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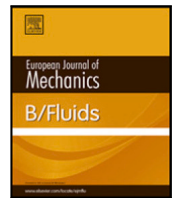
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Dynamics of liquid metal channel flow in a transverse magnetic field with buoyancy forces

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ABSTRACT

In this study, Large Eddy Simulations (LES) are employed to investigate the fundamental interplay between magnetic fields and thermal convection in a vertical, turbulent, liquid metal channel flow. The primary focus is on understanding the modifications to mean flow characteristics and higher-order statistical properties. This interaction leads to significant modifications to the turbulent flow structures, resulting in directional dependencies in the flow properties. The calculations are conducted at a friction Reynolds number of $Re_\tau \approx 395$, Hartmann number (Ha) in the range of $0 \leq Ha \leq 15$, and Richardson number (Ri) varying between 0 and 0.05. We reveal that the presence of a wall-normal magnetic field in this particular setup suppresses turbulent kinetic energy, while buoyancy forces counteract this effect, promoting turbulence and introducing asymmetry into the statistical properties of the flow. Moreover, the magnetic field induces a preferential alignment of both vortical structures and near-wall streaks. The correlation analysis discovers a stretch of the longitudinal scales, while the spectral analysis indicates that both the magnetic field and buoyancy forces affect the distribution of turbulent kinetic energy at different scales. Notably, at $Ha = 15$, the two-dimensional (2D) energy spectrum function has a slope similar to that predicted by the κ^{-3} law in the inertial range. This deviation of mechanism from “classical” turbulence scaling suggests that the applied magnetic field alters the energy cascade, primarily due to enhanced Joule dissipation.

1. Introduction

The combined effects of magnetic field and mixed convection, or magneto-convection, in liquid metals is a subject of considerable interest due to its relevance in various industrial applications such as metallurgy [1], liquid metal batteries [2], semiconductor crystal growth [3], liquid metal blankets for nuclear fusion reactors [4], and concentrated solar power applications [5]. This is due to liquid metals exhibiting properties such as excellent thermal and electrical conductivity, low viscosity, high surface tension and radiation resistance. However, understanding the entwining behaviours between magnetic field and buoyancy forces in such flows is critical to designing and optimising these systems. Liquid metals are characterised by low Prandtl numbers $Pr = \nu/\alpha$, where ν and α are kinematic viscosity and thermal diffusivity respectively. The typical Pr of liquid metals ($\ll 1$) indicates that the smallest temperature scales are much larger than the corresponding velocity scales [6]. This results in thicker thermal boundary layers, and heat transfer dominated by molecular conductivity in forced convection. Mixed convection for convective fluids has been extensively

studied [7,8], while experiments and numerical studies on liquid metals mixed convection have been relatively scarce, and only recently this topic has received more attention [9–14]. Specifically, studies on upward mixed convection of liquid metals have revealed enhanced heat transfer in buoyancy-aided flow [11], thinning of temperature boundary layer [9] (compared to forced convection), and alteration of secondary flow [13]. The distinctive features of liquid metal flows are further influenced by the presence of a magnetic field. Magneto-hydrodynamic (MHD) turbulence in liquid metals is well-known for its unique characteristics, including turbulence suppression, flow reorganisation, and the emergence of unconventional turbulent states [15–20]. Purely MHD flows have been investigated in channel [21,22], square [23,24] and pipe [25] flows, showing some peculiar characteristics. The main features can be summarised as: turbulence suppression on the components whose wave number is parallel to the magnetic field direction, vortical structures elongation and non-uniform streaks spacing.

The presence of buoyancy forces and their interaction with the magnetic field drastically alters the flow dynamics [26], therefore

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their interaction cannot be explained by treating the two phenomena separately. Mixed convection and MHD were investigated by Satake et al. [27], who found asymmetry due to the buoyancy forces in a channel with a wall-normal magnetic field. Rhodes et al. [28] investigated upward and downward flow in a duct wall finding that stable flows with strong jets attached to the hot wall in the upward flow, and reversal flow, higher temperatures (compared to the upward flow case), and flow instability in the heated region for the downward flow case. Fico et al. [29] found dependency of heat transfer characteristics on boundary conditions, and wall conductivity parameters. Zikanov et al. [30] demonstrated anomalous temperature fluctuations in vertical round tube with transversal magnetic field, while Belyaev et al. [31] experimentally showed the existence of different regimes characterised by strong temperature fluctuation in a downward pipe flow within an uniform magnetic field.

These studies show the complexity of the magneto-convective interaction, whose feature depends as well on the relative orientation of magnetic and buoyancy forces. On the other hand, the combined effects of Lorentz and buoyancy forces on turbulence is still not fully understood, with the majority of studies focused in deriving parameters for design purposes [32–36]. The objective of the present work is thus to characterise this interaction between buoyancy and magnetic forces, and their coupling with turbulence.

The Lorentz force interacts with the buoyancy-driven convection currents, leading to modifications in the velocity and temperature fields. One of the key phenomena arising from this interaction is anisotropy, where the flow properties become directionally dependent due to the influence of the magnetic field. The study of anisotropy in magneto-convection is essential as it provides insights into the fundamental mechanisms governing the stability and structure of the flow. Anisotropic behaviours can influence heat transfer rates, mixing efficiency, and the overall performance of liquid metal systems. In this context, this research aims to investigate the anisotropic nature of magneto-convective liquid metal in wall turbulence by examining the interactions between magnetic fields and buoyancy forces. Specifically, we focus on how the magnetic field modifies the energy distribution across different scales and directions within the flow, leading to increased flow anisotropy. By shedding light on these complex interactions, our study contributes to the broader understanding of magneto-convection and its applications. The findings have implications for optimising industrial processes involving liquid metals, and providing insights into natural phenomena governed by magneto-convective dynamics. The remainder of this paper is organised as follows: in Section 2 the governing equations and numerical algorithms used are described. In Section 3 the numerical results are presented showing the different effects of the magneto-convection interaction. In Section 4 the concluding remarks are summarised.

2. Governing equations

The fluid under consideration is incompressible, electrically conductive, and viscous Newtonian. The thermo-physical properties of the fluid are assumed constant, and computed based on the characteristics of lead lithium. The Oberbeck-Boussinesq approximation is imposed, which assumes constant fluid properties, with the exception of density, which varies linearly with temperature in the buoyancy force term. Utilising the assumption of a small magnetic Reynolds number (Re_m), the magnetic field influences the velocity field without reciprocal effects. This allows the use of the inductionless formulation for MHD.

For a spatially uniform, time-independent magnetic field applied to the flow, the Lorentz force is:

$$\mathcal{L}_F = \frac{1}{\rho} \mathbf{J} \times \mathbf{B}_0, \quad (1)$$

where $\mathbf{J}$ is the electric current density, which is computed through the Ohm's law:

$$\mathbf{J} = \sigma (-\nabla \phi + \mathbf{u} \times \mathbf{B}_0), \quad (2)$$

where ϕ is the electric potential and σ is the electric conductivity of the fluid. Using charge conservation ($\nabla \cdot \mathbf{J} = 0$), one can obtain the Poisson equation for electric potential:

$$\nabla^2 \phi = \nabla \cdot (\mathbf{u} \times \mathbf{B}_0), \quad (3)$$

The incompressible MHD equations at low magnetic Reynolds number are:

$$\nabla \cdot \mathbf{u} = 0, \quad (4)$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u} + \mathcal{L}_F + \mathbf{f}, \quad (5)$$

$$\frac{\partial \theta}{\partial t} + \mathbf{u} \cdot \nabla \theta = \nabla \cdot (\alpha \nabla \theta), \quad (6)$$

where $\mathbf{u}$, p , ρ , α , are respectively velocity vector, pressure, density and thermal diffusivity. The velocity vector components, $\mathbf{u}(u, v, w)$ are expressed in streamwise (x), wall normal (y) and spanwise (z) directions. The energy equation is expressed in terms of the “modified” temperature $\theta = T - T_b$, where T and T_b are temperature and bulk temperature, respectively. As mentioned at the beginning of this section, the Oberbeck-Boussinesq approximation is used, and the fluid density changes due to temperature variation

$$\rho = \rho_0(1 - \beta\theta), \quad (7)$$

with ρ_0 representing the reference density and β is the thermal expansion coefficient. The buoyancy force is then

$$\mathbf{f} = -\rho \mathbf{g}, \quad (8)$$

where $\mathbf{g}$ is the gravity vector, oriented conveniently without losing generality as $(-1, 0, 0)$. The strength of convection is given by the Richardson number, $Ri = Gr/Re^2$, and it is based on the definition of Grashof number (Gr):

$$Gr = \frac{g \Delta T \beta (2\delta)^3}{\nu^2}. \quad (9)$$

In the above, $\Delta T = T_h - T_c$ is the walls temperature difference, while the intensity of the magnetic field is given by the Hartmann number

$$Ha = B_0 \delta \sqrt{\sigma/\mu}, \quad (10)$$

which provides the ratio between Lorentz and viscous forces, where μ is the dynamic viscosity. No-slip boundary conditions are applied at the walls for the velocity field, with the walls considered electrically insulated so that a zero wall normal current density is imposed

$$J_n = -\frac{\partial \phi}{\partial n} \Big|_w = 0, \quad (11)$$

where n is indicating the wall normal direction. Periodic boundary conditions are applied for both streamwise (x) and spanwise (z) directions. Eqs. (3), (4), (5) and (6) are expressed in the unfiltered form, with the divergence of subgrid scale (SGS) term $-\nabla \tau^{sgs}$, not explicitly shown in the right hand side of Eq. (5). The SGS stresses are evaluated with wall-adapting local eddy-viscosity (WALE) model [37].

2.1. Mesh and simulation data

The simulations are carried out with a modified version of the open source code OpenFOAM [38], which uses the finite volume method to discretise the partial differential equations. The pressure-velocity coupling is solved using the pressure-implicit with splitting of operator (PISO) approach [39], and a maximum of four external loops are used at each time step to improve convergence. The electric currents and Lorentz force are estimated using a conservative interpolation following Ni et al. [40] approach. The discretised equations are integrated in time with backward second order implicit scheme. The spatial discretisation

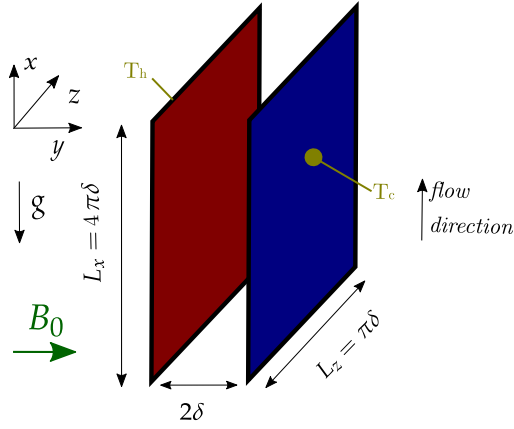


Fig. 1. Channel geometrical characteristics. The magnetic field B_0 is oriented in the positive y direction. The gravity vector g is oriented in negative x direction, while the flow is in positive x direction.

Table 1

Geometrical, physical and mesh characteristics. Mesh grid spacing values normalised with friction velocity u_τ .

Physical and geometrical data		Mesh characteristics	
Re_τ	395	N_x, N_y, N_z	(512, 256, 192)
Pr	0.0238		
L_x	$4\pi\delta$	Δx^+	9.8
L_y	2δ	$\Delta y^+ _{(max,min)}$	6.5, 0.025
L_z	$\pi\delta$	Δz^+	6.5

is made through central differences. The mesh is uniform in the streamwise (x) and spanwise (z) directions, while a non-uniform spacing is used in the wall-normal (y) direction, with the control volumes clustered towards the walls.

Table 1 summarises the geometrical, physical and mesh characteristics, where N_x , N_y and N_z are the number of cells in the axial, horizontal and vertical directions. The fine resolution in the y direction is necessary to capture the electric current loops within the thin Hartmann boundary layer, of which thickness is in the order of $\delta_b^* \approx 1/Ha$. The geometrical characteristics are shown in Fig. 1.

The time averages, denoted by $\langle \cdot \rangle$, are collected over a time period of $1000t^*$, where $t^* = \delta/U_b$ is the characteristic time. Wall coordinates, denoted using the superscript $(+)$, are normalised using the kinematic viscosity ν and the friction velocity $u_\tau = \sqrt{\tau_w/\rho}$, where τ_w is the wall shear stress. Unless otherwise specified, global coordinates (normalised with δ) are used in the next section, and the subscript $(*)$ is omitted. The MHD solver validation is discussed in the Appendix of Fico et al. [41], while the mesh resolution of this work is discussed in Appendix A. Note that, the mesh resolution is very close to DNS, resulting in negligible contributions from the SGS stresses.

3. Results and discussion

Table 2 summarises the investigated cases is presented, which covers Hartmann numbers $0 \leq Ha \leq 15$ and Richardson numbers $0 \leq Ri \leq 0.05$ at a fixed Reynolds number. The relatively low value of Ri are still sufficient to onset significant buoyancy effects, as suggested by Jackson [42]. Moreover, within the ranges studied, the flow exists in a state where no single force dominates, resulting in a highly unstable equilibrium among Lorentz forces, buoyancy forces, and inertia.

The first and second-order statistics in Section 3.1 offer insights into the mean flow characteristics and turbulence intensity in physical space. In Section 3.2, the analysis of flow structures using two-point correlation and spectral analysis helps reveal the spatial coherence and energy distribution across different scales, allowing for a deeper

investigation of how the flow structures are influenced by parameters such as the Hartmann and Richardson numbers. Further results relative to the temperature statistics are shown in Appendix B.

3.1. Turbulent statistical properties in the physical space

When the walls are electrically insulated, and for magnetic field oriented in the wall-normal direction, the velocity profile follows the typical Hartmann profile, characterised by a flattening of the velocity distribution and thinning of the boundary layer [43]. In Fig. 2a the mean velocity profile is plotted for hydrodynamic (A0), magnetic (C0) and magneto-convection cases (B03, C03). For the purely magnetic case, the typical Hartmann velocity profile appears. The magneto-convective cases are instead characterised by asymmetry of the velocity profile, with the maximum located closer to the hot wall ($y/\delta = 0$) due to the thermal boundary conditions.

The asymmetry is highlighted in Figs. 2b (hot wall) and 2c (cold wall), where the mean axial velocity profile for the magneto-convective cases are shown in semi-logarithmic plots. As mentioned in the Introduction, one of the key effects of the magnetic field is the alteration of the turbulent flow structures. In Fig. 3, the main components of the Reynolds stress tensor are shown in global coordinates. First, it can be noticed that all cases (except the hydrodynamic one) display suppression of the Reynolds stress components. Furthermore, the suppression is anisotropic, as wall-normal and spanwise Reynolds stress components are largely suppressed, in contrast to the streamwise component. Due to buoyancy, asymmetry is also observed in all three main components of the Reynold stress tensor for B02, B03 and C03 cases of Table 2. However, thermal convection is not only responsible for asymmetry in the statistics, but also for promoting turbulence. Indeed for a fixed Ha , the magneto-convective (B03, C03) cases show more turbulent kinetic energy compared to the purely magnetic cases (B0 and C0), which is clear by looking at the turbulent kinetic energy in Fig. 4a and b. At $Ri = 0.01$ the effect of buoyancy is minimal and the turbulent kinetic energy profile overlaps with the magnetic case (cases B0 and B02 of Table 2).

Due to the symmetry of the system, the mean velocity has a non-zero component only in the streamwise direction and varies solely with y . From the momentum equation (Eq. (5)), further insights can be gained regarding the distribution of turbulent kinetic energy by looking at the production of turbulent kinetic energy $P^+ = (-\langle u' v' \rangle \nu \partial \langle u \rangle / \partial y)^+$, which decreases with increasing Ha . This reduction is particularly pronounced near the heated wall, where buoyancy effects become significant, as shown in Fig. 5.

The suppression of turbulent kinetic energy production can be directly attributed to the influence of the magnetic field. The contributions to turbulent kinetic energy balance due to the magnetic field is referred as magnetic loss and gain, quantified, respectively, by the following expressions:

$$\mathcal{L}_{loss} = -\frac{\sigma B_0^2}{\rho} (\langle w'^2 \rangle + \langle u'^2 \rangle), \quad (12)$$

$$\mathcal{L}_{gain} = -\frac{\sigma}{\rho} \left(\langle u' \frac{\partial \phi'}{\partial z} \rangle - \langle w' \frac{\partial \phi'}{\partial x} \rangle \right). \quad (13)$$

As expected, larger losses are associated to larger Ha as shown in Fig. 6(a), whereas the magnetic gain is almost negligible (see Fig. 6(b)).

A qualitative effect of buoyancy and Lorentz forces on the turbulent structures is provided in Fig. 7, where turbulent structures are visualised in terms of wall normal velocity and axial velocity fluctuations. Increasing Ha , the turbulent structures are clamped together creating larger and cohesive vortical structures. This is particularly evident for the C03 (Fig. 7c) case in the $x-y$ and $y-z$ planes. Similarly, also the near-wall streaks are largely affected. The streaks spacing alteration is quite clear for the C03 case, with elongation of the streaks in the streamwise direction. The spacing is also less uniform (compared to the hydrodynamic case). This is an indication of large anisotropy in

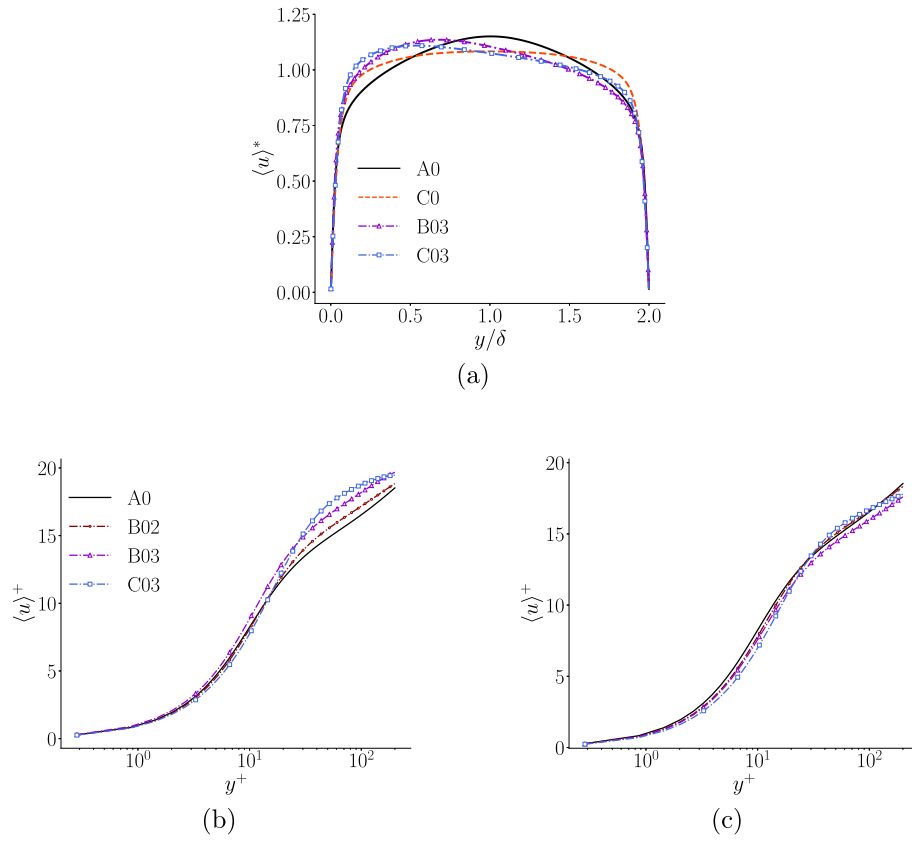


Fig. 2. Mean axial velocity profile in global coordinates (a). Mean axial velocity in semi-logarithmic plot at the hot wall (b), and cold wall (c).

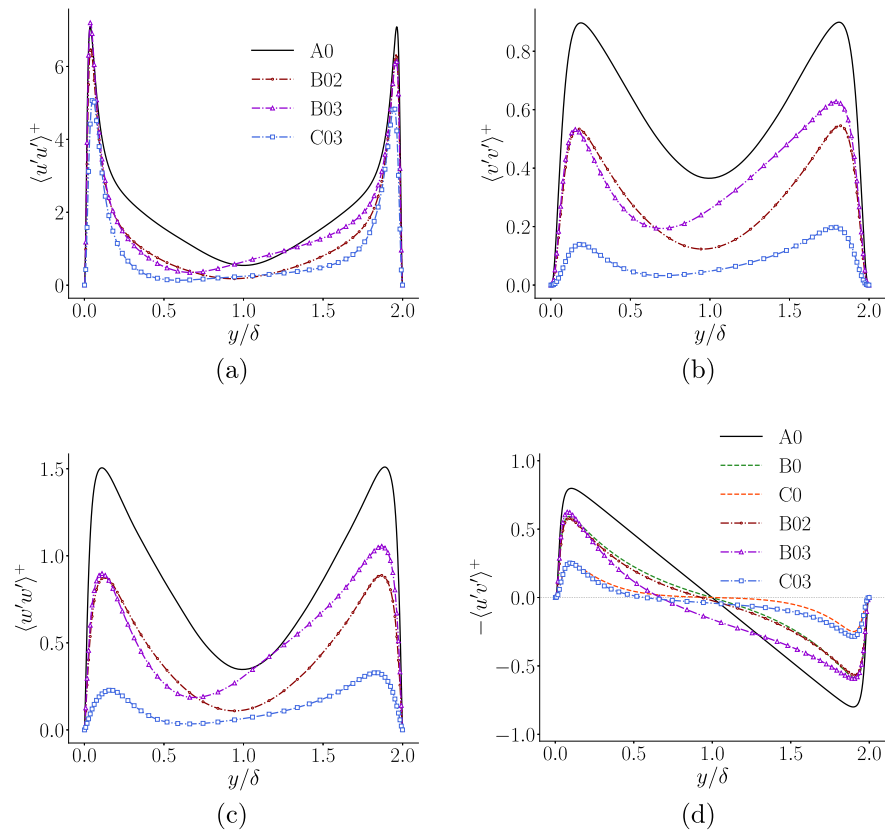
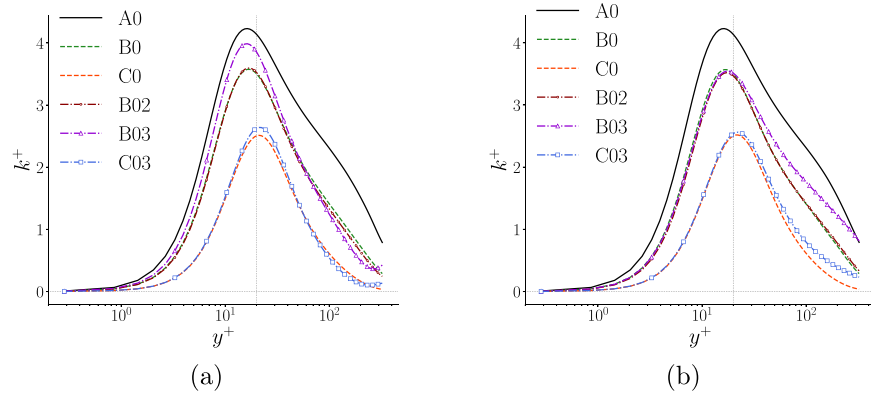
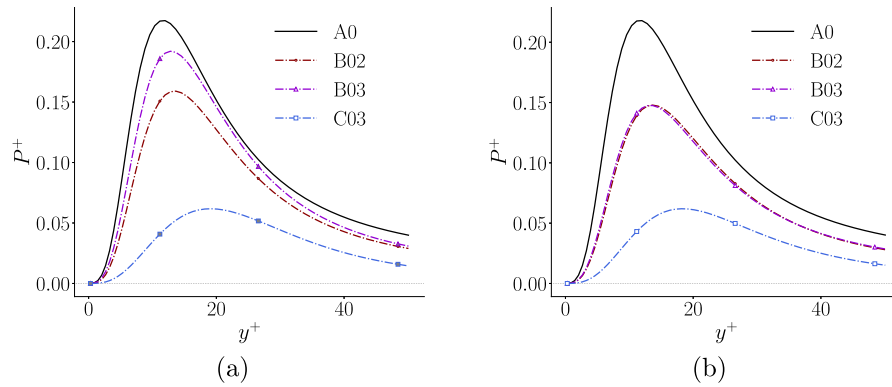
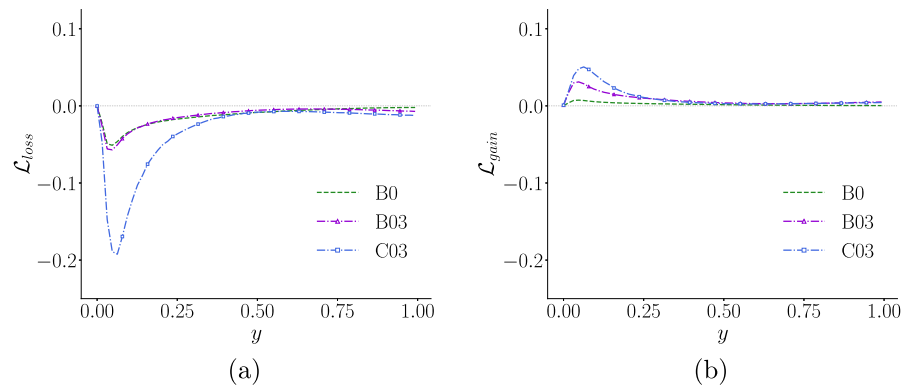


Fig. 3. Main components of Reynolds stress tensor in global coordinates; streamwise (a), wall-normal (b), spanwise components (c) and shear stress (d).

Table 2

Summary of the simulated cases, with friction Reynolds number and Prandtl number $Re_\tau \approx 395$, $Pr \approx 0.0238$. The cases cover different Hartmann numbers and Richardson numbers.

Ha	0	10	15	10	10	15
Ri	0	0	0	0.01	0.05	0.05
symbols/lines	—	---	---	---	---	---
Case denomination	A0	B0	C0	B02	B03	C03

**Fig. 4.** Turbulent kinetic energy at the hot wall (a) and cold wall (b).**Fig. 5.** Turbulent kinetic energy production at hot (a) and cold (b) walls in global coordinates.**Fig. 6.** Lorentz force contribution to turbulent kinetic energy budgets. Magnetic loss (a) and magnetic gain (b).

the flow patterns. Moreover, the intensity of the fluctuations decreases by increasing Ha , as expected by the observation of the Reynolds stress tensor components.

To visualise the anisotropy, the Reynolds stress anisotropy tensor is defined as:

$$b_{ij} = \frac{\langle u_i u_j \rangle}{2k} - \frac{1}{3} \delta_{ij}, \quad (14)$$

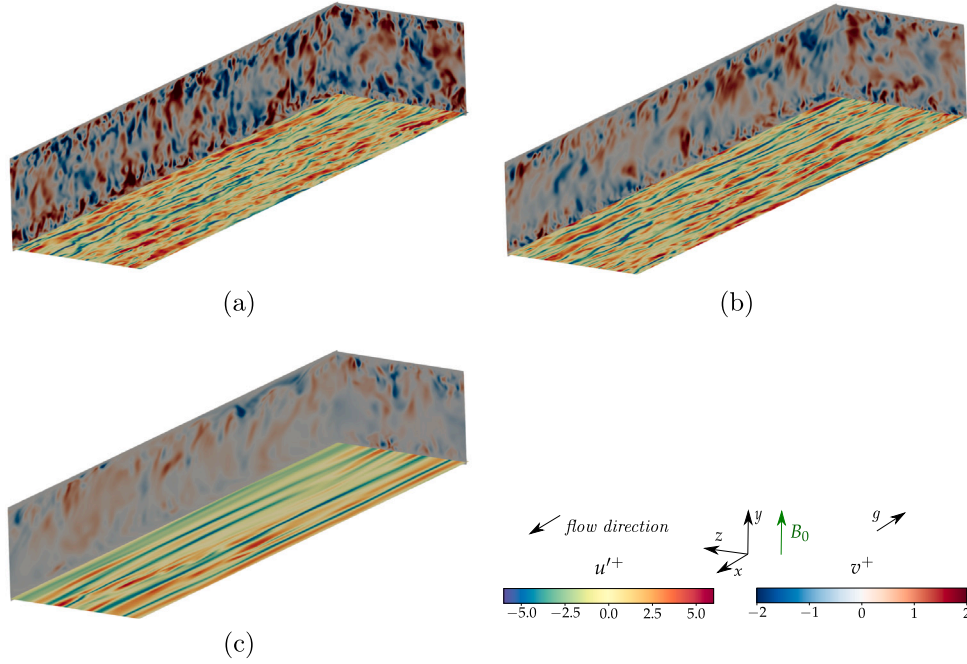


Fig. 7. Turbulent structures for A0 (a), B03 (b) and C03 (c) cases of Table 2. Wall-normal velocity in $x - y$ and $y - z$ planes; near wall streaks at $y^+ \approx 20$ (from the hot wall). Normalised with friction velocity.

with δ_{ij} being the Kronecker delta. The turbulence states can then be characterised by computing the invariants (I_1, I_2, I_3), of this tensor [44]. Note that I_1 is zero by definition for an incompressible flow. In Fig. 8, the invariant map [45] is shown for cases A0, at $Ri = 0.05$ and $Ha = 0$, C0 and C03. It can be noticed that there is no difference between the purely mixed convection (Fig. 8b) and the hydrodynamic (Fig. 8a) cases.

This indicates that, at least in this configuration, buoyancy forces do not affect the turbulent states, and their alteration is only due to the magnetic field. the near wall turbulent states for both magnetic cases C0 and C03 of Table 2 are closer to the 1-component state, while isotropic features are preserved at the centreline.

Overall, the above analysis indicates that the magnetic field and buoyancy forces influence the flow structure by suppressing turbulence (magnetic field), and inducing asymmetry (buoyancy) in the statistics. The Reynolds stress and invariant map analysis suggest an increased anisotropy in the flow; however, it does not provide any information on how the turbulent scales are affected. This aspect is discussed in the next section.

3.2. Correlation and spectral analysis

In order to quantitatively investigate the turbulent structures, observed in Section 3.1, an analysis of the two-point velocity correlations is presented in this section. The correlation function contains information about vortical structures present in turbulent flows and provides an overview of the scales of the coherent structures. Since the integral scales are related to the correlation functions, this analysis also provides an indication of the largest eddies. This approach enables to quantify the effects of buoyancy forces and the magnetic field on the elongation of the coherent turbulent structures. The two-point correlation function in the axial (x) direction is

$$R_{ij} = \frac{\langle u'_i(x, t) u'_j(x + r, t) \rangle}{\langle u'_i(x, t) u'_j(x, t) \rangle}, \quad (15)$$

where the variable r indicates the generic two-point separation distance. The same procedure can be done for the z direction. In Fig.

9 the two-point velocity correlations in the streamwise and spanwise directions in the buffer layer are shown for $y^+ \approx 18$. The two-point correlation is closely related to the integral scale:

$$L_{uu} = \int_0^\infty R_{uu}(r) dr, \quad (16)$$

with R_{uu} being the longitudinal correlation function, and L_{uu} the longitudinal integral scale. Thus, the larger area below the correlation function for the magneto-convective cases (B02, B03, C03) in Fig. 9 indicates larger scales for that component. The larger structures are mostly observed for the streamwise direction. This is consistent with the qualitative flow structures observed in Fig. 7, showing larger turbulent structures for the magneto-convective cases. Similar results are obtained in the logarithmic region of the boundary layer at $y^+ \approx 30$, and $y^+ \approx 62$ (not shown here). Furthermore, the correlation function, in Eq. (15), is linked to the spectral energy through to Fourier transform:

$$E_{ij}(\kappa) = \frac{1}{\pi} \int_{-\infty}^\infty R_{ij}(r) e^{-i\kappa r} dr, \quad (17)$$

where κ is the wave number. In the streamwise direction the wave number is $\kappa_x = n_x \kappa_{x0}$, where κ_{x0} is the lowest positive wave number, defined as $\kappa_{x0} = 2\pi/L_x$, with the integer $n_x \in [-N_x/2, N_x/2]$ for N_x sample points. The same can be done for the other homogeneous direction (z). The computation of the spectral energy is done for convenience by performing the one-dimensional (1D) spatial Fourier transform on the velocity fluctuations $u'_i = u_i - \langle u_i \rangle$, as:

$$u'_i(x, y, z, t) = \int_{-\infty}^\infty \hat{u}'_i(\kappa_x, y, z, t) e^{i\kappa_x x} d\kappa_x, \quad (18)$$

where the symbol $\hat{(\cdot)}$ indicates a variable in the Fourier space. The one-dimensional energy spectra for the $\hat{u}'_i$ component is then computed as $\langle E_{ii}(\kappa_x, y) \rangle = \langle \hat{u}'_i \hat{u}'_i^* \rangle_z / 2$, where $(^*)$ indicates the complex conjugate, and averaging is performed in time, and for one of the homogeneous directions (in this case z). In Fig. 10 the 1D energy spectra is plotted against the streamwise and the spanwise wave-number at $y^+ \approx 30$. For all the streamwise components the energy decreases with increasing

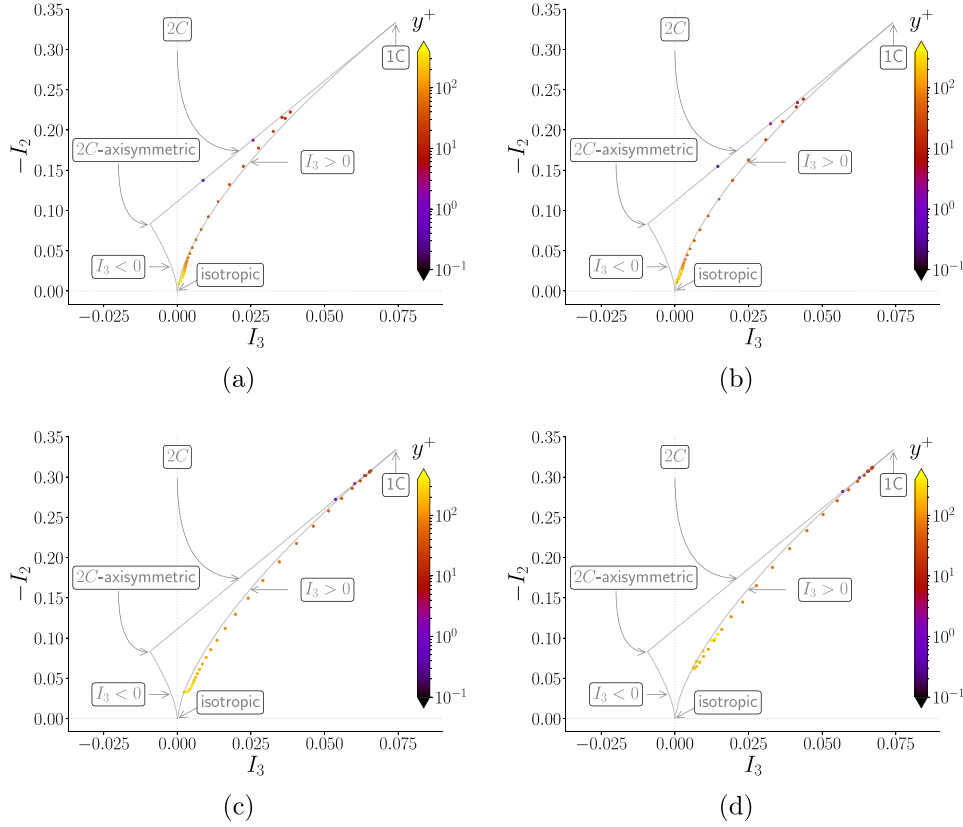


Fig. 8. Invariant map: (A0) hydrodynamic (a), mixed convection with $Ri \approx 0.05$ and $Ha = 0.0$ (b), (C0) purely magnetic (c) and (C03) magneto-convective (d) cases. The data points indicate y^+ positions from the hot wall ($y/\delta = 0$).

Ha. It is interesting to observe that the 1D energy spectrum along the spanwise direction is the most affected by the magnetic field, with case C0 of Table 2 predicting a spectral energy one order of magnitude smaller than the hydrodynamic case (A0). This is valid for all components at this wall location ($y^+ \approx 30$). Similar results are obtained also at $y^+ \approx 18$ (not shown here). Moreover, as suggested by the TKE analysis in physical space in Fig. 4 and at $Ha = 15$, the magneto-convective case C03 displays a higher spectral energy content than the C0 case, indicating that buoyancy forces have an effect on the energy distribution across the scales. Thus, at least for the configuration under study, it can be inferred that the buoyancy forces have a significant effect on the magnitude of the energy spectrum, although its slope remains mainly unaffected. Note that B0 and B02 cases of Table 2, completely overlap with B03 case, and for this reason are not shown here.

The 1D energy spectrum analysis provides information about the distribution of energy along a specific direction, making it useful for determining the dominant wavelengths in that direction. However, to obtain a more comprehensive view of the energy distribution across different scales and directions, it is beneficial to compute the two-dimensional (2D) energy spectrum. This can be achieved by applying a 2D Fourier transform to the velocity fluctuations:

$$u'_i(x, y, z, t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \hat{u}'_i(\kappa_x, y, \kappa_z, t) e^{i\kappa_x x} e^{i\kappa_z z} d\kappa_x d\kappa_z. \quad (19)$$

The 2D energy spectrum is then computed as:

$$\langle E(\kappa_x, y, \kappa_z) \rangle = (\langle \hat{u}' \hat{u}'^* \rangle + \langle \hat{v}' \hat{v}'^* \rangle + \langle \hat{w}' \hat{w}'^* \rangle) / 2. \quad (20)$$

This formulation retains dependence on the streamwise and spanwise wave numbers, as well as the wall-normal coordinate (still in physical space). As a result, it captures the energy distribution across the planar

$(x - z)$ scales and the wall-normal (y) direction. The information about the direction of the Fourier modes is removed by integrating over all wave-numbers κ of magnitude $\kappa = \sqrt{\kappa_x^2 + \kappa_z^2}$, with the 2D energy spectrum function $E(\kappa, y)$ depending only on the wave number magnitude κ and wall-normal direction [46]. In Fig. 11 the 2D energy spectrum function is shown at $y^+ \approx 18, 30, 62, 104$. For C0 and C03 cases (characterised by $Ha = 15$), and contrarily to the A0, B02 and B03 cases, the energy spectrum follows a different power scaling in the inertial range, that is closer to the κ^{-3} than the $\kappa^{-5/3}$ law [16,47]. This trend is followed also at $y^+ \approx 30$, but it is less pronounced at $y^+ \approx 62$ (Fig. 11c) and $y^+ \approx 104$ (Fig. 11d). Moreover, at $y^+ \approx 18$, the energy at larger scales is slightly higher than for the hydrodynamic case.

This deviation in power scaling appears to stem from the enhanced role of Joule dissipation under the applied magnetic field. According to Messadek and Moreau [48], such spectral behaviour reflects a dynamic equilibrium between local energy transfer and the Hartmann damping timescale.

Moving towards the channel centreline, the energy decreases, and the slope of the energy spectrum approaches the $\kappa^{-5/3}$ law, as shown for the C03 case in Fig. 12. Moreover, the spectra at the cold and hot walls are qualitatively similar, with differences observed only in their magnitudes.

Overall, larger longitudinal scales for the magneto-convective cases are confirmed by the correlation analysis. In addition, the spectral analysis indicates that both the magnetic and buoyancy forces affect the energy distribution at different scales, with the former affecting both magnitude and decay distribution and the latter only influencing the magnitude. These findings are significant because they reveal the distinct roles that magnetic and buoyancy forces play in shaping turbulent structures. Understanding how energy is distributed across scales

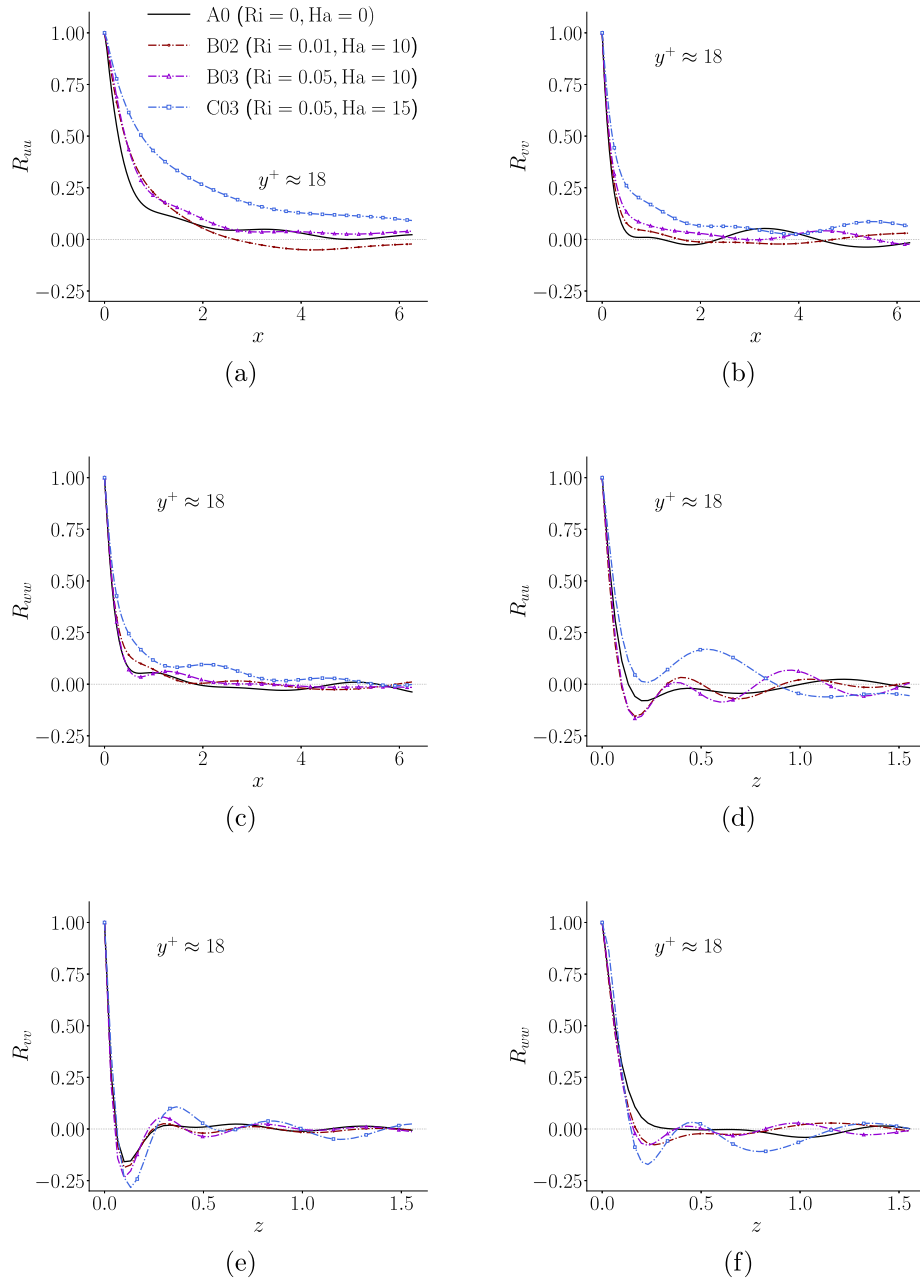


Fig. 9. Two points correlations at $y^+ \approx 18$ for hydrodynamic (A0) and magneto-convective (B02, B03, C03) cases along the streamwise (a–c) and spanwise (d–f) directions.

allows for better predictions of heat transfer and momentum transport in such flows.

4. Concluding remarks

A liquid metal flow in a one-sided heated channel within a transversal magnetic field has been studied to uncover the fundamental mechanisms using highly-resolved LES. The magnetic field effects and the heat transfer characteristics have been examined at $Pr \approx 0.0238$ and $Re_\tau \approx 395$. The Hartmann number was varied from $Ha = 0$ to $Ha = 15$, and Ri from 0 to 0.005. The results demonstrate significant changes in coherent turbulent structures as these parameters are varied. An asymmetry of mean statistics is observed for all the cases when $Ri > 0$, most likely due to the directionally imposed heating. Physical space and spectral analyses of the turbulent kinetic energy confirms

the turbulence suppression and enhanced anisotropic effects, with the spectral analysis further revealing how the energy is distributed among scales based on 1D and 2D spectra.

The key highlights of the conclusions are:

- larger scales (especially longitudinal) are observed in the magneto-convective cases compared to the hydrodynamic case, indicating that the magnetic field elongates flow structures and suppresses small-scale turbulence;
- the magnetic field significantly affects the energy distribution at all scales, with energy spectra inertial slope following a power scaling close to κ^{-3} at $Ha = 15$, reflecting a modified energy cascade due to enhanced Joule dissipation;
- the buoyancy forces in the current setup promote turbulence mostly by affecting the magnitude of the energy at scale without influencing the spectra shape.

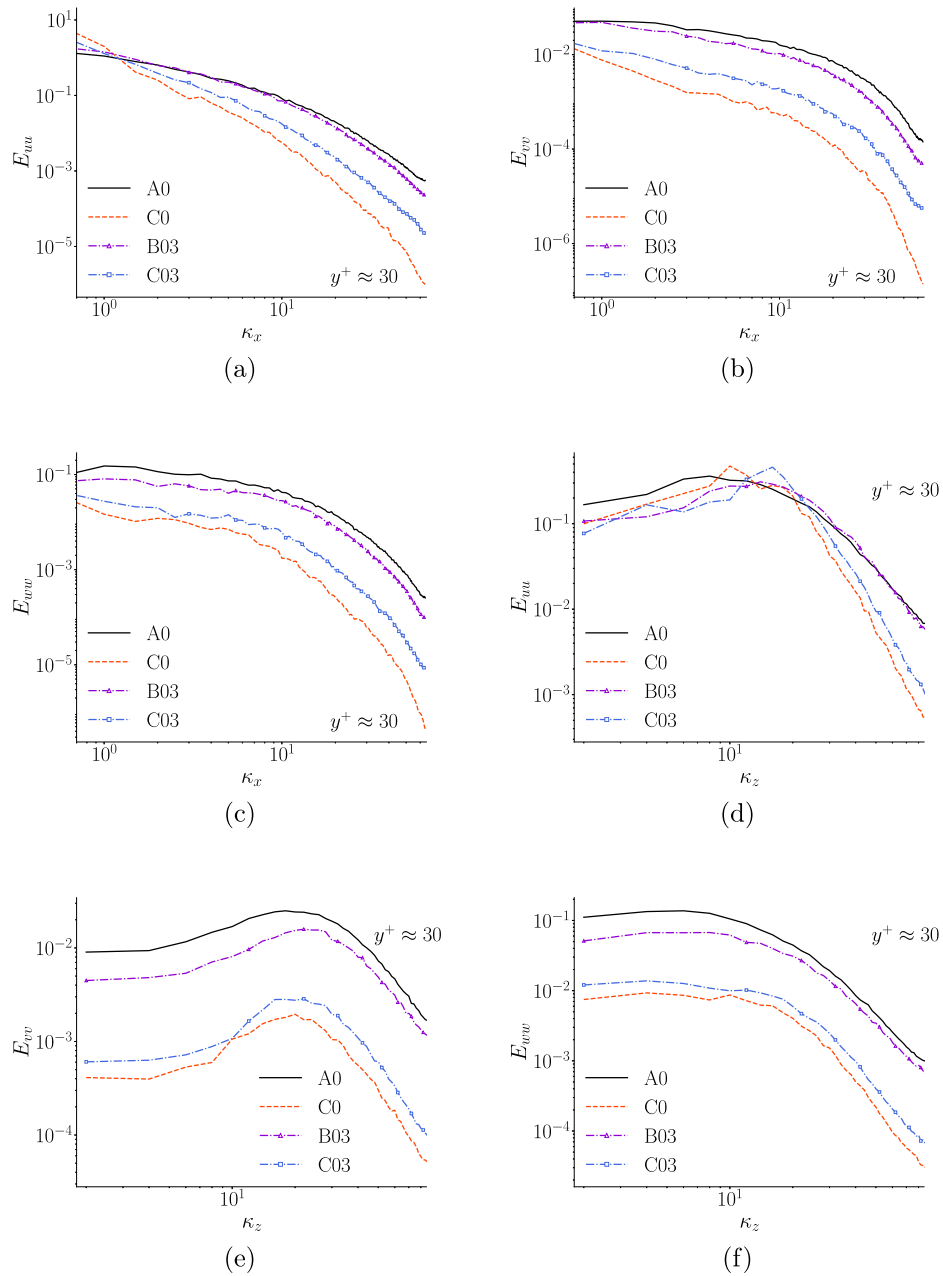


Fig. 10. One-dimensional energy spectra at $y^+ \approx 30$ (from the “hot wall”). Streamwise (a–c) and spanwise (d–f).

Additionally, the difference in power law scaling of the energy spectra observed in the C03 case becomes less significant as the distance from the wall increases, with the κ^{-3} scaling being most pronounced in the near-wall region, suggesting increased dissipation due to the influence of the magnetic field.

This study offers new insights into the interplay between magneto-convection and turbulence, highlighting the influence of transverse magnetic field and one-sided heating on flow structure and energy distribution. The findings hold significance for practical applications, such as optimising thermal management in nuclear fusion blankets, where the direction of buoyancy forces and magnetic field orientation critically influence the flow behaviour. While this work focused on a specific configuration with a transverse magnetic field and upward flow, it lays the groundwork for future research. For example, further studies are necessary to explore the effects of different orientations and configurations, shedding light on the broader phenomenology of magneto-convection in wall turbulence.

CRediT authorship contribution statement

Francesco Fico: Writing – original draft, Visualization, Software, Investigation, Formal analysis, Conceptualization. **Hao Xia:** Writing – review & editing, Supervision, Funding acquisition, Conceptualization. **Ivan Langella:** Writing – review & editing, Supervision, Funding acquisition.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

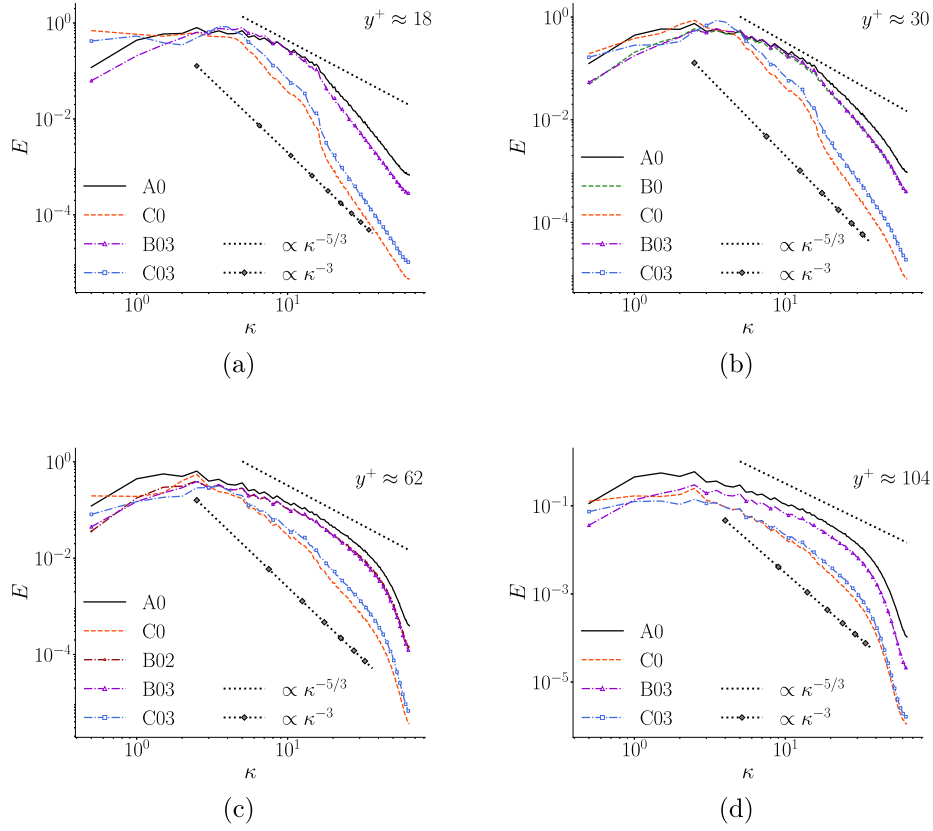


Fig. 11. 2D energy spectrum function at $y^+ \approx 18$ (a), $y^+ \approx 30$ (b), $y^+ \approx 62$ (c) and $y^+ \approx 104$ (d).

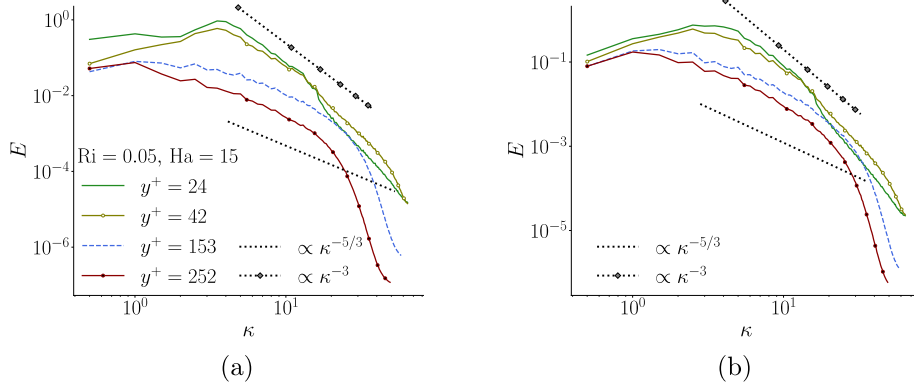


Fig. 12. 2D energy spectra for C03 case at various wall locations. Hot wall (a) and cold wall (b).

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Appendix A. Mesh resolution and validation of numerical simulation

In Section 2 it was mentioned that the mesh is fine enough to resolve over 98% of the kinetic energy. As one can notice in Fig. A.13a, the ratio of resolved k_{res} versus total ($k_{res} + k_{sgs}$) turbulent kinetic energy is presented, with k_{sgs} indicating the subgrid scale kinetic energy estimated as $k_{sgs} = v_{sgs}^2 / (C_k \Delta)^2$, where $C_k = 0.094$ is a constant. As shown

in the figure, the minimum ratio is around 98%, indicating a high resolution of the turbulent scales. In Fig. A.13b the root mean square of velocity fluctuations are compared to the results of Moser et al. [49]. Nearly perfect overlap is obtained for all quantities. Moreover, the comparison for 1D energy spectra in streamwise and spanwise direction is shown, respectively in Fig. A.13c and d also indicating a very good match with the DNS [49].

Appendix B. Thermal statistical properties

Temperature statistics are beyond the primary scope of this study. However, for completeness, the heat fluxes are reported here for the magnetoconvective cases listed in Table 2. The corresponding results in Fig. B.14, illustrate the impact of the applied magnetic field on heat transfer. Differences between cases highlight the relative influence of buoyancy and Lorentz forces on thermal transport. In addition to

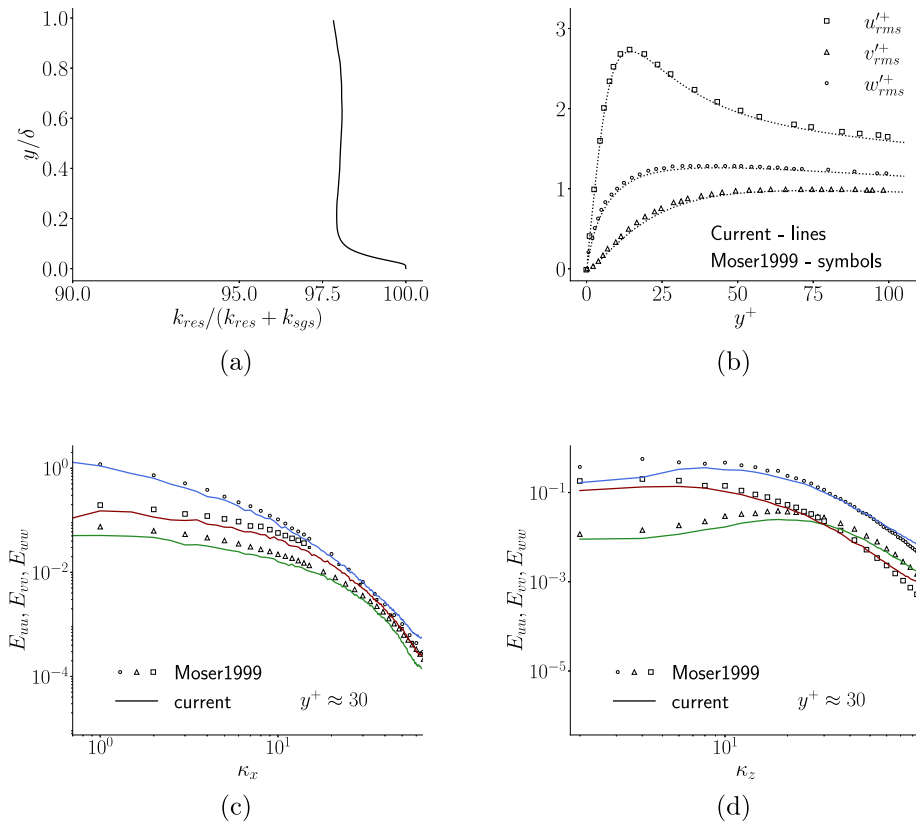


Fig. A.13. Kinetic energy ratio (a). Comparison with DNS from Moser et al. [49]: r.m.s of velocity fluctuations (b), 1D energy spectra along x (c) and 1D energy spectra along z (d).

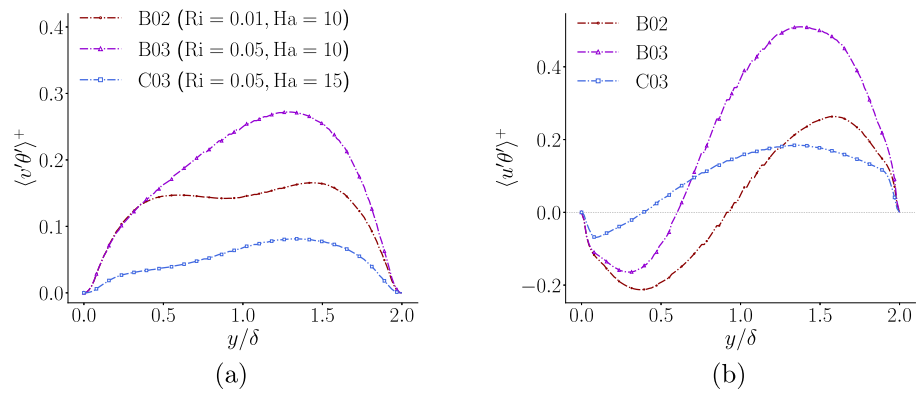


Fig. B.14. Heat fluxes: normal (a) and axial (b).

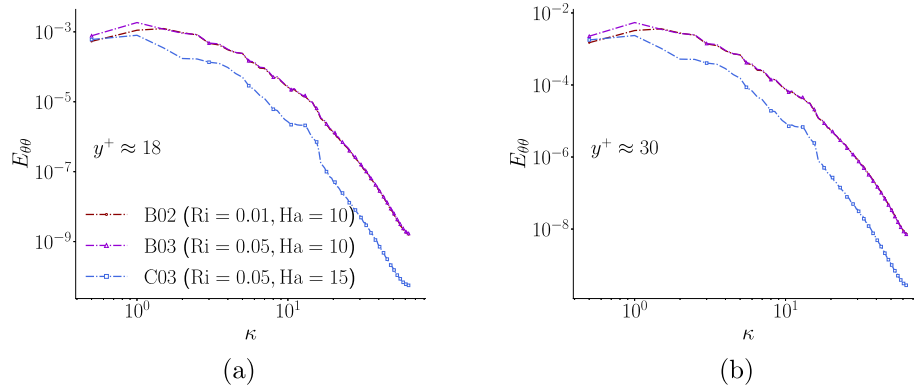


Fig. B.15. 2D temperature spectra at $y^+ \approx 18$ (a) and $y^+ \approx 30$ (b).

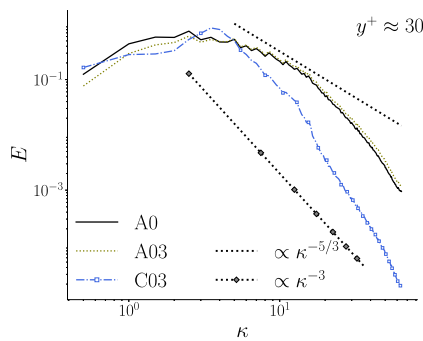


Fig. B.16. 2D energy spectra at $y^+ \approx 30$.

heat fluxes, the spectral characteristics of temperature fluctuations are examined to provide further insight into the magnetoconvective behaviour. Similarly to the velocity spectra, the 2D temperature spectra are shown in Fig. B.15 for B02, B03 and C03 cases at various wall locations. The temperature spectra of B02 and B03 cases completely overlap for most of the wave numbers, in a similar way to that observed for the energy spectra, suggesting that magnetic field has a relatively stronger effect than the thermal convection. Finally, Fig. B.16 compares the 2D energy spectra at $y^+ \approx 30$ against a purely mixed convection case ($Ha = 0, Ri = 0.05$). Consistent with the previous discussion, the energy spectra mostly overlap with those of the purely hydrodynamic case.

Data availability

Data will be made available on request.

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