \quad (2)$$

In the calculation, we will consider the self-charging model for the electrostatic part of the action. Here QP tunneling is responsible for the last two terms; the third one represents a renormalization to the mutual capacitance that leads to the appearance of an effective long-range electrostatic interaction which is temperature dependent (δC_α derives from the electronic Green's functions, see the Appendix). δC_α reduces for $T=0$ to $\delta C=3g/16\Delta$, where $g=R_0/R_N$ and, in the samples realized experimentally, is much larger than the nearest-neighbor geometrical capacitance, so that the choice $C_{ij}=C\delta_{ij}$ seems appropriate. The last term arising reflects an interference between Cooper pair and QP currents (hereafter, we will refer to it as QPPI). In the classical theory, it is encountered as a phase-dependent resistance.[17] By an inspection of (2), it is clear that QPPI could lead to an off-diagonal charging dependent enhanced effective E_J or a phase-dependent depression of QP dissipation. δC_γ is temperature dependent and reduces to δC for $T=0$. It contributes to the Euclidean action as an effective long-range electrostatic interaction; stability is not affected by the sign of QPPI because of the presence of QP terms. The physics of QPPI is not well understood: Its effects are not easily detectable in single-junction devices, and even the sign of the coefficient is a subject of controversy for theoreticians and experimentalists.[18] It is worth mentioning that very recently a Hamiltonian with the same structure of (2) has been constructed,[19] differing in the sign and the magnitude of the coupling constants of the two terms, where QPPI is described by a nonstandard combination of "coordinates" (ϕ) and "canonical momenta" ($C\dot\phi/4$). However, we will follow the formulation of the problem given in Ref. 16 because, according to this last work, the well-known classical limits are readily obtained.

A very natural procedure consists in performing a Hubbard-Stratonovich transformation to (2), in order to decouple the cosine and the derivative in the QPPI term.[20] This fluctuating field is sharp sufficiently far from the transition line and then, by using a saddle-point approximation, it is possible to show that if the system is in the phase coherent state, QPPI results in a renormalization of both the dissipative and the Josephson coupling constants. However, this approach is not reliable in working out the phase diagram due to the increasing fluctuations of the renormalized coupling constants.

The purpose of this paper is to work out the phase diagram arising from the effective action (2), as compared with the case in which QPPI is neglected. What follows is divided into four parts. First, we describe the SCHA, adopted in order to simulate both the Josephson and the QPPI terms by a quadratic trial potential. Then, we describe the resulting phase diagram for $T=0$: If QPPI is neglected, a critical resistance, slowly verying in the interesting range of the parameters, is obtained, whereas including QPPI determines appreciable modifications on the phase diagram. In the third part, the limiting case $C \ll \delta C$ is worked out for $T\neq 0$, and we find that reentrant behavior occurs if the temperature dependence of the QP strength (discussed in the Appendix) is accounted for and that QPPI preserves this feature. The final section is dedicated to the concluding remarks.

II. THE SELF-CONSISTENT HARMONIC APPROXIMATION

The SCHA consists in simulating the action (2) by a trial harmonic action,

$$S_H[\phi(\tau)]=\int_0^{\beta\hbar}d\tau\left[\sum_i\frac{1}{8e^2}C\dot\phi_i^2(\tau)+\sum_{\langle ij\rangle}\tfrac{1}{2}m\,\phi_{ij}^2(\tau)+\sum_{\langle ij\rangle}\frac{1}{8e^2}\delta C_\alpha\dot\phi_{ij}^2(\tau)\right], \quad (3)$$

using the Gibbs-Bogoliubov inequality to estimate the best upper bound for the free energy[21]

$$F^*=F_H+\beta^{-1}\langle S-S_H\rangle_H , \quad (4)$$

which is the minimum with respect to the variational stiffness m of (4). F_H is the free energy, and $\langle\ \rangle_H$ means the average value, both calculated with (3). When $m\neq 0$, $\langle\cos\phi_{ij}(\tau)\rangle_H$ is nonvanishing. Thus the assumption of this quantity as the (short-range) order parameter reduces the problem to determine the condition under which minimization of (3) yields a nonvanishing solution for m.

The action (2) completely defines the problem only if the allowed paths are prescribed, i.e., the set of the allowed states is specified.[22] We then assume that small leakage currents through the substrate exist, so that the states $|\phi\rangle$ and $|\phi+2\pi\rangle$ are distinguishable. The boundary condition for the allowed paths is $\phi(0)=\phi(\beta)$. Then, the trial action (3) reads

$$S_H[\phi_{kn}]=\sum_{k,n}S_{kn}|\phi_{kn}|^2 , \quad (5)$$

where

$$S_{kn}=(1/8\beta e^2)(C+\delta C_\alpha z_k)(\omega_n^2+\Omega_k^2) ,$$

$$\Omega_k^2=\frac{4me^2z_k}{C_\alpha+\delta C_\alpha z_k} .$$

Here, $z_k=\sum_{i=1}^{z}[1-\cos(\mathbf{k}\cdot\mathbf{a}_i)]$ is the usual dispersion for nearest-neighbor pairs, a_i are vectors joining nearest-neighbor pairs, and k is a vector of the first Brillouin zone.

The quadratic action can be expressed in terms of coordinates and momenta for the quantum harmonic oscillator in the following way:[20]

$$H=\tfrac{1}{2}\sum_k (P_k P_{-k}+\Omega_k^2 Q_k Q_{-k}) , \tag{6}$$

where

$$P_k=(1/2e)(C+\delta C_\alpha z_k)^{1/2}\dot{\phi}_k ,$$

and

$$Q_k=(1/2e)(C+\delta C_\alpha z_k)^{1/2}\phi_k .$$

Then the standard harmonic oscillator results[8,20] for the correlators between the spatial Fourier transforms,

$$\begin{aligned}\langle Q_k Q_{-k}\rangle &=(1/2)\frac{1}{\Omega_k}\operatorname{cotanh}(\tfrac{1}{2}\beta\Omega_k) ,\\ \langle P_k P_{-k}\rangle &=\Omega_k^2\langle Q_k Q_{-k}\rangle ,\end{aligned} \tag{7}$$

can be used in evaluating the averages involved in the problem.

The trial free energy (4) is now readily computed:

$$\begin{aligned}F^*&=F_H-\frac{Nz}{2}\left[E_J\langle\cos\phi\rangle+\frac{\delta C_\gamma}{24e^2}\langle\dot{\phi}^2\cos\phi\rangle+\tfrac{1}{2}m\langle\phi^2\rangle\right]\\ &=F_H-\frac{Nz}{2}\left[E_J e^{-\langle\phi^2\rangle/2}+\frac{\delta C_\gamma}{24e^2}\langle\dot{\phi}^2\rangle e^{-\langle\phi^2\rangle/2}+\tfrac{1}{2}m\langle\phi^2\rangle\right] .\end{aligned} \tag{8}$$

Here, we have dropped the subscript H in the averages. Moreover, these are not dependent on the bond $\langle ij\rangle$ and on the time, so we dropped the relative arguments of ϕ. N is the number of sites of the array, and a generating functional procedure was adopted to evaluate

$$\langle\dot{\phi}^2\cos\phi\rangle=-\left[\frac{\partial^2}{\partial a^2}\langle e^{i(a\dot{\phi}+\phi)}\rangle\right]_{a=0}=\langle\dot{\phi}^2\rangle e^{-\langle\phi^2\rangle/2}$$

together to the known properties of the Gaussian averages.

By minimizing (8) with respect to m, the self-consistency equation for the stiffness is found:

$$e^{-\langle\phi^2\rangle/2}-x=\frac{E_J\delta C_\gamma}{6e^2}\left[2\left[\frac{\partial}{\partial x}\langle\phi^2\rangle\right]^{-1}\left[\frac{\partial}{\partial x}\langle\dot{\phi}^2\rangle\right]-\langle\dot{\phi}^2\rangle\right]e^{-\langle\phi^2\rangle/2} , \tag{9}$$

where $x=m/E_J$, and where the correlation functions are

$$\begin{aligned}\langle\phi^2\rangle&=\frac{2}{Nz}x^{-1/2}\sum_k\left[\frac{e^2z_k}{E_J(C+\delta Cz_k)}\right]^{1/2}\operatorname{cotanh}\left[\beta E_J\left[\frac{e^2z_kx}{E_J(C+\delta Cz_k)}\right]^{1/2}\right] ,\\ \langle\dot{\phi}^2\rangle&=\frac{2}{Nz}x^{1/2}\sum_k\left[\frac{e^2z_k}{E_J(C+\delta Cz_k)}\right]^{3/2}\operatorname{cotanh}\left[\beta E_J\left[\frac{e^2z_kx}{E_J(C+\delta Cz_k)}\right]^{1/2}\right] .\end{aligned} \tag{10}$$

Here, the summations are performed over the first Brillouin zone, excluding $k=0$.

The right-hand side of Eq. (9) arises from QPPI and results in an enhancement of the Josephson contribution [$\propto\exp(-\langle\phi^2\rangle/2)$ in the left-hand side] favoring then phase coherence. We will discuss first the $T=0$ phase diagram, and then we will analyze the effect of the temperature to discuss the reentrant behavior of the phase diagram. In the limit in which the QP tunneling is disregarded, Eq. (9) reduces to the limit studied in Ref. 6.

III. THE $T=0$ PHASE DIAGRAM

If $T=0$, the correlation functions reduce to

$$\begin{aligned}\langle\phi^2\rangle&=x^{-1/2}(CE_J/e^2)^{-1/2}f_{1/2}(\lambda) ,\\ \langle\dot{\phi}^2\rangle&=x^{1/2}(CE_J/e^2)^{-3/2}f_{3/2}(\lambda) ,\end{aligned} \tag{11}$$

where $\lambda=C/\delta C$, and the lattice properties are carried by

$$f_{1/2}(\lambda)=\frac{2}{Nz}\sum_k\left[\frac{\lambda z_k}{\lambda+z_k}\right]^{1/2} ,$$

$$f_{3/2}(\lambda)=\frac{2}{Nz}\sum_k\left[\frac{\lambda z_k}{\lambda+z_k}\right]^{3/2} .$$

Since δC and E_J are related (at $T=0$, we have $\delta C=3g/16\Delta$, and $E_J=g\Delta/2$), we obtain the phase diagram by considering the values of g and of the reduced inverse charging energy Δ/U for which a nonvanishing x first appears, and the results are shown in Fig. 1. When QPPI is neglected, the SCHA yields a slowly varying critical g, of order 1 in the parameter range of interest ($\Delta/U\approx10^{-1}\div10^{-2}$). The including of QPPI appreciably changes the phase diagram.

In the limiting case in which $\lambda = C/\delta C \rightarrow 0$, the only relevant lattice parameter is z: The critical g value is $g_c = 2e(\frac{2}{3})^{1/2}(1/z) \approx 4.44/z$, when QPPI is neglected and is depressed to $g_c \approx 4.10/z$ if it is included. This partially agrees with the Ferrell and Mirashem predictions. Indeed, they found that neglecting QPPI, a MF value for $g_c = 3^{-1/2}(2/z) \approx 4.61/z$ (actually, their cluster calculation estimated that the fluctuations determine a slightly higher value), whereas including QPPI enhances the critical value of g by a factor of ~ 1.08. Our QPPI corrections have about the same magnitude but opposite sign. Probably, the discrepancy is related to the fact that the effective one-body action obtained in Ref. 13 does not fully take into account the long-range properties of Coulomb interaction. (A word of caution seems in order, however, insofar as the chosen approximation plays a role in the determination of the actual phase boundary).

The shortcomings are mainly due to the fact that a harmonic action is sometimes a poor approximation of the original problem. At the transition point, a jump of the variational stiffness appears and, consequently, a fictitious jump of the order parameter $\langle \cos\phi \rangle$, so it is believed that SCHA poorly describes the region near the transition point. In this regime, paths characterized by $\langle \phi^2 \rangle^{1/2} \gtrsim \pi$ become important, and ϕ feels the periodicity of the potential which the harmonic approximation destroys.

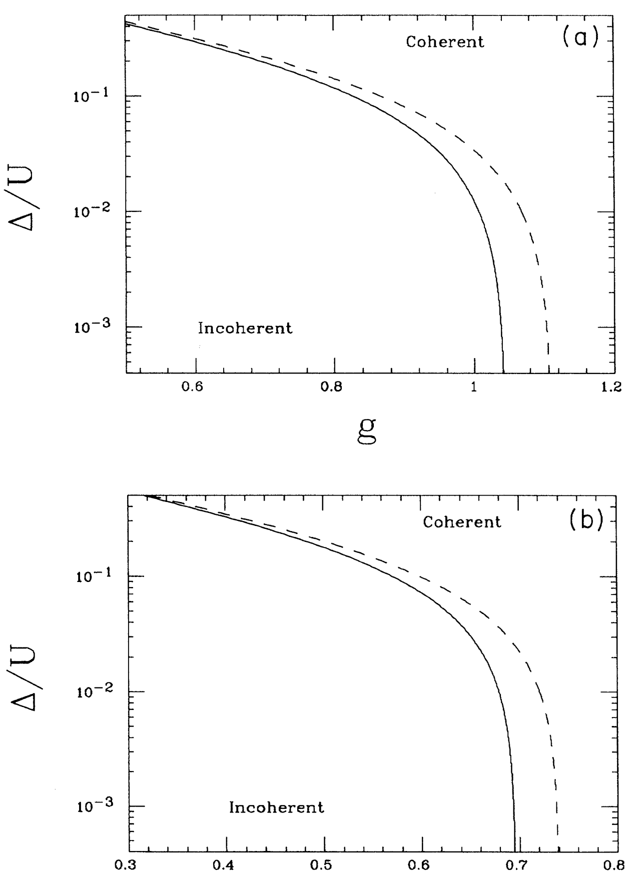

FIG. 1. The $T=0$ phase diagram is plotted both for the full problem and for the case in which QPPI is neglected (dashed lines) for (a) square ($D=2, z=4$) and (b) cubic ($D=3, z=6$) lattices; $g = R_0/R_N$, and U and Δ are the charging energy and the superconducting gap, respectively.

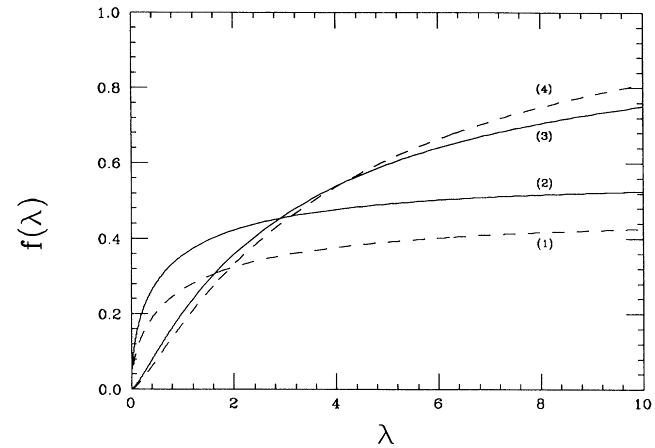

FIG. 2. The auxiliary functions $f_{1/2}(\lambda)$ (curves 1 and 2) and $f_{3/2}(\lambda)$ (curves 3 and 4) are plotted for square (solid line) and cubic (dashed line) lattices.

A periodic ("scalloped") trial potential has been employed in Ref. 23 to improve the SCHA predictions for the action (1). The main results are that a jump in the variational stiffness does not imply a jump in the order parameter, and that SCHA is accurate if the resulting fluctuations are small, $\langle \phi^2 \rangle^{1/2} \lesssim \pi/4$. On the critical line, $x = e^{-2}$, and $\langle \phi^2 \rangle^{1/2} = 2$ if QPPI is neglected; whereas if it is included, the average fluctuations are lowered to $\langle \phi^2 \rangle^{1/2} = \pi/2$, and the approach resulting is more believable.

Figure 2 illustrates the functions $f_{1/2}(\lambda)$ and $f_{3/2}(\lambda)$. Usually, for calculational convenience, the dispersion for

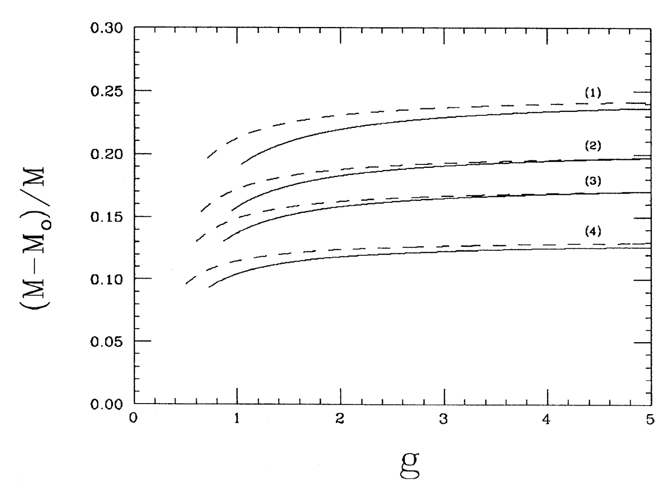

FIG. 3. The relative renormalization of the variational stiffness $(M - M_0)/M$ is plotted against g for Δ/U equal to (1) 0.02, (2) 0.06, (3) 0.1, and (4) 0.2, respectively ($T=0$).

nearest-neighbor pairs is approximated by its long-wavelength limit $z_k = a^2k^2$ (for square or cubic lattices). This is correct for λ small, but can be less accurate for $C \gg \delta C$: In the self-charging limit for cubic lattices, the critical threshold ZE_JC increases from the exact value 0.63 to 2.60.

For completeness, we show in Fig. 3 the variational stiffness renormalization if QPPI is included, for square and cubic lattices for different values of Δ/U.

IV. THE $T>0$ PHASE DIAGRAM

In the limit $\delta C \gg C$ the summations in (10) drop out and the only relevant parameter is the coordination number . In Fig. 4, the phase diagrams zg versus T^* are shown for various cases. T^* is the ratio of T to the "classical" transition temperature $zE_J/2k_B$. The adopted procedure is surely meaningful for $d=3$, where a ferromagnetic order is expected to set up. For two-dimensional arrays, the transition is of the Kosterlitz-Thouless-Berezinskii (KTB) type, at least for high temperatures, and the possible crossover at zero temperature to a $d=3$ behavior is still an open question.[9,14] In any case, in principle the SCHA allows the exploration of temperatures higher than reported here, but in the present case, QP tunneling can be modeled by an effective capacitance renormalization only for small enough temperatures; thus the approach based on (2) is not reliable. We have estimated that this happens for $T > T_C/\pi$ and, correspondingly, for $T^* > 0.73/g$.

The dashed curves are obtained by assuming that $\delta C_\alpha = \delta C_\gamma = \delta C$. The absence of reentrance found in Ref. 8 is here confirmed (curve 4), even allowing for a QPPI-type term (curve 2). MF theory predicts in this limit reentrant behavior, but entirely neglects fluctuation effects. The inclusion of QPPI seems to render more stable the superconductive state when the temperature increases.

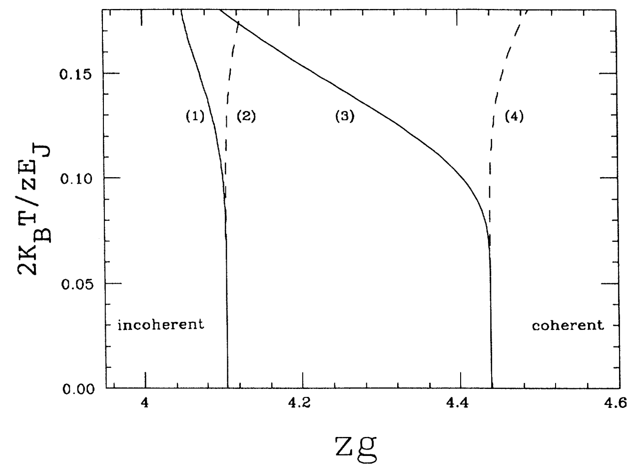

FIG. 4. The phase diagram in the g-T plane is shown: (1) accounting for the quasiparticle renormalization of the capacitance and the pair-interference term taking into account their temperature dependence (see the Appendix), (2) neglecting their temperature dependence, (3) accounting only for temperature dependent QP; and (4) neglecting in the latter case the dependence on the temperature. $\delta C \gg C$.

The situation changes when we include the temperature dependence of δC_α and δC_γ (see the Appendix for the derivation), which in the low temperature limit reads

$$\begin{Bmatrix}\delta C_\alpha(T)\\ \delta C_\gamma(T)\end{Bmatrix} = \delta C\left[\tanh\left[\frac{\beta\Delta}{2}\right] + \left[\frac{\beta\Delta}{2}\right]\cosh^{-2}\left[\frac{\beta\Delta}{2}\right] + \begin{Bmatrix}2\\ -6\end{Bmatrix}\left[\frac{\beta\Delta}{2}\right]^2 \tanh\left[\frac{\beta\Delta}{2}\right]\cosh^{-2}\left[\frac{\beta\Delta}{2}\right]\right], \tag{12}$$

where the upper and lower line to QP and QPPI, respectively. In this case, the transition is reentrant both neglecting (curve 3) and including QPPI (curve 1), but the phenomenon is somewhat suppressed in the latter case. On the other hand, we verified that for MF treatment, accounting for the temperature dependence of the QP strength, slightly changes the situation, the main role being played by temperature-independent off-diagonal charging (see also Ref. 15). If a KTB point of view is followed,[14] the temperature dependence (12) determines both reentrant and quasireentrant $R(T)$ curves. The plotted curves, in the reentrant case, will eventually bend again to approach the classical limit: We cannot extend our calculation to this region because the Eqs. (12) for the capacitance renormalizations are no longer valid.

Finally the temperature behavior of the capacitance renormalizations is shown in Fig. 5. The QP renormalization increases with temperature when QPPI renormalization has a rapid decrease.

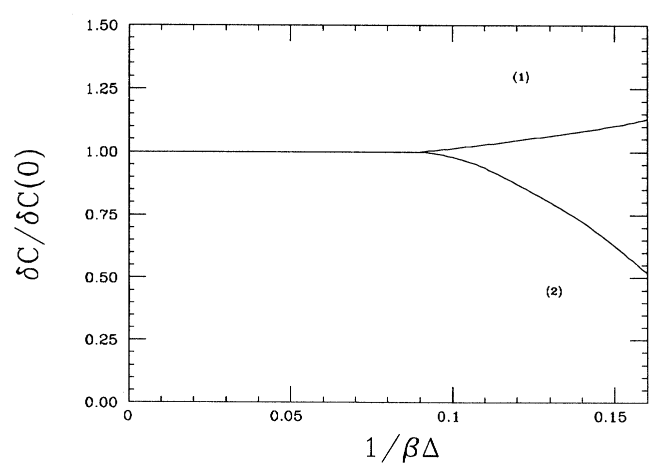

FIG. 5. The renormalization of the capacitance due to (1) QP and (2) QPPI is plotted against the reduced temperature $1/\beta\Delta$.

V. SUMMARY AND CONCLUSIONS

We have shown that QP tunneling tends to suppress quantum fluctuations, and that when it is the only relevant reactive response to voltage fluctuations, a resistance threshold is obtained of ~ 5.86 kΩ. If QPPI is included, the resistance threshold increases to ~ 6.34 kΩ, showing that the effect of QPPI is relavant. However, a proper treatment of QPPI would require accounting for both effective E_J enhancement and δC depression and for their fluctuational behavior. It should be very interesting to study the electromagnetic response, especially to microwave fields, as in classical experiments on QPPI. At finite temperature it is shown that reentrant behavior occurs if the temperature dependence of tunneling is considered, and that QPPI preserves this feature, even if it has the tendency to suppress the response against increase of the temperature. Finally, improved calculations seem to be in order to ascertain if the relative sign of QP and QPPI strengths can change with temperature.

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APPENDIX

The QP particle tunneling across a junction is usually described in terms of the normal Green's function, whereas Josephson tunneling and interference terms involve the anomalous Gor'kov propagators of the superconducting electrodes via the functions[14,16,24]

$$\alpha(\tau)=gG(\tau)G(-\tau)\ ,$$

$$\beta(\tau)=gF(\tau)F(-\tau)\ ,$$

where $G(\tau)$ and $F(\tau)$ are the momentum integrated ("quasiclassical") Green's functions of a superconductor.

The effective capacitances are defined in terms of the Fourier transforms:

$$\delta C_\alpha=-\frac{2}{\omega_n^2}[\alpha(\omega_n)-\alpha(\omega_n=0)]\ ,$$
$$\tfrac{1}{3}\delta C_\gamma=-\frac{2}{\omega_n^2}[\beta(\omega_n)-\beta(\omega_n=0)]-\frac{2}{\omega_n^2}\gamma(\omega_n)\ , \qquad \text{(A1)}$$

where

$$\alpha_n-\alpha_0=g\beta^{-1}\sum_m\left[\frac{\nu_m+\omega_n}{(\nu_m+\omega_n)^2+\Delta^2]^{1/2}}\frac{\nu_m}{(\nu_m^2+\Delta^2)^{1/2}}-\frac{\nu_m^2}{\nu_m^2+\Delta^2}\right]\ ,$$
$$\gamma_n g\Delta^2\beta^{-1}\sum_m\left[\frac{1}{[(\nu_m+\omega_n)^2+\Delta^2]^{1/2}}\frac{1}{(\nu_m^2+\Delta^2)^{1/2}}-\frac{1}{\nu_m^2+\Delta^2}\right]\ . \qquad \text{(A2)}$$

Here, $\nu_m=[(2m-1)\pi]/\beta$ are fermionic Matsubara frequencies, and $m=-\infty$ to $+\infty$.

The validity of the effective capacitances approach (2) rests on the fact that $\alpha_n-\alpha_0$ and γ_n are well approximated by their second-order expansions.

By expanding (A2) in powers of ω_n, we have

$$\begin{Bmatrix}\alpha_n-\alpha_0\\ \gamma_n\end{Bmatrix}=g/\beta^{-1}\omega_n^2\sum_{m=1}^{\infty}\begin{Bmatrix}-3\nu_m^2\\ 2\nu_m^2-1\end{Bmatrix}\frac{1}{(\nu_m^2+1)^3}\ , \qquad \text{(A3)}$$

where units of Δ are now used. Then, expanding in partial fractions, the sums in Eq. (A4) can be calculated:

$$\sum_{m=1}^{\infty}\begin{Bmatrix}1\\ \nu_m^2\end{Bmatrix}\frac{1}{(\nu_m^2+1)^3}=\frac{i\delta}{16}\begin{Bmatrix}3-3\delta\dfrac{\partial}{\partial\delta}+\delta^2\dfrac{\partial^2}{\partial\delta^2}\\ 1-\delta\dfrac{\partial}{\partial\delta}+\delta^2\dfrac{\partial^2}{\partial\delta^2}\end{Bmatrix}\sum_{m=1}^{\infty}\left[\frac{1}{m-\frac{1}{2}+i\delta}-\frac{1}{m-\frac{1}{2}+i\delta}\right]\ , \qquad \text{(A4)}$$

where $\delta=\beta/2\pi$. The summation yields

$$-\Psi(\tfrac{1}{2}+i\delta)+\Psi(\tfrac{1}{2}-i\delta)=-i\pi\tanh\pi\delta\ , \qquad \text{(A5)}$$

where $\Psi(z)$ are digamma functions.[25] Thus by inserting (A5) into (A4) and (A3), and using the definition (A1), the capacitances (12) are found, if we restore usual energetic units.

This result is valid for small ω_n and, consequently, small temperatures. We can estimate that when $1/\beta\Delta<1/1.76\pi$, then $T<T_C/\pi$, where T_C is the critical temperature for single-grain superconductivity.

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