Shaping a timber reciprocal frame structure

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Content

1. Abstract

This research paper will be about a non-conventional building structure called the reciprocal frame. In this paper the principle of the reciprocal frame structures is thoroughly described. This will be done by explaining firstly the technical aspects of the reciprocal frame. By looking into what the technical possibilities and limitations are. Apart from that, this paper will then describe what aspects a reciprocal frame has and how these could be assembled. This has many possibilities so in order to gain insight in the differences between the possibilities, this paper will have a closer look at what distinguishes them. This paper will also consist of a summation of the possible architectural implementations of this structural principle. In order to have a full picture and be able to make connections between these aspects, the investigated subjects will be shown through existing examples.

This research is done in order to put together a set of tools to be able to make a design for the assignment which consists of a reciprocal frame. We can conclude that this ancient structural principle has many possibilities both structurally and architecturally. It offers to not only be implemented in the traditional ways but also in contemporary ways.

2. Introduction

2.1 Background

This project focusses on a particular location in the Netherlands. This location lies within an area which is referred to as IBA parkstad in Limburg. This is an area in the Netherlands which contains a large history and has had a very diverse development. This is mainly due to the mines that used to be in this area and makes up a part of national labour history which unfortunately has been masked for quite a long time which has led to a stagnation in development and economy. In the past couple of years this area had been under the attention of the IBA. The IBA, which has developed several other industrial area's such as the Ruhr area, aims to make this area rise again within its own identity. *(IBA open oproep, 2014)*

2.2Problem statement

Like mentioned above the emphasis of the approach for this particular IBA project is to provide and bring out the identity of the area in Limburg. The people in the area are rather unsure of this identity which gives designers a lot of freedom.
The reason for this uncertainty is the dynamism of the landscapes of this area and the constant variation between rural area and dwellings. There is also a dynamism in the hightlevels in this area. This description to me is actually already a definition to this area in its essentiality. It is an area with fragmentation and constant change. To be able to showcase this and observe this, it is aimed within this assignment to be about a space where this could be done. It will be an opportunity for the area to educate visitors about the history of the area and show what the impacts are today. The research in this paper will be used in the process of designing this space. *(Intergemeentelijke structuurvisie parkstad Limburg, 2009)*

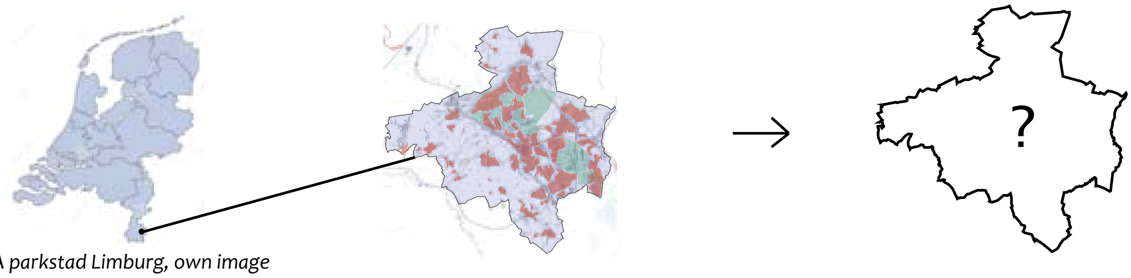


Fig.1 IBA parkstad Limburg, own image

2.3 Research question

The focus of the research will be on a structural principle called, the reciprocal frame. This structural principle has many faces to explore and is also characterised with being dynamic. This is essentially a construction principle that is based on patterns and repetition. The way it is constructed and assembled however provides another dimension to the total and creates a variety of possibilities for an architectural expression. This expression is in the history of the principle based on the context. Not necessarily in a literal and direct way, but one of the many aspects is derived from its context. These aspects of this structural principle was a reason to explore the possibilities within the context of IBA parkstad in Limburg.

The research question that focusses on this technical aspect and will be provided with a conclusion in this paper is described as follows: *In what ways is the shape and appearance of a building structure determined within the requirements for a reciprocal frame structure, in a low-cost way?*

The aim of this question to find out what are influences on the shape and formation of a structure based on the reciprocal frame principle in order to gain insight in the matter and therefore assemble a set of tools to be able to make a design with the reciprocal frame principle for the elaborated assignment.

In the following pages of this report the reciprocal frame will be generally explained and different variations of different aspects will be compared to each other in order to gain more knowledge and insight on how it works. After this there will be more insight in how to use this principle when designing a building or building structure.

3. Methods

To investigate the topic of technical research three methods of research will be used. These methods are literature studies, case studies and researching by design and they will be further explained below.

3.1 Literature studies

To fully understand what a reciprocal frame is and what this construction principle consists of, literature studies have been done. These consisted of books, articles, websites and other academic resources in order to properly gain knowledge on the subject and dive into the facts. The book written by O.P. Larson was used in particular, because of the broad knowledge that it provided to be able to understand how the system works. It is also the source which is mostly providing practical knowlegde and facts about variaties of built examples, as opposed other scientific resources. Understanding how the system of the reciprocal frame works was also necessary to further investigate the examples and be able to analyse these and be able to draw conclusions for further development.

3.2 Case studies

A certain number of examples have been analysed in relevant ways to gain more understanding in how the reciprocal frame can be implemented in the architectural or structural design of a building. These examples consist of various implementations and various appearances. This is done to be able to make connections and draw conclusions about the certain aspects of the reciprocal frame. The limitations and possibilities will then be more visible. This way of researching will give more insight in the opportunities that the reciprocal frame will create.

3.3 Research by design

To slightly put the gained knowledge into practice, this method has been interpreted as testing small models of basic types of reciprocal frames and analyse the differences and the way they work in a physical way. Although this will not represent the bigger picture of the subject, it will give an oppurtunity to have more grip on the subject.

4. Results

4.1 Reciprocal frames in general

The reciprocal frame is a construction principle which is based on pattern formation and repetition. The most simplified way to create a reciprocal frame is like the assembly which is illustrated below.

Fig.2 Basic reciprocal principle.
(Larsen O.P., 2008)

This principle has basic rules which the frame needs to meet to be able to be referred to as a reciprocal frame. The first one of these rules is that the assembly needs to consist of at least three beam/rod elements (*a*). Another rule is that each of these beam elements need to support another beam element and at the same time be supported by a beam element (*b*). The last rule is that the support of the elements is taking place along the span of each element (*c*). This will prevent nodes from being formed and the beams to come together in one point. *(Material Swarm Articulations, 2011)*

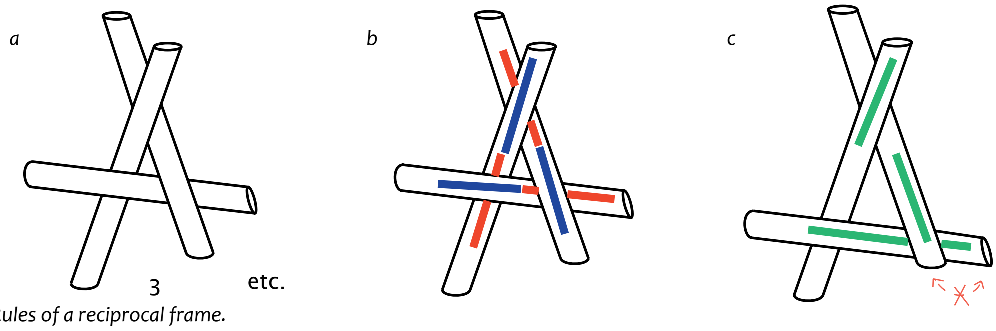


Fig.3 Rules of a reciprocal frame.
(own image)

This single frame/pattern can then be repeated to make an entire circuit, where all the elements are connected within the same rules. The circuits can be created in different ways, meaning not only by repeating one pattern but also several patterns which will create much more possibilities for assembling. The combination of several patterns will create tesselations.

This construction principle finds its origins in the far east and is mainly developed into roof structures to span sizes that exceed the beams themselves. Nowadays it is also still realized in Japanese architecture. In Europe this principle has been introduced in the 13th century, in gothic architecture. Also Leonardo da Vinci experimented with this principle to construct bridges, roofs and ceilings.

4.2 Anatomy of a reciprocal frame

Within the above mentioned principles it is possible to create numerous variations and the differences in these varieties is influenced by certain aspects of the frame. *(Larsen O.P., 2008)* These aspects are listed below:

- Number of beams (n)
- Radius through the outer support (ro)
- Radius through beam intersection points (ri)
- Vertical rise from the outer supports to the beam intersection points (H)
- Vertical spacing of centrelines of the beams at their intersection points (h2)
- Length of the beams on the slope (L)

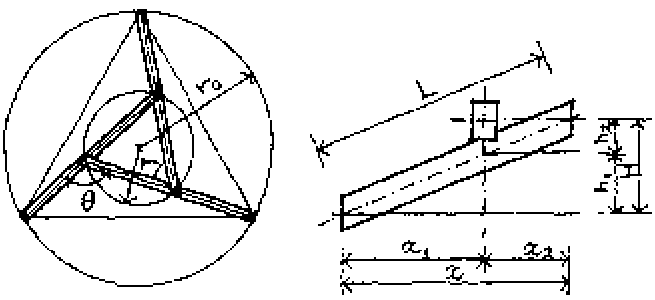


Fig.4 Anatomy of a reciprocal frame.
(Larsen O.P., 2008)

$$\theta = \frac{360}{n} \quad (4.1)$$

$$x_2 = 2r_i \sin\frac{\theta}{2} \quad (4.2)$$

$$x_1 = \left[r_0^2 - \left(r_i \cos\frac{\theta}{2}\right)^2\right]^{\frac{1}{2}} - \frac{x_2}{2} \quad (4.3)$$

$$x = x_1 + x_2 \quad (4.4)$$

$$h_1 = H\frac{x_1}{x} \quad (4.5)$$

$$h_2 = H - h_1 \quad (4.6)$$

$$L = (x^2 + H^2)^{\frac{1}{2}} \quad (4.7)$$

Fig.5 Formula for basic reciprocal frame.
(Larsen O.P., 2008)

A formula can be derived from these aspects and accordingly it is possible to make an assumption for a range of ratio's that are most suitable for a durable construction with a certain frame. This formula however, is only applicable to one type of frame, the most basic one that is shown above. Still it is suitable keep these ratio's in mind for a preliminary design. A graph with these ratio's for different parameters is put out and illustrated below. *(Larsen O.P., 2008)*

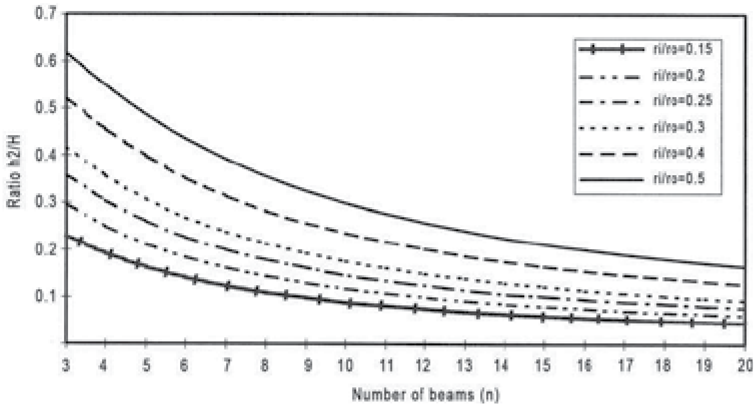


Table 1. Graph for ri/ro ratio (Larsen O.P., 2008)

According to these ratio's there are certain set of parameters that will depend on each other to form an equilibrium in the frame and that are considered optimal. A range of these parameters is listed in table below and in this table it is also taken into consideration how the beams are proportioned (depth x height). Although this is also applicable for one type of reciprocal frame, it is still considered suitable to keep these parameters as guideline for preliminary designs of other types of reciprocal frames. *(Larsen O.P., 2008)*

Reciprocal Frame Design

		Beam Depth (D)	200 mm × 60 mm			400 mm × 90 mm			600 mm × 120 mm			800 mm × 180 mm		
		S(o)	Span 3 m r(o) = 1.5 m			Span 6 m r(o) = 3 m			Span 9 m r(o) = 4.5 m			Span 12 m r(o) = 6 m		
		S(i)	0.5 m	1 m	1.5 m	1 m	2 m	3 m	2 m	4 m	6 m	2.5 m	5 m	8 m
Number of Rafters	3	Roof Pitch °	30.3°	16.5°	11.7°	30.3°	***1**–16.5°	11.7°	24.2°	13.3°	9.8°	24.6°	13.9°	9.8°
		Rafter Pitch °	27.1°	13.1°	8.4°	27.1°	***1**–13.1°	8.4°	20.7°	9.8°	6.5°	21.3°	10.5°	6.5°
	4	Roof Pitch °	32.1°	***2**–18.8°	13.1°	35°	18.8°	13.1°	25.9°	14.8°	10.5°	20.6°	15.5°	10.5°
		Rafter Pitch °	29.5°	***2**–15.7°	10.3°	32.2°	15.7°	10.3°	23°	11.8°	7.8°	17.6°	12.5°	7.8°
	5	Roof Pitch °	38.8	21	14.9	40.8	***3A**–21.4	15	30.8	***3B**–16.8	11.5	32.2	17.9	11.3
		Rafter Pitch °	36.5	18.4	12.4	38.4	***3A**–18.7	12.5	28.2	***3B**–14.2	9.4	29.6	15.2	9.2
	6	Roof Pitch°	45.3	24.2	16.7	46.8	***4**–24.6	17	36.9	19.1	13.1	39.3	20.2	12.9
		Rafter Pitch °	43.3	21.8	14.6	44.8	***4**–22.2	14.9	34.5	16.9	11.5	37	17.8	11.3
	9	Roof Pitch °	81.6	33.7	22.8	76.9	35.6	22.8	53.4	25.8	16.2	59.5	***5**–27.7	16.2
		Rafter Pitch °	81.3	32	21.7	76.3	33.9	21.7	52	24.5	16	58.2	***5**–26.3	16
	12	Roof Pitch °	78.1	82.8	29.9	79.2	45.8	29	79.8	***6**–34.4	20.6	77.4	36.7	16.7
		Rafter Pitch °	77.7	82.6	29.8	78.9	44.8	28.9	79.4	***6**–34	21.9	77	36.1	16.6

Table 2. Parameters (Larsen O.P., 2008)

What can be determined is that this principle is based on rotation and a suggestion of movement within an entity. In what direction this rotation takes place largely influences the appearance of the frame. The size of the beams is a factor as well, because the span of the total frame depends mostly on this.

Another factor that determines the slope of the beams and therefor affects the appearance and shape, but is not mentioned in the table or graph, is the way the beams are proportioned in section (height x width). The beams of a reciprocal frame can be higher than it is wide or the other way around. The larger the height of the beam of its width, the steeper its slope will get. And the flatter a beam, so smaller height than width, the less steep its slope will be. *(Kohlhammer T., Kotnik T., 2011)*

4.3 Structural behaviour of a reciprocal frame

Each element within an assembly will act as single beam carrying the loads of the supported and supporting elements and any additional dead loads. The centres of the patterns, which form the circulation centres as the suggested rotation of the elements are centered here, create the centres of gravity. There is no structural hierarchy within a frame assembly, meaning each beam element has the same function. Illustrated below is the way the forces are repeatedly passed on to the next connections point within an assembly. *(Larena A. B., Ménendez D. G., 2014)*

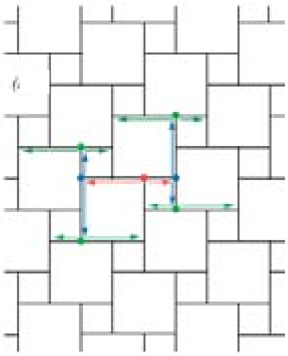


Fig.6 Force distribution.
(Kohlhammer T., Kotnik T., 2011)

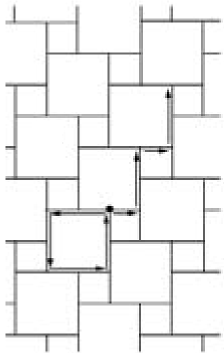


Fig.7 Iterative force flow.
(Kohlhammer T., Kotnik T., 2011)

The basic spatial frame type is illustrated below to show the difference in behaviour. The left illustration shows a frame which is not steep at all and has a large inner circle. The right illustration shows a frame which is much steeper and has a smaller inner circle. The number of beams is the same and the direction of rotation is the same as well. The axial forces in the different frames are distributed differently. The left frame has less compression in the entire structure than the right frame. This is because the compression is created where the beam is supporting another beam on top and the outer support. The tension exists in the parts where the beam is supporting another beam as reaction to the compression and at the inner support. When we look at the left frame which is less steep, we see that the compression forces have a larger area to be distributed. *(Kohlhammer T., Kotnik T., 2011)*

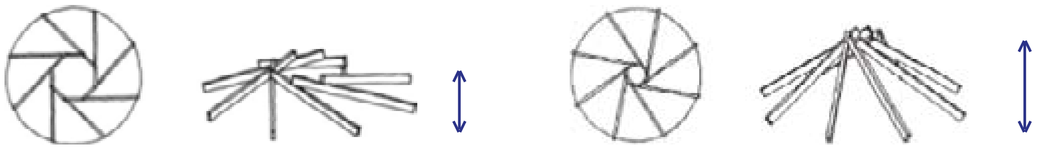


Fig.8 Slope differences in frames.
(Larsen O.P., 2008)

Because of the same reasons, the **shear forces** in the right frame are larger as well. This is also determined by the slope of the beams and the size of the inner opening.

The **bending moments** in a reciprocal frame are transferred through the connections and exist around the circulation centres. This means that a reciprocal frame assembly forms a closed circuit of forces which on its own is stiff and stable. If the assembled structure will be caried by secondary vertical structure however, this will also have to be stable and stiff on its own. The reciprocal frame structure will not provide stability and stiffness for a combined structure.
The bending moments generally increase with the size of the inner opening that forms the circulation centre. However a bigger slope of the beams can also increase it. *(Larsen O.P., 2008)*

Fig.9 Bending moments within frame.
(own image)

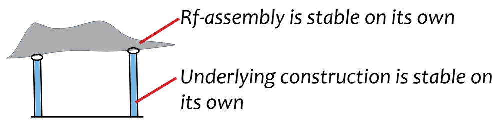


Fig.10 Stability on its own.
(own image)

The ratio of the height x width of the beams, the entire shape for that matter, that was mentioned before is also of influence for the behaviour of the frame. The direction and angle in which the next beam will be pushed to or pulled towards is for a big part dependant on this ratio/shape of the beam. *(Fu C., Goswami P., Song P., Zheng J.)*

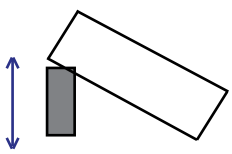


Fig.11 High beam, higher slope
(own image)

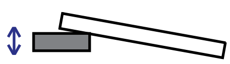


Fig.12 Low beam, low slope
(own image)

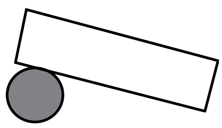


Fig.13 Round beam, slope is dependant on fixation
(own image)

4.4 Architectural possibilities of a reciprocal frame

The reciprocal frame has a variety of ways to be implemented into a design. In this paragraph the most common ways are explained. The most known applied way of a reciprocal frame is by making the roof out of a reciprocal frame. This is also the traditional way of implementing this principle in Japan. It offers a way to span quite a large distance with less height in the materials used. Apart from that it offers a big esthetical asset to the inner space, as the roof is mostly left open in the more traditional buildings in Japan.

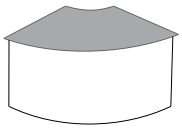


Fig.14 Cone, solid underlying structure (own image)

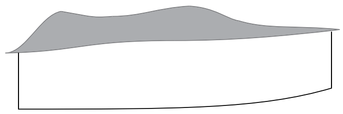


Fig.15 Free-form, solid underlying structure (own image)

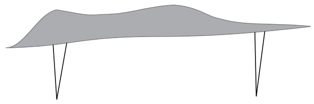


Fig.16 Free-form, underlying column structure (own image)

The cone shaped or cupola shaped roof is mostly seen in the older examples and the more contemporary implementations show much more possibilities to make a free-form structure.

Another way to implement the reciprocal frame in an architectural design, is to make an entire structure to form a shell or skin for a building. This is more contemporary implementation of this structure principle and does not yet have many examples. But it is certainly a field that is being explored mainly theoretically and through parametric design. The challenge with implementation is to be making something that is closed off from the outside. So to actually offer shelter is hard to realise without losing the esthetics that it has to offer. This way of implementing could also be seen as making a facade element out of this structural principle.

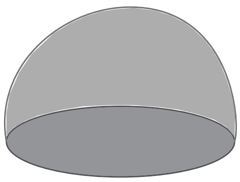


Fig.17 Free-form entire outer structure (own image)

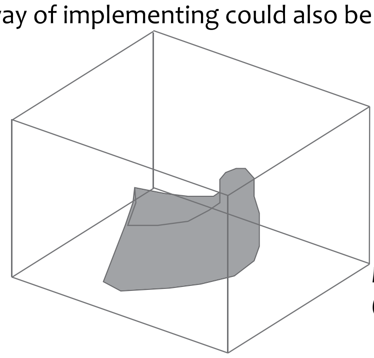


Fig.18 Free-form entire structure, interior element (own image)

As an extension of the previous mentioned way of implementing this structural principle, is to make the reciprocal frame into an interior element. Using it as a kind of room divider is very appropriate way to implement the reciprocal frame. It will add an estethic element to a space that is suitable for this. Inside a space that already offers shelter, it can stay a frame without having to add a skin to it. This means the esthetical value will be kept as it is.

4.5 Space structures in general

In order to be able to gain more insight in the limitations and possibilities of the reciprocal frame, it is briefly put in an broader spectrum to compare it with other similar types of structures. In the first table structures based on tension will be discussed and the second table will discuss structures based on compression.

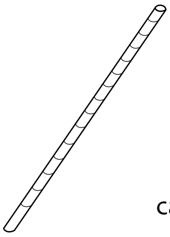
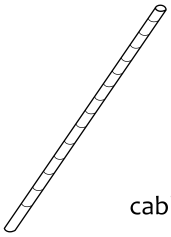
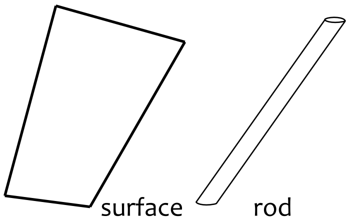
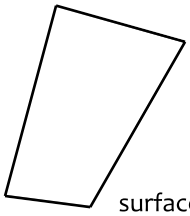
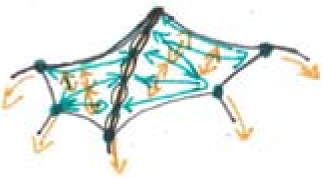
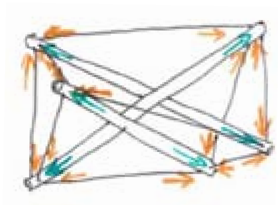
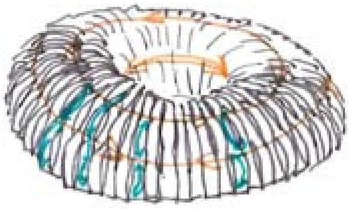
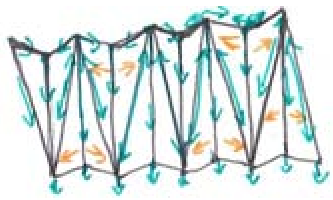
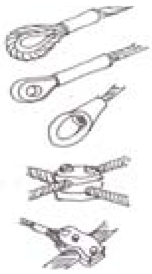
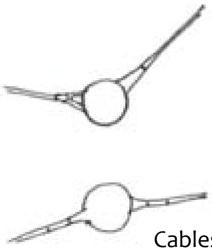
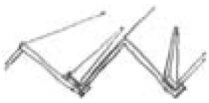
	Tension based			
	Cable net structure	Tensegrity structure	Bend actice structure	Pleated structure
Examples				
Dominant element	 cable	 cable	 surface rod	 surface
Common shapes	Hyperbolic, cone, arch	Hyperbolic, cone, arch	Hyperbolic, cone, arch, wave, sphere	Hyperbolic, arch, wave
Force flow	 Distributed through entire span, towards connection points. Then to the foundation.	 Distributed through elements, that push and pull on each other. Equilibrium achieved in the struc- ture.	 Distributed through entire span, towards connection points horizontally. Then vertically to the foundation.	 Distributed through entire span horizontally, towards folding sides. Then vertically to the foundation.
Typical joints	 Cables and struts	 Cables and struts	  Bending with wood pieces in between, connected with bolts	 Folded as entity  Glued/laminated
Structural space and span	long span long structural elements low structure space	long span long structural elements low structure space	small - medium span short structural elements low structure space	small - medium span short and long structural ele- ments medium-high structure space

Table 3. Space structures tension (own image)

Compression based

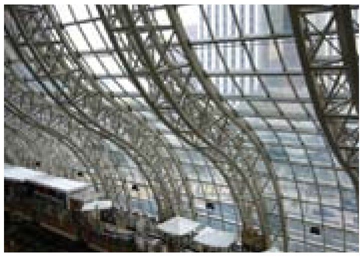
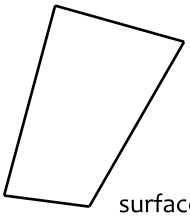
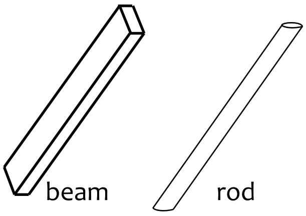
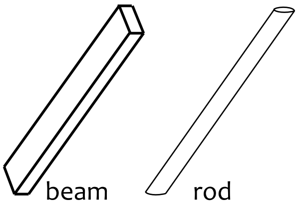
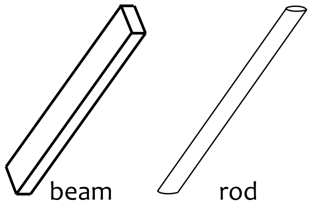
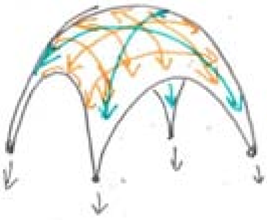
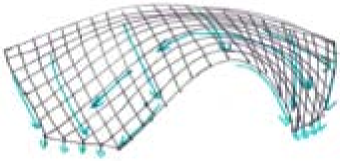
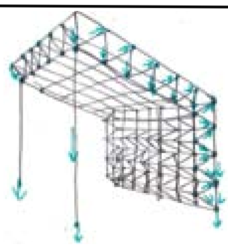
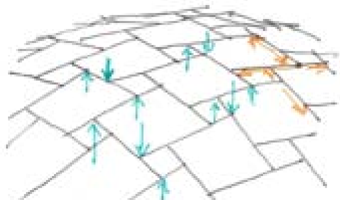
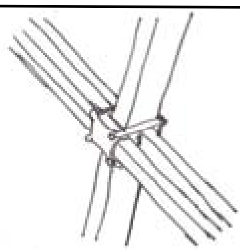
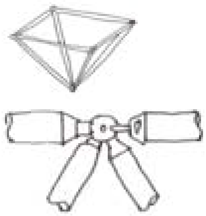
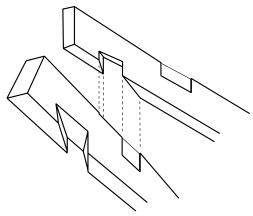
	Shell structure	Grid shell structure	Space frame structure	Reciprocal frame structure
Examples				
Dominant elemant	 surface	 beam rod	 beam rod	 beam rod
Common shapes	Hyperbolic, cone, arch, wave, sphere	Hyperbolic, cone, arch, wave, sphere	Hyperbolic, cone, arch, wave, sphere	Hyperbolic, cone, arch, wave, sphere
Force flow	 Distributed through entire span, towards connection points. Then to the foundation.	 Distributed through entire span, towards connection points. Then to foundation.	 Distributed through entire span, towards connection points diagonally. Then vertically to the foundation.	 Distributed through entire span iteratively, towards beam ends. Then vertically to the foundation.
Typical joints	 hinged joint	 Attached connector, with bolts	 Nodes, steel connetors	 Notched
Structural space and span	long span long structural elements medium-high structure space	long span short and long structural ele-ments low structure space	medium- long span short structural elements high structure space	short-medium span short structural elements low structure space

Table 4. Space structures compression (own image)

Pro's and cons:
The reciprocal frame structure can be categorised as a space structure that is based on compression, as seen in the previous table, because of the way each element is stacked on top of each other. But it does not only carry characteristics of compression based structures, it embodies both tensile and compression aspects.
Where generally compression based structures rely only on compression to transport forces and loads, the reciprocal frame can does not have this limitation. It tranport forces and loads through tension as well.
In the same the reciprocal frame structure distiguishes itself from tension based structures. This aspect of the reciprocal frame structure is both an advantage, as described, but it is also a disadvantage.
It forms a limitation because the elements used are requered to be able to adapt to both of these forms of loadtranfer. This means that in other structure types that are based on one type of loadtranfer, the elements and materials used can be optimized much more to the structural demands of the design. This means a certain hierarchy is taking place within these other structures, where each element has its own function within the structure meaning also different features. Within a reciprocal frame each element is meant to have the same function, so each element has the same features. This eliminates the existence of a hierarchy within the structure. *(Danz C., 2014)*
The ambition with this characteristic is to be able to distribute the loads within the structure as evenly as possible eventhough the shape of the structure might also vary from one part of the structure to another.

The way of assembling a reciprocal frame structure is also a factor that differs from other space structures. As generally other space structures consist of elements that span from one end of the total span to the other, the reciprocal frame elements span from one beamelement to another. This divides the total span in smaller spans overall, resulting in beams that are relatively smaller in height for the same total span.
In total the reciprocal frame is also designed to be built in a follow up manner, where other structures usually have contiuous elements. This means that the reciprocal frame structure is assembled step by step because the elements are interdependant. An advantage of this is that mistakes during the build up are immidiatly detected and fixed. A disadvantage that this has is that the duration of assembling will probably take a little longer as it requires much accuracy.

4.6 Joining methods for timber reciprocal frames

In this paragraph a range of different joining methods, that are used in built examples, are explained more in detail to be able to distinguish the way this type of structure could be assembled. The built examples that showcase these different types joining and assembling are mainly materialized with timber.

The most common way the timber beams are connected to each other is by notching. This means that the angle at which the beams come together is predetermined and the beams will be pre-cut accordingly. This is done in a way that the beams can be interlocked with each other in the directions of its rotation and as a result it is fixed without needing attributes, though it is still an option to do so for more security. This way of connecting requires a very accurate pre-cutting of the beams and it is important to also be accurate and careful when assembling the structure on site. If the notching is not done carefully, the rest of the structure will not fit as properly. Below are some examples of this type of connection. *(Larsen O.P., 2008)*

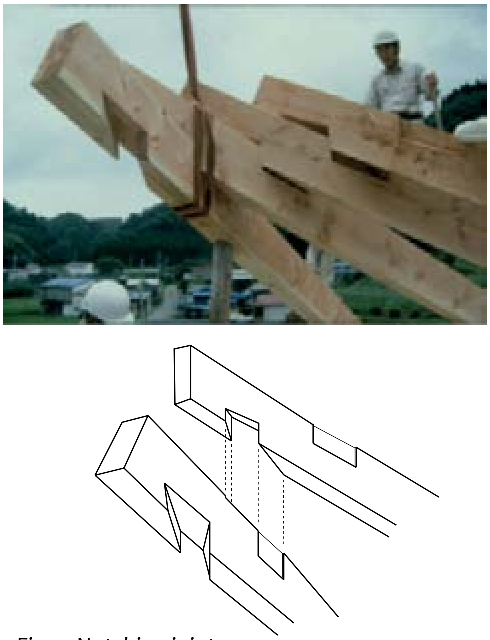


Fig.19 Notching joint
(Larsen O.P., 2008)

Fig.20 Notching joint
(Miller, 2014)

Another way of connecting traditionally that is similar to the above described method is by using friction and stacking the beams on top of each other. This also requires a big amount of accuracy, especially in the pre-cutting part. The cuts that are made in the beams for this method are meant to enable the beams on top to have grip on the lower beam. The assembly is therefore most stable when completed, because it will function like a puzzle that relies on every element used. This is again a method which does not use metal fixating elements such as bolts or screws. The aesthetic outcome and effortless look that this gives is an asset to this type of structure. *(Larsen O.P., 2008)*

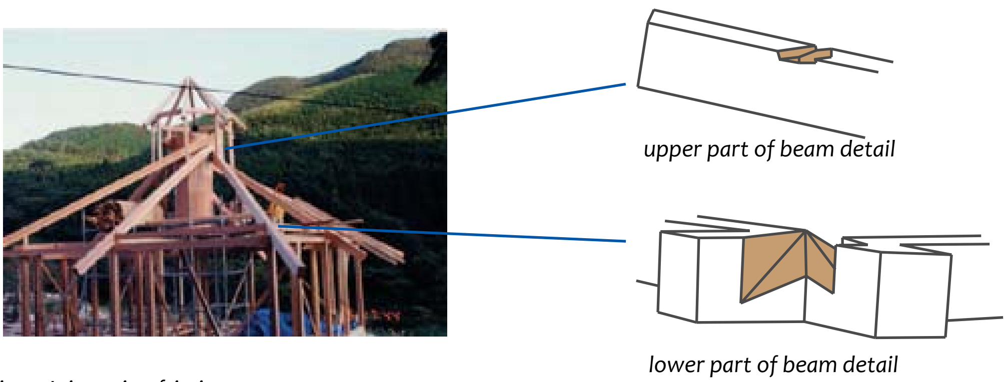


Fig.21 Joint using friction
(Larsen O.P., 2008)

Apart from traditional methods, the reciprocal frames can of course be connected using metal fixators or other attributes to fix the beams to each other. Some of the ways to does this will be shown in the following examples. In most of the cases that uses attributes to connect the beams to each other, the beams are round because the shape is not as much a factor to achieve the desired outcome. The attributes are in some of these cases also used to give a slope to the beams, but it is eliminated as much as possible, as it is not a tradional appearance of the reciprocal frame.

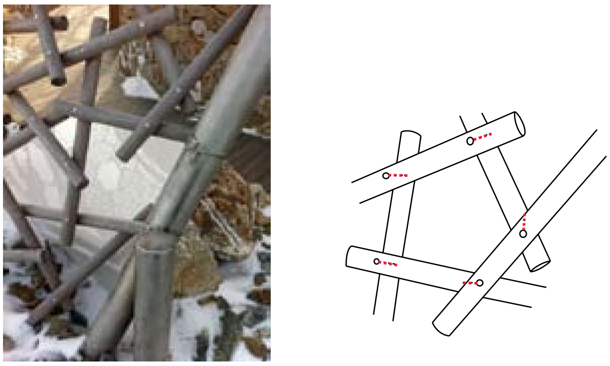


Fig.22 Bolted timber round beams
(Goto K., Kidokoro R., Matsuo T., 2011)

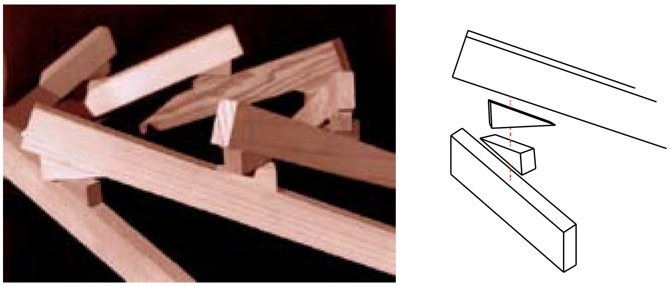


Fig.23 Wooden parts between beams, where they are fixed on
(Larsen O.P., 2008)

Fig.24 Steel anchor to fix round beams
(Larsen O.P., 2008)

4.7 Transition methods for flow of forces within reciprocal frame structures

This paragraph will describe a few variations of the way the forces are transported throughout the structure. The reciprocal frame has to somehow connect with a secondary structure or transport it to the foundation on its own. A few different ways to enable this have been explored to gain insight in this aspect.

One way to this is to assemble the entire structure as a reciprocal frame structure. This means that the shape of the structure has to transform as an entity from horizontal direction to a vertical direction in order to make the connection with the foundation of the structure. This connection will be made with the outside branches of the structures that are oriented downwards.

Fig.25 Example 1
(Larsen O.P., 2008)

Fig.26 Example 2
(Larsen O.P., 2008)

Fig.27 Example 1
(Larsen O.P., 2008)

Fig.28 Example 2
(Larsen O.P., 2008)

Another way to finish of the structure of a building, which a reciprocal frame is part of, is to connect it to a secondary structure that will transport the forces. This will be in the form of vertical structure (either solid or columns) that will have a surface for the outer ends of the reciprocal frame to rest on. In the examples that are shown on the left it is visible that there are beams between the frame and the vertical structure which creates a surface to be able to rest on.

Fig.29 Example 1
(Danz C., 2014)

Fig.30 Example 2
(ETH Zurich, 2015)

Other than the previously mentioned ways to be able to transport forces within the structure to the foundation, there is a way of doing this which has not yet been implemented in any existing building or structure. Although, theoretically this should be possible. By making columns, or vertical structural elements, that also are assembled according to the principle of the reciprocal frame. The examples shown on the left are small and big models of experimental structures. The challenge that lies in this approach is the way these customised columns would be connected to the roof element of the structure in a way that the appearance would not be unbalanced.
(Parigi D., Pugnale A., 2012)

Fig.31 Example 3
(Larsen O.P., 2008)

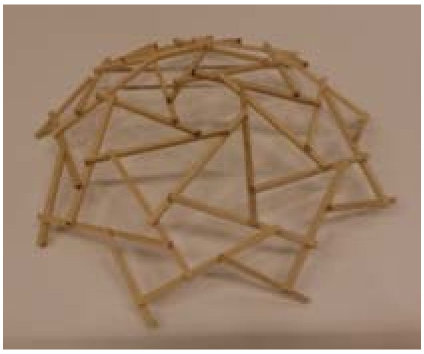
Fig.32 Example 4
(Ricegallery, 2003)

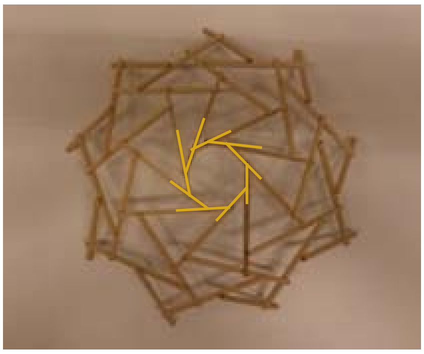
4.8 Model studies of type variations

To be able to put this information into perspective three different types of possible reciprocal frames have been investigated. Every type will be described i this paragraph, showing also the aspects that influence the shape and outcome of the assemblies.

Type 1: One centre where all the beams rotate around.
Type 2: Multiple centres with rotating beams around it and one repetitive pattern
Type 3: Multiple centres with rotating beams around it and multiple repetitive patterns to create a tessellation

Type 1:





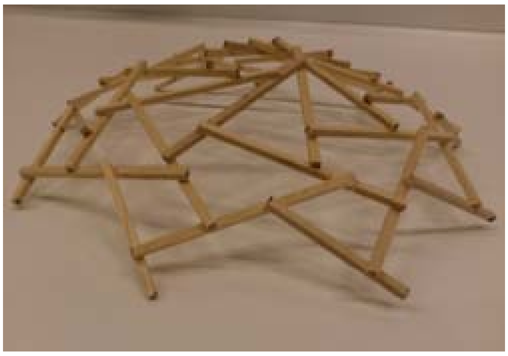
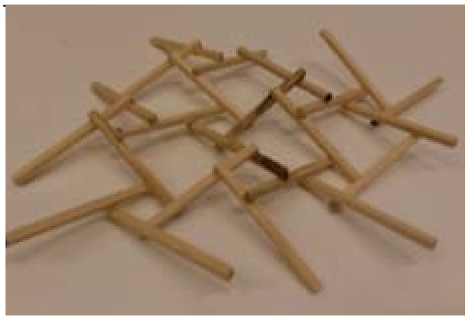
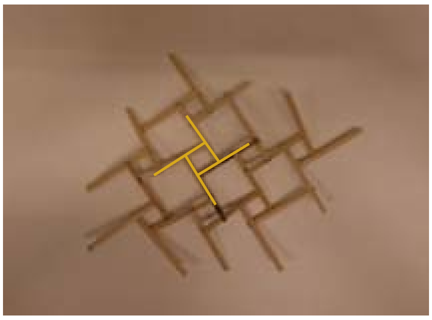


Fig.33,34,35 Model 1
(own image)

In this first example there is one centre where the assembly circulates around. The shape of the middle opening is formed by 9 beams that are equal in length and also in height x width ratio. The beams are stacked on top of each other. Every next row of beams is also stacked on top of each other which makes the outer ends of the assembly forced to push down. This results in a cupola shape that is shown in the pictures above. Although in this example the length of the beams is equal for every row, this is not obligatory. The lengths of the beams can vary, thus different non symmetrical shapes are possible in plan.





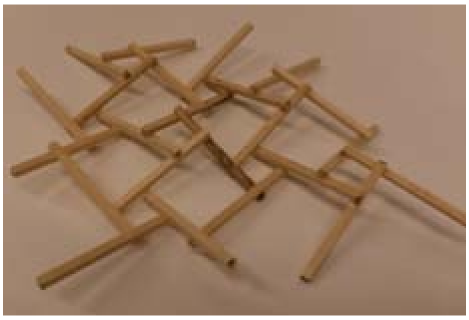
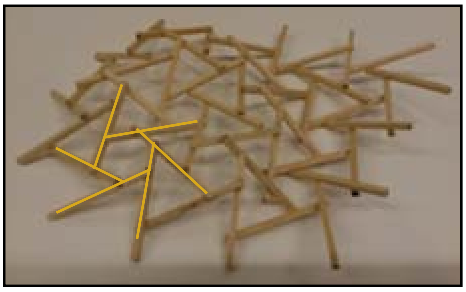
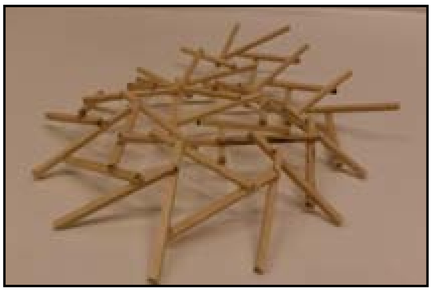


Fig.36,37,38 Model 2
(own image)





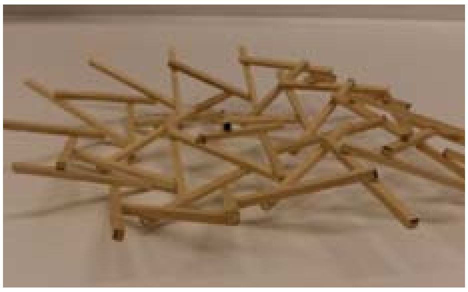


Fig.39,40,41 Model 3
(own image)

This type of assembly is formed by multiple circulation centres that are a repetition of one pattern/shape. In the first row of pictures of the models, you can see a pattern created with four beams that are stacked on top of each other. There is also a variation of stacking on top and under each other in the model. While one centre is created through stacking on top of each other, the next one is created by stacking under/over each other. The effect that this has is also visible in the model. The shape is either shifted upwards or downwards due to the beams pushing the following beam up or down.
In the second row of pictures is shown an assembly of a pattern that is created with six beams stacked on top of each other. The same counts for this assembly regarding the shape of the upwards or downwards pushed beams.

Type 3:

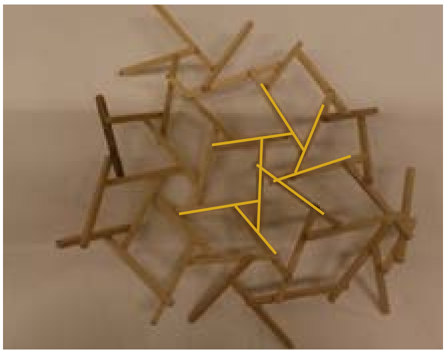
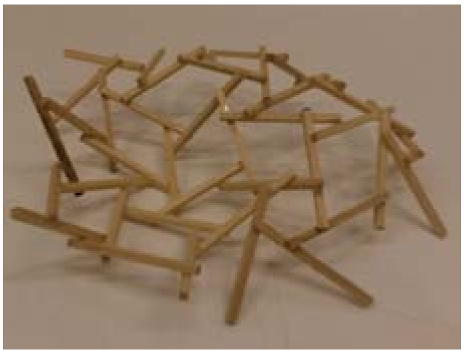
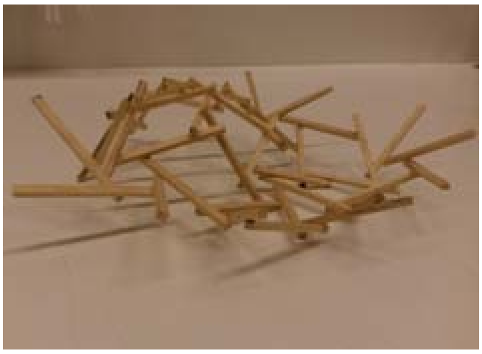




Fig.42,43,44 Model 4
(own image)

This type of assembly is formed by multiple circulation centres that are a repetition of multiple patterns/shapes. In this case the frame consists of two different patterns/shapes that form the circulation centres. The first kind is created out of six beam elements and the second kind is created with three beam elements. These are all connected with each other through stacking over and under each other. In principle this works similar to type 2, but because there are more than one kind of pattern/shape, the curvature can be manipulated to make it more or less steep, while alternating from pattern/shape.

In all the examples, the effect of stacking up on each other or under each other is clearly visible. This causes the curvature in the assemblies to alternate between upwards going and downwards and determines its shape. In the picture below you can see where the beams are being pushed down or pushed up.

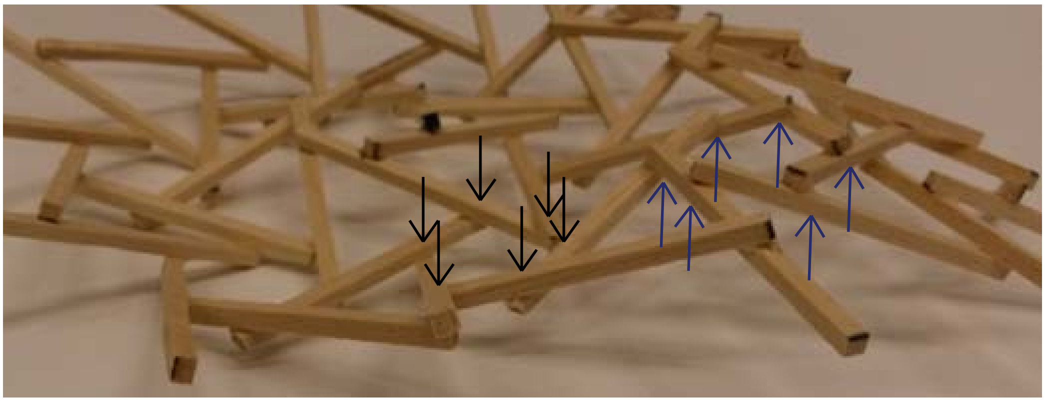


Fig.45 Close up model
(own image)

Digital fabrication:
After making these models, I have found that it is difficult to assemble these type structures in a traditional way and with the exact angles of the beams implemented, which is why I simply stacked them on top of each other. To be able to make the connections easier in real life, the field of digital fabrication can offer lots of solutions. The angles in the beams that are made for the connections, can be precisely cut with the help of cnc-milling for treating wood, or even 3D printing when other materials are used. Eventhough the angles in the connections are mostly unique (if it's not symmetrical), the amount of labour it will cost to make these angles into the material used will be a lot less. Therefore it will also be time-efficient.

4.9 Case studies

The following existing examples are studied in order to gain more insight in the practicality of the reciprocal frame structure. The subjects discussed previously will now have a chance to be connected and after these studies a conclusion can be drawn about how these separate subjects are related to each other.

4.9.1 The stone mason museum - Yasufumi Kijima

This museum is located in a village in Japan called Toh-yoh, which is set in an a mountainous region. This region is known for its stone structures and a local natural stone. As a result of this richness, this museum was built to emphasize this asset of the region. Stone was therefore an obvious material to be used in the design of this building, but the architect wanted to also make a roof out of timber and combine a range of materials in his design. To have a slender light weighted structure for the roof, the reciprocal frame principle has been implemented in the design. *(Larsen O.P., 2008)*

Implementation of RF in entity:
The reciprocal frame in this structure forms the roof of the building. The plan of the building shows that organic forms are good possibility, as this one is. The base of the plan is a circular shape.

in plan

Fig.46 Plan
(Larsen O.P., 2008)

Type of RF structure:
The type of frame is one with multiple circulation centres. The middle centre is considered the main circulation centre. The rest is assembled at the extensions of the main centre.

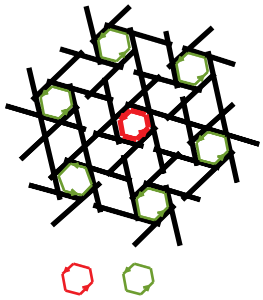


Fig.47 Type of RF
(own image)

Connection details:
The joints for this frame are supported by metal attributes, that are pinned in the timber round beam elements.

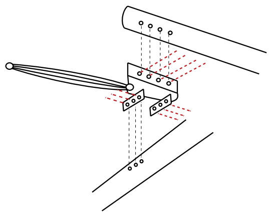


Fig.48 Detail
(Larsen O.P., 2008)

Transition and force flow:
The frame is resting on a solid masonry wall structure that follows the organic shape of the plan.

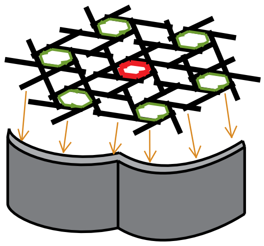


Fig.49 Force flow
(own image)

To summarize, this project shows organic form in plan are a possibility and that reciprocal frame is appropriate to implement as a roof structure, which is supported by a solid underlying structure. The shape of the roof is determined by stacking the round beams on top of each other and continuing to do this until the outer ends of the structure is reached. With the detailing, the natural way the beams would move upwards due to the stacking is influenced as less as possible. The reciprocal frame is only visible on the inside.

4.9.2 Bunraku puppet theatre complex - Kazuhiro Ishii

This is a complex of several buildings that is set in a rural landscape and is used as a theatre and exhibition spaces. The complex is designed by Kazuhiro Ishii. The two main buildings of the complex, the auditorium and the exhibition, have a reciprocal frame implemented in the structure. Although, the roof of the auditorium is debatable whether or not it is considered a reciprocal frame, the same techniques in joining the beam elements have been used by the designer to give the impression that it is a reciprocal frame. Both of these structures will be studied. *(Larsen O.P., 2008)*

Fig.50 Theatre complex
(Larsen O.P., 2008)

Exhibition space

Implementation of RF in entity:
The reciprocal frame forms the roof of this building.
It has a circular shape in plan. The roof is sloping upwards along its circular shape.

in plan

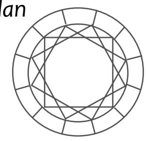


Fig.51 Plan
(Larsen O.P., 2008)

Type of RF structure:
It is a type of reciprocal frame with one circulation centre. The rotation of the frame is however repeated in a mirrored motion to make the structure more stable, creating two frames with a one circulation centre.

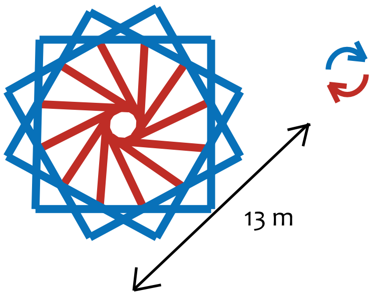
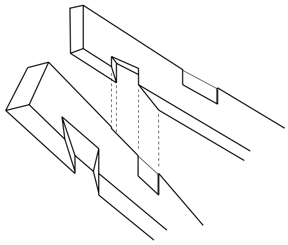


Fig.52 Type of RF
(own image)

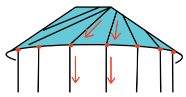
Connection details:
The joints of the beams are unique for every piece and they are based on friction and pressure. No attributes like bolt are used. The angles at which the pieces fit together are cut out first, based on Japanese traditional carpentry, to be able to assemble structure one by one on site.

Fig.53 Detail
(Larsen O.P., 2008)



Transition and force flow:
The reciprocal frame of the roof transitions into a column structure that follows the circular plan. The forces are then transported by these beams to the foundation.

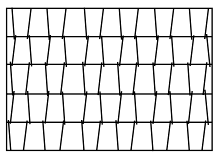
Fig.54 Force flow
(Larsen O.P., 2008)



To summarize, this building is another example of a roof structure that is assembled according to the reciprocal frame principle. It shows us that it is possible to have an underlying column structure holding the frame. The height of the beams makes the frame steeper and want to move upwards at a bigger angle. This is influenced as less as possible by joining the beams through notching. The shape of the roof follows the frame, that is assembled with one circulation centre. The frame is only visible from the inside. *(Larsen O.P., 2008)*

Auditorium

Implementation of RF in entity:
This frame is also the roof of the building, but like mentioned before is a case which is debated whether or not it could be considered a reciprocal frame. The reciprocity is more in the section, which creates the dynamic outcome. The frame has a square shaped plan.



in plan

Fig.55 Plan
(Larsen O.P., 2008)

Type of RF structure:
This frame cannot be categorised in the types that are mentioned in the previous paragraph. It is in fact a grid that is assembled in a way that shows characteristics of a reciprocal frame.

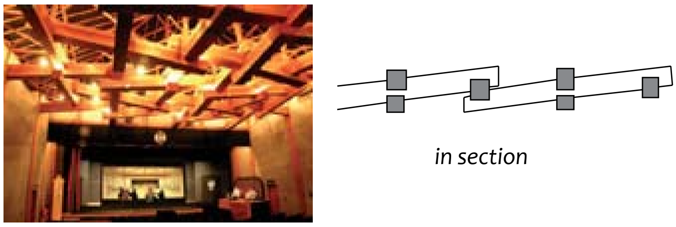


Fig.56 Type of RF
(Larsen O.P., 2008)

Connection details:
The joints of the beams are unique for every piece, like the previous example. No attributes like bolt are used. The angles at which the pieces fit together are cut out first, based on Japanese traditional carpentry, to be able to assemble structure one by one on site.

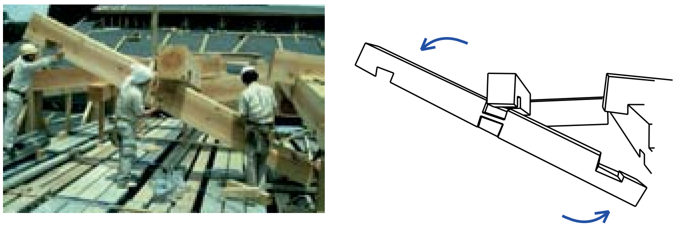


Fig.57 Detail
(Larsen O.P., 2008)

Transition and force flow:
The frame of this roof transitions into a column structure along the square shaped plan. The forces are then transported by these beams to the foundation.

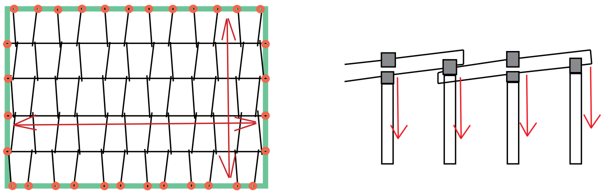


Fig.58 Force flow
(own image)

This frame shows us another interpretation of the reciprocal aspect of the a reciprocal frame. The frame is flat in essence and puts in the dynamic aspect of a traditional reciprocal frame by the way it is assembled and the joints are made. The circulation centres in the frame itself are non-existent, but is seen in the section as the crossbeams rotate around the longitudinal beams. This frame is only visible from the inside.

4.9.3 Rokko observatory - Hiroshi Sambuichi

This observatory is also located in Japan in Kobe to be exact. This is a more recent realisation of the reciprocal frame principle. The difference between the previous projects is immediately visible. The role and the appearance are different in this project. The building has a "shell" in the form of a reciprocal frame that can be seen as a barrier between outside and semi-outside. There is a centre in the composition which is climatised and forms the actual barrier with the outside. The reciprocal frame consists of plate material in some places which have pv-cells in them, so it is also used to generate energy. *(Goto K., Kidokoro R., Matsuo T., 2011)*

Implementation of RF in entity:
The reciprocal frame forms outer shell of the building, which is irregularly cupola shaped. In plan the building has a circular shape and it has centre which is asymmetrically placed and is a climatised space. The frame gives the impression that it is closed, but in reality it is not. It is literally a frame, with actual openings.

Fig.59 Plan
(Goto K., Kidokoro R., Matsuo T., 2011)

Type of RF structure:
The base of this frame is one with a one circulation centre that is repeated in the assembly. The pattern is derived from a voronoi shape. Inside these centres of the base, there is an assembly of a frame with multiple circulation centres, which does not have a loadbearing function.

10

base of frame *secondary frame*

Fig.60 Type of RF
(Goto K., Kidokoro R., Matsuo T., 2011)

Connection details:
The joints of the beams are done with metal bolts, that connect the beams to each other. This counts for the timber round beams within the circulation centres of the base. The base of the frame consists of steel tubes, that are welded together. The inner frame beams are connected to the base with metal strings.

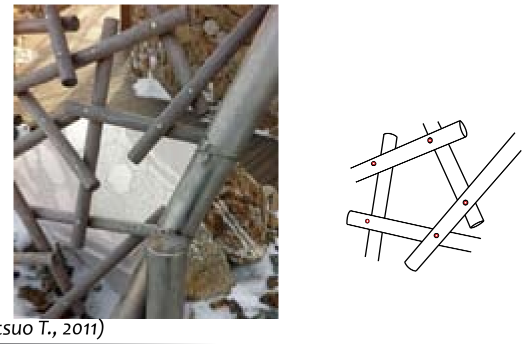


Fig.61 Detail
(Goto K., Kidokoro R., Matsuo T., 2011)

Transition and force flow:
Because the entire frame forms the entire shell, the forces are distributed throughout the entire frame to the foundation and connects with the separate structure of the centre of the building at the top. The red zone shown in the figure illustrates the compression, whereas the blue zone has the least amount of pressure, according to a study done by Arup.

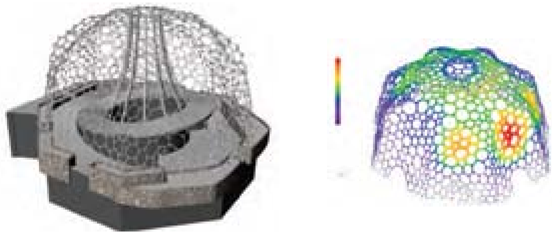


Fig.62 Force flow
(Goto K., Kidokoro R., Matsuo T., 2011)

This contemporary implementation of the reciprocal frame shows us the possibilities of making a free-form structure with type a frame that has multiple circulation centres. Although the beams of the base of the frame are steel, the way the curvature is formed is the essence of this project. The roundness of the beams and the joints that don't influence the directions of the beams, show us the possibilities of making a free-form structure this way. The change of direction in the shape of the structure is due to transitioning between stacking the beams on top and under each other. The reciprocal frame in this project is visible from both inside and outside.

4.9.4 Experimental pavilion - Shigeru Ban

This structure is an experimental pavilion made at a Rice University, in the United States as a prototype for an exhibition space that will be built in Forest Park in St. Louis (also in United States). The real to be built structure will be 10 times the size of this prototype. By making this prototype the designers have experimented with the overlapping of the beam elements in its orientation, which can change the curvature in the assembly. *(Ricegallery, 2003)*

Implementation of RF in entity:
This experimental frame forms a roof (or canopy), that does not have a particular shape, it can be formed into any desired shape. This is possible as a result of the flat beam elements that do not make big slope when stacked up on each other.

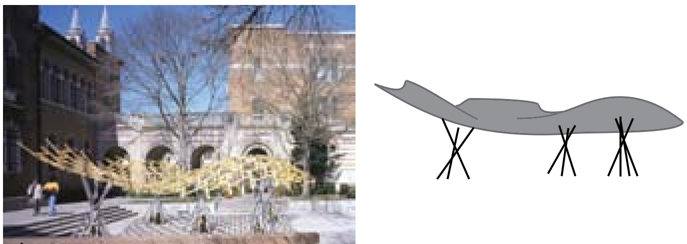


Fig.63 Plan
(Ricegallery, 2003)

Type of RF structure:
This type of reciprocal frame is one with multiple circulation centres of the same polygon, that is repeated throughout the assembly.

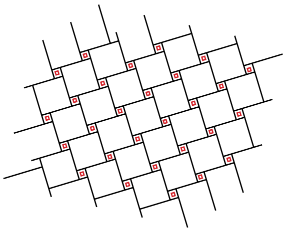


Fig.64 Type of RF
(own image)

Connection details:
The joints in this frame are done with metal bolts, which is easily done due to the flat shape of the beams. This also enables guiding the surface of the flat assembly to have a more flowy shape.

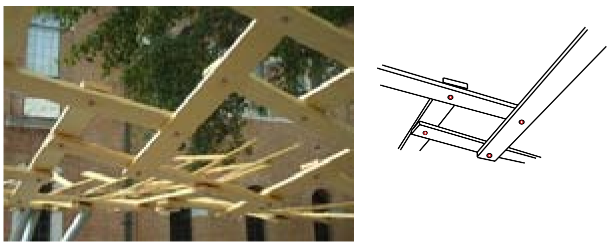


Fig.65 Detail
(Ricegallery, 2003)

Transition and force flow:
The frame is being held up by steal "columns" that are assembled within a rotation as well, these create one column. The connections with these columns are made within some of the circulation centres of the frame.

Fig.66 Transition
(Ricegallery, 2003)

This experimental structure shows us that making a free-from structure can also be achieved by using flat beams. The way these beams are stacked is much less of a factor for its shape. It also shows us that the upholding structure can be designed to fit in the principles of the reciprocal frame, because the steel tubes also have circulation centre and find support in each other although they do still need to be fixated and held together in the middle. This fixation happens in the foundations and in between the intersecting circulation centres of the reciprocal frame. The frame will be seen from both inside and outside.

4.9.5 Interior element - Student project

This is a project made by architecture students at the University of Nottingham, England. It was designed as an element within a space that functions as an exhibition space. It was meant to showcase the work of the students throughout their studies and at the same time introduced them with building a full size model as an ending to their studies at the university. It was mainly designed through the use of software. The lights that are placed inside the assemblies, makes them seem like little nests inside the bigger space. *(Miller, 2014)*

Implementation of RF in entity:
As mentioned before the assembly is designed as an interior separation element. It exists entirely out of an assembly of a reciprocal frame structure. In plan it is irregularly shaped, but has a sleek surface look without bumps etc.

Fig.67 Plan
(Miller, 2014)

Type of RF structure:
This type of reciprocal frame is one with multiple circulation centres of the same polygon, that is repeated throughout the assembly. The patterns are rather close to each other which makes it seem like there are multiple patterns, but that is not the case.

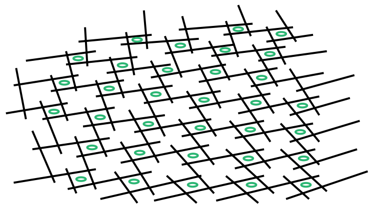


Fig.68 Type of RF
(own image)

Connection details:
The joining method that is used for this assembly is by notching the timber beam elements. The angle at which they will be glided into each other is pre-cut so that it can directly be assembled on site.

Fig.69 Detail
(Miller, 2014)

Transition and force flow:
The entire frame forms the entire shell, so the forces are distributed throughout the entire frame to the foundation. In section the assembly has an arc shape, so along this arc the forces will flow downwards.

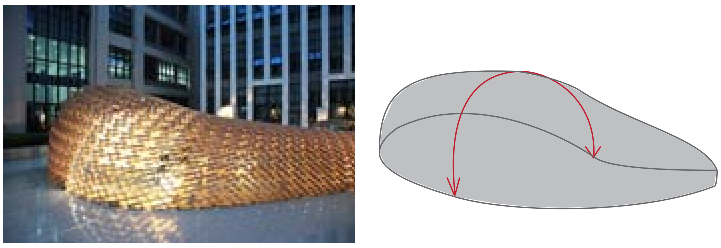


Fig.70 Force flow
(Miller, 2014)

To summarize, this structure shows us that the reciprocal frame can be used to create a space within a space. We can see that the way the joints are made, have determined the enabling the curvature in the entity. Even though the pre-cut high beams are not precisely cut to fit exactly into each other, but is rather general and has lots of tolerance space, but the way it is assembled is a bigger factor in the shape of the outcome. The beams are rather close to each other and are very dependent on each other, which makes the fixation stable. Reciprocal frame is visible from both the inside and the outside.

5. Conclusion & Discussion

This paper has summed a range of information that has been retrieved about the reciprocal frame. This exploration has been done to provide the research question with an answer. The question was as follows: *In what ways is the shape and appearance of a building structure determined within the requirements for a reciprocal frame structure, in a low-tech way?*

As it must now be clear, there are quite a few things that influence the shape of a reciprocal frame. Firstly the shape of the beams that are used to assemble the reciprocal frame is a big factor in the slopes created within the frame. So whether they are square, round or high or low. It all creates a different slope or gives different possibilities to influence its slope. The next important aspect within determining the shape is the way these beams are connected to each other. This also influences the shape and the curvature within the frame. Another important factor is the way the beams are assembled and this aspect has two layers. The first layer is the order in which the beams are stacked, so whether they are stacked up on each other or under each other. This will influence the circulation centres and their directions, which then influences the direction of the beams and the assembly. The second layer of this aspect is about what the type of frame is assembled. As described in this paper, there are many kinds of reciprocal frames. These can be categorised into three types, which all have different consequences for the shape of the structure/assembly.

When it comes to appearance, the above mentioned aspects are of course also a factor. But there are a few other things that determine the appearance of reciprocal frame structure besides these aspects. Firstly the form of implementation is the biggest factor in the appearance of these structures. While it has mainly been used as a roof structure until now, we can see that once this function changes, the appearance differs. For example the interior element or the outer shell frame that have been described in the previous chapter, it becomes clear that the reciprocal frame does not have to be a uniform element. Free-form structures are possible once the implementation form changes. Another difference that comes to sight within this division of use, is the fact that the roof structures are mainly only visible from the inside of the buildings. With the other forms of using these frames, it has been made visible completely.
These relations that this information has is put in a table as seen below.

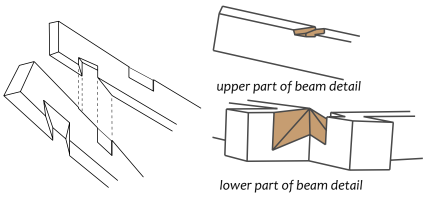
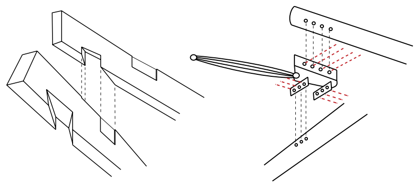
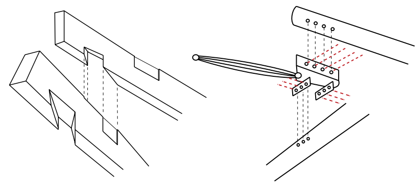
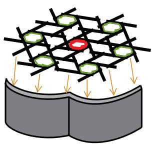
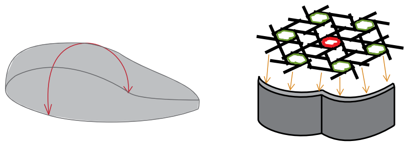
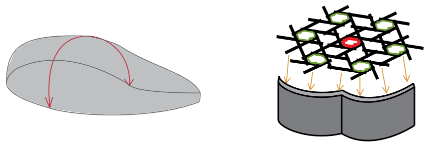
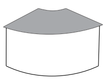
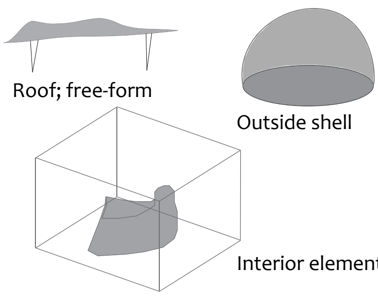
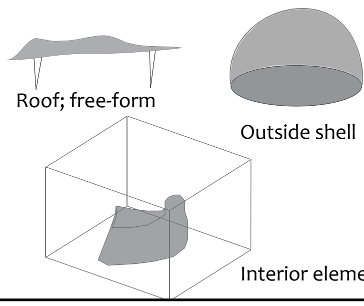
	Type 1	Type 2	Type 3
Type of joint	 upper part of beam detail; lower part of beam detail; Notching; Friction	 Notching; Attachment	 Notching; Attachment
Type of transition	 Underlying construction	 Entity; Underlying construction	 Entity; Underlying construction
Type of implementation	 Roof; circuliar shape	 Roof; free-form; Outside shell; Interior element	 Roof; free-form; Outside shell; Interior element

Table 5. Possibilities
(own image)

Now that all these conclusions have been drawn, the insight in this subject is clearer and more grounded to be able to design a building or space that implements a reciprocal frame.

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